Assignment 4
Phys 512 Computational Physics

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Q1:

The χ^2 the script gives for the parameters dialed in is 15267.937150261656 for 2501 degrees of freedom. This gives a probability of 0.0, which means I don't have enough precision to calculate how unlikely that this fit produced those points. So it's a bad fit.

The χ^2 you get for the closer parameters is 3272.2053559202204 for 2501 degrees of freedom. The probability that gives is 1.1493178719983565e-23. They say that $p < 0.01$ should be considered a bad fit, so I guess this isn't acceptable either even if it looks fine visually.

Q2:

To get this to work, I had to restrict τ to above 0.01.

The parameters I got are:

```
6.899999999999181455e+01
2.200000001292655091e-02
1.199999999833980674e-01
6.000000058478853482e-02
2.160832069508629684e-09
9.500000000199629868e-01
```

The problem is that my curvature matrix is going crazy for some reason. It's getting diagonal values of 10^{25}. Maybe my derivatives are really bad or something. Anyway the square root of the diagonals give the errors:

```
7.225667387430744141e+12
3.017120953036480957e+12
5.796136405545593750e+12
1.250247880447217285e+12
1.045272120905254883e+13
6.863767322006249023e+12
```

Anyway this means I can't use the curvature matrix to estimate my standard deviations for next questions, so that's not good. I made it only save the curvature matrix in the case that the χ^2 improved in case the curvature matrix is fine in that case, but it still isn't.

Q2: Bonus

I couldn't get it to converge. I could get parameters for $\Omega_b h^2 = 0.1$, but after that, even using the new parameters, I couldn't go any lower.

The parameters I got are

```
6.899339101834704024e+01
2.200119851629198098e-02
3.615389107876695873e-02
1.864143050610296623e-09
9.804538922944847634e-01
```

The χ^2 I ended up with is 4290.304484068948. This is quite a bit worse than with a higher dark matter density, so that indicates that that the dark matter density of 0.12 is a better choice.

Q3:
My parameter estimations are:

Parameter	Value
H_0	68.2 ± 0.6
$\Omega_b h^2$	0.0226 ± 0.0003
$\Omega_C h^2$	0.118 ± 0.001
τ	0.068 ± 0.003
A_s	$(2.16 \pm 0.01) \times 10^{-9}$
n_2	0.957 ± 0.005

In whatever units they're in.
Looking at how τ varies with the steps:

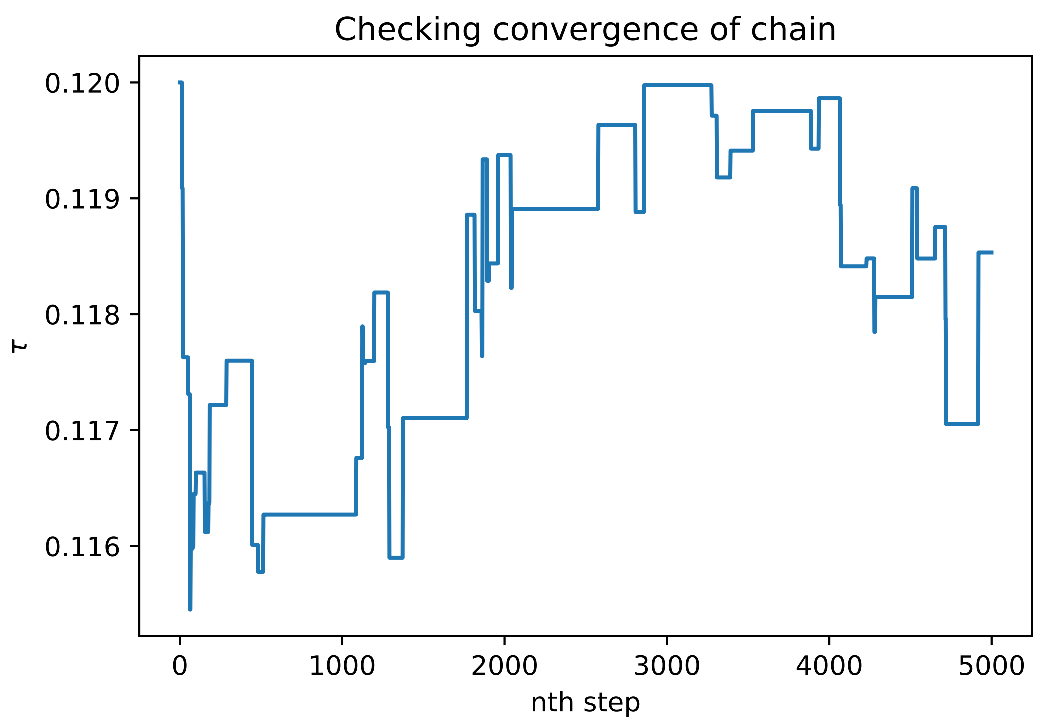

Figure 1: It's very plateau-y. For some reason there's long step intervals where τ doesn't change at all even though it should be theoretically changing about half the time.

I don't think it converges very well. That's not really what a Markov chain is supposed to look like.
Well I'll solve for Ω_Λ anyway.

$$\begin{aligned}\Omega_\Lambda &= 1 - \Omega_C - \Omega_b \\ &= 1 - \frac{\Omega_C h^2}{h^2} - \frac{\Omega_b h^2}{h^2} \\ &= 1 - \frac{0.118}{\left(\frac{68.2}{100}\right)^2} - \frac{0.0226}{\left(\frac{68.2}{100}\right)^2} \\ &= 0.697715\end{aligned}$$

And finding the error:

$$
\begin{aligned}
\Omega_\Lambda &= 1 - -\Omega_C - \Omega_b \\
&= 1 - \frac{100^2}{H_0^2}(\Omega_C h^2 + \Omega_b h^2) \\
\sigma(\Omega_\Lambda) &= 100^2 \sigma\left(\frac{1}{H_0^2}(\Omega_C h^2 + \Omega_b h^2)\right) \\
&= 100^2\left(\frac{1}{H_0^2}(\Omega_C h^2 + \Omega_b h^2)\right)\sqrt{\left(\frac{\sigma(\frac{1}{H_0^2})}{\frac{1}{H_0^2}}\right)^2 + \left(\frac{\sigma(\Omega_C h^2 + \Omega_b h^2)}{\Omega_C h^2 + \Omega_b h^2}\right)^2} \\
&= 100^2\left(\frac{1}{H_0^2}(\Omega_C h^2 + \Omega_b h^2)\right)\sqrt{H_0^4\left(\frac{1}{H_0^2}\frac{\sigma(H_0^2)}{H_0^2}\right)^2 + \frac{(\sqrt{\sigma(\Omega_C h^2)^2 + \sigma(\Omega_b h^2)^2})^2}{(\Omega_C h^2 + \Omega_b h^2)^2}} \\
&= 100^2\left(\frac{1}{H_0^2}(\Omega_C h^2 + \Omega_b h^2)\right)\sqrt{\left(\frac{\sigma(H_0^2)}{H_0^2}\right)^2 + \frac{\sigma(\Omega_C h^2)^2 + \sigma(\Omega_b h^2)^2}{(\Omega_C h^2 + \Omega_b h^2)^2}} \\
&= 100^2\left(\frac{1}{H_0^2}(\Omega_C h^2 + \Omega_b h^2)\right)\sqrt{\left(\frac{\sqrt{2}H_0\sigma(H_0)}{H_0^2}\right)^2 + \frac{\sigma(\Omega_C h^2)^2 + \sigma(\Omega_b h^2)^2}{(\Omega_C h^2 + \Omega_b h^2)^2}} \\
&= 100^2\left(\frac{1}{H_0^2}(\Omega_C h^2 + \Omega_b h^2)\right)\sqrt{2\frac{\sigma(H_0)^2}{H_0^2} + \frac{\sigma(\Omega_C h^2)^2 + \sigma(\Omega_b h^2)^2}{(\Omega_C h^2 + \Omega_b h^2)^2}}
\end{aligned}
$$

And we can plug in numbers:

$$
\begin{aligned}
\Omega_\Lambda &= 0.302285\sqrt{1.54797 \times 10^{-4} + \frac{1.09 \times 10^{-6}}{0.019768}} \\
&= 0.00438
\end{aligned}
$$

So our final estimate for Ω_Λ is 0.698 ± 0.004

Q3:
My parameter estimations from the tau constraint chain are:

Parameter	Value
H_0	68.85 ± 0.05
$\Omega_b h^2$	0.0229 ± 0.0001
$\Omega_C h^2$	0.1178 ± 0.0009
τ	0.0613 ± 0.0008
A_s	$(2.14 \pm 0.01) \times 10^{-9}$
n_2	0.955 ± 0.001

In whatever units they're in.
Looking at how τ varies with the steps:

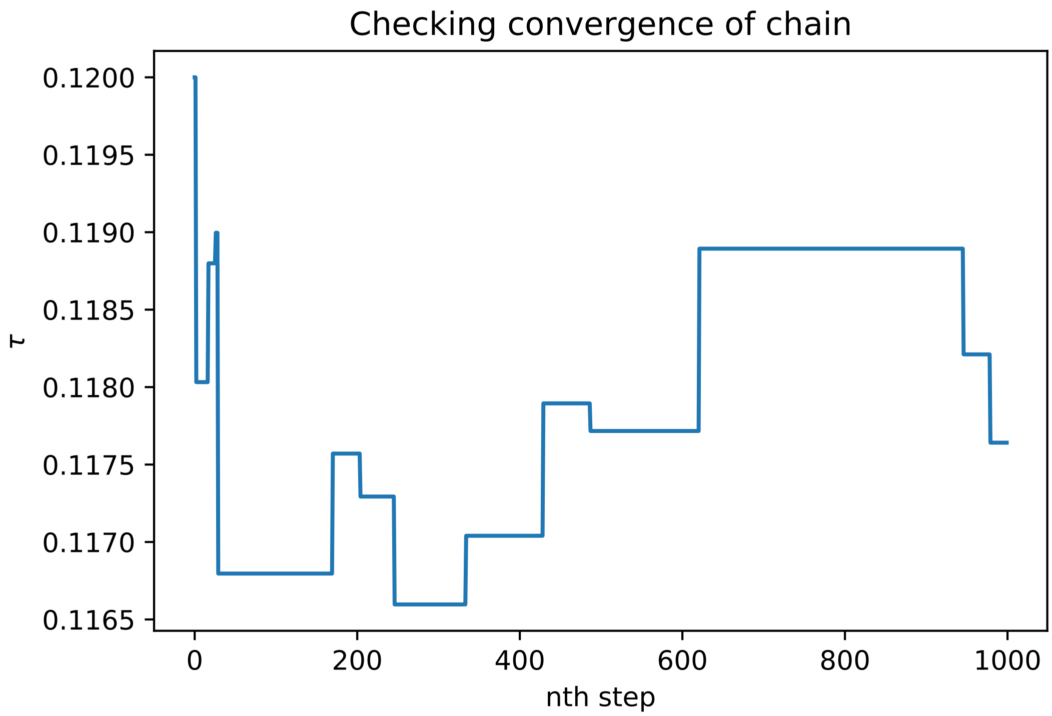

Figure 2: It's also very plateau-y. I had to take a lot fewer steps this time, but it looks kind of like the first one.

Importance sampling:
Our old probability was just $e^{-\frac{1}{2}\chi^2}$, but our new probability is $e^{-\frac{1}{2}\chi^2}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(\tau-\tau_0)^2}$. Dividing them gives our weights of $\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(\tau-\tau_0)^2}$. I guess we have to normalize that too.
My parameter estimations from the importance sampling are:

Parameter	Value
H_0	68.9 ± 0.6
$\Omega_b h^2$	0.0227 ± 0.0003
$\Omega_C h^2$	0.117 ± 0.001
τ	0.0655 ± 0.003
A_s	$(2.15 \pm 0.01) \times 10^{-9}$
n_2	0.954 ± 0.005

These are actually closer to the chain from question 4 than the chain from question 3.