Original Article

A decision making method based on interval type-2 fuzzy sets: An approach for ambulance location preference

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ABSTRACT

Selecting the best solution to deploy an ambulance in a strategic location is of the important variables that need to be accounted for improving the emergency medical services. The selection requires both quantitative and qualitative evaluation. Fuzzy set based approach is one of the well-known theories that help decision makers to handle fuzziness, uncertainty in decision making and vagueness of information. This paper proposes a new decision making method of Interval Type-2 Fuzzy Simple Additive Weighting (IT2 FSAW) as to deal with uncertainty and vagueness. The new IT2 FSAW is applied to establish a preference in ambulance location. The decision making framework defines four criteria and five alternatives of ambulance location preference. Four experts attached to a Malaysian government hospital and a university medical center were interviewed to provide linguistic evaluation prior to analyzing with the new IT2 FSAW. Implementation of the proposed method in the case of ambulance location preference suggests that the 'road network' is the best alternative for ambulance location. The results indicate that the proposed method offers a consensus solution for handling the vague and qualitative criteria of ambulance location preference.

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1. Introduction

Ambulance services and paramedic are very significant in society as they could save hundreds of lives daily by responding to emergency calls. They operate from stations that normally located in diverse areas to a medical center or hospital. With the assistance of control centers that responded to emergency calls, ambulances are dispatched when required. Ambulances are not always based in a building, but often at a very rudimentary location, such as parking lots [1]. More importantly, they are periodically redeployed to ensure a better coverage at all times. According to a study [2], a facility (ambulance) that is near a request point provides a better quality of coverage to that demand point than a facility situated a long way from that request point. Generally, ambulances do not patrol on streets between calls, but once they dispatched to the scene of an incident, they may be diverted to a more important call. Study of ambulance is part of emergency medical services (EMS). EMS is defined by the National Board of Health and Welfare as "health care provided by healthcare professionals within or adjacent to the ambulance" [3]. EMS is one the essential measures to optimize the safety of patients and also for sustaining human well-being. EMS is known as pre-hospital treatment and also known as services provided by hospital for emergency services to transport patients with illness and injuries to the hospital immediately to reduce patients' mortality, disability or suffering [1,4]. Management of the efficient EMS system is very critical and deserves particular attention by system planners [5]. The availability of an ambulance at a location may influence a chance of survival despite the minute difference of ambulance's arrival. Ambulance location is in a stochastic environment where the request calls arrive at the control center in a random manner. Travel time for a certain journey may contain randomness; the service time at the request call's scenes and hospitals is also uncertain. The above mentioned typical process of ambulance request and ambulance fulfillment processes as well the uncertain environments complicate the process of determining the strategic location for ambulances.

Recently, many efforts have been made to strengthen emergency management, particularly with respect to the placement of ambulances. A parallel tabu search heuristic [6,7], and stochastic

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optimization model [8,9] and local search heuristic [10] are among the methods used in redeployment problem for a fleet of ambulances. Recently, a study [11] proposed a dynamic ambulance management model for relocating idle ambulances that incorporate different performance measures related to response times. Study [12] used approximate dynamic programming (ADP) to solve ambulance dispatching and relocation problem. The ADP model has reduced the response time in the city of Vienna, the capital of Austria to 12.89%. This improvement is due to the main source for improvement which are the dispatching and relocation decisions. Study [13] adapted maximum expected coverage location problem model to determine the strategic location for a mobile ambulance in Shah Alam Selangor. Study [14] successfully implemented maximal covering location problem using local OpenStreetMap geodata together with Dijkstra, Quick Hull and Greedy Adding algorithms to solve the ambulance location selection in Johor Bahru Malaysia. Very recently, [15] analysed relocation strategies of ambulances using Double Standard Model to ensure a fair comparison of their performance. The above review supports the assertion that ambulance location management is tantamount to operational research problems. According to [16], emergency management is often conceptualized as a complex multi-objective optimization problem where an emergency situation is solved with limited resources. Therefore, the ambulance location problem is indeed a problem where multiple qualitative resources need to be considered concurrently.

It is shown that the existing literatures mostly deal with a specific operation, including ambulance placement with the aim to make an improvement for EMS. The methods used for the improvement varied from typical operations research methods to computational intelligence methods depending on respective research frameworks. However, the methods of multi-criteria decision making (MCDM) in the case of ambulance locations are rarely discussed despite the multiplicity in ambulance locations and qualitative criteria. Simple additive weighting (SAW) is one of the weighted based MCDM methods. This method is also known as a weighted linear combination or scoring method. One of the advantages of SAW method is proportional linear transformation of the raw data where the relative order of magnitude of the standardized scores remains equal. This method was used by [17] to accelerate the mechanism through eliminating unnecessary trade-offs. By using SAW method, a combined consequence of issues having high variation was obtained. As an extension of SAW, another weighted based MCDM method is fuzzy simple additive weighting (FSAW). The FSAW method depends on fuzzy numbers rather than the crisp numbers. It utilizes trapezoidal fuzzy numbers to show any imprecision in scores and weights. The FSAW method applies fuzzy weighting to approach experts' preferences. In literature, the FSAW has been successfully applied in diverse applications (see, for example, [18–23]. However, in many practical cases, it is pretty challenging for experts to express their preferences using one layer fuzzy membership function of type-1 fuzzy sets (T1 FS). Most of the existing FSAW methods are built from linguistic terms based on T1 FS. In reality, some decision might not be given as an exact relative crisp scale. The linguistic interval scales are used instead. The authors decide to use interval type-2 fuzzy sets (IT2 FS) scale for expressing linguistic evaluation due to the fact that the IT2 FS provides more flexibility to present uncertainties than T1 FS. Furthermore, IT2 FS can be used as an indicator of uncertainty where the larger length of the interval may capture more room of uncertainty that happened during the process of information gathering from the decision makers [24].

In this paper, the authors are motivated with the advantages of the FSAW and IT2 FS and desirous of exploring the possible merger of these two entities in MCDM framework. This proposed method can be seen as a FSAW framework where interval type-2 fuzzy numbers are used in linguistic scales. The introduction of IT2 FS in the FSAW gives a new look to the FSAW framework. The proposed method of interval type-2 FSAW is an extension of interval fuzzy additive weighting where T1 FS is substituted with IT2 FS. An IT2 FS is described by the footprint of uncertainty where this footprint is bounded by the lower membership function and upper membership function [25]. This footprint adds a new description of uncertainty. In addition, rather than the direct computation of typical defuzzification method, our proposed method applies the fuzzy ranking method as to reduce T2 FS to T1 FS. This proposed approach allows each decision matrix is made with trapezoidal IT2 FS as a measurement scale instead of using classical trapezoidal fuzzy numbers to represent the judgment scales and the weights of criteria. Unlike the FSAW method, we used the fuzzy ranking method based on IT2 FS as a reduction method. By incorporating fuzzy ranking method, this method offers a more detailed and comprehensive procedure. Although we used the IT2 FS linguistic scales and fuzzy ranking method, the proposed method is made without loss of generality the FSAW procedure. The proposed method could be employed to solve ambulance placement problem where limited qualitative resources and stochastic environment are present. The proposed method also inclusively considers experts' ambiguities, uncertainties and vagueness in evaluating ambulance location. Specifically, this paper aims to develop a new FSAW decision making method based on IT2FS (IT2 FSAW) and its application to a case of ambulance location preference in EMS management. The remainder of the paper is organized as follows. In Section 2, we present some basic notations and definitions that are needed in this paper. In Section 3, we present the procedures of the proposed method based on fuzzy SAW, IT2 FS and fuzzy ranking method. The proposed method is then applied to the case study for evaluating ambulance location preference. The implementation of this application is presented in Section 4. Section 5 concludes.

2. Preliminaries

This section introduces the elementary definitions and concepts of type-2 fuzzy set theory and fuzzy ranking method.

Definition 1 [26]. A type-2 fuzzy set $\widetilde{A}$ in the universe of discourse X is characterized by a type-2 membership function $\mu_{\widetilde{A}}(x, u)$ where

$$\widetilde{A} = \{((x, u), \mu_{\widetilde{A}}(x, u)) | \forall x \in X, \forall u \in J_x\}, \tag{1}$$

$x \in X, u \in J_x, J_x$ represents the main membership of x, $J_x \in [0, 1]$, $\mu_{\widetilde{A}}(x, u)$ represents the secondary grade of (x, u) and $0 \leqslant \mu_{\widetilde{A}}(x, u) \leqslant 1$. The type-2 fuzzy set $\widetilde{A}$ also can be denoted as:

$$\widetilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\widetilde{A}}(x, u)/(x, u), \tag{2}$$

where $x \in X, u \in J_x, J_x \subseteq [0, 1]$ and $\int$ represents the unification over all admissible x and u. A type-2 membership function is three-dimensional and the third dimension (i.e., $\mu_{\widetilde{A}}(x, u)$) which offers a degree of freedom in managing uncertainties.

Definition 2 [26]. Let $\widetilde{A}$ be a type-2 fuzzy set in the universe of discourse X denoted by the type-2 membership function $\mu_{\widetilde{A}}$. If all the secondary grades $\mu_{\widetilde{A}}(x, u)$ of $\widetilde{A}$ are equal to 1, then $\widetilde{A}$ is called an interval type-2 fuzzy set. It symbolically shown as:

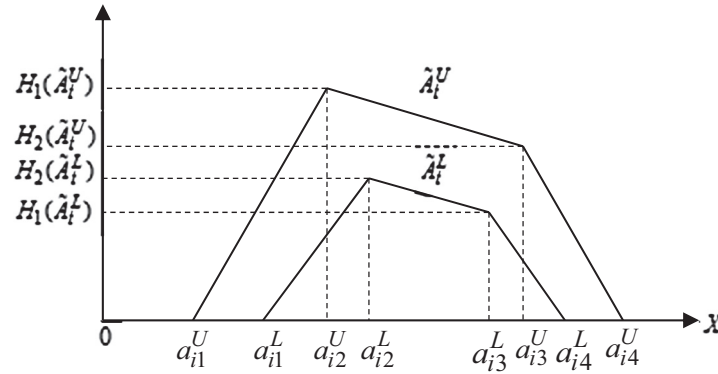$$\widetilde{A} = \int_{x \in X} \int_{u \in J} 1/(x, u), \tag{3}$$

Fig. 1. The UMF of IT2 trapezoidal fuzzy set and the LMF of IT2 trapezoidal fuzzy set.

where $x \in X$, $u \in J_x$, $J_x \subseteq [0, 1]$ and $\int$ denotes the union over all admissible x and u.

Definition 3 [26]. Vagueness of an interval type-2 fuzzy set $\widetilde{A}$ can be denoted by the unification of the main memberships in a circumscribed area. This area is called as "footprint of uncertainty". It is shown as follows:

$$FOU(\widetilde{A}) = \cup_{x \in X} J_x, \tag{4}$$

$$\widetilde{A}^u(x) = \overline{FOU(\widetilde{A})}, \quad \forall x \in X \tag{5}$$

$$\widetilde{A}^L(x) = \underline{FOU(\widetilde{A})}, \quad \forall x \in X \tag{6}$$

The $FOU(\widetilde{A})$ represents the "footprint of uncertainty" of the interval type-2 fuzzy set $\widetilde{A}$. The $\widetilde{A}^U$ and $\widetilde{A}^L$ represent the upper membership function (UMF) and the lower membership function (LMF) that enclosed $FOU(\widetilde{A})$, respectively, $\widetilde{A}^U(x) \in [0, 1]$, $\widetilde{A}^L(x) \in [0, 1]$, $\widetilde{A}^L(x) \leqslant \widetilde{A}^U(x)$ and $x \in X$.

A type-1 fuzzy set be a distinct case of interval type-2 fuzzy sets because the membership function of uncertainties (i.e. $FOU(\widetilde{A})$) is disappeared.

Definition 4 [27]. The UMF of an IT2 FS and the LMF of an IT2 FS are type-1 membership functions respectively.

The UMF of IT2 trapezoidal fuzzy set and the LMF of IT2 trapezoidal fuzzy set are shown in Fig. 1.

Fig. 1 shows a trapezoidal IT2 FS,

$$\widetilde{\widetilde{A}}_i = (\widetilde{A}_i^U, \widetilde{A}_i^L)$$
$$= ((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(\widetilde{A}_i^U), H_2(\widetilde{A}_i^U), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(\widetilde{A}_i^L), H_2(\widetilde{A}_i^L)).$$

where $H_j(\widetilde{A}_i^U)$ represents the membership value of the element $a_{i(j+1)}^U$ in the upper trapezoidal membership function $\widetilde{A}_i^U$, $1 \leqslant j \leqslant 2$, $H_j(\widetilde{A}_i^L)$ represents the membership value of the element $a_{i(j+1)}^L$ in the lower trapezoidal membership function $\widetilde{A}_i^L$, $1 \leqslant j \leqslant 2$, $H_1(\widetilde{A}_i^U) \in [0, 1]$, $H_2(\widetilde{A}_i^U) \in [0, 1]$, $H_1(\widetilde{A}_i^L) \in [0, 1]$, $H_2(\widetilde{A}_i^L) \in [0, 1]$, and $1 \leqslant i \leqslant n$.

Definition 5 [28]. Let $\widetilde{A}_s^U$ and $\widetilde{A}_t^U$ be upper trapezoidal MF of the IT2 FS $\widetilde{A}_s$ and $\widetilde{A}_t$, respectively. The two IT2 FSs are shown in Fig. 2, where $\widetilde{A}_s^U = (a_{s1}^U, a_{s2}^U, a_{s3}^U, a_{s4}^U; H_1(\widetilde{A}_s^U), H_2(\widetilde{A}_s^U))$ and $\widetilde{A}_t^U = (a_{t1}^U, a_{t2}^U, a_{t3}^U, a_{t4}^U; H_1(\widetilde{A}_t^U), H_2(\widetilde{A}_t^U))$.

In the interest of defining the likelihood $p(\widetilde{A}_s^U \geqslant \widetilde{A}_t^U)$ of $\widetilde{A}_s^U \geqslant \widetilde{A}_t^U$, the strength E_{ts} of $\widetilde{A}_t^U$ over $\widetilde{A}_s^U$ by considering the difference between a_{sk}^U and a_{tk}^U, where $1 \leqslant k \leqslant 4$, and by considering the difference between $H_k(\widetilde{A}_s^U)$ and $H_k(\widetilde{A}_t^U)$, where $1 \leqslant k \leqslant 2$. The strength E_{ts} of $\widetilde{A}_t^U$ over $\widetilde{A}_s^U$ is as follows:

$$E_{ts} = \frac{N_{ts}}{D_{ts}}$$
$$= \frac{\sum_{k=1}^{4} \max(a_{tk}^U - a_{sk}^U, 0) + (a_{t4}^U - a_{s1}^U) + \sum_{k=1}^{2} \max(H_k(\widetilde{A}_t^U) - H_k(\widetilde{A}_s^U), 0)}{\sum_{k=1}^{4} |a_{tk}^U - a_{sk}^U| + (a_{s4}^U - a_{s1}^U) + (a_{t4}^U - a_{t1}^U) + \sum_{k=1}^{2} |H_k(\widetilde{A}_t^U) - H_k(\widetilde{A}_s^U)|}, \tag{7}$$
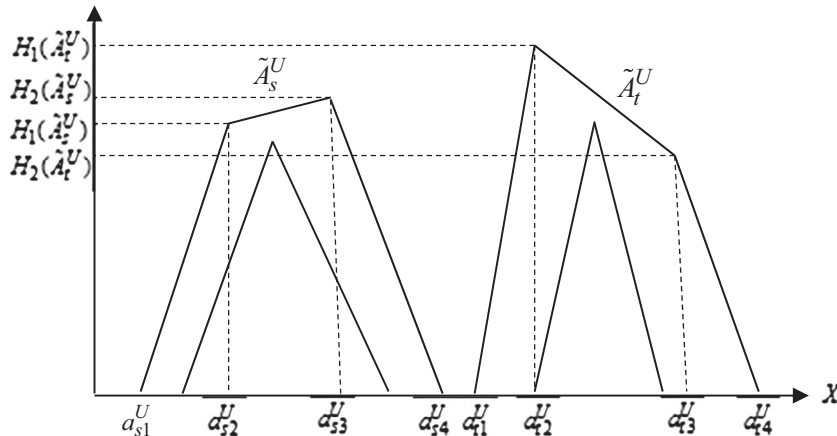

Fig. 2. Two interval triangular type-2 fuzzy sets $\widetilde{A}_s$ and $\widetilde{A}_t$..

where D_{ts} represents the summation of the absolute difference between a_{tk}^U and a_{sk}^U, where $1 \leqslant k \leqslant 4$, the absolute difference between a_{s4}^U and a_{s1}^U, the absolute difference between a_{t4}^U and a_{t1}^U, and the absolute difference between $H_k(\widetilde{A}_t^U)$ and $H_k(\widetilde{A}_s^U)$, where $1 \leqslant k \leqslant 2$; N_{ts} denotes summation of the difference between a_{tk}^U and a_{sk}^U, where $1 \leqslant k \leqslant 4$, the difference between a_{t4}^U and a_{s1}^U, and the difference between $H_k(\widetilde{A}_t^U)$ and $H_k(\widetilde{A}_s^U)$, where $1 \leqslant k \leqslant 2$. Because the strength E_{ts} of $\widetilde{A}_t^U$ over $\widetilde{A}_s^U$ might not lie between 0 and 1, in order to let the likelihood $p(\widetilde{A}_s^U \geqslant \widetilde{A}_t^U)$ of $\widetilde{A}_s^U \geqslant \widetilde{A}_t^U$ lie between 0 and 1, the likelihood $p(\widetilde{A}_s^U \geqslant \widetilde{A}_t^U)$ of $\widetilde{A}_s^U \geqslant \widetilde{A}_t^U$ is defined as follows:

$$p(\widetilde{A}_s^U \geqslant \widetilde{A}_t^U) = \max(1 - \max(E_{ts}, 0), 0) = \max\left(1 - \max\left(\frac{N_{ts}}{D_{ts}}, 0\right), 0\right)$$

$$= \max\left(1 - \max\left(\frac{\sum_{k=1}^{4}\max(a_{tk}^U - a_{sk}^U, 0) + (a_{t4}^U - a_{s1}^U) + \sum_{k=1}^{2}\max(H_k(\widetilde{A}_t^U) - H_k(\widetilde{A}_s^U), 0)}{\sum_{k=1}^{4}|a_{tk}^U - a_{sk}^U| + (a_{s4}^U - a_{s1}^U) + (a_{t4}^U - a_{t1}^U) + \sum_{k=1}^{2}|H_k(\widetilde{A}_t^U) - H_k(\widetilde{A}_s^U)|}, 0\right), 0\right)$$

(8)

If $E_{ts} \leqslant 0$, then $p(\widetilde{A}_s^U \geqslant \widetilde{A}_t^U) = 1$, where $E_{ts} \leqslant 0$, means that $\widetilde{A}_s^U$ dominates $\widetilde{A}_t^U$ absolutely; if $0 < E_{ts} < 1$, then $0 < p(\widetilde{A}_s^U \geqslant \widetilde{A}_t^U) < 1$; if $E_{ts} \geqslant 1$, then $p(\widetilde{A}_s^U \geqslant \widetilde{A}_t^U) = 0$, where $E_{ts} \geqslant 1$ means that $\widetilde{A}_t^U$ dominates $\widetilde{A}_s^U$ absolutely; if $0 < E_{ts} < 1$, then, the greater the value of E_{ts}, the lesser the likelihood $p(\widetilde{A}_s^U \geqslant \widetilde{A}_t^U)$ of $\widetilde{A}_s^U \geqslant \widetilde{A}_t^U$. It should be noted that the likelihood $p(\widetilde{A}_s^U \geqslant \widetilde{A}_t^U)$ of $\widetilde{A}_s^U \geqslant \widetilde{A}_t^U$ has the following properties:

i. $0 \leqslant p(\widetilde{A}_s^U \geqslant \widetilde{A}_t^U) \leqslant 1$,
ii. $p(\widetilde{A}_s^U \geqslant \widetilde{A}_t^U) + p(\widetilde{A}_t^U \geqslant \widetilde{A}_s^U) = 1$,
iii. $p(\widetilde{A}_s^U \geqslant \widetilde{A}_s^U) = 0.5$.

These preliminaries are being used in the development of the new proposed IT2 FSAW.

3. The proposed method

The fuzzy simple additive weighting (FSAW) was introduced to handle uncertainty in linguistic judgment [29–32], where fuzzy numbers are employed instead of the crisp numbers. In evaluation processes of decision making, the fuzzy set theory is germane to subjective judgment and quantitative assessment compared to classical evaluation method which is applying crisp values [33,34]. The FSAW utilizes trapezoidal fuzzy numbers to show any fuzziness in scores and weights. Unlike FSAW, the interval type-2 trapezoidal fuzzy numbers are utilized in the proposed method. The proposed method encompasses three stages. In the first stage, the evaluations of alternatives with respect to criteria and weights of criteria are expressed in IT2 FS. In the second stage, the weight for each criterion is computed with the aim to construct weighted decision matrices. In the third stage, ranking of alternatives is determined using the fuzzy ranking method. The six-step procedure that considered the three stages is proposed as follows.

Step 1: Establish a committee of experts who are familiar and experienced with an MCDM problem. Identify the criteria to be used in the MCDM evaluation. Introduce the linguistic variables to access criterion importance or degree of importance for individual criteria.
Step 2: Construct the decision matrix Y_p of the p-th decision maker, and build the aggregated fuzzy rating (AFR) matrix respectively.

$$Y_p = (\tilde{f}_{ij}^p)_{m \times n} = \begin{matrix} & z_1 & z_2 & \cdots & z_n \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{matrix} \begin{bmatrix} \tilde{\tilde{f}}_{11}^p & \tilde{\tilde{f}}_{12}^p & \cdots & \tilde{\tilde{f}}_{1n}^p \\ \tilde{\tilde{f}}_{21}^p & \tilde{\tilde{f}}_{22}^p & \cdots & \tilde{\tilde{f}}_{2n}^p \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}_{m1}^p & \tilde{\tilde{f}}_{m2}^p & \cdots & \tilde{\tilde{f}}_{mn}^p \end{bmatrix}$$

(9)

$$\overline{Y} = (\tilde{\tilde{f}}_{ij})_{m \times n},$$

(10)

where $\tilde{\tilde{f}}_{ij} = \left(\frac{\tilde{\tilde{f}}_{ij}^1 \oplus \tilde{\tilde{f}}_{ij}^2 \oplus \ldots \oplus \tilde{\tilde{f}}_{ij}^k}{k}\right) f_1, f_2, \ldots, f_m$ represent the criteria and $z_1, z_2, \ldots, z_n$ represents alternative. $\tilde{\tilde{f}}_{ij}$ is an IT2 FS, $1 \leqslant i \leqslant m$, $1 \leqslant j \leqslant n$, $1 \leqslant p \leqslant k$ and k represents the number of decision makers.

Step 3: Create the weighting matrix W_p of the criteria of the p-th decision maker and compute aggregated fuzzy weight (AFW) $\overline{W}$,
Let $\tilde{\tilde{w}}_i^p = (a_i, b_i, c_i, d_i)$, $i = 1, 2, \ldots, m$ be the linguistic weight specified to the subjective criteria $C_1, C_2, \ldots, C_h$, and objective criteria $C_{h+1}, C_{h+1}, \ldots, C_n$ by decision maker D_t.

$$W_p = (\tilde{w}_i^p)_{1 \times m} = \begin{matrix} & f_1 & f_2 & \cdots & f_m \\ & \end{matrix}\begin{bmatrix} \tilde{\tilde{w}}_1^p & \tilde{\tilde{w}}_2^p & \cdots & \tilde{\tilde{w}}_m^p \end{bmatrix},$$

(11)

$$\overline{W} = (\tilde{w}_i)_{1 \times m},$$

(12)

where $\tilde{\tilde{w}}_i = \frac{\tilde{w}_i^1 \oplus \tilde{w}_i^2 \oplus \ldots \oplus \tilde{w}_i^k}{k}$ is an interval type-2 fuzzy set, $1 \leqslant i \leqslant m$, $1 \leqslant p \leqslant k$. k represents the number of experts.

Step 4: Defuzzify the fuzzy weights of every criteria by normalizing the weights and build the weight vector. In order to defuzzify the weights of fuzzy criteria, the 'signed distance formula' is used [35].
Defuzzification of $\overline{W}$ is represented as:

$$d(\overline{W}_j) = \frac{1}{4}(\tilde{w}_j^1 + \tilde{w}_j^2 + \tilde{w}_j^3 + \tilde{w}_j^4), \quad j = 1, 2, \ldots, n$$

(13)

The crisp value of the normalized weight for criteria represented as $\widetilde{\widetilde{W}}$ is set by:

$$\widetilde{\widetilde{W}}_j = \frac{d(\overline{W}_j)}{\sum_{j=1}^{n} d(\overline{W}_j)}, \quad j = 1, 2, \ldots, n$$

(14)

where $\sum_{j=1}^{n} \widetilde{\widetilde{W}}_j = 1$. Therefore, the weight vector $W = [\widetilde{\widetilde{W}}_1, \widetilde{\widetilde{W}}_2, \ldots, \widetilde{\widetilde{W}}_n]$ is constructed.

Step 5: Construct the weighted decision matrix D by multiplying the fuzzy rating matrix with their individual weight vectors.

$$D = \overline{Y} \otimes W = \begin{bmatrix} \tilde{\tilde{f}}_{11} & \tilde{\tilde{f}}_{12} & \cdots & \tilde{\tilde{f}}_{1n} \\ \tilde{\tilde{f}}_{21} & \tilde{\tilde{f}}_{22} & \cdots & \tilde{\tilde{f}}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}_{m1} & \tilde{\tilde{f}}_{m2} & \cdots & \tilde{\tilde{f}}_{mn} \end{bmatrix} \otimes \begin{bmatrix} \widetilde{\widetilde{W}}_1 \\ \widetilde{\widetilde{W}}_2 \\ \cdot \\ \cdot \\ \cdot \\ \widetilde{\widetilde{W}}_n \end{bmatrix}$$

$$= \begin{bmatrix} \tilde{\tilde{f}}_{11} \otimes \widetilde{\widetilde{W}}_1 \oplus \tilde{\tilde{f}}_{12} \otimes \widetilde{\widetilde{W}}_2 \oplus \ldots \tilde{\tilde{f}}_{1n} \otimes \widetilde{\widetilde{W}}_n \\ \tilde{\tilde{f}}_{12} \otimes \widetilde{\widetilde{W}}_1 \oplus \tilde{\tilde{f}}_{22} \otimes \widetilde{\widetilde{W}}_2 \oplus \ldots \tilde{\tilde{f}}_{2n} \otimes \widetilde{\widetilde{W}}_n \\ \cdot \\ \cdot \\ \cdot \\ \tilde{\tilde{f}}_{m1} \otimes \widetilde{\widetilde{W}}_1 \oplus \tilde{\tilde{f}}_{m2} \otimes \widetilde{\widetilde{W}}_2 \oplus \ldots \tilde{\tilde{f}}_{mn} \otimes \widetilde{\widetilde{W}}_n \end{bmatrix} = \begin{bmatrix} \widetilde{\widetilde{A}}_1 \\ \widetilde{\widetilde{A}}_2 \\ \cdot \\ \cdot \\ \cdot \\ \widetilde{\widetilde{A}}_i \end{bmatrix}$$

(15)

Step 6: Calculate the ranking value $Rank(\widetilde{\widetilde{A}}_i)$ of the IT2 FS $\widetilde{\widetilde{A}}_i$, where $1 \leqslant i \leqslant n$ using Eq. (16–18) and Eq. (20).

The upper and lower fuzzy preference matrix P^U, P^L can be attained as follows:

$$P^U = \begin{bmatrix} p(\widetilde{A}_1^U \geqslant \widetilde{A}_1^U) & p(\widetilde{A}_1^U \geqslant \widetilde{A}_2^U) & \cdots & p(\widetilde{A}_1^U \geqslant \widetilde{A}_n^U) \\ p(\widetilde{A}_2^U \geqslant \widetilde{A}_1^U) & p(\widetilde{A}_2^U \geqslant \widetilde{A}_2^U) & \cdots & p(\widetilde{A}_2^U \geqslant \widetilde{A}_n^U) \\ \vdots & \vdots & \ddots & \vdots \\ p(\widetilde{A}_n^U \geqslant \widetilde{A}_1^U) & p(\widetilde{A}_n^U \geqslant \widetilde{A}_2^U) & \cdots & p(\widetilde{A}_n^U \geqslant \widetilde{A}_n^U) \end{bmatrix} \quad (16)$$

$$P^L = \begin{bmatrix} p(\widetilde{A}_1^L \geqslant \widetilde{A}_1^L) & p(\widetilde{A}_1^L \geqslant \widetilde{A}_2^L) & \cdots & p(\widetilde{A}_1^L \geqslant \widetilde{A}_n^L) \\ p(\widetilde{A}_2^L \geqslant \widetilde{A}_1^L) & p(\widetilde{A}_2^L \geqslant \widetilde{A}_2^L) & \cdots & p(\widetilde{A}_2^L \geqslant \widetilde{A}_n^L) \\ \vdots & \vdots & \ddots & \vdots \\ p(\widetilde{A}_n^L \geqslant \widetilde{A}_1^L) & p(\widetilde{A}_n^L \geqslant \widetilde{A}_2^L) & \cdots & p(\widetilde{A}_n^L \geqslant \widetilde{A}_n^L) \end{bmatrix} \quad (17)$$

Calculate the ranking value of the upper trapezoidal MF, $Rank(\widetilde{A}_i^U)$, the ranking value of the lower trapezoidal MF, $Rank(\widetilde{A}_i^L)$, and the ranking value of IT2FS, $Rank(\widetilde{\widetilde{A}}_i)$.

$$Rank(\widetilde{A}_i^U) = \frac{1}{n(n-1)} \left(\sum_{k=1}^{n} p(\widetilde{A}_i^U \geqslant \widetilde{A}_k^U) + \frac{n}{2} - 1 \right) \quad (18)$$

$$Rank(\widetilde{A}_i^L) = \frac{1}{n(n-1)} \left(\sum_{k=1}^{n} p(\widetilde{A}_i^L \geqslant \widetilde{A}_k^L) + \frac{n}{2} - 1 \right) \quad (19)$$

$$Rank(\widetilde{\widetilde{A}}_i) = \frac{Rank(\widetilde{A}_i^U) + Rank(\widetilde{A}_i^L)}{2} \quad (20)$$

where $1 \leqslant i \leqslant n$, and $\sum_{i=1}^{n} Rank(\widetilde{A}_i^U)$, $\sum_{i=1}^{n} Rank(\widetilde{A}_i^L)$, and $\sum_{i=1}^{n} Rank(\widetilde{\widetilde{A}}_i)$ are all equal to 1. The values of $Rank(\widetilde{\widetilde{A}}_i)$, indicate the preferences of alternatives. The proposed method is applied to the case of ambulance location preference.

4. Application to ambulance location preference

A committee of four experts has been identified and was invited to provide a qualitative evaluation using linguistic variables pertaining to EMS. The experts were interviewed in four different sessions in order to tap their evaluation regarding the preference of ambulance deployment with respect to the criteria in EMS. The linguistic evaluation is truly practical, especially in the presence of qualitative criteria [18]. The experts in this case are a medical officer at a public university medical center, an emergency department officer and two medical officers attached to the emergency department of a government funded hospital in Kuala Terengganu, Malaysia. All the four experts have more than five years of experience in ambulance management and emergency department. Profiles of the experts are presented in Table 1.

The experts are required to evaluate the relative measurement of alternatives with respect to criteria using interval type-2 linguistic terms. The alternatives of the case are road network (A_1), petrol station (A_2), parking lot (A_3), public clinic (A_4), and highway (A_5). Apart from the alternatives, several qualitative criteria also play their parts in emergency management. The selected criteria are response time (C_1), demand (C_2), coverage area (C_3), and ambulance workload (C_4). These alternatives and criteria are retrieved from several related research. For example, [36] used a real-world road network from Teleatlas which is reachable by car to test a formulated mixed integer program. Study [1] suggested parking lot as a possible ambulance location. Study [14] used road

Table 1
Personal profiles of experts.

Experts	Designation	Sector	Experience in EMS	Qualification
D_1	Medical officer	Hospital of public university	9 years	MD
D_2	Medical officer	Government hospital	5 years	MBBS
D_3	Medical officer	Government hospital	7 years	MBBS
D_4	Emergency department officer	Government hospital	15 years	B.Sc
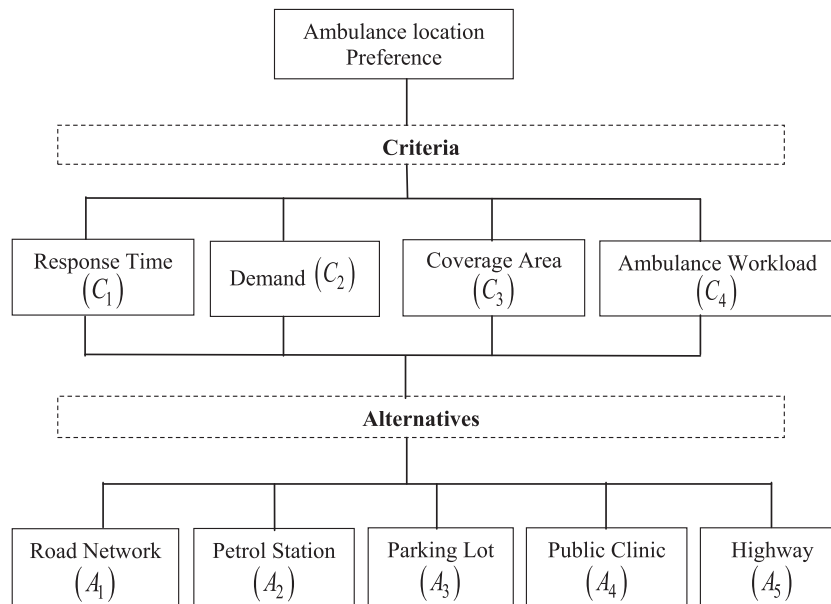
Fig. 3. The hierarchical structure of the case study.

network and petrol station as possible location sites to place an ambulance. Study [13] used government buildings and petrol stations as their landmarks in order to determine the strategic location for mobile ambulance. Studies [37,38] investigated the location of the ambulance cases along the highway with the purpose to reduce the mean user response time and the inequality of the ambulance workloads. The hierarchy structure of the decision making problem is depicted in Fig. 3.

The proposed IT2 FSAW method to determine the best ambulance location is composed in the following steps.

Step 1: A team of experts comprises three medical officers (D_1, D_2, D_3) and an emergency department officer (D_4) were asked to evaluate the alternative for ambulance location preference. The experts utilized the linguistic terms in Table 2 to rate the alternatives with admiration to each criterion. The linguistic variables are presented as IT2 FS.

Based on the linguistic terms in Table 3, the experts also provide fuzzy ratings of alternatives with respect to each subjective criterion.

Table 4 shows the ratings of alternatives with respect to individual subjective criteria evaluated by four experts.

Weights of the criteria evaluated by the expert team can be seen in Table 5.

Step 2: By using the information in Table 4, and the formulation in Eq. (9), the decision matrices Y_1, Y_2, Y_3 and Y_4 for the alternatives A_1, A_2, A_3, A_4 and A_5 are obtained.

$$Y_1 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array}\begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 \\ VG & G & G & G & G \\ VG & G & F & VG & G \\ VG & VG & F & VG & G \\ P & P & F & VP & P \end{bmatrix},$$

$$Y_2 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array}\begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 \\ G & G & G & G & G \\ G & G & G & G & G \\ MG & G & MG & G & F \\ P & P & F & P & F \end{bmatrix}$$

$$Y_3 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array}\begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 \\ G & G & G & G & G \\ VG & G & G & G & MG \\ G & G & G & G & F \\ G & P & MP & P & MG \end{bmatrix},$$

$$Y_4 = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array}\begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 \\ G & MG & VG & VG & G \\ MG & MG & F & G & MG \\ G & G & MG & G & F \\ MG & F & F & MG & F \end{bmatrix}$$

Table 2
Linguistic terms and their respective IT2 FS for importance weight of each criteria [27,28,38].

Linguistic terms	Interval type-2 fuzzy sets
Very Low (VL)	((0.0, 0,0.1;1,1), (0,0,0,0.05;0.9,0.9))
Low (L)	((0.0,0.1,0.1,0.3;1,1), (0.05,0.1,0.1,0.2;0.9,0.9))
Medium Low (ML)	((0.1,0.3,0.3,0.5;1,1), (0.2,0.3,0.3,0.4;0.9,0.9))
Medium (M)	((0.3,0.5,0.5,0.7;1,1), (0.4,0.5,0.5,0.6;0.9,0.9))
Medium High (MH)	((0.5,0.7,0.7,0.9;1,1), (0.6,0.7,0.7,0.8;0.9,0.9))
High (H)	((0.7,0.9,0.9,1;1,1), (0.8,0.9,0.9,0.95;0.9,0.9))
Very High (VH)	((0.9,1,1,1;1,1), (0.95,1,1,1;0.9,0.9))

Table 3
Linguistic terms and their corresponding IT2 FS for importance weight of alternatives with respect to criteria.

Linguistic terms	Interval type-2 fuzzy sets
Very Poor (VP)	((0,0,0,0.1;1,1), (0,0,0,0.05;0.9,0.9))
Poor (P)	((0.0,0.1,0.1,0.3;1,1), (0.05,0.1,0.1,0.2;0.9,0.9))
Medium Poor (MP)	((0.1,0.3,0.3,0.5;1,1), (0.2,0.3,0.3,0.4;0.9,0.9))
Fair (F)	((0.3,0.5,0.5,0.7;1,1), (0.4,0.5,0.5,0.6;0.9,0.9))
Medium Good (MG)	((0.5,0.7,0.7,0.9;1,1), (0.6,0.7,0.7,0.8;0.9,0.9))
Good (G)	((0.7,0.9,0.9,1;1,1), (0.8,0.9,0.9,0.95;0.9,0.9))
Very Good (VG)	((0.9,1,1,1;1,1), (0.95,1,1,1;0.9,0.9))

Table 4
Linguistic evaluation of the ambulance location alternatives with respect to the criteria.

Criteria	Alternatives	Decision makers			
		D_1	D_2	D_3	D_4
C_1	A_1	VG	VG	VG	P
	A_2	G	G	VG	P
	A_3	G	F	F	F
	A_4	G	VG	VG	VP
	A_5	G	G	G	P
C_2	A_1	G	G	MG	P
	A_2	G	G	G	P
	A_3	G	G	MG	F
	A_4	G	G	G	P
	A_5	G	G	F	F
C_3	A_1	G	VG	G	G
	A_2	G	G	G	P
	A_3	G	G	G	MG
	A_4	G	G	G	P
	A_5	G	MG	F	MG
C_4	A_1	G	MG	G	MG
	A_2	MG	MG	G	F
	A_3	VG	F	MG	F
	A_4	VG	G	G	MG
	A_5	G	MG	F	F

Table 5
Importance of weights for criteria.

Criteria	Decision makers			
	D_1	D_2	D_3	D_4
C_1	VH	VH	VH	MH
C_2	VH	VH	H	MH
C_3	H	VH	VH	H
C_4	VH	VH	H	MH

Construct the aggregated fuzzy ratings matrix, $\overline{Y}$ using Eq. (10) and the information in Table 2. It is presented in the following matrix.

$$\overline{Y} = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array}\begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 \\ \tilde{\tilde{f}}_{11} & \tilde{\tilde{f}}_{12} & \tilde{\tilde{f}}_{13} & \tilde{\tilde{f}}_{14} & \tilde{\tilde{f}}_{15} \\ \tilde{\tilde{f}}_{21} & \tilde{\tilde{f}}_{22} & \tilde{\tilde{f}}_{23} & \tilde{\tilde{f}}_{24} & \tilde{\tilde{f}}_{25} \\ \tilde{\tilde{f}}_{31} & \tilde{\tilde{f}}_{32} & \tilde{\tilde{f}}_{33} & \tilde{\tilde{f}}_{34} & \tilde{\tilde{f}}_{35} \\ \tilde{\tilde{f}}_{41} & \tilde{\tilde{f}}_{42} & \tilde{\tilde{f}}_{43} & \tilde{\tilde{f}}_{44} & \tilde{\tilde{f}}_{45} \end{bmatrix}$$

where $\tilde{\tilde{f}}$ is the average of linguistic evaluation.

Step 3: Acquire the weighting matrix, W_1, W_2, and W_3, using the information in Table 4 and substitute it into Eq. (11).

$$W_1 = \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} VH & VH & H & VH \end{bmatrix}$$

$$W_2 = \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} VH & VH & VH & VH \end{bmatrix}$$

$$W_3 = \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} VH & H & VH & H \end{bmatrix}$$

$$W_4 = \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} MH & MH & H & MH \end{bmatrix}$$

Evaluate the aggregated fuzzy weights (AFW) $\overline{W}$, using Eq. (12).

$$\overline{W} = \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{bmatrix} \tilde{\tilde{w}}_1 & \tilde{\tilde{w}}_2 & \tilde{\tilde{w}}_3 & \tilde{\tilde{w}}_4 \end{bmatrix}$$

where

$\tilde{\tilde{w}}_1 = (0.80, 0.93, 0.93, 0.975; 1, 1), (0.86, 0.93, 0.93, 0.95; 0.9, 0.9),$

$\tilde{\tilde{w}}_2 = (0.75, 0.90, 0.90, 0.975; 1, 1), (0.83, 0.90, 0.90, 0.9375; 0.9, 0.9),$

$\tilde{\tilde{w}}_3 = (0.80, 0.95, 0.95, 1; 1, 1), (0.88, 0.95, 0.95, 0.975; 0.9, 0.9),$

$\tilde{\tilde{w}}_4 = (0.75, 0.90, 0.90, 0.975; 1, 1), (0.83, 0.90, 0.90, 0.9375; 0.9, 0.9).$

Step 4: Compute the defuzzified values of AFWs using Eq. (13). Eq. (14) is used to normalize weight for individual criterion. Therefore the weight vector is obtained.

$$W = \left[\widetilde{\widetilde{W}}_1, \widetilde{\widetilde{W}}_2, \widetilde{\widetilde{W}}_3, \widetilde{\widetilde{W}}_4 \right]$$

where

$\widetilde{\widetilde{W}}_1 = (0.2522), (0.2519),$

$\widetilde{\widetilde{W}}_2 = (0.2452), (0.2451),$

$\widetilde{\widetilde{W}}_3 = (0.2574), (0.2580),$

$\widetilde{\widetilde{W}}_4 = (0.2452), (0.2451).$

Step 5: In this step, weighted decision matrix D can be constructed using Eq. (15).

$$D = \left[\tilde{\tilde{d}}_1 \quad \tilde{\tilde{d}}_2 \quad \tilde{\tilde{d}}_3 \quad \tilde{\tilde{d}}_4 \right]$$

where;

$\tilde{\tilde{d}}_1 = (0.6268, 0.7895, 0.7895, 0.8955; 1, 1), (0.7082, 0.7896, 0.7896, 0.8426; 0.9, 0.9),$

$\tilde{\tilde{d}}_2 = (0.5347, 0.7099, 0.7099, 0.8404; 1, 1), (0.6225, 0.7101, 0.7101, 0.7753; 0.9, 0.9),$

$\tilde{\tilde{d}}_3 = (0.5263, 0.7200, 0.7200, 0.8697; 1, 1), (0.6231, 0.7199, 0.7199, 0.7948; 0.9, 0.9),$

$\tilde{\tilde{d}}_4 = (0.5967, 0.7533, 0.7533, 0.8529; 1, 1), (0.6752, 0.7535, 0.7535, 0.8032; 0.9, 0.9),$

$\tilde{\tilde{d}}_5 = (0.4940, 0.6879, 0.6879, 0.8440; 1, 1), (0.5909, 0.6878, 0.6878, 0.7659; 0.9, 0.9).$

Step 6: Construct the upper and lower fuzzy preference matrix using Eq. (16) and Eq. (17).

$$P^U = \begin{bmatrix} 0.5000 & 0.7574 & 0.7231 & 0.6625 & 0.7841 \\ 0.2426 & 0.5000 & 0.4563 & 0.3370 & 0.5795 \\ 0.2769 & 0.5437 & 0.5000 & 0.3846 & 0.6104 \\ 0.3375 & 0.6630 & 0.6154 & 0.5000 & 0.7086 \\ 0.2159 & 0.4205 & 0.3896 & 0.2914 & 0.5000 \end{bmatrix}$$

$$P^L = \begin{bmatrix} 0.5000 & 0.8880 & 0.8502 & 0.7666 & 0.9184 \\ 0.1120 & 0.5000 & 0.4177 & 0.2234 & 0.6530 \\ 0.1498 & 0.5823 & 0.5000 & 0.2801 & 0.6975 \\ 0.2334 & 0.7766 & 0.7199 & 0.5000 & 0.8368 \\ 0.0816 & 0.3470 & 0.3025 & 0.1632 & 0.5000 \end{bmatrix}$$

Ranking values $Rank(\widetilde{\widetilde{A}}_i)$ and ranking of alternatives can be obtained using Eq. (18), Eq. (19) and Eq. (20). It is shown in Table 6.

According to the framework of IT2 FSAW, $Rank(\widetilde{\widetilde{A}}_1) > Rank(\widetilde{\widetilde{A}}_4) > Rank(\widetilde{\widetilde{A}}_3) > Rank(\widetilde{\widetilde{A}}_2) > Rank(\widetilde{\widetilde{A}}_5)$, the preference order of the alternatives A_1, A_2, A_3, A_4 and A_5 is $A_1 > A_4 > A_3 > A_2 > A_5$, where the symbol ">" means superior to. The road network A_1 (0.3896), represents slightly higher than other alternatives. Therefore, the A_1 is the best ambulance location for deployment of an ambulance. The order of the rest alternatives is public clinic, parking lot, petrol station and highway. This preference order is obtained from the implementation of the proposed method with the linguistic data. It is better to have a comparable analysis as to check the

Table 6
Final evaluation results.

Alternatives	$Rank(\tilde{A}_i^U)$	$Rank(\tilde{A}_i^L)$	$Rank(\widetilde{\widetilde{A}}_i)$
A_1	0.3689	0.4103	0.3896
A_2	0.2596	0.2422	0.2509
A_3	0.2763	0.2675	0.2719
A_4	0.3187	0.3389	0.3288
A_5	0.2348	0.1995	0.2172

Table 7
Ranking order under different methods.

Method	Ranking values for alternatives	Preference order
FSAW [20]	$A_1 = 0.7453, A_2 = 0.6883, A_3 = 0.6789,$ $A_4 = 0.7295, A_5 = 0.6455$	$A_1 > A_4 > A_2 > A_3 > A_5$
IT2-FTOPSIS [38]	$A_1 = 0.9187, A_2 = 0.7596, A_3 = 0.7351,$ $A_4 = 0.3861, A_5 = 0.7291$	$A_1 > A_2 > A_3 > A_5 > A_4$
The proposed IT2 FSAW	$A_1 = 0.3896, A_2 = 0.2509, A_3 = 0.2719,$ $A_4 = 0.3288, A_5 = 0.2172$	$A_1 > A_4 > A_3 > A_2 > A_5$

consistency of the preference order. The similar linguistic data sets are iterated to two other MCDM methods. Specifically, the proposed method is comparable with FSAW and IT2-FTOPSIS. The summary of the preference orders of the proposed method against the other two methods is presented in Table 7.

The preference orders of the proposed IT2-FSAW method and the other two methods are slightly inconsistent. The FSAW utilized linguistic terms represented by T1 FS where its membership functions are totally crisp thereby, it is weak in handling uncertainty. The limitation of T1 FS saw the preference order obtained using FSAW method is slightly inconsistent with the result obtained using the proposed method. As IT2 FSAW, the IT2-FTOPSIS also used the similar IT2 FS, but they differ in weight determination for each criterion. This might explain the small inconsistency in preference orders among the three methods.

5. Conclusions

Locating ambulance is one of the most important issues in the emergency management. Due to rapid growth of population, many researchers initiate work to identify the most strategic location in deploying ambulances, which in return, can benefit the residents. Hence, ambulance services are urged to seek and recognize the suitable and strategic location for the benefit of people. At the same time, the services also may minimize the loss of precious life. A good ambulance location can guarantee the arrival of an ambulance to the emergency call scene within a defined time threshold. Because of time pressure, lack of experience and personality, experts often evaluate the criteria and alternatives in the case of ambulance location using the linguistic variable. In this paper, we have proposed an interval type-2 fuzzy simple additive weighting to evaluate ambulance location selection. The linguistic scale of interval type-2 fuzzy set was used contrary to crisp numbers and type-1 fuzzy numbers to express the experts' evaluation for alternatives with respect to criteria and the weights of each criterion. The use of interval type-2 fuzzy sets makes the decision process considerably more practical. Fuzzy simple additive weighting was employed to compute the weights of criteria and fuzzy ranking method was adopted to rank the alternatives of strategic ambulance location. The proposed method has the ability to capture the vagueness of human thinking style and effectively solved multi-criteria decision making problems of ambulance location selection. Particularly, it provides emergency departments with a flexible manner to evaluate the strategic area for locating ambulance under fuzzy environments. It was consensual

agreed on road networks as the most influential alternative in ambulance location selection. Perhaps a reliable validating mechanism, such as sensitivity analysis could be introduced to investigate the stability of the suggested method. The implementation of the suggested method to other real applications is still open for further investigation.

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