Exercise 6

Philipp Hanslovsky, Robert Walecki

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6.3.1

The covariance is a symmetric bilinear form. Therefore the following rules apply:

$$Cov(X,Y) = Cov(Y,X) \tag{1}$$
$$Cov(aX+b,Y) = a * Cov(X,Y) \tag{2}$$
$$Cov(X+Y,Z) = Cov(X,Z) + Cov(Y,Z) \tag{3}$$

3 can be generalized to:

$$Cov(\sum_i X_i, Y) = \sum_i Cov(X_i, Y) \tag{4}$$

Furthermore the covariance is a generalization of the variance:

$$Var(X) = Cov(X,X) \tag{5}$$

The relation between covariance and correlation is given by:

$$\rho_{ij} = \frac{cov(X_i, X_j}{\sigma_i \sigma_j} \overset{i.d.}{=} \frac{cov(X_i,X_j)}{\sigma^2} \tag{6}$$

With this preconditions we get:

$$Var(\frac{1}{B}\sum_{i=1}^{B} X_i) \overset{5}{=} Cov(\frac{1}{B}\sum_{i=1}^{B} X_i, \frac{1}{B}\sum_{i=1}^{B} X_i) \tag{7}$$
$$\overset{2,3}{=} \frac{1}{B^2}\sum_{i,j=1}^{B}(X_i, X_j) \tag{8}$$
$$\overset{5}{=} \frac{1}{B^2} sum_{i=1}^{B} Var(X_i) + \frac{2}{B^2}\sum_{i=1}^{B-1}\sum_{j=i+1}^{B} cov(X_i, X_j) \tag{9}$$
$$\overset{6,i.d.}{=} \frac{1}{B^2} B\sigma^2 + \frac{2\rho\sigma^2}{B^2}\sum_{i=1}^{B-1}\sum_{j=i+1}^{B} 1 \tag{10}$$
$$= \frac{\sigma^2}{B} + \frac{2\rho\sigma^2}{B^2}\sum_{i=1}^{B-1} B - i \tag{11}$$
$$= \frac{\sigma^2}{B} + \frac{2\rho\sigma^2}{B^2}\left(B\,(B-1) - \frac{B\,(B-1)}{2}\right) \tag{12}$$
$$= \frac{\sigma^2}{B} + \frac{\rho\sigma^2}{B}(B-1) \tag{13}$$
$$= \rho\sigma^2 + \frac{(1-\rho)\sigma^2}{B} \tag{14}$$