

# Aircraft Survivability Modeling Evaluation and Optimization for Multi-UAV Operational Scenarios

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This project attempts to provide an advanced perspective for aircraft survivability in a modern threat age. Traditional and modern approaches are analyzed to quantify emerging threats and modern countermeasures. A sensitivity analysis is performed on the models and identifies sensitive aircraft survivability parameters. A simple aircraft survivability scenario of a C-130J Hercules against a single MANPADS is modeled and simulated. The scenario undergoes verification and validation Monte Carlo analysis. A MDAO approach is discussed to incorporate aircraft design with the produced aircraft survivability results. In whole, the project provides analytics of a modern perspective on aircraft survivability amongst the reality of emerging threats.

#### I. Nomenclature

 $r_k$  = firing rate

 $s_1$  = detection envelope time  $s_2$  = lethal envelope time T = simulation tool vector x = vector of design variables y = vector output of the CA

 $\sigma$  = uncertainty

#### **II.** Introduction

The focus of this project is the modeling, design, and operation of unmanned aerial vehicles with an emphasis on evaluation and optimization towards metrics of aircraft survivability. This chapter presents an introduction and motivational background to the topics of modeling and simulation, aircraft survivability, and aerospace engineering. Modeling and simulation is widely used in the aerospace community for design decision support regarding tradeoffs among performance, cost, and aircraft survivability. Aircraft survivability describes the capability of an aircraft to survive an encounter with an enemy, and aircraft survivability is understood to be an important metric for combat mission analysis. By understanding an aircraft's survivability, decision making for war game scenarios can be supported by engineering modeling and simulation. Given the importance for aircraft survivability, the aerospace industry strives to design and manufacture aircraft with high survivability to provide to their customers. The motivation came to support the aircraft survivability effort and improve the aircraft survivability approach with modeling and simulation. In result, modeling simulation can improve aircraft survivability to provide more useful metrics for practical combat scenario applications.

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## III. Overview

## A. Modeling and Simulation

A model is a representation of behavior, structure or information, and can be virtual or physical. Models typically have inputs and outputs, and have utility in describing physical phenomenon [2] [63]. A simulation is a representation of a model within a time-based sequence to, often represented by states [32] [63]. Having the ability to abstractly represent a phenomena though modeling and simulation can provide insight into the realistic capabilities of the aircraft. This project uses modeling and simulation to better understand aircraft survivability under novel perspectives and modern applications.

In order to have utility for decision-making, a model must undergo a process of verification and validation to understand whether a model fit for purpose. Verification is the activity of reasonably arguing a model's proper implementation with respect to the model description and solution [37] [51]. Typically, verification references established theories and seeks to comparing measurements of the model to established theories while describing explanations [40]. For instance, a model of turbulent flow could be verified through comparison to established theories suggesting that phenomena [46] [49]. Validation is the activity of deterministically arguing the amount of accuracy in a model representing the physical world under the intended uses [37] [51]. Validation often uses measured information from the model to provide an argument for agreement between experimental evidence and modeling and simulation metrics [42]. An example of a validation comparison would be comparing the point of turbulent flow separation from simulation, and from a representative airfoil in a wind tunnel experiment [3] [46] [49]. Metrics of validation can include modeling uncertainty, pure error estimate, and experimental error [30] [38]. Validation often relies on verification for support [48]. These metrics and physical comparables are typical accepted verification and validation evidence in the modeling and simulation community. This approach uses common verification and validation methods to demonstrate predictive to and convince the modeling and simulation community of findings.

#### **B.** Aircraft Survivability

The term "aircraft combat survivability" (ACS) is defined in *The Fundamentals of Aircraft Combat Survivability Analysis and Design, Second Edition* as "the capability of an aircraft to avoid or withstand a man-made hostile environment" [1]. Aircraft combat survivability is one of the most important metrics of aircraft performance and design [21]. Survivability is the ability of an aircraft to avoid or endure an artificially hostile environment and has a relationship with killability, susceptibility, and vulnerability [1]. Where killability is an aircraft's inability to avoid or endure an artificially hostile environment and is comprised of the product of susceptibility and vulnerability [1]. Also, susceptibility is the aircraft's inability to avoid hostile attacks [1]. Lastly, vulnerability is the aircraft's inability to withstand hostile attacks [1]. An understanding of aircraft survivability has been demonstrated to have considerable impact on military tactics and strategic decision making in combat [22].

The purpose of survivability *modeling* is to provide decision-makers with relevant, credible evidence, conveyed with some degree of certainty or inferential weight, about the survivability of an aircraft [1]. To model an aircraft's survivability for purposes of design, numerous methods have been developed that can be incorporated into design, refinement, maintenance, and operations stages of the aircraft lifecycle [54]. Some important modeling methods include the methods of Ball and shot-line geometrics for precision shots on subsystems, shown in Figure 1, and consider armored air vehicles [1] [29] [58] [59]. Many of the design tools that are available today implement Ball-type and shot-line methods, including BRAWLER, AFSIM, etc. [21] [39]. All the tools surveyed in this section are highly proprietary and typically require specific reasoning and/or clearance to acquire.

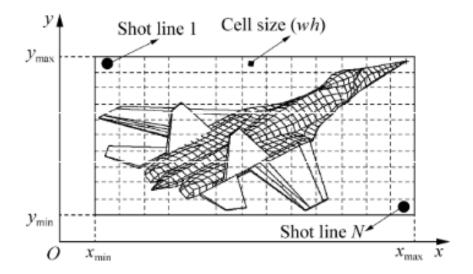


Fig. 1 Traditional Shot-line Geometric Approach [58]

For the shot-line geometrics approach, attacks effectiveness on the air vehicle are evaluated by the accumulation of attacks effects on the aircraft subsystems. A shot-line is measured to the subsystems within the shot-line path and the attacks effects are relative to subsystem armoring, subsystem redundancies, attack effectiveness, and various other parameters [57]. Measuring shot-line geometrics is effective for aircraft design scenarios, yet somewhat unpredictable in a mission level engagement. For this project, shot-line geometrics is recognized as specific attacks at a subsystem level to system level. In result, shot-line geometrics become out of scope due to the focus of this survivability approach is system level to mission level. Instead of shot-line geometrics, other higher level and generic methods are used.

The literary reviews discovered the state of being for aircraft survivability and an opportunity for a modeling and simulation application. As the research progressed, Ball's method was seen as prominent and appropriate for almost every application. Shot-line geometrics were also discovered as effective and useful detailed aircraft survivability approaches, specifically for subsystems. In this project, the system level application suggests the shot-line geometrics as out of scope. With more review progression, other aircraft survivability tools were recognized including BRAWLER and AFSIM. As the tools were discovered, they were noted to be difficult to require due to institutional withholdings. Next, Wang's method was discovered as an effective opportunity to include the range and time threatened by an enemy entity. After that, understandings of emerging and modern threats were discovered in the form of digital pheromones, loyal wingman, and swarming [26] [44]. These in whole have been the basis of the aircraft survivability improvement approach.

Relative to these traditional AS methods, new AS performance metrics and AS concepts have been developed to keep pace with emerging aircraft tactics and technologies [4]. For example, a traditional AS metric of performance is "hits on target", the number of munition hits that an aircraft can incur before failure [1]. New AS concepts understand that modern anti-aircraft munitions are far more effective than traditional munitions and that there may be ways to avoid being hit by enemy fire at all [9] [13] [45]. The modern threats today include MANPADS, AIMs, RIMS, etc. where the traditional threats have been flak, small arms, etc. <sup>6</sup> The newer modern methods discovered take into account more advanced ways to improve aircraft survivability.

The newer survivability methods are similar in nature with specific differences in practice. Depicted in the next few figures is each survivability application in a universal depiction language. The white UAV near the right of the images represents an air vehicle to be analyzed. Near the center of the images an anti-air emplacement represents a hostile entity. Surrounding the hostile entity, an orange circle illustrates the detection range and a red circle shows the lethal envelope. Lastly, the blue dashed arrow line(s) across the image represents the air vehicle flight path. The figures aid the depiction of each modern aircraft survivability application.

Firstly, the lethal envelop developed by Wang considers an aircraft threatened only when within range of a hostile entity [56]. Figure 2, shows one single vehicle traversing a combat space. Within the combat space is a hostile entity centered. The air vehicle travels past and directly above the hostile entity. As the aircraft approaches and leaves the hostile entity, the aircraft enters and leaves the detection range and lethal envelope [14]. Within the

detection range, the aircraft is able to be observed by the hostile entity. Within the lethal envelope, the aircraft is vulnerable to hostile entity attacks [16]. Scenarios similar to Figure 2 are simulated and iterated to observe the aircraft survivability. The lethal envelope approach represents a simplistic, bare-bones analysis for aircraft survivability. Today, methods have been developed to improve an aircraft's chances of surviving hostile entity encounters.

Modern methods to reduce aircraft killability as a whole have been considered. For example, the digital pheromone approach, described in Figure 3, seeks to sense and avoid hostile areas [19] [43] [52]. The loyal wingman approach, shown in Figure 4, seeks to intercept enemy fire and reduce hits on the aircraft [24] [25]. Swarm approaches, illustrated in Figure 5, considers aircraft survivability as a system rather than one vehicle [55]. Each of these methods is a step toward a more modern and relevant aircraft survivability analysis.

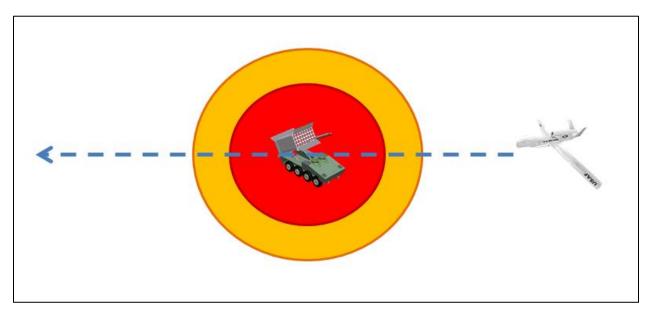


Fig. 2 Simple Lethal Envelope Scenario [56]

Digital pheromones are identifiers of area allegiance and are typically communicated to system entities. In Figure 3, a digital pheromone scenario is depicted. Similar to the lethal envelope approach, there are familiar elements: lethal envelope, detection range, flight path, etc [23]. In the digital pheromone scenario, the green area is the area denoted as safe by the air vehicle. The red box near the center of the image is the hostile area denoted by the air vehicle. Distinguishing between the two safe and unsafe areas provides the air vehicle with the opportunity to avoid an unsafe encounter, increasing survivability [10] [11]. For aircraft survivability, digital pheromones plays the role of locating potential hostile areas and deciding how best to avoid [27]. Certain areas are assigned their hostility type: hostile, neutral, or safe and are communicated to the navigating aircraft. With the area being known, the vehicle can choose the navigation route minimizing exposure to enemy hits [61]. By knowing and/or avoiding hostile areas, the aircraft is less likely to take enemy attacks, in result improving the aircraft survivability.

In AirSurF, digital pheromones utilized to various scenarios to determine aircraft survivability when navigating the least exposure to hostiles. The framework assigns hostility to square areas throughout the scenario for the vehicle to discover and decide. The methodology has the vehicle finding the hostile area, checking the surroundings, and deciding how to progress. The vehicle will often avoid hostile areas unless there is no other navigation option [62]. By avoiding enemy hostile areas, the vehicle can reduce the amount of hits it will receive, improving survivability.

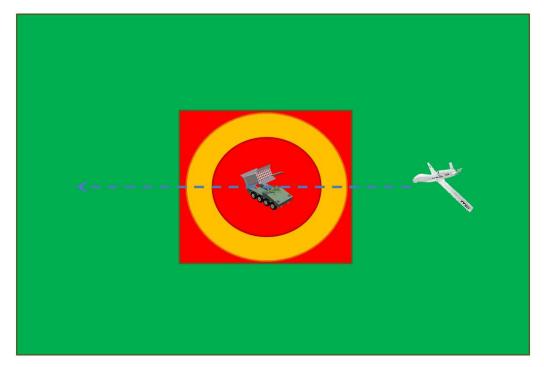


Fig. 3 Digital Pheromone Approach [44]

Loyal wingmen are dedicated air vehicles to protecting an escort vehicle either offensively and/or defensively [26]. Figure 4 shows a loyal wingman scenario where multiple vehicles are escorting an air vehicle. The air vehicles on each side of the centered air vehicle are loyal wingmen, intended to protect the centered air vehicle. Protecting the centered vehicle has many applications, defensively and offensively. Loyal wingmen are capable to intercepting hostile entity attacks as well as neutralizing hostile entity threats [50]. The loyal wingman is a newer concept in reference to unmanned aerial vehicles. Autonomous countermeasures with loyal wingman defending an escort vehicle are being explored. Possible solutions include intercepting incoming attacks and/or deploy countermeasures to enemy threats/entities [25]. Each consideration is investigated with these approaches of modeling and simulations.

Another capability of the loyal wingman is to have offensive measures. The loyal wingman is often able to attack the hostile enemy to eliminate any possible future attacks. The elimination of threats, in result, reduces the hits of the escort vehicle. Countermeasures can include munitions payloads, jamming, lasers, etc. Countermeasures will be likely option to simulate with.

Currently, instances of the loyal wingman include Boeing developing loyal wingman UAV from Royal Australian Air Force. The vehicle is described as four to six vehicles operating in conjunction to an escorted vehicle with performance similar to a fighter with sensor applications to conduct reconnaissance, surveillance, intelligence, and electronic warfare. A loyal wingman could provide information to the escorted vehicle for decision making as well as combat hostile entities defensively and offensively with electronic warfare. An instance of defensive electronic warfare combat could be disabling an incoming missile with jamming [34]. Where, an instance of offensive electronic warfare could be disabling an anti-air ground installation with directed energy. Each capability could be invaluable for supporting an escort vehicle [24].

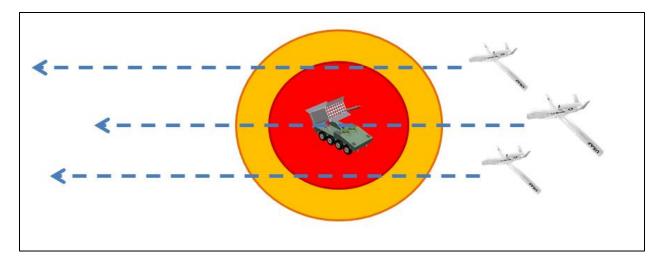


Figure 4. Loyal Wingman Approach [24]

Swarms are systems in multi-vehicle configurations. Depicted in Figure 5, swarms can be seen as multiple vehicles encounter hostiles as a system. Often swarms have self-awareness with the vehicles in the system. The traditional approaches for swarming are a collection of vehicles working to a common objective cooperatively. Vehicles in swarms are often expendable to fulfill the decided upon objective. By implementing swarming, the system will have a higher chance for survivability due to multiple vehicles enduring attacks rather than one vehicle.

Swarm is a system level of vehicle composition. Traditionally, aircraft survivability is of a single vehicle. With the swarm, survivability is measured in reference to the entire system of systems. There are two means to a swarm approach. A one-hit fail system or a system comprised of multiple-hit vehicles. The swarm provides robustness to enduring enemy assaults.

Swarms may also utilize countermeasures to better ensure the survivability of the system and the completion of the mission objective. Utilizing swarms to attack enemy entities can vary from ranged targeting to vehicles delivering their equipped payloads with onboard system navigation.

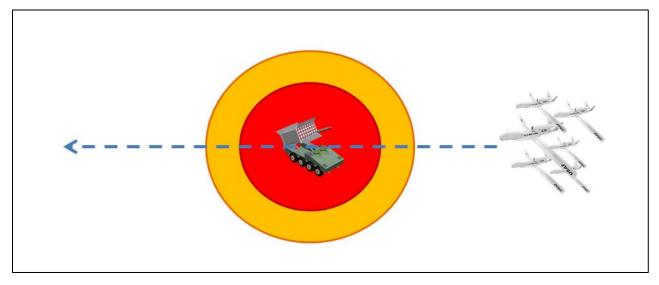


Figure 5. Swarm Approach [43]

## C. Overview Summary

Based on these observations, there exists a considerable gap in the understanding of the broader design space around the design of UAS for survivability. As aircraft survivability threats and technologies have advanced, aircraft survivability modeling concepts must do so as well. No research to date has defined the tradeoffs between the aircraft performance characteristics of UAS and modern survivability concepts including modern metrics, tactics and technologies. None of the survivability software suites are available for public inspection, validation and use under open-source science concepts. None of the aircraft survivability design concepts have been demonstrated to have utility to a UAS design process. This effort will seek to advance the state of knowledge in this field by addressing these gaps.

## IV. Approach

#### A. Introduction

The current aircraft survivability approach is driven to model hits on target and location of hit analysis (shot-line geometrics) [1] [33] [60]. The approach defined in this research seeks to provide more considerations to various important aircraft survivability factors. This approach considers the time within engagement range of an enemy entity (lethal envelope) [12] [56]. Also, various other advanced capabilities to engage with emerging threats have been applied. These considerations are digital pheromones, loyal wingman, and swarming [19] [24] [55]. All the combined analytic considerations provide a more robust and realistic understanding of aircraft survivability while preserving the value of traditional aircraft survivability approaches.

### B. UAS-specific Aircraft Survivability Models

This research effort establishes the methods and framework for physical and empirical parametric modeling of UAS-specific aircraft survivability. Information from Robert E. Ball's approach regarding number of hits on aircraft relative to aircraft survivability has been gathered to develop a model that can be simulated and integrated with various other approaches (i.e. lethal envelope, digital pheromones, loyal wingman, swarm, etc.) [1] [24] [55]. Traditionally, aircraft survivability approaches apply to manned and unmanned air vehicles. With the considerations of advanced aircraft survivability counters: loyal wingman, digital pheromones, and swarming, this approach is for unmanned aircraft. Via literature research and conference interfacing, strengths and weaknesses of current aircraft survivability analyses have been identified. Also, a new aircraft survivability methodology for a more robust, modern, and realistic approach has been developed. A modeling and simulation framework for analyzing the new aircraft survivability methodology has been developed and implemented with verification and validation evidence. Sensitivity analysis to identify aircraft design parameters closely related to aircraft survivability has been used. The new aircraft survivability approach with the old in relation to aircraft design has been compared and contrasted.

## C. Tactics and Performance

The AS community would like to be able to consider the modeling of AS in early design stages of a UAV/UAS design process.<sup>3</sup> Design considerations for UAS are modeled in a framework that allows for representation of the identified modern aircraft survivability tactics and scenarios. Tactics and performance of modern UAS are represented parametrically in an integrated and optimizable system model of Aircraft Survivability. There are a number of challenges that are associated with executing this representation. Many of the concepts that are defined as making up modern UAS performance and operation have not been represented outside of the academic literature, so their efficacy and impact on aircraft design has not been quantified. The interactions between the components of the simulation are complex, multi-domain and time dependent. The software must be constructed to be open-source and computationally efficient to be able to allow optimizability and adoption by the community. A baseline aircraft survivability framework and toolkit has been created. Open-Source implementation for AS community and military users has been created. Also, modern UAS-relevant tactics and performance models (Digital Pheromone method, Loyal Wingman, Swarm) has been implemented [1] [24] [55].

The integratability and optimizeability of the model will be supported if the models developed can be used to predict and optimize the survivability performance of a UAV under baseline and modern scenarios. If the design model can be used within a UAV design process to conceptually design and develop a UAS that meets design requirements, then the optimizeability and design process utility of the model will be supported.

#### D. Verification and Validation

With design models there exists a fundamental tradeoff between the fidelity of the design model and its usability in a computational design process. If the model is too refined, then the computational cost becomes too great for use in early stages of design. If the model is computationally efficient, but cannot predict the relevant design tradeoffs, then the model is of no value to designing among those tradeoffs. This research question seeks to understand the level of validation and verification that can be achieved using the models proposed. The proposed aircraft survivability software and methods are validated and verified.

Direct validation of aircraft survivability models has been complicated by the lack of data regarding "experimental" aircraft survivability datasets. For this effort, new datasets for aircraft survivability are unable to be gathered, and a suite of analyses and comparisons to establish the level of trust that can be placed in the proposed models by practitioners has been developed. In order to have utility for design purposes, the design model must be of the correct scale in order to capture relevant design characteristics, but must not be bloated with irrelevant contributing analyses. This research question seeks to identify sensitive and relevant flight performance parameters for quantifying aircraft survivability metrics.

#### E. Aircraft Design

The final aspect of this research is to demonstrate the utility of the models and considerations of aircraft survivability in improving the design of UAVs. The conditions under which the proposed UAV design parameters can be demonstrated to have benefit relative to a baseline design process are being defined. This project asserts that the design of UAS can be improved towards metrics of aircraft survivability by including tactics, missions, and behavioral modeling of the UAS groups with a deeper understanding of aircraft survivability. This research question seeks to build a direct comparison of UAV/UAS design with and without these detailed aircraft survivability models. The results of this research question will quantify the differences between the aircraft design methods proposed in this work, and a default aircraft sizing and synthesis algorithm that uses naïve surrogates metrics of performance to approximate survivability. As seen by the analytics, aircraft cruise speed is very sensitive to aircraft survivability. Almost all aircraft design parameters are related and cruise speed indirectly influences various parameters including weight, thrust, lift, fuel efficiency, etc. Therefore, the conditions to where the proposed UAV design parameters can be demonstrated to have benefit relative to a baseline design process are endless. Sensitive aircraft survivability parameters with system sensitivity analysis are being identified. Within an aircraft sizing algorithm, sensitive aircraft survivability parameters are being related to generic aircraft design parameters. With identified related aircraft design parameters, design changes utilizing each of the traditional aircraft survivability and the new approach are being made. Traditional aircraft survivability analysis is being compared and contrasted to the newer approach in relation to aircraft design.

#### V. Results

## A. Modeling and Simulation Development

This research has first developed a modeling framework to simulate the tactics and performance of modern UAS parametrically using a system model of aircraft survivability. The proposed approaches apply the traditional survivability methodologies including hits on target and the lethal envelope, as well as more modern analyses including digital pheromones, swarming, and loyal wingman. These applications listed are diagramed in the various figures below. For hits on target, the aircraft survivability decreases as hits on targets increase. For lethal envelope, as the vehicle is within the envelope, the UAS is vulnerable to hits. The more modern approaches take into account dynamic scenarios for specialized encounters. Specific computational and conceptual models were developed and simulated to accurately measure aircraft survivability and identify related sensitive parameters.

The overall objective of the framework is to provide reasonable air vehicle design parameter feedback in regards to survivability to the user. In Figure 7, a high-level outline describing inputs and outputs to significant subsystems of the framework is diagramed. The initial input to the simulation is the performance of the air vehicle system and a specific scenario definition. These characteristics are input to one of the aircraft survivability analyses (as specified by the user). Outputs from the survivability analysis are the three primary survivability metrics (survivability, susceptibility, and killability). The air vehicle design analysis outputs significant aircraft design parameters related to aircraft survivability and sensitivity analysis is performed on the output parameters. The approach can be described as a stochastic multidisciplinary design and optimization (MDO) method. The output of this modeling is a uncertainty informed understanding of the tradeoffs between air vehicle design and survivability in a specific scenario.

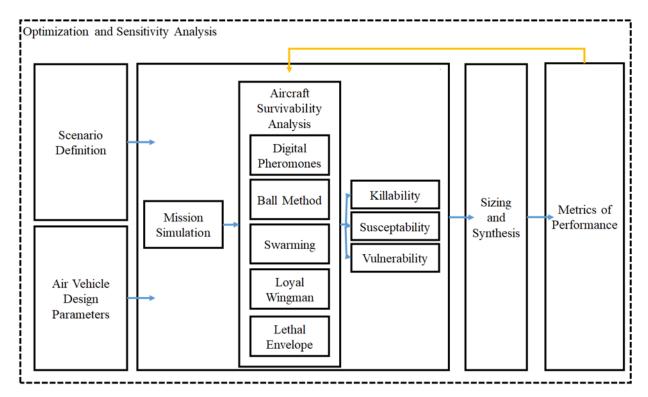


Fig. 7: High-level Simulation Framework Flow Chart. Under the control of the user interface, the user can input scenario and design parameters for execution under one of the 5 survivability analyses. Under the option of optimizer control, the optimizer measures metrics of performance and optimizes the design of the aircraft to minimize objectives.

At highest level, the modeling and simulation tool has been organized into system architecture. Shown in Figure 8, the framework is comprised of analysis, scenarios, simulations, and a user interface (UI). Each item performs an important role for the overall functionality of the framework as shown in attributes and operations. Together, the simulation analyzes a scenario simulation specified with the user interface. There are more complexities within each block contributing to the entirety of the aircraft survivability framework.

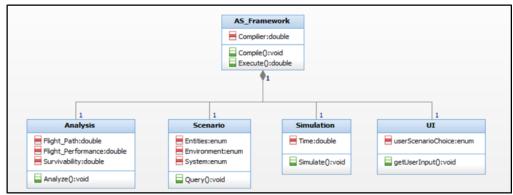


Fig. 8 Architecture for the Modeling and Simulation

Aircraft survivability can be a challenging metric to quantify, this project provides an approach to calculating aircraft killability, the opposite of survivability. In doing so, previous predicted generic metrics and information to a detailed model are applied utilizing simulation with physical and statistical analytics. For this demonstration, verification and validation methods are used to predict the average killability for a Lockheed Martin C130J Super Hercules in a mission where the aircraft is hit at least once. Figure 9 shows an image of the C130J Super Hercules in

flight. The C130J often enters hostile environments due to the larger landing approach of the aircraft [28]. By predicting the C130J Super Hercules average killability, an understanding of a blind environment mission can be quantified.



Fig. 9 Lockheed Martin C130J Super Hercules

In an aircraft survivability modeling sense, scenarios describe when and where the aircraft are exposed to hostile environments [6]. When the opposition is mostly known, having a metric to predict the likelihood of aircraft survivability of any encounter has value. The realistic hostile environments are when an aircraft is exposed to enemy environment operations, ambush tactics, and guerilla warfare. For the United States military, these environments have been encountered regularly in recent times [28]. As aircraft are valuable assets, having an understanding of the likelihood of aircraft survival is effective for military strategy, planning, support, and go-no-go decisions.

The model used is an aircraft survivability calculation from Analytic Model for Aircraft Survivability Assessment of a One-on-One Engagement by Xu Wang, Bi-Feng Song, and Yi-Fan Hu published in the *Journal of Aircraft 2009* [56]. There are multitudes of considerations to generate the one-on-one engagement model and are too lengthy to be listed here, see the publication for any conceptual clarification. Some of the more relevant considerations are the number of hits on the aircraft, aircraft velocity, and lethal area [7]. In the Air Force Institute of Technology publication, an A-10 Warthog was arbitrarily chosen as the conceptual model aircraft, given its documented exposure to hits in operation [36]. Figure 10 and Equation (1) illustrates some aspects of the model used, where Figure 10 shows the geometry of the combat scenario and Equation (1) is the survivability equation. With this model embedded into the AirSurF simulation, aircraft survivability can be determined as a function of aircraft performance characteristics and scenarios characteristics.

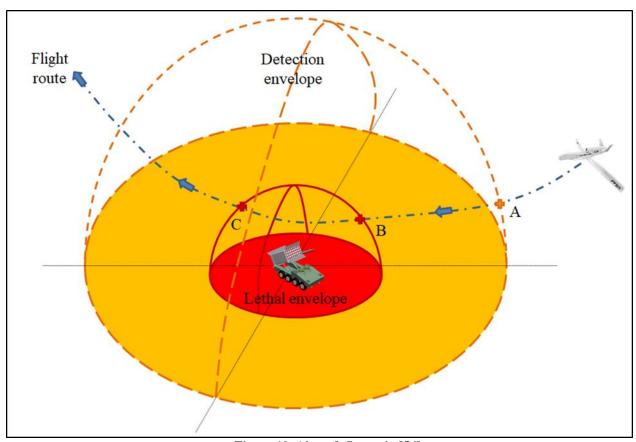


Figure 10. Aircraft Scenario [56]

$$\begin{split} P_{S} &= e^{-(s_{2}-a)r_{d}} + \frac{1}{2}\sqrt{q_{SSK}}r_{d}e^{-r_{d}(s_{2}-a)} \\ &\times \left[ \left( 1 + \sqrt{q_{SSK}} \right) \frac{e^{(s_{2}-s_{1})\beta} - 1}{\beta} - \left( 1 - \sqrt{q_{SSK}} \right) \frac{e^{(s_{2}-s_{1})\gamma} - 1}{\gamma} \right] \\ &+ \sqrt{q_{SSK}} \left( 1 - e^{-(s_{1}-a)r_{d}} \right) e^{-2r_{k}(s_{2}-s_{1})} \left[ \sinh \left( 2r_{k}\sqrt{q_{SSK}}(s_{2}-s_{1}) \right) + \sqrt{q_{SSK}} \cosh \left( 2r_{k}\sqrt{q_{SSK}}(s_{2}-s_{1}) \right) \right] \\ &+ \sqrt{q_{SSK}} \cosh \left( 2r_{k}\sqrt{q_{SSK}}(s_{2}-s_{1}) \right) \right] \\ &= a < s_{1} \end{split}$$

$$\beta = r_d - 2r_k \left(1 - \sqrt{q_{SSK}}\right) \tag{2}$$

$$\gamma = r_d - 2r_k \left( 1 + \sqrt{q_{SSK}} \right) \tag{3}$$

Equation (1) defines the Aircraft Survivability Equation,  $P_s$ , for a lethal envelope scenario. Within the equation are a plethora of variables, including  $s_1$ , the time the aircraft spends within the detection envelope. Variable  $s_2$  is the time the aircraft spends within the lethal envelope. Variable a is the time the hostile entity acquires the presence of the aircraft, tracks, and obtains a firing solution. The reciprocal of the mean time of detection is represented by rd.

Variable  $q_{SSK}$  is the single shot survivability. The reciprocal of the mean time of detection is represented by  $r_d$ . The listed variables can calculate total aircraft survivability.

The data acquired is from The Fundamentals of Aircraft Combat Analysis and Design by Ball, Robert E. from the AIAA Educations Series 1985 in which the means to the data acquisition is unknown [1]. From either experiments or an analytical model, the data will be treated as experimental. The provided data is aircraft killability in relation to aircraft exposure to hits. The range of hits of an aircraft is provided as 1 to 30 for redundant aircraft and 1 to 18 for nonredundant aircraft. The C130J computational model is likely to have many redundancies. However, both the redundant and nonredundant killabilities were measured in the likelihood of the C130J being a hybrid aircraft of both redundant and nonredundant subsystems. As aircraft survivability is less experimental and more predictive, the provided data is used and compared to the referenced engagement model. Input data and parameters are extensive including the lethal area, time detected, aircraft velocity, etc. The variations of inputs for the Monte Carlo simulation are the aircraft velocity and the number of hits. Where the variation of input for the MDA simulation is the detection time, detection and lethal area time, and reload fire rate. As seen in Equation (3), redundant or nonredudant aircraft killability greatly increases per hit. The exposure to multiple or one hit(s) is compared and used in the prediction of a C130J Super Hercules average killability.

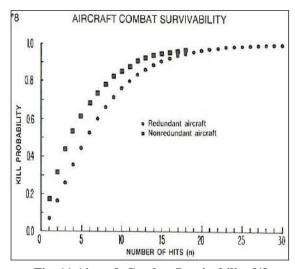


Fig. 11 Aircraft Combat Survivability [1]

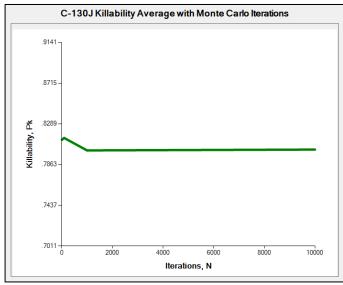


Fig. 12 C-130J Killability Average

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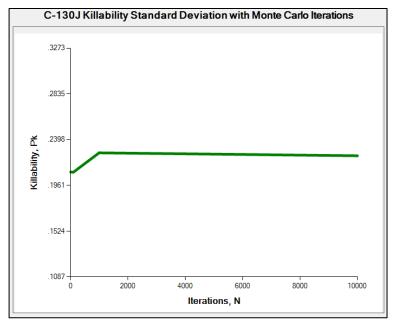


Fig. 13 C130J Killability Standard Deviation

The observed model and scenario would be described as Normal mode, consisting of traditional approaches and no emerging threat methods. Normal Mode's traditional aircraft survivability analysis is comprised of Ball's hits on target and Wang's lethal envelope. Specifically in the example scenario, the survivability implications of a C-130J entering the lethal envelope of a MANPADS, is shown. In Figure 12, the killability average can be seen and in Figure 13 the killability standard deviation is shown, both with Monte Carlo iterations. As seen from the Monte Carlo simulation, the average killability of a C130J Super Hercules for any mission exposed to hostilities is ~0.80 with a standard deviation of ~0.22. Other scenarios were developed to consider more threats and different air vehicles.

To conduct the study, a framework tool was developed and referred to as AirSurF. The user selects the scenario, mode, and scenario specifics. Figure 14 shows the User Interface (UI) to selecting the scenario mode. As shown, each mode represents one or multiple emerging threat considerations. The AirSurF utility is intended to be open-source with this project. Having a tool to conduct the analytics was irreplaceable for acquiring results.

```
Select scenario mode

1) Normal
2) Digital Pheromone
3) Loyal wingman (In development)
4) Swarm (In development)
5) Blended (DP/LW) (In development)
6) Blended (DP/S) (In development)
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Fig. 14 AirSurF User Interface

#### **B.** Countermeasures Modeling

In traditional aircraft survivability, the ability to withstand hits is measured without considering the capability of neutralized an enemy threat. Many air vehicles are equipped with defensive munitions. In a case-by-case scenario, the munitions may be used to eliminate enemy threats. By utilizing countermeasures to remove the possibility of attacks hitting the aircraft, the overall aircraft survivability will improved. Countermeasures are an AirSurF simulation option for specific scenarios.

## C. Verification and Validation

Supporting the Monte Carlo analysis, CDFs and their inverses are calculated. Figure 15 and Figure 16 show the CDF and inverse CDF of the measured number of hits. Whereas Figure 17 and Figure 18, show the CDF and inverse CDF of the Experimental Error in regards to the simulation. In addition, the sensitivity is seen and considered from the Kline McClintock calculations comprised of the experimental and validation errors. The largest error into to the Monte Carlo killability prediction is the model error. The tool error is relatively small. There, the killability variance can be seen as sensitive to the number of hits on the aircraft. With analysis, a variance in the aircraft cruise speed would influence the aircraft killability slightly. Cruise speeds for aircraft are consistent and vary little. Otherwise, the aircraft killability is slightly sensitive to aircraft velocity uncertainty [15]. Figure 19, 20, and 21 show the histograms supporting the aircraft killability Monte Carlo analysis. Another MDA simulation sheds more information to the C130J killability.

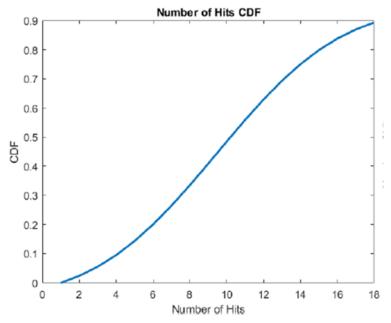


Fig. 15 Number of Hits CDF

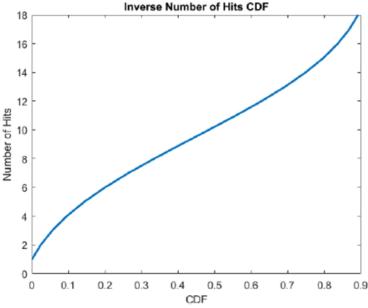
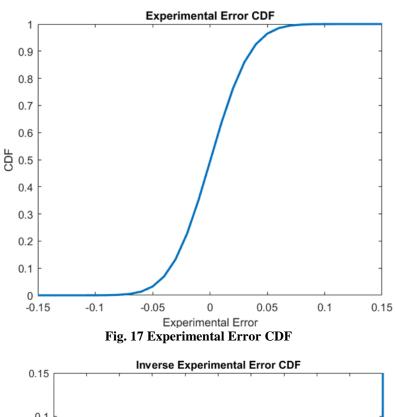


Fig. 16 Inverse Number of Hits CDF



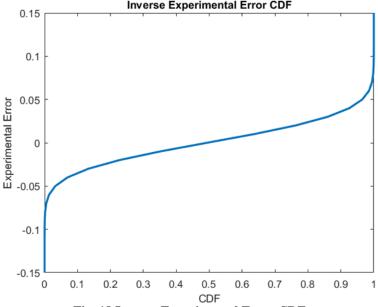


Fig. 18 Inverse Experimental Error CDF

As can been seen from the killability error, the model and provided killability of an aircraft are somewhat similar. The decent size error is enough to suggest the model is verifiable. Figure 17 and 18 display the cdf and inverse cdf of the experimental error. Some other uncertainties could be the various other inputs (i.e. lethal area, time detected, etc.) [17]. For comparison, many inputs were chosen to emulate one hostile entity for the model verification and the C130J simulation. Uncertainty in the hostile entity capabilities could greatly influence the aircraft killability (i.e. large lethal envelope, fast fire rate, etc.). With the moderate killability and experimental error, the model can be stated as somewhat verified.

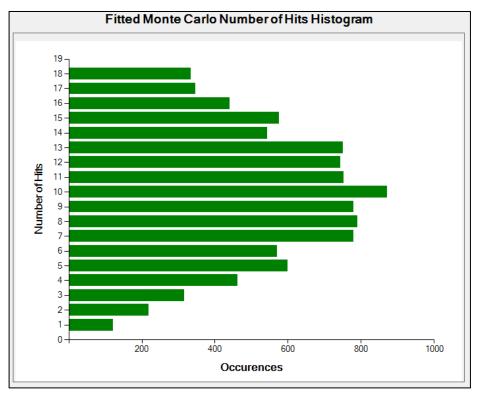


Fig. 19 Monte Carlo Number of Hits Histogram

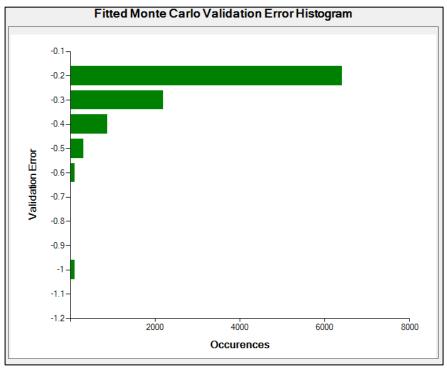


Fig. 20 Monte Carlo Validation Error Histogram

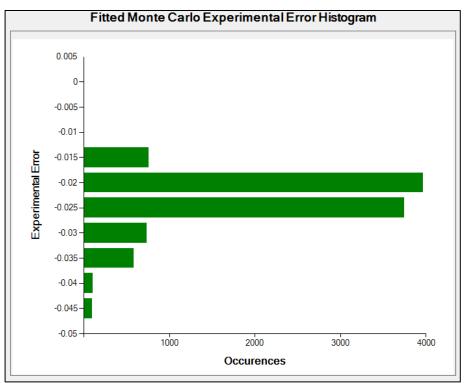


Fig. 21 Monte Carlo Experimental Error Histogram

The histograms of the Monte Carlo method describe various distributions for input and output parameters. In Figure 19, the Number of Hits Histogram is a relatively normal distribution. Also, the Validation Error, seen in Figure 20, seems to be an exponential distribution skewed right. The Experimental Error is more so a normal distribution, skewed left. Each distribution was used to conduct the Monte Carlo application on the model. By observing the distributions, some insight to the Monte Carlo approach is illuminated.

## D. Error Propagation and Estimation of Bias Error

This project intends to report model uncertainties effectively with minimal concessions to inaccuracy. An accepted method of uncertainty measurement is the ISO/ANSI approach, where Type A uncertainties are estimated with sampling and Type B uncertainties are estimated without sampling. Kline McClintock approach enables the combination of the two estimations. The Kline McClintock merge of ISO/ANSI method is used for uncertainty measurement. Without correlation and dependence, the first-order Kline-McClintock propagation of uncertainty for a x = f(u,v) equation is shown in Equation (4). The listed propagation of uncertainty approach is effective for simple and easily differentiable equations. For more challenging equations, the derivative can be approximated or Monte Carlo utilized. In the case of type B uncertainties, the uncertainty can be estimated with the un-sampled uncertainties using the sum of the square of the standard deviation [32]. The Kline-McClintock equation can be expanded out to support uncertainty propagation.

$$\sigma_x^2 \cong \sigma_u^2 \left(\frac{\partial x}{\partial u}\right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v}\right)^2 + \cdots$$
(4)

The first order Kline-McClintock equation shown in Equation (4) equation provides a basis to applying uncertainty propagation measurement. Where x is the vector of design variables, y is the vector output of the CA, and  $\sigma^2$  is a variance. An expansion of equations follows to make the Kline-McClintock more effective for simpler applications. In Figure 22, a model input and output in reference to the x = f(u,v) equation is displayed. For a more practical equation, Equation (5) is another simplified form the Kline McClintock equation in first-order. In the shown equations, L is represented as an uncertain input and all the models are deterministic. Now to consider

uncertainty, Figure 23 is the previous model diagram with a uncertainty update. With uncertainty taken into account, the Equation (4) Kline-McClintock equation is updated as shown in Figure 22. To again simplify the equation, Equation (7) reduces into another first order Kline McClintock equation with uncertainty [30]. Variable T represents a simulation tool vector. The Kline McClintock approach producing uncertainty shown supports the reported model sensitivity results.

$$\sigma_{v}^{2} = \sigma_{L}^{2} \left(\frac{\partial T}{\partial L}\right)^{2}$$

$$L \pm \sigma_{L}$$

$$V \pm \sigma_{v}$$

$$V \pm \sigma_{v}$$

$$(5)$$

Fig. 22 Model input and outputs [30]

$$\sigma_v^2 = \sigma_T^2 \left(\frac{\partial v}{\partial T}\right)^2 + \sigma_L^2 \left(\frac{\partial v}{\partial L}\right)^2 \tag{6}$$

$$\sigma_v^2 = \sigma_T^2 + \sigma_L^2 \left(\frac{\partial T}{\partial L}\right)^2 \tag{7}$$

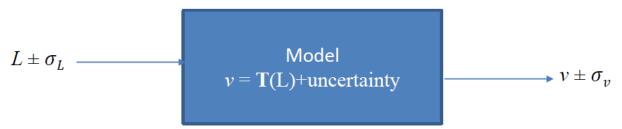


Fig. 23 Model input and outputs with uncertainty [30]

The Kline-McClintock approach in conjunction with the Mont Carlo iterations produced relatively reasonable le results for uncertainty. Each result for uncertainty supports the model sensitivity analysis. The estimated killability model average relative error is reported at 4.54% +/- 8.67% @ 90% CI. Survivability is a difficult metric to quantify the report relative error is acceptable in general nonetheless reasonable for aircraft survivability [18]. The measured number of hits uncertainty average relative error is reported at 0% +/- 0.169% @ 90% CI. A minuscule average relative error is self-evident as acceptable. The reported total tool variance is 0.00826, a small variance. Finally, total Kline McClintock uncertainty is reported as 2.63, again a small and reasonable result. In whole, the uncertainty approach was appropriate for producing uncertainty measurements effectively. The results produced were verified as accurate at conference discussions with industry community. With the appropriate approach reviewed and community support, the model sensitivity analysis results are viewed as acceptable.

## E. System Sensitivity Analysis Approach

To support the MDO objective, a multidisciplinary analysis (MDA) is applied to the complex system at a system level. Design points are specified as a number of design variables input to the MDA. Decomposed into disciplinary

contributing analyses (CA), the MDA connects to form a design system matrix (DSM). With the MDA connected, the DSM structures information flow between CAs to form a system of coupled compatibility equations as the CA perform the analyses. Compatibility equations are typically used for iterative schemes as applied in this project's approach, ensuring CA variable values are compatible. The aircraft performance is improved with an embedded DSM within an optimization routine, varying input design variables and minimizing a cost function (OEC). [45]

In the MDA approach, there are a few uncertainties accounted for: uncertainty from design variables and uncertainty from the CAs. Often, design variables add to system uncertainty from measurement inaccuracies. For the CAs, there are two sources of uncertainty: assumptions and deterministic computing models. CAs are essentially low fidelity approximations to reduce computational expenses and their discrepancies are measured as an uncertainty output. Also, the deterministic models are not capable of capturing stochastic variation of a component performance with accuracy. These uncertainties within the models are referred as computational noise [8] [53]. By involving feedforwards and feedbacks within the DSM, uncertainty can be increased or decreased nonlinearly at the converged design performance.

This project's approach to uncertainty propagation uses a Monte Carlo with MDA system sensitivity analysis. The MDA approach takes design variables and CAs and assigns them to uncertainties from uncertainty distributions. An approximation of a design point is acquired through iterations. This approach uses state variable vector output of the analysis (y), simple finite deviations (Δy), vector of design variables (x), and simulation tool vector (T) [35]. Also, n represents the number of CAs and m represents the number of inputted design variables. From there, the Local Sensitivity Vector (LSV), as seen in Equation (8), is generated [38]. The LSV represents partial derivatives of the CAs with respect to the design variables. By having the LSV, the total propagated uncertainty can be estimated.45

$$[LSM][GSV] = [LSV] \equiv \begin{bmatrix} I_1 & -\frac{\partial T_1}{\partial y_2} & \cdots & -\frac{\partial T_1}{\partial y_n} \\ -\frac{\partial T_2}{\partial y_1} & I_2 & & \vdots \\ \vdots & & \ddots & -\frac{\partial T_{n-1}}{\partial y_n} \\ -\frac{\partial T_n}{\partial y_1} & \cdots & -\frac{\partial T_n}{\partial y_{n-1}} & I_n \end{bmatrix} \begin{bmatrix} \frac{dy_1}{dx} \\ \frac{dy_2}{dx} \\ \vdots \\ \frac{dy_n}{dx} \end{bmatrix} = \begin{bmatrix} \frac{\partial T_1}{\partial x} \\ \frac{\partial T_2}{\partial x} \\ \vdots \\ \frac{\partial T_n}{\partial x} \end{bmatrix}$$
(8)

From acquiring the total propagated uncertainty, individual contributions of uncertainty can be identified. In Equation (9), the equation for total propagated variance can be seen. In Equation (9),  $\sigma$ s represent uncertainties for specific variables. With the total propagated variance, individual propagated uncertainty can be acquired [38]. Shown in Equation (10), the propagated contribution of uncertainty for either specific CAs or uncertainty of the inputs is illustrated. The listed uncertainty approaches are used to generate the system sensitivity analysis.

$$\begin{bmatrix} \sigma y_1^2 \\ \sigma y_2^2 \\ \vdots \\ \sigma y_n^2 \end{bmatrix} = (LSM^{-1})^2 \begin{pmatrix} \sigma T_1^2 \\ \sigma T_2^2 \\ \vdots \\ \sigma T_n^2 \end{bmatrix} + (LSV)^2 \begin{pmatrix} \sigma x_1^2 \\ \sigma x_2^2 \\ \vdots \\ \sigma x_m^2 \end{pmatrix}$$
(9)

$$\bar{\sigma}_j = \frac{\sqrt{\sum_{i=1}^{n+m} \sigma_i^2}}{\sum_{i=1}^{n+m} \sigma_i} \sigma_j$$
(10)

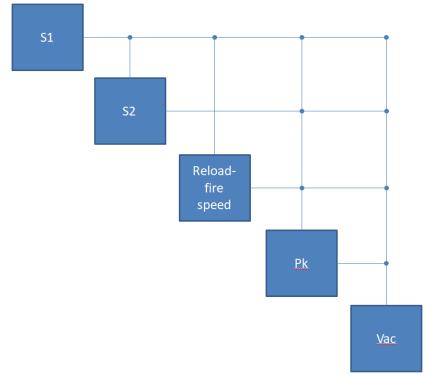


Fig. 24 Design System Matrix

With the sensitivity analysis, comes the application where variance errors are related to Design and CAs. In Figure 24, the MDAO approach is mapped displaying different CA variables to other CA variables to be applied in an iterative fashion. Each relationship between the CA variables is diagramed as circular nodes. The cascading process shows S1, S2, and Reload-fire speed dependent on Vac and influencing Pk. For instance, after Pk is calculated, Vac can be calculated and input to the previous variables, establishing an iterative approach. The diagram is executed until fulfilling a satisfactory convergence criterion. In Figure 25, each CA variables is listed with inputs and outputs as well as relevant fractional errors. With the application, came reasonable system sensitivity results.

CA Number	1	tool1	<tool 1=""></tool>	
x inputs	CA inputs	CA outputs	fractional error in design variable	Fractional error in CA output
Vac		s1	0.05	0.1
CA Number	2	tool2	<tool 2=""></tool>	
x inputs	CA inputs	CA outputs	fractional error in design variable	Fractional error in CA output
Vac		s2	0.05	0.1
CA Number	3	tool3	<tool 3=""></tool>	
x inputs	CA inputs	CA outputs	fractional error in design variable	Fractional error in CA output
Vac		reloadFire	0.05	0.1
CA Number	4	tool4	<tool 4=""></tool>	
x inputs	CA inputs	CA outputs	fractional error in design variable	Fractional error in CA output
numberHits	reloadFire	Pk	0.05	0.2
	s1			
	s2			

Fig. 25 Design and CA Variance Errors

The next efforts of this project are to define exactly how the Vac is going to be attained from Pk. Most likely, the aircraft will be sized using traditional aircraft sizing techniques. In aircraft design, Vac is an important and present variable. By acquiring Pk and desiring a target Pk, the Vac can be modified, thus altering the entire aircraft design. From there, iterations can improve and alter the aircraft design to improve Pk. In result, aircraft design can benefit from a modern perspective of aircraft survivability plugged into current aircraft design applications.

## F. System Sensitivity Results

In a multidisciplinary analysis (MDA) simulation, the s1 (detection time), s2 (detection and lethal envelope time), time to reload and fire, and killability are calculated where the C130J velocity and the average number of hits are design variables. In a system sensitivity analysis using a modified MATLAB script developed by Blake Moffitt and Dr. Thomas Bradley, the s2 contributing analysis has the largest time variance compared to s1 and the reload and fire time [38]. The time variances can be seen in Figure 26. Where the killability has a large influence from the s2 variance as well in reference to Figure 27. The output declares the killability has the largest uncertainty fraction of .4924 from the accumulation of uncertainties. The errors from each design variable were chosen in reference to variation in aircraft speed, time accuracy, and the verification and validation errors as seen in Figure 27. With the sensitivities considered and presented, the MDA is less optimistic than the Monte Carlo methodology claiming a killability of 0.7862 when exposed to 10 hits. Each simulation has a decent indication of likelihood for the C130J killability from exposure to hits. Having system sensitivity in regards to aircraft survivability measured enables an iterative relationship with aircraft design.

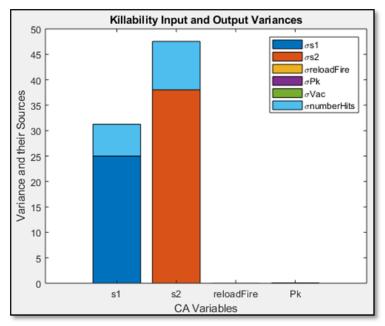


Fig. 26 Killability Variances

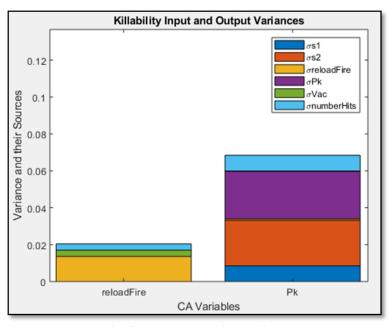


Fig. 27 Reload and Fire Variance

## VI. Future Work

#### A. Aircraft Survivability Related to Aircraft Design

The overall aircraft sizing approach is intended to intake aircraft survivability parameters and output important aircraft design metric values. By having a model and system sensitivity analysis, the aircraft survivability parameters can be related to aircraft design parameters in an iterative fashion. The established iteration relation can form a more solidified output in a Monte Carlo methodology with convergence [5]. Together aircraft survivability parameters' iteratively produced can identify important and sensitive aircraft design metrics [20]. Having modern aircraft survivability analysis related to aircraft design, produces effective and reasonable aircraft design sensitivity analysis.

With modern aircraft survivability analysis, various aircraft design parameters can be reviewed and better understood. Some important parameters for aircraft design are cruise speed, endurance, wing area, mass of aircraft, etc. [31]. The relation of aircraft survivability to aircraft design can improve the aircraft design process by provided more effective aircraft design analysis [41]. For instance, the duration of an air vehicle within the lethal envelope has a direct effect on the aircraft survivability. The design parameter of cruise speed is directly related to time within the lethal envelope. The iterative approach will more effectively gather evidence to support each aircraft design parameter's sensitivity to aircraft survivability [47]. An aircraft survivability centric approach to aircraft design improves an aircraft design to one of the most coveted aircraft metrics, aircraft survivability.

#### VII. Research Contributions

The anticipated research contributions of this project effort include:

- 1) A computational tool for UAS group sizing and synthesis that allows for the evaluation of aircraft survivability in tradeoff with aircraft performance metrics
- 2) The comparison of modern UAS communication and coordination tactics using metrics of aircraft survivability
  - 3) A rigorous verification and validation based uncertainty estimation for aircraft survivability modeling
- 4) A comparison of aircraft-survivability realized UAS design to designs utilizing traditional naïve methods of aircraft survivability.

#### VIII. Conclusion

As threats have advanced, aircraft survivability has opportunities to consider newer challenges. Looking back on the past, Ball opened the world to reliable aircraft survivability analytics and is still effective as well as useful today. Now, that effort can be shepherded forward to combat newer threats with accurate representation of aircrafts' encounters. With that thought, aircraft survivability can adopt modern analytics while preserving Ball's reliable approach. Ball shows the likelihood of survivability with relation to hits on the vehicle. Other methods now take into consideration the air vehicle being exposed to hostile fire in a variety of complex encounters. Combining the two, many scenarios encountering threats can be simulated. Also, more advanced technologies can be considered to reduce the effectiveness to hostile entity exposure. Some of these methods include sensing and avoiding hostile areas, intercepting enemy with the dedicated aircraft, and swarming systems. Have a modern perspective considered, aircraft design can integrate modern aircraft survivability to generate aircraft of modern design. Together, the integration of modern methods with Ball's traditional approach can make aircraft be accurately designed to aircraft survivability in an effective and advanced way.

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