Designing Surprise Bags for Surplus Foods

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Food Waste Problem

25%

OF FRESHWATER USED ANNUALLY

goes into food that ultimately goes to waste

10%

OF GLOBAL GREENHOUSE GAS EMISSIONS

are caused by food waste

11.5%

OF ALL LAND IN THE WORLD

is used annually to produce food that ultimately goes to waste.

$1.1

TRILLION

of money is lost through food waste every year

2.4

BILLION PEOPLE

do not have access to adequate, nutritious food

783

MILLION PEOPLE

are affected by hunger every day

Source: WWF 2024, FAO 2023.

Food Waste in Groceries and Dinning

Starbucks Barista Shows Shocking Amount of Food Wasted After Each Shift

BY **KATE UNDERWOOD**

PUBLISHED JUNE 14 2023, 9:54 A.M. ET

SOURCE: AIDANSTOCKIN/TIKTOK

Business

Here's how much food Walmart throws away over 12 days

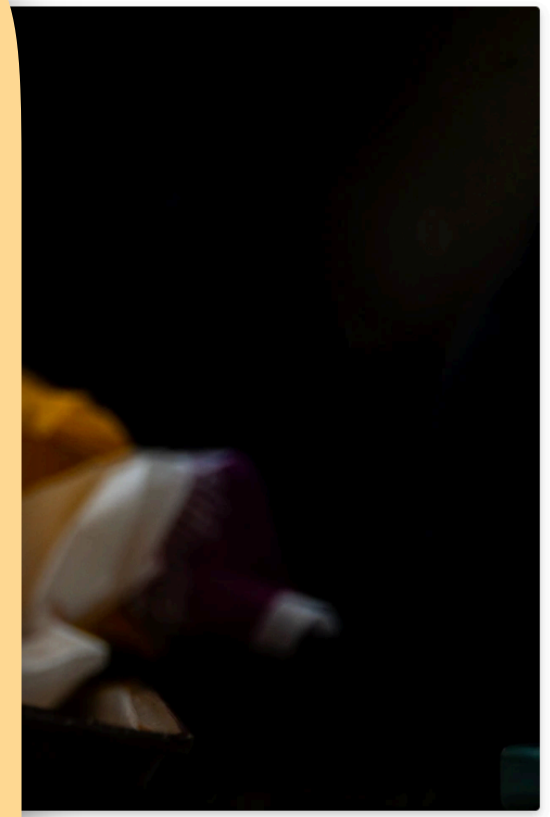
Marketplace finds discarded food well before the best-before to consume

Melissa Mancini, Nelisha Vellani · CBC News ·

EXCLUSIVE

McDonald's, El Pollo Loco Face Investor Bids to Cut Food Waste

December 5, 2024, 11:16 AM EST



ce food waste.

In 2023,

74 million tonnes

of surplus food are generated in U.S., accounting for 31% of U.S. food supply, but only

11%

was recovered through donations or animal feed

(Source: ReFED 2025)

Emerging Platforms

Emerging platforms are combating food waste by

connecting stores' *surplus foods* with *individual customers*

Too Good To Go

North America, Europe & Australia

Yindii

Hong Kong, Singapore & Thailand

X-bag

China (Mainland)

Too Good To Go: A Win-Win-Win Dynamic

- Founded in Denmark in 2015
- World's largest B2C marketplace for surplus food
- Partnering with both independent stores and major chains (e.g., Starbucks, 7-Eleven, Circle K, Whole Foods)

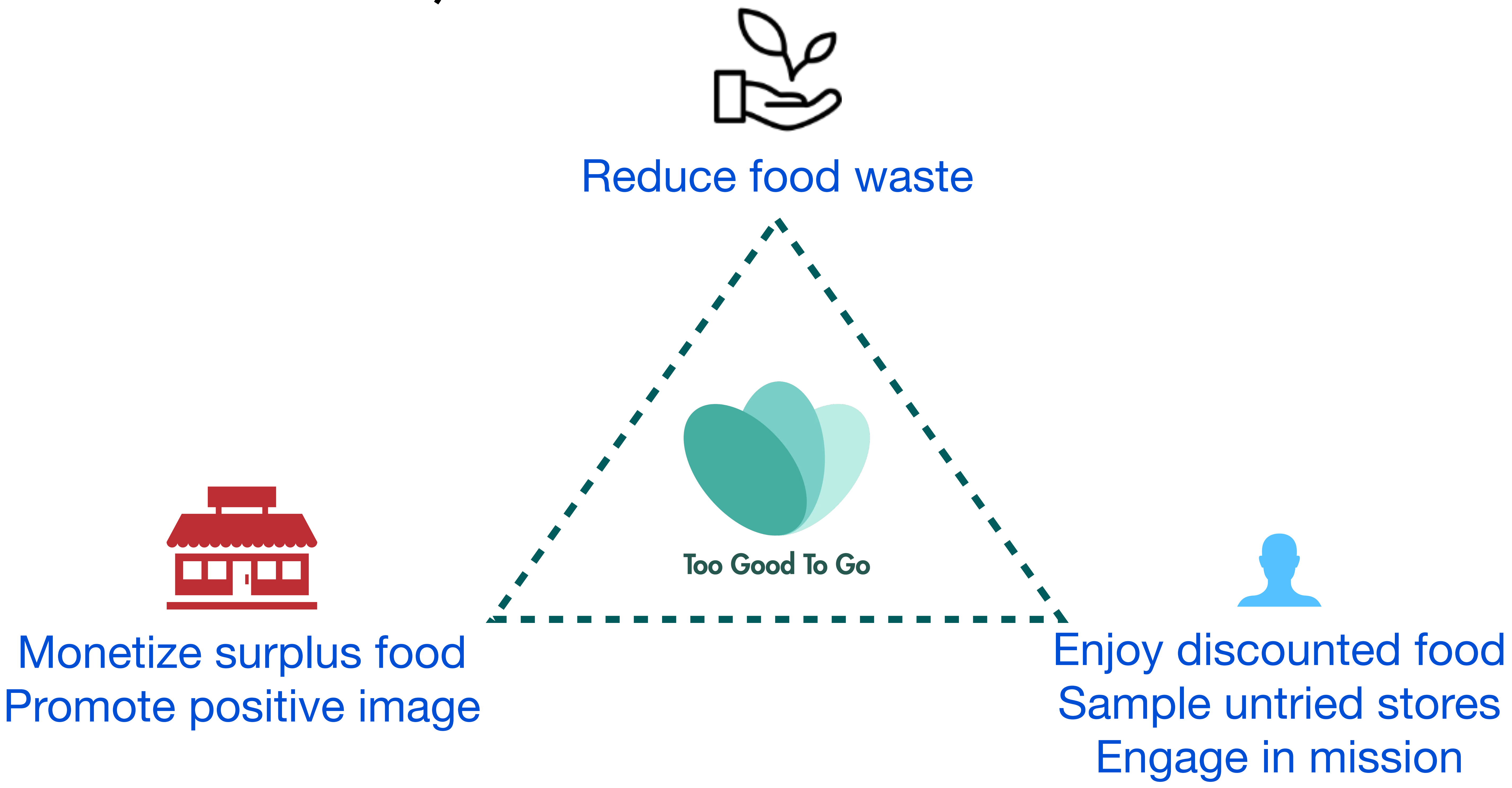
19
Countries

+100M
Registered Users

+175k
Partner Stores

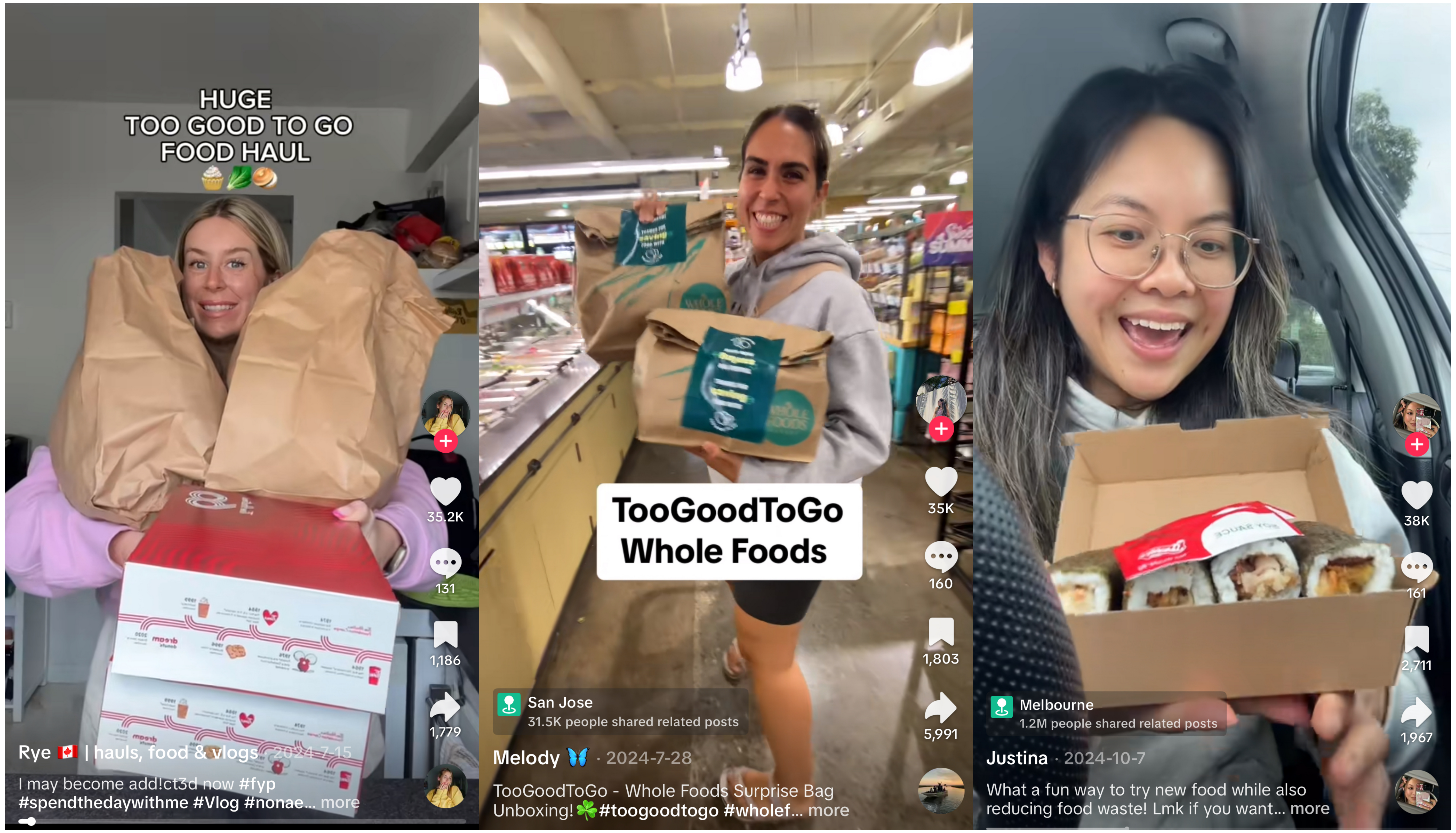
+350M
Meals Saved

+2B
Lbs of Co2e Avoided



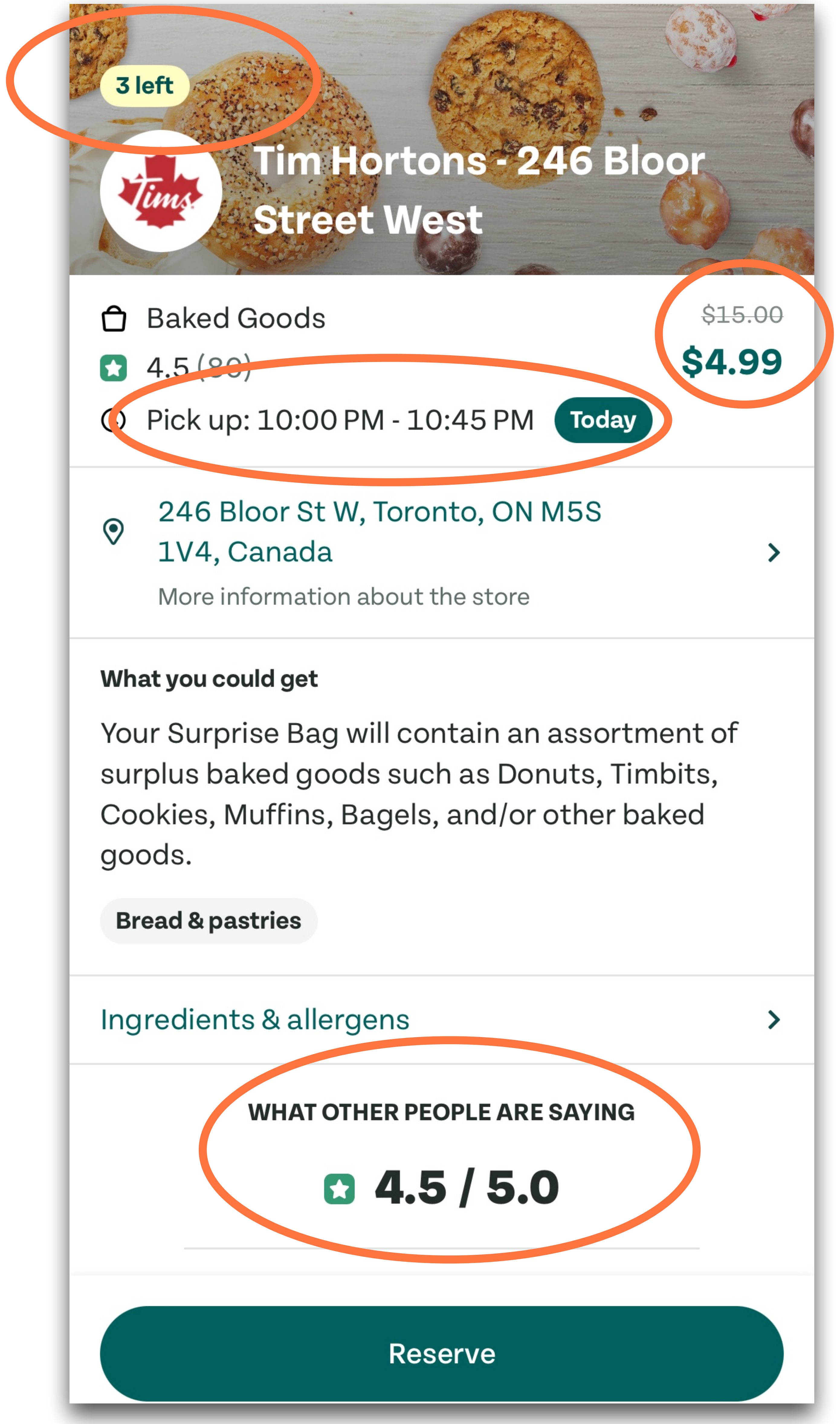
Surprise Bags

Too Good To Go: *"food waste varies on a day-to-day basis, so this is our way of making sure retailers have the flexibility to sell genuine surplus – whatever that ends up being."*



Source: TikTok

Workflow of TGTG Platform



Participating businesses list surprise bags of surplus foods

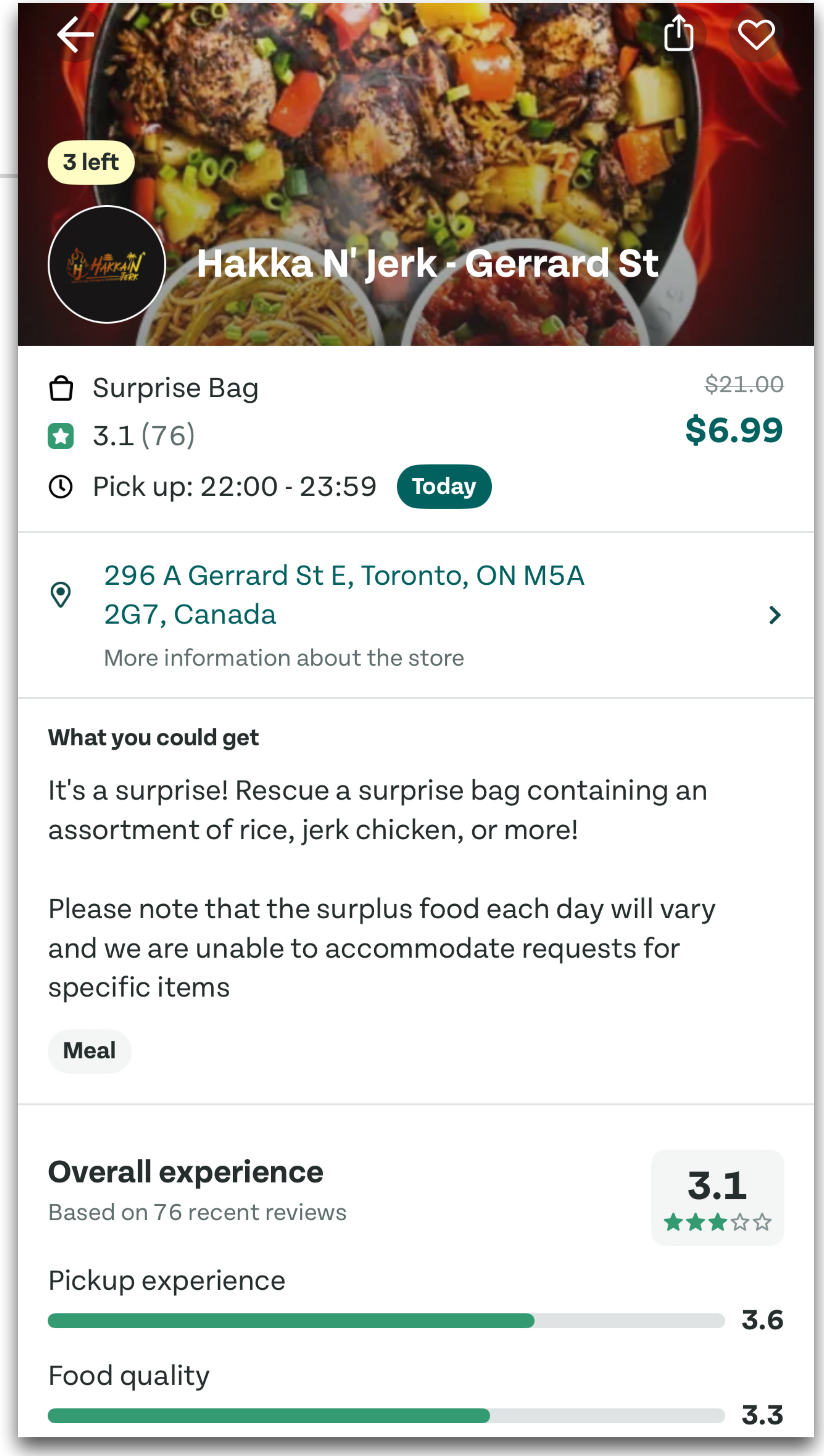
Customers reserve bags at highly discounted price

Customers self pick up during designated time slots

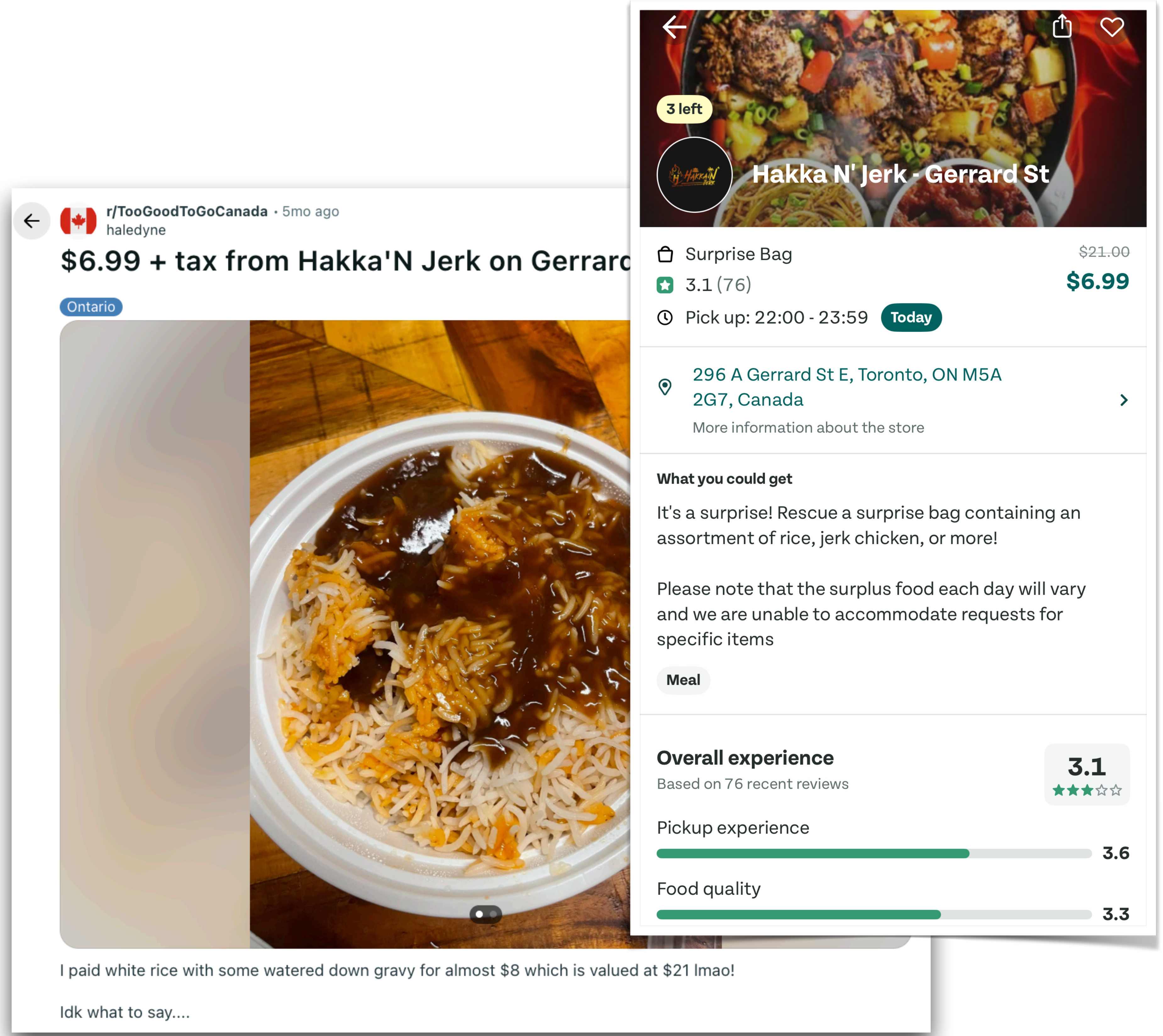
Customers leave a rating and aggregate rating is revealed

“Would You Buy This Bag?”

- You’re browsing TGTG in your neighborhood. You found this surprise bag
- You’ve never bought from this store before
- Would you buy it?



Reveal on Social Media



"I paid white rice with some watered down gravy for almost $8 which is valued at $21 lmao!
Idk what to say...." ☹️

I'm done ~200 orders with them lol. Treat them as a walking/cycling exercise each time. I love the fact that their star ratings are honest and accurate (can't take for granted these days), as anything above 3.7 stars should be solid. Bodegas/convenience stores tend to be inconsistent.

17 Award Share ···

"...anything above 3.7 stars should be solid..."

At least we can always rate them, I avoid those below 4.3.

I hope your next one is better!

3 Reply Award Share ···

"...avoid those below 4.3..."

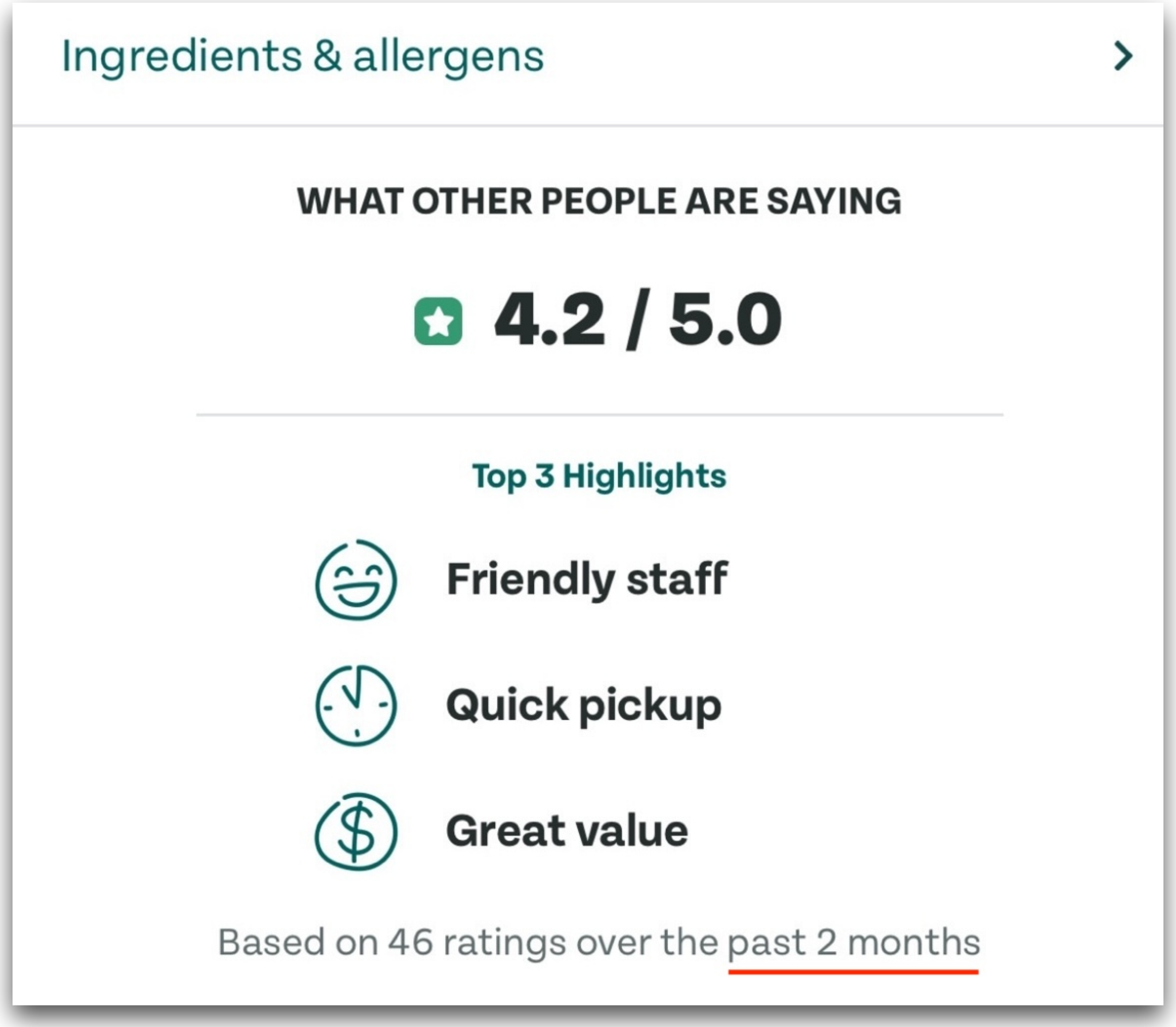
Always check the ratings for the store before ordering. Scores below 4 usually aren't worth it. I've had no problem and have definitely scored deals

1 Reply Award Share ···

"Always check the rating for the store before ordering..."

Why Ratings Matter So Much?

1. **Uncertainty + Surprise**
   - Store is untried
   - Contents unknown before purchase
   - Ratings = only quality signal
2. **Low switching cost**
   - Many stores on platform
   - No loyalty program



Store’s Operational Tensions

- **Commitment vs. Realization mismatch**
  - Must announce #bags 12-24 hours before pickup
  - Actual surplus unknown at commitment time
- **“Surprise” creates uncertainty, so ratings drive demand**
  - “You just need to watch out for store ratings on the app. The ratings are good indicators of what you end up getting” – A Reddit user
- **Challenge: Balance short-term revenue vs. long-term reputation**

Platform's Perspective: Why This Matters?

- **Two-Sided Market Challenge**
- **Platform Viability Depends on:**
  - Store retention: Stores must find TGTG profitable enough to continue
  - Customer satisfaction: Users need good experience to return
  - Network effects: More stores → More users → More stores
- **Platforms need to help stores optimize operations!**

Research Questions

Operational Questions (This paper)

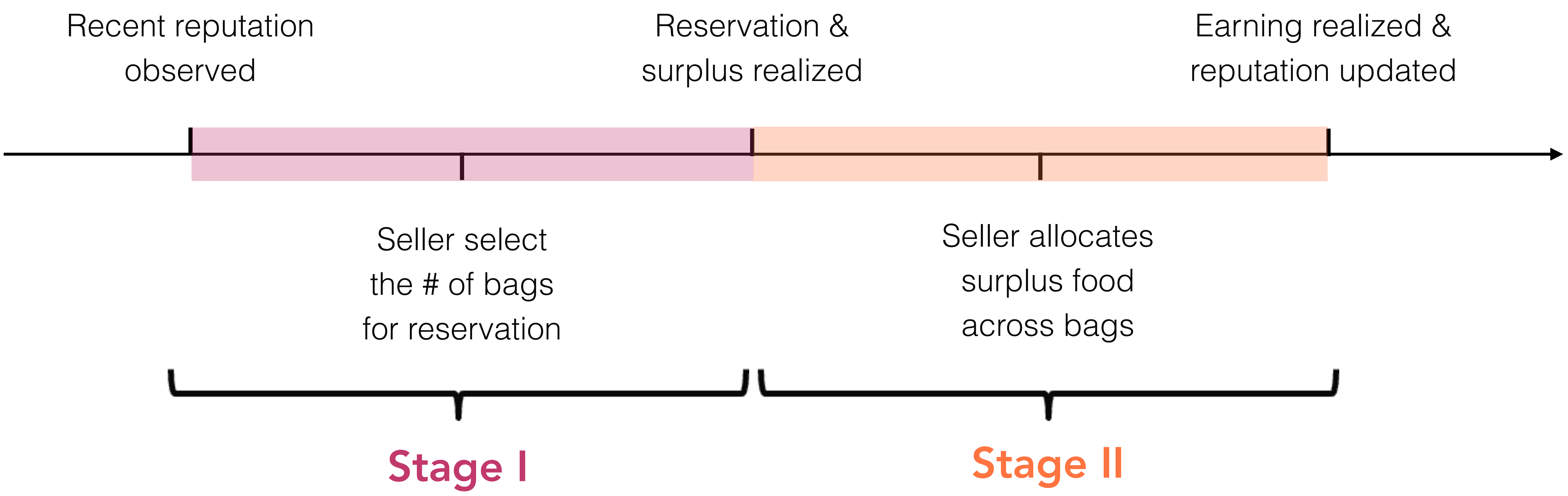
- Quantity: How many surprise bags should be offered?
- Allocation: How much total food should be included?
- Distribution: How should food be allocated across bags?

Strategic Questions (Future)

- More on this later…

Infinite-Horizon DP with Two-Stage Decision

Sequence of events in one period

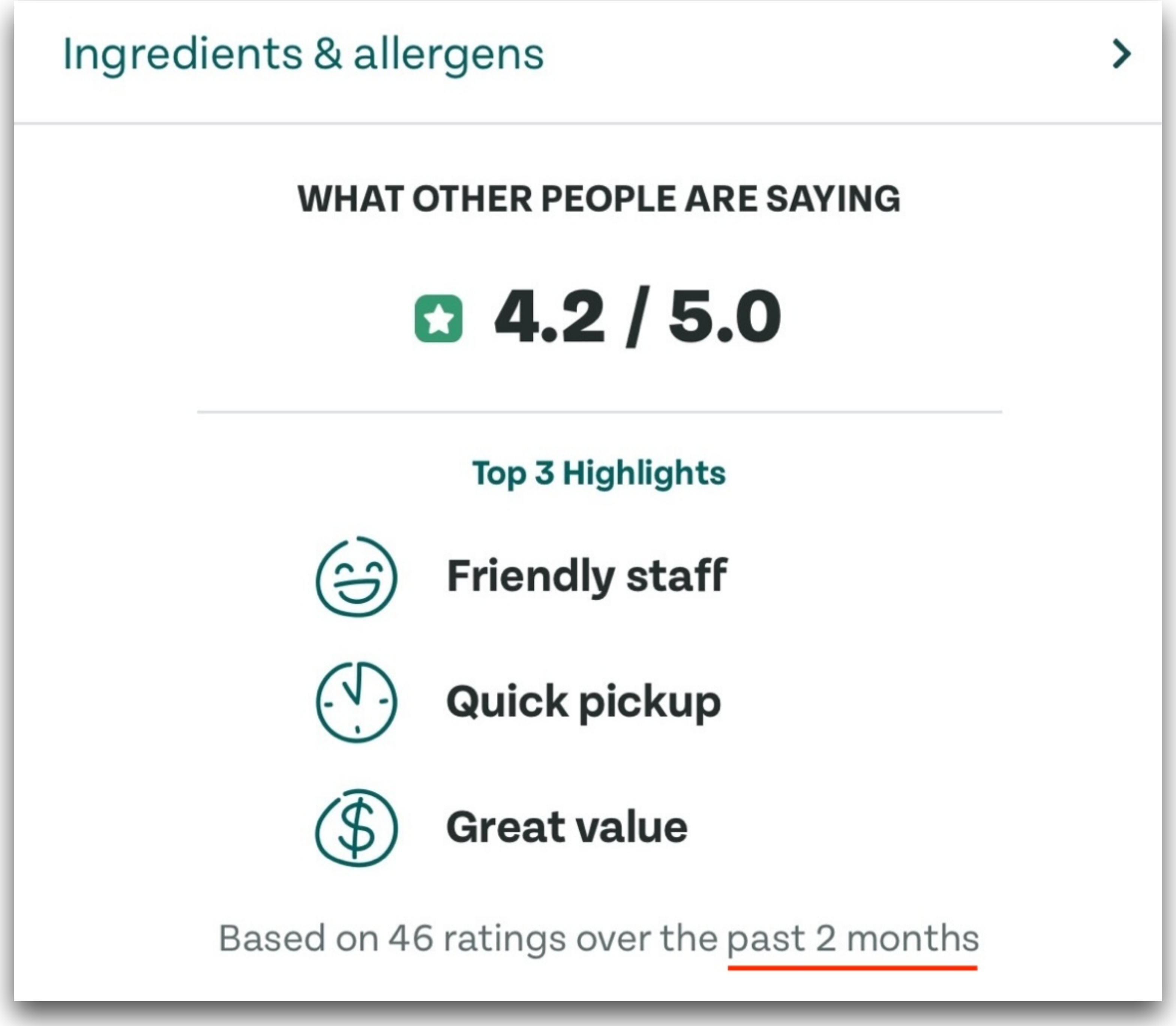


Objective

store’s long-run earnings ↔ platform’s revenue ↔ food waste reduction

Model: Demand and Leftover

- **Reputation** r_t
  - State variable
- **Demand** $D(r_t)$
  - Demand is concave and increasing in reputation
- **Leftover value** Q_t
  - Random variable with c.d.f. $F(\cdot)$
  - Realized in Stage II

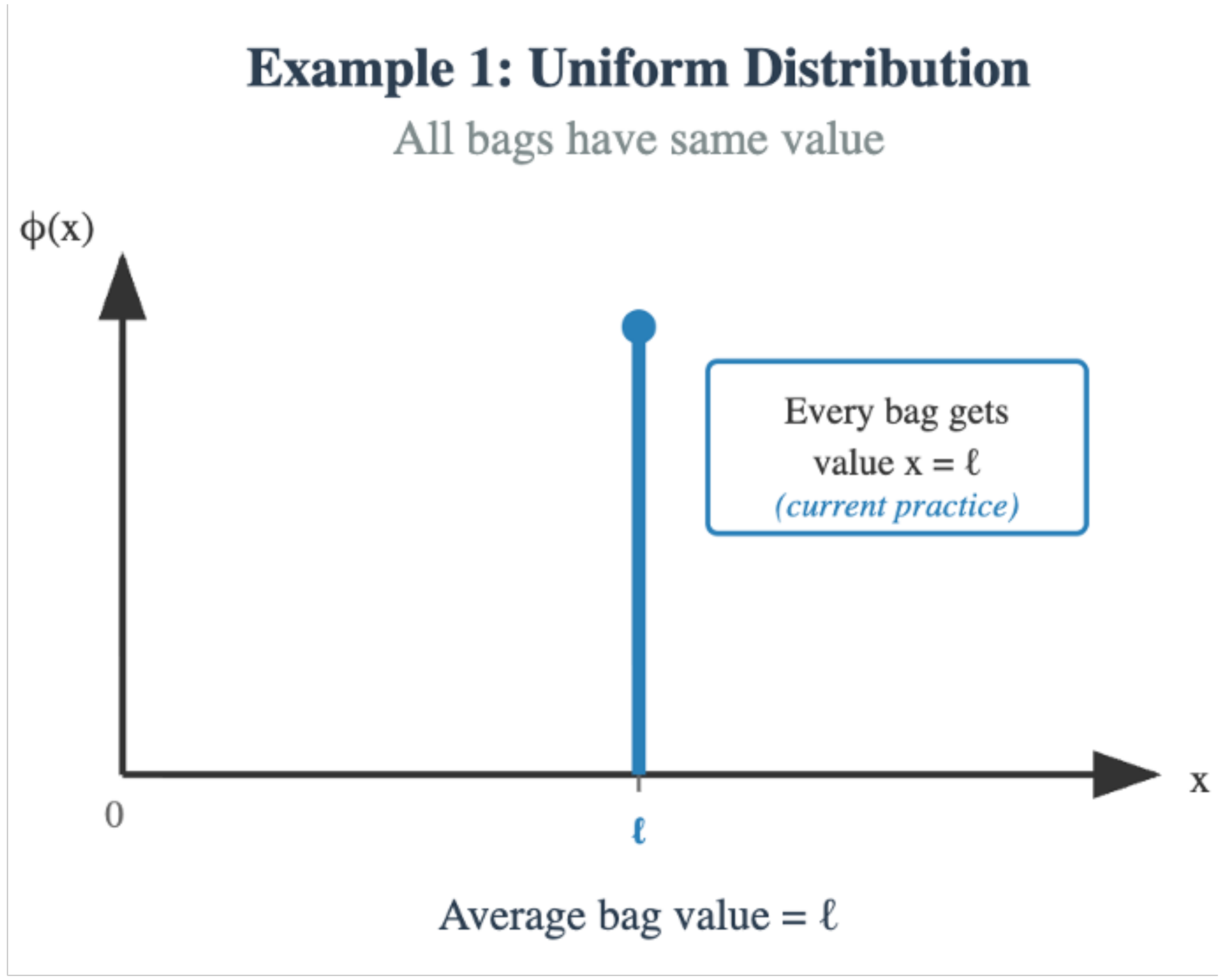


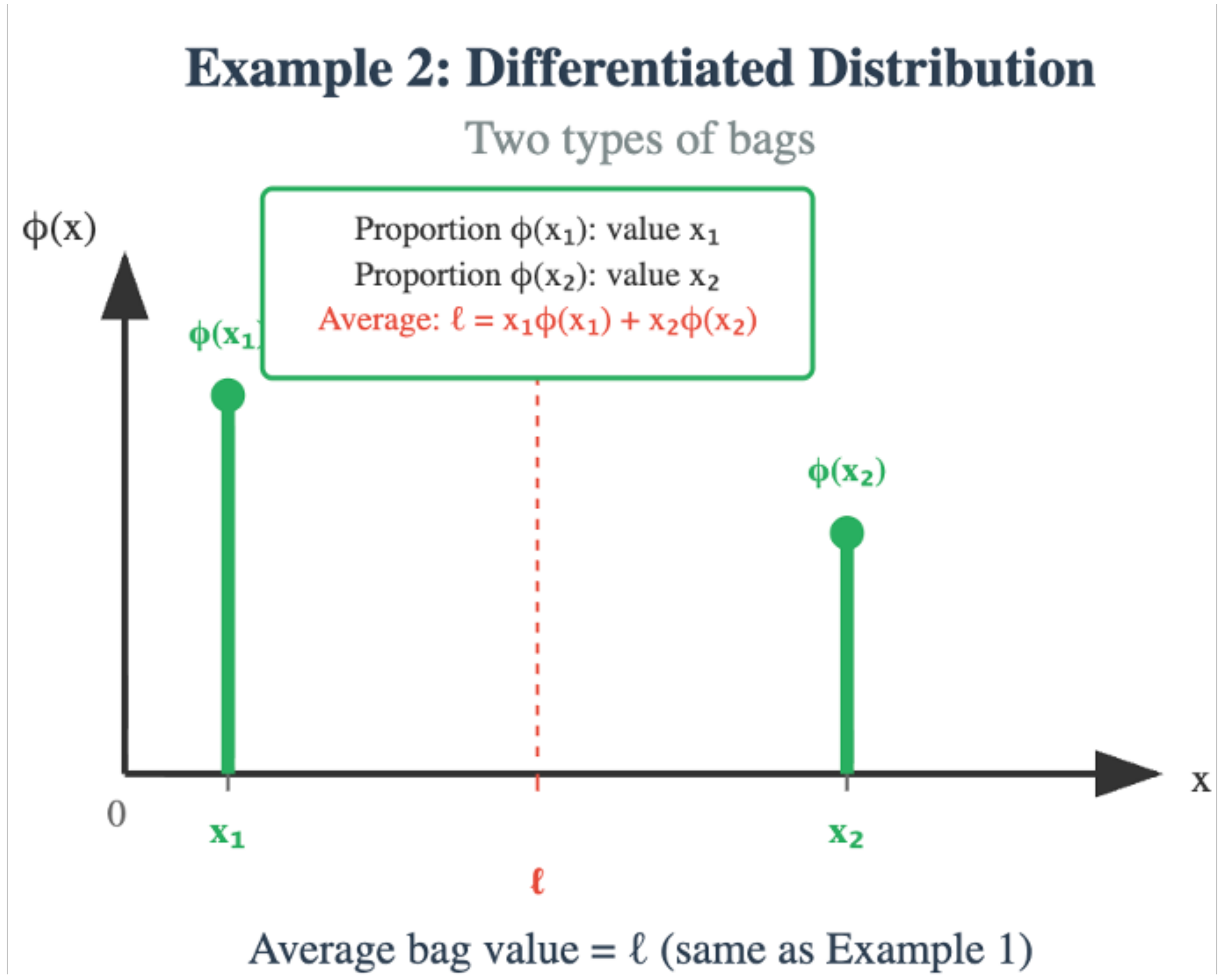
Model: Store's Decisions at Time t

Stage I: Number of bags for reservation n_t

Stage II: Distribution of bag value: $\phi_t(x)$ where x is bag value

— Proportion (density) of bag values





Bag Value

Longo's [2 Bloor East, Toronto] ($7 for $38.85 worth of Dairy Bag)…I'll definitely be getting this bag again if I'm looking to restock on some dairy or craving for some dips 🤩

Eataly - Baked goods (Bay/Bloor) $5.99… Very good haul for me, my breakfast is set for a while now 😋

I ordered a $9.99 meat and seafood bag from Food Basics…only received $14.97 worth of food when I was supposed to receive $30 😡

"Your Surprise Bag Experience"

Scenario

You buy a Whole Foods bakery bag for $5 with posted value of $15.

How would you rate each outcome? (1-5 stars)

Outcome A:

- Actual value = $4
- 2 day-old donuts
- 1 small coffee

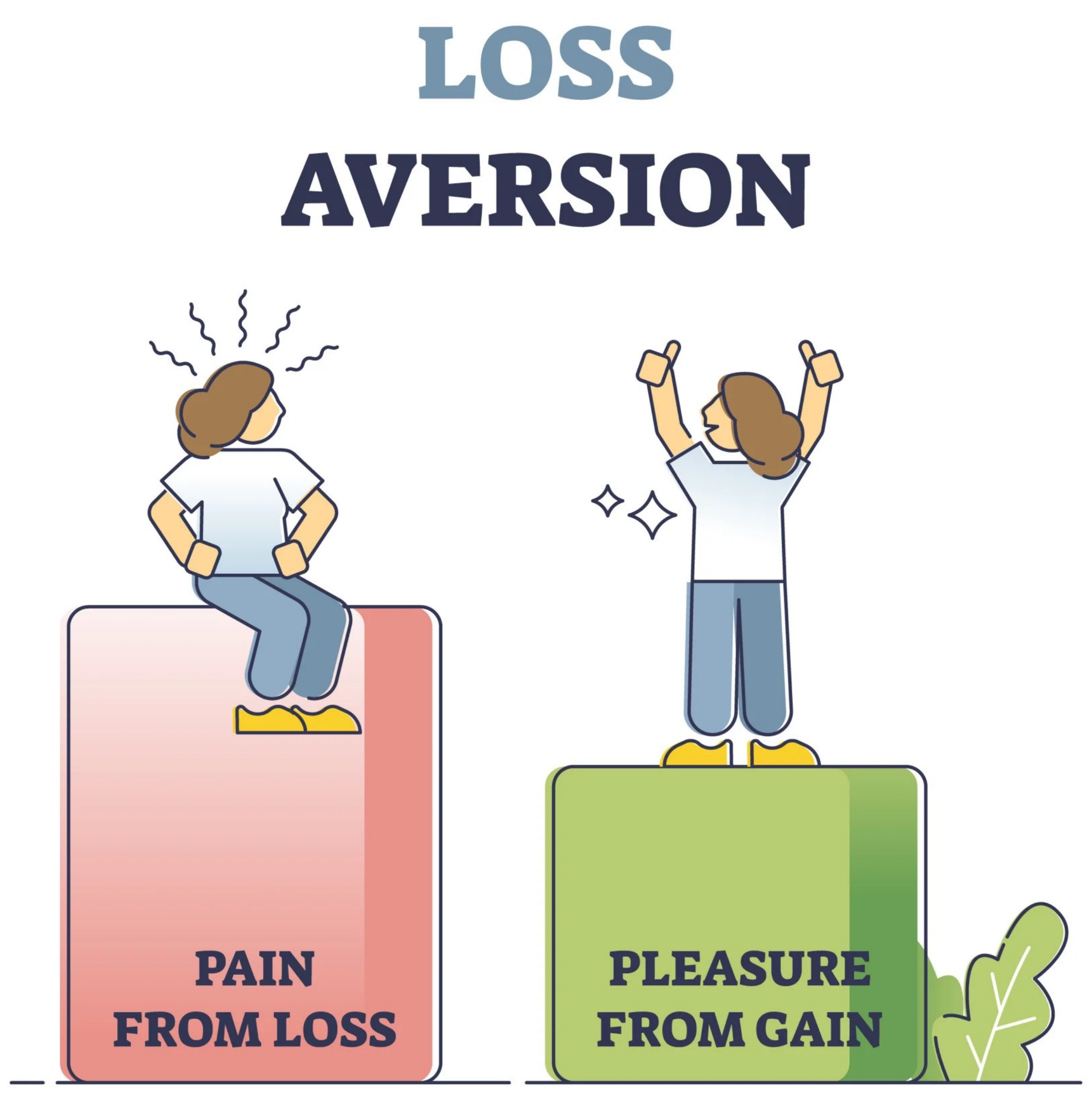
Outcome B:

- Actual value = $12
- 3 muffins
- 2 donuts

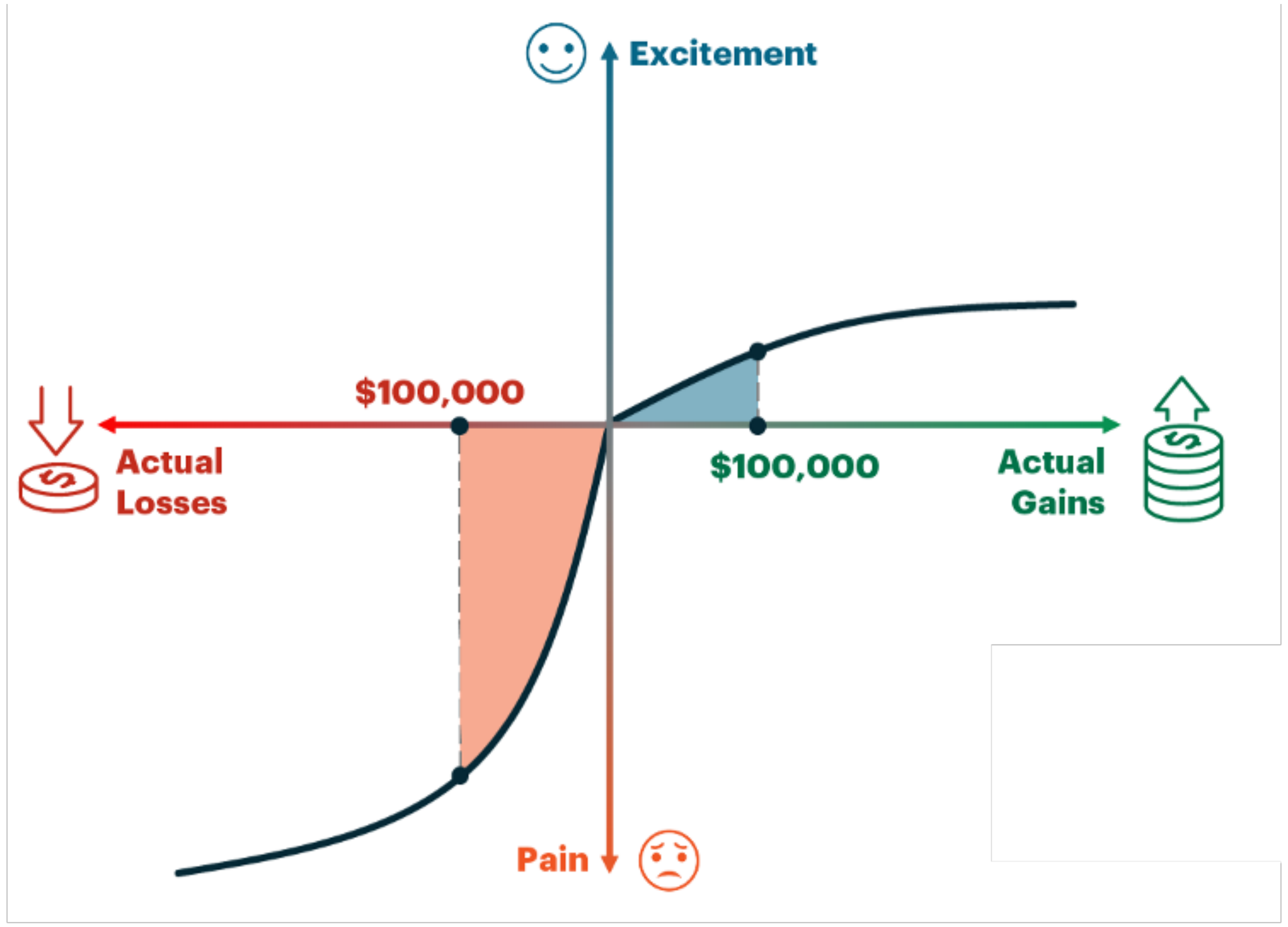
Outcome C:

- Actual value = $16
- 2 sandwiches
- 1 muffin

Prospect Theory: Utility from Gains and Losses



Source: Simply Psychology, Challenger 2019 B2B Buyer Study



Example: Utility from gain ($y \geq 0$) and loss ($y \leq 0$)

$$v_0(y) = y^\alpha \cdot \mathbf{1}_{y \geq 0} - \lambda(-y)^\alpha \cdot \mathbf{1}_{y<0},$$

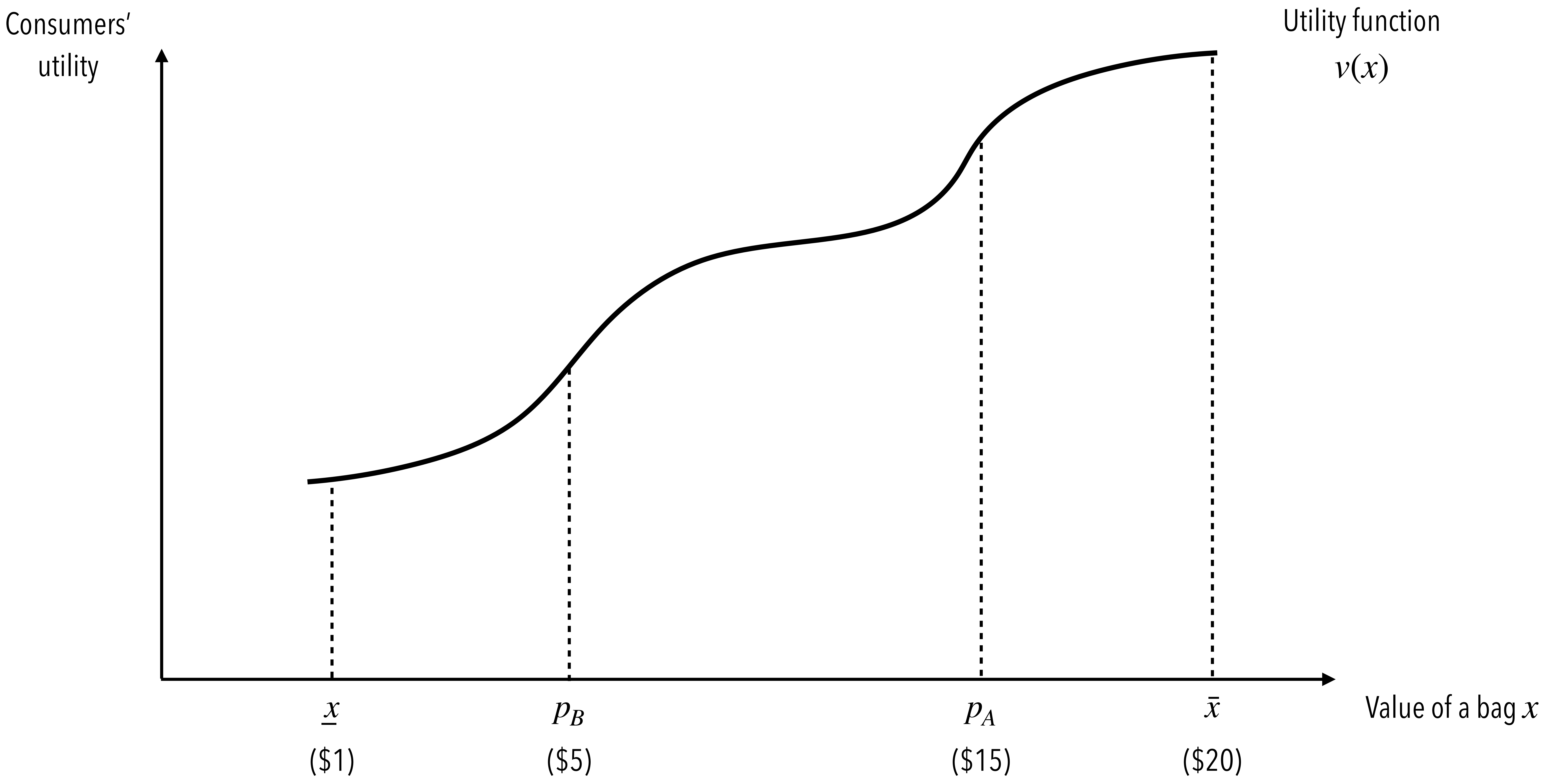
- $\alpha \in (0,1)$ captures diminishing sensitivity, $\lambda > 1$ captures loss aversion

Model: Customer's Utility from a Bag

- **Bag value:** x
- **Consumer utility:** $v(x)$
    - General increasing differentiable function with bounded slope
    - Potentially <u>non-concave</u>
    - Two reference prices: Posted value (p_A) and Paid price (p_B)
    - Total utility:

$$v(x) = (1-\phi)\underbrace{v_0(x - p_A)}_{\text{Reference effect from posted value}} + \phi \underbrace{v_0(x - p_B)}_{\text{Reference effect from paid price}}$$

Example: Non-Concave Utility Function



Model: Reputation Dynamics

- **Customer's expected utility at time t**

$$V[\phi_t(\cdot)] := \int_0^\infty v(x)\phi_t(x)\mathrm{d}x$$

- **Reputation r_t**
  - Dynamics via exponential smoothing

$$r_{t+1} = \delta V[\phi_t(\cdot)] + (1-\delta) r_t$$

  - $\delta \in (0,1)$ is smoothing factor
  - Small δ means reputation is less responsive to recent changes

Model: Store's Profit & Food Waste

- **Supplementary cost** $C[\phi_t(\,\cdot\,), n_t, Q_t]$
  - A store may want to distribute more than surplus food available

$$C[\phi(\,\cdot\,), n_t, Q_t] = c \cdot (n_t \cdot \underbrace{\int_0^\infty x\phi_t(x)\mathrm{d}x}_{\text{Total distributed}} - Q_t)^+$$

- **Store's revenue and food waste**
  - Exogenous price per bag p
  - Store's pay-off in a period

$$R[\phi_t(\,\cdot\,), n_t, Q_t] = p \cdot n_t - C[\phi_t(\,\cdot\,), n_t, Q_t]$$

  - Store's waste in a period

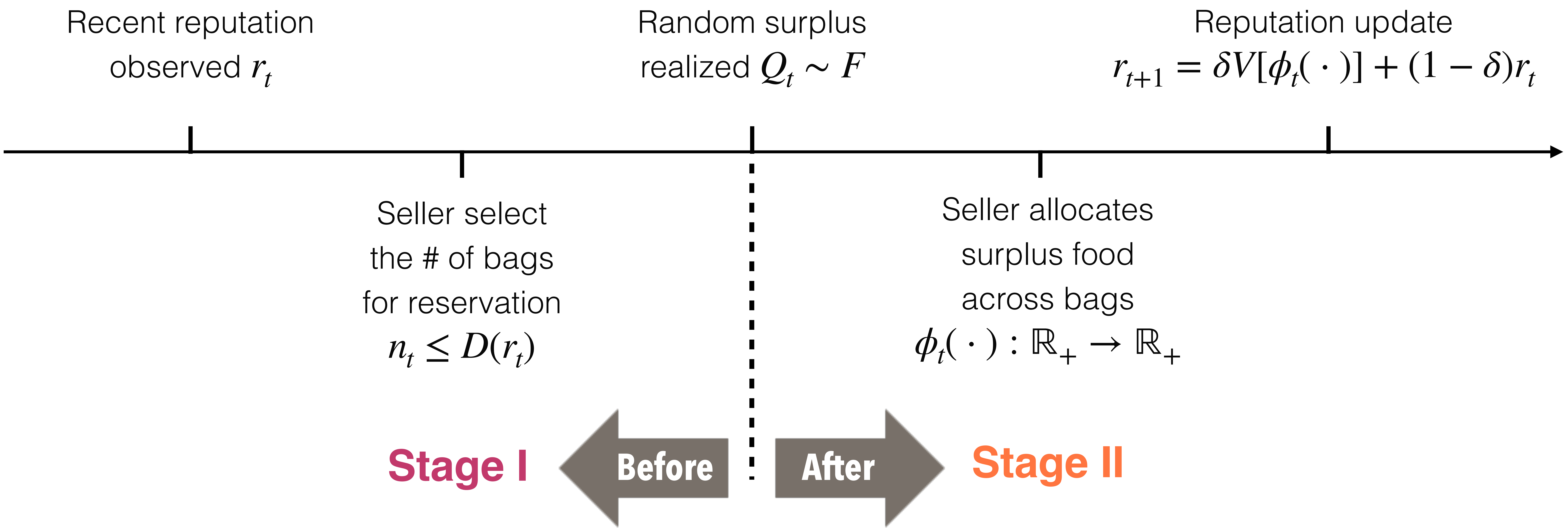
$$W_t = Q_t - n_t \cdot \int_0^\infty x\phi_t(x)\mathrm{d}x$$

Sequence of Events in One Period

An admissible policy π consists of two decision stages

Stage I How many bags? $r \mapsto n^{\pi}$

Stage II How to allocate surplus food across bags? $(r, n, Q) \mapsto \phi^{\pi}(\,\cdot\,)$



Store's Optimal Policy

An admissible policy π consists of two decision stages

Stage I How many bags? $r \mapsto n^\pi$

Stage II How to allocate surplus food across bags? $(r, n, Q) \mapsto \phi^\pi(\,\cdot\,)$

$$\max_{\pi \in \Pi} \quad \lim_{T \to \infty} \mathbb{E}_{Q_t \sim F} \sum_{t=0}^{T} \beta^t R\left[\phi_t^\pi(\,\cdot\,), n_t^\pi, Q_t\right]$$

(Cumulative payoffs)

$$\text{s.t.} \quad 0 \le n_t^\pi \le D(r_t),$$

(Upper bound reservation number)

$$\int_0^\infty \phi_t^\pi(x)\mathrm{d}x = 1,$$

(Distribution normalization)

$$r_{t+1} = \delta V[\phi_t^\pi(\,\cdot\,)] + (1-\delta) r_t \,.$$

(Reputation update)

DP Formulation

$$J^*(r) = \max_{n \le D(r)} \left[p \cdot n + \mathbb{E}_Q \left(\boxed{\max_{\phi(\cdot)}} \left[-C[\phi(\,\cdot\,), n, Q] + \beta J^* \left(\delta V[\boxed{\phi(\,\cdot\,)}] + (1-\delta) r \right) \right] \right) \right]$$

- Stage II is optimization over functions! But…
- If $\mathbb{E}[\phi(\,\cdot\,)] = \ell$ is given, then myopic cost is:

$$C[\phi(\,\cdot\,), n, Q] = (n\ell - Q)^+$$

- So, given ℓ, store always chooses distribution ϕ that maximizes future reputation or average customer satisfaction $V(\phi(\,\cdot\,))$.

DP with Reduced Action Space

- **Reduction:** Stage II decision can be reduced from a function $\phi(\cdot)$ to a scalar ℓ
- Note that ℓ uniquely determines an **optimal conditional distribution**

$$\boxed{\phi^*(\cdot \mid \ell) = \arg\max_{\phi(\cdot)} V(\phi(\cdot))}$$

$$J^*(r) = \max_{n \le D(r)} \left[p \cdot n + \mathbb{E}_Q \left(\max_{\boxed{\ell \ge 0}} \left[-C(\ell, n, Q) + \beta J^* \left(\delta V[\phi^*(\cdot \mid \ell)] + (1-\delta) r \right) \right] \right) \right]$$

Optimal Conditional Distribution Problem

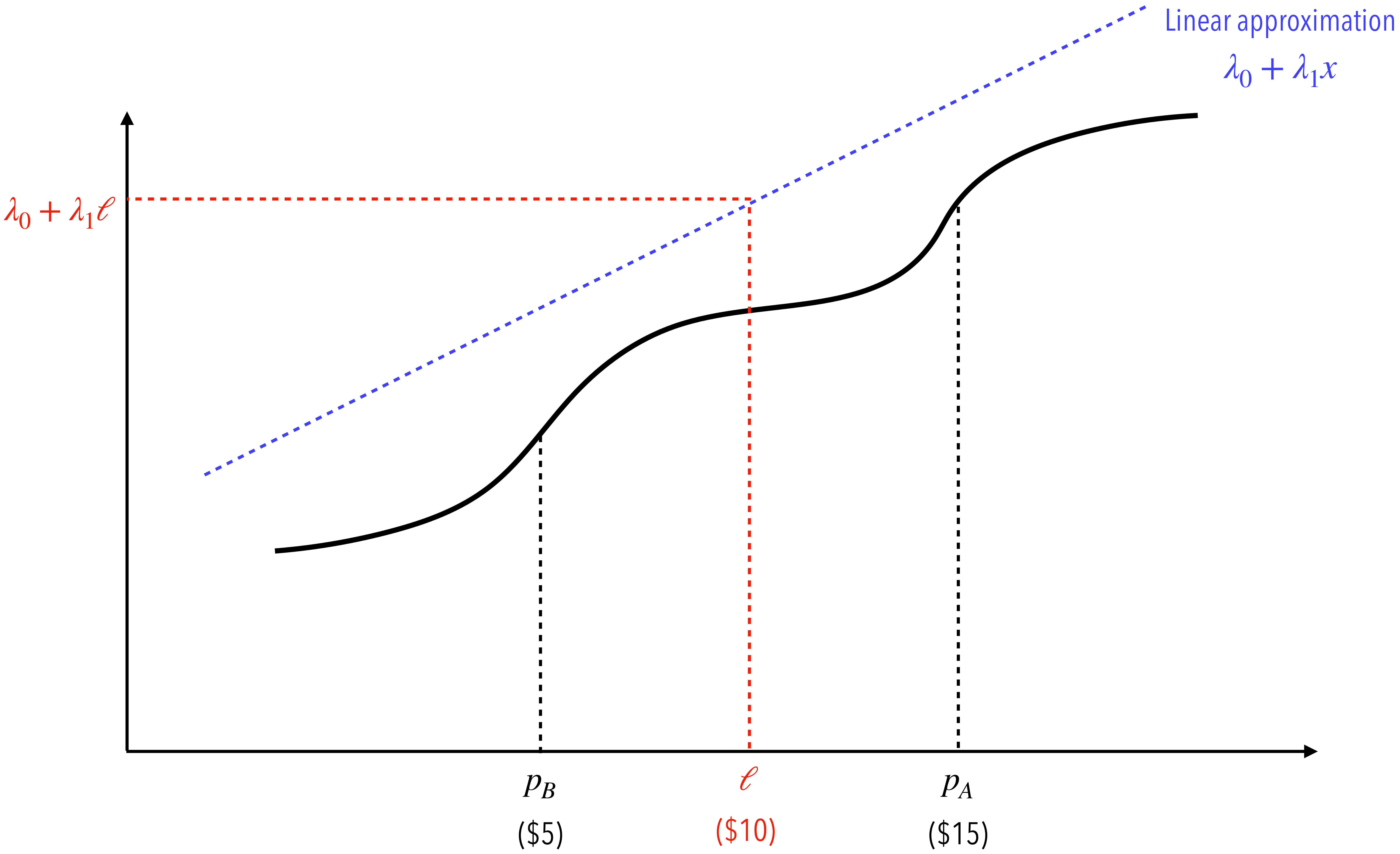
$\phi^*(\,\cdot\mid\ell)$ is the solution to a semi-infinite linear program

$$\max_{\phi(\cdot)\geq 0} \quad V[\phi(x)] = \int_0^\infty v(x)\phi(x)\mathrm{d}x$$
$$\text{s.t.} \quad \int_0^\infty \phi(x)\mathrm{d}x = 1$$
$$\int_0^\infty x\phi(x)\mathrm{d}x = \ell$$

Its dual problem is

$$\min_{\lambda_0,\lambda_1} \quad \lambda_0 + \ell\lambda_1$$
$$\text{s.t.} \quad \lambda_0 + x\lambda_1 \geq v(x), \quad \forall x \geq 0$$

Dual of Optimal Conditional Distribution Problem



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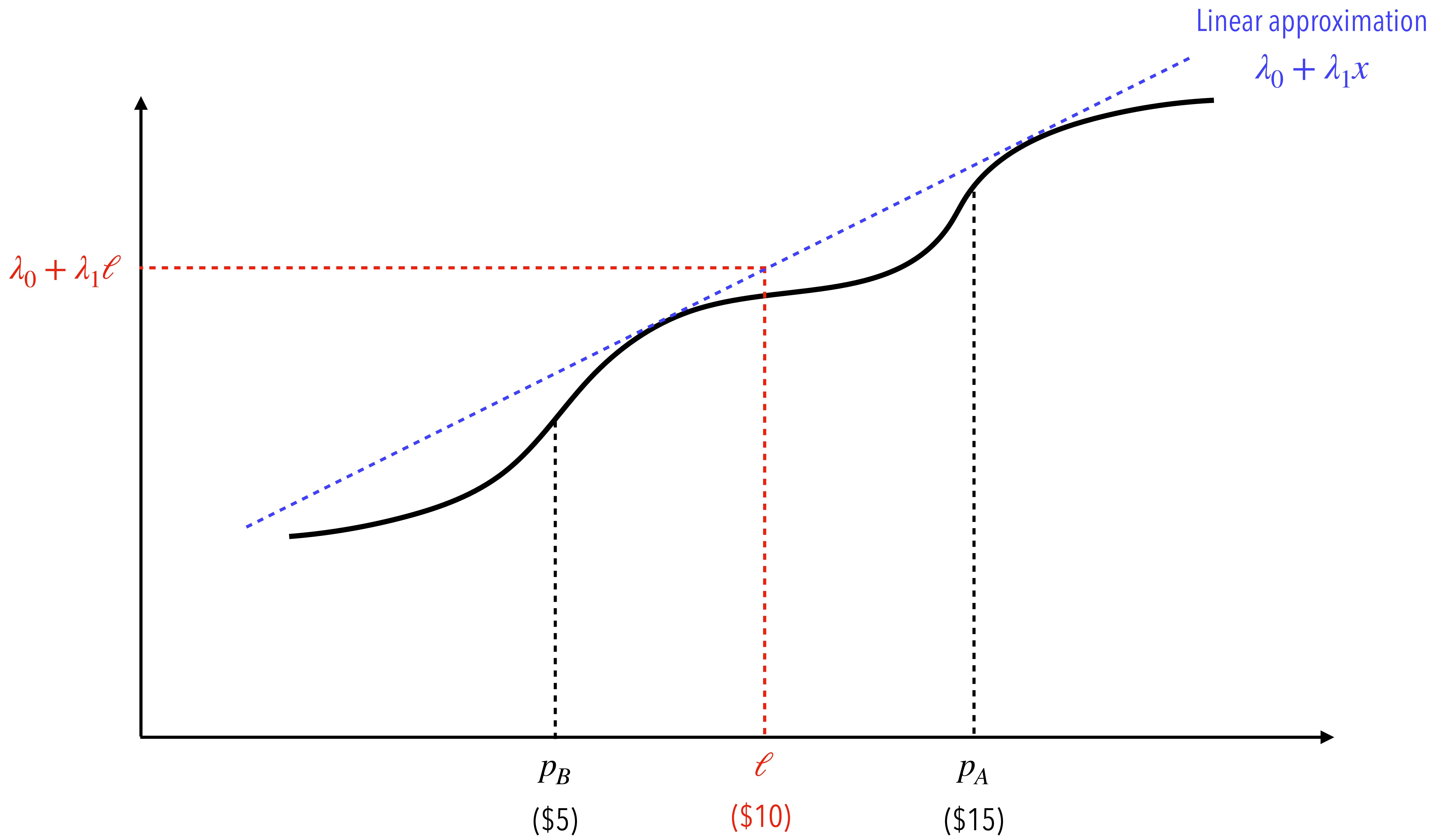


Optimal Conditional Distribution Problem

$\phi^*(\,\cdot\,|\,\ell)$ is the solution to a semi-infinite linear program

$$\begin{aligned} \max_{\phi(\cdot)\geq 0} \quad & V[\phi(x)] = \int_0^\infty v(x)\phi(x)\mathrm{d}x \\ \text{s.t.} \quad & \int_0^\infty \phi(x)\mathrm{d}x = 1 \\ & \int_0^\infty x\phi(x)\mathrm{d}x = \ell \end{aligned}$$

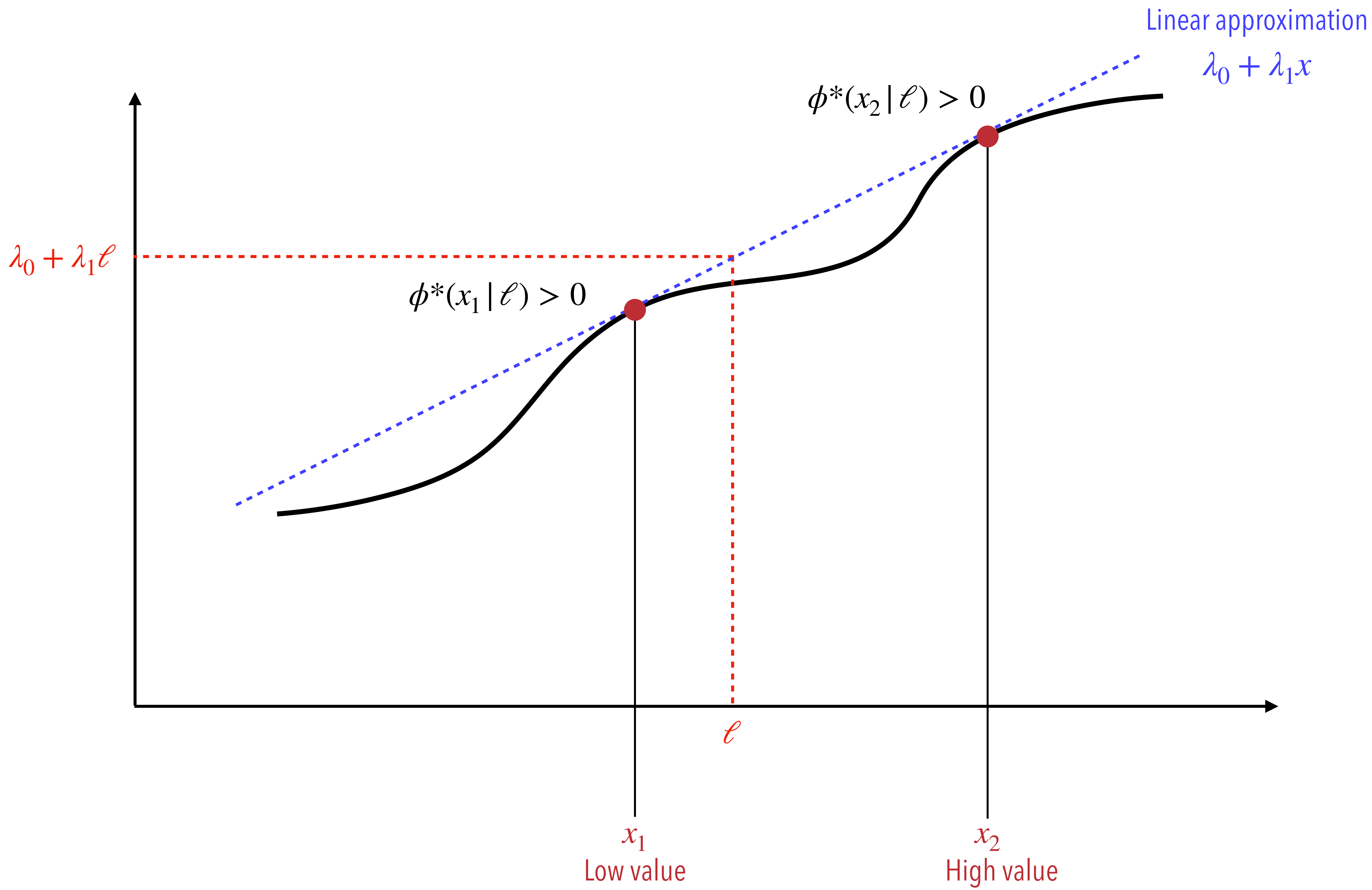
Its dual problem is

$$\begin{aligned} \min_{\lambda_0,\lambda_1} \quad & \lambda_0 + \ell\lambda_1 \\ \text{s.t.} \quad & \lambda_0 + x\lambda_1 \geq v(x), \quad \forall x \geq 0 \end{aligned}$$

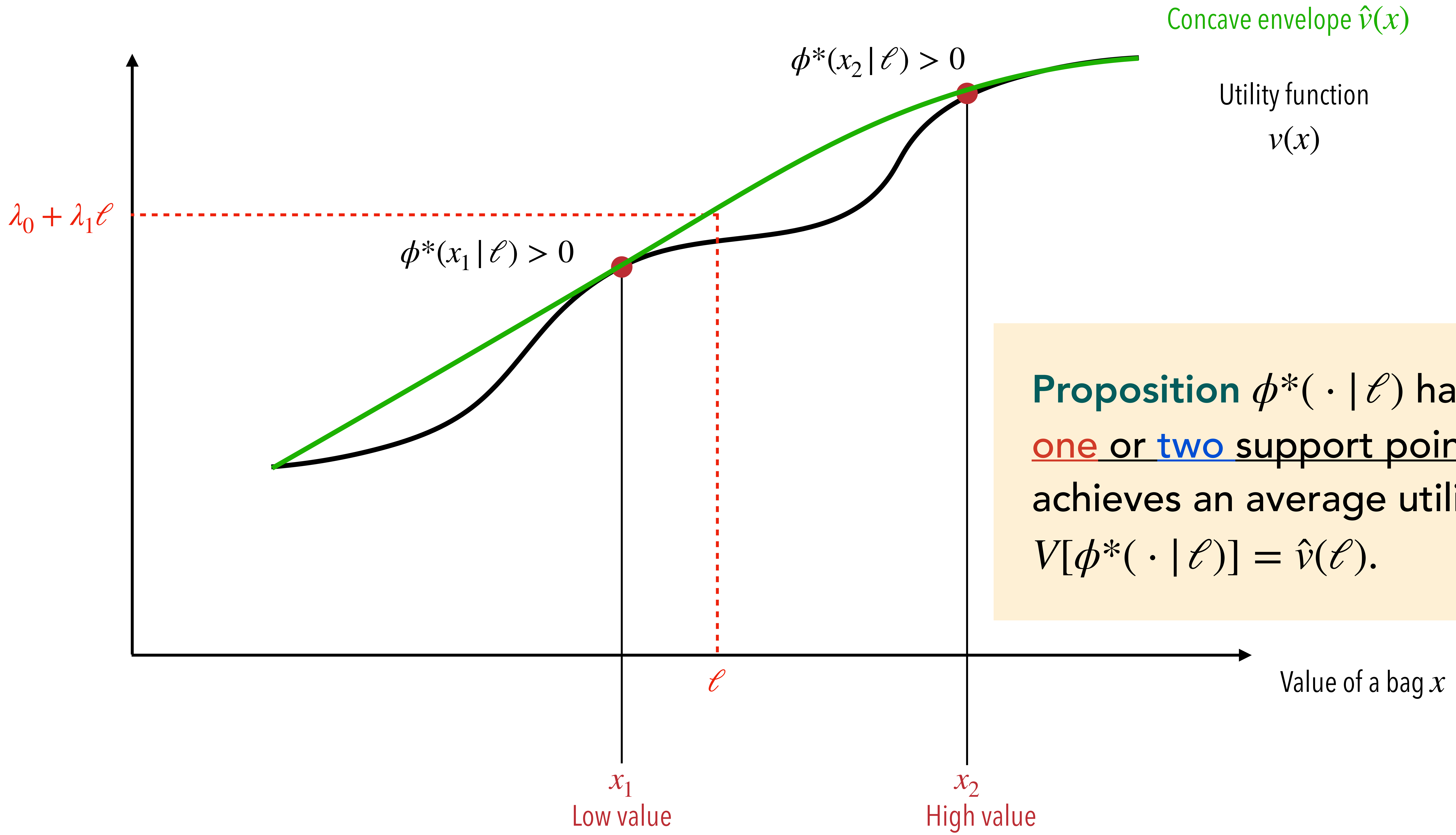
Complementary Slackness condition: For any $x \geq 0$, we have

$$\phi^*(x\,|\,\ell)\cdot(\lambda_0^* + x\lambda_1^* - v(x)) = 0$$

Complementary Slackness Condition

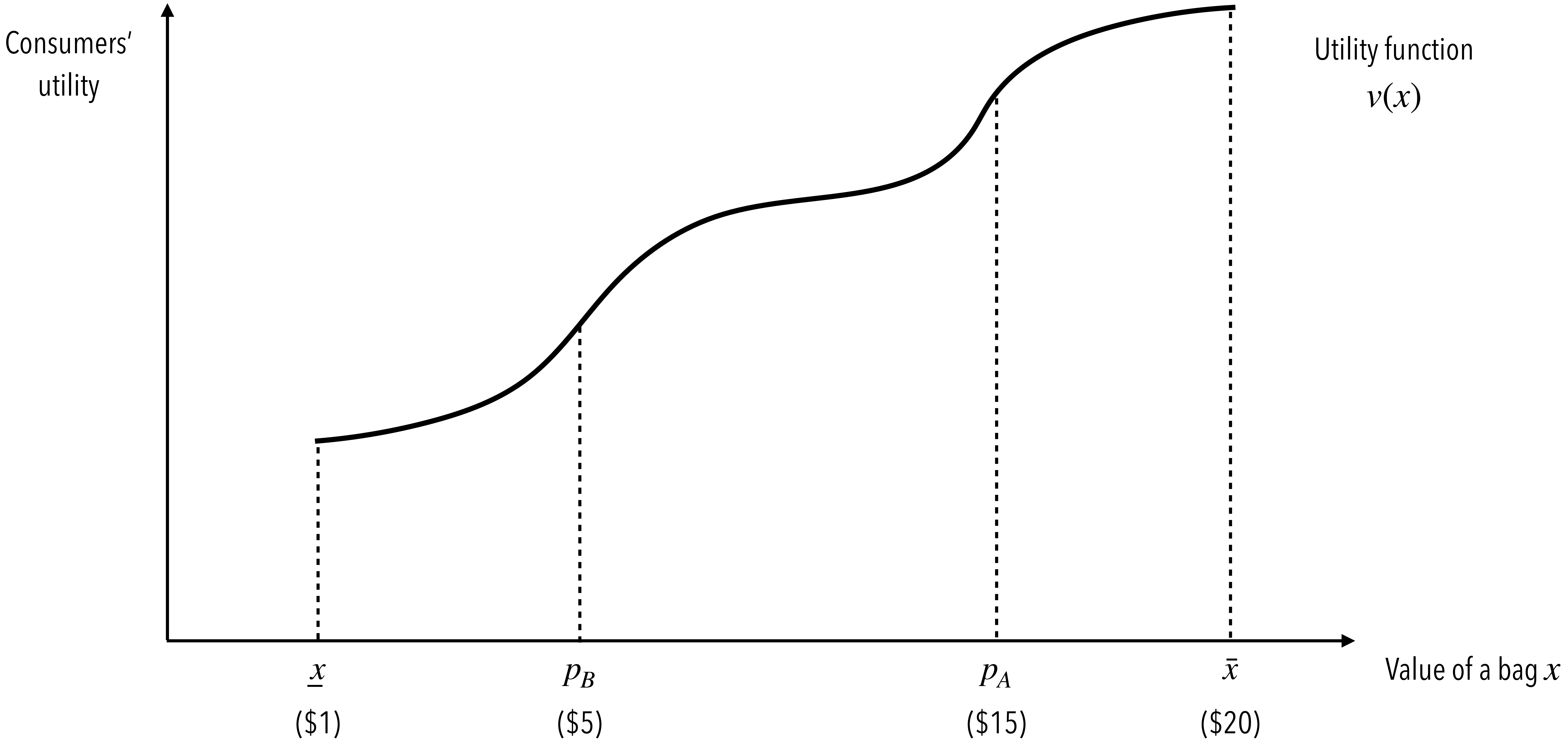


Structure of $\phi^*(\cdot \mid \ell)$



Proposition $\phi^*(\cdot \mid \ell)$ has either one or two support points and achieves an average utility $V[\phi^*(\cdot \mid \ell)] = \hat{v}(\ell)$.

Example: From Prospect Theory



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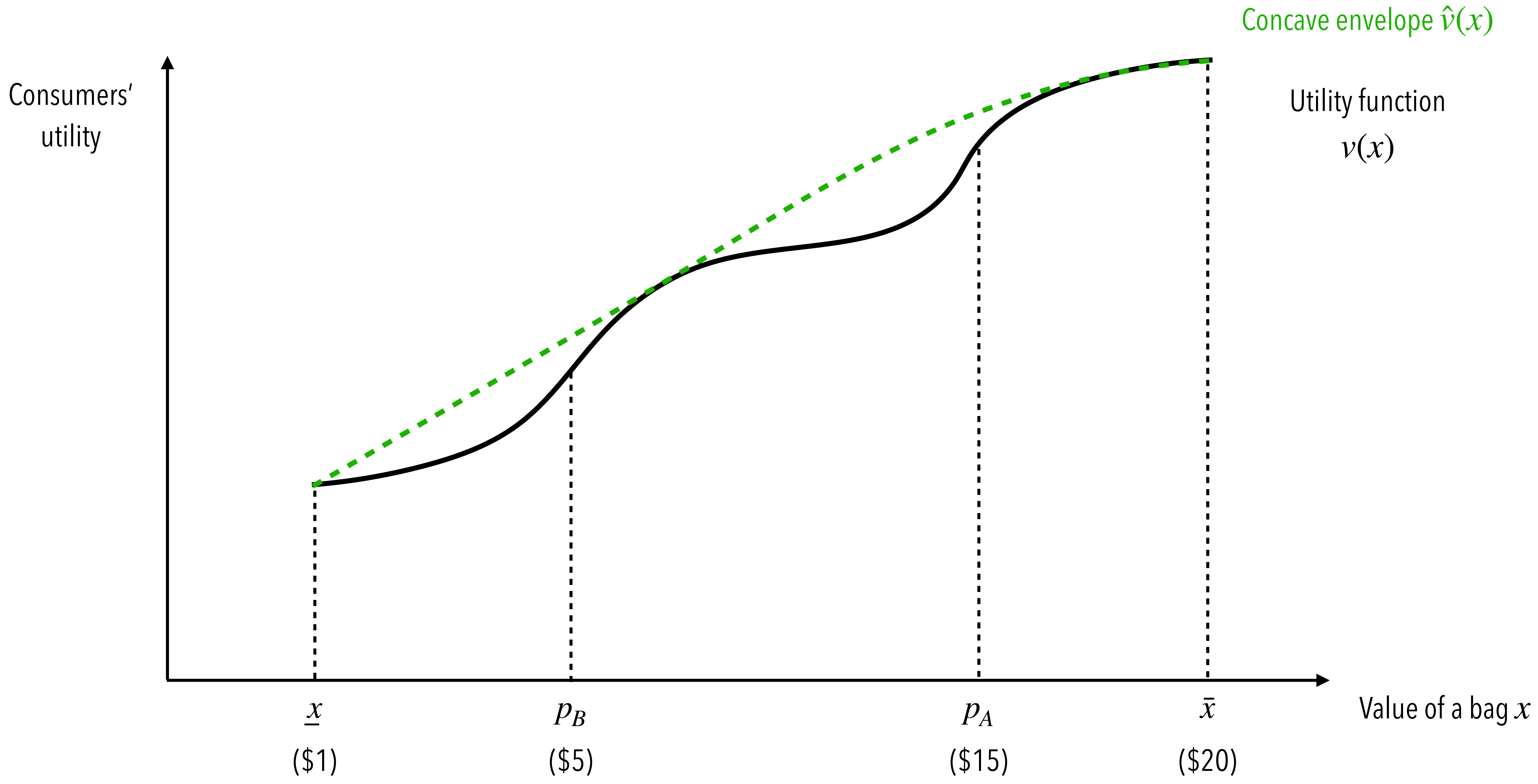
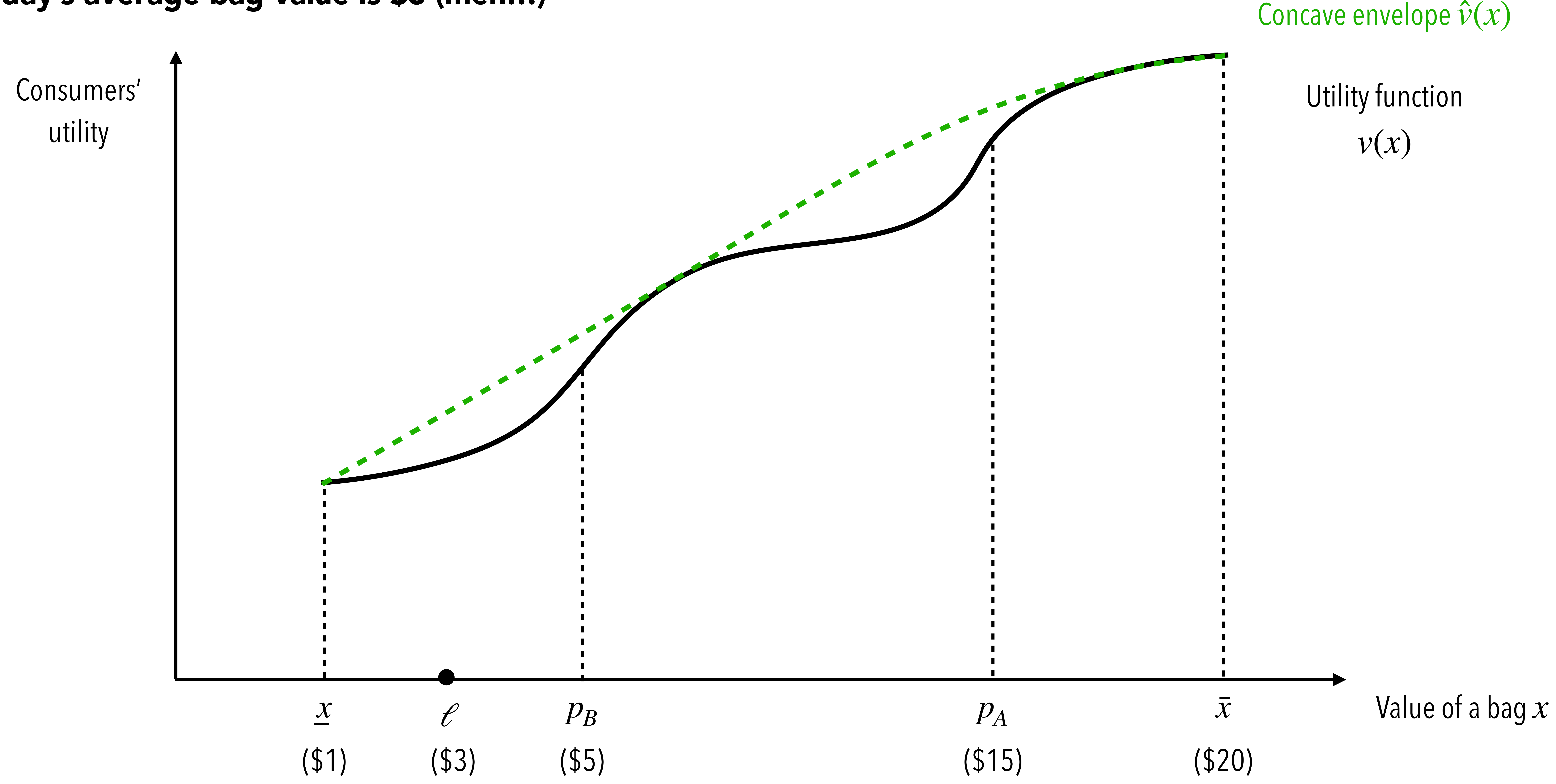


Example: From Prospect Theory

Case 1: Today's average bag value is \$3 (meh...)



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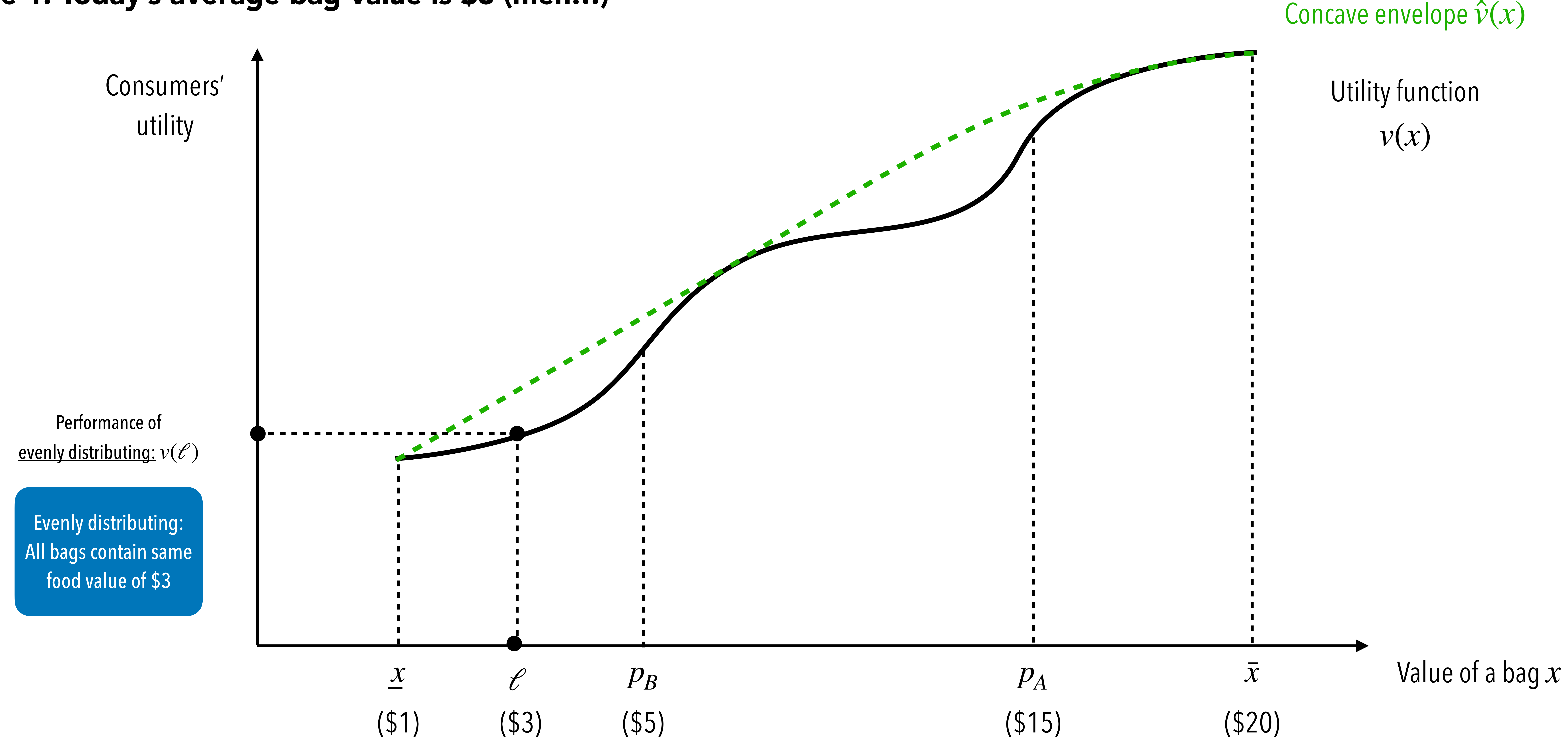

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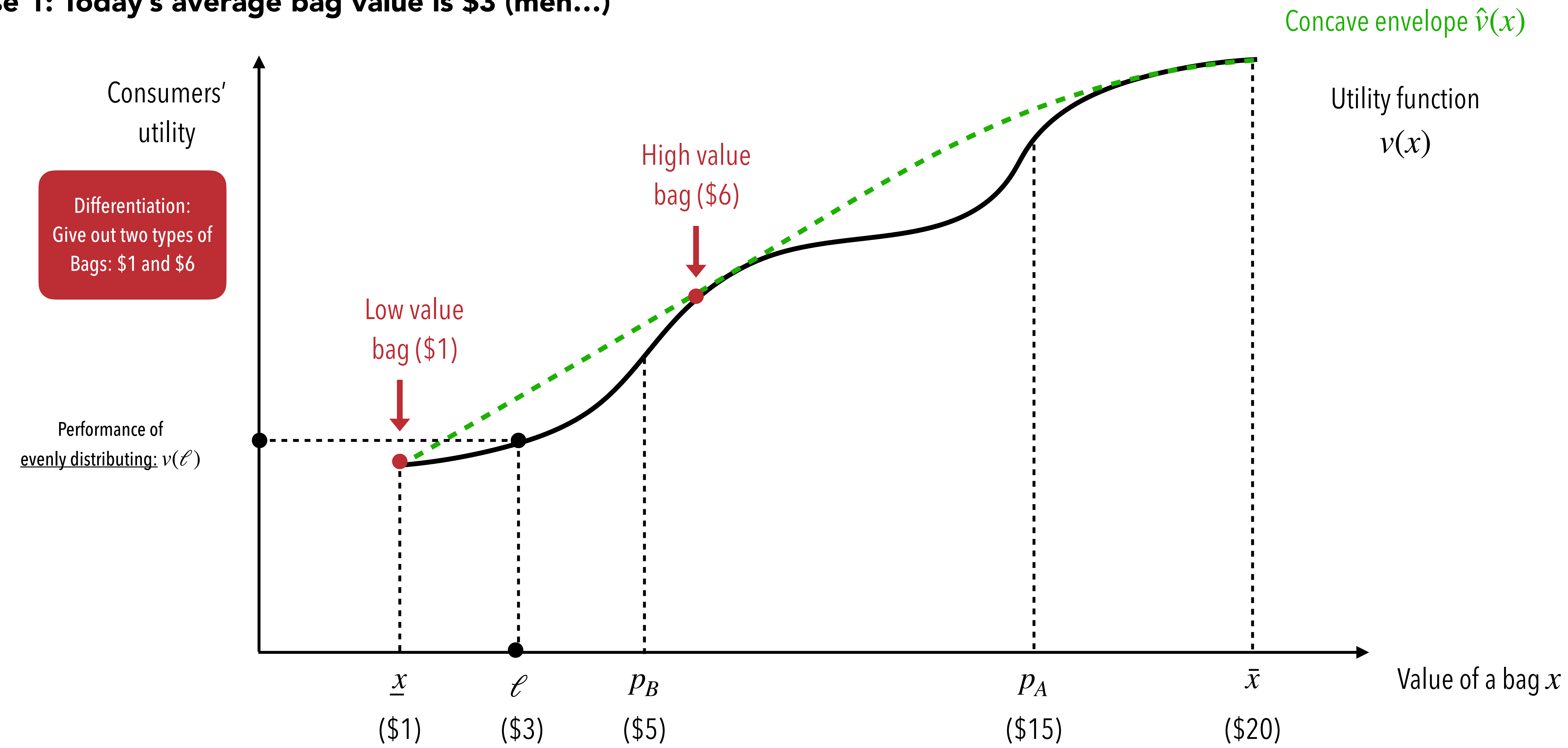
Differentiation:
Give out two types of
Bags: $1 and $6



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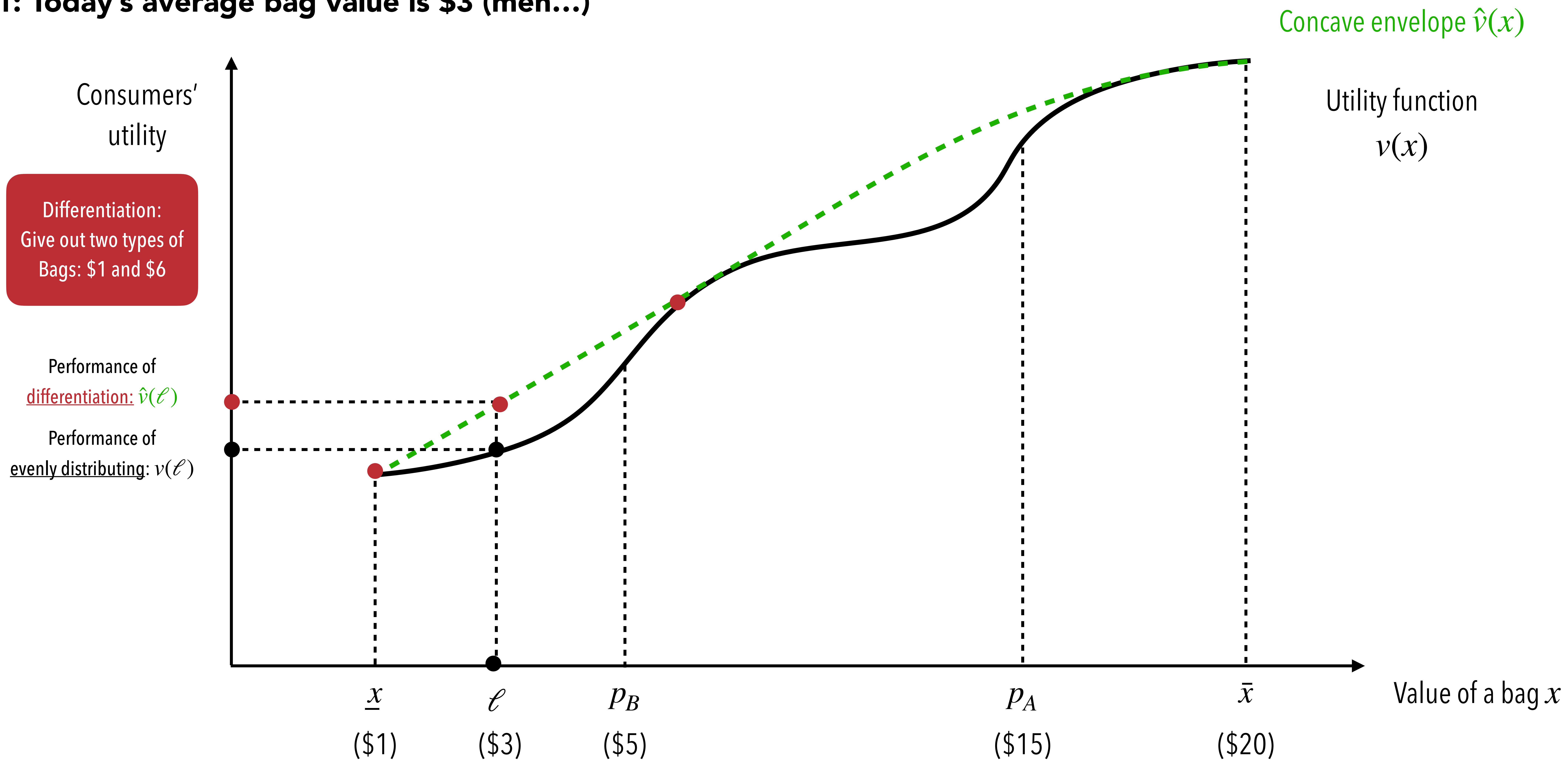

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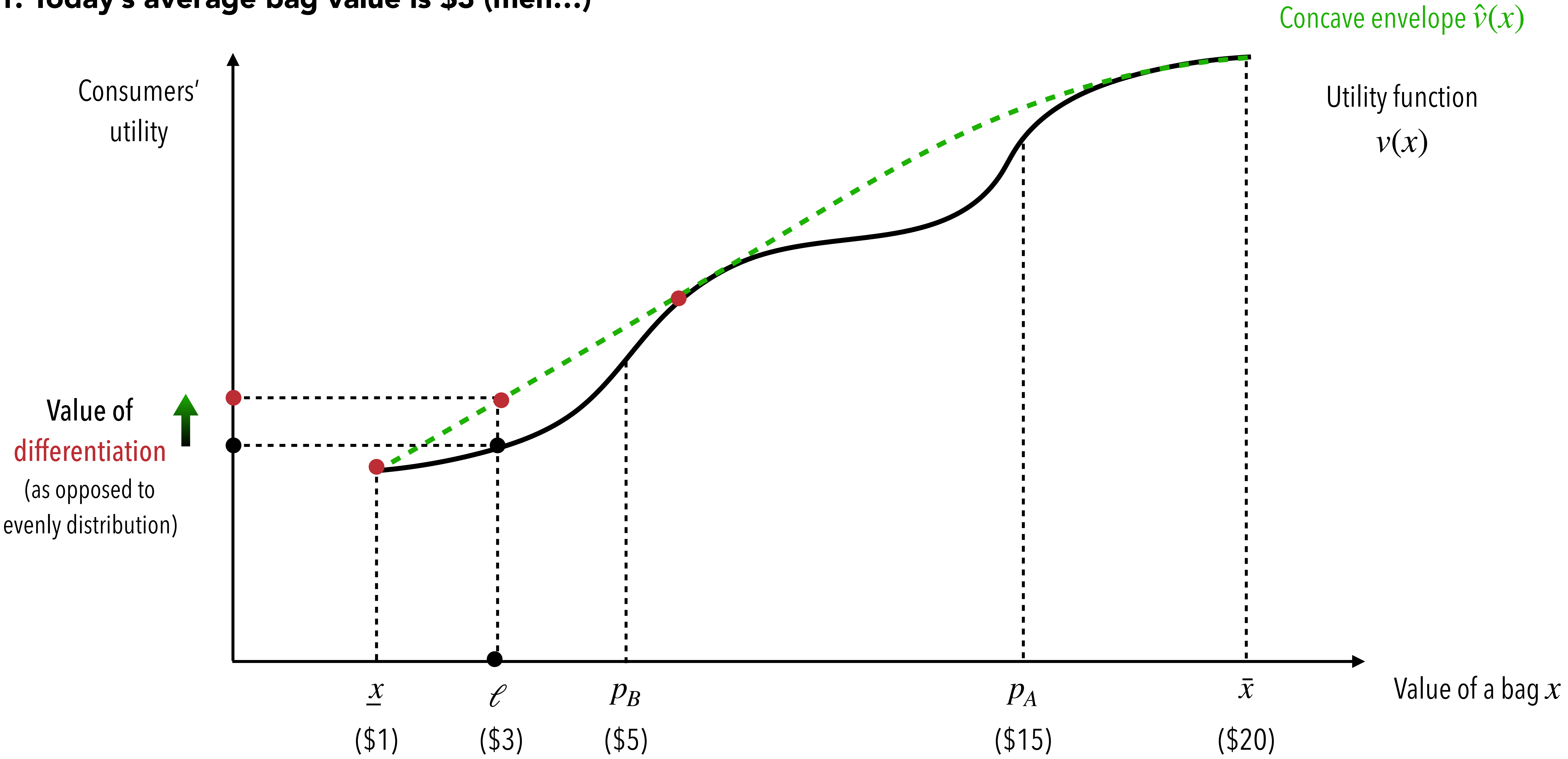
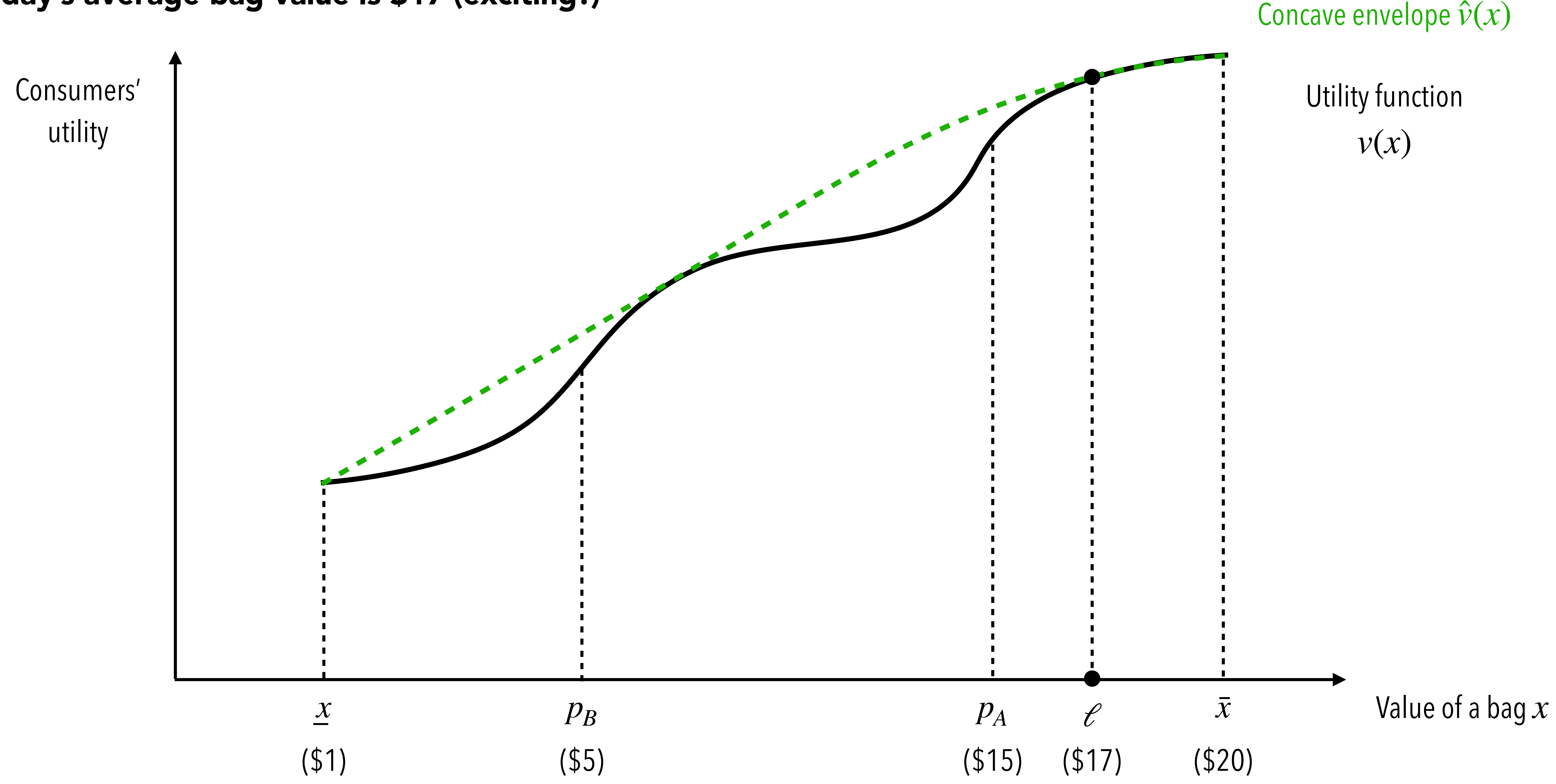


Example: From Prospect Theory

Case 2: Today's average bag value is \$17 (exciting!)



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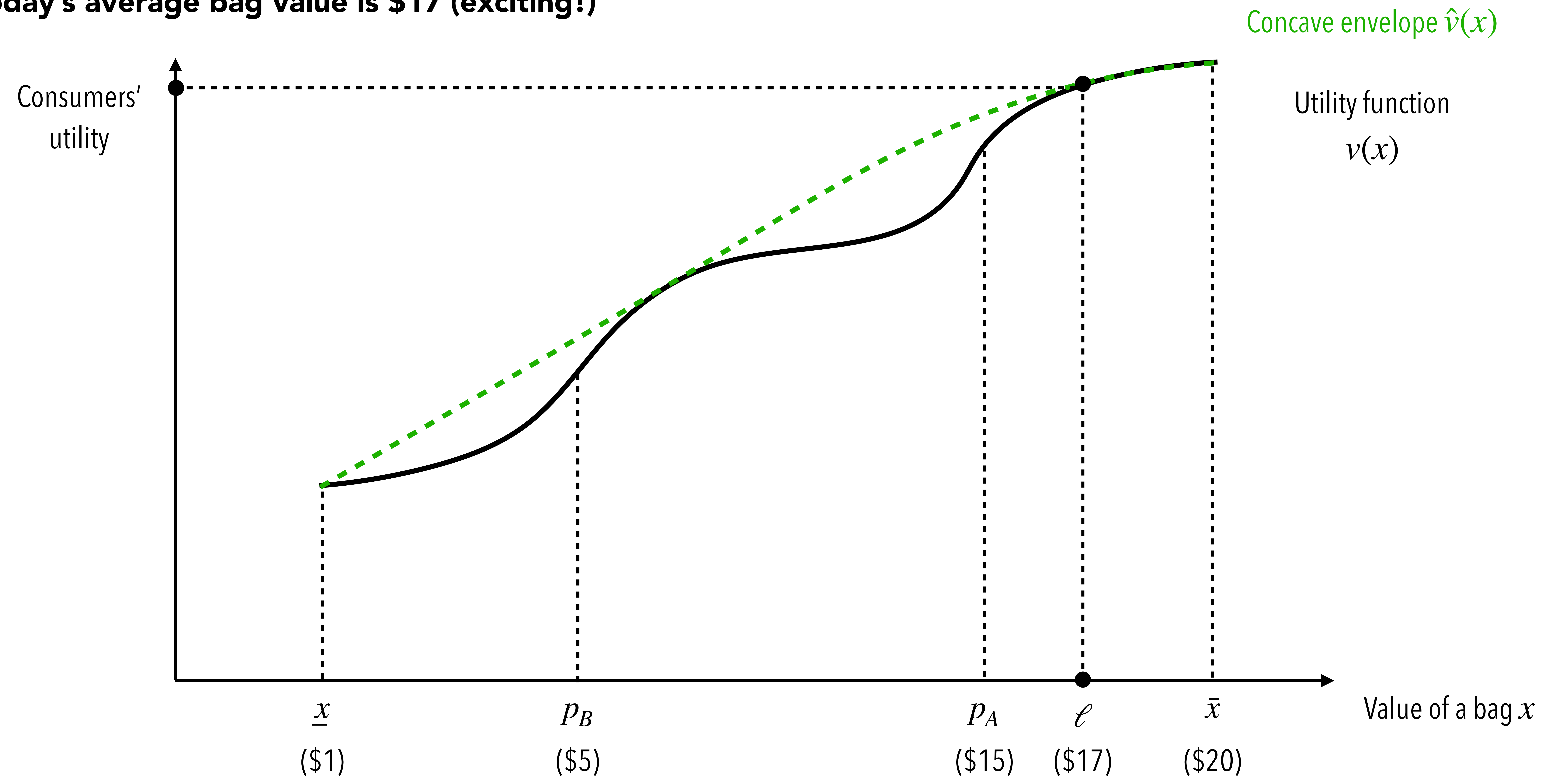

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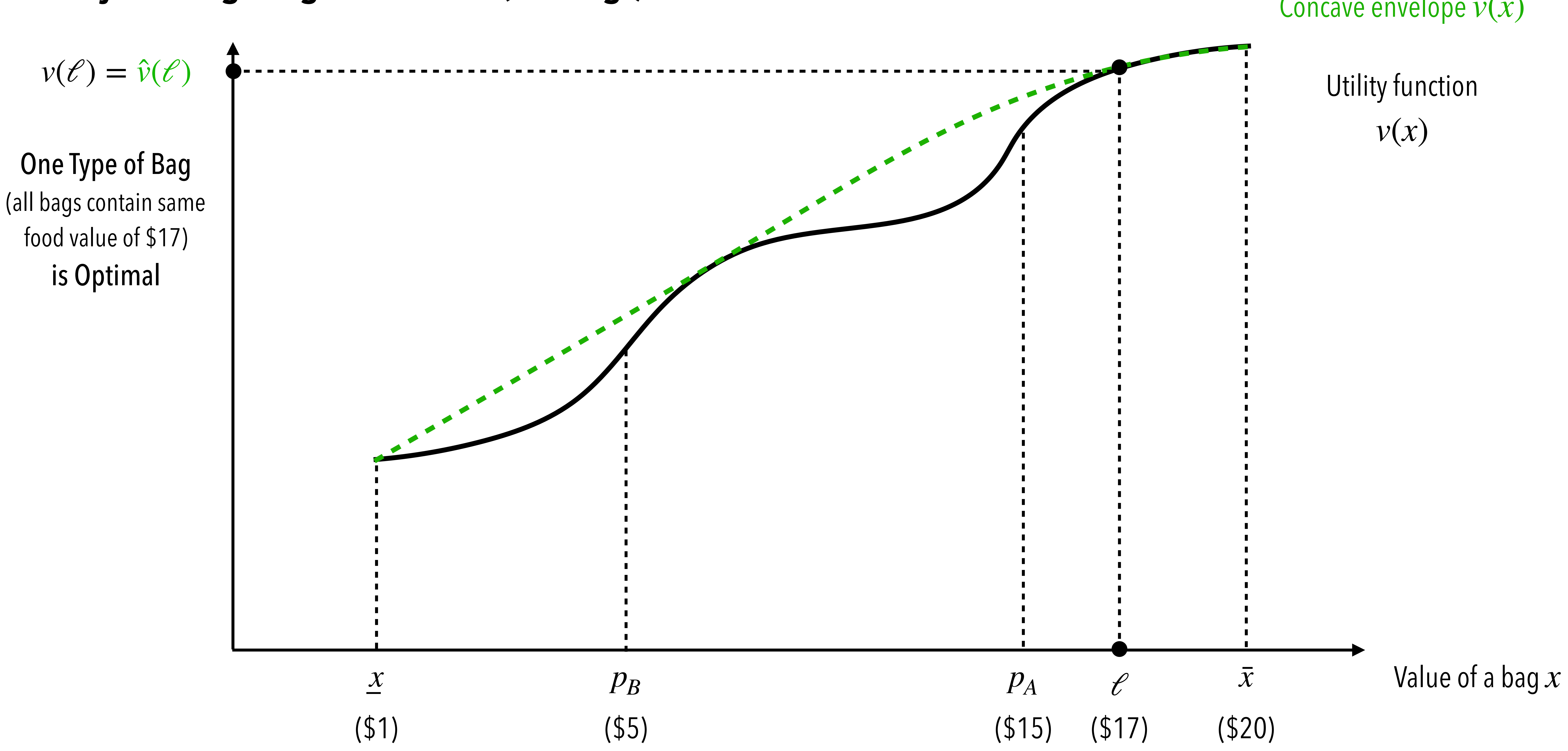


Optimal Policy

Reduced DP Formulation

7:59AM Observe today's rating

9PM Observe today's surplus

$$J^*(r) = \max_{n \le D(r)} \left[pn + \mathrm{E}_Q \left(\max_{\ell \ge 0} \{ \beta J^*(\delta \hat{v}(\ell) + (1-\delta) r) - c(n\ell - Q)^+ \} \right) \right]$$

8AM
How many bags?

9:10PM
What's each bag's value?

Avg. Customer satisfaction

Supplementary cost (in case bags contain more than surplus)

Properties of Optimal Policy

- $n^*(r)$ increase with r
- $\ell^*(r, Q)$ decreases with r and increases with Q
- For fixed r and Q, $n^*(r)$ and $l^*(r, Q)$ increases as p increases or c decreases

Deterministic Fluid Approximation

- **Deterministic fluid approximation policy π_D**
- **Replaces random $Q \sim F$ with expectation $\bar{Q} = \mathbb{E}_F[Q]$**
  - Bellman equation is

$$J^{\pi_D}(r) = \max_{n \le D(r), \ell \ge 0} \; pn - C(\ell, n, \bar{Q}) + \beta J^{\pi_D}\left[\delta \hat{v}(\ell) + (1-\delta) r\right]$$

- **Convex relaxation of single-period payoff (McCormick envelope)**

$$C(n, \ell, Q) = c(n\ell - Q)^{+} \ge c\left[\psi(n, \ell) - Q\right]^{+} := \underline{C}(n, \ell, Q)$$

- **Putting together, the Bellman equation is reduced to**

$$J^{MD}(r) = \max_{n \le D(r), \ell \ge 0} \; pn - \underline{C}(\ell, n, \bar{Q}) + \beta J^{MD}\left[\delta \hat{v}(\ell) + (1-\delta) r\right]$$

Structure of Relaxed Policy

Proposition $n^{MD}(r)$ increases in r and $\ell^{MD}(r)$ decreases in r. Specifically,

- If $p \geq c$, then $n^{MD}(r) = D(r)$.
- If $p < c$, we have $n^{MD}(r) = D(r) - \left[\ell^{MD}(r)D(r) - \bar{Q}\right]^+$, where $\ell^{MD}(r)$ is the solution to $\max_{\ell \in [0,1]} \; p\bar{Q} + (1-\ell)\bar{n}p + \beta J^{MD}\left[\delta\hat{v}(\ell) + (1-\delta)r\right]$.

Implications

- With small supplementary cost ($p \geq c$), the store maximizes immediate profits by distributing the maximum number of bags.
- With large supplementary cost ($p < c$), the store reduces the number of bags to avoid overly supplementing food.

Performance Bound of Relaxed Policy

Proposition It holds that $J^{\pi_D} \le J^*(r) \le J^M(r) \le J^{MD}(r)$, and

$$J^*(r) - J^{\pi_D}(r) \le J^{MD}(r) - J^{\pi_D}(r) \le \frac{c}{1-\beta} \cdot \left[\frac{\sigma}{2} + \left(\sqrt{D(r)} - \sqrt{\bar{Q}/\bar{x}}\right)^2\right]$$

Remark: The performance bound is tighter when

- Time discount factor β is small
- Supplementary cost c is small
- Standard deviation of surplus σ is small
- Average surplus amount $\bar{Q}$ is aligned with $xD(r)$

Steady State Analysis

Proposition Under the optimal policy with deterministic surplus, the steady-state bag value converges to ℓ^* is the unique solution to

$$\frac{cD[\hat{v}(\ell^*)]}{(p - c\ell^*)D'[\hat{v}(\ell^*)]} = \frac{\beta\delta\hat{v}'(\ell^*)}{1 - \beta(1-\delta)}.$$

Implication: Stores do not need to maintain a perfect rating.

It optimizes to converge to a higher rating if

- Higher price-to-cost ratio
- Faster reputation updating
- Focus more on long-term perspective
- Greater demand increase from better bag value

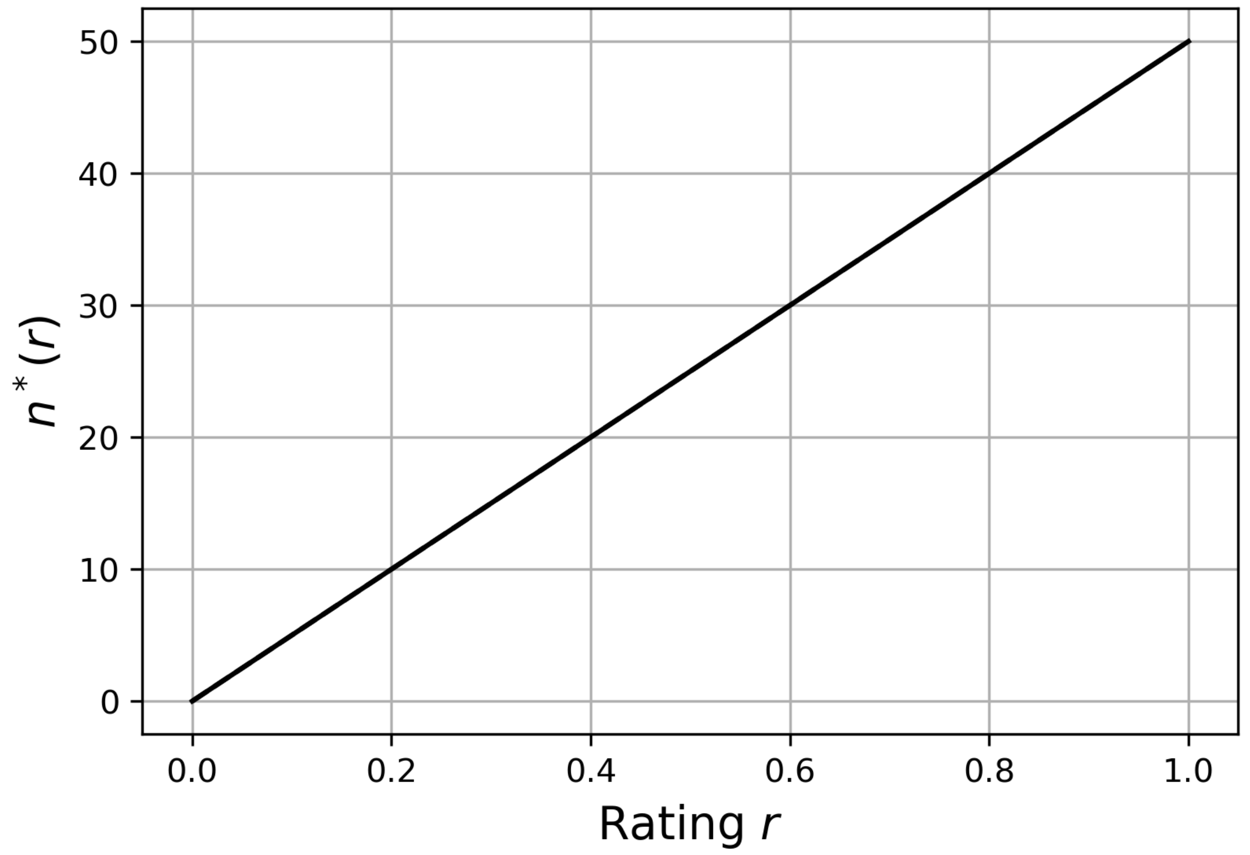
Illustration of Optimal Policy

Experiment setup

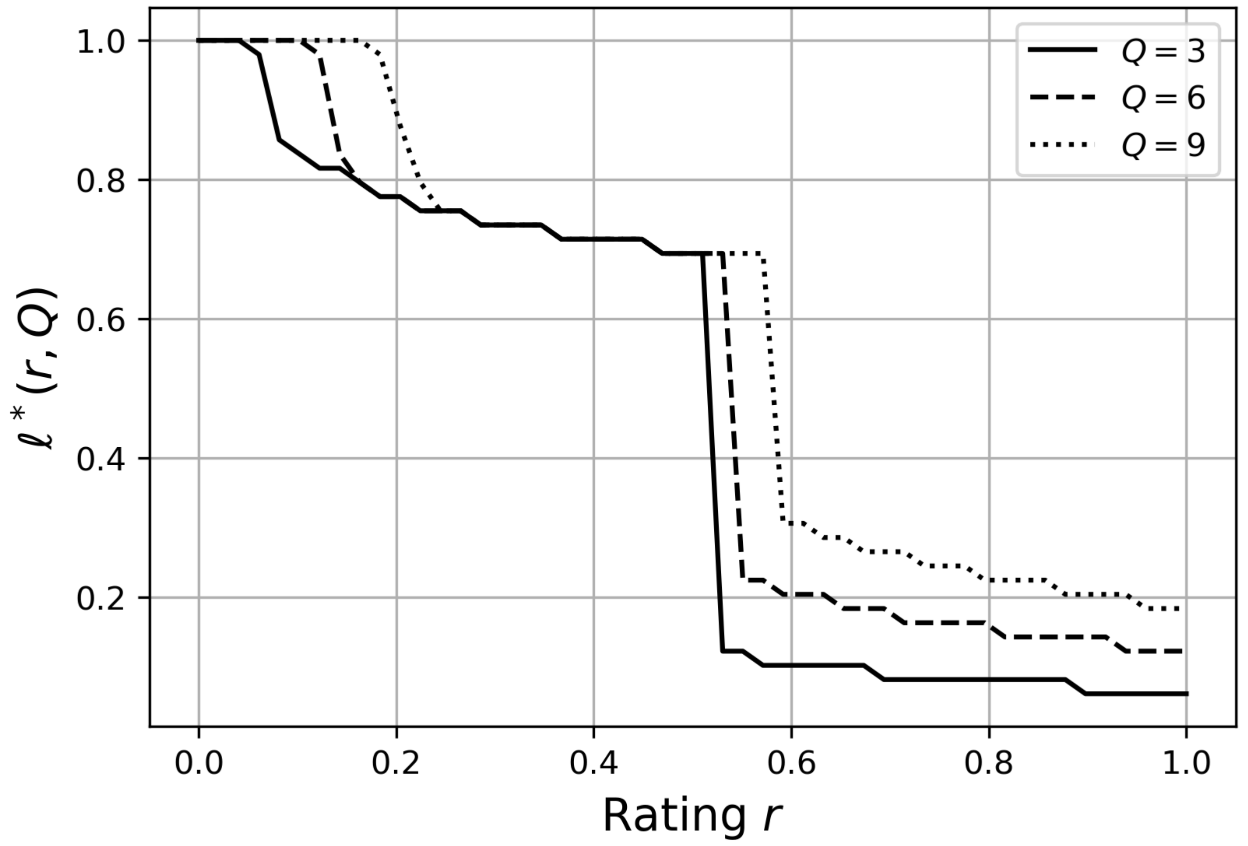
- We assume a simple logistic utility function with one reference point, given by
$$v(x) = \frac{1}{1 + e^{-10(x-0.5)}}.$$

- Its upper concave envelope can be expressed as
$$\hat{v}(x) = \begin{cases} 1.25x & \text{if } x \in [0, 0.676]; \\ \frac{1}{1 + e^{-10(x-0.5)}} & \text{if } x \in (0.676, 1]\,. \end{cases}$$

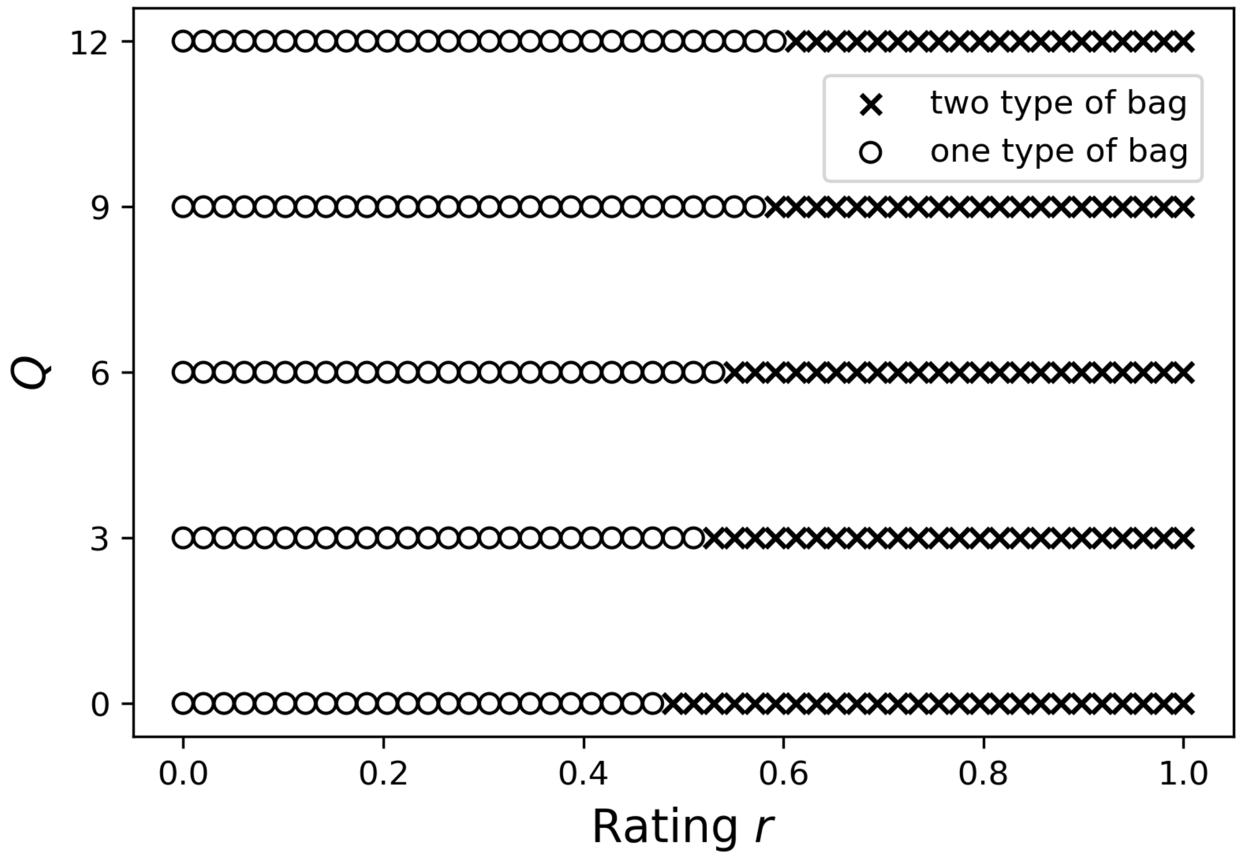
Optimal policy



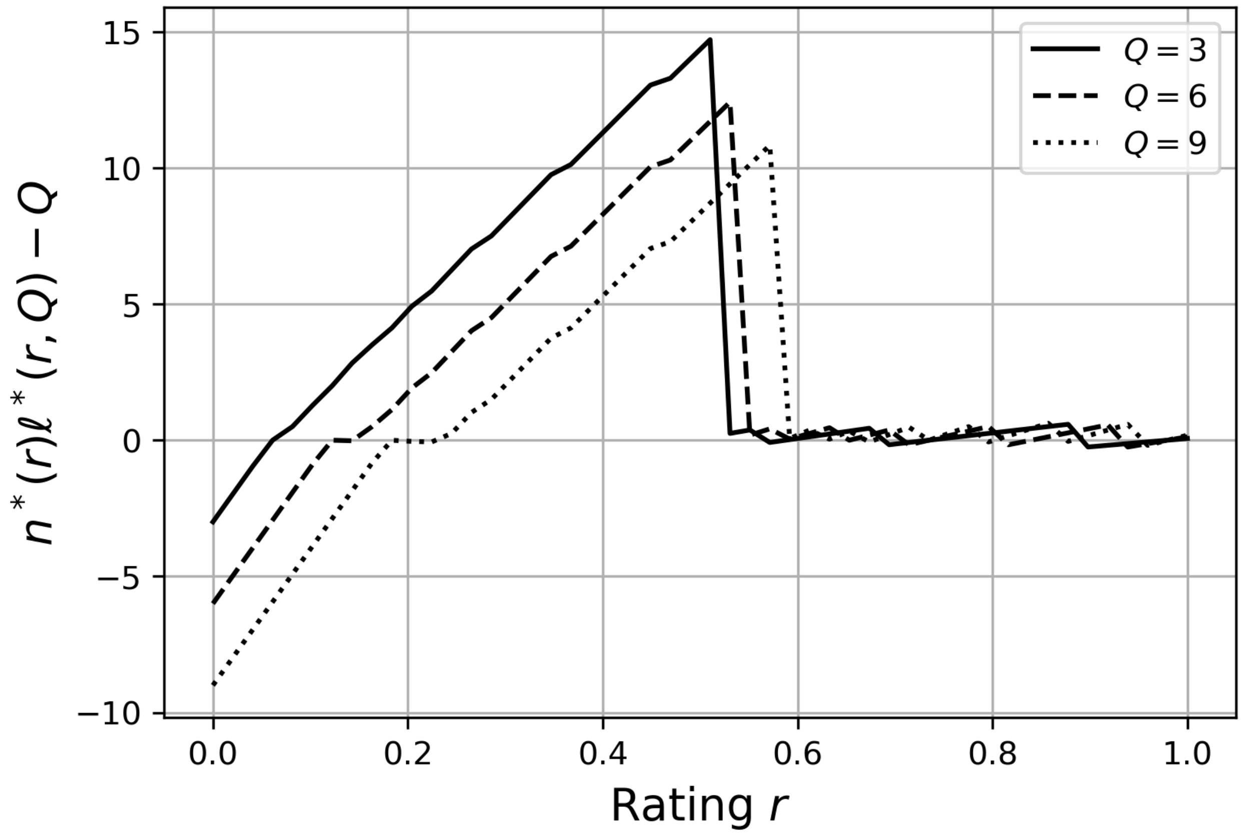
(a) Optimal bag number $n^*(r)$



(b) Optimal average bag value $\ell^*(r, Q)$



(c) Number of support points of $\phi^*(\cdot \mid \ell^*(r, Q))$



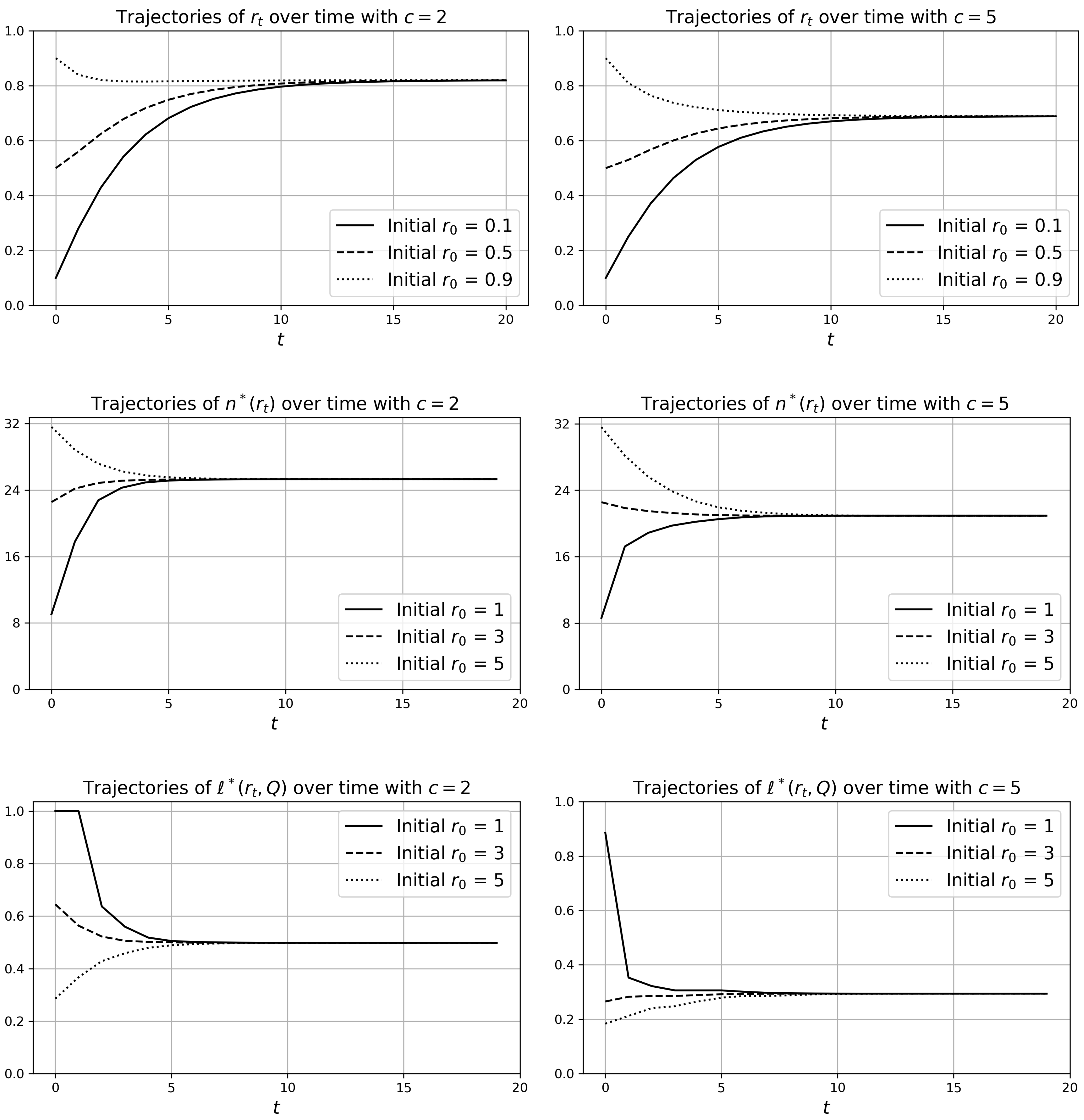
(d) Supplementary at optimality $n^*(r)\ell^*(r, Q) - Q$

Steady State Convergence

Reputation trajectory

Bag number trajectory

Average bag value trajectory



Left: Low c/p ratio Right: High c/p ratio

Policy Comparison: Revenue

Average revenues under different policies

Parameters $(c, \delta), Q \sim F$	Optimal	2-LA	DFA	Naive
$(5, 0.3), Q \sim U[0, 12]$	844.12 ± 13.91	841.82 ± 14.39	793.92 ± 10.44	787.78 ± 25.27
$(5, 0.6), Q \sim U[0, 12]$	931.70 ± 14.03	921.89 ± 14.07	870.41 ± 9.04	867.68 ± 27.23
$(10, 0.3), Q \sim U[0, 12]$	827.59 ± 21.78	826.44 ± 21.89	796.61 ± 23.44	794.27 ± 32.37
$(10, 0.6), Q \sim U[0, 12]$	910.14 ± 18.10	900.15 ± 18.10	858.10 ± 19.05	880.94 ± 25.72
$(5, 0.3), Q \sim U[3, 9]$	850.48 ± 10.77	849.39 ± 11.20	829.41 ± 7.32	849.96 ± 11.43
$(5, 0.6), Q \sim U[3, 9]$	921.94 ± 7.18	891.93 ± 7.20	916.91 ± 5.87	890.39 ± 7.64
$(10, 0.3), Q \sim U[3, 9]$	856.73 ± 8.80	855.63 ± 8.76	801.22 ± 10.99	859.46 ± 8.78
$(10, 0.6), Q \sim U[3, 9]$	936.56 ± 9.60	926.56 ± 9.60	889.64 ± 13.54	909.09 ± 9.61

Note: $\pm$ indicates the half-width of the 95% confidence interval for the estimated means of each metric.

Summary

- **DP with Two-Stage Decision Per Period**
  - Match random surplus with consumer demand driven by the store's dynamic rating
- **Optimal Food Distribution Across Bags**
  - Up to 2 types of bags is all we need
- **Practical Approximation Policies**
  - Provide clear, near-optimal structures with performance guarantees
- **Steady-State Analysis**
  - Perfect rating may not be optimal, it varies by the store type

Thank you for your attention!

Questions and comments are welcome!

Paper "Designing Surprise Bags for Surplus Foods." Available at SSRN: https://ssrn.com/abstract=5002233

Supplementary Slides

Operational Complexities

- **Non-stationary demand**
  - Sales are driven by customer ratings, which are influenced by the uncertain quantity and quality of surplus food
- **Unpredictability**
  - Amount of surplus foods are unknown until the point of sale, making it difficult to predict customer satisfaction
- **Decision under uncertain supply**
  - Stores must list available surprise bags 24 hours in advance without knowing the exact surplus quantity or value
- **Allocate content across surprise bags**
  - The common approach is to evenly distribute surplus items across all bags, ensuring a similar monetary value. However, this may not always maximize consumer satisfaction

Research Question

- **Backfires of existing heuristic approaches**
  - Cancel reservation *ex post* → Consumer complaints and penalty by platforms
  - Supplement the surplus with regular items from standard sales → High opportunity costs and additional labor costs

From Store View to Platform view

Revenue from Surprise Bags
Provides stores with salvage value for unsold surplus, essential for the long-term viability of the TGTG platform.

Platform's Role
TGTG is developing recommendation systems to help stores strategically manage surplus inventory and surprise bag design

Balancing Trade-offs
Recommendations need to balance store earnings and consumer satisfaction

Research Question

Research Question

How many surprise bags should be offered?

How much total food should be included in the bags?

How should the food be allocated across each bag?

Our Research Contribution

Supports the development of these systems by exploring optimal bag design strategies that ensure long-term profitability and satisfaction

Literature Review

- **Key drivers of food waste in the supply chain**
  - Cosmetic standards: designated channels for selling "ugly" foods
  - Expiration date: perishable inventory control, product display strategies, markdown pricing and promotion, technology-driven innovation to increase traceability (blockchain, AI)
- **Traditional channels of surplus foods**
  - Donation to food bank requires scale of similar products and involved volunteers with decentralized efforts, which do not scale well
- **Innovative surplus food management as of TGTG** ***(New and quickly growing!)***
  - Focus on clearance: Yang and Yu (2024) show how surplus food sales reduce waste and boost profits, but may lead to increased consumer-side waste
  - Our difference: We explore store reputation and the trade-offs between short-term profits and long-term reputation building in the context of TGTG

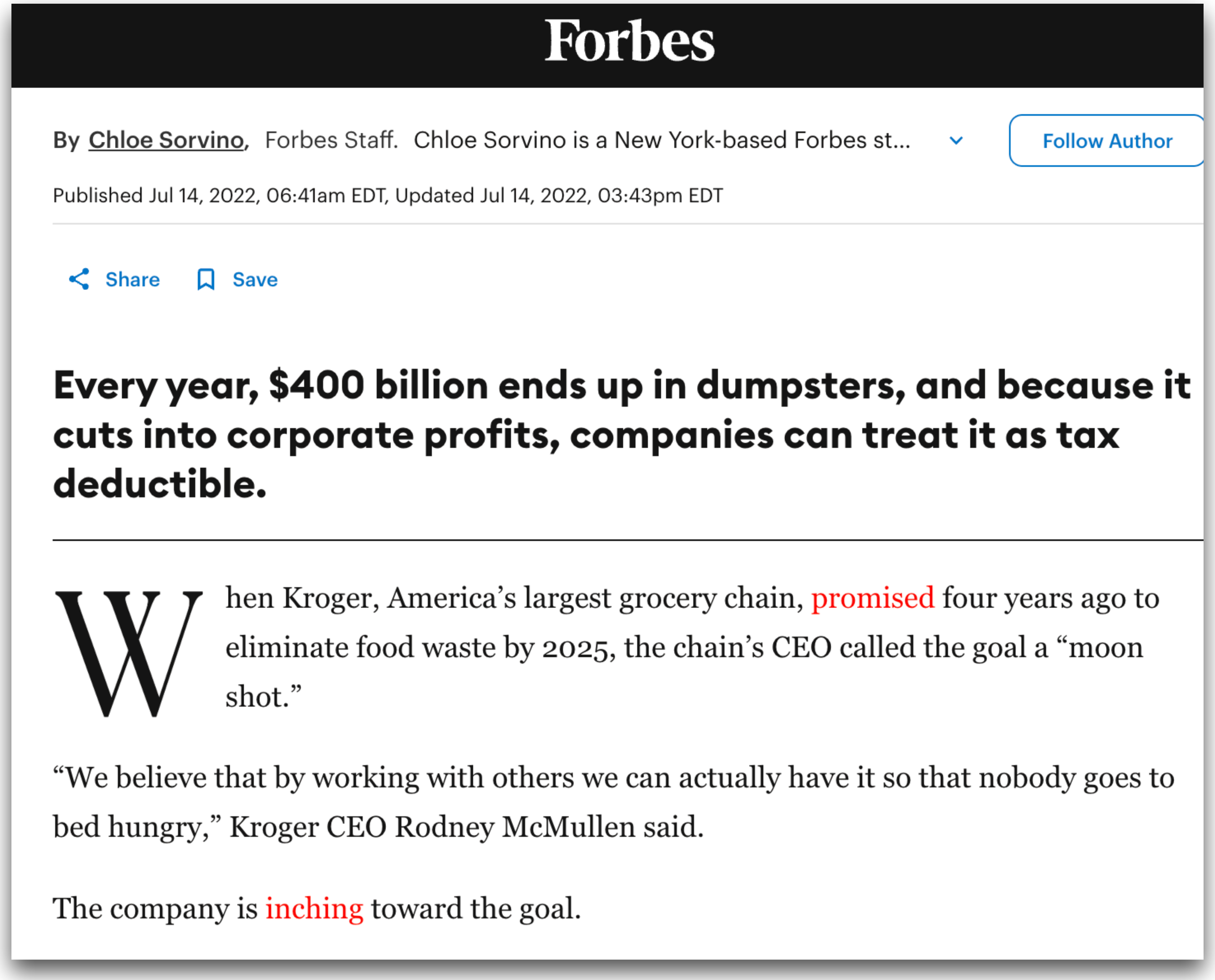
Literature Review

- **Opaque selling**
  - Research on opaque selling (where consumers cannot fully observe product attributes) contributes to understanding consumer behavior in surprise bag sales

- **Reputation management**
  - Reputation is central in our model, as customer satisfaction influences future demand. Previous research examines the role of ratings and pricing in shaping reputation

- **Positioning of our work**
  - Our paper is one of the first to examine opaque selling to reduce food waste, and the first to explore the optimal dynamic design of probabilistic goods in this context.

Side Facts..



Forbes

By Chloe Sorvino, Forbes Staff. Chloe Sorvino is a New York-based Forbes st...

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Published Jul 14, 2022, 06:41am EDT, Updated Jul 14, 2022, 03:43pm EDT

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Every year, $400 billion ends up in dumpsters, and because it cuts into corporate profits, companies can treat it as tax deductible.

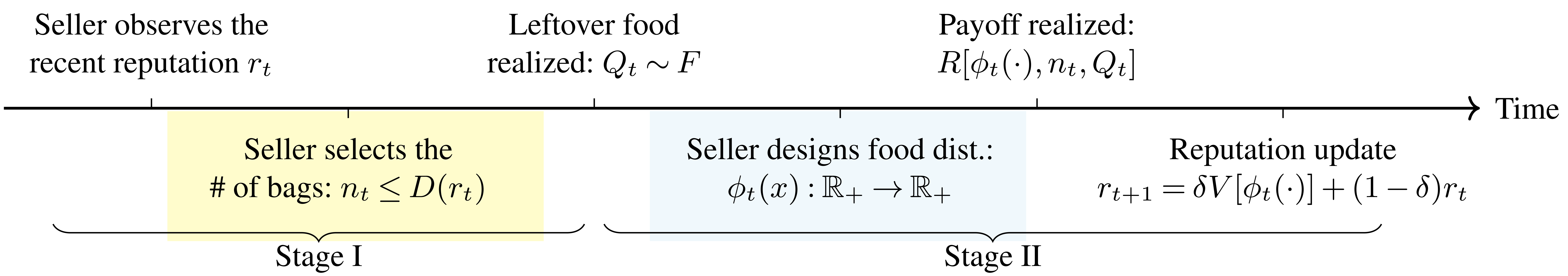
When Kroger, America's largest grocery chain, promised four years ago to eliminate food waste by 2025, the chain's CEO called the goal a "moon shot."

"We believe that by working with others we can actually have it so that nobody goes to bed hungry," Kroger CEO Rodney McMullen said.

The company is inching toward the goal.

Two-Stage Decision in Each Periods

Sequence of events in period t



- **Stage I:** **Upon observing the current reputation r_t, the store selects the number of bags to be distributed, n_t, which must satisfy $n_t \leq D(r_t)$, before the realization of random surplus Q_t**
- **Stage II:** **After observing the leftover food Q_t, the store determines the food value distribution across bags, represented by the function $\phi_t(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$, based on (r_t, n_t, Q_t)**

Lookahead Approximation

- **Optimize over finite K horizons**

$$V_t^*(r) = \max_{n \le D(r)} \quad p \cdot n + \mathbb{E}_Q \left[\max_{\ell \ge 0} \{ -C(\ell, n, Q) + \beta V_{t-1}^*(\delta \hat{v}(\ell) + (1-\delta) r) \} \right] \ (1 \le t \le K),$$
$$V_0^*(r) = 0$$

- **Let $\tilde{\pi}^{(K)} = \{\tilde{n}^{(K)}, \tilde{\ell}^{(K)}\}$ denote the K-LA policy**

Proposition Under the 1-LA policy (i.e., myopic policy), the optimal bag number is $\tilde{n}^{(1)}(r) = D(r)$ and the optimal bag value is $\tilde{\ell}^{(1)}(r, Q) = \min\{\bar{x}, Q/D(r)\}$.

Performance Bound of Lookahead Approximation

Proposition The revenue gap between the optimal policy and the K-LA policy is bounded by $J^*(r) - \tilde{J}^{(K)}(r) \leq \dfrac{\beta^K p\kappa\delta\zeta}{(1-(1-\delta)\beta)(1-\beta^K)}$.

- **Remark: the performance bound is tighter when**
  - Time discount factor β is small
  - Weight on new consumers' utility in reputation updating δ is small
  - Number of lookahead periods K is more
  - Maximal sensitivity of demand in response to reputation ζ is low
  - Maximal sensitivity of consumers' utility to food value κ is small

Optimal Bag Design

An admissible policy π consists of two decision stages

Stage I How many bags? $r \mapsto n^\pi$

Stage II How to allocate surplus food across bags? $(r, n, Q) \mapsto \phi^\pi(\cdot)$

$$\max_{\pi \in \Pi} \quad \lim_{T \to \infty} \mathbb{E}_{Q_t \sim F} \left[\sum_{t=0}^{T} \beta^t R\left[\phi_t^\pi(\cdot), n_t^\pi, Q_t\right] \right]$$ (Cumulative payoffs)

subject to $0 \le n_t^\pi \le D(r_t),$ (Upper bound reservation number)

$$\int_0^\infty \phi_t^\pi(x)\mathrm{d}x = 1,$$ (Distribution normalization)

$$r_{t+1} = \delta V[\phi_t^\pi(\cdot)] + (1-\delta) r_t \,.$$ (Reputation update)

Model: Consumer Utility and Food Distribution

- **Bag value** x
  - Combines quantity, freshness, cosmetic quality, etc.
- **Consumer utility** $v(x)$
  - General increasing differentiable function with bounded slope
- **Allocation distribution** $\phi_t(\cdot)$
  - Proportion (density) of distributed bags with value x
- **Population average utility** $V[\phi_t(\cdot)]$

$$V[\phi_t(\cdot)] := \int_0^\infty v(x)\phi_t(x)\mathrm{d}x$$

Model: Payoff

- **Supplementary cost $C[\phi_t(\cdot), n_t, Q_t]$**
  - A store may want to distribute more than surplus food available

$$C[\phi(\cdot), n_t, Q_t] = c \cdot (n_t \cdot \int_0^\infty x\phi_t(x)\mathrm{d}x - Q_t)^+$$

- **Store's revenue and food waste**
  - Exogenous price per bag p
  - Store's pay-off in a period $R_t[(\phi_t(\cdot), n_t, Q_t]$

$$R[\phi_t(\cdot), n_t, Q_t] = p \cdot n_t - C[\phi_t(\cdot), n_t, Q_t]$$

  - Store's waste in a period W_t

$$W_t = Q_t - n_t \cdot \int_0^\infty x\phi_t(x)\mathrm{d}x$$

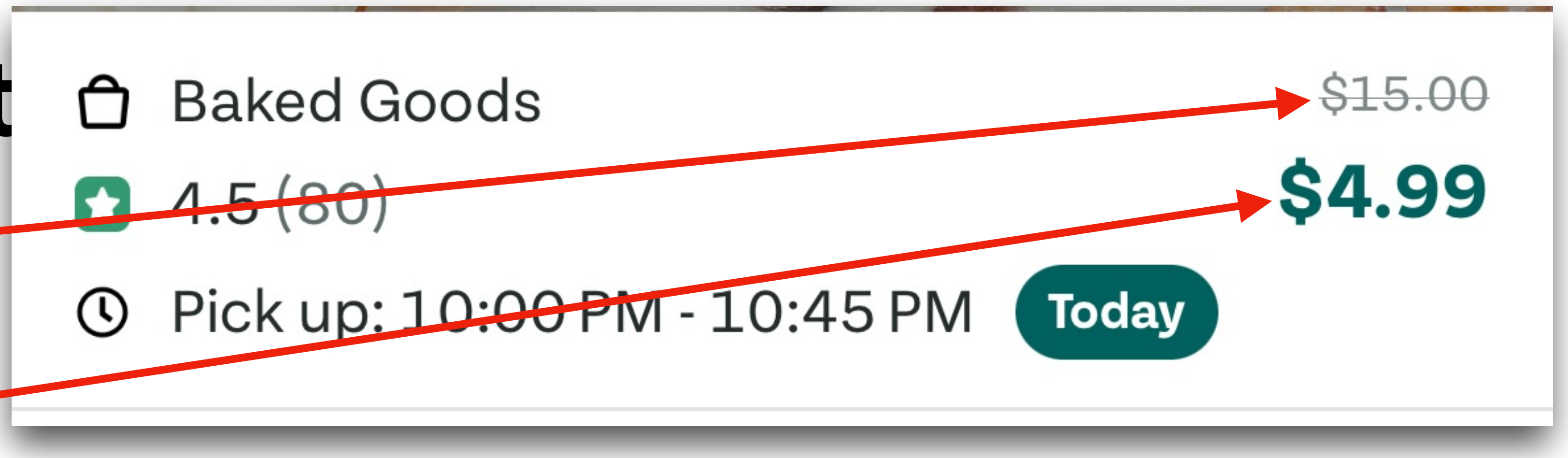
Example of Non-Concave Utility: From Prospect Theory

- **Base utility function v_0**

$$v_0(y) = y^\alpha \cdot \mathbf{1}_{y \geq 0} - \lambda(-y)^\alpha \cdot \mathbf{1}_{y<0},$$

  - $\alpha \in (0,1)$ captures diminishing sensitivity, $\lambda > 1$ captures loss aversion

- **Consumers' utility is influenced by two reference point**
  - The posted value of the surprise bag contents p_A
  - The price paid for the surprise bag p_B



- **Total utility**

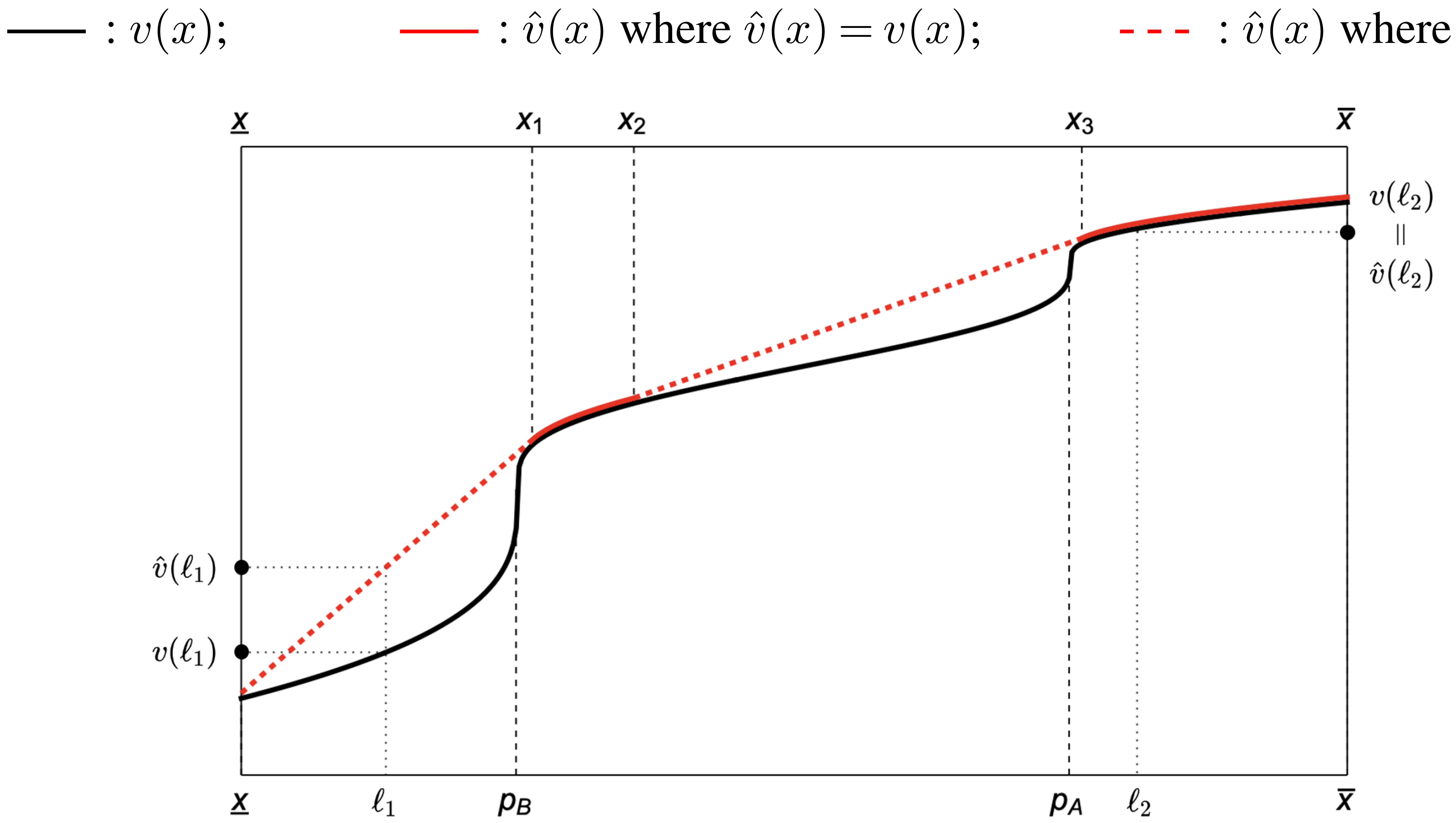
$$v(x) = (1 - \phi)v_0(x - p_A) + \phi v_0(x - p_B)$$

  - $v_0(x - p_A)$ captures the reference effects from posted value p_A
  - $v_0(x - p_B)$ captures the reference effects from price paid p_B

Distribution with Non-Concave Utility Function

- **When $v(\cdot)$ is non-concave, (at most) two types of bags are needed!**

Consumer utility function $v(x)$ and its upper concave envelope $\hat{v}(x)$



Note. $\hat{v}(x) = v(x)$ for $x \in \{\underline{x}\} \cup [x_1, x_2] \cup [x_3, \bar{x}]$, and $\hat{v}(x) > v(x)$ for $x \in (\underline{x}, x_1) \cup (x_2, x_3)$. Note that $x_1 > p_B$ and $x_3 > p_A$.

Action Space Reduction

DP formualtion

$$J^*(r) = \max_{n \le D(r)} \left[p \cdot n + \mathbb{E}_Q \left(\max_{\phi(\cdot)} \left[-C[\phi(\,\cdot\,), n, Q] + \beta J^* \left(\delta V[\phi(\,\cdot\,)] + (1-\delta) r \right) \right] \right) \right]$$

Key Property The store should choose a food distribution that maximizes customer satisfaction given any total supplementary cost, or average bag value, equivalently.

DP with action space reduction

Stage II decision can be reduced from distribution $\phi(\,\cdot\,)$ to average bag value ℓ, which uniquely determines a *conditional optimal food distribution* $\phi^*(\,\cdot\,|\,\ell)$.

$$J^*(r) = \max_{n \le D(r)} \left[p \cdot n + \mathbb{E}_Q \left(\max_{\ell \ge 0} \left[-C(\ell, n, Q) + \beta J^* \left(\delta V[\phi^*(\,\cdot\,|\,\ell)] + (1-\delta) r \right) \right] \right) \right]$$

Structure of $\phi^*(\,\cdot\,|\,\ell\,)$

$\phi^*(\,\cdot\,|\,\ell\,)$ maximizes consumer satisfaction, conditional on average bage value ℓ

$$\max_{\phi(\cdot):\mathbb{R}_+\to\mathbb{R}_+} \quad V[\phi(x)] = \int_0^\infty v(x)\phi(x)\mathrm{d}x$$

$$\textbf{subject to} \quad \int_0^\infty \phi(x)\mathrm{d}x = 1, \quad \textbf{and} \quad \int_0^\infty x\phi(x)\mathrm{d}x = \ell\,.$$

Denote the upper concave envelope of consumer utility function v by $\hat{v}$.

Proposition $\phi^*(\,\cdot\,|\,\ell\,)$ is supported by <u>either one or two Dirac points</u> and achieves a population's average utility at $V[\phi^*(\,\cdot\,|\,\ell\,)] = \hat{v}(\ell\,)$.

A store needs at most two types of bags!

One type of bag $\longrightarrow$ Distributing surplus food evenly across all bags

Two types of bag $\longrightarrow$ Differentiate bags into high and low values