The Verification and Analysis of Chaff Clouds' RCS Distribution Model

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Abstract— **The distribution model of target's RCS fluctuation and the verification method of the distribution model are studied. The probability distribution of chaff clouds' RCS is fitted based on the dynamic measurement data, and the goodness of fit is verified by χ^2 verification. Some problem is indicated to select and apply the verification method goodness of fit, which can be used for the test of chaff bomb.**

I. Introduction

Chaff is a kind of passive jamming material in common use. Chaff jamming is an effective method to jam radar. The chaff clouds' RCS is the basal electricity capability guide line of chaff jamming, which is shaped by chaff bomb in space. The deterministic method and the statistical method can be used to study chaff clouds' electromagnetic scattering. The deterministic method is very complex, because it's difficult to determinate chaff movement and calculational capability is limited to large numbers of chaff. As chaff clouds is a kind of dipole aggregate by large numbers of stochastic moving chaff, the statistical method is reasonable to study chaff clouds' scattering characteristics[1,2].

When chaff is moving in space, amplitude and phase of the scattering signal by a piece of chaff are stochastic variable. Usually, scattering amplitude change is too small to ignore, all chaff can be seemed as equal. The phase change has notable infection, which makes scattering signal fluctuation[3]. Reference1 to 6 analyses measure technique and measure precision of chaff clouds' electromagnetic scattering characteristic, and studies probability distribution of chaff clouds' RCS. The probability distribution of chaff clouds' RCS is fitted based on the dynamic measurement data, and the goodness of fit is verified by χ^2 verification. Some problem is indicated to select and apply the verification method goodness of fit.

II. The Distribution Model of RCS Fluctuation

The study to RCS fluctuation model comes through two phases. In the first phase, non- fluctuation model and five kinds Swerling model are studied. In the second phase, χ^2 distribution model, logarithm normal distribution model and Rice distribution model are studied. The χ^2 distribution model includes Swerling model.

A. χ^2 Distribution Model

The density function of χ^2 distribution is[7,8]

$$p(\sigma)=\frac{1}{\Gamma(k)}\frac{k}{\bar{\sigma}}\left(\frac{k\sigma}{\bar{\sigma}}\right)^{k-1}\exp\{-\frac{k\sigma}{\bar{\sigma}}\},\ \sigma>0 \qquad (1)$$

$\bar{\sigma}$ is the mean value of σ , k is a double freedom degree value, $2k$ is the freedom degree value of χ^2 distribution.

As $k=1$, formula (1) is two freedom degrees χ^2 distribution, namely Swerling Ⅰ distribution.

As $k=N$ (N is pulse integration number in one scan), formula (1) is 2N freedom degrees χ^2 distribution, namely Swerling Ⅱ distribution.

The target of Swerling Ⅰ and Ⅱ type can be seemed as an aggregate, which includes large numbers of statistic independence scattering object. Swerling Ⅰ type is slow fluctuation, and Swerling Ⅱ type is quick fluctuation.

As $k=2$, formula (1) is four 4 freedom degrees χ^2 distribution, namely Swerling Ⅲ distribution.

As $k=2N$, formula (1) is 4N freedom degrees χ^2 distribution, namely Swerling Ⅳ distribution.

The target of Swerling Ⅲ and Ⅳ type can be seemed as an aggregate, which includes a large scattering object and large numbers of statistic independence small scattering object. Swerling Ⅲ type is slow fluctuation, and Swerling Ⅳ type is quick fluctuation.

As $k=\infty$, σ is a constant quantity, namely Marcum distribution. The target is non-fluctuation.

B. Logarithm Normal Distribution Model and Rice Distribution Model

The density function of logarithm normal distribution is[8]

$$p(\sigma)=\frac{1}{\sigma\sqrt{4\pi\ln\rho}}\exp\{-\frac{\ln^2(\frac{\sigma}{\sigma_0})}{4\ln\rho}\},\ \sigma>0 \qquad (2)$$

In formula (2), σ_0 is median of σ , ρ is the mean-median ratio of σ , namely $\bar{\sigma}/\sigma_0$.

The logarithm normal distribution model shows target, which is composed by scattering objects with large electric size and anomalistic shape. The RCS of target is often bigger than median σ_0 , though the probability is small, with augment

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of ρ , the "tail" of the density function curve will become very long.

The density function of Rice distribution is[8]

$$p(\sigma)=\frac{1}{\psi_0}\exp(-s-\frac{\sigma}{\psi_0})I_0(2\sqrt{\frac{s\sigma}{\psi_0}}),\ \sigma>0 \quad (3)$$

In formula (3), s shows the RCS ratio of stabilization object to combined object by many Rice scattering units. ψ_0 is the mean value of σ 's part with Rice distribution. $I_0()$ is zero rank and first type Bessel function.

Rice distribution shows target, which is composed by a fixed amplitude RCS object and many Rice scattering units. s shows the scale of stabilization object in composed object.

III. THE VERIFICATION METHOD GOODNESS OF FIT

To analyse data, it is supposed usually that data obeys a kind distribution. For judging rationality of the suppose, it is need to verify by stylebook. The several distribution verification methods can be used, includes χ^2 verification, Kolmogorov-Smirnov verification (for short K-S verification), Jarque-Bera verification, and Lilliefors verification [9~11]. Jarque_Bera verification and Lilliefors verification are applied in the case that mean and square difference of the normal distribution suppose verification are all unknown. Because χ^2 verification statistic don'ts depend on collectivist distribution form, it can be used in many cases. On the other hand, the pertinence is not good. K-S verification compares experience distribution $F_n(x)$ with suppose distribution $F_0(x)$ by point to point, which is more sensitive than χ^2 verification and needs small stylebook amount [9]. But, K-S verification is not applicable for complex suppose. As common target RCS statistic models, when there are unknown parameter in $F_0(x)$, K-S verification can't be applied [9,10].

The χ^2 verification method is used to verify the goodness of fit of chaff clouds' RCS distribution model in this paper. It's definition is [9,10]:

Prescribe $\{F_\theta,\theta\in\Theta\}$ as a distribution group, the suppose waits for verification.

H : the distribution of the collectivist X is

$$F\in\{F_\theta,\theta\in\Theta\} \quad (4)$$

To explaining conformation and application of χ^2 statistic, it can be supposed that there is only one distribution F_0 in the distribution of the collectivist $\{F_\theta,\theta\in\Theta\}$. Well then, formula (4) is translated to

$$H:\ F=F_0 \quad (5)$$

Divide the collectivist value bound m into independence son sets as $S_1,S_2,\cdots,S_m$, when the dimension value of the collectivist is one, $S_1,S_2,\cdots,S_m$ are independence zones.

$$p_i=P_{F_0}(X\in S_i)\ ,i=1,\cdots,m. \quad (6)$$

For simple stylebook $X_1,\cdots,X_n$, it is marked as

n_i is the number of $X_1,\cdots,X_n$ falling in S_i , $i=1,\cdots,m.$ (7)

Then $(\frac{n_i}{n}-p_i)^2$ shows the windage of the frequency of stylebook falling in S_i on the probability of S_i . K Pearson uses weight sum

$$K_n=\sum_{i=1}^{m}n(\frac{n_i}{n}-p_i)^2\Big/p_i=\sum_{i=1}^{m}(n_i-np_i)^2\Big/np_i \quad (8)$$

to show the goodness of stylebook $X_1,\cdots,X_n$'s fit distribution. It is proved that as suppose (5) is right,

$$K_n\xrightarrow{d}\chi^2(m-1)\ ,\ n\to\infty\ . \quad (9)$$

All appearance, as K_n is rather small, the fit of F_0 is good, whereas, it is not good. So, about a certain level α , the verification can be established as when $K_n>\chi^2_\alpha(m-1)$ H is refused, otherwise H is accepted.

In $\{F_\theta,\theta\in\Theta\}$, θ is k dimensions parameter, Θ is a unempty area in R^k . Then, p_i in formula (8) is relational with θ .

$$p_i(\theta)=P_{F_\theta}(X\in S_i)\ ,\ i=1,\cdots,m,\ \ \theta\in\Theta \quad (10)$$

An estimate $\hat{\theta}_n$ of θ can be based on stylebooks. Use $\hat{\theta}_n$ to replace θ in $p_i(\theta)$, which is applied into K_n, the result is

$$\hat{K}_n=\sum_{i=1}^{m}n(\frac{n_i}{n}-p_i(\hat{\theta}_n))^2\Big/p_i(\hat{\theta}_n)=\sum_{i=1}^{m}(n_i-np_i(\hat{\theta}_n))^2\Big/np_i(\hat{\theta}_n) \quad (11)$$

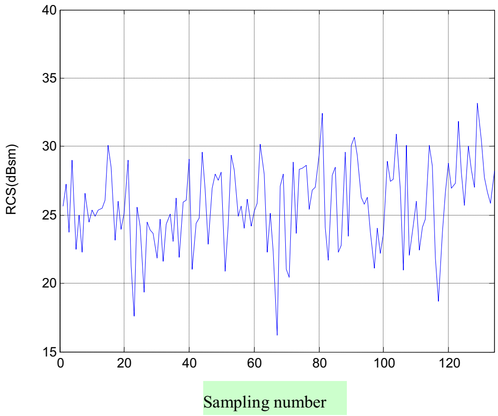
$\hat{K}_n$ is used as the goodness of fit. It can be proved, if suppose (4) is right, when $m>k+1$, there is

$$\hat{K}_n\xrightarrow{d}\chi^2(m-k-1)\text{ , as }n\to\infty\ . \quad (12)$$

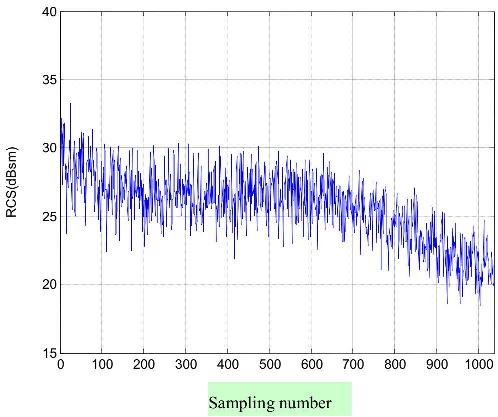
It should be noticed, that the value choosing of m is random, but m should be related to stylebook amount n . When n is more big, m should be more big. Whereas, When n is more small, m should be more small. In application, np_i can't be too small, because n should be enough big and p_i can't be too small as an obligatory condition.

IV. THE ANALYSIS OF MEASUREMENT DATA

Chaff clouds' scattering characteristic on and X-Band has been measured by dynamic method in test field. Fig 1(a) is the S-Band RCS measurement data. Fig 1(b) is the X-Band RCS measurement data.



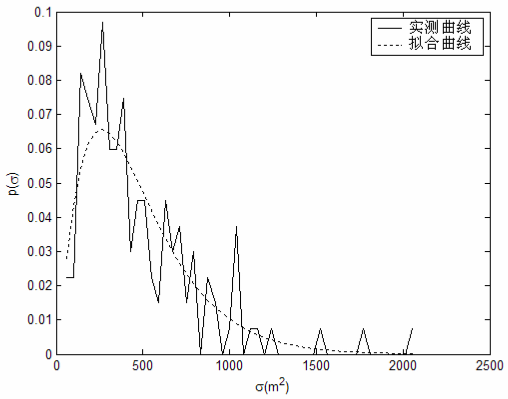
(a) S-Band



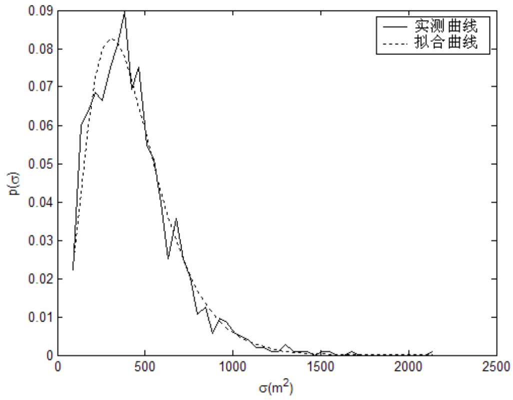
(b) X-Band

Fig. 1 Chaff clouds' RCS measurement data

Fig 2(a) is the S-Band RCS probability density measurement data and fit result. Fig 2(b) is the X-Band RCS probability density measurement data and fit result. The fit distribution type is χ^2 distribution as formula (1).



(a) S-Band



(b) X-Band

Fig. 2 Chaff clouds' RCS probability density measurement data and fit result

The χ^2 verification method is used to calculate the goodness of fit $\hat{K}_n$. The result lists in table 1. As verification salience level $\alpha = 0.1$ and $\alpha = 0.01$, the result of $\chi_\alpha^2(m-k-1)$ lists in table 2. For there are two unknown parameters in the fit χ^2 distribution, $k=2$ is chosen when $\chi_\alpha^2(m-k-1)$ is calculated. In addition, $m=10$ is chosen based on stylebook amount of the measurement data.

TABLE I

S-BAND AND X-BAND χ^2 VERIFICATION STATISTIC VALUE $\hat{K}_n$

	S-Band	X-Band
$\hat{K}_n$	10.20	17.37

TABLE III

VERIFICATION LIMIT $\chi_\alpha^2(m-k-1)$

	$\alpha = 0.1$	$\alpha = 0.01$
$\chi_\alpha^2(m-k-1)$	12.01	18.48

Based on TABLE I and TABLE II, verification result can be acquired as TABLE IIIII. From TABLE IIIV, it is seemed that chaff clouds' RCS obey χ^2 distribution on S-Band and X-Band as verification salience level α is 0.01,and chaff clouds' RCS obey χ^2 distribution on S-Band as verification salience level α is 0.1,and the suppose that RCS fluctuation model is χ^2 distribution is refused on X-Band when verification salience level α is 0.1.

TABLE IIV

CHAFF CLOUDS' RCS DISTRIBUTION VERIFICATION RESULT

	$\alpha = 0.1$	$\alpha = 0.01$
S-Band	Accept	Accept
X-Band	Refuse	Accept

V. Conclusions

Chaff is a kind of radar passive jamming material, which is used most early and effectively. The jamming effect should be tested by dynamic measurement. The probability distribution of chaff clouds' RCS is fitted based on the dynamic measurement data，and the goodness of fit is verified by χ^2 verification. Some problem is indicated to select and apply the verification method goodness of fit，which can be used for the test of chaff bomb and the analysis of dynamic target scattering characteristic.

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