Exercise 11

Philipp Hanslovsky, Robert Walecki

July 15, 2012

11.1.1

The question can be formulated as a LP:

$$\arg\max_x c^T x \tag{1}$$
$$s.t. \tag{2}$$
$$e^T x = 40 \tag{3}$$
$$x \leq d \tag{4}$$
$$x = (x1, x2)^T, c = (2.5, 3.0)^T, e = (1/2, 1/1.4)^T, d = (60, 40)^T \tag{5}$$

x is the amount of cherries and apples plucked. c is the profit. e is the effort to pluck 1kg. d is the maximum demand.

11.1.2

The optimal solution is selling 60kg of cherries and 14kg of apples. The profit then amounts to 192€.

11.2.2

$$\hat{x} = \arg\min_x ||y - Ax|| = \arg\min_x \sum_i |y_i - \sum_j a_{ij} x_j| \tag{6}$$

y contains the observed values at coordinates given by A. x are the regression parameters to be optimized. This problem can be translated to a linear program:

$$\hat{x} = \arg\min_{v,x} \mathbb{I}^T v \tag{7}$$
$$s.t. \tag{8}$$
$$-v + Ax \leq y \tag{9}$$
$$-v - Ax \leq y \tag{10}$$

$\mathbb{I}$ is a vector of ones of the same dimension as v. This can be rewritten as:

$$\arg\min_{\tilde{x}} c^T \tilde{x} \tag{11}$$
$$s.t. \tag{12}$$
$$\tilde{A}\tilde{x} \leq \tilde{y} \tag{13}$$
$$\tilde{A} = \begin{pmatrix} A & -\mathbb{1} \\ -A & -\mathbb{1} \end{pmatrix}, \tilde{x} = (x, v)^T, \tilde{y} = (y, -y)^T \tag{14}$$

$\mathbb{1}$ is the identity matrix, the others as before.