5.1. A roadway has 3 lanes. A vehicle is travelling in the middle lane (i.e., 2nd) and has the options of either ~~leaving~~ travelling in the same lane or changing either to the 1st or 3rd lanes. These decisions are governed by utilities of the lanes (U_L) and gaps (U_g). If the vehicle has decided to leave the current lane, the decisions of choosing among the other two lanes are governed by the utilities of gaps (U_g) in those lanes. On which lane would the vehicle like to travel probably?

$U_L = 3.467 - 0.0757 \times \text{Relativespeed} - 0.0064 \times \text{Frontgap}$

$U_g = 5.567 - 0.03 \times \text{Leadgap} - 0.0129 \times \text{Laggap}$

Lane No.	Relativespeed (m/s)	Front gap (m)	Lead gap (m)	Lag gad (m)
1	5	8	5	3
2	3	—	—	—
3	8	—	9	6

Solution: $U_{L_1} = 3.467 - 0.0757 \times 5 - 0.0064 \times 8 = 3.0373$

$U_{L_2} = 3.467 - 0.0757 \times 3 - 0.0064 \times 0 = 3.2399$

$U_{L_3} = 3.467 - 0.0757 \times 8 - 0.0064 \times 0 = 2.8614$

$U_{L_2} > U_{L_1} > U_{L_3} \Rightarrow$ travelling in the same lane

$e^{U_{L_1}} = e^{3.0373} = 20.85$

$e^{U_{L_2}} = e^{3.2399} = 25.53$

$e^{U_{L_3}} = e^{2.8614} = 17.49$

$P(L_1) = \frac{e^{U_{L_1}}}{e^{U_{L_1}} + e^{U_{L_2}} + e^{U_{L_3}}} = \frac{20.85}{63.87} = 0.326$

$P(L_2) = \frac{e^{U_{L_2}}}{e^{U_{L_1}} + e^{U_{L_2}} + e^{U_{L_3}}} = \frac{25.53}{63.87} = 0.400$

$P(L_3) = \frac{e^{U_{L_3}}}{e^{U_{L_1}} + e^{U_{L_2}} + e^{U_{L_3}}} = \frac{17.49}{63.87} = 0.274$

$P(L_2) > P(L_1) > P(L_3) \Rightarrow$ the vehicle would like to travel on 2nd lane.

5.2. The arrival rate is 800 vph on a certain highway subject to Poisson Distribution, average 4s service time for each vehicle on check point subject to Negative Exponential Distribution. Please estimate the: (1) average length in system, (2) average length in queue, (3) average waiting time in queue, and (4) average time in system.

Solution: $N=1$

Arrival rate: $\lambda = 800$ vph (vehicles/hour)

Deperature rate: $\mu = 4$ spv (seconds/vehicle) $= 900$ vph (vehicles/hour)

$\rho = \frac{\lambda}{\mu} = \frac{800 \text{ vph}}{900 \text{ vph}} = \frac{8}{9}$

(1) average length in system: $L = \frac{\rho}{(1-\rho)} = \frac{\frac{8}{9}}{(1-\frac{8}{9})}$ vehicle $= \frac{\frac{8}{9}}{\frac{1}{9}}$ vehicle $= 8$ vehicle

(2) average length in queue: $L_q = \frac{\rho^2}{(1-\rho)} = \frac{(\frac{8}{9})^2}{(1-\frac{8}{9})}$ vehicle $= \frac{64}{9}$ vehicle $= 7.11$ vehicle

(3) average waiting time in queue: $\bar{w} = \frac{1}{\mu}\left(\frac{\lambda}{\mu-\lambda}\right)$

$$= \frac{1}{900}\left(\frac{800}{900-800}\right) h = \frac{2}{225} h = 32 s$$

(4) average time in system: $\bar{t} = \frac{1}{\mu-\lambda}$

$$= \frac{1}{900-800} h = \frac{1}{100} h = 36 s$$

5.3. A gas station, 4 gas pumps total, vehicle arrival rate is 2400 vph subject to Pisson Distribution, average service time is 5 s subject to Negative Exponential Distribution. Please estimate the: (1) average length in queue, (2) average waiting time in queue, and (3) average time in system.

Solution: $N=4$

Arrival rate: $\lambda = 2400$ vph (vehicles / hour)

Deperature rate: $\mu = 5$ spv (seconds / vehicle) $= 720$ vph (vehicles / hour)

$$\rho = \frac{\lambda}{\mu} = \frac{2400 \text{ vph}}{720 \text{ vph}} = \frac{10}{3}$$

$$P_0 = \frac{1}{\sum_{n=0}^{N-1} \frac{\rho^n}{n!} + \frac{\rho^N}{N!(1-\rho/N)}} = \frac{1}{1+\frac{10}{3}+\frac{(\frac{10}{3})^2}{2}+\frac{(\frac{10}{3})^3}{6}+\frac{(\frac{10}{3})^4}{24(1-\frac{10}{12})}} = \frac{27}{1267}$$

(1) average length in queue: $L_q = \frac{P_0 \rho^{N+1}}{N! N}\left[\frac{1}{(1-\rho/N)^2}\right]$

$$= \frac{\frac{27}{1267} \times (\frac{10}{3})^5}{4! \times 4}\left[\frac{1}{(1-\frac{10}{12})^2}\right] \text{ vehicle} = \frac{12500}{3801} \text{ vehicle}$$

$$= 3.289 \text{ vehicle}$$

(2) average waiting time in queue: $\bar{w} = \frac{\rho + L_q}{\lambda} - \frac{1}{\mu}$ hour

$$= \frac{\frac{10}{3}+3.289}{2400} - \frac{1}{720} \text{ hour} = 4.9335 \text{ (second)}$$

(3) average time in system: $\bar{t} = \frac{\rho + L_q}{\lambda} = \frac{\frac{10}{3}+3.289}{2400}$ hour

$$= 9.9335 \text{ (second)}$$