# Assignment 2
# Phys 512 Computational Physics

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**Q2:**

I'm going to do this assignment out of order. Question 1 says to use "your integrator" without initializing what "your integrator" is, so I might as well use the one I'm writing in question 2.

For the initial call of the integrator, we compute the function 5 times. There's no way around that. But each subdivision uses three of five the y values calculated for the previous iteration, so we only need to calculate two more for each subdivision.

I made it count how many function calls each one gave. This is the output:

```
For a function that gets more complicated closer to 0, it saved 960 calls
For exp, it saved 192 calls
For a fourth order polynomial, it saved 762 calls
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(By the way the first function is this)

$$f(x) = \sin\left(30e^{-x^2}x\right)$$

I guess 960 is a lot if you're integrating a lot of functions or if your function is really complicated. And maybe it increases by a lot for way smaller tolerance.

**Q1**:

First I'll figure out what the integral's going to be.

Define the $z$ axis to be between the test point and the center of the sphere shell, and we're going to split the sphere shell into rings centered on that axis for maximum symmetry. Let's assume the shell has uniformly distributed charge with $\sigma$ being the charge per area.

Then the line charge $\lambda$ of each ring of infinitesimal height $dz$ is $\frac{R}{r}\sigma dz$. We can plug that in the equation for the electric field of a point on the axis of the ring a distance $d$ away from it. I'll ignore the direction vector since that would only depend on the sign of the charge anyway.

$$\begin{aligned} E_{ring(z)}(d) &= \frac{1}{4\pi\epsilon_0}\frac{\lambda(2\pi r(z))d}{(r(z)^2+d^2)^{3/2}} \\ &= \frac{\sigma R}{2\epsilon_0}\frac{d}{(r(z)^2+d^2)^{3/2}}dz \end{aligned}$$

If the shell's center is at the origin, then the radius of each ring is $r(z) = \sqrt{R^2 - z^2}$, $-R \leq z \leq R$.

Also, this relative distance $d$ is $z_0 - z$ where $z_0$ is the position of your test point

Then the total electric field at is

$$E_{total}(z_0) = \frac{\sigma R}{2\epsilon_0}\int_{-R}^{R}\frac{(z_0 - z)}{(R^2 - z^2 + d^2)^{3/2}}dz$$

For computing, we'll use units of charge such that $\frac{\sigma}{2\epsilon_0} = 1$ and units of length such that $R = 1$. This leaves

$$E_{total}(z_0) = \int_{-1}^{1}\frac{(z_0 - z)}{(1 - z^2 + (z_0 - z)^2)^{3/2}}dz$$

Integrating and plotting the both integrals gives:

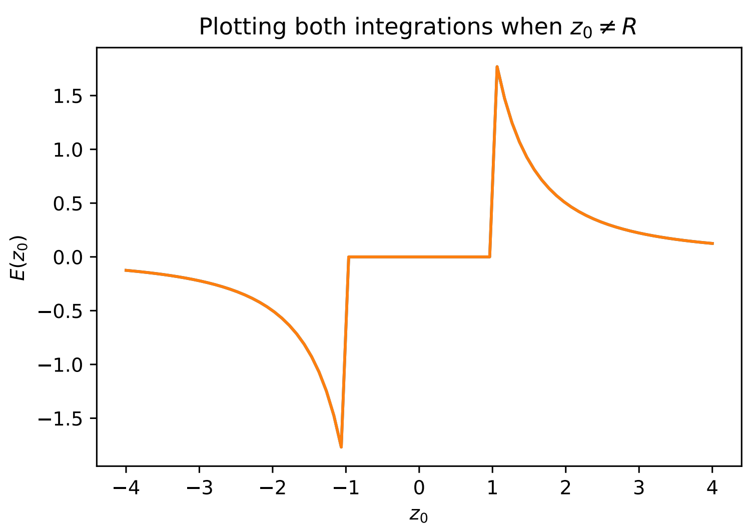


Figure 1: Both integrals are on top of each other

There is a singularity in this integral when $z_0 = 1$. Because then on the bottom we have $1 - z^2 + (1 - z)^2$. So when $z = -1$ or $z = 1$, we get $0/0$. The top and bottom don't cancel because of the $3/2$ power.

Quad doesn't care about this singularity but my adaptive integral crashes my Python when I try it.

**Q3:**

I don't know how many terms I need. I couldn't tell the difference visually after 4, but my printouts were only fully accurate when I went to 20 terms, so I'll stick with that.

The function works.

Bonus:

Legendre gave me similar error.

The RMS of chebyshev is: 1.357785312221 99e-15

The RMS of legendre is: 1.5218386748013602e-15

So same order, but Chebyshev was a little better.