MCEN 5125

Homework 5

Hanwen Zhao

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1 Promblem 1: Exercise 5

Consider the linear program

$$\begin{aligned} minimize\ c_1x_1 + c_2x_2 + c_3x_3 \\ subject\ to\ \ x_1 + x_2 \geqslant 1 \\ x_1 + 2x_2 \leqslant 3 \\ x_1 \geqslant 0,\ x_2 \geqslant 0,\ x_3 \geqslant 0 \end{aligned}$$

Give the optimal value and the optimal set for the following values of c: c = (-1,0,1), c = (0,1,0), c = (0,0,-1).

First, we can try to solve this simply problem by using intuitions,

- for the first set of c = (-1,0,1), we want to make x_3 as big as possible since c_1 is negative, therefore $x = [3;0;0]$ would be a optimal solution.
- for the second set of c = (0,1,0), we want to make x_2 as small as possible since c_2 is 1, therefore $x = [1;0;0]$ would be a optimal solution.
- for the third set of c = (0,0,-1), there is no optimal solution since there is no limitation on x_3.

We can verify our solution in MATLAB, reorganize the equations in inequality form:

$$\begin{aligned} -x_1\ -\ x_2\ &\leqslant -1 \\ x_1\ +\ 2x_2\ &\leqslant 3 \\ -x_1\ &\leqslant 0 \\ -x_2\ &\leqslant 0 \\ -x_3\ &\leqslant 0 \end{aligned}$$

we can reform the problem in matrix form:

$$A1 = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

and

$$b = \begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

By using linprog in MATLAB, we have

$$x = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

for the first set of c, and

$$x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

for the second set of c, and no solution for the third set of c.

2 Problem 2: Exercise 9

Formulate the following problems as LPs:

1. $|x|_\infty$

In addition, each rope has the maximum tension it can take, so the tension on each rope mush smaller than the maximum tension. Since we are solving this problem in Equality Form, we need to introduce some slack, now we have the following equations:

$$\begin{aligned} T_1 + S_1 &= 120 \\ T_2 + S_2 &= 160 \\ T_3 + S_3 &= 100 \\ T_4 + S_4 &= 100 \end{aligned}$$

where S are slack variables.
Also for all tension on ropes must greater than 0, we can do this in linprog by setting the lower bound.

2.1 Building Matrix

Now we can start building our matrix:

$$x = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ W_1 \\ W_2 \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix}$$

From statics equations, we have:

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & -4 & 4 & 0 & 0 \\ 1 & 1 & -1 & 0 & -1 & 0 \\ -8 & 8 & -4 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For maximum tension constrains, we have:

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

finally,

$$A = \begin{bmatrix} A1 \\ A2 \end{bmatrix}$$

and b equal to:

$$b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 120 \\ 160 \\ 100 \\ 100 \end{bmatrix}$$

We are solving for maximum of $W_1 + W_2$,

$$f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

As the result, we have:

$$W_1 = 170$$
$$W_2 = 200$$

3 Appendix

3.1 MATLAB Code: Diet Problem