Spring 2022 UGBA 141 Final Reference Sheet

1. Newsvendor

   (a) The order quantity Q^* satisfies

   $$\text{Prob}(D \leq Q^*) = \text{Critical ratio} = \frac{G}{G+L},$$

   where G is gain if stock additional unit and sell it, and L is loss if stock additional unit and don't sell it.

2. Supply Chain

   (a) Let Q denote the order quantity, and let D denote the random demand with expectation/mean μ.

   - Leftover inventory $= \max\{Q - D, 0\} = Q - \min\{Q, D\}$
   - Lost sales $= \max\{D - Q, 0\} = D - \min\{Q, D\}$
   - Sales $= \min\{Q, D\}$

   The following two fundamental equalities hold.

   $$\text{Expected lost sales} + \text{Expected sales} = \text{Expected demand } \mu$$

   $$\text{Expected sales} + \text{Expected leftover inventory} = \text{Expected order quantity } Q.$$

   (b) Read the "Standard Normal Inventory/Loss Table"

   $$\text{Expected leftover inventory} = \text{Demand standard deviation } \sigma \times I(z)$$

   where z is the ratio of $(Q - \mu)/\sigma$, and $I()$ is the standard normal inventory function read from the table.

   $$\text{Expected lost sales} = \text{Demand standard deviation } \sigma \times L(z)$$

   where z is the ratio of $(Q - \mu)/\sigma$, and $L()$ is the standard normal inventory function read from the table.

   (c) Expected profit of Newsvendor is

   $$G \times \text{Expected sales} - L \times \text{Expected leftover inventory}$$

   where gain G and loss L need to be interpreted based on contexts by taking into consideration costs, prices, salvage value, and shipping costs whenever needed. Remember that alternatively, G can be viewed as underage cost and L can be viewed as overage cost.

   (d) Optimal Buy-back price is equal to

   $$\text{Shipping cost} + \text{Price} - \frac{(\text{Price - Wholesale price}) \times (\text{Price - Salvage value})}{\text{Price - Cost}},$$

   where 'Price' refers to retailing price, and 'Cost' refers to production cost.

3. Queue

   (a) Coefficients of variation

   - Coefficients of variation for the arrival process CV_a= Std of interarrival time / average interarrival time
   - Coefficients of variation for the processing CV_p= Std of processing time / average processing time

(b) Implied utilization = Demand / Capacity = $p/(a \times m)$. Utilization = Flow rate/ Capacity. When Implied utilization < 1, Demand = Flow rate, then utilization = implied utilization.

(c) Time in queue / Waiting time

$$\text{Time in queue} = \left(\frac{p}{m}\right) \times \left(\frac{\text{Utilization}^{\sqrt{2m+2}-1}}{\text{1-Utilization}}\right) \times \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

where a is average interarrival time. p is the average processing time, and m is the number of servers. Time in system = time in queue + processing time (p)

(d) Number of customers = Time in system * flow rate ($\frac{1}{a}$)