homework13

1. 证明下列定解问题解唯一

$$\begin{cases} u_{xx} + u_{yy} = x^2 - y^2, & x^2 + y^2 < 1, \\ u|_{x^2+y^2=1} = 0. \end{cases}$$

2. 求变分问题 u 满足的边值问题:

$$J[u(x,y)] = \int_{x^2+y^2\le 1} \left[|\nabla u|^2 - 2xyu \right] dxdy, \; u|_{x^2+y^2=1} = xy.$$

3. 设

$$J[v] = \iiint_\Omega \frac{1}{2}\left[\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] dxdydz + \iint_\Gamma \left[\frac{1}{2}\sigma v^2 - gv\right] ds$$

考察变分问题: 求 $u \in V = \left\{ v \in C^2(\Omega) \cap C^1(\bar{\Omega}) \right\}$ 使得

$$J[u] = \min_{v\in V} J[v]$$

试推导 u 满足的边值问题。