> **Note**
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What is Logic?

Logic is the study of reasoning and principles that guide valid arguments. It forms the foundation for mathematics, computer science, and philosophy by allowing us to determine the truth or falsehood of statements based on rules and structure.

Proposition

A **proposition** is a declarative statement that is either **true** or **false** but not both.

Examples:

- "The sky is blue." (True)
- "2 + 2 = 5." (False)

Exp. Question:

Is the statement "It is raining" a proposition?

Solution:
Yes, because it can be either true or false depending on the weather.

Conjunction (AND)

The **conjunction** of two propositions (p) and (q) is the statement ($p \wedge q$), which is true if and only if both (p) and (q) are true.

Truth Table:

(p)	(q)	($p \wedge q$)
T	T	T
T	F	F
F	T	F
F	F	F

Examples:

- (p): "It is sunny." (T)
- (q): "It is warm." (T)
- ($p \wedge q$): "It is sunny and it is warm." (T)

Negation (NOT)

The **negation** of a proposition (p) is the statement (~p), which is true if (p) is false and false if (p) is true.

Truth Table:

(p)	(~p)
T	F
F	T

Example:

- (p): "It is raining." (T)

- (~p): "It is not raining." (F)

Disjunction (OR)

The **disjunction** of two propositions (p) and (q) is the statement (p ∨ q), which is true if either (p) or (q) or both are true.

Truth Table:

(p)	(q)	(p ∨ q)
T	T	T
T	F	T
F	T	T
F	F	F

Example:

- (p): "It is sunny." (T)
- (q): "It is raining." (F)
- (p ∨ q): "It is sunny or it is raining." (T)

Conditional (Implication)

A **conditional** statement (or implication) (p → q) is the statement "If (p), then (q)", which is false only when (p) is true and (q) is false.

Truth Table:

(p)	(q)	(p → q)
T	T	T
T	F	F
F	T	T
F	F	T

Example:

- (p): "You study." (T)
- (q): "You pass the exam." (T)
- (p → q): "If you study, then you pass the exam." (T)

Biconditional

The **biconditional** statement (p ↔ q) means "p if and only if q." It is true when both (p) and (q) are either true or false.

Truth Table:

(p)	(q)	(p ↔ q)
T	T	T
T	F	F
F	T	F
F	F	T

Example:

- (p): "It is a square." (T)
- (q): "It has four equal sides." (T)
- (p ↔ q): "It is a square if and only if it has four equal sides." (T)

Well-Formed Formula (WFF)

A **Well-Formed Formula** is a syntactically correct expression made up of logical symbols and propositions that follows the rules of the logical language.

Examples:

- (p ∧ (q ∨ ~r)) (WFF)
- (p ∧ ∨ q) (Not WFF)

Compound Statement

A **compound statement** is formed by combining two or more propositions using logical connectives like AND, OR, and NOT.

Example:

- ((p ∧ q) ∨ ~r): A compound statement where (p), (q), and (r) are propositions.

Tautology

A **tautology** is a compound statement that is always true regardless of the truth values of the individual propositions.

Example:

- (p ∨ ~p) is a tautology because it is always true whether (p) is true or false.

Logical Equivalence

Two statements (p) and (q) are **logically equivalent** if (p ↔ q) is a tautology.

Example:

- (p ->q) is logically equivalent to (~p ∨ q).

Truth Table Example for Logical Equivalence:

(p)	(q)	(~p)	(~p ∨ q)	(p → q)
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T