Scatter Communications with Radar Chaff*

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Summary—**The first part of this paper is concerned with finding an analytical expression for the scattering cross section of chaff oriented randomly within a vertical cone. The dipoles are allowed to take on all the angles within this cone. A vertically-polarized receiver is assumed off on the horizon and the transmitter on the ground below the chaff. The cross section is a function of the conical angle of the configuration and the angle between a normal to the ground and the incident electric field from the transmitter. Fig. 2 is a plot of the scattering cross section as a function of these two angles.**

Half-wave chaff randomly distributed within a conical angle about a vertical is not the most effective ensemble, but is a practical one at the lower frequencies. Cutting all these half-wave dipoles into very short ones makes it practical to place them in a horizontal position which has an ensemble gain over the conical but a reradiation loss, since short dipoles are less effective scatters than half-wave ones. The second part of this paper compares the reradiation loss and horizontal ensemble gain.

I. Introduction

THIS PAPER investigates the possibility of establishing moderate range communications for short intervals of time between non-line of sight points by using a large number of metallic half-wave dipoles (chaff), placed at a modest altitude somewhere between, used as the scattering element.

The initial chaff altitude has to be sufficient to allow line of sight propagation from it to both antennas. Chaff falls at the rate of several hundred feet per minute, and additional height of this amount above that needed for line of sight is required. For points separated some 200 miles, the line of sight altitude is 20,000 feet for smooth earth.

The transmission loss depends directly on the scattering ability of the chaff, which in turn depends on the length of the chaff dipoles and the arrangement of these dipoles in the ensemble. Half-wave resonant dipoles are considered first. The ensemble configuration is a random orientation of chaff within a vertical cone. The dipoles are allowed to take on all the angles within this cone. This particular configuration prompted Spogen[1] to suggest the possibility of using chaff dipoles much shorter than a half wave, but oriented in a horizontal plane. The hope was that the scattering loss due to the short dipoles would be balanced out by the gain of the superior configuration.

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II. Transmission Equation

The transmission equation gives the relationship between the power radiated by the transmitter antenna and the maximum power available at the receiver antenna terminals. It is similar to the radar equation, except that here the transmitter and receiver are at separate locations and hence can be at different distances from the scattering target.

The transmission equation can be derived as follows: The effective transmitter radiated power P_2 incident to the chaff is

$$P_2 = \frac{P_1 G_1 \bar{\sigma}}{4\pi r_1^2}, \tag{1}$$

where

P_1 = transmitter-radiated power,
r_1 = effective distance from transmitter antenna to chaff,
$\bar{\sigma}$ = average cross section of the chaff,
G_1 = average transmitter antenna gain in the direction of the chaff.

The cross section of the chaff σ[2] is by definition that area intercepting that amount of power which, when scattered isotropically, produces an echo equal to that observed from the target, for chaff σ is a fluctuating quantity. Here we use the average value and discuss the amount and frequency of this fluctuation in the section on chaff characteristics.

From the definition of σ, the incident power to this area as given in (1) must be scattered isotropically[3] to give the correct amount of power at the receiver. Thus, the power available at the receiving antenna terminal is

$$P_3 = \frac{P_2 A_3}{4\pi r_2^2} \tag{2}$$

where

A_3 = effective area of the receiving antenna,
r_2 = effective distance between the chaff and the receiving antenna.

² Isotropic scattering is merely a convenient and standard way of defining σ. Any type of scattering could be used to define σ, though each type gives a different value. The isotropic scattering used here to define σ in no way forces or constrains the real chaff actually to scatter energy isotropically. In fact, it does not; see (23).

³ D. E. Kerr, "Propagation of Short Radio Waves," McGraw-Hill Book Co., Inc., New York, N. Y., p. 33; 1951.

The effective area A of an antenna is related to its gain G as follows:

$$A = \frac{\lambda^2 G}{4\pi} \cdot \tag{3}$$

Eqs. (1)–(3) can be combined to give the transmission equation

$$P_3/P_1 = \frac{G_1 G_3 \lambda^2 \bar{\sigma}}{(4\pi)^3 (r_1 r_2)^2} \cdot \tag{4}$$

The net transmission attenuation can be calculated from the above equation once the system parameters of antenna gains, signal frequency, chaff effective area, and distances from chaff to both antennas are given. This attenuation is the amount that the available power P_3 at the receiver terminals is below the power P_1 radiated by the transmitting antenna.

III. Chaff Characteristics

A. Scattering Cross Section

The scattering cross section can be expressed analytically as

$$P_3 = \frac{P_2 \sigma}{4\pi r^2} \tag{5}$$

where

P_2 = incident power per square meter to the chaff,
P_3 = scattered power per square meter at the receiver,
r = distance from chaff to receiver.

For a given target, σ generally depends on its orientation with respect to both the incident wave and the receiver. This point is examined in more detail later. For now, it is only necessary to realize that σ is a function in spherical coordinates of the two angles θ and ϕ.

For a single chaff dipole the scattered power per square meter at a receiver distance away r is

$$P_3 = \frac{P_d G'}{4\pi r^2}, \tag{6}$$

where

P_d = power in the dipole collected from the incident wave and then reradiated,
G' = gain of the dipole in the direction of the receiver.

For a short-circuited resonant half-wave dipole,[4]

$$P_d = 4AP_2 \tag{7}$$

where

$A = \lambda^2 G/4\pi$ = antenna effective area,
λ = wavelength.

Substitution of (7) into (6) gives

$$P_3 = \frac{4AP_2G'}{4\pi^2} \cdot \tag{8}$$

Equating (8) and (5) and solving for σ gives

$$\sigma = 4AG_d = \frac{\lambda^2 G G'}{\pi} \cdot \tag{9}$$

Eq. (5) gives the value for σ only for resonant dipoles. Since σ is a function of angles, it should also be written as

$$\sigma(\theta, \phi) = \frac{\lambda^2}{\pi} G(\theta, \phi) G'(\theta', \phi'). \tag{10}$$

The dipoles in the chaff ensemble are assumed to be in a state of motion, fluttering and dispersion, such that the total scattered signal has a Rayleigh amplitude as a function of time. Under these conditions, the average power from the ensemble is just the sum of the average power per dipole. This average power is a time average found by allowing the dipole to assume all the allowable positions in the configuration. For half-wave dipoles the scattering cross section equation using spherical coordinates is

$$\bar{\sigma} = \frac{N\lambda^2}{A\pi} \int_0^{\phi} \int_0^{\theta} G(\theta, \phi) G'(\theta', \phi') \sin\theta \, d\theta \, d\phi, \tag{11}$$

and

$$A = \int_0^{\phi} \int_0^{\theta} \sin\theta \, d\theta \, d\phi, \tag{12}$$

where $G(\theta, \phi)$ and $G'(\theta', \phi')$ are the respective dipole gains in the direction of the transmitter and receiver, and N represents the number of dipoles in the ensemble.

B. Physical Properties

Chaff[5] has been used during war time as a means of establishing a large reflecting area to radar waves. The chaff was made from thin aluminum foil strips approximately one-half-wavelength long. Packages, each containing several thousand dipoles, were released from bombers and allowed to disperse in the air and float downward.

The first chaff which was made bent when packaged bundles of it were thrown into the slip stream of the airplane. Such bending caused some of the strips to become tangled into small masses that reflected little of the

[4] J. D. Kraus, "Antennas," McGraw-Hill Book Co., Inc., New York, N. Y., p. 47; 1950.

[5] D. Fink, "Radar countermeasures," *Electronics*, vol. 19, pp. 92–97; January, 1946.

radar wave and also fell quite rapidly. In addition, adjacent strips of the smooth foil stuck to one another, preventing rapid dispersal. To solve these problems, the foil was embossed and then crimped along its length. This chaff proved to be highly dispersive and fell at the rate of about 150 feet per minute. It was designed for 450 to 600 Mc and thus was 10 to $11\frac{1}{2}$ inches long. The weight of 1000 dipoles was only 2 ounces.

Studies made during the war on the 450- to 600-Mc chaff show that its rate of fall depends on its orientation in space. Horizontal dipoles fall about two thirds as fast as vertical dipoles. It is thought that dipoles shaped regularly fall in a near horizontal position. Dipoles that are irregular initially, or bent in the dispersal, are thought to fall predominantly in a near vertical position. Kuiper[6] gives rates of fall from 150 to 500 feet per minute depending on the type of chaff as well as its orientation in space.

The 450- to 600-Mc chaff dispersed about equally into the vertical and horizontal positions. This is in marked contrast with the results for two-inch dipoles at 3000 Mc.[7] The horizontal dipoles (3000 Mc) returned more power than the vertical dipoles by a factor of 12 to 15 db. This increase is thought to occur because the two-inch dipoles are quite regular and tend to resist bending, compared to the 10- to $11\frac{1}{2}$-inch chaff, and hence fall predominantly in a horizontal position.

C. Location of the Chaff

The optimum location of the chaff depends upon the criteria used. One criterion is to locate the chaff so as to minimize the product r_1r_2 in (4) for a given transmitter-receiver separation.

Assume that for a given transmitter-receiver separation the sum of r_1+r_2 is essentially constant when the chaff is placed somewhere between. Then the product r_1r_2 decreases as the chaff is moved from a central point toward one of the antennas. This is easy to demonstrate with numbers. Let r_1 and r_2 each be 5; their sum is 10, and their product 25. Now, let r_1 be 7 and r_2 be 3; their sum is still 10, but their product is now only 21. When r_1 is increased still more to 9 and r_2 reduced to 1, the product is now only 9. Under these conditions, the chaff should be as near one antenna as possible.

The earth's curvature makes it necessary to elevate the chaff. Thus, the optimum chaff location seems to be directly over one of the antennas at an altitude sufficient for line of sight propagation to the other antenna. From a practical viewpoint, this makes the chaff placement rather straight forward.

The chaff altitude depends on the rate at which it falls, the desired communication time, as well as the height needed for line-of-sight communications. The elevation needed for line-of-sight conditions, using a 4/3 earth-radius correction factor, is

$$h = D^2/2, \tag{13}$$

where

h = altitude in feet,
D = line-of-sight distance in miles.

A 200-mile separation between transmitter and receiver requires a minimum chaff altitude of 20,000 feet above a smooth earth. Irregular earth at the antenna farthest from the chaff, as well as the chaff rate of fall, makes it necessary to increase this altitude by an appropriate amount.

IV. Calculation of the Scattering Cross Section

Fig. 1 shows the positions of the two antennas with respect to the chaff origin. The chaff dipoles are assumed to be randomly oriented within the cone about the z axis bounded by a constant value of θ. The scattering cross section for a single fixed dipole within the cone is

$$\sigma(\theta, \phi) = \frac{\lambda^2}{\pi} G(\theta, \phi)G'(\theta', \phi'), \tag{14}$$

where $G(\theta, \phi)$ accounts for the chaff dipole gain including polarization with respect to the transmitting antenna and $G'(\theta', \phi')$ with respect to the receiving antenna. The angles θ and ϕ are the angles the chaff dipole makes with the x, y, z coordinates, while the angles θ' and ϕ' are with respect to the x', y', z' coordinates.

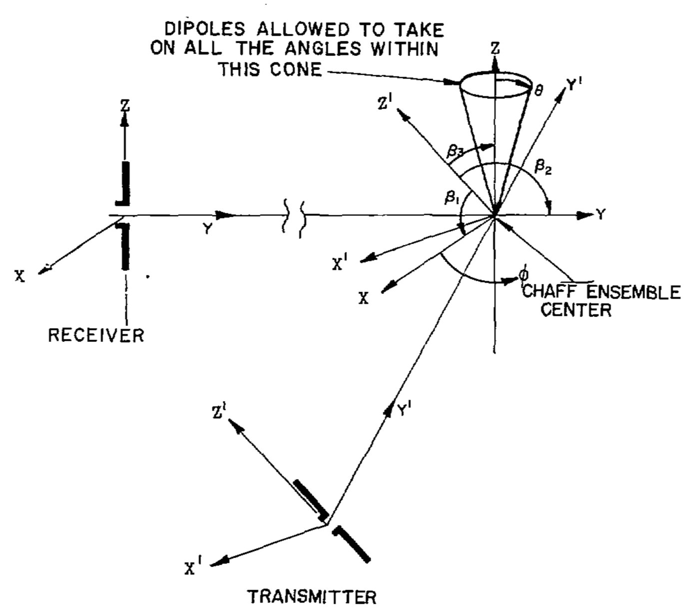

Fig. 1—Basic coordinate system.

[6] G. P. Kuiper, "A Study of Chaff Echoes at 515 Mc," Radio Res. Lab., Harvard University, Cambridge, Mass., Rept. 411-73, p. 1; December 19, 1943.
[7] *Ibid.*, p. 15.

For a half-wave chaff dipole, the gain in the direction of the transmitting antenna is

$$G(\theta, \phi) = 1.64 \left[\frac{\cos(\pi/2 \sin\theta)}{\cos\theta}\right]^2. \quad (15)$$

This expression for antenna gain does not lend itself to analytical integration or to a transformation in coordinates. The pattern for a short dipole is

$$G(\theta, \phi) = 1.5 \cos^2\theta. \quad (16)$$

This expression is manageable analytically without great effort. The pattern of a short dipole is slightly wider than that for a half-wave dipole, but the error involved is not appreciable, probably being in the order of a few tenths of a decibel. The gain of a chaff dipole in the direction of the receiving antenna is

$$G'(\theta', \phi') = 1.5 \cos^2\theta'. \quad (17)$$

To evaluate the integral for $\bar{\sigma}$ in (11), it is necessary to transform one set of coordinates to the other. Here the primed coordinates are transformed back to the original coordinates. That is, $G'(\theta', \phi')$ is expressed as a function of θ and ϕ, the unprimed coordinates. This transformation[8] is standard and goes as follows: Let R be the length of a chaff dipole. Its projections on the original coordinates are

$$R_x = R \sin\theta \cos\phi, \quad (18a)$$

$$R_y = R \sin\theta \sin\phi, \quad (18b)$$

$$R_z = R \cos\theta. \quad (18c)$$

Adding primes to all the values except R makes (18) valid for the primed coordinate system. That is,

$$R_x' = R \sin\theta' \cos\phi', \quad (19a)$$

$$R_y' = R \sin\theta' \sin\phi', \quad (19b)$$

$$R_z' = R \cos\theta'. \quad (19c)$$

The projection on the z' axis can also be expressed as

$$R_z' = R_x \cos\beta_1 + R_y \cos\beta_2 + R_z \cos\beta_3. \quad (20)$$

Eqs. (17), (18), (19c) and (20) yield the following expression for $G(\theta', \phi')$ in terms of the unprimed coordinates and the rotational angles β_1, β_2, and β_3:

$$G'(\theta', \phi') = 1.5[(\sin\theta \cos\phi \cos\beta_1)^2 + (\sin\theta \sin\phi \cos\beta_2)^2 + (\cos\theta \cos\beta_3)^2 + \text{cross-product terms}]. \quad (21)$$

Each cross-product term contains one of the following: $\sin\phi \cos\phi$, $\cos\phi$, or $\sin\phi$. Each of these integrates to zero when ϕ goes from 0° to 360°, as it does for the conical configuration of chaff considered here. Thus, the cross-product terms are omitted in the work that follows.

Substitution of (16) and (21) into (11) yields a solution for the scattering cross section. The integrals with respect to ϕ are

$$\int_0^{2\pi} \cos\phi^2 = \pi, \qquad \int_0^{2\pi} \sin\phi^2 = \pi.$$

Thus the expression becomes

$$\bar{\sigma} = \frac{N\lambda^2(1.5)^2}{A} \int_0^{\theta} [\sin^3\theta \cos^2\theta \cos^2\beta_1 + \sin^3\theta \cos^2\theta \cos^2\beta_2 + 2\cos^4\theta \sin\theta \cos^2\beta_3] d\theta. \quad (22)$$

For the particular cone angle of 90° representing spherically random chaff, the scattering cross section becomes

$$\bar{\sigma} = 0.048 N\lambda^2[1 + 2\cos^2\beta_3], \quad (23)$$

since

$$\cos^2\beta_1 + \cos^2\beta_2 + \cos^2\beta_3 = 1. \quad (24)$$

The scattering cross section for other values of θ is shown in Fig. 2 in db relative to the value of (13) when $\cos\beta_3$ is unity.

V. Calculation of the Transmission Loss

The transmission loss is found from (4). Letting the antenna gains be unity and the scattering cross that for random spherical chaff, the normalized loss L_n is

$$L_n = \frac{0.14 N\lambda^4}{(4\pi)^3 (r_1 r_2)^2}.$$

This normalized loss is shown in Fig. 3. Actual scattering cross sections with respect to random spherical chaff are given in Fig. 2. The ordinate values are added or subtracted from the normalized loss to give the actual loss exclusive of antenna gains.

VI. Horizontal Chaff

Half-wave chaff randomly distributed within a conical angle about a vertical is not the most effective configuration but is a practical one for, say, 100 Mc where a half-wave dipole is about 5 feet long. Cutting all these half-wave dipoles into very short ones makes it practical to place them in a horizontal position, which has a configuration gain over the conical but a reradiation loss since short dipoles are less effective scatterers than half-wave ones.

The configuration or scattering cross section is considered first; then the reradiation difference, and finally a comparison.

A. Scattering Cross Section

In Fig. 1, let the chaff be randomly oriented in the $x-z$ plane; now rotate the drawing so that the $x-z$ plane becomes horizontal. This makes the angle ϕ always zero, and the integral [see (11)] for the average cross section reduces to

$$\bar{\sigma} = \frac{N\lambda^2}{A\pi} \int_0^{\pi} G(\theta, \phi) G'(\theta', \phi') d\theta, \quad (25)$$

[8] O. W. Eshbach, "Handbook of Engineering Fundamentals," John Wiley and Sons, Inc., New York, N. Y., pp. 2, 72; 1936.

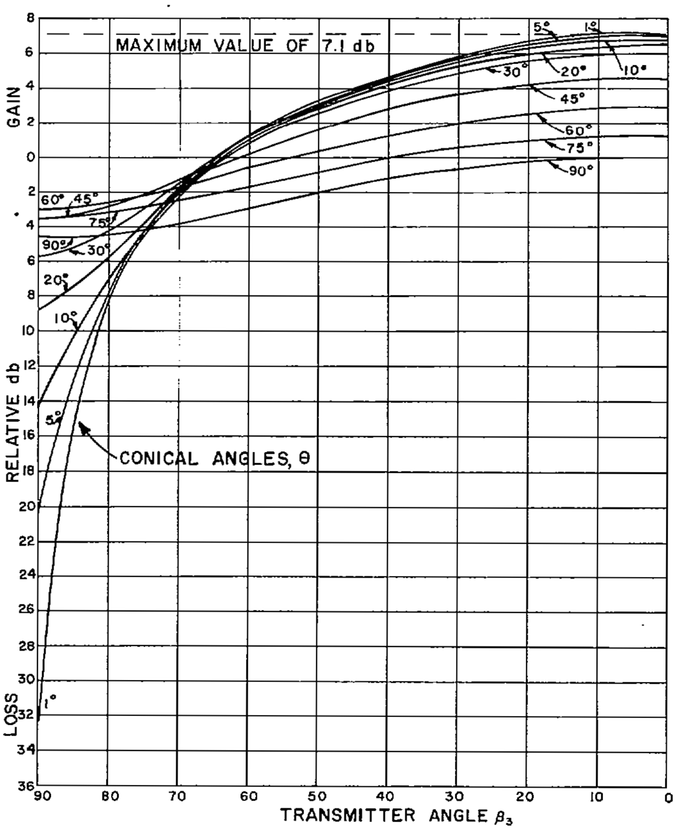

Fig. 2—Scattering cross section ($\bar{\sigma}/(N\lambda^4)$) vs conical and transmitter angles.

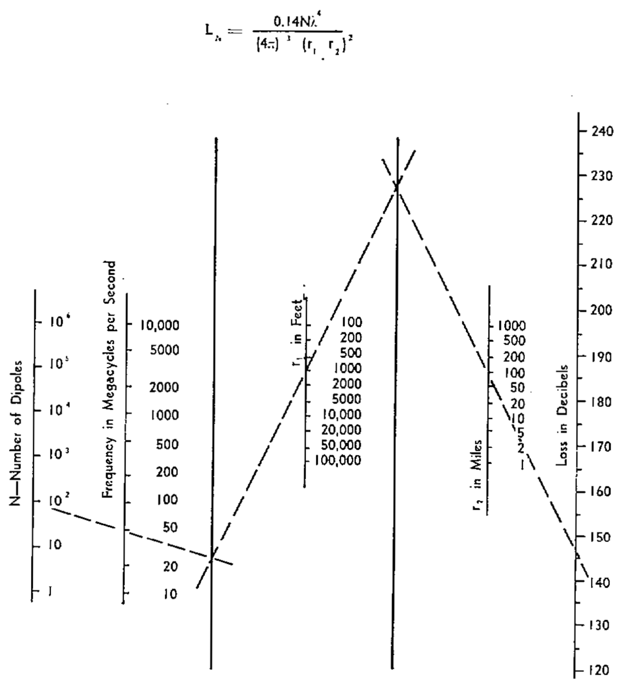

Fig. 3—Normalized transmission loss.

and

$$A = \int_0^{\pi} d\theta = \pi.$$

Substitution of (16) and (21) into (25) yields

$$\bar{\sigma} = \frac{N\lambda^2(1.5)^2}{\pi^2} \int_0^{\pi} [(\sin\theta\cos\theta\cos\beta_1)^2 + (\cos^2\theta\cos\beta_3)^2 + (2\sin\theta\cos^3\theta\cos\beta_1\cos\beta_3)]d\theta. \quad (26)$$

The third term integrates to zero and the first two are

$$\int_0^{\pi} \sin^2\theta\cos^2\theta\cos^2\beta_1 d\theta = \frac{\pi}{8}\cos^2\beta_1$$

$$\int_0^{\pi} \cos^4\theta\cos^2\beta_3 d\theta = \frac{3\pi}{8}\cos^2\beta_3.$$

Thus the integral is

$$\bar{\sigma} = 0.09\lambda^2\cos^2\beta_1 + 0.27\lambda^2\cos^2\beta_3. \quad (27)$$

With the use of (24), this becomes

$$\bar{\sigma} = 0.27\lambda^2 - 0.27\lambda^2\cos^2\beta_2 - 0.18\lambda^2\cos^2\beta_1, \quad (28)$$

which has a maximum value of

$$\bar{\sigma} = 0.27\lambda^2 \quad (29)$$

when both β_1 and β_2 are 90°. This value compares with the zero ordinate value of $0.14\lambda^2$ in Fig. 2; thus it represents about a 2.9-db gain over the zero scale in Fig. 2.

B. Short Dipole Reradiation

A short dipole compared to a half-wavelength is not too effective at absorbing and reradiating electromagnetic energy. The object of this section is to find quantitatively the amount of this decrease compared to a half-wave resonant dipole. Orientation is not a factor in this calculation; therefore let both dipoles be rotated so as to intercept the maximum power from an incident plane wave. The current at the midpoint of the dipole is

$$I_{sc} = \frac{V_{oc}}{z_a},$$

where

$V_{oc} = EL'$,
E = incident electric field strength,
L' = effective length of the dipole,
Z_a = dipole impedance, $R + jX$.

The power reradiated by the dipole is

$$P_d = I_{sc}^2 R$$

$$P_d = \frac{(EL')^2 R}{(R+jX)^2} \cdot \quad (30)$$

Let L_1 be the length of a half-wave resonant dipole and L_2 the length of a short dipole. In comparing the reradiation ability, assume that the short dipoles outnumber the half-wave dipoles so that the same total footage of each type exists. That is, for every half-wave dipole let there be m short dipoles when $m = L_1/L_2$. A half-wave resonant dipole has zero reactance and a radiation resistance of about 70 ohms. With these values, the radiated power ratio H/S becomes

$$\frac{H}{S} = \frac{(L_1')^2}{(L_2')^2} \frac{(R_2+jX_2)^2}{m70R_2}. \tag{31}$$

The effective lengths are

$$L_1' = \frac{2L_1}{\pi} = 0.64L, \qquad L = \lambda/2, \tag{32a}$$

$$L_2' = \frac{2L_2}{3} = 0.67L, \qquad L \ll \lambda/2. \tag{32b}$$

Effective lengths are defined in Jordan[9] with current distributions as given by Schelkunoff and Friis.[10] Eq. (32b) represents the limiting value for infinitely short dipoles.

The ratio now becomes

$$\frac{H}{S} = \frac{m(R_2 + X_2)^2}{76R_2}. \tag{33}$$

The radiation resistance of short dipoles is

$$R = 80\pi^2 \left(\frac{L'}{\lambda}\right)^2. \tag{34}$$

Substitution of (32b) for the effective length gives

$$R = 350 \left(\frac{L_2}{\lambda}\right)^2. \tag{35}$$

The reactance of a dipole depends on its shape as well as its length-to-width ratio. Jordan[11] and Schelkunoff[12] both show curves of reactance vs length for several length-to-width ratios. The curve in Jordan labeled $r_0/r_\lambda = 0.001$ and the one in Schelkunoff for $a/b = 70.7$ are about the same. Both of these represent for a flat dipole a length-to-width ratio in the range of 20 to 35. Smaller ratios reduce the reactance but are not thought to be practical. Table I lists the reactances, resistances, and the H/S ratio for various lengths L_2 of short flat dipoles.

[9] E. C. Jordan, "Electromagnetic Waves and Radiating Systems," Prentice-Hall, Inc., New York, N. Y., p. 333; 1950.
[10] S. A. Schelkunoff and H. T. Friis, "Antennas, Theory and Practice," John Wiley and Sons, Inc., New York, N. Y., p. 242; 1952.
[11] Jordan, *op. cit.*, p. 364.
[12] S. A. Shelkunoff, "Advanced Antenna Theory," John Wiley and Sons, Inc., New York, N. Y., p. 118; 1952.

C. Comparison of Reradiation Loss and Horizontal Ensemble Gain for Short Dipoles

The reradiation loss of short dipoles is merely a measure of their decreased ability to scatter electromagnetic energy compared to half-wave dipoles. The same footage of material is used in both cases; thus, short dipoles one-tenth the length of a half-wave dipole are ten times more numerous in their ensemble. This loss is listed in Table I for various lengths of short dipoles.

The short dipoles are assumed to be randomly distributed in a horizontal pattern, whereas the half-wave dipoles have a random distribution within a fixed angle about a vertical axis. The scattering ability of the horizontal ensemble is 8.6 db better than a 30° conical ensemble and increases rapidly with a decrease in polar angle. Other values of conical angles are shown in Fig. 2 for $\beta_3 = 90°$, remembering that the horizontal ensemble has a value 2.9 db above the zero axis.

Fig. 4 is a plot of the reradiation loss as a function of short dipole length. The horizontal ensemble gain over various conical angles is also indicated on the curve. For example, short dipoles 0.2λ long have a reradiation loss of 26.8 db and the horizontal ensemble has a gain of this same amount over a conical ensemble of somewhat less than 5°.

TABLE I
RERADIATION LOSS FACTORS

L_2	m	R_2	X_2	H/S	
0.4λ	1.25	56.0	140	6.5	8.2 db
0.2λ	2.5	14.0	450	480	26.8 db
0.15λ	3.3	7.9	560	1700	32.3 db
0.10λ	5.0	3.5	830	13×10^3	41.1 db
0.05λ	10.0	0.9	1600	440×10^3	56.4 db

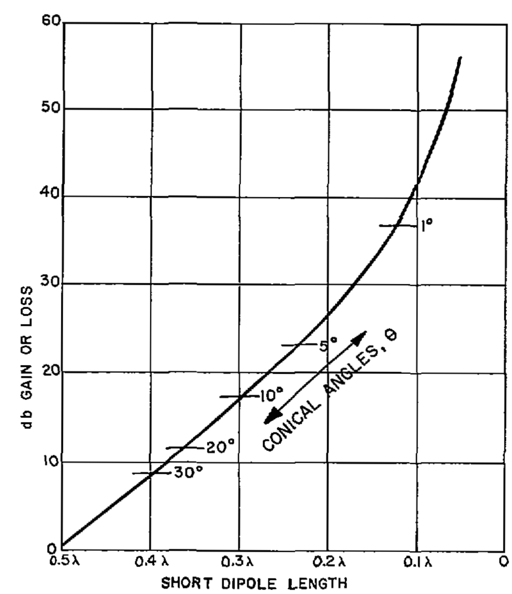

Fig. 4—Reradiation loss and horizontal ensemble gain for short dipoles, where $\beta_3 = 90°$.

VII. Conclusions

The first part of this report was concerned with finding an analytical expression for the scattering cross section of chaff oriented randomly within a vertical cone. A vertically polarized receiver is assumed off on the horizon and the transmitter on the ground below the chaff. The cross section is a function of the conical angle of the configuration and the angle between a normal to the ground and the incident electric field from the transmitter. Fig. 2 is a plot of the scattering cross section as a function of these two angles. The values are all relative to the same number of dipoles placed in a spherical random ensemble. Fig. 3 is a nomogram for finding the transmission loss for a spherical random ensemble. The necessary corrections to the conical ensemble are obtained from Fig. 2.

Half-wave chaff randomly distributed within a conical angle about a vertical is not the most effective ensemble but is a practical one at the lower frequencies. Cutting all these half-wave dipoles into very short ones makes it practical to place them in a horizontal position which has an ensemble gain over the conical but a reradiation loss since short dipoles are less effective scatters than half-wave ones. The second part of this report compared the reradiation loss and horizontal ensemble gain. Fig. 4 is a plot of the loss as a function of the short dipole length. The horizontal ensemble gain over various conical angles is also indicated on the curve.

Diffraction by a Slit*

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Summary—**The electric field diffracted by a strip caused by an incident cylindrical wave with E parallel to the edge, at various angles of incidence, is measured in a parallel plane medium. The field is compared with that computed from geometrical optics currents and with the addition of equivalent line currents at the edges. The edge "line currents" improve the geometrical optics current field particularly at oblique incidence.**

I. Introduction

A SOLUTION to the problem of the diffraction of electromagnetic waves by a strip was first given by Morse and Rubinstein.[1] In their method of solution the strip is considered as the limiting case of an elliptical cylinder, and an eigenfunction solution is developed. This result is expressed in the form of an infinite series involving Mathieu functions.

Because of the inadequate tabulation of Mathieu functions and also because of the slowness of convergence for large arguments, the above solution has been cumbersome to use, and satisfactory approximate techniques have been sought. Thus, from an analysis of the induced currents on a strip of width λ/π and $2\lambda/\pi$ caused by normal plane wave incidence and with E parallel to the edge, Moullin and Phillips[2,3] characterized the diffracted field as due to geometrical optics currents plus equivalent line currents at the edge. In this formulation they followed an approach originally suggested by Braunbek.[4,5]

Moullin and Phillips noted that the currents near the edges of a strip are very similar to that near the edge of a half plane, provided the strip is $2\lambda/\pi$ or greater in width. By examining the "deviation from geometrical optics" currents for the half plane, the author[6] developed a more generalized "edge current" which permits consideration of oblique incidence. We summarize this development below.

Considering the geometry in Fig. 1, the Sommerfeld

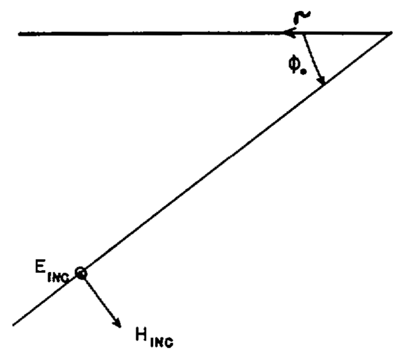

Fig. 1—Half-plane geometry.

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[1] P. M. Morse and P. J. Rubinstein, "The diffraction of waves by ribbons and by slits," *Phys. Rev.*, vol. 54, pp. 895–898; December, 1938.

[2] E. B. Moullin and F. M. Philips, "On the current induced in a conducting ribbon by the incidence of a plane electromagnetic wave," *Proc. IEE*, vol. 99, pp. 137–150; July, 1952.

[3] E. B. Moullin, "On the current induced in a conducting ribbon by a current filament parallel to it," *Proc. IEE*, vol. 101, pp. 7–17; February, 1953.

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