

## Cheatsheet 2: Quality

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## 1. Control charts

In building control charts, we usually have access to data from multiple periods. The sample from one period may contains multiple data points, and the number of data points in one sample is called sample size. For each sample, we can compute its mean  $\bar{X}$ , range  $R$ , proportion of defects  $p$ , and count of defects  $c$ . Taking the average of  $\bar{X}$ ,  $R$ ,  $p$ , and  $c$  across multiple periods, we get  $\bar{\bar{X}}$ ,  $\bar{R}$ ,  $\bar{p}$ , and  $\bar{c}$ .

The sample size affects the UCL and LCL computation of mean charts,  $R$ -charts, and  $p$ -charts. The dependence of  $p$ -charts on the sample size is clear from the table below. For mean charts and  $R$ -charts, the control chart constants  $A_2, D_3, D_4$  are decided by the sample size (See the Table of Control Chart Constants in bcourses/Files/05 Quality I/).

	Mean Charts	R-Charts	p-Charts	c-Charts
CL	$\bar{\bar{X}}$	$\bar{R}$	$\bar{p}$	$\bar{c}$
UCL	$\bar{\bar{X}} + A_2 \bar{R}$	$D_4 \bar{R}$	$\bar{p} + 3 * \sqrt{\frac{\bar{p}(1-\bar{p})}{\text{sample size}}}$	$\bar{c} + 3\sqrt{\bar{c}}$
LCL	$\bar{\bar{X}} - A_2 \bar{R}$	$D_3 \bar{R}$	$\bar{p} - 3 * \sqrt{\frac{\bar{p}(1-\bar{p})}{\text{sample size}}}$	$\bar{c} - 3\sqrt{\bar{c}}$

## 2. Capability analysis

- (a) For centered process ( $\bar{X} = \frac{USL+LSL}{2}$ ), the process capability measure is  $C_p$ , calculated as

$$C_p = \frac{USL - LSL}{6\hat{\sigma}}.$$

- (b) For off-centered process ( $\bar{X} \neq \frac{USL+LSL}{2}$ ), the process capability measure is  $C_{pk}$ , calculated as

$$C_{pk} = \min \left\{ \frac{USL - \bar{X}}{3\hat{\sigma}}, \frac{\bar{X} - LSL}{3\hat{\sigma}} \right\}$$

- (c)  $\hat{\sigma}$  is estimated standard deviation. If  $\hat{\sigma}$  is not given, we pool all data points together to compute the standard deviation using formula

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^N (X_i - \bar{X})^2}{N - 1}},$$

where there is a slight abuse of notation as the mean is named  $\bar{\bar{X}}$  in the mean-charts.

## References

- [TC2006] C. TERWIESCH and G. CACHON, Matching supply with demand: An introduction to operations management (Chapter 7), McGraw-Hill 2006
- [SG2018] R. SCHROEDER and S. M. GOLDSTEIN, Operations Management in the Supply Chain (Chapter 8 and 9), McGraw-Hill 2018