Modeling Quasielastic Scattering in 1GeV/c Pi+ Liquid Argon Interactions with Geant4

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Abstract

Geant4 has been a popular interaction simulation for high energy physics since its conception because of its ability to model interactions between various types of particles and target matter. Quasielastic scattering is an interaction in which the incident particle interacts with one particle in the target nucleus instead of the entire target nucleus. This study aims to review how accurately and to what extent Geant4 is able to replicate quasielastic scattering in π+ interactions with a thin liquid argon target. To do this, the data from Geant4 was analyzed in ROOT to produce graphs that are evidence for key characteristics of quasielastic scattering. It can be seen that Geant4 is able to model quasielastic scattering in π+ liquid argon interactions. Further research is necessary to experimentally validate Geant4 and expand its capabilities to other models and other interactions.

Introduction

Scattering Models

Multiple different types of scattering can be observed when particles and matter are interacting with each other. The most common type is elastic scattering, where the incident particle interacts with the entire nucleus [1]. The incident particle then transfers energy to the target nucleus and scatters according to following equations:

$$\sum_{i=1}^{N} \gamma_i m_i c^2 = \sum_{i=1}^{N} \gamma'_i m_i c^2 \quad (1)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad (2)$$

where m_i is the mass of particle i and v is the velocity of particle i. In the type of interaction covered by this paper, elastic scattering is defined by two bodies as the only two particles involved in the interaction are the incident particle and target nucleus. After the interaction, these become the scattered particle and the recoil particle, respectively.

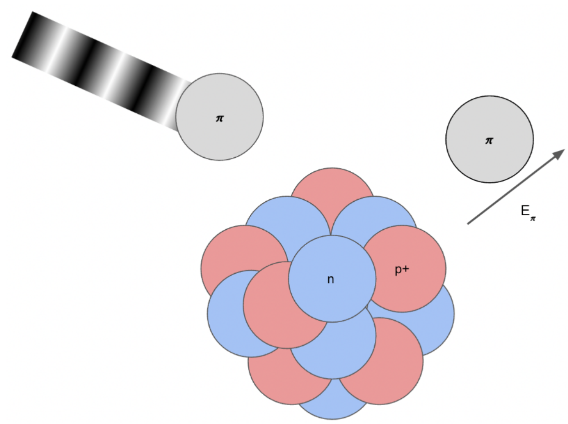

Figure 1: A representation of elastic scattering.

Another type of interaction (and the focus of this paper) is quasielastic scattering. In this method, the incident particle interacts with only one nucleon in the target nucleus, leaving the other nucleons as spectators to the interaction, and scatters with some outgoing energy. The nucleon with which the incoming particle interacted will transfer its energy to the spectators before becoming the recoil particle [2].

The deBroglie wavelength (given by the following formula) is what dictates the size of particle that is able to be studied:

$$\lambda = \frac{h}{p} \qquad (3)$$

where λ is the deBroglie wavelength, h is Planck's constant ($4.1356676696 \cdot 10^{-15}$ eV·s), and p is the momentum of the particle (in eV/c). Given an incoming momentum of 1 GeV/c, a pion will have a deBroglie wavelength of approximately $1.1 \cdot 10^{-15}$ m. The size of a proton is approximately 10^{-15} m [3], which makes this an appropriate choice to study protons and neutrons. The pion is the incoming particle of choice because it participates in nuclear interactions. Similar to the size of a proton, the strong nuclear force studied in this interaction acts over a distance of 10^{-15} m [4], making this momentum an appropriate choice. It is necessary to keep the wave/particle duality in mind because without a wave-like interaction, there would be no minima or maxima in the differential cross section of the interaction. The most correct model of particle scattering assumes a planar wave scattering off a nuclear potential.

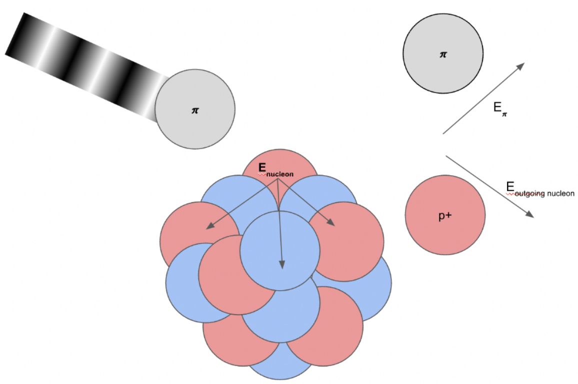

Figure 2: A representation of quasielastic scattering.

For $\pi+$ quasielastic scattering, the mechanics are defined in the following equation:

$$E_\pi^{QE} = \frac{m_p^2 - (m_n - E_b)^2 - m_\pi^2 + 2(m_n - E_b)E_\pi}{2(m_n - E_b - E_\pi + |\vec{p}_\pi| \cos\theta} \qquad (4)$$

where E_π^{QE} is the energy (MeV) of the incoming pion, E_b is the binding energy of a single nucleon in the target nucleus (4 MeV), E_π is the energy of the outgoing pion (MeV), m_p is the mass of a proton (938.28 MeV/c^2), m_n is the mass of a neutron (939.57 MeV/c^2), m_π is the mass of a pion (139.57 MeV/c^2), $\vec{p}_\pi$ is the momentum of the outgoing pion (MeV/c), and θ is the angle between the incoming pion direction and the outgoing pion direction. The mass of the argon nucleus is 37211.32 MeV/c^2. The formula was adapted from the neutrino scattering quasielastic hypothesis [5]. This quasielastic hypothesis makes the key assumption that each outgoing pion is the direct result of an incoming pion and both pions are linked via quasielastic scattering.

Geant 4

We used simulated data created by Geant4 [6]. More information about the process will be discussed in the Methods section. Geant4 is used to simulate interactions between a variety of particles and targets. Its ease of changing initial conditions make it a popular choice for high energy physicists to model particle propagation in detectors. However, being that it is a simulation, Geant4 needs to be validated with experimental data. The remainder of this paper serves as a study in how effectively Geant4 models quasielastic scattering.

Methods

Geant4 was used to model $\pi+$ interactions with a liquid argon target. The incident particle was given a momentum of 1 GeV/c along the z axis. A thin (1cm) liquid argon target was placed with its center at $\vec{r} = (0, 0, 0)$ where $\vec{r}$ is the three dimensional position vector in the (x, y, z)

plane. The macro is then run through Geant4 using the Bertini cascade model, which produces an output file with the information from the simulation. Geant4 only distinguishes between elastic and inelastic scattering. Only the interactions labeled as inelastic were saved, meaning that when Geant4 stores the inelastic scattering process, many other subprocesses are also saved (including pion absorption, pion charge exchange, and quasielastic scattering. After Geant4 had produced the model of the interaction, the output file was analyzed in ROOT to access the TTree with the saved quantities of each particle [7, 8]. An abbreviated version of the TTree can be found below:

Table 1: An abbreviated version of the TTree output when analyzed in ROOT.

Row	Instance	Event ID	PDGcode
0	0	243	211
0	1	243	1.00e+09
0	2	243	22
0	3	243	2112
0	4	243	2112
0	5	243	2212

Using the information stored in the TTree about the run, the simulated data was then analyzed again using C scripts and ROOT to plot various quantities as shown in the results and discussion section.

Results and Discussion

To determine the likelihood of quasielastic scattering, the first graphs to examine are the counts of each outgoing particle.

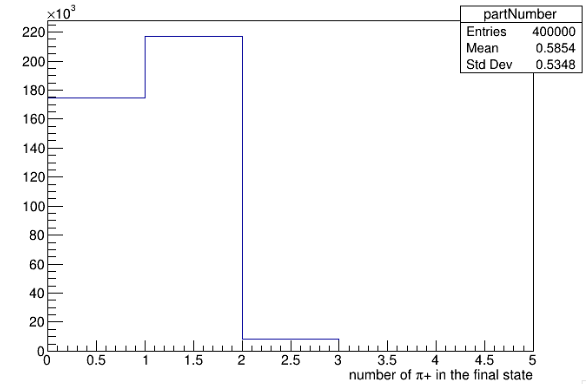

Figure 3: A histogram of the number of $\pi+$ in the final state.

This graph shows that the majority of events stored by Geant4 have only one outgoing pion. While the existence of outgoing pions on their own is not sufficient evidence for quasielastic scattering as pions may be produced independent of the incoming pion, it is one condition that is necessary for quasielastic scattering to occur. Another example of this is shown below:

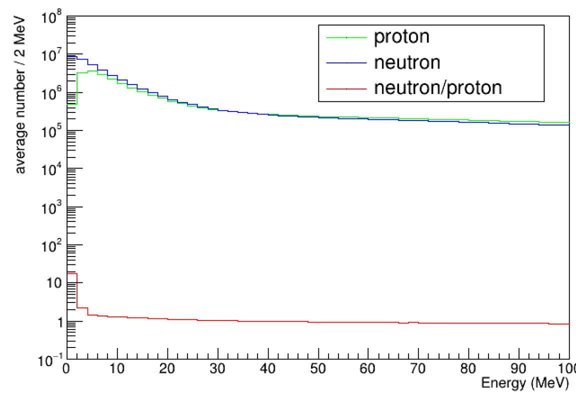

Figure 4: A graph showing the number of outgoing protons and neutrons and the ratio between the two.

This graph shows a peak for protons at approximately 4 MeV, which is the same value as the energy necessary to surpass the Coulomb barrier. Similar to Figure 3, outgoing protons and neutrons may have been produced by means other than quasielastic interactions, but it is necessary to have outgoing nucleons to claim the existence of quasielastic scattering in Geant4.

A feature of quasielastic scattering is a low level of energy loss as compared to other inelastic processes. The energy loss for each outgoing pion can be calculated using the following formula:

$$E_{loss} = E_{incident} - E_{scattered} \quad (5)$$

The following histogram shows the number of particles per 4 MeV and the corresponding energy loss.

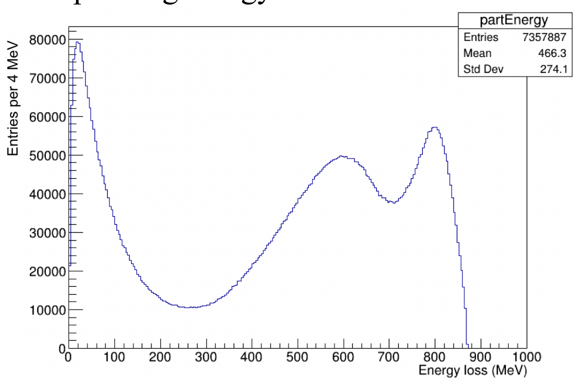

Figure 5: A histogram of the number of particles for a certain energy loss.

As shown in the figure, there is a peak at a very low energy. This is consistent with the low energy peak in figure 4, suggesting that the energy lost by the pions was given to protons or neutrons upon the initial interaction. These similarly energetic peaks suggest that there may be quasielastic scattering taking place in these interactions.

Yet another graph that highlights the possibility of quasielastic scattering is of the outgoing pion momentum vs. angle as compared to similar elastic interactions. The following formulas were used to calculate the outgoing elastic pion momentum as a function of angle [9, 10]:

$$p_c = p\frac{(s+m_a^2-m_b^2)cos(\theta)\pm 2W\sqrt{m_b^2-m_a^2 sin^2(\theta)}}{2(s+p^2 sin^2(\theta))} \quad (6)$$

$$s = m_a^2 + m_b^2 + 2m_b E_{lab} \quad (7)$$

$$W = E_{lab} + m_b \quad (8)$$

where p_c is the momentum of the outgoing pion (MeV/c), p is the incoming pion momentum in the lab frame (MeV/c), m_a is the pion mass (MeV/c^2), m_b is the target particle mass (MeV/c^2), θ is the angle between the z axis and the outgoing pion momentum vector (rad), and E_{lab} is the energy of the incoming pion in the lab frame (MeV).

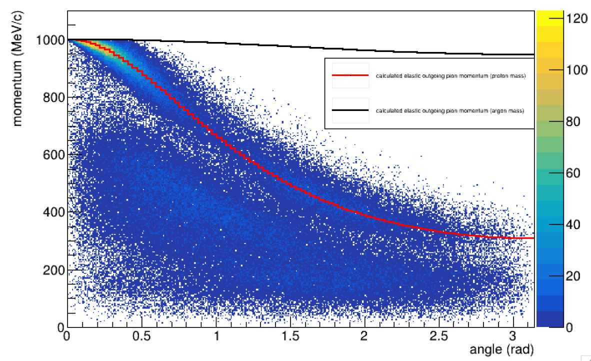

Figure 6: A graph showing the outgoing pion vs. outgoing momentum. The red line shows the outgoing pion momentum as a function of angle for elastic scattering with a proton. The black line shows the outgoing pion momentum as a function of angle for elastic scattering with an argon nucleus.

To create this figure, the number of events was reduced from the total (12546723 events) to 400000 events in order to best portray the banding in the graph. The figure shows two areas of banding. Out of the two, the upper right band is considered the band corresponding to quasielastic scattering while the other events recorded correspond to other inelastic processes. This is because the upper right band is similar in shape to the elastic approximation with a proton. Because the pion only interacts with one nucleon instead of the nucleus during a quasielastic interaction, the elastic approximation with the proton as the target particle is more accurate than the argon nucleus as a target particle.

The final proof of quasielastic scattering is through inverse kinematics. By using formula 4, the calculated incoming pion's energy can be compared to the actual incoming pion energy to determine the accuracy of the quasielastic scattering hypothesis.

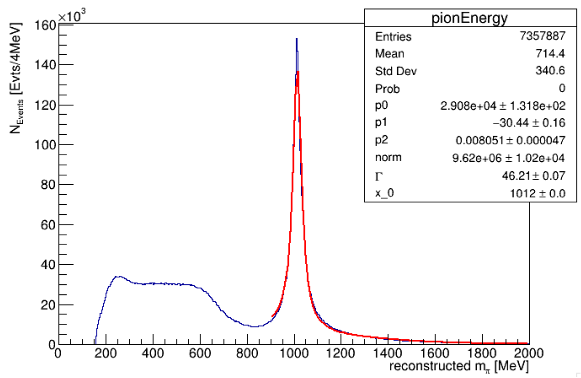

Figure 7: A histogram showing the calculated incoming pion energy. The red line represents a fit to a lorentian representing the signal and a second order polynomial representing the background.

This plot shows a peak at the energy expected for quasielastic scattering on top of the combinatorial background. This background results from the fact that not all interactions shown in the plot stem from quasielastic interactions, but also interactions for which equation 4 is not valid. Because the peak is centered at the expected energy for quasielastic scattering, it should be assumed that there are substantial quasielastic processes. It should be noted that there is various information included in this peak. First, the exact position and the width of the peak can be used to extract other nuclear properties. Furthermore, the number of events can be translated to the total cross section for nuclear quasielastic $\pi+$ liquid argon interactions using the following formula:

$$\sigma = \frac{R}{I} \qquad (9)$$

In this formula, σ is the cross section of the reaction (b), R is the number of reactions per nucleus, and I is the number of incident particles unit area (b^{-1}). The following cross sections for $\pi+$ liquid argon interactions are the result:

Table 2: A table showing the cross sections for various interactions.

Interaction	Cross Section (mb)
Elastic	239
Inelastic	481
Total	720
Quasielastic	<94 (19.5% of inelastic)
Charge exchange	<15.8 (3.3% of inelastic)

To obtain the cross section for the charge exchange interaction, the following graph was used:

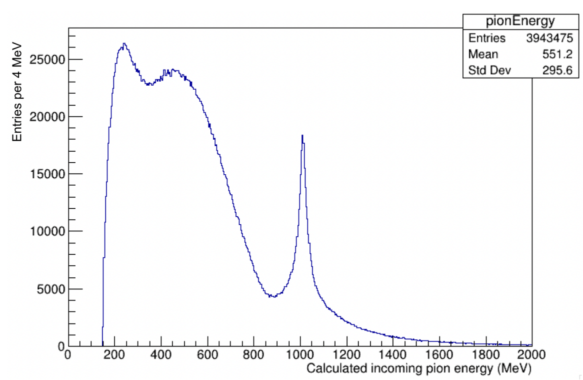

Figure 8: A histogram of the incoming $\pi+$ energy calculated from the outgoing π^0 energy.

Because the incoming pion has a charge of +1 and the outgoing pion is neutral, there must be a charge exchange happening when there is an outgoing pion. To calculate the charge exchange cross section, the number of events under the peak were calculated and compared to the total number of events in accordance with equation 9.

Conclusion

This study sought to examine the extent to which Geant4 (specifically the Bertini cascade model) models quasi elastic scattering. This was done by using the data from Geant4 to create several graphs that highlight significant conditions necessary for quasielastic scattering. The quasielastic cross section was then calculated to be <94 mb.

Despite Geant4's relative ability to model quasielastic scattering, there are still more improvements to be made. The first step is to validate Geant4 simulations with experimental data for different targets and incoming particles. Another method of improvement would be to compare with other models inherent in Geant4. Additional studies compare the results obtained with the Bertini cascade model to different models like the binary cascade (BIC) or intranuclear cascade lyon (INCL++).

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