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UGBA 141 PRODUCTION AND OPERATIONS MANAGEMENT
SPRING 2022

Problem Set 5 - Solution

Due Tuesday, April 26, 2022, 11:59pm PT on bCourses

Q1. Kristen, now happily married, owns land that she uses to grow and harvest trees. The table below provides data on the number of trees harvested per year and the average age of the trees harvested in the year.

Year	Trees harvested	Average age (years) of harvested trees
2013	4000	5
2014	8000	3
2015	7000	3
2016	5000	4

On average, how many <u>trees</u> does she have growing on her land?

R = (4K+8K+7K+5K)/4 = 6000

T = (4*5 + 8*3 + 7*3 + 5*4)/(4+8+7+5) = 3.54

I = R x T = 6000 x 3.54 = 21250

The Minute Clinic of West Berkeley (Q2-3)

The Minute Clinic of West Berkeley sees walk in patients (no appointments) with routine medical needs. Patients arrive at the rate of 1 every 2.5 minutes with a standard deviation of 2.5 minutes. Each patient is seen by one nurse practitioner for on average 16 minutes with a standard deviation of 24 minutes.

Q2. What is the minimum number of nurse practitioners needed (integer answer required) to ensure that the queue of patients doesn't continue to grow throughout the day? (Note, the number of nurse practitioners must be an integer.)

Util = p / a x m < 1 p/a < m

16/2.5 = 6.4 < m → m = 7

Q3. Suppose the Minute Clinic of West Berkeley has 8 nurses. On average, how many patients are waiting to see a nurse practitioner?

U = p/(a x m) = 0.8

Tq = (p /m) x (u^(sqrt(2x(m+1))-1)/(1-u))x(CVa^2+CVp^2)/2 = 7.88

Iq = 1/a x Tq = 3.15

Q4. After taking UGBA141, you and your team come up with the business idea of a private home security service. This private home security service guarantees to either dispatch one of their own five guards immediately if one of their customers sends in an alarm or, in the case that all five guards are responding to other calls, direct the alarm to the local police. The company receives 12 calls per hour, evenly distributed over the course of the day.

The local police charges the home security company $500 for every call that the police responds to. It takes a guard, on average, 90 minutes to respond to an alarm.

What fraction of the time are incoming alarms directed to the police?

We have m=5, an interarrival time of a=5 minutes and a service time of p=90 minutes.

We compute r=p/a=18; we then use the Erlang loss table to find that the probability that all 5 servers are utilized, $P_5(18)=0.7402$

Berkeley Public Library (for Q5-7)

Berkeley Public Library (BPL) is open 24 hours a day, and—due to its proximity to UC Berkeley—experiences customers arriving around the clock. A recent analysis done by the manager indicates that there are 30 customers arriving every hour, with a standard deviation of interarrival times of 2 minutes. This arrival pattern is consistent and is independent of the time of day. The checkout is currently operated by one staff, who needs on average 1.7 minutes to check out a customer. The standard deviation of this checkout time is 3 minutes, primarily as a result of customers checking out different numbers of books.

Q5. When there is no checkout, the staff is sorting returned books, of which there are always plenty waiting to be sorted. How many books can the staff sort over an 8-hour shift (assume no breaks) if it takes exactly 1.5 minutes to sort a single book?

utilization = 1.7 min / 2 min = 0.85

idle time = 0.15 * 8 hrs = 72 min / 1.5 min per sort = 48 sorts

Q6. What is the average number of customers who are at the checkout desk, either waiting or currently being served (within 1 customer)?

Using R = minimum of 1/a and 1/p, we calculate R = 0.5. Thus, the average number of customers in line waiting I = R*Tq = 0.5*19.82 = 9.9 customers. In addition, an average of .85 customers (equal to the utilization u) are being served at any given time. So the average number of customers at the check-out desk = 9.9 + 0.85 = 10.75.

Q7. Now assume for this question only that 10% of the customers do not check out a book at all and therefore do not have to go through checkout. What is the average time a customer has to wait in line before getting served by the checkout staff, not including the actual checkout time (within 1 minute)? Assume that the coefficient of variation for the arrival process remains the same as before.

Because only 90% of customers go through checkout, the inter-arrival time of paying customers changes: 27 customers per hour = 1 customer / 2.22 min
waiting time = 1.7 min * [(1.7/2.22)/(1-1.7/2.22)] * [(1^2 + 1.765^2)/2] = *11.38 min*

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Cal Performances (for Q8-10)

Jeremy, the director at Cal Performances, is planning his pricing strategy for a musical to be held in a 100-seat theater. He sets the full price at $80 and estimates demand at this price to be normally distributed with mean 40 and standard deviation 30. Jeremy also decides to offer student-only advance sale tickets discounted 50% off the full price. Demand for the discounted student-only tickets is usually abundant and occurs well before full-price ticket sales.

Q8. Suppose Jeremy sets a 50-seat booking limit for the student-only tickets. What is the number of full-price tickets that Jeremy expects to sell?

The high-fare protection level = Capacity – Low-fare booking limit = 100 – 50 = 50. Next, z = $(Q - \mu)/\sigma$ = (50 – 40)/30 = 10/30 = 0.333. $L(0.333) = 0.2555$. Expected lost sales = $\sigma \times L(z) = 30 \times 0.2555 = 7.665$. Expect sales (high fare) = μ - Expected lost sales = 40 - 7.665 = 32.335.

Q9. Based on a review of the show in another city, Jeremy updates his demand forecast for full-price tickets to be normal with mean 60 and standard deviation 40, but he does not change the prices. What is the optimal protection level for full-price seats?

Critical Ratio = $C_u/(C_u + C_o) = (r_h - r_l) / r_h = (80 - 0.5 \times 80) / 80 = 0.5$. $F(0) = 0.5$, i.e. z = 0. $Q = \mu + z \times \sigma = \mu = 60$.

Q10. Jeremy realizes that having many empty seats negatively affects the attendees' value from the show. Hence, he decides to change the discount given on student-only tickets from 50% off the full price to 55% off the full price and he continues to set his protection level optimally. (The demand forecast for full-price tickets remains as in Q8, normal with mean 60 and standard deviation 40.)

How will this change in the student-only discount price affect the expected number of empty seats? (Will they increase, decrease, or remain the same, or is it not possible to determine what will happen?)

An increase in the discount leads to lower student ticket prices which increases the critical ratio. Critical Ratio = $C_u/(C_u + C_o) = (r_h - r_l) / r_h = (80 - 0.45 \times 80) / 80 = 0.55$. Thus, the optimal protection level, Q will increase. This means fewer seats available for students, demand from whom is abundant and more seats that could potentially go empty if not enough full-price demand materialize. Thus, the expected number of empty seats will increase.