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HOMEWORK
DISCRETE MATHEMATICS
PROBLEM SET 02

Problem 1

x irrational -> 1/x irrational
if 1/x is rational
=> 1/x = a/b (a,b Z)
=> x = b/a
=> x is rational
-Q=>-P = P=>Q

Problem 2

Prove that given a nonnegative integer n, there is a unique nonnegative integer m such that $m^2 \leq n < (m+1)^2$.

m2≤n<m+1^2

m≤ $\sqrt{n}$ <m+1

n is a nonnegative integer. We can then take the square root of n, which is $\sqrt{n}$. $\sqrt{n}$ is a real number and any real number lies between two consecutive integers (or is equal to an integer). Thus there exists an integer m such that:

$$m \leq \sqrt{n} < m + 1$$

Since $\sqrt{n}$ is nonnegative, the integer m also has to be nonnegative.

Square each side of the inequality:

$$m^2 \leq n < (m+1)^2$$

We have then proven that there exists a nonnegative integer m such that the inequality holds.

Problem 3

Show that p_1, p_2, p_3, p_4, p_5 can be shown to be equivalent by proving that the conditional statements $p_1 \to p_4$, $p_4 \to p_2$, $p_2 \to p_5$, $p_5 \to p_3$, $p_3 \to p_1$ are true.

Assume p1 is true:
p1->p4 => p4 true
p4 -> p2 =>p2 true
…..
=> p5 true
Assume p1 is false:
p3->p1 => p3 is false
…..
p4 false

Problem 4

Prove that there is no positive integer n such that $n^2 + n^3 = 100$.

Assume there exits n

=> n^3 < 100 => = {1,2,3,4}

if n =1 f(n)=2

…..

n=4 f(n)=16+64=80

=> there is no positive integer n

Problem 5

Prove that if x^3 is irrational and $x \geq 0$ then the square root of x is irrational.

if $\sqrt{x}$ is rational => $\sqrt{x}$= ab

x= (a/b)^2= a^2/b^2

x^3= a^6/b^6 => x^3 rational

Problem 6

Prove that if m is a power of 3 and n is a power of 3 then $m + n$ is never a power of 3.

m=3^x (x,y are integers)

n=3^y

m+n = 3^x + 3^y

=> m+n=3x(1+3y-x)

1+3^(y-x)) is indivisible by 3

=> m+n is not a power of 3

Problem 7

Assume that a and b are both integers and that $a \neq 0$ and $b \neq 0$. Explain why $\frac{(b-a)}{(ab^2)}$ must be a rational number.

$\frac{b-a}{ab^2} = \frac{1}{ab} - \frac{1}{b^2}$

a,b C Z => 1/ab is rational and 1/b^2 is rational

Problem 8

Prove by contraposition: For all positive integers n, r and s, if $rs \leq n$, then $r \leq \sqrt{n}$ or $s \leq \sqrt{n}$.

if r2>n and s2>n => (rs)2>n^2 => rs > n

Problem 9

Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

$\sqrt{2}+\sqrt{3}=\frac{a}{b}$ (a,b Z)

$5+2\sqrt{6}$= a^2/b^2

$2\sqrt{6}$= a^2/b^2-5

$\sqrt{6} = \frac{a^2-5b^2}{2b^2}$ (contradiction)

Therefore, $\sqrt{2}+\sqrt{3}$ is irrational

Problem 10

Prove that $\forall n \in Z$, if $n > 2$ then there is a prime number p such that $n < p < n!$. (*Hint*: Use the theorem: "Any integer $n > 1$ is divisible by a prime number". Prove that: $p|(n!-1)$, if $p \leq n$, then $p = 1 \rightarrow$ contradiction. Therefore, $n < p \leq n! - 1$).

P: n>2 Q: n<p<n!

if n>2 => n<p<n!

assume –Q: n!≤p≤n

=> n!≤n

=> n =1 or n =2

=> 0<n ≤2 (contraposition)

=> -P

-Q => -P

Therefore, P => Q