03/24/2023 Recitation #3 Handout

1. Environment

A mechanism design environment $\mathcal{E} = \left\{N, \{\Theta_i\}_{i\in N}, p, X, \{u_i\}_{i\in N}\right\}$

- $N = \{1, 2, \cdots, n\}$ set of agents
- Θ_i set of agent i's types (Write $\Theta = \prod_{i\in N} \Theta_i$)
- $p \in \Delta(\Theta)$ common prior distribution
- X set of outcomes (or social alternatives)
- $u_i : X \times \Theta \to \mathbb{R}$ agent i's utility function

Notes: (1) $\mathcal{E}$ has *independent types*, if p is a product distribution; (2) $\mathcal{E}$ has *private values*, if we can write $u_i(x, \theta) = u_i(x, \theta_i)$.

2. Social Choice Function/Correspondence

- A social choice function (SCF) $f : \theta \to x$ assigns each type profile θ an outcome x.
- A social choice correspondence (SCC) $F : \Theta \rightrightarrows X$ assigns each type profile θ a set of outcomes.

3. Mechanisms A mechanism $\Gamma = \left\{\{S_i\}_{i\in N}, g\right\}$

- S_i set of agent i's actions (Write $S = \prod_{i\in N} S_i$)
- $g : S \to X$ outcome function

Notes: Γ is a direct mechanism if $S_i = \Theta_i$, $\forall i \in N$.

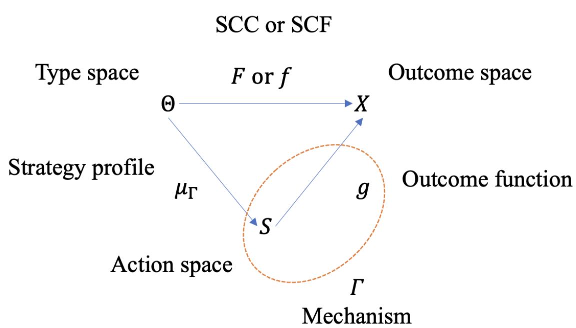

Figure 1: The Mount-Reiter Diagram

$\mathcal{E}$ and Γ defines a Bayesian game $\mathcal{G} = \left\{N, \{S_i\}_{i\in N}, \{\Theta_i\}_{i\in N}, \{p_i\}_{i\in N}, \{v_i\}_{i\in N}\right\}$, where agents' first-order beliefs p_i are derived from p[1] and the utility functions are defined by $v_i(s, \theta) = u_i(g(s), \theta)$.

[1] Many applications of Bayesian games employ the common prior assumption—the assumption that the players' first-order beliefs $p_i : \Theta_i \to \Delta(\Theta_{-i})$ are conditional probabilities generated from some p.

4. Implementation

We need to impose some assumption on μ_Γ, i.e., to specify the equilibrium concept we are using: dominant strategies, NE, BNE, rationalizability...

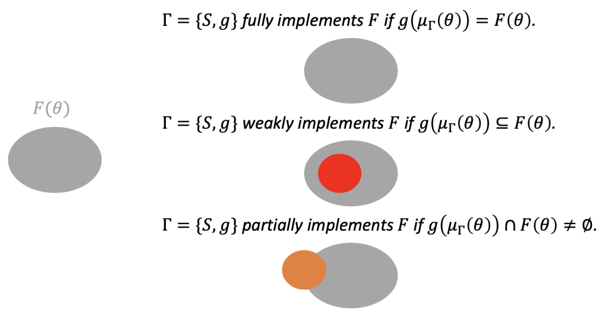

Figure 2: Implementation

Our goal is to find a mechanism that implements F. It might not be an easy task, since the set of all possible mechanisms is very large.

5. Revelation Principle

We will show that one can focus on mechanisms of a particular simple kind.
To fix ideas, let's consider implementing a social choice function f.

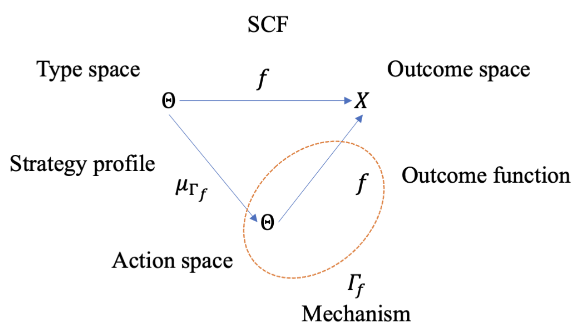

Figure 3: Γ_f

- Γ_f is an equivalent direct mechanism for SCF f.
- F is incentive-compatible (IC) under equilibrium concept μ, if $\forall\theta \in \Theta$, $\theta \in \mu_{\Gamma_f}(\theta)$.

Theorem 1. *If there is a* $\Gamma = \left\{\{S_i\}_{i\in N}, g\right\}$ *that implements* f *in dominant strategies, then the direct mechanism* $\Gamma_f = \left\{\{\Theta_i\}_{i\in N}, f\right\}$ *truthfully implements* f *in dominant strategies. Thus, a social choice function is dominant strategy implementable by some mechanism if and only if it is dominant strategy incentive-compatible!*

Sketch of the Proof. Simply use $g(\mu_{\Gamma(1)}(\theta_1), \mu_{\Gamma(2)}(\theta_2), ..., \mu_{\Gamma(n)}(\theta_n)) = f(\theta_1, ..., \theta_n), \quad \forall\theta.$ □