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UGBA 141 PRODUCTION AND OPERATIONS MANAGEMENT
SPRING 2022

Problem Set 4 - Solution

Due Tuesday, April 5, 2022 11:59pm PT on bCourses

Kristen's Wedding (for Q1-4)

Kristen is done with cookies and construction. She is now planning her wedding! Kristen and her fiancé have signed a contract with a caterer that calls for them to tell the caterer the number of guests that will attend the reception a week before the actual event. This "final number" will determine how much they have to pay the caterer; they must pay $60 per guest that they commit to. If, for example, they tell the caterer that they expect 90 guests, they must pay $5,400 (= 90 × $60) even if only, say, 84 guests show up. The contract calls for a higher rate of $85 per extra guest for the number of guests beyond what the couple commits to. Thus, if Kristen and her fiancé commit to 90 guests but 92 show up, they must pay $5,570 (the original $5,400 plus 2 × $85).

Q	*f(Q)*	*F(Q)*	*I*(*Q*)	*L(Q)*
84	0.0303	0.0303	0.00	16.00
85	0.0303	0.0606	0.03	15.03
86	0.0303	0.0909	0.09	14.09
87	0.0303	0.1212	0.18	13.18
88	0.0303	0.1515	0.30	12.30
89	0.0303	0.1818	0.45	11.45
90	0.0303	0.2121	0.64	10.64
91	0.0303	0.2424	0.85	9.85
92	0.0303	0.2727	1.09	9.09
93	0.0303	0.3030	1.36	8.36
94	0.0303	0.3333	1.67	7.67
95	0.0303	0.3636	2.00	7.00
96	0.0303	0.3939	2.36	6.36
97	0.0303	0.4242	2.76	5.76
98	0.0303	0.4545	3.18	5.18
99	0.0303	0.4848	3.64	4.64
100	0.0303	0.5152	4.12	4.12
101	0.0303	0.5455	4.64	3.64
102	0.0303	0.5758	5.18	3.18
103	0.0303	0.6061	5.76	2.76
104	0.0303	0.6364	6.36	2.36
105	0.0303	0.6667	7.00	2.00
106	0.0303	0.6970	7.67	1.67
107	0.0303	0.7273	8.36	1.36
108	0.0303	0.7576	9.09	1.09
109	0.0303	0.7879	9.85	0.85
110	0.0303	0.8182	10.64	0.64
111	0.0303	0.8485	11.45	0.45
112	0.0303	0.8788	12.30	0.30
113	0.0303	0.9091	13.18	0.18
114	0.0303	0.9394	14.09	0.09
115	0.0303	0.9697	15.03	0.03
116	0.0303	1.0000	16.00	0.00

Q = Number of guests that show up to the wedding
$f(Q)$ = Density function = Prob{Q guests show up}
$F(Q)$ = Distribution function = Prob{Q or fewer guests show up}
$L(Q)$ = Loss function = Expected number of guests above Q

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The problem Kristen faces is that she still does not know the exact number of guests to expect. Despite asking that friends and family members reply to their invitations a month ago, some uncertainty remains: Her brother may—or may not—bring his new girlfriend; her fiancé's college roommate may—or may not—be able to take a vacation from work; and so forth. Kristen has determined that the expected number of guests (i.e., the mean number) is 100, but the actual number could be anywhere from 84 to 116:

Q1. How many guests should Kristen commit to with the caterer?

The overage cost is $C_o = \$60$ because ordering a plate for a guest that doesn't show up costs $60. The underage cost is $C_u = \$85 - 60 = \$25$, because not committing to a guest that does show up costs an extra $25. The critical ratio is 25 / (60 + 25) = 0.2941. From the table, $F(92) = 0.2727$ and $F(93) = 0.3030$, so the optimal number of guests to commit to is 93.

Q2. Suppose Kristen commits to 105 guests. What is Kristen's expected bill?

Expected number of extra guests is $L(105) = 2$, so her bill is $60 × 105 + $85 × 2 = $6,470.

Q3. Suppose that the caterer is willing to alter the contract so that if fewer than the number of guests they commit to show up, they will get a partial refund. In particular, they only have to pay $45 for each "no-show." For example, if they commit to 90 but only 84 show, they will have to pay 84 × $60 = 6 × $45 = $5,310. Now how many guests should she commit to?

The overage cost is $C_o = \$45$ because committing to a guest that doesn't show only costs $45. The underage cost is still $C_u = \$85 - 60 = \$25$, because not committing to a guest that does show up costs an extra $25. The critical ratio is 25 / (45 + 25) = 0.3571. From the table, $F(94) = 0.3333$ and $F(95) = 0.3636$, so the optimal number of guests to commit to is 95.

Q4. The caterer offers Kristen another option. She could pay $70 per guest, no matter how many guests show up; that is, she wouldn't have to commit to any number before the wedding. Should Kristen prefer this option or the original option ($60 per committed guest and $85 each guest beyond the commitment)?

With the original plan we need to evaluate the expected bill. Expected number of extra guests is $L(93) = 8.36$, so her bill is \$60 × 93 + \$85 × 8.36 = \$6,291. At \$70 per guest, her expected bill is \$70 × 100 = \$7000. So go with the original option contract.

Pegasus Books (Q5-8)

Pegasus Books (PB) is deciding on how many copies of a book to purchase at the start of the upcoming selling season for the bookstore. The book retails at \$28.00. The publisher sells the book to PB for \$20.00. PB will dispose of all of the unsold copies of the book at 75% off the retail price, at the end of the season. PB estimates that demand for this book during the season is normal with a mean of 100 and a standard deviation of 42. The publisher's variable cost per book is \$7.50.

Q5. How many books should PB order to maximize his expected profit?

Use the newsvendor model. The overage cost is C_o = cost – salvage value = \$20 - \$28 /4 = \$13. The underage cost is C_u = price - cost = \$28 - \$20 = \$8. The critical ratio is 8 / (13 + 8) = 0.3810. Look up the critical ratio in the Standard Normal Distribution Function table to find the appropriate z statistic = -0.30. The optimal order quantity is $Q = \mu + z \times \sigma = 100 - 0.30 \times 42 = 87$.

The publisher is thinking of offering the following deal to PB. At the end of the season, the publisher will buy back unsold copies at a pre-determined price of \$15.00. However, PB would have to bear the costs of shipping unsold copies back to the publisher at \$1.00 per copy.

Q6. How many books should PB order to maximize their expected profits given the buy-back offer?

The underage cost is a lost sale that costs PB the gross margin, C_u = \$8. PB buys each book for \$20 and then returns leftover books for a net salvage value of \$15 - \$1 (due to the shipping cost) = \$14. So their overage cost is now C_o = cost – salvage value = \$20 - \$14 = \$6. The critical ratio is 8 / (6 + 8) = 0.5714. Look up the critical ratio in the Standard Normal CDF table to find the appropriate z statistic = 0.18. The optimal order quantity is $Q = mu + z \times sigma = 100 + 0.18 \times 42 = 108$.

Q7. Given the order quantity in Q6, what is PB's expected profit?

Expected leftover inventory = sigma x l(z) = 20.8. Expected sales = Q – 20.8 = 87.2. Profit = price x Expected sales + salvage value x Expected leftover inventory – Q x cost = \$28 x 87.2 + \$14 x 20.8 – 108 x \$20 = \$573.

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Q8. Suppose the publisher continues to charge $20 per book and PB still incurs a $1 cost to ship each book back to the publisher. What price should the publisher pay PB for returned books to maximize the supply chain's profit (the sum of the publisher's profit and PB's profit)?

The formula in the lecture gives the buyback price that coordinates the supply chain (that is, maximizes the supply chain's profit).

$$\text{Buy-back price} = \text{Shipping cost} + \text{Price} - (\text{Price} - \text{Wholesale price}) \times \left(\frac{\text{Price} - \text{Salvage value}}{\text{Price} - \text{Cost}}\right)$$

That buyback price is $1 + $28 – ($28 - $20) x ($28 - $7)/($28 - $7.5) = $20.80. Note, the publisher's buyback price is actually higher than the wholesale price because the publisher needs to subsidize PB's shipping cost to return books: PB's net loss on each book returned is $20 – (20.80 – 1) = $0.20.