Intertemporal Pricing via Nonparametric Estimation

—The Value of Reference Effects and Consumer Heterogeneity

Hansheng Jiang[1] Junyu Cao[2] Zuo-Jun Max Shen[1]

[1]Department of IEOR, University of California, Berkeley
[2]McCombs School of Business, University of Texas at Austin

MSOM Data Driven Research Challenge
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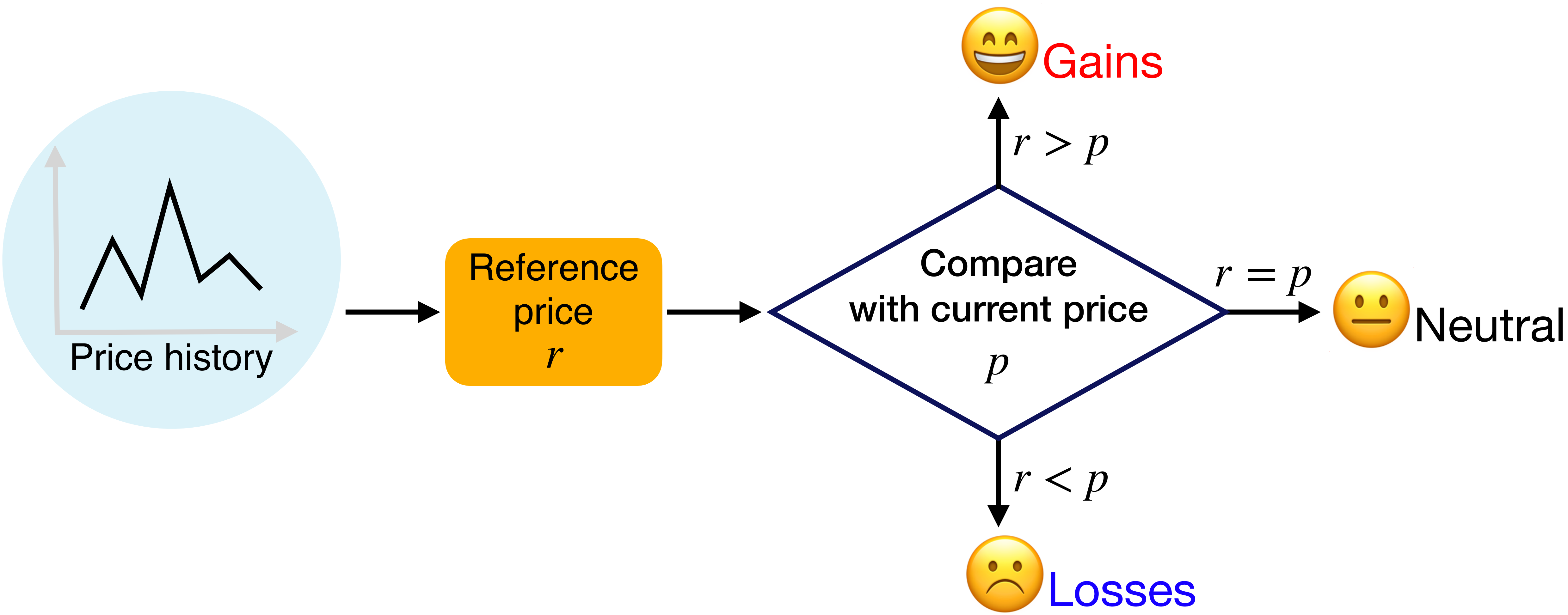
Reference Effects

- **Retailing Markets**
  - Frequent consumer interactions
  - Repeatedly purchased products
- **Consumers**
  - Purchases depend on current prices and **reference prices**



Real Examples

Fig 1. Snapshot of JD.com webpage for certain fruits

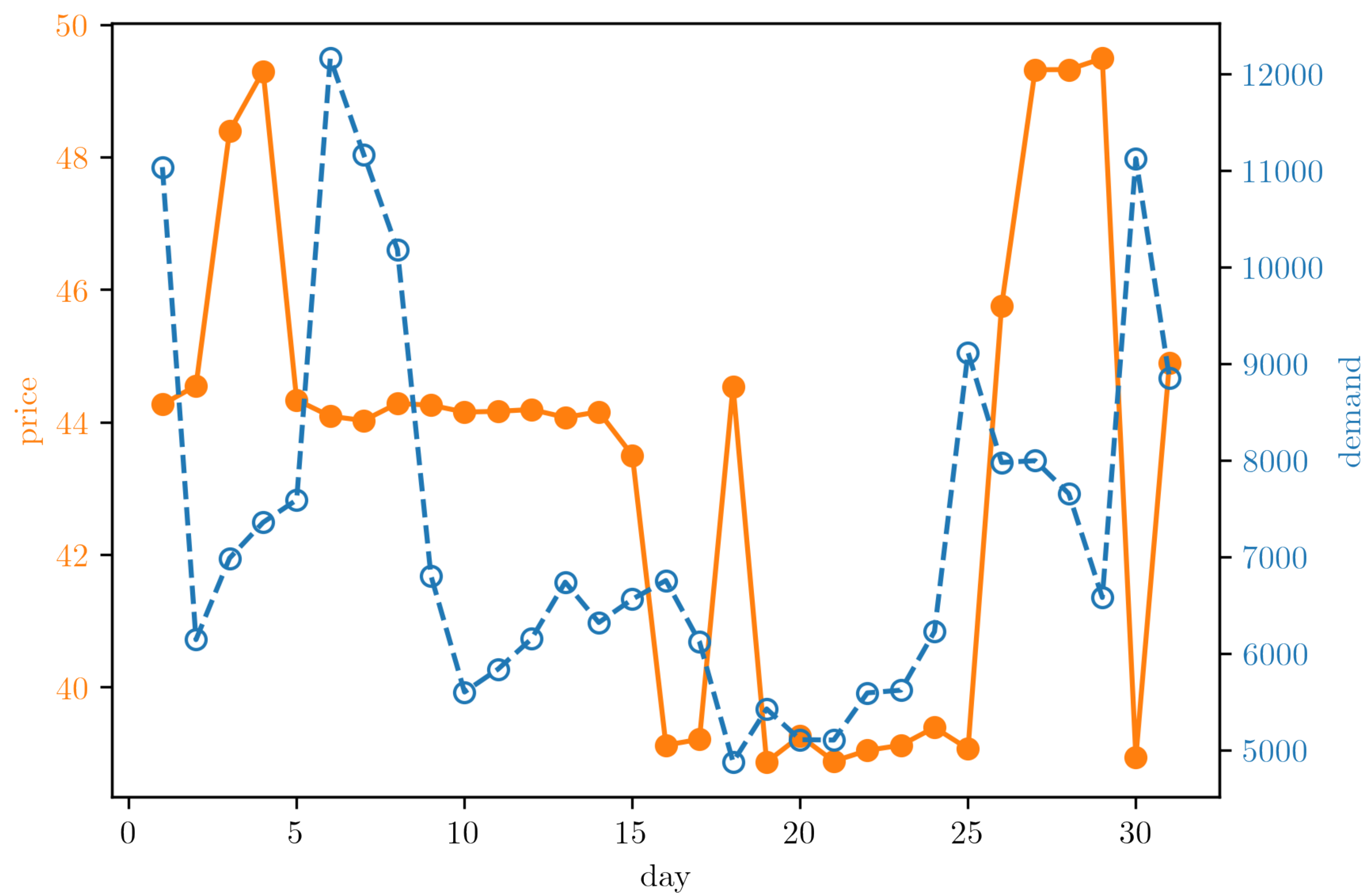


Fig 2. Price and demand (by day) of a product in MSOM-JD dataset

Reference Effects Impact Optimal Pricing Policies

- Optimal pricing policy might not be a fixed price!

- **For a homogeneous market** (Kopalle et al., 1996; Popescu and Wu, 2007)
  - Cyclic pricing policy is optimal if only gain-seeking consumers
  - Constant pricing policy is optimal if only loss-averse consumers

- In practice, consumers are likely heterogeneous

- Reference effects describe consumer behaviors and are therefore naturally modeled in the individual level

JD.COM 京东

How should online retailers like JD.com optimize their pricing policies using historical transaction data?

- How to predict the demand more accurately under heterogeneous reference effects?
- How to translate knowledge of consumer heterogeneity into better pricing policies?

Contents

Contents

Demand under Heterogeneous Reference Effects

- Denote responsiveness parameters (a, b, c_+, c_-) by $\boldsymbol{\theta}$
- Consumer utility function

price

reference price

$$u_t(\boldsymbol{\theta}) = a - bp_t + c_+(r_t - p_t)_+ + c_-(r_t - p_t)_-$$

- The purchase probability is

$$\mathrm{P}(r_t, p_t \mid \boldsymbol{\theta}) := \frac{\exp\{u_t(\boldsymbol{\theta})\}}{1 + \exp\{u_t(\boldsymbol{\theta})\}}$$

Accounting for heterogeneity in $\boldsymbol{\theta}$

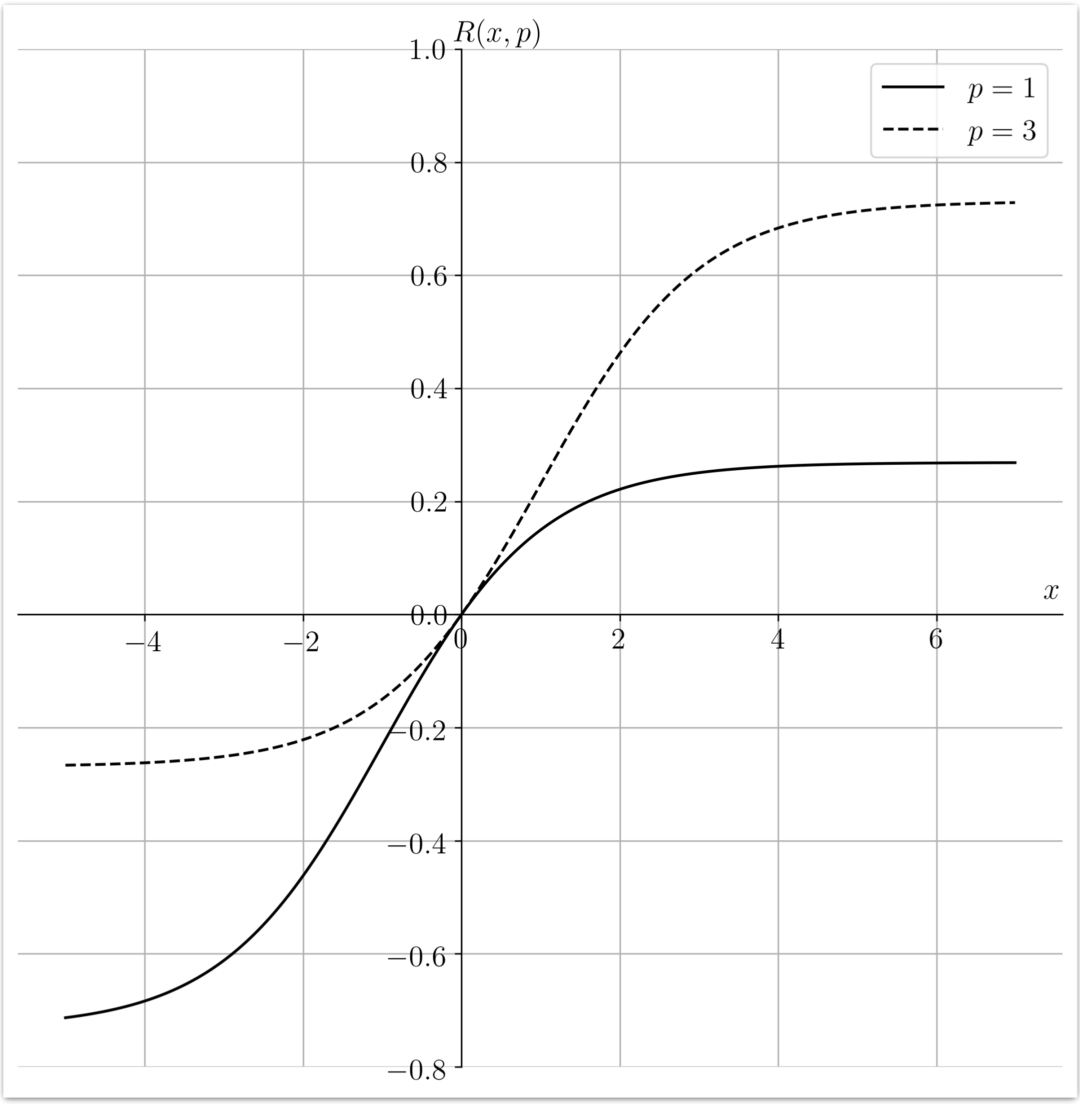
$$\boldsymbol{\theta} \sim G^*$$

$$\mathrm{P}^{G^*}(r_t, p_t) := \int_{\boldsymbol{\theta} \in \Theta} \mathrm{P}(r_t, p_t \mid \boldsymbol{\theta}) \mathrm{d}G^*(\boldsymbol{\theta})$$

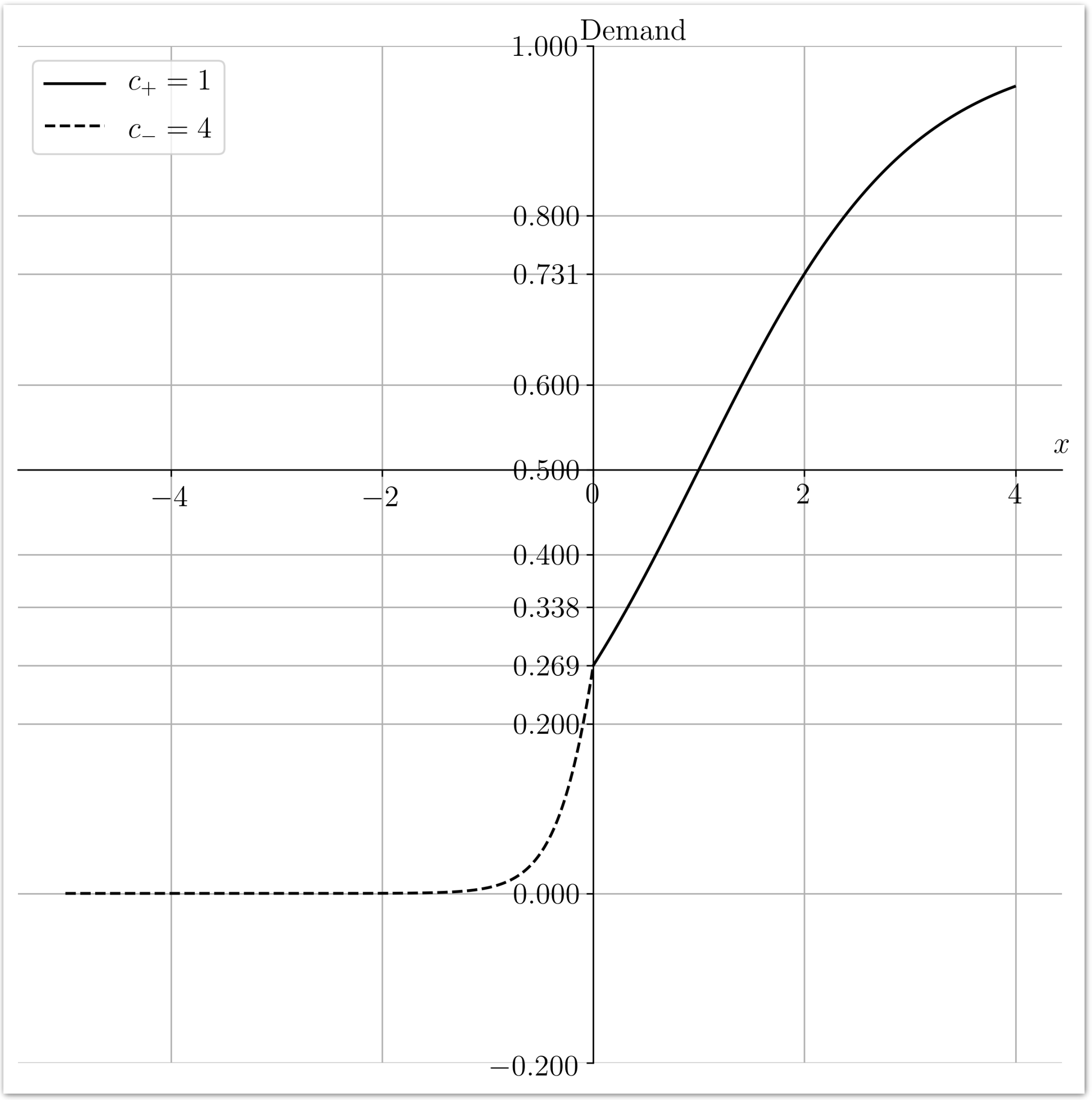
- No parametric assumption is imposed on G^*

Illustration of Logit Demand

(a) Reference effects $R(x, p), x = r - p$



(b) Example of "regionally" loss-averse/gain-seeking



Properties

(i) Diminishing sensitivity; (ii) Decreasing curvature

Revenue Maximization Goal

- **Single period revenue**

$$\Pi(r_t, p_t) := p_t \cdot \mathrm{P}^{G*}(r_t, p_t)$$

- **Long-term discounted revenue**
  - Given initial reference price r_0 and price range $\mathscr{P}$,

$$V(r_0) = \underset{p_t \in \mathscr{P}}{\text{maximize}} \sum_{t=1}^{\infty} \beta^t \, \Pi(r_t, p_t)$$

$$\text{subject to } r_t = (1-\alpha)p_{t-1} + \alpha r_{t-1}$$

    - $\alpha \in [0,1]$ — memory parameter (Greenleaf,1995)
    - $\beta \in [0,1]$ — discount factor

Contents

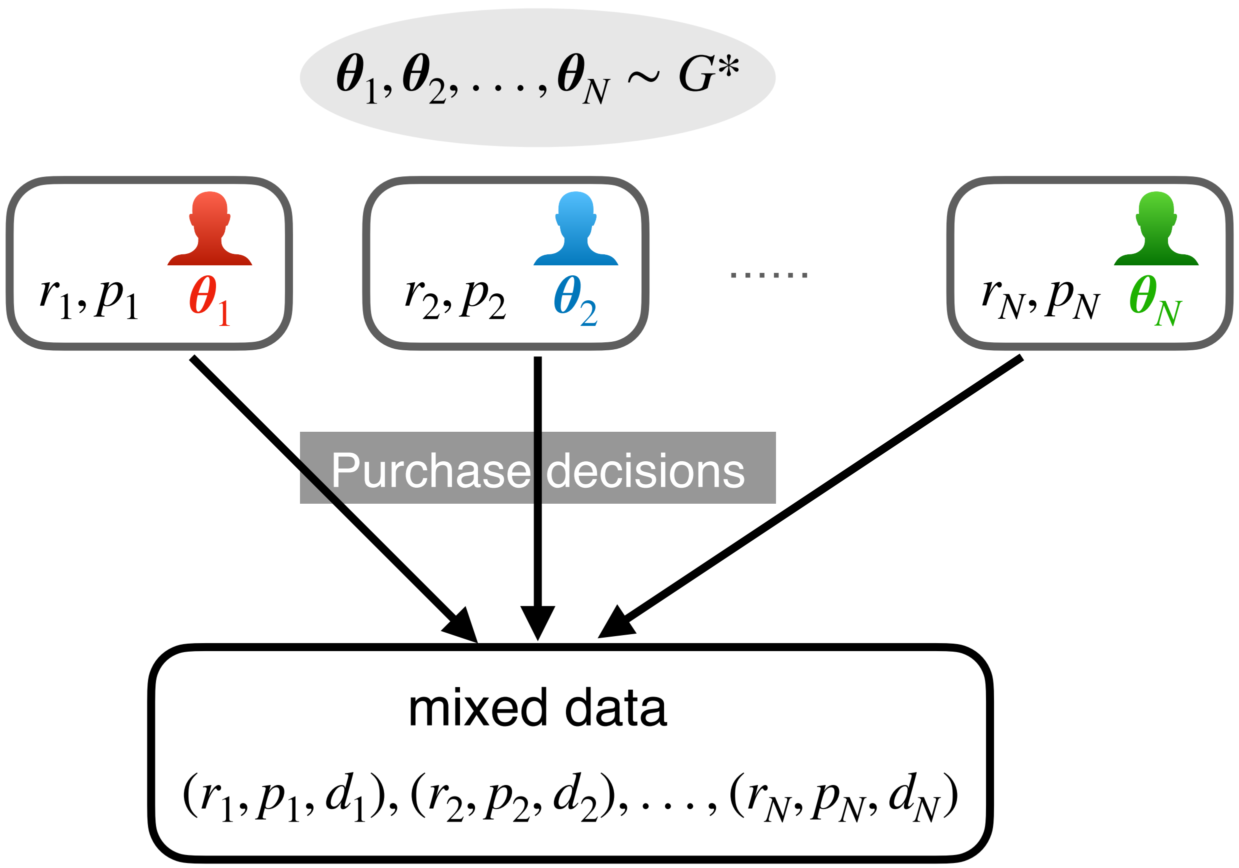
Contents

Demand Estimation

- Consumer purchase probability is

$$\mathrm{P}(d_n = 1) = \int_{\boldsymbol{\theta}\in\Theta} \mathrm{P}(r_n, p_n \mid \boldsymbol{\theta})\mathrm{d}G^*(\boldsymbol{\theta})$$

- Recover G^* from transaction data



Our approach

- Nonparametric maximum likelihood estimation (NPMLE) (Kiefer and Wolfowitz 1956)
- Do not need any parametric assumptions on G^*

Computation

- Use the framework of conditional gradient method, built on prior work (Jagabathula et al. 2020)
- Propose alternating minimization algorithm for solving subproblem step

Contents

Contents

Sub-optimality of Constant Pricing Policy

Proposition The constant pricing policy is **not** optimal if $c_+ \leq c_-$ and utility parameters a, b satisfy that $a < \log\left\{(1-\beta)/\beta\right\}$, where c_- is sufficiently large.

- **Implications**
  - Constant pricing policy may not be optimal even if all consumers are loss-averse/neutral!
    - Condition $a < \log\left\{(1-\beta)/\beta\right\}$ is true for products with relatively low purchase probabilities (Note: purchase probabilities typically < 5% for products in the MSOM-JD dataset)
  - Compared to similar results
    - Do not need the simplified assumption $\alpha = 0$ as in Hu and Nasiry (2017)
    - Individual level demand model with arbitrary number of consumer segments, in contrast to aggregate level linear demand model with two segments in Chen and Nasiry (2020)

Computation of Optimal Pricing Policy

- **Algorithm**

Modified policy iteration algorithm

Initialize $V^0 = 0, k = 1$
Repeat
    Policy improvement
        Generate new pricing policy π_k based on value function V^{k-1}
    Approximate policy evaluation
        Calculate the value function V^k according to policy π_k
    $k \leftarrow k + 1$
Until convergence

- Per iteration complexity reduced to $1/\epsilon^2$ from $1/\epsilon^3$

Discretization Guarantee For any $r \in \mathscr{P}_\epsilon$,

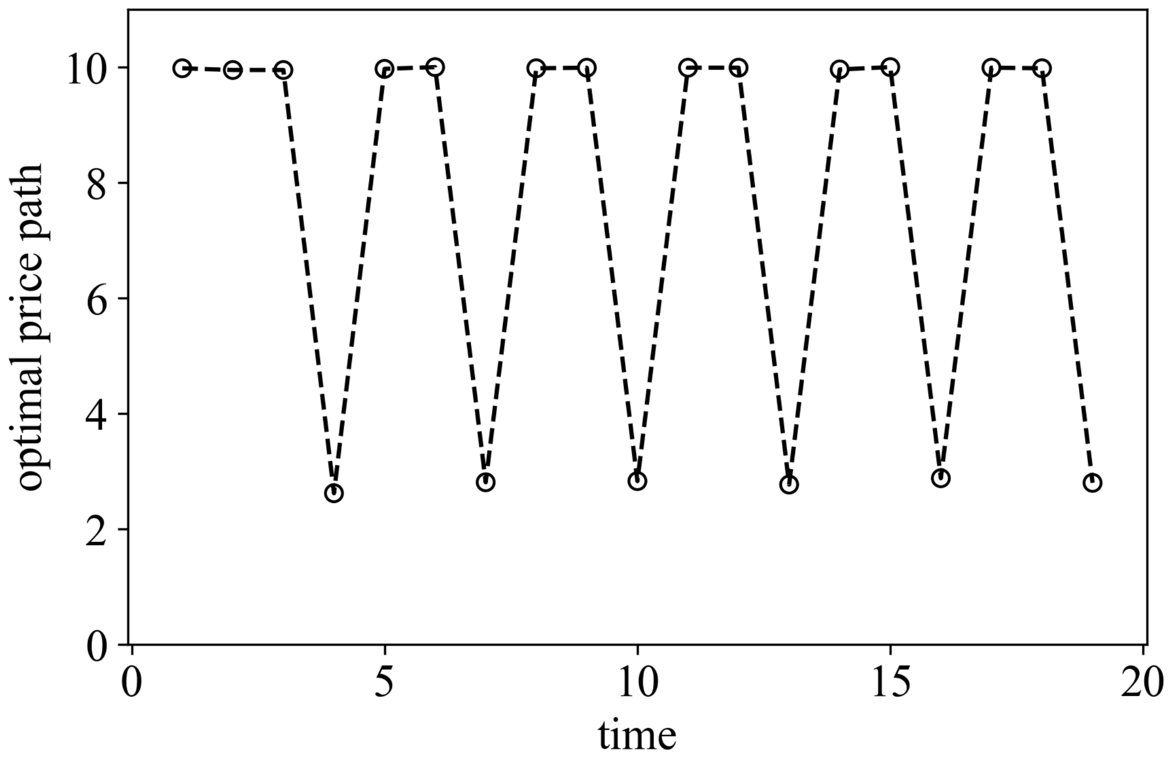
$$0 \leq V(r) - V_\epsilon(r) \leq \left[\frac{C_1}{4(1-\alpha)(1-\beta)} + \frac{\beta C_2}{4(1-\alpha)(1-\alpha\beta)} \right] \epsilon.$$

- We also proved a performance guarantee for the myopic pricing policy

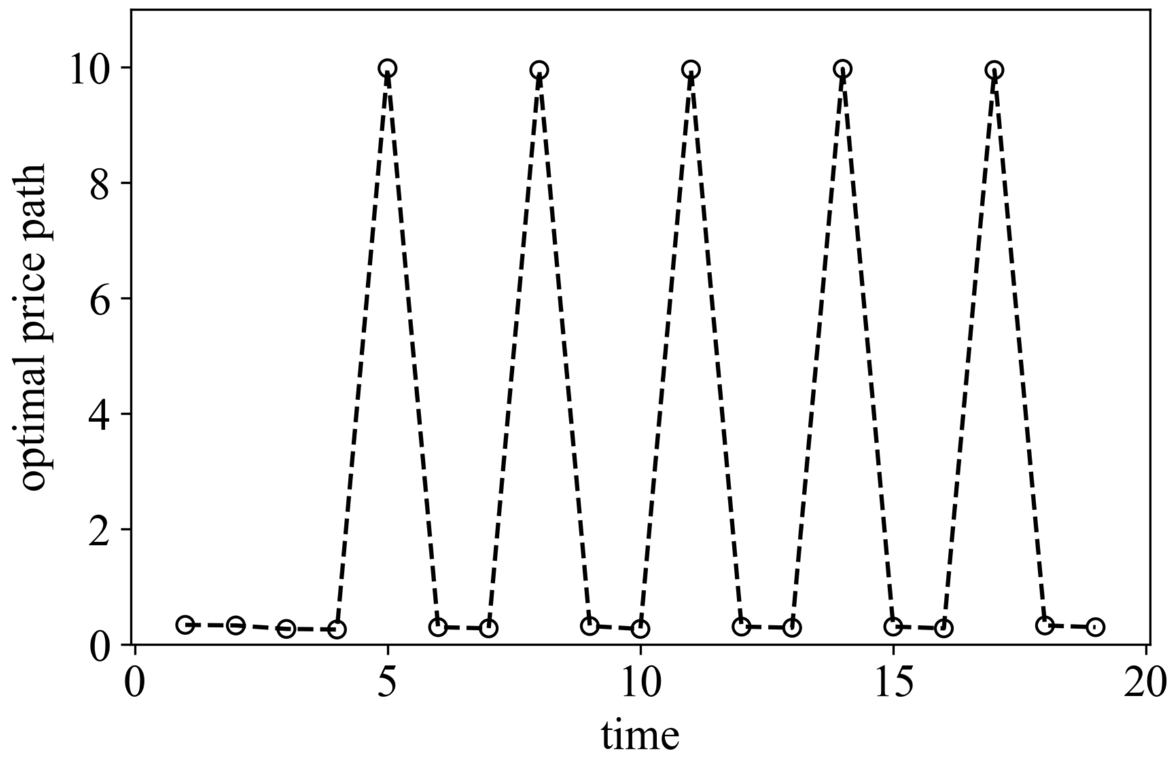
Numerical Example 1

Local loss-aversion does not preclude price variations

(a) $r_0 = 4.85, (a, b, c_+, c_-) = (-5,0,1,5)$



(b) $r_0 = 6.64, (a, b, c_+, c_-) = (2,10,0.5,1)$



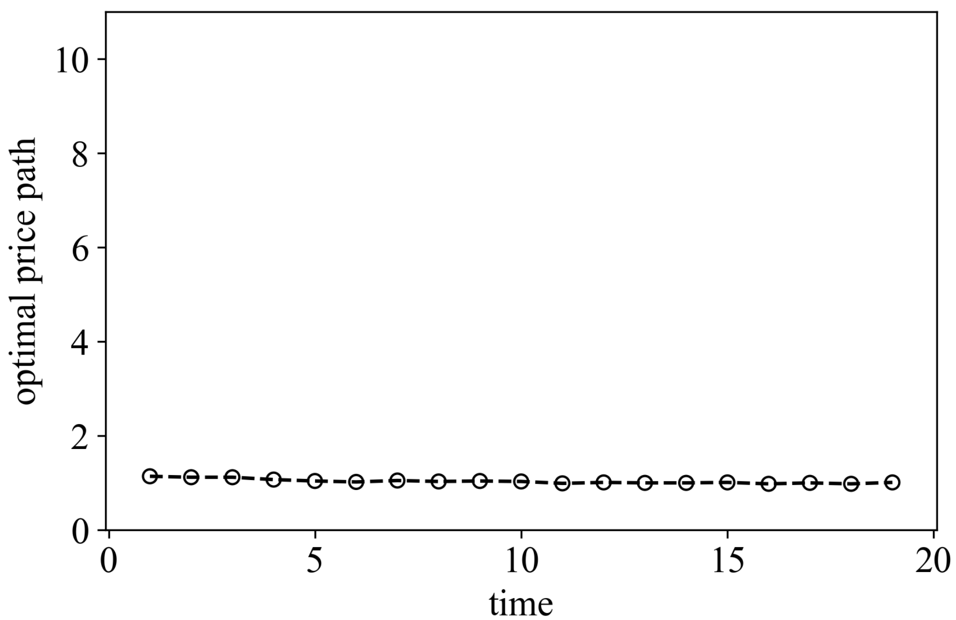
Notes.
r_0 initial reference price
utility functions $u_t(\boldsymbol{\theta}) = a - bp_t + c_+(r_t - p_t)_+ + c_-(r_t - p_t)_-$

Numerical Example 2

Constant optimal pricing + constant optimal pricing $\neq$ constant optimal pricing

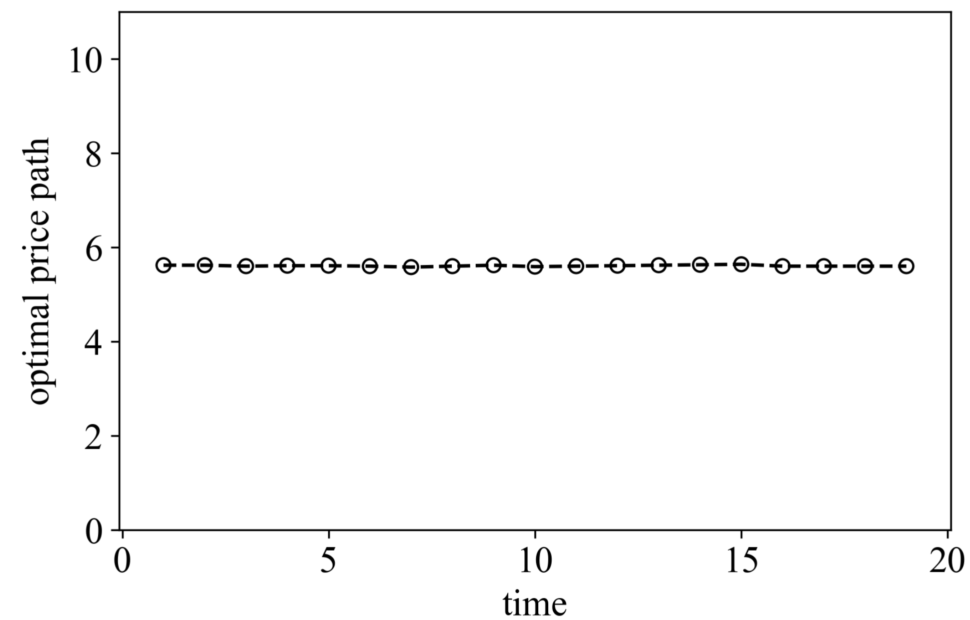
(a) Homogeneous, consumer A only

$(a_A, b_A, c_{A+}, c_{A-}) = (2,2,0.2,0.2)$

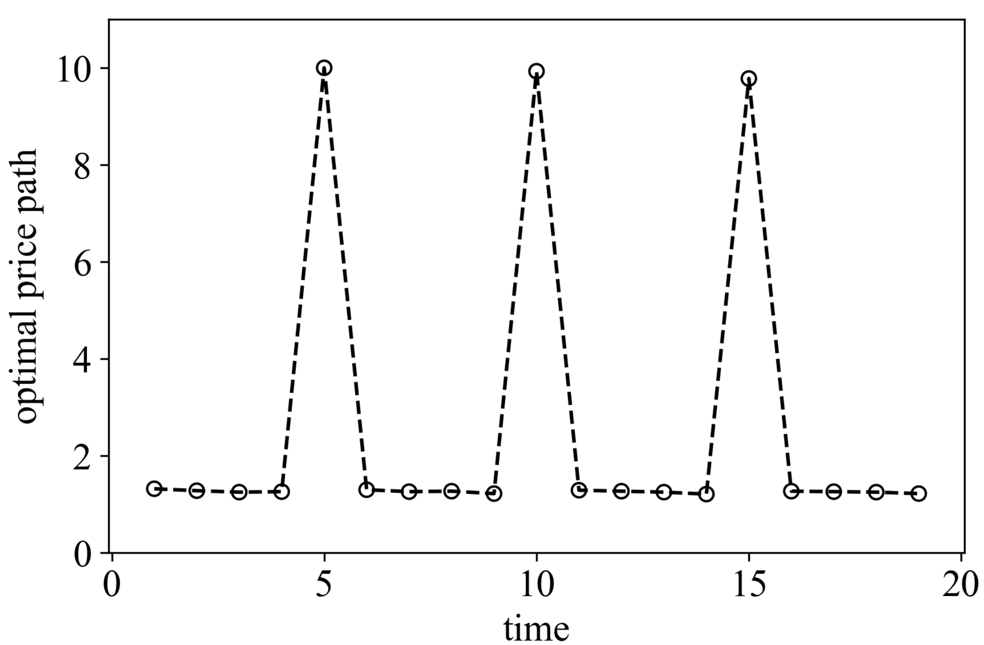


(b) Homogeneous, consumer B only

$(a_B, b_B, c_{B+}, c_{B-}) = (-1,0.2,0,0)$



(c) Heterogeneous, 50% consumer A, 50% consumer B



Notes.

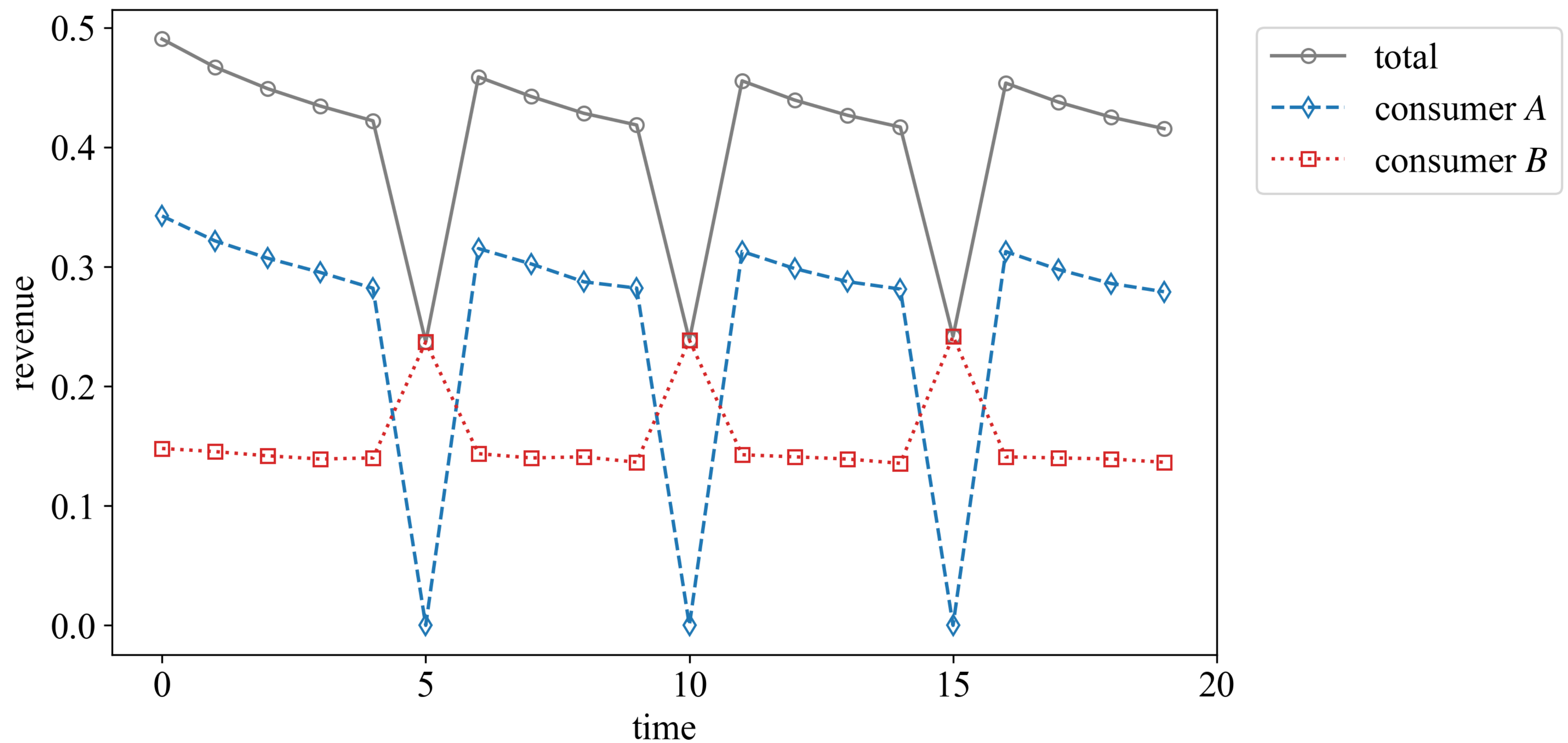
$r_0 = 5.0$ initial reference price

utility functions $u_t(\boldsymbol{\theta}) = a - bp_t + c_+(r_t - p_t)_+ + c_-(r_t - p_t)_-$

Intuition on Numerical Example 2

Constant optimal pricing + constant optimal pricing $\neq$ constant optimal pricing

Fig. Per period revenue from the whole market and two consumer segments respectively



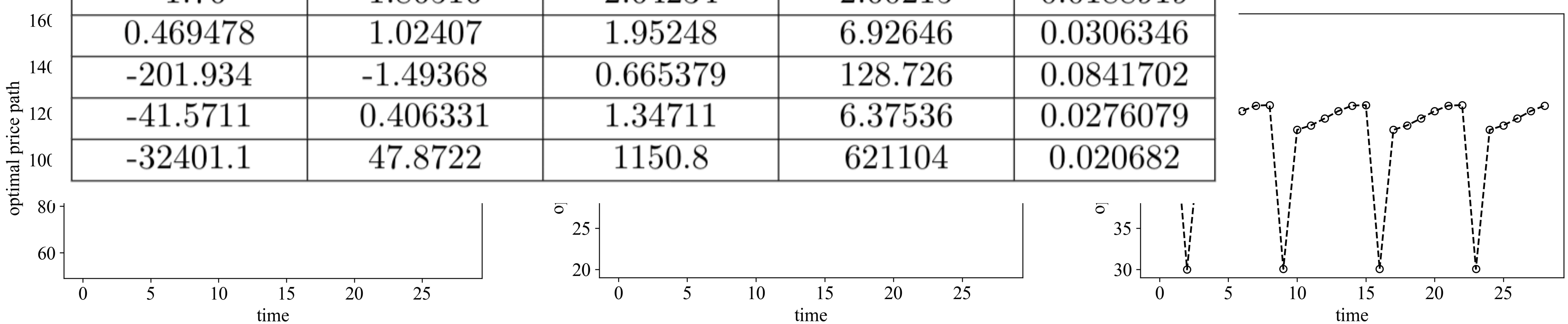
Contents

Contents

Empirical Study on MSOM-JD Data

- Data preprocessing
  - Pricing information and purchase decisions extracted from data tables, `clicks` and `orders`
- Price endogeneity: (i) control function method (ii) extension to multiple products
- Estimation results exhibit consumer heterogeneity

Responsiveness parameters $\boldsymbol{\theta}$				Proportion
a	b	c_+	c_-	ϖ
-14260.2	-27.5264	29.7306	30.5553	0.818013
-1.76	1.86516	2.64234	2.60215	0.0188919
0.469478	1.02407	1.95248	6.92646	0.0306346
-201.934	-1.49368	0.665379	128.726	0.0841702
-41.5711	0.406331	1.34711	6.37536	0.0276079
-32401.1	47.8722	1150.8	621104	0.020682



(c) Product 3

Compare with Aggregate Demand Model

- Piece-wise Linear Demand Model

$$D_t = A - Bp_t + C_+(r_t - p_t)_+ + C_-(r_t - p_t)$$

- Most common model in the literature
  (Greenleaf 1995, Chen et al. 2016, Hu et al. 2016, Chen and Nasiry 2020)

- Comparison of prediction error

Model\Metric	SKU1		SKU2		SKU3	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Linear	0.0482	0.0381	0.0249	0.018	0.0974	0.0746
Nonparametric MLE	0.0429	0.0365	0.0214	0.0165	0.0940	0.0744

- Comparison of long-term revenue in simulations

Policy\Revenue	SKU1		SKU2		SKU3	
	Mean	Median	Mean	Median	Mean	Median
Linear*	384.650	321.771	457.999	470.118	128.979	123.677
Optimum*	2237.29	2135.24	658.171	673.325	394.544	393.766

Conclusion

- We study intertemporal pricing in the presence of reference effects and consumer heterogeneity, which is motivated by practical challenges in retailing businesses
  - An integrated prediction and optimization framework
    - Incorporate reference effects into individual demand
    - Learn consumer heterogeneity via nonparametric estimation
    - Compute optimal pricing policies by modified policy iteration algorithm
    - Theoretically show sub-optimality of constant pricing policy
  - Managerial insights
    - Empirical evidences of heterogeneous consumer behaviors from MSOM-JD dataset
    - Heterogeneous reference effects offer a strong motive for promotions and price fluctuations

Thank You!

Questions or comments?

Supplementary slides

References

- Chen N, Nasiry J (2020) Does loss aversion preclude price variation?Manufacturing & Service OperationsManagement22(2):383–395.
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Myopic Pricing Policy

$$p_{\mathrm{m}}(r_t) = \arg\max_{p \in \mathscr{P}} \Pi(r_t, p)$$

- Likely sub-optimal but computationally efficient

Proposition For any initial reference price r,

$$0 \le V^*(r) - V_{\mathrm{m}}(r) \le \frac{\beta(1-\alpha)}{(1-\alpha\beta)(1-\beta)} \eta(G) p_H$$

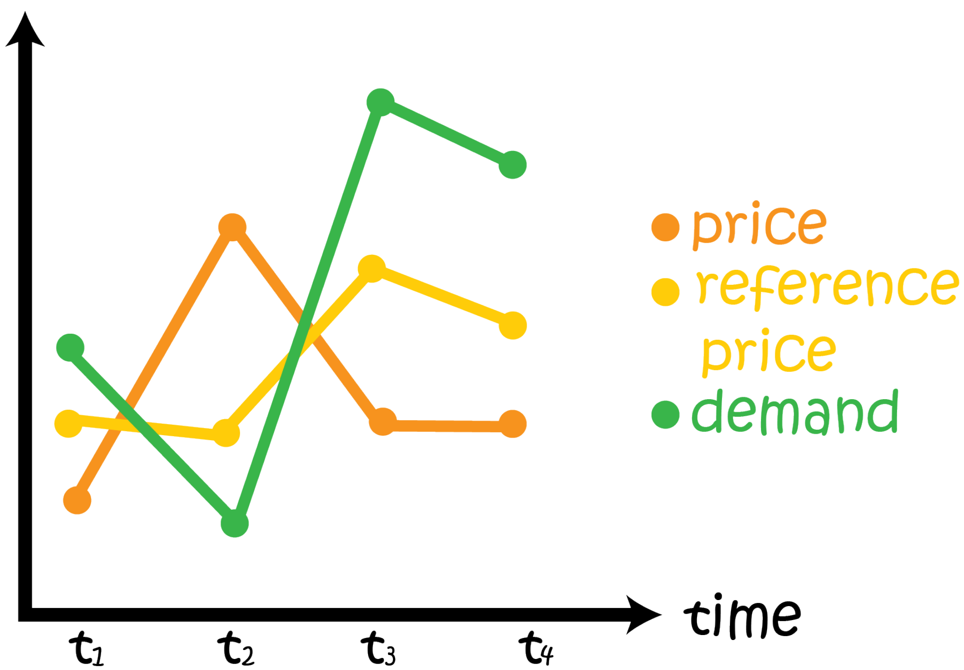
where $\eta(G) = \min\left(1, \sup_{(a,b,c_+,c_-) \in \mathrm{supp}(G)} \frac{\max(c_+, c_-)}{b + c_-}\right)$.

Literature Review

- **On consumer heterogeneity affecting optimal pricing policies**
  - Chen and Nasiry (2019) consider a market consisting of two segments
    - Loss-averse within each segment
    - Heterogeneous in the aggregate level
    - Optimal pricing policy might not be constant
- **On individual and aggregate level reference effects**
  - Hu and Nasiry (2017) consider consumers with heterogeneous valuations for a product
    - Individual level consumer gain-seeking/loss-averse behaviors might not translate into the aggregate level
- **Our work**

  We take a systematic approach to learn heterogeneous reference effects in the individual level, and study how they affect optimal pricing policies.

Reference Effects Impact Optimal Pricing Policies

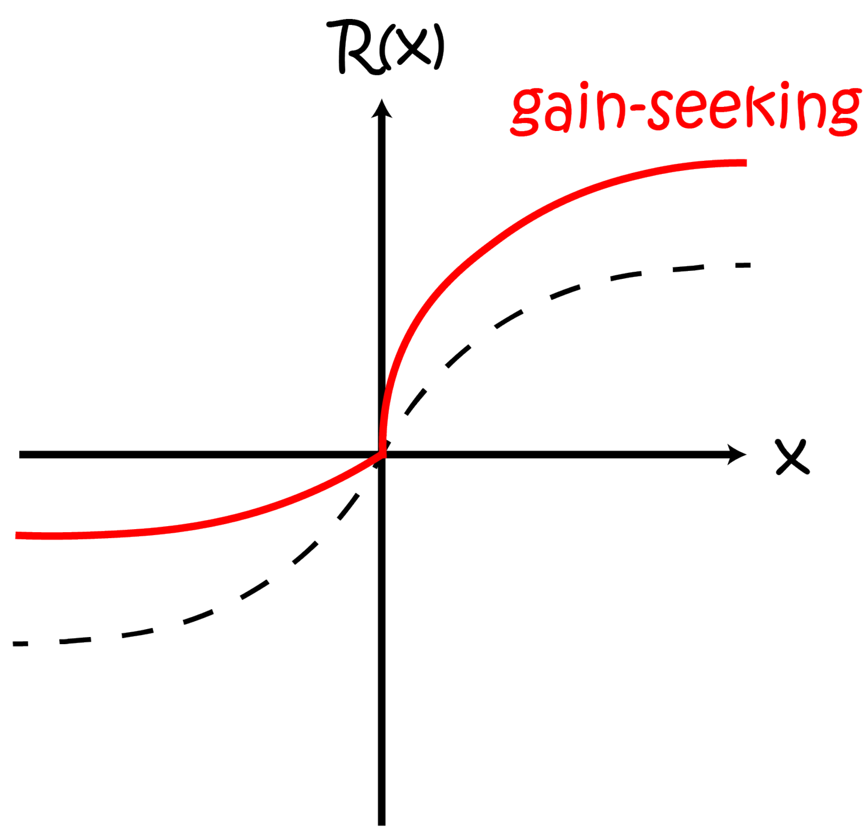


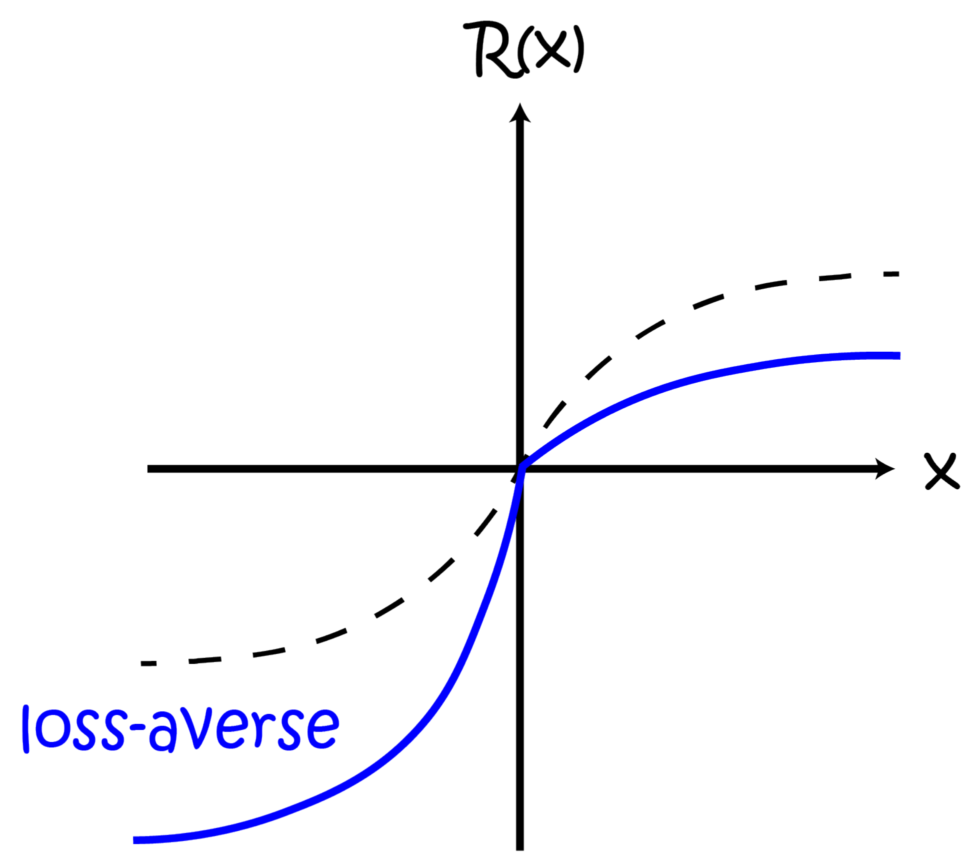
- **Is cyclic pricing policy optimal?**
  - Promotion stimulates demand of gain-seeking consumers
  - Continuing low prices lead to low reference prices and decreases future demand after some time
- **Is constant pricing policy optimal?**
  - Price variation antagonizes loss-averse consumers and diminishes their demand
- **Assuming homogeneous market**
  - Gain-seeking → cyclic optimal
  - Loss-averse → constant optimal

(Kopalle et al., 1996; Popescu and Wu, 2007)

Reference Effects

- Reference discrepancy x: reference price r - current price p
- Reference effect $R(p)$: incurred demand change
- Frequent consumers perceive gains if $x > 0$ and losses if $x < 0$
- Consumers respond differently under reference effects





Related Literature

Popescu and Wu (2007) Dynamic pricing strategies with reference effects. *Operations Research*

Chen et al. (2016) Efficient algorithms for the dynamic pricing problem with reference price effect. *Management Science*

Hu et al. (2016) Dynamic pricing with gain-seeking reference price effects. *Operations Research*

Aggregate Model

Kahneman and Tversky (1979) Prospect theory: An analysis of decision under risk. *Econometrica*

Hu and Nasiry (2017) Are markets with loss-averse consumers more sensitive to losses? *Management Science*

Kopalle PK et al. (2012) The impact of household level heterogeneity in reference price effects on optimal retailer pricing policies. *Journal of Retailing*

Individual Model