homework 5

1. 求解下列定解问题

$$\begin{cases} u_{xx} - u_{yy} + \sin x = 0, \quad y > 0, -\infty < x < +\infty \\ u(x,0) = 0, \quad u_y(x,0) = 4x \end{cases}$$

2. 证明下面定解问题解的唯一性:

$$\begin{cases} u_{tt} - a^2 u_{xx} + x u_x = f(x,t), & (0 \le x \le l, t > 0) \\ (u_x - \sigma_1 u)\big|_{x=0} = \lambda(t), u|_{x=l} = \mu(t), & \sigma_1 > 0 \\ t = 0 : u = \varphi(x), u_t = \psi(x) \end{cases}$$

3. 利用能量积分方法, 证明下述初边值问题解的唯一性:

$$\begin{cases} u_{tt} = c^2 u_{xx}, & 0 < x < L, t > 0 \\ u(x,0) = \phi(x), u_t(x,0) = \psi(x), & 0 \leqslant x \leqslant L \\ (-\alpha u_x + \beta u)\big|_{x=0} = p(t), & t \geqslant 0 \\ (\alpha u_x + \beta u)\big|_{x=L} = q(t), & t \geqslant 0 \end{cases}$$

其中常数 $\alpha, \beta > 0$.