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***HOMEWORK***
***DISCRETE MATHEMATICS***
***PROBLEM SET 07***

***Problem 1:***

Packets 2,1,3. (2 lost)

Consider P(2) = 2, P(3) = 1, P(5) = 3

P(x) go through (2,2) , (3,1) , (5,3)

=> P(x) = $2 * \frac{(x-3)(x-5)}{(2-3)(2-5)} + 1 * \frac{(x-2)(x-5)}{(3-2)(3-5)} + 3 * \frac{(x-2)(x-3)}{(5-2)(5-3)}$

=> P(x) = $\frac{2}{3}x^2 - \frac{13}{3}x + 8$

Send P(1) = $\frac{13}{3}$ , P(2) = 2, P(3) = 1, P(4) = $\frac{4}{3}$ , P(5) = 3

Receive P(2) = 2, P(3) = 1, P(5) = 3

=> The corrected values of the error packages are P(1) = $\frac{13}{3}$ and P(4) = $\frac{4}{3}$

***Problem 2:***

a) Error rate is 1/5 => k = 1 => E(x) degree 1 => E(x) = x + $b_0$

n + 2k = 5 => n = 3 => P(x) degree (n-1) = 2

Q(x) = P(x) * E(x) => Q(x) degree 3 => Q(x) = $a_3x^3 + a_2x^2 + a_1x + a_0$

$R(x_1) = R(1) = 2$ $R(x_4) = R(4) = 5$

$R(x_2) = R(2) = 1$ $R(x_5) = R(5) = 3$

$R(x_3) = R(3) = 4$

Have: $Q(x) = R(x)*E(x)$

=> $Q(1) = R(1)*E(1)$

…….

$Q(5) = R(5)*E(5)$

=> $a_3 + a_2 + a_1 + a_0 = 2 + 2b_0 \equiv a_3 + a_2 + a_1 + a_0 + 5b_0 = 2$

$a_3 + 4a_2 + 2a_1 + a_0 = 2 + b_0 \equiv a_3 + 4a_2 + 2a_1 + a_0 + 6b_0 = 2$

$6a_3 + 2a_2 + 3a_1 + a_0 = 5 + 4b_0 \equiv 6a_3 + 2a_2 + 3a_1 + a_0 + 3b_0 = 5$

$a_3 + 2a_2 + 4a_1 + a_0 = 6 + 5b_0 \equiv a_3 + 2a_2 + 4a_1 + a_0 + 2b_0 = 6$

$6a_3 + 4a_2 + 5a_1 + a_0 = 1 + 3b_0 \equiv 6a_3 + 4a_2 + 5a_1 + a_0 + 4b_0 = 1$

=> $a_3 = \frac{-1}{20} \equiv 1 \pmod 7$

$a_2 = \frac{7}{4} \equiv 0 \pmod 7$

$a_1 = \frac{-9}{4} \equiv 3 \pmod 7$

$a_0 = \frac{351}{20} \equiv 6 \pmod 7$

$b_0 = -3 \equiv 4 \pmod 7$

=> Q(x) =$x^3 + 3x + 6$ and E(x) = $x - 3$ => E(3) = 0 => packet $x_3$ at which error occurred

=> P(x) = R(x) = Q(x) / E(x) = $x^2 + 3x + 12 \equiv x^2 + 3x + 5 \pmod 7$

=>The corrected value of the error packet is: P(3) = $23 \equiv 2 \pmod 7$

b) Error rate is 1/5 => k = 1 => E(x) degree 1 => E(x) = $x + b_0$

$n + 2k = 5 \Rightarrow n = 3$ => P(x) degree (n-1) = 2

Q(x) = P(x) * E(x) => Q(x) degree 3 => Q(x) = $a_3x^3 + a_2x^2 + a_1x + a_0$

$R(x_1) = R(1) = 3$ $R(x_4) = R(4) = 2$

$R(x_2) = R(2) = 2$ $R(x_5) = R(5) = 5$

$R(x_3) = R(3) = 1$

Have: Q(x) = R(x)*E(x)

=> Q(1) = R(1)*E(1)

…….

Q(5) = R(5)*E(5)

=> $a_3 + a_2 + a_1 + a_0 = 3(1 + b_0) \equiv a_3 + a_2 + a_1 + a_0 + 4b_0 = 3$

$a_3 + 4a_2 + 2a_1 + a_0 = 2(2 + b_0) \equiv a_3 + 4a_2 + 2a_1 + a_0 + 5b_0 = 4$

$6a_3 + 2a_2 + 3a_1 + a_0 = 1(3 + b_0) \equiv 6a_3 + 2a_2 + 3a_1 + a_0 + 6b_0 = 3$

$a_3 + 2a_2 + 4a_1 + a_0 = 2(4 + b_0) \equiv a_3 + 2a_2 + 4a_1 + a_0 + 5b_0 = 1$

$6a_3 + 4a_2 + 5a_1 + a_0 = 5(5 + b_0) \equiv 6a_3 + 4a_2 + 5a_1 + a_0 + 2b_0 = 4$

=> $a_3 = \frac{3}{10} \equiv 1 \pmod 7$

$a_2 = \frac{7}{10} \equiv 0 \pmod 7$

$a_1 = \frac{-4}{5} \equiv 2 \pmod 7$

$a_0 = 4 \pmod 7$

$b_0 = \frac{-3}{10} \equiv -1 \equiv 6 \pmod 7$

=> Q(x) = $x^3 + 2x + 4$ and E(x) = $x - 1$ => E(1) = 0 => packet $x_1$ at which error occurred

=> P(x) = R(x) = Q(x) / E(x) = $x^2 + x + 3$

=>The corrected value of the error packet is: P(1) = 5

c) Error rate is 1/5 => k = 1 => E(x) degree 1 => E(x) = x + $b_0$

n + 2k = 5 => n = 3 => P(x) degree (n-1) = 2

Q(x) = P(x) * E(x) => Q(x) degree 3 => Q(x) = $a_3x^3 + a_2x^2 + a_1x + a_0$

$R(x_1) = R(1) = 5$ $R(x_4) = R(4) = 2$

$R(x_2) = R(2) = 1$ $R(x_5) = R(5) = 5$

$R(x_3) = R(3) = 1$

Have: Q(x) = R(x)*E(x)

=> Q(1) = R(1)*E(1)

……

Q(5) = R(5)*E(5)

=> $a_3 + a_2 + a_1 + a_0 = 5(1 + b_0) \equiv a_3 + a_2 + a_1 + a_0 + 2b_0 = 5$

$a_3 + 4a_2 + 2a_1 + a_0 = 1(2 + b_0) \equiv a_3 + 4a_2 + 2a_1 + a_0 + 6b_0 = 2$

$6a_3 + 2a_2 + 3a_1 + a_0 = 1(3 + b_0) \equiv 6a_3 + 2a_2 + 3a_1 + a_0 + 6b_0 = 3$

$a_3 + 2a_2 + 4a_1 + a_0 = 2(4 + b_0) \equiv a_3 + 2a_2 + 4a_1 + a_0 + 5b_0 = 1$

$6a_3 + 4a_2 + 5a_1 + a_0 = 5(5 + b_0) \equiv 6a_3 + 4a_2 + 5a_1 + a_0 + 2b_0 = 4$

=> $a_3 = \frac{37}{100} \equiv 1 \pmod 7$

$a_2 = \frac{1}{20} \equiv 6 \pmod 7$

$a_1 = \frac{-3}{4} \equiv 1 \pmod 7$

$a_0 = \frac{653}{100} \equiv 1 \pmod 7$

$b_0 = \frac{-3}{5} \equiv -2 \equiv 5 \pmod 7$

=> $Q(x) = x^3 + 6x^2 + x + 1$ and $E(x) = x - 2$ => $E(2) = 0$ => packet $x_2$ at which error occurred

=> $P(x) = R(x) = Q(x) / E(x) = x^2 + 8x + 17 \equiv x^2 + x + 3$

=> The corrected value of the error packet is: $P(2) = 9 \equiv 2 \pmod 7$

d) Error rate is 1/5 => k = 1 => E(x) degree 1 => $E(x) = x + b_0$

$n + 2k = 5$ => $n = 3$ => P(x) degree $(n-1) = 2$

$Q(x) = P(x) * E(x)$ => Q(x) degree 3 => $Q(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

$R(x_1) = R(1) = 5$ $R(x_4) = R(4) = 2$

$R(x_2) = R(2) = 2$ $R(x_5) = R(5) = 5$

$R(x_3) = R(3) = 3$

Have: $Q(x) = R(x)*E(x)$

=> $Q(1) = R(1)*E(1)$

…….

$Q(5) = R(5)*E(5)$

=> $a_3 + a_2 + a_1 + a_0 = 5(1 + b_0) \equiv a_3 + a_2 + a_1 + a_0 + 2b_0 = 5$

$a_3 + 4a_2 + 2a_1 + a_0 = 2(2 + b_0) \equiv a_3 + 4a_2 + 2a_1 + a_0 + 5b_0 = 4$

$6a_3 + 2a_2 + 3a_1 + a_0 = 3(3 + b_0) \equiv 6a_3 + 2a_2 + 3a_1 + a_0 + 4b_0 = 2$

$a_3 + 2a_2 + 4a_1 + a_0 = 2(4 + b_0) \equiv a_3 + 2a_2 + 4a_1 + a_0 + 5b_0 = 1$

$6a_3 + 4a_2 + 5a_1 + a_0 = 5(5 + b_0) \equiv 6a_3 + 4a_2 + 5a_1 + a_0 + 2b_0 = 4$

=> $a_3 = \frac{-3}{25} \equiv 1 \pmod 7$

$a_2 = \frac{4}{5} \equiv 5 \pmod 7$

$a_1 = \frac{-7}{10} \equiv 0 \pmod 7$

$a_0 = \frac{341}{50} \equiv 5 \pmod 7$

$b_0 = \frac{-9}{10} \equiv -3 \equiv 4 \pmod 7$

=> Q(x) = $x^3 + 5x^2 + 5$ and E(x) = $x - 3$ => E(3) = 0 => packet $x_3$ at which error occurred

=> P(x) = R(x) = Q(x) / E(x) = $x^2 + 8x + 24 \equiv x^2 + x + 3$

=> The corrected value of the error packet is: P(3) = 15 $\equiv$ 1 (mod 7)