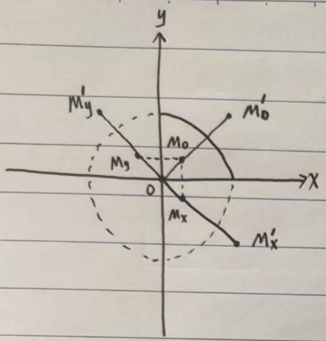
1、解:



反演点：$M_0 \to M_0'$

$M_x \to M_x'$

$M_y \to M_y'$

对称点：$M_0 \leftrightarrow M_x$ 关于边界 I_x (x轴)

$M_0 \leftrightarrow M_y$ 关于边界 I_y (y轴)

$$G(M,M_0)=\frac{1}{2\pi}\left(\ln\frac{1}{r_{MM_0}}-\ln\frac{1}{r_{MM_x}}-\ln\frac{1}{r_{MM_y}}-\ln\frac{R}{r_0 r_{MM_0'}}+\ln\frac{R}{r_x' r_{MM_x'}}+\ln\frac{R}{r_y' r_{MM_y'}}\right)$$

其中 $r_0=|OM_0'|$, $r_x'=|OM_x'|$, $r_y'=|OM_y'|$

$$\begin{cases} \Delta u=0 & \left[u=u(r,\theta),\ 0\le r\le 1,\ 0\le\theta\le\frac{\pi}{2}\right] \\ u(r,\frac{\pi}{2})=r-r^2 \\ u(r,0)=r(1-r)^2 \\ u|_{r=1}=0 \end{cases}$$

$$u(\cancel{r,\theta})=\frac{1}{4\pi}\int_0^{\frac{\pi}{2}}\cancel{\frac{(1-r^2)}{(1+r^2-2r)^{3/2}}}\cancel{\sin\theta}d\theta$$

$$u(r,\theta)=1-4r\cos\theta+3r^2\cos^2\theta+3r^2\cos\theta\sin\theta+r\sin\theta-3r^3\cos\theta\sin^2\theta-r^2\sin^2\theta$$

$$=r\cos\theta(1-r\cos\theta)^2+(3r\cos\theta+1)r\sin\theta(1-r\sin\theta)$$

$$u(x,y)=x(1-x)^2+(3x+1)y(1-y)$$