Assignment 6
Phys 512 Computational Physics

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Q1:
a)
Window function:
I chose the Tucky Window because it's flat in the middle and relatively simple. `en.wikipedia.org/wiki/Window_function#Tukey_window`
It is defined by (for some $0 < \alpha < 1$):

$$\text{Tukey Window}(n) = \begin{cases} \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{\alpha N}\right)\right) & 0 \leq n < \frac{\alpha N}{2} \\ 1 & \frac{\alpha N}{2} \leq n \leq N - \frac{\alpha N}{2} \\ \frac{1}{2}\left(1 - \cos\left(\frac{2\pi(N-n)}{\alpha N}\right)\right) & N - \frac{\alpha N}{2} < n < N \end{cases}$$

I'm going to pick $\alpha = 0.5$ because why not. We want a flat window like this because the information we want to keep is in the center of the data. If we curve too much in the middle, it might reduce the magnitude of the hit.

For the smoothing/dealing with lines, our goal is to reduce the big spikes. Usually when you want to reduce a spike by smoothing, we take a median of the surrounding points, which discards the spike completely. The question is whether this method will discard way too much information at the small level. I guess I'll just commit to it and see what happens.

To be more specific, I'm taking the median of 10 points around each point in the nft. It looks like this is pretty good for reducing spikes. Plotting the unsmoothed and smooth looks like:

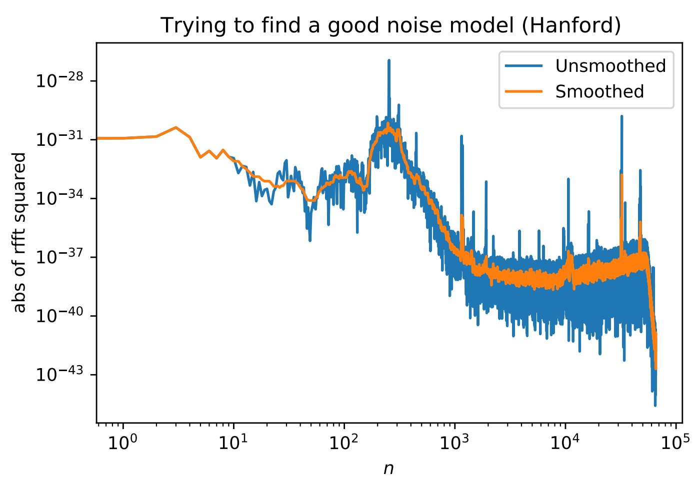

Figure 1: There's a visible difference pretty much everywhere between the smoothed and unsmoothed, which is concerning. Hopefully there's not too much information being lost here. On the bright side, those spikes are completely gone or a lot smaller.

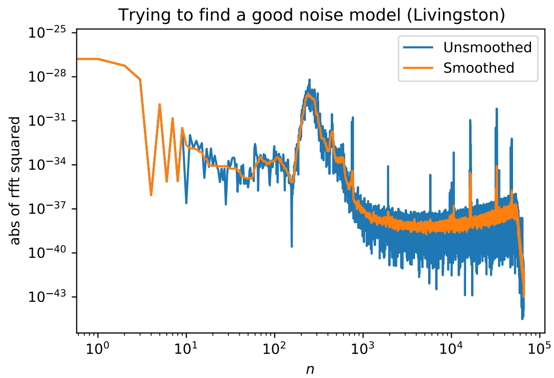

Figure 2: Here's the Livingston one. It's pretty similar.

b) Here are the plots I get:

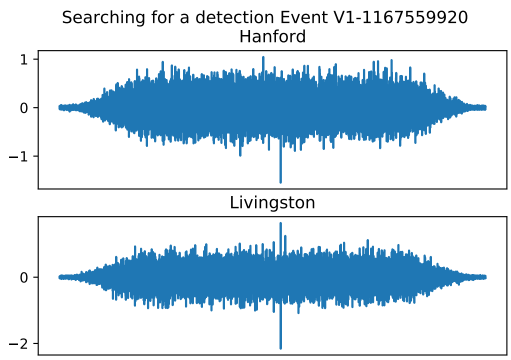

Figure 3: I think there's a detection here. There's a big spike downward for Hanford and spikes visible for Livingston

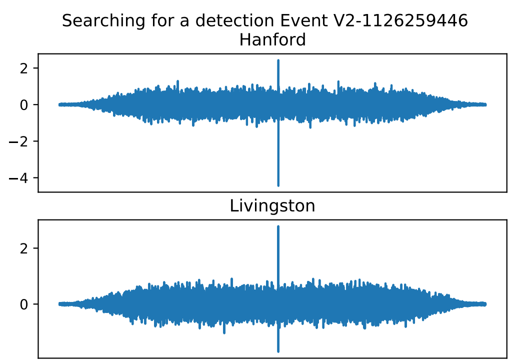

Figure 4: This is the nicest detection. You can easily see spikes for both.

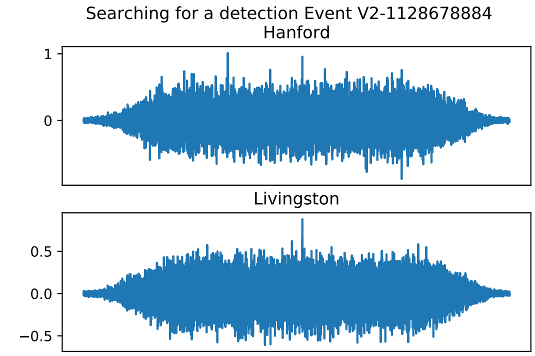

Figure 5: This one is a little more sketchy. Hanford has a few spikes that are the same height as the one in the middle.

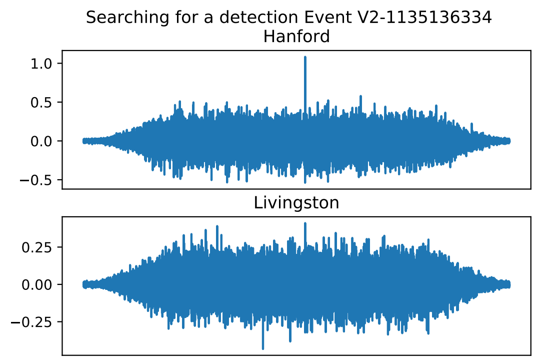

Figure 6: This one's also sketchy, but the spike for Hanford is pretty high.

c)

For the SNR of the combined event, I'm using that $\mathrm{SNR}^2 = \chi^2$, which means I can find the combined SNR by adding in quadrature.

The SNRs:

Event	Hanford	Livingston	Combined
V1-1167559920	9.5	11.6	14.9
V2-1126259446	21.4	17.8	27.8
V2-1128678884	7.81	8.48	11.5
V2-1135136334	12.2	7.54	14.3

Wow the second one is pretty good. Even the third, which is the worst, is more than 5σ.

d) For the analytic noise, I'll use

$$\text{SNR} = \sigma_{\text{analytic}} = \sqrt{\sum \frac{\text{fft(template)}^2}{\text{Noise}}}$$

I just kind of took this from the Slack. This is how I interpret what it's supposed to be anyway. There's also something called the mode that's supposed to be in this equation, bu I have no clue what that could mean, so I'm going to ignore it. Also I divided my SNR^2 by N because that got me down to the right order of magnitude.

The SNRs I get seem to be different by a factor.

Anyway the comparison table is this:

Event	Analytic	From Scatter
V1-1167559920 Hanford	36.3	9.5
V1-1167559920 Livingston	45.5	11.6
V2-1126259446 Hanford	43.3	21.4
V2-1126259446 Livingston	38.0	17.8
V2-1128678884 Hanford	29.5	7.81
V2-1128678884 Livingston	26.5	8.48
V2-1135136334 Hansford	20.5	12.2
V2-1135136334 Livingston	14.1	7.54

One reason that they're off be that our noise model isn't great, so that would mean that this method of calculating the SNR would be inaccurate. Another problem could be that there's some factor difference that I don't know about.

e) For e, I'm saying this is whenever the cumulative sum (the sum is same as above) reaches have the value of the analytic SNR^2. Here are the indexes in the frequency array when that happens for each event:

Event	Index
V1-1167559920 Hanford	2980
V1-1167559920 Livingston	2425
V2-1126259446 Hanford	3190
V2-1126259446 Livingston	3685
V2-1128678884 Hanford	2439
V2-1128678884 Livingston	3044
V2-1135136334 Hansford	2279
V2-1135136334 Livingston	3415

These are all way under half.

f) I couldn't figure out how to do this one. You want to find the uncertainty in your horizontal shift of your matched filter. Your matched filter indirectly gives the χ^2 value for every time t. It must be possible to get some information from the peak of the matched filter that tells you the uncertainty in your Δt, where Δt is the horizontal shift in the matched filter.

We know that the distance between the two in space (Δx) and time (Δt) are related linearly:

$$\Delta x = c\Delta t$$

This implies that their relative uncertainties are the same.

$$\frac{\sigma(\Delta x)}{\Delta x} = \frac{\sigma(\Delta t)}{\Delta t}$$

Once you figure out $\sigma(\Delta t)$, you can solve for $\sigma(\Delta x)$:

$$\sigma(\Delta x) = \frac{\sigma(\Delta t)}{\Delta t}(\text{few thousand km})$$