Project 5

1) $$\frac{dy}{dt} = \phi(y,t) = -200{,}000\,y + 199{,}000\,y^{2/3}e^{-t} + e^{-t}$$

Backwood Euler

$$y_{i+1} = y_i + \phi(y_{i+1}, t_{i+1})\,h.$$

$$y_{i+1} = y_i + h\left(-200{,}000\,y_{i+1} + 199{,}000\,y_{i+1}^{2/3}e^{-t_{i+1}} + e^{-t_{i+1}}\right).$$

→ Nonlinear eqⁿ in y_{i+1}.

Use Newton's method.

Let $f(y_{i+1}) = y_{i+1} - y_i - h\left(-200{,}000\,y_{i+1} + 199{,}000\,y_{i+1}^{2/3}e^{-t_{i+1}} + e^{-t_{i+1}}\right).$

Now we need to find the solution to $f(y_{i+1}) = 0$.

Newton's method gives us the equation.

$$\boxed{y_{i+1}^{new} = y_{i+1} - \frac{f(y_{i+1})}{f'(y_{i+1})}}$$

Final solution gives the real y_{i+1}.

Then continue to the next time step.

3). Second Order ODE

$$m\frac{d^2y}{dt^2} = -mg \pm c\left(\frac{dy}{dt}\right)^2$$

Define $V = \frac{dy}{dt}$.

We have a system of 2 1st order ODEs.

$$V = \frac{dy}{dt} \longrightarrow (1)$$

$$m\frac{dv}{dt} = -mg \pm cV^2 \longrightarrow (2)$$

Use these 2 ODEs to use finite difference methods and solve the problem.