1. Given $s = 0.30/(60-u)$, where s is the spacing in miles (mi) and u is the speed in miles per hour (r
drive the relationship u-k, q-k, and u-q. Also, estimate the capacity (i.e. q_{max}) of the roadway.

Solution:

$$u_f = 60, \quad k_j = \frac{60}{0.30} = 200$$

① u-k

$$u = u_f - \frac{u_f}{k_j}k = 60 - \frac{60}{200}k = 60 - 0.30k$$

② q-k

$$q = uk = (60 - 0.30k)k = -0.30k^2 + 60k$$

③ u-q

$$q = uk = u\frac{60-u}{0.30} = -\frac{10}{3}u^2 + 200u$$

④ q_{max}

$$q_{max} = \frac{1}{4}u_f k_j = \frac{1}{4} \times 60 \times 200 \text{ vph} = 3000 \text{ vph}$$

2. The optimum speed and the optimum density for the high-speed freeway data set is assumed to be 40 m
per hour and 50 vehicles per mile per lane, repectively, determine the equation for the Greenberg, Und
and Northwestern models, and construct flow-speed-density relationship in a fashion similar to Figure 10.7.

Solution:

$$u_0 = 40, \quad k_0 = 50, \quad u_f = 40 \times 2 = 80, \quad k_j = 50 \times 2 = 100$$

① Greenberg

$$u = u_0 \ln\left(\frac{k_j}{k}\right) = 40\ln\left(\frac{100}{k}\right)$$

$$q = uk = 40\ln\left(\frac{100}{k}\right)k = 40k\ln\left(\frac{100}{k}\right)$$

$$q = uk = u\frac{100}{e^{\frac{u}{40}}} = 100u\,e^{-\frac{u}{40}}$$

② Underwood

$$u = u_f e^{-\frac{k}{k_0}} = 80e^{-\frac{k}{50}}$$

$$q = uk = 80e^{-\frac{k}{50}}k = 80k\,e^{-\frac{k}{50}}$$

$$q = uk = u\ln\left(\frac{u}{80}\right)(-50) = -50u\ln\left(\frac{u}{80}\right)$$

③ Northwestern

$$u = u_f e^{-\frac{1}{2}\left(\frac{k}{k_0}\right)^2} = 80e^{-\frac{1}{2}\left(\frac{k}{50}\right)^2}$$

$$q = uk = 80e^{-\frac{1}{2}\left(\frac{k}{50}\right)^2}k = 80k\,e^{-\frac{1}{2}\left(\frac{k}{50}\right)^2}$$

$$q = uk = u\sqrt{\ln\left(\frac{u}{80}\right)(-2)50^2} = 50u\sqrt{-2\ln\left(\frac{u}{80}\right)}$$

3. A helicopter pilot recorded the travel time of five vehicles on a 2-mile segment of a highway. Estimated the time-mean speed and space-mean speed of the vehicles.

Vehicles	Travel Times (sec)
1	126
2	163
3	149
4	139
5	193

Solution:

① time-mean speed

$$v_1 = \frac{d}{t_1} = \frac{2\times5280}{126} \text{ ft/s} = 83.81 \text{ ft/s}$$

$$v_2 = \frac{d}{t_2} = \frac{2\times5280}{163} \text{ ft/s} = 64.79 \text{ ft/s}$$

$$v_3 = \frac{d}{t_3} = \frac{2\times5280}{149} \text{ ft/s} = 70.87 \text{ ft/s}$$

$$v_4 = \frac{d}{t_4} = \frac{2\times5280}{139} \text{ ft/s} = 75.97 \text{ ft/s}$$

$$v_5 = \frac{d}{t_5} = \frac{2\times5280}{193} \text{ ft/s} = 54.72 \text{ ft/s}$$

$$TMS = \bar{v} = \frac{v_1+v_2+v_3+v_4+v_5}{5} = \frac{83.81+64.79+70.87+75.97+54.72}{5} \text{ ft/s}$$

$$= 70.03 \text{ ft/s}$$

② space-mean speed

$$SMS\ \bar{t} = \frac{t_1+t_2+t_3+\cancel{\frac{2\times5280}{}}t_4+t_5}{5} = \frac{126+163+149+139+193}{5} \text{ s}$$

$$= 154 \text{ s}$$

$$SMS = \frac{d}{\bar{t}} = \frac{2\times5280}{154} \text{ ft/s} = 68.57 \text{ ft/s}$$

4. A line of vehicles are in car-following mode and all vehicles are traveling at 44 feet per second with distance headways of 114 feet. The lead vehicle suddenly decelerates at a rate of 4.6 feet per second² until it stops completely. Calculate and plot the projectory of the lead and only the first following vehicle for every second of time until both vehicles are stopped. Use the GM first car-following model. Consider three different drivers in the following vehicle with the following characteristics.

Driver	Reaction Time Δt	Sensitivity Parameter λ
1	1.0	0.17
2	1.55	0.37
3	2.2	0.74

Solution:

Model 1 $\quad \ddot{X}_{n+1}(t+\Delta t) = \lambda[\dot{X}_n(t) - \dot{X}_{n+1}(t)]$

Driver 1 $\quad \ddot{X}_2(t+\Delta t) = 0.17[\dot{X}_1(t) - \dot{X}_2(t)]$ $\quad \dot{X}_1(t) = 44 - 4.6t$

Driver 2 $\quad \ddot{X}_2(t+\Delta t) = 0.37[\dot{X}_1(t) - \dot{X}_2(t)]$ $\quad \dot{X}_1(t) = 44 - 4.6t$

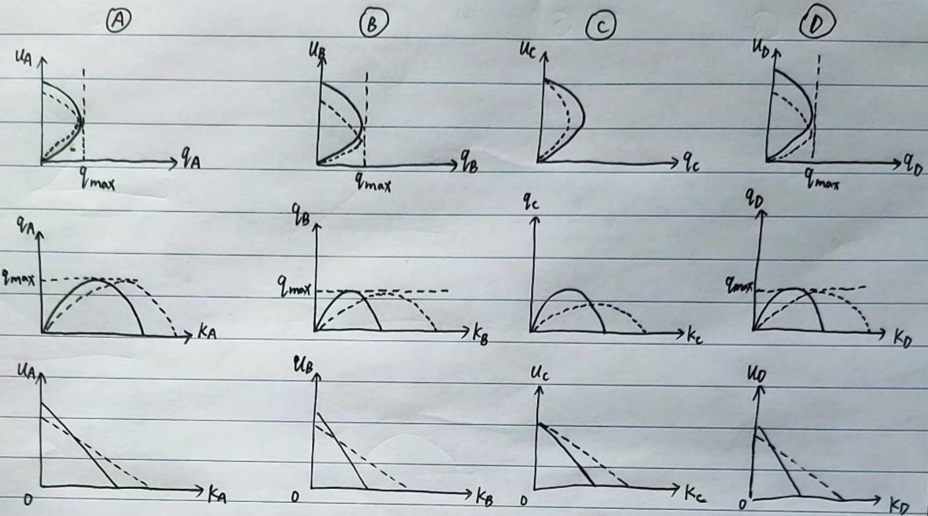
Driver 3 $\quad \ddot{X}_2(t+\Delta t) = 0.74[\dot{X}_1(t) - \dot{X}_2(t)]$ $\quad \dot{X}_1(t) = 44 - 4.6t$

For more information, please infer the following charts and figures.

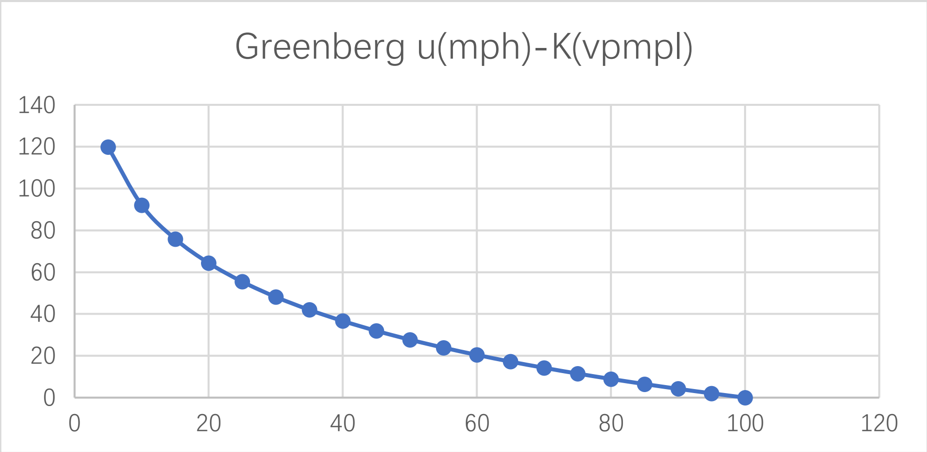
5. Question: ---.

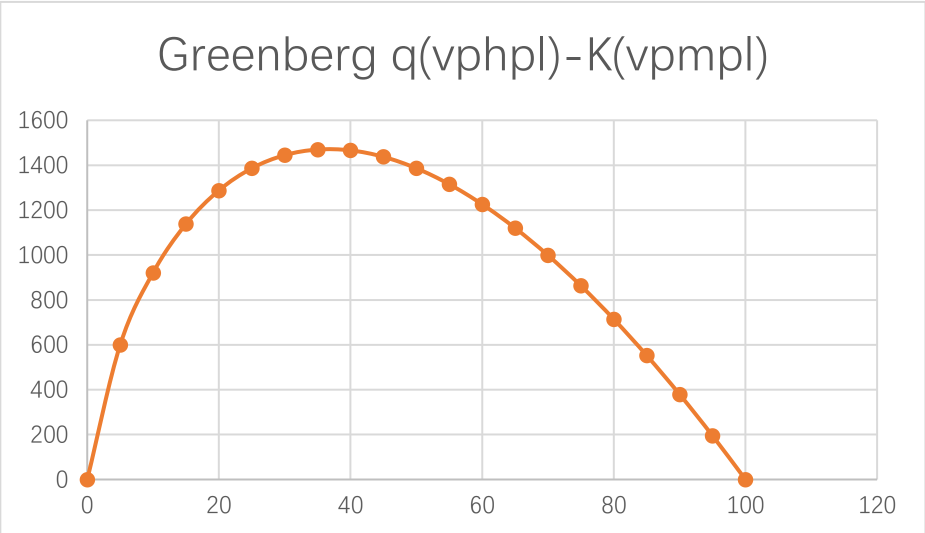
Solution: ———— solid line (the former figure)

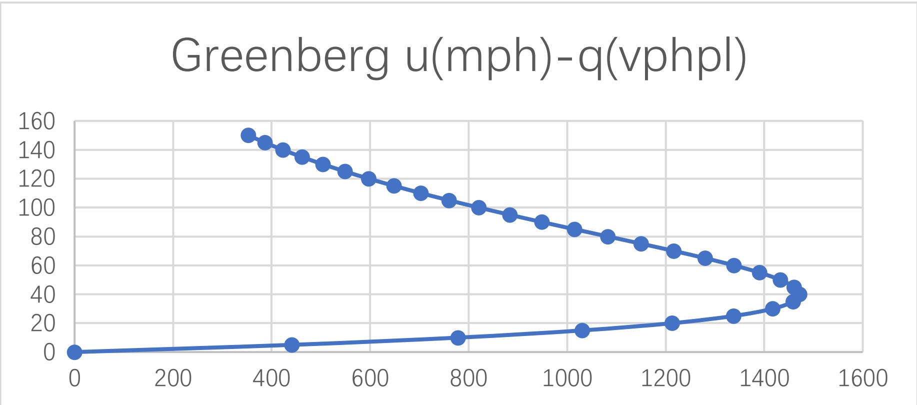
------ dashed line (the latter figure)

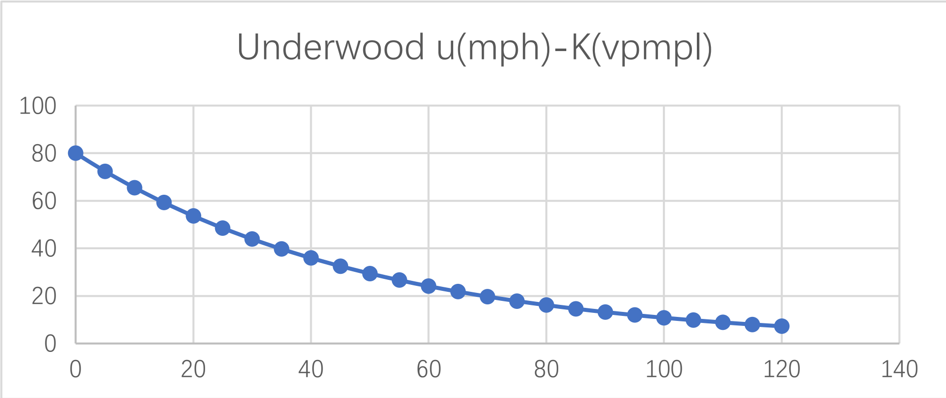


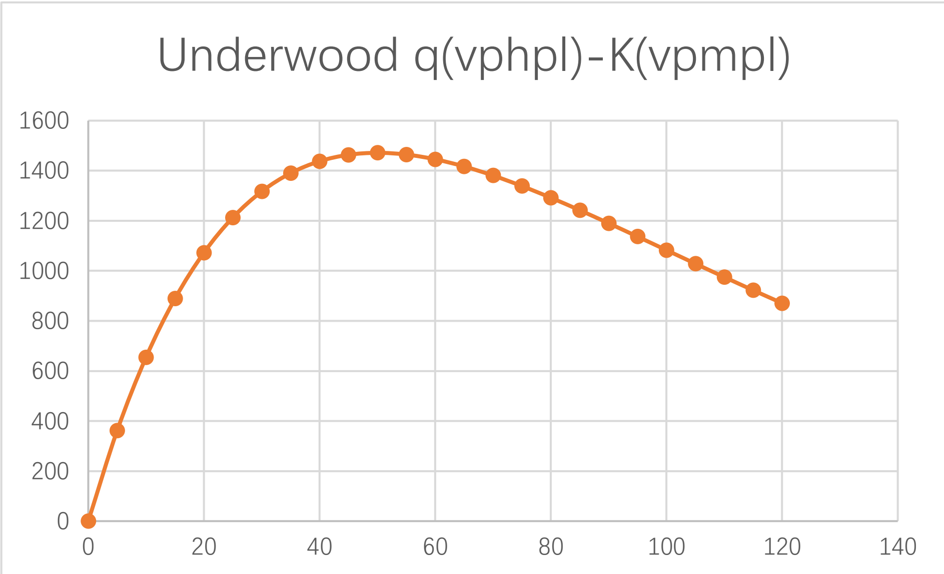
Figures for Question 2:

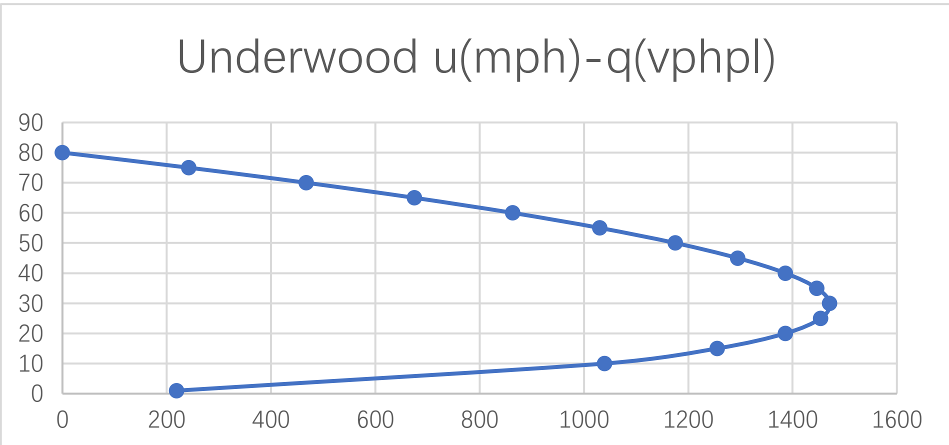


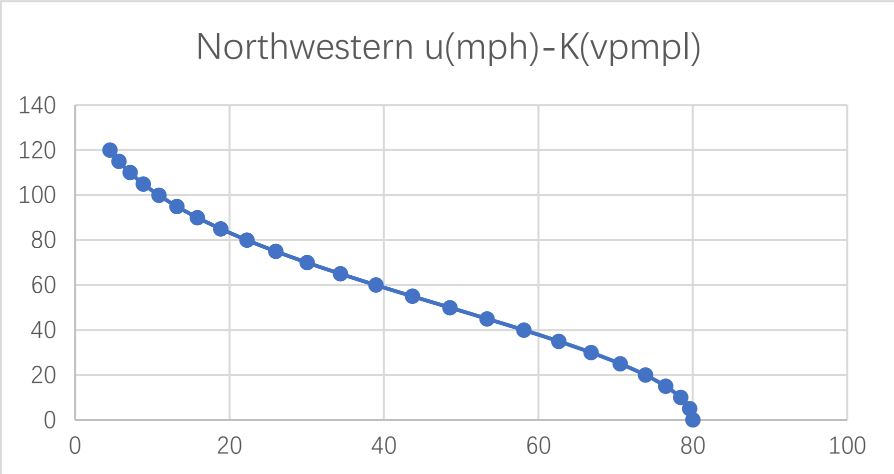


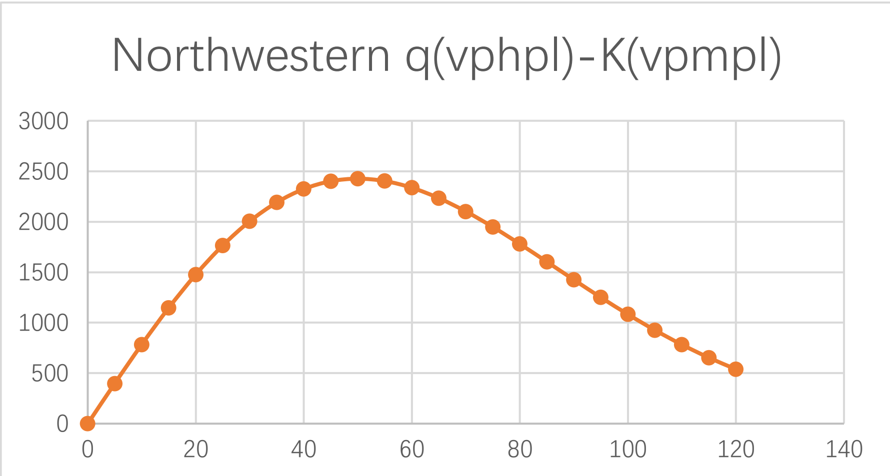


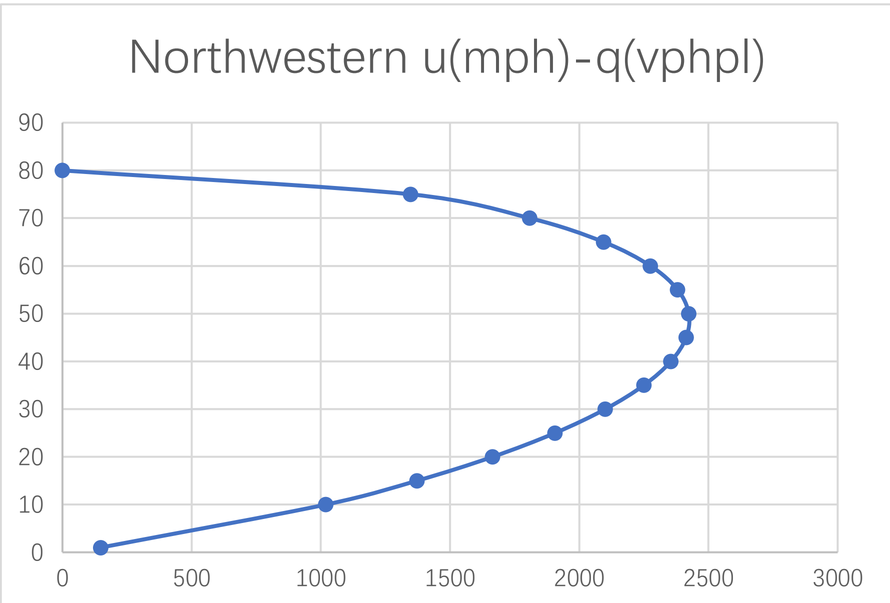

Underwood u(mph)-K(vpmpl)
100
80
60
40
20
0
0
20
40
60
80
100
120
140

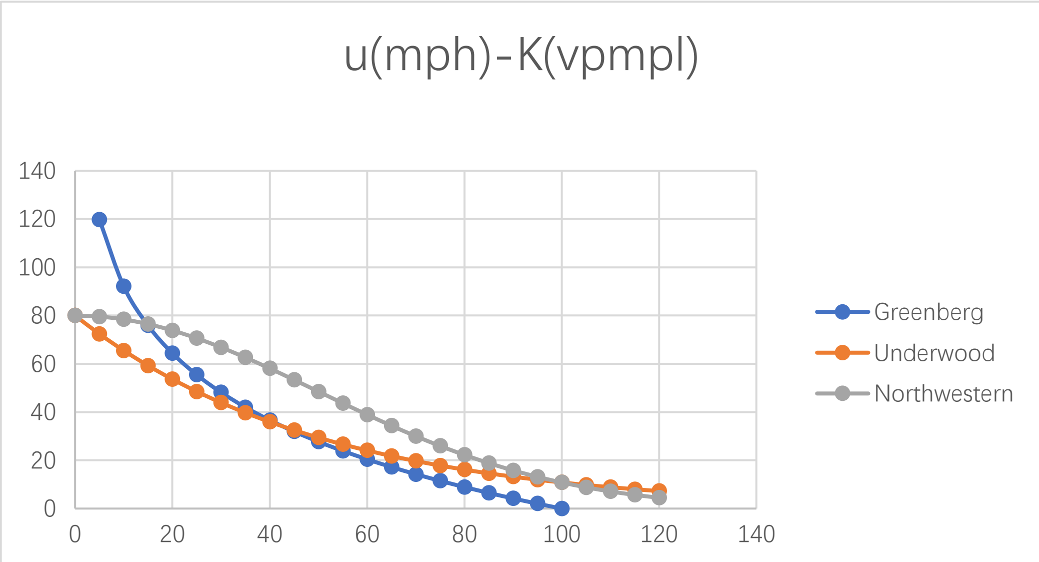

Underwood q(vphpl)-K(vpmpl)
1600
1400
1200
1000
800
600
400
200
0
0
20
40
60
80
100
120
140

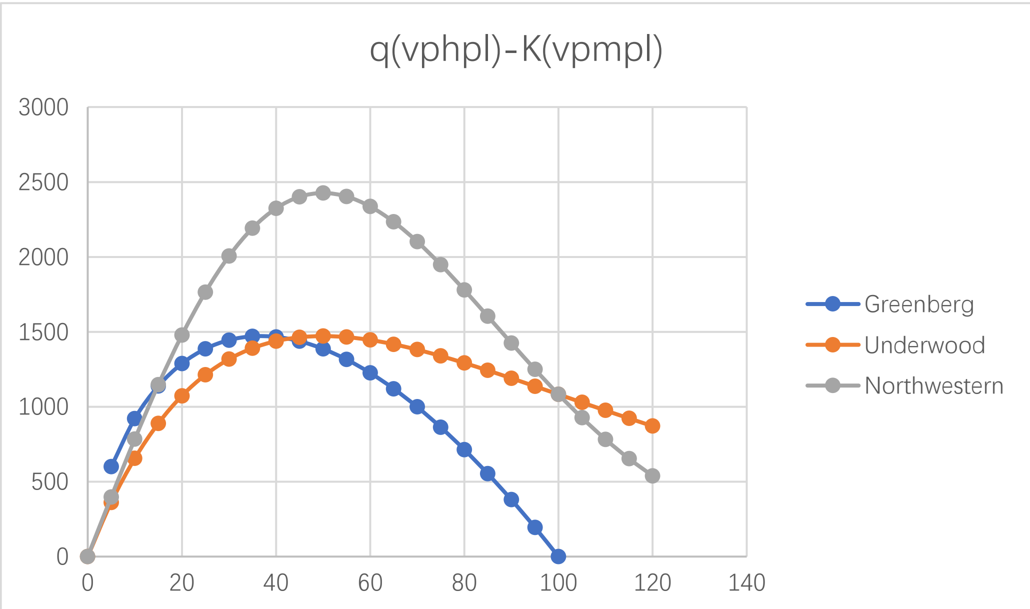

Underwood u(mph)-q(vphpl)
90
80
70
60
50
40
30
20
10
0
0
200
400
600
800
1000
1200
1400
1600

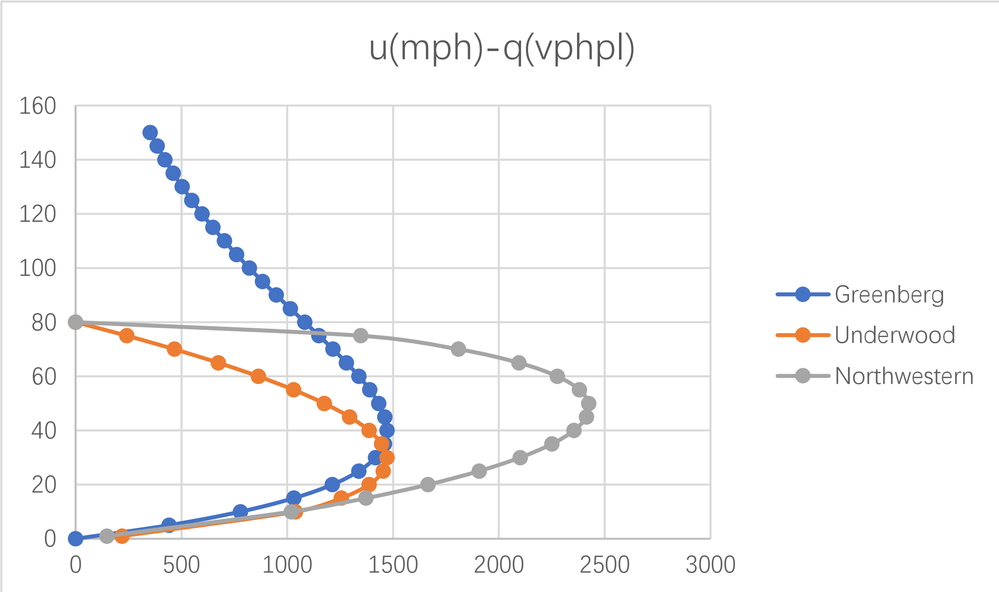

Northwestern u(mph)-K(vpmpl)
140
120
100
80
60
40
20
0
0
20
40
60
80
100


Northwestern q(vphpl)-K(vpmpl)
3000
2500
2000
1500
1000
500
0
0
20
40
60
80
100
120
140


Northwestern u(mph)-q(vphpl)
90
80
70
60
50
40
30
20
10
0
0
500
1000
1500
2000
2500
3000

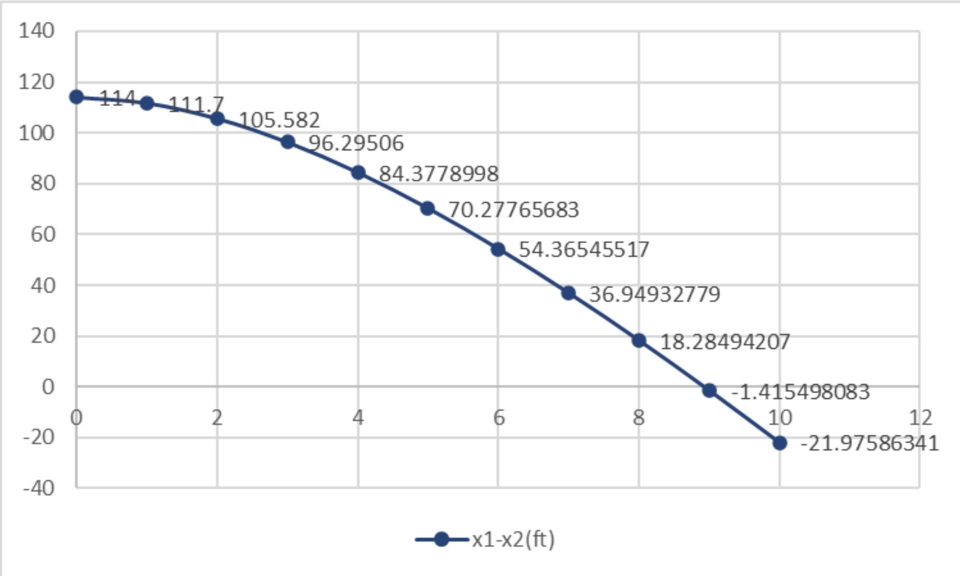

u(mph)-K(vpmpl)
140
120
100
80
60
40
20
0
0
20
40
60
80
100
120
140
Greenberg
Underwood
Northwestern


q(vphpl)-K(vpmpl)
3000
2500
2000
1500
1000
500
0
0
20
40
60
80
100
120
140
Greenberg
Underwood
Northwestern

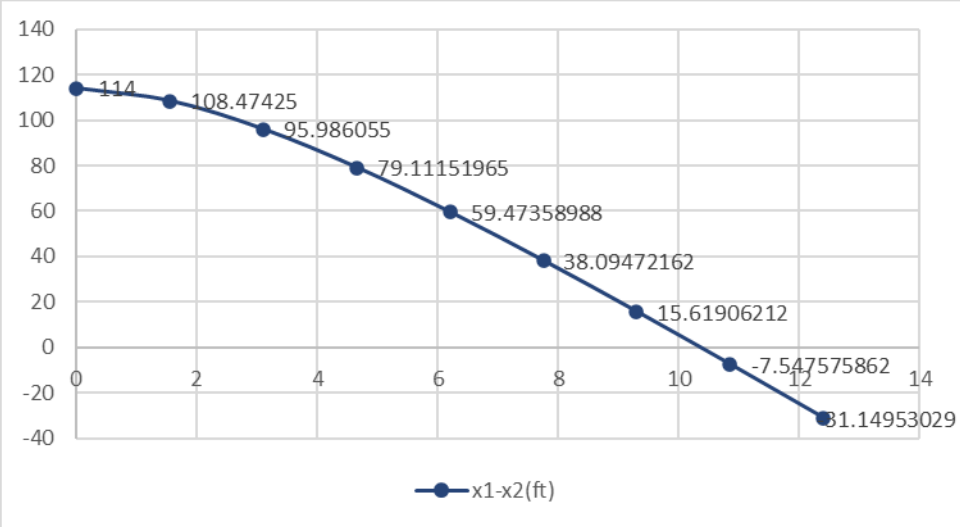

u(mph)-q(vphpl)
160
140
120
100
80
60
40
20
0
0
500
1000
1500
2000
2500
3000
Greenberg
Underwood
Northwestern

Charts and Figures for Question 4:

t(s)	x1'(ft/s)	x2'(ft/s)	α(1/s)	x2''(ft/s/s)	Δx1(ft)	Δx2(ft)	x1-x2(ft)
0	44	44	0.17	0	0	0	114
1	39.4	44		-0.782	41.7	44	111.7
2	34.8	43.218		-1.43106	37.1	43.218	105.582
3	30.2	41.78694		-1.96978	32.5	41.78694	96.29506
4	25.6	39.81716		-2.41692	27.9	39.81716	84.3779
5	21	37.40024		-2.78804	23.3	37.40024	70.27766
6	16.4	34.6122		-3.09607	18.7	34.6122	54.36546
7	11.8	31.51613		-3.35174	14.1	31.51613	36.94933
8	7.2	28.16439		-3.56395	9.5	28.16439	18.28494
9	2.6	24.60044		-3.74007	4.9	24.60044	-1.4155
10	-2	20.86037		-3.88626	0.3	20.86037	-21.9759



t(s)	x1'(ft/s)	x2'(ft/s)	α(1/s)	x2''(ft/s/s)	x1(ft)	x2(ft)	x1-x2(ft)
0	44	44	0.37	0	0	0	114
1.55	36.87	44		-2.6381	62.67425	68.2	108.4743
3.1	29.74	41.3619		-4.3001	51.62275	64.11095	95.98606
4.65	22.61	37.0618		-5.34716	40.57125	57.44579	79.11152
6.2	15.48	31.71463		-6.00681	29.51975	49.15768	59.47359
7.75	8.35	25.70782		-6.42239	18.46825	39.84712	38.09472
9.3	1.22	19.28543		-6.68421	7.41675	29.89241	15.61906
10.85	-5.91	12.60122		-6.84915	-3.63475	19.53189	-7.54758
12.4	-13.04	5.752067		-6.95306	-14.6863	8.915704	-31.1495



t(s)	x1'(ft/s)	x2'(ft/s)	α(1/s)	x2''(ft/s/s)	x1(ft)	x2(ft)	x1-x2(ft)
0	44	44	0.74	0	0	0	114
2.2	33.88	44		-7.4888	85.668	96.8	102.868
4.4	23.76	27.52464		-2.78583	63.404	60.55421	105.7178
6.6	13.64	21.39581		-5.7393	41.14	47.07077	99.78702
8.8	3.52	8.769354		-3.88452	18.876	19.29258	99.37044
11	-6.6	0.223406		-5.04932	-3.388	0.491493	95.49095
13.2	-16.72	-10.8851		-4.31783	-25.652	-23.9472	93.78616