HA NOI UNIVERSITY **FINAL EXAM**
Faculty of Information Technology **Academic year: 2023-2024**

Course: ***CALCULUS*** *Exam duration: 90 Minutes*
Student's name: ..... *ID: .....*

Exam code : 01

Instructions:

- This exam should be taken without text or notes or electronic devices.
- The use of calculators is not allowed.

Part I*. (7 point) Short answer: just write a final answer without explanation.*

(1). Row reduce the matrix to reduced echelon form $D = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$

(2). Let $\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 7$. Find the determinant of the matrix:

$$A = \begin{bmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{bmatrix}.$$

(3). For what values of x, if any, is the function $f(x) = \dfrac{x^2-16}{x^2-5x+4}$ discontinuous.

(4). Evaluate $I = \int \dfrac{dx}{x^2-3x-4}$.

(5). Evaluate $\lim_{n\to+\infty} \dfrac{x^4}{e^x}$.

(6). Find the second-order partial derivative $f_{xy}(x,y)$ of $f(x,y) = \ln(x^2 + y^2)$.

(7). Find solutions of the Cauchy problem: $\begin{cases} y' + 2xy = x; \\ y(0) = 1. \end{cases}$

Part II. *(3 point) Full answer: full credit is awarded only for well-presented, correct solutions in which all of your work is shown.*

(8). Determine α such that the system of equations is consistent and find the general solutions:

$$\begin{cases} x_1 - x_2 + 2x_3 = 1 \\ x_1 + x_2 + 5x_3 + x_4 = \alpha \\ 2x_1 - 2x_2 + 3x_3 + x_4 = 2\alpha + 2 \\ x_2 + x_3 + x_4 = \alpha \end{cases}$$

(9). Locate all relative extrema and saddle points of $f(x,y) = \dfrac{x^2 y}{2} - xy + y^2$.