Smoothness-Adaptive Dynamic Pricing with Nonparametric Demand Learning

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Naturally, the estimation method and pricing performances correspond to demand assumptions

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- **Price-setting monopolist** decision variable $p_t \in \mathcal{P}$

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- **Regret** $R^\pi(T) = \mathbb{E}^\pi \left[\sum_{t=1}^T \{p^* f(p^*) - p_t f(p_t)\} \right]$

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- Linear demand $f(p) = a + b \cdot p$, where a, b are unknown parameters

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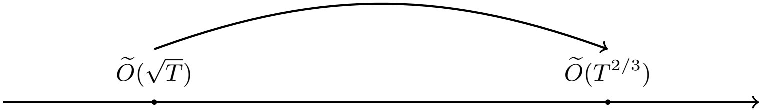

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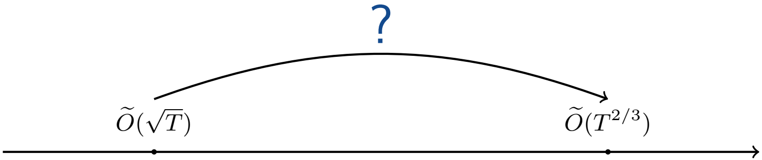

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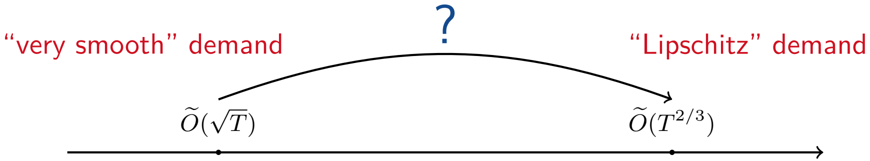

Regret Under Nonparametric Demand

- **Hölder-smooth demand function** $f(p) \in \mathcal{H}(\beta, L)$ for some $\beta, L > 0$ if

$$\left| f^{(w(\beta))}(p) - f^{(w(\beta))}(p') \right| \leq L \cdot \left| p - p' \right|^{\beta - w(\beta)}$$

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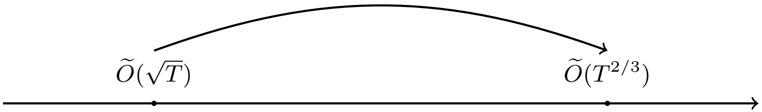

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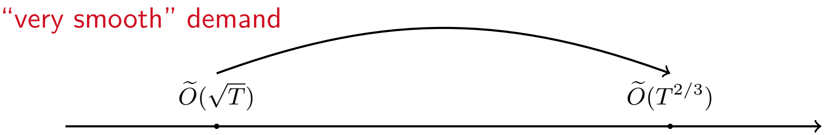

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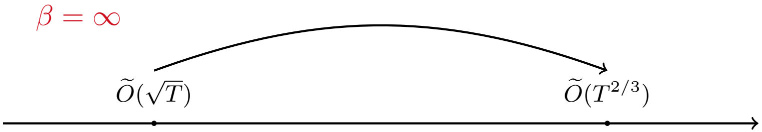

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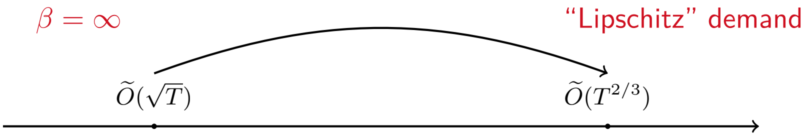

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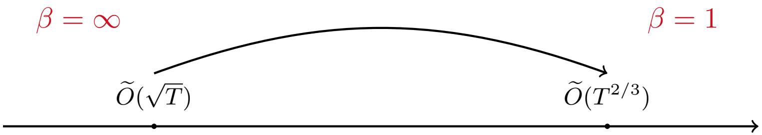

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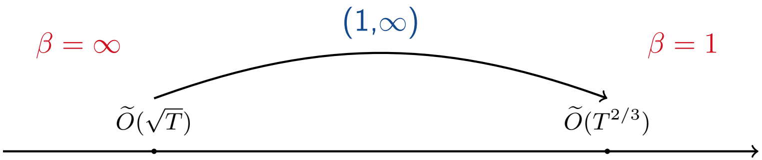

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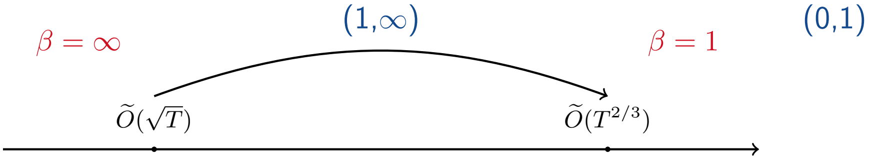

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- **Optimal minimax regret rate**
  Our Smoothness-Adaptive Dynamic Pricing (`SADP`) algorithm enjoys an optimal regret bound $\widetilde{O}(T^{\frac{\beta+1}{2\beta+1}})$

Role of Smoothness β

- If smoothness parameter β **is known**, optimal regret rate is achievable by our Hölder-Smooth Dynamic Pricing (HSDP) Algorithm

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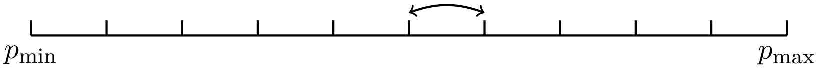
`HSDP` Algorithm Design



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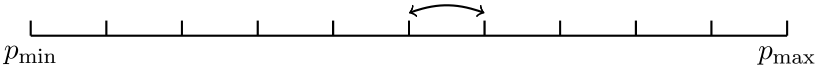

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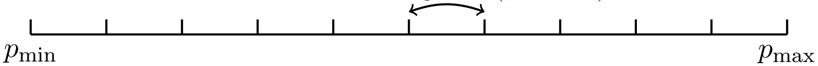
`HSDP` **Algorithm Design**

interval length $O(T^{-\frac{1}{2\beta+1}})$



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Characterizing Adaptivity Challenge

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Theorem

Fix any two positive Hölder smoothness parameters $\alpha > \beta > 0$*, and parameters* $L(\alpha), L(\beta) > 0$*. Suppose that there is a policy* π *achieves the optimal regret* $\widetilde{O}\left(T^{\frac{\alpha+1}{2\alpha+1}}\right)$ *over* $\mathbb{E}[d|p] = f(p) \in \mathcal{H}(\alpha, L(\alpha))$ *, then there exists a constant* $C > 0$ π *such that*

$$\sup_{f \in \mathcal{H}(\beta, L(\beta))} R^{\pi}(T) \geq \Omega\left(T^{\frac{\beta+1}{2\beta+1} + \frac{\beta(\alpha-\beta)}{2(2\beta+1)^2(2\alpha+1)}}\right).$$

Self-Similarity Condition

Definition

A function $g : [a, b] \to \mathbb{R}, [a, b] \subseteq [0, 1]$ is self-similar on $[a, b]$ with parameters $\beta, l \in \mathbb{Z}^+, M_1 \in \mathbb{R}_{\geq 0}, M_2 \in \mathbb{R}_+$ if for some positive integer $c > M_1$ it holds that

$$\max_{V \in \mathcal{V}_c} \sup_{p \in V} \left| \Gamma_l^V g(p) - g(p) \right| \geq M_2 \cdot 2^{-c\beta},$$

where we define

$$\mathcal{V}_c = \left\{ \left[a + \frac{i}{2^c}, a + \frac{i+1}{2^c} \right] \cap [0, 1], i = 0, 1, \ldots 2^c - 1 \right\}$$

for any positive integer c.

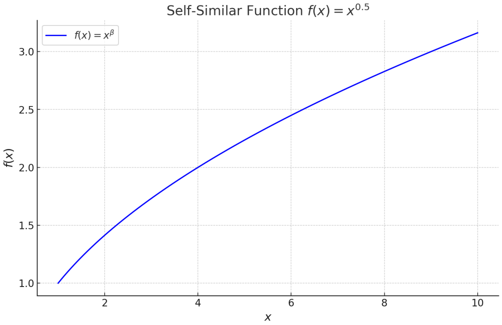
Examples of Self-Similar Functions

Example

Let f be any function with continuous first-order derivative uniformly bounded by C_1. We define the function class $\mathcal{F}$ as

$$\mathcal{F}(f) = \{f : x \mapsto c_0 \cdot x^{\beta} + f : c_0 \in \mathbb{R}, |c_0| \geq C_1\},$$

then all function in $\mathcal{F}(f)$ is self-similar with parameters $, l = 0$ for some constants M_1, M_2 depending on C_1 and C_2.



Self-Similarity Condition Doesn't Decrease Complexity

Theorem

For any positive parameters $\beta, M_1, L > 0$, *there exists a constant* $M_2 > 0$ *satisfying that*

$$\inf_{\pi} \sup_{f \in \mathcal{H}(\beta,L) \cap \mathcal{S}(\beta,w(\beta),M_1,M_2)} R^{\pi}(T) \geq \Omega(T^{\frac{\beta+1}{2\beta+1}}).$$

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- Self-similarity does not change the minimax regret rate and therefore does not lower the problem difficulty for any admissible dynamic pricing policy π

Towards Smoothness-Adaptivity

Main idea behind learning smoothness parameter

- Employ two distinct levels of granularity to estimate the demand function, indexed by 1 and 2 respectively
- The difference of two estimates tells information on β

Lemma (Estimation Accuracy)

With $0 < \beta \leq \beta_{\max}$, we can obtain an estimator $\hat{\beta}$ in $2T^{\lfloor \frac{1}{2}+k_1 \rfloor}$ periods for $k_1 = \frac{1}{2\beta_{\max}+2}$, such that with probability at least $1 - O\left(e^{-C\ln^2(T)}\right)$,

$$\hat{\beta} \in \left[\beta - \frac{4\left(\beta_{\max}+1\right)\ln\left(\ln\left(T\right)\right)}{\ln\left(T\right)}, \beta\right].$$

Smoothness-Adaptive Dynamic Pricing Algorithm

Algorithm Smoothness-Adaptive Dynamic Pricing (`SADP`)

Input: Time horizon T, Hölder smoothness range $[\beta_{\min}, \beta_{\max}]$, minimum price $p_{\min}$, maximum demand $d_{\max}$;

Set $k_2 = \frac{1}{2\beta_{\max}+2}, k_2 = \frac{1}{4\beta_{\max}+2}$, $K_i = 2^{\lfloor k_i \log_2(T) \rfloor}$, for $i = 1, 2$;
for $i = 1, 2$ **do**
  Set trial time $T_i = T^{\lfloor \frac{1}{2} + k_i \rfloor}$;
  Fit local polynomial regression $\hat{f}_i$, respectively, on collected data from pulling T_i times uniformly selected prices;
end for
Let $\hat{\beta} = -\frac{\ln(\max \|\hat{f}_2 - \hat{f}_1\|_\infty)}{\ln(T)} - \frac{\ln(\ln(T))}{\ln(T)}$;
Call HSDP with $\hat{\beta}$;

Theorem

The cumulative regret of SADP is upper bounded by $\tilde{O}\left(T^{\frac{\beta+1}{2\beta+1}}\right)$.

Numerical Illustration

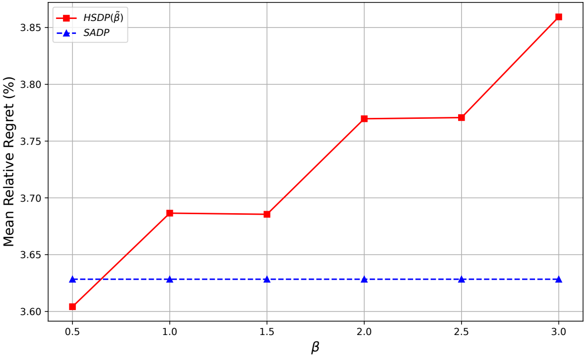

Figure: The comparison of mean relative regret over 30 experiments.

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Paper available at arxiv.org/abs/2310.07558

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Thank You!

Questions or comments? Contact: hansheng.jiang@rotman.utoronto.ca