



**HANU**  
HANOI UNIVERSITY

FACULTY OF INFORMATION TECHNOLOGY  
DEPARTMENT OF COMPUTER SCIENCE

# **HOMEWORK**

## **Discrete Mathematics**

### **TUT-03: Problem Set 03**

Instructor: Dr. Hung Ta

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**Problem 1**

Prove that  $\forall n \in \mathbb{N}^+$

$$1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

**Problem 2**

Prove that  $\forall n \in \mathbb{N}, n > 1$

$$1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2} < 2 - \frac{1}{n}$$

**Problem 3**

Prove that  $\forall n \in \mathbb{N}^+$

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

**Problem 4**

Prove that  $\forall n \in \mathbb{N}^+$

$$3 + 9 + 27 + \dots + 3^n = \frac{1}{2}(3^{n+1} - 3)$$

**Problem 5**

Prove that  $\forall n \in \mathbb{N}^+$

$$1 + 3 + 5 + \dots + (2n-1) = n^2$$

**Problem 6**

Prove that  $\forall n \in \mathbb{N}, n \geq 2$

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2^n - 1} < n$$

**Problem 7**

Prove that  $\forall n \in \mathbb{N}, n \geq 2$

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}$$

**Problem 8**

Prove that  $\forall n \in \mathbb{N}$

$$\frac{1}{2} + \frac{3}{4} + \frac{5}{6} + \dots + \frac{(2n+1)}{(2n+2)} > \frac{1}{\sqrt{3n+4}}$$

**Problem 9**

Prove that  $\forall n \in \mathbb{N}$

$$1 + 3 + 6 + 10 + \dots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$$

**Problem 10**

Prove that  $\forall n \in \mathbb{N}^+$

$$\underbrace{\sqrt{a^2 + \sqrt{a^2 + \sqrt{a^2 + \dots + \sqrt{a^2}}}}}_n \leq |a| + 1$$

**References**

- [1] K. H. Rosen, *Discrete Mathematics and Its Applications*, McGraw-Hill, 7th edition, 2011.
- [2] S. S. Epp, *Discrete Mathematics with Applications*, Cengage-Learning, 4th edition, 2010.
- [3] T. W. Judson and R. A. Beezer, *Abstract Algebra: Theory and Applications*, Free Software Foundation, 2017, [Online; accessed 08-September-2017].