HANU
HANOI UNIVERSITY
FACULTY OF INFORMATION TECHNOLOGY
DEPARTMENT OF COMPUTER SCIENCE

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DISCRETE MATHEMATIC
LEC-01:
Statements and Propositions

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Lecture 1

- Propositions
- Quantifiers
- Quantifiers and Negation

Propositions

A proposition is a ***statement*** *which is either* ***true*** *or* ***false***

<u>Examples:</u>

- $\sqrt{3}$ is irrational
- $1 + 3 = 5$
- FIT was established in Nov. 2006

Propositions

A proposition is a ***statement*** *which is either* ***true*** *or* ***false***

Examples (NOT propositions):

- $10 + 5$
- $x^2 + 3x = 5$
- Mr. fit often visits F-IT Online. (What is “often?”)
- Bill Gates is popular. (What is “popular?”)

Propositions

Propositions may be joined together to form more complex statements.

- **Conjunction**
- **Disjunction**
- **Negation**
- **Implication**

Conjunction

$P \wedge Q$ (P "and" Q)

True *when* **both** *P and Q are* **True**

P	Q	$P \wedge Q$
True	True	True
True	False	False
False	True	False
False	False	False

PROPOSITIONS

Disjunction

$P \vee Q$ (P “or” Q)

True when **one** of P or Q is **True**

P	Q	$P \vee Q$
True	True	True
True	False	True
False	True	True
False	False	False

PROPOSITIONS

Negation

¬ *P* (“not” *P*)

True when ***P*** is **False**

P	¬ *P*
True	False
False	True

PROPOSITIONS

P: 3 is odd

Q: 4 is odd

R: 5 is even

$$P \wedge R$$
$$P \vee R$$
$$\neg Q$$

$P \vee \neg P$ (tautology)

$P \wedge \neg P$

(contradiction)

PROPOSITIONS

Remark

A tautology is a proposition that's always TRUE.

A contradiction is a proposition that's always FALSE.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

P R O P O S I T I O N S

Implication

$$P \Rightarrow Q \quad (P \text{ “implies” } Q)$$

False *only when* ***P*** **is True** *and* ***Q*** **is False**

<u>Examples:</u>

- **If you stand in the rain, then you’ll get wet.**
- **If you got 6.0 in IELTS, you didn’t have to take the test again**

PROPOSITIONS

Implication

$P \Rightarrow Q$ (P "implies" Q)

False *only when* ***P*** **is True** *and* ***Q*** **is False**

P	*Q*	*P* =>*Q*
True	True	True
True	False	False
False	True	True
False	False	True

PROPOSITIONS

Implication

P	Q	P =>Q
True	True	True
True	False	False
False	True	True
False	False	True

P=>Q is always true when P is false (vacuously true)

- If pigs can fly, then horses can read
- If 14 is odd then 1 + 2 = 18
- If fishes lay eggs on tree, then I will marry you

Implication

P=>Q

(1) If P, then Q

(2) Q if P

(3) P only if Q

(4) P is sufficient for Q

(5) Q is necessary for P

PROPOSITIONS

Implication

P=>Q is true

Q=>P is true

P if and only if Q

P iff Q

Examples:

- **P: “3 is odd”, R: “6 is even”**
- **P if and only if R (logically equivalent)**

PROPOSITIONS

Implication

Contrapositive

$$\neg Q \Rightarrow \neg P$$

<u>Example:</u>

- If you got 6.0 in IELTS, you didn't have to take the test again

  Contrapositive:

  If you had to take IELTS again, you didn't get 6.0 in IELTS

P R O P O S I T I O N S

Implication

Converse

$$Q \Rightarrow P$$

Example:

- If you got 6.0 in IELTS, you didn't have to take the test again

Converse:

If you didn't have to take the test again, you must have got 6.0 in IELTS

Remark

Contrapositives: $p \rightarrow q$ *and* $\neg q \rightarrow \neg p$

- *Ex. "If it is noon, then I am hungry."*

  "If I am not hungry, then it is not noon."

Converses: $p \rightarrow q$ *and* $q \rightarrow p$

- *Ex. "If it is noon, then I am hungry."*

  "If I am hungry, then it is noon."

Inverses: $p \rightarrow q$ *and* $\neg p \rightarrow \neg q$

- *Ex. "If it is noon, then I am hungry."*

  "If it is not noon, then I am not hungry."

PROPOSITIONS

Implication

P	Q	¬P	¬Q	$P \to Q$	$Q \to P$	$\neg Q \to \neg P$	$P \leftrightarrow Q$
					Converse	Contrapositive	
T	T	F	F	**T**	*T*	**T**	T
T	F	F	T	**F**	*T*	**F**	F
F	T	T	F	**T**	*F*	**T**	F
F	F	T	T	**T**	*T*	**T**	T

The ***contrapositive*** *of* $P \to Q$ *has the same truth values, while the* ***converse*** *does not.*

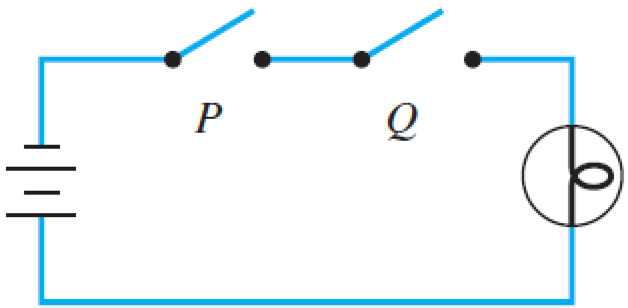
PROPOSITIONS

Distributivity: *p* ▸ (*q* ◂ *r*) (*p* ▸ *q*) ◂ (*p* ▸ *r*)

p	*q*	*r*	*q* ◂ *r*	*p* ▸ (*q* ◂ *r*)	*p* ▸ *q*	*p* ▸ *r*	(*p* ▸ *q*) ◂ (*p* ▸ *r*)
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

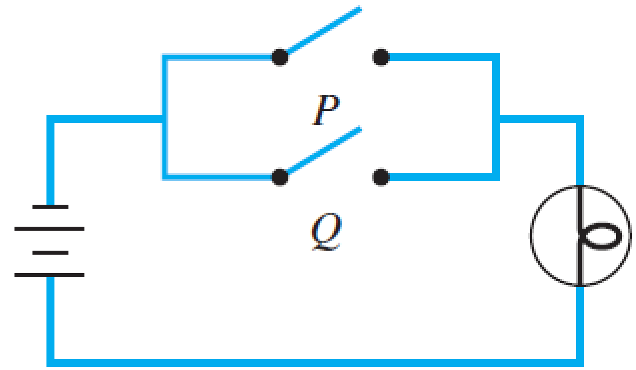
PROPOSITIONS

Application: Digital Logic Circuits



Switches "in series"

(a)



Switches "in parallel"

(b)

Switches		Light Bulb
P	***Q***	**State**
closed	closed	on
closed	open	off
open	closed	off
open	open	off

Switches		Light Bulb
P	***Q***	**State**
closed	closed	on
closed	open	on
open	closed	on
open	open	off

Quantifiers

Universal quantifier

$$\forall$$

(For all)

<u>Example:</u>

$$\forall n \in \mathbb{Z}^+, (n^2 + n + 41) \text{ is prime}$$

Quantifiers

Existential quantifier

$$\exists$$

(There exists)

<u>Example:</u>

$$\exists\ x \in \mathbb{Z}, x < 0$$

Quantifiers and Negation

$$\neg(\forall x, P(x)) \equiv \exists x, \neg P(x)$$
$$\neg(\exists x, P(x)) \equiv \forall x, \neg P(x)$$

Example:

$$\neg((\forall x < 3), x^2 \leq 2) \equiv (\exists x < 3), x^2 > 2$$

Question and Answer

- This proposition is true or false?

$$2 + 3 \geq 5$$

Question and Answer

- If *a* is in [5,10], then *a* is greater than or equal to 5.
- Is 5 in [5,10] ?
- 5 is greater than or equal to 5
- 5 is greater than 5 OR 5 is equal to 5

Question and Answer

- Contruct the truth table for the following proposition:

$$(p \to q) \land (\neg p \to r)$$

Homework

- Problem set 1