

# Designing Surprise Bags for Surplus Foods

Hansheng Jiang (UToronto Rotman)

Joint work with

Fan Zhou (CUHK-SZ), Andrea Li (Too Good To Go), Joline Uichanco (Michigan Ross)

INFORMS Revenue Management and Pricing Conference

Columbia University

July 16, 2025

# Food Waste Problem

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**OF FRESHWATER USED ANNUALLY**

goes into food that  
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Source: WWF 2024, FAO 2023.

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**783**

**MILLION PEOPLE**

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Source: WWF 2024, FAO 2023.

# Food Waste in Groceries and Dining

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## Starbucks Barista Shows Shocking Amount of Food Wasted After Each Shift

BY KATE UNDERWOOD

PUBLISHED JUNE 14 2023, 9:54 A.M. ET



SOURCE: AIDANSTOCKIN/TIKTOK

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Marketplace finds discarded food well before the best-before date; retailer calls it unfit to consume

[Melissa Mancini](#), [Nelisha Vellani](#) · CBC News ·

Posted: Oct 25, 2016 5:00 AM EDT | Last Updated: October 25, 2016



Bottles of water and still-cold frozen food: Marketplace found garbage bins full of food at Walmart. (CBC)

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EXCLUSIVE

## McDonald's, El Pollo Loco Face Investor Bids to Cut Food Waste

December 5, 2024, 11:16 AM EST



Clara Hudson  
Reporter



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In 2023,

**74 million tonnes**

of surplus food are generated in U.S., accounting for 31% of U.S. food supply, but only

**11%**

was recovered through donations or animal feed

(Source: ReFED 2025)



ce food waste.

# Emerging Platforms

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Emerging platforms are combating food waste by

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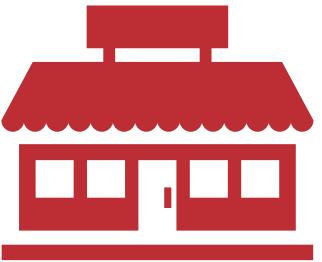
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Surplus food

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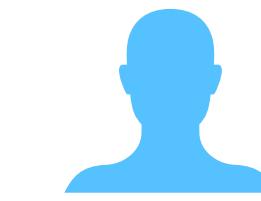
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Surplus food



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**Too Good To Go**

North America, Europe & Australia

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**X-bag**

China (Mainland)

# Too Good To Go: A Win-Win-Win Dynamic

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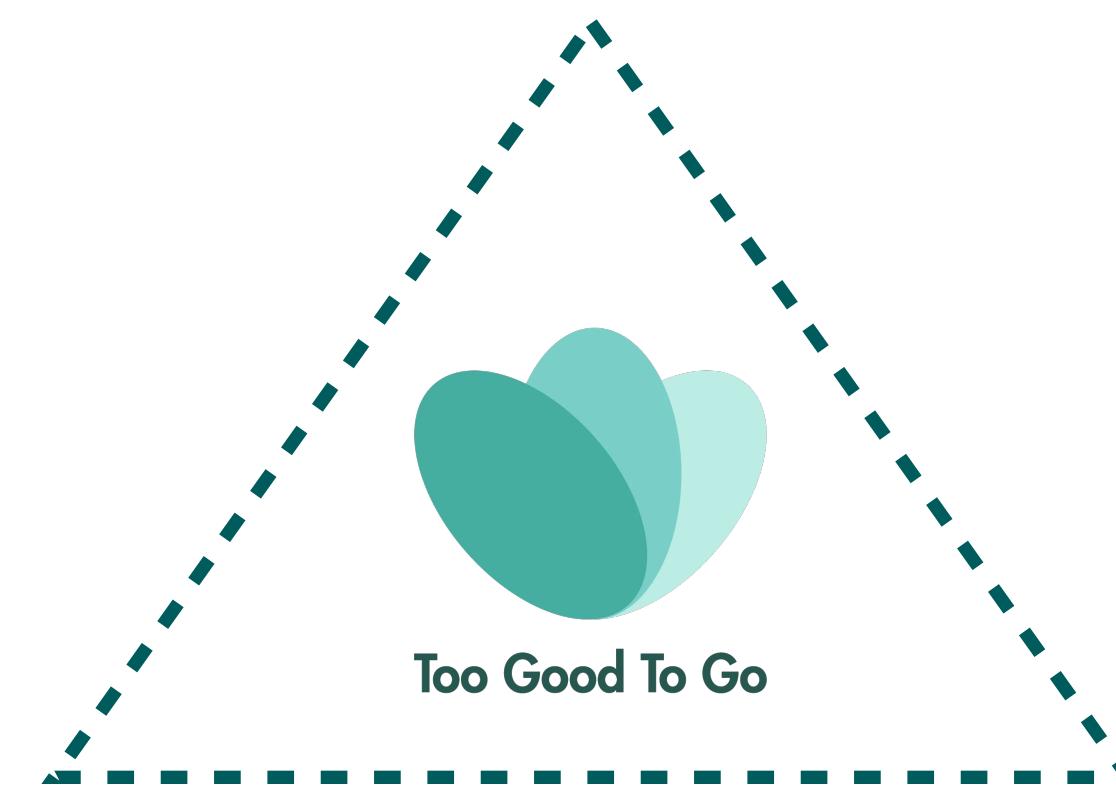
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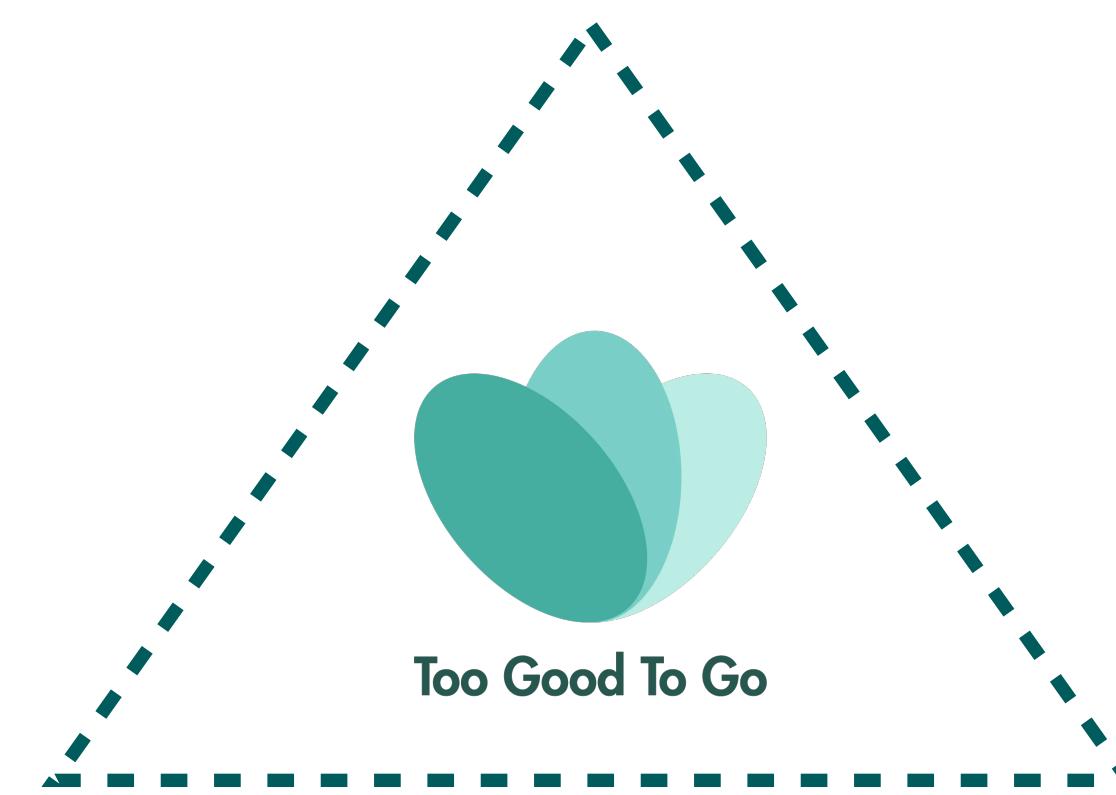
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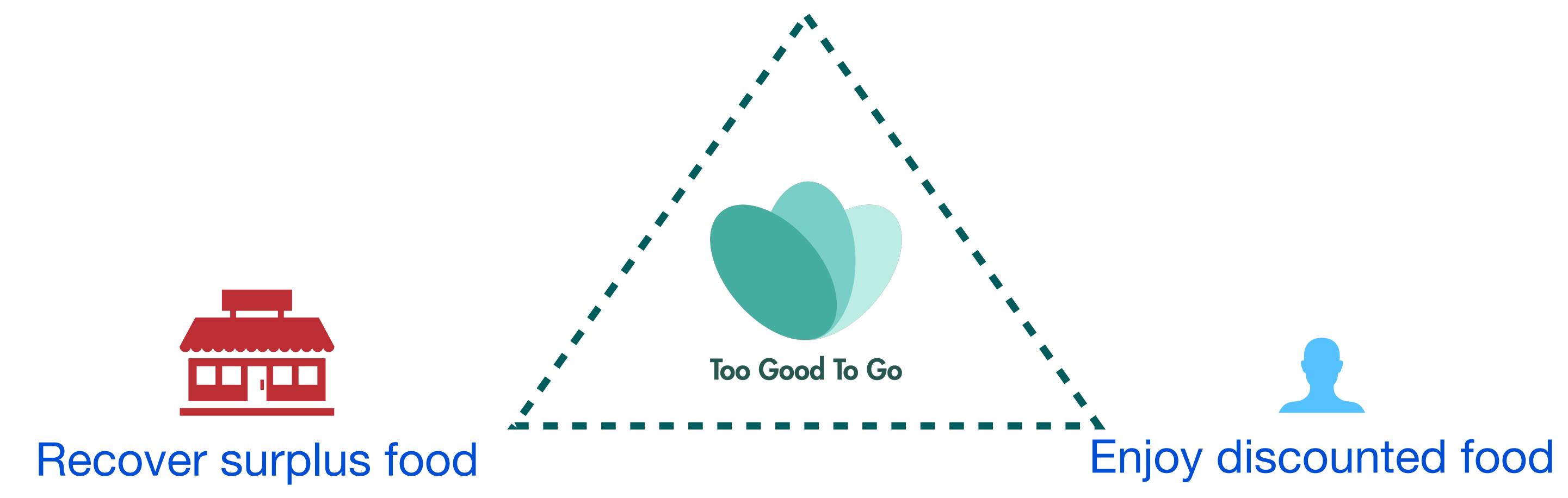
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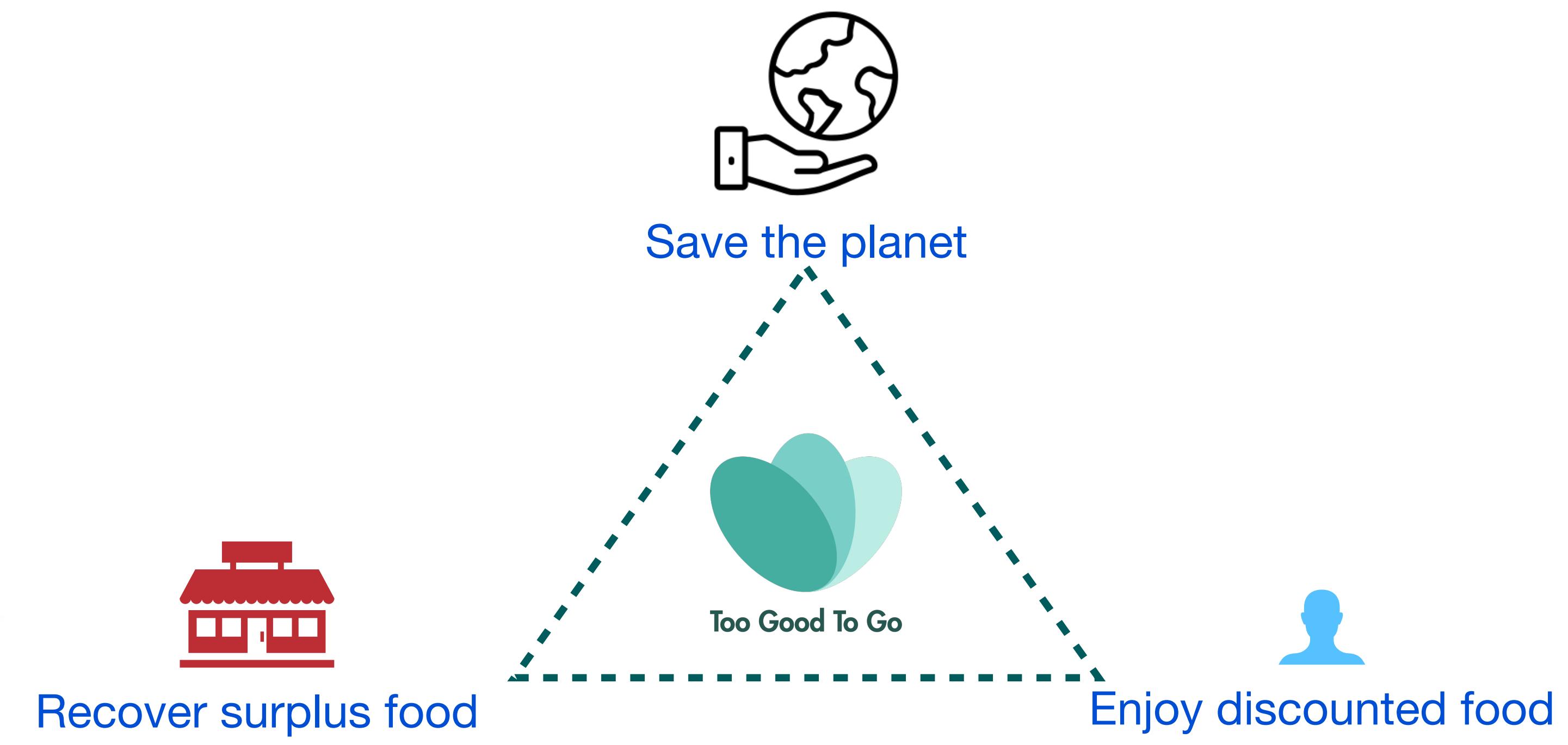
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# Surprise Bags

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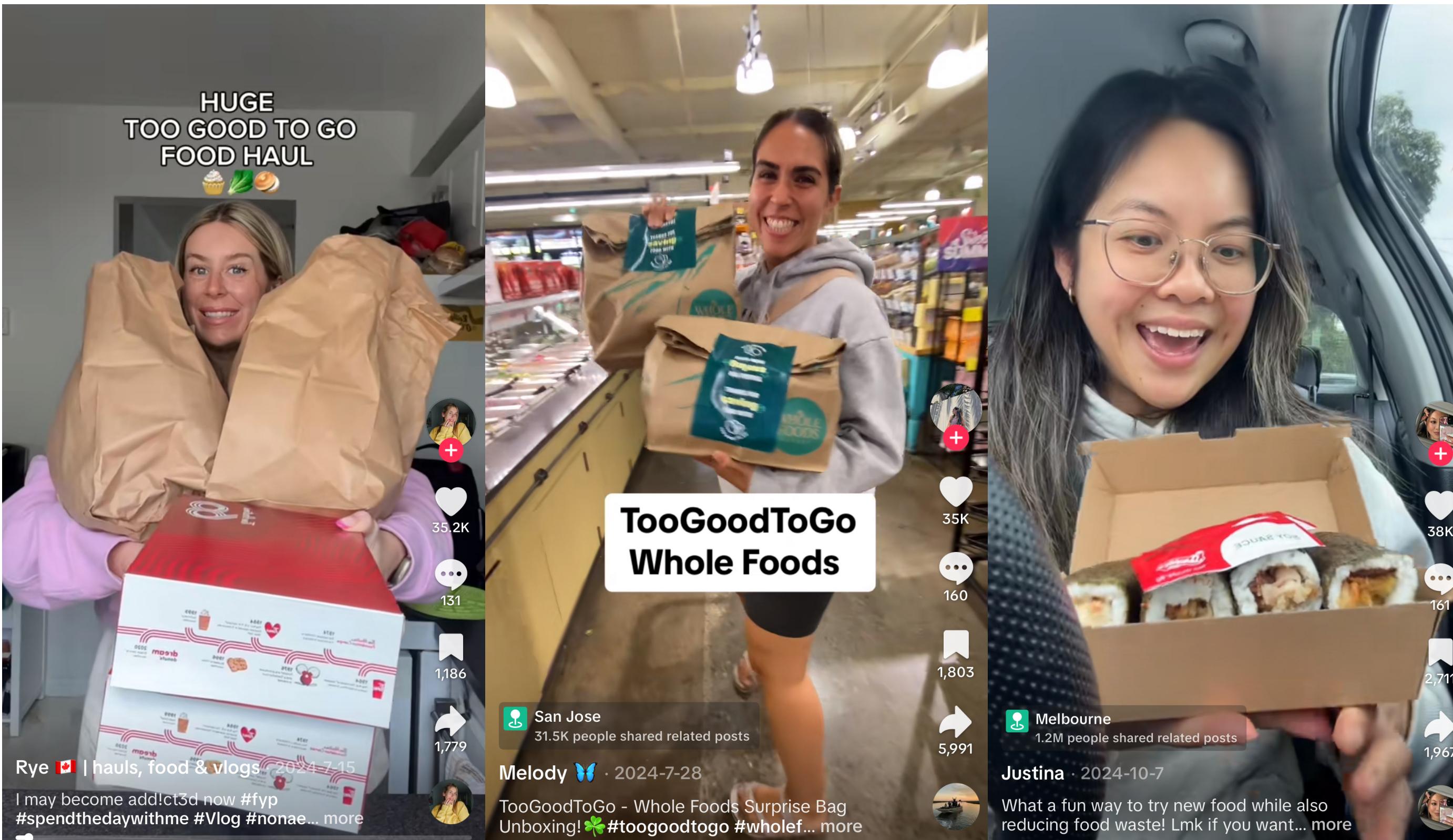
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# Workflow of TGTG Platform

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The screenshot displays a product listing on the TGTG platform. At the top, there is a photograph of various baked goods, including donuts and cookies. A yellow callout bubble in the top-left corner of the photo says "3 left". Below the photo, the store name "Tim Hortons - 246 Bloor Street West" is displayed, along with the Tims logo.

**Baked Goods**      \$15.00  
★ 4.5 (80)      **\$4.99**  
🕒 Pick up: 10:00 PM - 10:45 PM      **Today**

**Location:** 246 Bloor St W, Toronto, ON M5S 1V4, Canada      [More information about the store](#)

**What you could get:**  
Your Surprise Bag will contain an assortment of surplus baked goods such as Donuts, Timbits, Cookies, Muffins, Bagels, and/or other baked goods.

**Bread & pastries**

**Ingredients & allergens**      [More](#)

**WHAT OTHER PEOPLE ARE SAYING**

★ **4.5 / 5.0**

**Reserve**

# Workflow of TGTG Platform

A screenshot of a mobile application interface for the TGTG platform. At the top, there's a photograph of various baked goods like donuts and cookies. A blue circle highlights a yellow button labeled "3 left". Below the image, the store name "Tim Hortons - 246 Bloor Street West" is displayed, along with the Tim Hortons logo.

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**WHAT OTHER PEOPLE ARE SAYING**  
★ **4.5 / 5.0**

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Participating businesses list surprise bags of surplus foods

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The screenshot shows a listing for a Tim Hortons store at 246 Bloor Street West. The listing includes a photo of various baked goods, a price of \$4.99, a rating of 4.5 stars from 80 reviews, and a pick-up time of 10:00 PM - 10:45 PM on Today. It also shows that there are 3 items left. The listing includes sections for 'What you could get' (surplus baked goods like donuts, timbits, cookies, muffins, bagels), 'Bread & pastries', 'Ingredients & allergens', and 'WHAT OTHER PEOPLE ARE SAYING' with a rating of 4.5 / 5.0. A large green 'Reserve' button is at the bottom.

3 left

Tim Hortons - 246 Bloor Street West

Baked Goods \$15.00  
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4.5 (80)

Pick up: 10:00 PM - 10:45 PM Today

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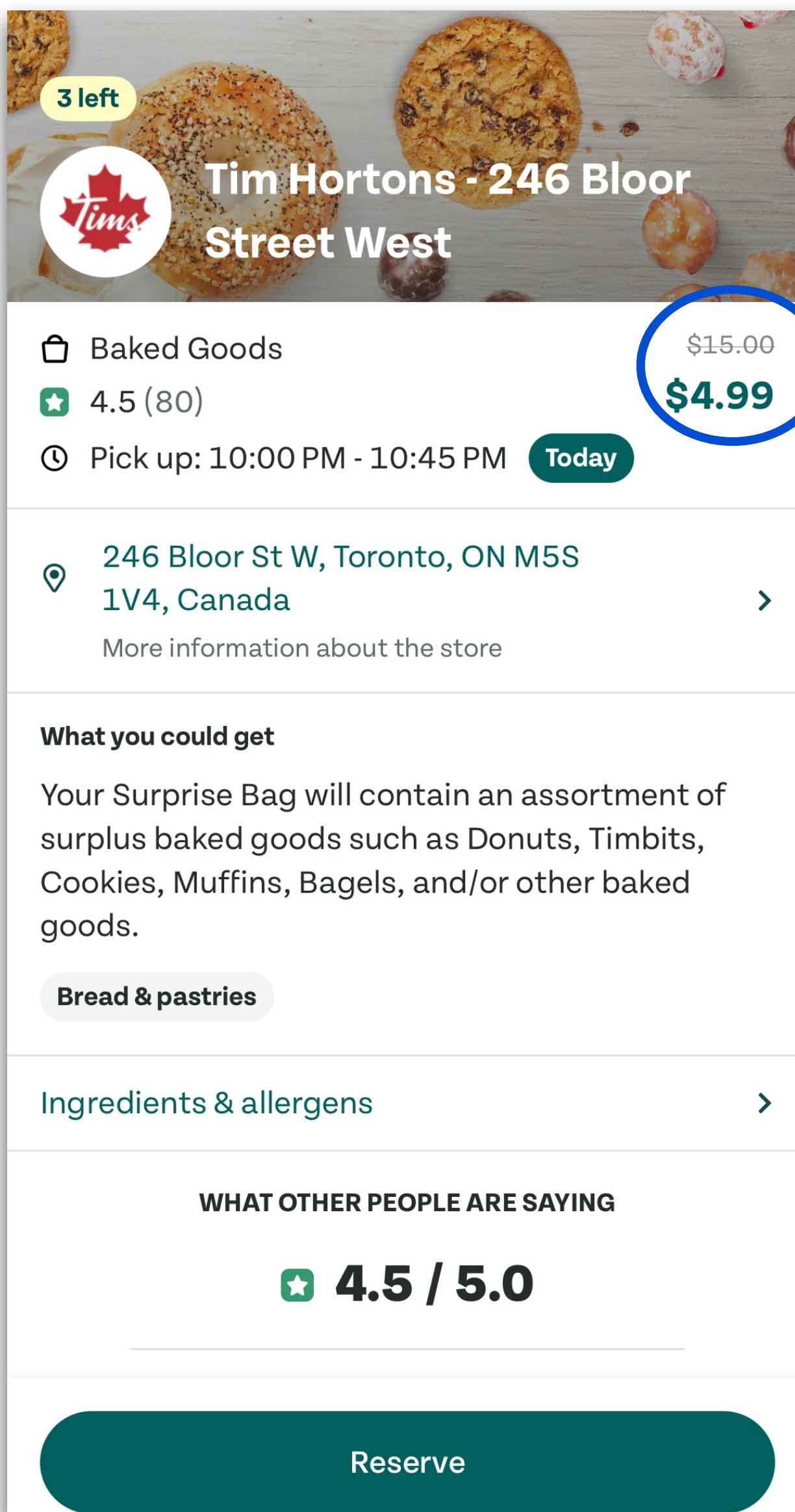
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4.5 / 5.0

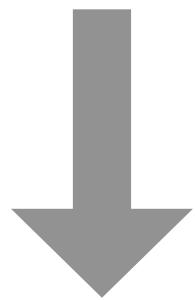
Reserve

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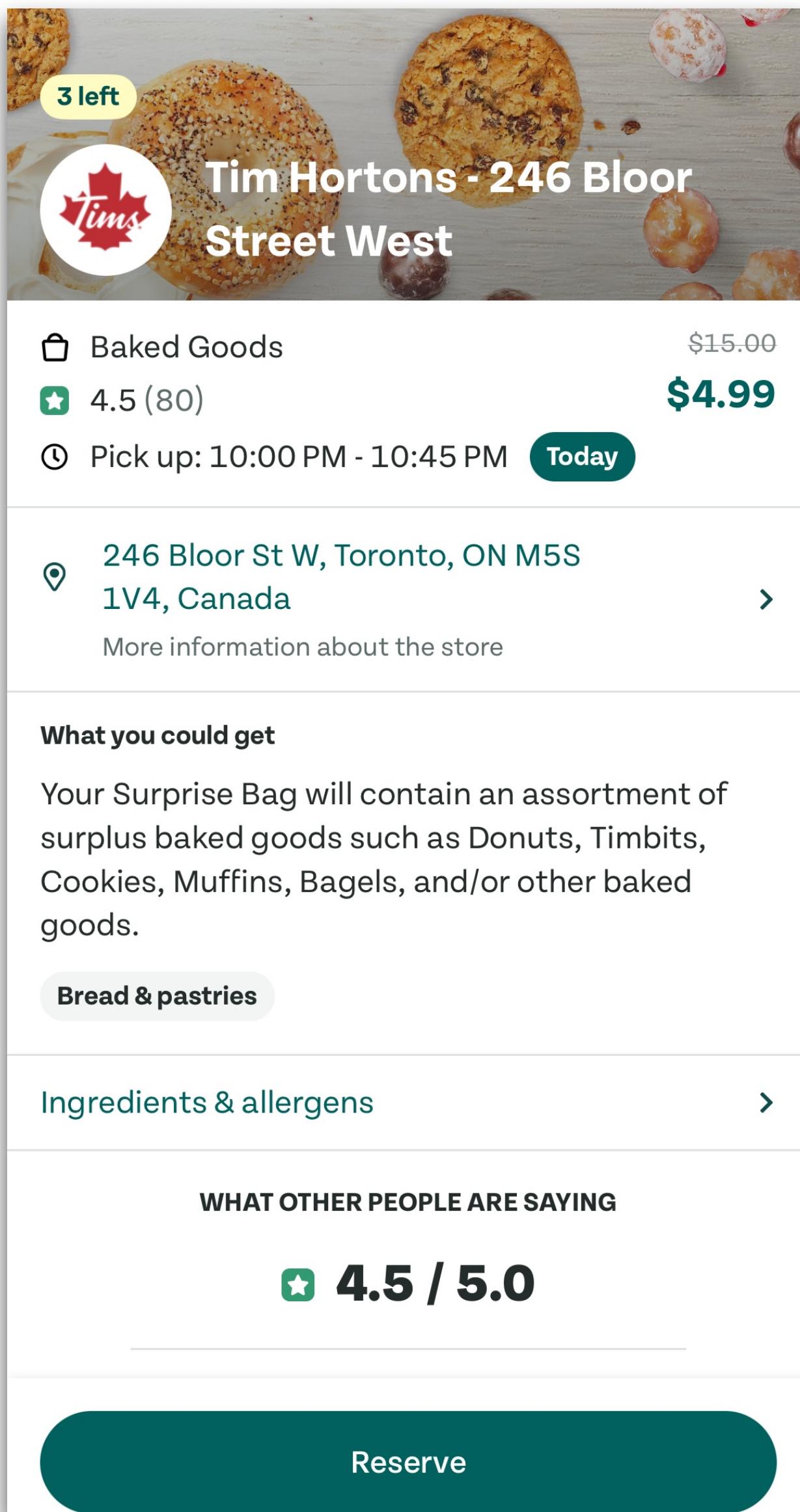


Participating businesses list  
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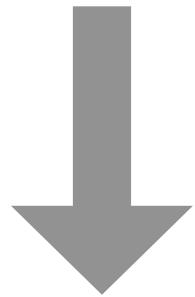


Customers reserve bags at  
highly discounted price

# Workflow of TGTG Platform

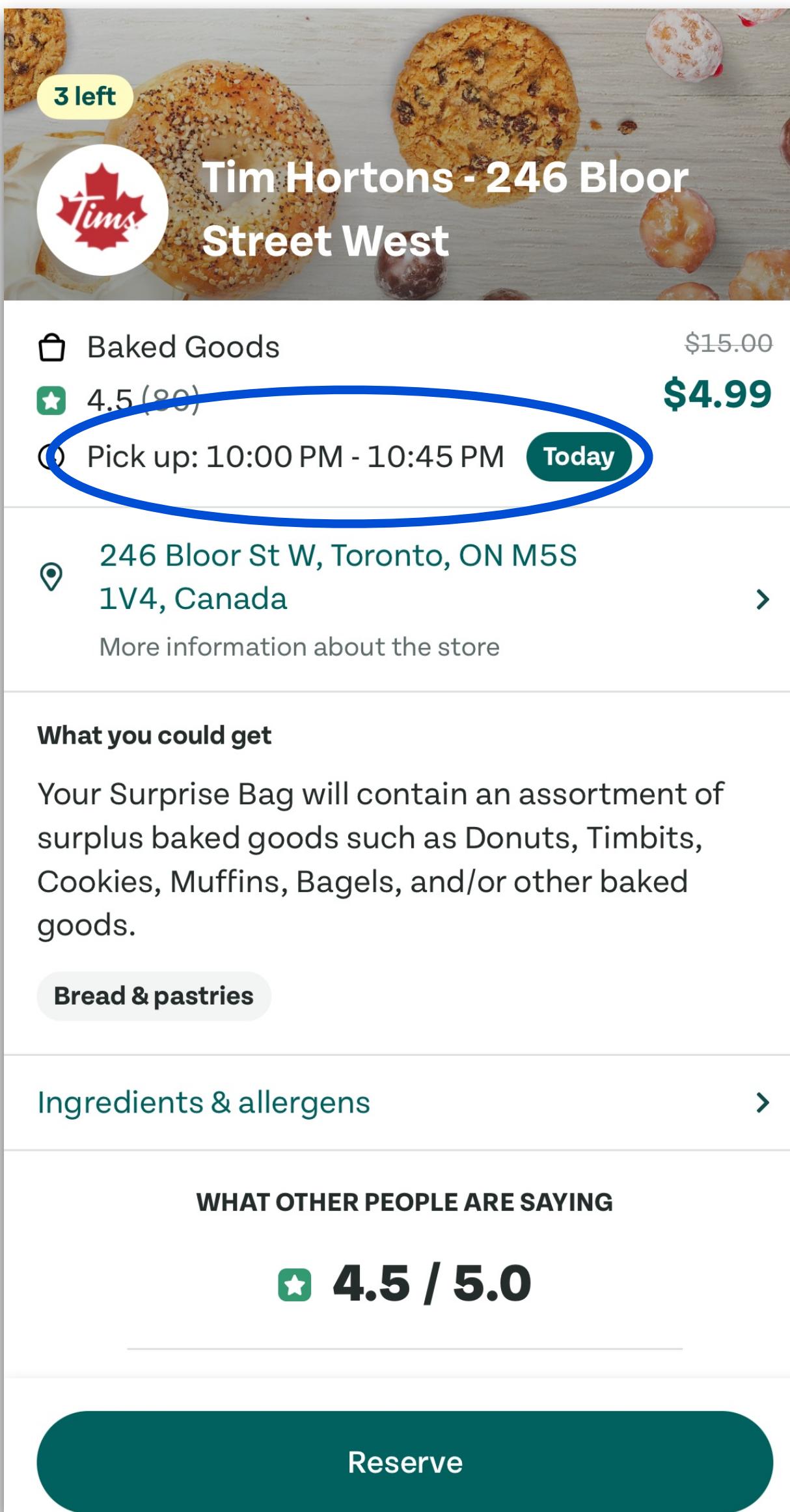


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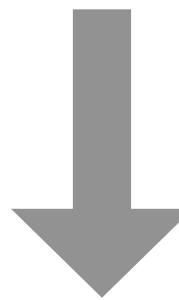


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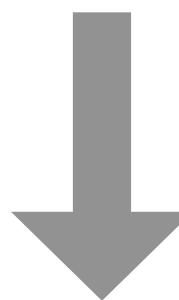
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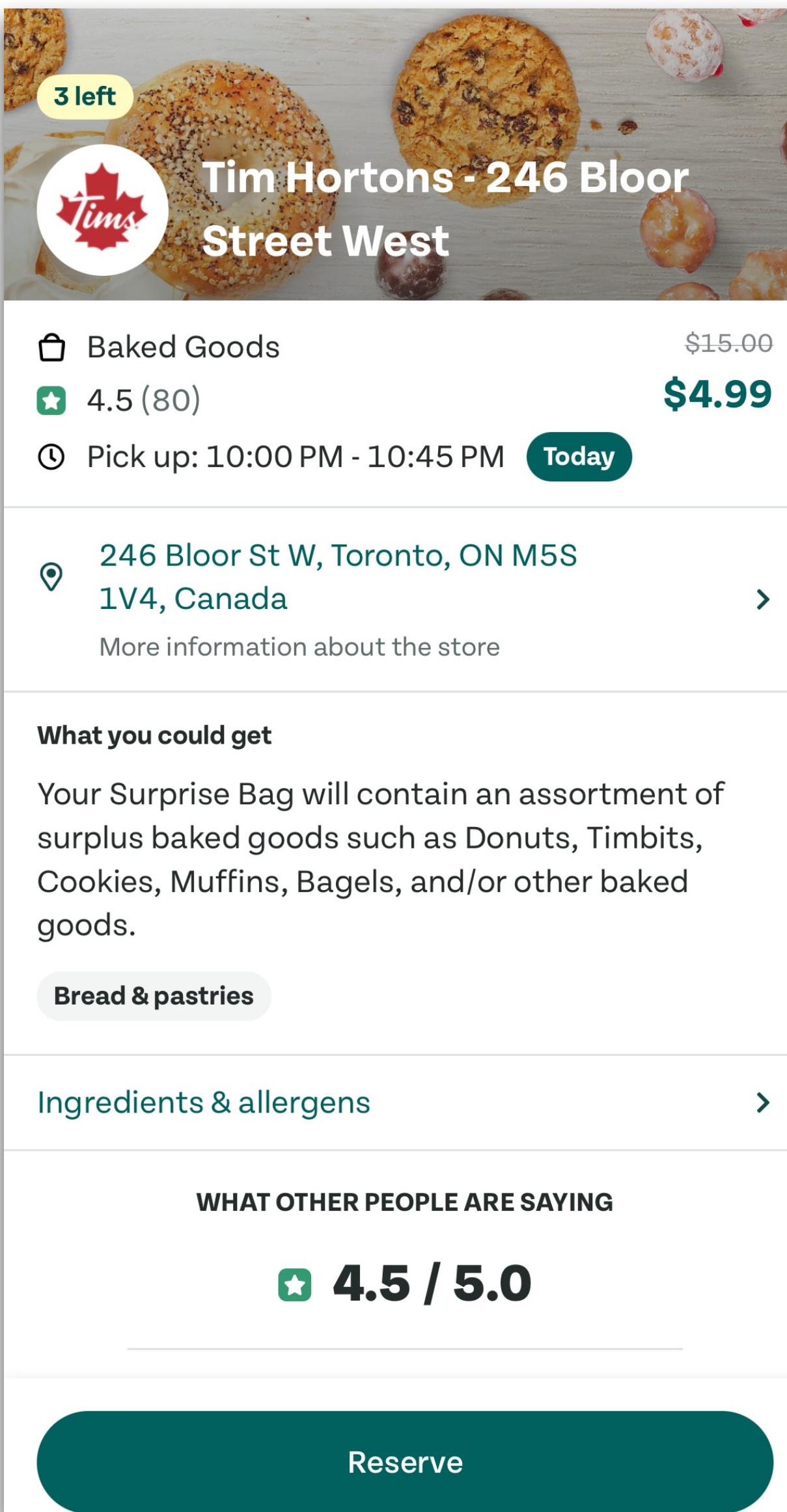


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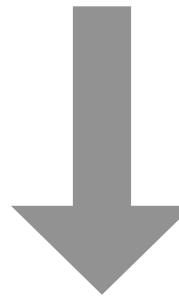


Customers self pick up during designated time slots

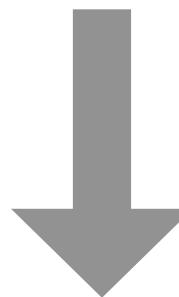
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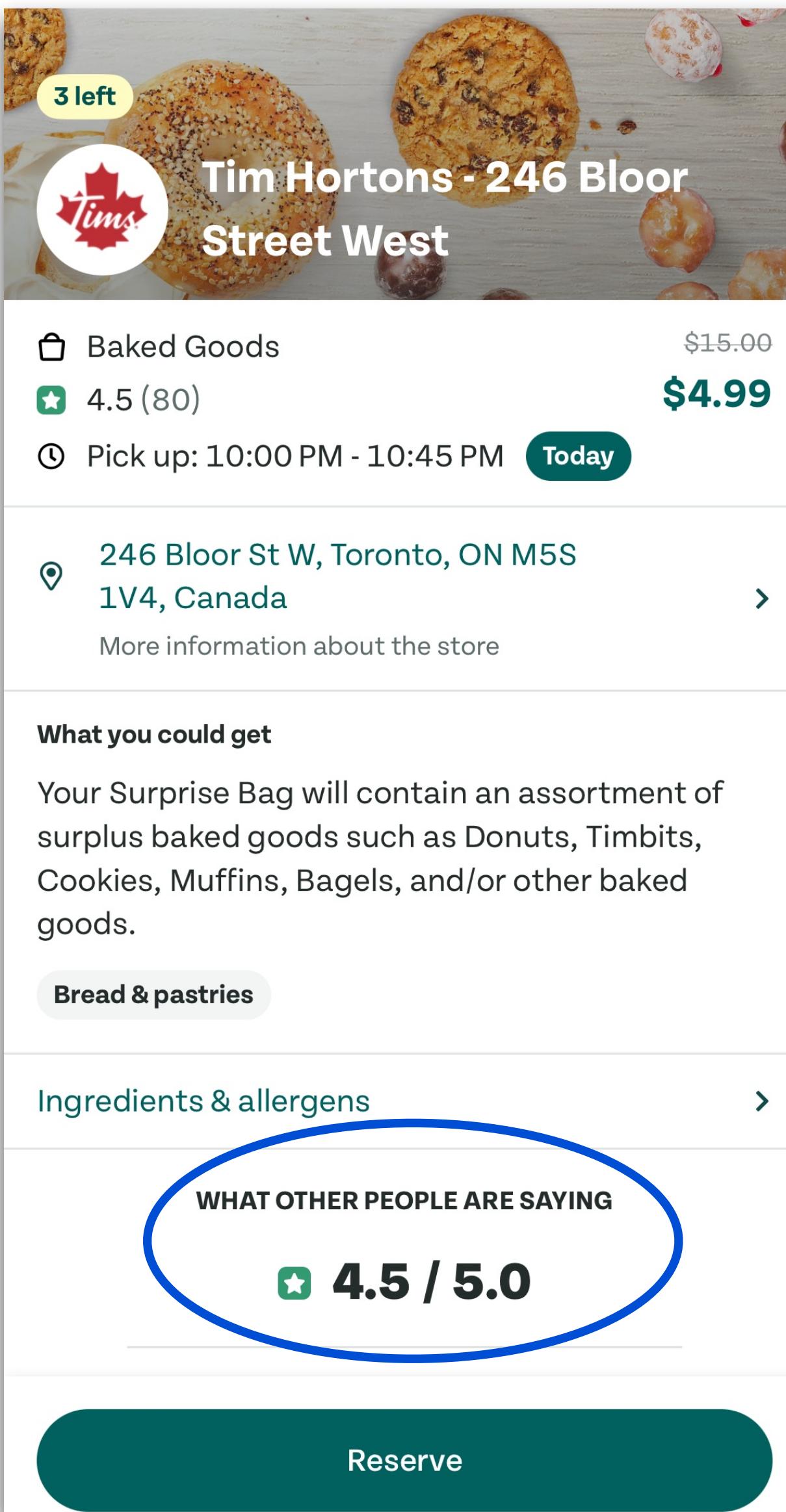


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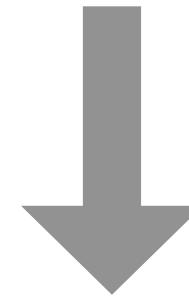


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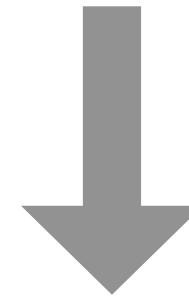
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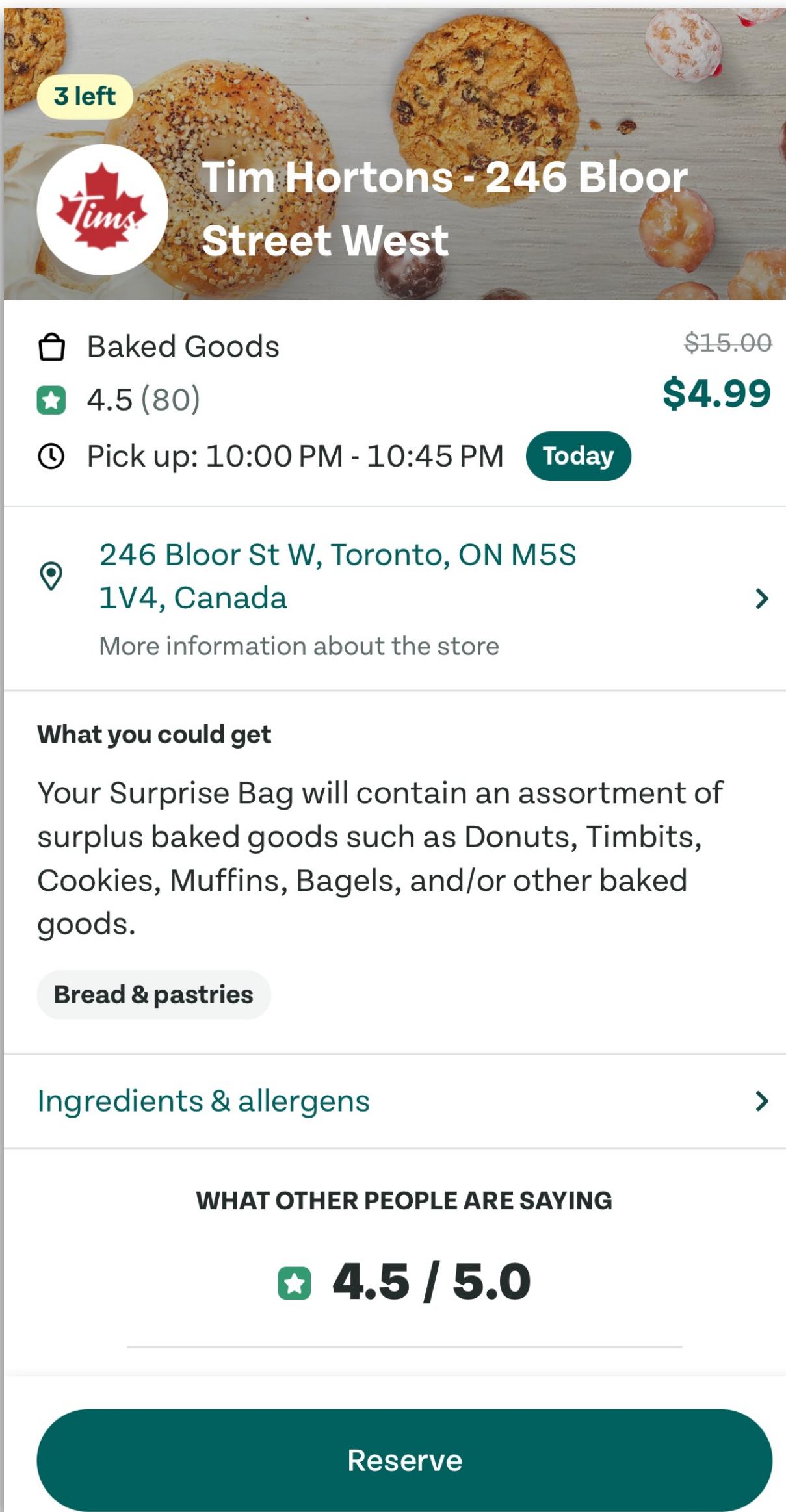


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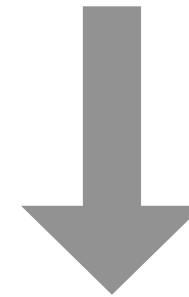


Customers leave a rating and aggregate rating is revealed

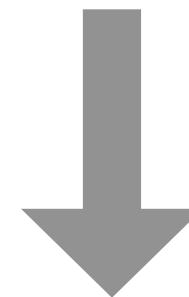
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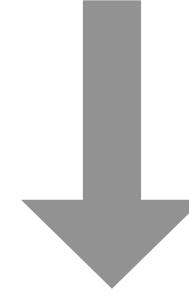
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# Operational Challenges

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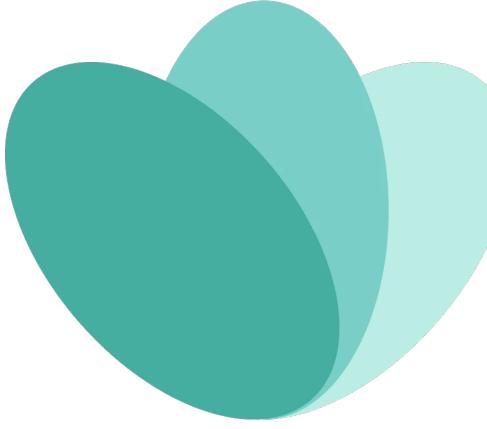
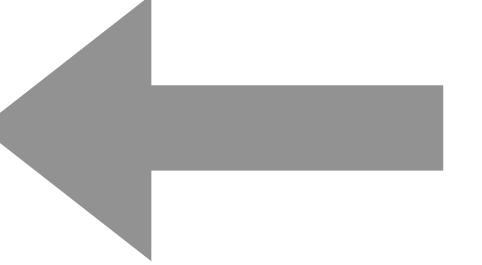
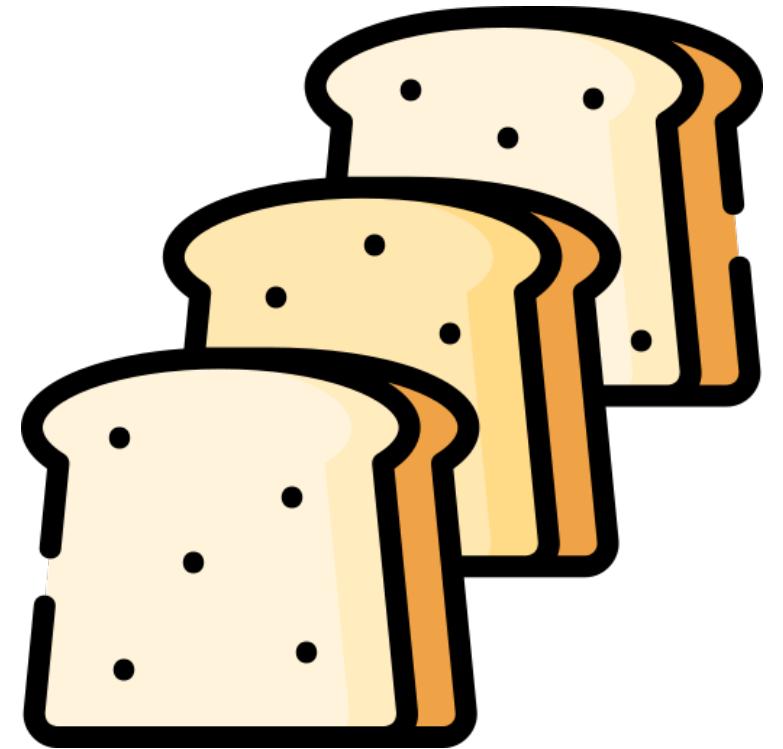
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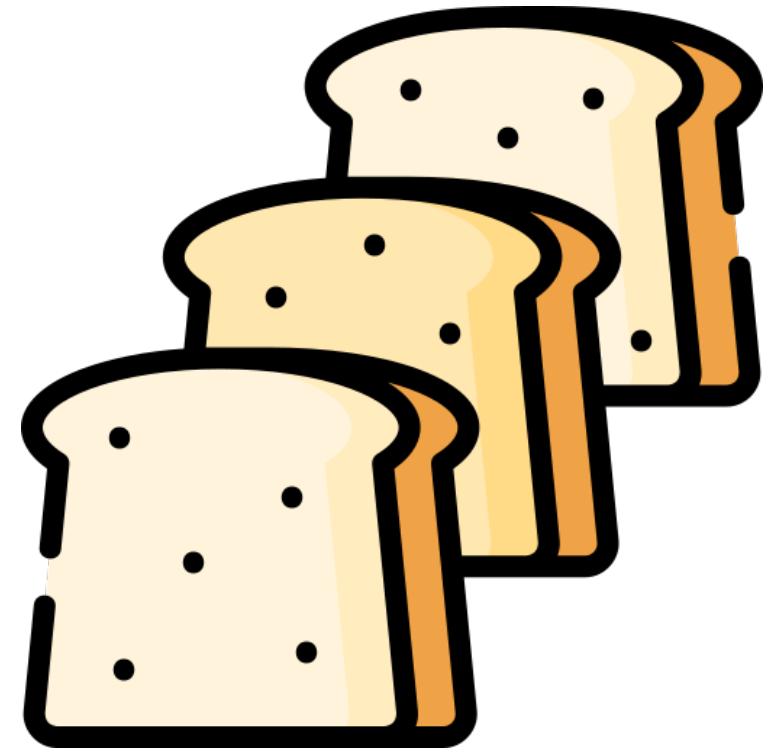


Too Good To Go

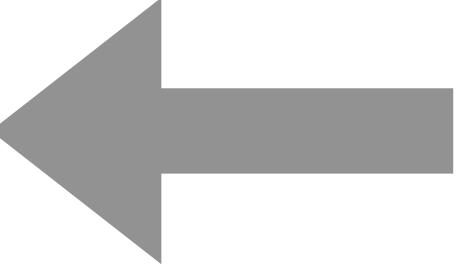
Random surplus food

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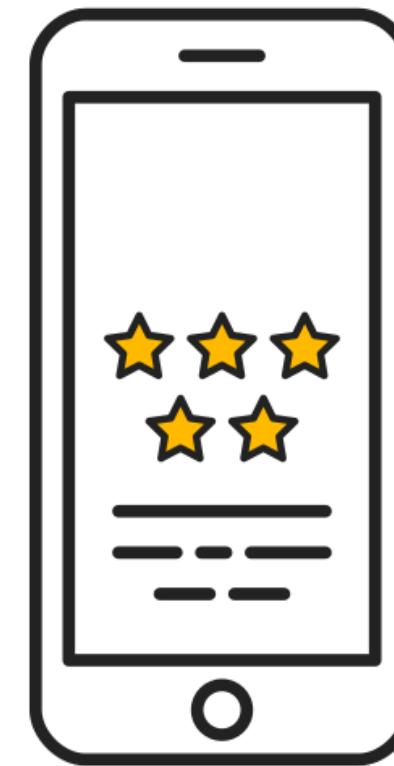
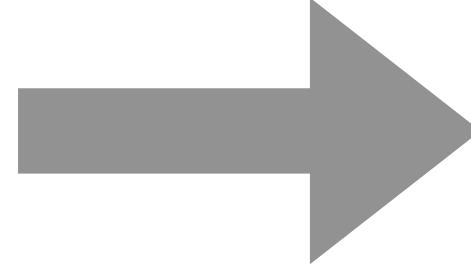
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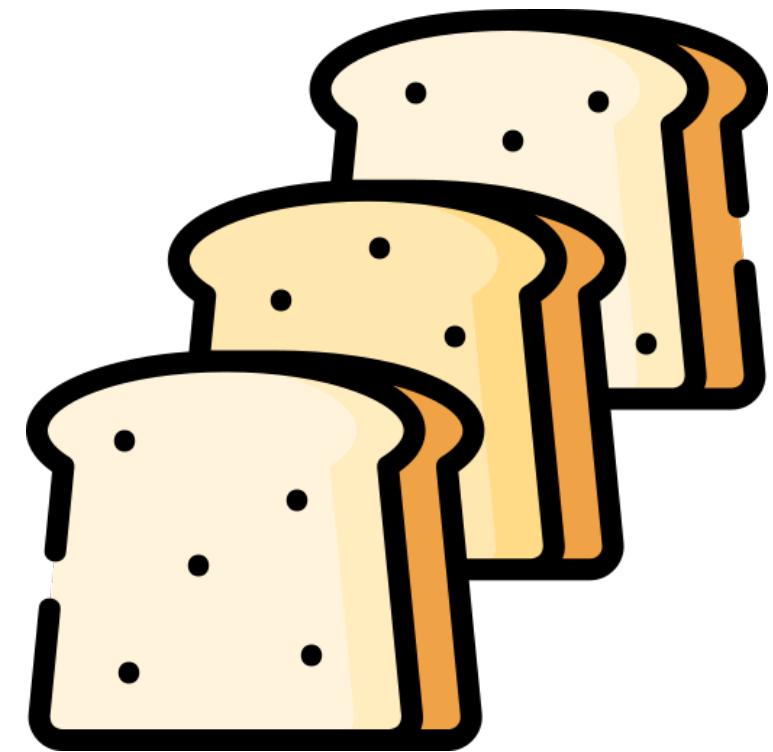
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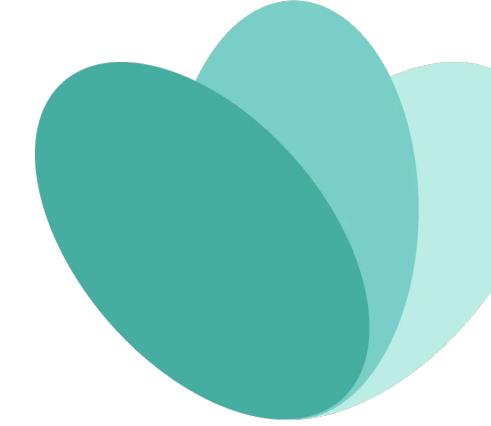
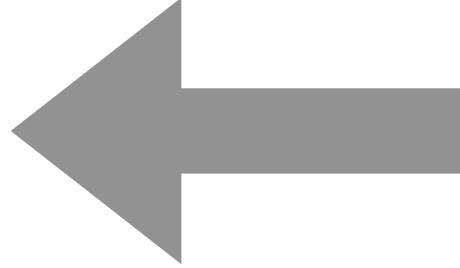
Rating-driven demand

# Operational Challenges

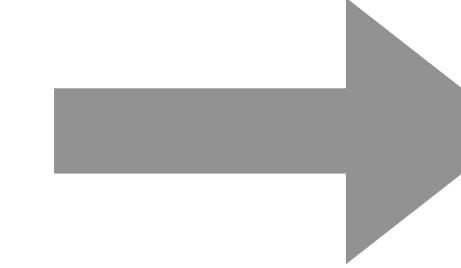
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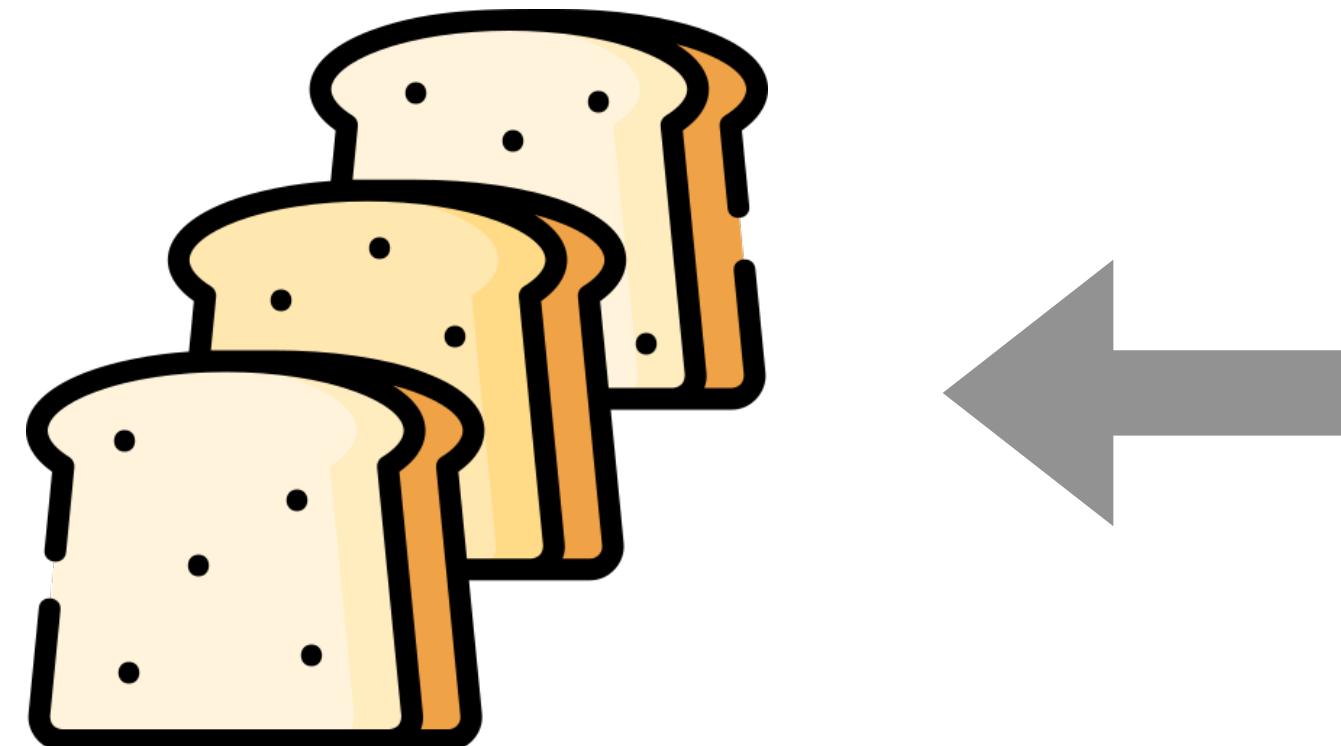
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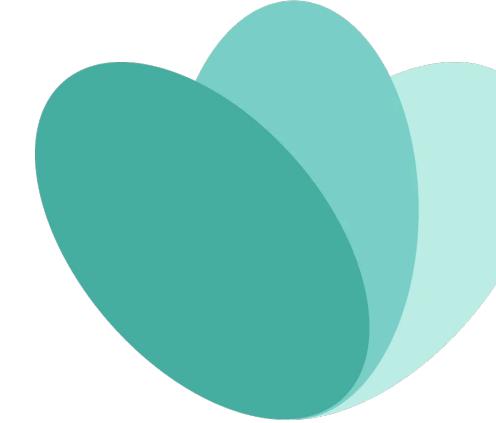
Rating-driven demand

- Announcing bag availability before knowing actual surplus, and unpredictable bag contents might hurt consumers' satisfaction, damaging future sales

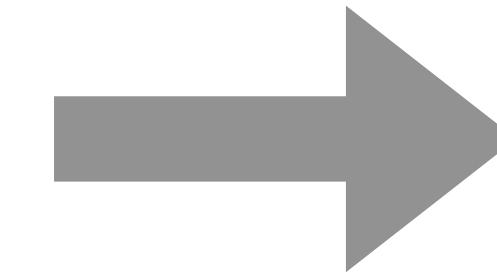
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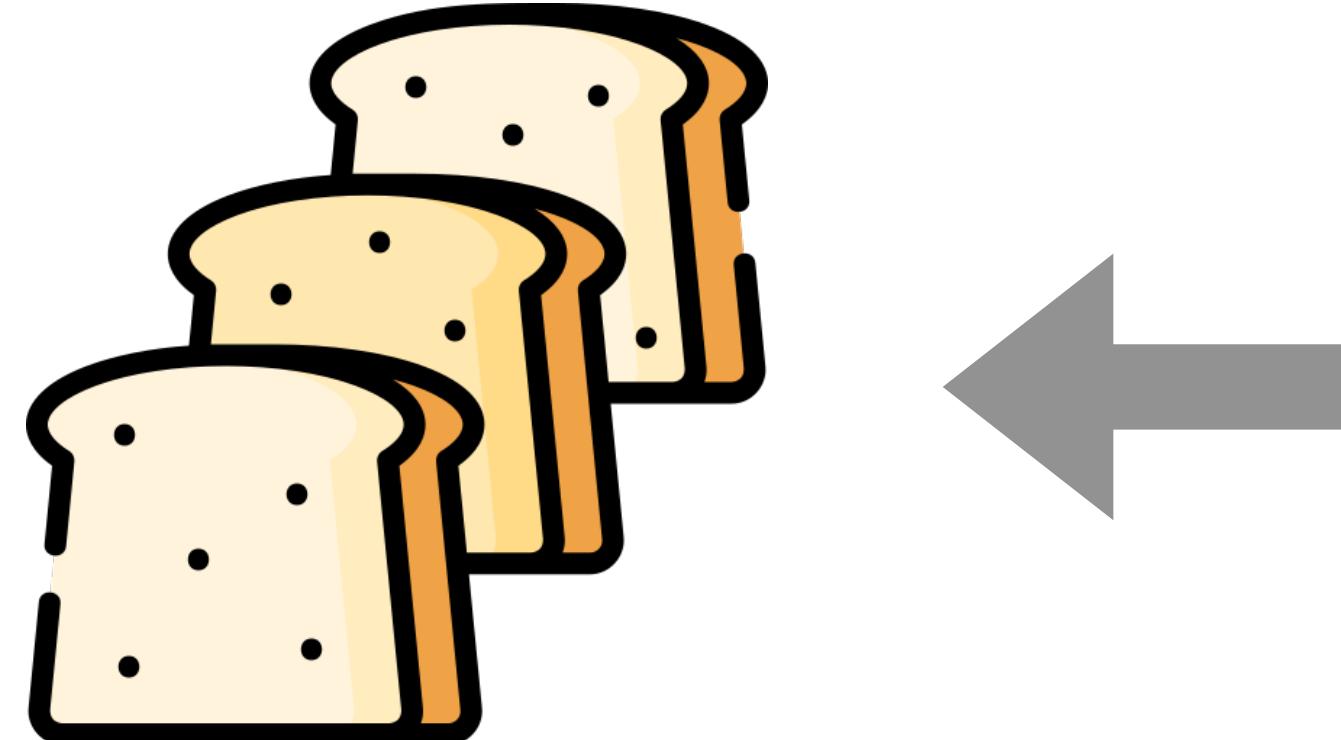
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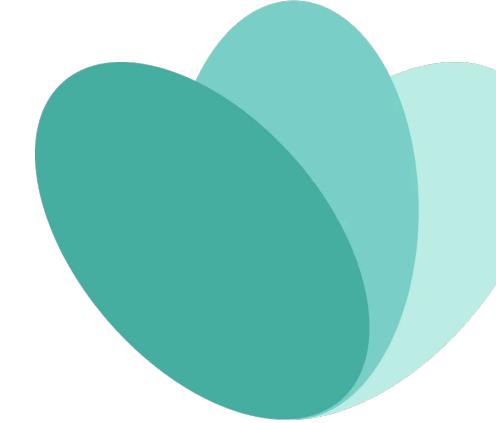
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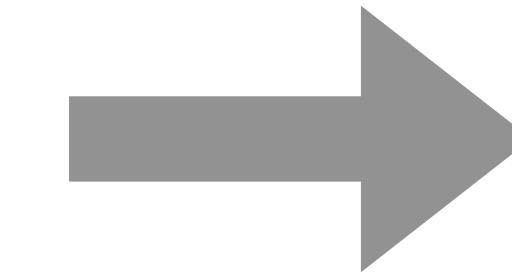
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Random surplus food



Too Good To Go



Rating-driven demand

- Announcing bag availability before knowing actual surplus, and unpredictable bag contents might hurt consumers' satisfaction, damaging future sales
- Supplementing surprise bags with food from regular sales is feasible, but expensive
- Evenly distributing unsold items is common, yet its effect on satisfaction is unclear

# Motivation & Research Questions

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## Role of the TGTG platform



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- Help stores manage surplus inventory and bag design



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- Ensure stores' long-term earnings, which is critical for the platform's viability and food waste reduction



# Motivation & Research Questions

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- How many surprise bags should be offered?
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## Our research question

- How many surprise bags should be offered?
- How much total food should be included?
- How should food be allocated across bags?



# Infinite-Horizon DP with Two-Stage Decision

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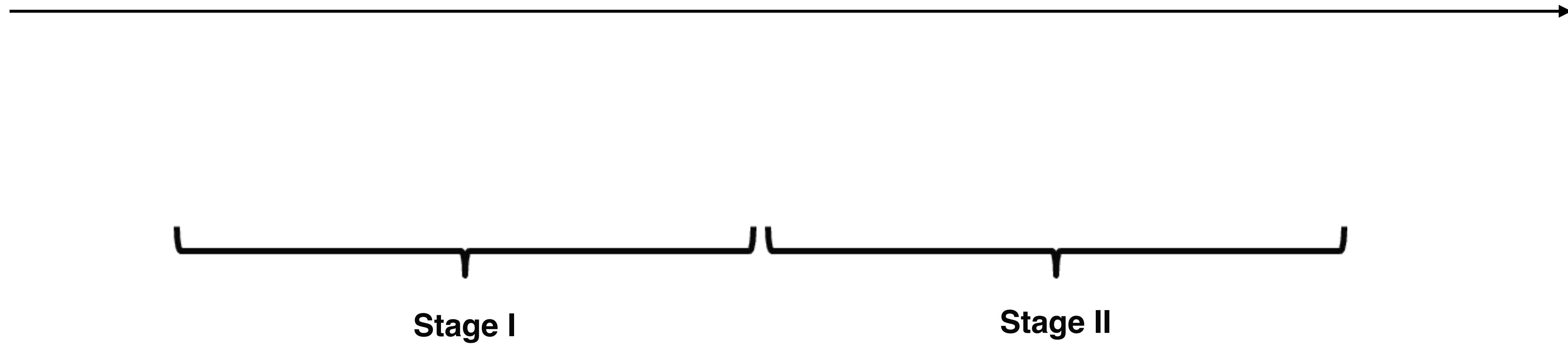
**Sequence of events in one period**



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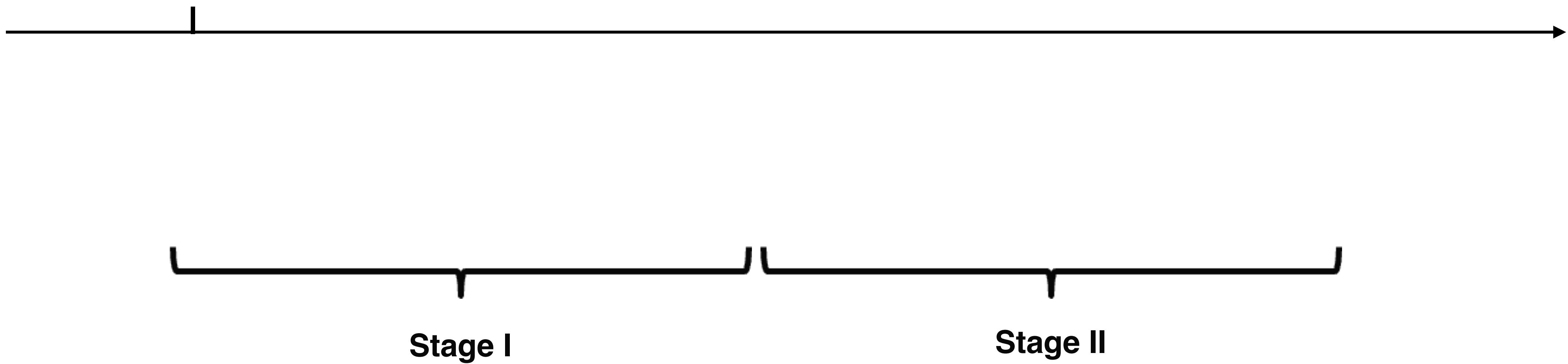


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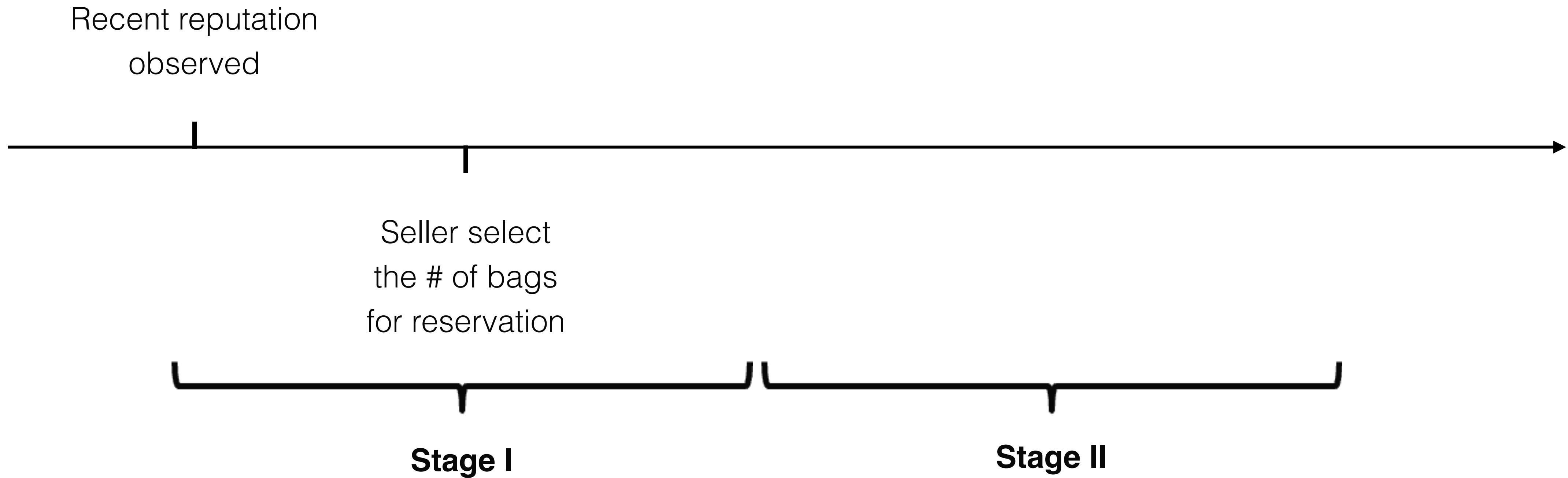
## Sequence of events in one period

Recent reputation  
observed



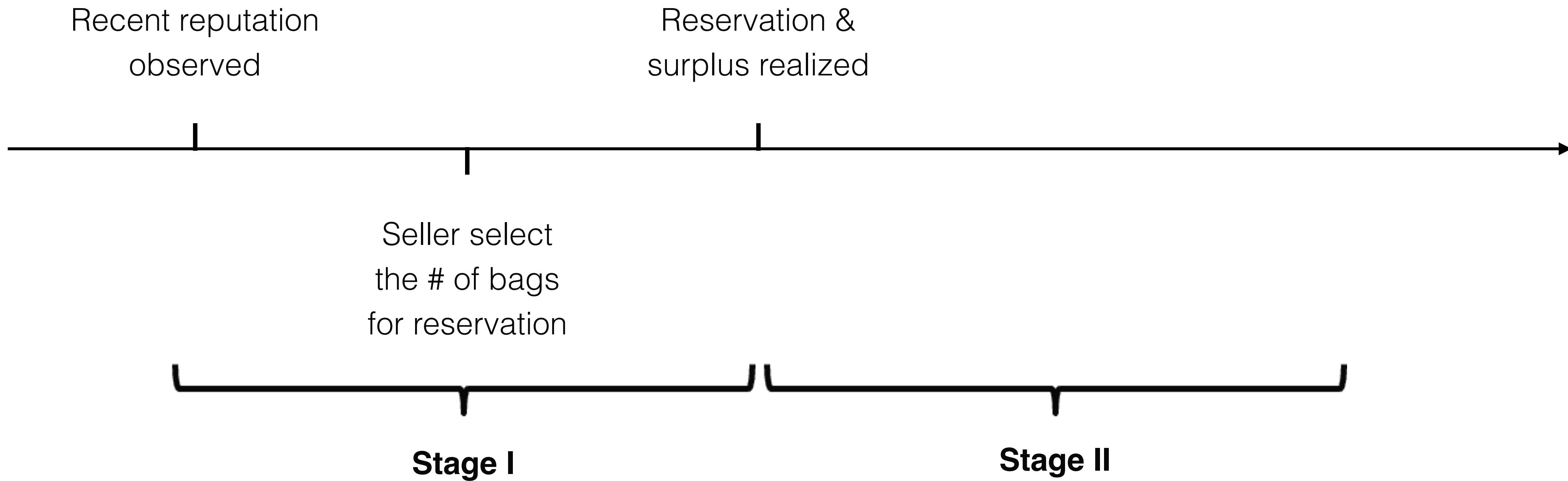
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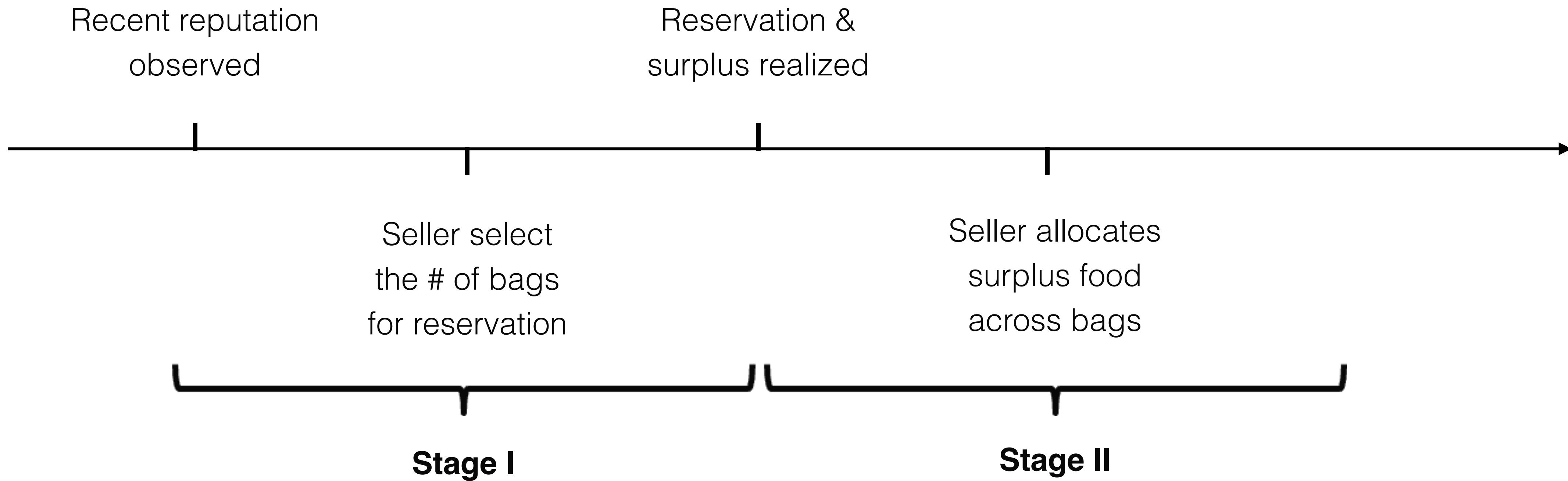
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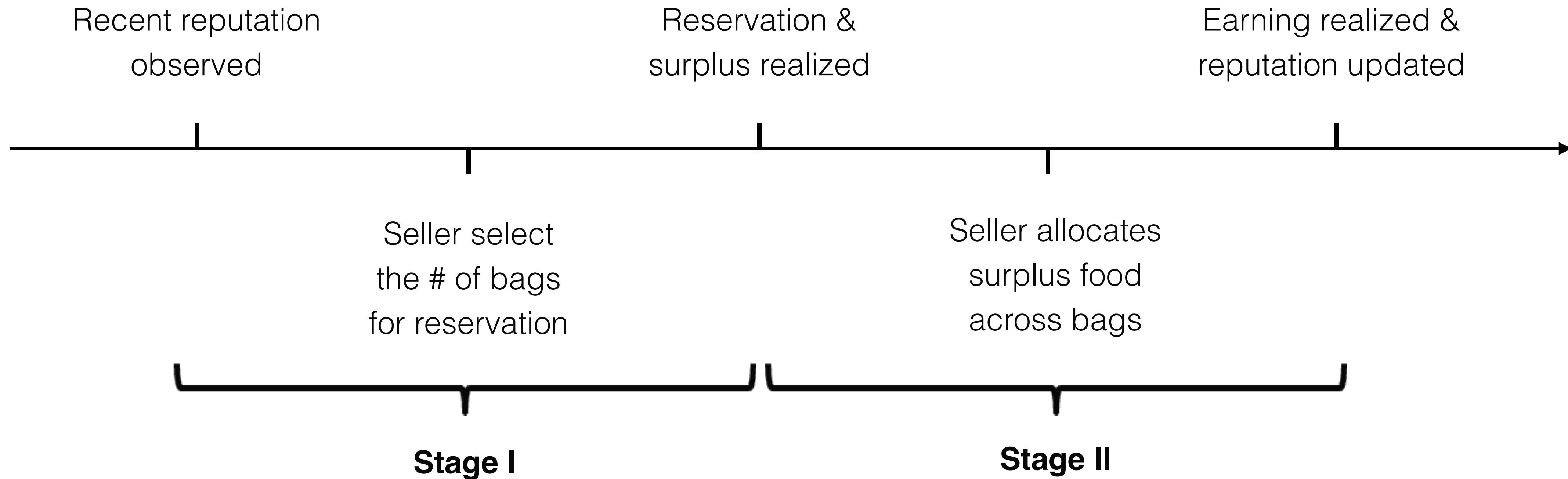
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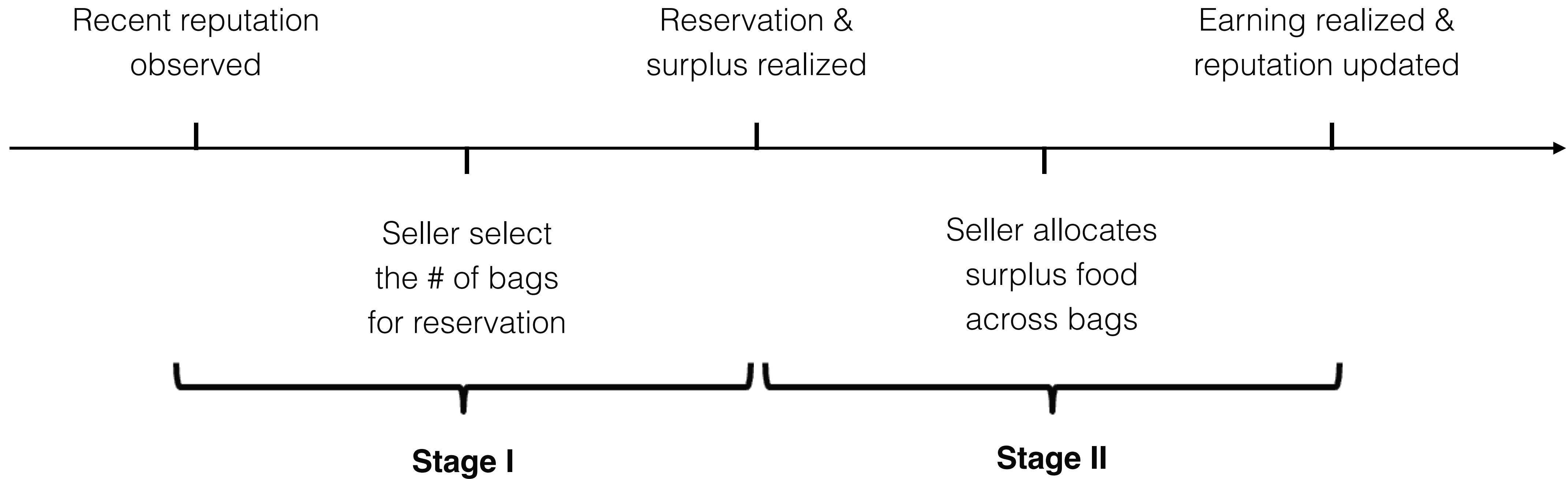
# Infinite-Horizon DP with Two-Stage Decision

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# Infinite-Horizon DP with Two-Stage Decision

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## Objective

store's long-run earnings  $\leftrightarrow$  platform's revenue  $\leftrightarrow$  food waste reduction

# Model: Consumer Utility and Food Distribution

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# Model: Consumer Utility and Food Distribution

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# Model: Reputation, Demand, Reservation

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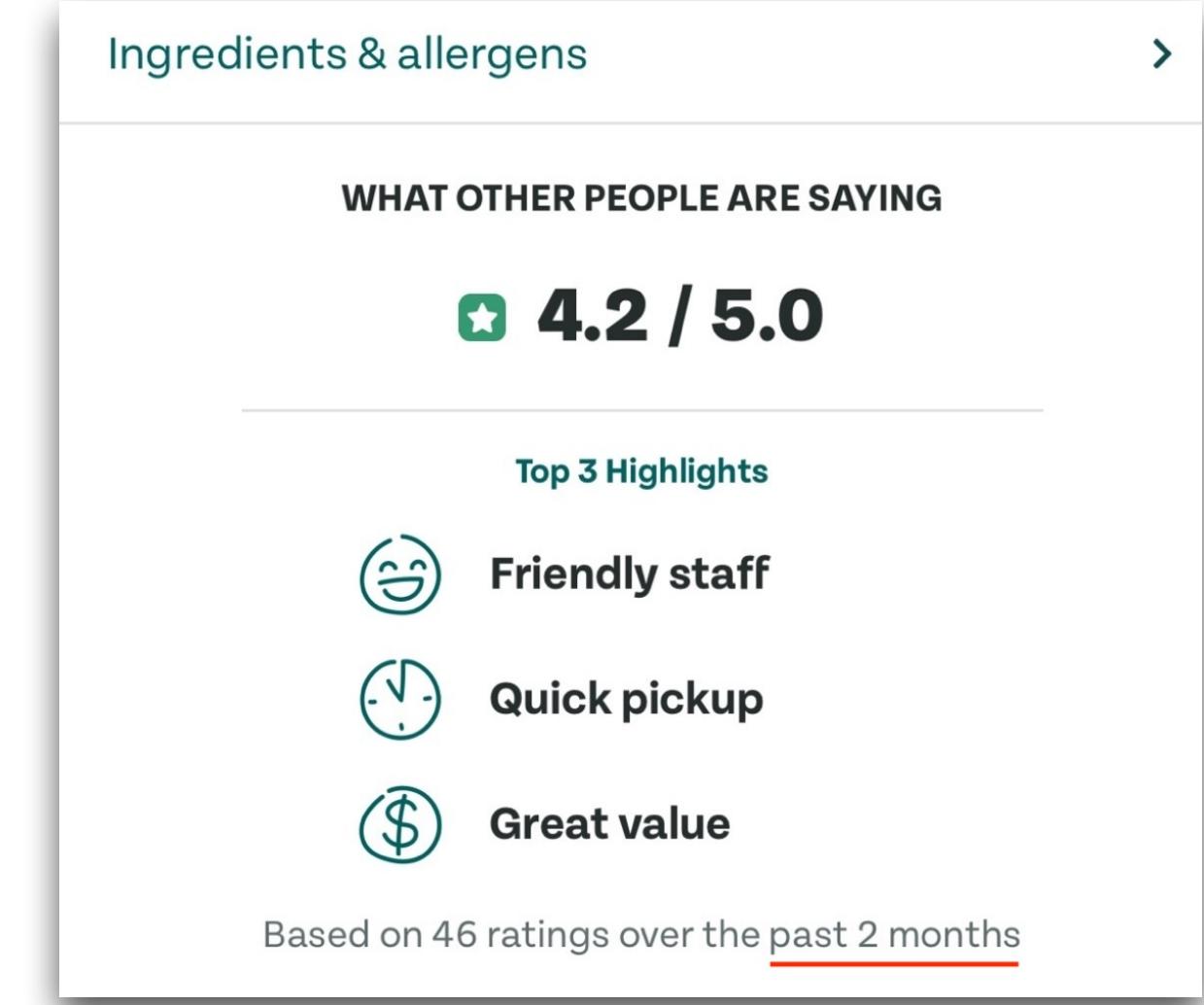
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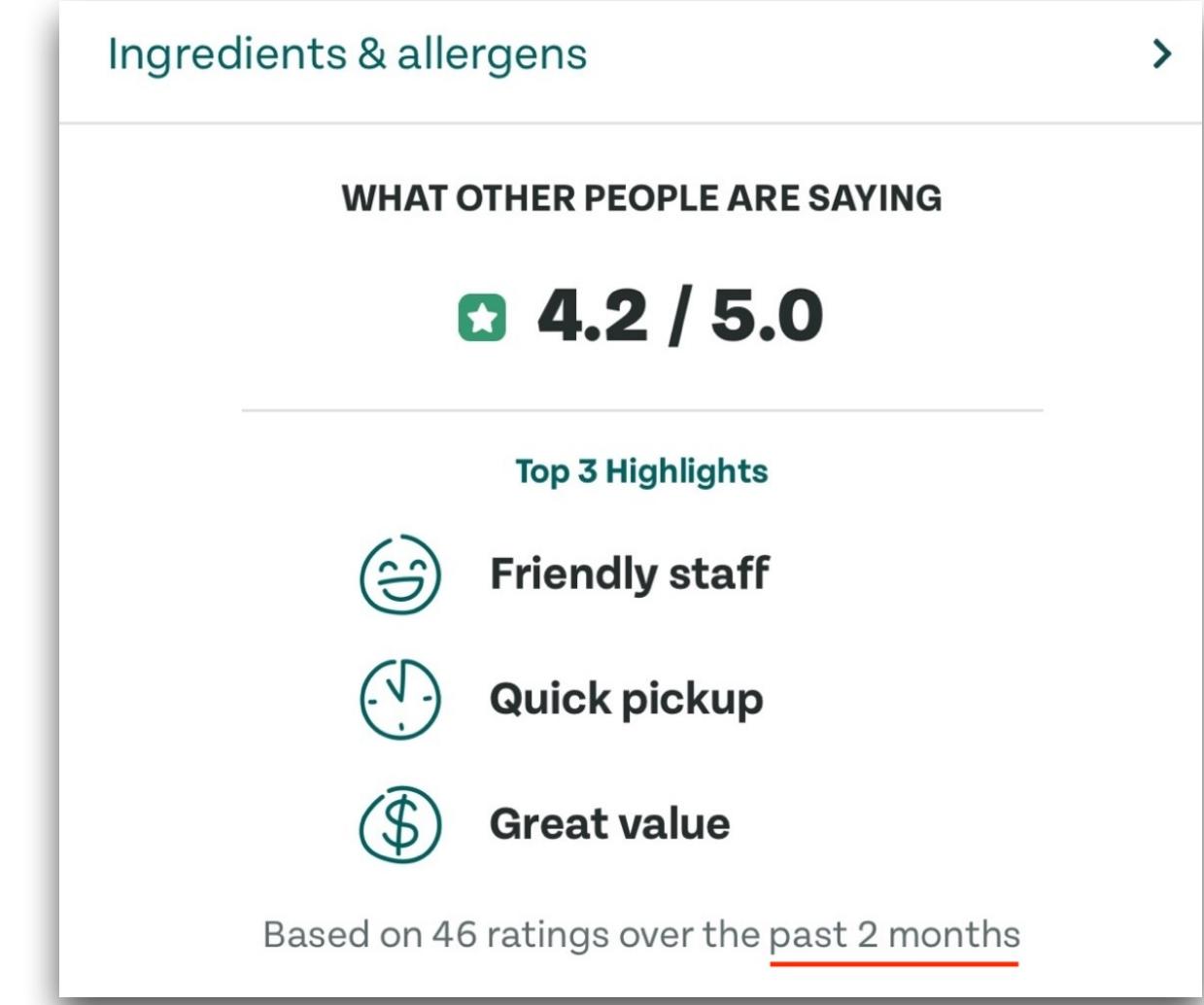
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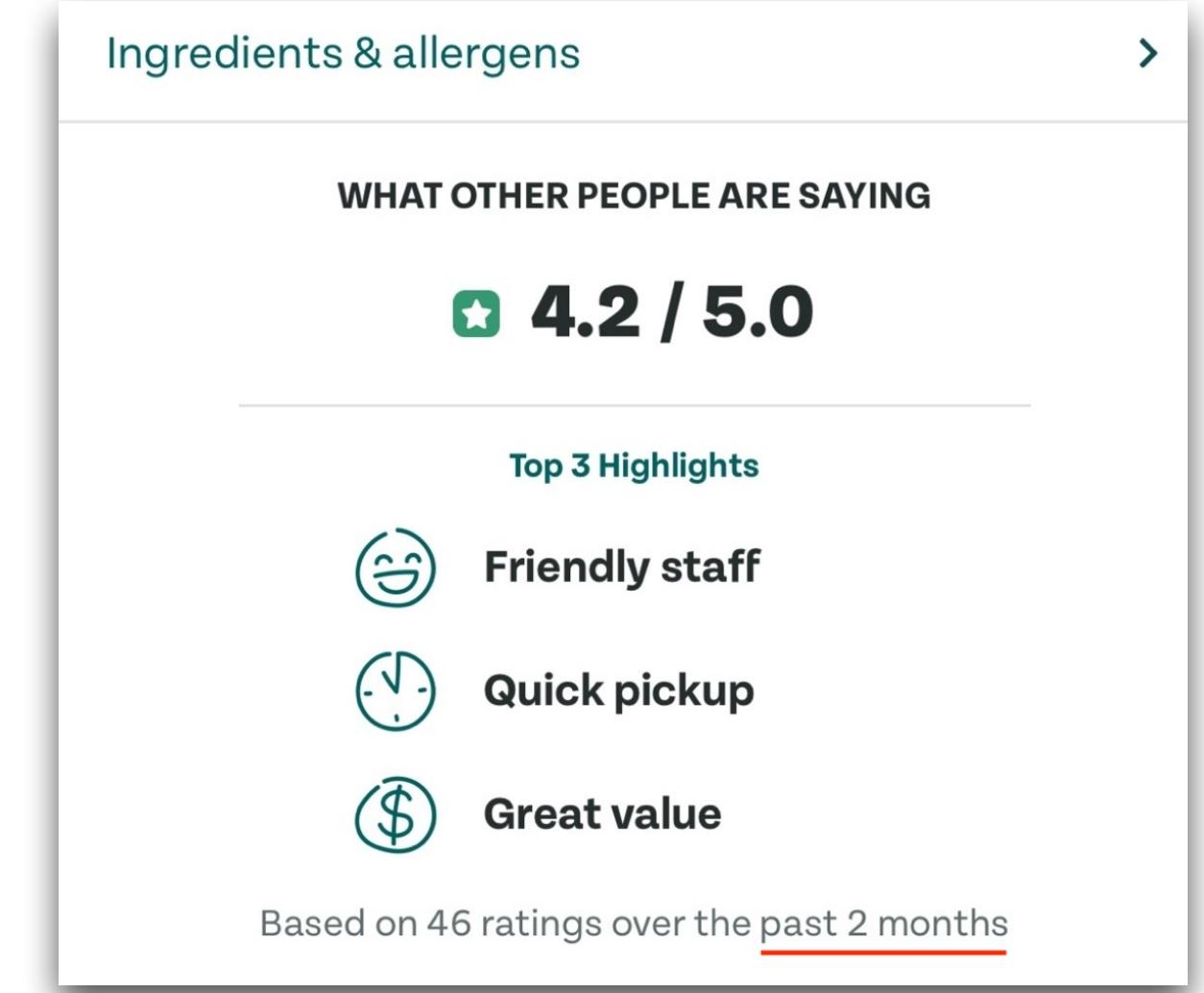
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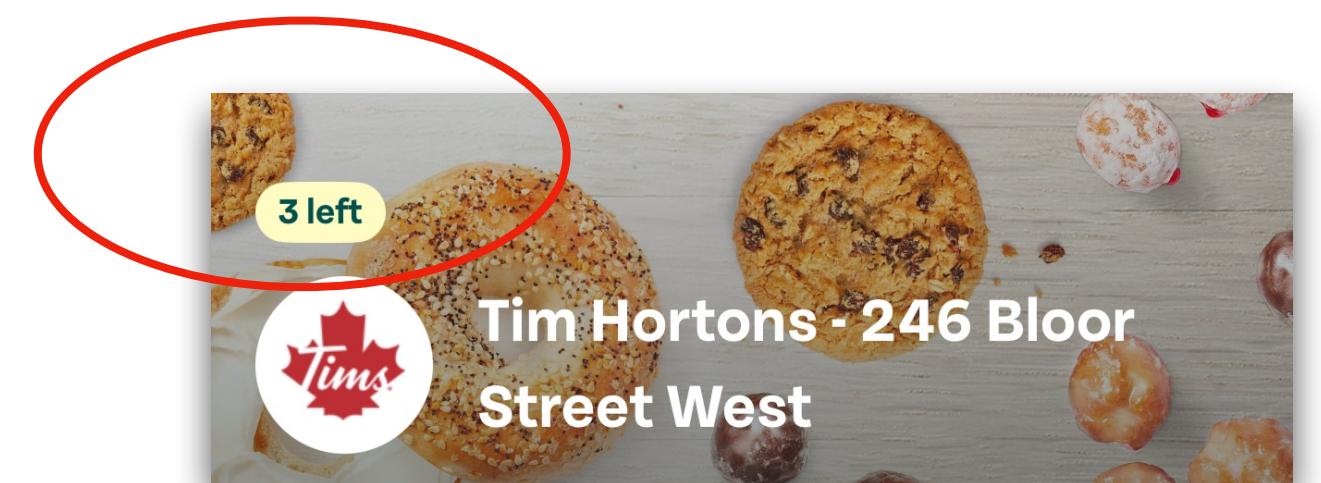
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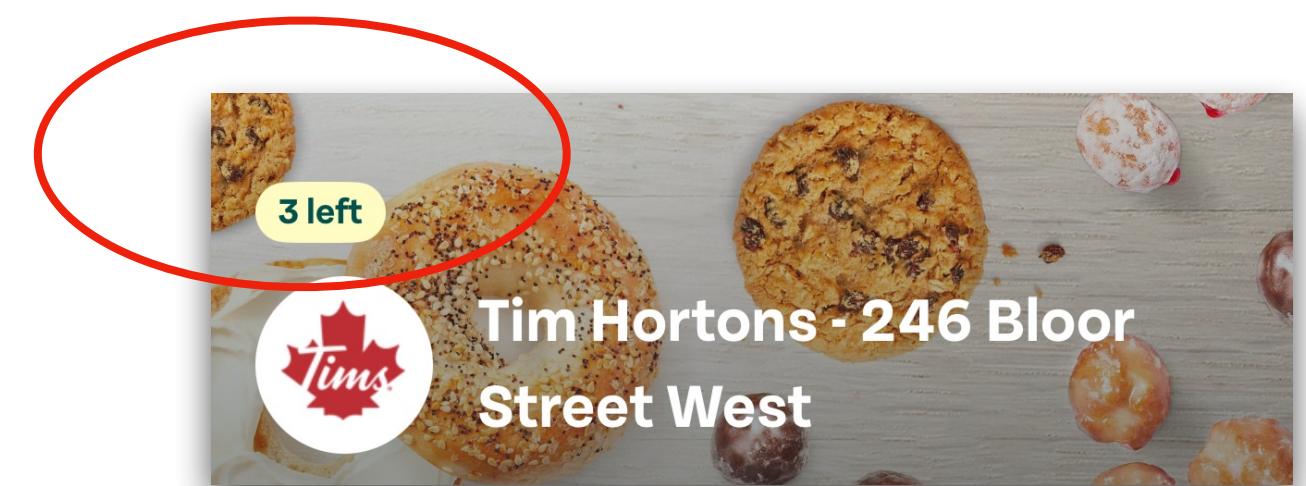
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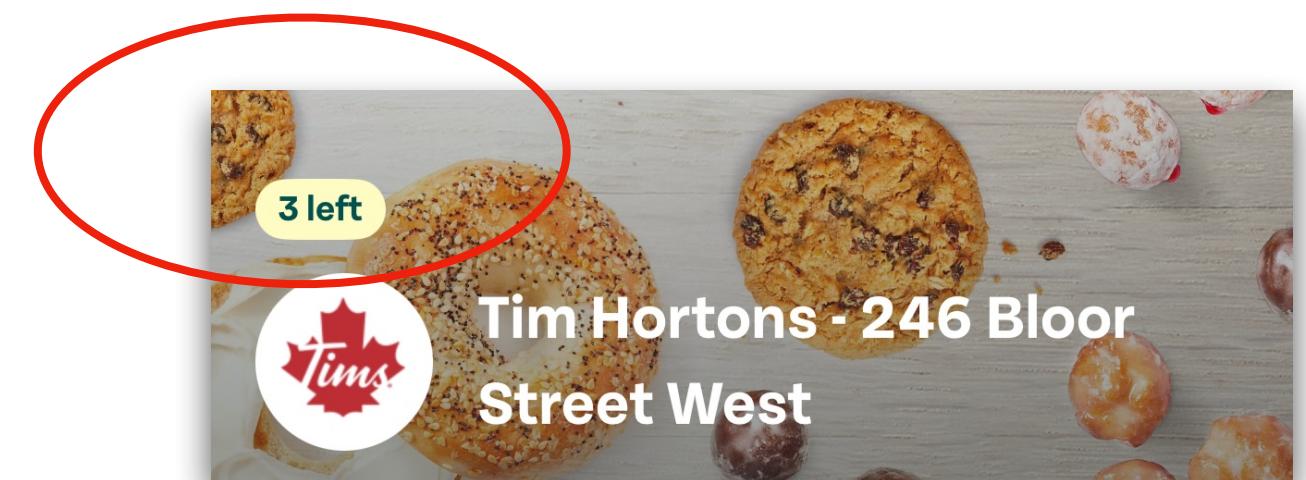
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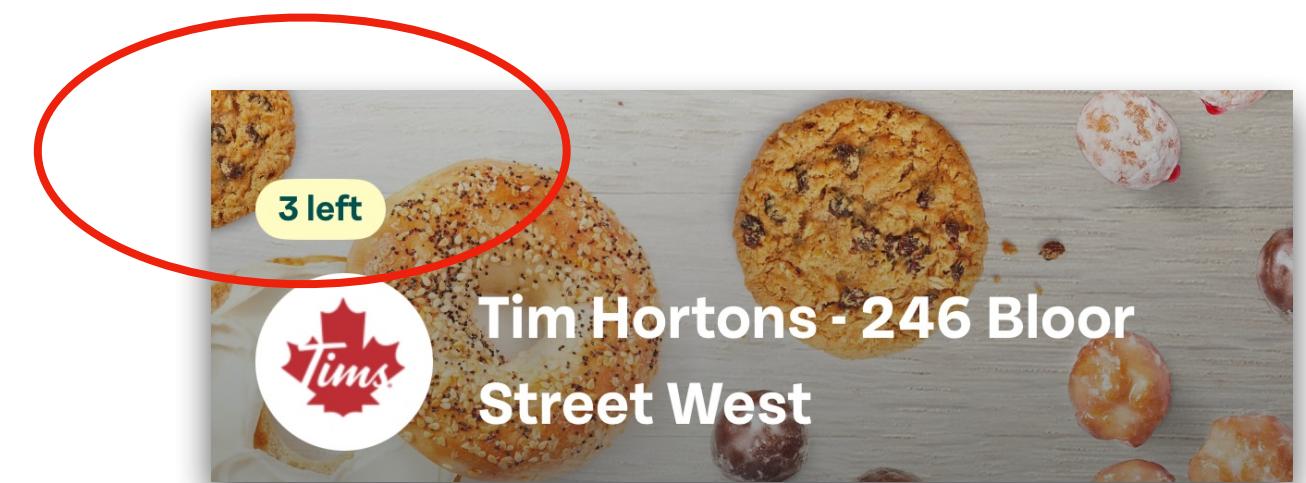
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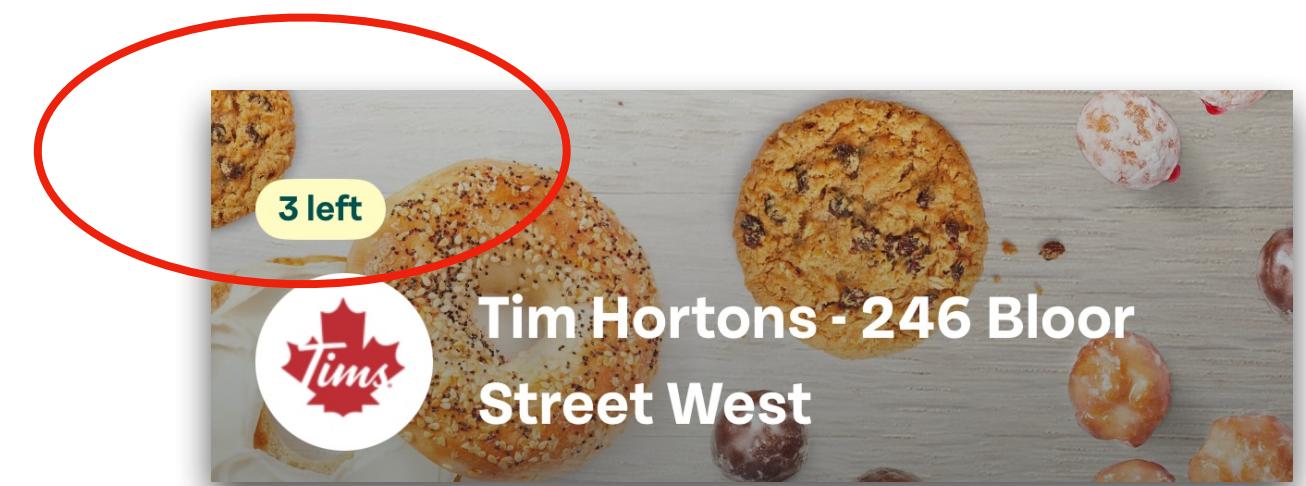
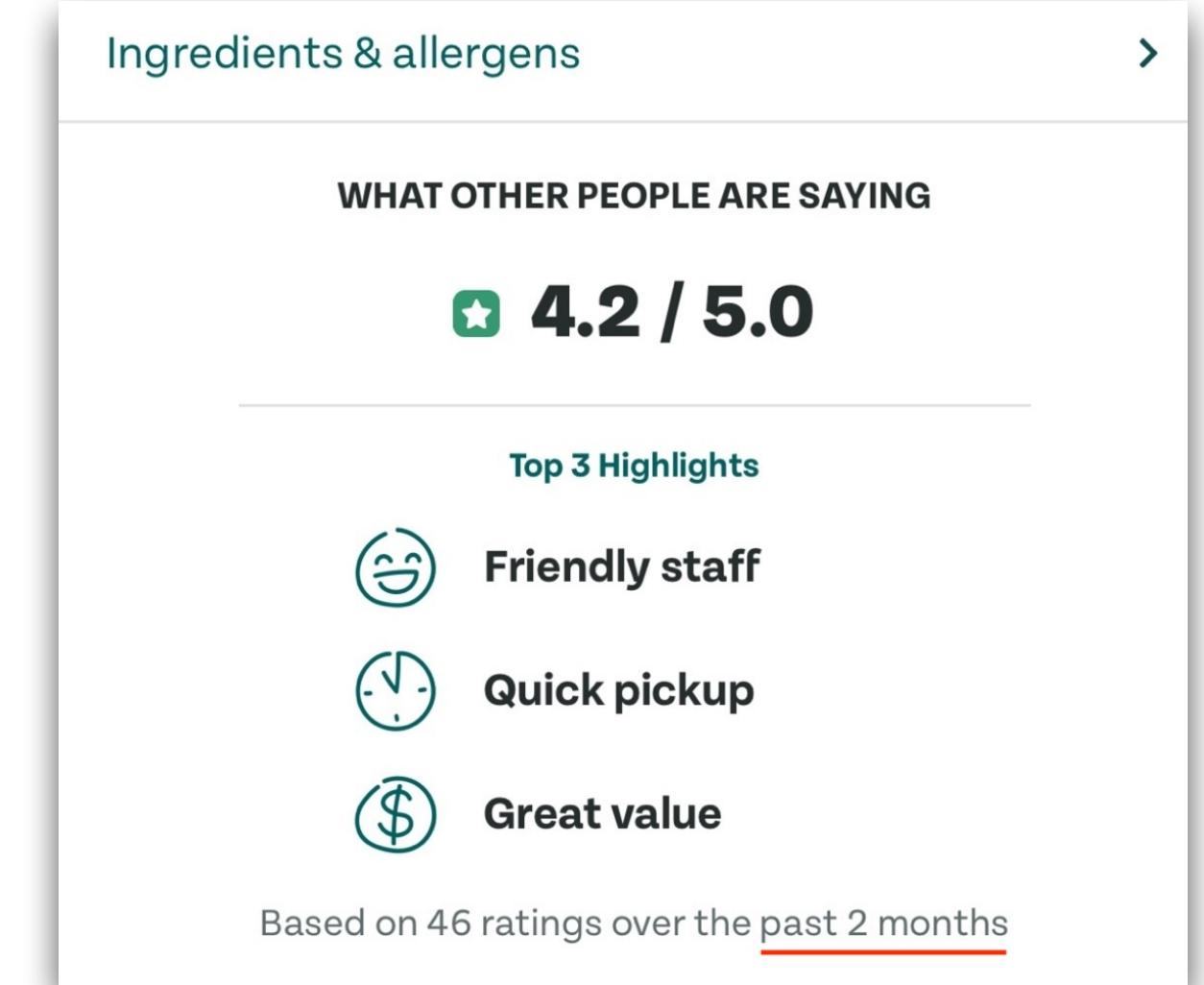
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  - Realized after announcing  $n_t$



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# Admissible Policy

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**Stage I**

**Stage II**

# Admissible Policy

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**Stage II**

# Admissible Policy

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Recent reputation  
observed  $r_t$

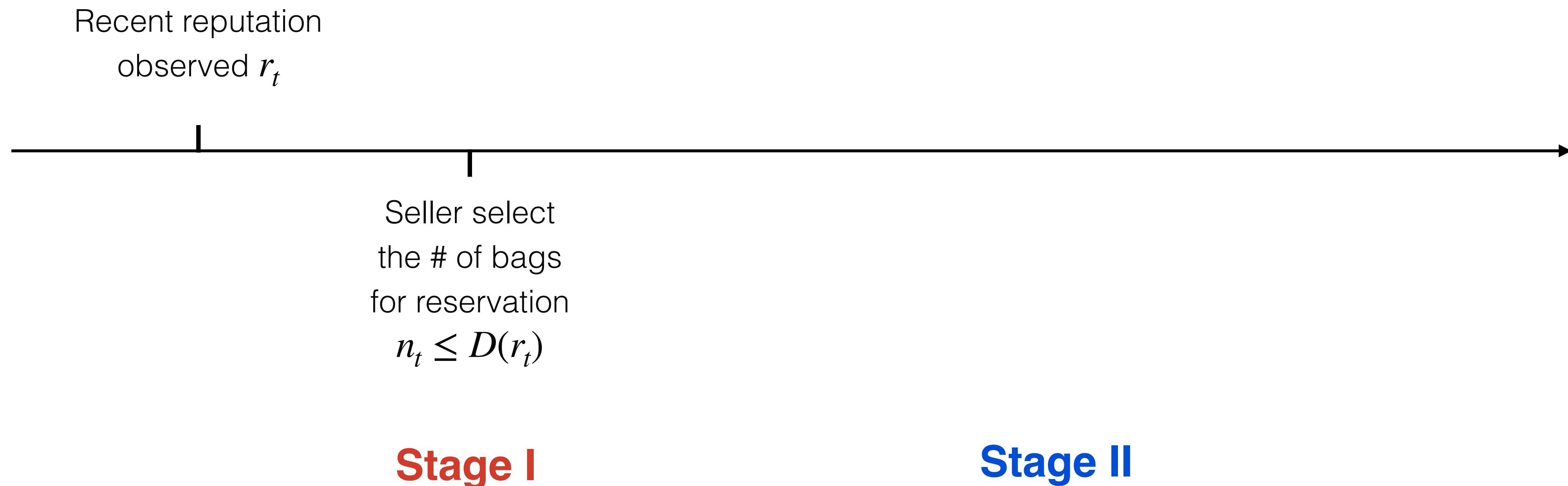


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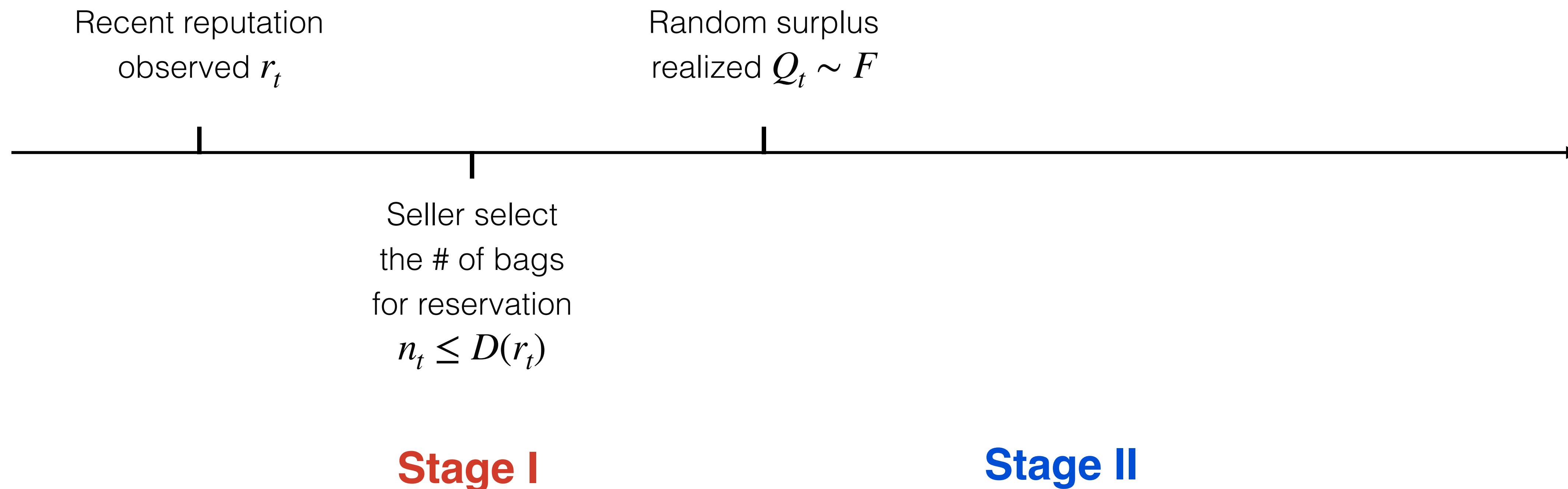
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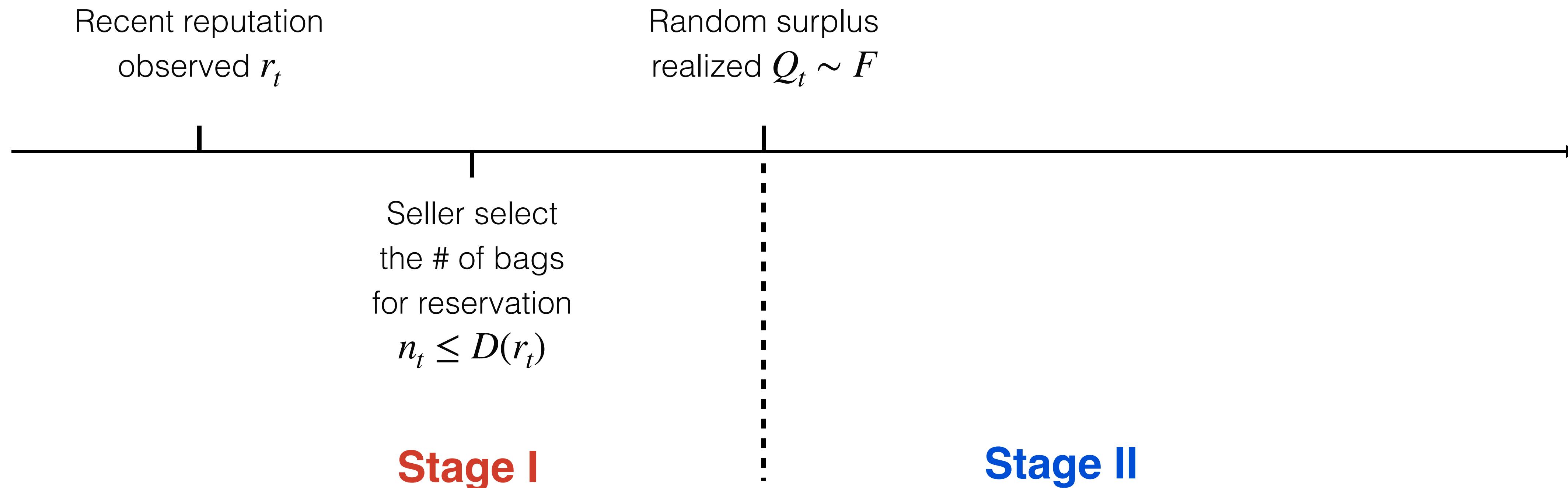
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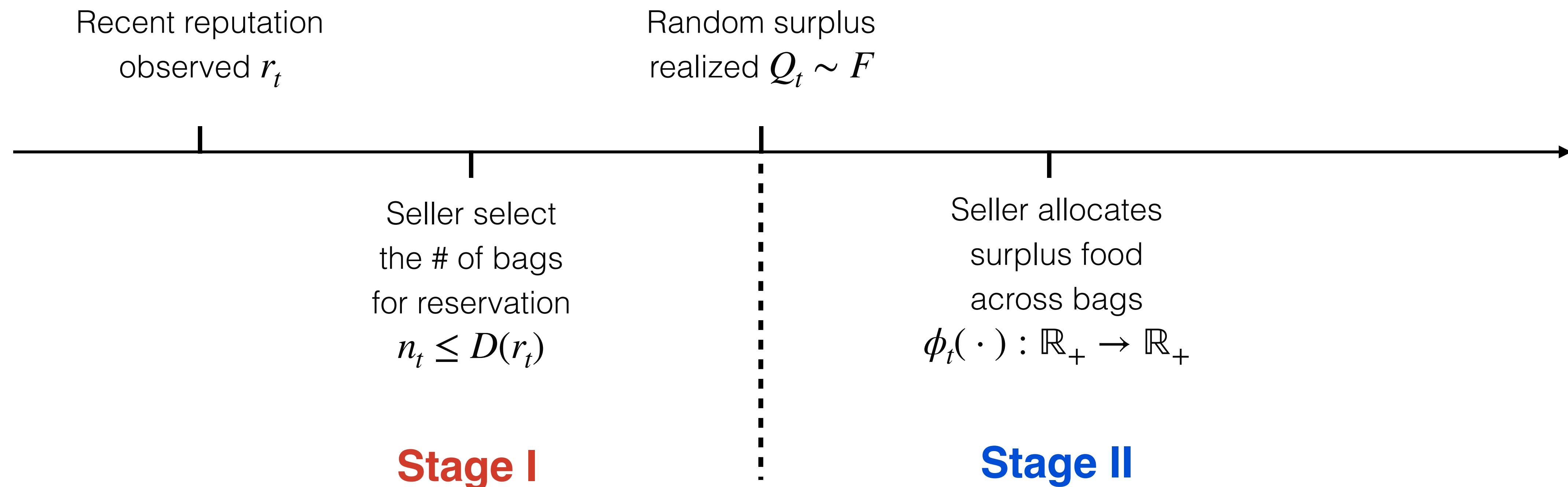
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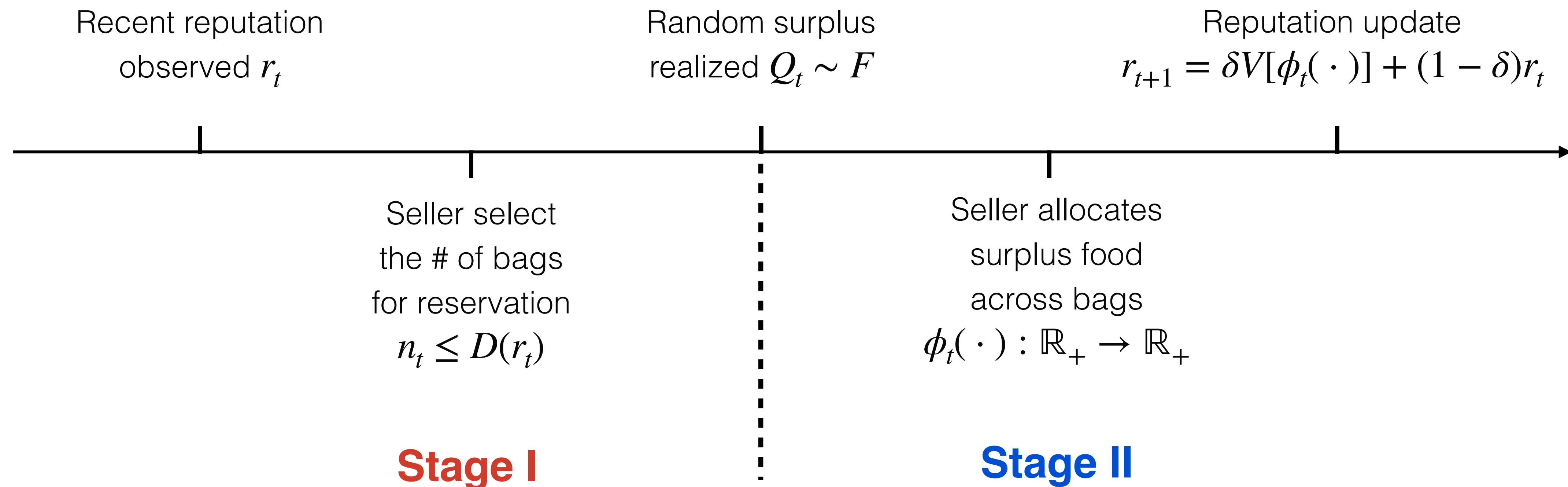


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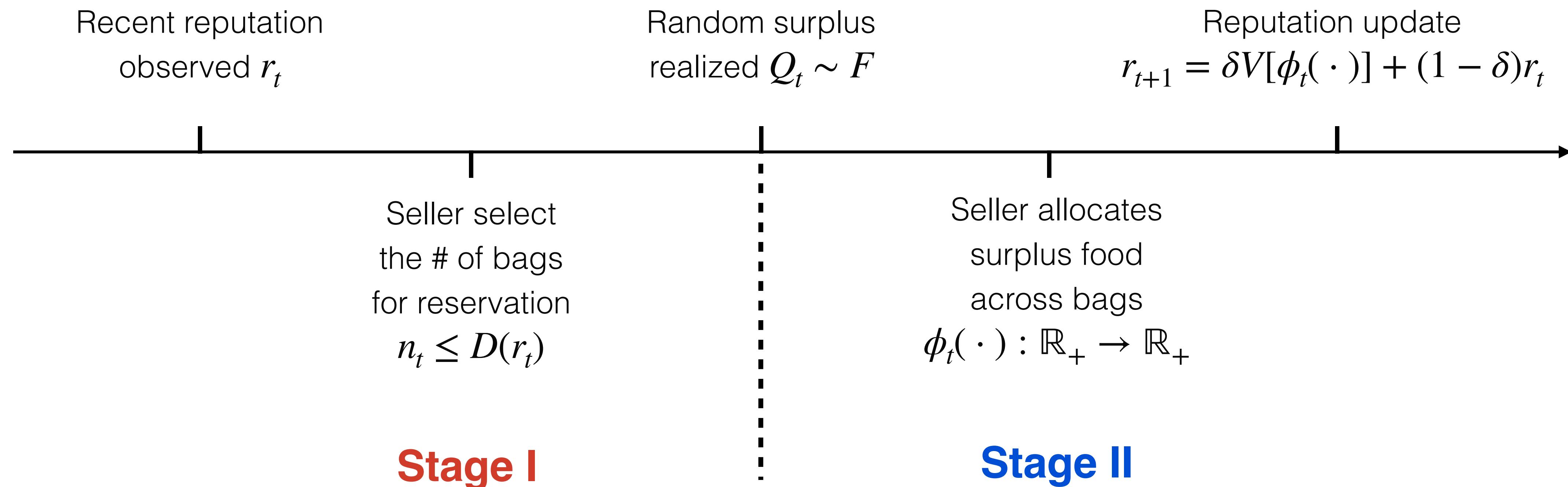


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An admissible policy  $\pi$  consists of two decision stages

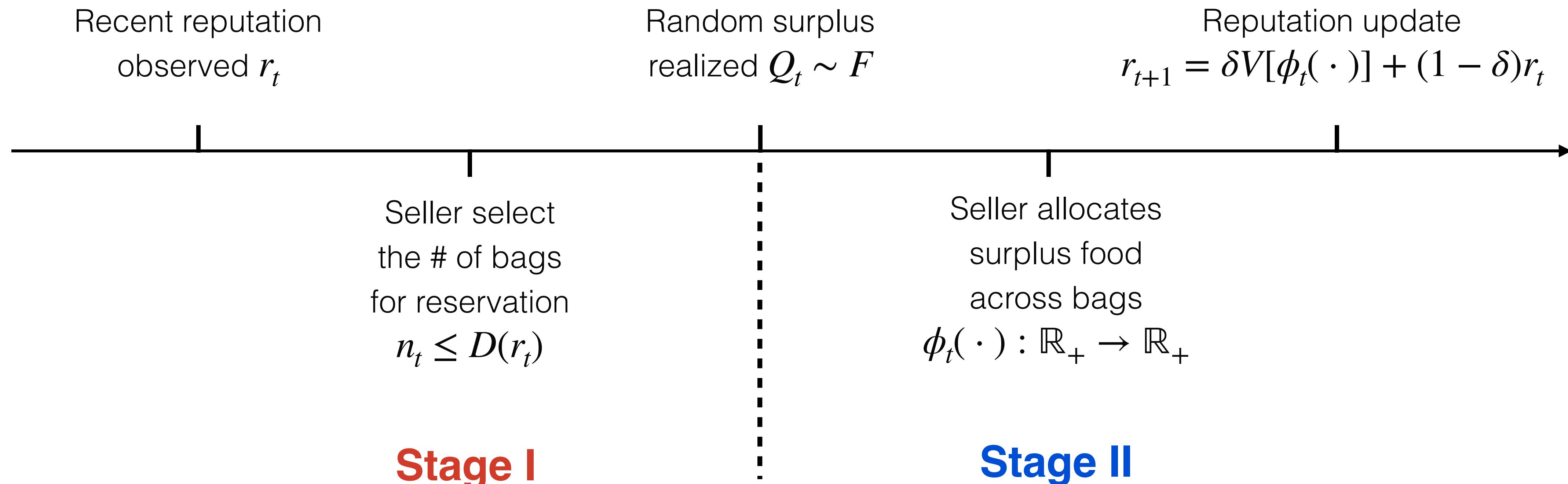


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**Stage I** How many bags?

$$r \mapsto n^\pi$$



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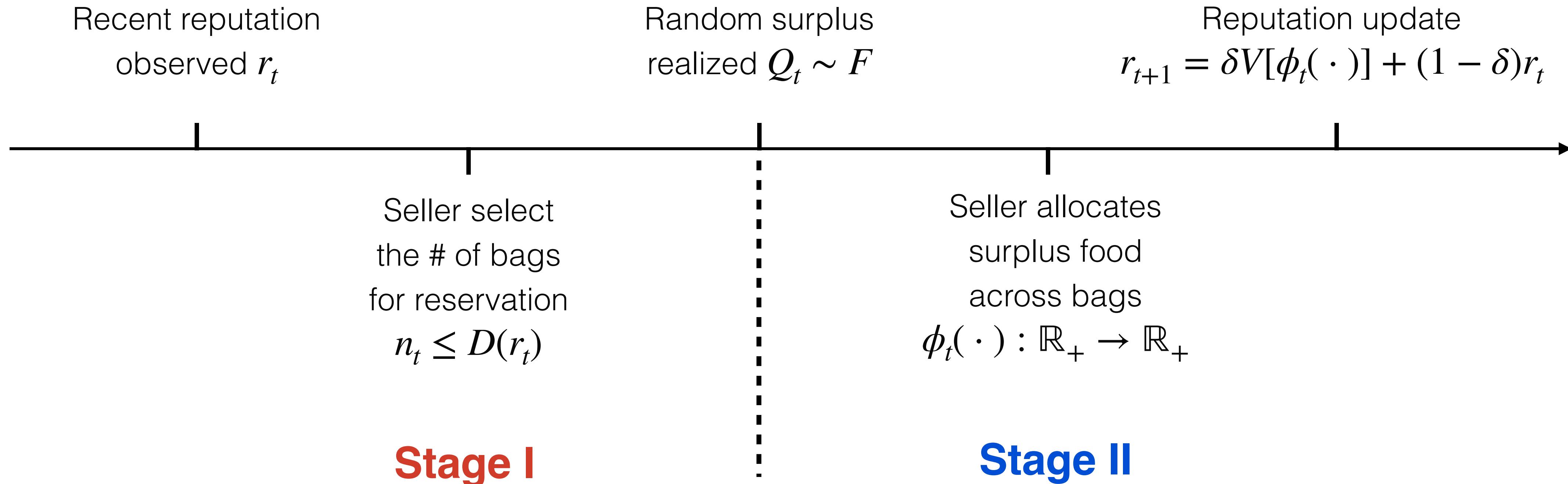
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**Stage II** How to allocate surplus food across bags?

$$(r, n, Q) \mapsto \phi^\pi(\cdot)$$



# Action Space Reduction

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## DP formualtion

$$J^*(r) = \max_{n \leq D(r)} \left[ p \cdot n + \mathbb{E}_Q \left( \max_{\phi(\cdot)} \left[ -C[\phi(\cdot), n, Q] + \beta J^* \left( \delta V[\phi(\cdot)] + (1 - \delta)r \right) \right] \right) \right]$$

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Stage II decision can be reduced from distribution  $\phi(\cdot)$  to average bag value  $\ell$ , which uniquely determines a *conditional optimal food distribution*  $\phi^*(\cdot | \ell)$ .

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Distributing surplus food evenly across all bags

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Differentiate bags into high and low values

# Example of Non-Concave Utility: From Prospect Theory

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- Base utility function  $v_0$

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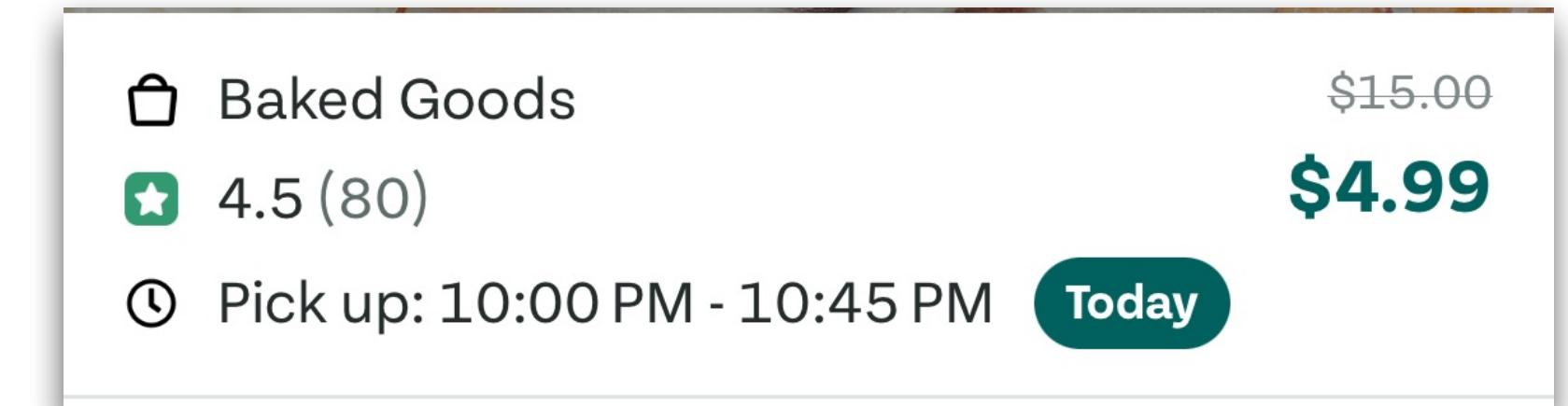
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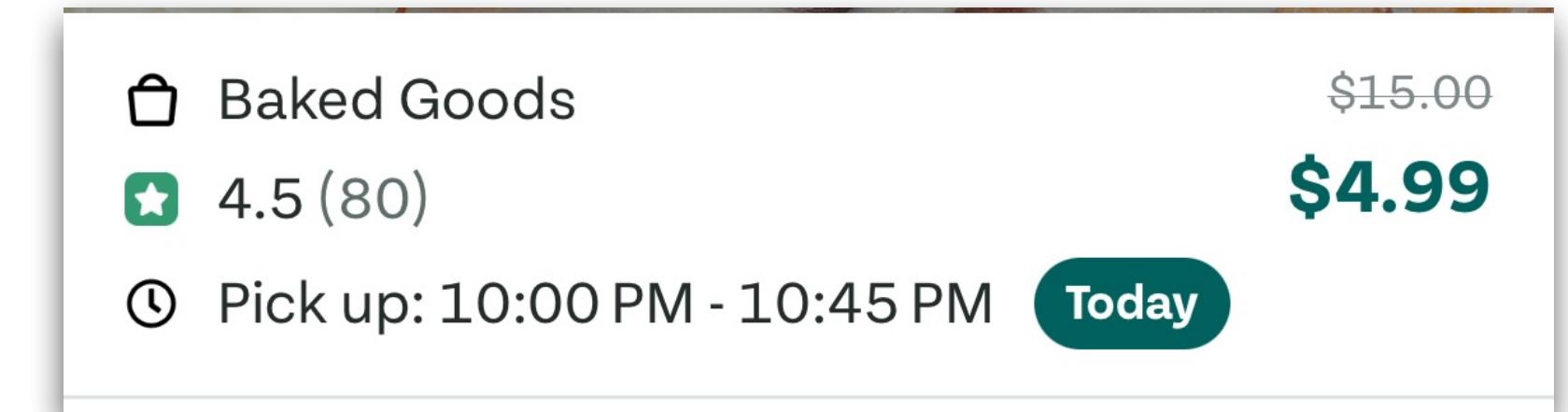


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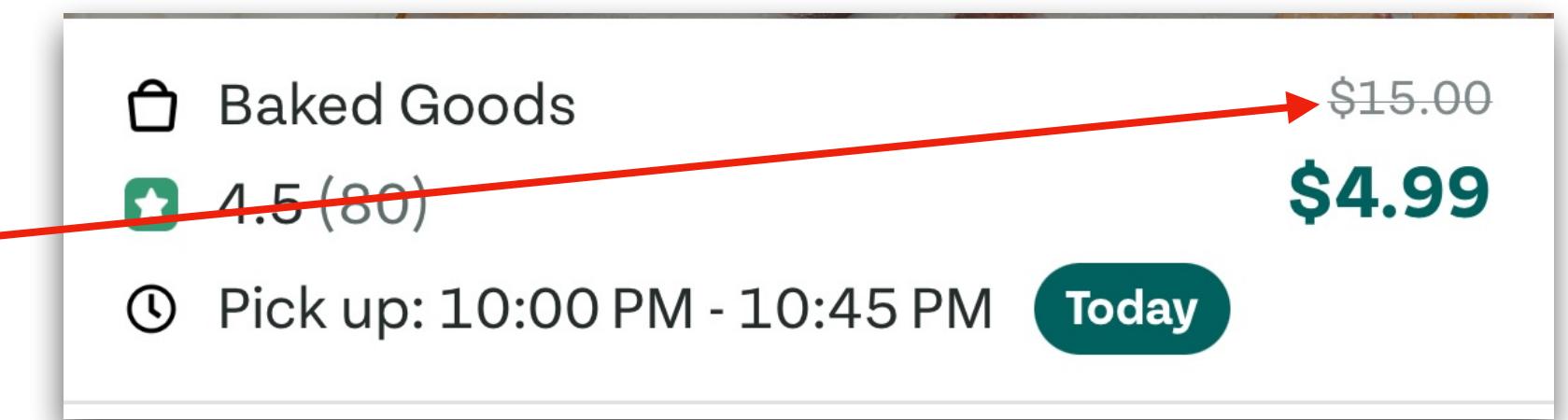


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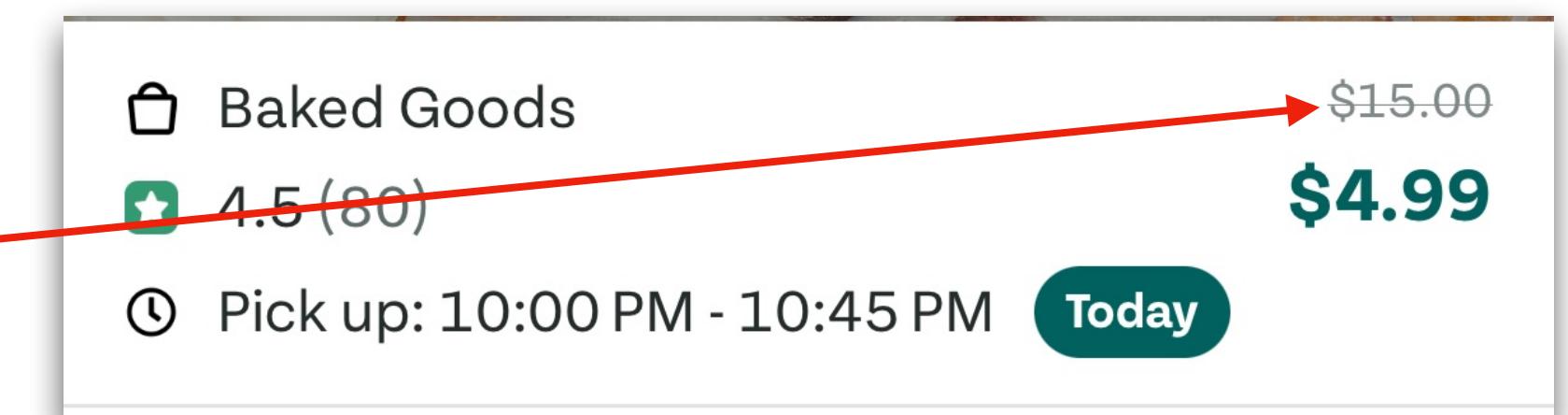
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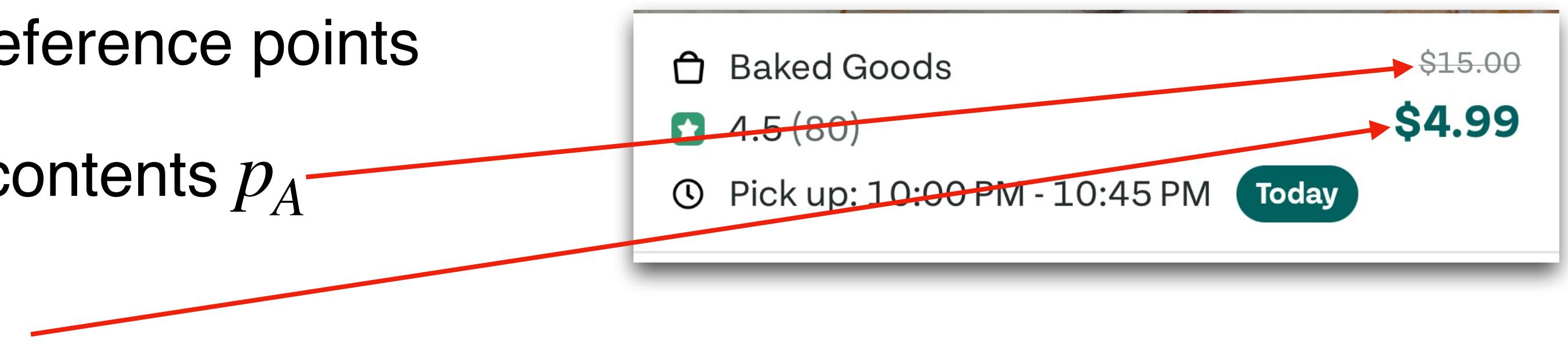


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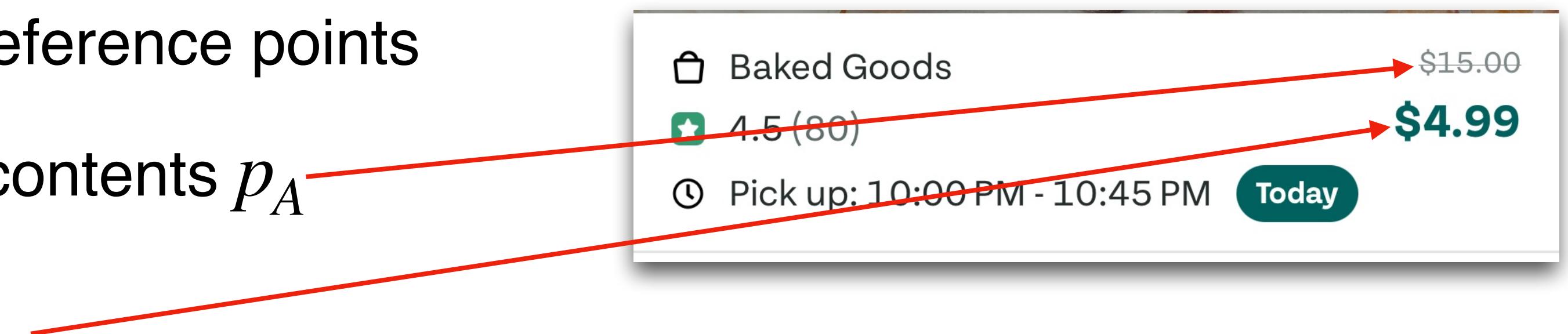


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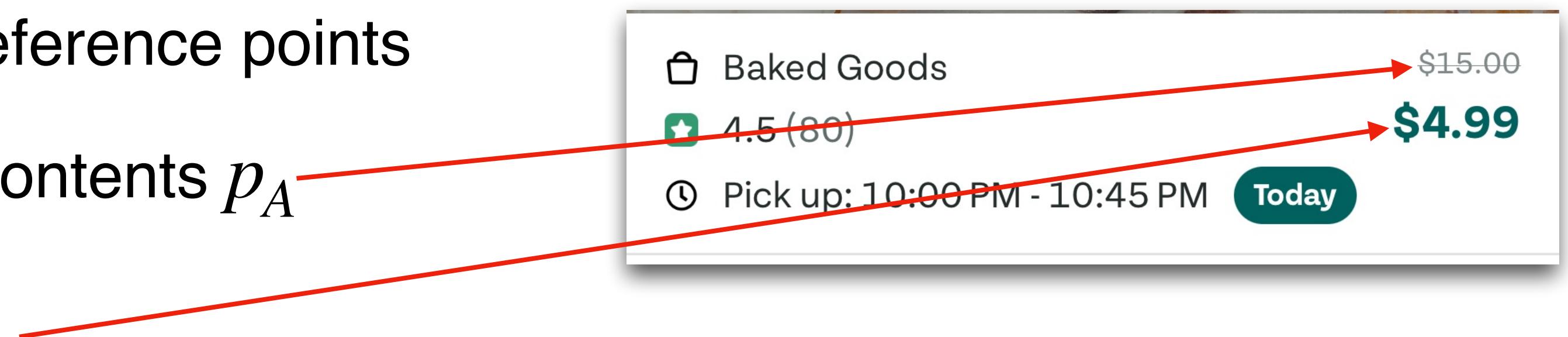
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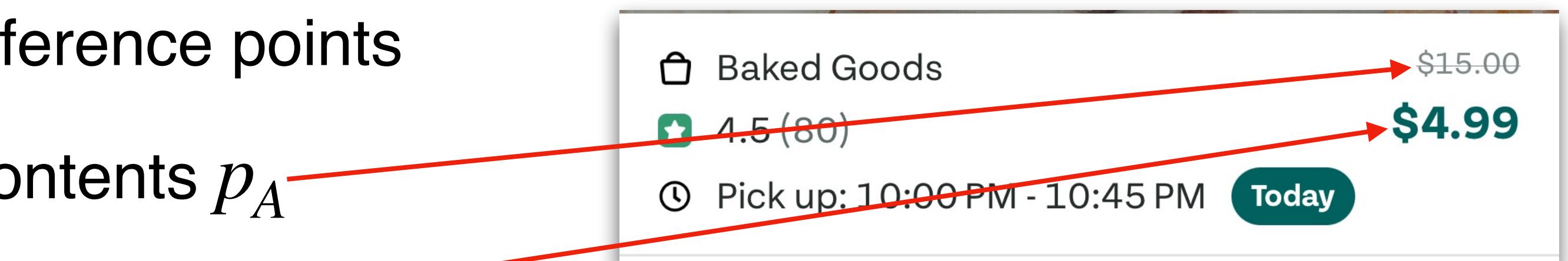
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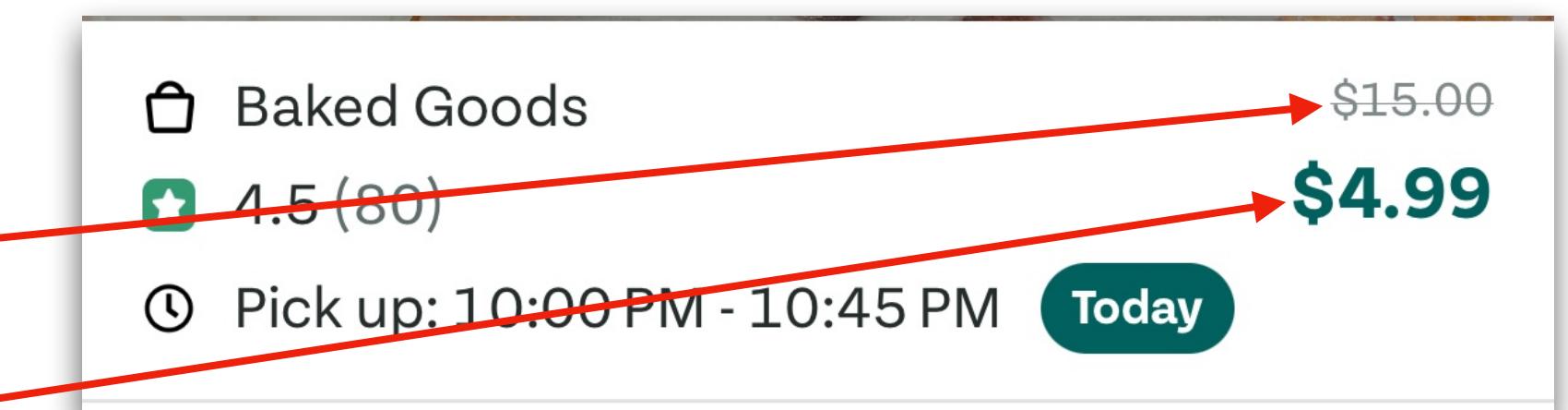
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# Distribution with Non-Concave Utility Function

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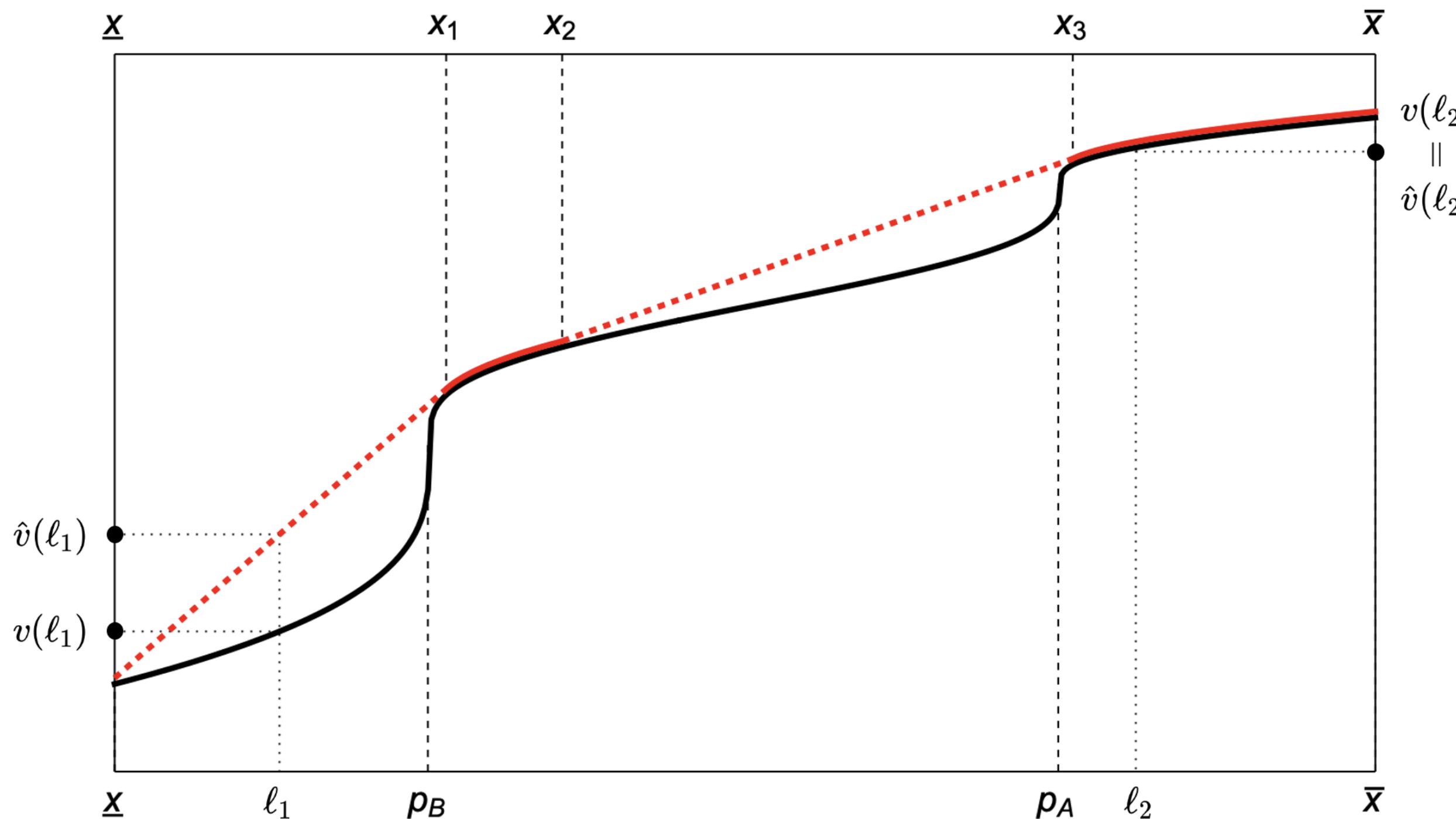
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**Consumer utility function  $v(x)$  and its upper concave envelope  $\hat{v}(x)$**

— :  $v(x)$ ; — :  $\hat{v}(x)$  where  $\hat{v}(x) = v(x)$ ; - - - :  $\hat{v}(x)$  where  $\hat{v}(x) \neq v(x)$ .



Note.  $\hat{v}(x) = v(x)$  for  $x \in \{\underline{x}\} \cup [x_1, x_2] \cup [x_3, \bar{x}]$ , and  $\hat{v}(x) > v(x)$  for  $x \in (\underline{x}, x_1) \cup (x_2, x_3)$ . Note that  $x_1 > p_B$  and  $x_3 > p_A$ .

# Deterministic Fluid Approximation

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**Proposition**  $n^{MD}(r)$  increases in  $r$  and  $\ell^{MD}(r)$  decreases in  $r$ . Specifically,

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**Remark:** the performance bound is tighter when

- Time discount factor  $\beta$  is small
- Supplementary cost  $c$  is small
- Standard deviation of surplus  $\sigma$  is small
- Average surplus amount  $\bar{Q}$  is aligned with  $xD(r)$

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**Implication:** Stores do not need to maintain a perfect rating; it optimizes to converge to a higher rating if

- Higher price-to-cost ratio
- Faster reputation updating
- Focus more on long-term perspective
- Greater demand increase from better bag value

# Illustration of Optimal Policy

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$$v(x) = \frac{1}{1 + e^{-10(x-0.5)}}.$$

- Its upper concave envelope can be expressed as

$$\hat{v}(x) = \begin{cases} 1.25x & \text{if } x \in [0, 0.676]; \\ \frac{1}{1 + e^{-10(x-0.5)}} & \text{if } x \in (0.676, 1]. \end{cases}$$

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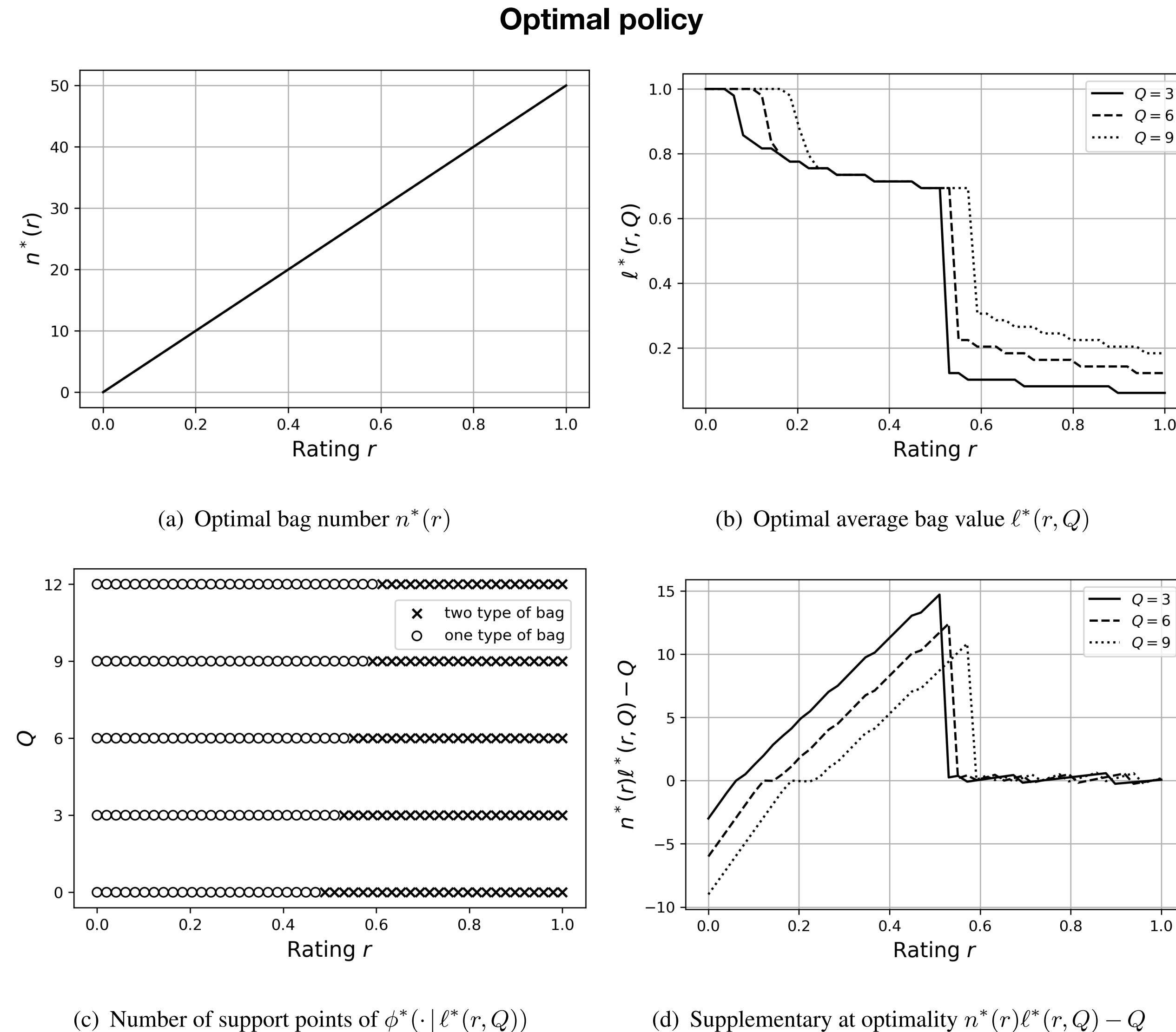
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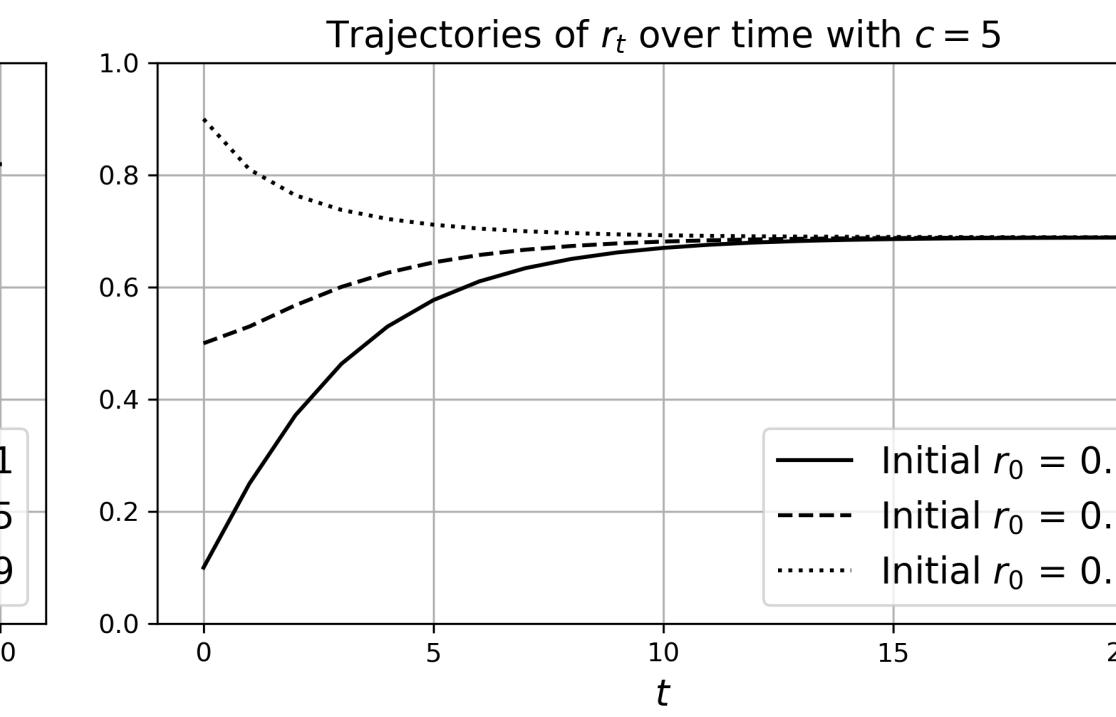
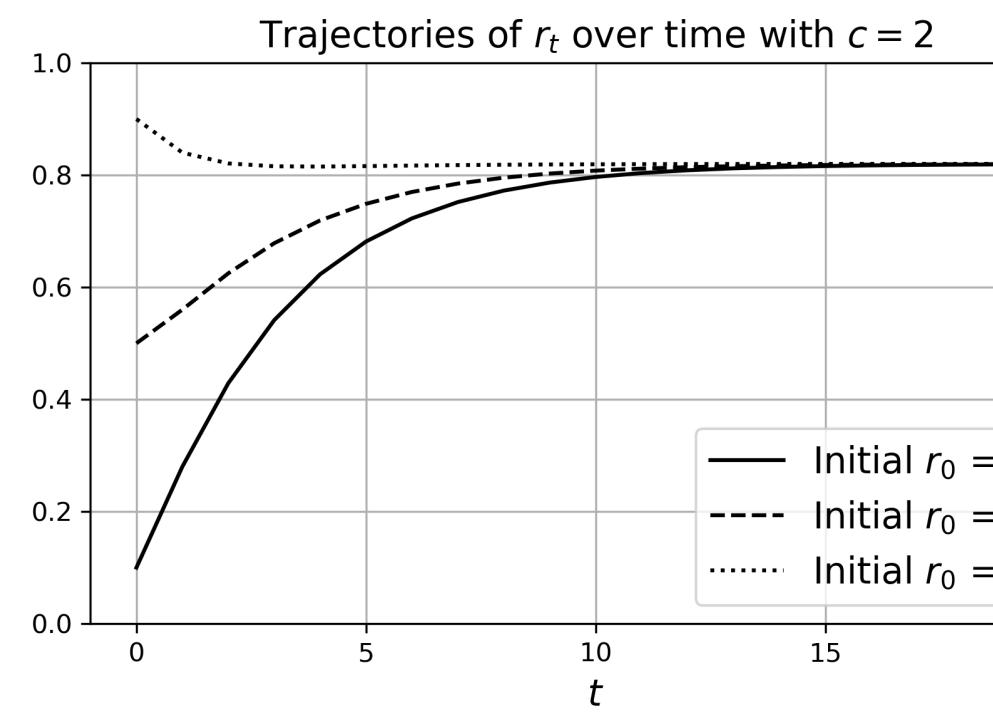
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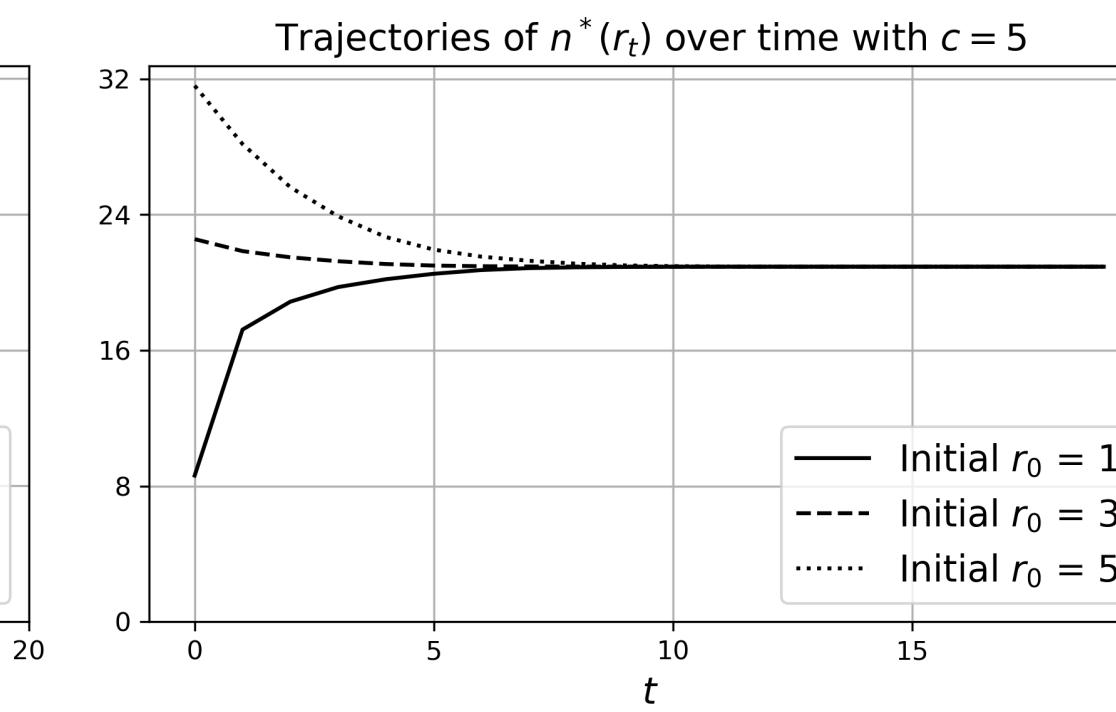
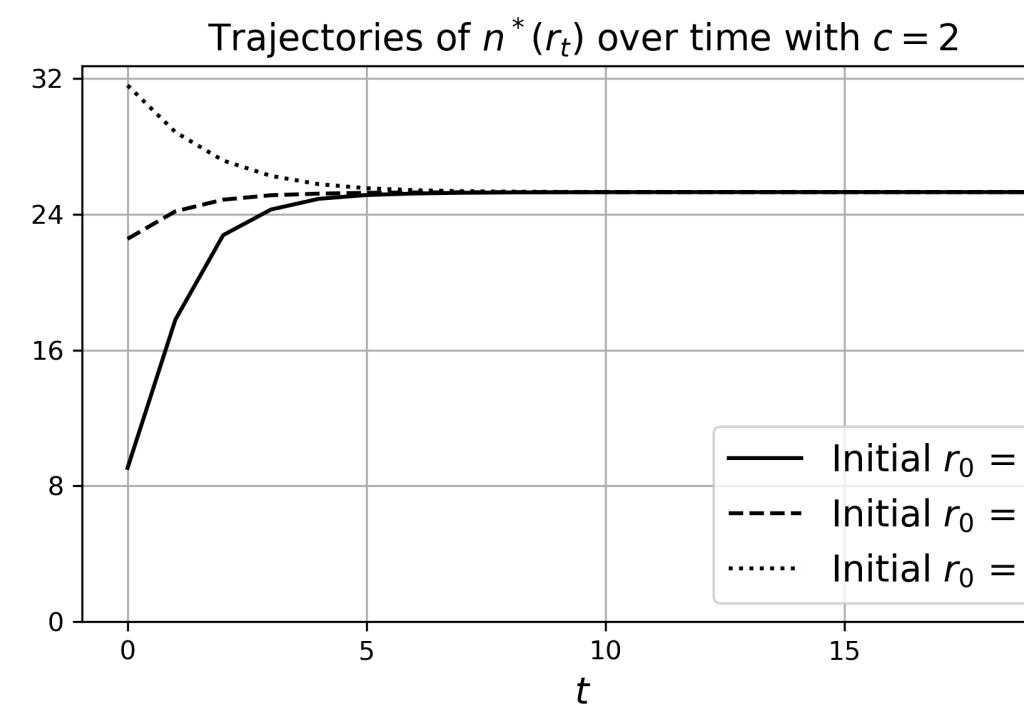


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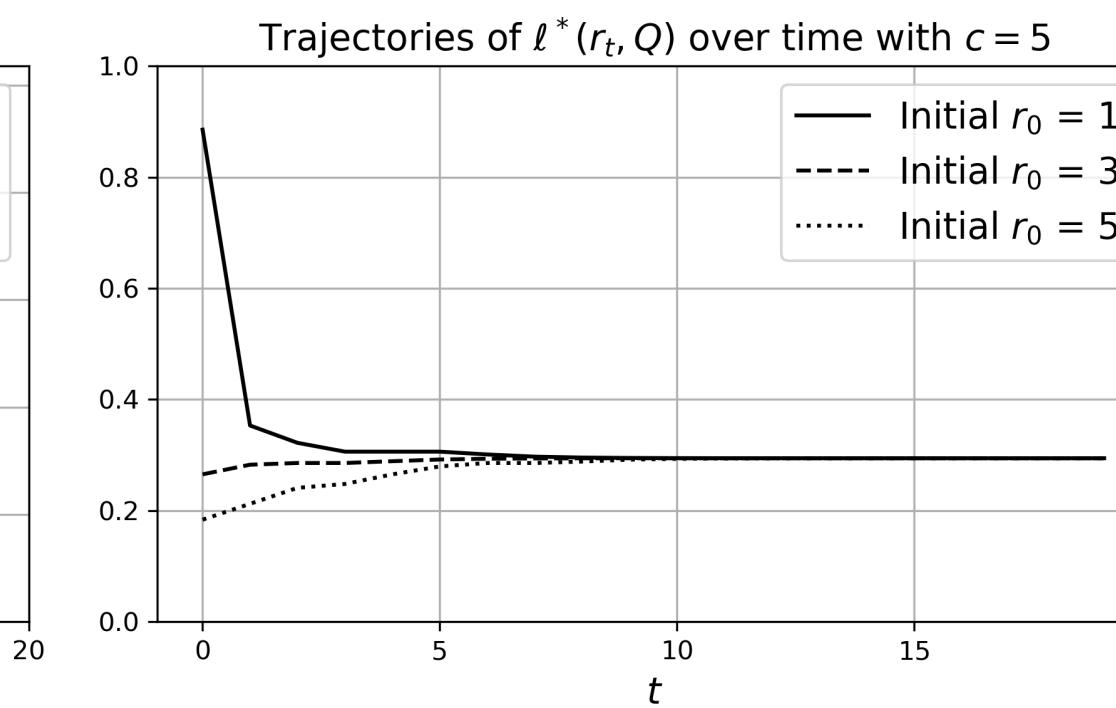
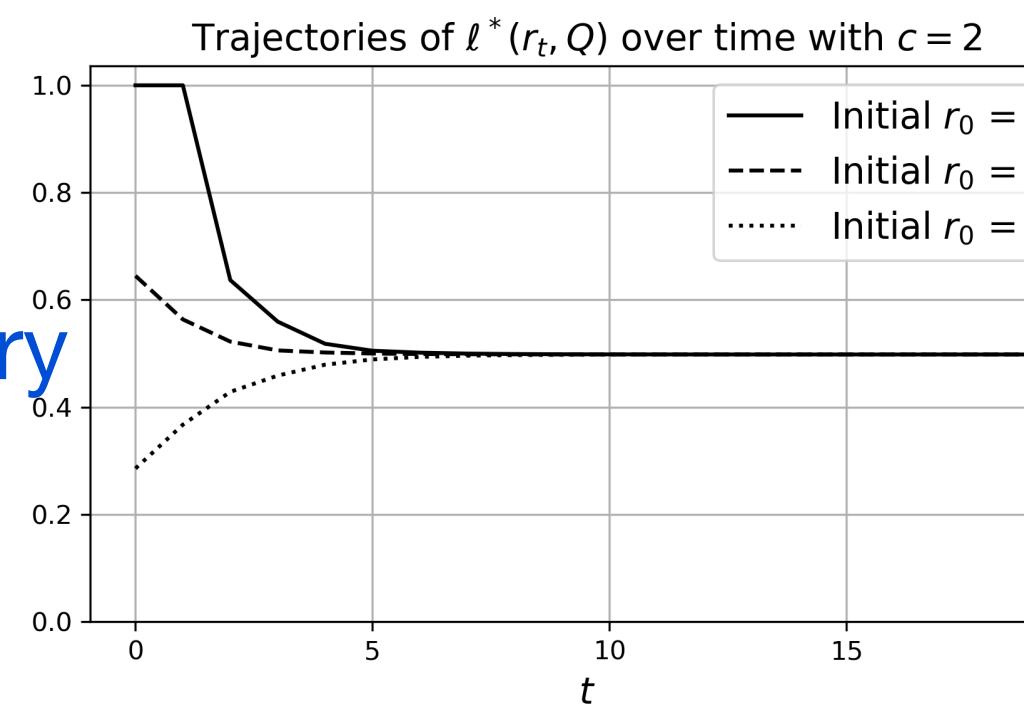
Reputation trajectory



Bag number trajectory



Average bag value trajectory



Left: Low  $c/p$  ratio

Right: High  $c/p$  ratio

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  - Match random surplus with consumer demand driven by the store's dynamic rating
- **Optimal Food Distribution Across Bags**
  - Up to 2 types of bags is all we need
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**Thank you for your attention!**

Questions and comments are welcome!

Paper "Designing Surprise Bags for Surplus Foods." Available at SSRN: <https://ssrn.com/abstract=5002233>