03/03/2023 Recitation #1 Handout

1. Pareto Optimality It is impossible to make anyone better-off without hurting someone.

2. Utility Possibility Set & Utility Possibility Frontier

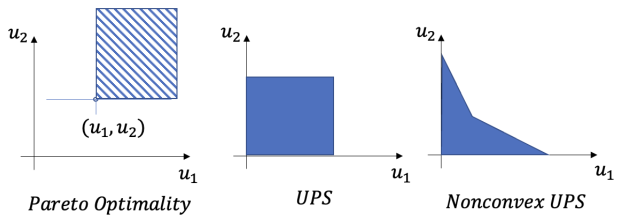

Figure 1: Pareto optimality, UPS & UPF

We define UPF to be the set of utility vectors that correspond to Pareto optimal allocations.
Note: If UPS is a square, then UPF only contains a single point.

3. Social Welfare Function

A social welfare function aggregates individual preferences into social preferences. $W = F(u_1, u_2)$

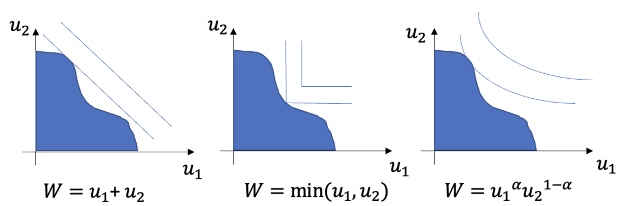

Figure 2: SWF

Question: Which SWF is reasonable?

- Convex UPS:

  maximizing $\sum_i u_i$ can achieve ONE Pareto optimal allocation;

  allowing for all possible weights ($\sum_i \lambda_i = 1$ and $\lambda_i \geq 0$, $\forall i$) and maximizing $\sum_i \lambda_i u_i$ can achieve ALL Pareto optimal allocations.

  (Intuition: supporting hyperplane theorem)

- Quasi-linear:

  maximizing $\sum_i u_i$ can achieve ALL Pareto optimal allocations.

  The following example shows that this statement is not true if we restrict allocations to be non-negative.

Example 1.

$$
\begin{aligned}
&u_1(x_1, m_1) = \sqrt{x_1} + m_1 \\
&u_2(x_2, m_2) = \sqrt{x_2} + m_2 \\
&\left\{((x_1, m_1), (x_2, m_2)) \in \mathbb{R}_+^2 \times \mathbb{R}_+^2 : x_1 + x_2 = 2, m_1 + m_2 = 2\right\}
\end{aligned}
$$

Note that $a_1 = ((x_1, m), (x_2, m_2)) = ((2, 2), (0, 0))$ *is P.O. This is the allocation where all resources go to agent 1. Clearly, giving more to agent 2 will hurt agent 1.* $a_2 = ((1, 1), (1, 1))$ *yields the higher sum of* u_1 *and* u_2*.*

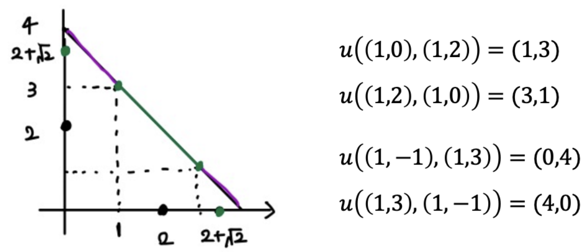

Figure 3: $\sum_i u_i$ cannot achieve all P.O. allocations