Final Project

The Water Table as a Subdued Replica of Topography

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Introduction

This project explores the validity of assuming groundwater is a subdued replica of topography within the scope of East-Central Florida, around St. Johns River. The study site is a 120 square mile region centered on the town of Geneva. Because groundwater is naturally influenced by the terrain within which it is contained, the water table often follows terrain contours (Fetter, 2001; Toth, 1963; Haitjema & Mitchell-Bruker, 2005). In Florida, this idea has been applied as a generalized assumption about the unconfined surficial aquifer to construct groundwater models where actual water table elevation data was difficult to obtain at sufficient detail (Rios, 2010; Arthur, Baker, Cichon, Wood, & Rudin, 2005; Sepulveda N. , 2002). Else, it has been used as supplementary data in estimating water tables (Sepulveda N. , 2003; Cooper, Zhang, & Selch, 2015).

Is the assumption that groundwater can be modeled using topographic information reasonable? To test this, a simple groundwater model will be constructed using only topographic and hydrographic information as input. The groundwater model will be a steady-state groundwater model based on the Jacobi iterative solution to the finite difference expression of the Laplace equation, and will be scripted and packaged as an easy to use tool in ArcGIS using the Python programming language. Hydrography will be used to determine boundaries in the model. Topographic information from LiDAR will be used as the “initial guesses” of unknown groundwater head for this model. If the assumption that groundwater is a subdued replica of topography is valid, such a model would prove to be an accurate representation of groundwater. It is hypothesized that the particular model proposed here would accurately provide slope information that could be used in groundwater velocity calculations, but would not be able to predict the water table heads accurately.

Most areas of Florida are covered with sufficiently dense water level monitoring stations and have official, publicly available groundwater models to enable evaluation of this test. The St. John’s River Water Management District (SJRWMD) and the South Florida Water Management District (SFWMD) provide water level measurement data to the public for free, and groundwater models based on this data by request. The latest groundwater model for the study area was produced by SJRWMD and SFWMD, and is called the East Central Florida Transient Model (ECFT). ECFT is a model based on MODFLOW, which is based on a much more complex solution to the finite difference equation, incorporating monitoring station measurements, recharge, evaporation, hydraulic conductivity, etc. LiDAR data for this region can be obtained for free from the National Oceanic and Atmospheric Association (NOAA). In this study, LiDAR elevation data will be used as input to create a simple groundwater model based on the Jacobi iterative solution to the finite difference equation, and the performance of this model will be assessed by comparing the results with the official ECFT groundwater model. Because LiDAR will provide all input for the initial conditions of the model (hydrography will be used to delimit boundary conditions), this model assumes that groundwater is a subdued replica of topography.

Next is an overview of the governing characteristics and general behavior of groundwater, followed by a literature review of Florida based models using topography to model groundwater, and a description of the Jacobi iterative solution of the finite difference equation.

A Theoretic Background That Relates Water Table and Groundwater Flow to Topography

Groundwater is defined by Fetter as all the water contained in interconnected pores located below the water table in an unconfined aquifer, or water located in a confined aquifer (Fetter, 2001). The water table is the surface of water at which pore water pressure is equal to atmospheric pressure, and everything beneath the water table is considered to be in the saturated zone, where all pores are filled by water. The unsaturated vadose zone lies above the water table and below the ground surface. An aquifer is any geologic unit that can store and transmit water at high rates, and multiple aquifers are often found layered one on top of another, separated by confining layers with little or no permeability. An aquifer with continuous layers of highly permeable material extending from the land surface down to some confining layer is called an unconfined aquifer. A confined aquifer is one where a continuous layer of permeable material is sandwiched between two confining layers. In this research, the unconfined aquifer is of interest.

As demonstrated by Henry Darcy in 1856, Groundwater generally behaves as you would expect: it flows from points of high elevation to points of low elevation, and flows slower through materials with less conductive properties (Darcy, 1856). This concept is expressed mathematically in Darcy's well-known law:

$$Q = -KA\left(\frac{dh}{d\ell}\right) \qquad \textbf{\textit{Equation 1}}$$

Where Q is discharge in volume per unit time, K is the hydraulic conductivity or permeability of a porous material, A is the cross sectional area through which groundwater flow occurs, and $\frac{dh}{d\ell}$ is the hydraulic gradient (change in head h divided by change in length ℓ between two points).

This work by Darcy was later expanded on by Hubbert to include such things as fluid viscosity and pressure (Hubbert, The Theory of Ground-Water Motion, 1940; Hubbert, Dary's Law and the Field Equations of the Flow of Underground Fluids, 1957). Under Hubbert's additions, discharge becomes a function of intrinsic conductivity K_i, dynamic viscosity u, and specific weight γ, in addition to cross sectional area A and hydraulic gradient $\frac{dh}{d\ell}$.

$$Q = -\frac{K_i \gamma A}{u}\left(\frac{dh}{d\ell}\right) \qquad \textbf{\textit{Equation 2}}$$

And head (potential) is defined as (Wang & Anderson, 1982):

$$h = \frac{P}{p_w g} + z \qquad \textbf{\textit{Equation 3}}$$

Where P is water pressure, p_w is the density of water, g is the acceleration of gravity, and z is the elevation head. Head h defined by this equation is the water table elevation. It takes further mathematics to model groundwater, and some of the best current methods utilize a numerical solution of the Laplace equation (Wang & Anderson, 1982).

Following is a quick overview of the Laplace equation paraphrased from the description provided by Wang (1982), for more details refer to his book *Introduction to Groundwater Modeling.* Darcy's law can be modified to yield a vector, the darcy velocity q, in a single direction. Usually, we take this to be the x-axis, and we denote this as q_x.

$$q_x = -K\frac{dh}{dx}$$ ***Equation 4***

If a surface of hydraulic head, h, is defined in more than one dimensions, such as $h(x, y, z)$, Darcy's law still applies, such that we end up with darcy velocity vectors along the y, and z axis as well. This can be expressed using the del (gradient) operator of vector calculus (Wang & Anderson, 1982).

$$q = -K\nabla h = -K\frac{\partial h}{\partial x}, -K\frac{\partial h}{\partial y}, -K\frac{\partial h}{\partial z}$$ ***Equation 5***

Note that we now use partial derivatives because head is now a three-dimensional function (Wang & Anderson, 1982). When we then apply continuity rules to this equation, requiring that the amount of water flowing into an elemental volume is equal to the volume of water flowing out, we end up with the Laplace equation (***equation 6***).

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$ ***Equation 6***

The Laplace equation governs groundwater flow through an isotropic, homogeneous aquifer under steady state conditions, and has been very instrumental in modeling groundwater in two or three dimensions (Wang & Anderson, 1982). One application of this formula is the finite difference equation in two dimensions (***equation 8***) for determining groundwater head of a two dimensional surface. The finite difference equation in two dimensions will be used later on to model groundwater based on topographic input data.

With these developments in groundwater research, the basic principle of groundwater flow discovered by Darcy stand the test of time, although we now say that groundwater flows from points of high pressure to points of low pressure, and flows slower through materials depending on the permeability of a material and the fluid's viscosity. Toth, in *A Theoretical Analysis of Groundwater Flow in Small Drainage Basins* (1963), uses the principles discovered by Darcy and Hubbert to develop a two dimensional (X, Z) mathematical model for a small drainage basin wherein groundwater is influenced by topography (represented by a slightly tilted, gently sloping sinusoidal wave). Upon analyzing the results, he presents several theories relating topography to groundwater flow. His theory classifies groundwater flow within a small drainage basin into three different flow systems (**Figure 1**); local, intermediate, and regional. All three are tied to the topography of the basin, such that (Toth, 1963):

1. A local flow system is bounded by topographic highs at either side
2. An intermediate flow system incorporates several topographic highs and lows but does not necessarily include the highest and lowest elevations of the drainage basin.
3. A regional system occupies the highest elevations at the water divide, and discharges at the lowest elevation at the bottom of the basin.

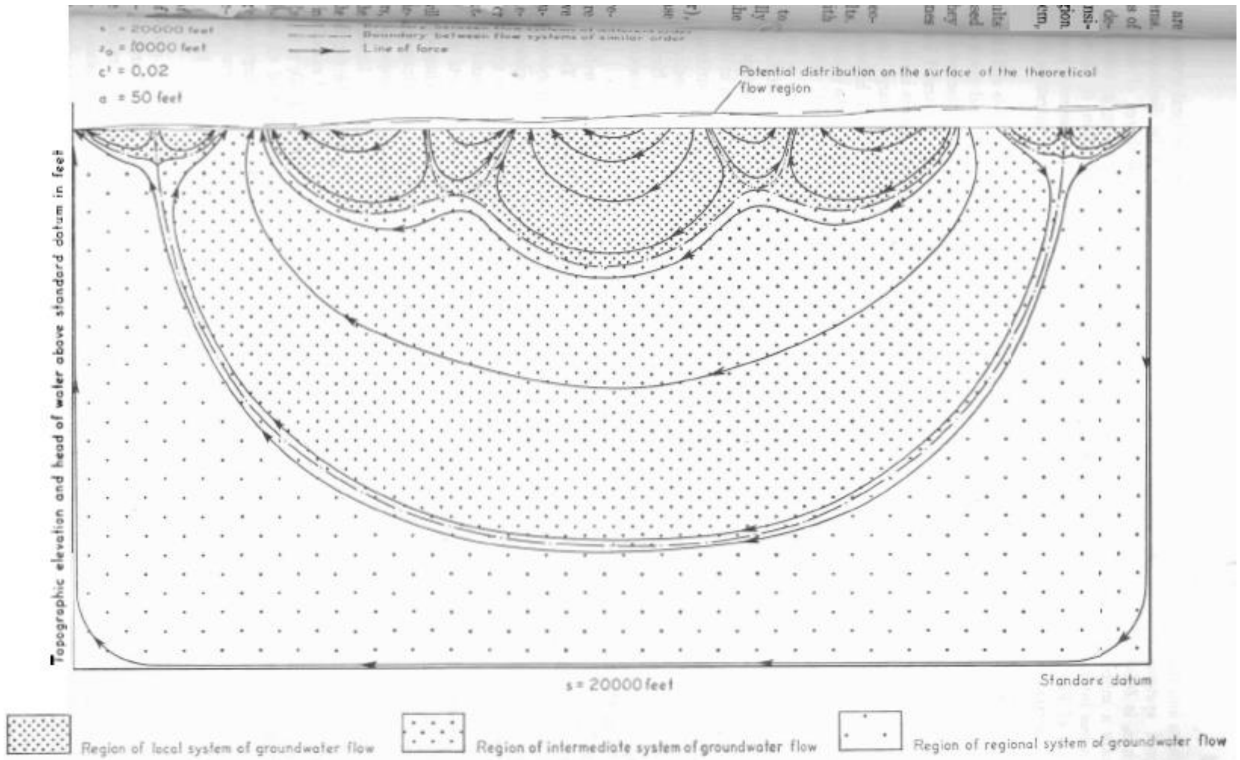


Figure 1: *Toth's theoretical flow pattern and boundaries between different flow systems, showing local, intermediate, and regional flow (Toth, 1963).*

Thus we can observe, as Fetter notes, that unconfined groundwater will generally flow away from higher topography (higher pressure) to discharge at lower topography (lower pressure), such that the water table shape will generally follow the topography (Fetter, 2001). However, it is also stated that exceptions occur, for example, in arid regions.

When is it Valid to assume that the Water Table is a Subdued Replica of the Water Table

As noted in the last paragraph of the previous section, the water table does not always follow topography. Haitjema suggests that the concept of the water table being related to the overlying topography may be valid for some regions, but not universally (Haitjema & Mitchell-Bruker, 2005). To argue this point, Haitjema notes that several studies have found that the water table is poorly correlated to topography or entirely unrelated in their study area. He also shows that Toth's specification of the water table as a subdued replica of the topography can lead to false assumptions. He contrasts Toth's approach of specifying the water table in advance (**Figure 2**) with an approach wherein the water table head is obtained through solution of the governing equation (**Figure 3**), and finds the latter to be a more realistic representation in many situations. While both generally follow the topography, Toth's approach leads to multiple points of discharge between each mound in the simulated landscape – a result of specifying the water table as a replica of the surface topography. The following graphs showcase Haitjema's results:

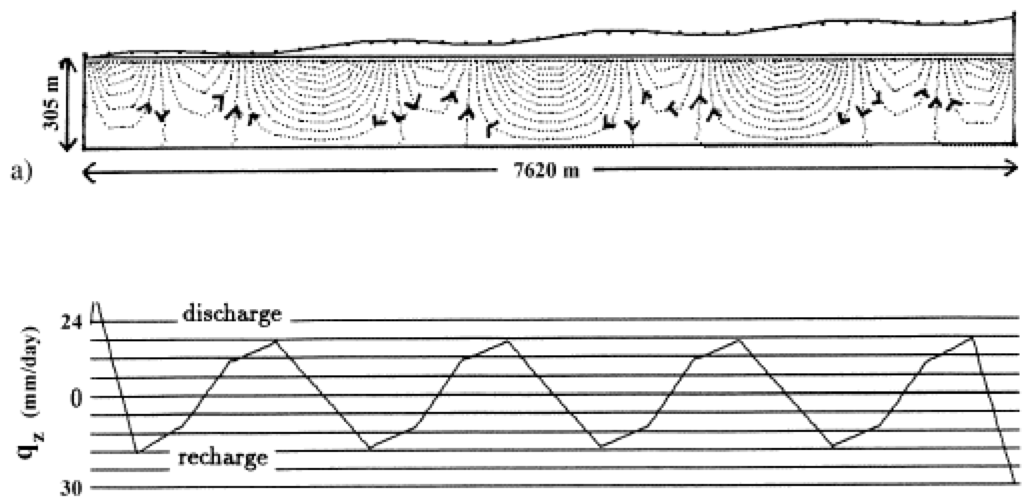


Figure 2: *Toth's approach leads to multiple areas of discharge for every mound occurring in the water table, which is assumed to be known in advance as a replica of topography. Graphic from "Are Water Tables a Subdued Replica of Topography?" (Haitjema & Mitchell-Bruker, 2005).*

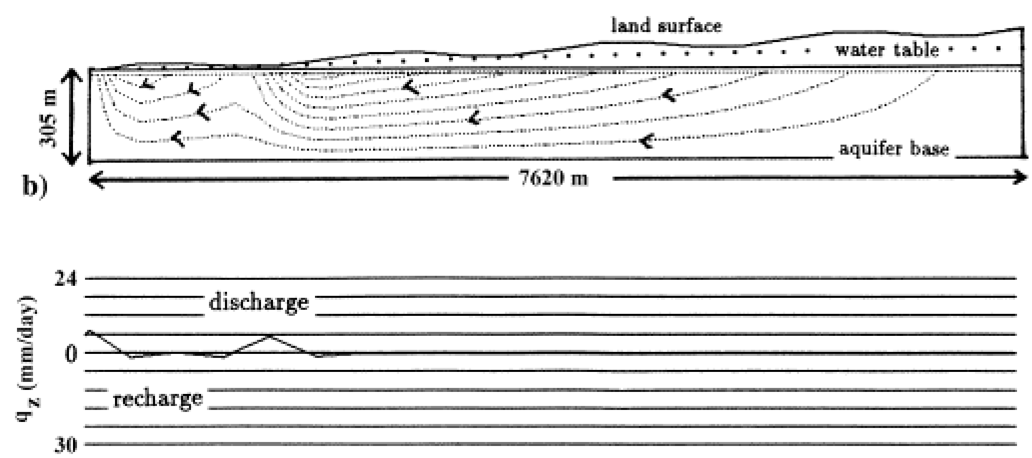


Figure 3: *In an approach to modeling groundwater flow where the water table follows from solution, the water table only reaches the surface to discharge near the valley bottom, and the discharge limits the rise of the water table. Graphic from "Are Water Tables a Subdued Replica of Topography?" (Haitjema & Mitchell-Bruker, 2005).*

Haitjema sets out to discover under what circumstances we can assume that the water table is a replica of topography, and uses the Dupuit-Forchheimer approximation to show that water table mounding is dependent on the distance between hydrological boundaries (surface waters), and the ratio of recharge rate R and hydraulic conductivity K (Haitjema & Mitchell-Bruker, 2005). The mounding of groundwater, such as that shown in **Figure 2** occurs when the recharge rate is high and the hydraulic conductivity is low.

Note that Haitjema's research does not debunk Toth (Haitjema & Mitchell-Bruker, 2005). It shows that generally, the water table and topography are always in some way related. Some areas, however, are more influenced by topography than others, and this depends on various factors, most prominently the rate of recharge and the hydraulic conductivity. In any case, it is likely not correct to assume that the water table is completely a replica of topography, because the gradient of the water table will likely be less than the gradient of the topography (Haitjema & Mitchell-Bruker, 2005). The validity of such an assumption is heavily dependent on the local aquifer properties and recharge conditions. Next is a review of literature wherein such an assumption has been applied to model groundwater in Florida.

Literature Review

South Florida Groundwater Models that assume a Topographic Relationship

When data is sparse and to improve accuracy, or to simplify the process of creating a groundwater model, several researchers have utilized the assumption that the water table is either a replica or closely related to the overlying topography to generate water table elevation maps (Arthur, Baker, Cichon, Wood, & Rudin, 2005; Rios, 2010; Sepulveda N. , 2002; Sepulveda N. , 2003; Cooper, Zhang, & Selch, 2015; Wang, et al., 2013). Two approaches have been made here, with one approach simply applying several filtering processes to a bare-earth digital terrain model (DTM) (Rios, 2010; Wang, et al., 2013), and the other being a spatial regression modeling method developed by Sepulveda that incorporates terrain elevation (Arthur, Baker, Cichon, Wood, & Rudin, 2005; Sepulveda N. , 2002; Sepulveda N. , 2003; Cooper, Zhang, & Selch, 2015).

The method presented by Rios is intended to be a simplified groundwater model for use in a screening model for nitrogen loading of surficial water features through subsurface discharge of septic tank plumes (Rios, 2010). It is meant to be as easy as possible to implement by the layman and therefore lacks in technical complexity. First, a rule of thumb index presented by Hiatjema is used to verify that the study region's water table is likely a replica of the overlying topography. Next, a DTM is processed by first applying a smoothing filter. The smoothing filter applied was a 7x7 cell averaging kernel, otherwise known as a moving window average. Any sinks within the smoothed filter are then removed using the ArcGIS "Fill" tool. Then the ArcGIS "flow direction" tool is used to assign a slope to flat areas, because the filling of sinks produces flat areas which would otherwise produce a stagnant body of groundwater. The entire task of processing the DTM is accomplished within a larger nitrogen loading toolset programmed for ArcGIS, named ArcNLET. The end user must simply select the DTM used as input and specify the number of iterations run by the smoothing filter. The author suggests using a coarse resolution DTM because the water table will usually not exactly follow the water table. This instinct is collaborated by the conceptual understanding we have gained in the previous sections from Darcy, Toth, Haitjema, and Fetter.

The tool ArcNLET was used at two neighborhoods in Jacksonville, Florida to estimate the mass loading of nitrogen into the Lower St. Johns River Basin (Wang, et al., 2013). Actual water table elevation measurements taken from several groundwater monitoring wells run by SJWMD are used to calibrate the estimated water table. The DTM is processed through the smoothing filter with varying iterations as described above until the estimated head correlates well to averaged observations from groundwater monitoring wells. This method achieves high correlation coefficients of more than 0.9.

Sepulveda takes a different approach entirely to estimating water table elevation, but also relies on topographic information. He uses a multiple linear regression in conjunction with a "minimum water table" (Sepulveda N. , 2003). The minimum water table is interpolated strictly from measured altitude at drains such as streams and lakes by fitting quantic polynomials of continuous first and second derivatives between neighboring nodes of measured elevation at lakes, river stages, or the ocean shoreline. Two independent variables are used in the regression model: the minimum water table ($MINWT_i$) and the vertical distance between land surface altitude (LSA_i) and the minimum water table ($LSA_i - MINWT_i$). The dependent variable is the water table (WT_i).

$$WT_i = \beta_1 MINW_i + \beta_2 (LSA_i - MINWT_i) + \varepsilon_i \qquad \textit{Equation 7}$$

Where β_1 and β_2 are the dimensionless regression coefficients, and ε_i is the residual error. The land surface elevation (LSA_i) variable is based on a DTM. The water table elevation at any location i can then be calculated using the formula above. As such, this method is a topography-controlled method **Figure 4**.

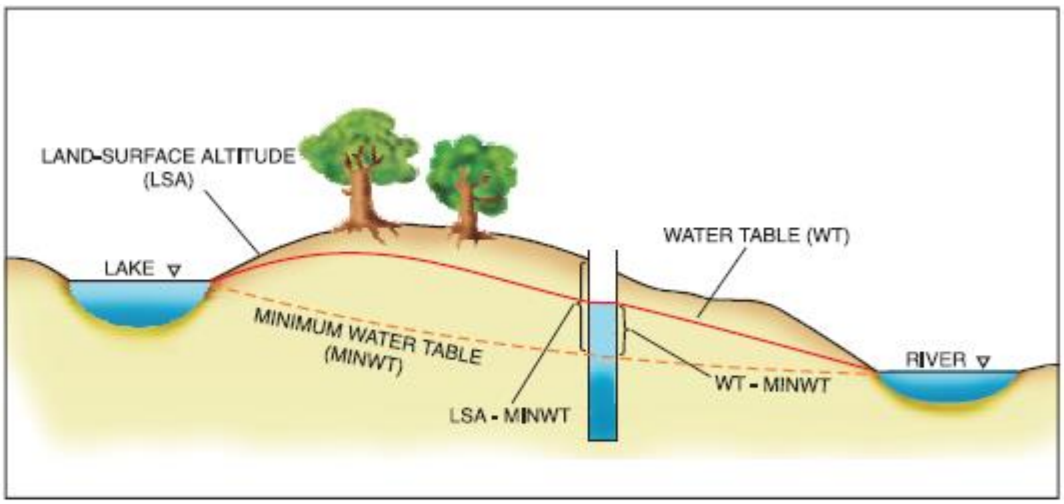


Figure 4: A graphic taken from "Simulation of Groundwater Flow in the Intermediate and Floridan Aquifer Systems in Peninsular Florida" (Sepulveda N. , 2002) showing the concept of a "minimum water table" and the relationship between terms the variables used in the multiple linear regression model developed for estimating the water table elevation of the Unconfined Surficial Aquifer.

This topography-following method was applied in a groundwater model of the confined Floridan Aquifer System that encompasses most of peninsular Florida (Sepulveda N. , 2002). Although the Floridan Aquifer System is confined, the ground-following method was used to estimate the overlying unconfined surficial aquifer. Within that study, the simulated water table elevations are not directly used in the active simulation, instead they are used as one of the hydraulic properties for determining leakage rates to and from the Flroidan Aquifer. Sepulveda found this method to reasonably address both topography and recharge controlled situations by using the minimum water table as a control and constraining β_1 in areas where the minimum water table, the land-surface altitude, and the water table coincide. When applied in peninsular Florida, the method achieved a weighted average residual error of 3.53ft. This method was applied in the Florida Aquifer Vulnerability Assessment (FAVA) in 2005 (Arthur, Baker, Cichon, Wood, & Rudin, 2005), and achieved good, although not best, results in an evaluation conducted by Cooper et al (2015).

Topographic information from LiDAR Data

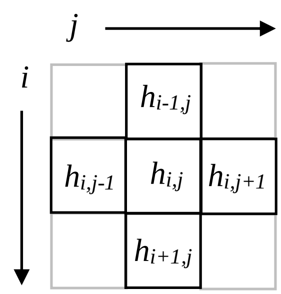
Common to both these methods is the need for topographic data. This data will typically come in the form of a LiDAR derived DTM. LiDAR is an active remote sensing system that works by sending out laser pulses and measuring the precise timing of the reflected return pulse (Petrie & Toth, 2008). Coupled with an airborne platform, this technology is capable of providing highly accurate topographic data for large areas and at high resolution. Data is provided in the form of a dense point cloud, and can include multiple returns. The last return is of interest to this study, as the last return usually comes from bare earth in vegetated areas, but may also include buildings and sometimes vegetation. Because the last return can also include buildings and vegetation, further filtering of the point cloud is necessary (Kraus & Pfeifer, 1998; Shan & Toth, 2008). LiDAR data provided by vendors is often filtered and classified into different groups of points, such that only ground points or only surface points can be selected. Ground points can then be used to create digital terrain models of bare-earth topography (Shan & Toth, 2008).

Jacobi Iterative Method of the Finite Difference Expression of the Laplace Equation

As discussed earlier, the Laplace equation governs groundwater flow through an isotropic, homogeneous aquifer under steady state conditions. The finite difference expression of Laplace's equation can be derived by approaching Laplace's equation numerically (Wang & Anderson, 1982). Here, a continuous surface is broken into a grid of cells where each cell is of equal dimensions. The two-dimensional form of the finite difference equation solved for head is as follows:

$$h_{i,j} = \frac{(h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1})}{4} \qquad \textit{\textbf{Equation 8}}$$

Where the value of $h_{i,j}$ at any point is the average value of the head of its nearest four cardinal neighbors (Wang & Anderson, 1982).



This equation is often called the five-point operator because the groundwater heads are approximated by moving a “star” of five points through the problem domain (Wang & Anderson, 1982). Jacobi Iteration is a simple method that applies the finite difference equation to solve the values of head for a two dimensional surface. Using this method, a matrix of known values and initial trial values (initial guesses) of head is used to begin the iteration process. The author does not specify how such guesses may be obtained, only that a “best guess” should be used. One concludes that any cell with an unknown head value thus receives a “best guess” value that can be obtained through whatever method obtains the best possible educated guess of the actual head for that cell. Cells with known value are held constant as per the Dirichlet boundary condition (Wang & Anderson, 1982). The finite difference equation is then solved in iterative steps for the entire raster grid by running the five point moving star across each cell in the grid. The particular order doesn’t matter, since each iterative step iterates over the previous iteration’s results, such that newly computed values at the current iteration level are not used until the next iteration step. The iteration process continues until the difference between the values generated by the current step and the previous step is less than a preset convergence criterion. Boundaries of a model where the head is unknown are called Neumann boundary conditions. Here, a no-flow condition is modeled by creating a fictitious row/column that mirrors the value of the adjacent row/column that is within the model domain. This is akin to inserting a glass pane along the external boundary of the model.

Methods

Next are a description of the study area for this project, the data used, and the analysis of the data done to determine whether topography can be used to generate a reasonable groundwater model.

Study Area

The site chosen for the model is a 120 square mile area in East-Central Florida surrounding the town Geneva. It is bordered by natural hydraulic boundaries on all sides. St. Johns River is to the North, Lake Harney and St. Johns River to the East, Econlockhatcheee River to the South, and Lake Jesup to the West. Several smaller lakes, ponds, and wetlands are contained within the area, making it ideal for a groundwater model.

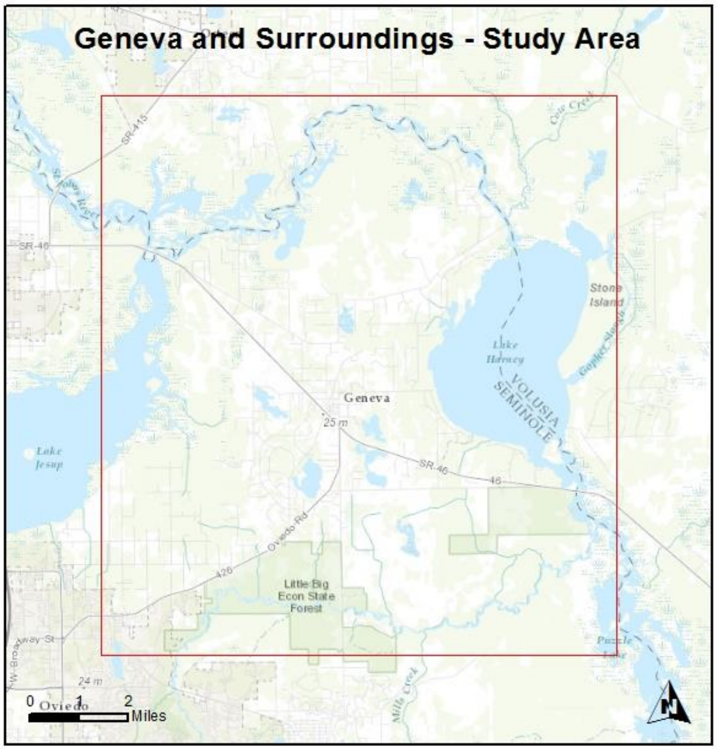


Figure 5: Study Area

The study area lies within the Surficial Aquifer System, which is generally under unconfined (water-table) conditions and is made up of mostly unconsolidated sand, shelly sand, and shell (Florida Department of Environmental Protection, 2015). It is stated that water mostly enters the aquifer through rainfall and generally flows from higher elevation towards the coast or streams where it is discharged. This suggests that the water table in this area may indeed be a subdued replica of topography, and makes this study area a good candidate for testing the assumption that groundwater can be modeled using topographic information. Between the years 2000 and 2010, the aquifer has received recharge from an average daily rainfall of 0.13 inches. Note that St. Johns River actually flows due North, and discharges near Jacksonville, Florida.

Data

From the literature review it was determined that the following data is required in order to study whether a certain area is topographically controlled:

- Topographic Information
- Surficial Water Features
- A Groundwater Model of Higher Accuracy

With the help of LiDAR data, a DTM will be created to supply the topographic information required by this project. LiDAR data is made publicly available by NOAA, and can be downloaded in the .las format for most of Florida from online data retrieval tools. NOAA provides the *digital coast data access viewer* at http://coast.noaa.gov/dataviewer/# to explore and download LiDAR datasets interactively. A .las dataset from a LiDAR mission flown February 8th - 11th, 2009 was obtained using the above source. This data has been filtered by the vendor into five classes, with the second class, ground points, being of interest here. The dataset has an overall vertical accuracy of one foot at the 95% confidence level, and is referenced to the NAVD88 vertical datum. It was converted to a DTM in ArcGIS using the “Las Dataset To Raster” tool with a five foot cell size. Due to the prolonged time (four days) the groundwater model would have taken to run at this resolution, the raster cell size was changed to 95 feet. Water table elevation measurements are routinely taken by the SJRWMD, and are made publicly available via the *Hydrologic data* online tool at http://webapub.sjrwmd.com/agws10/hdsnew/map.html. After processing the data, it became apparent that only four well measurements fell within the time period of February 8th - 11th, 2009, and no surface water body measurements were available for that time period. This limited quantity of measurements available for the study area means it cannot be used to validate the accuracy of the model or as input to the model. Surficial water features are recorded in GIS databases maintained by SJRWMD and can be downloaded for free through the districts online GIS data website at http://floridaswater.com/gisdevelopment/docs/themes.html. The most recent groundwater model created for the study area is the East Central Florida Transient model (ECFT). EFCT is a model based on MODFLOW, which is based on the finite difference solution and uses well data, recharge, evaporation, general head boundaries, etc. for input (South Florida Water Management DIstrict, 2006). This model is being jointly developed by SFWMD and SJRWMD, and a groundwater elevation raster derived from this model for January 2006 was obtained as the groundwater model of higher accuracy. It is unfortunate that a model for February 2009 was not available, as this will certainly significantly affect the comparative results of this project. Both models are conveniently set during the dry season so that steady-state conditions are most reasonable to assume.

Analysis

To test whether the assumption that groundwater can be modeled using topographic information is reasonable, a steady-state groundwater model based on the Jacobi iterative solution to the finite difference expression of the Laplace equation is produced using a DTM and hydrographic boundaries as the only input. This methodology is similar to that used by Rios (2010), in that it also uses a moving window. However, because our method is based upon the widely used and scientifically grounded finite difference equation (five-point star with convergence vs. 7x7 moving window with a pre-set number of iterations), the results will be a hydraulically correct surface that won't require any of the post-processing of Rios's approach. This method is also less complex than the method described by Sepulveda (2002), because it does not require fitting quantic polynomials and regressions, and can be easily run in ArcGIS using the tool that was scripted specifically for this purpose. The project was broken down into the following steps:

1. Script a Jacobi Iteration tool for ArcGIS using the Python programming language.
2. Establish surface of initial guesses, and internal hydraulic boundary conditions.
3. Run the Jacobi Iteration tool to create water table elevation model (WTEM).
4. Validate the results against the official EFCT model.

Scripting a Jacobi Iteration Tool for ArcGIS Using the Python Programming Language

A script that can operate on raster files in the ArcGIS environment was written in the Python programming language. The script was integrated into ArcGIS using the arcpy module from ESRI, and formatted into a tool that can be run from within ArcGIS like a standard tool in a toolbox. The tool requires two raster datasets for input, a raster of "initial guesses," and a raster of hydrographic boundaries with specified head (**Figure 1**). The tool also requires as input a convergence tolerance (the minimum difference between two consecutive iterations) and the maximum allowed number of iterations.

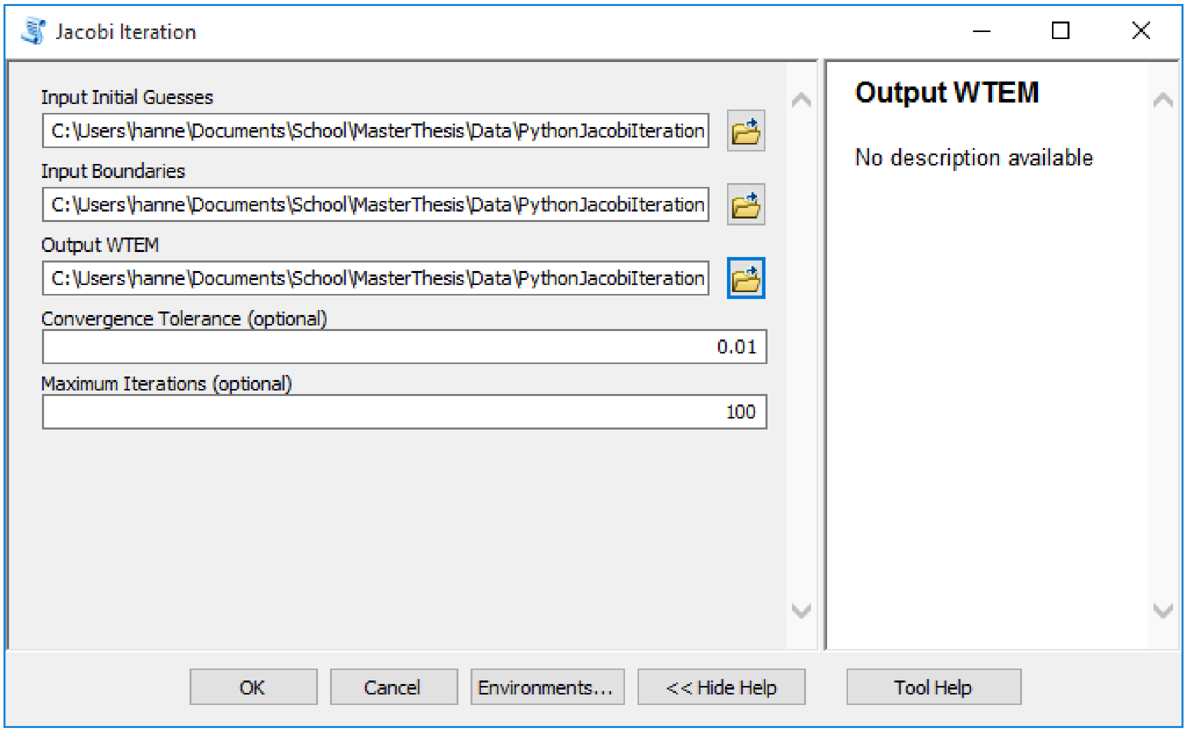
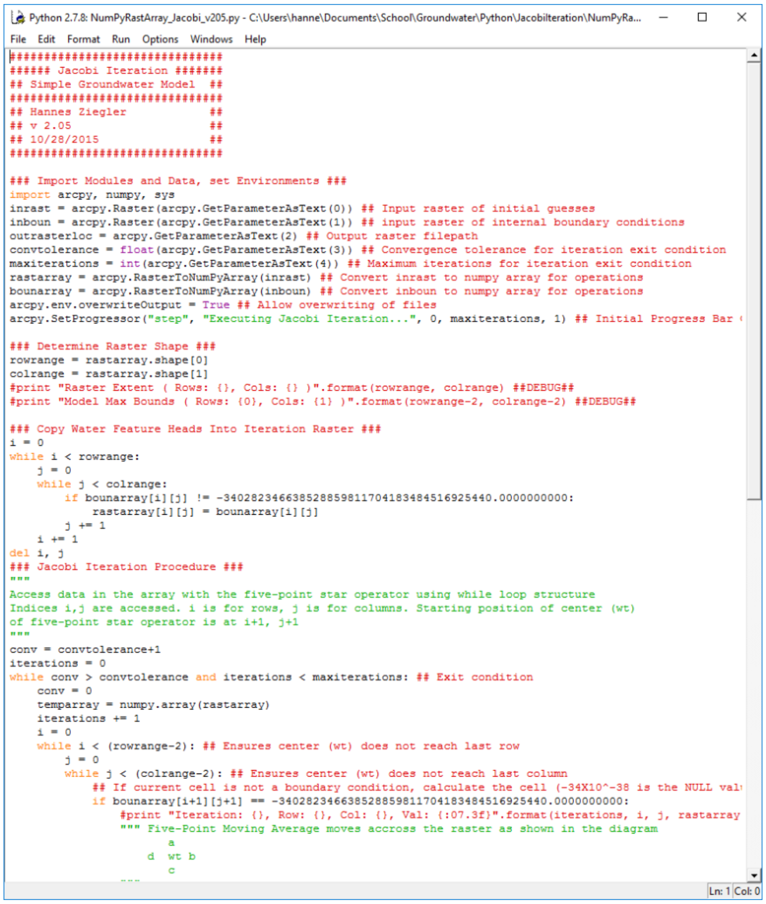


Figure 6: Example of the Jacobi Tool dialog box.

The script is programmed to complete successfully if the solution converges (the convergence tolerance is met), and will break out of the iteration process if the maximum number iterations is met, producing a warning to increase the maximum number of iterations or reduce the convergence criterion. The finite difference equation in two dimensions as stated in ***equation 8*** forms the basis of script's operations. The tool functions as follows: The input raster of initial guesses, which in this case will be a DTM, is used as the initial conditions from which to begin iteration. The boundary conditions, which



```
Python 2.7.8: NumPyRastArray_Jacobi_v205.py - C:\Users\hanne\Documents\School\Groundwater\Python\JacobiIteration\NumPyRa...
File Edit Format Run Options Windows Help
###############################
###### Jacobi Iteration #######
## Simple Groundwater Model  ##
###############################
## Hannes Ziegler            ##
## v 2.05                    ##
## 10/28/2015                ##
###############################

### Import Modules and Data, set Environments ###
import arcpy, numpy, sys
inrast = arcpy.Raster(arcpy.GetParameterAsText(0)) ## Input raster of initial guesses
inboun = arcpy.Raster(arcpy.GetParameterAsText(1)) ## input raster of internal boundary conditions
outrasterloc = arcpy.GetParameterAsText(2) ## Output raster filepath
convtolerance = float(arcpy.GetParameterAsText(3)) ## Convergence tolerance for iteration exit condition
maxiterations = int(arcpy.GetParameterAsText(4)) ## Maximum iterations for iteration exit condition
rastarray = arcpy.RasterToNumPyArray(inrast) ## Convert inrast to numpy array for operations
bounarray = arcpy.RasterToNumPyArray(inboun) ## Convert inboun to numpy array for operations
arcpy.env.overwriteOutput = True ## Allow overwriting of files
arcpy.SetProgressor("step", "Executing Jacobi Iteration...", 0, maxiterations, 1) ## Initial Progress Bar

### Determine Raster Shape ###
rowrange = rastarray.shape[0]
colrange = rastarray.shape[1]
#print "Raster Extent ( Rows: {}, Cols: {} )".format(rowrange, colrange) ##DEBUG##
#print "Model Max Bounds ( Rows: {0}, Cols: {1} )".format(rowrange-2, colrange-2) ##DEBUG##

### Copy Water Feature Heads Into Iteration Raster ###
i = 0
while i < rowrange:
    j = 0
    while j < colrange:
        if bounarray[i][j] != -340282346638528859811704183484516925440.0000000000:
            rastarray[i][j] = bounarray[i][j]
        j += 1
    i += 1
del i, j
### Jacobi Iteration Procedure ###
"""
Access data in the array with the five-point star operator using while loop structure
Indices i,j are accessed. i is for rows, j is for columns. Starting position of center (wt)
of five-point star operator is at i+1, j+1
"""
conv = convtolerance+1
iterations = 0
while conv > convtolerance and iterations < maxiterations: ## Exit condition
    conv = 0
    temparray = numpy.array(rastarray)
    iterations += 1
    i = 0
    while i < (rowrange-2): ## Ensures center (wt) does not reach last row
        j = 0
        while j < (colrange-2): ## Ensures center (wt) does not reach last column
            ## If current cell is not a boundary condition, calculate the cell (-34X10^-38 is the NULL val
            if bounarray[i+1][j+1] == -340282346638528859811704183484516925440.0000000000:
                #print "Iteration: {}, Row: {}, Col: {}, Val: {:07.3f}".format(iterations, i, j, rastarray
                """ Five-Point Moving Average moves accross the raster as shown in the diagram
                        a
                    d  wt b
                        c
Ln: 1 Col: 0
```

Figure 7: The Jacbi iteration tool script.

represents surficial waterbodies with known head, are copied onto the raster of initial guesses. The five-point star of the finite difference equation in two dimensions is moved across the raster from left to right while skipping over areas specified by the hydrographic boundaries. This is essentially treating hydrographic boundaries of known head as a Dirichlet boundary condition. The external boundaries of the model are treated as Neumann boundary conditions of no-flow by making border cells along the outer peripherals of the model take on the same value as the adjacent cell across a line of symmetry. At each iteration level, a new matrix is written, which is then used as input in the subsequent iteration level and discarded thereafter, so that no value from the current iteration level is ever used in the computations. During each iteration level, the greatest convergence value is tracked, and once this convergence value is smaller than or equal to the convergence tolerance, the current iteration is permanently saved and exported as a new raster. This new raster is called the water table elevation model (WTEM) and provides the groundwater head at each cell in the raster.

Establish Surface of Initial Guesses, and Internal Hydraulic Boundary Conditions

The Jacobi iteration method requires as input an initial guess for each cell. This guess, as stated earlier, can be the obtained through any method that presents the best educated guess one can make. Since in this project we would like to test whether topographic information can be used to model groundwater, the initial guess will be simply the DTM. The internal boundary conditions are delineated by a polygon shapefile of hydrography obtained from SJRWMD. Any areas in the DTM that had no data values were converted to polygon features using the “Is Null” and the “Raster to Polygon” tools. The resulting polygon features were then copied into the hydrography shapefile in an edit session, and certain boundaries were edited where features overlapped, and others were merged.

Because none of the obtained measurements from monitoring stations were surface water measurements, measurements of internal boundaries had to be determined using a different approach. The DTM provides the only other means of obtaining the elevations of surface water bodies. Here we have to make a difference between lakes and rivers/canals. Lakes are assumed to be flat, while rivers/canals are assumed to have some downstream gradient. Water levels were queried from the DTM using a random sample of points. This process involved creating a new shapefiles of random points across lakes and dissected sections of rivers using the “Create Random Points” and “Extract Values To Points” tools. Here, the number of random points per polygon feature was made to be proportional to the shape’s area by creating a new field in the hydrography shapefile and using this field to specify the number of random points to be placed within each feature. For lakes, the number of points was calculated using the formula Int([AREA]/1000) in field calculator. The “Spatial Join” tool was then used to join the extracted features back into the lakes, with the merge rule set to mode such that the mode value was used as the elevation of the lake. Where there was no mode value, the lowest value was chosen. For Econlockhatchee River, the number of points to randomly place within each section of the river was calculated in field calculator using the formula Int([AREA]/1000). The “Spatial Join” tool was used to join the extracted values back into the river sections, and the merge rule was set to minimum such that the lowest value was used as the elevation of the river section. The gradient of St. Johns River was very low within the study area, ranging from -0.3 to -0.83. Because the water level measurements were very inconsistent, elevation was assigned to the split sections of St. Johns River based on visually querying and evaluating the water levels. For Lake Harney and Lake Jesup, no elevation data was available from either measuring stations or the DTM.

The elevation of Lake Harney was estimated by averaging the known upstream and downstream elevations of St. Johns River. For Lake Jesup, only one adjacent measurement of St. Johns River was available where St. Johns River opens onto Lake Jesup, and it read -0.83. The closest values from SJRWMD measuring stations found were 1/29/2009 and 2/23/2009 (about 10 days before the time period in sought, which is 2/8-11/2009). These measurements were similar to the LiDAR measurement from the nearest part of St. Johns River. 1/29/2009 was -0.8 and 2/23/2009 was -0.65, while the nearest LiDAR measurement of St. Johns River was -0.83. The value of -0.83 was chosen as the elevation of Lake Jessup because no data on the flow direction of the lake could be obtained, and thus to stay on the safe side, it was assumed to be stagnant or very slow. The Hydrography shapefile was then converted to a raster using the "Polygon To Raster" tool in ArcMap. All inputs to the model are now ready (***Figure 8***).

Running the Jacobi Iteration Tool to Create a WTEM

The Jacobi Iteration tool was now run using the input DTM, the produced internal boundaries, and a convergence tolerance of 1 (***Figure 9***).

Validating the Results Against the Official EFCT Model

In the resulting WTEM, the heads were not expected to accurately model the water table elevations. However, the gradient should be the same or very similar to the actual water table. As the measured water table rises, the simulated topography following water table should rise as well. To test this, we will use a correlation. A strong and significant correlation would indicate that the values of the Jacobi model rise and fall together with the values of the EFCT model. In addition, a line of regression fitted through the data can provide additional insight into the performance of the model. Here, particular attention is given to the coefficient of the independent variable, because a coefficient close to one will indicate that a one unit rise in the Jacobi model is equal to a one unit rise in the EFCT model. This would mean that the shapes (and thus slopes) of the two surfaces are identical, and that calculating a velocity

Internal Boundaires

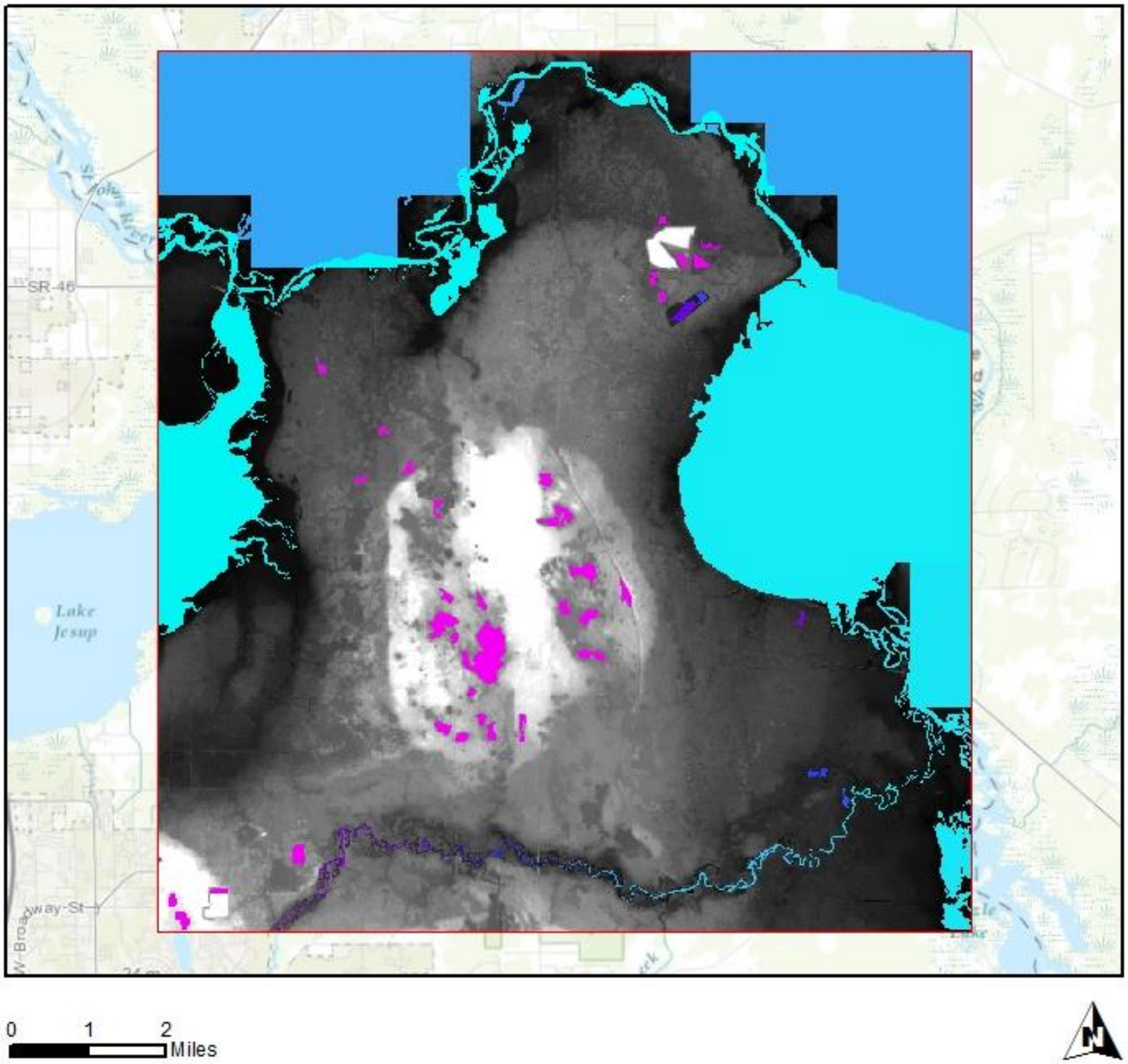


Figure 8: DTM and internal boundaries to be used as input to the Jacbi Iteration Tool.

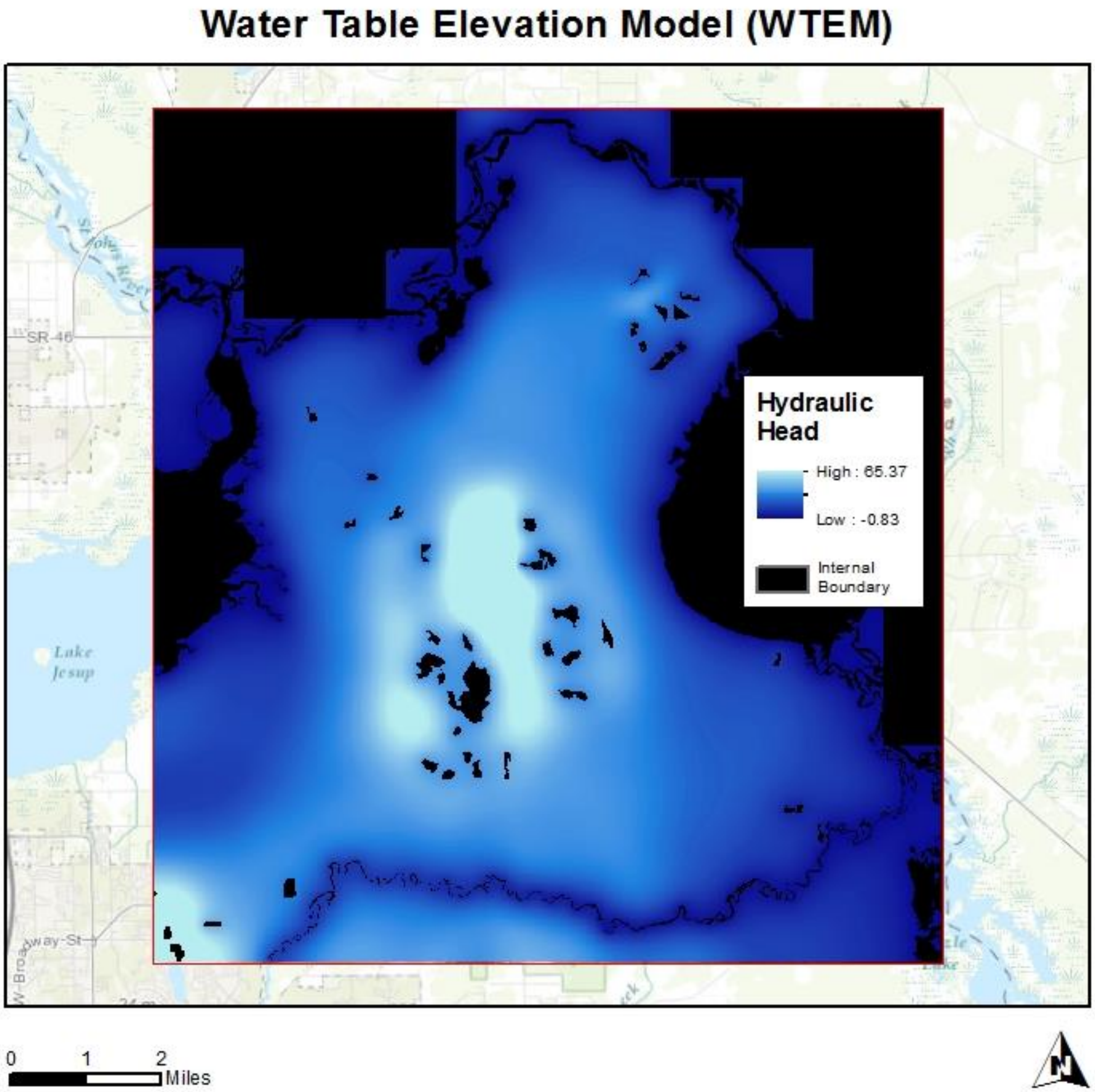


Figure 9: The WTEM resulting from running the Jacboi Iteration tool on the DTM and internal boundary datasets.

vector on either surface would yield the same result (Wang, et al., 2013). Finally, to test the accuracy of the model, a Root Mean Square Error (RMSE) is calculated (Cooper, Zhang, & Selch, 2015).

To perform the above tests, a random sample of water table elevation values had to first be obtained from both surfaces. The ECFT model has a cell size of 1,250 by 1,250 feet, while the Jacboi model has a cell size of 95 by 95 feet. The first step was to gather a sizeable sample from the datasets so that they can be statistically analyzed. Since lakes and no data areas should not be included in the sample, a polygon shapefile was created that delineates the area from which the random sample should be taken. The "Erase" tool in ArcMap is used to erase inner boundaries from the study area boundary to create the masked polygon. Next, the "Create Random Points" tool is used to create 250 random points to be used for sampling. The minimum distance between points is constrained to 1,251 feet so as to prevent two or more points from being placed in the same cell. Next, the tool "Extract Values To Points" is used to extract values from the two models. The resulting data was then converted to an excel format using the "Table To Excel" tool, and the assessment criterion of correlation, regression, and RMSE were calculated according to standard statistical procedures.

Results and Discussion

The Jacobi Iteration method produced a highly accurate groundwater model using only elevation data from a DTM, and hydrography as input. Although the EFCT model was set during a different year, the two models agreed well. The correlation coefficient is a statistically significant 0.96, indicating a very strong relationship between the elevation of the water of the EFCT model, and the water table of the Jacabi model. A fitted regression line through the data has a coefficient of 1.07 on the independent variable x. This Indicates that the relationship between the EFCT model and the Jacobi model have a near one-to-one ratio – as one rises or falls the other rises or falls by a corresponding amount. The two models effectively have the same or similar shape, and thus the same or similar slope. This is particularly important because calculating groundwater velocity (magnitude and direction) using Darcy’s law will yield the similar results for both the EFCT model and the Jacobi model.

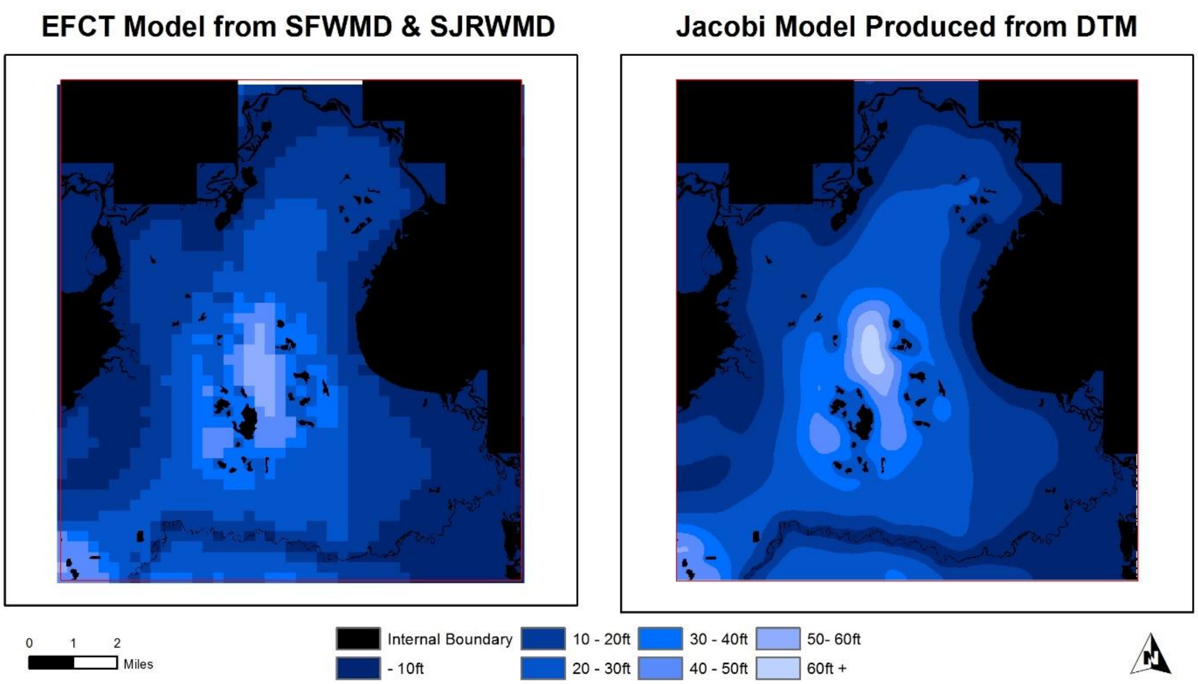


Figure 10: A comparison between the EFCT model produced by SFWMD and the Jacobi model produced using only topographic and hydrographic data. The data is classified into seven discrete ranges, and the boundaries these classes create visually show how well the two models compare. Note that the EFCT model has a cell size of 1,250x1,250 feet, while the Jacobi model has a cell size of 95x95 feet.

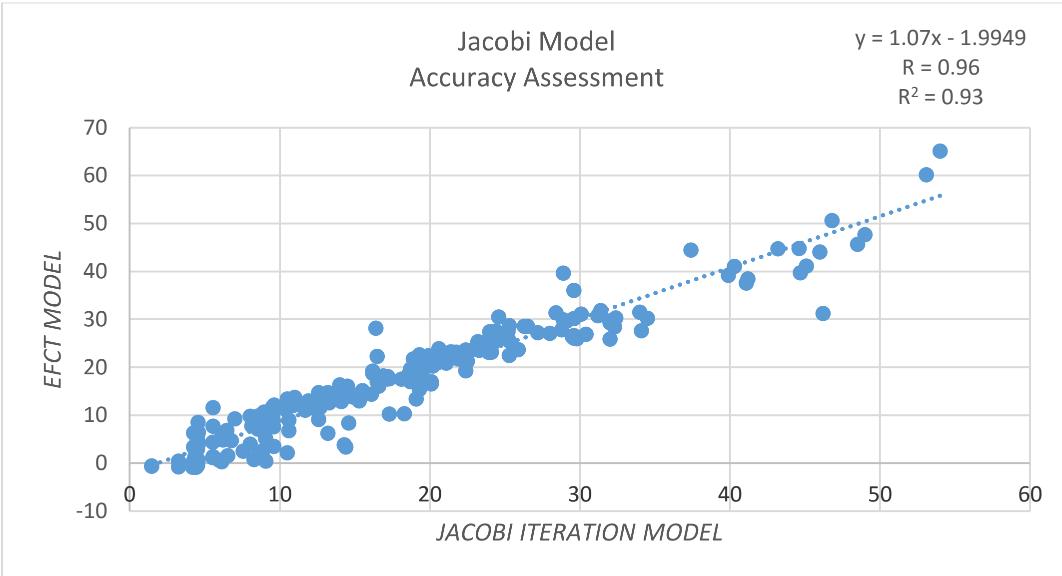
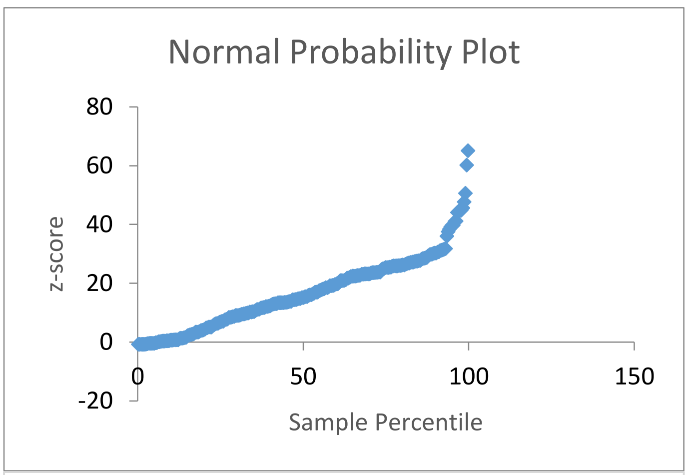


Figure 11: A regression line fitted through the plotted data of the EFCT model as the dependent variable and the Jacobi model as the independent variable. The plot shows a strong linear relationship, and a slope near one. The plot shows that values in the EFCT model are easily predicted by the Jacobi model.



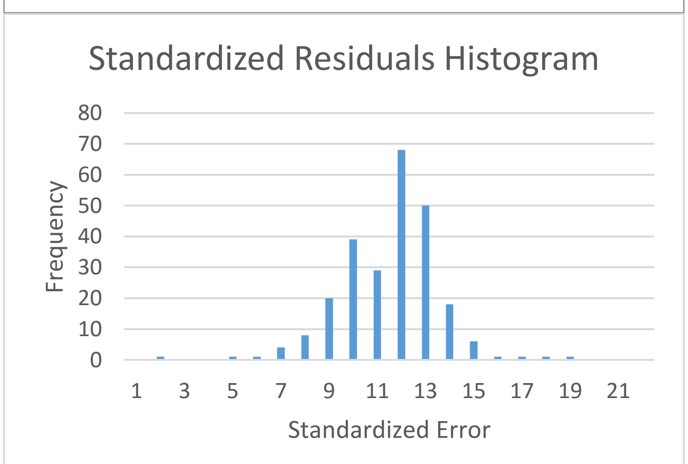


Figure 12: Above: A normal probability plot showing a straight line with some outliers towards the higher range of values. Below: A histogram of standardized residuals exhibiting normal distribution.

A normal probability plot shows that the residuals are normally distributed, with some significant outliers toward the high values in the distribution. A histogram of normalized residuals also shows that the residuals have a normal distribution, and the skew and kurtosis values of -0.4 and 2.78 respectively, are within rule-of-thumb thresholds of normal distribution.

The RMSE of the Jacobi model as compared to the EFCT model is 3.52 feet. This level of accuracy was surprising because it was originally hypothesized that the Jacobi model would only accurately model groundwater shape, but not actual head. In general, corresponding values of the two models actually agree quite well (***Figure 13***).

ECFT Model	Jacobi Model
0.385306001	4.320000172
44.71590042	43.20000076
11.02460003	11.69999981
15.48149967	14.19999981
14.40849972	15.60000038
18.03269958	17.20000076
21.93460083	20.29999924
28.41160011	32.29999924
17.00110054	16.5
8.719269753	9.529999733
30.27459908	32.40000153
30.51040077	24.60000038
13.31060028	13.80000019

Figure 13: A table displaying a selection of hydraulic heads. In general, the Jacobi model agrees well with the official EFCT model from SFWMD and SJRWMD.

Conclusions

The Jacobi iterative solution to the finite difference equation produces an accurate WTEM from only topographic and hydrographic data. Not only is the water table produced by the model similar in shape to the actual water table prediceted by the official EFCT model, it also successfully predicts the water table elevations with relatively acceptable accuracy. Because the model can be easily implemented in ArcMap using the Jacobi Iteration tool (created in this project), the method is also simple enough for the layman to apply. It requires only topographic and hydrographic data, which can be easily obtained for free for most of the U.S. from government agencies. This solves the issue of data acquisition and complexity that other methods have also solved using topographic data as supplementary information when other data was sparse. This solution is arguably simpler than the methods applied by Rios (2010) and Sepulveda (2003), and furthermore, is scientifically grounded in tried and proven physical laws that govern groundwater flow. Assessment of the results indicate the the groundwater surface created from the Jacobi model is at least as good - if not better than - those created by either of the alternate methods. Wang et al. (2013) applied Rio’s (2010) methods and achieved a correlation coefficient of 0.93 and a slope of linear regresssion close to one when plotted against observd (well) groundwater measurements. The Jacboi model achieved a stronger correlation coefficient of 0.96 and a slope of linear regression close to one when plotted against the official EFCT groundwater model. In addition, the model applied by Wang et al. cannot be relied on for accurate groundwater head.

Sepulveda (2002) achieved a weighted average residual error of 3.53 feet, which compares nicely with the root mean square error of 3.52 feet achieved by the Jacobi model. Because the EFCT model was set during a different year than the Jacobi model produced for this study, it is possible that the results may prove to be even more accurate if validated against a WTEM from the EFCT for the same time period. Nevertheless, these results show that it is valid, at least for the East-Central Florida region, to assume that groundwater is a subdued replica of topography. One can reasonably assume that groundwater can be modeled using only topographic information, and supplement their data with topographic information. In conclusion, the Jacobi method applied in this project is recommended in regions where the aquifer generally is a subdued replica of topography.

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