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A General Effectiveness Methodology for Aircraft Survivability Assessments

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ABSTRACT

The quantification of aircraft survivability in modern battlefield environments is a complex mathematical problem. In general, consideration must be given to the quantification of aircraft vulnerability to individual weapon systems, single encounter aircraft survivability, and the mathematical mapping of single encounter aircraft survivability into mission attrition. A methodology for quantifying the impacts of electronic warfare (EW) upon aircraft survivability is realized by the General Effectiveness Methodology (GEM) which is based upon a hierarchy of computer models. This paper describes this hierarchy of computer simulation tools which extensively employs probability theory to estimate the various engagement events such as aircraft detection, acquisition, missile launch, missile intercept, and probability of aircraft kill.

IN THE PAST COMBAT AIRCRAFT DESIGNS have been primarily driven by aerodynamic performance requirements. However, the complexity and potential lethality of hostile weapon systems to be deployed in large numbers in modern operational scenarios has invoked additional survivability requirements into airborne designs for combat aircraft. To achieve acceptable survivability on the battlefield, future combat aircraft designs will likely feature stealth technology coupled with integrated avionics and electronic combat (EC) systems.

The purpose of this paper is to present a General Effectiveness Methodology (GEM) to quantify combat aircraft survivability in complex operational scenarios. Factors impacting aircraft survivability ranging from structural design considerations to electronic combat assets to concepts of employment are examined to illustrate the application of the GEM. Particular attention is given to demonstrating how the various mathematical formulations and computer simulation tools which comprise the GEM can be utilized to characterize aircraft vulnerability, single engagement survivability, and mission attrition.

AIRCRAFT VULNERABILITY

METHODOLOGY - The characterization of combat aircraft vulnerability is the first key element of the GEM. At a macroscopic level, one would expect aircraft vulnerability to vary with the type of weapon encountered (surface to air missile, gun, etc.) as well as the location of vulnerable areas on the airframe structure. At a somewhat more microscopic level aircraft vulnerability to hostile weapon systems can also be shown to be a function of the endgame encounter geometry principally (i.e., the final approach of a missile to its target). The following discussion describes the GEM mathematical formulations and computer simulation tools used to characterize aircraft vulnerability to various types of hostile weapon systems. The methodology follows that of the Enhanced Surface to Air Missile Simulation (ESAMS) computer program endgame methodology (1).

Fuzing and warhead modeling is accomplished in three segments: fuzing, blast effects, and fragmentation effects. Missile debris effects are not considered. Radar fuzing is modeled using the target glitter points (i.e., good radar energy reflectors), fixed fuzing angles, and fuzing ranges that depend on the missile type but not the target type. When a glitter point is within given range and angle limits of the fuze system, the fuze detects target presence, and the warhead detonates after a given time delay. Blast effects are assessed using look-up tables given for each combination of target type, missile system, and target range at time of warhead detonation.

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The calculation of fragmentation kill is more sophisticated. The striking velocity of the fragments is calculated using the initial static velocity of the fragments, the missile velocity, and the distance to intercept point of the target. The striking velocity does not include the velocity of the target except as it affects the distance to intercept. Once the striking velocity has been calculated, the vulnerable area is looked up as a function of target type, striking angle, and striking velocity. However, the vulnerable area does not depend upon the mass of the striking fragment.

The number of fragments expected to strike each vulnerable component is calculated as a function of vulnerable area, the percentage of the area presented, and the density of the fragments. The probability of component kill is calculated assuming that the fragments are randomly distributed over the presented area. The probability of kill due to fragments is then computed by combining the probabilities for each of the components. The total probability of kill is computed by combining the probability of kill due to blast and the probability of kill due to fragments.

TARGET VULNERABILITY - The vulnerable areas of an aircraft exposed to a weapon system are dependent upon the endgame angle of attack. A vulnerability table is indexed by elevation and azimuth in the target body reference system and by the striking velocity of the warhead fragments. For given look angles from the aircraft to the warhead detonation point and a given striking velocity of the warhead fragments, a vulnerable area is computed by linear interpolation between individual entries in the vulnerability table.

FUZING - Fuzing of missile warheads is calculated by determining the proximity of target glitter points to the missile warhead fuzing pattern. The target's glitter points are transformed into an inertial reference system, a vector from the missile to each glitter point determined, and the corresponding look angle computed. This look angle is measured from the missile velocity vector (missile roll-axis and velocity vector assumed coincident). The blast point is determined by examining the target's glitter points until one of them is in a correct position to fuze the warhead.

When one of the glitter points meets the fuzing criteria, there is a small time delay before the warhead detonates and the warhead fragment sprayout occurs. This situation is depicted in Figure 1. The position of the missile is updated to determine the blast point. This, combined with an update of target position, is used to determine the blast vector.

Prior to the detailed fragment spray flyout, the blast kill probability zones are determined. The size of each of these blast kill zones is determined by the blast point/target geometry. The blast kill tables contain entries which give maximum radii from the blast point to the target center of gravity as a function of azimuth and elevation look angles defining a given probability of kill region to blast effects (PKB) as illustrated in Figure 2.

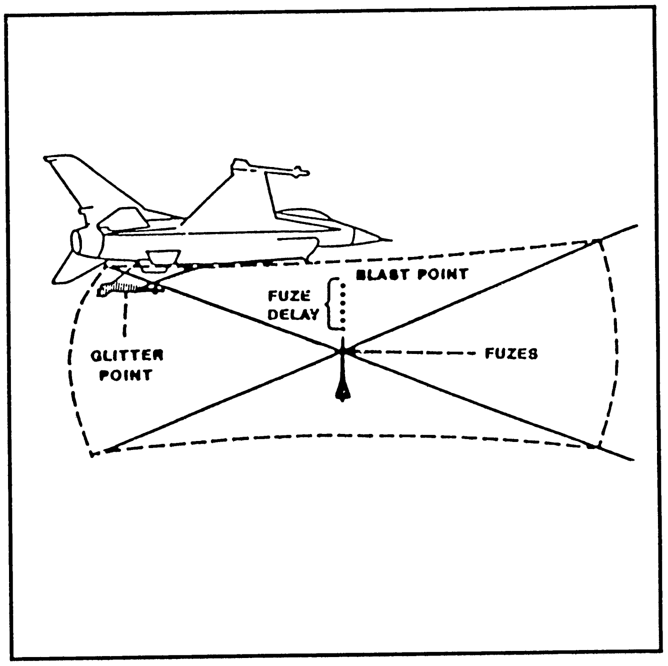

Figure 1. Proximity fuzing dynamics

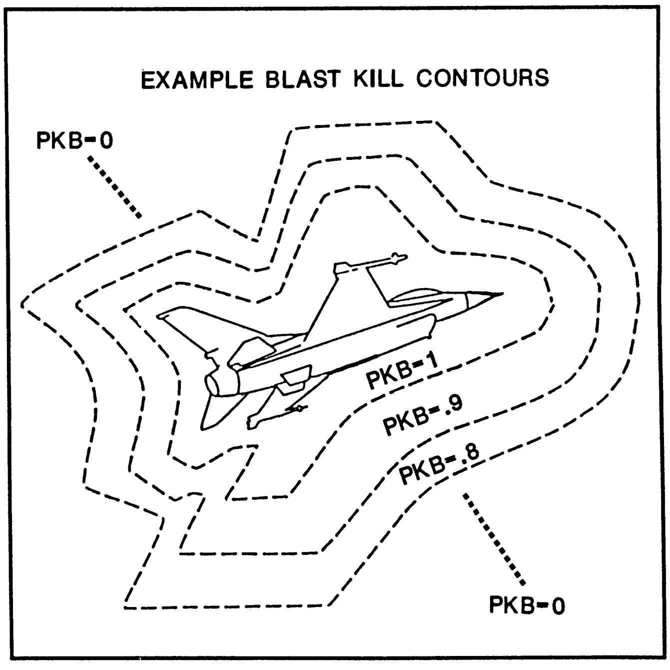

Figure 2. Aircraft kill contours due to warhead blast effects

Table 1 lists the inputs to and the outputs from the fuzing model.

P_K EVALUATION - Three factors affect the probability of kill (P_K) of an aircraft target by a missile warhead:

1. Blast kill
2. Blast and Fragment kill
3. Near Miss (P_K = missile reliability when inside 100% lethal zone)

Table 2 lists inputs and outputs of the P_K model. The detonation of the warhead sends out pellets in a circular band centered at the blast point. A pellet's total velocity is the vector sum of the velocity of the missile and the velocity provided by the warhead detonation. The pellets in different parts of the spray will have different velocities. The angular limits of the pellet sprayout are determined with respect to the missile velocity vector. The motion of the pellet spray and of the vulnerable components of the target are modeled until the spray has flown out a distance equal to or exceeding the current distance from the blast point to the vulnerable component as illustrated in Figure 3.

Fragment kill depends upon the look angles from the blast point to components, striking velocity, fragment density, and the percentage of each component inside the pellet spray. Some of the vulnerable components are so important to the target that their destruction results in destruction of the target. Others are less critical - their destruction will not directly result in target kill. The probability of target kill by fragments is a combination of events determined by target configuration. The probability of target kill by fragments, PKF, can be found from:

$$PKF = 1 - (1-PKC_1)\ (1-PKC_2 \cdot PKC_3)\ (1-PKC_4 \cdot PKC_5)\ (1-PKC_6 \cdot PKC_7)\ (1-PKC_8 \cdot PKC_9)\ (1-PKC_{10} \cdot PKC_{11})\ (1-PKC_{12} \cdot PKC_{13})\ (1-PKC_{14} \cdot PKC_{15})$$

Here, PKC_i is the probability that the pellet spray destroys the ith component.

The probability of kill due to blast is solely a function of the distance of the target from the blast point at the instant of blast. This method uses a linear interpolation using the blast kill zones. The formula used is:

$$PKB = \frac{B_{K+1} - D_{ms}}{B_{K+1} - B_K}\ (0.1) + (1-K/10)$$

where,

D_{ms} = Miss distance (distance between target and blast point

B_K, B_{K+1} = Defined limits of blast region based on D_{ms}

Combining the missile reliability with the probability of blast kill and the probability of fragment kill, the final probability of target kill, given that it is engaged by a missile, is:

$$P_K = P_{rel}\ (1 - (1-PKF)\ (1-PKB)).$$

TABLE 1. FUZING MODEL

Inputs to Fuzing Model:

- Target position vector
- Target Euler angles (pitch, roll, yaw)
- Distance vector from target trackpoint to target center-of-gravity
- Target glitter points
- Missile position vector
- Missile velocity vector
- Maximum angle from missile body axis at which a glitter point will fuze warhead
- Maximum distance from missile at which glitter point will fuze warhead
- Time between warhead fuzing and explosion
- Missile reliability (P_{rel})

Outputs from Fuzing Model:

- Blast point
- Blast kill probability distances

TABLE 2. P_K MODEL

Input to P_K Model

- Speed imparted to spray by warhead explosion
- Static spray angles
- Coefficient of drag for warhead fragment
- Reference area for warhead fragment
- Mass of a warhead fragment
- Number of warhead fragments
- Atmospheric density
- Blast point
- Missile velocity magnitude
- Target velocity vector
- Center of gravity for target vulnerable components
- Vulnerable areas for target vulnerable components - function of blast-point look-angles and warhead fragment striking velocity
- Presented areas for target vulnerable components - function of target-to-blast point look-angles
- Blast kill probability zones
- Missile reliability

Output from P_K Model

- Probability of kill

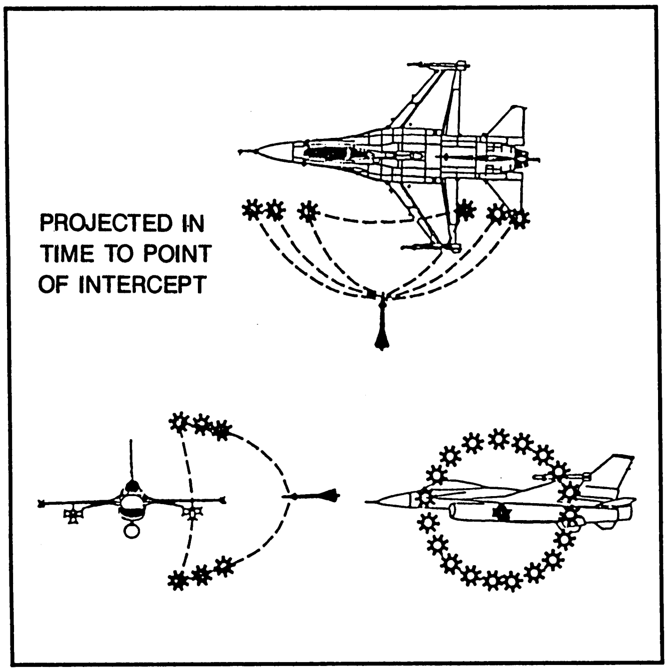

Figure 3. Fragmentation effects on the probability of aircraft kill

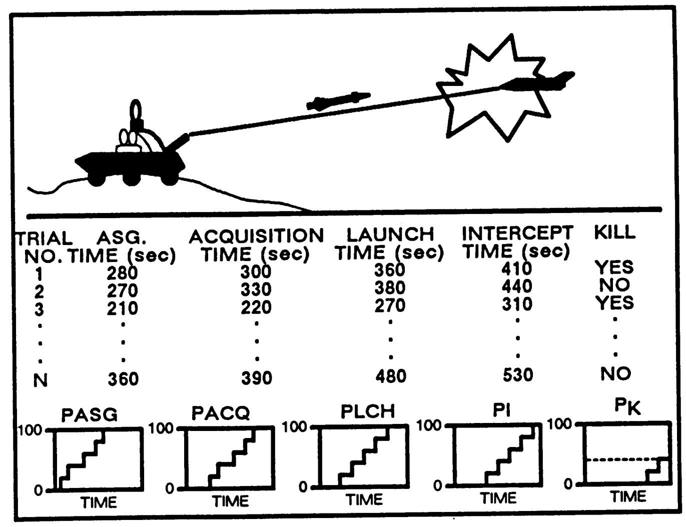

Figure 4. Experimental basis for single encounter aircraft survivability methodology

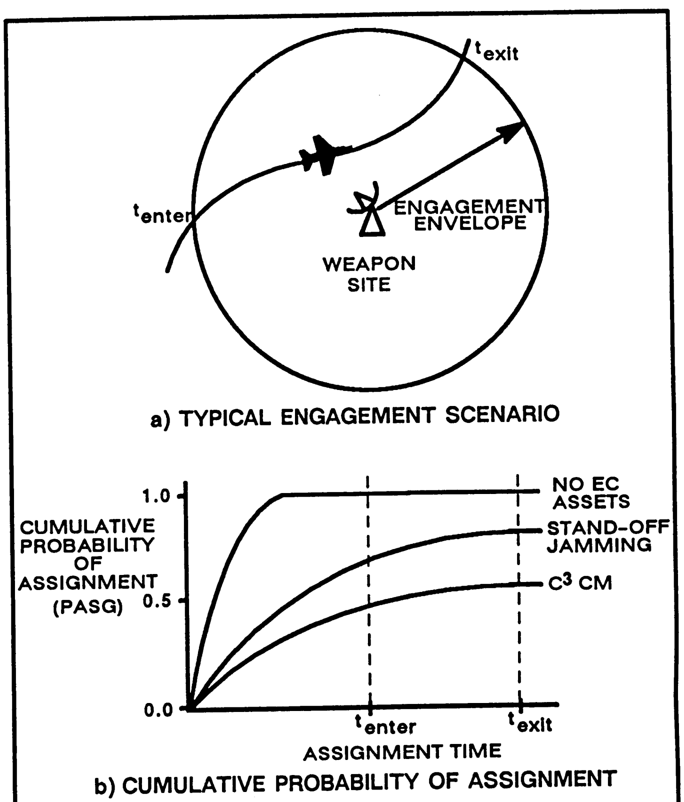

Figure 5. Graphical illustration of the potential impacts of EC support jamming

SINGLE ENCOUNTER AIRCRAFT SURVIVABILITY

The second major element of the GEM is the construction of single encounter aircraft survivability data bases to support subsequent mission analyses. Single encounter aircraft survivability is a scenario dependent measure of effectiveness (MOE) which varies as a function of airframe characteristics, weapon system, engagement geometry, the employment of electronic combat assets, and penetration tactics. The computation of single encounter aircraft survivability within the GEM is based upon mathematical models of each of the events which must be executed by individual weapon systems to engage aircraft. The events include target assignment, target detection, target acquisition, weapon employment, weapon intercept and target kill. The GEM characterizes the cumulative probability of occurrence of each of these events as a function of time as illustrated in Figure 4.

PROBABILITY OF ASSIGNMENT - The cumulative probability of weapon assignment (PASG) is the mathematical function which describes in a probabilistic fashion the distribution as a function of time of the initiation of all possible aircraft engagements for a given deterministic set of environmental conditions. The cumulative PASG function may be altered by the application of EC support assets such as stand-off jamming (SOJ) and command, control, and communciation countermeasures (C^3CM) as illustrated in Figure 5. However, these alterations in the cumulative PASG function will impact single encounter kill probability only when the maximum value of PASG achievable with respect to a defined baseline condition is either reduced or sufficiently delayed in time to impact the completion of other events (e.g., weapon launch, weapon intercept) (2). For example, if the cumulative PASG for a selected baseline condition reaches a maximum value of one (1.0) one minute prior to the time at which the deployment of a weapon could first occur, a thirty (30) second delay in assignment of the weapon system would not impact subsequent events. However, if delays greater than sixty (60) seconds are achieved, then subsequent events would be impacted.

PROBABILITY OF AIRCRAFT DETECTION - Given the quantification of weapon system assignment, the next events modeled in the GEM are the detection, acquisition, and tracking of an aircraft by the weapon system. The single scan probability of target detection (PDET) is calculated in the GEM as a function of signal-to-noise ratio (SNR) for the probability of false alarm (PFA) operating point corresponding to that of a particular individual weapon system. The false alarm operating point will vary as a function of detection processing (e.g., human observer versus electronic device), as well as threat type (e.g., beam agile systems can operate with a higher false alarm rate than non-beam agile systems since the cost of processing a false alarm is not as severe). It should be noted that for a given false alarm probability, the PDET versus SNR may differ among various weapon systems due to the utilization of different signal processing schemes (e.g. phase versus amplitude detectors), number of pulses integrated, type of integration (e.g. coherent versus non-coherent), assumptions on target ensemble and temporal statistics, etc., as noted by numerous authors (3,4,5). The GEM uses table lookup and interpolation from PDET versus SNR tables computed offline for the various target/threat combinations to be evaluated.

PROBABILITY OF AIRCRAFT ACQUISITION- Sequential samples of single scan PDET are converted to a <u>non-conditional</u> cumulative probability of acquisition (PAQ(t)) using empirically derived mathematical transformations derived from experimental data.

PAQ(t) = f(PDET(t), PDET(t-1),...PDET(t-k))

where,

PAQ(t) = Cumulative probability of acquisition at time t

f = Mathematical function or transform

PDET(t) = Probability of detection at time t

It should be noted that such mathematical transforms are equivalent to the "operator factor" defined by other authors (6,7). The familiar "M out of N" detector is a possible mathematical structure here when available data supports this simplification of the more general process of processing multiple, sequential detections to form an acquisition decision.

The cumulative probability of acquisition (PACQ(t)) is calculated by weighing PAQ(t) by PASG(t). Thus,

PACQ(t) = PAQ(t) X PASG(t)

where,

PACQ(t): Cumulative probability of acquisition

PASG(t): Cumulative probability of target assignment

It should be noted that the total probability formula reduces to the above form due to the zero probability of the conditional cumulative probability of acquisition given <u>no</u> target assignment.

PROBABILITY OF LAUNCH - The cumulative probability of missile launch (PLCH) or projectile fire (PFIRE) is derived from the cumulative PACQ function based upon either engineering estimates or experimental data describing the distribution of time delays between target acquisition and weapon employment (i.e. probability density function of launch/fire delay times). The probability density function serves as the weighting function on the cumulative PACQ function for the purpose of modeling the cumulative PLCH/PFIRE. The implementation of the model takes the form depicted in Figure 6 where the individual weighting factors, W_i's, are sampled values from the probability density function used to describe the time delays between target acquisition and weapon launch/fire.

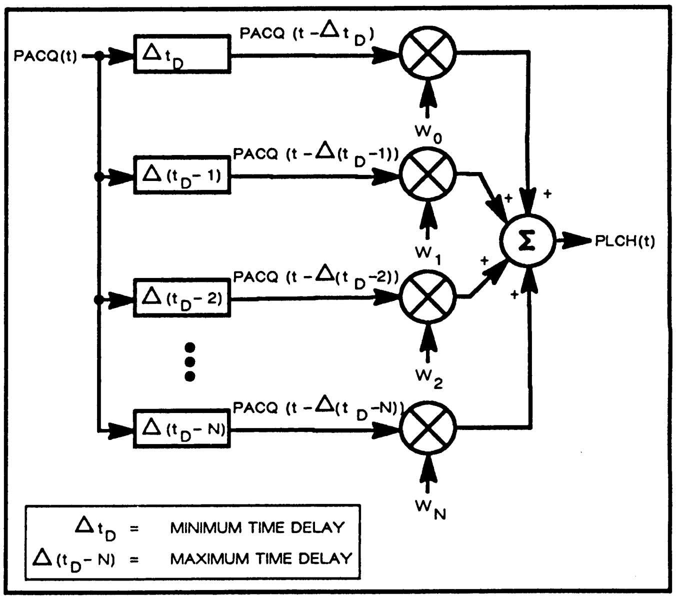

Figure 6. Graphical illustration of the mathematical model for mapping the cumu lative probability of acquisition versus time to the cumulative probabillity of launch versus time

PROBABILITY OF INTERCEPT - The cumulative probability of intercept (PI) is a time-delayed version of the cumulative PLCH/PFIRE function with the time-delay corresponding to the missile/projectile time of flight (TOF). Since a missile/projectile TOF is actually dependent on the target/threat relative geometry at the time of launch/fire, the TOF is a time-dependent variable. For this reason, the cumulative PLCH/PFIRE functions are not exact replicas (constant time delay versus time) of the cumulative PACQ function. This detail can become important and is incorporated in the GEM. In practice, the computation of the missile/projectile time of flight is a by-product of missile/projectile fly-out models used to determine missile/projectile miss distance (1).

CONDITIONAL PROBABILITY OF KILL GIVEN INTERCEPT - Missile/projectile miss distance data as a function of aspect angle at intercept with respect to the targeted aircraft provides the necessary index into the aircraft vulnerability database to compute the conditional probability of kill given intercept (PKGI). The incremental change in the probability of aircraft kill (Δ PK) may now be computed from the PKGI and the derivative of the cumulative PI function (i.e. the PI density function) as:

$$\Delta PK(t) = PKGI \times \Delta PI(t)$$
$$= PKGI \times (PI(t) - PI(t-\Delta t))$$

where,

Δ PK(t): Incremental change in PK at time t,
PKGI: Conditional probability of kill given intercept, and
Δ PI(t): Incremental change in the probability of intercept at time t.

PROBABILITY OF KILL - The cumulative probability of aircraft kill (PK) for a single encounter is simply the accumulation of the Δ PK's as indicated in Figure 7. It is important to note that each final cumulative PK value corresponds to a particular engagement sequence and represents the encounter PK for a specific aircraft flying a specific profile against a specific threat weapon system. For these reasons the GEM is referred to as a site specific analysis tool.

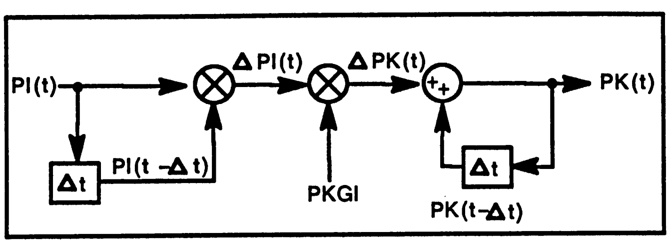

Figure 7. Mathematical model of the single encounter cumulative probability of aircraft kill

PROBABILITY OF KILL VERSUS ASSIGNMENT TIME - The GEM requires the computation of the cumulative PK for all possible engagements. The process described above is repeated for all possible assignment times which allow any non-zero cumulative PK result as illustrated in Figure 8. PK versus assignment time data is also generated for various combinations of electronic combat assets. For example, as shown in Figure 8, a single encounter aircraft survivability baseline may be established by examining survivability without the employment of any electronic combat assets (i.e., "dry"). Single encounter survivability analyses may then be repeated to examine reductions in PK potentially realizable via employment of various electronic combat assets. For example, stand-off support jamming is shown to reduce the engagement time available at this weapon site, thus reducing the single encounter PK from its maximum value in this case. Self-protection EC assets are seen to reduce the PK over a significant portion of the engagement envelope of the weapon site. Impacts of EC system reliability on single encounter aircraft survivability may also be quantified. It is perhaps of interest to note that EC system reliability impacts on single encounter aircraft survivability are easily derived from the curves describing PK versus assignment time for the "dry" (i.e., no EC assets) and "wet" (i.e., EC assets) conditions using the total probability formula:

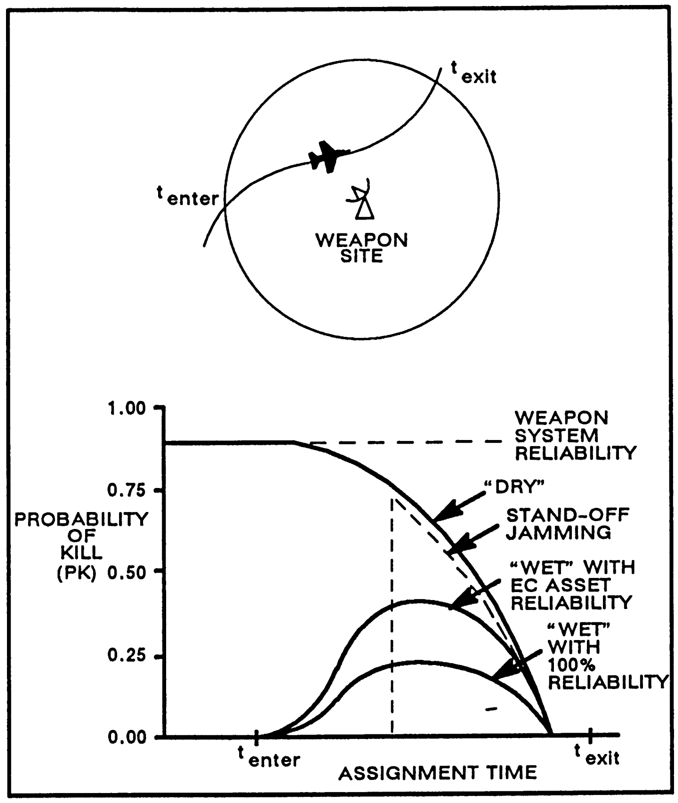

Figure 8. Probability of kill as a function of weapon assignment for various combinations of electronic combat assets

$$PK = (PK/EC)\ PEC + (PK/NEC)\ PNEC$$

where,

PK: PK for a specified EC reliability value
PK/EC: Cumulative PK given EC assets
PEC: Probability of EC assets (i.e., reliability)
PK/NEC: Cumulative PK given no EC assets (i.e., "dry")
PNEC: Probability of no EC assets (i.e., non-reliability).

Single encounter aircraft survivability data in terms of PK versus assignment time for various combinations of EC assets, concepts of employment, and penetration tactics furnish the input database required to conduct mission analyses.

MISSION ANALYSES

The determination of the survivability of an aircraft in a complex mission scenario or the determination of the probability of success of a mission can follow from the single site engagement outcomes discussed above. As each individual encounter is defined in terms of the engagement time, the simulation of individual aircraft probability of kill for a multi-threat scenario will depend upon the ability to model threat assignment times. As the single site engagement outcomes depend upon the application of EW techniques in specific ways, the mission outcomes will depend on the application of EW techniques as well.

The scenario data required for this approach must be available and include the aircraft flight profiles, the weapon system geographical locations, and weapon system technical and operational parameters. Typical elements of an example scenario are depicted in Figure 9, where the complexity of the potential engagements are indicated. The one-on-one engagement results are generated for each aircraft-threat encounter which is possible to occur, under all the conditions of the analysis, such as with and without self-protection jamming, with and without escort jamming, and so on. These results are determined for a given flight path, as a function of engagement time, as discussed.

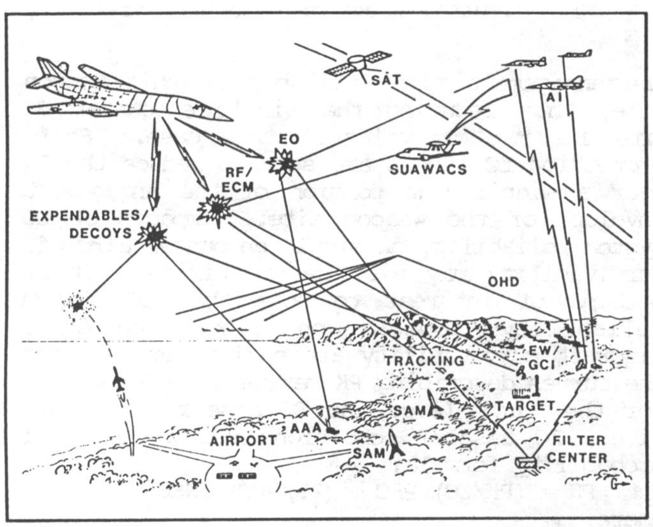

Figure 9. Elements of operational scenario

When these data bases, including the one-on-one data, are available, the aircraft survivability and mission success analyses can begin. As the simulated flight of aircraft penetrate the hostile air space, the engagements occur as assignments are made by the command and control net. These assignments are the results of the rules of engagement used by the hostile forces, and will often result in other than optimum weapon system assignments, for the models include the effects of deceptive jamming as well as noise jamming.

The probability of kill for the ith aircraft in the scenario, Pki, is:

$$PKi = 1- \pi(1-PKij)$$

where,

PKij: Probability of kill of the ith aircraft by the jth weapon system

PKi: Accumulated kill of the ith aircraft for multiple engagements

A simplified example of accumulated aircraft attrition is shown in Figure 10. When the Pki exceeds a threshold, T, the aircraft is considered killed and removed from the scenario. For many engagement conditions, a value of T equal to 0.6 will remove aircraft as attrited exactly as a Monte Carlo simulation would predict the mean number of aircraft attrited for the same scenario (9).

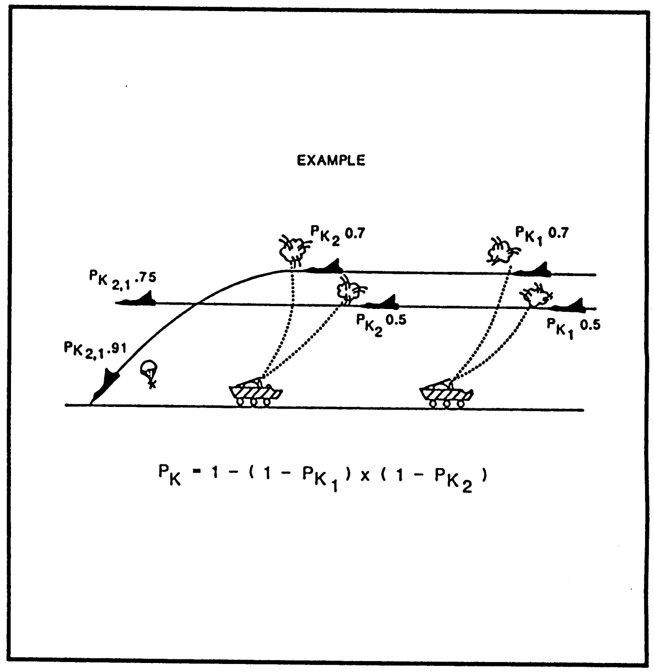

Figure 10. Cumulative aircraft attrition from multiple engagements

This mechanism permits the analysis of a variety of options, including tactics such as maneuvers and altitude selection. So long as the option results in a change in the individual site Pkij curve as a function of assignment time, or influences the assignment time through the command and control net, the simulation can provide a measure of the effectiveness of that option in terms of aircraft survivability.

The final product from exercise of the GEM is shown in Figure 11 which shows a plot of hypothetical results from several analysis iterations to determine the sensitivity of friendly force attrition to a wide range of alternative electronic combat asset employments.

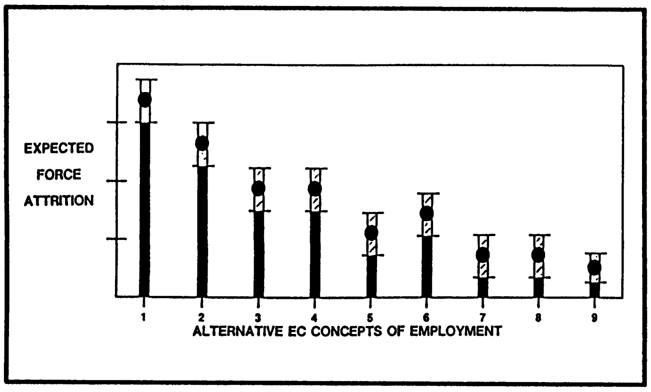

Figure 11. Quantitative assessment of electronic combat employment alternatives

SUMMARY

The General Effectiveness Methodology (GEM) is an analysis procedure, formally realized in a hierarchy of computer simulation tools, which is useful for quantifying the impact on aircraft attrition of alternative electronic combat concepts of employment. Aircraft vulnerability to individual weapon systems is contained in three dimensional probability of kill tables indexed by azimuth angle, elevation angle, and miss distance. Single encounter survivability is determined for all possible engagements to calculate probability of kill as a function of time when the weapon system initiates the engagement. Single encounter probability of kill databases are generated for all potential engagement pairings of aircraft and weapon systems within a specified scenario to support subsequent mission analyses. The mission analysis procedure uses a threshold on probability technique for aircraft removal to obtain single pass estimates of expected aircraft attrition which reproduce multi-run Monte Carlo results.

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