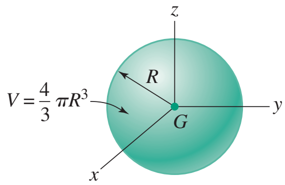
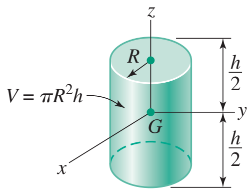
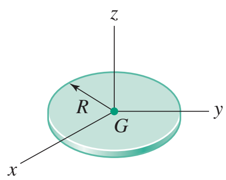
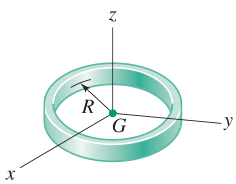
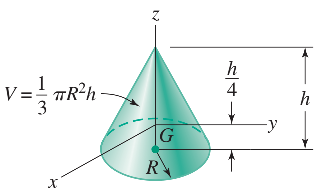
TABLE A.2 Mass Moments of Inertia *J* and Radii of Gyration *k* for Selected Homogeneous Solid Bodies Rotating About Selected Axes, as Sketched[1]

Description of Body		Mass Moment of Inertia, J	Radius of Gyration, k
Case 1. Sphere		$J_x = J_y = J_z = \frac{2}{5}mR^2$	$k_x = k_y = k_z = \sqrt{\frac{2}{5}}R$
Case 2. Cylinder		$J_z = \frac{1}{2}mR^2$ $J_x = J_y = \frac{1}{12}m(3R^2 + h^2)$	$k_z = \sqrt{\frac{1}{2}}R$ $k_x = k_y = \sqrt{\frac{1}{12}(3R^2 + h^2)}$
Case 3. Thin circular disk		$J_z = \frac{1}{2}mR^2$ $J_x = J_y = \frac{1}{4}mR^2$	$k_z = \sqrt{\frac{1}{2}}R$ $k_x = k_y = \sqrt{\frac{1}{4}}R$
Case 4. Thin ring		$J_z = mR^2$ $J_x = J_y = \frac{1}{2}mR^2$	$k_z = R$ $k_x = k_y = \sqrt{\frac{1}{2}}R$
Case 5. Cone		$J_z = \frac{3}{10}mR^2$ $J_x = J_y = \frac{3}{80}m(4R^2 + h^2)$	$k_z = \sqrt{\frac{3}{10}}R$ $k_x = k_y = \sqrt{\frac{3}{80}(4R^2 + h^2)}$

[1]Extracted from Hibbler, R. C., *Engineering Mechanics: Dynamics*, 2nd ed., Macmillan, New York, 1978. Also note that m is the mass of the rotating body.

For items from Table 3.1 that have been identified as special needs in any given application, Table 3.2 then provides performance evaluation indices that form the basis for comparison and selection of candidate materials. The procedure for accomplishing this task is discussed further in 3.4.

To use the performance evaluation indices for comparing candidate materials, it is necessary to find quantitative materials data for each of the key parameters that comprise the pertinent evaluation indices. There are many sources for such data.[2] To illustrate the procedure suggested here, a short compilation of selected materials, and their rank-ordered properties relating to the performance evaluation indices of Table 3.2, are given in Tables 3.3 through 3.20. Each rank-ordered table is *arranged with the best material for a given*

TABLE 3.3 Strength Properties of Selected Materials[1]

		Ultimate Tensile Strength, S_u		Yield Strength, S_{yp}	
Material	Alloy	ksi	MPa	ksi	MPa
Ultra-high-strength steel	AISI 4340	287	1979	270	1862
Stainless steel (age hardened)	AM 350	206	1420	173	1193
High-carbon steel	AISI 1095[2]	200	1379	138	952
Graphite-epoxy composite	—	200	1379	—	—
Titanium	Ti-6Al-4V	150	1034	128	883
Ceramic	Titanium carbide (bonded)	134	924	—	—
Nickel-based alloy	Inconel 601	102	703	35	241
Medium-carbon steel	AISI 1060 (HR)[3]	98	676	54	372
	AISI 1060 (CD)[4]	90	621	70	483
Low-carbon, low-alloy steel	AISI 4620 (HR)	87	600	63	434
	AISI 4620 (CD)	101	696	85	586
Stainless steel (austenitic)	AISI 304 (annealed)	85	586	35	241
Yellow Brass	C 26800 (hard)	74	510	60	414
Commercial Bronze	C 22000 (hard)	61	421	54	372
Low-carbon (mild) steel	AISI 1020 (CD)	61	421	51	352
	AISI 1020 (annealed)	57	393	43	296
	AISI 1020 (HR)	55	379	30	207
Phosphor Bronze	C 52100 (annealed)	55	379	24	165
Gray cast iron	ASTM A-48 (class 50)	50[5]	345	—	—
	ASTM A-48 (class 40)	40	276	—	—
Aluminum (wrought)	2024-T3 (heat treated)	70	483	50	345
	2024 (annealed)	27	186	11	76
Aluminum (perm. mold cast)	356.0 (solution treated; aged)	38	262	27	186
Magnesium (extruded)	ASTM AZ80A-T5	50	345	35	241
Magnesium (cast)	ASTM AZ63A	29	200	14	97
Thermosetting polymer	Epoxy (glass reinforced)	—	—	10	69
Thermoplastic polymer	Acrylic (cast)	—	—	7	48

[1]See, for example, ref. 1–10.
[2]Quenched and drawn to Rockwell C-42.
[3]Hot rolled
[4]Cold drawn
[5]Ultimate *compressive* strength is 170 ksi, 1172 MPa

[2]See, for example, refs. 1–10 and 16–19.

TABLE 3.4 Strength/Weight Ratios for Selected Materials

Material	Weight Density, w lb/in³	Weight Density, w kN/m³	Approx. Ultimate Strength/Wt Ratio, $\frac{S_u}{w}$ in × 10³	Approx. Ultimate Strength/Wt Ratio, $\frac{S_u}{w}$ m × 10³	Approx. Yield Strength/Wt Ratio, $\frac{S_{yp}}{w}$ in × 10³	Approx. Yield Strength/Wt Ratio, $\frac{S_{yp}}{w}$ m × 10³
Graphite-epoxy composite	0.057	15.47	3500	89.14	—	—
Ultra-high-strength steel	0.283	76.81	1000	25.76	950	24.24
Titanium	0.160	43.42	950	23.81	800	20.34
Stainless steel (age hardened)	0.282	76.53	750	18.55	600	15.59
Aluminum (wrought)	0.100	27.14	700	17.80	500	12.71
Titanium carbide	0.260	70.56	500	14.65	—	—
Aluminum (perm. mold cast)	0.097	26.33	400	9.95	300	7.06
Medium-carbon steel	0.283	76.81	350	8.80	200	4.84
Nickel-based alloy	0.291	78.98	350	8.90	100	3.05
Stainless steel (austenitic)	0.290	78.71	290	7.45	120	3.06
Yellow Brass	0.306	83.05	250	6.14	200	4.99
Low-carbon steel	0.283	76.81	200	5.48	150	4.58
Commercial Bronze	0.318	86.31	200	4.88	150	4.31
Gray cast iron (class 50)	0.270	73.28	200	4.71	—	—
Epoxy (glass reinf.)	0.042	11.40	—	—	250	6.85
Acrylic (cast)	0.043	11.67	—	—	150	4.11

TABLE 3.5 Strength at Elevated Temperatures for Selected Materials

Material	Temperature °F	Temperature °C	Ultimate Tensile Strength, $(S_u)_\Theta$ ksi	Ultimate Tensile Strength, $(S_u)_\Theta$ MPa	Yield Strength, $(S_{yp})_\Theta$ ksi	Yield Strength, $(S_{yp})_\Theta$ MPa
Ultra-high-strength steel (4340)	−200	−129	313	2158	302	2082
	RT[1]	RT	287	1979	270	1862
	400	204	276	1882	235	1620
	800	427	221	1524	186	1283
	1200	649	103	710	62	428
Stainless steel (AM 350)	RT	RT	206	1420	173	1193
	400	204	185	1376	144	993
	800	427	179	1234	119	821
	1000	538	119	821	83	572
Titanium (Ti-6Al-4V)	−200	−129	187	1289	155	1069
	RT	RT	150	1034	128	883
	400	204	126	869	101	696
	800	427	90	821	75	517
	1000	538	81	559	59	407
Titanium carbide	RT	RT	134	924	—	—
	1500	816	94	648	—	—
	1800	982	72	496	—	—
Inconle (601)	RT	RT	102	703	35	241
	400	204	94	648	31	214
	800	427	84	579	28	193
	1200	649	66	455	23	159
	1600	871	20	138	12	83

(Continued)

TABLE 3.5 ***(Continued)***

	Temperature		Ultimate Tensile Strength, $(S_u)_\Theta$		Yield Strength, $(S_{yp})_\Theta$	
Material	°F	°C	ksi	MPa	ksi	MPa
Low-carbon steel (1020)	−200	−129	97	669	83	572
	RT	RT	61	421	51	352
	400	204	61	421	51	352
	800	427	45	310	38	262
	900	482	29	200	24	166
Aluminum (2024-T3)	−200	−129	74	510	54	372
	RT	RT	70	483	50	345
	400	204	52	356	39	269
	800	427	4	28	4	28
Magnesium (AZ80A-T5)	−200	−129	63	434	53	365
	RT	RT	50	345	35	241
	200	93	43	297	19	131
	400	204	22	152	11	76

[1]Room temperature.

index at the top, followed in order of decreasing desirability by several other materials. Although suitable for solving problems in this textbook, real-world design tasks often require more comprehensive data gleaned from the literature and/or other data banks if the design is to be competitive.

TABLE 3.6 Stress Rupture Strength Levels Corresponding to Various Rupture Times and Temperatures for Selected Materials

		Temperature		Rupture Time, t (hr): 10		100		600		1000		10,000	
Material	Alloy	°F	°C	ksi	MPa	ksi	MPa	ksi	MPa	ksi	MPa	ksi	MPa
Stainless steel	AM 350	800	427	—	—	184	1269	—	—	182	1255	—	—
Iron-based superalloy	A-286	1000	538	120	827	100	590	—	—	80	552	76	524
		1200	649	78	538	68	469	—	—	50	345	34	234
		1350	732	50	345	35	241	—	—	21	145	14	97
		1500	816	21	145	11	76	—	—			—	—
Cobalt-base superalloy	X-40	1500	816	61	421	56	386	—	—	51	352	—	—
Inconel	601	1200	649	—	—	—	—	—	—	28	193	—	—
		1400	760	—	—	—	—	—	—	9.1	63	—	—
		1600	871	—	—	—	—	—	—	4.2	29	—	—
		1800	982	—	—	—	—	—	—	2	14	—	—
Carbon steel	1050	750	399	—	—	52.5	362	49	338	—	—	—	—
		930	499	—	—	22.4	154	18	124	—	—	—	—
Aluminum	Duralumin	300	149	—	—	38	262	32.5	224	—	—	—	—
		480	249	—	—	11.2	77	8.3	57	—	—	—	—
		660	349	—	—	3.1	21	2.7	19	—	—	—	—
Brass	60/40	300	149	—	—	47	324	42.5	293	—	—	—	—
		480	249	—	—	15.7	108	9	62	—	—	—	—

TABLE 3.7 Creep Limited Maximum Stress Corresponding to Various Strain Rates and Temperatures for Selected Materials

		Temperature		Strain Rate, $\dot{\varepsilon}$, $\mu\varepsilon$/hr									
				0.4		1		4		10		40	
Material	Alloy	°F	°C	ksi	MPa	ksi	MPa	ksi	MPa	ksi	MPa	ksi	MPa
Stainless steel	AM 350	800	427	—	—	91	627	—	—	—	—	—	—
Chromium steel (Q&T)	13% Cr	840	449	23.5	162	—	—	33.6	232	—	—	41.5	286
Manganese steel	1.7% Mn	840	449	23.5	162	—	—	27	186	—	—	36	248
Carbon steel (forged)	1030	930	499	13	90	—	—	16.3	112	—	—	19	131
Stainless steel	304	1000	538	—	—	—	—	—	—	10	69	—	—
		1300	704	—	—	—	—	—	—	8	55	—	—
		1500	816	—	—	—	—	—	—	5	35	—	—
Phospher Bronze		440	227	10	69	—	—	15.7	108	—	—	21.3	147
Magnesium	HZ32A-T5	400	204	—	—	—	—	—	—	—	—	10	69
		500	260	—	—	—	—	—	—	—	—	8	55
		600	316	—	—	—	—	—	—	—	—	5	35
Aluminum	Duralumin	440	227	5.6	39	—	—	7.4	51	—	—	9.7	67
Brass	60/40	440	227	1.02	7	—	—	2.7	19	—	—	5.6	39

TABLE 3.8 Coefficients of Thermal Expansion for Selected Materials

		Coefficient of Thermal Expansion α		Temperature Range of Validity	
Material	Alloy	$\times 10^{-6}$ in/in/°F	$\times 10^{-6}$ m/m/°C	°F	°C
Ceramic	Titanium carbide (bonded)	4.3–7.5	7.74–13.5	68–1200	20–649
Titanium	Ti-6Al-4V	5.3	9.5	68–1000	20–538
Gray cast iron	ASTM A-48 (class 50)	6.0	10.8	32–212	0–100
Steel	Most	6.3	11.3	0–200	−18–93
Stainless steel	AM 350	6.3	11.3	—	—
Nickel-base alloy	Inconel 601	7.6	13.7	80–200	27–93
	Inconel 600	9.3	16.7	80–1500	27–816
Cobalt base superalloy	X-40	9.2	16.6	70–1800	21–982
Stainless steel	304	9.6	17.3	32–212	0–100
Graphite-epoxy composite	—	10.0	18.0	—	—
Commercial bronze	C 22000	10.2	18.4	68–572	20–300
Iron-based superalloy	A-286	10.3	18.5	70–1000	21–538
Yellow brass	C 26800	11.3	20.3	68–572	20–300
Aluminum (cast)	356	11.9	21.4	68–212	20–100
(wrought)	2024-T3	12.9	23.2	68–212	20–100
(wrought)	2024-T3	13.7	24.7	68–572	20–300
Magnesium	Most	14.0	25.2	68	20
Magnesium	Most	16.0	28.8	68–750	20–399
Thermosetting polymer	Epoxy (glass reinf.)	10–20	36.0	—	—
Thermoplastic polymer	Acrylic	45	81.0	—	—

TABLE 3.9 Stiffness Properties of Selected Materials

Material	Young's Modulus of Elasticity, *E*		Shear Modulus of Elasticity, *G*		Poisson's Ratio, *v*
	Msi	GPa	Msi	GPa	
Tungsten carbide	95	655	—	—	0.20
Titanium carbide	42–65 (77°F)	290–448 (25°F)	—	—	0.19
Titanium carbide	33–48 (1600–1800°F)	228–331 (871–982°C)	—	—	
Molybdenum	47 (RT)[1]	324	—	—	0.29
Molybdenum	33 (1600°F)	227.5 (871°C)	—	—	—
Molybdenum	20 (2400°F)	137.9 (1316°C)	—	—	—
Steel (most)	30	207	11.5	79	0.30
Stainless steel	28	193	10.6	73	0.31
Iron based superalloy (A-286)	29.1 (RT)	201	—	—	0.31
	23.5 (1000°F)	162 (538°C)	—	—	—
	22.2 (1200°F)	153 (649°C)	—	—	—
	19.8 (1500°F)	137 (816°C)	—	—	—
Cobalt-base superalloy	29	200	—	—	—
Inconel	31	214	11.0	76	—
Cast iron	13–24	90–166	5.2–8.5	36–89	0.21–0.27
Commercial Bronze (C 22000)	17	117	6.3	43	0.35
Titanium	16	110	6.2	43	0.31
Phospher bronze	16	110	6.0	41	0.35
Aluminum	10.3	71	3.9	27	0.33
Magnesium	6.5	45	—	—	0.29
Graphite-epoxy composite	6.0	41	—	—	—
Acrylic thermoplastic	0.4	2.8	—	—	0.4

[1]Room temperature.

TABLE 3.10 Ductility of Selected Materials

Material	Alloy	Percent Elongation, *e*, in 2-in (50 mm) Gage Length
Phosphor bronze C	C 52100	70
Inconel	601	50(RT)
Inconel	601	50 (1000°F, 538°C)
Inconel	601	75 (1400°F, 760°C)
Stainless steel	AISI 304	60
Copper	Oxygen-free	50
Silver		48
Gold		45
Aluminum (annealed)	1060	43
Low-carbon low-alloy steel	AISI 4620 (HR)[1]	28
	AISI 4620 (CD)[2]	22
Low-carbon steel	AISI 1020 (HR)	25
	AISI 1020 (CD)	15
Aluminum (wrought)	2024-T3	22
Stainless steel	AM 350	13
Medium-carbon steel	AISI 1060 (HR)	12
	AISI 1060 (CD)	10
Ultra-high-strength steel	AISI 4340	11

TABLE 2.9 Cumulative Distribution Function *F(X)* for the Standard Normal Distribution, Where

$$F(X) = \int_{-\infty}^{X} \frac{1}{\sqrt{2\pi}} e^{(-t^2/2)} dt$$

X	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.8	$.9^{4}28$	$.9^{4}31$	$.9^{4}33$	$.9^{4}36$	$.9^{4}38$	$.9^{4}41$	$.9^{4}43$	$.9^{4}46$	$.9^{4}48$	$.9^{4}50$
3.9	$.9^{4}52$	$.9^{4}54$	$.9^{4}56$	$.9^{4}58$	$.9^{4}59$	$.9^{4}61$	$.9^{4}63$	$.9^{4}64$	$.9^{4}66$	$.9^{4}67$
4.0	$.9^{4}68$	$.9^{4}70$	$.9^{4}71$	$.9^{4}72$	$.9^{4}73$	$.9^{4}74$	$.9^{4}75$	$.9^{4}76$	$.9^{4}77$	$.9^{4}78$
4.1	$.9^{4}79$	$.9^{4}80$	$.9^{4}81$	$.9^{4}82$	$.9^{4}83$	$.9^{4}83$	$.9^{4}84$	$.9^{4}85$	$.9^{4}85$	$.9^{4}86$
4.2	$.9^{4}87$	$.9^{4}87$	$.9^{4}88$	$.9^{4}88$	$.9^{4}89$	$.9^{4}89$	$.9^{4}90$	$.9^{5}00$	$.9^{5}02$	$.9^{5}07$

Figure 4.4
Initially curved beam subjected to pure bending moment.

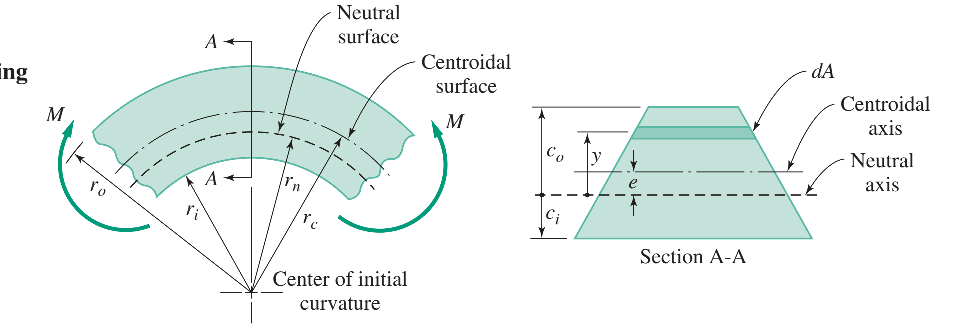


TABLE 4.3 Integral Evaluations and Widely Distributed Stress Concentration Factors k_i as a Function of $\frac{r_c}{c}$ for Curved Beams with Various Cross Sections[1]

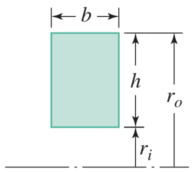
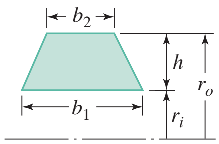
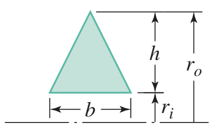
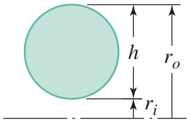
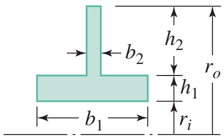
Cross Section		$\int \frac{dA}{r}$	$\frac{r_c}{c}$	k_i
1. Rectangular		$b \ln \frac{r_o}{r_i}$	1.20	2.888
			1.40	2.103
			1.60	1.798
			1.80	1.631
			2.00	1.523
			3.00	1.288
			4.00	1.200
			6.00	1.124
			8.00	1.090
			10.00	1.071
2. Trapezoidal		$\left(\frac{b_1 r_o - b_2 r_i}{h} \ln \frac{r_o}{r_i}\right) - b_1 + b_2$	when $b_2/b_1 = 1/2$	
			1.20	3.011
			1.40	2.183
			1.60	1.859
			1.80	1.681
			2.00	1.567
			3.00	1.314
			4.00	1.219
			6.00	1.137
			8.00	1.100
			10.00	1.078
3. Triangular (base inward)		$\left(\frac{b r_o}{h} \ln \frac{r_o}{r_i}\right) - b$	1.20	3.265
			1.40	2.345
			1.60	1.984
			1.80	1.784
			2.00	1.656
			3.00	1.368
			4.00	1.258
			6.00	1.163
			8.00	1.120
			10.00	1.095

TABLE 4.3 ***(Continued)***

Cross Section		$\int \frac{dA}{r}$	$\frac{r_c}{c}$	k_i
4. Circular		$2\pi\left\{\left(r_i + \frac{h}{2}\right) - \left[\left(r_i + \frac{h}{2}\right)^2 - \frac{h^2}{4}\right]^{1/2}\right\}$	1.20	3.408
			1.40	2.350
			1.60	1.957
			1.80	1.748
			2.00	1.616
			3.00	1.332
			4.00	1.229
			6.00	1.142
			8.00	1.103
			10.00	1.080
5. Inverted tee		$b_1 \ln \frac{r_i + h_1}{r_i} + b_2 \ln \frac{r_o}{r_i + h_1}$	(when $b_2/b_1 = 1/4$ and $h_1/(h_1 + h_2) = 1/4$)	
			1.20	3.633
			1.40	2.538
			1.60	2.112
			1.80	1.879
			2.00	1.731
			3.00	1.403
			4.00	1.281
			6.00	1.176
			8.00	1.128
			10.00	1.101

[1]Extracted from ref. 4. Note that k_i is to be applied to a nominal "straight-beam" bending stress in which $c_i = c_o = c$; hence the "straight-beam" parameter r_c/c is tabulated.

The (hyperbolic) stress distribution from inner to outer curved beam surfaces is given by

$$\sigma = -\frac{My}{eA(r_n + y)} \tag{4-10}$$

where r_n is the radius of curvature of the neutral surface and a *positive* moment is defined as one tending to *straighten* the beam. The extreme values of stress at the inner and outer surfaces, where y is equal to c_i and c_o, respectively, are

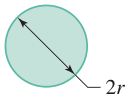
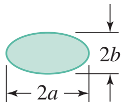
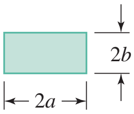
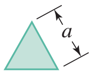
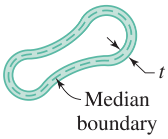
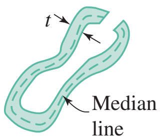
$$\sigma_i = \frac{Mc_i}{eAr_i} \tag{4-11}$$

and

$$\sigma_o = -\frac{Mc_o}{eAr_o} \tag{4-12}$$

Thus if a positive moment is defined as one tending to straighten the beam, tensile stresses result at the inner radius of the beam, r_i, and compressive stresses result at the outer radius r_o. Equations (4-11) and (4-12) are, strictly speaking, applicable only for the case of pure bending. If, as illustrated in Figure 4.5, the bending moment is produced by a system of forces whose resultant line of action does not pass through the centroid of the critical cross section, the moment of the forces, M, should be computed about the *centroidal axis*,

TABLE 4.5 Parameters for Determining Shearing Stress and Angular Deformation for Bars of Various Cross-Sectional Shape Subjected to Torsional Moments

Shape and Dimensions of Cross Section		Q, in^3	K, in^4	Location of Maximum Shearing Stress
1. Solid circular		$\dfrac{\pi r^3}{2}$	$\dfrac{\pi r^4}{2}$	On entire boundary
2. Solid elliptical		$\dfrac{\pi a b^2}{2}$	$\dfrac{\pi a^3 b^3}{a^2 + b^2}$	At ends of minor axis
3. Solid rectangular		$\dfrac{8a^2b^2}{3a + 1.8b}$ (for $a \geq b$)	$ab^3\left[\dfrac{16}{3} - 3.36\,\dfrac{b}{a}\left(1 - \dfrac{b^4}{12a^4}\right)\right]$ (for $a \geq b$)	At midpoint of each longer side
4. Solid equilateral triangle		$\dfrac{a^3}{20}$	$\dfrac{\sqrt{3}\, a^4}{80}$	At midpoint of each side
5. Any thin tube of uniform thickness t U = length of median boundary. A = area enclosed by median boundary.		$2At$	$\dfrac{4A^2t}{U}$	Nearly uniform throughout
6. Any open thin tube of uniform thickness t U = length of medium line		$\dfrac{U^2t^2}{3U + 1.8t}$	$\dfrac{Ut^3}{3}$	Nearly uniform throughout

L is bar length, and G is shear modulus of elasticity. Expressions for Q and K are provided for several cross-sectional shapes in Table 4.5, and for numerous other shapes in the literature.[11]

Example 4.6 Design of Torsionally Loaded Round and Square Shafts

Experimental power measurements made on a new-style rotary garden tiller indicate that under full load conditions the internal combustion engine must supply 4.3 horsepower, steadily, to the mechanical drive train. Power is transmitted through a solid 0.50-inch-diameter round steel shaft rotating at 1800 rpm. It is being proposed to replace the round

[11] See, for example, refs. 4 and 7.