FACULTY OF INFORMATION TECHNOLOGY
DEPARTMENT OF COMPUTER SCIENCE

HOMEWORK

Discrete Mathematics

TUT-02: Problem Set 02

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Problem 1

Prove that if x is irrational then $\frac{1}{x}$ is irrational.

Problem 2

Prove that given a nonnegative integer n, there is a unique nonnegative integer m such that $m^2 \leq n < (m+1)^2$.

Problem 3

Show that p_1, p_2, p_3, p_4, p_5 can be shown to be equivalent by proving that the conditional statements $p_1 \to p_4$, $p_4 \to p_2$, $p_2 \to p_5$, $p_5 \to p_3$, $p_3 \to p_1$ are true.

Problem 4

Prove that there is no positive integer n such that $n^2 + n^3 = 100$.

Problem 5

Prove that if x^3 is irrational and $x \geq 0$ then the square root of x is irrational.

Problem 6

Prove that if m is a power of 3 and n is a power of 3 then $m + n$ is never a power of 3.

Problem 7

Assume that a and b are both integers and that $a \neq 0$ and $b \neq 0$. Explain why $\frac{(b-a)}{(ab^2)}$ must be a rational number.

Problem 8

Prove by contraposition: For all positive integers n, r and s, if $rs \leq n$, then $r \leq \sqrt{n}$ or $s \leq \sqrt{n}$.

Problem 9

Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Problem 10

Prove that $\forall n \in Z$, if $n > 2$ then there is a prime number p such that $n < p < n!$. (*Hint*: Use the theorem: "Any integer $n > 1$ is divisible by a prime number". Prove that: $p|(n!-1)$, if $p \leq n$, then $p = 1 \rightarrow$ contradiction. Therefore, $n < p \leq n! - 1$).

References

[1] K. H. Rosen, *Discrete Mathematics and Its Applications*, McGraw-Hill, 7th edition, 2011.

[2] S. S. Epp, *Discrete Mathematics with Applications*, Cengage-Learning, 4th edition, 2010.

[3] T. W. Judson and R. A. Beezer, *Abstract Algebra: Theory and Applications*, Free Software Foundation, 2017, [Online; accessed 08-September-2017].