Housing Rent, Inelastic Housing Supply, and International Business Cycles*

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Abstract

Despite its distinctive features—such as large expenditure shares and inelastic supply—housing service has received scant attention in the international macroeconomics literature. To fill this gap, I examine the role of housing in international business cycles for eurozone countries. I show that housing rents exhibit larger variations than the prices of tradable and other nontradable, both in cross-country and time series. In addition, housing rents account for more than half of the Balassa-Samuelson effect and the negative Backus-Smith correlation. By simulating eurozone economies using a two-country model with a realistically calibrated housing sector and incomplete market, I show that the cross-country distribution of sectoral productivities, inelastic housing supply, and its interaction with the wealth effect via incomplete markets are key to understanding the empirical moments of real exchange rates. Compared with the standard model, the model with the housing sector generates larger variations of the real exchange rate, a stronger Balassa-Samuelson effect, and more realistic Backus-Smith correlations.

Keywords: Real Exchange Rates, Balassa- Samuelson Effect, Backus-Smith Puzzle, Housing Market, Housing Rents, International Business Cycles
JEL Classification: F41, F44, R31

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1 Introduction

Understanding international business cycles is of paramount interest to many economists, both for academic inquiry and policy recommendations. The real exchange rate, defined as the relative price levels across countries, is a crucial general equilibrium object in numerous international macroeconomic models. It significantly influences fundamental model mechanisms, including international risk sharing and trade, underscoring its importance. This suggests that a comprehensive understanding of the real exchange rate is a prerequisite for proper policy recommendations and legitimate academic studies. Unfortunately, our grasp of real exchange rates remains limited, as evidenced by the various international macroeconomic puzzles described by Itskhoki (2021).

One of the potential causes for the limited understanding of real exchange rates might stem from the abstraction of housing. Most international macroeconomic models neglect housing, assuming that a housing service is the same as other nontradable services. However, housing deserves separate attention. As one of the most important components of household consumption, housing service can significantly impact the aggregate economy. According to a recent EU Statistics of Income and Living Conditions Survey (EU-SILC), households in European countries allocate approximately 20% to 23% of their disposable income to housing rents. Moreover, housing service differs markedly from other goods and services. Its supply is highly inelastic due to long construction periods and heavy reliance on land as the primary input for construction, coupled with limited land availability, factors often exacerbated by urbanization and stringent land-use regulations. Ignoring its economic significance and unique characteristics can lead to a limited understanding of international business cycles.

This paper addresses this gap by focusing specifically on housing services and distinguishing them from other nontradable services. In particular, I investigate the role of housing rent in three empirical aspects of real exchange rates: cross-sectional and time-series variations, the Balassa-Samuelson effect, and the Backus-Smith correlation. To conduct this analysis, I use the Eurostat-OECD Purchasing Power Parity database, which contains detailed relative price information. This database covers 224 items and represents an entire consumption basket. Notably, it includes data on pure housing rent, excluding maintenance fees and utility costs. Furthermore, it not only tracks relative price changes but also provides relative price levels. By categorizing goods and services as tradable items, nontradable items, and housing services, I effectively decompose the aggregate real exchange rate into three components: tradable real exchange rate, nontradable real exchange rate, and rent real exchange rate. As a result, the aggregate real exchange rate becomes an expenditure-weighted sum of these three components; this creates an ideal environment for examining the role of housing rent in the dynamics of real exchange rates.

I focus on eurozone countries in which the nominal exchange rates among countries are set at one, which eliminates the influence of nominal exchange rates on real exchange rates. It is widely recognized that nominal exchange rates can be affected by monetary policies, financial shocks, and potentially nonfundamental shocks. If the real exchange rates were primarily driven by nominal exchange rates, this could complicate examination of the connection between the housing sector and real exchange rates. In addition, my goal is to investigate the real supply and demand aspects of the housing service, rather than delving into housing-related financial market features such as mortgages. These render the eurozone area an ideal environment for studying the role of housing on real exchange rates. This strategy aligns with the approach used by Berka et al. (2018), which has proven to be fruitful.

Descriptive statistics analysis reveals that the rent real exchange rate exhibits larger variations and persistence in both cross-section and time-series than those of tradables and nontradables. Furthermore, by conducting a variance decomposition of the aggregate real exchange rate, I find that the rent real exchange rate contributes to 33% of the aggregate real exchange rate variation across different countries (cross-sectional variation) and accounts for a range of 0%-60% of the aggregate real exchange rate variation over time (time-series variation), with the specific percentage varying by country. An intriguing observation is that in the time-series dimension, the rent real exchange rate displays very large fluctuations in countries significantly affected by demand shocks, such as Greece and Ireland. This implies that the inelastic nature of housing supply may be more important than previously thought.

Furthermore, I augment this panel data on sectoral real exchange rates with data on relative real GDP per capita to investigate the Balassa-Samuelson effect. This effect is the empirical regularity with which countries with higher real GDP per capita tend to exhibit higher price levels, which is well documented by Rogoff (1996). The Balassa-Samuelson hypothesis is the most well-known theory to explain such a phenomenon. It posits that higher relative productivity in the tradable sector pushes up production factor prices, which pushes up nontradable prices and the overall price level. Although this is considered to primarily be applicable between developed and developing countries in the long run, recent research by Berka et al. (2018) indicates that it also holds among eurozone countries in the short run. I extend their work by specifically examining the role of housing rent. My panel regression analysis reveals that a 1% higher real GDP per capita than the eurozone average corresponds to a 0.25% higher price level than the eurozone average. In addition, I further dissect the contribution of each sector's real exchange rate to this aggregate effect. Remarkably, even when accounting for the 16% expenditure weight associated with rent, 0.122% of the 0.25% total relative price increase can be attributed to the relative rent increase. This constitutes nearly half of the overall effect. Notably, this is due to the fact that a 1% higher relative real GDP per capita translates to a 0.76% higher relative rent level. These findings underscore the significance of housing services in shaping the Balassa-Samuelson effect.

I also incorporate relative real consumption data in my dataset to examine the Backus-Smith correlation, which is the time-series link between the growth of the real exchange rate and real relative consumption. Typically, empirical data reveal this correlation to be close to zero or even negative. This suggests that countries' consumption increases more than the foreign when their price levels increase more than the foreign, which implies the deviation from the perfect risk sharing. Contrary to this, Backus and Smith (1993) demonstrated that a standard two-country model with a complete market predicts this correlation to be 1. This became a significant puzzle when attempts to modify the model's prediction closer to data proved difficult, even under the assumption of an incomplete market. Several promising solutions have been proposed—yet no previous research has considered the impact of the rent real exchange rate. However, the rent real exchange rate turns out to be very important. My panel regression analysis for eurozone countries indicates that when a country's consumption grows 1% more than the eurozone average, its real exchange rate appreciates by 0.14%, which implies a negative Backus-Smith correlation.[1] Remarkably, 0.126% of this 0.14% appreciation stems from the rent real exchange rate. This is striking, because it is the number we get after we account for the relatively low expenditure weight of housing rent compared with that of tradables or nontradables. This again highlights the pivotal role of housing rent.

Motivated by these observations, I develop a two-country model that incorporates a realistically calibrated housing sector by combining two models from Berka et al. (2018) and Davis and Heathcote (2005). Also, I assume an incomplete market between countries to examine the role of the wealth effect studied by Corsetti et al. (2008), since an inelastic supply of housing naturally implies the importance of a demand shock via the wealth effect. As Itskhoki (2021) underscores, real exchange rates are shaped through general equilibrium forces. This requires examination of real exchange rates from a general equilibrium viewpoint. To achieve this, I simulate my model using sectoral productivity shocks—namely, those in the tradable, non-tradable, and construction sectors—directly calibrated from the EUKLEMS database. By simulating under varied calibrations of the housing market, I delve into the role housing plays in the dynamics of real exchange rates.

Simulations of my model reveal its capability to generate significant variations in the rent real exchange rate. To generate the substantial time-series variations, the incomplete market assumption turns out to be crucial. The inelastic supply of housing services—attributed to land being a primary input for construction and the minor flow of new housing relative to existing housing stock—reduces the volatility of the rent real exchange rate that arises from

[1] Devereux and Hnatkovska (2020) argue that following the introduction of the euro, the average Backus-Smith correlations across eurozone countries turned positive, increasing from -0.19 to 0.05, compared to the pre-euro period. They also found that eight out of the twelve eurozone countries exhibited positive Backus-Smith correlations after the introduction of the euro by using data from 2000 to 2007. However, in my dataset, which spans from 2000 to 2019, nine out of twelve countries show negative Backus-Smith correlations, with an average of -0.06. This is still negative and near zero, being starkly different from 1. This suggests that while the nominal exchange rate plays a significant role, there are other mechanisms at work that contribute to the negative Backus-Smith correlation.

the productivity shock under a complete market. However, it amplifies the effect of relative demand changes from the wealth effect of Corsetti et al. (2008) due to its inelastic supply. Consequently, the model generates substantial rent real exchange rate volatility similar to what we observe in the data, only under an incomplete market.

Model simulations also provide valuable insights for the Balassa-Samuelson effect. Simulations show that housing's heavy reliance on land as the primary input actually dampens the textbook Balassa-Samuleson hypothesis mechanism because land is not used in the tradable sector. However, the model still generates the strong Balassa-Samuelson effect via housing rents as in the data, and it comes from the cross-country distribution of sectoral productivities. The sectoral productivity levels of eurozone countries, directly calibrated from the EUKLEMS database, reveal a pattern in which countries with highly productive tradable sectors also tend to exhibit high productivity in the nontradable sector, yet they demonstrate relatively lower productivity levels in the construction sector. This amplifies the textbook Balassa Samuelson hypothesis mechanism via the rent real exchange rate. This observation aligns with the recent findings of stagnant productivity in the construction sector documented by Goolsbee and Syverson (2023). On the top of these findings, the wealth effect from an incomplete market amplifies the model-generated Balassa-Samuelson effect across all sectors, with outcomes that surpass empirical estimates.

Lastly, incorporating a housing sector improves my model's ability to address the Backus-Smith puzzle. Although the standard model cannot, my model accurately replicates the panel regression results in the data and generates most of the negative Backus-Smith correlation through the rent real exchange rate component. This is primarily due to inelastic housing supply. It renders the aggregate supply more inelastic, which causes the aggregate price level to be more responsive to relative demand shifts. Consequently, price levels rise more when relative consumption increases via demand shifts from wealth effect, which generates a stronger negative Backus-Smith correlation. Furthermore, the now more inelastic aggregate supply diminishes the impact of nontradable sector and construction sector productivity shocks, which typically act as potent supply shocks and generate a positive Backus-Smith correlation.

This paper builds on a large literature on the secular movements of the real exchange rate. The relative price of the nontradable sector was initially thought to be important in aggregate real exchange rate dynamics. However, Engel (1999) documented that the bulk of US real exchange rate variation comes from relative prices of the tradable sector, under a floating exchange rate regime. Several studies suggest that differences in the consumer prices of traded goods across countries are due to the distribution margin (e.g., Burstein et al. 2003, Burstein et al. 2005, Betts and Kehoe 2006). On the other hand, other studies analyze firms' pricing behaviors based on variable markups (e.g., Atkeson and Burstein (2008), Gopinath and Itskhoki (2010)). On the top of all these, Mussa (1986) documented large real exchange rate volatility under a floating exchange rate compared with that under a fixed exchange rate regime. My paper contributes to the literature by examining the importance of housing rent in real exchange rate movements under a fixed exchange rate regime.

My paper also intersects with the extensive literature on the Balassa-Samuelson effect. A foundational prediction of the Balassa-Samuelson hypothesis is that countries with higher real GDP per capita, which is employed as a proxy for tradable sector productivity, should exhibit more appreciated real exchange rates. Rogoff (1996) validated this by demonstrating a pronounced Balassa-Samuelson effect in a cross-sectional analysis of 1990 data. Bordo et al. (2017) identified a long-run correlation between relative income and real exchange rates across a panel of 14 countries in relation to the US. Several other studies have directly probed the nexus between real exchange rates and sectoral productivities rather than GDP per capita and resulted in a spectrum of outcomes (e.g., Lee and Tang 2007, Choudhri and Schembri 2014, Gubler and Sax 2019). In a recent contribution to the literature, Berka et al. (2018) examine eurozone countries' real exchange rates and sectoral productivity. Their findings suggest that the Balassa-Samuelson effect permeates the eurozone—even in the short run and within a time-series framework—when factoring in the labor wedge. I extend their work by distinctly focusing on housing rent and show that housing contributes to over half of the entire Balassa-Samuelson effect. By simulating the model with a realistically calibrated housing sector, I show that the cross-country distribution of sectoral productivities and the wealth effect are the key forces generating the Balassa-Samuelson effect in eurozone countries.

Lastly, this paper builds on literature on the Backus-Smith puzzle. While the correlation between real exchange rates and relative consumption is negative or near zero in the data, Backus and Smith (1993) found that a standard two-country model with a complete market predicts this correlation will be 1. Because this model prediction largely depends on the complete market assumption, Chari et al. (2002) constructed a two-country model with an incomplete market under monetary policy shocks. However, they again generated a correlation closer to 1. Later, Corsetti et al. (2008) generated a negative correlation under an incomplete market by assuming either very persistent productivity shocks or very low substitutabiltiy between home and foreign tradable goods. Similarly, Benigno and Thoenissen (2008) generated this under an incomplete market and emphasize the role of the Balassa-Samuelson mechanism. Other papers use home production (e.g., Karabarbounis 2014) or non-rational expectations (e.g., Lambrias (2020)) to resolve the Backus-Smith puzzle. Devereux and Hnatkovska (2020) point out that the role of the nominal exchange rate is important for a negative Backus-Smith correlation, and the most recent development in the literature is that of Itskhoki and Mukhin (2021), who resolve many international macroeconomic puzzles via financial frictions. My paper contributes to this literature by inspecting the role of housing rents in the Backus-Smith puzzle. While the importance of nominal exchange rate is unquestionable as proven by the huge change in the Backus-Smith correlation between pre-euro period and after-euro period (Devereux and Hnatkovska 2020), my paper shows that there are still negative, though dampened, Backus-Smith correlation remained among eurozone countries. Moreover, I show that, among all relative prices, rent real exchange rate is the one that contributes most. Lastly, I theoretically contribute to this literature by showing how a realistically calibrated housing sector can help the standard model produce improved predictions for the Backus-Smith puzzle.

The paper is structured as follows. Section 2 details the data sources and describes how I construct sectoral real exchange rates. I also state the basic properties of sectoral real exchange rates and conduct panel regression analyses to identify the role of the rent real exchange rate in both the Balassa-Samuelson effect and the Backus-Smith puzzle. In Section 3, I outline the model that incorporates a housing sector. Section 4 presents the calibration of the model and sectoral productivity shock processes. I also report the simulation analysis result to elucidate the role of the housing sector in real exchange rate dynamics. Finally, in Section 5, I offer conclusions and propose extensions and questions for future research.

2 Data: Housing Rents and Real Exchange Rates

2.1 Real Exchange Rates in Eurozone Countries

Data Source and Coverage I construct the aggregate and sectoral real exchange rates of eurozone countries using the Eurostat-OECD Purchasing Power Parity (Eurostat PPP) database, which contains the cross-country relative price levels of 224 items and covers a whole consumption basket of European countries. These include all types of goods and services, such as food, clothing, transportation, education, and health care services. Most importantly, they provide the relative prices of *Actual Rentals for Housing* and *Imputed Rentals for Housing*. These two relative prices are for housing rents that do not include any other costs, such as maintenance fees or utility costs. This enables cleaner identification of housing service prices. The full list of goods and services in the database is in the appendix.

Reporting frequency is annual. I use data from 2000 to 2019 to examine the period after introduction of the euro and before the COVID-19 outbreak. In addition, I only use data on the 12 countries that introduced the euro in 2000: Austria, Belgium, Germany, Greece, Spain, Finland, France, Ireland, Italy, Luxembourg, the Netherlands, and Portugal. This is because fixed exchange rate countries provide a better identification environment for studying the role of housing rent in the dynamics of real exchange rates by eliminating the noise from nominal exchange rates.

Superiority of the Eurostat PPP Database Before jumping into the actual data, it is worth emphasizing the quality of the data I use. As is well explained by Berka et al. (2018), the Eurostat PPP database offers a number of advantages compared with the datasets used in other research. First, to construct the Eurostat PPP database, each country conducts a national survey that covers all items in their consumption baskets. This implies that the database covers the overall price levels of the economy. Compared with research that uses price data from a single supermarket chain (Gopinath et al. 2011), from a single international retailer of household goods (Haskel and Wolf 2001, Baxter and Landry 2017), or from a few online retailers (Cavallo et al. 2014), the Eurostat PPP database offers better coverage. Second, the Eurostat PPP database guarantees better cross-country comparability. For example, though some research has used price data that cover a comprehensive set of items, such as the Economist Intelligence Unit survey (Engel and Rogers 2004, Crucini and Shintani 2008), such data lack the validity of cross-country comparability. In contrast, the Eurostat PPP database is organized under a single entity, Eurostat, which guarantees more homogeneous data collection procedures across countries (e.g., the selection of items and outlets where prices are measured). In addition, the Eurostat PPP database undergoes an internal review process every year to check the comparability and completeness of the dataset. The fact that I use only eurozone countries itself should also increase cross-country comparability, since they share similar spending patterns and cultural and legal backgrounds.

In particular, housing rent data in the Eurostat PPP database offers the most credible cross-country comparability. The housing rent level is notoriously difficult to measure for cross-country comparison. To overcome this, every year Eurostat asks all member countries to derive rent-level data based on their internal rent survey. Most reporting countries use rent survey data for their national account construction, which demonstrates how precise and detailed the data are. In addition, Eurostat provides members with detailed instructions on how to compute the rent price level.[2] Lastly, they conduct an annual internal review to examine the cross-comparability and consistency of the dataset. All of these procedures ensure that the cross-comparability of this dataset is superior.

Construction of Real Exchange Rates This database provides the price level index (PLI) for 224 items that cover a whole consumption basket. A PLI (p_{ijt}) for item i and country j is defined as the log relative price of item i in country j relative to that of the EU 15 average (geometric average).

$$p_{ijt} = log(\frac{P_{iEU15t}}{P_{ijt}}) \quad \text{where} \quad P_{iEU15t} = \prod_{k \in EU} P_{ikt}^{\frac{1}{15}}.$$

For example, if the croissant price is 1.2 euros in France but its EU 15 average price is 1 euro, the PLI of croissants in France is given as $0.079 = log(\frac{1.2}{1.0})$. Note that this contains information

[2] While collecting rent data, the quality of houses is also taken into account. They are classified by the number of rooms, type of house (apartment, single house, etc.), and features (central heating system, etc.) to derive a quality-based quantity index. For more information, refer to the OECD and Eurostat (2012)

not only on the relative growth rate of the prices but also the relative levels of the prices. For every item in the consumption basket, I can observe how expensive that item is in a certain country compared with the EU 15 average.

The database also contains the expenditure weight of each item for each country. As is usual, the expenditure weight (γ_{ijt}) for item i and country j is defined as follows:

$$\gamma_{ijt} = \frac{EXP_{ijt}}{\sum_{i=1}^{224} EXP_{ijt}}, \quad \sum_{i=1}^{224} \gamma_{ijt} = 1.$$

By using these PLIs and expenditure weights, I construct aggregate real exchange rates as follows. Note that since PLI is defined as the price level compared with the EU 15 average, this real exchange rate is between country j and the EU 15 average:

$$q_{j,t} = \sum_{i=1}^{224} \gamma_{ijt} p_{ijt} = log(\frac{\prod_{i=1}^{224} P_{iEU15t}^{\gamma_{ijt}}}{\prod_{i=1}^{224} P_{ijt}^{\gamma_{ijt}}}).$$

In this definition, the real exchange rate is effectively the expenditure-weighted geometric average of the relative prices of goods and services. Intuitively, this implies the relative overall price level of country j compared with that of the EU 15 average. Going one step further, I can decompose this aggregate real exchange rate into sectoral real exchange rates as follows:

$$q_{jt}^{T} = \frac{\sum_{i \in T} \gamma_{ijt} p_{ijt}}{\sum_{i \in T} \gamma_{ijt}} \quad \text{(159 Items)}, \quad (\sum_{i \in T} \gamma_{ijt} = \gamma_j^T),$$
$$q_{jt}^{NT} = \frac{\sum_{i \in NT} \gamma_{ijt} p_{ijt}}{\sum_{i \in NT} \gamma_{ijt}} \quad \text{(63 Items)}, \quad (\sum_{i \in NT} \gamma_{ijt} = \gamma_j^{NT}),$$
$$q_{jt}^{R} = \frac{\sum_{i \in H} \gamma_{ijt} p_{ijt}}{\sum_{i \in H} \gamma_{ijt}} \quad \text{(2 Items)}, \quad (\sum_{i \in H} \gamma_{ijt} = \gamma_j^R = 1 - \gamma_j^T - \gamma_j^{NT}).$$

In essence, these are again the expenditure-weighted geometric averages of the prices of a certain group of goods and services. It shows that I classify 159 items as tradable, 63 items as nontradable, and 2 items as housing rents. For this classification, I closely follow the approach of Berka et al. (2018).[3] Two housing-service related items are *Actual Rentals for Housing* and *Imputed Rentals for Housing*. By construction, I arrive at the following decomposition of the aggregate real exchange rate:

$$q_{jt} = \gamma_j^T q_{jt}^T + \gamma_j^{NT} q_{jt}^{NT} + \gamma_j^R q_{jt}^R,$$

where $\gamma_j^R = 1 - \gamma_j^T - \gamma_j^{NT}$. Note that $q_{jt} < 0$ implies that country j's overall price level is higher than the EU 15 average, and $\Delta q_{jt} < 0$ implies the appreciation of country j's real

[3]The classification procedure is detailed in the appendix.

exchange rate.

Properties of Real Exchange Rates and Expenditure Weights I start with the descriptive statistics in Table 1. The upper panel of the table provides information on each country and the lower panel provides information on the average of each country's statistics. Together, they provide information on the overall characteristics of real exchange rates in the eurozone.

The first table in the upper panel shows how each country's sectoral price level compares with the EU 15 average. Ireland is in the first row, with a 13.2% higher price level than the EU 15 average, and Portugal is in the last row, with a 24.4% lower price level than the EU 15 average. An interesting observation is that countries with a higher overall price level have higher prices for nontradable services and also more expensive rent.

To understand the overall characteristics of sectoral real exchange rates, see the lower panel of Table 1. The first important observation is the large volatility of the rent real exchange rate (q^R) in both the cross-section and time series. As is well known, housing service supply is inelastic, and thus any demand shocks likely generate a large price response rather than a quantity response. In addition, because q^R is a component of q, the large volatility of q^R implies that it should significantly contribute to the overall volatility of q. This huge volatility of q^R compared with q^T and q^{NT} is still the case, even though I compute the standard deviations of the differences and compare Δq^R with Δq^T or Δq^{NT}. Lastly, q^R is also the most persistent compared with other sectoral productivities. Because of the slow response of supply to any shock, price changes via certain shocks are likely to last longer unless the shock itself is temporary.

Figure 1 shows movements of the sectoral real exchange rates of all eurozone countries. I set the ranges of the y-axes in all graphs the same to facilitate comparison across sectors. An interesting pattern is that the variation of q^T is very small in both the cross-section and time series, even compared with other nontradable items. In addition, it even shows the sign of the convergence of price levels as the year progresses. Most importantly, we can clearly see the large variation of q^R in both the cross-section and time series, as captured in Table 1. These volatile dynamics of q^R are most prominent in countries that experienced significant demand shocks during the eurozone crisis (e.g., Portugal, Ireland, and Greece).

Figure 2 shows the dynamics of the sectoral expenditure weights of all countries. Again, I set the ranges of the y-axes in all graphs the same to facilitate comparison across sectors (except for the total expenditure weight, which is by definition 1.) A notable pattern is that the expenditure weights of tradables are decreasing, while the expenditure weights for nontradables and housing rents are increasing. Also, note that rent expenditure weights are roughly in the range of 15%-20%, which is a substantial weight for only two items. Though there is cross-country heterogeneity in expenditure weights, overall heterogeneity is not that signifi-

Table 1: Descriptive Statistics of Real Exchange Rates

	Mean				*Standard deviation*				*Autocorrelation(1)*			
Country	$\bar{q}$	$\bar{q^T}$	$\bar{q^{NT}}$	$\bar{q^R}$	$std(q)$	$std(q^T)$	$std(q^{NT})$	$std(q^R)$	$\rho(q)$	$\rho(q^T)$	$\rho(q^{NT})$	$\rho(q^R)$
Ireland	-0.132	-0.102	-0.140	-0.187	0.034	0.021	0.035	0.128	0.737	0.500	0.731	0.866
Finland	-0.124	-0.093	-0.138	-0.187	0.021	0.028	0.034	0.022	0.823	0.919	0.725	0.681
Luxembourg	-0.047	0.080	-0.059	-0.425	0.040	0.015	0.087	0.039	0.954	0.692	0.965	0.564
France	0.002	0.023	0.002	-0.057	0.014	0.027	0.034	0.030	0.536	0.813	0.801	0.888
Belgium	0.005	0.006	0.006	-0.003	0.012	0.019	0.017	0.028	0.677	0.736	0.774	0.899
Netherlands	0.010	0.027	0.010	-0.038	0.026	0.015	0.035	0.055	0.866	0.585	0.770	0.954
Austria	0.028	0.017	-0.047	0.273	0.015	0.014	0.018	0.053	0.715	0.690	0.732	0.920
Germany	0.030	0.033	0.029	0.020	0.023	0.015	0.028	0.068	0.912	0.644	0.885	0.979
Italy	0.068	0.008	0.100	0.222	0.018	0.018	0.021	0.049	0.693	0.723	0.416	0.682
Spain	0.162	0.147	0.176	0.172	0.032	0.025	0.047	0.070	0.858	0.877	0.814	0.869
Greece	0.211	0.134	0.254	0.364	0.050	0.041	0.062	0.200	0.863	0.916	0.839	0.944
Portugal	0.244	0.118	0.313	0.641	0.016	0.022	0.045	0.121	0.530	0.607	0.768	0.965

Aggregate	$std(mean_i)$	$mean(std_i)$	$mean(autocorr_i)$
q	0.119	0.025	0.764
q^T	0.079	0.022	0.725
q^{NT}	0.144	0.039	0.768
q^R	0.286	0.072	0.851

$q_{it} = ln(P_{EU15t}/P_{it})$, $q^T_{it} = ln(P^T_{EU15t}/P^T_{it})$, $q^{NT}_{it} = ln(P^{NT}_{EU15t}/P^{NT}_{it})$, $q^R_{it} = ln(P^R_{EU15t}/P^R_{it})$, where P_{EU15t} is the aggregate price level of 15 European countries. The data period is from 2000 to 2019 and data are at annual frequency. Countries in the upper panel are sorted in the order of price levels.

cant. In addition, the relative size of each sector's expenditure weight is roughly same across countries, which implies that we do not need to be particularly concerned, and average expenditure weights across time can be used without affecting the data more than slightly.

Variance Decomposition While the q^R shows very large cross-country and time-series variations, it also has the lowest expenditure weight compared with tradables and nontradables. Thus, it might be the case that in the end, relative rents do not affect the dynamics of the aggregate real exchange rate much. To get a sense of the quantitative importance of the relative rents, I conduct the following variance decomposition. Any variance of $Var(q)$ can be decomposed as follows:

$$Var(q) = Cov(q, \gamma^T q^T + \gamma^{NT} q^{NT} + \gamma^R q^R) = \gamma^T Cov(q, q^T) + \gamma^{NT} Cov(q, q^{NT}) + \gamma^R Cov(q, q^R).$$

By dividing both sides by $Var(q)$, I arrive at the following:

$$1 = \gamma^T Corr(q, q^T)\frac{std(q^T)}{std(q)} + \gamma^{NT} Corr(q, q^{NT})\frac{std(q^{NT})}{std(q)} + \underbrace{\gamma^R Corr(q, q^R)\frac{std(q^R)}{std(q)}}_{\text{Share of } q^R \text{ in } RER \text{ Variation}}.$$

According to this decomposition, the contribution of the rent real exchange rate can be calculated as $\gamma^R Corr(q, q^R)\frac{std(q^R)}{std(q)}$. Note that this measure also takes the expenditure weight

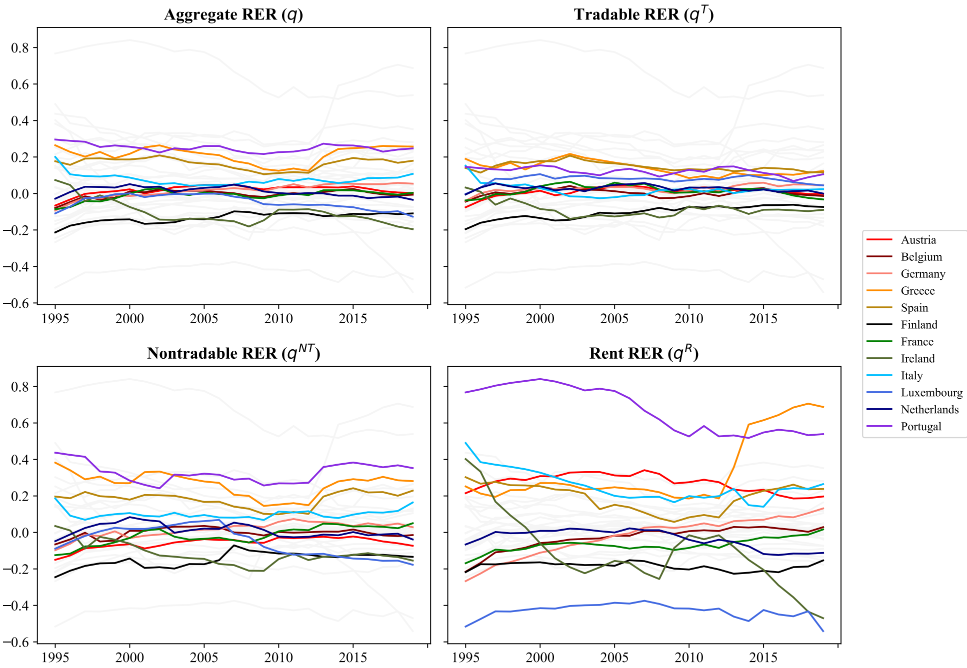

Figure 1: Properties of Real Exchange Rates

into account. I apply this decomposition to both cross-section and time-series variation. For the cross-section, I decompose the cross-country variation of the average aggregate real exchange rate, $Var(\bar{q})$. This exercise demonstrates how important housing rent is in accounting for price-level differences across countries. The left panel in Figure 3 shows the result. We can see that the rent real exchange rate accounts for 33% of the total variations. Given the 16% expenditure weight, it's very substantial number.

On the top of it, for time-series variation, I apply this decomposition to each country j's time-series variation, $Var(q_{jt})$. This exercise shows how rent real exchange rate affects the time-series variation of total real exchange rate in each country. The right panel in Figure 3 shows the results. Though there are countries in which the contributions of the real exchange rates are lower than their expenditure shares, we see that there are countries in which more than half of the total variation comes from rent real exchange rates. Again, given the 15%-20% expenditure weight of housing rents, this shows the significant role of rent real exchange rates. In particular, countries that experienced large demand shocks but have no rent control exhibit larger roles for q^R.

This is in contrast to the findings of Engel (1999), whereby the relative price of the tradable is the one that drives the time-series standard deviation of the real exchange rate. This is probably because Engel (1999) examined the floating exchange rate regime and I examine the fixed exchange rate regime. However, it still shows that nontradable services and housing

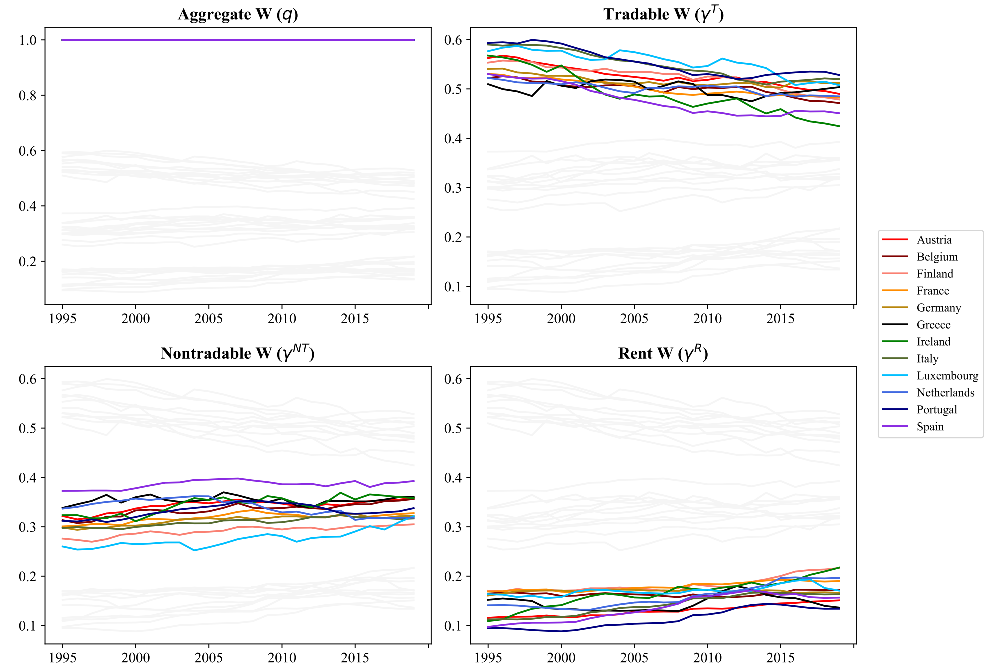

Figure 2: Properties of Expenditure Weights

rent deserve more attention in the real exchange rate literature. In addition, the rent expenditure weights provided by the Eurostat PPP database much lower than what we see in the household survey.[4] This implies that any empirical implications of q^R I find with the Eurostat PPP database can be considered to be a lower bound.

2.2 Housing Rent, the Balassa-Samuelson Effect, and the Backus-Smith Puzzle

Now I combine my panel dataset of sectoral real exchange rates with those of real GDP per capita and real consumption to explore the Balassa-Samuelson effect and the Backus-Smith correlation. Both datasets are from the national account of each country and are in real terms to measure volume changes.[5]

To render these variables consistent with my definition of real exchange rates, I define

[4]For European countries, households' actual expenditure weights on rents are much larger than Eurostat PPP data imply if I use EU-SILC household survey data. The rent expenditure weight is higher for poorer households. I provide graphical descriptions of households' expenditure weights on rents in the appendix.

[5]For real GDP per capita, I use *real GDP per capita in PPP-adjusted EU15* and, for real relative consumption, I use *real final consumption expenditure of households, chain-linked volumes (2010), million euro.* Some prior research has used per capita consumption for the Backus-Smith correlation, and the results do not change much when I use per capita consumption. Tables based on per capita consumption are provided in the appendix.

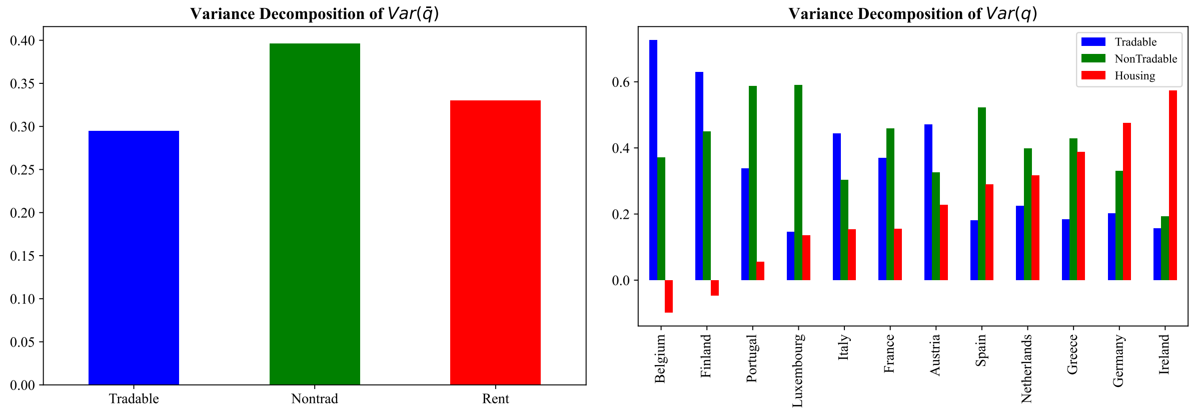

Figure 3: Variance Decompositions of RER

relative real GDP per capita (y) and relative real consumption (c) as follows:

$$y_{jt} = log(Y_{jt}/Y_{EU12t}),$$
$$c_{jt} = log(C_{jt}/C_{EU12t}).$$

Note that Y_{EU12t} and C_{EU12t} are the geometric averages of 12 eurozone countries' real GDP per capita and real aggregate consumption. Δy_{jt} (Δc_{jt}) represents the relative growth rate of real GDP per capita (real aggregate consumption) relative to that of the EU 12 average. Unlike the case of real exchange rates, $\Delta y_{jt} > 0$ ($\Delta c_{jt} > 0$) implies that country j's real GDP per capita (real aggregate consumption) is growing faster than the average of the 12 eurozone countries.

Cross-sectional Variation VS Time-series Variation To examine the Balassa-Samuelson effect, Rogoff (1996) used the cross-section data of countries in 1990, exploiting cross-sectional-level variation. On the other hand, Corsetti et al. (2008) calculated each country's time-series correlation between its real exchange rate growth and relative consumption growth, effectively exploiting the time-series dimension of the data. While these two moments concern variations in different dimensions, what I have is panel data that capture both cross-section and time-series variations. Running a pooled OLS might prevent me from observing the empirical patterns of interest.

To avoid such problems, I run the following four regressions:

$$\bar{q}_j = \beta \bar{x}_t + \eta_t \qquad \text{(Country Average)}, \tag{1}$$
$$q_{jt} = \alpha_t + \beta x_{jt} + \epsilon_{jt} \qquad \text{(Time Fixed Effect)}, \tag{2}$$
$$\Delta q_{jt} = \Delta \beta x_{jt} + \epsilon_{jt} \qquad \text{(Growth Rate)}, \tag{3}$$
$$q_{jt} = \alpha_j + \beta x_{jt} + \epsilon_{jt} \qquad \text{(Country Fixed Effect)}. \tag{4}$$

The regressions in Equation (1) and (2) capture cross-sectional-level variations. By averaging out across time in each country or by using the time-fixed effect, I remove time-series variations and capture only cross-country variations. The Balassa-Samuelson effect is expected to appear in these two regressions and the Backus-Smith correlation is expected not to appear.

On the other hand, the regressions in Equation (3) and (4) capture time-series variations. By using it in growth rate form or by using the country-fixed effect, I remove the cross-country level variations.[6] The Balassa Samuelson effect may appear here as Berka et al. (2018) finds out, and the Backus-Smith correlation is expected to appear for these regressions.

Note that fixed effects are not used to remove any potential endogeneity stemming from unobserved heterogeneity. I am primarily interested in the correlation relationship, and these fixed effects are used to capture the variations of interest in different dimensions. In addition, using these regressions, I can check the significance of β under either time- or country-clustered standard deviations, exploiting all data more systematically than calculating correlation for each country. I will apply all of these regressions to both the Balassa Samuelson effect and the Backus-Smith correlations, and see from which variations those relationships emerge.

Regression-based Decomposition Before directly jumping into the actual regression, I explain how I capture the role of rent real exchange rate for any empirical patterns. In fact, regression analysis provides very intuitive decomposition of the relationships. For example, if I am interested in the relationship between a variable of interest (x) and the aggregate real exchange rate (q), I can perform the following regression analysis:[7]

$$q_{jt} = \alpha + \beta x_{jt} + \epsilon_{jt}.$$

In this regression, β, which is $\frac{Cov(q,x)}{Var(x)}$, summarizes the relationship between q and x. Also, to examine the relationship between x and each sector's real exchange rate (q^T, q^{NT}, q^R), I can perform the following regression analysis:

$$\begin{aligned} q^T_{jt} &= \alpha + \beta^T x_{jt} + \epsilon_{jt}, \\ q^{NT}_{jt} &= \alpha + \beta^{NT} x_{jt} x + \epsilon_{jt}, \\ q^R_{jt} &= \alpha + \beta^R x_{jt} + \epsilon_{jt}. \end{aligned}$$

[6]There is no significant cross-country heterogeneity in growth rates of aggregate and sectoral real exchange rates.

[7]This applies to all four regression forms discussed in the previous paragraph because all regressions are effectively OLS regressions in levels, growth rates, or demeaned values.

Then, given that $q = \gamma^T q^T + \gamma^{NT} q^{NT} + \gamma^R q^R$ and the linearity of the OLS estimator, the following holds:

$$\beta = \gamma^T \beta^T + \gamma^{NT} \beta^{NT} + \gamma^R \beta^R.$$

This procedure effectively decomposes the relationship between x and q represented by β into the weighted sum of the relationship between x and each sector's real exchange rate. This regression-based decomposition yields intuitive assessment of the role of each sector's real exchange rate. $\gamma^R \beta^R$ will be the contribution of q^R to the aggregate empirical relationship summarized by β. By using this decomposition, I will examine how much the rent real exchange rate contributes to the Balassa-Samuelson effect and the Backus-Smith correlation.

Housing Rents and the Balassa-Samuelson Effect First, I examine the Balassa-Samuelson effect in eurozone countries. As a motivating figure, I generate a scatter plot in which the average relative real GDP per capita of each country ($\bar{y}_j$) is on the x-axis and each country's average aggregate real exchange rate ($\bar{q}_j$) and sectoral real exchange rates ($\bar{q}_j^T, \bar{q}_j^{NT}, \bar{q}_j^R$) are on the y-axis in Figure 4.

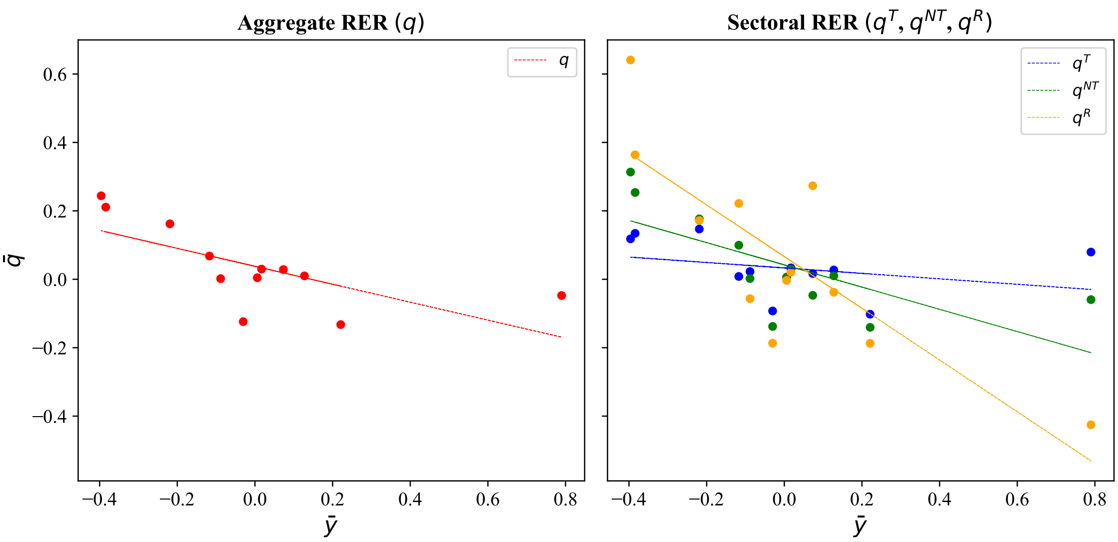

Figure 4: The Balassa-Samuelson Effect in Eurozone Countries

As we can clearly see in the left panel, countries with higher relative GDP per capita show higher relative price levels ($q < 0$). This implies that the Balassa-Samuelson effect exists in eurozone countries. Then, the right panel shows where such a relationship comes from. We can see that the relative price levels of tradables (q^T, denoted by the blue line) do not increase much, even if the country has a high GDP per capita. On the other hand, rent real exchange rates (q^R, denoted in yellow) exhibit a steep increase and decrease depending on the country's relative GDP per capita, which implies the important role of relative rents in the Balassa-Samuelson effect.

To examine the data more systematically, by using four types of regressions stated in equation (1), (2), (3), and (4) for all real exchange rates, I examine the relationship between aggregate and sectoral real exchange rates (q, q^T, q^{NT}, q^R) and relative real GDP per capita (y). Results of the regressions are reported in Table 2.

Table 2: Balassa-Samuelson Effect Regressions

Cross-section					
		$\bar{q}$	$\bar{q}^T$	$\bar{q}^{NT}$	$\bar{q}^R$
Country Average	$\bar{y}$	-0.26*	-0.08	-0.33*	-0.76***
		(0.14)	(0.13)	(0.18)	(0.19)
	R^2	0.43	0.08	0.45	0.64
	N	12	12	12	12
		q	q^T	q^{NT}	q^R
Time Fixed Effect	y	-0.26***	-0.07***	-0.31***	-0.75***
		(0.01)	(0.01)	(0.01)	(0.03)
	R^2	0.45	0.08	0.45	0.64
	N	240	240	240	240

Time-series					
		Δq	Δq^T	Δq^{NT}	Δq^R
Growth Rate	Δy	0.07*	0.11***	0.13**	-0.17**
		(0.04)	(0.03)	(0.07)	(0.08)
	R^2	0.02	0.04	0.02	0.02
	N	240	240	240	240
		q	q^T	q^{NT}	q^R
Country Fixed Effect	y	-0.11***	0.08*	-0.08	-0.67***
		(0.04)	(0.05)	(0.10)	(0.22)
	R^2	0.07	0.07	0.02	0.25
	N	240	240	240	240

$q = ln(P_{EU15t}/P_{it})$, $q^T = ln(P^T_{EU15t}/P^T_{it})$, $q^{NT} = ln(P^{NT}_{EU15t}/P^{NT}_{it})$, $q^R = ln(P^R_{EU15t}/P^R_{it})$ where P_{EU15t} is the price level of 15 European countries' average. $c = ln(C_{it}/C_{EU12t}), y = ln(Y_{it}/Y_{EU12t})$ where C_{EU12t}, Y_{EU12t} are geometric averages of C, Y over 12 eurozone countries. Data period is from 2000 to 2019 and data are at annual frequency. All standard error are computed using a panel-corrected standard errors method under the assumption of period correlation (cross-sectional clustering). Parentheses below estimates include standard deviations. * means 10% significance, ** means 5% significance, *** means 1% significance.

In the first two regressions that capture cross-sectional variations, as expected, β is estimated as -0.26 statistically significantly. This implies that a country with 1% higher relative GDP per capita has the 0.26% higher relative price levels, which proves the existence of the Balassa-Samuelson effect in eurozone countries. Another interesting observation is that q^R has very large coefficients compared with the other two sectoral real exchange rates, while

q^T's coefficient is not even significant in the country-average regression.

In the last two regressions that capture time-series variations, evidence on the Balassa-Samuelson effect in the time-series dimension is mixed.[8] When a country's relative GDP per capita increases with time, it does not necessarily accompany the increase in the relative price level in the growth rate regression. One strong result is that the rent real exchange rate appreciates when relative GDP per capita increases in both regressions.

Given that the cross-sectional pattern is stronger, I conduct the regression-based decomposition described in the previous paragraph for two regressions for cross-sectional variations. Left panel in Figure 5 shows the result. Each bar is each sectoral real exchange rate's β multiplied by the sectoral expenditure weight γ, so the sum of the blue, green, and red bar should be equal to the black bar.

Because β^T is not significant, this decomposition shows that almost half of the Balassa-Samuelson effect actually comes from the rent real exchange rate. While the rent expenditure weight is less than half the expenditure weight of other nontradables, its contribution to the Balassa-Samuelson effect is more than that. This implies the important role of housing rent in the Balassa-Samuelson effect.

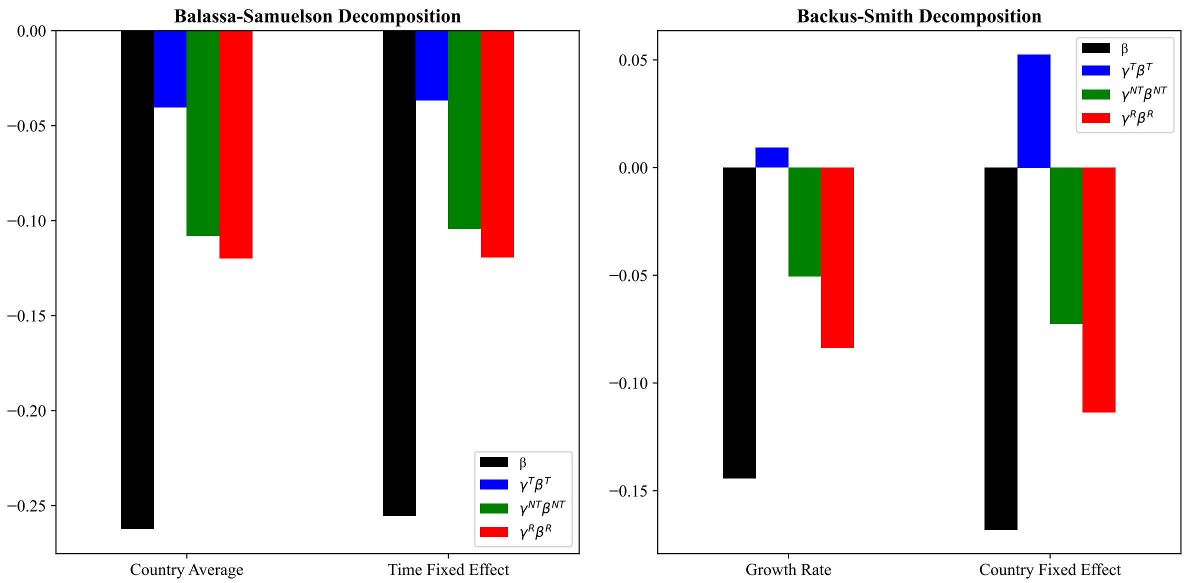

Figure 5: β Decompositions for the Balassa-Samuelson Effect and the Backus-Smith Puzzle

[8] Berka et al. (2018) show that the Balassa-Samuelson hypothesis actually also works in dynamics. The difference between their regressions and mine is that they directly use sectoral productivity levels from the EUKLEMS in the data and not relative real GDP per capita. Their regressions work with the data until 2007. However, once the time period after 2008 is included, they no longer work. This is likely because, after 2008, the economy was primarily driven by financial sector shock and not productivity shocks, as also explained by Berka et al. (2018).

Housing Rents and Backus-Smith Correlations For the last empirical analysis, I will examine the role of housing rents in the Backus-Smith correlation, which is one of the most important international macroeconomics puzzles (Backus and Smith 1993, Chari et al. 2002, Corsetti et al. 2008, Benigno and Thoenissen 2008, Karabarbounis 2014, Itskhoki and Mukhin 2021, Itskhoki 2021). Again, by exploiting the regression-based decomposition, I will examine which sectoral real exchange rate is responsible for the negative Backus-Smith correlations in the data.[9]

Before moving to the regression analysis, following the literature, I calculate the Backus-Smith correlation of the aggregate and sectoral real exchange rates for each country in the eurozone. Table 3 shows these correlations for each country. Though not all countries show negative correlations, nine countries out of 12 show the near-zero or negative correlations between real relative consumption and real exchange rates, which manifests the presence of negative Backus-Smith correlations among eurozone countries. On average, the Backus-Smith correlations are -0.059, which are far from the 1 implied by the standard model.[10]

However, again, this is not a systematic way to examine the correlation between relative consumption and the real exchange rate. Consequently, as I did for the Balassa-Samuleson effect, I run the four regressions to examine the relationship between changes in aggregate and sectoral real exchange rates (q, q^T, q^{NT}, q^R) and real relative consumption (c). Table 4 reports the results.

Unlike the case of the Balassa-Samuelson effect, now the cross-sectional regressions do not show any significant results. However, this time, regressions that capture time-series variations show significant results. In the time-series dimension, when a country's aggregate consumption grows 1% more than EU 12 average, the country's price level gets more expensive by 0.14% than other countries. In addition, we can see that q^R is the one that has significant β^R estimates in both regressions, and the estimated sizes are very large.

Given that the time-series pattern is stronger, I conduct regression-based decomposition again for two regressions for time-series variations. The right panel in Figure 5 shows the results. In both cases, while the tradable real exchange rate component shows positive corre-

[9] Corsetti et al. (2012) conducted an analysis similar to mine to identify which sectoral prices drive the Backus-Smith correlation in the data. Due to a lack of data, they first get data on P^T and estimate the changes in nontradable price P^{NT} as a residual by subtracting P^T from aggregate price changes. Here, I have sectoral real exchange rates based on item-level relative price data. My approach should be more precise and reliable—but their analyses are superior, in the sense that they use quarterly frequency data. Also, they use the producer price index or prices in dock, which allow them to separate the effect of the distribution margin from tradable good prices.

[10] Devereux and Hnatkovska (2020) ran fixed-effect regressions for a very large number of sub-regions across Japan, the US, and many other European countries. They found that the negative Backus-Smith correlation exists between countries while the risk-sharing is working between regions in a same country, showing that the border effect is important for the negative Backus-Smith correlation. On the top of that, they argue that nominal exchange rate accounts for one third of such border effect. As they argued, the role of the nominal exchange rate is important. If I include the data for periods when there was no euro (1995-1999), the Backus-Smith correlations get much more negative. The negative Backus-Smith correlations that I found in periods after the introduction of euro can be understood as a remaining part of the border effect.

Table 3: Backus-Smith Correlations

	$Corr(\Delta c, \Delta q)$	$Corr(\Delta c, \Delta q^T)$	$Corr(\Delta c, \Delta q^{NT})$	$Corr(\Delta c, \Delta q^R)$
Austria	-0.066	-0.031	0.131	-0.489
Belgium	-0.029	0.047	-0.087	-0.118
Finland	0.246	0.481	-0.027	-0.020
France	0.307	0.219	0.467	-0.162
Germany	-0.205	-0.012	-0.122	-0.551
Greece	-0.075	0.090	-0.110	-0.080
Ireland	-0.418	-0.218	-0.242	-0.541
Italy	0.135	0.048	0.288	0.011
Luxembourg	-0.082	0.302	-0.159	-0.260
Netherlands	-0.039	-0.149	0.176	-0.299
Portugal	-0.275	-0.183	-0.137	0.052
Spain	-0.203	0.114	-0.272	-0.235
Average	-0.059	0.059	0.008	-0.224

$q = ln(P_{EU15t}/P_{it})$, $q^T = ln(P^T_{EU15t}/P^T_{it})$, $q^{NT} = ln(P^{NT}_{EU15t}/P^{NT}_{it})$, $q^R = ln(P^R_{EU15t}/P^R_{it})$ where P_{EU15t} is the price level of 15 European countries' average. $c = ln(C_{it}/C_{EU12t}), y = ln(Y_{it}/Y_{EU12t})$ where C_{EU12t}, Y_{EU12t} are geometric averages of C, Y over 12 eurozone countries. Data period is from 2000 to 2019 and data are at annual frequency. Results from using consumption per capita are in the appendix.

lations, nontradables and the rent real exchange rate show negative correlations. In particular, the red bar is the biggest even though the γ^R is only 0.16. This shows how important the rent real exchange rate is important in understanding the Backus-Smith correlation.

3 Model: Inelastic Housing Supply and Real Exchange Rates

Motivated by the empirical evidence that suggests the crucial role of housing rent, to explore its role in the international business cycles I construct a standard two-country DSGE model and extend it in several dimensions. First, I assume that both the home and a foreign country have tradable, nontradable, and construction sectors. In addition, there is a distribution margin for consuming retail tradable goods, as in Berka et al. (2018). Second, I incorporate housing in the model following Davis and Heathcote (2005). Each country needs to accumulate housing stock so that they can obtain housing services from it. Importantly, to produce houses, they need to use residential-zoned land as a production input, which is under fixed supply. In addition, it takes one period to build houses. Lastly, I assume an incomplete market between two countries so that the two countries can insure their risks only through noncontingent bonds, as in Corsetti et al. (2008). Because I primarily use data on eurozone countries, which use the same currency (euro), I have not modeled any monetary component in the model. I present only the home country, and the foreign economy is same as the home country because this model has a symmetric structure.

Table 4: Backus-Smith Correlation Regressions

Cross-section					
		$\bar{q}$	$\bar{q}^T$	$\bar{q}^{NT}$	$\bar{q}^R$
Country Average	$\bar{c}$	0.03	0.00	0.03	0.07
		(0.02)	(0.02)	(0.02)	(0.05)
	R^2	0.07	0.01	0.07	0.09
	N	12	12	12	12
		q	q^T	q^{NT}	q^R
Time-FE	c	0.02	0.00	0.03	0.07
		(0.02)	(0.02)	(0.02)	(0.06)
	R^2	0.07	0.01	0.06	0.09
	N	240	240	240	240

Time-series					
		Δq	Δq^T	Δq^{NT}	Δq^R
Growth Rate	Δc	-0.14**	0.02	-0.15***	-0.53***
		(0.07)	(0.05)	(0.06)	(0.23)
	R^2	0.03	0.00	0.01	0.06
	N	240	240	240	240
		q	q^T	q^{NT}	q^R
Country-FE	c	-0.16**	0.10*	-0.21	-0.72**
		(0.07)	(0.06)	(0.14)	(0.37)
	R^2	0.09	0.05	0.06	0.17
	N	240	240	240	240

$q = ln(P_{EU15t}/P_{it})$, $q^T = ln(P^T_{EU15t}/P^T_{it})$, $q^{NT} = ln(P^{NT}_{EU15t}/P^{NT}_{it})$, $q^R = ln(P^R_{EU15t}/P^R_{it})$ where P_{EU15t} is the price level of 15 European countries' average. $c = ln(C_{it}/C_{EU12t}), y = ln(Y_{it}/Y_{EU12t})$ where C_{EU12t}, Y_{EU12t} are geometric averages of C, Y over 12 eurozone countries. Data period is from 2000 to 2019 and data are at annual frequency. All standard error are computed using a panel-corrected standard errors method under the assumption of period correlation (cross-sectional clustering). Parentheses below estimates include standard deviations. * means 10% significance, ** means 5% significance, *** means 1% significance.

3.1 Households

The home country's infinitely lived representative household solves the following optimization problem, in which C_t in equation (5) is a home household's aggregate consumption bundle.

$$U = \sum_{t=0}^{\infty} E_t[\beta^t(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi})], \quad \beta < 1, \tag{5}$$

$$s.t. \quad P_tC_t + D_{t+1}/R_{t+1} + P_{RI,t}I_{RI,t} = W_tN_t + P_{R,t}H_{t-1} + P_{l,t}l_t + D_t - \frac{\phi^c}{2}D^2_{t+1}, \tag{6}$$

$$H_t = (1-\delta)H_{t-1} + I_{RI,t}. \tag{7}$$

The household supplies labor and earns a wage, owns the houses and uses the houses for housing service consumption, and supplies a fixed amount of residential-zoned land every period. A constant amount of land becomes available to the household every period and they sell it to a real estate developer.[11] In addition, under the incomplete market assumption, households save and borrow through noncontingent international bonds, but there is a convex cost of owning them. This is one of the methods for guaranteeing the unique steady state of a two-country model suggested by Schmitt-Grohé and Uribe (2003). Lastly, through residential investment, they need to accumulate houses while the house is under constant depreciation. Note that the labor disutility function is separable from the consumption utility function.[12]

The aggregate consumption bundle is defined as the CES aggregation of a housing service $(C_{R,t})$ and non-housing consumption bundle $(C_{NR,t})$, as in equation (8).

$$C_t = (\gamma_R^{\frac{1}{v}} C_{R,t}^{1-\frac{1}{v}} + (1-\gamma_R)^{\frac{1}{v}} C_{NR,t}^{1-\frac{1}{v}})^{\frac{v}{v-1}}. \tag{8}$$

Housing service is assumed to be proportional to the housing stock (H_t), which implies that $C_{R,t} = H_{t-1}$. v is the elasticity of substitution between housing service and the non-residential consumption bundle and γ_R is the relative weight of housing service in the aggregator.

The non-residential consumption bundle is defined over a tradable consumption bundle $(C_{T,t})$ and nontradable consumption bundle $(C_{NT,t})$, as in equation (9).

$$C_{NR,t} = ((1-\gamma_{NT})^{\frac{1}{\theta}} C_{T,t}^{1-\frac{1}{\theta}} + \gamma_{NT}^{\frac{1}{\theta}} C_{NT,t}^{1-\frac{1}{\theta}})^{\frac{\theta}{\theta-1}}. \tag{9}$$

θ is the elasticity of substitution between tradable consumption bundle $(C_{T,t})$ and nontradable consumption $(C_{NT,t})$, while γ_{NT} is the relative weight of nontradable consumption. The tradable consumption bundle again has additional layers, as in equation (10):

$$C_{T,t} = (\omega_H^{\frac{1}{\lambda}} C_{H,t}^{1-\frac{1}{\lambda}} + (1-\omega_H)^{\frac{1}{\lambda}} C_{F,t}^{1-\frac{1}{\lambda}})^{\frac{\lambda}{\lambda-1}}. \tag{10}$$

$C_{T,t}$ is defined as an aggregation of a (retail) home-tradable consumption bundle $(C_{H,t})$ and a (retail) foreign-tradable consumption bundle $(C_{F,t})$. ω_H is the relative weight of $C_{H,t}$, and ω_H larger than 0.5 implies homebias in tradable consumption. λ is the elasticity of substitution between the home tradable consumption bundle and foreign tradable consumption bundle.

Both (retail) home and foreign-tradable consumption bundles are defined as the aggregation between each tradable good $(I_{H,t}, I_{F,t})$ and distribution margin services $(V_{H,t}, V_{F,t})$, as in

[11]The constant land supply assumption is standard in the literature. (Davis and Heathcote 2005, Favilukis et al. 2012, Kaplan et al. 2020) The assumption is that government zoned land as residential-zoned land through an administrative procedure and provides that to households. Also, the deconstruction of old houses can offer new available land to households.

[12]While many papers studying real exchange rate dynamics, such as Chari et al. (2002), use a separable utility like my model, others, such as Karabarbounis (2014), use non-separablility between consumption and leisure to generate the Backus-Smith correlation even under a complete market. This breaks the one-to-one relationship between relative price and consumption under a complete market, whereby leisure also affects marginal utilities.

equation (11) and equation (12).

$$C_{H,t} = ((1-\chi_{NT})^{\frac{1}{\phi}} I_{H,t}^{1-\frac{1}{\phi}} + \chi_{NT}^{\frac{1}{\phi}} V_{H,t}^{1-\frac{1}{\phi}})^{\frac{\phi}{\phi-1}}, \tag{11}$$

$$C_{F,t} = ((1-\chi_{NT})^{\frac{1}{\phi}} I_{F,t}^{1-\frac{1}{\phi}} + \chi_{NT}^{\frac{1}{\phi}} V_{F,t}^{1-\frac{1}{\phi}})^{\frac{\phi}{\phi-1}}. \tag{12}$$

In other words, for households to consume the traded goods, this requires the use of non-tradable services.[13] This distribution margin is justified by the distribution cost incurred by local input, such as labor for transporting goods. $\chi_{NT,t}$ defines the relative importance of the distribution margin and ϕ is the elasticity of substitution between tradable goods and the distribution margin.

This consumption structure implies the aggregate price index P_t and non-resident consumption price index as $P_{NR,t}$, as in equation (13) and equation (14).

$$P_t = (\gamma_R P_{R,t}^{1-v} + (1-\gamma_R) P_{NR,t}^{1-v})^{\frac{1}{1-v}}, \tag{13}$$

$$P_{NR,t} = ((1-\gamma_{NT}) P_{T,t}^{1-\theta} + \gamma_{NT} P_{NT,t}^{1-\theta})^{\frac{1}{1-\theta}}. \tag{14}$$

Note that $P_{R,t}$ is the housing rent, which is my major focus. Equation (13) and equation (14) imply that the aggregate price level is a weighted average of the price of the tradable, the nontradable, and the rent.

The tradable consumption bundle price index $P_{T,t}$ is defined as in equation (15), while the non-tradable consumption bundle price $P_{NT,t}$ is just the price of nontradable goods.

$$P_{T,t} = (\omega_H \tilde{P}_{H,t}^{1-\lambda} + (1-\omega_H) \tilde{P}_{F,t}^{1-\lambda})^{\frac{1}{1-\lambda}}. \tag{15}$$

Because I assume the presence of the distribution margin, I know that the retail price of home tradable $\tilde{P}_{H,t}$ and foreign tradable $\tilde{P}_{F,t}$ should contain distribution margins. These are well denoted in equations (16) and (17).[14]

$$\tilde{P}_{H,t} = ((1-\chi_{NT}) P_{H,t}^{1-\phi} + \chi_{NT} P_{NT,t}^{1-\phi})^{\frac{1}{1-\phi}}, \tag{16}$$

$$\tilde{P}_{F,t} = ((1-\chi_{NT}) P_{F,t}^{1-\phi} + \chi_{NT} P_{NT,t}^{1-\phi})^{\frac{1}{1-\phi}}. \tag{17}$$

[13]The importance of the distribution margin has been examined in numerous studies such as Burstein et al. (2003) and MacDonald and Ricci (2005). This is important as a component that drives the relative prices of tradables and aggregate real exchange rates.

[14]This implies that not only the terms of trade ($\frac{P_t^F}{P_t^H}$) but also the nontradable real exchange rate ($q_t^{NT} = \frac{P_{N,t}^*}{P_{N,t}}$) affects the tradable real exchange rate (q_t^T).

These price indices are combined to generate sectoral real exchange rates, as follows:

$$Q_t = \frac{P_t^*}{P_t} \quad (q_t = log(Q_t)), \tag{18}$$

$$Q_t^T = \frac{P_{T,t}^*}{P_{T,t}} \quad (q_t^T = log(Q_t^T)), \tag{19}$$

$$Q_t^{NT} = \frac{P_{NT,t}^*}{P_{NT,t}} \quad (q_t^{NT} = log(Q_t^{NT})), \tag{20}$$

$$Q_t^R = \frac{P_{R,t}^*}{P_{R,t}} \quad (q_t^R = log(Q_t^R)). \tag{21}$$

3.2 International Asset Market

I assume an incomplete market so that both countries' households can insure themselves against the shock only via noncontingent bonds. As famously noted by Corsetti et al. (2008), introducing an incomplete market generates wealth effects from the tradable sector productivity shock. Of the methods for ensuring a stationary equilibrium in a two-country incomplete market model suggested by Schmitt-Grohé and Uribe (2003), I chose the convex portfolio adjustment cost. Assuming symmetric economies, optimality conditions for international saving and borrowing D_t, D_t^* is as in equation (22).

$$R_t = E_t[\frac{1}{\beta}(\frac{C_t^{-\sigma}}{C_{t+1}^{-\sigma}})(\frac{P_{t+1}}{P_t})(\phi^c D_{t+1} + 1)] = E_t^*[\frac{1}{\beta^*}(\frac{(C_t^*)^{-\sigma}}{(C_{t+1}^*)^{-\sigma}})(\frac{P_{t+1}^*}{P_t^*})(\phi^c D_{t+1}^* + 1)]. \tag{22}$$

Once I ignore the ϕ^c, which will be calibrated as tiny, I see that the relationship between relative consumption and the real exchange rate holds under expectation, not state by state. This allows me to deviate from perfect risk sharing and provides an environment for generating the negative Backus-Smith correlation.

3.3 Intermediate Good Production

Moving toward to the production side, I have three sectors—tradable, nontradable, and construction—as in equations (23), (24), and (25).

$$Y_{H,t} = A_{H,t} N_{H,t}^{\alpha^H}, \tag{23}$$

$$Y_{N,t} = A_{N,t} N_{N,t}^{\alpha^N}, \tag{24}$$

$$Y_{CR,t} = A_{CR,t} N_{CR,t}^{\alpha^{CR}}. \tag{25}$$

There are no adjustment costs for labor reallocation, and I assume there is no non-residential capital for brevity. The foreign country has a symmetric production structure.

Each sector has its own sectoral productivity. I assume they are AR(1) processes, as follows:

$$ln(A_{H,t}) = ln(\bar{A}_H) + \rho_H(ln(A_{H,t-1}) - ln(\bar{A}_H)) + \epsilon_{H,t}, \tag{26}$$
$$ln(A_{N,t}) = ln(\bar{A}_N) + \rho_N(ln(A_{N,t-1}) - ln(\bar{A}_N)) + \epsilon_{N,t}, \tag{27}$$
$$ln(A_{CR,t}) = ln(\bar{A}_{CR}) + \rho_{CR}(ln(A_{CR,t-1}) - ln(\bar{A}_{CR})) + \epsilon_{CR,t}. \tag{28}$$

3.4 Housing Construction

To construct new houses ($I_{RI,t}$), real estate developers in each country combine land and construction goods. τ implies the share of residential zoned-land for the housing production.

$$I_{RI,t} = Y_{CR,t}^{1-\tau} l_t^{\tau}. \tag{29}$$

The law of motion for housing stock is stated in equation (30). As is clear in the law of motion, it takes one period to build new houses, and new houses become available for consumption only after one period.[15] In addition, housings depreciate by δ_h per period. Later, in the calibration section, I show how δ_h is connected to the residential structure depreciation δ_s.

$$H_{t+1} = (1 - \delta_h)H_t + I_{RI,t}. \tag{30}$$

In my model, $Y_{CR,t}$ is effectively the residential investment, which does not include the land component.[16] In addition, I only focus on residential buildings because I cannot observe commercial rents in the data. The construction sector in my model is a residential building construction sector.

I assume that land is supplied in the fixed amount every period. Following Davis and Heathcote (2005), I do not attempt to model the supply of residential-zoned land, which requires consideration of infrastructure development and the zoning process. I assume that through deconstruction of existing buildings and the government's new zoning assignment, a constant amount of residential zoned land is supplied.

$$l_t = \bar{l}. \tag{31}$$

The Role of Land in Housing Supply Elasticity The role of land in housing production is easy to see if I fix the price of construction goods. If we assume that $P_t^{CR} = \bar{P}^{CR}$ and $l = \bar{l}$, the

[15] For a model with longer periods to build houses and its impact on macro variables, refer to Bahadir and Mykhaylova (2014).

[16] Davis and Heathcote (2005) allow the real estate developer to combine manufacturing goods, services, and construction goods to generate residential investment. On the other hand, Kaplan et al. (2020) model a housing sector in which households combine lands and labor to produce housing.

real estate developer's first-order conditions imply the following:

$$Y_t^{CR} = (\frac{\bar{P}^{CR}}{l^{\tau}(1-\tau)P_t^{RI}})^{-\frac{1}{\tau}}. \tag{32}$$

Substituting this in the production function of the real estate developer, I can calculate the housing supply function as below.

$$I_t^H = (P_t^H)^{\frac{1-\tau}{\tau}}(1-\tau)^{\frac{1-\tau}{\tau}}(\bar{P}^{CR})^{\frac{\tau-1}{\tau}}\bar{l}. \tag{33}$$

This implies the following housing supply elasticity:

$$\frac{\partial ln(I^H)}{\partial ln(P^H)} = \frac{1-\tau}{\tau}. \tag{34}$$

This implies that the larger the land input share, the more housing supply elasticity decreases, which implies a steeper supply curve. Note that this is the supply for the new housing flow, not the total housing service supply. For the housing service supply, in the short run, the elasticity is 0 because it takes one period to build a house. In addition, depending on δ, new housing flows might be very small compared to the total housing stock. This makes aggregate housing service supply even more inelastic.

3.5 Relative Output and Relative Consumption

To study the Balassa-Samuelson effect and the Backus-Smith correlation, I need to define the relative real output per capita (y_t) and relative real consumption growth (Δc_t) in the model. First, I define an output (per capita) (Y_t) as follows:[17]

$$Y_t = P_t C_t + P_{RI,t} I_{RI,t} + P_{H,t}(Y_{H,t} - C_{H,t}) - P_{F,t}(Y_{F,t}^* - C_{F,t}). \tag{35}$$

Then, I construct relative output per capita y as follows:

$$y_t = ln(Y_t) - ln(Y_t^*). \tag{36}$$

Also, I use the home country aggregate consumption bundle as a numeraire by normalizing $P_t = 1$. For relative consumption growth (Δc_t), I use each country's aggregate consumption as follows.

$$\Delta c_t = \Delta(ln(C_t) - ln(C_t^*)). \tag{37}$$

[17]Note that $P_{H,t} = P_{H,t}^*$ and $P_{F,t} = P_{F,t}^*$. In my model, I do not assume any other frictions such as variable mark-up. The law of one price holds for every good in the model.

3.6 Equilibrium of the Model and Solution Methods

Since the equilibrium definition of my model is very standard, for the sake of brevity I skip the definition of model equilibrium. The model is solved using the first-order approximation with Dynare.

4 Quantitative Analysis: Model-simulated Real Exchange Rates

In this section, by using the structural model, I provide a more detailed quantitative analysis of the relationship between housing and the real exchange rate empirically examined in Section 2. As well explained by Itskhoki (2021), the real exchange rate is a general equilibrium object. This implies that proper examination of the relationship between housing and the real exchange rate also requires a general equilibrium perspective. Given this, I solve and simulate a model-produced sample with the same dimensions as the data. My simulation procedure closely follows the strategy of Berka et al. (2018). Although my model has only two countries, I can map the simulated data onto the empirical data by treating the model home country as the relevant EU country, and assuming the model foreign country as the EU average. This gives me simulated panel data on 8 countries for a 20-year period.[18] Using these simulated data, I replicate the empirical analysis I did in the earlier section and explore the role of the housing sector in real exchange rate determination.

4.1 Model Calibration

To render my simulation analysis quantitatively realistic, proper calibrations of both on the model parameters and shock processes are important. Note that my calibration strategy targets to match the housing-related moments and productivity shock processes are directly from data. Empirical moments of the real exchange rates are not targeted in my calibration. Table 5 provides model calibration details.

Non-housing Parameters The upper panel of Table 5 shows my calibration for non-housing parameters. I use $\beta = 0.99$, assuming quarterly frequency in the model, and match the long-run real interest rate among eurozone countries. For the coefficient of relative risk aversion and Frisch elasticity of labor supply, I set $\sigma = 2$ and $\psi = 1$, which are standard values used in DSGE modeling. For the relative weight between the tradable and nontradable, I set $\gamma^{NT} = 0.4$ to match the expenditure shares of each in the data. The elasticity of substitution between the tradable and the nontradable is set as $\theta = 0.7$, following Berka et al. (2018). Given

[18] For the model simulation, I use only eight eurozone countries, which provide the industry-level productivity data in EUKLEMS 2023. These are Austria, Belgium, Spain, Finland, France, Germany, Italy, and the Netherlands. The calibration will also be based on these countries.

Table 5: Model Calibration

Parameters	Variable	Value	Reference
1. Non-Housing Parameters			
Household			
Discount factor, yearly	β	0.99	
Relative risk aversion	σ	2	
Macro Frisch elasticity	ψ	1	
Non-Residential Consumption Aggregator			
Non-Tradable weight	γ^{NT}	0.4	Berka et al. (2018)
ES between traded and non-traded	θ	0.7	Berka et al. (2018)
Tradable Consumption Aggregator			
Home-bias	ω^{H}	0.5	No Homebias
ES between retail H and F	λ	8	Corsetti et al. (2010)
Distribution Margin			
Distribution Margin Weight	χ^{NT}	0.32	Goldberg and Campa (2010)
ES between retail and distribution service	ϕ	0.25	Berka et al. (2018)
Production			
Elasticity of Labor	α	1	Berka et al. (2018)
International Financial Market			
Portfolio Adjustment Cost	ϕ^{C}	0.001	Benigno and Thoenissen (2008)
2. Housing Parameters			
Residential Consumption			
Housing Service Weight	γ^{R}	0.25	
ES between housing and non-housing	v	0.85	Davidoff and Yoshida (2013)
Residential Building Production			
Land Input Share	τ	0.35	Combes et al. (2021)
Depreciation Rate of Residential Structure	δ^{S}	0.0037	

the presence of a distribution margin that generates home bias by itself and the homogeneity of eurozone countries, I set $\omega^H = 0.5$, which implies no home bias in the retail level. Elasticity between the home tradable and the foreign tradable (which is also called trade elasticity) is set as $\lambda = 8$, following Berka et al. (2018). Trade elasticity has been known to be small in the short run, at lower than 1, and large in long run, at larger than 1.[19]. Because my trade elasticity, λ, is the elasticity between the retail home and foreign goods and both of these retail contain the domestic distribution margin, it is not exactly the same as the trade elasticities used in other research.[20] For weights for the distribution margin, I use the estimates of Goldberg and Campa (2010) and calculate the average of eight eurozone countries' distribution margin for household consumption. This implies $\chi^{NT} = 0.32$. Lastly, regarding the portfolio adjustment

[19] Corsetti et al. (2008) use 0.5 for their first case and 4 for their second case. Cross-country estimates imply elasticity larger than 1 (Broda and Weinstein 2006), while the time-series estimates based on the response of import quantities to the exchange rate suggests elasticity less than 1 (Feenstra et al. 2018, Amiti et al. 2022)

[20] Corsetti et al. (2010) show that this implies a lower elasticity of substitution between traded wholesale goods, due to the presence of distribution services.

cost, following Benigno and Thoenissen (2008), I set ϕ^C equal to 0.001.[21]

Housing Parameters The lower panel of Table 5 shows my calibration for housing parameters. Calibrating housing-related parameters is difficult, and papers in the literature use different values. These parameters include the weight of residential consumption (i.e., housing service) in the aggregate consumption CES aggregator (γ^R), the elasticity of substitution between residential consumption and non-residential consumption (v), the land input share in the housing production function (τ), and the residential structure depreciation rate (δ^S).

Several papers provide information on these parameters. For example, Combes et al. (2021) show that τ in France is roughly 35% using detailed French housing construction data, which include house prices, land sizes, and land prices. However, papers that study the US housing market, such as Kaplan et al. (2020) and Favilukis et al. (2012), use 0.25 and 0.1 for τ, which are very different from 0.35. Also, for v, Davidoff and Yoshida (2013) suggest that this should range between 0.4 and 0.9 using aggregate time-series data under a non-homothetic preference assumption. However, this is contradicted by Davis and Ortalo-Magne (2021), who show that the housing rent expenditure share is constant across regions over time and suggest using Cobb-Douglas specifications.

Given the absence of consensus on these parameters, I decided to use the strategy of Davis and Heathcote (2005). While I use parameter values that are in ranges suggested by the literature, I choose parameter values that replicate the housing-related empirical moments of eight countries in the steady state. For example, Davis and Heathcote (2005) choose parameters that match empirical moments such as households' working hours, the value of residential structure capital over GDP, and the capital and labor input share in each sector.

I target five empirical moments related to the housing sector: (1) the value of residential structure capital stock over GDP ($RCOY$), (2) residential investment over GDP ($RIOY$), (3) share of construction sector labor over total labor ($NConRatio$), (4) household expenditure share on housing rents (REW), and (5) new housing construction flow over housing stock ($HFoHS$).

This requires that I identify proper model counterparts for those empirical moments. First, for the residential structure capital over GDP ($RCOY$), I define the net stock of residential structure, S, as follows under the assumption that residential structure depreciates by δ_s per period. Under the steady state, I can define S as follows:[22]

$$S = \sum_{k=1}^{\infty}(1-\delta_s)^k Y_{CR_{t-k}}. \tag{38}$$

[21] Rabanal and Tuesta (2010) provides estimates of 0.007 for quarterly data, but there is no big difference in model-simulated results.

[22] Land is not included in residential capital stock, according to the most recent national accounting system.

In the steady state, $P_{CR}S = \frac{P_{CR}Y_{CR}}{\delta_s}$ and $P_{RI}H = \frac{P_{RI}I_{RI}}{\delta_h}$ hold from the law of motion for housing stock and residential capital structure. Also, $P_{RI}I_{RI} = \frac{P_{CR}Y_{CR}}{(1-\tau)}$ holds from the optimal condition of the real estate developer. Combining all these, under the steady state, the following holds:[23]

$$\frac{P_{CR}S}{P_{RI}H} = \frac{(P_{CR}Y_{CR})/\delta_s}{P_{CR}Y_{CR}/((1-\tau)*\delta_h)} = \frac{(1-\tau)*\delta_h}{\delta_s} = \frac{(1-\tau)*(1-(1-\delta_s)^{1-\tau})}{\delta_s}. \tag{39}$$

Consequently, residential structure capital stock over GDP $(RCOY)$ in my model will be defined as

$$RCOY = \frac{(1-\tau)*(1-(1-\delta_s)^{1-\tau})}{\delta_s}\frac{P_{RI}H}{PY}. \tag{40}$$

Second, I define the residential investment over GDP $(RIOY)$ as follows:

$$RIOY = \frac{P_{CR}Y_{CR}}{PY}, \tag{41}$$

because Y_{CR} is the residential investment in my model. Third, the share of labor in the construction sector over total labor $(NConRatio)$ is defined as

$$NConRatio = \frac{N_{CR}}{N_H + N_N + N_{CR}}. \tag{42}$$

It is especially important to match this moment because it determines the size of the effect of the construction sector productivity shock on the aggregate economy via the labor market in the model. Fourth, the household expenditure weight on housing rents (REW) is defined as follows:

$$REW = \frac{P_R C_R}{PC} = \frac{P_R H}{PC}, \tag{43}$$

under the assumption that housing service flow is proportional to the housing stock.

Lastly, I define new housing flow over housing stock ($HFoHS$) in quantities as follows:

$$HFoHS = I_H/H. \tag{44}$$

To replicate these moments, I set $\gamma^R = 0.25$. In addition, I set elasticity of substitution between housing and non-housing consumption as $v = 0.85$. This is also chosen to match the increasing patterns of rent expenditure weights over time in all eurozone countries in Figure 2. For the land input share in housing production, in the end, I set $\tau = 0.35$ following Combes et al. (2021). This is much larger than the values used by Kaplan et al. (2020) or Favilukis et al.

[23] As explained by Davis and Heathcote (2005), $1-\delta_h = (1-\delta_s)^{1-\tau}$.

(2012), who study the US housing market. However, it is well known that eurozone countries have substantially lower housing supply elasticities compared with that of the US. In addition, while there is large heterogeneity across eurozone countries' housing supply elasticities,[24] a recent estimate suggests that France's housing supply elasticity is in the middle among the eurozone countries. This again makes using $\tau = 0.35$ proper. Last, for the residential structure depreciation rate, I use $\delta^S = 0.0037$, which implies a 1.48% depreciation rate every year.

Table 6 shows how my model performs in terms of replicating these moments of average of eurozone countries in the steady state. As can be seen in the table, the model successfully replicates most of the moments. In particular, it replicates the fact that housing is very inelastically supplied ($HFoHS$), the housing rent accounts for a substantial portion of the aggregate expenditure (REW), and the residential construction sector accounts for a small portion of the aggregate labor market ($NConRatio$).

Table 6: Housing Sector Moments: Data vs Model Steady State

	Data	**Model**
Supply side		
Residential Capital over GDP ($RCOY$)	1.457	1.403
Residential Investment over GDP ($RIOY$)	0.029	0.021
Labor Share of Construction Sector ($NConRatio$)	0.017	0.025
Housing Flow over Housing Stock ($HFoHS$)	0.009	0.009
Demand side		
Housing Rent Expenditure Share (REW)	0.161 (0.212)	0.170
Tradable Expenditure Share	0.516	0.497
Nontradable Expenditure Share	0.328	0.331

Data period for 8 Eurozone countries is (2000-2019). Note that the construction sector in my model is effectively the residential construction sector, not the total construction sector. According to the European Construction Industry Federation, 50.4% of total construction is estimated to be residential construction in 2022. So, I use the half of the value of the corresponding construction sector for construction sector-related variables when I match the empirical moments of the construction sector in my model.

Sectoral Productivity Shocks In my simulation, I use sectoral productivity shocks as the main drivers of international business cycles. To estimate each sector's productivity, I closely follow the estimation procedure used by Berka et al. (2018) and extend their estimates up to 2019. The appendix details the procedure I used to estimate these processes. Here, I provide a brief description.

To estimate sectoral productivity shock processes, I use the GGDC 1997 database and EUK-

[24] There is substantial heterogeneity across countries regarding the long-run price elasticity of the housing supply. Caldera and Åsa Johansson (2013) estimate elasticities across countries. European countries such as Austria (0.234), Belgium (0.315), Finland (0.988), France (0.363), Germany (0.428), Ireland (0.631), Italy (0.258), the Netherlands (0.186), and Spain (0.452) show much lower estimates than the US (2.014)

LEMS 2023. GGDC 1997 provides all industries' productivity levels.[25] Then, for each country, by dividing each industry's productivity by that of the geometric average of the 11 European countries that provide productivity data in EUKLEMS 2023, I calculate each industry's relative productivity level against that of the average of 11 European countries. Then, using the EUKLEMS 2023 database, which provides the industry-level growth rate for each country, I calculate the productivity growth rate of each industry relative to that of 11 European countries' average.

By combining these relative levels and relative growth rates, I can construct panel data for each industry's relative productivity of each country against the 11 European countries' average. Lastly, by classifying these industries into tradable, nontradable, and construction sectors, and by averaging with the value-added of each sector as a weight, I obtain panel data for sectoral productivity levels (tradable sector, nontradable sector, and construction sector) relative to those of the 11 European countries' average.[26]

The log of these estimated relative sectoral productivities are presented in Figure 6. We can see that both the tradable and construction sectors have larger cross-country and time-series variations than that of the nontradable sector.

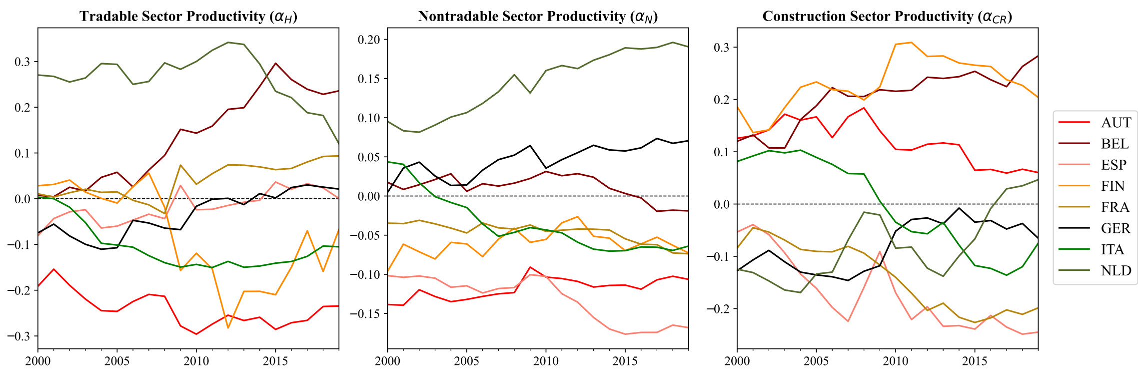

Figure 6: Sectoral Productivities

Given this panel data, I estimate sectoral productivity shock processes for each country. Note that what I have estimated are the productivity level of each sector of a certain country relative to that of the 11 European countries' average. Thus what I will estimate are the following relative sectoral productivity shock processes, where $\alpha_{Y,jt} = ln(\frac{A_{Y,jt}}{A_{Y,EUt}})$ for $Y \in$

[25]This is a given industry's productivity relative to that industry in the US. But this cancels out if I divide that productivity by that of the Europe 11 average (relative to the US).

[26]The industry classifications of EUKLEMS are a bit different from those of GGDC 1997 database. However, they are closely related. Consequently, as I explain in detail in appendix, I generated a sectoral concordance table and use that accordingly. I have a total 12 tradable industries, 9 nontradable industries, and 1 construction industry.

$\{H, N, CR\}$. Data are from 2000 to 2019 at annual frequency.

$$\alpha_{H,it} - \bar{\alpha}_H = \rho_H(\alpha_{H,it-1} - \bar{\alpha}_H) + \epsilon_{h,it},$$
$$\alpha_{N,it} - \bar{\alpha}_N = \rho_N(\alpha_{N,it-1} - \bar{\alpha}_N) + \epsilon_{n,it},$$
$$\alpha_{CR,it} - \bar{\alpha}_{CR} = \rho_{CR}(\alpha_{CR,it-1} - \bar{\alpha}_{CR}) + \epsilon_{cr,it}.$$

Note that while my data are at annual frequency, the model simulation will be conducted at quarterly frequency, so I convert estimated parameters to quarterly frequencies. In particular, I take the quadratic root of ρ to render it at quarterly frequency. Also, the variance-covariance matrix of the shock processes is estimated under the assumption that the shock is i.i.d at quarterly frequency. In addition, I allow covariance relationships among the productivity processes of all sectors and countries. Table 7 reports the results of the estimation.

Several interesting patterns emerge. First, as the productivity itself is defined as relative

Table 7: Estimated Sectoral TFP Processes

	A. Cross-section			B. Time-series					
	Mean values			AR(1) Coefficients			Standard Deviations		
	$\bar{a}_H$	$\bar{a}_N$	$\bar{a}_{CR}$	ρ_H	ρ_N	ρ_{CR}	σ_H	σ_N	σ_{CR}
AUT	-0.241	-0.118	0.119	0.918	0.894	0.966	2.367	0.936	2.344
BEL	0.135	0.011	0.205	0.983	0.976	0.971	2.700	0.907	2.017
ESP	-0.018	-0.132	-0.172	0.873	0.987	0.945	2.409	0.951	3.499
FIN	-0.080	-0.060	0.231	0.939	0.769	0.946	6.198	1.262	2.804
FRA	0.040	-0.046	-0.139	0.925	0.997	0.989	2.716	0.583	1.862
GER	-0.034	0.046	-0.080	0.973	0.905	0.962	2.198	1.206	2.228
ITA	-0.106	-0.036	-0.003	0.951	0.959	0.987	1.326	0.708	2.402
NLD	0.264	0.145	-0.080	0.990	0.986	0.986	2.919	1.153	3.359
AVG	-0.005	-0.024	0.010	0.944	0.934	0.969	2.854	0.963	2.564

productivity, it is natural that the cross-country averages of $\alpha_H, \alpha_N, \alpha_{CR}$ are close to 0. Also, the relative productivities of both the tradable sector and construction sector show much larger cross-sectional and time-series variations compared with those of the nontradable sector. This aligns with estimates from previous research.

Simulation Procedure Given the calibrated sectoral productivity shock processes and the model calibration explained above, I simulate the eight countries whose productivity data are available. The simulated periods are 80 quarters, as in the data (2000-2019). In my simulation, the core assumption is that each of these eight countries is a home country and the eurozone average is a foreign country. Under this assumption, because the sectoral productivity shock processes are estimated in units of each country's sectoral productivity relative to that of the eurozone average, the simulated shocks will only be fed into the home country, while the foreign country does not receive any shocks during the simulation. Only the transmissions of home country shocks affect the foreign country. After each simulation, I collect only the home country's aggregate real exchange rates, sectoral real exchange rates, relative real GDP per capita, and relative real consumption. This gives me simulated panel on such variables, and this simulated panel is comparable to what I have in actual data.

Given the panel data from each simulation, I replicate the empirical analysis that I performed in the empirical section, and I repeat the whole procedure 500 times. This repetition gives me the distributions of the parameters of interest, such as cross-sectional and time-series variations of the real exchange rates, Balassa-Samuelson regression coefficients, and Backus-Smith regression coefficients. Such distributions will be compared with actual data estimates. Also, this procedure will be repeated in a different calibration environment to understand how the housing market affects real exchange rate dynamics.

One thing to note is that were during this simulation, none of the empirical moments of the real exchange rate were targeted. This simulation exercise should be understand as exploring how far we can go in explaining the role of housing rent in the real exchange rate by combining the standard model of the housing market and the two-country international business cycle model.

4.2 Simulation Result: Model-generated Real Exchange Rates

In this subsection, I provide the simulation results. Following the same order as in the empirical section, I present results in the order of cross-section and time-series variations of RER, the Balassa-Samuleson effect, and the Backus-Smith correlation.

4.2.1 Housing and Variations of RER

Table 8 compares cross-section and time-series variations of model-generated real exchange rates under different calibrations with those of the actual data. While the upper panel provides the estimates for cross-sectional variations, the lower panel provides estimates for time-series variations.

Data and the Baseline Model Column (1) shows the variations of RER in the data and column (2) shows model-generated variations of RER under the baseline calibration. In the upper panel for the cross-section, we see that my baseline model can generate substantial cross-sectional variations. In addition, the relative sizes of variations of sectoral real exchange rates are also consistent with the data, and show the largest variations in q^R. Moving to the lower panel, we see that the baseline model produces substantial time-series variation, which is comparable to the data. However, one limitation is that my model generates much stronger variations in the nontradable real exchange rate than that of the rent real exchange rate compared with the data. This may be because my model does not have housing-finance

Table 8: Simulated Cross-sectional and Time-series Variations of RER

	(1) Data	(2) Bond Baseline	(3) Arrow-Debreu Baseline	(4) Arrow-Debreu ($\tau = 0.01$)	(5) Arrow-Debreu ($\delta = 0.99$)	(6) Arrow-Debreu (τ=0.01, δ=0.99)	(7) Arrow-Debreu (7) + ($\bar{A}_j^{CR}/\bar{A}_{EU}^{CR} = 1$)
Cross-section							
$\sigma_j(q_{jt})$	0.121	0.085	0.053	0.059	0.057	0.067	0.053
$\sigma_j(q_{jt}^T)$	0.081	0.039	0.028	0.027	0.028	0.027	0.027
$\sigma_j(q_{jt}^{NT})$	0.149	0.121	0.087	0.084	0.085	0.084	0.083
$\sigma_j(q_{jt}^R)$	0.297	0.197	0.134	0.212	0.149	0.214	0.126
Time-series							
$\sigma_t(q_{jt})$	0.025	0.033	0.022	0.023	0.025	0.028	0.028
$\sigma_t(q_{jt}^T)$	0.022	0.018	0.013	0.013	0.013	0.012	0.012
$\sigma_t(q_{jt}^{NT})$	0.039	0.054	0.041	0.039	0.039	0.038	0.038
$\sigma_t(q_{jt}^R)$	0.072	0.038	0.009	0.053	0.050	0.075	0.076

component or any direct demand shock in the model. (e.g., cyclicality in fiscal policies in Greece or foreign capital inflows in Ireland.)

Role of an Incomplete Market To examine why my model generates such variations, first, I assume a complete market instead of an incomplete market and simulate the data. An incomplete market generates a deviation from perfect risk-sharing, which opens room for the demand shocks to drive international business cycles. For example, if the home country receives a tradable sector productivity shock and gets wealthier than the other, its aggregate demand increases more than that of the other country. This demand differential will make both consumption and price increase more than those of the other country, which results in real appreciation. The result is reported in column (3). Compared with column (2), we can see a large decrease in all variations. This proves that the wealth effect that arises from the incomplete market boosts the variations in all dimensions. The most striking change comes from the time-series variation of the rent real exchange rate, $\sigma_t(q_{jt}^R)$, which drops from 0.038 to 0.009. This is a much larger drop compared with those of other sectoral real exchange rates and arises from the inelastic supply of housing service.

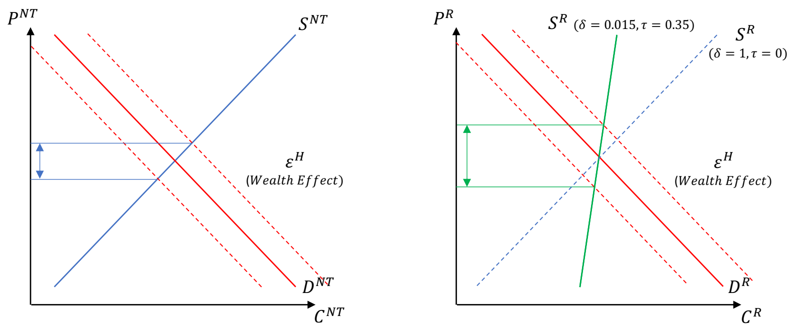

Figure 7: Wealth Effect on Nontradable (Left) and Housing Service (Right) Market

With Figure 7, this is easier to understand. If housing construction does not require land ($\tau = 0$), and housing flow in each period is equal to the housing stock ($\delta = 1$), the housing service supply curve would be same as that of the nontradables as the blue-dotted line show in the right panel of Figure 7. However, in my model, because of the large reliance on the land input, which is under fixed supply, even though the rent (P^R) increases, the supply cannot increase much. In addition, because housing service comes from the housing stock—which is much larger than the per period new housing flow—even P^R increases, and again supply cannot increase in a short time. This makes the housing service supply much more inelastic than that of the nontradable, which results in a steeper supply curve. Then, even though the home country has slightly higher demand than the foreign, the rent real exchange rate can respond very strongly. Once we assume a complete market we eliminate this channel, which disproportionately reduces the time-series standard deviations of q^R.

Housing and Other Nontradables These characteristics of housing services also affect the responses of the nontradable sector and rent real exchange rate to productivity shocks on nontradable and construction sector. In my model, a positive nontradable sector productivity shock directly generates a large supply increase, and shifts the supply curve substantially. However, the rent real exchange rate does not shift much in response to the positive construction sector shock, as in Figure 8.

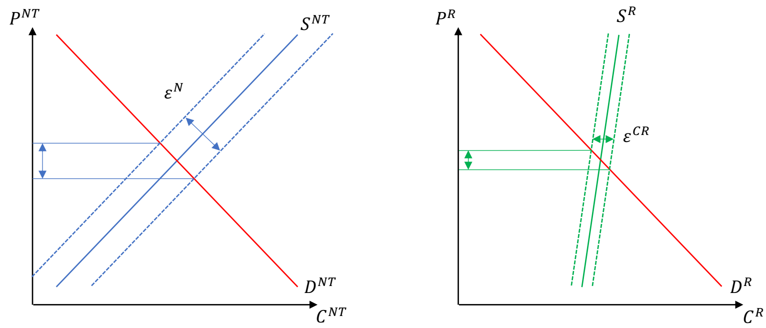

Figure 8: Productivity Shocks on the Nontradable (Left) and Housing Service (Right) Market

The intuition is again simple. Even though production in construction goods (e.g., cement) increases via the productivity increase, land to build on is limited.[27] Consequently, the housing quantity itself cannot increase much. In addition, given that the housing stock is much larger than the flow, any temporal relative productivity gain cannot increase the housing stock and housing services meaningfully in a short time. This explains why the rent real exchange rate time-series standard deviation is so low under a complete market.

This becomes clearer when we simulate the model with different values of τ and δ under a complete market. To examine the role of land, I change the land input share (τ) from 0.35 to 0.01 so that a housing construction no longer requires land, while still assuming a complete market. The result is reported in column (4). We can see that both cross-section and time-series variations of q^R increase once we set $\tau = 0.01$. With high τ, land dampens the effect of the construction sector productivity shock, as explained in Figure 8, but now the land is not used when $\tau = 0.01$. In addition, when τ is high, housing service is less exposed to the conventional Balassa-Samuelson hypothesis mechanism. This is because, though the positive tradable sector productivity shock may push up the wage, the marginal production cost of the housing service not only depends on wages but also the price of land, which is not exposed to the tradable sector productivity shock. However, once τ is set to 0.01, such dampening effect disappears, which generates the additional fluctuations.

Moving to the second characteristic, I change the depreciation rate (δ) from 0.00375 to 0.99 so that housing stock is not accumulated. This again renders housing service similar to other nontradables. Unlike in the case of τ, it increases only the time-series variations of the real exchange rate significantly. The larger stock compared with the small flows cause any temporal change in the productivity shock to affect the housing service supply only small, and in turn the relative price changes little. However, with high δ, such dampening effect disappears. In column (6), I combine two of these changes, and the effect gets stronger only in the time

[27]Urbanization and stringent land-use regulations in cities put effective limits on land supply in almost every city.

series.

Note that columns (4), (5), and (6) are under a complete market. So this is mainly about the productivity increase, the Balassa-Samuelson hypothesis mechanism, and the substitution effect within the country given the aggregate constant relative demand of two countries. These patterns together imply that specific characteristics of the housing service dampen the textbook Balassa-Samuelson hypothesis mechanism and the substitution effect. On the other hand, it amplifies the reponse of q^R to the wealth effect.

Role of the Cross-sectional Distribution of Sectoral Productivities A remaining puzzling observation is that, even though I render housing service similar to other nontradable services by setting τ as 0.01 and δ as 0.99, we still observe more substantial variations in q^R than q^{NT} or q^T in both dimensions. It turns out that the larger time-series variation simply comes from the larger standard deviations of the construction sector productivity shocks, as in Table 7. However, cross-sectional variation requires additional analysis. In column (7), I remove the differences in relative construction sector productivity across countries by setting relative $\frac{A_j^{CR}}{A_{EU}^{CR}}$ as 1 for all j. This dampens a substantial amount of the cross-sectional variations. However, it still shows substantial cross-sectional variations in q^R. It turns out that this comes from the cross-country distributions of sectoral productivities, which will be explored in more depth in the section on the Balassa-Samuelson effect.

Role of the Housing Market Given such sources of variations of the real exchange rates, I examine the role of the housing market here. Table 9 shows the simulated variations of real exchange rates under different weights of housing in aggregate consumption.

Table 9: Simulated Cross-sectional and Time-series Variations of RER: Role of Housing

	(1) **Data**	(2) **Bond** ($\gamma^R = 0.01$)	(3) **Bond** ($\gamma^R = 0.25$)	(4) **Bond** ($\gamma^R = 0.45$)
Cross-section				
$\sigma_j(q_{it})$	0.121	0.073	0.085	0.106
$\sigma_j(q_{it}^T)$	0.081	0.039	0.039	0.039
$\sigma_j(q_{it}^{NT})$	0.149	0.122	0.121	0.123
$\sigma_j(q_{it}^R)$	0.297	0.198	0.197	0.200
Time-series				
$\sigma_t(q_{it})$	0.025	0.032	0.033	0.034
$\sigma_t(q_{it}^T)$	0.022	0.017	0.018	0.018
$\sigma_t(q_{it}^{NT})$	0.039	0.053	0.054	0.055
$\sigma_t(q_{it}^R)$	0.072	0.035	0.038	0.039

With column (3) as the baseline calibration, I study what happens if housing either becomes less important ($\gamma^R = 0.01$) or more important ($\gamma^R = 0.45$) in the aggregate consumption bundle. Simulations show that more weights in housing service increase both the cross-sectional and time-series variations of the aggregate real exchange rate even though the latter shows only small changes. This demonstrates how housing can be an important source of variations of the real exchange rates. This mainly arises from its importance in the aggregation consumption and inelastic supply. In addition, a large portion is from the cross-country distribution of sectoral productivities. This implies that in each economy, each sector's characteristics—such as the necessity of specific inputs, supply elasticity, distribution of sectoral productivities, and its importance in consumption—may have a substantial impact on the model's predictions on aggregate real exchange rate behaviors.

4.2.2 Housing and the Balassa-Samuelson Effect

In the previous section, we examined unconditional variations of the real exchange rates, and one striking feature of the model prediction was that cross-sectional variations of the rent real exchange rate were still substantially larger than the other nontradables, even though I rendered housing service very similar to the other nontradables by setting $\tau = 0.01, \delta = 0.99$ and removing the cross-sectional heterogeneity in the relative productivities of the construction sector. This turns out to be related to the Balassa-Samuelson effect. To study this, I examine the relationship between real GDP per capita (y) and real exchange rates (q, q^T, q^{NT}, q^R). In particular, I replicate the country average cross-sectional regressions I did using the model-simulated real exchange rate and relative GDP per capita.

$$
\begin{aligned}
\bar{q}_j &= \alpha + \beta \bar{y}_j + u_j, \\
\bar{q}_j^T &= \alpha^T + \beta^T \bar{y}_j + u_j^T, \\
\bar{q}_j^{NT} &= \alpha^{NT} + \beta^{NT} \bar{y}_j + u_j^{NT}, \\
\bar{q}_j^R &= \alpha^R + \beta^R \bar{y}_j + u_j^R.
\end{aligned}
$$

Table 10 shows the simulated β for each sector. Note that while column (1) reports the regression result with data, other columns are from model simulations. Columns other than the data column contain the mean value of the 500 simulations, and the parentheses below contain the 0.1 quantile and 0.9 quantile of the 500 simulations.

Data and the Baseline Model Column (1) and column (2) show how my baseline model performs compared with the data. My model overestimates the relationships between relative GDP per capita and all sectoral real exchange rates by a factor of two. One thing the model matches well is the relative importance of sectoral real exchange rates with respect to the overall Balassa Samuelson effect. Both in the data and the baseline model simulation, we see that β^R is the largest and should make the largest contribution to the aggregate β.

Table 10: Simulated Balassa-Samuelson Regressions

	(1) Data	(2) Bond Baseline	(3) Arrow-Debreu Baseline	(4) Arrow-Debreu ($\gamma^R = 0.01$)	(5) Arrow-Debreu ($\tau = 0.01$)	(6) Arrow-Debreu ($\delta = 0.99$)	(7) Arrow-Debreu (τ=0.01, δ=0.99)	(8) Arrow-Debreu (7) + ($\bar{A}^{CR}_J / \bar{A}^{CR}_{EU} = 1$)
Bal/Sam								
β	-0.26*	-0.57*	-0.17*	-0.06	-0.18*	-0.18*	-0.22*	-0.12
	(0.14)	(-0.76,-0.36)	(-0.27,-0.06)	(-0.17,0.04)	(-0.31,-0.05)	(-0.31,-0.03)	(-0.38,-0.06)	(-0.29,0.04)
β^T	-0.08	-0.21*	-0.04	-0.03	-0.04	-0.03	-0.02	-0.02
	(0.13)	(-0.32,-0.09)	(-0.11,0.03)	(-0.09,0.04)	(-0.11,0.04)	(-0.11,0.05)	(-0.09,0.07)	(-0.10,0.06)
β^{NT}	-0.33*	-0.65*	-0.13	-0.11	-0.12	-0.09	-0.05	-0.06
	(0.18)	(-0.96,-0.31)	(-0.34,0.09)	(-0.29,0.07)	(-0.34,0.12)	(-0.33,0.16)	(-0.28,0.22)	(-0.32,0.18)
β^R	-0.76***	-1.46*	-0.62*	-0.49*	-0.86*	-0.63*	-0.93*	-0.45
	(0.19)	(-1.64,-1.26)	(-0.82,-0.43)	(-0.67,-0.33)	(-1.19,-0.49)	(-0.95,-0.32)	(-1.38,-0.45)	(-0.89,0.01)

Role of an Incomplete Market To examine the role of an incomplete market, I simulate the model again but with the complete market. The result is reported in column (3). We can see that all β for sectoral real exchange rates become smaller, while only the β^R is significantly estimated. This again shows the role of the wealth effect in driving prices. Under an incomplete market, increased tradable sector productivity also increases the country's wealth and demand, which adds additional force for a price hike. This amplifies the model-generated Balassa-Samuelson effect. Assuming a complete market yields a better match with the overall data patterns. However, the role of nontradables seems to be underestimated compared with the data. This implies that the truth should lie between column (2) and column (3).

Role of the Housing Sector and Characteristics of Housing Having a better match with the data under the complete market assumption, I conduct a calibration where only γ^R changes from 0.25 to 0.01, which renders housing less important in the aggregate consumption bundle in column (4). It turns out that the model cannot generate the Balassa-Samuelson effect. This implies that there is a substantial role for housing service in generating the Balassa-Samuelson effect in the model.

To examine where it comes from, I again simulate the model by rendering housing similar to other nontradables by setting τ as 0.01 in column (5), δ as 0.99 in column (6) or τ as 0.01 and δ as 0.99 simultaneously in column (7). Other than these, the calibrations are the same as in column (3). In the case of columns (5) and (7) compared with column (3), we observe small, though not significant, increases in the model-generated Balassa Samuelson effect via q^R. This likely arises from the increased Balassa-Samuelson hypothesis mechanism, because the construction sector is fully exposed to it since they only use labor, and not land, which is not used in the tradable sector. On the other hand, low δ has no significant impact on the model-generated Balassa-Samuelson effect. This is because we are focusing on cross-sectional regressions by using the mean of each country, effectively comparing the steady states of all countries. Whatever δ is, the relative importance of land is fixed as τ. Thus the Balassa-Samuelson effect does not change. As we will see in the Backus-Smith puzzle section, the major contribution of δ is to the time-series dimension of the real exchange rate, since it defines the relative sizes of the housing stock and housing flows in the model dynamics.

Role of the Cross-sectional Distribution of Sectoral Productivities The above exercises leave a puzzling fact, as was the case with the cross-sectional variations of RER. It turns out that the unique characteristics of housing we are interested in actually dampen the Balassa-Samuelon effect via q^R in the model. Even though I set τ as 0.01 and δ as 0.99, which renders the housing service very similar to the other nontradable sector, we still observe a stronger Balassa-Samuelson effect via the rent real exchange rate, not via the nontradable real exchange rate.

It turns out that this strong role of housing rent is from the distribution of sectoral productivities. The productivity shock processes I directly calibrated from the EUKLEMS 2023 database show that $Corr(ln(\frac{\bar{A}_j^H}{\bar{A}_{EU}^H}), ln(\frac{\bar{A}_j^{NT}}{\bar{A}_{EU}^{NT}})) = 0.76$, while $Corr(ln(\frac{\bar{A}_j^H}{\bar{A}_{EU}^H}), ln(\frac{\bar{A}_j^{CR}}{\bar{A}_{EU}^T})) = -0.23$ for the eight countries I use for the simulation. Figure 9 shows how the tradable sector of each country is related to that of the nontradable or construction sector.[28] We can see very strong positive and negative correlation relationships among sectoral productivities. Figure 9 implies that a country with higher relative tradable sector productivity has higher relative nontradable sector productivity but relatively lower construction sector productivity. This causes the model to generate the Balassa-Samuelson effect primarily through the housing rents. Though the wage goes up due to high tradable sector productivity, high nontradable sector productivity prevents the nontradable real exchange rate from appreciating. However, even though the wage accounts for only 1-τ portion of the housing construction cost, because of the lower construction sector productivity the Balassa-Samuelson effect works strongly through the rent real exchange rate rather than other nontradables.

[28]This figure also includes three non-eurozone countries: Sweden, Denmark, and the UK. These are mean relative productivities that are directly calibrated from the data without any inference.

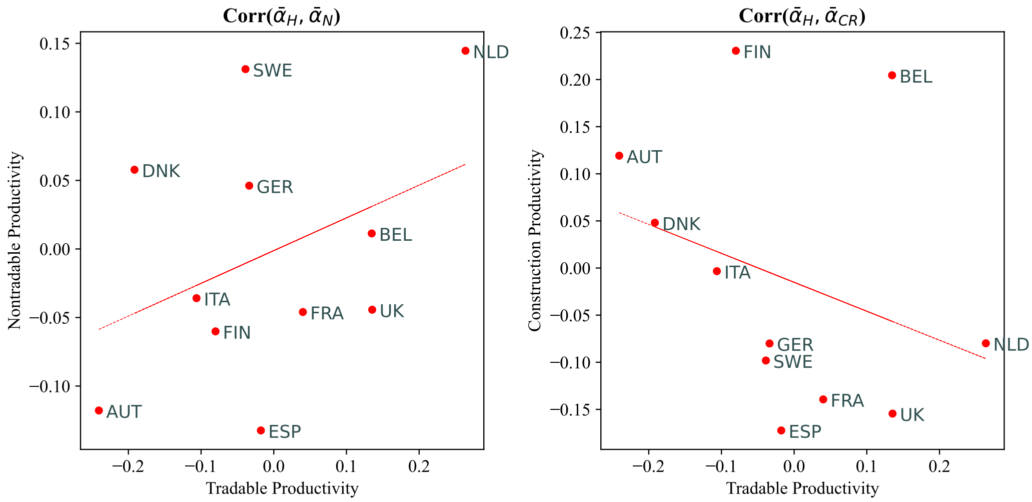

Figure 9: Cross-country Distributions of Sectoral Productivities

Once I remove the cross-country-level heterogeneity of construction sector productivities, the model-generated Balassa-Samuelson effect disappears, and renders β^R not significant, as in column (8). However, our point estimate of β^R still has a very lower value than that of β^{NT}. This is because, again, $Corr(ln(\frac{\bar{A}_j^H}{\bar{A}_{EU}^H}), ln(\frac{\bar{A}_j^{NT}}{\bar{A}_{EU}^{NT}})) = 0.76$. If I set $ln(\frac{\bar{A}_j^{NT}}{\bar{A}_{EU}^{NT}}) = 0$ for all countries, it would cause β^{NT} to be similar to that of β^R in column (7).

This connects to recent literature on stagnant productivity in the construction sector. Goolsbee and Syverson (2023) show that construction sector productivity has been decreasing in the US, but also in the EU 27 area as a whole. If the construction sector is the sector whose productivity does not grow in all countries while other nontradable sector productivity increases, the different growth rates of tradable sector productivity across countries should generate a strong Balassa-Samuelson effect via the rent real exchange rate. Because I have only eleven country observations of productivities, this naturally calls for more research on a more granular level to link the Balassa-Samuelson effect and construction sector productivities across regions.

Role of the Housing Market Table 11 shows how the model-generated Balassa-Samuelson effect changes as γ^R changes. We can clearly see that the model generates a stronger Balassa-Samuelson effect as the housing weight increases.

In summary, comparative statics simulations show that my model is able to generate a strong Balassa-Samuelson effect, especially via the housing sector, as in the data. Being different from our priors, the unique characteristics of housing service dampen the textbook Balassa-Samuelson hypothesis mechanism. A major portion of the Balassa-Samuelson effect

Table 11: Simulated Balassa-Samuelson Regressions: Role of Housing

	(1) Data	(2) Bond ($\gamma^R = 0.01$)	(3) Bond ($\gamma^R = 0.25$)	(4) Bond ($\gamma^R = 0.45$)
Balassa/Samuleson				
β	-0.26*	-0.45*	-0.57*	-0.72*
	(0.14)	(-0.63,-0.26)	(-0.76,-0.36)	(-0.93,-0.52)
β^T	-0.08	-0.24*	-0.21*	-0.19*
	(0.13)	(-0.34,-0.14)	(-0.32,-0.09)	(-0.31,-0.08)
β^{NT}	-0.33*	-0.75*	-0.65*	-0.61*
	(0.18)	(-1.06,-0.44)	(-0.98,-0.31)	(-0.94,-0.26)
β^R	-0.76***	-1.51*	-1.46*	-1.43*
	(0.19)	(-1.69,-1.31)	(-1.65,-1.26)	(-1.65,-1.21)

comes from the wealth effect through an incomplete market and the cross-sectional distribution of sectoral productivities.

4.2.3 Housing and the Backus-Smith Puzzle

Lastly, I examine the role of housing in the Backus-Smith puzzle via model simulation. Using model-simulated data, I replicate the following four panel regressions as in the empirical analysis section:

$$\begin{aligned}
\Delta q_{jt} &= \alpha + \beta \Delta c_{jt} + e_{jt}, \\
\Delta q_{jt}^T &= \alpha^T + \beta^T \Delta c_{jt} + e_{jt}^T, \\
\Delta q_{jt}^{NT} &= \alpha^{NT} + \beta^{NT} \Delta c_{jt} + e_{jt}^{NT}, \\
\Delta q_{jt}^R &= \alpha^R + \beta^R \Delta c_{jt} + e_{jt}^R.
\end{aligned}$$

Table 12 presents replication results under different calibrations. First, column (2) clearly shows that under a complete market, my model cannot replicate the negative β. It generates β closer to σ, which is calibrated as 2 in my model. This is because the complete market condition implies that $ln(C_t/C_t^*) = \frac{1}{\sigma} ln(P_t^*/P_t)$ for every state and time. So, this exactly shows the Backus-Smith puzzle found by Backus and Smith (1993). Moving to column (3), I assume an incomplete market but a very small housing expenditure share in the model by setting γ^R as 0.01. Though the model generates 0.38, which is much smaller than the case of column (1), it is still very far from its data counterpart, being statistically significantly different from 0. This again shows why the Backus-Smith puzzle is hard to resolve, even under the incomplete market assumption.

One of my main findings appears in column (4), where I set γ^R as 0.25, which is my baseline calibration generating a realistic rent expenditure share of 17% in the model. Now the model can replicate negative β, whose point estimate is very close to that of the data. In addition, such negative correlation comes largely from the β^R, whose value is -1.29. Though my model β^R is much bigger than its data counterpart, my model replicates the data pattern in which q^R accounts for a major portion of the negative Backus-Smith correlation.

Table 12: Simulated Backus-Smith Puzzle Regressions: Role of Housing

	(1) Data	(2) **Arrow-Deberu** ($\gamma^R = 0.25$)	(3) **Bond** ($\gamma^R = 0.01$)	(4) **Bond** ($\gamma^R = 0.25$)	(5) **Bond** ($\gamma^R = 0.45$)
Backus/Smith					
β	-0.14**	1.99*	0.38*	-0.12	-0.51
	(0.07)	(1.98,2.02)	(0.09,0.68)	(-0.53,0.28)	(-1.14,0.07)
β^T	0.02	1.21*	0.19	0.05	0.04
	(0.05)	(1.20,1.23)	(0.04,0.37)	(-0.18,0.27)	(-0.31,0.36)
β^{NT}	-0.15**	3.75*	0.68*	0.22	0.18
	(0.06)	(3.72,3.79)	(0.18,0.90)	(-0.46,0.90)	(-0.87,1.15)
β^R	-0.53**	0.82*	-0.79^*	-1.29^*	-1.69^*
	(0.23)	(0.81,0.83)	(-1.09,-0.51)	(-1.69,-0.89)	(-2.31,-1.11)

To understand why a realistically calibrated housing sector helps the model to generate a negative Backus-Smith correlation, we need to understand how model-generated real exchange rates and relative consumptions respond to each sectoral productivity shock. Note that these sectoral productivity shocks are the only sources of the business cycles in my model. The upper panel of the Figure 10 shows the impulse response functions (IRFs) of the aggregate real exchange rate (q) and the relative consumption (c) to a one standard deviation shock to each sector's relative productivity.

One notable observation is that the tradable sector shock decreases q (appreciates the real exchange rate) and increases c (increases the relative consumption) while the nontradable sector shock and construction sector shock increase q (depreciate the real exchange rate) and increase c. Given that c moves in the same direction for all types of shocks, a sign of the model-generated $Corr(\Delta c, \Delta q)$ will depend on the relative size of the effect of tradable sector shock on q compared with those of the nontradable and construction sector shock.

Since the mechanism behind the effect of the nontradable sector shock and construction shock are similar, it becomes easier to understand once we combine these two forces into one force as in lower panel of Figure 10. We can consider two forces in the model, in which the tradable sector productivity shock generates $Corr(\Delta c, \Delta q) < 0$ and the sum of the nontradable and construction sector productivity shocks generate $Corr(\Delta c, \Delta q) > 0$.

Given these characteristics of the shocks, showing how the IRFs of q and c to such shocks change under different housing weights is the most straightforward way to check the role of housing in the Backus-Smith correlation. Figure 11 shows how these model responses change when γ^R changes from 0.01 to 0.45 in the model. Comparing the dotted line ($\gamma^R = 0.01$)

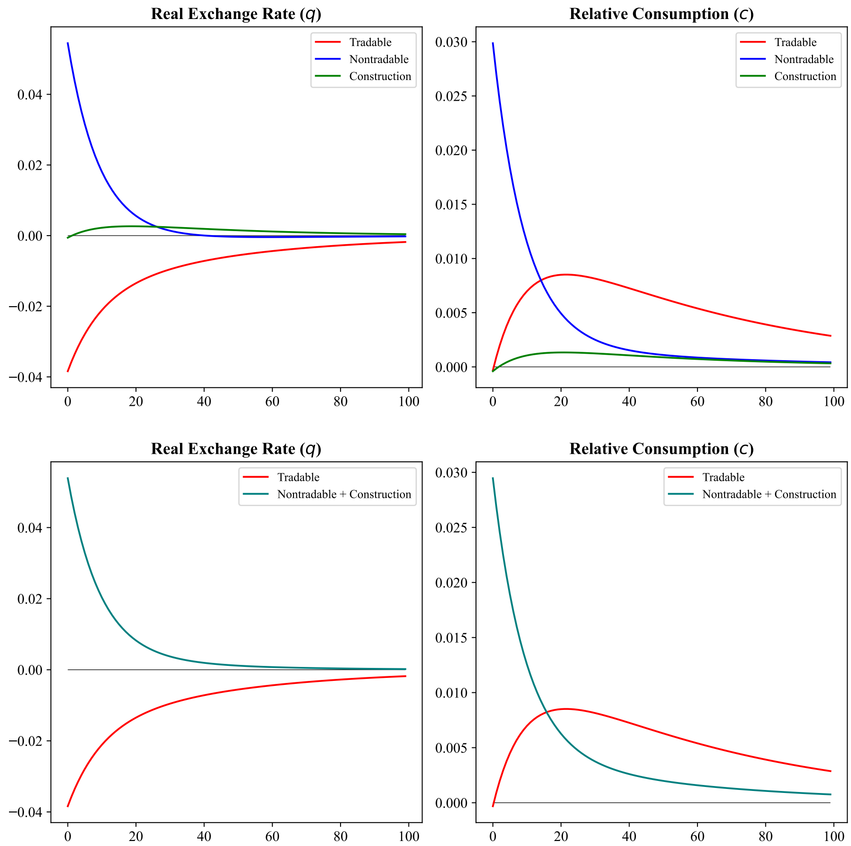

Figure 10: Model q and c Responses to Sectoral Productivity Shocks

with the solid line ($\gamma^R = 0.45$), we observe that the IRFs of q to all shocks are dragged down to lower values. In particular, while the effects of the nontradable and construction sector shocks get smaller, the effect of the tradable sector shock is sustained and the IRFs of q to that get more persistent. This means that under higher γ^R, the tradable sector shock effect gets amplified and the nontradable and construction sector shocks effect gets smaller, which causes the model to generate a more negative aggregate correlation between Δq and Δc.

To understand why this happens, we need to know that Δq is a expenditure-weighted average of $\Delta q^T, \Delta q^{NT}$, and Δq^R in the model, since we used the first-order approximation method for the model solution. This implies that as γ^R gets larger, Δq will be affected more by Δq^R but less by Δq^T and Δq^{NT}. This implies that understanding the response of each sectoral real exchange rate's IRFs is important. Figure 12 shows the IRFs of sectoral real exchange rates to both the tradable sector shock (ϵ_h) or the sum of the nontradable and construction sector shocks $(\epsilon_n + \epsilon_{cr})$ under both a complete and incomplete market.

Housing and Tradable Sector Productivity Shock First, we focus on the effect of the tradable sector shock. All sectoral real exchange rates appreciate to the positive tradable sec-

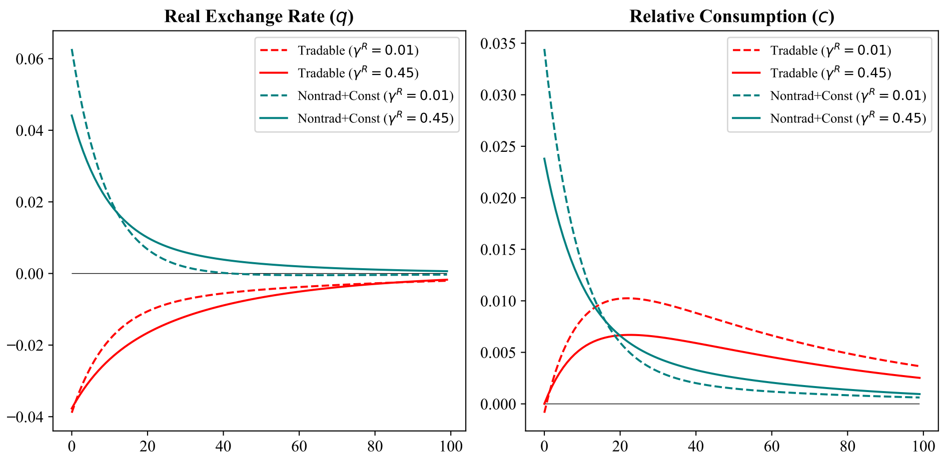

Figure 11: Role of Housing in IRFs of q and c

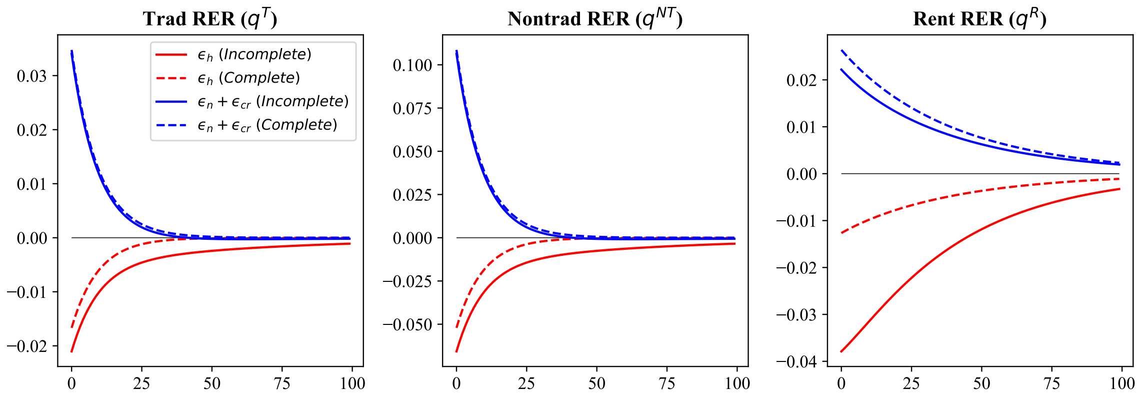

Figure 12: Impulse Response Functions of Sectoral Real Exchange Rates

tor productivity shock. Two forces generate such appreciations. The first mechanism is the well-known textbook Balassa-Samuelson hypothesis. When the tradable sector gets a positive productivity shock, while the price of the tradable is sustained due to foreign demand, it increases the marginal product of labor, and pushes up wages. This increased wage again increases the marginal production cost of nontradables and housing construction via the labor market. Note that q^T also appreciates due to the distribution margin. The other mechanism is the wealth effect generated under an incomplete market. If the home country receives a positive tradable sector productivity shock, this makes the home country wealthier than the foreign country, and causes it to consume more than the foreign country. This pushes up the home country's demand for all goods and services more than the foreign country, which results in the home country's appreciation.[29]

[29]To make this wealth effect work, the calibration should be within a certain parameter region such as high substitutability between tradable goods. My calibration is within such region, and for more detail, refer to Corsetti

While the Balassa-Samuleson hypothesis channel works under any risk-sharing assumption, the wealth effect only works under an incomplete market. This implies that the difference between IRFs under an complete market and incomplete market can be interpreted as the wealth effect. In Figure 12, it becomes clear that the q^R responds much more to the wealth effect than to other sectoral real exchange rates. The difference between the red dotted line and the red solid line is asymmetrically larger for q^R compared with q^T and q^{NT}. This arises from the fact that housing service supply is more inelastic than tradables or other nontradables, as we can see in Figure 7. On the top of this, we see that the q^R's impulse response function is most persistent. This is due to slower adjustment in the aggregate supply of the housing service.

On the other hand, comparing the dotted IRF of q^{NT} and q^R, we see that the textbook Balassa-Samuelson hypothesis mechanism works much less for q^R: q^{NT} appreciates almost 5%, while q^R only appreciates by 1%. This stems from residential-zoned land. While the land is important for housing production, it is not used in the tradable sector. Consequently, even though the tradable sector receives a positive productivity shock, it does not push up the price of land. Naturally, this leads to a smaller increase in the production cost of housing compared with that of the nontradables.

In summary, the unique characteristics of the housing service generate asymmetry between responses of q^R and those of q^T or q^{NT} to the change in the tradable sector productivity shock. This implies that when γ^R increases the response of q arises more from q^R, which generates the stronger response to the tradable sector shock and more negative Backus-Smith correlation forces.

Housing and Nontradable/Construction Sector Productivity Shock I move now to IRFs of sectoral real exchange rates to nontradable/construction sector shocks. While it is understandable that q^T shows little response to the productivity shocks, since its productivity is not directly affected, it is striking that q^R is not depreciating as much as q^{NT}.

Again, this recalls Figure 8. The large land input share (τ) and low depreciation rate of housing stock (δ) decrease the effect of the construction sector productivity shock on the supply of housing services. Consequently, the supply increase is limited and the price of housing service does not decrease much. If $\tau = 0$ and $\delta^S = 1$, the responses of q^R would be exactly same as those of q^{NT}.

In one sentence, the unique characteristics of housing service generate the asymmetry in which response of q^R to the productivity shock on its sector are much smaller than that of q^{NT}. This implies that when γ^R increases, q's response is more driven by that of q^R, which makes q responds less to sum of productivity shocks in nontradable and construction sector. In aggregate, this causes the model to generate smaller positive Backus-Smith correlation

et al. (2008).

forces.

Housing, Inelastic Supply, and the Backus-Smith Puzzle In general equilibrium, two roles of housing explained above work simultaneously when the γ^R gets larger in the model. Consequently, the housing sector causes the model to generate a more negative Backus-Smith correlation in aggregate.

Table 13 shows how simulation results change when I change housing-related parameters. As the complete market case has no meaning, I show only bond-economy cases. Column (2) is the baseline case where γ^R is 0.25, which generates the results similar to those in the data. When I change the τ to 0.01, in column (3), the model generates a stronger negative Backus-Smith correlation. Given very low δ, its effect in dampening the nontradable/construction sector shock is limited. On the other hand, the tradable sector shock gets amplified via a stronger Balassa-Samuelson hypothesis mechanism with smaller land-input share. Altogether, this leads to a stronger negative Backus-Smith correlation.

Column (4) is the case in which I set δ as 0.99 so that housing service has elastic supply like other nontradables. As we can clearly see, the model generates positive β and even positive β^R. In this case, housing service no longer shows dampened responses to the construction sector productivity shocks, since their prices decrease directly once they receive the positive productivity shocks. In addition, q^R's large response to the wealth effect via the tradable sector shock will also go away. Consequently, the model completely lose its capacity to generate a negative β estimate. In column (5), once I assume both low τ and high δ, the model gets further from the negative β. In column (3), because of low δ, τ could not play its role in dampening the response of q^R to the construction productivity shock. Now, given the high δ, it was doing its job by dampening the construction sector shocks but once τ is also set low, the model generates a stronger positive β than column (4). Lastly, in column (6), we can see that the cross-country distribution of sectoral productivities has no role in the Backus-Smith puzzle, since we are interested in more time-series dynamics and not cross-country level dynamics. These simulations show that the inelastic housing supply, dampened productivity shocks' supply effects, and their interaction with the wealth effect are the key mechanism for generating the negative Backus-Smith correlation in my model.

Lastly, I want to emphasize that this is a more general result than it seems. In this paper, I use housing service as a representative example because of its economic significance and unique characteristics. However, any goods and services can work similarly in the model, as long as they account for a large expenditure share (share of overall price level), their supply is very inelastic, and they require unique production factors that are immune to productivity gains.

Table 13: Simulated Backus-Smith Puzzle Regressions: Role of Housing

	(1) Data	(2) Bond Baseline	(3) Bond ($\tau = 0.01$)	(4) Bond ($\delta = 0.99$)	(5) Bond (τ=0.01, δ=0.99)	(6) Bond (7) + ($\bar{A}_j^{CR}/\bar{A}_{EU}^{CR} = 1$)
Backus/Smith						
β	-0.14**	-0.12	-0.32	0.62*	1.18*	1.21*
	(0.07)	(-0.53,0,28)	(-0.71,0.07)	(0.21,1.04)	(0.83,1.58)	(0.81,1.63)
β^T	0.02	0.05	-0.09	0.29*	0.30*	0.32*
	(0.05)	(-0.18,0.27)	(-0.29,0.12)	(0.09,0.49)	(0.15,0.47)	(0.13,0.50)
β^{NT}	-0.15**	0.22	-0.20	0.95*	0.95*	1.00*
	(0.06)	(-0.46,0.90)	(-0.84,0.44)	(0.36,1.57)	(0.47,1.48)	(0.43,1.56)
β^R	-0.53**	-1.29*	-1.44*	0.86*	3.48*	3.52*
	(0.23)	(-1.69,-0.89)	(-1.92,-0.95)	(0.17,1.57)	(2.72,4.27)	(2.73,4.38)

Discussion of Corsetti et al. (2008) and Benigno and Thoenissen (2008) It is worthwhile to discuss the relationship between my findings and the lessons of Corsetti et al. (2008) and Benigno and Thoenissen (2008).

First, Corsetti et al. (2008) resolve the Backus-Smith puzzle under an incomplete market by making either the tradable goods very non-substitutable and pushing up the terms of trade or by making the productivity shock itself very persistent under high substitutability of the tradable goods, which causes the wealth effect itself to be stronger.

My work is the same as Corsetti et al. (2008), in the sense that it uses the wealth effect via an incomplete market. However, how I amplify the relative price responses to such a shock differ from their methods. Rather than affecting the terms of trade or amplifying the size of demand increase itself, I make the response of the aggregate price level to a given demand shock stronger by making the aggregate supply more inelastic. This is achieved by incorporating inelastically supplied housing service in the aggregate consumption with a significant expenditure share. This aggregate supply curve, which becomes more inelastic, causes the aggregate price level to increase much more to a given wealth effect, which in turn causes more negative Backus-Smith correlation.

Second, Benigno and Thoenissen (2008) resolve the Backus-Smith puzzle by pushing up the nontradable prices via the Balassa-Samuelson hypothesis mechanism, given the tradable sector shocks. Different with them, my model focuses more on dampening the effect of nontradable and construction sector productivity shocks by using the unique characteristics of housing. While the land input share dampens a bit of the supply effect from the construction sector shock, what matters most in my paper is the fact that housing is capital that is directly consumed. Its stock is much larger than the flow, and it needs to be accumulated throughout the long investment. This setup renders any positive productivity gain in the construction sector not capable of increasing the supply, which generates a weaker supply shock effect. This removes the forces generating the positive Backus-Smith correlation from the model.

This again differs from the sector-specific capital model. In the sector-specific capital model, though those sector-specific capitals are hard to adjust, these capitals are not the only input for producing those sectors' output. These sectors also use labor, which makes the supply a bit more elastic. However, the housing service comes only from the housing stock, and not additional labor. This causes the supply to be dramatically inelastic, which helps the model improve on the Backus-Smith puzzle.

5 Conclusion

This paper has examined the role of the housing sector in international business cycles, with a specific focus on its role in real exchange rate dynamics. By using disaggregated relative price level data from eurozone countries, I show that the relative rent is the most volatile component of the aggregate real exchange rate. Moreover, I find out that the rent real exchange rate contributes to over half of the Balassa-Samuelson effect and the negative Backus-Smith correlation within eurozone countries.

Building on these empirical findings regarding the significance of the rent real exchange rate and the housing sector, I construct a two-country, three-sector model in which the realistically calibrated housing sector is incorporated. Simulation of the model using sectoral productivity shocks, directly calibrated from the EUKLEMS database yields several insights into the roles of rent real exchange rates and the housing sector. The inclusion of a realistically calibrated housing sector enables the model to generate greater cross-sectional and time-series variations. This is attributed to variations in sectoral productivity levels and a larger standard deviation of shocks to construction sector productivity.

Furthermore, unique housing characteristics, such as the role of land and the substantial stock compared with the relatively small flow, have been identified as factors that mitigate the textbook Balassa-Samuelson hypothesis mechanism. Still, my model demonstrates that the strong Balassa-Samuelson effect via q^R stems from the negative correlation between relative tradable sector productivity and that of the construction sector. Also, the wealth effect via incomplete markets amplifies the Balassa-Samuelson effect across all sectors. Finally, the model that incorporates the housing sector yields improved predictions for the Backus-Smith correlation. The inelastic housing supply intensifies the model's response to wealth effects (demand shocks) and mitigates its response to positive nontradable and construction sector productivity shocks. These mechanisms have shifted the response of aggregate real exchange rates in a negative direction for all types of shocks, which aids the model in generating a Backus-Smith correlation that closely aligns with the empirical data.

These implications not only shed light on the role of the housing sector in real exchange rate dynamics within eurozone countries, but also offer broader insights about the functioning of international business cycle models. Although housing rent has been used as a representative example due to its economic significance, the underlying principles revealed in this study are applicable to any goods and services characterized by limited productivity growth, reliance on unique production inputs not used in other sectors, or a substantial stock relative to the flow. With respect to addressing the Balassa-Samuelson effect, further exploration of stagnant construction sector productivity—which is observed in the recent literature—will be crucial. In the context of the Backus-Smith puzzle, it would be valuable to investigate heterogeneity among countries in terms of expenditure weights and the relative economic importance of goods and services subject to inelastic supply.

A Additional Backus-Smith Correlations Table

Table 14: Backus-Smith Correlations (per capita consumption)

	$Corr(\Delta c, \Delta q)$	$Corr(\Delta c, \Delta q^T)$	$Corr(\Delta c, \Delta q^{NT})$	$Corr(\Delta c, \Delta q^R)$
Austria	-0.048	-0.000	0.095	-0.445
Belgium	0.033	0.085	-0.009	-0.095
Finland	0.202	0.443	-0.052	-0.044
France	0.193	0.063	0.337	0.028
Germany	-0.145	-0.004	-0.022	-0.555
Greece	-0.207	-0.065	-0.235	-0.135
Ireland	-0.597	-0.359	-0.297	-0.718
Italy	0.082	-0.205	0.429	0.144
Luxembourg	-0.051	0.076	-0.037	-0.102
Netherlands	-0.138	-0.381	0.031	0.143
Portugal	-0.207	-0.231	-0.005	0.087
Spain	-0.024	0.183	-0.104	-0.110
Average	-0.075	-0.033	0.011	-0.150

$cc^* = ln(C_{it}/C_{EU12t})$ where C_{EU12t} is a geometric means of C over 12 eurozone countries. C is final consumption expenditure per capita.

B OECD Eurostat PPP Basic Headings

For the quality of the data, refer to the metadata[30] provided by Eurostat.

Table 15: Eurostat PPP Basic Headings

Class	Name	Class	Name	Class	Name
T	Rice	T	Electricity	T	Hotels, motels, inns and similar accommodation...
T	Flours and other cereals	T	Natural gas and town gas	T	Holiday centres, camping sites, youth hostels ...
T	Bread	T	Liquefied hydrocarbons (butane, propane, etc.)	T	Accommodation services of other establishments
T	Other bakery products	T	Liquid fuels	T	Electric appliances for personal care
T	Pizza and quiche	T	Solid fuels	T	Non-electrical appliances
T	Pasta products and couscous	T	Heat energy	T	Articles for personal hygiene and wellness, es...
T	Breakfast cereals	T	Household furniture	T	Jewellery
T	Other cereal products	T	Garden furniture	T	Clocks and watches
T	Beef and veal	T	Lighting equipment	T	Other personal effects
T	Pork	T	Other furniture and furnishings	NT	Cleaning, repair and hire of clothing
T	Lamb and goat	T	Carpets and other floor coverings	NT	Repair and hire of footwear
T	Poultry	T	Furnishing fabrics and curtains	NT	Services for the maintenance and repair of the...
T	Other meats	T	Bed linen	NT	Water supply
T	Edible offal	T	Table linen and bathroom linen	NT	Refuse collection
T	Dried, salted or smoked meat	T	Other household textiles	NT	Sewage collection
T	Other meat preparations	T	Refrigerators, freezers and fridge-freezers	NT	Other services relating to the dwelling n.e.c.
T	Fresh or chilled fish	T	Clothes washing machines, clothes drying machi...	NT	Repair of furniture, furnishings and floor cov...
T	Frozen fish	T	Cookers	NT	Repair of household textiles
T	Fresh or chilled seafood	T	Heaters, air conditioners	NT	Repair of household appliances
T	Frozen seafood	T	Cleaning equipment	NT	Domestic services by paid staff
T	Dried, smoked or salted fish and seafood	T	Other major household appliances	NT	Cleaning services
T	Other preserved or processed fish and seafood-...	T	Small electric household appliances	NT	Hire of furniture and furnishings
T	Milk, whole, fresh	T	Glassware, crystal-ware, ceramic ware and chin...	NT	Other domestic services and household services
T	Milk, low fat, fresh	T	Cutlery, flatware and silverware	NT	Medical services
T	Milk, preserved	T	Non-electric kitchen utensils and articles	NT	Dental services
T	Yoghurt	T	Repair of glassware, tableware and household u...	NT	Paramedical services
T	Cheese and curd	T	Major tools and equipment	NT	General hospitals
T	Other milk products	T	Small tools and miscellaneous accessories	NT	Mental health and substance abuse hospitals
T	Eggs	T	Cleaning and maintenance products	NT	Speciality hospitals
T	Butter	T	Other non-durable small household articles	NT	Nursing and residential care facilities
T	Margarine and other vegetable fats	T	Pharmaceutical products	NT	Maintenance and repair of personal transport e...
T	Olive oil	T	Other medical products	NT	Other services in respect of personal transpor...
T	Other edible oils	T	Therapeutic appliances and equipment	NT	Passenger transport by train
T	Other edible animal fats	T	New motor cars	NT	Passenger transport by underground and tram
T	Fresh or chilled fruit	T	Second-hand motor cars	NT	Passenger transport by bus and coach
T	Frozen fruit	T	Motor cycles	NT	Passenger transport by taxi and hired car with...
T	Dried fruit and nuts	T	Bicycles	NT	Passenger transport by sea and inland waterway
T	Preserved fruit and fruit-based products	T	Animal drawn vehicles	NT	Combined passenger transport
T	Fresh or chilled vegetables other than potatoe...	T	Tyres	NT	Other purchased transport services
T	Frozen vegetables other than potatoes and othe...	T	Spare parts for personal transport equipment	NT	Postal services
T	Dried vegetables, other preserved or processed...	T	Accessories for personal transport equipment	NT	Wired telephone services
T	Potatoes	T	Diesel	NT	Wireless telephone services
T	Crisps	T	Petrol	NT	Internet access provision services
T	Other tubers and products of tuber vegetables	T	Other fuels for personal transport equipment	NT	Bundled telecommunication services
T	Sugar	T	Lubricants	NT	Other information transmission services
T	Jams, marmalades and honey	T	Passenger transport by air	NT	Repair of audio-visual, photographic and infor...
T	Chocolate	T	Telephone and telefax equipment	NT	Maintenance and repair of other major durables...
T	Confectionery products	T	Equipment for the reception, recording and rep...	NT	Recreational and sporting services
T	Edible ices and ice cream	T	Equipment for the reception, recording and rep...	NT	Cinemas, theatres, concerts
T	Artificial sugar substitutes	T	Portable sound and vision devices	NT	Museums, libraries, zoological gardens
T	Sauces, condiments	T	Other equipment for the reception, recording a...	NT	Television and radio licence fees, subscriptions
T	Salt, spices and culinary herbs	T	Photographic and cinematographic equipment and...	NT	Hire of equipment and accessories for culture
T	Baby food	T	Personal computers	NT	Photographic services
T	Ready-made meals	T	Accessories for information processing equipment	NT	Other cultural services
T	Other food products n.e.c.	T	Software	NT	Education - HH
T	Coffee	T	Calculators and other information processing e...	NT	Restaurants, cafés and dancing establishments
T	Tea	T	Pre-recorded recording media	NT	Fast food and take away food services
T	Cocoa and powdered chocolate	T	Unrecorded recording media	NT	Canteens
T	Mineral or spring waters	T	Other recording media	NT	Hairdressing for men and children
T	Soft drinks	T	Major durables for outdoor recreation	NT	Hairdressing for women
T	Fruit and vegetable juices	T	Musical instruments and major durables for ind...	NT	Personal grooming treatments
T	Spirits	T	Games and hobbies	NT	Prostitution
T	Wine	T	Toys and celebration articles	NT	Repair of jewellery, clocks and watches
T	Beer	T	Equipment for sport, camping and open-air recr...	NT	Social protection
T	Tobacco	T	Garden products	NT	Life insurance
T	Narcotics	T	Plants and flowers	NT	Insurance connected with the dwelling
T	Clothing materials	T	Pets and related products	NT	Insurance connected with health
T	Garments for men	T	Veterinary and other services for pets	NT	Insurance connected with transport
T	Garments for women	T	Games of chance	NT	Other insurance
T	Garments for infants (0 to 2 years) and childr...	T	Books	NT	FISIM
T	Other articles of clothing and clothing access...	T	Newspapers	NT	Other financial services n.e.c.
T	Footwear for men	T	Magazines and periodicals	NT	Other services n.e.c.
T	Footwear for women	T	Miscellaneous printed matter	H	Actual rentals for housing
T	Footwear for infants and children	T	Stationery and drawing materials	H	Imputed rentals for housing
T	Materials for the maintenance and repair of th...	T	Package holidays		

30 https://ec.europa.eu/eurostat/cache/metadata/en/prc$_{p}$$pp_{e}$$sms.htmrelatedmd$1678716803148

C Relative Sectoral Productivity Estimation

In this section, I explain how I estimate the relative sectoral productivities of Eurozone countries. To estimate the sector-specific productivity shock process (i.e., tradable sector, nontradable sector and construction sector), I closely follow the procedure used by Berka et al. (2018) with a few modifications. The following is the overall procedure for calculating relative sectoral productivities.

1. Make a proper industry concordance between Groningen Growth and Development Centre (GGDC) 1997 and EUKLEMS & INTANProd 2023 in order to use the two datasets together.
2. Calculate the 1997 relative productivity level (against the 12 European countries) of each industry in each country by using the GGDC 1997 database.
3. Calculate the relative productivity growth of each industry in each country using EUKLEMS & INTANProd 2023 from 1995 to 2019.
4. Combining the levels and growth rates from the first and second steps and construct panel data on the relative productivity for each industry in each country from 1995 to 2019.
5. Aggregate industries into tradable, nontradable and construction sectors using industry-level value-added as weights.
6. Estimate the $AR(1)$ process for each sector and each country using the generated relative sectoral productivities from 2000 to 2019.

C.1 Sectoral Concordance

GGDC 1997 and EUKLEMS & INTANProd 2023 use different industry classification systems, as shown in Table 16 and Table 17. To align the two systems, I proceed as followings. First, I create concordance between industries as in Table 18. In some cases, to make mutually exclusive but informative connections, one industry in the GGDC industry classification system has been matched with two industries in the EUKLEMS classification system, or vice versa. When two industries in the GGDC are matched with one industry in the EUKLEMS, initial productivity levels are aggregated based on the sectoral output weights in that year. On the other hand, when two industries in the EUKLEMS are matched with one industry in the GGDC, each year's productivity growth rates are aggregated using the 2000-2019 average of the relative value-added weight of each sector.[31]

[31]This value-added weighted average allows the possibility that production inputs are not perfectly substitutable across industries, which is a more realistic approach than input-based aggregation methods.

C.2 Relative Productivity Level: The GGDC 1997 Database

I calculate industry-level relative productivity across European countries using the GGDC 1997 TFP database. For industry i and country j, the GGDC database provides

$$C_{ij1997} = \frac{A_{ij1997}}{A_{iUS1997}}. \tag{45}$$

Among all productivity measures, I use **multi-factor productivity from the sectoral output-based** approach, which is used by Berka et al. (2018). The GGDC 1997 database provides data on a large set of industries. Table 16 shows the set of industries for which data are available. Of all available industries, to make a proper connection with the EUKLEMS 2023 database, I use only a subset of industries. Table 17 provides the set of industries for which the data are available for the EUKLEMS 2023 database, and Table 18 shows how I create a concordance between the GGDC 1997 database and the EUKLEMS 2023 database.

According to the concordance in Table 18, I need to aggregate *(13) "Wood and products of wood and cork"* and *(14) "Pulp, paper, paper products, printing and publishing."* Also, I need to aggregate *(17) "Rubber and plastics products"* and *(18) "Other non-metallic mineral products."* To aggregate these industries, I use the relative weights computed by their relative sectoral output. To aggregate industries $i1$ and $i2$ into an industry named i, when $SO_{i1j1997}$ is the output for industry $i1$ in country j in 1997, I calculate the following:

$$C_{ij1997} = \frac{SO_{i1j1997}}{SO_{i1j1997} + SO_{i2j1997}} \frac{A_{i1j1997}}{A_{i1US1997}} + \frac{SO_{i2j1997}}{SO_{i1j1997} + SO_{i2j1997}} \frac{A_{i2j1997}}{A_{i2US1997}}. \tag{46}$$

This gives me cross-sectional data on relative productivities for all industries in Table 18 for all countries.

Given these, for each industry i, I calculate the simple average of Austria, Belgium, Denmark, Finland, France, Germany, Italy, the Netherlands, Sweden, the United Kingdom, and Spain as follows:[32]

$$C_{iEU1997} = \frac{1}{12} \sum_j C_{ij1997} \sim \frac{A_{iEU1997}}{A_{iUS1997}}. \tag{47}$$

Then $C_{iEU1997}$ will be the average productivity of Europe for industry i relative to that of the US. Finally, by dividing C_{ij1997} by $C_{iEU1997}$, we have the relative productivity of industry i in country j against the European average.

$$\tilde{A}_{ij1997} = \frac{C_{ij1997}}{C_{iEU1997}}. \tag{48}$$

[32]These are the countries that provide industry-level productivity growth data in EUKLEMS 2023. For consistency, I calculate the European average using only these countries.

C.3 Productivity Growth Rate: The EUKLEMS & INTANProd 2023 Databases

For growth rates, I use the EUKLEMS & INTANProd 2023 release to construct the industry-level productivity growth rate. While Berka et al. (2018) use the March 2011 updated version of EUKLEMS, I use the latest version of the database to extend the periods. This database covers the period from 1995 to 2020 for 27 European countries: the UK, the US, and Japan, and covers 42 industries.[33]

From the dataset, I use the value-added-based TFP index ($VATFP\_I$) and value-added in current prices (VA_CP) to calculate the industry-level productivity growth rate. $VATFP\_I$ shows how the productivity of a certain industry increases or decreases throughout the years. I set $VATFP\_I_{ij1997} = 100$ for all i and j.

As in the case of the GGDC 1997 database, I need to aggregate some industries to make a proper connection between the EUKLEMS & INTANProd 2023 and the GGDC database. In particular, for the EUKLEMS database, I need to aggregate *(12) "Manufacture of computer, electronic and optical products"* and *(13) "Manufacture of electrical equipment"*. Also, I need to aggregate *(17) "Electricity, gas, steam and air conditioning supply"* and *(18) "Water supply; sewerage, waste management and remediation activities"*. To aggregate these, I use the relative sizes of the time-series average of weights based on value-added in current prices.

$$
\begin{aligned}
VATFP\_I_{ijt} = & \frac{1}{T}\sum_t [\frac{VA\_CP_{i1jt}}{VA\_CP_{i1jt} + VA\_CP_{i2jt}}] VATFP\_I_{i1jt} \\
& + \frac{1}{T}\sum_t \frac{VA\_CP_{i2jt}}{VA\_CP_{i1jt} + VA\_CP_{i2jt}} VATFP\_I_{i2jt}.
\end{aligned} \tag{49}
$$

Then, for each industry, I calculate the European average index as follows:

$$
VATFP\_I_{iEUt} = exp(\frac{1}{12}\sum_j ln(VATFP\_I_{ijt})). \tag{50}
$$

Several observations are missing in the dataset. For example, the US has missing observation for *"Electricity, gas, steam and air conditioning supply"* but has one with a different name (likely because of the different classification system). I use the one with the different name. Spain has missing observations on *"Manufacture of computer, electronic and optical products"*, *"Manufacture of electrical equipment"*, and *"Computer, electronic, optical products; electrical equipment"*. I supplement these with the growth rates of the closest industries. Lastly, Belgium has missing observations for the growth rate of all industries from 1995 to 1998. I supplement these using the European average only for missing periods.

Lastly, given the calculated growth rate for each industry across countries, by using each

[33] This release is unique in the sense that it tried to incorporate intangible capital into growth accounting, which had not been tried before. In this appendix, I briefly discuss the parts that are related to my analysis. For more information, please refer to Corrado et al. (2023) and O'Mahony and Timmer (2009).

country's and Europe 12 countries' average growth rate, I calculate the relative productivity growth rate for each industry as follows:

$$\tilde{A}_{ijt} = \tilde{A}_{ij1997} \frac{VATFP\_I_{ijt}}{VATFP\_I_{iEUt}}. \tag{51}$$

C.4 Aggregation of Industries into Sectors

Once each industry's productivity (relative to the US) throughout the years is constructed, I again aggregate these relative productivities of those industries into tradable, nontradable, and construction sector productivities. When I aggregate industries into a sector, I use the value-added weighted average of all industries' relative productivities, which are in one of three sectors (tradable/nontradable/construction.) Table 18 shows which industry belongs to which sector.[34] This value-added approach is based on the *bottom-up* approach explained by Corrado et al. (2023). Since this allows the imperfect substitution of inputs between sectors, which is more realistic, I proceed using the bottom-up method.[35]

In addition, I use a statistical module rather than an analytical module for comparability with previous research. However, productivity series generated from the statistical module and analytical module (which incorporates intangible capital in calculating industry productivity) show correlation higher than 0.98 between them, which implies that this should not be a big issue.

Given $\tilde{A}_{ijt}$ for all industry i, for country j, we have the following relative sectoral productivities. Note that the construction sector is equal to the construction industry.

$$\tilde{A}_{Tjt} = \frac{\sum_{i \in T}(\bar{VA}_{ij}\tilde{A}_{ijt})}{\sum_{i \in T}\bar{VA}_{ij}}, \quad \tilde{A}_{NTjt} = \frac{\sum_{i \in NT}(\bar{VA}_{ij}\tilde{A}_{ijt})}{\sum_{i \in NT}\bar{VA}_{ij}}, \quad \tilde{A}_{CRjt} = \tilde{A}_{CRjt}. \tag{52}$$

By taking log to these $\tilde{A}_{Tjt}$, $\tilde{A}_{NTjt}$, and $\tilde{A}_{CRjt}$, we have

$$a_{Tjt} = log(\tilde{A}_{Tjt}), \quad a_{NTjt} = log(\tilde{A}_{NTjt}), \quad a_{CRjt} = log(\tilde{A}_{CRjt}). \tag{53}$$

[34] For the US, for industry 13, I use *Electricity, gas, steam; water supply, sewerage, waste management*. For other countries, I use *Electricity, gas, steam and air conditioning supply, Water supply; sewerage, waste management and remediation activities*. This difference comes from different industry classification systems in the US and Europe.

[35] Corrado et al. (2023, p. 36) explain how the industry aggregation can be implemented. There are two methods, *direct calculation* and *bottom-up*. The *Direct calculation* approach assumes a perfect mobility of input across industries, that labor and capital earn the same compensation in all industries, and that all industries have the same value-added function. So they aggregate all capital input, labor input, and value-added to calculate the TFP growth of the aggregate sector. The *Bottom-up* appraoch assumes that inputs are not perfectly mobile. Consequently, it adds capital input, labor input, and value-added as averages calculated with the weights of capital income, labor income, and value-added of a certain industry with respect to total industry. The aggregate TFP calculated using this approach reflects the value-added weighted contribution of industry-level TFP.

C.5 Estimation of Sectoral Productivities

Given these a_{Tjt}, a_{NTjt}, a_{CRjt} time series for all countries, I estimate the following AR(1) process as home country productivity relative to foreign country productivity using data from 2000 to 2019:

$$\alpha_{T,jt} - \bar{\alpha}_T = \rho_T(\alpha_{T,jt-1} - \bar{\alpha}_T) + \epsilon_{h,jt},$$
$$\alpha_{N,jt} - \bar{\alpha}_N = \rho_N(\alpha_{N,jt-1} - \bar{\alpha}_N) + \epsilon_{n,jt},$$
$$\alpha_{CR,jt} - \bar{\alpha}_{CR} = \rho_{CR}(\alpha_{CR,jt-1} - \bar{\alpha}_{CR}) + \epsilon_{cr,jt}.$$

After we estimate $\bar{a}_{Tj}$, ρ_{Tj}, $\bar{a}_{NTj}$, ρ_{NTj}, $\bar{a}_{CRj}$, ρ_{CRj}, we convert it to quarterly frequency parameters as follows: $\bar{a}_{Tj} = \bar{a}^q_{Tj}$, $\bar{a}_{NTj} = \bar{a}^q_{NTj}$, $\bar{a}_{CRj} = \bar{a}^q_{CRj}$, and $\rho^q_{Tj} = \rho^{(1/4)}_{Tj}$, $\rho^q_{NTj} = \rho^{(1/4)}_{NTj}$, $\rho^q_{CRj} = \rho^{(1/4)}_{CRj}$.

Lastly, by using the estimated residuals of all processes, I estimate the covariance-variance matrix for all sectors and countries (3 sectors $\times$ 8 industries.) I allow any potential positive correlation between productivity shocks across countries and sectors. Following Berka et al. (2018), I assume that $var(\epsilon_{h,jt}) = var(\epsilon^q_{h,jt})$ for all sector s.

C.6 Comparison with Berka et al. (2018)

First, the industry classification used here and the one used by Berka et al. (2018) are very similar. However, there are some differences due to differences in the versions of the EUKLEMS database used. First, I combine "*Wood and of wood and cork*" with "*Pulp, paper, paper printing, and publishing*" to obtain "*Manufacturing—Wood, paper, printing and reproduction.*" Second, "*Chemical, rubber, plastics and fuel*" is decomposed into "*Chemicals and chemical products*" and "*Rubber, plastics products, and other non-metallic products*". Lastly, I add "*Education*" and "*Health and social work*" to the industry list since they account for a certain portion of households' expenditure.

There are other differences as well too. First, Berka et al. (2018) used output-based productivity growth data, while my growth rate data are value-added-based. I use such measures because the EUKLEMS 2023 release only provides value-added-based productivity growth rates. Second, for aggregating industries' growth rate into that of the sector, I use the period-average of each industry's relative weight calculated based on the value-added of sectors, while Berka et al. (2018) use the relative weights based on sectoral output in 1995. Lastly, I use only 11 countries for the EU average, as those are the only available countries.

Table 16: Sectors in the GGDC 1997 TFP Level Database

	GGDC Industry Classification
1	**TOTAL INDUSTRIES**
2	*MARKET ECONOMY*
3	ELECTRICAL MACHINERY, POST AND COMMUNICATION SERVICES
4	Electrical and optical equipment
5	Post and telecommunications
6	GOODS PRODUCING, EXCLUDING ELECTRICAL MACHINERY
7	TOTAL MANUFACTURING, EXCLUDING ELECTRICAL
8	Consumer manufacturing
9	Food products, beverages and tobacco
10	Textiles, textile products, leather and footwear
11	Manufacturing nec; recycling
12	Intermediate manufacturing
13	Wood and products of wood and cork
14	Pulp, paper, paper products, printing and publishing
15	Coke, refined petroleum products and nuclear fuel
16	Chemicals and chemical products
17	Rubber and plastics products
18	Other non-metallic mineral products
19	Basic metals and fabricated metal products
20	Investment goods, excluding hightech
21	Machinery, nec
22	Transport equipment
23	OTHER PRODUCTION
24	Mining and quarrying
25	Electricity, gas and water supply
26	Construction
27	Agriculture, hunting, forestry and fishing
28	MARKET SERVICES, EXCLUDING POST AND TELECOMMUNICATIONS
29	DISTRIBUTION
30	Trade
31	Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of fuel
32	Wholesale trade and commission trade, except of motor vehicles and motorcycles
33	Retail trade, except of motor vehicles and motorcycles; repair of household goods
34	Transport and storage
35	FINANCE AND BUSINESS, EXCEPT REAL ESTATE
36	Financial intermediation
37	Renting of m&eq and other business activities
38	PERSONAL SERVICES
39	Hotels and restaurants
40	Other community, social and personal services
41	Private households with employed persons
42	*NON-MARKET SERVICES*
43	Public admin, education and health
44	Public admin and defence; compulsory social security
45	Education
46	Health and social work
47	Real estate activities

Table 17: Sectors in the EUKLEMS 2023 Release

	EUKLEMS 2023 Industry Classification
1	**Agriculture, forestry and fishing**
2	**Mining and quarrying**
3	**Manufacturing**
4	Manufacture of food products; beverages and tobacco products
5	Manufacture of textiles, wearing apparel, leather and related products
6	Manufacture of wood, paper, printing and reproduction
7	Manufacture of coke and refined petroleum products
8	Manufacture of chemicals and chemical products
9	Manufacture of basic pharmaceutical products and pharmaceutical preparations
10	Manufacture of rubber and plastic products and other non-metallic mineral products
11	Manufacture of basic metals and fabricated metal products, except machinery and equipment
12	Manufacture of computer, electronic and optical products
13	Manufacture of electrical equipment
14	Manufacture of machinery and equipment n.e.c.
15	Manufacture of motor vehicles, trailers, semi-trailers and of other transport equipment
16	Manufacture of furniture; jewellery, musical instruments, toys; repair and installation of machinery and equipment
17	**Electricity, gas, steam and air conditioning supply**
18	**Water supply; sewerage, waste management and remediation activities**
19	**Construction**
20	**Wholesale and retail trade; repair of motor vehicles and motorcycles**
21	Wholesale and retail trade and repair of motor vehicles and motorcycles
22	Wholesale trade, except of motor vehicles and motorcycles
23	Retail trade, except of motor vehicles and motorcycles
24	**Transportation and storage**
25	Land transport and transport via pipelines
26	Water transport
27	Air transport
28	Warehousing and support activities for transportation
29	Postal and courier activities
30	**Accommodation and food service activities**
31	**Information and communication**
32	Publishing, motion picture, video, television programme production; sound recording, programming and broadcasting activities
33	Telecommunications
34	Computer programming, consultancy, and information service activities
35	**Financial and insurance activities**
36	**Real estate activities**
37	**Professional, scientific and technical activities**
38	**Administrative and support service activities**
39	**Public administration and defence; compulsory social security**
40	Public administration, defence, education, human health and social work activities
41	**Education**
42	**Human health and social work activities**
43	Human health activities
44	Residential care activities and social work activities without accommodation
45	**Arts, entertainment and recreation**
46	Arts, entertainment, recreation; other services and service activities, etc.
47	**Other service activities**
48	**Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use**
49	**Activities of extraterritorial organisations and bodies**

Table 18: Sectoral Concordance

	EUKLEMS Industry	GGDC Industry	Industry Name	Sector
1	1	27	Agriculture, hunting, forestry and fishing	T
2	2	24	Mining and quarrying	T
3	4	9	Manufacturing - Food products, beverages and tobacco	T
4	5	10	Manufacturing - Textiles, textile products, leather and footwear	T
5	6	13 14	Manufacturing - Wood, paper, printing and reproduction	T
6	7	15	Manufacturing - Coke, refined petroleum products and nuclear fuel	T
7	8	16	Manufacturing - Chemicals and chemical products	T
8	10	17 18	Manufacturing - Rubber, plastics products, and other non-metallic products	T
9	11	19	Manufacturing - Basic metals and fabricated metal products	T
10	12 13	4	Manufacturing - Computer, electrical and optical equipment	T
11	14	21	Manufacturing - Machinery, nec	T
12	15	22	Manufacturing - Transport equipment	T
13	17 18	25	Electricity, gas, water supply, and waste management	NT
14	19	26	Construction	C
15	20	30	Wholesale trade, retail trade, and repair of vehicles	NT
16	24	34	Transport and storage	NT
17	30	39	Hotels and restaurants	NT
18	31	5	Information, post and telecommunications	NT
19	35	35	Finance and Business activities	NT
20	36	47	Real estate activities	NT
21	39	44	Public admin and defense; compulsory social security	NM
22	41	45	Education	NT
23	42	46	Health and social work	NT

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