

Truth table for the conjunction operation (AND)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$p \wedge q$ is **true** only when *both* p and q are **true**

false for all other combinations

Truth table for the disjunction operation (OR)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$p \vee q$ is **true** when *either* of p or q is **true**

$p \vee q$ is **false** only when p and q are both false

Truth table for the exclusive or (OR)

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Truth table for the negation operation (NOT)

p	$\neg p$
T	F
F	T

$\neg p$ is simply the opposite of p.

Evaluating compound propositions

$p : T$ $q : F$ $r : T$

$p \wedge \neg(q \vee r)$

$T \wedge \neg(F \vee T)$

$T \wedge \neg T$

$T \wedge F$

F

$p \wedge \neg(q \vee r)$

To evaluate $p \wedge \neg(q \vee r)$,

fill in the given truth values for p , q , and r

Then evaluate the operations
in the required order.

The compound proposition $p \wedge \neg(q \vee r)$
evaluates to false

Truth table with three variables

p	q	r	$(p \vee r) \wedge \neg q$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

Truth table for the conditional operation (IF)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth table for the biconditional operation (ONLY IF)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

English expressions of the conditional operation

Consider the propositions:

p: You mow Mr. Smith's lawn.

q: Mr. Smith will pay you.

If p, then q.	If you mow Mr. Smith's lawn, then he will pay you.
If p, q.	If you mow Mr. Smith's lawn, he will pay you.
q if p	Mr. Smith will pay you if you mow his lawn.
p implies q.	Mowing Mr. Smith's lawn implies that he will pay you.
p only if q.	You will mow Mr. Smith's lawn only if he pays you.
p is sufficient for q.	Mowing Mr. Smith's lawn is sufficient for him to pay you.
q is necessary for p.	Mr. Smith's paying you is necessary for you to mow his lawn.

The converse, contrapositive, and inverse

Proposition:	$p \rightarrow q$	Ex: If it is raining today, the game will be cancelled.
Converse:	$q \rightarrow p$	If the game is cancelled, it is raining today.
Contrapositive:	$\neg q \rightarrow \neg p$	If the game is not cancelled, then it is not raining today.
Inverse:	$\neg p \rightarrow \neg q$	If it is not raining today, the game will not be cancelled.

Truth table for tautology $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Truth table for contradiction $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

Truth table to show: $\neg p \vee \neg q \equiv \neg(p \wedge q)$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T