UGBA 141 Discussion 4

Agenda: Review and Practice of Quality Module

— Control charts
— Capability analysis

Feb 11, 2022
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Logistics

- New GSI Office Hours
  - Tuesday 1-2 pm and Friday 2-3pm at Chou N455

Recap: Key Concepts in Statistical Process Control

- $\overline{X}$: average/mean of a sample
- R: range of a sample = maximum value - minimum value
- p: fraction of defects
- c: number of defects
- **Control limits: Upper and Lower**

Summary

range percentage of d. count of d.

	Mean Charts	R-Charts	p-Charts	c-Charts
CL	$\overline{\overline{X}}$	$\overline{R}$	$\overline{p}$	$\overline{c}$
UCL	$\overline{\overline{X}} + A_2\overline{R}$	$D_4\overline{R}$	$\overline{p} + 3 * \sqrt{\dfrac{\overline{p}(1-\overline{p})}{\text{sample size}}}$	$\overline{c} + 3\sqrt{\overline{c}}$
LCL	$\overline{\overline{X}} - A_2\overline{R}$	$D_3\overline{R}$	$\overline{p} - 3 * \sqrt{\dfrac{\overline{p}(1-\overline{p})}{\text{sample size}}}$	$\overline{c} - 3\sqrt{\overline{c}}$

- A_2, D_3, D_4 are control chart constants, see 'Table of Control Chart Constants.pdf' on bcourse

Table of Control Chart Constants

Sample Size = m	A_2	A_3	d_2	D_3	D_4	B_3	B_4
2	1.880	2.659	1.128	0	3.267	0	3.267
3	1.023	1.954	1.693	0	2.574	0	2.568
4	0.729	1.628	2.059	0	2.282	0	2.266
5	0.577	1.427	2.326	0	2.114	0	2.089
6	0.483	1.287	2.534	0	2.004	0.030	1.970
7	0.419	1.182	2.704	0.076	1.924	0.118	1.882
8	0.373	1.099	2.847	0.136	1.864	0.185	1.815
9	0.337	1.032	2.970	0.184	1.816	0.239	1.761
10	0.308	0.975	3.078	0.223	1.777	0.284	1.716
11	0.285	0.927	3.173	0.256	1.744	0.321	1.679
12	0.266	0.886	3.258	0.283	1.717	0.354	1.646
13	0.249	0.850	3.336	0.307	1.693	0.382	1.618
14	0.235	0.817	3.407	0.328	1.672	0.406	1.594
15	0.223	0.789	3.472	0.347	1.653	0.428	1.572
16	0.212	0.763	3.532	0.363	1.637	0.448	1.552
17	0.203	0.739	3.588	0.378	1.622	0.466	1.534
18	0.194	0.718	3.640	0.391	1.608	0.482	1.518
19	0.187	0.698	3.689	0.403	1.597	0.497	1.503
20	0.180	0.680	3.735	0.415	1.585	0.510	1.490
21	0.173	0.663	3.778	0.425	1.575	0.523	1.477
22	0.167	0.647	3.819	0.434	1.566	0.534	1.466
23	0.162	0.633	3.858	0.443	1.557	0.545	1.455
24	0.157	0.619	3.895	0.451	1.548	0.555	1.445
25	0.153	0.606	3.931	0.459	1.541	0.565	1.435

Practice Problem: $\overline{X}$-Charts and R-Charts

- A cereal manufacturer fills cereal boxes to an average weight of 20 ounces and has an average range of 2 ounces when the filling process is in control. A sample size of 10 boxes is used in evaluating the process.

  a. What are the CL, UCL, and LCL for the x and R charts?

$\bar{x}$ Chart

CL = 20

UCL = 20 + .308 × 2 = 20.616

LCL = 20 − .308 × 2 = 19.384

R chart

CL = 2

UCL = 1.777 × 2 = 3.554

LCL = 0.223 × 2 = 0.446

  b. A sample with the following 10 measurements was just taken: 20, 21, 19, 18, 19, 21, 22, 20, 20, 19. Is the process still in control?

sample mean 199 / 10 = 19.9 ⇒ mean is in control

sample range 22 − 18 = 4. ⇒ range is out of control

Practice Problem: p-Charts

- A company that makes golf tees controls its production process by periodically taking a sample of 100 tees from the production line. Each tee is inspected for defective characteristics. Control limits are developed using three standard deviations from the mean. During the last 16 samples taken, the proportion of defective items per sample was recorded as follows: .01 .02 .01 .03 .02 .01 .00 .02 .00 .01 .03 .02 .03 .02 .01 .00

  a. Determine the mean proportion defective, the UCL, and the LCL.

$$CL = \frac{.01 + .02 + .01 + .03 + .02 + .01 + .00 + .02 + .00 + .01 + .03 + .02 + .03 + .02 + .01 + .00}{16}$$

$$= .015$$

$$UCL = 0.15 + 3\sqrt{\frac{.015(1-.015)}{100}} = 0.0515$$

$$LCL = 0.15 - 3\sqrt{\frac{.015(1-.015)}{100}} = -0.0215 < 0 \Rightarrow \text{truncate, } LCL = 0$$

  b. Does it appear that the process for making tees is in statistical control?

All 16 points are in control limits [LCL, UCL]

process is in statistical control

Capability Analysis

- **Process capability measures**
  - Centered process

$$C_p = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}}$$

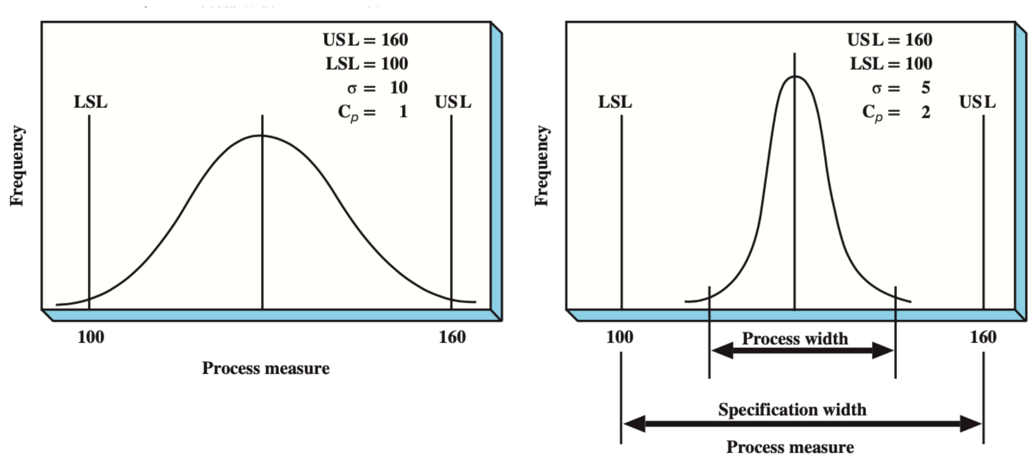
  - Off-centered process

$$C_{pk} = \min\left\{ \frac{\text{USL} - \overline{X}}{3\hat{\sigma}}, \frac{\overline{X} - \text{LSL}}{3\hat{\sigma}} \right\}$$

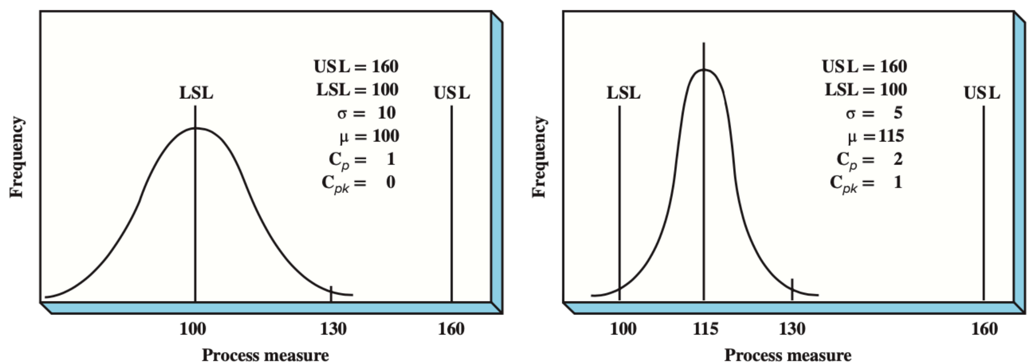
  - $\hat{\sigma}$ is sample standard deviation $\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2}$

Examples of Process Capability Measures

- Centered



- Off-centered

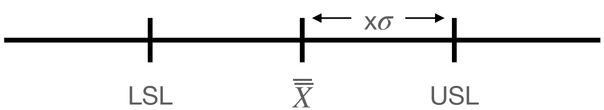


(a)

(b)

Probability of Defect

- Convert Process capability measures to probability of defect



$x\sigma$	C_p	P{defect}	ppm
1σ	0.33	0.317	317,000
2σ	0.67	0.0455	45,500
3σ	1.00	0.0027	2,700
4σ	1.33	0.0001	63
5σ	1.67	0.0000006	0.6
6σ	2.00	2×10^{-9}	0.00

Six Sigma = Zero Defect

Practice Problem: C_{pk} and C_p

- The operations manager of an insurance claims- processing department wants to determine the claims-processing capability. Claims usually take a minimum of four days to handle. The company has a commitment to handle all claims within 10 days. On average, claims are processed in 8 days and processing has a standard deviation of 1 day

a. Compute C_p and C_{pk} for the claims-processing department. Based on these computations, should the claims department improve its process?

$$C_p = \frac{10-4}{6\times 1} = 1 \quad , \quad C_{pk} = \min\left\{\frac{10-8}{3\times 1}, \frac{8-4}{3\times 1}\right\} = 0.667$$

b. Using the same data, recompute C_{pk}, but use an average claims-processing time of seven days instead of eight days.

$$C_{pk} = \min\left\{\frac{10-7}{3\times 1}, \frac{7-4}{3\times 1}\right\} = 1$$

c. Using the original data, recompute C_{pk}, but use a standard deviation of $2/3$ of a day. Which change made the most improvement—the change in mean in part b or the change in standard deviation? Can you explain the results?

$$C_{pk} = \min\left\{\frac{10-8}{3\times 0.667}, \frac{8-4}{3\times 0.667}\right\} = 1$$

Explain: to increase C_{pk}, one can either shift mean to center of LSL, USL or decrease the standard deviation.

How to Read Standard Normal Table

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899