HANU
HANOI UNIVERSITY
FACULTY OF INFORMATION TECHNOLOGY
DEPARTMENT OF COMPUTER SCIENCE

HOMEWORK

Discrete Mathematics

TUT-01: Problem Set 01

Instructor: Ms.c Nguyen Thi Huong

Release Date: September 7, 2021

Semester: Fall 2021

Problem 1

Let p and q be the propositions "*Swimming at the New Jersey shore is allowed*" and "*Sharks have been spotted near the shore*" respectively. Express each of these compound propositions as an English sentence.

a. $\neg q$

b. $p \wedge q$

c. $\neg p \vee q$

d. $p \rightarrow \neg q$

e. $\neg q \rightarrow p$

f. $\neg q \rightarrow \neg p$

g. $p \leftrightarrow \neg q$

h. $\neg p \wedge (p \vee \neg q)$

Problem 2

In an island, there are two kinds of inhabitants: knights, who always tell the truth and knaves, who always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusions?

a. A says "*At least one of us is a knave*" and B says nothing.

b. A says "*The two of us are both knights*" and B says "*A is knave*".

c. A says "*I am a knave or B is a knight*" and B says nothing.

d. Both A and B say "*I am a knight.*"

e. A says "*We are both knaves*" and B says nothing.

Problem 3

Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

Problem 4

Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

Problem 5

Show that $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.

Problem 6

Let $P(x)$ be the statement "*x can speak Russian*" and let $Q(x)$ be the statement "*x knows the computer language C++.*" Express each of these sentences in terms of $P(x)$, $Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a. There is a student at your school who can speak Russian and who knows C++.

b. There is a student at your school who can speak Russian but who does not know C++.

c. Every student at your school either can speak Russian or knows C++.

d. No student at your school can speak Russian or knows C++.

Problem 7

Translate these specifications into English where $F(p)$ is "*Printer p is out of service*", $B(p)$ is "*Printer p is busy*", $L(j)$ is "*Printer job j is lost*", and $Q(j)$ is "*Printer job j is queued.*"

a. $\exists p\ (F(p) \wedge B(p)) \to \exists j\ L(j)$

b. $\forall p\ B(p) \to \exists j\ Q(j)$

c. $\exists j\ (Q(j) \wedge L(j)) \to \exists p\ F(p)$

d. $(\forall p\ B(p) \wedge \forall j\ Q(j)) \to \exists j\ L(j)$

Problem 8

What are the truth values of these statements?

a. $\exists! x\ P(x) \to \exists x\ P(x)$.

b. $\forall x\ P(x) \to \exists! x\ P(x)$

c. $\exists! x\ \neg P(x) \to \neg \forall x\ P(x)$

Problem 9

Let $L(x, y)$ be the statement "*x loves y*", where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements.

a. Everybody loves Jerry. $(\forall x, L(x, \text{"}Jerry''))$

b. Everybody loves somebody.

c. There is somebody whom everybody loves.

d. Nobody loves everybody.

e. There is somebody whom Lydia does not love.

f. There is somebody whom no one loves.

g. There is exactly one person whom everybody loves.

h. There are exactly two people whom Lynn loves.

i. Everyone loves himself or herself.

j. There is someone who loves no one besides himself or herself.

Problem 10

Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a. $\forall n \exists m \ (n^2 < m)$

b. $\forall n \exists m \ (n < m^2)$

c. $\forall n \exists m \ (n + m = 0)$

d. $\exists n \forall m \ (nm = m)$

e. $\exists n \exists m \ (n^2 + m^2 = 5)$

f. $\exists n \exists m \ (n^2 + m^2 = 6)$

g. $\exists n \exists m \ (n + m = 4 \wedge n - m = 1)$

h. $\exists n \exists m \ (n + m = 4 \wedge n - m = 2)$

i. $\forall n \forall m \exists p \ (p = (m + n)/2)$.

Problem 11

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "*It is not the case that.*")

1. Every student in this class has taken exactly two mathematics classes at this school.
2. Someone has visited every country in the world except Libya.
3. No one has climbed every mountain in the Himalayas.
4. Every movie actor has either been in a movie with Kevin Bacon or has been in a movie with someone who has been in a movie with Kevin Bacon.

References

[1] K. H. Rosen, *Discrete Mathematics and Its Applications*, McGraw-Hill, 7th edition, 2011.

[2] S. S. Epp, *Discrete Mathematics with Applications*, Cengage-Learning, 4th edition, 2010.

[3] T. W. Judson and R. A. Beezer, *Abstract Algebra: Theory and Applications*, Free Software Foundation, 2017, [Online; accessed 08-September-2017].