#college-studies #computer IT

> ✎ **Note**
>
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What is Logic?

Logic is the study of reasoning and principles that guide valid arguments. It forms the foundation for mathematics, computer science, and philosophy by allowing us to determine the truth or falsehood of statements based on rules and structure.

Proposition

A **proposition** is a declarative statement that is either **true** or **false** but not both.

Examples:

- "The sky is blue." (True)
- "2 + 2 = 5." (False)

Exp. Question:

Is the statement "It is raining" a proposition?

Solution:
Yes, because it can be either true or false depending on the weather.

Conjunction (AND)

The **conjunction** of two propositions (p) and (q) is the statement ($p \wedge q$), which is true if and only if both (p) and (q) are true.

Truth Table:

(p)	(q)	($p \wedge q$)
T	T	T
T	F	F
F	T	F
F	F	F

Examples:

- (p): "It is sunny." (T)
- (q): "It is warm." (T)
- ($p \wedge q$): "It is sunny and it is warm." (T)

Negation (NOT)

The **negation** of a proposition (p) is the statement (~p), which is true if (p) is false and false if (p) is true.

Truth Table:

(p)	(~p)
T	F
F	T

Example:

- (p): "It is raining." (T)
- (~p): "It is not raining." (F)

Disjunction (OR)

The **disjunction** of two propositions (p) and (q) is the statement (p ∨ q), which is true if either (p) or (q) or both are true.

Truth Table:

(p)	(q)	(p ∨ q)
T	T	T
T	F	T
F	T	T
F	F	F

Example:

- (p): "It is sunny." (T)
- (q): "It is raining." (F)
- (p ∨ q): "It is sunny or it is raining." (T)

Conditional (Implication)

A **conditional** statement (or implication) (p → q) is the statement "If (p), then (q)", which is false only when (p) is true and (q) is false.

Truth Table:

(p)	(q)	(p → q)
T	T	T
T	F	F
F	T	T
F	F	T

Example:

- (p): "You study." (T)
- (q): "You pass the exam." (T)
- (p → q): "If you study, then you pass the exam." (T)

Biconditional

The **biconditional** statement (p ↔ q) means "p if and only if q." It is true when both (p) and (q) are either true or false.

Truth Table:

(p)	(q)	(p ↔ q)
T	T	T
T	F	F
F	T	F
F	F	T

Example:

- (p): "It is a square." (T)
- (q): "It has four equal sides." (T)
- (p ↔ q): "It is a square if and only if it has four equal sides." (T)

Well-Formed Formula (WFF)

A **Well-Formed Formula** is a syntactically correct expression made up of logical symbols and propositions that follows the rules of the logical language.

Examples:

- (p ∧ (q ∨ ~r)) (WFF)
- (p ∧ ∨ q) (Not WFF)

Compound Statement

A **compound statement** is formed by combining two or more propositions using logical connectives like AND, OR, and NOT.

Example:

- ((p ∧ q) ∨ ~r): A compound statement where (p), (q), and (r) are propositions.

Tautology

A **tautology** is a compound statement that is always true regardless of the truth values of the individual propositions.

Example:

- (p ∨ ~p) is a tautology because it is always true whether (p) is true or false.

Logical Equivalence

Two statements (p) and (q) are **logically equivalent** if (p ↔ q) is a tautology.

Example:

- (p ->q) is logically equivalent to (~p ∨ q).

Truth Table Example for Logical Equivalence:

(p)	(q)	(~p)	(~p ∨ q)	(p → q)
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

identity law

Here's an expanded version with additional examples and Venn diagrams using Mermaid syntax to visualize set operations:

Set Theory

Definition of Set

A **set** is a well-defined collection of distinct objects, considered as an object in its own right.

Properties of Sets

1. **No Repetition of Values**:
   In a set, each element is unique and cannot be repeated. This property ensures that sets are distinct collections of items.

   Examples:
   - (A = {1, 2, 3, 3, 4}) is simplified to (A = {1, 2, 3, 4}).
   - (B = {apple, banana, apple}) simplifies to (B = {apple, banana}).

2. **Order Does Not Matter**:
   Elements in a set can be listed in any order without affecting the set's identity.

   Examples:
   - (C = {1, 2, 3}) is the same as (C = {3, 2, 1}).
   - (D = {red, blue, green}) is identical to (D = {green, blue, red}).

- **Finite Set**: A set with a limited number of elements.
  Example: (A = {1, 2, 3})
- **Infinite Set**: A set with an unlimited number of elements.
  Example: (B = {x | x ≥ 0})
- **Singleton Set**: A set with only one element.
  Example: (C = {5})
- **Empty/Null Set**: A set with no elements, denoted by (\ ∅) or ({}).
  Example: (D = {})
- **Subset**: A set (A) is a subset of set (B) if all elements of (A) are in (B).
  Example: If (A = {1, 2}) and (B = {1, 2, 3}), then (A ⊆ B).
- **Proper Subset**: A subset that is not equal to the original set.
  Example: (A = {1, 2}) is a proper subset of (B = {1, 2, 3}).
- **Superset**: A set that contains all elements of another set.
  Example: (B \⊇ A)
- **Universal Set**: A set that contains all possible elements under consideration.
  Example: (U = {1, 2, 3, 4, 5})
- **Complement Set**: The set of all elements not in the given set.
  Example: If (U = {1, 2, 3, 4}) and (A = {1, 2}), then (A' = {3, 4}).
- **Union Set**: A set containing all elements of two sets, denoted by (A ∪ B).
  Example: (A = {1, 2}), (B = {2, 3}), then (A ∪ B = {1, 2, 3}).
- **Intersection Set**: A set containing only the elements common to both sets, denoted by (A ∩ B).
  Example: (A = {1, 2}), (B = {2, 3}), then (A ∩ B = {2}).

Representation of Sets

Table of Differences: Roster Form vs. Set Builder Form

Aspect	Roster Form	Set Builder Form
Definition	Lists all elements explicitly in curly braces.	Describes elements based on a defining property or rule.
Notation	A={1,2,3}	A = {x | x is a positive integer}
Usage	Best for small or explicitly known sets.	Useful for sets with an infinite or large number of elements.
Example	B={2,4,6,8}	B={x ∣ x=2n, where n∈N}
Simplicity	Direct and easy to understand for finite sets.	More abstract, requiring understanding of the property.
Flexibility	Less flexible for large or infinite sets.	Highly flexible for describing complex or infinite sets.

Introduction to Matrix

A matrix is a rectangular array of numbers or symbols arranged in rows and columns. It serves as a fundamental tool in various fields such as mathematics, physics, engineering, and computer science. The elements within a matrix are organized in a grid-like format and are often used to represent linear transformations, systems of equations, and data structures. Each matrix is typically defined by its dimensions, which indicate the number of rows and columns it contains. For instance, a matrix with 3 rows and 4 columns is called a (3 \times 4) matrix. Matrices are utilized to simplify complex calculations, model real-world phenomena, and solve problems in a systematic manner.

Operations on matrices are essential for manipulating and analyzing these arrays. The primary operations include matrix addition, subtraction, and multiplication. Matrix addition and subtraction involve combining two matrices of the same dimensions by performing element-wise operations. Matrix multiplication, however, is more intricate and involves a series of dot products between rows and columns. Understanding these operations is crucial for tasks such as solving linear systems, transforming geometric data, and optimizing algorithms.

Addition of Matrices

Definition: Matrix addition involves adding corresponding elements from two matrices of the same dimensions. The resulting matrix will have the same dimensions as the original matrices.

Rules:

1. **Dimension Matching:** Both matrices must have the same number of rows and columns.
2. **Element-wise Addition:** Add each element of the first matrix to the corresponding element of the second matrix.

Example: If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, then their sum C is:

$$C = A + B = \begin{pmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

Subtraction of Matrices

Definition: Matrix subtraction involves subtracting corresponding elements from two matrices of the same dimensions. The resulting matrix will have the same dimensions as the original matrices.

Rules:

1. **Dimension Matching:** Both matrices must have the same number of rows and columns.
2. **Element-wise Subtraction:** Subtract each element of the second matrix from the corresponding element of the first matrix.

Example: If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, then their difference C is:

$$C = A - B = \begin{pmatrix} 1-5 & 2-6 \\ 3-7 & 4-8 \end{pmatrix} = \begin{pmatrix} -4 & -4 \\ -4 & -4 \end{pmatrix}$$

Multiplication of Matrices

Definition: Matrix multiplication involves combining two matrices through a process of dot products between rows of the first matrix and columns of the second matrix. For two matrices A and B to be multiplied, the number of columns in A must equal the number of rows in B. The resulting matrix C will have dimensions equal to the number of rows in A and the number of columns in B.

Rules:

1. **Dimension Compatibility:** The number of columns in the first matrix must match the number of rows in the second matrix.
2. **Dot Product:** Each element in the resulting matrix is computed by taking the dot product of the corresponding row from the first matrix and column from the second matrix.

Example: If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, then their product C is:

$$C = A \times B = \begin{pmatrix} (1 \cdot 5 + 2 \cdot 7) & (1 \cdot 6 + 2 \cdot 8) \\ (3 \cdot 5 + 4 \cdot 7) & (3 \cdot 6 + 4 \cdot 8) \end{pmatrix} = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$$