“The Difficulty of Easy Projects” by Wioletta Dziuda, A. Arda Gitmez &Mehdi Shadmehr

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1 Setup

1. N citizens; the project succeeds if $\geq q$ citizens contribute
2. The contributor pays c_i, which is her private information; everyone gets u if the project succeeds
3. A pure strategy of citizen i is $c_i \to \{0, 1\}$.
4. The best response is a threshold strategy.
5. Equilibrium threshold $c_q^*(u)$ is determined by

$$c_q^*(u) = \binom{N-1}{q-1} (F(c_q^*(u)))^{q-1}(1 - F(c_q^*(u)))^{N-q}u \equiv Pr\{piv|q, c_q^*(u)\}u$$

.

6. The probability of success is

$$S_q(c_q^*(u)) = \sum_{k=q}^{N} \binom{N}{k} (F(c_q^*(u)))^k(1 - F(c_q^*(u)))^{N-k}$$

7. Main question: When

$$S_{q+1}(c_{q+1}^*(u)) - S_q(c_q^*(u)) > 0$$

2 Decomposition

$$S_{q+1}(c^*_{q+1}(u)) - S_q(c^*_q(u)) = \underbrace{[S_{q+1}(c^*_{q+1}(u)) - S_{q+1}(c^*_q(u))]}_{\text{indirect effect???}} + \underbrace{[S_{q+1}(c^*_q(u)) - S_q(c^*_q(u))]}_{\text{direct effect}<0} \quad (1)$$

1. It is clear that the direct effect is negative, because fixing c^*_q, the probability of at least $q+1$ citizens contributing is lower than the probability of at least q citizens contributing.

2. Question: When $[S_{q+1}(c^*_{q+1}(u)) - S_{q+1}(c^*_q(u))] > 0$ and dominate the direct effect?

   Remark: Comparative statics wrt $q \in \mathcal{N}$, so we cannot take derivatives.

   (1) The question now becomes when the function $S_{q+1}(c)$ is increasing in c.

   For each term in $S_{q+1}(c)$, the change with respect to c has the same sign as $k[1 - F(c)] - (N-k)F(c) = k - NF(c)$. We have $k - NF(c) > 0 \iff c < F^{-1}(k/N)$. The probability of k people vote increases with higher cutoff c if the probability to contribute for the marginal citizen is smaller than $\frac{k}{N}$. This condition is not affected by varying u.

   In a $N = 2$ example, there's only one term $k = 2 = N$ in S_{q+1}, so it's increasing.

   (2) Question: When $c^*_{q+1}(u) - c^*_q(u) > 0$?

   (3) Summary:

$c^*_{q+1}(u) - c^*_q(u)$	$\partial S_{q+1}(c)/\partial c$	$[S_{q+1}(c^*_{q+1}(u)) - S_{q+1}(c^*_q(u))]$
+	-	-
-	+	-
+	+	+
-	-	+

   The effect of u enters in the ranking of equilibrium cutoffs. I simulated a model with $N = 10$ and normal distribution $N(1,1)$, and found that the last two cases appear when u is large. The difference is very elastic wrt the change of u!

3 Asymptotics

The paper proves results when $u \to +\infty$. (I think it is a clever move for two reasons: (1) It reduces computation burden, i.e., avoids splitting into many subcases that may blur the insight; (2) The simplification has strong motivation, because the benefit of public goods is often large.)

It shows that under common distribution assumptions, lowering the bar of agreement can indeed reduce the probability of passing the new policy, especially when the benefit is large! They clarify that unanimity, a high probability of pivotality, or a very high probability of participation are not the driving force behind these results.

On page 5, they say: "For the indirect effect to dominate, it must be that as the equilibrium threshold increases, the expected number of new participants (those whose participation costs c_is are just above the old equilibrium threshold c_q^*) is sufficiently large. When $F(c)$ increases more quickly, it means there are more such citizens; and a bounded support or log-concavity of $1 - F(c)$ means that $F(c)$ increases sufficiently fast."