Exercise 11

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11.1.1

The question can be formulated as a LP:

$$\arg\max_x c^T x \tag{1}$$
$$s.t. \tag{2}$$
$$e^T x = 40 \tag{3}$$
$$x \leq d \tag{4}$$
$$x = (x1, x2)^T, c = (2.5, 3.0)^T, e = (1/2, 1/1.4)^T, d = (60, 40)^T \tag{5}$$

x is the amount of cherries and apples plucked. c is the profit. e is the effort to pluck 1kg. d is the maximum demand.

11.1.2

The optimal solution is selling 60kg of cherries and 14kg of apples. The profit then amounts to 192€.

11.1.3

The dual is derived from the Lagrangian:

$$g(\lambda, \nu) = \sup_x L(x, \lambda, \nu) \tag{6}$$
$$\mathcal{L} = c^T x + \lambda^T (d - x) + \nu(e^T x - 40) = x^T (c - \lambda + \nu e) + \lambda^T d - 40\nu, \lambda \geq 0 \tag{7}$$
$$g(\lambda, \nu) = \begin{cases} \lambda^T d - 40\nu & (c - \lambda + \nu e) = 0 \\ -\infty & otherwise \end{cases} \tag{8}$$

It is obvious that the infinite infimum is not of any use. The dual problem is the minimization of g:

$$\arg\min_{\lambda,\nu} d^T \lambda - 40\nu \tag{9}$$
$$s.t. \tag{10}$$
$$c - \lambda + \nu e = 0 \tag{11}$$
$$\lambda \geq 0 \tag{12}$$

The dual of the dual can be derived the same way:

$$\mathcal{L} = d^T\lambda - 40\nu + k^T(c - \lambda + \nu e) - l^T\lambda = \lambda^T(d - k - l) + \nu(-40 + e^T k), l \geq 0 \tag{13}$$
$$\arg\max_k g = \arg\max_k \inf_{\lambda,\nu} L = \arg\max_k c^T k \tag{14}$$
$$s.t. \tag{15}$$
$$k - d \leq 0 \tag{16}$$
$$e^T k = 0 \tag{17}$$

Comparing the dual of the dual with the primal it becomes obvious that $k = x$ and the dual of the dual is the primal. Note: In order to minimize the primal and to maximize the dual, it is neccessary to multiply both problems with -1. That does not change a

11.2.1

The least squares method does not fit the data very well. The outliers distort the data resulting in a too steep slope (see fig. 1).

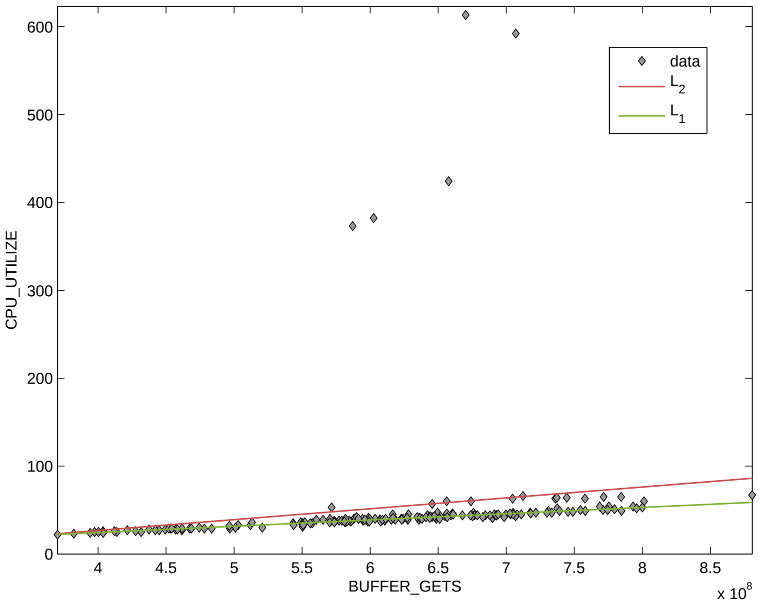

Figure 1: data and fits for different regression methods

11.2.2

$$\hat{x} = \arg\min_x ||y - Ax|| = \arg\min_x \sum_i |y_i - \sum_j a_{ij} x_j| \tag{18}$$

y contains the observed values at coordinates given by A. x are the regression parameters to be optimized. This problem can be translated to a linear program:

$$\hat{x} = \arg\min_{v,x} \mathbb{I}^T v \tag{19}$$

$$s.t. \tag{20}$$

$$-v + Ax \leq y \tag{21}$$

$$-v - Ax \leq y \tag{22}$$

$\mathbb{I}$ is a vector of ones of the same dimension as v. This can be rewritten as:

$$\arg\min_{\tilde{x}} c^T \tilde{x} \tag{23}$$

$$s.t. \tag{24}$$

$$\tilde{A}\tilde{x} \leq \tilde{y} \tag{25}$$

$$\tilde{A} = \begin{pmatrix} A & -\mathbb{1} \\ -A & -\mathbb{1} \end{pmatrix}, \tilde{x} = (x, v)^T, \tilde{y} = (y, -y)^T, c = (0, \mathbb{I})^T \tag{26}$$

$\mathbb{1}$ is the identity matrix, the others as before.

11.2.3

The L_1 fit is more robust against outliers. The line is closer to the points that are actual data and not outliers (see fig. 1). With least squares larger distances (i.e. outliers) are weighted more strongly.