

# Sum Rule

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**Proposition 1.**  $\forall f, g$ , both differentiable at  $c$ ,  $\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$ .

*Proof.*

$$\begin{aligned}\frac{d}{dx}f(x) + \frac{d}{dx}g(x) &= \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c} + \lim_{x \rightarrow c} \frac{g(x) - g(c)}{x - c} \\ &= \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c} + \frac{g(x) - g(c)}{x - c} \\ &= \lim_{x \rightarrow c} \frac{f(x) - f(c) + g(x) - g(c)}{x - c} \\ &= \lim_{x \rightarrow c} \frac{(f(x) + g(x)) - (f(c) + g(c))}{x - c} \\ &= \frac{d}{dx}(f(x) + g(x))\end{aligned}$$

□