

homework 9

1. 证明下面定解问题解的唯一性:

$$\begin{cases} u_{tt} = a^2 (u_{xx} + u_{yy}) + f(t, x, y), & (t > 0, (x, y) \in \Omega \subset \mathbb{R}^2) \\ t = 0 : u = \varphi(x, y), u_t = \psi(x, y) \\ \left(\frac{\partial u}{\partial n} + \sigma u \right) \Big|_{\partial \Omega} = g(t, x, y) \end{cases}$$

2. 求解 Cauchy 问题

$$\begin{cases} u_t - u_{xx} = 0 \\ t = 0 : u = \sin x \end{cases}$$

3. 设 u 为下列初边值问题的解

$$\begin{cases} u_t - a^2 \Delta u = 0, G = \Omega \times [0, T] \\ \frac{\partial u}{\partial n} \Big|_{\partial \Omega \times [0, T]} = 0 \\ u|_{t=0} = \varphi(x, y, z) \end{cases}$$

其中 Ω 为有界域 ($\Omega \subset \mathbb{R}^3$), 试证明, 对 $\forall t \in (0, T)$:

$$\int_{\Omega} u(x, y, z, t) dx dy dz = \int_{\Omega} \varphi(x, y, z) dx dy dz.$$