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Class: DMA-B05

HOMEWORK
DISCRETE MATHEMATICS
PROBLEMSET 06

Problem 1:

$$\begin{aligned} P(x) &= (1) \cdot \Delta_1(x) + (2) \cdot \Delta_2(x) + (3) \cdot \Delta_3(x) \\ &= 1 * \frac{(x-4)(x-5)}{(3-4)(3-5)} + 2 * \frac{(x-3)(x-5)}{(4-3)(4-5)} + 3 * \frac{(x-3)(x-4)}{(5-3)(5-4)} \\ &= \frac{x^2 - 9x + 20}{2} - 2 * \frac{x^2 - 8x + 15}{1} + 3 * \frac{x^2 - 7x + 12}{2} \\ &= x - 2 \end{aligned}$$

Problem 2:

$$\begin{aligned} P(x) &= (4) \cdot \Delta_1(x) + (3) \cdot \Delta_2(x) + (5) \cdot \Delta_3(x) \\ &= 4 * \frac{(x-1)(x-3)}{(2-1)(2-3)} + 3 * \frac{(x-2)(x-3)}{(1-2)(1-3)} + 5 * \frac{(x-2)(x-1)}{(3-2)(3-1)} \\ &= \frac{-4(x^2 - 4x + 3)}{1} + \frac{3(x^2 - 5x + 6)}{2} + \frac{5(x^2 - 3x + 2)}{2} \\ &= x + 2 \end{aligned}$$

Problem 3:

$$\begin{aligned} P(x) &= (-2) \cdot \Delta_1(x) + (7) \cdot \Delta_2(x) + (20) \cdot \Delta_3(x) \\ &= -2 * \frac{(x-2)(x-3)}{(1-2)(1-3)} + 7 * \frac{(x-1)(x-3)}{(2-1)(2-3)} + 20 * \frac{(x-1)(x-2)}{(3-1)(3-2)} \\ &= -x^2 + 5x - 6 - 7(x^2 - 4x + 3) + 10(x^2 - 3x + 2) \\ &= 2x^2 + 3x - 7 \end{aligned}$$

Problem 4:

a) If official 3, 4 and 5 get together, they know $P(3)=1$, $P(4)=6$, $P(5)=3$

$\Rightarrow P(x)$ goes through $(3,1)$, $(4,6)$ and $(5,3) \Rightarrow P_1(x) = -4x^2 + 33x - 62$

$\Rightarrow P(0) = 0 + 0 - 62 = -62 \Rightarrow$ the secret S1 is -62

b) If official 1, 2 and 5 get together, they know $P(1)=2$, $P(2)=2$, $P(5)=3$

$\Rightarrow P(x)$ goes through $(1,2)$, $(2,2)$ and $(5,3) \Rightarrow P_2(x) = \frac{1}{12}x^2 - \frac{1}{4}x + \frac{13}{6}$

$\Rightarrow P(0) = \frac{13}{6} \Rightarrow$ the secret S2 is $\frac{13}{6}$