After getting superpixels, the next goal will be grouping them together and using their outer boundary as the closure for an object. Based on the method proposed by Levinshtein et. al. [6], finding the optimal grouping of some adjacent superpixels can be reduced to a mathematical optimization problem. Intuitively, we try to pick out several superpixels, whose boundaries have strong edge support. As an interested object is usually composed of a bunch of adjacent superpixels, we should lay some emphasis on the spatial coherence of them. Levinshtein et. al. proposed a cost function, which is a ratio of a novel learned boundary gap measure to the area of the selected superpixels to solve this problem. Additionally, using parametric maxflow, we can get a few possible optimal groupings of superpixels.

First, let be a binary variable for the i-th superpixel, with 1 indicating that it belongs to the grouping of superpixels and 0 indicating that it belongs to ground. The superpixels of the whole image is represented by vector . Based on Stahl and Wang [18], Levinshtein et. al. [6] defines the cost function to be . The numerator measures the cost of the gap measurement around the outer boundary of the group of selected superpixels, while the denominator is the area of the group of selected superpixels. More specifically, , where measures the total number of pixels on the outer boundary of selected superpixels, and measures the total number of pixels on both the outer boundary and an edge detected by previous steps. The more boundary pixels on an edge, the smaller the value of . This makes sense, because the grouping of the selected superpixels is a better estimation of a contour closure for an object if the boundary of them are supported by the edges detected by previous methods. The above cost function is written in the concise form, but it can also be written in more specific form. Moreover, Levinshtein et. al. also showed a few methods to determine if a pixel is considered on an edge. However, we do not list all these details here, since we think explanation of intuition is more important. If you are interested in the details, please refer to the original paper [6].

The next issue is how to optimize the above cost function. In fact, for binary variables, we can think of ratio minimization problem as a parametric maxflow problem. Kolmogorov et al. [19] showed that under certain constraints on the ratio , the energy can be submodular and can thus be optimized globally in polynomial time by the min-cuts algorithm. The method in [19] can not only optimize the ratio R(x), but also find all intervals of λ (and the corresponding ***X***). The smallest breakpoint will correspond to the optimal ratio , and consecutively larger breakpoints , , . . . will correspond to other ratio optimization for other intervals [19]. Therefore, with the parametric maxflow method applied to optimizing closure cost function, we can obtain a set of optimal closure solutions, and usually the first solution will be the optimal grouping of superpixels.