in wave shape as long as peak value and period are the same. When practical, however, it is better to account for each and every stress reversal by using a good *cycle counting method* such as the rain flow method discussed in the section "Cumulative Damage Concepts and Cycle Counting" below.

The magnitude of any *nonzero-mean stress* has an important influence on fatigue response and is discussed in detail in the next section. Likewise, the accumulation of fatigue damage caused by cyclic loading, or *cumulative damage*, is fully discussed in a later section.

Example 5.10 Estimating Fatigue Properties

A wrought carbon-steel alloy is known to have the static properties $S_u = 76,000$ psi, $S_{yp} = 42,000$ psi, and e (2 inches) = 18 percent, but fatigue properties cannot be located for the material. It is necessary to quickly estimate the fatigue properties for the preliminary design of a machine part for which the fluctuating loads will induce a stress spectrum with cyclic amplitudes in both the finite-life range and the infinite-life range.

- **a.** How could the basic small-polished-specimen "*mean S-N* curve" be estimated for this material?
- **b.** How could the R = 99.9 percent reliability S-N curve be estimated for the material?

Solution

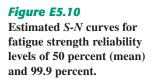
a. Using the estimation guidelines for wrought-ferrous alloys with $S_{ut} < 200$ ksi (since $S_{ut} = S_u = 76,000$ psi),

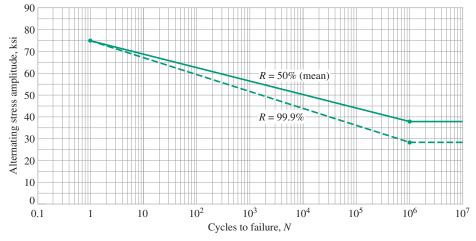
$$S'_f = 0.5(S_u) = 0.5(76,000) = 38,000 \text{ psi}$$

 $S'_{N=1} = S_u = 76,000 \text{ psi}$

Plotting these values on semilog coordinates and connecting them in accordance with the guideline procedures gives the mean (R = 50 percent) S-N curve shown in Figure E5.10.

b. To obtain an estimate of the R = 99.9 percent reliability *S-N* curve, the expression for standard normal variable *X* given in (5-59) may be utilized. Since the R = 99.9 percent





been published, 15 and several typical charts for selecting proper C values for through-the-thickness cracks are given in Figures 5.17 through 5.21. Figure 5.22 gives a chart for *surface flaw shape parameter Q* to be used in finding stress-intensity factors for part-through thumbnail-shaped surface cracks. The stress-intensity factor, K, calculated from (5-50), is a single-parameter measure of the seriousness of the stress field around the crack tip. The magnitude of K associated with the onset of rapid crack extension (initiation of brittle fracture) has been designated as *critical stress intensity*, K_c . Thus failure by brittle fracture may be predicted to occur for through-the-thickness cracks if (FIPTOI)

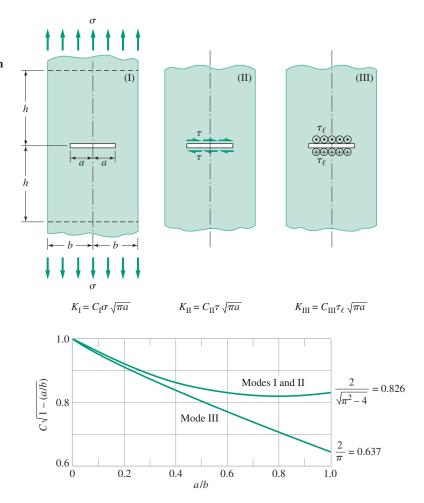
$$K = C\sigma\sqrt{\pi a} \ge K_c \tag{5-51}$$

or, for thumbnail-shaped surface cracks if

$$K = \frac{1.12}{\sqrt{Q}} \quad \sigma \sqrt{\pi a} \ge K_c \tag{5-52}$$

For a given cracked plate, for example the case shown in Figure 5.19, the stress-intensity factor K increases proportionally with gross nominal stress σ , and also is a function of

Figure 5.17 Stress-intensity factors K_I , K_{II} and K_{III} , for center-cracked test specimen. (Source: ref. 11, Del Research Corp.)



¹⁵See, for example, ref. 7.

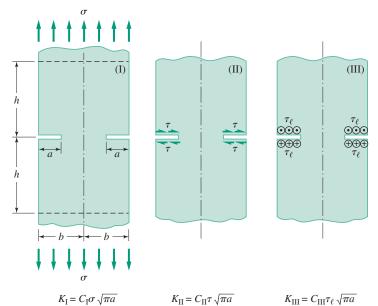


Figure 5.18

Stress-intensity factors K_I , K_{II} and K_{III} , for double-edge notch test specimen. (Source: ref. 7, Del Research Corp.)

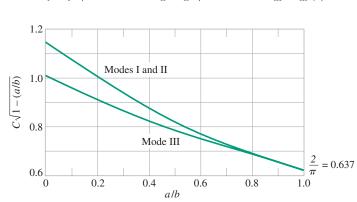
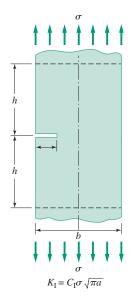
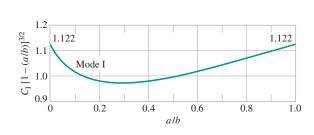


Figure 5.19

Stress-intensity factors K_I , for single-edge notch test specimen. (Source: ref. 7, Del Research Corp.)

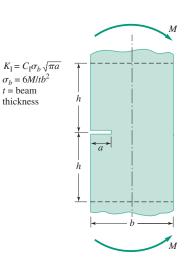


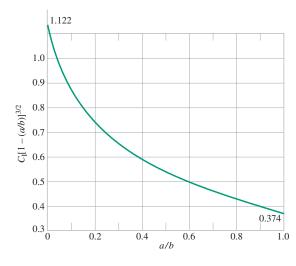


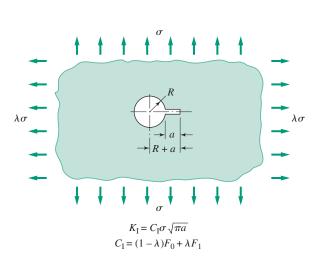
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Figure 5.20

Stress-intensity factors K_I , for single through-thethickness edge crack under pure bending moment. (Source: ref. 7, Del Research Corp.)







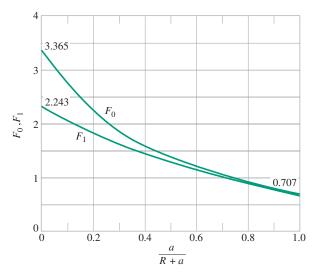


Figure 5.21 Stress-intensity factors K_I , for a through-the-thickness crack emanating from a circular hole in an infinite plate under biaxial tension. (Source: ref. 7, Del Research Corp.)

Figure 5.22 Surface flaw shape parameter. (From ref. 9; adapted by permission of Pearson Education, Inc., Upper Saddle River, N.J.)

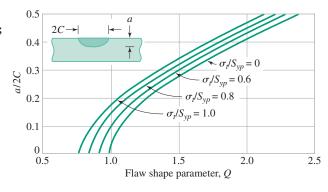


TABLE 5.2 Yield Strength and Plane Strain Fracture Toughness Data for Selected Engineering Alloys¹

Alloy	Form	Test Temperature		S_{yp}		K_{Ic}	
		°F	°C	ksi	MPa	ksi√in	MPa√m
AISI 1045 steel	Plate	25	-4	39	269	46	50
AISI 1045 steel	Plate	0	-18	40	276	46	50
4340 steel (500°F temper)	Plate	70	21	217-238	1495-1640	45-57	50-63
4340 steel (800°F temper)	Forged	70	21	197-211	1360-1455	72-83	79–91
D6AC steel (1000°F temper)	Plate	70	21	217	1495	93	102
D6AC steel (1000°F temper)	Plate	-65	-54	228	1570	56	62
18 Ni maraging steel (300)	Plate	600	316	236	1627	80	87
18 Ni maraging steel (300)	Plate	70	21	280	1931	68	74
18 Ni maraging steel (300)	Plate	-100	-73	305	2103	42	46
A 538 steel				250	1722	100	111
2014-T6 aluminum	Forged	75	24	64	440	28	31
2024-T351 aluminum	Plate	80	27	54–56	370–385	28–40	31–44
6061-T651 aluminum	Plate	70	21	43	296	26	28
6061-T651 aluminum	Plate	-112	-80	45	310	30	33
7075-T6 aluminum		_		75	517	26	28
7075-T651 aluminum	Plate	70	21	75-81	515-560	25-28	27-31
7075-T7351 aluminum	Plate	70	21	58-66	400-455	28-32	31-35
Ti-6Al-4V titanium	Plate	74	23	119	820	96	106

¹From refs. 10–12.

As long as the crack-tip plastic zone is in the regime of *small-scale yielding*,²¹ this estimation procedure provides a good design approach. If the plastic zone size ahead of the crack tip becomes so large that the small-scale yielding condition is no longer satisfied, an appropriate *elastic-plastic fracture mechanics (EPFM)* procedure would give better results. For example, a *failure assessment diagram*²² might be utilized; however such EPFM procedures are beyond the scope of this text.

To utilize (5-52) as a design or failure prediction tool, the stress-intensity factor K must be determined for the particular loading and geometry of the part or structure under consideration. The critical stress intensity is set equal to K_{Ic} if the minimum thickness criterion (5-53) is met, otherwise K_c is estimated from (5-54). If failure is predicted to occur using (5-52), redesign becomes necessary.

Example 5.9 Brittle Fracture

Two "identical" support straps of forged 2014-T6 aluminum, shown in Figure E5.8, have been inspected and found to contain through-the-thickness cracks. While the total crack length of 5 mm is the same for both straps, one strap (case A) involves two edge cracks, each with 2.5-mm length, opposite each other; while the other strap (case B) involves a single crack of 5-mm length at the center. The straps are of rectangular cross section 50 mm

²¹Small-scale yielding means that the crack-tip plastic zone size is small compared to the dimensions of the crack.

²²See ref. 1, pp. 70–76.

Influencing Factor	Symbol	Approximate Range	"Typical" Value ¹
Material composition		_	Specific S-N data required
Heat treatment	_	_	Specific S-N data required
Operating temperature	_	_	Specific S-N data required
Grain size and direction	k_{gr}	0.4-1.0	1.0
Welding	k_{we}	0.3-0.9	0.8
Geometrical discontinuity	k_f	0.2-1.0	Reciprocal of K_f ; see 4.8
Surface condition	k_{sr}	0.2-0.9	0.7
Size effect	k_{s_7}	0.5 - 1.0	0.9
Residual surface stress	k_{rs}	0.5-2.5	Specific data required; see 4.11
Fretting	k_{fr}	0.1–0.9	0.35 if fretting exists, 1.0 if no fretting; also see 2.11
Corrosion	k_{cr}	0.1 - 1.0	Specific data required
Operating speed	k_{sp}	0.9-1.2	1.0
Strength reliability required	k_r	0.7 - 1.0	0.9; also see Table 2.3
Configuration of stress-time pattern	<u>-</u>	_	See later section titled Cumulative Damage Concepts and Cycle Counting
Nonzero-mean stress		_	See later section titled Nonzero-Mean Stress
Damage accumulation —		_	See later section titled Cumulative Damage Concepts and Cycle Counting

¹These "typical" values may be used for solving problems in this text or for making preliminary design estimates when actual conditions are poorly known. However, any critical design situation would require a literature/database search or supporting laboratory experiments to establish more accurate values.

wrought-ferrous alloys, the mean S-N curve may be estimated for polished specimens by plotting on a semilog plot of strength versus log life the following:

- 1. Plot $S'_N = S_{ut}$ at N = 1 cycle.
- **2.** Plot $S'_f = 0.5S_{ut}$ at $N = 10^6$ cycles if $S_{ut} \le 200$ ksi Plot $S'_f = 100$ ksi at $N = 10^6$ cycles if $S_{ut} > 200$ ksi
- **3.** Connect (1) with (2) by a straight line.
- 4. Construct a horizontal straight line from (2) toward very long lives.

This estimated mean (50 percent reliability) S-N curve may be supplemented if desired by using the standard normal variable X of Table 5.4 to calculate and construct estimated S-N curves with higher reliabilities.

TABLE 5.4 Strength Reliability Factors as a Function of Reliability Level

Reliability <i>R</i> (percent)	Corresponding Standard Normal Variable <i>X</i> (see Table 2.9)	Strength Reliability Factor k_r 0.90	
90	1.282		
95	1.645	0.87	
99	2.326	0.81	
99.9	3.090	0.75	
99.995	3.891	0.69	

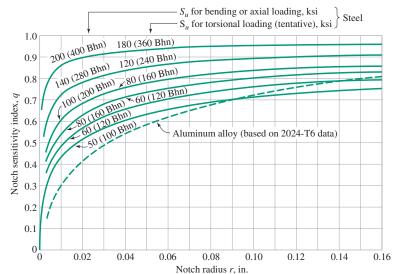


Figure 5.46

Curves of notch sensitivity index versus notch radius for a range of steels and an aluminum alloy subjected to axial, bending, and torsional loading. (After ref. 15; reprinted with permission of the McGraw-Hill Companies.)

Figure 5.46 for a range of steels and an aluminum aloy. For finer-grained materials, such as quenched and tempered steels, q is usually close to unity. For coarser-grained materials, such as annealed or normalized aluminum alloys, q approaches unity if the notch radius exceeds about one-quarter inch. In view of these facts it is tempting to recommend the use of $K_f = K_t$ as a simplifying assumption. Doing so, however, would ignore several important notch sensitivity effects, including:

- Under fatigue loading, an alloy steel with superior *static* properties will often be found not to have superior fatigue properties when compared to a plain carbon steel, because of the difference in notch sensitivities.
- There is a tendency to improperly assess the effects of tiny scratches and cavities unless notch sensitivity effects are recognized.
- Serious errors in applying the results from models to large structures may be made if notch sensitivity effects are not recognized.
- In critical design situations, inefficiencies may accrue if notch sensitivity effects are not considered.

Based on (5-91), an expression for fatigue stress concentration factor may be written as

$$K_f = q(K_t - 1) + 1 (5-92)$$

The theoretical elastic stress concentration factor K_t may be determined, on the basis of geometry and loading, from handbook charts such as those depicted in Figures 5.4 through 5.12. The notch sensitivity index q may also be read from charts, such as the one shown in Figure 5.46.

For uniaxial states of cyclic stress it is sometimes convenient to use K_f as a "strength reduction factor" rather than as a "stress concentration factor." This may be done by dividing the fatigue limit by K_f rather than multiplying the applied nominal cyclic stress times K_f . Although conceptually it is more correct to think of K_f as a stress concentration factor, computationally it is often simpler to use K_f as a strength reduction factor when the cyclic stresses are uniaxial. For multiaxial states of stress, however, K_f must be used as a stress concentration factor.

The fatigue stress concentation factor (or strength reduction factor) determined from (5-92) is strictly applicable only in the high-cycle fatigue range (lives of 10^5-10^6 cycles

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1. (25 pts) For the following static plane stress state:

$$\sigma_{xx} = -11 \text{ ksi}, \ \sigma_{yy} = 7 \text{ ksi}, \ \sigma_{xy} = 0$$

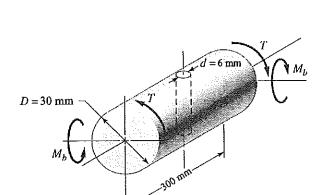
- (a) If the stress state is for a part that is made of a brittle material that has the properties $S_{ut} = 25$ kpsi and $S_{uc} = 70$ kpsi, determine the factor of safety guarding against failure.
- (b) If the stress state is for a part that is made of a ductile material that has a yield strength of 30 ksi, determine the factor of safety guarding against yield.

If more than one theory can be applied to each case, use all of them.

principal stresses:
$$\sqrt{12} + ksi$$
, $\sqrt{12} = 0$, $\sqrt{13} = 11$ ksi $\sqrt{13} = 6$ pts

 $\sqrt{13} = min\left(\frac{5ut}{\sqrt{1}}, \frac{5ut}{10s1}\right) = min\left(\frac{25}{7}, \frac{70}{11}\right) = min\left(\frac{3.5}{7}, \frac{5}{7}, \frac{6.36}{9}\right)$
 $\sqrt{10} = \frac{7}{7} =$

2. (15 pts) A rod made of AISI 1020 cold-drawn steel with a radial hole as shown is subjected to a static bending moment M_b and a torque $T = 2M_b$. What is the allowable value of M_b if a safety factor of 1.7 against yield is desired? Use distortion energy theory.



bending:
$$\frac{M_b}{52} = \frac{10^3 \times 10^2 \times 10^3 \times 10^3$$

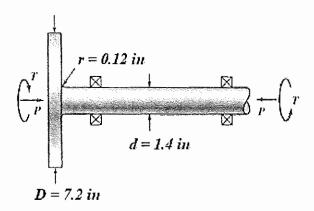
The material is duelite -> Stress Concentration factors are not applied.

DET:
$$\nabla_{q} = \frac{1}{\sqrt{2}} \left[2\nabla_{nn}^{2} + 6\nabla_{ny}^{2} \right] = \sqrt{\nabla_{nn}^{2} + 3}\nabla_{ny}^{2} = M_{b}(mm^{3}) \sqrt{(5.7119 \times 10^{3})^{2}}$$

$$\nabla_{q} = 9.7247 \times 10 (mm^{3}) M_{b}$$

$$n = \frac{Syp}{10eq} \Rightarrow 1.7 = \frac{352 \, N/mm^2}{(4.7247 \, \text{xip mm}^3) \, \text{Hb}} \Rightarrow M_b = 2.12 \, 92 \, \text{xip} \, N_c \, mm$$

3. The shaft shown in the figure is subjected to a static axial load P and a cyclic torque that fluctuates between T=0 and T=(1.5in)P. The shaft is made of a kind of steel with reported properties of $S_{ut}=145$ ksi, $S_{yp}=120$ ksi, and a fully modified fatigue limit of $S_f=60$ ksi.



(a) (21 pts) Find mean and alternating stress components acting on the shaft at critical location due to the axial and torsional loads in terms of P. Appropriately account for stress concentration effects. The theoretical stress concentration factors for the fillet are 2.2 and 1.5 for the axial and torsional loading, respectively.

(c) (5 pts) Find the equivalent completely reversed stress in terms of P.

(1) (3 pts) Find the allowable load P such that the shaft will have an unlimited fatigue life with a factor of safety of 2.7.

The with a factor of safety of 2.7.

$$\frac{Sf}{f} > 2.7 = \frac{60 \text{ kso}}{3.520 \text{ in}^2 \text{ P}} \Rightarrow P = 5.69 \text{ kips}$$

$$\frac{3.520 \text{ in}^2 \text{ P}}{12.497 \text{ in}^2 \text{ P}} = \frac{146 \text{ kso}}{146 \text{ kso}}$$

(4 pts) For the load found in part (c), what is the safety factor guarding against

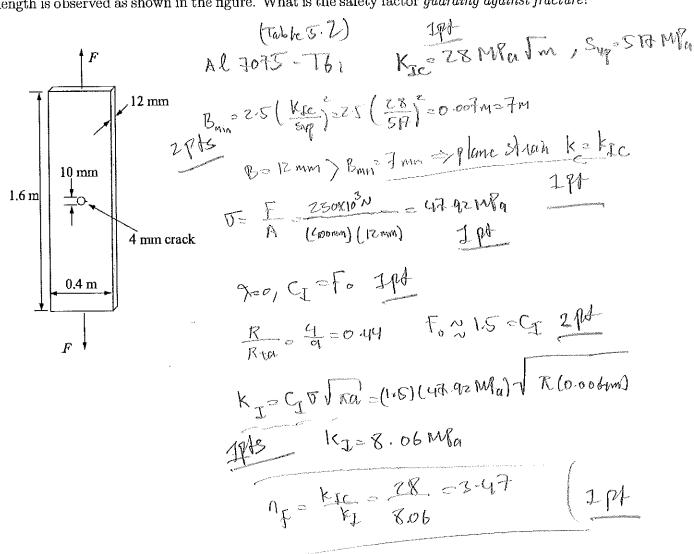
Veg. man
$$(6)$$
 (4 pts) For the load found in part (c), what is the salety factor guarding again yield?

Veg. man $(0.6496) + 3(2.784)$ (5.69 kips) = 24.69 klps

 $\frac{3.945}{\sqrt{2}}$
 $\frac{3.945}{\sqrt{2}}$
 $\frac{3.945}{\sqrt{2}}$
 $\frac{120 \text{ kiv}}{\sqrt{2}} = 4.3$

4. Short answer/multiple choice.

(a) (10 pts) An aircraft component is made of 7075-T6 aluminum and is rectangular as shown (1.6 m × 0.4 m) with a thickness of 12 mm. In the center of the component is a circular fastener hole of diameter 10 mm. When the aircraft is in service, the component is subjected to a peak axial tensile load of 230 kN during landing of the aircraft. During a routine inspection, a through-the-thickness crack emanating from the circular hole of 4 mm length is observed as shown in the figure. What is the safety factor guarding against fracture?



(b) (10 pts) A low-alloy steel link is to be made from a solid cylindrical bar subjected to a completely reversed axial σ_a . The material properties are $S_{ut}=690$ MPa, $S_{yp}=524$ MPa and the elongation in 50-mm is 25%. The bar will be lathe-turned and a reliability of 99% at 500,000 cycles of life is desired. What should be value of σ_a ?

at 500,000 cycles of the 18 desired. What should be value of
$$\sigma_{a}$$
?

$$\begin{cases}
5 = 0.5 \text{ Sut} = 0.5 \text{ (690) Mla} = 345 \text{ Mla} \\
5 = 0.75 \text{ Ml}
\end{cases}$$

$$\begin{cases}
5 = 0.5 \text{ Sut} = 0.75 \text{ Ml}
\end{cases}$$

$$\begin{cases}
6 = 0.75 \text{ Ml}
\end{cases}$$

$$\begin{cases}
7 = 0.75$$

Section 5.6: Fatigue Failure Analysis

- Cycle counting
- Counting cycles for a fluctuating load (step (a)) is more challenging. We use a method called *Rain Flow Cycle Counting*.
- Rules
 - 1. If cycles are counted over a duty cycle block that is repeated, the cycle counting should start by initiating the first drop at either the most negative valley or the most positive peak.
 - 2. Rain allowed to flow on roof and drip down to next slope until it comes to valley (peak) more negative (positive) than from which it initiated.
 - 3. Rain flow must always stop if it meets rain from a roof above.
 - 4. Combine drops to form cycles. A cycle consists of 2 drops; 1 from a peak and 1 from a valley. Pair drops that either run off (highest peak and most negative valley) or end on same roof.

Section 5.6: Fatigue Failure Analysis

- Fatigue Modifying Factors
- Modifying factors relate S_f' and S_N' to S_f and S_N

$$S_f = k_{qr}k_{we}k_fk_{sr}k_{sz}k_{rs}k_{fr}k_{cr}k_{sp}k_rS_f'$$

- See Table 5.3 for description of modifying factors and "typical values"
- Full influence at very high number of cycles and little to no effect at low cycles
 - Apply modification to S_f^\prime for ferrous and titanium alloys
 - Apply modification to S_N' at $N \ge 10^6$ for nonferrous alloys
 - Interpolate to lower number of cycles

For this class:

 $k_{gr}=1$ (grain structure effect) $k_{we}=1$ if no weld, $k_{we}=0.8$ if weld $k_f=1$ (stress concentration factor should always be applied to stress, never used to reduce strength!) k_{sr} , see Fig. 5.33 (surface finish effect) $k_{sz}=0.9$ (size effect, may find eq. in Handbook or Codes/Standards) $k_{rs}=1$ (residual stress effect) or Fig. 4.22 for 7075-T6 notched bar $k_{fr}=1$ if no fretting, else $k_{fr}=0.35$ $k_{cr}=1$ (corrosion effect) $k_{sp}=1$ (speed effect, 200-7000cpm, decrease for slower, increases for faster)

 k_r , see Table 5.4 or Eq. (5-60) (reliability)

Section 5.6: Fatigue Failure Analysis

- Estimating S-N curves

