Learning While Repositioning in On-Demand Inventory Sharing Networks

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Joint work with
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Vehicle Sharing Networks

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Features

- On-demand: customers reserve a vehicle when they want
- One-way: rent from one location and return the vehicle to *any other* location in the service network
- Examples: bikes, scooters, cars

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Source: Generated by Midjourney

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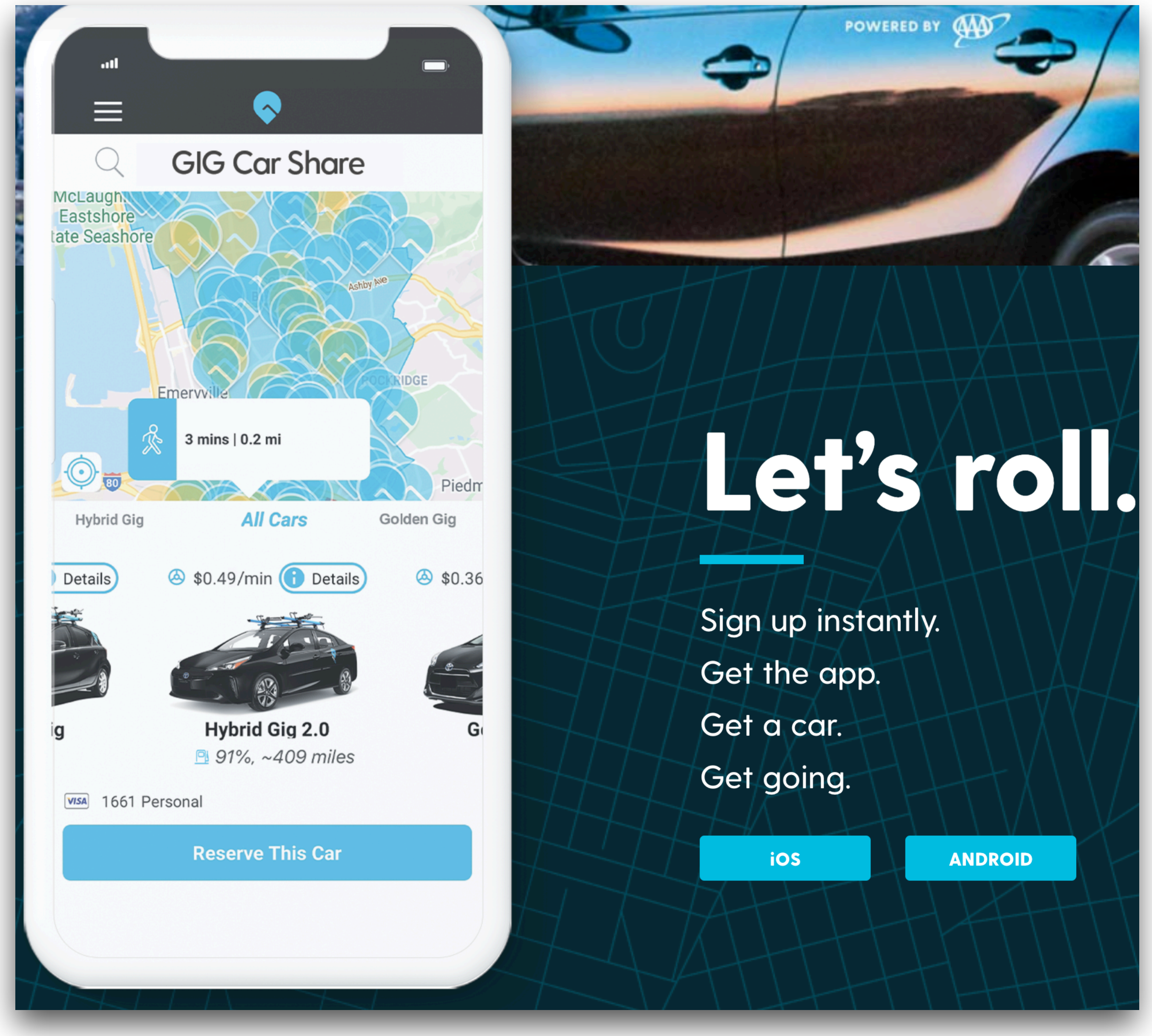
Benefits

- Increased flexibility and convenience for customers
- Competitive transportation costs for customers
- Environmental friendly
  - May reduce overall vehicle ownerships and produce less carbon emissions
  - Help to promote adoption of EVs with cleaner energy

Source: Generated by Midjourney

Emerging Platforms and Programs

Emerging Platforms and Programs



Source: gigcarshare.com

"GIG Car Share: Carsharing service in parts of the San Francisco Bay Area, Sacramento, and Seattle, created by A3 Ventures

The company operates a fleet of Toyota Prius Hybrid vehicles and all-electric Chevrolet Bolts. It offers one-way point-to-point rentals."

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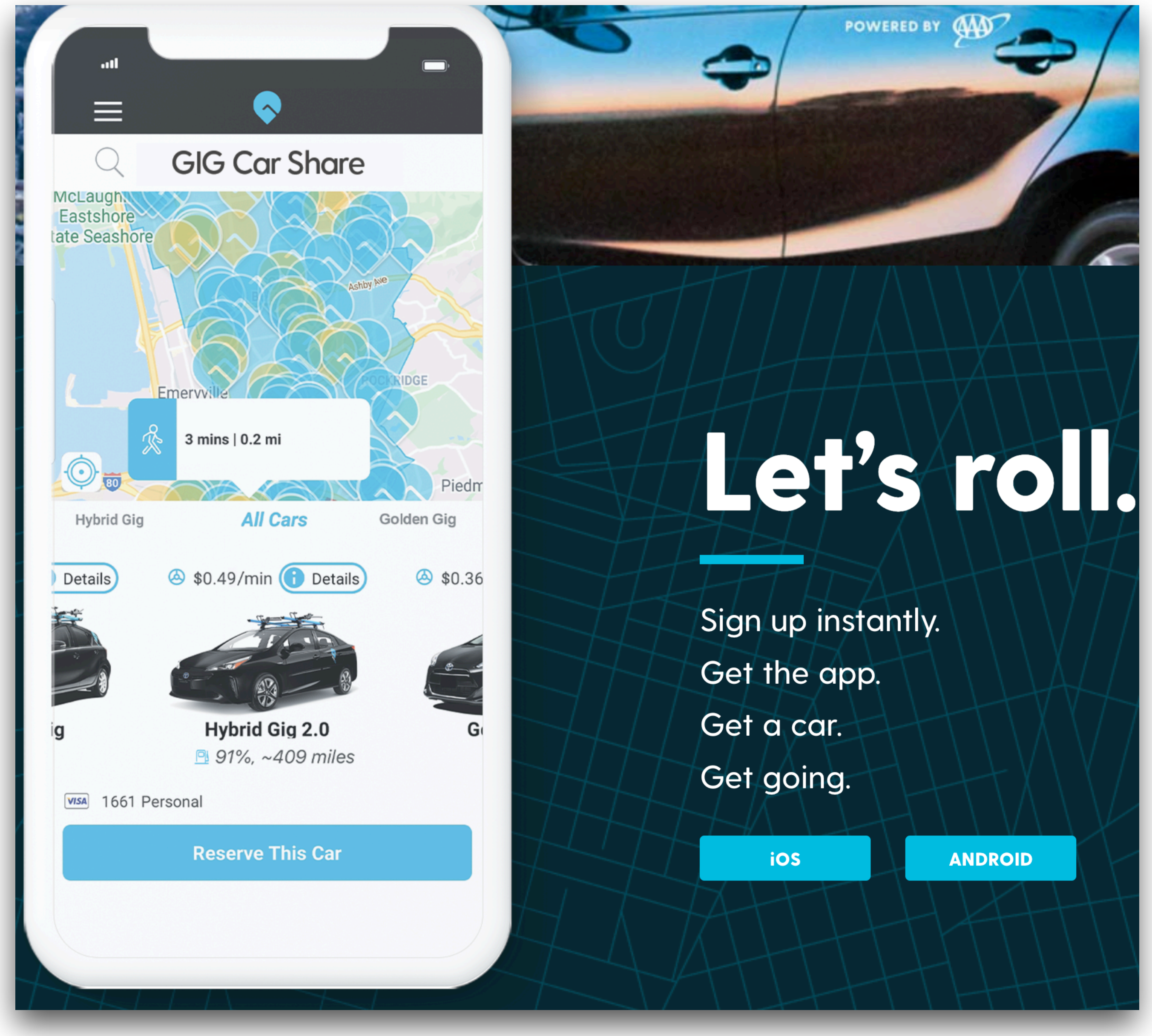

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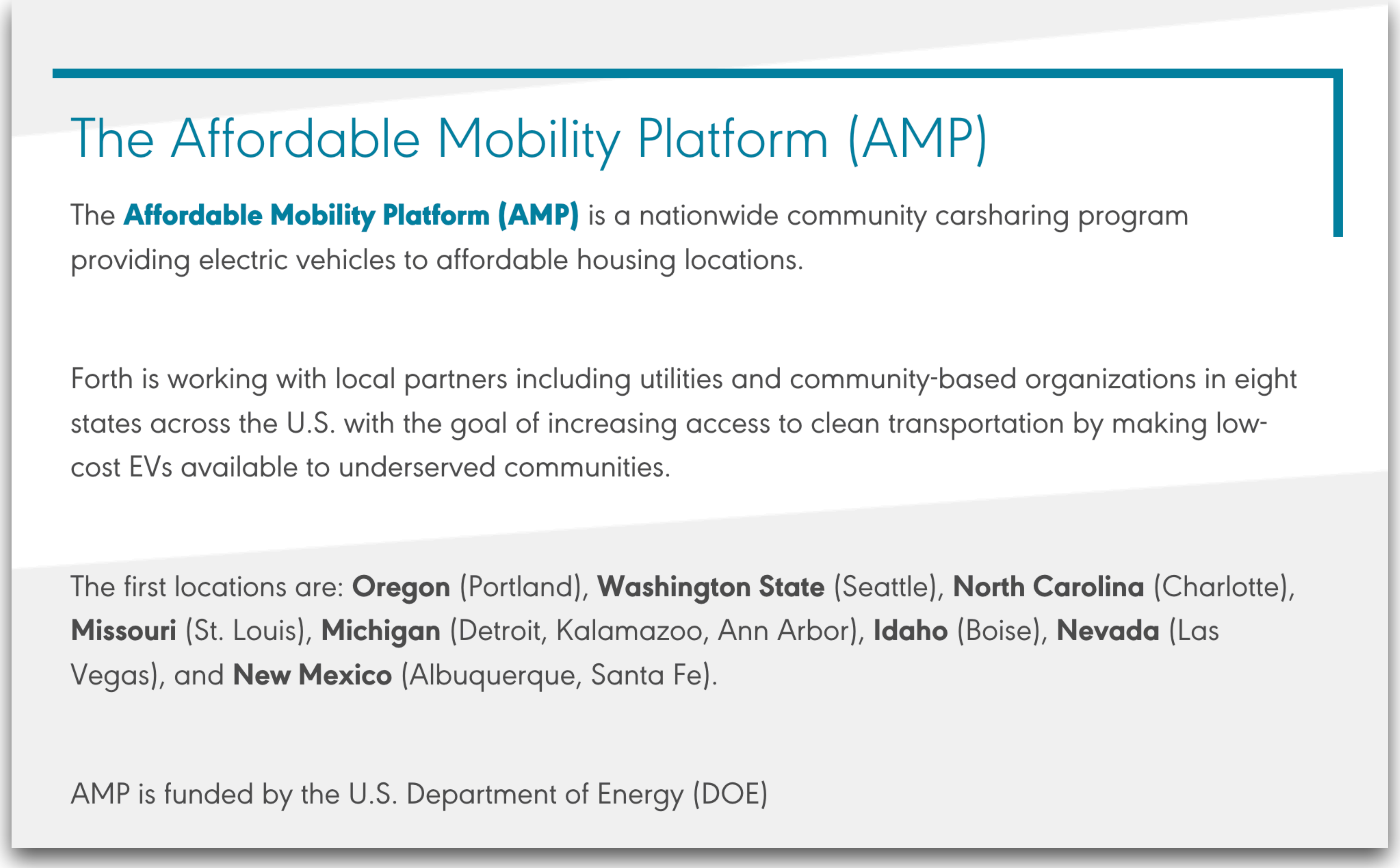

The Affordable Mobility Platform (AMP)

The **Affordable Mobility Platform (AMP)** is a nationwide community carsharing program providing electric vehicles to affordable housing locations.

Forth is working with local partners including utilities and community-based organizations in eight states across the U.S. with the goal of increasing access to clean transportation by making low-cost EVs available to underserved communities.

The first locations are: **Oregon** (Portland), **Washington State** (Seattle), **North Carolina** (Charlotte), **Missouri** (St. Louis), **Michigan** (Detroit, Kalamazoo, Ann Arbor), **Idaho** (Boise), **Nevada** (Las Vegas), and **New Mexico** (Albuquerque, Santa Fe).

AMP is funded by the U.S. Department of Energy (DOE)

Source: forthmobility.org/community-carsharing

Nationwide community carsharing program addressing lack of public transportation and providing cleaner transportation option for low-income community

Motivation

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Numerous **operational challenges** of vehicle sharing networks

- Service region design
- Fleet sizing
- Trip pricing
- Infrastructure planning

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Focus of this talk: **Inventory Repositioning**

- Lost demand due to lack of vehicles in high utilization zone
- Low utilization zone with oversupply of vehicles

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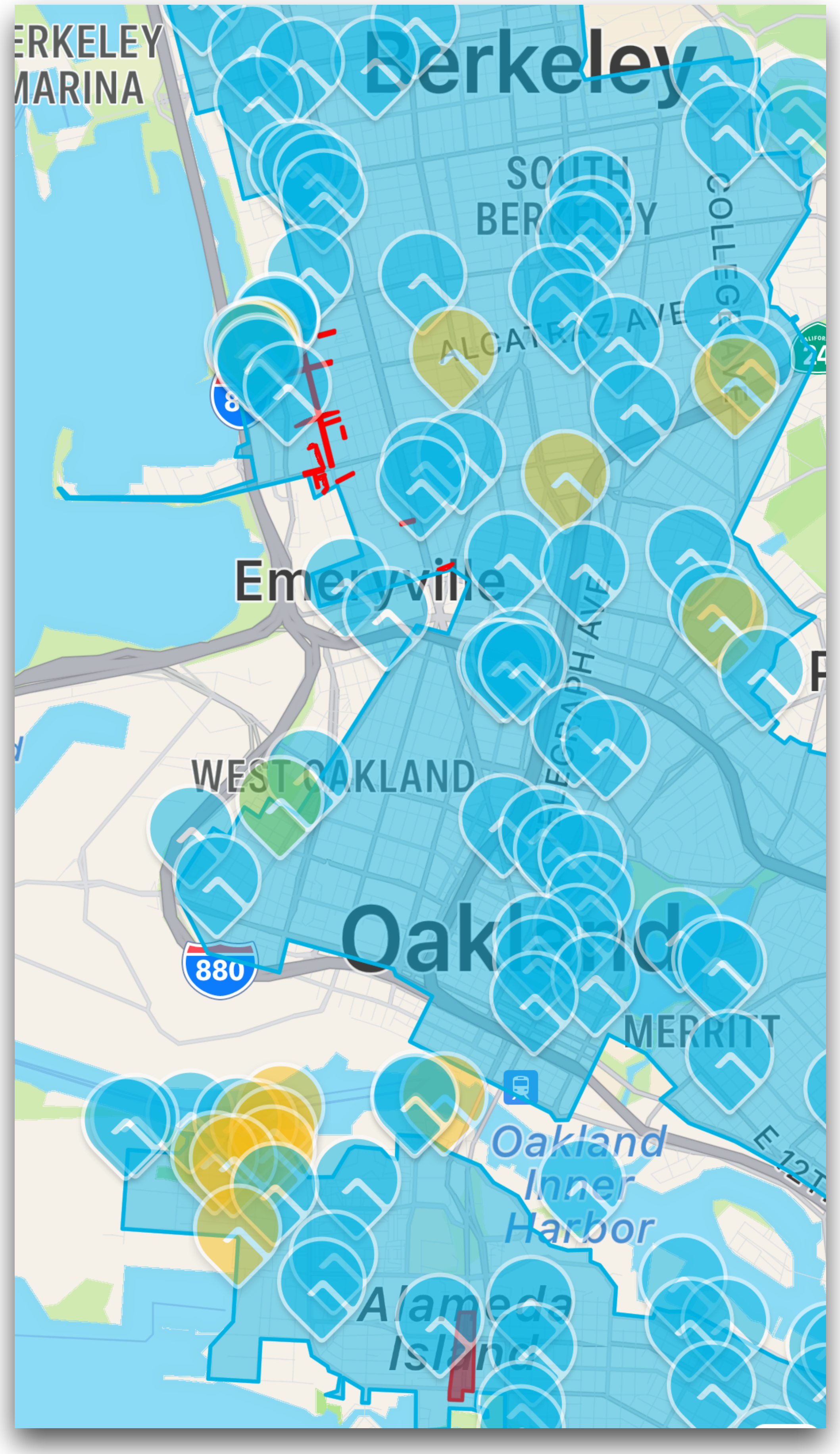

Screenshot of GIG Car Share App

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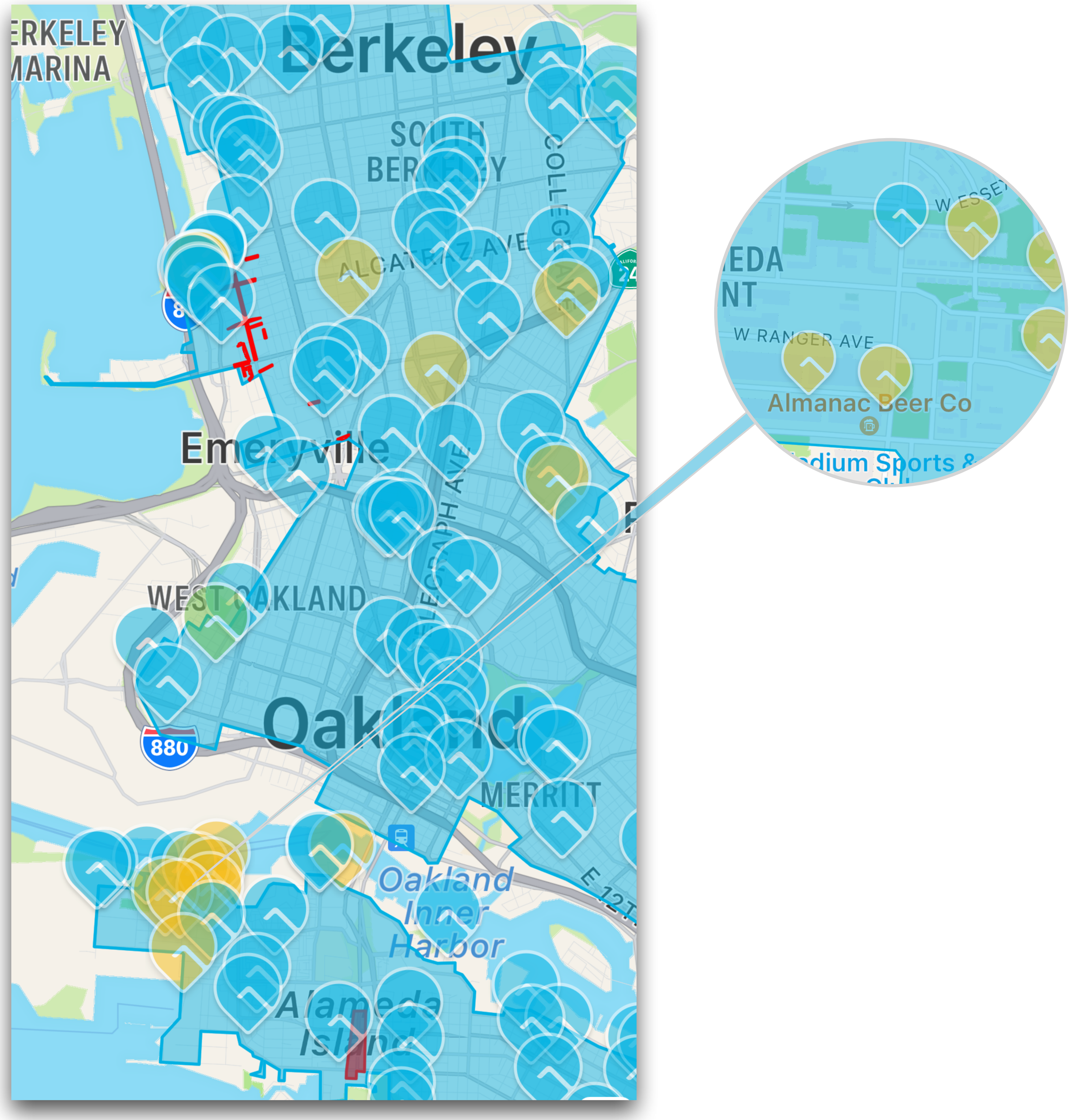

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Matching Supply with Demand in Network

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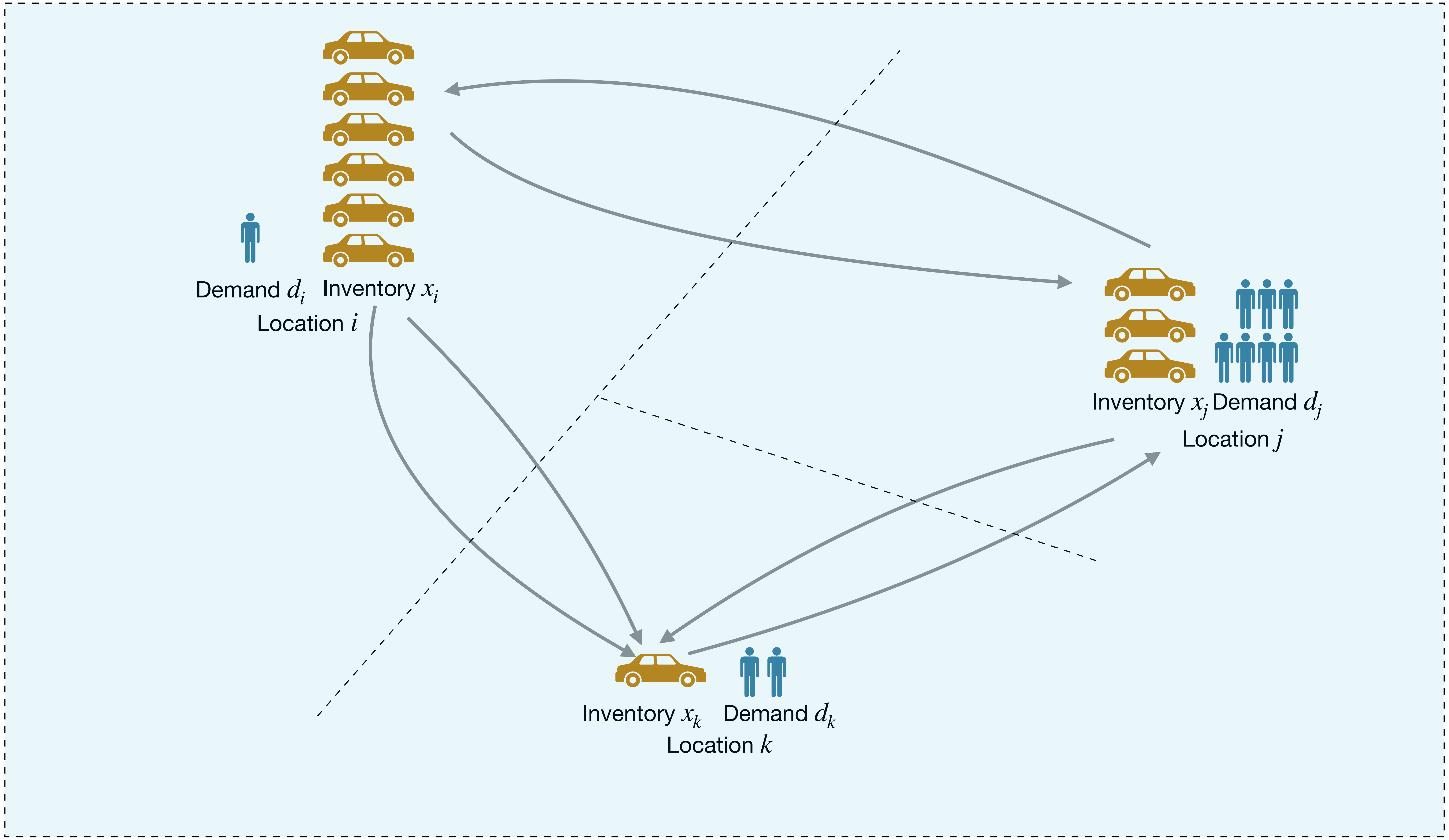

Illustration of 3 locations in a n-location service region

Inventory Dynamics as MDP

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Markov Decision Process

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At period $t = 1, 2, \ldots$

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At period $t = 1,2,\ldots$

I) Service provider reviews the current inventory level $\boldsymbol{x}_t$ (State)

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  - Censored demand $\min(\boldsymbol{d}_t, \boldsymbol{y}_t)$

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$$\boldsymbol{x}_{t+1} = (\boldsymbol{y}_t - \boldsymbol{d}_t)^+ + \boldsymbol{P}^T \min(\boldsymbol{y}_t, \boldsymbol{d}_t)$$

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Single-period cost of policy π

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$$M_t = \min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \cdot \xi_{ij}$$

Obtained by solving minimum cost flow problem

$$\text{s.t.} \sum_{i=1}^{n} \xi_{ij} - \sum_{k=1}^{n} \xi_{jk} = y_{t,j} - x_{t,j}$$

$$L_t = \sum_{i} \sum_{j} l_{ij} \cdot P_{ij}(d_i - y_i)^+$$

P_{ij} origin-to-destination probability

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Long-run average cost of policy π

$$\lambda^\pi = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[C_t^\pi]$$

Designing Repositioning Policy

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Optimal Policy

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Optimal Policy

$$\min_{\pi} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[C_t^{\pi}] \,, T \rightarrow \infty$$

- Optimal policy is computationally expensive in general even when the demand distribution is known or fully observed

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- **Definition:** Repositioning to base-stock level $\boldsymbol{S} = (S_1, \ldots, S_n)$ regardless of the current state $\boldsymbol{x}_t$

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$$\mathbf{S}^{\star} \in \arg \min_{\mathbf{S} \in \Delta_{n-1}} \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}^{\mathbf{S}}[C_t]$$

Asymptotic Optimality of Base-Stock Policy

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Theorem (*informal*) The ratio of the *optimal* base-stock repositioning policy's long-run average cost to the optimal repositioning policy's long-run average cost approaches 1 when the ratio of unit lost sales cost to unit repositioning cost $l_{ij}/c_{ij} \to \infty$.

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Repositioning can be done in bulk; Minimizing user dissatisfication; Need for market growth

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Lost sales cost occurred individually at each location — the opposite of "risk pooling"

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Performance Metric The regret compared with the base-stock repositioning policy incurred by algorithm A with optimal base-stock level $S^\star$

$$\mathrm{Regret}(\mathrm{A}, T) = \sum_{t=1}^{T} \mathbb{E}[C_t^{\mathrm{A}}] - \sum_{t=1}^{T} \mathbb{E}[C_t^{S^\star}]$$

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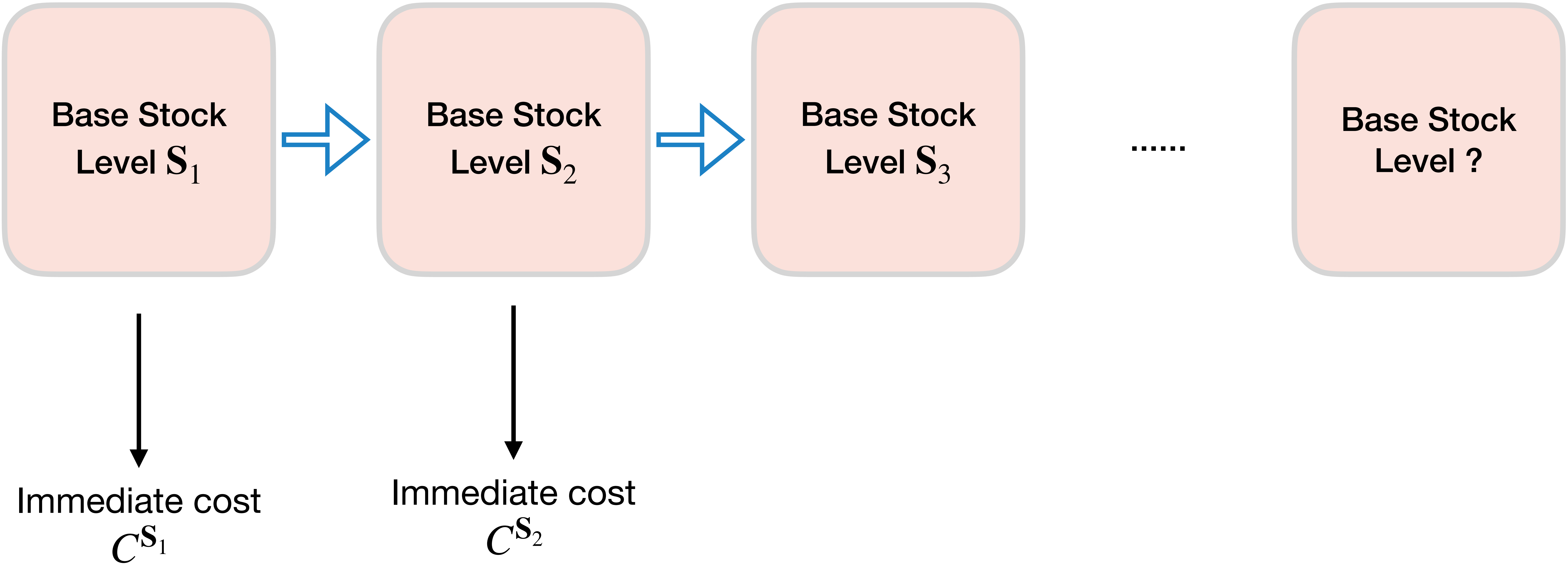

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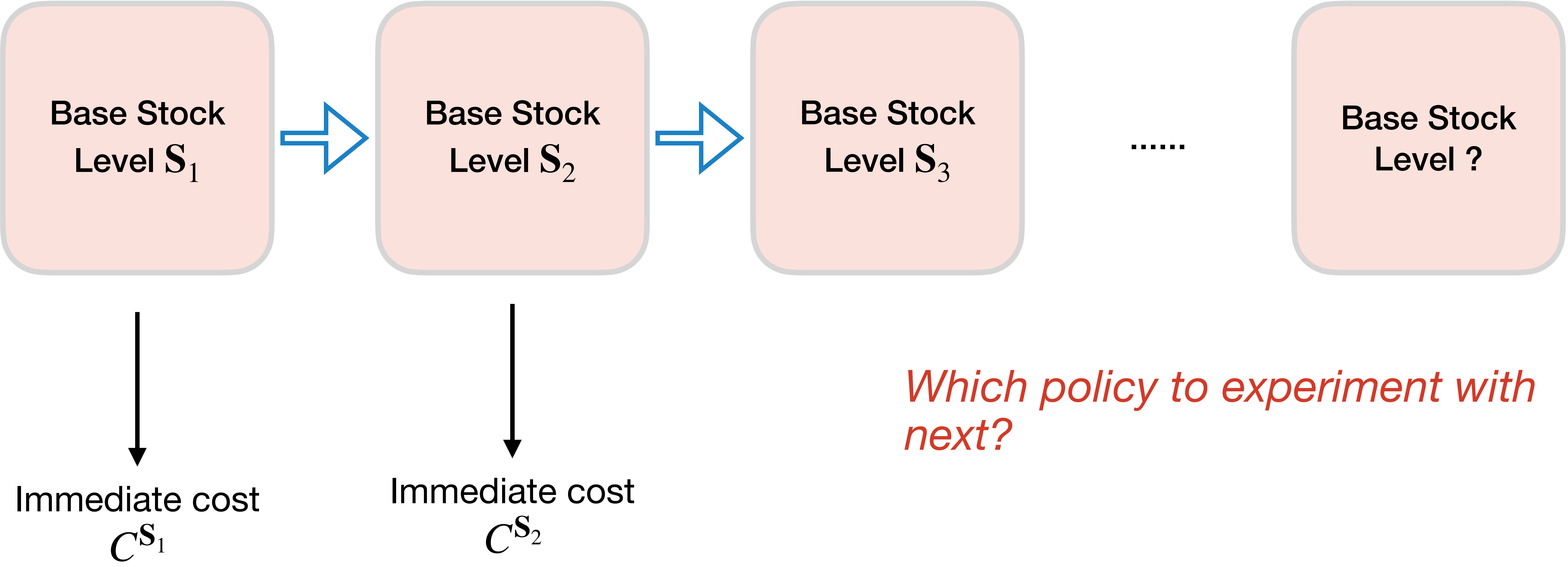

Learning While Repositioning

Base Stock Level S_1 → Base Stock Level S_2 → Base Stock Level S_3 …… Base Stock Level ?

Immediate cost C^{S_1}

Immediate cost C^{S_2}

Which policy to experiment with next?

Compared with classical MAB literature:
demand is censored
immediate single period cots $\neq$ long run average cost
potential policies are not finite
….

Lipschitz Bandits-based Repositioning

`LipBR` Algorithm

Lipschitz Bandits-based Repositioning

LipBR Algorithm

Algorithm Design Idea

- Establish Lipschitz property of the long-run average cost wrt policy
- Discretize the policy space Δ_{n-1} by covering, and bound the covering number by $O(\epsilon^{1-n})$ for accuracy ϵ
- Concentration inequalities of single period costs versus long-run average costs
- Monitor pseudo costs $\tilde{C}$ in regret definition to address unobservable lost sales cost
- Regret $\approx \sqrt{KT} + K\epsilon$, where $K = O(\epsilon^{1-n})$ and $\epsilon = O(T^{-1/(n+1)})$

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Can we bypass the curse of dimensionality and remove the power dependence on n?

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Offline Optimization Problem

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Two Reformulations Tackling NonConvexity

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Theorem (*informal*) With uncensored demand data, the dynamic learning algorithm can achieve $\tilde{O}(T^{\frac{1}{2}})$ regret.

Regret Analysis of Offline-Based Algorithm

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Concentration Inequality

With probability at least $1 - \frac{1}{T^2}$, it holds that

$$\sup_{\mathbf{s} \in \Delta_{n-1}} \left| \frac{1}{t} \sum_{s=1}^{t} \widetilde{C}_s^{\mathbf{s}} - \mathbb{E}[\widetilde{C}_1^{\mathbf{s}}] \right| \leq 6n^3 \left(\max_{i,j} c_{ij} + \max_{i,j} l_{ij} \right) \cdot \frac{\sqrt{\log T}}{\sqrt{t}}$$

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One time learning $\tilde{O}(T^{\frac{2}{3}})$

1. Explore for $nT^{2/3}$ time periods by placing sufficient inventory in n locations respectively

Regret Analysis of Offline-Based Algorithm

Concentration Inequality

With probability at least $1 - \frac{1}{T^2}$, it holds that

$$\sup_{\mathbf{S}\in\Delta_{n-1}} \left| \frac{1}{t}\sum_{s=1}^{t} \widetilde{C}_s^{\mathbf{S}} - \mathbb{E}[\widetilde{C}_1^{\mathbf{S}}] \right| \le 6n^3 \left(\max_{i,j} c_{ij} + \max_{i,j} l_{ij} \right) \cdot \frac{\sqrt{\log T}}{\sqrt{t}}$$

One time learning $\tilde{O}(T^{\frac{2}{3}})$

1. Explore for $nT^{2/3}$ time periods by placing sufficient inventory in n locations respectively
2. Solve the offline problem using data collected in $nT^{2/3}$ time periods

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Dynamic learning $\tilde{O}(T^{\frac{1}{2}})$ (if demand is uncensored)

- At each period, solve the offline problem and update the policy

Online Learning with Optimal Regret

Algorithm Design

At iteration t

1. Consider the dual optimal solution $\lambda_{t,i}$ to the constraints $w_{t,i} \leq \min\{d_{t,i}, S_i\}$
2. $g_{t,i} = \lambda_i \mathbf{1}_{\{\min\{d_{t,i},S_{t,i}\}=S_{t,i}\}}$ is a sub-gradient
3. Gradient descent $\widetilde{\mathbf{S}}_t = \mathbf{S}_t - \frac{1}{\sqrt{t}}\mathbf{g}_t$
4. Project $\widetilde{\mathbf{S}}_t$ onto Δ_{n-1} to obtain $\mathbf{S}_{t+1}$

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Theorem (*informal*) The online stochastic gradient **OSG** based algorithm achieves a regret of $\tilde{O}(T^{\frac{1}{2}})$ and this rate even holds for adversarial data. This rate matches the theoretical lower bound.

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More Extensions

- Incorporating other controls such as pricing and special incentive programs
- More practical challenges in inventory monitoring
  - Seasonal or non-stationary demand
  - New infrastructure such as charging stations

Thanks for your attention!

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Supplementary slides

Numerical Illustration