UGBA 141
Discussion 11

Agenda: - Review and practice for Queue I and II
- Prepare for Queue III
- Discuss ways of reducing variation

April 8, 2022
Hansheng Jiang

Reminder

- HW4 is due tonight (Friday)

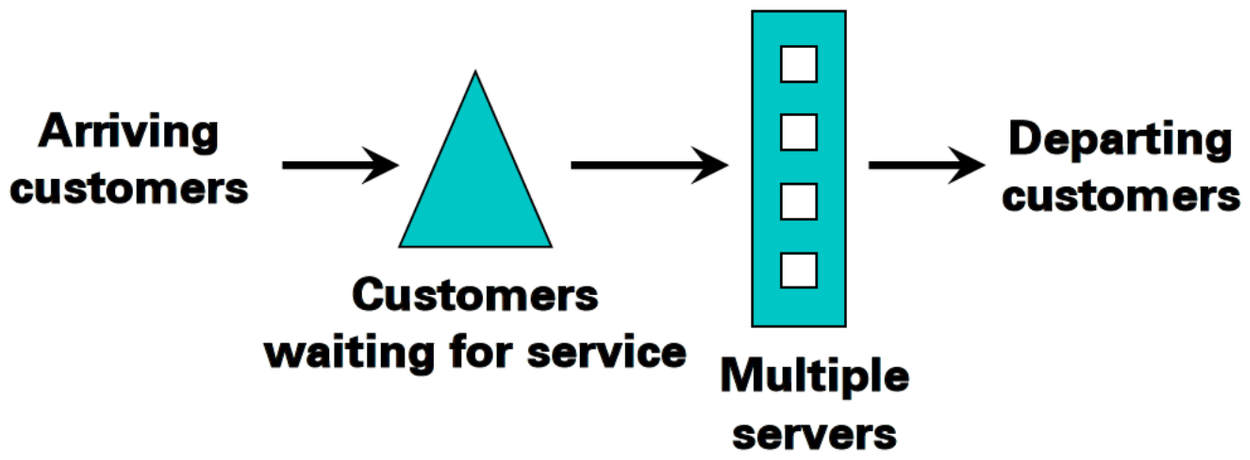
- Please come up with a new team name for Littlefield Simulation if not already
  - We will update the Team ID based on the new team name on Saturday

- Read the Littlefield handout + watch the walkthrough video (it's only 6 minutes)
  - To do well, analyze historical data (processing times are on bCourses, demand is inside the platform)

- No class on Mon (4/11)/Wed(4/13)

Review: Queue Metrics

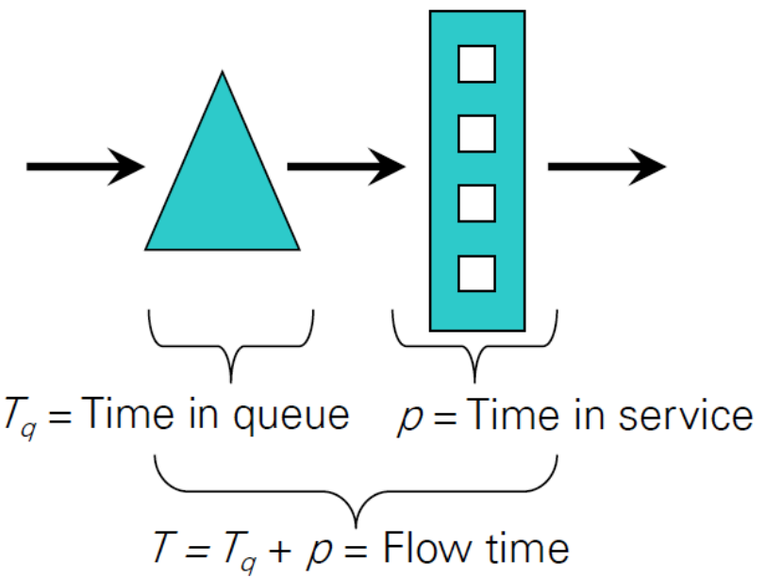
- a = average interarrival time, p = average processing time
- m = number of servers
- Coefficients of variation: CV_a, CV_p
- Utilization in a multi-server queue



  - Implied utilization = $p/(a \times m)$
  - When implied utilization<1, utilization = implied utilization

Review: Time in Queue/System & #Customers

- Waiting time/ time in queue



$$Time\ in\ queue = \left(\frac{p}{m}\right) \times \left(\frac{Utilization^{\sqrt{2(m+1)}-1}}{1\text{-}Utilization}\right) \times \left(\frac{CV_a^2 + CV_p^2}{2}\right)$$

- Time in system = time in queue + processing time (p)
- Using Little's law: Number of customers = Time in system * Flow rate ($1/a$)

Practice Problem: Wait Time

American Airline (Q1-Q4)

Customers arrive to the 5 check-in kiosks at American Airlines at the rate of 100 per hour. The standard deviation of the interarrival times is 1.9 min. Each customer takes on average 2.5 minutes to check-in at a kiosk with a standard deviation of 2.5 minutes.

Q1. What is the utilization (%) of the kiosks?

Q2. If American wants to avoid very long queues, what is the minimum number of kiosks that must be installed and operating?

Practice Problem: Wait Time

American Airline (Q1-Q4)

Customers arrive to the 5 check-in kiosks at American Airlines at the rate of 100 per hour. The standard deviation of the interarrival times is 1.9 min. Each customer takes on average 2.5 minutes to check-in at a kiosk with a standard deviation of 2.5 minutes.

Q3. On average, how long does a customer wait (min) before they start the check-in process with a kiosk?

Q4. On average, how many customers are in the check-in process (with a kiosk or waiting to use a kiosk).

Review: Loss System

- Define $r = \frac{p}{a}$
- Demand lost once all m servers are busy, and the probability of this event if denoted by $P_m(r)$
- Read Erlang Loss Table to find $P_m(r)$ for given m and r

Practice Problem: Loss Systems

Loss System (Q5-Q6)

Flow units arrive at a rate of 60 units per hour. A resource has $m = 4$ and a processing time of 3 minutes. There is no inventory space to buffer flow units unless they can start service immediately.

Q5. What is the implied utilization of this process?

Q6. What fraction of incoming flow units will be lost?

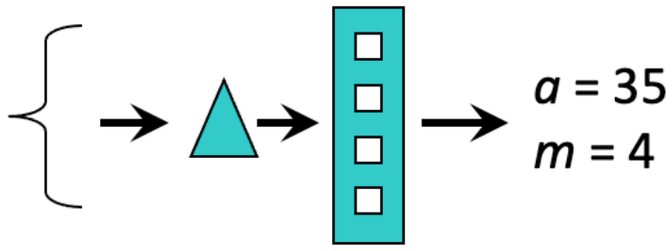
Read Erlang Loss Table

0.30	0.2308	0.0335	0.0033	0.0003	0.0000	0.0000	0.000
0.33	0.2500	0.0400	0.0044	0.0004	0.0000	0.0000	0.000
0.40	0.2857	0.0541	0.0072	0.0007	0.0001	0.0000	0.000
0.50	0.3333	0.0769	0.0127	0.0016	0.0002	0.0000	0.000
0.60	0.3750	0.1011	0.0198	0.0030	0.0004	0.0000	0.000
0.67	0.4000	0.1176	0.0255	0.0042	0.0006	0.0001	0.000
0.70	0.4118	0.1260	0.0286	0.0050	0.0007	0.0001	0.000
0.75	0.4286	0.1385	0.0335	0.0062	0.0009	0.0001	0.000
0.80	0.4444	0.1509	0.0387	0.0077	0.0012	0.0002	0.000
0.90	0.4737	0.1757	0.0501	0.0111	0.0020	0.0003	0.000
1.00	0.5000	0.2000	0.0625	0.0154	0.0031	0.0005	0.000
1.10	0.5238	0.2237	0.0758	0.0204	0.0045	0.0008	0.000
1.20	0.5455	0.2466	0.0898	0.0262	0.0063	0.0012	0.000
1.25	0.5556	0.2577	0.0970	0.0294	0.0073	0.0015	0.000
1.30	0.5652	0.2687	0.1043	0.0328	0.0085	0.0018	0.000
1.33	0.5714	0.2759	0.1092	0.0351	0.0093	0.0021	0.000
1.40	0.5833	0.2899	0.1192	0.0400	0.0111	0.0026	0.000
1.50	0.6000	0.3103	0.1343	0.0480	0.0142	0.0035	0.000
1.60	0.6154	0.3299	0.1496	0.0565	0.0177	0.0047	0.001
1.67	0.6250	0.3425	0.1598	0.0624	0.0204	0.0056	0.001
1.70	0.6296	0.3486	0.1650	0.0655	0.0218	0.0061	0.001
1.75	0.6364	0.3577	0.1726	0.0702	0.0240	0.0069	0.001
1.80	0.6429	0.3665	0.1803	0.0750	0.0263	0.0078	0.002
1.90	0.6552	0.3836	0.1955	0.0850	0.0313	0.0098	0.002
2.00	0.6667	0.4000	0.2105	0.0952	0.0367	0.0121	0.003
2.10	0.6774	0.4156	0.2254	0.1058	0.0425	0.0147	0.004
2.20	0.6875	0.4306	0.2400	0.1166	0.0488	0.0176	0.005
2.25	0.6923	0.4378	0.2472	0.1221	0.0521	0.0192	0.006
2.30	0.6970	0.4449	0.2543	0.1276	0.0554	0.0208	0.006
2.33	0.7000	0.4495	0.2591	0.1313	0.0577	0.0220	0.007
2.40	0.7059	0.4586	0.2684	0.1387	0.0624	0.0244	0.008
2.50	0.7143	0.4717	0.2822	0.1499	0.0697	0.0282	0.010
2.60	0.7222	0.4842	0.2956	0.1612	0.0773	0.0324	0.011
2.67	0.7273	0.4923	0.3044	0.1687	0.0825	0.0354	0.013
2.70	0.7297	0.4963	0.3087	0.1725	0.0852	0.0369	0.014
2.75	0.7333	0.5021	0.3152	0.1781	0.0892	0.0393	0.015
2.80	0.7368	0.5078	0.3215	0.1837	0.0933	0.0417	0.016
2.90	0.7436	0.5188	0.3340	0.1949	0.1016	0.0468	0.019
3.00	0.7500	0.5294	0.3462	0.2061	0.1101	0.0522	0.021
3.10	0.7561	0.5396	0.3580	0.2172	0.1187	0.0578	0.024
3.20	0.7619	0.5494	0.3695	0.2281	0.1274	0.0636	0.028
3.25	0.7647	0.5541	0.3751	0.2336	0.1318	0.0666	0.030
3.30	0.7674	0.5587	0.3807	0.2390	0.1362	0.0697	0.031

Which System is More Effective?

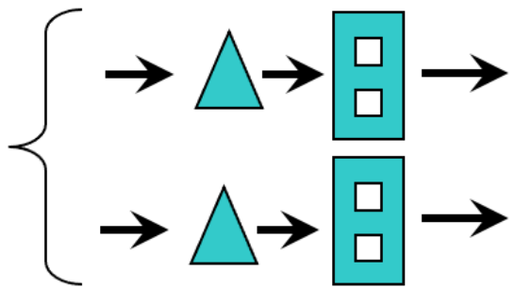
Pooled system:

One queue, four servers



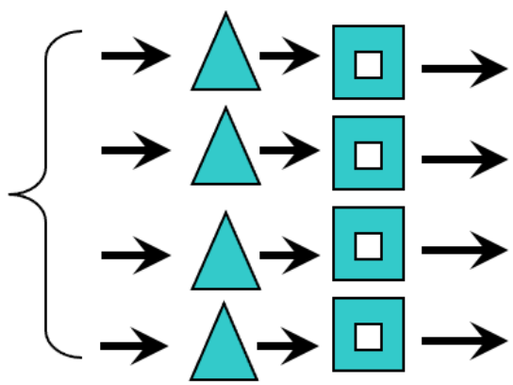
Partially pooled system:

Two queues, two servers with each queue.



Parallel queue system:

Four queues, one server with each queue.



Across these three types of systems:

Variability is the same: $CV_a = 1$, $CV_p = 1.25$

Total demand is the same: 1/35 customers per second.

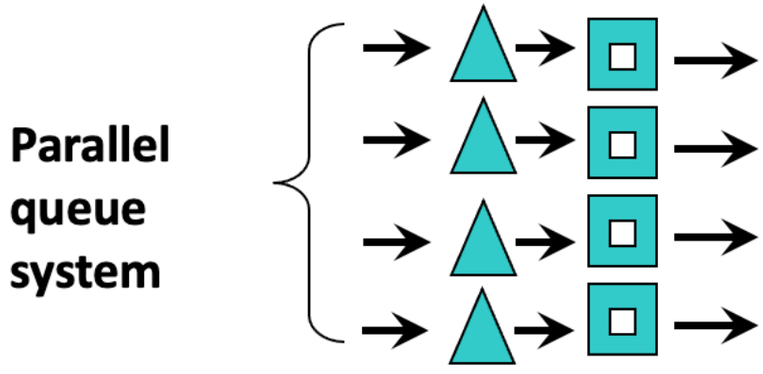
Processing time is the same: $p = 120$

Utilization is the same: p / a x m = 85.7%

The probability a server is busy is the same = 0.857

Prepare for Queue III

- Calculate for each of three systems
  - Time in Queue
  - Total Number of Customers
- Example



- What can you observe from the calculated results?

Reducing Variability

- Ways of reducing arrival variability
  - Appointment systems
  - Early-bird specials
  - Price discounts during off-peak dates
  - Price changes in transportation (train/airline)
    - Surge pricing in Uber/Lyft

- Ways of reducing processing time variability
  - In service context, heavy investment in training employees or make it more automated for customers
  - In manufacturing context, variation usually reflects quality problem, which indicates necessary improvements