Restoring Rotation Invariance of Diffusion MRI Estimators in the Presence of Missing or Corrupted Measurements

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Synopsis

A natural requirement of estimated tissue microstructure anisotropy features is that they are rotation invariant. To attain rotation invariance the strategy has so far been to measure in as many uniformly distributed directions as can be afforded and simply compute the projection on an appropriate angular basis functions. However, in he presence of missing samples this approach is sub-optimal. In following we show that attaching carefully chosen weights to each measurement can achieve a significantly improved rotation invariance.

Purpose

In many clinical situations some measurements are corrupted or simply not obtainable, due to, for example. subject motion, [RR1 RR2]. We demonstrate that, by introducing measurement weights, an improved precision and rotation invariance of tissue microstructure anisotropic features can be obtained. Our analysis targets the estimation of the angular dependence of the signal strength for a given b-value. We show that optimal measurement weights can be found for any single shell encoding schemes in the presence of missing or corrupted measurements.

Methods

Studying anisotropic behaviour a natural approach is to express the directional dependance as a function on the surface of a sphere using a spherical harmonics (SPH's) as a basis. The SPH's provide an orthogonal basis on the sphere. A continuous signal, $f(x)$, can then be represented as:

$$c_{nk} = \int b_{nk}(x)\, f(x)\, dx \qquad \text{or (using a single index)} \qquad c_m = \int b_m(x)\, f(x)\, dx$$

where n is the spherical harmonics degree, k indicates the order and x is a vector on the unit sphere. For the present purpose all SPH's are basis functions of equal merit and the single index expression above is more appropriate. Let y_i be the set of unit vectors specifying the measured directions and a_i the weight attached the corresponding measurement. The spherical harmonics components, c_m, of the weighted sampling pattern is then given by:

$$c_m = \sum_i b_m(y_i)\; a_i$$

Optimal sets of weights a_{mi} are readily found by solving the following least squares problem, here expressed in matrix notation:

$$\mathbf{a}_m = \operatorname{argmin}[\; (\mathbf{Ba} - \mathbf{c}_m)^T \mathbf{W} (\mathbf{Ba} - \mathbf{c}_m)\;] = (\mathbf{B}^T \mathbf{W} \mathbf{B})^{-1} (\mathbf{W}\mathbf{B})^T \mathbf{c}_m$$

where $\mathbf{a}_m$ is a vector holding the optimal weights for the m:th SPH ; $\mathbf{B}$ is a matrix containing the values of the spherical harmonics basis sampled at the orientations given by y_i; $\mathbf{W}$ is a diagonal matrix that contains the weight attached to a goodness of fit for each spherical component; and $\mathbf{c}_m$ is the ideal response vector for the m:th SPH, holding the desired projection value for each spherical component.

Results

It clearly demonstrates that an optimal weighting yields an improved rotational invariance, which can be applied to existing directional schemes.

Discussion and Conclusion

We have demonstrated a straight forward scheme for optimizing the weighting of directions when analysing microstructure anisotropy. The main limitation of this study is that we have not investigated if weighting yields a relevant benefit in actual data, since the effects of rotational variance may be small compared to other sources of variance and bias, such as signal noise. On the other hand the cost of using our optimized weighting scheme is negligible. In conclusion, we have provided a method for optimal weighting of signals for restoring estimate rotation invariance in the presence of corrupted data.

Acknowledgements

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References

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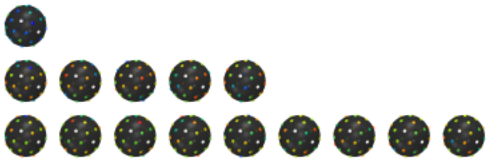
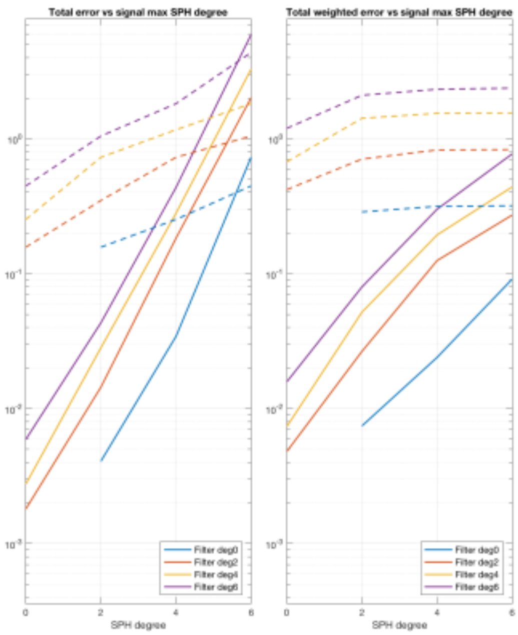
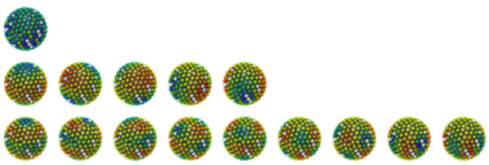
Figures



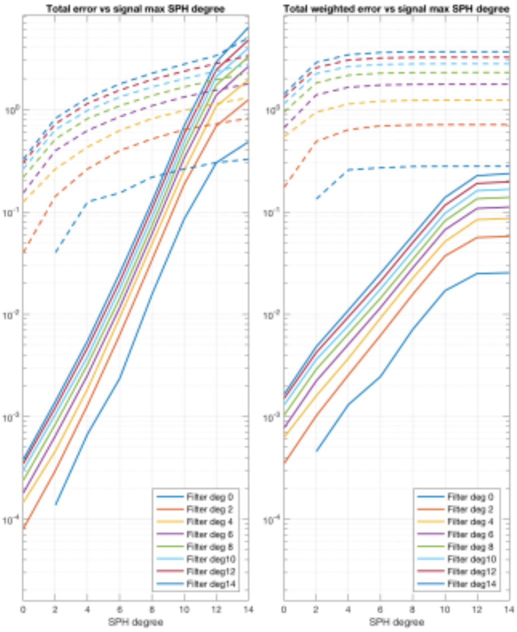
The figure shows the result of the weight optimization for the case of 21 uniformly distributed orientations (i.e. 42 diametrically positioned points on the sphere) with two missing measurements. The rows show results for spherical harmonic degrees 0, 2 and 4. Colors indicate filter weight values, blue is most positive and red is most negative. The missing measurement locations are shown in white. The top plot shows the the weights for the zero degree spherical harmonics. For a complete uncorrupted acquisition all weights would be identical. It clearly shows how the weights around the missing measurements are increased in order to compensate for the loss.



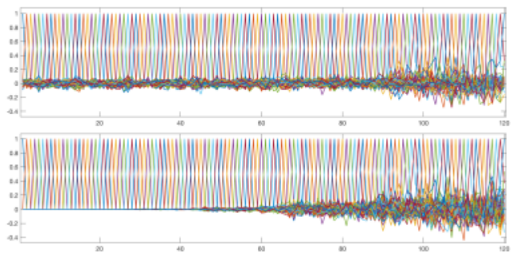
Estimated error distribution for the case of 21 uniformly distributed orientations with two missing measurements. The error is given as a function of the maximum spherical harmonic degree of the measured signal and the degree of the measurement filter. The left plot shows the result for a signal with equal energy for all spherical harmonics up to the degree indicated on the x-axis. The right plot shows the result using the much more realistic case where the energy decreases as specified by W . The dashed lines show the errors using unaltered spherical harmonic function values. The continuous lines show the result using the optimized weights.



The figure shows the result of the weight optimization for the case of 120 uniformly distributed orientations with 12 missing measurements. The rows show results for spherical harmonic degrees 0, 2 and 4. Colors indicate filter weight values, blue is most positive and red is most negative. The missing measurement locations are shown in white. The top plot shows the the weights for the zero degree spherical harmonics. For a complete uncorrupted acquisition all weights would be identical. It clearly shows how the weights around the missing measurements are increased in order to compensate for the loss.



Estimated error distribution for the case of 120 uniformly distributed orientations with 12 missing measurements. The error is given as a function of the maximum spherical harmonic degree of the measured signal and the degree of the measurement filter. The left plot shows the result for a signal with equal energy for all spherical harmonics up to the degree indicated on the x-axis. The right plot shows the result using the much more realistic case where the energy decreases as specified by W . The dashed lines show the errors using unaltered spherical harmonic function values. The continuous lines show the result using the optimized weights.



Spherical harmonics filter response plots for the 120 case. Shows how much signal each weight vector picks up from the input signal spherical harmonic components. Top: Unaltered SPH weights. Bottom: Optimized filters. Ideally the result should be 1 at the targeted harmonic and 0 elsewhere. The optimized weight filters (bottom) are much closer to the ideal for the lower degree filters and that is where it matters since actual signal will have most of the energy there (see figure 4).