Visualization of the Higgs Interaction in the Standard Model and Beyond the Standard Model Physics

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We use visualization to investigate the Higgs Interaction in the Standard Model and Beyond the Standard Model Physics. The Higgs discovery at the Large Hadron Collider has allowed for a precise determination of the Higgs Potential. We visualize this potential and the Higgs interactions with the fermions of the Standard Model as well as beyond standard model such as interactions with Dark matter and heavy neutrinos. We animate these potentials with respect to energy and temperature (different energies and temperatures per frame) to see how the potential changes shape as one moves up in energy or temperature.

I. Introduction

In 1960s, Peter Higgs and his colleagues theorized the existence of a zero spin and electrically neutral boson. Named after the theoretical physicist, the Higgs boson allows other particles to gain mass by interacting with the Higgs field where the spontaneous symmetry breaking occurs. As a result, the existence of Higgs boson also explains mass of the particle, whereas massless nature is expected based on their symmetric interactions. All though the unstable massive nature of the Higgs boson delayed the discovery at the LHC of CERN located in Geneva, Switzerland detected the Higgs boson, confirming the prediction. In order to solve and examine the equations, Mathematica was the primary software used for the visualization. We examined Higgs potential to study the Higgs interaction, using the experimental data from the 2012 discovery.

In this project we create visualizations to investigate the Higgs Interaction in the Standard Model and Beyond the Standard model physics. The Higgs discovery at the Large Hadron Collider at CERN has allowed for a precise determination of the parameters of the Higgs Potential [1]. We visualize this potential and the Higgs interactions with the fermions of the Standard Model (six quarks and six leptons). To do this we use the measured value of the Yukawa couplings to the quarks and leptons and visualize the interaction with respect to the Higgs field. The interaction with the Higgs field and the fermions give rise to their mass so we see how the effective mass varies as one moves around the Higgs potential away from its minimum. The coupling strength of the Higgs field is not constant but varies with energy and we solve the renormalization group equations for this coupling to determine the shape of the Higgs potential with energy [2]. We study the Higgs potential with respect to energy to see how the potential changes shape as one moves up in energy. The high energy behavior has a sensitive dependence on the top quark coupling. We study the Higgs potential at finite temperature and illustrate the change in shape and phase transition of the model at High temperature where the shape changes from a Mexican Hat potential to a bowl shape [3].

Beyond Standard Model Physics refers to fields and particles that are not contained in the Standard Model of quarks, leptons, gluons, weak bosons and the photon. These include a possible additional Higgs bosons associated with grand unification, dark matter particles and heavy neutrinos associated with the seesaw mechanism for generating small neutrino masses. We visualize the interaction of the Higgs field with a dark matter scalar particle and study the effect of the potential of a coupling between the Higgs which generate interactions between matter and dark matter [3], the so called Higgs portal.

A. Visualizing Higgs Field

As the massless particles enter the Higgs field, the field possessed by the zero spin and electrically neutral Higgs boson, spontaneous symmetry breaking occurs as the result of the particle's interaction with the scalar Higgs field. The potential, which is the function Higgs field and Higgs coupling and quadratic constant is visualized below.

B. Self Interacting Higgs Potential

While the fundamental particles of the standard model gains mass through the Higgs couplings, the Higgs boson self-interacts. The potential defined with self interaction is described as below.

$$\begin{aligned} &V(H) = m^2 \mid H^2 \mid + \lambda \mid H \mid^4 \\ &m^2 = -(90Gev)^2 \\ &\lambda = 0.13 \end{aligned} \qquad (1)$$

Where m^2 is the quadratic term and λ is the self-interacting coupling constant. The function describes potential as a function of the scalar Higgs field. For the 2 dimensional graph H is defined as a one dimensional scalar field. The potential plots as below:

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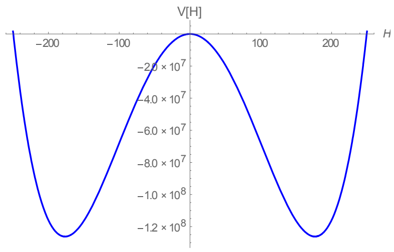

Figure 1. Two dimensional Mexican Hat potential standard model

The plot demonstrates two dimensional Mexican hat potential. The name of the Mexican Hat potential is derived from the distinctive shape of the symmetric plot with one maxima and two local minima. The function, which has a shape of traditional Mexican sombrero often appears as a function of the Higgs potential. Once the Higgs field is defined as a function of scalar x and y field, the name becomes more relative.

$$|H| = \sqrt{x^2 + y^2} \tag{2}$$

The scalar Higgs field H is related to x and y by Pythagorean theorem relations.

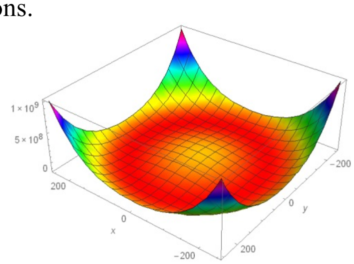

Figure 2. Three dimensional Mexican Hat potential for standard model

Figure 2 shows three dimensional plot of the standard model Higgs potential. The plot is symmetric by its axis where the minima is found around the local maxima, the top of the Mexican hat. Let the graph 1 and 2 visualize the path of a hypothetical ball located on the origin, where each path of the ball can be traced to find the instantaneous potential. At the top of the hat unstable equilibrium is expected where any external disturbance can break the system's symmetry through the spontaneous symmetry break. The most stable state is found at the nonzero minima where the ball, traveling down from the top will eventually reach as a most stable equilibrium state. Unlike the initial symmetry breaking, once the ball reaches the stable nonzero minima higher disturbance is required to change the stable state. Even if the ball leaves the minima trait, the system will easily restore back to the stable equilibrium state. The ball will travel out of the circular minima, and return back to the trail. The plot's minima indicate the minimum Higgs potential, which is universally found to provide mass to the massless particles through spontaneous symmetry breaking.

II. Standard Model Yukawa Coupling

Once the particles enter the Higgs field, the coupling between fermion and the Higgs field cause change in potential. The interaction between scalar Higgs field and Dirac fermion field is described by Yukawa coupling.

A. *Yukawa coupling constant*

The Yukawa coupling constant is defined as:

$$g = \sqrt{2}\left(\frac{m_t}{v}\right) \tag{3}$$

Where v=246Gev and m_t is mass of the top quark.
The Yukawa constant calculation for all fermion flavor is presented as following:

TABLE I. YUKAWA COUPLING PER FERMION

	Yukawa Coupling per Fermion		
	Flavor	***Mass(Gev)***	***Yukawa Constant***
Quarks	Up	3.3×10^{-3}	1.89712×10^{-5}
	Down	5.8×10^{-3}	3.33432×10^{-5}
	Strange	1.01×10^{-1}	5.80632×10^{-4}
	Charm	1.27	7.30102×10^{-3}
	Bottom	4.67	2.68471×10^{-2}
	Top	172	9.888×10^{-1}
Leptons	Electrons	5.11×10^{-4}	2.93766×10^{-6}
	Electron Neutrino	2.2×10^{-9}	1.26474×10^{-11}
	Muons	1.057×10^{-1}	6.07652×10^{-4}
	Muon Neutrino	1.7×10^{-4}	9.77302×10^{-7}
	Tau	1.777	1.02157×10^{-2}
	Neutrino Tau	1.55×10^{-2}	8.9107×10^{-5}

B. *Standard model Yukawa Coupling Higgs potential*

Once the Yukawa coupling is added, the new definition of potential is defined as following.

$$V(H,\psi) = m^2 |H^2| + \lambda |H|^4 + g|H|(\psi^\dagger \psi) \tag{4}$$

The function now has parameters of H and ψ has additional Yukawa term to the standard model potential. Unlike the previous definition of H, the H is defined merely as a scalar parameter without x and y dependency.

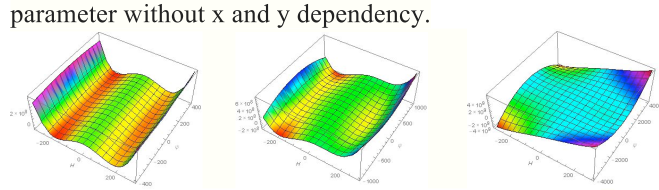

Figure 3. Evolution of the potential with a constant H and increasing ψ

Mexican hat potential is distorted as higher fermionic field is coupled to the Higgs field.

III. Effective Potential at Finite Temperature

A. *Effective Mass*

The effective mass is the second derivative of the effective temperature potential at the potential's minimum. The effective mass is thus dependent upon temperature T defined as:

$$m_{eff}^{\,2}(h,T) = m^2(h) + \Pi(T) \tag{5}$$

B. *Effective Potential at Finite Temperature*

Where $\Pi(T)$ is the self-energy loop correction. During the phase transition, where the symmetry is restored and effective mass reaches zero, the effective potential is estimated to be:

$$V_{eff}(h,T) \approx \frac{1}{2}(-\mu^2 + cT^2)h^2 - \frac{eT}{12\pi}(h^2)^{3/2} + \frac{\lambda}{4}h^4 \tag{6}$$

With the definitions

$$e = \frac{\left(6M_w{}^3 + 3M_z{}^3\right)}{v^3} \tag{7}$$

$$c = \frac{\left(6M_t{}^2 + 6M_W{}^2 + 3M_Z{}^2 + \frac{3}{2}M_H{}^2\right)}{12v^2} \tag{8}$$

$$\mu = \frac{M_H}{\sqrt{2}} \tag{9}$$

$$\lambda = \frac{M_H{}^2}{v\sqrt{2}} \tag{10}$$

The potential is visualized in both plane and concentric 3D plot.

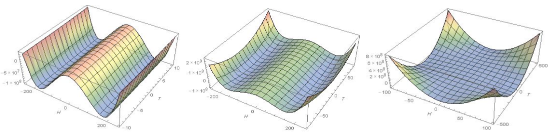

Figure 4. V_{eff} on 3D plane as function of fixed value H and varying T plane.

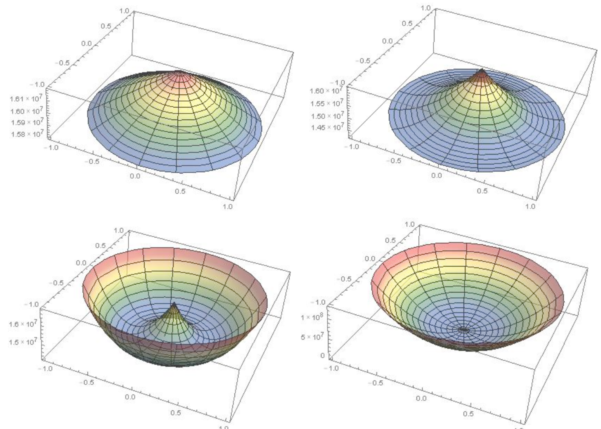

Figure 5. V_{eff} on 3D rotational plane as function of fixed value H and varying T plane with themperature increasing fro top left to bottom right.

As indicated from both figures, unstable potential at the origin becomes gradually stablized at high T and the symmetry is restored.

IV. One loop effective potential

The one loop correction in Feynman diagram refers to quantum corrections to the classical Feynman diagram. From the correctionm effevctive potential is calculated as a function of energy.

A. One Loop Effective Potential

$$kV^{(1)} = \frac{H^2}{4}(\overline{\ln} H - \frac{3}{2}) + \frac{3G^2}{4}(\overline{\ln} G - \frac{3}{2}) - 3T^2(\overline{\ln} T - \frac{3}{2}) + \frac{3W^2}{2}(\overline{\ln} W - \frac{5}{6}) + \frac{3Z^2}{4}(\overline{\ln} Z - \frac{5}{6}) \tag{11}$$

Parameters are defined by:

$$\begin{aligned} k &= 16\pi^2 \\ H &= m^2 + \frac{\lambda}{2}\phi^2 \\ T &= \frac{h^2}{2}\phi^2 \\ G &= m^2 + \frac{\lambda}{6}\phi^2 \\ W &= g_1{}^2\phi^2 \\ Z &= \frac{(g_1{}^2 + g_2{}^2)}{4}\phi^2 \end{aligned} \tag{12}$$

$$\overline{\ln} X = \ln(\frac{X}{\mu^2}) + \gamma - \ln(4\pi)$$

The complicated expressions for the one loop corrected potential as function of energy scale μ is graphed below:

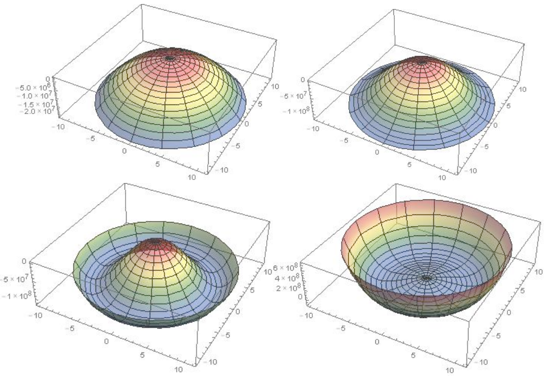

Figure 6. one loop effective potential as a functhion of H fron with increasing energy from left to right, top to bottom

As with the finite temperature potential the symmetry is restored at high energy scales.

B. Electroweak and Strong Interactions

The term g refers to the Coupling constants for the electroweak and strong interactions.

$$\begin{aligned} &\alpha_i{}^{-1}(x) = \alpha_i{}^{-1}(M_Z) - \frac{b_i}{4\pi}x \\ &\alpha_i{}^{-1}(M_z) = \{58.962, 29.61, 8.3\} \\ &b_i = \{\frac{21}{5}, -3, -7\} \\ &x(\mu) = \log(\frac{\mu^2}{Mz^2}) \\ &M_Z = 91.19 GeV \\ &g_i{}^2 = 4\pi\alpha_i \end{aligned} \tag{13}$$

Here g has alpha dependency where parameter x is a function of energy scale μ. Comparing the values of inverse alpha for all three couplings the plot revealed intersection between the the couplings at which potential Unification of couplings could take place. Complete or Grand Unification, if it occurs, would take place at enormous enrgies of $10^{15} GeV$ as indcated in the below plot.

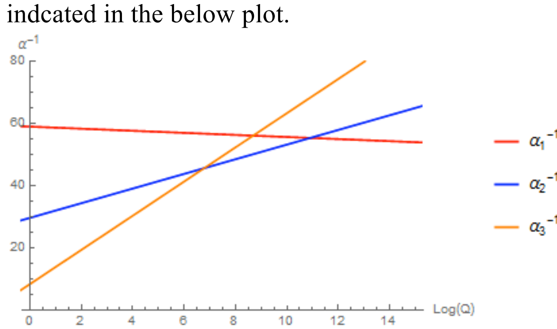

Figure 7 plot of for α_i^{-1} from different gauge group

V. RENORMALIZATION OF THE HIGGS POTENTIAL

So far we defined the self-interacting constant λ as a discrete constant rather than continuous function. To examine the self-interaction function, differential equations were solved to find the numerical solutions to the renormalization group differential equations. Two senarios were proposed to evaluate the λ, for the standard model with and without the strong interaction [4].

A. Standard Model

The coupled equations without strong interaction is defined as:

$$\frac{d\lambda}{dlog(Q)} = \frac{9}{8}\frac{\lambda^2}{\pi^2} + \frac{\lambda f_t^2}{4\pi^2} - \frac{f_t^4}{8\pi^2} \qquad (14)$$

$$\frac{df_t}{dlog(Q)} = \frac{5 f_t^3}{32\pi^2} \qquad (15)$$

The numerical solution to this coupled differentil yield the following plot.

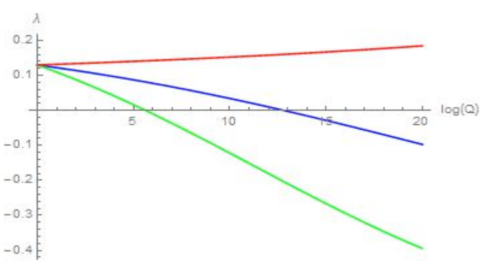

Figure 8 Evolution of λ by value of log(Q). Each equation of λ has different f_t of 1,1.2, and 0(from top to bottom graph) In this graph initial condition of λ is set to 0.13.

B. Standard Model with Strong Interaction

Once the strong interaction is considered into the system, the coupling constant g is coupled to the previous equations as well as the number of colors, Nc. The equation is presented as following.

$$\frac{d\lambda}{dlog(Q)} = \frac{9}{8}\frac{\lambda^2}{\pi^2} + \frac{N_c\lambda f_t^2}{4\pi^2} - \frac{N_c f_t^4}{8\pi^2} \qquad (15)$$

$$\frac{dg_s}{dlog(Q)} = \frac{(2N_c+3)f_t^3}{32\pi^2} - \frac{3{g_s}^2 f_t(\frac{{N_c}^2-1}{2N_c})}{8\pi^2} \qquad (16)$$

$$\frac{df_t}{dlog(Q)} = \frac{(2N_c+3)f_t^3}{32\pi^2} - \frac{3{g_s}^2 f_t(\frac{{N_c}^2-1}{2N_c})}{8\pi^2} \qquad (17)$$

Note that if Nc=3 for the standard model.

Next the Higgs potential was visualized both with and without strong interaction effects. The Higgs potential.

$$V(H, Log(Q)) = m^2 |H^2| + \lambda(Log(Q)) |H|^4 \qquad (18)$$

with strong interaction shows a significant effect with large warping to the shape of the effective potential.

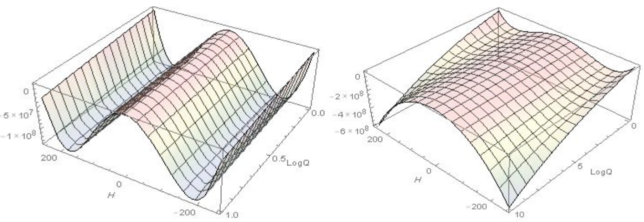

Figure 9. Evolution of the Higgs potential with change of λ without strong interaction

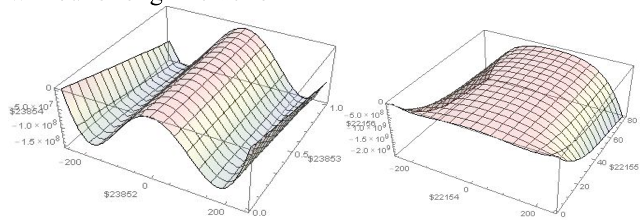

Figure 10. Evolution of the Higgs potential with change of λ with strong interaction

VI. BEYOND THE STANDARD MODEL

Finally We examined the Higg's coupling to an scalar field X, which can represent coupling to the dark matter.

$$V(H,X,Q) = m^2(H)^2 + \lambda(H)^4 + M^2{}_X(Q)(X*X) + \frac{K}{6}(X*X)^2 + Q|H|^2(X*X) \qquad (19)$$

$$M^2{}_X(Q) = \frac{-(K+Q)}{24}\frac{(\lambda * v^2)}{c}\left(1 - \frac{\lambda}{2c}\left(\frac{e}{6\pi * \lambda}\right)^2\right)^{-1}$$

$$K = 1$$

$$\lambda = 0.13 \qquad (20)$$

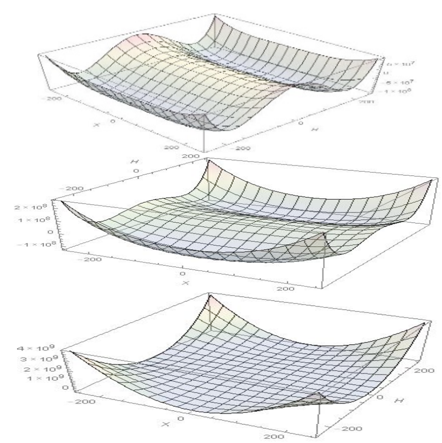

Figure 11. Evolution of the Higgs potential with change of Q for the beyond standard model

The figures above show the effect of increasing the parameter Q from top to botton. The dark matter field interaction strongly modifies the Mexican hat shape finally leading to a bowl shape for large Q. Thus the effects of spontaneous symmetry breaking are muted for large interactions with the dark matter field X.

Acknowledgment

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