Sum Rule

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Proposition 1. *$\forall f, g$, both differentiable at c, $\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$.*

Proof.

$$\begin{aligned}
\frac{d}{dx}f(x) + \frac{d}{dx}g(x) &= \lim_{x\to c}\frac{f(x)-f(c)}{x-c} + \lim_{x\to c}\frac{g(x)-g(c)}{x-c} \\
&= \lim_{x\to c}\frac{f(x)-f(c)}{x-c} + \frac{g(x)-g(c)}{x-c} \\
&= \lim_{x\to c}\frac{f(x)-f(c)+g(x)-g(c)}{x-c} \\
&= \lim_{x\to c}\frac{(f(x)+g(x))-(f(c)+g(c))}{x-c} \\
&= \frac{d}{dx}(f(x)+g(x))
\end{aligned}$$

□