

CS526 O2  
Homework Assignment 2

**Problem 1 (20 points).** This is practice for analyzing running time of algorithms. Express the running time of the following methods, which are written in pseudocode, using the *big-oh* notation. Assume that all variables are appropriately declared. You must justify your answers. If you show only answers, you will not get any credit even if they are correct.

(1)

```
method1(int[ ] a) // returns integer
x = 0;           //c1
y = 0;           //c2
for (i=1; i<n; i++) { //n-1 // n is the number of elements in array a
    if (a[i] == a[i-1]) { //c3
        x = x + 1;
    }
    else {
        y = y + 1;
    }
}
return (x - y);    //c4
```

Answer: x takes c1 constant time, y takes c2 constant time, for loop and loop body of the for loop runs n-1 linear times, loop body c3 takes constant time, return statement takes c4 constant time.

We do not count constant time here and we only take the longest runtime with the worst scenario.

Total running time:

$$f(n) = c1 + c2 + c3(n-1) + c4$$

$$f(n) = c3(n-1)$$

$$f(n) = c3n - c3$$

$$\underline{F(n) = O(n)}$$

(2)

```
method2(int[ ] a, int[ ] b) // assume equal-length arrays
    x = 0;           //c1
    i = 0;           //c2
    while (i < n) { //c3(n-1) // n is the number of elements in each array
        y = 0;       //c4
        j = 0;       //c5
        while (j < n) { //c6(n-1)
            k = 0     //c7
            while (k <= j) { //c8(n)
                y = y + a[k]; //c9(1)
                k = k + 1;    //c10(1)
            }
            j = j + 1; //c11(1)
        }
    }
```

```

    }
    if (b[i] == y) {    // c12(1)
        x++;
    }
    i = i +
    1;
    //c13(1)
}
return x;    //c14(1)

```

Answer: x takes c1 constant time, i takes c2 constant time, while loop (i < n) and loop body for the while loop runs c3(n-1) linear times, loop body y takes c4 constant time, j takes c5 constant time, while loop (j < n) runs c6(n-1) linear times, loop body k takes c7 constant time, while loop (k <= j) takes c8(n) linear time, loop body c9, c10, c11 takes constant time, if statement (b[i] == y) runs c12(1) linear time with b[i] == y max n-1 time, if statement body c13 linear time, return statement takes c14 constant time.

We do not count constant time here and we only take the longest runtime with the worst scenario.

Total running time:

$$f(n) = (c1 + c2 + c3(n-1) + c14) * (c4 + c5 + c6(n-1) + c12 + c13) * (c7 + c8(n) + c11) * (c9 + c10)$$

$$f(n) = c3(n-1) * c6(n-1) * c8(n) = n*n*n$$

$$f(n) = n^3$$

**F(n) = O(n^3)**

(3)

```

// n is the length of array a
// p is an array of integers of length 2
// initial call: method3(a, n-1, p)
// initially p[0] = 0, p[1] = 0
method3(int[] a, int i, int[] p)
    if (i == 0) {    //c1 - O(1)
        p[0] = a[0];
        p[1] = a[0];
    }
    else {
        method3(a, i-1, p);    //c2(n-1) - O(n-1)
        if (a[i] < p[0]) {    //c3 - O(1)
            p[0] = a[i];
        }
        if (a[i] > p[1]) {    // c4 - O(1)
            p[1] = a[i];
        }
    }
}

```

Answer: if else statement ( $i == 0$ ) with its body runs  $c_1$  constant time, recursive method3 runs  $c_2(n-1)$  linear time as the input size is reduced 1 with each recursive call, this function is being called recursively  $n$  times before reaching the base case. If statement ( $a[i] < p[0]$ ) runs  $c_3$  constant time, if statement ( $a[i] > p[1]$ ) runs  $c_4$  constant time.

We do not count constant time here and we only take the longest runtime with the worst scenario.

Total running time:

$$f(n) = c_1 + c_2(n-1) + c_3 + c_4$$

$$f(n) = c_2(n-1)$$

$$f(n) = c_2n - c_2$$

$$f(n) = n$$

$$\underline{F(n) = O(n)}$$

(4)

```
// initial call: method4(a, 0, n-1) // n is the length of array a
public static int method4(int[] a, int x, int y)
{
    if (x >= y) //c1 - O(1)
    {
        return a[x];
    }
    else
    {
        z = (x + y) / 2; // c2 - O(1) - integer division
        u = method4(a, x, z); //c3(n) - O(n)
        v = method4(a, z+1, y); //c4(n+1) - O(n)
                                //O(2^n)
        if (u < v) return u;    //c5 - O(1)
        else return v;
    }
}
```

Answer: if else statement ( $x \geq y$ ) with its body runs  $c_1$  constant time and it splits into two possible sequences, in the else statement where  $z$  variable takes  $c_2$  constant time to get the integer division. The variable  $u$  takes a recursive call which runs  $c_3$  linear time since the function is being called recursively  $n$  times before reaching the base case, and the variable  $v$  takes a recursive call which runs  $c_4$  linear time. With  $c_3$  in the same block, it's  $O(2^n)$  exponential times, since each function call calls itself twice unless it has been recursed  $n$  times, and  $v$  has to wait until  $u$  resolves its recursive call and unwind before  $v$  starts its recursive call. The if else statement ( $u < v$ ) with its body runs  $c_5$  constant time. We do not count constant time here and we only take the longest runtime with the worst scenario.

Total running time:

$$f(n) = c_1 + c_2 + (2^n) + c_5$$

$$\underline{F(n) = O(2^n)}$$

**Problem 2 (20 points)** This problem is about the stack and the queue data structures that are described in the textbook.

(1) Suppose that you execute the following sequence of operations on an initially empty stack. Using Example 6.3 in the textbook as a model, complete the following table.

Operation	Return Value	Stack Contents
push(10)		(10)
pop( )	10	()
push(12)		(12)
push(20)		(12, 20)
size( )	2	(12, 20)
push(7)		(12, 20, 7)
pop( )	7	(12, 20)
top( )	20	(12, 20)
pop( )	20	(12)
pop( )	12	()
push(35)		(35)
isEmpty( )	false	(35)

(2) Suppose that you execute the following sequence of operations on an initially empty queue. Using Example 6.4 in the textbook as a model, complete the following table.

Operation	Return Value	Queue Contents (first $\leftarrow$ Q $\leftarrow$ last)
enqueue(7)		(7)
dequeue( )	7	()
enqueue(15)		(15)
enqueue(3)		(15, 3)
first( )	15	(15, 3)
dequeue( )	15	(3)
dequeue( )	3	()
first( )	null	()
enqueue(11)		(11)
dequeue( )	11	()
isEmpty( )	true	()

enqueue(5)		(5)
------------	--	-----

**Problem 3 (60 points)** The goal of this problem is: (1) practice of using and manipulating a doubly linked list and (2) practice of designing and implementing a small recursive method. Write a program named *Hw2\_p3.java* that implements the following requirement:

- This method receives a doubly linked list that stores integers.
- It reverses order of all integers in the list.
- This must be done a ***recursive*** manner.
- The signature of the method must be:

```
public static void reverse(DoublyLinkedList<Integer> intList)
```

- You may want to write a separate method with additional parameters (refer to page 214 of the textbook).
- You may not use additional storage, such as another linked list, arrays, stacks, or queues. The rearrangement of integers must occur within the given linked list.

An incomplete *Hw2\_p3.java* is provided. You must complete this program.

You must use the *DoublyLinkedList* class that is posted along with this assignment. You must not use the *DoublyLinkedList* class that is included in textbook's source code collection.

Note that you must not modify the given *DoublyLinkedList* class and the *DoubleLinkNode* class.

### **Deliverable**

No separate documentation is needed for the program problem. However, you must include the following in your source code:

- Include the following comments above each method:
- Brief description of the method
- Input arguments
- Output arguments
- Include inline comments within your source code to increase readability of your code and to help readers better understand your code.

You must submit the following files:

- *Hw2\_p1\_p3.pdf*. This file must include the answers to problems 1, 2, and 3.

Combine all files into a single archive file and name it *LastName\_FirstName\_hw2.EXT*, where *EXT* is an appropriate archive file extension, such as *zip* or *rar*.

### **Grading**

Problem 1 (20 points):

- Up to 4 points will be deducted for each wrong answer.

Problem 2-(1) (10 points):

- Up to 8 points will be deducted if your answer is wrong.

Problem 2-(2) (10 points):

- Up to 8 points will be deducted if your answer is wrong.

Problem 3 (60 points):

- If your program does not compile, 32 points are deducted.
- If your program compiles but causes a runtime error, 24 points are deducted.
- If there is no output or output is completely wrong, 20 points are deducted.
- If your program is partly wrong, up to 20 points are deducted