Optimal Trade Policies and Labor Markets*

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Abstract

We provide a general formula for optimal unilateral policies in multi-sector, general-equilibrium models with various widely adopted labor market specifications. Sector-specific tariffs are summarized by a matrix of partial supply elasticities and the share of Home's net import in foreign incomes, reflecting Home's import market power. Sector-specific export taxes depend on trade elasticities and Home's market share in foreign consumption, reflecting Home's export market power. Home imposes higher tariffs or export taxes on sectors with larger market powers. Furthermore, for models belonging to our defined *CES supply system*, the differential tariffs across sectors can be expressed by a constant related to the elasticity of Foreign labor market supply and the sectorial share of Home's net import in foreign incomes, even though endogenous supply elasticities are not constant. Due to direct targeting, Home's unilateral optimal tariffs are directly influenced by the elasticity of Foreign labor market supply, rather than its own labor market conditions. However, in the event of a trade war, when Foreign retaliates, the change in Home's welfare depends on Foreign's market power hence Home's own labor market.

Keywords: Optimal Policies, Tariffs, Labor Market, Perfectly Mobile Labor, Sector Specific

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Labor, Ricardo-Roy Model

1 Introduction

A large number of quantitative models analyze the impact of trade on labor markets. A natural question is: what are the policy implications of such models? In this paper, we study optimal policies in the extended multi-sector trade models, encompassing various labor market specifications that are widely used in quantitative works. We develop a general formula for optimal policies in these models, regardless of the labor market conditions. We then apply this formula to a range of labor market specifications, whether they are efficient or inefficient, and prescribe the specific optimal policies for each scenario. Moreover, we show for the models belong to our defined *CES supply system*, relative tariffs across sectors only depend on a constant parameter and the sectorial share of Home's net import in foreign incomes, even though the *endogenous* supply elasticities in own and other sectors are not constant and vary through general equilibrium price changes.

Characterizing the optimal policies within general equilibrium (GE) quantitative trade frameworks has been challenging, given the complex interdependency of demand and supply across goods and countries in general equilibrium. The traditional optimal trade policy literature breaks the interdependency by using simple general equilibrium models or partial equilibrium models reduces the problem to a single-good monopolist/monopsonist, and leads to the prediction that the optimal tariff should be equal to the inverse of the (own-price) elasticity of the foreign export supply curve. Intuitively, the optimal tariff reflects how effectively Home can affect foreign prices by suppressing foreign demand. This is the essential idea of optimal tariffs role of terms-of-trade manipulation. See Bagwell and Staiger (1999), Broda, Limao, and Weinstein (2008) and Feenstra (2015).

In recent decades, contemporary general equilibrium trade models have emphasized the interlinkage between goods and countries. A tariff in one sector will affect demand and supply in Foreign and Home, and affect other sectors' demand and supply through general equilibrium effects. Optimal tariffs need to consider all these interdependencies. Given the challenge, most existing work studies counterfactual alternative policies instead of optimal ones. Recently, researchers have started to examine optimal trade policies within contemporary general equilibrium trade frameworks. For example, Costinot, Donaldson, Vogel, and Werning (2015) uses a canonical Ricardian model Dornbusch, Fischer, and Samuelson

(1977) and demonstrates that optimal export taxes should increase with sectors' comparative advantage, but tariffs should be uniform across sectors. The property of optimal export taxes reflects Home government's larger monopoly power in sectors with greater comparative advantage, whereas uniform tariffs are driven by wages are equalized across sectors, and the relative import prices of two goods are not manipulable. Home government has no incentive to use differential tariffs to manipulate the relative import prices.

It may seem that the existing theoretical results derived from standard Ricardian models are difficult to reconcile with the observed policies with heterogeneous tariffs across sectors. Caliendo and Parro (2021) points out that the observed wide range of tariff changes across products during the recent trade war contradicts the theoretical result of uniform import tariffs in the Ricardian models.[1] With this seeming inconsistency between theory and data, does it mean terms of trade manipulation in the Ricardian model play no role in explaining differential tariffs across sectors? The answer is no, and here we derive how different labor market structures and other model assumptions drive the Home government's incentives of using different trade policies to manipulate terms of trade in general equilibrium.

We first use a simple example to prove that as long as labor is imperfectly mobile across sectors, it is optimal for Home government to use sector-specific import tariffs. For example, in the special case when labor is sector-specific, suppose Home imposes higher tariffs on a sector Home imports, since sectorial labor is fixed, the sectorial wage in Foreign decreases, and Foreign's price of the goods decreases, Home's terms of trade improve. Hence, optimal tariffs should vary across sectors in this case. A tariff reduces the demand that Foreign faces and depresses the sector wage and price. The larger Home's share in the Foreign sector, the greater the impact of tariffs on Foreign prices, and the higher the optimal tariff.

It is worth noting that quantitative models have been widely studying imperfectly mobile labor and the associated impact of trade. However, our knowledge of optimal trade policies in these frameworks falls behind. Thus, our paper makes two contributions. First, our paper bridges the gap between the quantitative and theoretical literature by providing a theoretical formula for both the general case and the specific cases of labor markets that have been widely used in the quantitative literature. Second, our paper bridges the gap

[1] Costinot et al. (2015). Beshkar and Lashkaripour (2020) show that optimal import tariffs are uniform even under the presence of input-output linkages as far as export taxes are available.

between the traditional literature with simplified general equilibrium and the contemporary trade models that emphasize the interdependency of sectors. We show that even with interdependency, the optimal policies in the contemporary trade models follow a unified mechanism with the traditional literature. And the terms of trade incentives can justify different trade policy patterns. Our paper is primarily theoretical and helps understand the potential economic incentives of Home government trade policies in the current trade models.

Specifically, we first study the unilateral policies of a Home country in the extended multi-sector model under different labor market specifications. Home government chooses policies, consumption, production, trade, and prices subject to technical feasibility and resource constraints. The economic objective of the policy is classic: to exploit terms of trade benefit and to correct distortions. Our first finding is that optimal policies are tied to the Lagrange multipliers on the resource constraints, regardless of the specific labor market or production technology assumptions. The multipliers on foreign-goods-market constraint reflect the impact of foreign goods supply on Home's utility and how Home government internalizes such impact and uses policies to exploit Home's market power. The multipliers on domestic markets reflect how Home internalizes the impact of domestic supply on Home utility and uses policies to correct domestic distortions.

We further unpack these multipliers and connect optimal policies with Home's market powers and domestic labor distortions. First, we show that Home's optimal import tariff in a sector depends on the entire matrix of foreign partial supply elasticity and the proportion of Home net imports in foreign income within that sector. Second, given other taxes, export tax in a sector depends on trade elasticities and the significance of foreign demand for Home goods. Lastly, domestic taxes are directly associated with Home's own partial supply elasticities, which reflects the Bhagwati-Johnson principle of targeting. All these Foreign and Home elasticities and trade shares are jointly determined in the general equilibrium and are affected by optimal policies. The optimal tax formula is general. However, these specifications, whether they are efficient or inefficient, can affect labor allocation across sectors and endogenous supply elasticities, and thus the market power of Home, thereby influencing Home optimal policies in different labor market specifications.

Moreover, we introduce the *CES supply system*. A labor market specification in the Ricardian model generates a CES supply system with a constant elasticity κ if a sector's partial supply elasticities with respect to wages in any two other sectors are the same, though the two elasticities may not be constant, and the supply elasticity with respect to the sector's own wage relative to the elasticity to any other sector's wages is constant, given by κ. It turns out that many widely studied labor market specifications belong to this CES supply system, such as a specification with perfectly mobile labor across sectors ($\kappa \to \infty$), a specification with sector-specific labor ($\kappa = 1$), and imperfectly substitute labor as in a Ricardo-Roy model with one factor.

Under our CES supply system, the interlinks across sectors break down because any two sectors' wages have an equal impact on a third sector's supply in the foreign country. As a result, we can further simplify the general optimal tax formula and obtain the following insights. First, a sector j's tariff relative to a sector s's tariff can be summarized by two sufficient statistics: i) the inverse of κ and ii) Home net imports over foreign's income in sector j relative to that in sector s. Together, the two reflect Home's import market power in sector j relative to sector s. In sum, Home tends to impose higher tariffs in a sector when Home is a large buyer of foreign goods in that sector, and the tariff gap across sectors decreases with κ. Second, given other taxes, a sector's export tax depends on i) the inverse of trade elasticity and ii) Home's market shares in foreign consumption in that sector, together reflecting Home's export market power. Home uses high export taxes on sectors where Home is a large seller to Foreign. Lastly, optimal domestic taxes are zero. This implies that labor markets that generate the CES supply system must be efficient.

We apply our general formula for optimal policies to specific cases of labor markets. The first one we consider is the widely adopted model with perfectly mobile labor across sectors. This labor market generates a CES supply system with κ approaching infinity. In this case, Home would not use sector-specific tariffs and can further set all tariffs to zero. This is because, with labor freely mobile across sectors, foreign supply is completely elastic. Thus, Home has no market power to change foreign supply prices, resulting in zero tariffs. In contrast, with imperfectly substitutable labor across sectors, imposing a tariff decreases Home's demand for foreign goods and further depresses foreign wages and prices in that sector. The larger Home's purchase share in the foreign sector, the greater the impact of tariffs on foreign prices, and the higher the tariff. We also apply the general formula to imperfectly substitutable labor with finite labor market elasticity of substitution κ.

We further study two examples of labor market specifications that do not generate a CES supply system. One is the Ricardo-Roy model with multiple types of labor, and the other involves inefficient labor markets where workers select into sectors based on both wage and labor market wedges. Optimal policies still follow the general formula. Under the Ricardo-Roy model, there are no labor market frictions, and Home does not use any domestic tax. In contrast, in the model with an inefficient labor market, Home chooses domestic taxes that are negatively related to sectors' wages to correct domestic inefficiency. Given that the supply system is not CES, optimal tariffs consider the impacts of each sector's tariff on all other sectors. Thus, the optimal tariffs are determined jointly and depend on the whole supply elasticity matrix and the vector of net import shares in the foreign country.

We then extend the implications of labor market specifications on supply curves to how other factors affect endogenous supply elasticities in GE. Fajgelbaum, Goldberg, Kennedy, Khandelwal, and Taglioni (2024) shows that inelastic supply curves can be micro-founded by both returns to scale in production and elasticity of factor mobility across products and sectors. An example of optimal trade policies in a multi-sector, multi-country GE model under productivity-driven supply curves is Bai, Jin, and Lu (2023) (BJL hereafter), whose finding is consistent with the current paper: optimal import tariffs are related to foreign supply elasticities. However, the current paper has an upward-sloping curve under a fixed technology and imperfectly substitutable labor across sectors, but BJL has a downward-sloping foreign supply curve under endogenous technology and perfectly mobile labor. Thus, the two papers have different implications on the sign of tariffs but follow the same formula.

Furthermore, we show BJL is another example of the CES supply system. Similarly, the multi-sector Krugman and multi-sector Melitz-Pareto models are the CES supply system as well because they are isomorphic to BJL at the steady state. See Kucheryavyy, Lyn, and Rodríguez-Clare (2023) for the isomorphic gravity equations for different returns to scale models. We can apply our optimal policy formula to these two models (see Appendix I).

In summary, both imperfectly substitutable labor and endogenous technology can provide an inelastic foreign supply, and our formula applies.

Notice that the *CES supply system* is not equivalent to each sector's supply elasticity is constant. It is not equivalent to each sector's export supply elasticity is κ either. Similar to the logic of "CES demand system" in Arkolakis, Costinot, and Rodríguez-Clare (2012), where welfare depends on two sufficient statistics: the import share and the trade elasticity of that system. In their general equilibrium trade models, a CES import demand system is conceptually distinct from the assumption of CES preferences. The import demand obviously depends on preferences, it also takes into account the supply side as this affects the allocation of expenditures to domestic production. In fact, CES preferences are neither necessary nor sufficient to obtain a CES import demand system. The *CES supply system* does not imply a *CES export supply system*. We further show all the assumptions on the preference and demand side that affect export supplies and the interdependency of optimal tariffs.

Lastly, we derive Nash optimal policies. Due to direct targeting, Home's unilateral optimal tariffs directly relate to foreign labor market supply elasticity, instead of its own. However, in a trade war, Foreign retaliate, the welfare change of Home would depend on Home labor market supply elasticity. The less flexible Home's labor market, the more Foreign's market power. Hence, Home's labor market affects Home's welfare in a trade war. In a quantitative exercise, we show both the US and China loss in a trade war and the welfare loss depends on each labor market elasticities.

Our approach of providing a general formula for optimal trade policies in general-equilibrium models can be traced back to the classic work of Dixit (1985), which sets up the general problem of optimal taxes in an open economy as a fictitious planning problem and derives the associated first-order conditions under a foreign offer curve. To be clear, there is no direct formula from Dixit to apply to different current trade setups. The Dixit formula itself is a different system of implicit equations in different setups. What the foreign offers curves are and what the foreign export supply elasticities were not known in these workhorse trade models under various labor market specifications. We use the method of FOCs. Moreover, we derive explicitly the optimal policies. We characterize the optimal policies in all different specifications of the labor market and provide formulae for sector-specific import tariffs and export taxes in both general and specific cases. The explicit formula will make calculations of optimal taxes extremely efficient.

Our paper is related to the literature on optimal trade policies. Characterizing optimal trade policy in current trade modes is challenging, but the literature has introduced new methods and new insights. For example, optimal trade policies in recent general equilibrium trade models are theoretically derived in: Costinot et al. (2015), which studies trade policy in the canonical Ricardian model Dornbusch, Fischer, and Samuelson (1977). Costinot, Rodríguez-Clare, and Werning (2020) characterizes optimal firm-level trade policy in a two-country-single-sector Melitz model. Lashkaripour and Lugovskyy (2023) studies policies in multi-industry, multi-country trade models where misallocation occurs due to scale economies. Bai, Jin, and Lu (2023) studies optimal dynamic policies with endogenous technology through innovation. See Caliendo and Parro (2021) for a review of optimal trade policies in general equilibrium models.

The paper proceeds as follows. Section 2 extends the two-country, multi-sector Ricardian framework to incorporate general labor market specifications and details the model environments. Section 3 derives optimal policies for general labor market specifications and, in particular, if the supply system satisfies CES. Section 4 gives a list of labor market examples and explains the specific optimal policies for each scenario. It also extends to other factors beyond the labor market and discusses demand-side assumptions. Section 5 derives Nash optimal policies. Section 6 quantifies the unilateral and Nash optimal policies for the US and China and the associated welfare change. Section 7 concludes.

2 Theoretical Framework

We study optimal trade and domestic policies in a two-country multi-sector model with various labor market specifications. Preference is Cobb-Douglas across the consumptions of different sectors. Within each sector, the technology and preference assumptions can be either Eaton-Kortum or Armington. Both yield the same gravity equations with trade elasticity θ.

The world has $N = 2$ countries and J sectors. Country n has a measure $\bar{L}_n$ of labor. Let country 1 be Home, and country 2 be Foreign. Consumers in all countries have the same utility over final goods C given by $u(C) = C$. Final goods is a Cobb-Douglas function across the consumption of different sector $j \in J$ goods, $C_n = \Pi_{j \in J} \left(C_{nj}\right)^{\beta_j}$, where the constant β_j captures the share of sector j. All sectors have a linear production function and use labor to produce. Let T_{nj} denote the technology level of sector j at country n. All goods are tradable under an iceberg trade cost d_{ni} between country n and i, and θ governs the trade elasticity.

We consider a variety of labor market specifications that have been extensively studied in the trade and labor literature. The labor market specification determines wages and labor in equilibrium. Let w_{nj} and L_{nj} be the wage and labor in sector j of country n, and let Ω represent the equilibrium labor conditions. For example, when labor is perfectly mobile across sectors, we can define $\Omega = \{(w_{nj}, L_{nj}) : \sum_j L_{nj}(\boldsymbol{w}) = \bar{L}_n,\ w_{nj} = w_n\}$. Section 4 provides specific examples of Ω and the associated optimal policies.

Suppose Home imposes unilateral trade and domestic policies, but Foreign is passive and has zero taxes. Specifically, Home government imposes sector-specific import tariffs τ_j^m, export taxes τ_j^x, and domestic sales tax τ_j^d on its imports, exports, and domestic sales, respectively. Let P_n and x_n be country n's final goods price and expenditures, respectively, with $x_n = P_n C_n$.[2] Below, we define the world market equilibrium under Home's policies.

Definition 1 (World Market Equilibrium)**.** *The world market equilibrium consists of an allocation of labor $\{L_{nj}\}$, consumption $\{C_n\}$, expenditures $\{x_n\}$, prices $\{P_n\}$, and wages $\{w_{nj}\}$ such that consumers maximize utility, firms maximize profits, and the market clearing conditions hold, taking as given Home government's policies $\left\{\tau_j^m, \tau_j^x, \tau_j^d\right\}$:*

1. *Expenditures are given by*

$$x_1 = \sum_{j=1}^{J} w_{1j} L_{1j} + \sum_{j=1}^{J} \beta_j \left[\frac{\tau_j^x}{1+\tau_j^x} \pi_{21,j} x_2 + \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1 \right], \tag{1}$$

$$x_2 = \sum_{j=1}^{J} w_{2j} L_{2j}. \tag{2}$$

[2]This indicates transfers are allowed if there are different factors.

2. *Trade shares satisfy, for each sector j*

$$\pi_{11,j} = \frac{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}, \tag{3}$$

$$\pi_{12,j} = \frac{T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}, \tag{4}$$

$$\pi_{21,j} = \frac{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}w_{2j}^{-\theta}}, \tag{5}$$

$$\pi_{22,j} = \frac{T_{2j}(w_{2j})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}w_{2j}^{-\theta}}. \tag{6}$$

3. *Consumer prices are given by*

$$P_1 = \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}\right]^{-\frac{\beta_j}{\theta}}, \tag{7}$$

$$P_2 = \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}w_{2j}^{-\theta}\right]^{-\frac{\beta_j}{\theta}}. \tag{8}$$

4. *Goods market clearing conditions, for each j*

$$w_{1j}L_{1j} = \beta_j \left[\frac{1}{1+\tau_j^d}\pi_{11,j}x_1 + \frac{1}{1+\tau_j^x}\pi_{21,j}x_2\right], \tag{9}$$

$$w_{2j}L_{2j} = \beta_j \left[\frac{1}{1+\tau_j^m}\pi_{12,j}x_1 + \pi_{22,j}x_2\right]. \tag{10}$$

5. *The labor market clearing conditions $\Omega(\{w_{nj}, L_{nj}\})$ hold.*

Without loss of generality, we normalize Home's final goods price $P_1 = 1$. Note that in the world market equilibrium, the market clearing conditions and definition of expenditures imply that the balanced trade condition holds.

3 General Formula of Optimal Policy

This section presents a general formula for optimal policy that is applicable to various labor market conditions. Section 4 examines specific labor market assumptions and their corresponding optimal policies. It is worth noting that under the multi-sector trade framework, sector-specific export taxes, tariffs, and sales taxes are the finest policy instruments as they adequately address the externalities arising from terms of trade and potential labor market frictions.

Definition 2. *Home government's problem is to choose sector-specific* $\{\tau_j^d, \tau_j^x, \tau_j^m\}$ *to maximize domestic consumers' consumption,* $\max x_1/P_1$*, subject to the world market equilibrium given by Definition 1.*

3.1 Lagrange Multipliers and Optimal Policies

We use Lagrange multiplier methods to derive the general formula for optimal policies. Let γ_{1j} and γ_{2j} be the Lagrange multipliers on the goods market clearing constraint of Home (9) and Foreign (10).

lemma 1 (Optimal Policies and Multipliers). *Irrespective of labor market specifications, optimal policies for the problem in Definition 2 take the form of*

$$1+\tau_j^m = -\gamma_{2j}, \quad 1+\tau_j^x = \frac{\gamma_{1j}}{\gamma_{2j}}\left(1+\frac{1}{\theta\pi_{22,j}}\right), \quad 1+\tau_j^d = -\gamma_{1j}. \tag{11}$$

Proof. Appendix A.

There are two potential reasons why Home government would like to utilize taxes: to manipulate terms of trade and to address domestic distortions in labor markets. First, Home, as a whole, is a monopoly on the goods it exports and may have buyer powers in the goods it imports. Individual agents do not consider the impact of their imports or exports on the prices they pay, but Home government internalizes this and aims to use trade policies to manipulate the terms of trade. γ_{2j} reflects how changing Foreign goods demand affects Home utility, hence Home government's consideration of exploiting buyer's power. When γ_{2j} is positive, Home government would like to relatively subsidize imports of sector j goods. Second, when there are domestic distortions, Home government seeks to use domestic policies to correct these distortions. Domestic sales taxes τ_j^d depend on multiplier γ_{1j} or how changing domestic goods demand matters for Home utility. It reflects the Home government's consideration of correcting domestic distortions. When γ_{1j} is positive, Home government relatively subsidizes sector j.

3.2 A First Look at Labor Market Specifications and Optimal Policies

In order to guide through the general results and illustrate how the labor market specification matters, we first use the simplest two labor market specifications possible for comparison.

Perfectly mobile labor across sectors. This type of model has been widely adopted in the literature, for example, Costinot, Donaldson, and Komunjer (2012) extends Eaton and Kortum (2002) to multi-sectors. In this case, the equilibrium wages are equalized across sectors, and the labor market is cleared in equilibrium, leading to $\Omega = \{(w_{nj}, L_{nj}) : \sum_j L_{nj}(\boldsymbol{w}) = \bar{L}_n,\ w_{nj} = w_n\}$.

With mobile labor, we can sum over labor markets (9)-(10) across sectors (See Appendix D). Now, the goods market clearing conditions become:

$$w_1\bar{L}_1 = \sum_{j=1}^{J} \beta_j \left[\frac{1}{1+\tau_j^d}\pi_{11,j}x_1 + \frac{1}{1+\tau_j^x}\pi_{21,j}x_2\right], \quad (\gamma_1)$$

$$w_2\bar{L}_2 = \sum_{j=1}^{J} \beta_j \left[\frac{1}{1+\tau_j^m}\pi_{12,j}x_1 + \pi_{22,j}x_2\right], \quad (\gamma_2)$$

where $x_2 = w_2\bar{L}_2$.

lemma 2. *When labor is perfectly mobile across sectors, the optimal policies for the Home country take the form of*

$$1+\tau_j^d = -\gamma_1, \quad 1+\tau_j^m = -\gamma_2, \quad 1+\tau_j^x = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\theta\pi_{22,j}}\right). \tag{12}$$

Proof. Appendix A and D. Since now the multipliers γ_1 and γ_2 are not sector-specific, the optimal policies become $\tau_j^d = -\gamma_1 - 1 = \tau^d$, and $\tau_j^m = -\gamma_2 - 1 = \tau^m$. Thus, domestic tax or tariffs are uniform across sectors. Furthermore, the Appendix D shows tax neutrality, and both the uniform tariff and domestic tax can be normalized to zero.

Immobile labor/specific factors. In this case, goods market clearing conditions (9) and (10) are:

$$w_{1j}\bar{L}_{1j} = \beta_j \left[\pi_{11,j}x_1 + \frac{1}{1+\tau_j^x}\pi_{21,j}x_2 \right] \quad (\gamma_{1j}, \quad J) \tag{13}$$

$$w_{2j}\bar{L}_{2j} = \beta_j \left[\frac{1}{1+\tau_j^m}\pi_{12,j}x_1 + \pi_{22,j}x_2 \right] \quad (\gamma_{2j}, \quad J) \tag{14}$$

lemma 3. *When labor is totally immobile across sectors, the optimal policies for the Home country take the form of*

$$\tau_j^d = 0, \quad \tau_j^m - \tau_i^m = \frac{\beta_j x_1 - w_{1j}\bar{L}_{1j}}{w_{2j}\bar{L}_{2j}} - \frac{\beta_i x_1 - w_{1i}\bar{L}_{1i}}{w_{2i}\bar{L}_{2i}}, \quad 1+\tau_j^x = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\theta\pi_{22,j}}\right). \tag{15}$$

Proof. Appendix E. We prove that τ_j^d is uniform across sectors. Furthermore, as demonstrated in the Appendix E, tax neutrality holds, allowing us to normalize the uniform domestic tax to zero. The optimal import tariff τ_j^m increases with the ratio of Home's net imports to Foreign's production.

3.3 Optimal Policies

In general, the terms-of-trade gains of trade policy come from Home's ability to change import and export prices by affecting the demand Foreign face and the supply of Home goods to Foreign. In other words, Home's gain from manipulating terms of trade depends on its buyer power in Foreign goods and its monopoly power in exporting to Foreign. Furthermore, these Home powers are reflected in Foreign export supply and import demand elasticities, respectively. In the general equilibrium models, these elasticities are endogenous as goods supplies are interlinked across sectors. Below, we show that the matrix of partial supply elasticities and Home's net import shares in Foreign sector are sufficient for import tariffs, and trade elasticity and Home's export shares in Foreign consumption determine export taxes.

Let Y_{nj} denote the income of sector j in country n. We can define the *partial supply elasticity* between sector i and j as $\frac{\partial \ln Y_{ni}}{\partial \ln w_{nj}}\frac{Y_{ni}}{Y_{nj}}$, which is the product of the partial elasticity of a sector's income with respect to another sector's wage, multiplied by the relative income share of these two sectors. These partial supply elasticities show how changes in one sector's wage affect the relative supply of other sectors, assuming equilibrium wages are exogenous. Moreover, these elasticities are determined by labor market specifications given the preferences, technology, and market structure assumptions. Let matrix Λ_n consist of these elasticities in country n, with its jth-row-ith-column entry as $\frac{\partial \ln Y_{ni}}{\partial \ln w_{nj}} \frac{Y_{ni}}{Y_{nj}}$.

We now present the other determinant of optimal policies: Home net imports as a share of Foreign production, for example, $\frac{\beta_j x_1 - (1+\tau_j^d) Y_{1j}}{Y_{2j}}$ for sector j. Here, $\beta_j x_1$ and $(1+\tau_j^d) Y_{1j}$ are Home's total expenditure and its production on own goods in sector j, respectively, and the difference between the two is the Home's net spending on Foreign goods in sector j. Let the vector Ψ_1 contain all these shares, with the jth entry of Ψ_1 as $\frac{\beta_j x_1 - (1+\tau_j^d) Y_{1j}}{Y_{2j}}$. These shares and the partial supply elasticities capture Home's buyer powers across sectors.

Definition 3. *Matrix Λ_n consist of partial supply elasticities in country n, with its jth-row-ith-column entry as $\frac{\partial \ln Y_{ni}}{\partial \ln w_{nj}} \frac{Y_{ni}}{Y_{nj}}$. Vector Ψ_1 contains Home's net import spending share in Foreign income with the jth entry of $\frac{\beta_j x_1 - (1+\tau_j^d) Y_{1j}}{Y_{2j}}$.*

Different labor market specifications, whether elastic or inelastic, efficient or inefficient, can affect labor allocations and buyer power of Home, showing up in the partial supply elasticities and Home's net import shares in Foreign income. However, it is shown below that the optimal tax *formula* is independent of these specifications.

Proposition 1 (Optimal Policy Formula). *Home government's unilateral optimal policy satisfies,*

$$\text{Domestic tax vector:} \quad (\Lambda_1 - I)(1 + \boldsymbol{\tau^d}) = 0, \tag{16}$$

$$\text{Import tariff vector:} \quad \boldsymbol{\tau^m} = \Lambda_2^{-1} \Psi_1, \tag{17}$$

$$\text{Export tax:} \quad 1 + \tau_j^x = \frac{1 + \tau_j^d}{1 + \tau_j^m} \left(1 + \frac{1}{\theta \pi_{22,j}} \right), \quad \forall j, \tag{18}$$

where $\boldsymbol{\tau^d}$ and $\boldsymbol{\tau^m}$ are vectors of domestic taxes and import tariffs across sectors, respectively. I is an identity matrix.

Proof. Appendix B. First, domestic taxes are directly related to Home's partial supply elasticities Λ_1, which depends on Home's own labor market efficiency. Second, as we discussed above, import tariffs are linked to Foreign export supply elasticities, which in turn depends on Foreign's partial supply elasticity matrix Λ_2 and how important is Home purchase in Foreign, Ψ_1. Lastly, given other taxes, export tax is higher if the demand elasticity θ is smaller, i.e., demand is less elastic, and if $\pi_{22,j}$ is lower, which implies a greater Foreign demand for Home goods. The schedule of export taxes is consistent with Costinot et al. (2015), which shows that export taxes reflect the Home government's incentive to manipulate relative prices in its favor by exploiting its export market power. In equilibrium, all taxes are jointly determined.

Definition 4 (**CES Supply System**). *Let κ be a constant. A CES supply system with elasticity κ satisfies*

$$\frac{\partial \ln Y_s}{\partial \ln w_j}\frac{Y_s}{Y_j} - \frac{\partial \ln Y_s}{\partial \ln w_i}\frac{Y_s}{Y_i} = \begin{cases} 0 & \text{for } s \neq i, j \\ \kappa & \text{for } s = j. \end{cases} \tag{19}$$

In extended multi-sector Ricardian models, examples of CES supply systems include the standard case of perfectly mobile labor and the case with totally immobile labor across sectors, corresponding to $\kappa \to \infty$ and $\kappa = 1$, respectively. In the next section, we discuss a variety of labor market structures that generate different supply systems and their associated optimal policies.

Proposition 2 (**Optimal Policy under CES Supply System**). *Under a CES supply system with a parameter κ_1 and κ_2 for Home and Foreign respectively, Home government's unilateral optimal policy satisfies, for any sectors j, i,*

$$\textit{Domestic tax:} \quad \tau_j^d = 0, \tag{20}$$

$$\textit{Import tariff:} \quad \tau_j^m - \tau_i^m = \frac{1}{\kappa_2}\left(\frac{\beta_j x_1 - Y_{1j}}{Y_{2j}} - \frac{\beta_i x_1 - Y_{1i}}{Y_{2i}}\right), \tag{21}$$

$$\textit{Export tax:} \quad 1 + \tau_j^x = \frac{1}{1+\tau_j^m}\left(1 + \frac{1}{\theta \pi_{22,j}}\right), \tag{22}$$

Proof. Appendix C. Proposition 2 demonstrates that under a CES supply system, Home would not use sector-specific domestic tax. In Appendix D-G, we further establish that if the labor market condition Ω is homogenous of degree zero on wages, tax neutrality holds. In this case, one of the domestic taxes can be normalized to zero, and since Home optimally will not use sector-specific domestic tax, all domestic taxes are zero with a CES supply system.

Moreover, under a CES supply system, the partial elasticities of a sector's income with respect to wages in any other two sectors (weighted by relative income shares) are the same, and the partial elasticities of a sector's income with respect to its own wage relative to that with respect to the other sector's wage is constant at κ, i.e., $\frac{\partial \ln Y_j}{\partial \ln w_j} - \frac{\partial \ln Y_j}{\partial \ln w_i}\frac{Y_j}{Y_i} = \kappa$. Under this property, the difference of any two rows in the general import tariff vector (17) in Proposition 1 yields the formula (21).

Thus, the relative import tariffs between any two sectors only depend on the relative Home purchases of the two sectors' goods. A larger Home net import spending relative to Foreign income, $\frac{\beta_j x_1 - Y_{1j}}{Y_{2j}}$, results in greater Home buyer power and thus a greater import tariff τ_j^m. Importantly, the relative import tariffs $\tau_j^m - \tau_i^m$ does not depend on Home's import share in other sectors. Furthermore, instead of depending on the whole supply elasticity matrix, the relative import tariffs only depend on the constant κ_2. Lower κ_2 leads to higher incentives to impose tariffs on sectors with larger Home buyer powers.

4 Examples

In this section, we explore the labor market specifications that are widely used in the literature. We show how the labor market conditions Ω is determined in each case and how Proposition 1 and 2 on unilateral optimal policies apply to each case.

We consider three labor market specifications, all of which maintain the assumption that labor cannot move across countries. The first specification features a constant elasticity of labor substitution across sectors. We call these models *CES labor market*. This specification nests the models with perfectly mobile labor across sectors and models with sector-specific labor. The second specification is a *Ricardo-Roy model*. The third one involves *inefficient labor markets*, where worker selection into sectors depends on both wages and wedges.

4.1 CES Labor Market

Here, total labor is a CES aggregator over labor in different sectors, $L_n = [\sum_{j=1}^{J} \alpha_j{}^{-\frac{1}{\kappa_n - 1}} L_{nj}{}^{\frac{\kappa_n}{\kappa_n - 1}}]^{\frac{\kappa_n - 1}{\kappa_n}}$, where $\{\alpha_j\}$ are constants and $\kappa_n - 1 \geq 0$ captures the elasticity of substitution across labor in different sectors. This implies a labor supply curve as

$$L_{nj} = \alpha_j \left(\frac{w_{nj}}{W_n}\right)^{\kappa_n - 1} \bar{L}_n, \quad \text{with } W_n = \left[\sum_{j=1}^{J} \alpha_j w_{nj}{}^{\kappa_n}\right]^{\frac{1}{\kappa_n}}. \tag{23}$$

When $\kappa \to \infty$, $L_n = \sum_{j=1}^{J} L_{nj}$ and labor is perfectly substitutable across sectors, which is widely adopted in the literature. For example, Costinot, Donaldson, and Komunjer (2012) extends Eaton and Kortum (2002) to multi-sectors. In this case, the equilibrium wages are equalized across sectors, leading to $\Omega = \{(w_{nj}, L_{nj}) : \sum_j L_{nj}(\boldsymbol{w}) = \bar{L}_n,\ w_{nj} = w_n\}$. When $\kappa = 1$, the model is the same as the specific-factor model with fixed labor in each sector, and thus $\Omega = \{(w_{nj}, L_{nj}) : L_{nj} = \alpha_j \bar{L}_{nj},\ w_{nj} = w_{nj}(\boldsymbol{L})\}$.

In general, we can rewrite the labor supply curve (23) and obtain the equilibrium labor market condition as

$$\Omega = \left\{(w_{nj}, L_{nj}) : \frac{w_{nj} L_{nj}}{W_n \bar{L}_n} = \alpha_j \left(\frac{w_{nj}}{W_n}\right)^{\kappa_n},\ W_n = [\sum_{j=1}^{J} \alpha_j w_{nj}{}^{\kappa_n}]^{\frac{1}{\kappa_n}}\right\}. \tag{24}$$

We define the income in sector j, $Y_{nj} = w_{nj} L_{nj}$. Labor market condition (24) further implies a supply system,

$$Y_{nj} = \alpha_j \left(\frac{w_{nj}}{W_n}\right)^{\kappa_n} Y_n.$$

Let the share of sector j income in country n as $m_{nj} = Y_{nj}/Y_n$. We can show that the partial-supply-elasticity matrix Λ_n takes the form of

$$\Lambda_n = \begin{pmatrix} \kappa_n - (\kappa_n - 1)m_{n1} & \dots & -(\kappa_n - 1)m_{nj} & \dots & -(\kappa_n - 1)m_{nJ} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa_n - 1)m_{n1} & \dots & \kappa_n - (\kappa_n - 1)m_{nj} & \dots & -(\kappa_n - 1)m_{nJ} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa_n - 1)m_{n1} & \dots & -(\kappa_n - 1)m_{nj} & \dots & \kappa_n - (\kappa_n - 1)m_{nJ} \end{pmatrix}. \quad (25)$$

Take the difference between any two rows of Λ_n, say i and j, and call the resulting vector $d\Lambda_{n,ij}$. The condition (19) of the definition of CES supply system requires all entries of $d\Lambda_{n,ij}$ to be zero, except for the ith or jth entry, meaning that any two sectors' tariffs must have a symmetric impact on the third sector. It is clear that Λ_n matrix (25) satisfies this requirement. Thus, the model with constant labor elasticity is a CES supply system. We can then apply the Proposition 2 for optimal policies. See Appendix F for details. Furthermore, $\tau_j^d = \bar{\tau}^d$, in addition, Ω in (24) has the property of homogenous of degree zero in wages. Hence, tax neutrality holds, and we can set $\bar{\tau}^d = 0$. The optimal domestic taxes are zeroes for all sectors. Intuitively, without domestic frictions in the model, Home government has no incentives to use domestic taxes.

The optimal tariff formula equation (21) nests the case with perfectly mobile labor across sectors ($\kappa_2 \to \infty$). In such case, the optimal import tariffs can be set to zero, $\tau_j^m = 0$ for any j. This is because under completely elastic Foreign supply, Home has no market power to change foreign supply prices and, thus, no incentives to use tariffs. In contrast, with imperfect substitute labor, including sector-specific labor model, where $\kappa_2 < \infty$, imposing a tariff decreases the demand that Foreign faces and depresses the sector wage and price. The larger Home's share in the Foreign sector, the greater the impact of tariffs on Foreign prices, and the higher the tariff.

4.2 Ricardo-Roy models

Ricardo-Roy models, such as those in Costinot and Vogel (2010) and Costinot and Vogel (2015), extend the Ricardian trade model with more than one factor, as in the Roy model.

Here we adopt Galle, Rodríguez-Clare, and Yi (2023), which combines multi-sector EK Costinot, Donaldson, and Komunjer (2012) and Roy model as in Lagakos and Waugh (2013) and Hsieh, Hurst, Jones, and Klenow (2019).

There are G groups of workers in each country.[3] The total number of group-g workers in country n is fixed at $\bar{L}_{ng}$. A worker in group g of country n draws an efficiency unit z_j in sector j from a Fréchet distribution F_{njg} with shape parameter κ and scale parameter A_{njg}. A worker with the vector $\mathbf{z} = (z_1, z_2, \dots, z_J)$ chooses to work in a sector that gives her the highest income based on her productivity and wage in that sector. Let Ξ_{nj} be the set of workers choosing sector j, $\Xi_{nj} \equiv \{\mathbf{z}\colon w_{nj}z_j \geq w_{nk}z_k \text{ for all } k\}$ and $F_{ng}(\mathbf{z})$ be the joint probability distribution of $\mathbf{z}$ for group-g workers in country n.

In equilibrium, the share of workers in group g that apply to sector j in country n is given by

$$\lambda_{njg} = \frac{A_{njg}(w_{nj})^{\kappa_n}}{\sum_{s=1}^{J} A_{nsg}(w_{ns})^{\kappa_n}}.$$

The sum of efficiency units supplied to sector j in country n is $L_{njg} \equiv L_{ng} \int_{\Xi_{nj}} z_j dF_{ng}(\mathbf{z}) = \frac{\xi_n [\sum_{s=1}^{J} A_{nsg}(w_{ns})^{\kappa_n}]^{\frac{1}{\kappa_n}}}{w_{nj}} \lambda_{njg}\bar{L}_{ng}$, where $\xi_n \equiv \Gamma\left(1 - 1/\kappa_n\right)$. For notation simplification, we scale A_{nsg} by $\xi_n^{\kappa_n}$, i.e., setting A_{nsg} to $\xi_n^{\kappa_n} A_{nsg}$. Let $L_{nj} = \sum_g L_{njg}$ be the total efficiency labor worked in sector j. Hence, Ω satisfies

$$\Omega = \left\{ (w_{nj}, L_{nj}) : w_{nj}L_{nj} = \sum_{g=1}^{G} \left[\frac{A_{njg}(w_{nj})^{\kappa_n}}{W_{ng}^{\kappa_n}} W_{ng}\bar{L}_{ng} \right], \; W_{ng} = \left[\sum_{s=1}^{J} A_{nsg}(w_{ns})^{\kappa_n} \right]^{\frac{1}{\kappa_n}} \right\}.$$

When there is only one group, the equilibrium condition becomes $\Omega = \Big\{ (w_{nj}, L_{nj}) : \frac{w_{nj}L_{nj}}{W_n\bar{L}_n} = A_{nj} \left(\frac{w_{nj}}{W_n}\right)^{\kappa_n} \Big\}$, which is the same as the condition (24) under imperfect labor substitution with $\alpha_j = A_{nj}$. Thus, the one-group Ricardo-Roy model is a CES supply system.

Generally, the Ricardo-Roy model does not satisfy the conditions of the CES supply system. In particular, the matrix of partial supply elasticity of Foreign country takes the

[3] For example, there could be two groups of workers: skilled and unskilled groups.

form of

$$
\Lambda_2 = \begin{pmatrix}
\kappa_2 - (\kappa_2 - 1)\frac{\sum_g m_{21g} m_{21g} W_{2g} \bar{L}_{2g}}{Y_{21}} & \dots & -(\kappa_2 - 1)\frac{\sum_g m_{21g} m_{2jg} W_{2g} \bar{L}_{2g}}{Y_{21}} & \dots & -(\kappa_2 - 1)\frac{\sum_g m_{21g} m_{2Jg} W_{2g} \bar{L}_{2g}}{Y_{21}} \\
\dots & \dots & \dots & \dots & \dots \\
-(\kappa_2 - 1)\frac{\sum_g m_{2jg} m_{21g} W_{2g} \bar{L}_{2g}}{Y_{2j}} & \dots & \kappa_2 - (\kappa_2 - 1)\frac{\sum_g m_{2jg} m_{2jg} W_{2g} \bar{L}_{2g}}{Y_{2j}} & \dots & -(\kappa_2 - 1)\frac{\sum_g m_{2jg} m_{2Jg} W_{2g} \bar{L}_{2g}}{Y_{2j}} \\
\dots & \dots & \dots & \dots & \dots \\
-(\kappa_2 - 1)\frac{\sum_g m_{2Jg} m_{21g} W_{2g} \bar{L}_{2g}}{Y_{2J}} & \dots & -(\kappa_2 - 1)\frac{\sum_g m_{2Jg} m_{2jg} W_{2g} \bar{L}_{2g}}{Y_{2J}} & \dots & \kappa_2 - (\kappa_2 - 1)\frac{\sum_g m_{2Jg} m_{2Jg} W_{2g} \bar{L}_{2g}}{Y_{2J}}
\end{pmatrix},
$$

where $m_{2jg} = Y_{2jg}/Y_{2g} = \lambda_{2jg}$ is the share of income in sector j produced by group g workers in Foreign. Take the difference between any two rows of Λ_n, say i and j, and call the resulting vector $d\Lambda_{n,ij}$. Condition (19) of the CES supply system requires all entries of $d\Lambda_{n,ij}$ to be zero, except for the ith or jth entry. Here, none of the entries of $d\Lambda_{n,ij}$ are zero. Thus, the general Ricardo-Roy model fails to satisfy the CES supply requirement.

Specifically, sector i's income, represented by $w_i L_i$, is affected by the wage in sector j through the expression $-(\kappa_2 - 1)\frac{\sum_g m_{2jg} m_{2ig} W_{2g} \bar{L}_{2g}}{Y_{2j}} = -(\kappa - 1)\sum_g \frac{Y_{2jg}}{Y_{2j}} m_{2ig}$, which captures the relationship between the two income shares m_{2jg} and m_{2ig}. This relationship indicates how likely workers in group g within sector j apply to sector i. A higher covariance between these income shares results in greater elasticity, meaning that changes in sector j's wage will have a more pronounced effect on sector i's income. This heightened impact is due to the increased likelihood of worker mobility between the two sectors in response to change in j. This covariance is ultimately determined by the correlation of productivities $\{A_{2jg}\}$. In general, the covariance between sector j and i can differ from that between sector s and i. Thus, the impact of price changes of sector j and sector s on the third sector i are different, and thus, Ricardo-Roy with multi-factors is not a CES supply system.

We can apply Proposition 1 for optimal policies. Moreover, even if the condition for the CES supply system is not met, we can still prove that the Home government does not use any domestic taxes, i.e., $\tau_j^d = 0$ for any sector j. The reason is that the Ricardo-Roy model has no labor market frictions. See Appendix G for details.

4.3 Model with inefficient labor markets

Each inefficient labor market model calls for different policies. Here, we study a labor sectorial choice model with wedges.

Workers choose to work in a sector that maximizes their income, depending both on the wage and their wedge in the sector. A country n's worker draws a wedge ϵ_{nj} for working in sector j from a Fréchet distribution with κ_n governing the degree of dispersion across individuals. The law of large numbers ensures the share of workers in sector j given by $\lambda_{nj} = \Pr\left(\epsilon_{nj} w_{nj} \geq \max_s \{\epsilon_{ns} w_{ns}\}\right)$. Under the Fréchet distribution, the labor share becomes $\lambda_{nj} = \frac{(w_{nj})^{\kappa_n}}{\sum_{s=1}^{J}(w_{ns})^{\kappa_n}}$ and total employment in sector j of country n is $L_{nj} = \lambda_{nj}\bar{L}_n$. Hence, the equilibrium condition is given by

$$\Omega = \left\{(w_{nj}, L_{nj}) : \frac{Y_{nj}}{Y_n} = \frac{w_{nj}L_{nj}}{W_n L_n} = \frac{(w_{nj})^{\kappa_n+1}}{\sum_{s=1}^{J}(w_{ns})^{\kappa_n+1}},\ W_n = \frac{\sum_{s=1}^{J}(w_{ns})^{\kappa_n+1}}{\sum_{s=1}^{J}(w_{ns})^{\kappa_n}}\right\}. \tag{26}$$

In this case, a typical entry for Λ_2 is $\frac{\partial \ln Y_{2s}}{\partial \ln w_{2j}}\frac{Y_{2s}}{Y_{2j}} = -\kappa_2 m_{2s}\frac{W_2}{w_{2j}}$ for $s \neq j$, and the model does not satisfy the requirement for the CES supply system in Definition 4. Sector-specific tariffs are interlinked. For domestic taxes, with inefficiency in labor markets, Home government has incentives to fix domestic distortions. Suppose Foreign country has zero taxes, including its domestic taxes. The following Lemma shows Home's optimal policies in this environment.

lemma 4. *Under the inefficient labor market model, Home government's unilateral optimal policy satisfies, for any sectors j, i,*

$$\textit{Domestic tax:} \quad \frac{1+\tau_j^d}{1+\tau_i^d} = \frac{w_{1i}}{w_{1j}} \tag{27}$$

$$\textit{Import tariff:} \quad \tau_j^m - \tau_i^m = \frac{1}{\kappa_2+1}\left[\frac{\beta_j x_1 - (1+\tau_j^d)Y_{1j}}{Y_{2j}} - \frac{\beta_i x_1 - (1+\tau_i^d)Y_{1i}}{Y_{2i}}\right] + \frac{\kappa_2}{\kappa_2+1}\left(\frac{W_2}{w_{2j}} - \frac{W_2}{w_{2i}}\right)\sum_{s=1}^{J}\tau_s^m m_{2s}, \tag{28}$$

Proof. Appendix H. This model differs from Ricardo-Roy, where labor is allocated to sectors according to their revenue productivity, and expected income per worker is equalized, even if wages per efficiency unit of labor differ across sectors. Home government cares about workers' total consumption, which reaches the optimum when workers are allocated among sectors based on their productivity. However, in the market equilibrium, workers also relocate based on their sector wedges. Therefore, as shown in (27), Home government subsidizes sectors with higher wages that result from better technologies in those sectors.

The relative import tariffs look similar to the formula (21) in Proposition 2 under CES supply system. It is still the case that relative Home import shares, $\frac{\beta_j x_1-(1+\tau_j^d)Y_{1j}}{Y_{2j}}-\frac{\beta_i x_1-(1+\tau_i^d)Y_{1i}}{Y_{2i}}$, matter for the relative import tariffs across sectors. However, in this case, there is an extra term, which depends on Foreign country's relative wages in the two sectors, $\frac{1}{w_{2j}}-\frac{1}{w_{2i}}$. The reason is that inefficiencies are still present in Foreign without Foreign taxes. These inefficiencies affect Foreign supply elasticities, which in turn shapes Home government's incentives to use import tariffs.

We further consider a case in which Foreign uses domestic taxes to fix its domestic inefficiencies, but Foreign still does not invoke any trade policies. In this case, Foreign's domestic taxes satisfy $\frac{1+\tau_j^{d*}}{1+\tau_i^{d*}}=\frac{w_{2i}}{w_{2j}}$, and the second term at Home's relative import tariffs become zero. Now, the optimal import tariffs satisfy

$$\tau_j^m-\tau_i^m=\frac{1}{\kappa_2+1}\left(\frac{\beta_j x_1-(1+\tau_j^d)Y_{1j}}{(1+\tau_j^{d*})Y_{2j}}-\frac{\beta_i x_1-(1+\tau_i^d)Y_{1i}}{(1+\tau_i^{d*})Y_{2i}}\right),$$

which is the same as the efficient case under the CES supply system. See Appendix H.4 for details. Hence, trade policies depend on the Foreign labor market, while domestic policies depend on the domestic labor market.

4.4 Other Production/Supply and Demand Specifications

We then extend the implications of labor market specifications on supply curves to how other factors affect the endogenous supply elasticities in GE. An example of optimal trade policies in a multi-sector, multi-country GE model under productivity-driven supply curves is Bai, Jin, and Lu (2023) (BJL hereafter), whose finding is consistent with the current paper: optimal import tariffs are related to foreign supply elasticities. Furthermore, the steady-state BJL framework is isomorphic to the standard multi-sector Krugman and multi-sector

Melitz-Pareto models. This result is proven in Appendix I.2, and further details on the isomorphic gravity equations for models with varying returns to scale can be found in Kucheryavyy, Lyn, and Rodríguez-Clare (2023).

Building on BJL, we revisit their steady-state optimal policies while incorporating endogenous technology accumulation. In the BJL framework, total labor in each sector is divided into two components: production labor and researchers. Labor is assumed to be perfectly mobile across sectors, and technology endogenously depends on the number of researchers. In steady-state equilibrium, the ratio of production labor to researchers remains constant, implying that technology can be expressed as a function of production labor. Let T_{nj} denote the technology level of sector j in country n, which depends on an efficiency parameter ν_{nj} and the endogenous production labor in the sector, L^p_{nj}. Specifically, $T_{nj} = \nu_{nj} L^p_{nj}$.

We generalize the analysis to cases where labor is not necessarily perfectly mobile, and technology depends endogenously on production labor. Under any given labor market specification, technology may exhibit constant, decreasing, or increasing returns to scale as a result of external economies of scale (EES). Specifically, the technology of sector j in country n is a function of the sector's endogenous production labor, i.e.,

$$T_{nj} = \nu_{nj}(L^p_{nj})^{\epsilon}.$$

where ϵ is the scale elasticity, governing the strength of EES. This generalization provides a unified framework for analyzing labor market specifications under different economic structures. The BJL framework emerges as a special case when $\epsilon = 1$ and labor is perfectly mobile. In contrast, when $\epsilon = 0$, the technology are fixed and corresponds to the framework described in the earlier sections.

In the BJL framework, firms engage in Bertrand competition, where the lowest-cost producer of each good in each market captures the entire market by charging a markup just enough to keep the second-lowest-cost producer out of the market. In equilibrium, markups follow a Pareto distribution with parameter θ. Since all firms selling in the market charge a markup drawn from the same distribution, total profits of firms earned by firms in the market are a constant share of that market's total sales. Specifically, firms (both domestic and foreign) selling in the market earn profits equal to $1/(1+\theta)$ of total sales. The remaining $\theta/(1+\theta)$ share of total sales goes to production labor. Therefore, we can express the total sales of sector j in country n as $\frac{1+\theta}{\theta} w_n L^p_{nj}$.

Goods market clearing conditions are[4]

$$\frac{1+\theta}{\theta} w_{1j} L_{1j} = \beta_j \left[\frac{1}{1+\tau^d_j} \pi_{11,j} x_1 + \frac{1}{1+\tau^x_j} \pi_{21,j} x_2 \right], \quad (\gamma_{1j}, \quad J)$$

$$\frac{1+\theta}{\theta} w_{2j} L_{2j} = \beta_j \left[\frac{1}{1+\tau^m_j} \pi_{12,j} x_1 + \pi_{22,j} x_2 \right], \quad (\gamma_{2j} \quad J)$$

The total expenditures are given by

$$x_1 = \frac{1+\theta}{\theta} \sum_{j=1}^{J} w_{1j} L_{1j} + \sum_{j=1}^{J} \beta_j \frac{\tau^x_j}{1+\tau^x_j} \pi_{21,j} x_2 + \sum_{j=1}^{J} \beta_j \frac{\tau^m_j}{1+\tau^m_j} \pi_{12,j} x_1 + \sum_{j=1}^{J} \beta_j \frac{\tau^d_j}{1+\tau^d_j} \pi_{11,j} x_1,$$

$$x_2 = \frac{1+\theta}{\theta} \sum_{j=1}^{J} w_{2j} L_{2j}.$$

Consider a general case where labor has imperfect mobility with elasticity κ_n. The labor supply curve is given by (23) and the equilibrium labor market condition is given by (24). The following proposition shows Home's unilateral optimal policies in this environment.

Proposition 3. *Under the model with endogenous technology and imperfectly mobile labor, Home government's unilateral optimal policy satisfies, for any sectors j, i,*

$$\textit{Domestic tax:} \quad \tau^d_j = 0 \tag{29}$$

$$\textit{Import tariff:} \quad \tau^m_j - \tau^m_i = \left(\frac{1}{\kappa_2} - \frac{\epsilon(\kappa_2 - 1)}{\theta\kappa_2}\right) \left(\frac{\beta_j x_1 - Y_{1j}}{Y_{2j}} - \frac{\beta_i x_1 - Y_{1i}}{Y_{2i}} \right) \tag{30}$$

where $Y_{nj} = \frac{1+\theta}{\theta} w_{nj} L_{nj}$ denote the income of sector j in country n. Export taxes satisfy (18).

Proof. Appendix I.1. When foreign labor is perfectly mobile ($\kappa_2 \to \infty$) and $\epsilon = 1$, $\frac{1}{\kappa_2} - \frac{\epsilon(\kappa_2-1)}{\theta\kappa_2} = -\frac{1}{\theta}$, the optimal policies are the same as those derived in BJL. When $\epsilon = 0$, $\frac{1}{\kappa_2} - \frac{\epsilon(\kappa_2-1)}{\theta\kappa_2} = \frac{1}{\kappa_2}$, the optimal policies are the same as those derived in Proposition 2.

[4] To maintain consistency with the previous equations, we simplify L^p_{nj} to L_{nj}.

Other Demand Specifications Notice that the *CES supply system* is not equivalent to each sector's supply elasticity is constant. It is not equivalent to each sector's export supply elasticity is κ either. Obviously, export supply depends on domestic demand. The *CES supply system* does not imply a *CES export supply system*. The baseline with Cobb-Douglas demand across sectors and CES demand between countries helps to show tariffs depend on supply elasticity and net import share in Foreign production, nicely linked to a monopsony pricing; export taxes depend on demand elasticity and Foreign purchasing share. Here, we further discuss how all the assumptions on the preference and demand side affect export supplies and the interdependency of optimal tariffs.

Appendix J rewrites FOCs for wages, hence the system of equations for optimal tariffs as export supply with respect to all sectorial price changes. All partial export supply elasticities are also endogenous variables. In the case of Cobb-Douglas demand across sectors, the impacts of price changes on demand can be summarized by Home consumption of the goods. We extend to CES across sectors; optimal tariffs still can be written as differences across sectors with Ψ_1 taking into account Home and Foreign demand changes across sectors.

Cases when supply is CES but the export supply system is not CES involve demand across other buyers that do not satisfy CES. For example, with multi-country and Home manipulating the relative wage of each country, country 2's export supply to Home— E_{12} — depends on country 2's production supply and also country 2 itself and all other foreign countries' demand for country 2's goods. Wage changes in other countries affect country 2 asymmetrically. Specifically, when imposing tariffs, all countries' wages change jointly. Not only the wage in country 2 but also the wages in other countries influence country 2's export supply to country 1, denoted as E_{12}. The expression $-\pi_{i2} - \frac{\theta \sum_{n\neq 1} \pi_{n2}\pi_{ni}x_n}{x_i} = -\pi_{i2} - \theta \sum_{n\neq 1} \frac{\pi_{ni}x_n}{x_i}\pi_{n2}$, captures the extent to which the wage in country i affect E_{12}. This expression reflects the covariance of the shares π_{n2} and country i's supply share affects the elasticity. The higher wage in country i decreases all other countries' purchases from i and increases their purchases from country 2, hence decreasing 2's export supply to Home 1. The covariance illustrates how likely it is for other countries that import from country i to also buy from country 2, and a greater covariance between these shares results in an increased elasticity, indicating that changes in the country i's wage will have a more significant influence on country 2's selling to other countries hence export supply to country 1. This pronounced effect arises from the heightened probability that countries that would typically import from country i are now more inclined to import from country 2 as well. Ultimately, this covariance is shaped by the correlation between productivities and trade costs.

Even in this case the export supply system is not CES, the explicit FOCs will still help to solve the optimal tariffs more efficiently.

5 Nash Optimal Policies

Due to direct targeting, Home's unilateral optimal tariffs directly relate to foreign labor market supply elasticity, instead of its own. However, in a trade war, Foreign retaliates, and the welfare change of Home would depend on Home labor market supply elasticities which affect Foreign's market power.

Let τ_j^x and τ_j^{x*} denote the export taxes imposed by the Home and Foreign countries in sector j, respectively, in sector j. Similarly, τ_j^m and τ_j^{m*} represent the import tariffs imposed by the Home and Foreign countries in sector j, while τ_j^d and τ_j^{d*} denote the domestic taxes in sector j imposed by the Home and Foreign countries, respectively.

Home's government chooses $\{\tau_j^x, \tau_j^m, \tau_j^d\}$ to maximize domestic consumers' consumption, $\max u_1(x_1/P_1)$, while Foreign's government chooses $\{\tau_j^{x*}, \tau_j^{m*}, \tau_j^{d*}\}$ to maximize domestic consumers' consumption, $\max u_2(x_2/P_2)$, subject to the world market equilibrium and the other country's policies:

1. Expenditures are given by

$$x_1 = \sum_{j=1}^{J} w_{1j} L_{1j} + \sum_{j=1}^{J} \beta_j \left[\frac{\tau_j^x}{1+\tau_j^x} \frac{1}{1+\tau_j^{m*}} \pi_{21,j} x_2 + \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1 \right], (\gamma_{x1}, \gamma_{x1}^*) \tag{31}$$

$$x_2 = \sum_{j=1}^{J} w_{2j} L_{2j} + \sum_{j=1}^{J} \beta_j \left[\frac{\tau_j^{x*}}{1+\tau_j^{x*}} \frac{1}{1+\tau_j^{m}} \pi_{12,j} x_1 + \frac{\tau_j^{m*}}{1+\tau_j^{m*}} \pi_{21,j} x_2 + \frac{\tau_j^{d*}}{1+\tau_j^{d*}} \pi_{22,j} x_2 \right], (\gamma_{x2}, \gamma_{x2}^*) \tag{32}$$

2. Trade shares satisfy, for each sector j

$$\pi_{11,j} = \frac{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{x*})d_{12})^{-\theta}}, \tag{33}$$

$$\pi_{12,j} = \frac{T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{x*})d_{12})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{x*})d_{12})^{-\theta}}, \tag{34}$$

$$\pi_{21,j} = \frac{T_{1j}(w_{1j}(1+\tau_j^x)(1+\tau_j^{m*})d_{21})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)(1+\tau_j^{m*})d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta}}, \tag{35}$$

$$\pi_{22,j} = \frac{T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)(1+\tau_j^{m*})d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta}}. \tag{36}$$

3. Consumer prices are given by

$$P_1 = \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{x*})d_{12})^{-\theta} \right]^{-\frac{\beta_j}{\theta}}, (\gamma_{p1}, \gamma_{p1}^*) \tag{37}$$

$$P_2 = \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^x)(1+\tau_j^{m*})d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta} \right]^{-\frac{\beta_j}{\theta}}, (\gamma_{p2}, \gamma_{p2}^*) \tag{38}$$

4. Goods market clearing conditions, for each j

$$w_{1j} L_{1j} = \beta_j \left[\frac{1}{1+\tau_j^d} \pi_{11,j} x_1 + \frac{1}{1+\tau_j^x} \frac{1}{1+\tau_j^{m*}} \pi_{21,j} x_2 \right], (\gamma_{1j}, \gamma_{1j}^*) \tag{39}$$

$$w_{2j} L_{2j} = \beta_j \left[\frac{1}{1+\tau_j^m} \frac{1}{1+\tau_j^{x*}} \pi_{12,j} x_1 + \frac{1}{1+\tau_j^{d*}} \pi_{22,j} x_2 \right], (\gamma_{2j}, \gamma_{2j}^*) \tag{40}$$

5. The labor market clearing conditions $\Omega(\{w_{nj}, L_{nj}\})$ hold.

The multipliers $\{\gamma_{x1}, \gamma_{x2}, \gamma_{p1}, \gamma_{p2}, \gamma_{1j}, \gamma_{2j}\}$ correspond to the Home country's maximization problem, while the multipliers denoted with (*) correspond to the Foreign country's maximization problem.

Proposition 4 (Nash Optimal Policy Formula). *Home and foreign government's Nash optimal policy satisfies,*

$$
\begin{aligned}
&\textit{Home domestic tax vector:} && (\Lambda_1 - I)(1+\boldsymbol{\tau^d}) && = 0, && (41)\\
&\textit{Foreign domestic tax vector:} && (\Lambda_2 - I)(1+\boldsymbol{\tau^{d*}}) && = 0, && (42)\\
&\textit{Home import tariff vector:} && \boldsymbol{\tau^m} \cdot (1+\boldsymbol{\tau^{x*}}) && = \Lambda_2^{-1}\Psi_1, && (43)\\
&\textit{Foreign import tariff vector:} && \boldsymbol{\tau^{m*}} \cdot (1+\boldsymbol{\tau^{x}}) && = \Lambda_1^{-1}\Psi_2, && (44)\\
&\textit{Home export tax:} && (1+\tau_j^x)[(1+\tau_j^{x*})(1+\tau_j^{m*})\tau_j^m + 1] && = 1 + \frac{1}{\theta\pi_{22,j}}, && (45)\\
&\textit{Foreign export tax:} && (1+\tau_j^{x*})[(1+\tau_j^{x})(1+\tau_j^{m})\tau_j^{m*} + 1] && = 1 + \frac{1}{\theta\pi_{11,j}}. && (46)
\end{aligned}
$$

where $\boldsymbol{\tau^d}$, $\boldsymbol{\tau^m}$ and $\boldsymbol{\tau^x}$ are vectors of Home domestic taxes, import tariffs and export tax across sectors, respectively. $\boldsymbol{\tau^{d*}}$, $\boldsymbol{\tau^{m*}}$ and $\boldsymbol{\tau^{x*}}$ are vectors of Foreign domestic taxes, import tariffs and export tax across sectors, respectively. I is an identity matrix. Vector Ψ_1 contains Home's net import spending share in Foreign income with the jth entry of $\frac{\beta_j x_1 - (1+\tau_j^d)Y_{1j}}{Y_{2j}}$, and vector Ψ_2 contains Foreign's net import spending share in Home income with the jth entry of $\frac{\beta_j x_2 - (1+\tau_j^{d*})Y_{2j}}{Y_{1j}}$.

Proof. Appendix K.

Proposition 5 (Nash Optimal Policy under CES Supply System). *Under a CES supply system with elasticity, κ_1 and κ_2, Home and Foreign government's Nash optimal policy satisfies, for any sectors j, i*

$$
\begin{aligned}
&\textit{Home domestic tax:} && \tau_j^d = 0 && (47)\\
&\textit{Foreign domestic tax:} && \tau_j^{d*} = 0 && (48)\\
&\textit{Home import tariff:} && \tau_j^m(1+\tau_j^{x*}) - \tau_i^m(1+\tau_i^{x*}) = \frac{1}{\kappa_2}\left[\frac{\beta_j x_1 - Y_{1j}}{Y_{2j}} - \frac{\beta_i x_1 - Y_{1i}}{Y_{2i}}\right] && (49)\\
&\textit{Foreign import tariff:} && \tau_j^{m*}(1+\tau_j^{x}) - \tau_i^{m*}(1+\tau_i^{x}) = \frac{1}{\kappa_1}\left[\frac{\beta_j x_2 - Y_{2j}}{Y_{1j}} - \frac{\beta_i x_2 - Y_{2i}}{Y_{1i}}\right] && (50)
\end{aligned}
$$

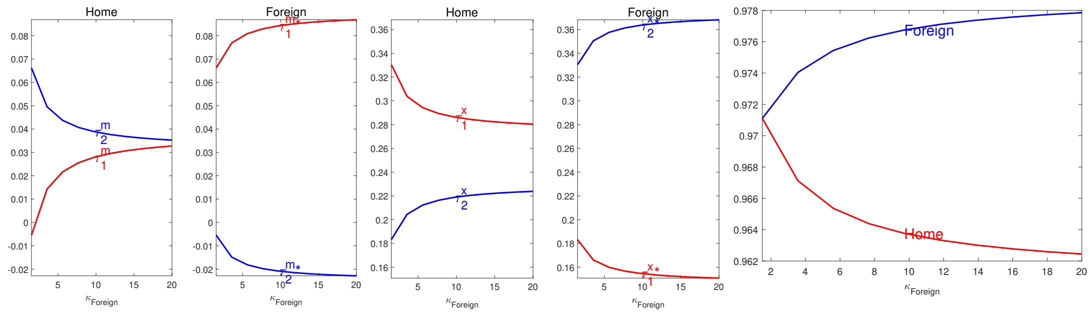
and export taxes satisfy (45) and (46).

Proof. Appendix K. Proposition 5 demonstrates that under a CES supply system, Home and Foreign would not use sector-specific domestic tax. In Appendix K, we further prove that the tax neutrality holds. In this case, one of the domestic taxes of each country can be normalized to zero, and since Home and Foreign optimally will not use sector-specific domestic tax, all domestic taxes are zero with a CES supply system.

Unlike formula (21), the existence of Foreign export tax implies that Home's relative import tariffs between any two sectors not only depend on the net import spending relative to Foreign income of these two sectors, but are also influenced by foreign export taxes of these two sectors.

Figure 1: Foreign has more flexible labor market

(a) Import Tariff (b) Export Tax (c) Welfare Change



Note: This figure plots the Nash optimal policies and welfare change when $\kappa_{Foreign}$ increases. The welfare change is calculated by the change in real consumption relative to the private equilibrium without any policy $\hat{c}_n = \frac{\hat{x}_n}{\hat{P}_n}$.

A Simple Case with Two Sectors. Figure 1 presents a numerical example where Foreign's labor becomes more flexible.[5] Specifically, we fix $\kappa_1 = 1.5$ and allow the foreign elasticity κ_2 to increase from 1.5 to 20. We then examine the resulting Nash optimal policy as well as the corresponding welfare changes relative to the private equilibrium in the absence of any policy interventions. As Foreign labor becomes more flexible, the disparity in import tariffs between the two sectors in the Home economy becomes smaller. This indicates that increased foreign labor flexibility reduces the differential tariffs across sectors in the Home country. In contrast, the disparity in import tariffs between the two sectors in the Foreign

[5]In this example, we assume that $\theta = 4, T_{11} = 3, T_{12} = 1, T_{21} = 1, T_{22} = 3, \alpha_1 = \alpha_2 = 0.5, d_{21} = d_{12} = 1.15, \bar{L}_1 = \bar{L}_2 = 1$.

country becomes larger. In terms of welfare, the changes reveal opposing trends for the two countries. Foreign lose less as its labor becomes more flexible, and Home has less market power to manipulate prices. The welfare of the Home country declines.

6 Quantitative Optimal Policies

In this quantitative analysis, the world consists of two countries: China and the U.S. We first compute each country's unilateral optimal policies and welfare change. Next, we calculate the Nash optimal policies and the welfare change. Using observable data, we extend the exact hat method to compute the counterfactual equilibrium under the optimal policies.

Let variables without 'prime' denote the observed variables, which include the trade matrix $\{\pi_{nm,j}\}$ and sectoral production $\{w_{nj}L_{nj}\}$. Variables denoted with 'prime' represent counterfactuals after implementing the optimal policies, and variables with 'hats' denote the ratios of prime variables to the observed ones.

Proposition 6 (**Welfare Change under Unilateral Optimal Policy**). *Given observable data on trade shares and sectoral production, $\{\pi_{ni,j}, w_{nj}L_{nj}\}$, along with parameter values $\{\theta, \kappa_n\}$, we can evaluate the economic impact of Home country's optimal policy. Specifically, this involves solving for the variables $\{\hat{w}_{nj}, \hat{L}_{nj}, \hat{P}_n, x'_n, \pi'_{nm,j}, 1+\tau_j^{m'}, 1+\tau_j^{x'}, 1+\tau_j^{d'}\}$, which constitute the solution to the system of equations detailed in Appendix L.1. Once the equilibrium is determined, the welfare effects of the Home's unilateral optimal policies for both countries can be fully characterized as $\hat{c}_n = \hat{x}_n / \hat{P}_n$.*

Proposition 7 (**Welfare Change under Nash Optimal Policy**). *Given observable data on trade shares and sectoral production, $\{\pi_{ni,j}, w_{nj}L_{nj}\}$, along with parameter values $\{\theta, \kappa_n\}$, we can evaluate the economic impact of country n's optimal policy. Specifically, this involves solving for the variables $\{\hat{w}_{nj}, \hat{L}_{nj}, \hat{P}_n, x'_n, \pi'_{nm,j}, 1+\tau_j^{m'}, 1+\tau_j^{x'}, 1+\tau_j^{d'}, 1+\tau_j^{m*'}, 1+\tau_j^{x*'}, 1+\tau_j^{d*'}\}$, which constitute the solution to the system of equations detailed in Appendix L.2. Once the equilibrium is determined, the welfare effects of the Nash optimal policies for both countries can be fully characterized as $\hat{c}_n = \hat{x}_n / \hat{P}_n$.*

In summary, the method outlined in Propositions 6 and 7 enables the calculation of optimal policies and equilibrium changes using bilateral trade and sector-level production data, without the need to explicitly recover technology T_{nj}, fundamental productivity α_j, or trade costs d_{nm}. We adapt the standard exact hat method but incorporate labor market clearing conditions with imperfect labor substitution and our FOCs for optimal policies.

6.1 Data and Measurement

To conduct the analysis, we need sectoral-level data on gross production and bilateral trade for each country. The data on bilateral trade flows are sourced from the United Nations' Statistical Division Commodity Trade (COMTRADE) database, while the annual gross production data are obtained from the OECD Structural Analysis Database (STAN) and National Accounts and Industrial Statistics Database (UNIDO) compiled by the United Nations. The gross production data are available for 2-digit level ISIC industries, while the 6-digit H.S. trade data is mapped onto two-digit ISIC industries, resulting in 20 two-digit manufacturing sectors for the year 2016. The product concordance between the 6-digit H.S. and 2-digit ISIC classifications is sourced from the World Integrated Trade Solution (WITS). We rescale China's production so that the ratio of the total gross production of the U.S. to China matches the ratio of their GDPs, with GDP data for both countries sourced from the World Development Indicators (WDI) of the World Bank.

The parameters comprise those that are common across countries and sectors—such as the Fréchet parameter θ and CES supply system elasticity κ_n. We set $\theta = 4$, consistent with the trade elasticity estimated by Simonovska and Waugh (2014), and $\kappa_n = 1.5$ following Galle, Rodríguez-Clare, and Yi (2023), who estimate values of κ ranging from 1.42 to 2.79.

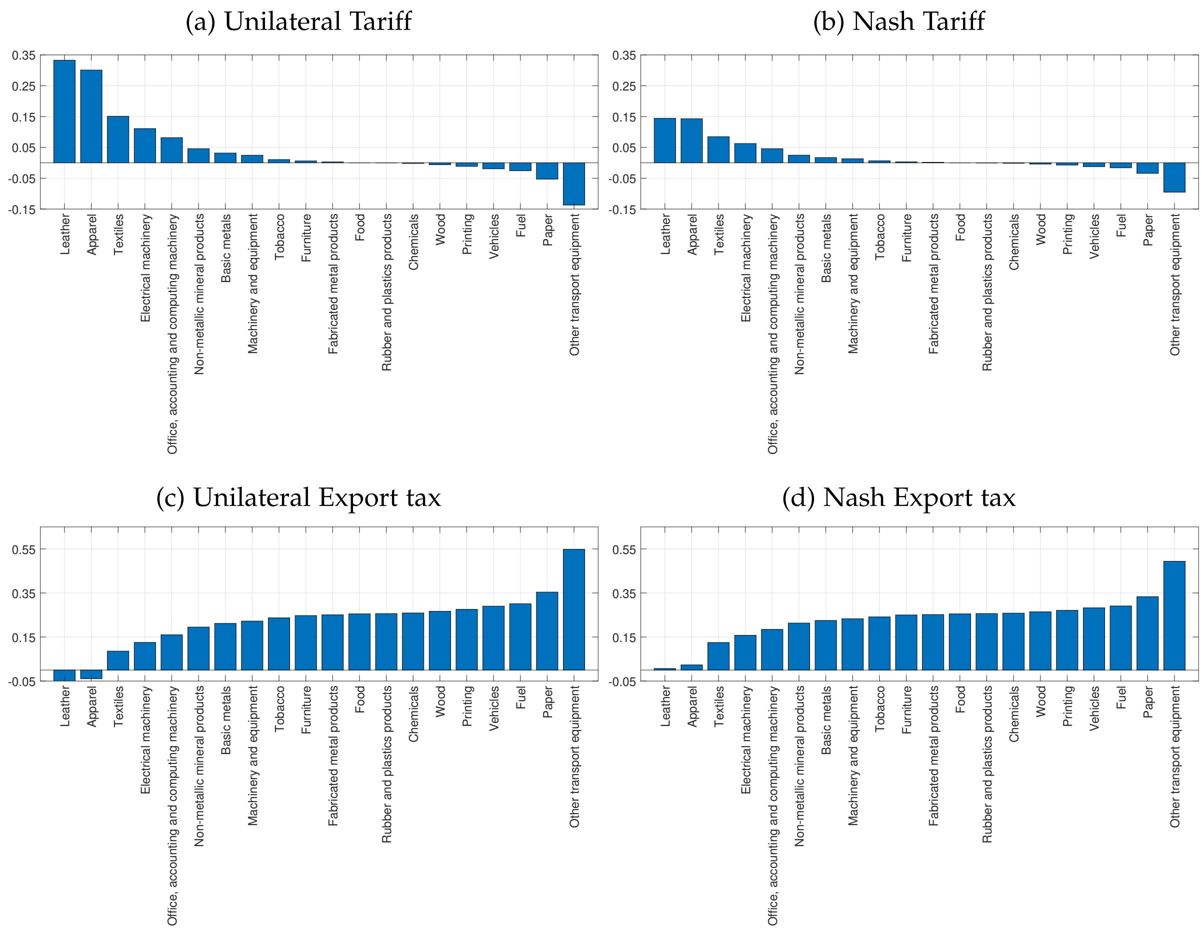
6.2 Unilateral Optimal Policies

Our first set of results elucidates policy consequences when Home government impose unilateral policies.

Figure 2(a), 2(c), 3(a), and 3(c) shows the unilateral optimal policies across sectors. We examine unilateral optimal policies of the U.S. and China in turn and order the sectors by U.S. net import spending relative to China's income before the policy. The import tariff of "Food" sector is normalized to zero. Our first finding is that export taxes and import tariffs are heterogeneous across sectors and exhibit a wide range of values. For instance, U.S. optimal import tariffs on China are lowest in the "Other transport equipment" sector and highest in the "Leather" sector. Similarly, China's optimal import tariffs on the U.S. are lowest in the "Leather" sector and highest in the "Other transport equipment" sector. All optimal taxes and tariffs are relative, as tax neutrality holds.

Second, these patterns suggest that the home country tends to impose higher tariffs on sectors where it has higher net import spending relative to foreign income, as shown in Tables 1 and 2 where the sectors are ordered by their net import spending relative to foreign income before the policy.

Figure 2: U.S. Optimal Policies



Note: This figure plots U.S. unilateral and Nash optimal trade policies.

The first two columns of Table 5 present the welfare changes resulting from the unilateral optimal policies implemented by the U.S. and China. In both scenarios, the country implementing the policy experiences welfare gains, while the other country suffers welfare

Table 1: U.S. net import spending relative to China income and Unilateral Optimal Policies

Sectors	US net import spending relative to China income		Optimal policies	
	Before policy	After policy	Export Tax	Import Tariff
Leather	0.47	0.48	-0.06	0.33
Apparel	0.44	0.44	-0.04	0.30
Textiles	0.26	0.21	0.09	0.15
Electrical machinery	0.20	0.15	0.13	0.11
Office, accounting and computing machinery	0.14	0.11	0.16	0.08
Non-metallic mineral products	0.08	0.05	0.20	0.05
Basic metals	0.05	0.03	0.21	0.03
Machinery and equipment	0.03	0.02	0.22	0.02
Tobacco	-0.00	0.00	0.24	0.01
Furniture	-0.02	-0.01	0.25	0.01
Fabricated metal products	-0.03	-0.01	0.25	0.00
Food	-0.03	-0.02	0.26	-0.00
Rubber and plastics products	-0.04	-0.02	0.26	0.00
Chemicals	-0.04	-0.02	0.26	-0.00
Wood	-0.05	-0.02	0.27	-0.01
Printing	-0.07	-0.03	0.28	-0.01
Vehicles	-0.10	-0.05	0.29	-0.02
Fuel	-0.12	-0.05	0.30	-0.03
Paper	-0.22	-0.10	0.35	-0.05
Other transport equipment	-0.61	-0.22	0.55	-0.14

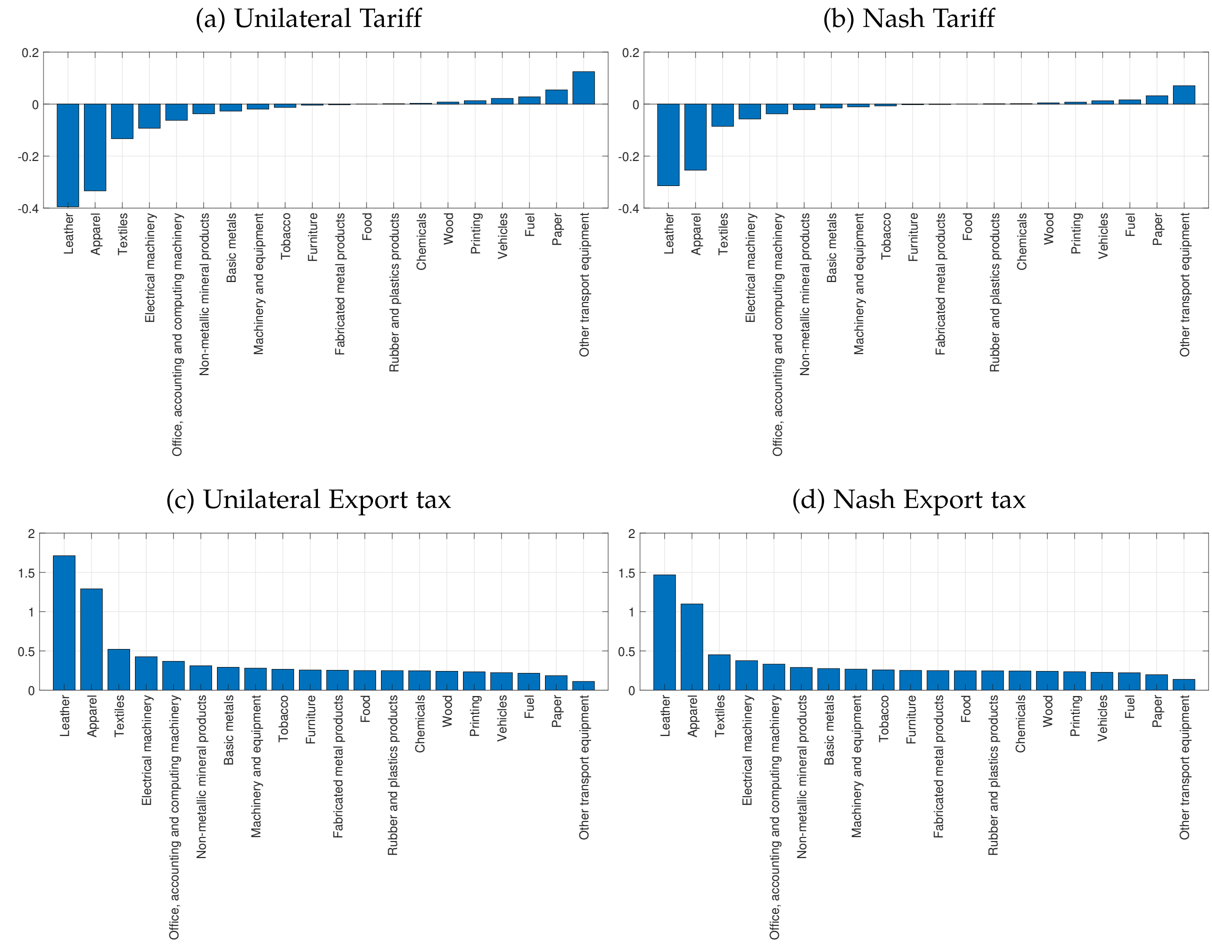
Note: Sectors are ordered by their net import spending relative to China's income before policy.

Table 2: China net import spending relative to U.S. income and Unilateral Optimal Policies

Sectors	China net import spending relative to US income		Optimal policies	
	Before policy	After policy	Export Tax	Import Tariff
Other transport equipment	0.25	0.21	0.11	0.12
Paper	0.14	0.10	0.19	0.05
Fuel	0.09	0.06	0.22	0.03
Vehicles	0.07	0.05	0.22	0.02
Printing	0.06	0.04	0.23	0.01
Wood	0.04	0.03	0.24	0.01
Chemicals	0.03	0.02	0.25	0.00
Rubber and plastics products	0.03	0.02	0.25	0.00
Food	0.03	0.02	0.25	-0.00
Fabricated metal products	0.02	0.02	0.25	-0.00
Furniture	0.02	0.01	0.26	-0.00
Tobacco	0.00	0.00	0.27	-0.01
Machinery and equipment	-0.03	-0.01	0.28	-0.02
Basic metals	-0.05	-0.02	0.29	-0.03
Non-metallic mineral products	-0.08	-0.04	0.31	-0.04
Office, accounting and computing machinery	-0.18	-0.07	0.37	-0.06
Electrical machinery	-0.29	-0.12	0.43	-0.09
Textiles	-0.47	-0.18	0.52	-0.13
Apparel	-2.39	-0.48	1.29	-0.33
Leather	-3.80	-0.57	1.71	-0.39

Note: Sectors are ordered by their net import spending relative to U.S. income before policy.

Figure 3: China's Optimal Policies



Note: This figure plots China's unilateral and Nash optimal trade policies.

losses. As Home's unilateral policies improve own terms of trade and worsen Foreign's, leading to welfare losses.

6.3 Nash Optimal Policies

In this section, we analyze the scenario in which the governments of the U.S. and China implement policies that result in a Nash equilibrium. Tables 3 and 4, along with Figures 2(b), 2(d), 3(b), and 3(d), present the optimal policies across various sectors. In this analysis, the import tariffs for the "Food" sector in both countries are normalized to zero.

Similar to the unilateral policy case, the export taxes and import tariffs under Nash equilibrium are heterogeneous across sectors, displaying a wide range of values. For example, the U.S.'s optimal import tariffs on goods from China are lowest in the "Other transport equipment" sector and highest in the "Leather" sector, reflecting the same sectoral pattern observed under unilateral policies. However, the magnitude of these policies differs under Nash equilibrium due to the strategic interaction between the two countries.

When comparing the Nash equilibrium policies to the unilateral optimal policies shown in panels (a) and (c) of these two figures, we observe a notable reduction in the dispersion of optimal policies across sectors under the Nash framework. This suggests that the range of variation in sector-specific policies is narrower when both countries consider each other's strategic responses. The underlying reason is that, under Nash equilibrium, the U.S. and China must take into account the interdependence of their policies. Each country's policy decisions are influenced not only by their own objectives but also by the anticipated reactions of the other. This mutual consideration leads to more moderated policies and reduces the policy divergence between sectors compared to the unilateral approach.

The welfare outcomes under Nash optimal policies, presented in the last column of Table 5, while unilateral policies yield higher welfare gains for Home, the countries in Nash equilibrium face retaliation, and resulting in lower welfare for both countries.

Figure 4 illustrates the welfare changes for both the U.S. and China under Nash equilibrium policies assuming China's labor market becomes more flexible (i.e., as κ_{CN} increases), while keeping original observables—such as the trade matrix and sectoral production—unchanged. As κ_{CN} rises, China's welfare improves significantly, even exceeding 1, which means the welfare level is higher than that in the initial state without any policies. In contrast, the U.S. experiences larger welfare losses as China's labor market flexibility increases.

Table 3: U.S. net import spending relative to China income and Optimal Policies in Nash

	US net import spending relative to China income		Optimal policies	
Sectors	Before policy	After policy	Export Tax	Import Tariff
Leather	0.47	0.51	0.01	0.14
Apparel	0.44	0.43	0.02	0.14
Textiles	0.26	0.17	0.13	0.08
Electrical machinery	0.20	0.12	0.16	0.06
Office, accounting and computing machinery	0.14	0.08	0.18	0.05
Non-metallic mineral products	0.08	0.04	0.21	0.02
Basic metals	0.05	0.02	0.23	0.02
Machinery and equipment	0.03	0.01	0.23	0.01
Tobacco	-0.00	-0.00	0.24	0.01
Furniture	-0.02	-0.01	0.25	0.00
Fabricated metal products	-0.03	-0.01	0.25	0.00
Food	-0.03	-0.01	0.26	0.00
Rubber and plastics products	-0.04	-0.01	0.26	-0.00
Chemicals	-0.04	-0.01	0.26	-0.00
Wood	-0.05	-0.02	0.26	-0.00
Printing	-0.07	-0.03	0.27	-0.01
Vehicles	-0.10	-0.03	0.28	-0.01
Fuel	-0.12	-0.04	0.29	-0.02
Paper	-0.22	-0.07	0.33	-0.03
Other transport equipment	-0.61	-0.17	0.49	-0.10

Table 4: China net import spending relative to U.S. income and Optimal Policies in Nash

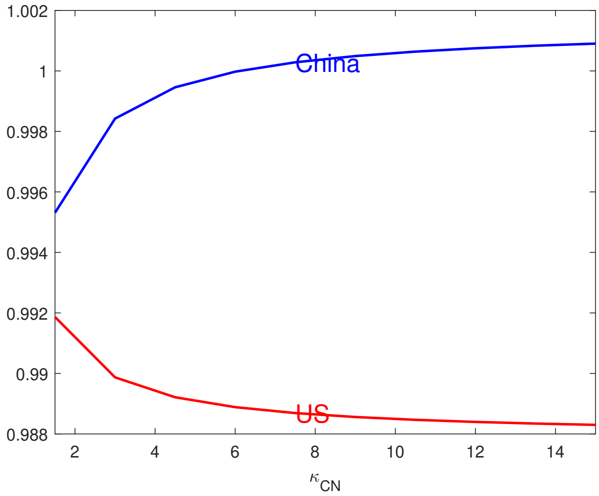
Sectors	China net import spending relative to US income		Optimal policies	
	Before policy	After policy	Export Tax	Import Tariff
Other transport equipment	0.25	0.17	0.14	0.07
Paper	0.14	0.08	0.20	0.03
Fuel	0.09	0.05	0.22	0.02
Vehicles	0.07	0.04	0.23	0.01
Printing	0.06	0.03	0.24	0.01
Wood	0.04	0.02	0.24	0.00
Chemicals	0.03	0.02	0.24	0.00
Rubber and plastics products	0.03	0.02	0.25	0.00
Food	0.03	0.01	0.25	0.00
Fabricated metal products	0.02	0.01	0.25	-0.00
Furniture	0.02	0.01	0.25	-0.00
Tobacco	0.00	0.00	0.26	-0.01
Machinery and equipment	-0.03	-0.01	0.27	-0.01
Basic metals	-0.05	-0.01	0.28	-0.01
Non-metallic mineral products	-0.08	-0.03	0.29	-0.02
Office, accounting and computing machinery	-0.18	-0.05	0.33	-0.04
Electrical machinery	-0.29	-0.09	0.38	-0.06
Textiles	-0.47	-0.13	0.45	-0.09
Apparel	-2.39	-0.38	1.10	-0.25
Leather	-3.80	-0.46	1.47	-0.31

Table 5: Welfare Change in Optimal Policies for U.S. and China

	U.S. Policy	China Policy	Nash
U.S.	1.006	0.987	0.992
China	0.989	1.007	0.995

Note: The welfare change is calculated by $\hat{c}_n = \frac{\hat{x}_n}{\hat{P}_n}$

Figure 4: Welfare change in Nash



Note: This figure plots the welfare change in Nash when China's labor market becomes more flexible. The welfare change is calculated by $\hat{c}_n = \frac{\hat{x}_n}{\hat{P}_n}$

7 Conclusion

We study optimal policies in a set of multi-sector models with different labor market specifications, which are widely used for quantitative investigations of the labor market impact of trade. We provide a general formula for optimal policies in models with efficient or inefficient labor markets and a specific one for the CES supply system.

We demonstrate that sector-specific optimal tariffs are tied to the matrix of foreign export supply elasticities, which in turn depends on the foreign partial supply elasticities and the share of Home's net import in foreign income, together reflecting Home's import market power. The sector-specific export taxes depend on goods market elasticities and Home's market share in foreign consumption, together reflecting Home's export market power. Home tends to impose higher tariffs on large importing sectors and higher export taxes on its comparative advantage sector, though all taxes are jointly determined. The outcome of a Nash equilibrium depend on the Home and Foreign's market powers.

Other factors, in addition to assumptions on production technology and preferences, can also influence trade and optimal trade policies. These factors include market structure and political and strategic considerations. While extensive literature has examined some of these factors, our paper specifically concentrates on examining the implications of supply-side assumptions for trade policy. Using a simple unified setup, we aim to provide a clear and explicit understanding of how different model specifications matter in optimal taxation.

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Online Appendix to "Optimal Trade Policies and Labor Markets "

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This appendix is organized as follows.

A Proof for Lemma 1: optimal policies and multipliers

Home government chooses $\{\tau_j^d, \tau_j^x, \tau_j^m, x_1, L_{1j}, L_{2j}, w_{1j}, w_{2j}\}$ to solve the following problem:

$$\max \frac{x_1}{P_1},$$

subject to world market equilibrium characterized by the following constraints:

$$w_{1j}L_{1j} = \beta_j \left[\frac{1}{1+\tau_j^d}\pi_{11,j}x_1 + \frac{1}{1+\tau_j^x}\pi_{21,j}x_2 \right], \quad (\gamma_{1j}, \quad J) \tag{A.1}$$

$$w_{2j}L_{2j} = \beta_j \left[\frac{1}{1+\tau_j^m}\pi_{12,j}x_1 + \pi_{22,j}x_2 \right], \quad (\gamma_{2j}, \quad J) \tag{A.2}$$

and the labor market clearing conditions $\Omega(\{w_{nj}, L_{nj}\}$ determined by the labor market specification, where $\{x_2, P_1, P_2\}$ and trade shares are given by

$$\begin{aligned}
x_2 &= \sum_{j=1}^{J} w_{2j}L_{2j}, \\
P_1 &= \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = 1, \\
P_2 &= \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}w_{2j}^{-\theta} \right]^{-\frac{\beta_j}{\theta}}, \\
\pi_{11,j} &= \frac{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}, \\
\pi_{12,j} &= \frac{T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}}, \\
\pi_{21,j} &= \frac{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}(w_{2j})^{-\theta}}, \\
\pi_{22,j} &= \frac{T_{2j}(w_{2j})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}(w_{2j})^{-\theta}}.
\end{aligned}$$

By Walras' Law, combining the J equations in (A.1) and the J equations in (A.2), we can derive the formula for x_1. As a result, it is not necessary to include this formula as a constraint.

$$x_1 = \sum_{j=1}^{J} w_{1j}L_{1j} + \sum_{j=1}^{J} \beta_j \frac{\tau_j^x}{1+\tau_j^x}\pi_{21,j}x_2 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^m}{1+\tau_j^m}\pi_{12,j}x_1 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^d}{1+\tau_j^d}\pi_{11,j}x_1,$$

Optimal conditions:

We use first-order conditions (FOCs) to characterize the Home government's problem and link the optimal policies with multipliers on goods market clearing conditions.

FOC over x_1

$$1 + \sum_{j=1}^{J} \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\pi_{11,j} + \sum_{j=1}^{J} \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\pi_{12,j} = 0$$

FOC over export tax τ_j^x

$$-\gamma_{1j}\beta_j\frac{1}{(1+\tau_j^x)^2}\pi_{21,j}x_2+\gamma_{1j}\beta_j\frac{1}{1+\tau_j^x}\frac{\partial\pi_{21,j}}{\partial\tau_j^x}x_2+\gamma_{2j}\beta_j\frac{\partial\pi_{22,j}}{\partial\tau_j^x}x_2=0$$

after plugging in derivatives and simplifying, the FOC over τ_j^x becomes

$$1+\tau_j^x=\frac{\gamma_{1j}(\theta\pi_{22,j}+1)}{\gamma_{2j}\theta\pi_{22,j}}. \tag{A.3}$$

FOC over import tariff τ_j^m

$$-\beta_j x_1\frac{\pi_{12,j}}{1+\tau_j^m}+\gamma_{1j}\beta_j\frac{1}{1+\tau_j^d}\frac{\partial\pi_{11,j}}{\partial\tau_j^m}x_1-\gamma_{2j}\beta_j\frac{\pi_{12,j}}{(1+\tau_j^m)^2}x_1+\gamma_{2j}\beta_j\frac{1}{1+\tau_j^m}\frac{\partial\pi_{12,j}}{\partial\tau_j^m}x_1=0$$

after plugging in derivatives and simplifying, the FOC over τ_j^m becomes

$$1+\tau_j^m=\frac{\gamma_{2j}\left(1+\theta\pi_{11,j}\right)}{\gamma_{1j}\frac{1}{1+\tau_j^d}\theta\pi_{11,j}-1}. \tag{A.4}$$

FOC over domestic tax τ_j^d

$$-\beta_j x_1\frac{\pi_{11,j}}{1+\tau_j^d}-\gamma_{1j}\frac{1}{(1+\tau_j^d)^2}\beta_j\pi_{11,j}x_1+\gamma_{1j}\frac{1}{1+\tau_j^d}\beta_j\frac{\partial\pi_{11,j}}{\partial\tau_j^d}x_1+\gamma_{2j}\beta_j\frac{1}{1+\tau_j^m}\frac{\partial\pi_{12,j}}{\partial\tau_j^d}x_1=0$$

after plugging in derivatives and simplifying, the FOC over τ_j^d becomes

$$1+\tau_j^d=\frac{\gamma_{1j}\left(1+\theta\pi_{12,j}\right)}{\gamma_{2j}\frac{1}{1+\tau_j^m}\theta\pi_{12,j}-1}. \tag{A.5}$$

Combining the optimal tariff (A.4) and domestic tax (A.5), we can get

$$1+\tau_j^m=-\gamma_{2j},\qquad 1+\tau_j^d=-\gamma_{1j}. \tag{A.6}$$

Combining the above two equations with equation (A.3), we get

$$1+\tau_j^x=\frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\theta\pi_{22,j}}\right). \tag{A.7}$$

□

B Proof for Proposition 1: general formula for optimal policies

FOC over w_{1j}

$$-\beta_j x_1 \frac{\pi_{11,j}}{w_{1j}} - \gamma_{1j} L_{1j} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{1j}} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial w_{1j}} x_2$$
$$+ \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{1j}} x_1 + \gamma_{2j}\beta_j \frac{\partial \pi_{22,j}}{\partial w_{1j}} x_2 + A_j / w_{1j} = 0 \tag{A.8}$$

Further

$$-\beta_j x_1 \pi_{11,j} - \gamma_{1j} w_{1j} L_{1j} - \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\theta \pi_{11,j}\pi_{12,j} x_1 - \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\theta \pi_{21,j}\pi_{22,j} x_2$$
$$+ \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\theta \pi_{12,j}\pi_{11,j} x_1 + \gamma_{2j}\beta_j \theta \pi_{22,j}\pi_{21,j} x_2 + A_j = 0$$

Where $A_j = \sum_{k=1}^J (-\gamma_{1k}) w_{1k} \frac{\partial L_{1k}}{\partial w_{1j}} w_{1j} = \sum_{k=1}^J (-\gamma_{1k}) \frac{\partial w_{1k} L_{1k}}{\partial w_{1j}} w_{1j} + \gamma_{1j} w_{1j} L_{1j}$.

FOC over w_{2j}

$$-\beta_j x_1 \frac{\pi_{12,j}}{w_{2j}} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{2j}} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial w_{2j}} x_2 + \sum_{k=1}^J \gamma_{1k}\beta_k \frac{1}{1+\tau_k^x}\pi_{21,k} L_{2j} \tag{A.9}$$
$$-\gamma_{2j} L_{2j} + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{2j}} x_1 + \gamma_{2j}\beta_j \frac{\partial \pi_{22,j}}{\partial w_{2j}} x_2 + \sum_{k=1}^J \gamma_{2k}\beta_k \pi_{22,k} L_{2j} + B_j / w_{2j} = 0$$

Further

$$-\beta_j x_1 \pi_{12,j} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\theta \pi_{11,j}\pi_{12,j} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\theta \pi_{21,j}\pi_{22,j} x_2 + \sum_{k=1}^J \gamma_{1k}\beta_k \frac{1}{1+\tau_k^x}\pi_{21,k} w_{2j} L_{2j}$$
$$-\gamma_{2j} w_{2j} L_{2j} - \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\theta \pi_{12,j}\pi_{11,j} x_1 - \gamma_{2j}\beta_j \theta \pi_{22,j}\pi_{21,j} x_2 + \sum_{k=1}^J \gamma_{2k}\beta_k \pi_{22,k} w_{2j} L_{2j} + B_j = 0$$

Where

$$B_j = \sum_{k=1}^J (cons_1 - \gamma_{2k}) w_{2k} \frac{\partial L_{2k}}{\partial w_{2j}} w_{2j} = \sum_{k=1}^J (cons_1 - \gamma_{2k}) \frac{\partial w_{2k} L_{2k}}{\partial w_{2j}} w_{2j} - (cons_1 - \gamma_{2j}) w_{2j} L_{2j},$$

with $cons_1 = \sum_{s=1}^J (\gamma_{1s}\beta_s \frac{1}{1+\tau_s^x}\pi_{21,s} + \gamma_{2s}\beta_s \pi_{22,s})$.

B.1 Proof of $A_j = 0$ and optimal domestic taxes

Combining FOCs over w_{1j} (A.8), τ_j^x(A.7), and τ_j^m (A.6), we get

$$-\beta_j x_1 - \gamma_{1j} w_{1j} L_{1j} - \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\pi_{12,j}x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\pi_{21}^j x_2 + A_j = 0$$

hence,

$$-\beta_j x_1 - \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\pi_{12,j}x_1 - \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\pi_{11,j}x_1 + A_j = 0.$$

Using the optimal policies $1+\tau_j^m = -\gamma_{2j}$ and $1+\tau_j^d = -\gamma_{1j}$, we arrive at

$$-\beta_j x_1 \pi_{12,j} + \beta_j x_1 \pi_{12,j} + A_j = 0.$$

which implies $A_j = 0$. Now let us revisit the definition of A_j:

$$\sum_{k=1}^{J}(-\gamma_{1k})\frac{\partial w_{1k}L_{1k}}{\partial w_{1j}}w_{1j} + \gamma_{1j}w_{1j}L_{1j} = 0.$$

Plugging the optimal formula for τ_j^d in (A.6) to the above equation, we can further show that

$$\sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial \ln(w_{1k}L_{1k})}{\partial \ln(w_{1j})}\frac{w_{1k}L_{1k}}{w_{1j}L_{1j}} - (1+\tau_j^d) = 0.$$

Hence, domestic taxes $\boldsymbol{\tau^d} = [\tau_1^d, ..., \tau_j^d, ..., \tau_J^d]'$ for J sectors satisfy

$$(\Lambda_1 - I)\cdot(1+\boldsymbol{\tau^d}) = 0,$$

where element of matrix Λ_n at row j and column k is given by $\frac{\partial \ln(Y_{nk})}{\partial \ln(w_{nj})}\frac{Y_{nk}}{Y_{nj}}$, and

$$\Lambda_1 - I = \begin{pmatrix} \frac{\partial ln(w_{11}L_{11})}{\partial ln(w_{11})}\frac{w_{11}L_{11}}{w_{11}L_{11}} - 1 & ... & \frac{\partial ln(w_{1j}L_{1j})}{\partial ln(w_{11})}\frac{w_{1j}L_{1j}}{w_{11}L_{11}} & ... & \frac{\partial ln(w_{1J}L_{1J})}{\partial ln(w_{11})}\frac{w_{1J}L_{1J}}{w_{11}L_{11}} \\ ... & ... & ... & ... & ... \\ \frac{\partial ln(w_{11}L_{11})}{\partial ln(w_{1j})}\frac{w_{11}L_{11}}{w_{1j}L_{1j}} & ... & \frac{\partial ln(w_{1j}L_{1j})}{\partial ln(w_{1j})}\frac{w_{1j}L_{1j}}{w_{1j}L_{1j}} - 1 & ... & \frac{\partial ln(w_{1J}L_{1J})}{\partial ln(w_{1j})}\frac{w_{1J}L_{1J}}{w_{1j}L_{1j}} \\ ... & ... & ... & ... & ... \\ \frac{\partial ln(w_{11}L_{11})}{\partial ln(w_{1J})}\frac{w_{11}L_{11}}{w_{1J}L_{1J}} & ... & \frac{\partial ln(w_{1j}L_{1j})}{\partial ln(w_{1J})}\frac{w_{1j}L_{1j}}{w_{1J}L_{1J}} & ... & \frac{\partial ln(w_{1J}L_{1J})}{\partial ln(w_{1J})}\frac{w_{1J}L_{1J}}{w_{1J}L_{1J}} - 1 \end{pmatrix}.$$

B.2 Proof of optimal import tariffs

Combining FOCs over w_{1j} (A.8) and w_{2j} (A.9) and the formula of B_j, we get

$$
\begin{aligned}
& -\beta_j x_1 - \gamma_{1j} w_{1j} L_{1j} + (cons_1 - \gamma_{2j}) w_{2j} L_{2j} + B_j = 0 \\
(\Rightarrow) & -\beta_j x_1 - \gamma_{1j} w_{1j} L_{1j} + (cons_1 - \gamma_{2j}) w_{2j} L_{2j} + \sum_{k=1}^{J} (cons_1 - \gamma_{2k}) \frac{\partial w_{2k} L_{2k}}{\partial w_{2j}} w_{2j} - (cons_1 - \gamma_{2j}) w_{2j} L_{2j} = 0 \\
(\Rightarrow) & \sum_{k=1}^{J} (cons_1 - \gamma_{2k}) \frac{\partial w_{2k} L_{2k}}{\partial w_{2j}} w_{2j} = \beta_j x_1 + \gamma_{1j} w_{1j} L_{1j}
\end{aligned}
$$

Finally, we get

$$
\sum_{k=1}^{J} (cons + \tau_k^m) \frac{\partial \ln(w_{2k} L_{2k})}{\partial \ln(w_{2j})} \frac{w_{2k} L_{2k}}{w_{2j} L_{2j}} = \frac{\beta_j x_1 - (1 + \tau_j^d) w_{1j} L_{1j}}{w_{2j} L_{2j}},
$$

where $cons = cons_1 + 1$ is common across all the sectors. Due to the tax neutrality, we can normalize it to zero. See details in Appendix D-G.

Hence, import tariffs $\boldsymbol{\tau^m} = [\tau_1^m, ..., \tau_j^m, ..., \tau_J^m]'$ for J sectors satisfy

$$
\Lambda_2 \boldsymbol{\tau^m} = \begin{pmatrix} \frac{\beta_1 x_1 - (1+\tau_1^d) w_{11} L_{11}}{w_{21} L_{21}} \\ ... \\ \frac{\beta_j x_1 - (1+\tau_j^d) w_{1j} L_{1j}}{w_{2j} L_{2j}} \\ ... \\ \frac{\beta_J x_1 - (1+\tau_J^d) w_{1J} L_{1J}}{w_{2J} L_{2J}} \end{pmatrix} = \Psi_1,
$$

where the element of matrix Λ_2 at row j and column i is given by $\frac{\partial \ln(w_{2i} L_{2i})}{\partial \ln(w_{2j})} \frac{w_{2i} L_{2i}}{w_{2j} L_{2j}}$, i.e.,

$$
\Lambda_2 = \begin{pmatrix}
\frac{\partial ln(w_{21}L_{21})}{\partial ln(w_{21})} \frac{w_{21}L_{21}}{w_{21}L_{21}} & ... & \frac{\partial ln(w_{2i}L_{2i})}{\partial ln(w_{21})} \frac{w_{2i}L_{2i}}{w_{21}L_{21}} & ... & \frac{\partial ln(w_{2J}L_{2J})}{\partial ln(w_{21})} \frac{w_{2J}L_{2J}}{w_{21}L_{21}} \\
... & ... & ... & ... & ... \\
\frac{\partial ln(w_{21}L_{21})}{\partial ln(w_{2j})} \frac{w_{21}L_{21}}{w_{2j}L_{2j}} & ... & \frac{\partial ln(w_{2i}L_{2i})}{\partial ln(w_{2j})} \frac{w_{2i}L_{2i}}{w_{2j}L_{2j}} & ... & \frac{\partial ln(w_{2J}L_{2J})}{\partial ln(w_{2j})} \frac{w_{2J}L_{2J}}{w_{2j}L_{2j}} \\
... & ... & ... & ... & ... \\
\frac{\partial ln(w_{21}L_{21})}{\partial ln(w_{2J})} \frac{w_{21}L_{21}}{w_{2J}L_{2J}} & ... & \frac{\partial ln(w_{2i}L_{2i})}{\partial ln(w_{2J})} \frac{w_{2i}L_{2i}}{w_{2J}L_{2J}} & ... & \frac{\partial ln(w_{2J}L_{2J})}{\partial ln(w_{2J})} \frac{w_{2J}L_{2J}}{w_{2J}L_{2J}}
\end{pmatrix}.
$$

□

C Proof of Proposition 2: optimal policies under CES supply system

Proof of domestic taxes As shown in Proposition 1, $\forall k, j$,

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{1s}}{\partial \ln w_{1j}} \frac{Y_{1s}}{Y_{1j}} (1+\tau_s^d) = (1+\tau_j^d), \tag{A.10}$$

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{1s}}{\partial \ln w_{1k}} \frac{Y_{1s}}{Y_{1k}} (1+\tau_s^d) = (1+\tau_k^d). \tag{A.11}$$

Subtracting (A.11) from (A.10) and using Definition 4, we have

$$0 + \left(\frac{\partial \ln Y_{1j}}{\partial \ln w_{1j}} - \frac{\partial \ln Y_{1j}}{\partial \ln w_{1k}} \frac{Y_{1j}}{Y_{1k}} \right) \tau_j^d + \left(\frac{\partial \ln Y_{1k}}{\partial \ln w_{1j}} \frac{Y_k}{Y_{1j}} - \frac{\partial \ln Y_{1k}}{\partial \ln w_{1k}} \right) \tau_k^d = \tau_j^d - \tau_k^d.$$

Thus, $\tau_j^d = \tau_k^d$, and Home does not use differential domestic taxes across sectors.

Proof of optimal import tariffs As shown in Proposition 1, $\forall k, j$, import tariffs satisfy

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{2s}}{\partial \ln w_{2j}} \frac{Y_{2s}}{Y_{2j}} \tau_s^m = \Psi_{1j}, \tag{A.12}$$

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{2s}}{\partial \ln w_{2k}} \frac{Y_{2s}}{Y_{2k}} \tau_s^m = \Psi_{1k}. \tag{A.13}$$

Subtracting (A.13) from (A.12) and using Definition 4, we have

$$0 + \left(\frac{\partial \ln Y_{2j}}{\partial \ln w_{2j}} - \frac{\partial \ln Y_{2j}}{\partial \ln w_{2k}} \frac{Y_{2j}}{Y_{2k}} \right) \tau_j^m + \left(\frac{\partial \ln Y_{2k}}{\partial \ln w_{2j}} \frac{Y_{2k}}{Y_{2j}} - \frac{\partial \ln Y_{2k}}{\partial \ln w_{2k}} \right) \tau_k^m = \Psi_{1j} - \Psi_{1k},$$

where Ψ_{1j} and Ψ_{1k} are the jth and kth row of vector Ψ_1. This implies $\tau_j^m - \tau_k^m = \frac{\Psi_{1j} - \Psi_{1k}}{\kappa_2}$. □

D Optimal policies with perfectly mobile labor across sectors

With perfectly mobile labor across sectors, the goods market clearing conditions are

$$w_{1j}L_{1j} = \beta_j \left[\frac{1}{1+\tau_j^d} \pi_{11,j} x_1 + \frac{1}{1+\tau_j^x} \pi_{21,j} x_2 \right], \quad (\gamma_{1j}, \quad J)$$
$$w_{2j}L_{2j} = \beta_j \left[\frac{1}{1+\tau_j^m} \pi_{12,j} x_1 + \pi_{22,j} x_2 \right], \quad (\gamma_{2j}, \quad J)$$

The labor market clearing conditions are

$$\sum_{j=1}^{J} L_{1j} = \bar{L}_1,$$
$$\sum_{j=1}^{J} L_{2j} = \bar{L}_2,$$

Since the technology T_{nj} is exogenous, the sectoral labor L_{nj} cannot affect the trade share by affecting T_{nj}. In this case, we can sum over goods markets across sectors. Now, the goods market clearing conditions become:

$$w_1\bar{L}_1 = \sum_{j=1}^{J} \beta_j \left[\frac{1}{1+\tau_j^d} \pi_{11,j} x_1 + \frac{1}{1+\tau_j^x} \pi_{21,j} x_2 \right], \quad (\gamma_1)$$
$$w_2\bar{L}_2 = \sum_{j=1}^{J} \beta_j \left[\frac{1}{1+\tau_j^m} \pi_{12,j} x_1 + \pi_{22,j} x_2 \right], \quad (\gamma_2)$$

where $x_2 = w_2\bar{L}_2$. By Walras' Law, the formula $x_1 = w_1\bar{L}_1 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^x}{1+\tau_j^x} \pi_{21,j} x_2 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1$ is redundant by combining two equations in market clearing conditions.

It is important to note that when technology T_{nj} is endogenous and depends on L_{nj}, as demonstrated in Appendix I, we cannot simply aggregate the goods market conditions.

Since the multipliers γ_1 and γ_2 are not sector-specific, the optimal policy following equations (A.6) becomes $1+\tau_j^m = -\gamma_2 = 1+\tau^m$ and $1+\tau_j^d = -\gamma_1 = 1+\tau^d$. Thus, domestic taxes or tariffs are also not independent of sectors. Furthermore, the following subsection shows tax neutrality, and both the uniform tariff and domestic tax can be normalized to zero.

D.1 Proof of tax neutrality

Given $\Gamma = \{(\tau^m + 1, \tau_j^x + 1, \tau^d + 1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}^m + 1, \check{\tau}_j^x + 1, \check{\tau}^d + 1 : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_n}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1 + \check{\tau}^m = \lambda(1 + \tau^m)$, $1 + \check{\tau}_j^x = \frac{1+\tau_j^x}{\mu}$ and $1 + \check{\tau}^d = \lambda \frac{1+\tau^d}{\mu}$, for any constants $\mu > 0$ and $\lambda > 0$ for any j. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_n}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_1 = \mu\frac{w_1}{\lambda}$, $\check{w}_n = \frac{w_n}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{w}_1 \bar{L}_1 = \sum_{j=1}^{J} \beta_j \left[\frac{\mu}{\lambda(1+\tau^d)} \pi_{11,j} x_1 + \frac{\mu}{1+\tau_j^x} \pi_{21,j} \frac{x_2}{\lambda} \right] = \frac{\mu}{\lambda} w_1 \bar{L}_1$$

$$\check{w}_2 \bar{L}_2 = \sum_{j=1}^{J} \beta_j \left[\frac{1}{\lambda(1+\tau^m)} \pi_{12,j} x_1 + \pi_{22,j} \frac{x_2}{\lambda} \right] = \frac{w_2}{\lambda} \bar{L}_2$$

$$\check{x}_2 = \frac{w_2}{\lambda} L_2 = \frac{x_2}{\lambda}$$

$$\check{P}_1 = \Pi_j \left[T_{1j} (\mu \frac{w_1}{\lambda} \lambda \frac{1+\tau^d}{\mu})^{-\theta} + T_{2j} (\frac{w_2}{\lambda} \lambda (1+\tau^m) d_{12})^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = P_1 = 1$$

$$\check{P}_2 = \Pi_j \left[T_{1j} (\mu \frac{w_1}{\lambda} \frac{1+\tau_j^x}{\mu} d_{21})^{-\theta} + T_{2j} (\frac{w_2}{\lambda})^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = \frac{P_2}{\lambda}$$

$$\check{\pi}_{11,j} = \frac{T_{1j} (\mu \frac{w_1}{\lambda} \lambda \frac{1+\tau^d}{\mu})^{-\theta}}{T_{1j} (\mu \frac{w_1}{\lambda} \lambda \frac{1+\tau^d}{\mu})^{-\theta} + T_{2j} (\frac{w_2}{\lambda} \lambda (1+\tau^m) d_{12})^{-\theta}} = \pi_{11,j}$$

$$\check{\pi}_{21,j} = \frac{T_{1j} (\mu \frac{w_1}{\lambda} \frac{1+\tau_j^x}{\mu} d_{21})^{-\theta}}{T_{1j} (\mu \frac{w_1}{\lambda} \frac{1+\tau_j^x}{\mu} d_{21})^{-\theta} + T_{2j} (\frac{w_2}{\lambda})^{-\theta}} = \pi_{21,j}$$

The same allocations satisfy the equilibrium conditions under $\{\check{\tau}^m + 1, \check{\tau}_j^x + 1, \check{\tau}^d + 1\}$, hence we proved the allocations and welfare are the same under $\{\tau^m + 1, \tau_j^x + 1, \tau^d + 1\}$ and $\{\check{\tau}^m + 1, \check{\tau}_j^x + 1, \check{\tau}^d + 1\}$. For any policy Γ, we can set $\frac{\mu}{\lambda} = 1 + \tau^d$ and obtain $\check{\tau}^d = 0$. Then $\gamma_1 = -1$. Second, by further adjusting λ, we can get $\tau^m = 0$. From equation (A.3), $\tau_j^x = \frac{1}{\theta \pi_{22}}$. Alternatively, we can normalize *cons* to zero instead of setting τ^m to zero. □

E Optimal policies with immobile/sector-specific labor

In this case, labor is fixed in each sector, and goods market clearing conditions are sector-specific:

$$w_{1j}\bar{L}_{1j} = \beta_j \left[\frac{1}{1+\tau_j^d} \pi_{11,j} x_1 + \frac{1}{1+\tau_j^x} \pi_{21,j} x_2 \right], \quad (\gamma_{1j}, \quad J)$$
$$w_{2j}\bar{L}_{2j} = \beta_j \left[\frac{1}{1+\tau_j^m} \pi_{12,j} x_1 + \pi_{22,j} x_2 \right], \quad (\gamma_{2j}, \quad J)$$

where $x_2 = \sum_{j=1}^{J} w_{2j}\bar{L}_{2j}$ and trade shares and prices satisfy equations (3) - (8).

By Walras' Law, $x_1 = \sum_{j=1}^{J} w_{1j}\bar{L}_{1j} + \sum_{j=1}^{J} \beta_j \frac{\tau_j^x}{1+\tau_j^x} \pi_{21,j} x_2 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1$ is redundant by combining $2J$ equations in market clearing conditions.

E.1 Proof of tax neutrality and $\tau_j^d = 0$

Given $\Gamma = \{(\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1) : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_{nj}}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We define that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. Hence tax neutrality reflects the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1 + \check{\tau}_j^m = \lambda(1 + \tau_j^m)$, $1 + \check{\tau}_j^x = \frac{1+\tau_j^x}{\mu_j}$ and $1 + \check{\tau}_j^d = \lambda \frac{1+\tau_j^d}{\mu_j}$, for any constants $\mu_j > 0$ and $\lambda > 0$ for any j. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_{nj}}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_{1j} = \mu_j \frac{w_{1j}}{\lambda}$, $\check{w}_{nj} = \frac{w_{nj}}{\lambda}$,

$\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{w}_{1j}\bar{L}_{1j} = \beta_j \left[\frac{\mu_j}{\lambda(1+\tau_j^d)} \pi_{11,j} x_1 + \frac{\mu_j}{1+\tau_j^x} \pi_{21,j} \frac{x_2}{\lambda} \right] = \frac{\mu_j}{\lambda} w_{1j} \bar{L}_{1j} \tag{A.14}$$

$$\check{w}_{2j}\bar{L}_{2j} = \beta_j \left[\frac{1}{\lambda(1+\tau_j^m)} \pi_{12,j} x_1 + \pi_{22,j} \frac{x_2}{\lambda} \right] = \frac{w_{2j}}{\lambda} \bar{L}_{2j} \tag{A.15}$$

$$\check{x}_2 = \sum_{j=1}^{J} \frac{w_{2j}}{\lambda} \bar{L}_{2j} = \frac{x_2}{\lambda} \tag{A.16}$$

$$\check{P}_1 = \Pi_j \left[T_{1j} (\mu_j \frac{w_{1j}}{\lambda} \lambda \frac{1+\tau_j^d}{\mu_j})^{-\theta} + T_{2j} (\frac{w_{2j}}{\lambda} \lambda (1+\tau_j^m) d_{12})^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = P_1 = 1 \tag{A.17}$$

$$P_2 = \Pi_j \left[T_{1j} (\mu_j \frac{w_{1j}}{\lambda} \frac{1+\tau_j^x}{\mu_j} d_{21})^{-\theta} + T_{2j} (\frac{w_{2j}}{\lambda})^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = \frac{P_2}{\lambda} \tag{A.18}$$

$$\check{\pi}_{11,j} = \frac{T_{1j} (\mu_j \frac{w_1}{\lambda} \lambda \frac{1+\tau_j^d}{\mu_j})^{-\theta}}{T_{1j} (\mu_j \frac{w_1}{\lambda} \lambda \frac{1+\tau_j^d}{\mu_j})^{-\theta} + T_{2j} (\frac{w_{2j}}{\lambda} \lambda (1+\tau_j^m) d_{12})^{-\theta}} = \pi_{11,j} \tag{A.19}$$

$$\check{\pi}_{21,j} = \frac{T_{1j} (\mu_j \frac{w_1}{\lambda} \frac{1+\tau_j^x}{\mu_j} d_{21})^{-\theta}}{T_{1j} (\mu_j \frac{w_1}{\lambda} \frac{1+\tau_j^x}{\mu_j} d_{21})^{-\theta} + T_{2j} (\frac{w_{2j}}{\lambda})^{-\theta}} = \pi_{21,j}. \tag{A.20}$$

The same allocations satisfy the equilibrium conditions under $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$, hence we proved the allocations and welfare are the same under $\{\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1\}$ and $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$. For any policy Γ, we can set $\mu_j = \lambda(1+\tau_j^d)$ and obtain $\check{\tau}_j^d = 0$. Home can also scale up τ_j^m and τ_j^x to any level; this only leads to change in w_{1j} and has no real impacts.

E.2 Proof of optimal import tariffs

In this case, the element of matrix Λ_2 at row j and column k is given by $\frac{\partial \ln(Y_{2k})}{\partial \ln(w_{2j})} \frac{Y_{2k}}{Y_{2j}}$, which is 1 if $j = k$ and 0 if $j \neq k$. Therefore, $\Lambda_2 = I$ becomes the identity matrix. Consider two rows, jth and kth,

$$\tau_j^m = \Psi_{1j}$$
$$\tau_k^m = \Psi_{1k}.$$

The difference of the two implies

$$\tau_j^m - \tau_k^m = \Psi_{1j} - \Psi_{1k}.$$

For any sector j and k,

$$
\begin{aligned}
\tau_j^m - \tau_k^m = & \frac{\beta_j x_1 - (1+\tau_j^d) w_{1j}\bar{L}_{1j}}{w_{2j}\bar{L}_{2j}} - \frac{\beta_k x_1 - (1+\tau_k^d) w_{1k}\bar{L}_{1k}}{w_{2k}\bar{L}_{2k}} \\
= & \frac{\beta_j x_1 - (1+\tau_j^d) Y_{1j}}{Y_{2j}} - \frac{\beta_k x_1 - (1+\tau_k^d) Y_{1k}}{Y_{2k}} \\
= & \frac{\beta_j x_1 - Y_{1j}}{Y_{2j}} - \frac{\beta_k x_1 - Y_{1k}}{Y_{2k}}.
\end{aligned}
$$

Thus, Home's import tariff imposed on good j increases with the share of Home net imports in Foreign income, relative to sector k. □

F Optimal policies with imperfectly substitutable labor across sectors

In this case, labor across sectors are aggregated with a CES function, i.e.,

$$
L_n = [\sum_{j=1}^{J} \alpha_j{}^{-\frac{1}{\kappa_n - 1}} L_{nj}{}^{\frac{\kappa_n}{\kappa_n - 1}}]^{\frac{\kappa_n - 1}{\kappa_n}}, \quad 0 < \kappa_n - 1 < \infty.
$$

In addition, goods market clearing conditions are (9) and (10), and the labor supply satisfies

$$
L_{1j} = \alpha_j (\frac{w_{1j}}{W_1})^{\kappa_1 - 1} L_1, \tag{A.21}
$$

$$
L_{2j} = \alpha_j (\frac{w_{2j}}{W_2})^{\kappa_2 - 1} L_2, \tag{A.22}
$$

where $W_n = [\sum_{j=1}^{J} \alpha_j w_{nj}{}^{\kappa}]^{\frac{1}{\kappa}}$. Furthermore, trade shares and prices satisfy equations (3) - (8).

F.1 Proof of tax neutrality

Given $\Gamma = \{(\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1) : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_{nj}}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1 + \check{\tau}_j^m = \lambda(1 + \tau_j^m)$, $1 + \check{\tau}_j^x = \frac{1+\tau_j^x}{\mu}$ and $1 + \check{\tau}_j^d = \lambda \frac{1+\tau_j^d}{\mu}$, for any constants $\mu > 0$ and $\lambda > 0$. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_{nj}}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_{1j} = \mu \frac{w_{1j}}{\lambda}$, $\check{w}_{nj} = \frac{w_{nj}}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{L}_{1j} = \alpha_j (\frac{\frac{\mu}{\lambda} w_{1j}}{\frac{\mu}{\lambda} W_1})^{\kappa_1 - 1} L_1 = L_{1j}, \qquad \check{L}_{2j} = \alpha_j (\frac{\frac{1}{\lambda} w_{2j}}{\frac{1}{\lambda} W_2})^{\kappa_2 - 1} L_2 = L_{2j}.$$

The proof for market clearing conditions, expenditures, prices, and trade shares equations are similar to (A.14)-(A.20).

The same allocations satisfy the equilibrium conditions under $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$. Hence, we proved the allocations and welfare are the same under $\{\tau_j^m + 1, \tau_j^x + 1, \tau_j^d + 1\}$ and $\{\check{\tau}_j^m + 1, \check{\tau}_j^x + 1, \check{\tau}_j^d + 1\}$. For any policy Γ, we can set $\frac{\mu}{\lambda} = 1 + \tau_1^d$ and obtain $\check{\tau}_1^d = 0$. This differs from the case with immobile labor, from the proof of tax neutrality, only one domestic tax is redundant. Then $\gamma_{1,1} = -1$. By further adjusting λ, we can eliminate one tariff. Alternatively, we can normalize *cons* to zero instead of setting one tariff to zero.

F.2 Proof of optimal domestic taxes and import tariffs

In this case, Λ_n satisfies Definition 4 and becomes

$$\Lambda_1 - I = \begin{pmatrix} \kappa_1 - (\kappa_1 - 1)\frac{w_{11}L_{11}}{W_1\bar{L}_1} - 1 & \dots & -(\kappa_1 - 1)\frac{w_{1j}L_{1j}}{W_1\bar{L}_1} & \dots & -(\kappa_1 - 1)\frac{w_{1J}L_{1J}}{W_1\bar{L}_1} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa_1 - 1)\frac{w_{11}L_{11}}{W_1\bar{L}_1} & \dots & \kappa_1 - (\kappa_1 - 1)\frac{w_{1j}L_{1j}}{W_1\bar{L}_1} - 1 & \dots & -(\kappa_1 - 1)\frac{w_{1J}L_{1J}}{W_1\bar{L}_1} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa_1 - 1)\frac{w_{11}L_{11}}{W_1\bar{L}_1} & \dots & -(\kappa_1 - 1)\frac{w_{1j}L_{1j}}{W_1\bar{L}_1} & \dots & \kappa_1 - (\kappa_1 - 1)\frac{w_{1J}L_{1J}}{W_1\bar{L}_1} - 1 \end{pmatrix}.$$

Consider two rows, jth and kth,

$$-(\kappa_1 - 1)\sum_{s=1}^{J} \frac{w_{1s}L_{1s}}{W_1\bar{L}_1}(1 + \tau_s^d) + (\kappa_1 - 1)(1 + \tau_j^d) = 0$$
$$-(\kappa_1 - 1)\sum_{s=1}^{J} \frac{w_{1s}L_{1s}}{W_1\bar{L}_1}(1 + \tau_s^d) + (\kappa_1 - 1)(1 + \tau_k^d) = 0$$

The difference of the two implies

$$(\kappa_1 - 1)(\tau_j^d - \tau_k^d) = 0.$$

This indicates $\tau_j^d = \tau_k^d$ for any k, j. Thus, the domestic tax is uniform across all sectors. All can be normalized to zero.

Let us revisit Λ_2:

$$\Lambda_2 = \begin{pmatrix} \kappa_2 - (\kappa_2 - 1)\frac{w_{21}L_{21}}{W_2\bar{L}_2} & ... & -(\kappa_2 - 1)\frac{w_{2j}L_{2j}}{W_2\bar{L}_2} & ... & -(\kappa_2 - 1)\frac{w_{2J}L_{2J}}{W_2\bar{L}_2} \\ ... & ... & ... & ... & ... \\ -(\kappa_2 - 1)\frac{w_{21}L_{21}}{W_2\bar{L}_2} & ... & \kappa_2 - (\kappa_2 - 1)\frac{w_{2j}L_{2j}}{W_2\bar{L}_2} & ... & -(\kappa_2 - 1)\frac{w_{2J}L_{2J}}{W_2\bar{L}_2} \\ ... & ... & ... & ... & ... \\ -(\kappa_2 - 1)\frac{w_{21}L_{21}}{W_2\bar{L}_2} & ... & -(\kappa_2 - 1)\frac{w_{2j}L_{2j}}{W_2\bar{L}_2} & ... & \kappa_2 - (\kappa_2 - 1)\frac{w_{2J}L_{2J}}{W_2\bar{L}_2} \end{pmatrix}.$$

Consider two rows, jth and kth,

$$-(\kappa_2 - 1)\sum_{s=1}^{J}\frac{w_{2s}L_{2s}}{W_2\bar{L}_2}\tau_s^m + \kappa_2\tau_j^m = \Psi_{1j}$$
$$-(\kappa_2 - 1)\sum_{s=1}^{J}\frac{w_{2s}L_{2s}}{W_2\bar{L}_2}\tau_s^m + \kappa_2\tau_k^m = \Psi_{1k}.$$

The difference of the two implies

$$\kappa_2(\tau_j^m - \tau_k^m) = \Psi_{1j} - \Psi_{1k}.$$

Thus, for any sector j and k,

$$\tau_j^m - \tau_k^m = \frac{1}{\kappa_2}\left[\frac{\beta_j x_1 - w_{1j}L_{1j}}{w_{2j}L_{2j}} - \frac{\beta_k x_1 - w_{1k}L_{1k}}{w_{2k}L_{2k}}\right].$$

□

G Ricardo-Roy models

In this case, goods market clearing conditions are given by (9) and (10), and labor supply satisfies

$$w_{1j}L_{1j} = \sum_g \frac{A_{1jg}w_{1j}^{\kappa_1}}{W_{1g}^{\kappa_1}}W_{1g}\bar{L}_{1g}, \quad w_{2j}L_{2j} = \sum_g \frac{A_{2jg}w_{2j}^{\kappa_2}}{W_{2g}^{\kappa_2}}W_{2g}\bar{L}_{ng},$$

where $W_{ng} = \left[\sum_{s=1}^{J} A_{nsg}(w_{ns})^{\kappa_n}\right]^{\frac{1}{\kappa_n}}$ and trade shares and prices satisfy equations (3) - (8).

G.1 Proof of tax neutrality

Given $\Gamma = \{(\tau_j^m+1, \tau_j^x+1, \tau_j^d+1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1 : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_{nj}}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1+\check{\tau}_j^m = \lambda(1+\tau_j^m)$, $1+\check{\tau}_j^x = \frac{1+\tau_j^x}{\mu}$ and $1+\check{\tau}_j^d = \lambda\frac{1+\tau_j^d}{\mu}$, for any constants $\mu > 0$ and $\lambda > 0$. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_{nj}}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_{1j} = \mu\frac{w_{1j}}{\lambda}$, $\check{w}_{nj} = \frac{w_{nj}}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{w}_{1j}L_{1j} = \sum_g \frac{A_{1jg}(\frac{\mu}{\lambda}w_{1j})^{\kappa_1}}{(\frac{\mu}{\lambda}W_{1g})^{\kappa_1}}\frac{\mu}{\lambda}W_{1g}\bar{L}_{1g} = \frac{\mu}{\lambda}w_{1j}L_{1j}$$
$$\check{w}_{2j}L_{2j} = \sum_g \frac{A_{2jg}(\frac{1}{\lambda}w_{2j})^{\kappa_2}}{(\frac{1}{\lambda}W_{2g})^{\kappa_2}}\frac{1}{\lambda}W_{2g}\bar{L}_{2g} = \frac{1}{\lambda}w_{2j}L_{2j}$$

The market clearing, expenditures, prices, and trade shares equations are the same as (A.14)-(A.20). The same allocations satisfy the equilibrium conditions under $\{\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1\}$, hence we proved the allocations and welfare are the same under $\{\tau_j^m+1, \tau_j^x+1, \tau_j^d+1\}$ and $\{\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1\}$. For any policy Γ, we can set $\frac{\mu}{\lambda} = 1+\tau_1^d$ and obtain $\check{\tau}_1^d = 0$. Then $\gamma_{1,1} = -1$. By further adjusting λ, we can eliminate one tariff. Alternatively, we can normalize *cons* to zero instead of setting one tariff to zero.

G.2 Proof of optimal domestic taxes

With one factor:

$$\Lambda_1 - I = \begin{pmatrix} \kappa_1 - (\kappa_1 - 1)\frac{m_{11}m_{11}W_1\bar{L}_1}{w_{11}L_{11}} - 1 & ... & -(\kappa_1 - 1)\frac{m_{11}m_{1j}W_1\bar{L}_1}{w_{11}L_{11}} & ... & -(\kappa_1 - 1)\frac{m_{11}m_{1J}W_1\bar{L}_1}{w_{11}L_{11}} \\ ... & ... & ... & ... & ... \\ -(\kappa_1 - 1)\frac{m_{1j}m_{11}W_1\bar{L}_1}{w_{1j}L_{1j}} & ... & \kappa_1 - (\kappa_1 - 1)\frac{m_{1j}m_{1j}W_1\bar{L}_1}{w_{1j}L_{1j}} - 1 & ... & -(\kappa_1 - 1)\frac{m_{1j}m_{1J}W_1\bar{L}_1}{w_{1j}L_{1j}} \\ ... & ... & ... & ... & ... \\ -(\kappa_1 - 1)\frac{m_{1J}m_{11}W_1\bar{L}_1}{w_{1J}L_{1J}} & ... & -(\kappa_1 - 1)\frac{m_{1J}m_{1j}W_1\bar{L}_1}{w_{1J}L_{1J}} & ... & \kappa_1 - (\kappa_1 - 1)\frac{m_{1J}m_{1J}W_1\bar{L}_1}{w_{1J}L_{1J}} - 1 \end{pmatrix}$$

$$= \begin{pmatrix} \kappa_1 - 1 - (\kappa_1 - 1)m_{11} & ... & -(\kappa_1 - 1)m_{1j} & ... & -(\kappa_1 - 1)m_{1J} \\ ... & ... & ... & ... & ... \\ -(\kappa_1 - 1)m_{11} & ... & \kappa_1 - 1 - (\kappa_1 - 1)m_{1j} & ... & -(\kappa_1 - 1)m_{1J} \\ ... & ... & ... & ... & ... \\ -(\kappa_1 - 1)m_{11} & ... & -(\kappa_1 - 1)m_{1j} & ... & \kappa_1 - 1 - (\kappa_1 - 1)m_{1J} \end{pmatrix}.$$

Consider two rows, jth and kth,

$$\begin{aligned} -(\kappa_1 - 1)\sum_{s=1}^{J} m_{2s}(1 + \tau_s^d) + (\kappa_1 - 1)(1 + \tau_j^d) = 0 \\ -(\kappa_1 - 1)\sum_{s=1}^{J} m_{2s}(1 + \tau_s^d) + (\kappa_1 - 1)(1 + \tau_k^d) = 0 \end{aligned}$$

The difference of the two implies

$$(\kappa_1 - 1)(\tau_j^d - \tau_k^d) = 0.$$

This indicates $\tau_j^d = \tau_k^d$ for any k, j. Thus, the domestic tax is uniform across all sectors.

With multiple factors:

$$\Lambda_1 - I = (\kappa_1 - 1)\begin{pmatrix} 1-\frac{\sum_g m_{11g}m_{11g}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} & \dots & -\frac{\sum_g m_{11g}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} & \dots & -\frac{\sum_g m_{11g}m_{1Jg}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g m_{1jg}m_{11g}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} & \dots & 1-\frac{\sum_g m_{1jg}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} & \dots & -\frac{\sum_g m_{1jg}m_{1Jg}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g m_{1Jg}m_{11g}W_{1g}\bar{L}_{1g}}{w_{1J}L_{1J}} & \dots & -\frac{\sum_g m_{1Jg}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{1J}L_{1J}} & \dots & 1-\frac{\sum_g m_{1Jg}m_{1Jg}W_{1g}\bar{L}_{1g}}{w_{1J}L_{1J}} \end{pmatrix}. \quad \text{(A.23)}$$

Due to tax neutrality, the domestic tax can only be determined at a relative level. We define a matrix E that satisfies

$$E = (\kappa_1 - 1)\begin{pmatrix} 1-\frac{\sum_g m_{11g}m_{11g}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} & \dots & -\frac{\sum_g m_{11g}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} & \dots & -\frac{\sum_g m_{11g}m_{1,J-1,g}W_{1g}\bar{L}_{1g}}{w_{11}L_{11}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g m_{1jg}m_{11g}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} & \dots & 1-\frac{\sum_g m_{1jg}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} & \dots & -\frac{\sum_g m_{1jg}m_{1,J-1,g}W_{1g}\bar{L}_{1g}}{w_{1j}L_{1j}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g m_{1,J-1,g}m_{11g}W_{1g}\bar{L}_{1g}}{w_{1,J-1}L_{1,J-1}} & \dots & -\frac{\sum_g m_{1,J-1,g}m_{1jg}W_{1g}\bar{L}_{1g}}{w_{1,J-1}L_{1,J-1}} & \dots & 1-\frac{\sum_g m_{1,J-1,g}m_{1,J-1,g}W_{1g}\bar{L}_{1g}}{w_{1,J-1}L_{1,J-1}} \end{pmatrix}.$$

Using this matrix, we can rewrite the system of equations (A.23) as

$$E \cdot \begin{pmatrix} \tau_1^d - \tau_J^d \\ \dots \\ \tau_j^d - \tau_J^d \\ \dots \\ \tau_{J-1}^d - \tau_J^d \end{pmatrix} = \begin{pmatrix} 0 \\ \dots \\ 0 \\ \dots \\ 0 \end{pmatrix}.$$

As $|E| \neq 0$, $\tau_j^d - \tau_J^d = 0$, and thus domestic taxes are uniform across all sectors. $\square$

G.3 Proof of optimal import tariffs

With one factor,

$$
\Lambda_2 = \begin{pmatrix}
\kappa_2 - (\kappa_2 - 1)\frac{m_{21}m_{21}W_2\bar{L}_2}{w_{21}L_{21}} & ... & -(\kappa_2 - 1)\frac{m_{21}m_{2j}W_2\bar{L}_2}{w_{21}L_{21}} & ... & -(\kappa_2 - 1)\frac{m_{21}m_{2J}W_2\bar{L}_2}{w_{21}L_{21}} \\
... & ... & ... & ... & ... \\
-(\kappa_2 - 1)\frac{m_{2j}m_{21}W_2\bar{L}_2}{w_{2j}L_{2j}} & ... & \kappa_2 - (\kappa_2 - 1)\frac{m_{22}m_{2j}W_2\bar{L}_2}{w_{2j}L_{2j}} & ... & -(\kappa_2 - 1)\frac{m_{2j}m_{2J}W_2\bar{L}_2}{w_{2j}L_{2j}} \\
... & ... & ... & ... & ... \\
-(\kappa_2 - 1)\frac{m_{2J}m_{21}W_2\bar{L}_2}{w_{2J}L_{2J}} & ... & -(\kappa_2 - 1)\frac{m_{2J}m_{2j}W_2\bar{L}_2}{w_{2J}L_{2J}} & ... & \kappa_2 - (\kappa_2 - 1)\frac{m_{2J}m_{2J}W_2\bar{L}_2}{w_{2J}L_{2J}}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
\kappa_2 - (\kappa_2 - 1)m_{21} & ... & -(\kappa_2 - 1)m_{2j} & ... & -(\kappa_2 - 1)m_{2J} \\
... & ... & ... & ... & ... \\
-(\kappa_2 - 1)m_{21} & ... & \kappa_2 - (\kappa_2 - 1)m_{2j} & ... & -(\kappa_2 - 1)m_{2J} \\
... & ... & ... & ... & ... \\
-(\kappa_2 - 1)m_{21} & ... & -(\kappa_2 - 1)m_{2j} & ... & \kappa_2 - (\kappa_2 - 1)m_{2J}
\end{pmatrix}.
$$

Again, consider two rows, jth and kth,

$$
\begin{aligned}
-(\kappa_2 - 1)\sum_{s=1}^{J} m_{2s}\tau_s^m + \kappa_2\tau_j^m &= \Psi_{1j} \\
-(\kappa_2 - 1)\sum_{s=1}^{J} m_{2s}\tau_s^m + \kappa_2\tau_k^m &= \Psi_{1k}.
\end{aligned}
$$

The difference of the two implies

$$
\kappa_2(\tau_j^m - \tau_k^m) = \Psi_{1j} - \Psi_{1k}.
$$

Thus, for any sector j and k

$$
\begin{aligned}
\tau_j^m - \tau_k^m &= \frac{1}{\kappa_2}\left[\frac{\beta_j x_1 - (1+\tau_j^d)w_{1j}L_{1j}}{w_{2j}L_{2j}} - \frac{\beta_k x_1 - (1+\tau_k^d)w_{1k}L_{1k}}{w_{2k}L_{2k}}\right] \\
&= \frac{1}{\kappa_2}\left[\frac{\beta_j x_1 - w_{1j}L_{1j}}{w_{2j}L_{2j}} - \frac{\beta_k x_1 - w_{1k}L_{1k}}{w_{2k}L_{2k}}\right].
\end{aligned}
$$

With multiple factors:

$$\Lambda_2 = \begin{pmatrix} \kappa_2 - (\kappa_2 - 1)\frac{\sum_g m_{21g}m_{21g}W_{2g}\bar{L}_{2g}}{w_{21}L_{21}} & \dots & -(\kappa_2 - 1)\frac{\sum_g m_{21g}m_{2jg}W_{2g}\bar{L}_{2g}}{w_{21}L_{21}} & \dots & -(\kappa_2 - 1)\frac{\sum_g m_{21g}m_{2Jg}W_{2g}\bar{L}_{2g}}{w_{21}L_{21}} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa_2 - 1)\frac{\sum_g m_{2jg}m_{21g}W_{2g}\bar{L}_{2g}}{w_{2j}L_{2j}} & \dots & \kappa_2 - (\kappa_2 - 1)\frac{\sum_g m_{2jg}m_{2jg}W_{2g}\bar{L}_{2g}}{w_{2j}L_{2j}} & \dots & -(\kappa_2 - 1)\frac{\sum_g m_{2jg}m_{2Jg}W_{2g}\bar{L}_{2g}}{w_{2j}L_{2j}} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa_2 - 1)\frac{\sum_g m_{2Jg}m_{21g}W_{2g}\bar{L}_{2g}}{w_{2J}L_{2J}} & \dots & -(\kappa_2 - 1)\frac{\sum_g m_{2Jg}m_{2jg}W_{2g}\bar{L}_{2g}}{w_{2J}L_{2J}} & \dots & \kappa_2 - (\kappa_2 - 1)\frac{\sum_g m_{2Jg}m_{2Jg}W_{2g}\bar{L}_{2g}}{w_{2J}L_{2J}} \end{pmatrix}$$

The supply system doesn't satisfy Definition 4; hence, optimal tariffs can not be written as Proposition 2. Optimal tariffs satisfy the general formula as in Proposition 1.

□

H Optimal policies under inefficient labor markets

In addition to wages, workers have wedges over different sectors. Workers then choose a sector to work in to maximize their utility. Let λ_{nj} denote the share of workers in sector j relative to the total number of workers in country n. The law of large numbers ensures the proportion of these workers who work in sector j is $\lambda_{nj} = \Pr\left(\epsilon_{nj} w_{nj} \geq \max_s \{\epsilon_{ns} w_{ns}\}\right)$. Assume that wedge ϵ_{nj} over sectors follow a Frechet distribution where κ_n governs the degree of dispersion across individuals. Then the share of workers in sector j relative to the total number of workers in country n is

$$\lambda_{nj} = \frac{\left(w_{nj}\right)^{\kappa_n}}{\sum_{k=1}^{J}\left(w_{nk}\right)^{\kappa_n}}$$

and total employment in sector j of country n is $L_{nj} = \lambda_{nj}\bar{L}_n$. Hence, the equilibrium requires

$$\Omega = \left\{ \left(w_{nj}, L_{nj}\right) : \frac{Y_{nj}}{Y_n} = \frac{w_{nj}L_{nj}}{W_n L_n} = \frac{w_{nj}^{\kappa_n+1}}{\sum_{s=1}^{J} w_{ns}^{\kappa_n+1}},\ W_n = \frac{\sum_{s=1}^{J} w_{ns}^{\kappa_n+1}}{\sum_{s=1}^{J} w_{ns}^{\kappa_n}} \right\}. \tag{A.24}$$

In this case, goods market clearing conditions are (9) and (10), and labor supply satisfy

$$L_{1j} = \lambda_{1j}\bar{L}_1 = \frac{\left(w_{1j}\right)^{\kappa_1}}{\sum_{k=1}^{J}\left(w_{1k}\right)^{\kappa_1}}\bar{L}_1, \quad L_{2j} = \lambda_{2j}\bar{L}_2 = \frac{\left(w_{2j}\right)^{\kappa_2}}{\sum_{k=1}^{J}\left(w_{2k}\right)^{\kappa_2}}\bar{L}_2,$$

and trade shares and prices satisfy equations (3) - (8).

H.1 Proof of tax neutrality

Given $\Gamma = \{(\tau_j^m+1, \tau_j^x+1, \tau_j^d+1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1 : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_{nj}}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1+\check{\tau}_j^m = \lambda(1+\tau_j^m)$, $1+\check{\tau}_j^x = \frac{1+\tau_j^x}{\mu}$ and $1+\check{\tau}_j^d = \lambda\frac{1+\tau_j^d}{\mu}$, for any constants $\mu > 0$ and $\lambda > 0$. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_{nj}}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_{1j} = \mu\frac{w_{1j}}{\lambda}$, $\check{w}_{nj} = \frac{w_{nj}}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{L}_{1j} = \frac{\left(\frac{\mu}{\lambda}w_{1j}\right)^{\kappa_1}}{\sum_{k=1}^J\left(\frac{\mu}{\lambda}w_{1k}\right)^{\kappa_1}}\bar{L}_1 = L_{1j}, \quad \check{L}_{2j} = \frac{\left(\frac{1}{\lambda}w_{2j}\right)^{\kappa_2}}{\sum_{k=1}^J\left(\frac{1}{\lambda}w_{2k}\right)^{\kappa_2}}\bar{L}_2 = L_{2j}.$$

The market clearing, expenditures, prices, and trade shares equations are the same as (A.14)-(A.20). The same allocations satisfy the equilibrium conditions under $\{\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1\}$, hence we proved the allocations and welfare are the same under $\{\tau_j^m+1, \tau_j^x+1, \tau_j^d+1\}$ and $\{\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1\}$. For any policy Γ, we can set $\frac{\mu}{\lambda} = 1+\tau_1^d$ and obtain $\check{\tau}_1^d = 0$. Then $\gamma_{1,1} = -1$. By further adjusting λ, by using tax neutrality, we can only eliminate one tariff. Alternatively, we can normalize *cons* to zero instead of setting one tariff to zero.

H.2 Proof of optimal domestic taxes

$$\Lambda_1 - I = \begin{pmatrix} \kappa_1 - \kappa_1 m_{11}\frac{W_1}{w_{11}} & \dots & -\kappa_1 m_{1j}\frac{W_1}{w_{11}} & \dots & -\kappa_1 m_{1J}\frac{W_1}{w_{11}} \\ \dots & \dots & \dots & \dots & \dots \\ -\kappa_1 m_{11}\frac{W_1}{w_{1j}} & \dots & \kappa_1 - \kappa_1 m_{1j}\frac{W_1}{w_{1j}} & \dots & -\kappa_1 m_{1J}\frac{W_1}{w_{1j}} \\ \dots & \dots & \dots & \dots & \dots \\ -\kappa_1 m_{11}\frac{W_1}{w_{1J}} & \dots & -\kappa_1 m_{1j}\frac{W_1}{w_{1J}} & \dots & \kappa_1 - \kappa_1 m_{1J}\frac{W_1}{w_{nJ}}. \end{pmatrix}$$

where the jth and kth rows are given by

$$-\kappa_1\frac{1}{w_{1j}}\sum_{s=1}^J m_{1s}W_1(1+\tau_s^d) + \kappa_1(1+\tau_j^d) = 0,$$
$$-\kappa_1\frac{1}{w_{1k}}\sum_{s=1}^J m_{1s}W_1(1+\tau_s^d) + \kappa_1(1+\tau_k^d) = 0,$$

respectively. Taking the differences of the two and rearranging terms, we get

$$\frac{1+\tau_j^d}{1+\tau_k^d} = \frac{w_{1k}}{w_{1j}}.$$

This indicates that the domestic tax imposed on a high-wage sector is lower than that on the low-wage sector. □

H.3 Proof of optimal import tariffs

In this case, Λ_2 is given by

$$\Lambda_2 = \begin{pmatrix} \kappa_2+1-\kappa_2 m_{21}\frac{W_2}{w_{21}} & \dots & -\kappa_2 m_{2j}\frac{W_2}{w_{21}} & \dots & -\kappa_2 m_{2J}\frac{W_2}{w_{21}} \\ \dots & \dots & \dots & \dots & \dots \\ -\kappa_2 m_{21}\frac{W_2}{w_{2j}} & \dots & \kappa_2+1-\kappa_2 m_{2j}\frac{W_2}{w_{2j}} & \dots & -\kappa_2 m_{2J}\frac{W_2}{w_{2j}} \\ \dots & \dots & \dots & \dots & \dots \\ -\kappa_2 m_{21}\frac{W_2}{w_{2J}} & \dots & -\kappa_2 m_{2j}\frac{W_2}{w_{2J}} & \dots & \kappa_2+1-\kappa_2 m_{2J}\frac{W_2}{w_{2J}} \end{pmatrix}.$$

Again, consider two rows, jth and kth,

$$-\kappa_2\frac{W_2}{w_{2j}}\sum_{s=1}^{J} m_{2s}\tau_s^m + (\kappa_2+1)\tau_j^m = \Psi_{1j}$$
$$-\kappa_2\frac{W_2}{w_{2k}}\sum_{s=1}^{J} m_{2s}\tau_s^m + (\kappa_2+1)\tau_k^m = \Psi_{1k}.$$

The difference of the two implies

$$(\kappa_2+1)(\tau_j^m-\tau_k^m) - \kappa_2\left(\frac{W_2}{w_{2j}}-\frac{W_2}{w_{2k}}\right)\sum_{s=1}^{J} m_{2s}\tau_s^m = \Psi_{1j}-\Psi_{1k}.$$

Thus, for any sector j and k, the following holds

$$\tau_j^m-\tau_k^m = \frac{1}{\kappa_2+1}\left[\frac{\beta_j x_1-(1+\tau_j^d)w_{1j}L_{1j}}{w_{2j}L_{2j}} - \frac{\beta_k x_1-(1+\tau_k^d)w_{1k}L_{1k}}{w_{2k}L_{2k}} + \kappa_2\left(\frac{W_2}{w_{2j}}-\frac{W_2}{w_{2k}}\right)\sum_{s=1}^{J}\tau_s^m m_{2s}\right]$$

The additional term reflects the inefficiency in the foreign labor market and non-CES supply system.

H.4 Optimal policies when foreign correct distortions

Now, suppose Foreign imposes domestic taxes that satisfy

$$\frac{1+\tau_j^{d*}}{1+\tau_k^{d*}} = \frac{w_{2k}}{w_{2j}}, \quad \forall j,k.$$

Home planner chooses $\{w_{nj}, x_n, \tau_j^x, \tau_j^m, \tau_j^d, L_{nj}\}$ to maximizes domestic welfare $\max x_1/P_1$ subject to the world market equilibrium characterized by

$$w_{1j}L_{1j} = \beta_j\left[\frac{1}{1+\tau_j^d}\pi_{11,j}x_1 + \frac{1}{1+\tau_j^x}\pi_{21,j}x_2\right] \quad (\gamma_{1j}, \quad J)$$

$$(1+\tau_j^{d*})w_{2j}L_{2j} = \beta_j\left[\frac{1}{1+\tau_j^m}\pi_{12,j}x_1 + \pi_{22,j}x_2\right] \quad (\gamma_{2j}, \quad J)$$

where

$$L_{1j} = \lambda_{1j}\bar{L}_1 = \frac{(w_{1j})^{\kappa_1}}{\sum_{k=1}^J (w_{1k})^{\kappa_1}}\bar{L}_1, \quad L_{2j} = \lambda_{2j}\bar{L}_2 = \frac{(w_{2j})^{\kappa_2}}{\sum_{k=1}^J (w_{2k})^{\kappa_2}}\bar{L}_2$$

$$x_2 = \sum_{j=1}^J w_{2j}L_{2j} + \sum_{j=1}^J \beta_j \frac{\tau_j^{d*}}{1+\tau_j^{d*}}\left[\frac{1}{1+\tau_j^m}\pi_{12,j}x_1 + \pi_{22,j}x_2\right] = \sum_{j=1}^J (1+\tau_j^{d*})w_{2j}L_{2j}$$

$$P_1 = \Pi_j\left[T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{d*})d_{12})^{-\theta}\right]^{-\frac{\beta_j}{\theta}} = 1$$

$$P_2 = \Pi_j\left[T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta}\right]^{-\frac{\beta_j}{\theta}}$$

$$\pi_{11,j} = \frac{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{d*})d_{12})^{-\theta}}$$

$$\pi_{12,j} = \frac{T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{d*})d_{12})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{d*})d_{12})^{-\theta}}$$

$$\pi_{21,j} = \frac{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta}},$$

$$\pi_{22,j} = \frac{T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta}}.$$

By Walras' Law, combining $2J$ equations in market clearing conditions, we can derive the formula for $x_1 = \sum_{j=1}^J w_{1j}L_{1j} + \sum_{j=1}^J \beta_j \frac{\tau_j^x}{1+\tau_j^x}\pi_{21,j}x_2 + \sum_{j=1}^J \beta_j \frac{\tau_j^m}{1+\tau_j^m}\pi_{12,j}x_1 + \sum_{j=1}^J \beta_j \frac{\tau_j^d}{1+\tau_j^d}\pi_{11,j}x_1$.

H.4.1 Proof of tax neutrality

Given $\Gamma = \{(\tau_j^m+1, \tau_j^x+1, \tau_j^d+1, \tau_j^{d*}+1) : \forall j\}$ and $\check{\Gamma} = \{(\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1, \check{\tau}_j^{d*}+1 : \forall j\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{in,j}, \frac{w_{nj}}{P_n}, \frac{x_n}{P_n}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1+\check{\tau}_j^m = \lambda(1+\tau_j^m)$, $1+\check{\tau}_j^x = \frac{1+\tau_j^x}{\mu}$, $1+\check{\tau}_j^d = \lambda\frac{1+\tau_j^d}{\mu}$ and $1+\check{\tau}_{2j}^d = 1+\tau_j^{d*}$, where $i \neq n$, for any constants $\mu > 0$ and $\lambda > 0$. We guess the allocations $\{\check{\pi}_{in,j}, \frac{\check{w}_{nj}}{\check{P}_n}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{ni,j} = \pi_{ni,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_{1j} = \mu\frac{w_{1j}}{\lambda}$, $\check{w}_{nj} = \frac{w_{nj}}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$. We then verify all equilibrium conditions hold.

$$\check{w}_{1j}L_{1j} = \beta_j\left[\frac{\mu}{\lambda(1+\tau_j^d)}\pi_{11,j}x_1 + \frac{\mu}{1+\tau_j^x}\pi_{21,j}\frac{x_2}{\lambda}\right] = \frac{\mu}{\lambda}w_{1j}L_{1j}$$

$$(1+\check{\tau}_j^{d*})\check{w}_{2j}L_{2j} = \beta_j\left[\frac{1}{\lambda(1+\tau_j^m)}\pi_{12,j}x_1 + \pi_{22,j}\frac{x_2}{\lambda}\right] = (1+\tau_j^{d*})\frac{w_{2j}}{\lambda}L_{2j}$$

$$\check{x}_2 = \sum_{j=1}^{J}(1+\tau_j^{d*})\frac{w_{2j}}{\lambda}L_{2j} = \frac{x_2}{\lambda}$$

$$\check{P}_1 = \Pi_j\left[T_{1j}(\mu\frac{w_{1j}}{\lambda}\lambda\frac{1+\tau_j^d}{\mu})^{-\theta} + T_{2j}(\frac{w_2}{\lambda}\lambda(1+\tau_j^m)(1+\tau_j^{d*})d_{12})^{-\theta}\right]^{-\frac{\beta_j}{\theta}} = P_1 = 1$$

$$\check{P}_2 = \Pi_j\left[T_{1j}(\mu\frac{w_{1j}}{\lambda}\frac{1+\tau_j^x}{\mu}d_{21})^{-\theta} + T_{2j}(\frac{w_{2j}}{\lambda}(1+\tau_j^{d*}))^{-\theta}\right]^{-\frac{\beta_j}{\theta}} = \frac{P_2}{\lambda}$$

$$\check{\pi}_{11,j} = \frac{T_{1j}(\mu\frac{w_1}{\lambda}\lambda\frac{1+\tau_j^d}{\mu})^{-\theta}}{T_{1j}(\mu\frac{w_1}{\lambda}\lambda\frac{1+\tau_j^d}{\mu})^{-\theta} + T_{2j}(\frac{w_{2j}}{\lambda}\lambda(1+\tau_j^m)(1+\tau_j^{d*})d_{12})^{-\theta}} = \pi_{11,j}$$

$$\check{\pi}_{21,j} = \frac{T_{1j}(\mu\frac{w_1}{\lambda}\frac{1+\tau_j^x}{\mu}d_{21})^{-\theta}}{T_{1j}(\mu\frac{w_1}{\lambda}\frac{1+\tau_j^x}{\mu}d_{21})^{-\theta} + T_{2j}(\frac{w_{2j}}{\lambda}(1+\tau_j^{d*}))^{-\theta}} = \pi_{21,j}$$

$$\check{L}_{1j} = \frac{\left(\frac{\mu}{\lambda}w_{1j}\right)^{\kappa_1}}{\sum_{k=1}^{J}\left(\frac{\mu}{\lambda}w_{1k}\right)^{\kappa_1}}\bar{L}_1 = L_{1j}$$

$$\check{L}_{2j} = \frac{\left(\frac{1}{\lambda}w_{2j}\right)^{\kappa_2}}{\sum_{k=1}^{J}\left(\frac{1}{\lambda}w_{2k}\right)^{\kappa_2}}\bar{L}_2 = L_{2j}$$

The same allocations satisfy the equilibrium conditions under $\{\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1, \check{\tau}_j^{d*}+1\}$, hence we proved the allocations and welfare are the same under $\{\tau_j^m+1, \tau_j^x+1, \tau_j^d+1, \tau_j^{d*}+1\}$ and $\{\check{\tau}_j^m+1, \check{\tau}_j^x+1, \check{\tau}_j^d+1, \check{\tau}_j^{d*}+1\}$. For any policy Γ, we can set $\frac{\mu}{\lambda} = 1+\tau_1^d$ and obtain $\check{\tau}_1^d = 0$. Then

$\gamma_{1,1} = -1$. By further adjusting λ, we can eliminate one tariff. Alternatively, we can normalize *cons* to zero instead of setting one tariff to zero.

H.4.2 Proof of optimal domestic taxes

In this case, Λ_1 is given by

$$\Lambda_1 - I = \begin{pmatrix} \kappa_1 - \kappa_1 m_{11}\frac{W_1}{w_{11}} & ... & -\kappa_1 m_{1j}\frac{W_1}{w_{11}} & ... & -\kappa_1 m_{1J}\frac{W_1}{w_{11}} \\ ... & ... & ... & ... & ... \\ -\kappa_1 m_{11}\frac{W_1}{w_{1j}} & ... & \kappa_1 - \kappa_1 m_{1j}\frac{W_1}{w_{1j}} & ... & -\kappa_1 m_{1J}\frac{W_1}{w_{1j}} \\ ... & ... & ... & ... & ... \\ -\kappa_1 m_{11}\frac{W_1}{w_{1J}} & ... & -\kappa_1 m_{1j}\frac{W_1}{w_{1J}} & ... & \kappa_1 - \kappa_1 m_{1J}\frac{W_1}{w_{nJ}} \end{pmatrix}$$

. Consider two rows, j and k

$$\begin{aligned} &-\kappa_1 \frac{1}{w_{1j}} \sum_{s=1}^{J} m_{1s} W_1 (1+\tau_s^d) + \kappa_1 (1+\tau_j^d) = 0, \\ &-\kappa_1 \frac{1}{w_{1k}} \sum_{s=1}^{J} m_{1s} W_1 (1+\tau_s^d) + \kappa_1 (1+\tau_k^d) = 0. \end{aligned}$$

Taking the difference of the two and reorganizing, we can show

$$\frac{1+\tau_j^d}{1+\tau_k^d} = \frac{w_{1k}}{w_{1j}}.$$

□

H.4.3 Proof of optimal import tariffs

In this case,

$$
\Lambda_2 = \begin{pmatrix}
\kappa_2 + 1 - \kappa_2 \frac{w_{21}L_{21}}{(1+\tau_1^{d*})w_{21}\bar{L}_2} & \dots & -\kappa_2 \frac{w_{2j}L_{2j}}{(1+\tau_1^{d*})w_{21}\bar{L}_2} & \dots & -\kappa_2 \frac{w_{2J}L_{2J}}{(1+\tau_1^{d*})w_{21}\bar{L}_2} \\
\dots & \dots & \dots & \dots & \dots \\
-\kappa_2 \frac{w_{21}L_{21}}{(1+\tau_j^{d*})w_{2j}\bar{L}_2} & \dots & \kappa_2 + 1 - \kappa_2 \frac{w_{2j}L_{2j}}{(1+\tau_j^{d*})w_{2j}\bar{L}_2} & \dots & -\kappa_2 \frac{w_{2J}L_{2J}}{(1+\tau_j^{d*})w_{2j}\bar{L}_2} \\
\dots & \dots & \dots & \dots & \dots \\
-\kappa_2 \frac{w_{21}L_{21}}{(1+\tau_J^{d*})w_{2J}\bar{L}_2} & \dots & -\kappa_2 \frac{w_{2j}L_{2j}}{(1+\tau_J^{d*})w_{2J}\bar{L}_2} & \dots & \kappa_2 + 1 - \kappa_2 \frac{w_{2J}L_{2J}}{(1+\tau_J^{d*})w_{2J}\bar{L}_2}
\end{pmatrix}
$$

Consider two rows:

$$
\begin{aligned}
&-\kappa_2 \frac{1}{(1+\tau_j^{d*})w_{2j}} \sum_{s=1}^{J} m_{2s} W_2 \tau_s^m + (\kappa_2+1)\tau_j^m = \Psi_{1j} \\
&-\kappa_2 \frac{1}{(1+\tau_k^{d*})w_{2k}} \sum_{s=1}^{J} m_{2s} W_2 \tau_s^m + (\kappa_2+1)\tau_k^m = \Psi_{1k}
\end{aligned}
$$

Taking the difference of the two and using Foreign condition $\frac{1}{(1+\tau_j^{d*})w_{2j}} = \frac{1}{(1+\tau_k^{d*})w_{2k}}$, we have

$$
(\kappa_2+1)(\tau_j^m - \tau_k^m) = \Psi_{1j} - \Psi_{1k}.
$$

Thus, for any sector j and k.

$$
\begin{aligned}
\tau_j^m - \tau_k^m &= \frac{1}{\kappa_2+1}\left(\frac{\beta_j x_1 - (1+\tau_j^d)w_{1j}L_{1j}}{(1+\tau_j^{d*})w_{2j}L_{2j}} - \frac{\beta_k x_1 - (1+\tau_k^d)w_{1k}L_{1k}}{(1+\tau_k^{d*})w_{2k}L_{2k}}\right) \\
&= \frac{1}{\kappa_2+1}\left[\frac{\beta_j x_1 - (1+\tau_j^d)Y_{1j}}{(1+\tau_j^{d*})Y_{2j}} - \frac{\beta_k x_1 - (1+\tau_k^d)Y_{1k}}{(1+\tau_k^{d*})Y_{2k}}\right].
\end{aligned}
$$

□

I Revisit steady-state optimal policies in Bai, Jin, and Lu (2023)

In this section, we revisit the steady-state optimal policies in BJL, an example of optimal trade policies in a multi-sector, multi-country GE model under productivity-driven supply curves. We first derive the optimal policies under these model. We then show that the multi-sector Krugman and multi-sector Melitz-Pareto models are isomorphic to BJL at the steady state.

In BJL framework, total labor in each sector is divided into two components: production labor and researchers. Labor is assumed to be perfectly mobile across sectors, and technology endogenously depends on the number of researchers. In steady-state equilibrium, the ratio of production labor to researchers remains constant, implying that technology can be expressed as a function of production labor. Let T_{nj} denote the technology level of sector j in country n, which depends on an efficiency parameter ν_{nj} and the endogenous production labor in the sector, L^p_{nj}. Specifically, $T_{nj} = \nu_{nj} L^p_{nj}$.

We consider a more general case where labor does not need to be perfectly mobile. Under any given labor market specification, technology may exhibit constant, decreasing, or increasing returns to scale as a result of external economies of scale (EES). Specifically, the technology of sector j in country n is a function of the sector's endogenous production labor, i.e.,

$$T_{nj} = \nu_{nj} (L^p_{nj})^{\epsilon}.$$

where ϵ is the scale elasticity, governing the strength of EES. BJL represents a special case when $\epsilon = 1$ and labor is perfectly mobile. When $\epsilon = 0$, the technology is exogenous and aligns with that described in the previous sections.

Firms engage in Bertrand competition, where the lowest-cost producer of each good in each market captures the entire market by charging a markup just enough to keep the second-lowest-cost producer out of the market. In equilibrium, markups follow a Pareto distribution with parameter θ. Since all firms selling in the market charge a markup drawn from the same distribution, total profits of firms earned by firms in the market are a constant share of that market's total sales. Specifically, firms (both domestic and foreign) selling in the market earn profits equal to $1/(1+\theta)$ of total sales. The remaining $\theta/(1+\theta)$ share of total sales goes to production labor. Therefore, we can express the total sales of sector j in country n as $\frac{1+\theta}{\theta} w_n L^p_{nj}$.

Now, goods market clearing conditions are[6]

$$\frac{1+\theta}{\theta} w_{1j} L_{1j} = \beta_j \left[\frac{1}{1+\tau_j^d} \pi_{11,j} x_1 + \frac{1}{1+\tau_j^x} \pi_{21,j} x_2 \right], \quad (\gamma_{1j}, \quad J)$$
$$\frac{1+\theta}{\theta} w_{2j} L_{2j} = \beta_j \left[\frac{1}{1+\tau_j^m} \pi_{12,j} x_1 + \pi_{22,j} x_2 \right] \quad (\gamma_{2j}, \quad J)$$

where $(1+\theta)/\theta$ captures firms' markup and $x_2 = \frac{1+\theta}{\theta} \sum_{j=1}^{J} w_{2j} L_{2j}$. By Walras' Law, combining the $2J$ equations in market clearing conditions, we can derive the formula for x_1. Therefore, it is not necessary to be included as a constraint.

$$x_1 = \frac{1+\theta}{\theta} \sum_{j=1}^{J} w_{1j} L_{1j} + \sum_{j=1}^{J} \beta_j \frac{\tau_j^x}{1+\tau_j^x} \pi_{21,j} x_2 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \sum_{j=1}^{J} \beta_j \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1$$

Labor market clearing condition implies,

$$L_{1j} = \alpha_j \left(\frac{w_{1j}}{W_1}\right)^{\kappa_1 - 1} L_1,$$
$$L_{2j} = \alpha_j \left(\frac{w_{2j}}{W_2}\right)^{\kappa_2 - 1} L_2,$$

where $W_n = [\sum_{j=1}^{J} \alpha_j {w_{nj}}^{\kappa_n}]^{\frac{1}{\kappa_n}}$. Lastly, trade shares and prices satisfy equations (3) - (8).

I.1 Proof of optimal domestic taxes and import tariffs

FOC over w_{1j}

$$\begin{aligned}
&[-\beta_j x_1 \frac{\pi_{11,j}}{w_{1j}} + \gamma_{1j} \beta_j \frac{1}{1+\tau_j^d} \frac{\partial \pi_{11,j}}{\partial w_{1j}} x_1 + \gamma_{1j} \beta_j \frac{1}{1+\tau_j^x} \frac{\partial \pi_{21,j}}{\partial w_{1j}} x_2 \\
&+ \gamma_{2j} \beta_j \frac{1}{1+\tau_j^m} \frac{\partial \pi_{12,j}}{\partial w_{1j}} x_1 + \gamma_{2j} \beta_j \frac{\partial \pi_{22,j}}{\partial w_{1j}} x_2] + (-\gamma_{1j}) \frac{1+\theta}{\theta} L_{1j} \\
&+ \sum_{k=1} \{ [\beta_k x_1 \frac{\pi_{11,k}}{\theta T_{1k}} + \gamma_{1k} \beta_j \frac{1}{1+\tau_k^d} \frac{\partial \pi_{11,k}}{\partial T_{1k}} x_1 + \gamma_{1k} \beta_k \frac{1}{1+\tau_k^x} \frac{\partial \pi_{21,k}}{\partial T_{1k}} x_2 \\
&+ \gamma_{2k} \beta_k \frac{1}{1+\tau_k^m} \frac{\partial \pi_{12,k}}{\partial T_{1k}} x_1 + \gamma_{2k} \beta_k \frac{\partial \pi_{22,k}}{\partial T_{1k}} x_2] \frac{T_{1k} \epsilon}{L_{1k}} + (-\gamma_{1k}) \frac{1+\theta}{\theta} w_{1k} \} \frac{\partial L_{1k}}{\partial w_{1j}} = 0
\end{aligned} \tag{A.25}$$

Since

$$\frac{\partial \pi_{nm,j}}{\partial L_{1j}} L_{1j} = \epsilon \frac{\partial \pi_{nm,j}}{\partial T_{1j}} T_{1j} = -\frac{\epsilon}{\theta} \frac{\partial \pi_{nm,j}}{\partial w_{1j}} w_{1j}$$

[6]To maintain consistency with the previous equations, we simplify L^p_{nj} to L_{nj}.

FOC over w_{1j} becomes

$$
\begin{aligned}
&[-\beta_j x_1 \frac{\pi_{11,j}}{w_{1j}} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{1j}} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial w_{1j}} x_2 \\
&+ \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{1j}} x_1 + \gamma_{2j}\beta_j \frac{\partial \pi_{22,j}}{\partial w_{1j}} x_2] + (-\gamma_{1j})\frac{1+\theta}{\theta} L_{1j} \\
&+ \sum_{k=1} \{-[-\beta_k x_1 \frac{\pi_{11,k}}{w_{1k}} + \gamma_{1k}\beta_j \frac{1}{1+\tau_k^d}\frac{\partial \pi_{11,k}}{\partial w_{1k}} x_1 + \gamma_{1k}\beta_k \frac{1}{1+\tau_k^x}\frac{\partial \pi_{21,k}}{\partial w_{1k}} x_2 \\
&+ \gamma_{2k}\beta_k \frac{1}{1+\tau_k^m}\frac{\partial \pi_{12,k}}{\partial w_{1k}} x_1 + \gamma_{2k}\beta_k \frac{\partial \pi_{22,k}}{\partial w_{1k}} x_2] \frac{\epsilon w_{1k}}{\theta L_{1k}} + (-\gamma_{1k})\frac{1+\theta}{\theta} w_{1k}\} \frac{\partial L_{1k}}{\partial w_{1j}} = 0
\end{aligned}
$$

Combining FOCs over w_{1j}, τ_j^x and τ_j^m, and using the optimal tax formua $1+\tau_j^m = -\gamma_{2j}$ and $1+\tau_j^d = -\gamma_{1j}$, we have

$$
\begin{aligned}
&\sum_{k=1} [(-\gamma_{1k})\frac{1+\theta}{\theta} w_{1k} \frac{\epsilon}{\theta} + (-\gamma_{1k})\frac{1+\theta}{\theta} w_{1k}] \frac{\partial L_{1k}}{\partial w_{1j}} = 0 \\
(\Rightarrow) &\frac{\epsilon+\theta}{\theta} \sum_{k=1} [(-\gamma_{1k})\frac{1+\theta}{\theta} w_{1k}] \frac{\partial L_{1k}}{\partial w_{1j}} = 0 \\
(\Rightarrow) &\sum_{k=1} (-\gamma_{1k})\frac{1+\theta}{\theta} w_{1k} \frac{\partial L_{1k}}{\partial w_{1j}} = 0 \\
(\Rightarrow) &\sum_{k=1} (-\gamma_{1k})\frac{\partial Y_{1k}}{\partial w_{1j}} w_{1j} - (-\gamma_{1j})\frac{1+\theta}{\theta} w_{1j} L_{1j} = 0 \\
(\Rightarrow) &\sum_{k=1} (1+\tau_k^d)\frac{\partial \ln(Y_{1k})}{\partial \ln(w_{1j})} Y_{1k} - (1+\tau_j^d)\frac{1+\theta}{\theta} w_{1j} L_{1j} = 0 \\
(\Rightarrow) &\sum_{k=1} (1+\tau_k^d)\frac{\partial \ln(Y_{1k})}{\partial \ln(w_{1j})} \frac{Y_{1k}}{Y_{1j}} - (1+\tau_j^d) = 0
\end{aligned}
$$

Hence, domestic taxes $\boldsymbol{\tau^d} = [\tau_1^d, ..., \tau_j^d, ..., \tau_J^d]'$ for J sectors satisfy

$$
(\Lambda_1 - I) \cdot (1+\tau^d) = 0
$$

Similar to the proof in Section F, under CES supply system, the domestic tax is uniform across sectors.

FOC over w_{2j}

$$
\begin{aligned}
&-\beta_j x_1 \frac{\pi_{12,j}}{w_{2j}} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{2j}} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial w_{2j}} x_2 + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{2j}} x_1 + \gamma_{2j}\beta_j \frac{\partial \pi_{22,j}}{\partial w_{2j}} x_2 \\
&+ (cons_1 - \gamma_{2j})\frac{1+\theta}{\theta} L_{2j} \\
&+ \sum_{k=1}\{[\beta_k x_1 \frac{\pi_{12,k}}{\theta T_{2k}} + \gamma_{1k}\beta_k \frac{1}{1+\tau_k^d}\frac{\partial \pi_{11,k}}{\partial T_{2k}} x_1 + \gamma_{1k}\beta_k \frac{1}{1+\tau_k^x}\frac{\partial \pi_{21,k}}{\partial T_{2k}} x_2 + \gamma_{2k}\beta_k \frac{1}{1+\tau_k^m}\frac{\partial \pi_{12,k}}{\partial T_{2k}} x_1 + \gamma_{2k}\beta_k \frac{\partial \pi_{22,k}}{\partial T_{2k}} x_2] \frac{T_{2k}\epsilon}{L_{2k}} \\
&+ (cons_1 - \gamma_{2k})\frac{1+\theta}{\theta} w_{2k}\} \frac{\partial L_{2k}}{\partial w_{2j}} = 0 \qquad \text{(A.26)}
\end{aligned}
$$

where $cons_1 = \sum_{s=1}^{J}(\gamma_{1s}\beta_s \frac{1}{1+\tau_s^x}\pi_{21,s} + \gamma_{2s}\beta_s \pi_{22,s})$.

Similarly,

$$
\frac{\partial \pi_{nm,j}}{\partial L_{2j}} L_{2j} = \epsilon \frac{\partial \pi_{nm,j}}{\partial T_{2j}} T_{2j} = -\frac{\epsilon}{\theta}\frac{\partial \pi_{nm,j}}{\partial w_{2j}} w_{2j}
$$

FOC over w_{2j} becomes

$$
\begin{aligned}
&-\beta_j x_1 \frac{\pi_{12,j}}{w_{2j}} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{2j}} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial w_{2j}} x_2 + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{2j}} x_1 + \gamma_{2j}\beta_j \frac{\partial \pi_{22,j}}{\partial w_{2j}} x_2 \\
&+ (cons_1 - \gamma_{2j})\frac{1+\theta}{\theta} L_{2j} \\
&+ \sum_{k=1}\{-[-\beta_k x_1 \frac{\pi_{12,k}}{w_{2k}} + \gamma_{1k}\beta_k \frac{1}{1+\tau_k^d}\frac{\partial \pi_{11,k}}{\partial w_{2k}} x_1 + \gamma_{1k}\beta_k \frac{1}{1+\tau_k^x}\frac{\partial \pi_{21,k}}{\partial w_{2k}} x_2 + \gamma_{2k}\beta_k \frac{1}{1+\tau_k^m}\frac{\partial \pi_{12,k}}{\partial w_{2k}} x_1 + \gamma_{2k}\beta_k \frac{\partial \pi_{22,k}}{\partial w_{2k}} x_2] \\
&\cdot \frac{\epsilon w_{2k}}{\theta L_{2k}} + (cons_1 - \gamma_{2k})\frac{1+\theta}{\theta} w_{2k}\} \frac{\partial L_{2k}}{\partial w_{2j}} = 0
\end{aligned}
$$

Combining FOCs over w_{1j} and w_{2j}, we get

$$
\begin{aligned}
& -\beta_j x_1 + (-\gamma_{1j})\frac{1+\theta}{\theta}w_{1j}L_{1j} + (cons_1 - \gamma_{2j})\frac{1+\theta}{\theta}w_{2j}L_{2j} \\
& + \sum_{k=1}\{[\beta_k x_1 - (-\gamma_{1k})\frac{1+\theta}{\theta}w_{1k}L_{1k}]\frac{\epsilon}{\theta L_{2k}} + (cons_1 - \gamma_{2k})\frac{1+\theta}{\theta}w_{2k}\}\frac{\partial L_{2k}}{\partial w_{2j}}w_{2j} = 0 \\
(\Rightarrow) & -\beta_j x_1 + (-\gamma_{1j})Y_{1j} + (cons_1 - \gamma_{2j})Y_{2j} + \sum_{k=1}\{[\beta_k x_1 - (-\gamma_{1k})Y_{1k}]\frac{\epsilon}{\theta} + (cons_1 - \gamma_{2k})Y_{2k}\}\frac{\partial L_{2k}}{\partial w_{2j}}\frac{w_{2j}}{L_{2k}} = 0 \\
(\Rightarrow) & -\beta_j x_1 + (-\gamma_{1j})Y_{1j} + (cons_1 - \gamma_{2j})Y_{2j} + \sum_{k=1}\{[\beta_k x_1 - (-\gamma_{1k})Y_{1k}]\frac{\epsilon}{\theta} + (cons_1 - \gamma_{2k})Y_{2k}\}\frac{\partial Y_{2k}}{\partial w_{2j}}\frac{w_{2j}}{Y_{2k}} \\
& -\{[\beta_j x_1 - (-\gamma_{1j})Y_{1j}]\frac{\epsilon}{\theta} + (cons_1 - \gamma_{2j})Y_{2j}\} = 0 \\
(\Rightarrow) & -\frac{\epsilon+\theta}{\theta}(\beta_j x_1 - (-\gamma_{1j})Y_{1j}) + \sum_{k=1}\{[\beta_k x_1 - (-\gamma_{1k})Y_{1k}]\frac{\epsilon}{\theta} + (cons_1 - \gamma_{2k})Y_{2k}\}\frac{\partial \ln(Y_{2k})}{\partial \ln(w_{2j})} = 0 \\
(\Rightarrow) & -\frac{\epsilon+\theta}{\theta}\frac{\beta_j x_1 - (1+\tau_j^d)Y_{1j}}{Y_{2j}} + \sum_{k=1}\{\frac{\beta_k x_1 - (1+\tau_k^d)Y_{1k}}{Y_{2k}}\frac{\epsilon}{\theta} + (cons_1 - \gamma_{2k})\}\frac{\partial \ln(Y_{2k})}{\partial \ln(w_{2j})}\frac{Y_{2k}}{Y_{2j}} = 0 \\
(\Rightarrow) & -\frac{\epsilon+\theta}{\theta}\frac{\beta_j x_1 - (1+\tau_j^d)Y_{1j}}{Y_{2j}} + \sum_{k=1}\{\frac{\beta_k x_1 - (1+\tau_k^d)Y_{1k}}{Y_{2k}}\frac{\epsilon}{\theta} + (cons + \tau_k^m)\}\frac{\partial \ln(Y_{2k})}{\partial \ln(w_{2j})}\frac{Y_{2k}}{Y_{2j}} = 0
\end{aligned}
$$

where $cons = cons_1 + 1$, which can be normalized to zero.

Therefore, $\forall k, j$, import tariffs satisfy

$$
\sum_{s=1}^{J}\frac{\partial \ln Y_{2s}}{\partial \ln w_{2j}}\frac{Y_{2s}}{Y_{2j}}(\tau_s^m + \Psi_{1s}\frac{\epsilon}{\theta}) = \frac{\epsilon+\theta}{\theta}\Psi_{1j}, \tag{A.27}
$$

$$
\sum_{s=1}^{J}\frac{\partial \ln Y_{2s}}{\partial \ln w_{2k}}\frac{Y_{2s}}{Y_{2k}}(\tau_s^m + \Psi_{1s}\frac{\epsilon}{\theta}) = \frac{\epsilon+\theta}{\theta}\Psi_{1k}. \tag{A.28}
$$

Hence, import tariff $\boldsymbol{\tau^m} = [\tau_1^m, ..., \tau_j^m, ..., \tau_J^m]'$ for J sectors satisfy

$$
\begin{aligned}
\Lambda_2(\tau^m + \Psi_1\frac{\epsilon}{\theta}) &= \frac{\epsilon+\theta}{\theta}\Psi_1 \\
(\Rightarrow)\Lambda_2\boldsymbol{\tau^m} &= \frac{\epsilon+\theta}{\theta}\Psi_1 - \frac{\epsilon}{\theta}\Lambda_2\Psi_1 \\
(\Rightarrow)\boldsymbol{\tau^m} &= \frac{\epsilon+\theta}{\theta}\Lambda_2^{-1}\Psi_1 - \frac{\epsilon}{\theta}\Psi_1 \\
(\Rightarrow)\boldsymbol{\tau^m} &= (\frac{\epsilon+\theta}{\theta}\Lambda_2^{-1} - \frac{\epsilon}{\theta})\Psi_1
\end{aligned}
$$

Subtracting (A.28) from (A.27) and using Definition 4, we have

$$0+\left(\frac{\partial \ln Y_{2j}}{\partial \ln w_{2j}}-\frac{\partial \ln Y_{2j}}{\partial \ln w_{2k}}\frac{Y_{2j}}{Y_{2k}}\right)(\tau_j^m+\Psi_{1j}\frac{\epsilon}{\theta})+\left(\frac{\partial \ln Y_{2k}}{\partial \ln w_{2j}}\frac{Y_{2k}}{Y_{2j}}-\frac{\partial \ln Y_{2k}}{\partial \ln w_{2k}}\right)(\tau_k^m+\Psi_{1k}\frac{\epsilon}{\theta})=\frac{\epsilon+\theta}{\theta}(\Psi_{1j}-\Psi_{1k})$$

$$(\Rightarrow)0+\kappa_2(\tau_j^m+\Psi_{1j}\frac{\epsilon}{\theta})-\kappa_2(\tau_k^m+\Psi_{1k}\frac{\epsilon}{\theta})=\frac{\epsilon+\theta}{\theta}(\Psi_{1j}-\Psi_{1k})$$

$$(\Rightarrow)\tau_j^m-\tau_k^m=\frac{\theta+\epsilon(1-\kappa_2)}{\theta\kappa_2}(\Psi_{1j}-\Psi_{1k})$$

where Ψ_{1j} and Ψ_{1k} are the jth and kth row of vector Ψ_1.

When $\kappa_2\to\infty$ and $\epsilon=1$, $\frac{1}{\kappa_2}+\frac{\epsilon}{\theta}\frac{1-\kappa_2}{\kappa_2}\to-\frac{1}{\theta}$, which is the same as the formula of optimal tariff in BJL. When $\epsilon=0$, $\frac{1}{\kappa_2}+\frac{\epsilon}{\theta}\frac{1-\kappa_2}{\kappa_2}=\frac{1}{\kappa_2}$, which is the same as the formula of optimal tariff in section F. □

I.2 Isomorphism between the multi-sector Krugman, multi-sector Melitz-Pareto models and the SS of BJL model

Kucheryavyy, Lyn, and Rodríguez-Clare (2023) prove the isomorphic gravity equations for models with varying returns to scale, including the multi-sector Krugman and multi-sector Melitz-Pareto models.

Multi-sector Krugman model Each sector contains a continuum of differentiated varieties. Preferences are structured as follows: Cobb-Douglas across sectors with weights β_j, CES across country bundles within a sector with an elasticity of substitution σ, and CES across varieties within a country bundle with the same elasticity σ. The corresponding demand function for a representative variety in sector j produced in country i and consumed in country n is:

$$q_{ni,j}=p_{ni,j}^{-\sigma}P_{n,j}^{\sigma-1}\beta_j x_n$$

where $p_{ni,j}$ is the price of the variety. $P_{ni,j}=M_{i,j}^{1/(1-\sigma)}p_{ni,j}$ is the price index in country n of country i varieties of sector j. $M_{i,j}$ is the measure of firms or measure of varieties produced in country i sector j. $P_{n,j}=(\sum_i P_{ni,j}^{1-\sigma})^{1/(1-\sigma)}$ is the sector price index in country n.

Let $A_{i,j}$ represent the exogenous productivity in sector j of country i, which is common across all firms in that sector. Similarly, let $F_{i,j}$ denote the fixed cost (in terms of labor) associated with producing any variety in sector j of country i. Under monopolistic competition, the price index in country n for varieties from sector j in country i can be expressed as:

$$P_{ni,j} = M_{i,j}^{1/(1-\sigma)} \left(\bar{\sigma} w_{i,j}(1+\tau_{ni,j}) d_{ni,j}/A_{i,j}\right)$$

where $\bar{\sigma} \equiv \sigma/\left(\sigma-1\right)$ is the markup. When $n \neq i$, $\tau_{ni,j}$ denotes the import tariff imposed by country n on sector j imports from country i. When $n = i$, $\tau_{nn,j}$ represents the domestic tax imposed by country n on sector j. Denoting the revenue of the representative firm from sector j in country i by $R_{i,j} \equiv \sum_n \frac{1}{1+\tau_{ni,j}} p_{ni,j} q_{ni,j}$, the corresponding profit is then given by $R_{i,j}/\sigma$.

$$\begin{aligned}
\frac{R_{i,j}}{\sigma} &= \frac{1}{\sigma} \sum_n \frac{1}{1+\tau_{ni,j}} \left(\frac{p_{ni,j}}{P_{n,j}}\right)^{1-\sigma} \beta_j x_n \\
&= \frac{M_{i,j}^{-1}}{\sigma} \sum_n \frac{1}{1+\tau_{ni,j}} \left(\frac{P_{ni,j}}{P_{n,j}}\right)^{1-\sigma} \beta_j x_n \\
&= \frac{M_{i,j}^{-1}}{\sigma} \sum_n \frac{1}{1+\tau_{ni,j}} \pi_{ni,j} \beta_j x_n \\
&= \frac{M_{i,j}^{-1}}{\sigma} w_{i,j} L_{i,j}
\end{aligned}$$

where sector-level trade shares are

$$\pi_{ni,j} = \left(\frac{P_{ni,j}}{P_{n,j}}\right)^{1-\sigma} = \frac{A_{i,j}^{\sigma-1} M_{i,j} \left(w_{i,j}(1+\tau_{ni,j}) d_{ni,j}\right)^{1-\sigma}}{\sum_l A_{l,j}^{\sigma-1} M_{l,j} \left(w_{l,j}(1+\tau_{nl,j}) d_{nl,j}\right)^{1-\sigma}}$$

Free entry then implies,

$$\begin{aligned}
\frac{R_{i,j}}{\sigma} - F_{i,j} w_{i,j} &= 0 \\
(\Rightarrow) M_{i,j} &= \frac{L_{i,j}}{\sigma F_{i,j}}
\end{aligned}$$

Total labor $L_{i,j}$ must be consistent with the amounts produced for each market $L_{i,j}^p$, plus the fixed cost of entry, and labor markets must clear, $\sum_j L_{i,j} = \bar{L}_i$.

$$\begin{aligned}
L_{i,j} &= M_{i,j} \sum_n q_{ni,j} d_{ni,j} / A_{i,j} + M_{i,j} F_{i,j} \\
&= M_{i,j} \sum_n \frac{1}{1+\tau_{ni,j}} p_{ni,j}^{-\sigma} P_{n,j}^{\sigma-1} d_{ni,j} \beta_j x_n / A_{i,j} + \frac{1}{\sigma} \sum_n \frac{1}{1+\tau_{ni,j}} \pi_{ni,j} \beta_j x_n / w_{i,j} \\
&= M_{i,j} \frac{1}{\bar{\sigma}} \sum_n \frac{1}{1+\tau_{ni,j}} p_{ni,j}^{1-\sigma} P_{n,j}^{\sigma-1} \beta_j x_n / w_{i,j} + \frac{1}{\sigma} \sum_n \frac{1}{1+\tau_{ni,j}} \pi_{ni,j} \beta_j x_n / w_{i,j} \\
&= \frac{1}{\bar{\sigma}} \sum_n \frac{1}{1+\tau_{ni,j}} \pi_{ni,j} \beta_j x_n / w_{i,j} + \frac{1}{\sigma} \sum_n \frac{1}{1+\tau_{ni,j}} \pi_{ni,j} \beta_j x_n / w_{i,j} \\
&= \sum_n \frac{1}{1+\tau_{ni,j}} \pi_{ni,j} \beta_j x_n / w_{i,j} \\
&= L_{i,j} \\
L_{i,j}^p &= \frac{\sigma-1}{\sigma} L_{i,j} \qquad \text{(A.29)}
\end{aligned}$$

(A.29) implies that production labor maintains constant share of total labor.

Trade shares are then

$$\begin{aligned}
\pi_{ni,j} = \left(\frac{P_{ni,j}}{P_{n,j}}\right)^{1-\sigma} &= \frac{A_{i,j}^{\sigma-1} F_{i,j}^{-1} L_{i,j} \left(w_{i,j}(1+\tau_{ni,j}) d_{ni,j}\right)^{1-\sigma}}{\sum_l A_{l,j}^{\sigma-1} F_{l,j}^{-1} L_{l,j} \left(w_{l,j}(1+\tau_{nl,j}) d_{nl,j}\right)^{1-\sigma}} \\
&= \frac{A_{i,j}^{\sigma-1} F_{i,j}^{-1} L_{i,j}^p \left(w_{i,j}(1+\tau_{ni,j}) d_{ni,j}\right)^{1-\sigma}}{\sum_l A_{l,j}^{\sigma-1} F_{l,j}^{-1} L_{l,j}^p \left(w_{l,j}(1+\tau_{nl,j}) d_{nl,j}\right)^{1-\sigma}}
\end{aligned}$$

with price index given by

$$\begin{aligned}
P_{n,j} &= \sigma^{1/(\sigma-1)} \bar{\sigma} \left[\sum_l A_{l,j}^{\sigma-1} F_{l,j}^{-1} L_{l,j} \left(w_{l,j}(1+\tau_{nl,j}) d_{nl,j}\right)^{1-\sigma}\right]^{1/(1-\sigma)} \\
&= (\sigma-1)^{1/(\sigma-1)} \bar{\sigma} \left[\sum_l A_{l,j}^{\sigma-1} F_{l,j}^{-1} L_{l,j}^p \left(w_{l,j}(1+\tau_{nl,j}) d_{nl,j}\right)^{1-\sigma}\right]^{1/(1-\sigma)}
\end{aligned}$$

The expressions for trade shares and sector price indexes in this Krugman model collapse to those in BJL by setting $\nu_{nj} = A_{n,j}^{\sigma-1} F_{n,j}^{-1}$.

Multi-sector Melitz-Pareto model To enter, firms must first make an initial investment, modeled as a fixed entry cost $F_{i,j}$ (measured in units of labor) in country i. Firms then can produce a variety in sector j, with their labor productivity drawn from a Pareto distribution $g_{i,j}(\varphi)$ characterized by a shape parameter $\theta > \sigma - 1$ and a location parameter $b_{i,j}$. $g_{i,j}(\varphi)$ has positive support over

$(0,\infty)$ and has a continuous cumulative distribution $G_{i,j}(\varphi)$. Firms from country i must pay a fixed cost $f_{n,j}$ (measured in units of labor) in country n to serve that market.

Let $\Omega_{ni,j}$ represent the set of varieties that country i sells to country n in sector j. The price index of these goods is given by $P_{ni,j} \equiv \left(\int_{\omega\in\Omega_{ni,j}} p_{ni,j}(\omega)^{1-\sigma}d\omega\right)^{\frac{1}{1-\sigma}}$. Let $M_{i,j}$ denote the total number of firms that enter in sector j in country i. $\varphi^*_{ni,j}$ denote the productivity cutoff, such that country i exports to country n all goods with productivity greater than $\varphi^*_{ni,j}$.

$$
\begin{aligned}
P_{ni,j}^{1-\sigma} &= M_{i,j}\int_{\varphi^*_{ni,j}}^{\infty}\left[p_{ni,j}(\varphi)\right]^{1-\sigma}dG_{i,j}(\varphi)\\
&= \theta b_{i,j}^{\theta}M_{i,j}\left[\bar{\sigma}w_{i,j}(1+\tau_{ni,j})d_{ni,j}\right]^{1-\sigma}\int_{\varphi^*_{ni,j}}^{\infty}\varphi^{\sigma-\theta-2}d\varphi\\
&= \frac{\theta b_{i,j}^{\theta}}{\theta-\sigma+1}M_{i,j}\left[\bar{\sigma}w_{i,j}(1+\tau_{ni,j})d_{ni,j}\right]^{1-\sigma}\left(\varphi^*_{ni,j}\right)^{\sigma-\theta-1}
\end{aligned}
\tag{A.30}
$$

where $\bar{\sigma} \equiv \sigma/(\sigma-1)$ is the markup, and $G_{i,j}(\varphi) = 1-(b_{i,j}/\varphi)^{\theta}$. When $n \neq i$, $\tau_{ni,j}$ denotes the import tariff imposed by country n on sector j imports from country i. When $n = i$, $\tau_{nn,j}$ represents the domestic tax imposed by country n on sector j.

Zero cutoff profit conditions determine the cutoff $\varphi^*_{ni,j}$

$$
\begin{aligned}
\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_{i,j}(1+\tau_{ni,j})d_{ni,j}}{\varphi^*_{ni,j}}\right)^{1-\sigma} & P_{n,j}^{\sigma-1}x_{n,j} = w_{n,j}f_{n,j}\\
(\Rightarrow)\varphi^*_{ni,j} &= \frac{\sigma}{\sigma-1}\frac{w_{i,j}(1+\tau_{ni,j})d_{ni,j}}{P_{n,j}}\left(\frac{\sigma w_{n,j}f_{n,j}}{x_{n,j}}\right)^{\frac{1}{\sigma-1}}\\
&= \frac{\sigma}{\sigma-1}\frac{w_{i,j}(1+\tau_{ni,j})d_{ni,j}}{P_{n,j}}\left(\frac{\sigma w_{n,j}f_{n,j}}{\beta_j x_n}\right)^{\frac{1}{\sigma-1}}
\end{aligned}
\tag{A.31}
$$

Since $1-G(\varphi^*_{ni,j})$ is the ex-ante probability of successful entry, it defines the aggregate productivity level $\tilde{\varphi}_{ni,j}$ as a function of the cutoff level $\varphi^*_{ni,j}$

$$
\begin{aligned}
\left(\tilde{\varphi}(\varphi^*_{ni,j})\right)^{\sigma-1} &= \frac{1}{1-G(\varphi^*_{ni,j})}\int_{\varphi^*_{ni,j}}^{\infty}\varphi^{\sigma-1}dG(\varphi)\\
&= \left(\frac{\varphi^*_{ni,j}}{b_{i,j}}\right)^{\theta}\frac{\theta b_{i,j}^{\theta}}{\theta-\sigma+1}\left(\varphi^*_{ni,j}\right)^{\sigma-\theta-1}\\
&= \frac{\theta}{\theta-\sigma+1}\left(\varphi^*_{ni,j}\right)^{\sigma-1}
\end{aligned}
$$

Denote $x(\tilde{\varphi}_{ni,j})$ as the average sales from country i to country n in sector j, conditional on successful entry, which is also tied to the cutoff level $\varphi^*_{ni,j}$.

$$\begin{aligned} x(\tilde{\varphi}_{ni,j}) &= \left[\frac{\tilde{\varphi}\left(\varphi^*_{ni,j}\right)}{\varphi^*_{ni,j}}\right]^{\sigma-1} x(\varphi^*_{ni,j}) \\ &= \frac{\theta}{\theta-\sigma+1} x(\varphi^*_{ni,j}) \\ &= \frac{\theta\sigma}{\theta-\sigma+1} f_{nj} \end{aligned}$$

Thus, the fixed costs constitute a constant share $\zeta = \frac{\theta-(\sigma-1)}{\sigma\theta}$ of sales.

For each country i and sector j, the expected value of entry must equal the entry cost. The expected value of entry is the ex-ante probability of successful entry multiplied by the expected profitability of producing the good, which is $(1/\sigma-\zeta)\sum_n x_{ni,j}$. The total labor consists of production labor $L^p_{i,j}$, and labor for entry $L^e_{i,j}$, satisfying $w_i L_{i,j} = (1-\zeta)\sum_n x_{ni,j}$, and $L_{i,j} = L^p_{i,j} + L^e_{i,j}$.

Free entry conditions can then be expressed as:

$$(1/\sigma-\zeta)\sum_n x_{ni,j} = w_{i,j} F_{i,j} M_{i,j}$$

$$(\Rightarrow)\frac{1/\sigma-\zeta}{1-\zeta} w_{i,j} L_{i,j} = w_{i,j} F_{i,j} M_{i,j}$$

$$(\Rightarrow)\frac{1}{1+\theta} L_{i,j} = F_{i,j} M_{i,j} = L^e_{i,j} \tag{A.32}$$

$$(\Rightarrow)\frac{\theta}{1+\theta} L_{i,j} = L^p_{i,j} \tag{A.33}$$

(A.32) and (A.33) imply that both production and entry labor maintain constant shares of total labor.

Plugging expressions for $\varphi^*_{ni,j}$ (A.31) and $M_{i,j}$ (A.32) into the expression $P_{ni,j}$ (A.30) yields

$$P^{1-\sigma}_{ni,j} = \mu^{-\theta}_{n,j} b^{\theta}_{i,j} F^{-1}_{i,j} L_{i,j} \left(w_{i,j}(1+\tau_{ni,j}) d_{ni,j}\right)^{-\theta} P^{1-\sigma+\theta}_{n,j}$$

where

$$\mu_{n,j} \equiv \frac{\sigma}{\sigma-1}\left[\frac{\theta}{\theta-\sigma+1}\cdot\frac{1}{1+\theta}\right]^{-\frac{1}{\theta}} \left(\frac{\sigma w_{n,j} f_{n,j}}{\beta_j x_n}\right)^{\frac{1}{\sigma-1}-\frac{1}{\theta}}$$

Combining $P_{n,j}^{1-\sigma} = \sum_i P_{ni,j}^{1-\sigma}$ with the result above for $P_{ni,j}$ yields price index

$$
\begin{aligned}
P_{n,j} &= \mu_{n,j} \left[\sum_l b_{l,j}^{\theta} F_{l,j}^{-1} L_{l,j} \left(w_{l,j}(1+\tau_{nl,j}) d_{nl,j} \right)^{-\theta} \right]^{-\frac{1}{\theta}} \\
&= \mu_{n,j} \left[\frac{\theta}{1+\theta} \right]^{\frac{1}{\theta}} \left[\sum_l b_{l,j}^{\theta} F_{l,j}^{-1} L_{l,j}^{p} \left(w_{l,j}(1+\tau_{nl,j}) d_{nl,j} \right)^{-\theta} \right]^{-\frac{1}{\theta}}
\end{aligned}
$$

The expression for trade shares can then be derived

$$
\begin{aligned}
\pi_{ni,j} = \left(\frac{P_{ni,j}}{P_{n,j}} \right)^{1-\sigma} &= \frac{b_{i,j}^{\theta} F_{i,j}^{-1} L_{i,j} \left(w_{i,j}(1+\tau_{ni,j}) d_{ni,j} \right)^{-\theta}}{\sum_l b_{l,j}^{\theta} F_{l,j}^{-1} L_{l,j} \left(w_{l,j}(1+\tau_{nl,j}) d_{nl,j} \right)^{-\theta}} \\
&= \frac{b_{i,j}^{\theta} F_{i,j}^{-1} L_{i,j}^{p} \left(w_{i,j}(1+\tau_{ni,j}) d_{ni,j} \right)^{-\theta}}{\sum_l b_{l,j}^{\theta} F_{l,j}^{-1} L_{l,j}^{p} \left(w_{l,j}(1+\tau_{nl,j}) d_{nl,j} \right)^{-\theta}}
\end{aligned}
$$

The expressions for trade shares and sector price indexes in this Melitz-Pareto model collapse to those in BJL by setting $\nu_{nj} = b_{n,j}^{\theta} F_{n,j}^{-1}$.

J Demand side assumptions

J.1 Optimal import tariffs under CES demand across sectors

Home government chooses $\{\tau_j^d, \tau_j^x, \tau_j^m, x_1, L_{1j}, L_{2j}, w_{1j}, w_{2j}\}$ to solve the following problem:

$$
\max \frac{x_1}{P_1},
$$

subject to world market equilibrium characterized by the following constraints:

$$
\begin{aligned}
w_{1j} L_{1j} - \beta_j^{\sigma} \frac{1}{1+\tau_j^x} \pi_{21,j} p_{2j}^{1-\sigma} P_2^{\sigma-1} x_2 &= \beta_j^{\sigma} \frac{1}{1+\tau_j^d} \pi_{11,j} p_{1j}^{1-\sigma} P_1^{\sigma-1} x_1, \quad (\gamma_{1j}, \quad J) \\
w_{2j} L_{2j} - \beta_j^{\sigma} \pi_{22,j} p_{2j}^{1-\sigma} P_2^{\sigma-1} x_2 &= \beta_j^{\sigma} \frac{1}{1+\tau_j^m} \pi_{12,j} p_{1j}^{1-\sigma} P_1^{\sigma-1} x_1, \quad (\gamma_{2j}, \quad J)
\end{aligned}
$$

and the labor market clearing conditions $\Omega(\{w_{nj}, L_{nj}\}$ determined by the labor market specification, where x_2 and trade shares satisfy equations (2) - (6). Prices are given by

$$
\begin{aligned}
p_{1j} &= \left[T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)d_{12})^{-\theta}\right]^{-\frac{1}{\theta}}, \\
p_{2j} &= \left[T_{1j}(w_{1j}(1+\tau_j^x)d_{21})^{-\theta} + T_{2j}w_{2j}^{-\theta}\right]^{-\frac{1}{\theta}}, \\
\left[\sum_{j=1}^{J} p_{1j}^{1-\sigma}\beta_j^\sigma\right]^{1/(1-\sigma)} &= P_1 = 1, \quad (\gamma_{p1}) \\
\left[\sum_{j=1}^{J} p_{2j}^{1-\sigma}\beta_j^\sigma\right]^{1/(1-\sigma)} &= P_2, \quad (\gamma_{p2})
\end{aligned}
$$

We define $E_{11j} = w_{1j}L_{1j} - \beta_j^\sigma \frac{1}{1+\tau_j^x}\pi_{21,j}p_{2j}^{1-\sigma}P_2^{\sigma-1}x_2$ as the supply of Home on its domestic market, and $E_{12j} = w_{2j}L_{2j} - \beta_j^\sigma\pi_{22,j}p_{2j}^{1-\sigma}P_2^{\sigma-1}x_2$ as the export supply of Foreign on Home's market. We use $C_{nj} = \beta_j^\sigma p_{nj}^{-\sigma}P_n^{\sigma-1}x_n$ to denote the consumption of sector j in country n.

FOC over P_1

$$\gamma_{p1} = \sigma x_1$$

FOC over P_2

$$\gamma_{p2} = (1-\sigma)\frac{x_2}{P_2} * cons_2$$

where $cons_2 = \sum_{s=1}^{J}\left[\gamma_{1s}\beta_s^\sigma\frac{1}{1+\tau_s^x}\pi_{21,s}p_{2s}^{1-\sigma}P_2^{\sigma-1} + \gamma_{2s}\beta_s^\sigma\pi_{22,s}p_{2s}^{1-\sigma}P_2^{\sigma-1}\right]$.

FOC over τ_j^d

$$
\begin{aligned}
&-\gamma_{p1}\beta_j^\sigma p_{1j}^{1-\sigma}\frac{\pi_{11,j}}{1+\tau_j^d} - \gamma_{1j}\frac{1}{(1+\tau_j^d)^2}\beta_j^\sigma p_{1j}^{1-\sigma}\pi_{11,j}x_1 + \gamma_{1j}\frac{1}{1+\tau_j^d}\beta_j^\sigma p_{1j}^{1-\sigma}\frac{\partial\pi_{11,j}}{\partial\tau_j^d}x_1 + \gamma_{2j}\beta_j^\sigma p_{1j}^{1-\sigma}\frac{1}{1+\tau_j^m}\frac{\partial\pi_{12,j}}{\partial\tau_j^d}x_1 \\
&+\gamma_{1j}(1-\sigma)\frac{1}{1+\tau_j^d}\beta_j^\sigma p_{1j}^{-\sigma}\pi_{11,j}\frac{\partial p_{1j}}{\partial\tau_j^d}x_1 + \gamma_{2j}(1-\sigma)\beta_j^\sigma p_{1j}^{-\sigma}\frac{1}{1+\tau_j^m}\pi_{12,j}\frac{\partial p_{1j}}{\partial\tau_j^d}x_1 = 0
\end{aligned}
$$

FOC over τ_j^m

$$
\begin{aligned}
&-\gamma_{p1}\beta_j^\sigma p_{1j}^{1-\sigma}\frac{\pi_{12,j}}{1+\tau_j^m} + \gamma_{1j}\beta_j^\sigma p_{1j}^{-\sigma}\frac{1}{1+\tau_j^d}\frac{\partial\pi_{11,j}}{\partial\tau_j^m}x_1 - \gamma_{2j}\beta_j^\sigma p_{1j}^{-\sigma}\frac{\pi_{12,j}}{(1+\tau_j^m)^2}x_1 + \gamma_{2j}\beta_j^\sigma p_{1j}^{-\sigma}\frac{1}{1+\tau_j^m}\frac{\partial\pi_{12,j}}{\partial\tau_j^m}x_1 \\
&+\gamma_{1j}(1-\sigma)\frac{1}{1+\tau_j^d}\beta_j^\sigma p_{1j}^{-\sigma}\pi_{11,j}\frac{\partial p_{1j}}{\partial\tau_j^m}x_1 + \gamma_{2j}(1-\sigma)\beta_j^\sigma p_{1j}^{-\sigma}\frac{1}{1+\tau_j^m}\pi_{12,j}\frac{\partial p_{1j}}{\partial\tau_j^m}x_1 = 0
\end{aligned}
$$

after plugging in derivatives and simplifying, the FOCs over τ_j^d and τ_j^m become

$$1+\tau_j^d = -\gamma_{1j}, \qquad 1+\tau_j^m = -\gamma_{2j}$$

FOC over τ_j^x

$$-\gamma_{p2}\beta_j^\sigma p_{2j}^{-\sigma}P_2^\sigma\frac{\partial p_{2j}}{\partial \tau_j^x} - \gamma_{1j}\beta_j^\sigma p_{2j}^{1-\sigma}P_2^{\sigma-1}\frac{1}{(1+\tau_j^x)^2}\pi_{21,j}x_2 + \gamma_{1j}\beta_j^\sigma p_{2j}^{1-\sigma}P_2^{\sigma-1}\frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial \tau_j^x}x_2 + \gamma_{2j}\beta_j^\sigma p_{2j}^{1-\sigma}P_2^{\sigma-1}\frac{\partial \pi_{22,j}}{\partial \tau_j^x}x_2$$
$$+\gamma_{1j}\beta_j^\sigma p_{2j}^{-\sigma}P_2^{\sigma-1}\frac{1}{1+\tau_j^x}\pi_{21,j}\frac{\partial p_{2j}}{\partial \tau_j^x}x_2 + \gamma_{2j}\beta_j^\sigma p_{2j}^{1-\sigma}P_2^{\sigma-1}\pi_{22,j}\frac{\partial p_{2j}}{\partial \tau_j^x}x_2 = 0$$

after plugging in derivatives and simplifying, FOC over τ_j^x becomes

$$1+\tau_j^x = \frac{\gamma_{1j}(\sigma+(\theta+1-\sigma)\pi_{22j})}{\gamma_{2j}(\theta+1-\sigma)\pi_{22j}+(\sigma-1)*cons_2}$$

FOC over w_{1j}

$$-\gamma_{p1}\beta_j^\sigma p_{1j}^{1-\sigma}\frac{\pi_{11,j}}{w_{1j}} - \gamma_{p2}\beta_j^\sigma p_{2j}^{1-\sigma}P_2^\sigma\frac{\pi_{21,j}}{w_{1j}} + \gamma_{1j}\beta_j^\sigma p_{1j}^{1-\sigma}\frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{1j}}x_1 - \sum_{k=1}^J \gamma_{1k}\frac{\partial E_{11,k}}{\partial w_{1j}} - \sum_{k=1}^J \gamma_{2k}\frac{\partial E_{12,k}}{\partial w_{1j}}$$
$$+\gamma_{2j}\beta_j^\sigma p_{1j}^{1-\sigma}\frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{1j}}x_1 + (1-\sigma)(\gamma_{1j}\beta_j^\sigma\frac{1}{1+\tau_j^d}\pi_{11,j}p_{1j}^{-\sigma}x_1 + \gamma_{2j}\beta_j^\sigma\frac{1}{1+\tau_j^m}\pi_{12,j}p_{1j}^{-\sigma}x_1)\frac{\partial p_{1j}}{\partial w_{1j}} = 0$$

Further

$$(\sigma-1)\beta_j^\sigma p_{1j}^{1-\sigma}\pi_{11,j}x_1 - \gamma_{p1}\beta_j^\sigma p_{1j}^{1-\sigma}\pi_{11,j} - \gamma_{p2}\beta_j^\sigma p_{2j}^{1-\sigma}P_2^\sigma\pi_{21,j} - \sum_{k=1}^J \gamma_{1k}\frac{\partial E_{11,k}}{\partial w_{1j}}w_{1j} - \sum_{k=1}^J \gamma_{2k}\frac{\partial E_{12,k}}{\partial w_{1j}}w_{1j} = 0$$
$$\Rightarrow \sum_{k=1}^J(1+\tau_k^m)\frac{\partial E_{12,k}}{\partial w_{1j}}w_{1j} = \beta_j^\sigma p_{1j}^{1-\sigma}\pi_{11,j}x_1 + \gamma_{p2}\beta_j^\sigma p_{2j}^{1-\sigma}P_2^\sigma\pi_{21,j} - \sum_{k=1}^J(1+\tau_k^d)\frac{\partial E_{11,k}}{\partial w_{1j}}w_{1j} \tag{A.34}$$

Similar to the proof in Appendix B, combining (A.34) with FOCs over τ_j^d and τ_j^x, we derive

$$(\Lambda_1 - I)(1+\tau^d) = 0$$
$$\Rightarrow \Lambda(1+\tau^d) = 1+\tau^d$$
$$\Rightarrow \sum_{k=1}^J(1+\tau_k^d)\frac{\partial \ln Y_{1k}}{\partial \ln w_{1j}}\frac{Y_{1k}}{Y_{1j}} = 1+\tau_j^d$$

FOC over w_{2j}

$$-\gamma_{p1}\beta_j^\sigma p_{1j}^{1-\sigma}\frac{\pi_{12,j}}{w_{2j}}-\gamma_{p2}\beta_j^\sigma p_{2j}^{1-\sigma}P_2^\sigma\frac{\pi_{22,j}}{w_{2j}}+\gamma_{1j}\beta_j^\sigma p_{1j}^{1-\sigma}\frac{1}{1+\tau_j^d}\frac{\partial\pi_{11,j}}{\partial w_{2j}}x_1-\sum_{k=1}^{J}\gamma_{1k}\frac{\partial E_{11,k}}{\partial w_{2j}}-\sum_{k=1}^{J}\gamma_{2k}\frac{\partial E_{12,k}}{\partial w_{2j}}$$
$$+\gamma_{2j}\beta_j^\sigma p_{1j}^{1-\sigma}\frac{1}{1+\tau_j^m}\frac{\partial\pi_{12,j}}{\partial w_{2j}}x_1+(1-\sigma)(\gamma_{1j}\beta_j^\sigma\frac{1}{1+\tau_j^d}\pi_{11,j}p_{1j}^{-\sigma}x_1+\gamma_{2j}\beta_j^\sigma\frac{1}{1+\tau_j^m}\pi_{12,j}p_{1j}^{-\sigma}x_1)\frac{\partial p_{1j}}{\partial w_{2j}}=0$$

Further

$$(\sigma-1)\beta_j^\sigma p_{1j}^{1-\sigma}\pi_{12,j}x_1-\gamma_{p1}\beta_j^\sigma p_{1j}^{1-\sigma}\pi_{12,j}-\gamma_{p2}\beta_j^\sigma p_{2j}^{1-\sigma}P_2^\sigma\pi_{22,j}-\sum_{k=1}^{J}\gamma_{1k}\frac{\partial E_{11,k}}{\partial w_{2j}}w_{2j}-\sum_{k=1}^{J}\gamma_{2k}\frac{\partial E_{12,k}}{\partial w_{2j}}w_{2j}=0$$
$$\Rightarrow\sum_{k=1}^{J}(1+\tau_k^m)\frac{\partial E_{12,k}}{\partial w_{2j}}w_{2j}=\beta_j^\sigma p_{1j}^{1-\sigma}\pi_{12,j}x_1+\gamma_{p2}\beta_j^\sigma p_{2j}^{1-\sigma}P_2^\sigma\pi_{22,j}-\sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial E_{11,k}}{\partial w_{2j}}w_{2j} \tag{A.35}$$

$$\begin{pmatrix}\frac{\partial E_{12,1}}{\partial\ln w_{21}} & ... & \frac{\partial E_{12,J}}{\partial\ln w_{21}}\\ ... & & ...\\ \frac{\partial E_{12,1}}{\partial\ln w_{2J}} & ... & \frac{\partial E_{12,J}}{\partial\ln w_{2J}}\end{pmatrix}\begin{pmatrix}1+\tau_1^m\\ ...\\ 1+\tau_J^m\end{pmatrix}=\begin{pmatrix}\beta_1^\sigma p_{11}^{1-\sigma}\pi_{12,1}x_1+\gamma_{p2}\beta_1^\sigma p_{21}^{1-\sigma}P_2^\sigma\pi_{22,1}\\ ...\\ \beta_J^\sigma p_{1J}^{1-\sigma}\pi_{12,J}x_1+\gamma_{p2}\beta_J^\sigma p_{2J}^{1-\sigma}P_2^\sigma\pi_{22,J}\end{pmatrix}-\begin{pmatrix}\frac{\partial E_{11,1}}{\partial\ln w_{21}} & ... & \frac{\partial E_{11,J}}{\partial\ln w_{21}}\\ ... & & ...\\ \frac{\partial E_{11,1}}{\partial\ln w_{2J}} & ... & \frac{\partial E_{11,J}}{\partial\ln w_{2J}}\end{pmatrix}\begin{pmatrix}1+\tau_1^d\\ ...\\ 1+\tau_J^d\end{pmatrix}$$

Further

$$\sum_{k=1}^{J}(1+\tau_k^m)\frac{\partial Y_{2k}}{\partial w_{2j}}w_{2j}-\sum_{s=1}^{J}(1+\tau_s^m)\frac{p_{2s}C_{2s}}{x_2}\pi_{22,s}\frac{\partial x_2}{\partial w_{2j}}w_{2j}+(1+\tau_j^m)p_{2j}C_{2j}\theta\pi_{21,j}\pi_{22,j}$$

$$-(1+\tau_j^m)(1-\sigma)p_{2j}C_{2j}\pi_{22,j}\pi_{22,j}=p_{1j}C_{1j}\pi_{12,j}+\frac{\gamma_{p2}}{x_2P_2}p_{2j}C_{2j}\pi_{22,j}+\sum_{s=1}^{J}(1+\tau_s^d)\frac{p_{2s}C_{2s}}{x_2}\frac{\pi_{21,s}}{1+\tau_s^x}\frac{\partial x_2}{\partial w_{2j}}w_{2j}$$

$$+(1+\tau_j^d)(1-\sigma)p_{2j}C_{2j}\frac{\pi_{21,j}\pi_{22,j}}{1+\tau_j^x}+\frac{1+\tau_j^d}{1+\tau_j^x}p_{2j}C_{2j}\theta\pi_{21,j}\pi_{22,j}$$

$$\Rightarrow\sum_{k=1}^{J}(cons_2+1+\tau_k^m)\frac{\partial Y_{2k}}{\partial w_{2j}}w_{2j}+(1+\tau_j^m)p_{2j}C_{2j}(\theta\pi_{21,j}-(1-\sigma)\pi_{22,j})\pi_{22,j}=p_{1j}C_{1j}\pi_{12,j}+\frac{\gamma_{p2}}{x_2P_2}p_{2j}C_{2j}\pi_{22,j}$$

$$+\frac{1+\tau_j^d}{1+\tau_j^x}p_{2j}C_{2j}(\theta+1-\sigma)\pi_{21,j}\pi_{22,j}$$

$$\Rightarrow\sum_{k=1}^{J}(cons_2+1+\tau_k^m)\frac{\partial Y_{2k}}{\partial w_{2j}}w_{2j}+(1+\tau_j^m)p_{2j}C_{2j}\theta\pi_{22,j}-(1+\tau_j^m)p_{2j}C_{2j}(\theta+1-\sigma)\pi_{22,j}\pi_{22,j}=p_{1j}C_{1j}\pi_{12,j}$$

$$+\frac{\gamma_{p2}}{x_2P_2}p_{2j}C_{2j}\pi_{22,j}+\frac{1+\tau_j^d}{1+\tau_j^x}p_{2j}C_{2j}(\theta+1-\sigma)\pi_{21,j}\pi_{22,j}$$

$$\Rightarrow\sum_{k=1}^{J}(cons_2+1+\tau_k^m)\frac{\partial Y_{2k}}{\partial w_{2j}}w_{2j}+(1+\tau_j^m)p_{2j}C_{2j}\theta\pi_{22,j}=p_{1j}C_{1j}\pi_{12,j}+\frac{1+\tau_j^d}{1+\tau_j^x}p_{2j}C_{2j}(\theta+1)\pi_{22,j}$$

$$\Rightarrow\sum_{k=1}^{J}(cons_2+1+\tau_k^m)\frac{\partial Y_{2k}}{\partial w_{2j}}w_{2j}+(1+\tau_j^m)p_{2j}C_{2j}(\theta+1-\sigma)\pi_{22,j}-(1+\tau_j^m)p_{2j}C_{2j}(1-\sigma)\pi_{22,j}=p_{1j}C_{1j}\pi_{12,j}$$

$$+\frac{1+\tau_j^d}{1+\tau_j^x}p_{2j}C_{2j}(\theta+1)\pi_{22,j}$$

$$\Rightarrow\sum_{k=1}^{J}(cons_2+1+\tau_k^m)\frac{\partial Y_{2k}}{\partial w_{2j}}w_{2j}+\frac{1+\tau_j^d}{1+\tau_j^x}p_{2j}C_{2j}(\sigma+(\theta+1-\sigma)\pi_{22,j})-\frac{\gamma_{p2}}{x_2P_2}p_{2j}C_{2j}-(1+\tau_j^m)p_{2j}C_{2j}(1-\sigma)\pi_{22,j}$$

$$=p_{1j}C_{1j}\pi_{12,j}+\frac{1+\tau_j^d}{1+\tau_j^x}p_{2j}C_{2j}(\theta+1)\pi_{22,j}$$

$$\Rightarrow\sum_{k=1}^{J}(cons_2+1+\tau_k^m)\frac{\partial Y_{2k}}{\partial w_{2j}}w_{2j}-(1+\tau_j^m)p_{2j}C_{2j}(1-\sigma)\pi_{22,j}=p_{1j}C_{1j}\pi_{12,j}+\frac{\gamma_{p2}}{x_2P_2}p_{2j}C_{2j}-\frac{1+\tau_j^d}{1+\tau_j^x}p_{2j}C_{2j}\sigma\pi_{21,j}$$

$$\Rightarrow\sum_{k=1}^{J}(cons_2+1+\tau_k^m)\frac{\partial Y_{2k}}{\partial w_{2j}}w_{2j}-(1+\tau_j^m)(1-\sigma)Y_{2j}+(1-\sigma)\pi_{12,j}p_{1j}C_{1j}=p_{1j}C_{1j}\pi_{12,j}+\frac{\gamma_{p2}}{x_2P_2}p_{2j}C_{2j}$$

$$-\sigma(1+\tau_j^d)Y_{1j}+\sigma p_{1j}C_{1j}\pi_{11,j}$$

$$\Rightarrow\sum_{k=1}^{J}(cons_2+1+\tau_k^m)\frac{\partial\ln Y_{2k}}{\partial\ln w_{2j}}\frac{Y_{2k}}{Y_{2j}}-(1+\tau_j^m)(1-\sigma)=\frac{\sigma p_{1j}C_{1j}+\frac{\gamma_{p2}P_2}{x_2}p_{2j}C_{2j}-(1+\tau_j^d)\sigma Y_{1j}}{Y_{2j}}\tag{A.36}$$

$$\Rightarrow\sum_{k=1}^{J}(cons_2+1+\tau_k^m)\frac{\partial\ln Y_{2k}}{\partial\ln w_{2j}}\frac{Y_{2k}}{Y_{2j}}-(1+\tau_j^m)(1-\sigma)=\frac{\sigma p_{1j}C_{1j}+(1-\sigma)cons_2\cdot p_{2j}C_{2j}-(1+\tau_j^d)\sigma Y_{1j}}{Y_{2j}}\tag{A.37}$$

where

$$
\begin{aligned}
cons_2 &= -\sum_{s=1}^{J}\left[(1+\tau_s^d)\beta_s^\sigma \frac{1}{1+\tau_s^x}\pi_{21,s}p_{2s}^{1-\sigma}P_2^{\sigma-1} + (1+\tau_s^m)\beta_s^\sigma \pi_{22,s}p_{2s}^{1-\sigma}P_2^{\sigma-1}\right] \\
&= \frac{1}{x_2}\sum_{s=1}^{J}\left[\beta_s^\sigma p_{1s}^{1-\sigma}x_1 - (1+\tau_s^d)Y_{1s} - (1+\tau_s^m)Y_{2s}\right] \\
&= \frac{1}{x_2}\sum_{s=1}^{J}\left[p_{1s}C_{1s} - (1+\tau_s^d)Y_{1s} - (1+\tau_s^m)Y_{2s}\right]
\end{aligned}
$$

Under CES supply system, the optimal domestic tax is uniform across sectors and can be normalized to zero. The optimal import tariffs for sectors j and k satisfy

$$
\tau_j^m - \tau_k^m = \frac{1}{\kappa-1+\sigma}\left[\frac{\sigma p_{1j}C_{1j} + \frac{\gamma_{p2}P_2}{x_2}p_{2j}C_{2j} - \sigma Y_{1j}}{Y_{2j}} - \frac{\sigma p_{1k}C_{1k} + \frac{\gamma_{p2}P_2}{x_2}p_{2k}C_{2k} - \sigma Y_{1k}}{Y_{2k}}\right]
$$

J.2 Optimal import tariffs under CD demand across sectors

Home government chooses $\{\tau_j^d, \tau_j^x, \tau_j^m, x_1, L_{1j}, L_{2j}, w_{1j}, w_{2j}\}$ to solve the following problem:

$$
\max \frac{x_1}{P_1},
$$

subject to world market equilibrium characterized by the following constraints:

$$
\begin{aligned}
w_{1j}L_{1j} - \beta_j\frac{1}{1+\tau_j^x}\pi_{21,j}x_2 &= \beta_j\frac{1}{1+\tau_j^d}\pi_{11,j}x_1, \quad (\gamma_{1j}, \quad J) \\
w_{2j}L_{2j} - \beta_j\pi_{22,j}x_2 &= \beta_j\frac{1}{1+\tau_j^m}\pi_{12,j}x_1, \quad (\gamma_{2j}, \quad J)
\end{aligned}
$$

and the labor market clearing conditions $\Omega(\{w_{nj}, L_{nj}\}$ determined by the labor market specification, where $\{x_2, P_1, P_2\}$ and trade shares satisfy equations (2) - (8).

We define $E_{11j} = w_{1j}L_{1j} - \beta_j\frac{1}{1+\tau_j^x}\pi_{21,j}x_2$ as the supply of Home on its domestic market, and $E_{12j} = w_{2j}L_{2j} - \beta_j\pi_{22,j}x_2$ as the export supply of Foreign on Home's market.

FOC over w_{1j}

$$
-\beta_j x_1\frac{\pi_{11,j}}{w_{1j}} + \gamma_{1j}\beta_j\frac{1}{1+\tau_j^d}\frac{\partial\pi_{11,j}}{\partial w_{1j}}x_1 - \sum_{k=1}^{J}\gamma_{1k}\frac{\partial E_{11,k}}{\partial w_{1j}} + \gamma_{2j}\beta_j\frac{1}{1+\tau_j^m}\frac{\partial\pi_{12,j}}{\partial w_{1j}}x_1 - \sum_{k=1}^{J}\gamma_{2k}\frac{\partial E_{12,k}}{\partial w_{1j}} = 0
$$

Further

$$
\begin{aligned}
& -\beta_j x_1 \pi_{11,j} - \sum_{k=1}^{J} \gamma_{1k} \frac{\partial E_{11,k}}{\partial w_{1j}} w_{1j} - \sum_{k=1}^{J} \gamma_{2k} \frac{\partial E_{12,k}}{\partial w_{1j}} w_{1j} = 0 \\
& \Rightarrow \sum_{k=1}^{J} (1+\tau_k^m) \frac{\partial E_{12,k}}{\partial w_{1j}} w_{1j} = \beta_j x_1 \pi_{11,j} - \sum_{k=1}^{J} (1+\tau_k^d) \frac{\partial E_{11,k}}{\partial w_{1j}} w_{1j}
\end{aligned}
\tag{A.38}
$$

Similar to the proof in Appendix B, combining (A.38) with FOCs over τ_j^d and τ_j^x, we derive

$$
\begin{aligned}
& (\Lambda_1 - I)(1+\tau^d) = 0 \\
& \Rightarrow \Lambda(1+\tau^d) = 1+\tau^d \\
& \Rightarrow \sum_{k=1}^{J} (1+\tau_k^d) \frac{\partial \ln Y_{1k}}{\partial \ln w_{1j}} \frac{Y_{1k}}{Y_{1j}} = 1+\tau_j^d
\end{aligned}
$$

FOC over w_{2j}

$$
-\beta_j x_1 \frac{\pi_{12,j}}{w_{2j}} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d} \frac{\partial \pi_{11,j}}{\partial w_{2j}} x_1 - \sum_{k=1}^{J} \gamma_{1k} \frac{\partial E_{11,k}}{\partial w_{2j}} + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m} \frac{\partial \pi_{12,j}}{\partial w_{2j}} x_1 - \sum_{k=1}^{J} \gamma_{2k} \frac{\partial E_{12,k}}{\partial w_{2j}} = 0
$$

Further

$$
\begin{aligned}
& -\beta_j x_1 \pi_{12,j} - \sum_{k=1}^{J} \gamma_{1k} \frac{\partial E_{11,k}}{\partial w_{2j}} w_{2j} - \sum_{k=1}^{J} \gamma_{2k} \frac{\partial E_{12,k}}{\partial w_{2j}} w_{2j} = 0 \\
& \Rightarrow \sum_{k=1}^{J} (1+\tau_k^m) \frac{\partial E_{12,k}}{\partial w_{2j}} w_{2j} = \beta_j x_1 \pi_{12,j} - \sum_{k=1}^{J} (1+\tau_k^d) \frac{\partial E_{11,k}}{\partial w_{2j}} w_{2j}
\end{aligned}
\tag{A.39}
$$

$$
\begin{pmatrix}
\frac{\partial E_{12,1}}{\partial \ln w_{21}} & \dots & \frac{\partial E_{12,J}}{\partial \ln w_{21}} \\
\dots & & \dots \\
\frac{\partial E_{12,1}}{\partial \ln w_{2J}} & \dots & \frac{\partial E_{12,J}}{\partial \ln w_{2J}}
\end{pmatrix}
\begin{pmatrix}
1+\tau_1^m \\ \dots \\ 1+\tau_J^m
\end{pmatrix}
=
\begin{pmatrix}
\beta_1 \pi_{12,1} x_1 \\ \dots \\ \beta_J \pi_{12,J} x_1
\end{pmatrix}
-
\begin{pmatrix}
\frac{\partial E_{11,1}}{\partial \ln w_{21}} & \dots & \frac{\partial E_{11,J}}{\partial \ln w_{21}} \\
\dots & & \dots \\
\frac{\partial E_{11,1}}{\partial \ln w_{2J}} & \dots & \frac{\partial E_{11,J}}{\partial \ln w_{2J}}
\end{pmatrix}
\begin{pmatrix}
1+\tau_1^d \\ \dots \\ 1+\tau_J^d
\end{pmatrix}
$$

Plug in derivatives,

$$
\begin{aligned}
&\sum_{k=1}^{J}(1+\tau_k^m)\left[\frac{\partial w_{2k}L_{2k}}{\partial w_{2j}}w_{2j}-\beta_k\pi_{22,k}\frac{\partial x_2}{\partial w_{2j}}w_{2j}\right]+(1+\tau_j^m)\beta_j\theta\pi_{21,j}\pi_{22,j}x_2=\beta_j\pi_{12,j}x_1+\sum_{k=1}^{J}(1+\tau_k^d)\beta_k\frac{\pi_{21,k}}{1+\tau_k^x}\frac{\partial x_2}{\partial w_{2j}}w_{2j}\\
&+\frac{1+\tau_j^d}{1+\tau_j^x}\beta_j\theta\pi_{21,j}\pi_{22,j}x_2\\
&\Rightarrow\sum_{k=1}^{J}(1+\tau_k^m)\frac{\partial Y_{2k}}{\partial w_{2j}}w_{2j}-\sum_{s=1}^{J}(1+\tau_s^m)\beta_s\pi_{22,s}\frac{\partial\sum_{k=1}^{J}Y_{2k}}{\partial w_{2j}}w_{2j}=\beta_j\pi_{12,j}x_1+\sum_{s=1}^{J}(1+\tau_s^d)\beta_s\frac{\pi_{21,s}}{1+\tau_s^x}\frac{\partial\sum_{k=1}^{J}Y_{2k}}{\partial w_{2j}}w_{2j}\\
&+\left[\frac{1+\tau_j^d}{1+\tau_j^x}-(1+\tau_j^m)\right]\beta_j\theta\pi_{21,j}\pi_{22,j}x_2\\
&\Rightarrow\sum_{k=1}^{J}(cons_1+1+\tau_k^m)\left[\frac{\partial\ln Y_{2k}}{\partial\ln w_{2j}}Y_{2k}\right]=\beta_j\pi_{12,j}x_1+\left[\frac{1+\tau_j^d}{1+\tau_j^x}-(1+\tau_j^m)\right]\beta_j\theta\pi_{21,j}\pi_{22,j}x_2\\
&\Rightarrow\sum_{k=1}^{J}(cons_1+1+\tau_k^m)\left[\frac{\partial\ln Y_{2k}}{\partial\ln w_{2j}}\frac{Y_{2k}}{Y_{2j}}\right]=\frac{\beta_j\pi_{12,j}x_1}{Y_{2j}}-\frac{1+\tau_j^d}{1+\tau_j^x}\beta_j\pi_{21,j}\frac{x_2}{Y_{2j}}\\
&\Rightarrow\sum_{k=1}^{J}(cons_1+1+\tau_k^m)\left[\frac{\partial\ln Y_{2k}}{\partial\ln w_{2j}}\frac{Y_{2k}}{Y_{2j}}\right]=\frac{\beta_j\pi_{12,j}x_1}{Y_{2j}}-(1+\tau_j^d)\frac{Y_{1j}}{Y_{2j}}+\frac{\beta_j\pi_{11,j}x_1}{Y_{2j}}\\
&\Rightarrow\sum_{k=1}^{J}(cons_1+1+\tau_k^m)\frac{\partial\ln Y_{2k}}{\partial\ln w_{2j}}\frac{Y_{2k}}{Y_{2j}}=\frac{\beta_jx_1-(1+\tau_j^d)Y_{1j}}{Y_{2j}}
\end{aligned}
\tag{A.40}
$$

where the transition from the fourth-to-last line to the third-to-last line utilizes the FOC over τ_j^x (18).

$$
\begin{aligned}
cons_1&=-\sum_{s=1}^{J}\left[(1+\tau_s^d)\beta_s\frac{1}{1+\tau_s^x}\pi_{21,s}+(1+\tau_s^m)\beta_s\pi_{22,s}\right]\\
&=\frac{1}{x_2}\sum_{s=1}^{J}\left[\beta_sx_1-(1+\tau_s^d)Y_{1s}-(1+\tau_s^m)Y_{2s}\right]
\end{aligned}
$$

Under CES supply system, the optimal domestic tax is uniform across sectors and can be normalized to zero. The optimal import tariffs for sectors j and k satisfy

$$
\tau_j^m-\tau_k^m=\frac{1}{\kappa}\left[\frac{\beta_jx_1-Y_{1j}}{Y_{2j}}-\frac{\beta_kx_1-Y_{1k}}{Y_{2k}}\right]
$$

K Nash optimal policies

Let τ_j^x and τ_j^{x*} denote the export taxes imposed by the Home and Foreign countries in sector j, respectively, in sector j. Similarly, τ_j^m and τ_j^{m*} represent the import tariffs imposed by the Home and Foreign countries in sector j, while τ_j^d and τ_j^{d*} denote the domestic taxes in sector j imposed by the Home and Foreign countries, respectively.

Country 1 (Home)'s government chooses $\{\tau_j^x, \tau_j^m, \tau_j^d\}$ to maximize domestic consumers' consumption, $\max u_1(x_1/P_1)$, while country 2 (Foreign)'s government chooses $\{\tau_j^{x*}, \tau_j^{m*}, \tau_j^{d*}\}$ to maximize domestic consumers' consumption, $\max u_2(x_2/P_2)$, subject to the world market equilibrium and the other country's policies:

1. Expenditures are given by

$$x_1 = \sum_{j=1}^{J} w_{1j}L_{1j} + \sum_{j=1}^{J} \beta_j \left[\frac{\tau_j^x}{1+\tau_j^x} \frac{1}{1+\tau_j^{m*}} \pi_{21,j} x_2 + \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1 \right], (\gamma_{x1}, \gamma_{x1}^*) \tag{A.41}$$

$$x_2 = \sum_{j=1}^{J} w_{2j}L_{2j} + \sum_{j=1}^{J} \beta_j \left[\frac{\tau_j^{x*}}{1+\tau_j^{x*}} \frac{1}{1+\tau_j^{m}} \pi_{12,j} x_1 + \frac{\tau_j^{m*}}{1+\tau_j^{m*}} \pi_{21,j} x_2 + \frac{\tau_j^{d*}}{1+\tau_j^{d*}} \pi_{22,j} x_2 \right], (\gamma_{x2}, \gamma_{x2}^*) \tag{A.42}$$

2. Trade shares satisfy, for each sector j

$$\pi_{11,j} = \frac{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{x*})d_{12})^{-\theta}}, \tag{A.43}$$

$$\pi_{12,j} = \frac{T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{x*})d_{12})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{x*})d_{12})^{-\theta}}, \tag{A.44}$$

$$\pi_{21,j} = \frac{T_{1j}(w_{1j}(1+\tau_j^x)(1+\tau_j^{m*})d_{21})^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)(1+\tau_j^{m*})d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta}}, \tag{A.45}$$

$$\pi_{22,j} = \frac{T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta}}{T_{1j}(w_{1j}(1+\tau_j^x)(1+\tau_j^{m*})d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta}}. \tag{A.46}$$

3. Consumer prices are given by

$$P_1 = \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^d))^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^m)(1+\tau_j^{x*})d_{12})^{-\theta} \right]^{-\frac{\beta_j}{\theta}}, (\gamma_{p1}, \gamma_{p1}^*) \tag{A.47}$$

$$P_2 = \Pi_j \left[T_{1j}(w_{1j}(1+\tau_j^x)(1+\tau_j^{m*})d_{21})^{-\theta} + T_{2j}(w_{2j}(1+\tau_j^{d*}))^{-\theta} \right]^{-\frac{\beta_j}{\theta}}, (\gamma_{p2}, \gamma_{p2}^*) \tag{A.48}$$

4. Goods market clearing conditions, for each j

$$w_{1j}L_{1j} = \beta_j \left[\frac{1}{1+\tau_j^d} \pi_{11,j} x_1 + \frac{1}{1+\tau_j^x} \frac{1}{1+\tau_j^{m*}} \pi_{21,j} x_2 \right], (\gamma_{1j}, \gamma_{1j}^*) \tag{A.49}$$

$$w_{2j}L_{2j} = \beta_j \left[\frac{1}{1+\tau_j^m} \frac{1}{1+\tau_j^{x*}} \pi_{12,j} x_1 + \frac{1}{1+\tau_j^{d*}} \pi_{22,j} x_2 \right], (\gamma_{2j}, \gamma_{2j}^*) \tag{A.50}$$

5. The labor market clearing conditions $\Omega(\{w_{nj}, L_{nj}\})$ hold.

The multipliers $\{\gamma_{x1}, \gamma_{x2}, \gamma_{p1}, \gamma_{p2}, \gamma_{1j}, \gamma_{2j}\}$ correspond to the Home country's maximization problem, while the multipliers denoted with (*) correspond to the Foreign country's maximization problem.

K.1 Home's optimal conditions

We use first-order conditions (FOCs) to characterize the Home government's problem and link the optimal policies with multipliers on goods market clearing conditions.

FOC over x_1

$$\begin{aligned} & u_{c1} + \sum_{j=1}^{J} \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\pi_{11,j} + \sum_{j=1}^{J} \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{1}{1+\tau_j^{x*}}\pi_{12,j} \\ & - \gamma_{x1}\left(1 - \sum_{j=1}^{J}\beta_j \frac{\tau_j^m}{1+\tau_j^m}\pi_{12,j} - \sum_{j=1}^{J}\beta_j \frac{\tau_j^d}{1+\tau_j^d}\pi_{11,j}\right) + \gamma_{x2}\sum_{j=1}^{J}\beta_j \frac{\tau_j^{x*}}{1+\tau_j^{x*}}\frac{1}{1+\tau_j^m}\pi_{12,j} = 0 \end{aligned}$$

FOC over x_2

$$\begin{aligned} & \sum_{j=1}^{J} \gamma_{2j}\beta_j \frac{1}{1+\tau_j^{d*}}\pi_{22,j} + \sum_{j=1}^{J} \gamma_{1j}\beta_j \frac{1}{1+\tau_j^{m*}}\frac{1}{1+\tau_j^{x}}\pi_{21,j} \\ & - \gamma_{x2}\left(1 - \sum_{j=1}^{J}\beta_j \frac{\tau_j^{m*}}{1+\tau_j^{m*}}\pi_{21,j} - \sum_{j=1}^{J}\beta_j \frac{\tau_j^{d*}}{1+\tau_j^{d*}}\pi_{22,j}\right) + \gamma_{x1}\sum_{j=1}^{J}\beta_j \frac{\tau_j^{x}}{1+\tau_j^{x}}\frac{1}{1+\tau_j^{m*}}\pi_{21,j} = 0 \end{aligned}$$

FOC over P_1

$$\gamma_{p1} = u_{c1}x_1$$

FOC over P_2

$$\gamma_{p2} = 0$$

FOC over export tax τ_j^x

$$\begin{aligned}
& -\gamma_{1j}\beta_j \frac{1}{(1+\tau_j^x)^2}\frac{1}{1+\tau_j^{m*}}\pi_{21,j}x_2 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{1}{1+\tau_j^{m*}}\frac{\partial \pi_{21,j}}{\partial \tau_j^x}x_2 \\
& + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^{d*}}\frac{\partial \pi_{22,j}}{\partial \tau_j^x}x_2 + \gamma_{x1}\beta_j \frac{1}{(1+\tau_j^x)^2}\frac{1}{1+\tau_j^{m*}}\pi_{21,j}x_2 + \gamma_{x1}\beta_j \frac{\tau_j^x}{1+\tau_j^x}\frac{1}{1+\tau_j^{m*}}\frac{\partial \pi_{21,j}}{\partial \tau_j^x}x_2 \\
& + \gamma_{x2}\beta_j \frac{\tau_j^{m*}}{1+\tau_j^{m*}}\frac{\partial \pi_{21,j}}{\partial \tau_j^x}x_2 + \gamma_{x2}\beta_j \frac{\tau_j^{d*}}{1+\tau_j^{d*}}\frac{\partial \pi_{22,j}}{\partial \tau_j^x}x_2 = 0
\end{aligned} \tag{A.51}$$

after plugging in derivatives and simplifying, the FOC over τ_j^x becomes

$$1+\tau_j^x = \frac{\gamma_{x1}-\gamma_{1j}}{\left(\gamma_{x2}-\gamma_{2j}\right)\frac{1+\tau_j^{m*}}{1+\tau_j^{d*}} + \gamma_{x1} - \gamma_{x2}}\frac{\theta\pi_{22,j}+1}{\theta\pi_{22,j}}. \tag{A.52}$$

FOC over import tariff τ_j^m

$$\begin{aligned}
& -u_{c1}\beta_j x_1 \frac{\pi_{12,j}}{1+\tau_j^m} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial \tau_j^m}x_1 - \gamma_{2j}\beta_j \frac{\pi_{12,j}}{(1+\tau_j^m)^2}\frac{1}{1+\tau_j^{x*}}x_1 + \gamma_{2j}\frac{1}{1+\tau_j^m}\frac{1}{1+\tau_j^{x*}}\frac{\partial \pi_{12,j}}{\partial \tau_j^m}x_1 \\
& + \gamma_{x1}\beta_j \frac{1}{(1+\tau_j^m)^2}\pi_{12,j}x_1 + \gamma_{x1}\beta_j \frac{\tau_j^m}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial \tau_j^m}x_1 + \gamma_{x1}\beta_j \frac{\tau_j^d}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial \tau_j^m}x_1 \\
& - \gamma_{x2}\beta_j \frac{\tau_j^{x*}}{1+\tau_j^{x*}}\frac{1}{(1+\tau_j^m)^2}\pi_{12,j}x_1 + \gamma_{x2}\beta_j \frac{\tau_j^{x*}}{1+\tau_j^{x*}}\frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial \tau_j^m}x_1 = 0
\end{aligned} \tag{A.53}$$

FOC over domestic tax τ_j^d

$$\begin{aligned}
& -u_{c1}\beta_j x_1 \frac{\pi_{11,j}}{1+\tau_j^d} - \gamma_{1j}\frac{1}{(1+\tau_j^d)^2}\beta_j\pi_{11,j}x_1 + \gamma_{1j}\frac{1}{1+\tau_j^d}\beta_j\frac{\partial \pi_{11,j}}{\partial \tau_j^d}x_1 + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{1}{1+\tau_j^{x*}}\frac{\partial \pi_{12,j}}{\partial \tau_j^d}x_1 \\
& + \gamma_{x1}\beta_j \frac{\tau_j^m}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial \tau_j^d}x_1 + \gamma_{x1}\beta_j \frac{1}{(1+\tau_j^d)^2}\pi_{11,j}x_1 + \gamma_{x1}\beta_j \frac{\tau_j^d}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial \tau_j^d}x_1 \\
& + \gamma_{x2}\beta_j \frac{\tau_j^{x*}}{1+\tau_j^{x*}}\frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial \tau_j^d}x_1 = 0
\end{aligned}$$

Combining the FOCs over optimal tariff and domestic tax, we can get

$$1+\tau_j^m = \frac{\gamma_{x1}-\gamma_{x2}+(\gamma_{x2}-\gamma_{2j})\frac{1}{1+\tau_j^{x*}}}{u_{c1}}, \qquad 1+\tau_j^d = \frac{\gamma_{x1}-\gamma_{1j}}{u_{c1}}. \tag{A.54}$$

FOC over w_{1j}

$$\begin{aligned}
&-u_{c1}\beta_j x_1 \frac{\pi_{11,j}}{w_{1j}} - \gamma_{1j}L_{1j} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{1j}}x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{1}{1+\tau_j^{m*}}\frac{\partial \pi_{21,j}}{\partial w_{1j}}x_2 \\
&+\gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{1}{1+\tau_j^{x*}}\frac{\partial \pi_{12,j}}{\partial w_{1j}}x_1 + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^{d*}}\frac{\partial \pi_{22,j}}{\partial w_{1j}}x_2 + \gamma_{x1}L_{1j} + \gamma_{x1}\beta_j \frac{\tau_j^x}{1+\tau_j^x}\frac{1}{1+\tau_j^{m*}}\frac{\partial \pi_{21,j}}{\partial w_{1j}}x_2 \\
&+\gamma_{x1}\beta_j \frac{\tau_j^m}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{1j}}x_1 + \gamma_{x1}\beta_j \frac{\tau_j^d}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{1j}}x_1 + \gamma_{x2}\beta_j \frac{\tau_j^{x*}}{1+\tau_j^{x*}}\frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{1j}}x_1 \\
&+\gamma_{x2}\beta_j \frac{\tau_j^{m*}}{1+\tau_j^{m*}}\frac{\partial \pi_{21,j}}{\partial w_{1j}}x_2 + \gamma_{x2}\beta_j \frac{\tau_j^{d*}}{1+\tau_j^{d*}}\frac{\partial \pi_{22,j}}{\partial w_{1j}}x_2 + A_j/w_{1j} = 0
\end{aligned} \tag{A.55}$$

Further

$$\begin{aligned}
&-u_{c1}\beta_j x_1 \pi_{11,j} - \gamma_{1j}w_{1j}L_{1j} - \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\theta\pi_{11,j}\pi_{12,j}x_1 - \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{1}{1+\tau_j^{m*}}\theta\pi_{21,j}\pi_{22,j}x_2 \\
&+\gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{1}{1+\tau_j^{x*}}\theta\pi_{12,j}\pi_{11,j}x_1 + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^{d*}}\theta\pi_{22,j}\pi_{21,j}x_2 + \gamma_{x1}w_{1j}L_{1j} \\
&-\gamma_{x1}\beta_j \frac{\tau_j^x}{1+\tau_j^x}\frac{1}{1+\tau_j^{m*}}\theta\pi_{21,j}\pi_{22,j}x_2 + \gamma_{x1}\beta_j \frac{\tau_j^m}{1+\tau_j^m}\theta\pi_{12,j}\pi_{11,j}x_1 - \gamma_{x1}\beta_j \frac{\tau_j^d}{1+\tau_j^d}\theta\pi_{11,j}\pi_{12,j}x_1 \\
&+\gamma_{x2}\beta_j \frac{\tau_j^{x*}}{1+\tau_j^{x*}}\frac{1}{1+\tau_j^m}\theta\pi_{12,j}\pi_{11,j}x_1 - \gamma_{x2}\beta_j \frac{\tau_j^{m*}}{1+\tau_j^{m*}}\theta\pi_{21,j}\pi_{22,j}x_2 + \gamma_{x2}\beta_j \frac{\tau_j^{d*}}{1+\tau_j^{d*}}\theta\pi_{22,j}\pi_{21,j}x_2 + A_j = 0
\end{aligned}$$

Where $A_j = \sum_{k=1}^J(\gamma_{x1}-\gamma_{1k})w_{1k}\frac{\partial L_{1k}}{\partial w_{1j}}w_{1j} = \sum_{k=1}^J(\gamma_{x1}-\gamma_{1k})\frac{\partial w_{1k}L_{1k}}{\partial w_{1j}}w_{1j} - (\gamma_{x1}-\gamma_{1j})w_{1j}L_{1j}$.

FOC over w_{2j}

$$
\begin{aligned}
&- u_{c1}\beta_j x_1 \frac{\pi_{12,j}}{w_{2j}} - \gamma_{2j} L_{2j} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{2j}} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{1}{1+\tau_j^{m*}}\frac{\partial \pi_{21,j}}{\partial w_{2j}} x_2 \\
&+ \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{1}{1+\tau_j^{x*}}\frac{\partial \pi_{12,j}}{\partial w_{2j}} x_1 + \gamma_{2j}\beta_j \frac{1}{1+\tau_j^{d*}}\frac{\partial \pi_{22,j}}{\partial w_{2j}} x_2 + \gamma_{x2} L_{2j} + \gamma_{x1}\beta_j \frac{\tau_j^x}{1+\tau_j^x}\frac{1}{1+\tau_j^{m*}}\frac{\partial \pi_{21,j}}{\partial w_{2j}} x_2 \\
&+ \gamma_{x1}\beta_j \frac{\tau_j^m}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{2j}} x_1 + \gamma_{x1}\beta_j \frac{\tau_j^d}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial w_{2j}} x_1 + \gamma_{x2}\beta_j \frac{\tau_j^{x*}}{1+\tau_j^{x*}}\frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial w_{2j}} x_1 \\
&+ \gamma_{x2}\beta_j \frac{\tau_j^{m*}}{1+\tau_j^{m*}}\frac{\partial \pi_{21,j}}{\partial w_{2j}} x_2 + \gamma_{x2}\beta_j \frac{\tau_j^{d*}}{1+\tau_j^{d*}}\frac{\partial \pi_{22,j}}{\partial w_{2j}} x_2 + B_j / w_{2j} = 0 \qquad \text{(A.56)}
\end{aligned}
$$

Further

$$
\begin{aligned}
&- u_{c1}\beta_j x_1 \pi_{12,j} - \gamma_{2j} w_{2j} L_{2j} + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^d}\theta \pi_{11,j}\pi_{12,j} x_1 + \gamma_{1j}\beta_j \frac{1}{1+\tau_j^x}\frac{1}{1+\tau_j^{m*}}\theta \pi_{21,j}\pi_{22,j} x_2 \\
&- \gamma_{2j}\beta_j \frac{1}{1+\tau_j^m}\frac{1}{1+\tau_j^{x*}}\theta \pi_{12,j}\pi_{11,j} x_1 - \gamma_{2j}\beta_j \frac{1}{1+\tau_j^{d*}}\theta \pi_{22,j}\pi_{21,j} x_2 + \gamma_{x2} w_{2j} L_{2j} \\
&+ \gamma_{x1}\beta_j \frac{\tau_j^x}{1+\tau_j^x}\frac{1}{1+\tau_j^{m*}}\theta \pi_{21,j}\pi_{22,j} x_2 - \gamma_{x1}\beta_j \frac{\tau_j^m}{1+\tau_j^m}\theta \pi_{12,j}\pi_{11,j} x_1 + \gamma_{x1}\beta_j \frac{\tau_j^d}{1+\tau_j^d}\theta \pi_{11,j}\pi_{12,j} x_1 \\
&- \gamma_{x2}\beta_j \frac{\tau_j^{x*}}{1+\tau_j^{x*}}\frac{1}{1+\tau_j^m}\theta \pi_{12,j}\pi_{11,j} x_1 + \gamma_{x2}\beta_j \frac{\tau_j^{m*}}{1+\tau_j^{m*}}\theta \pi_{21,j}\pi_{22,j} x_2 - \gamma_{x2}\beta_j \frac{\tau_j^{d*}}{1+\tau_j^{d*}}\theta \pi_{22,j}\pi_{21,j} x_2 + B_j = 0
\end{aligned}
$$

Where $B_j = \sum_{k=1}^J (\gamma_{x2} - \gamma_{2k}) w_{2k} \frac{\partial L_{2k}}{\partial w_{2j}} w_{2j} = \sum_{k=1}^J (\gamma_{x2} - \gamma_{2k}) \frac{\partial w_{2k} L_{2k}}{\partial w_{2j}} w_{2j} - (\gamma_{x2} - \gamma_{2j}) w_{2j} L_{2j}$.

K.2 Foreign's optimal conditions

We use first-order conditions (FOCs) to characterize the Foreign government's problem and link the optimal policies with multipliers on goods market clearing conditions.

FOC over x_1

$$
\begin{aligned}
&\sum_{j=1}^J \gamma_{1j}^* \beta_j \frac{1}{1+\tau_j^d}\pi_{11,j} + \sum_{j=1}^J \gamma_{2j}^* \beta_j \frac{1}{1+\tau_j^m}\frac{1}{1+\tau_j^{x*}}\pi_{12,j} \\
&- \gamma_{x1}^* \left(1 - \sum_{j=1}^J \beta_j \frac{\tau_j^m}{1+\tau_j^m}\pi_{12,j} - \sum_{j=1}^J \beta_j \frac{\tau_j^d}{1+\tau_j^d}\pi_{11,j}\right) + \gamma_{x2}^* \sum_{j=1}^J \beta_j \frac{\tau_j^{x*}}{1+\tau_j^{x*}}\frac{1}{1+\tau_j^m}\pi_{12,j} = 0
\end{aligned}
$$

FOC over x_2

$$\frac{u_{c2}}{P_2}+\sum_{j=1}^{J}\gamma_{2j}^{*}\beta_j\frac{1}{1+\tau_j^{d*}}\pi_{22,j}+\sum_{j=1}^{J}\gamma_{1j}^{*}\beta_j\frac{1}{1+\tau_j^{m*}}\frac{1}{1+\tau_j^{x}}\pi_{21,j}$$
$$-\gamma_{x2}^{*}\left(1-\sum_{j=1}^{J}\beta_j\frac{\tau_j^{m*}}{1+\tau_j^{m*}}\pi_{21,j}-\sum_{j=1}^{J}\beta_j\frac{\tau_j^{d*}}{1+\tau_j^{d*}}\pi_{22,j}\right)+\gamma_{x1}^{*}\sum_{j=1}^{J}\beta_j\frac{\tau_j^{x}}{1+\tau_j^{x}}\frac{1}{1+\tau_j^{m*}}\pi_{21,j}=0$$

FOC over P_1

$$\gamma_{p1}^{*}=0$$

FOC over P_2

$$\gamma_{p2}^{*}=\frac{u_{c2}x_2}{P_2^2}$$

FOC over export tax τ_j^{x*}

$$-\gamma_{2j}^{*}\beta_j\frac{1}{(1+\tau_j^{x*})^2}\frac{1}{1+\tau_j^{m}}\pi_{12,j}x_1+\gamma_{2j}^{*}\beta_j\frac{1}{1+\tau_j^{x*}}\frac{1}{1+\tau_j^{m}}\frac{\partial\pi_{12,j}}{\partial\tau_j^{x*}}x_1$$
$$+\gamma_{1j}^{*}\beta_j\frac{1}{1+\tau_j^{d}}\frac{\partial\pi_{11,j}}{\partial\tau_j^{x*}}x_1+\gamma_{x2}^{*}\beta_j\frac{1}{(1+\tau_j^{x*})^2}\frac{1}{1+\tau_j^{m}}\pi_{12,j}x_1+\gamma_{x2}^{*}\beta_j\frac{\tau_j^{x*}}{1+\tau_j^{x*}}\frac{1}{1+\tau_j^{m}}\frac{\partial\pi_{12,j}}{\partial\tau_j^{x*}}x_1$$
$$+\gamma_{x1}^{*}\beta_j\frac{\tau_j^{m}}{1+\tau_j^{m}}\frac{\partial\pi_{12,j}}{\partial\tau_j^{x*}}x_1+\gamma_{x1}^{*}\beta_j\frac{\tau_j^{d}}{1+\tau_j^{d}}\frac{\partial\pi_{11,j}}{\partial\tau_j^{x*}}x_1=0 \tag{A.57}$$

after plugging in derivatives and simplifying, the FOC over τ_j^{x*} becomes

$$1+\tau_j^{x*}=\frac{\gamma_{x2}^{*}-\gamma_{2j}^{*}}{\left(\gamma_{x1}^{*}-\gamma_{1j}^{*}\right)\frac{1+\tau_j^{m}}{1+\tau_j^{d}}+\gamma_{x2}^{*}-\gamma_{x1}^{*}}\frac{\theta\pi_{11,j}+1}{\theta\pi_{11,j}}. \tag{A.58}$$

FOC over import tariff τ_j^{m*}

$$-\frac{u_{c2}}{P_2}\beta_j x_2\frac{\pi_{21,j}}{1+\tau_j^{m*}}+\gamma_{2j}^{*}\beta_j\frac{1}{1+\tau_j^{d*}}\frac{\partial\pi_{22,j}}{\partial\tau_j^{m*}}x_2-\gamma_{1j}^{*}\beta_j\frac{\pi_{21,j}}{(1+\tau_j^{m*})^2}\frac{1}{1+\tau_j^{x}}x_2+\gamma_{1j}^{*}\frac{1}{1+\tau_j^{m*}}\frac{1}{1+\tau_j^{x}}\frac{\partial\pi_{21,j}}{\partial\tau_j^{m*}}x_2$$
$$+\gamma_{x2}^{*}\beta_j\frac{1}{(1+\tau_j^{m*})^2}\pi_{21,j}x_2+\gamma_{x2}^{*}\beta_j\frac{\tau_j^{m*}}{1+\tau_j^{m*}}\frac{\partial\pi_{21,j}}{\partial\tau_j^{m*}}x_2+\gamma_{x2}^{*}\beta_j\frac{\tau_j^{d*}}{1+\tau_j^{d*}}\frac{\partial\pi_{22,j}}{\partial\tau_j^{m*}}x_2$$
$$-\gamma_{x1}^{*}\beta_j\frac{\tau_j^{x}}{1+\tau_j^{x}}\frac{1}{(1+\tau_j^{m*})^2}\pi_{21,j}x_2+\gamma_{x1}^{*}\beta_j\frac{\tau_j^{x}}{1+\tau_j^{x}}\frac{1}{1+\tau_j^{m*}}\frac{\partial\pi_{21,j}}{\partial\tau_j^{m*}}x_2=0 \tag{A.59}$$

FOC over domestic tax τ_j^{d*}

$$
\begin{aligned}
&-\frac{u_{c2}}{P_2}\beta_j x_2 \frac{\pi_{22,j}}{1+\tau_j^{d*}} - \gamma_{2j}^* \frac{1}{(1+\tau_j^{d*})^2}\beta_j \pi_{22,j} x_2 + \gamma_{2j}^* \frac{1}{1+\tau_j^{d*}}\beta_j \frac{\partial \pi_{22,j}}{\partial \tau_j^{d*}} x_2 + \gamma_{1j}^* \beta_j \frac{1}{1+\tau_j^{m*}} \frac{1}{1+\tau_j^x} \frac{\partial \pi_{21,j}}{\partial \tau_j^{d*}} x_2 \\
&+ \gamma_{x2}^* \beta_j \frac{\tau_j^{m*}}{1+\tau_j^{m*}} \frac{\partial \pi_{21,j}}{\partial \tau_j^{d*}} x_2 + \gamma_{x2}^* \beta_j \frac{1}{(1+\tau_j^{d*})^2} \pi_{22,j} x_2 + \gamma_{x2}^* \beta_j \frac{\tau_j^{d*}}{1+\tau_j^{d*}} \frac{\partial \pi_{22,j}}{\partial \tau_j^{d*}} x_2 \\
&+ \gamma_{x1}^* \beta_j \frac{\tau_j^x}{1+\tau_j^x} \frac{1}{1+\tau_j^{m*}} \frac{\partial \pi_{21,j}}{\partial \tau_j^{d*}} x_2 = 0
\end{aligned}
$$

Combining the FOCs over optimal tariff and domestic tax, we can get

$$
1+\tau_j^{m*} = \frac{\gamma_{x2}^* - \gamma_{x1}^* + (\gamma_{x1}^* - \gamma_{1j}^*)\frac{1}{1+\tau_j^x}}{u_{c2}/P_2}, \qquad 1+\tau_j^{d*} = \frac{\gamma_{x2}^* - \gamma_{2j}^*}{u_{c2}/P_2}. \tag{A.60}
$$

FOC over w_{1j}

$$
\begin{aligned}
&-\frac{u_{c2}}{P_2}\beta_j x_2 \frac{\pi_{21,j}}{w_{1j}} - \gamma_{1j}^* L_{1j} + \gamma_{1j}^* \beta_j \frac{1}{1+\tau_j^d} \frac{\partial \pi_{11,j}}{\partial w_{1j}} x_1 + \gamma_{1j}^* \beta_j \frac{1}{1+\tau_j^x} \frac{1}{1+\tau_j^{m*}} \frac{\partial \pi_{21,j}}{\partial w_{1j}} x_2 \\
&+ \gamma_{2j}^* \beta_j \frac{1}{1+\tau_j^m} \frac{1}{1+\tau_j^{x*}} \frac{\partial \pi_{12,j}}{\partial w_{1j}} x_1 + \gamma_{2j}^* \beta_j \frac{1}{1+\tau_j^{d*}} \frac{\partial \pi_{22,j}}{\partial w_{1j}} x_2 + \gamma_{x1}^* L_{1j} + \gamma_{x1}^* \beta_j \frac{\tau_j^x}{1+\tau_j^x} \frac{1}{1+\tau_j^{m*}} \frac{\partial \pi_{21,j}}{\partial w_{1j}} x_2 \\
&+ \gamma_{x1}^* \beta_j \frac{\tau_j^m}{1+\tau_j^m} \frac{\partial \pi_{12,j}}{\partial w_{1j}} x_1 + \gamma_{x1}^* \beta_j \frac{\tau_j^d}{1+\tau_j^d} \frac{\partial \pi_{11,j}}{\partial w_{1j}} x_1 + \gamma_{x2}^* \beta_j \frac{\tau_j^{x*}}{1+\tau_j^{x*}} \frac{1}{1+\tau_j^m} \frac{\partial \pi_{12,j}}{\partial w_{1j}} x_1 \\
&+ \gamma_{x2}^* \beta_j \frac{\tau_j^{m*}}{1+\tau_j^{m*}} \frac{\partial \pi_{21,j}}{\partial w_{1j}} x_2 + \gamma_{x2}^* \beta_j \frac{\tau_j^{d*}}{1+\tau_j^{d*}} \frac{\partial \pi_{22,j}}{\partial w_{1j}} x_2 + A_j^*/w_{1j} = 0
\end{aligned} \tag{A.61}
$$

Further

$$
\begin{aligned}
&-\frac{u_{c2}}{P_2}\beta_j x_2 \pi_{21,j} - \gamma_{1j}^* w_{1j} L_{1j} - \gamma_{1j}^* \beta_j \frac{1}{1+\tau_j^d} \theta \pi_{11,j} \pi_{12,j} x_1 - \gamma_{1j}^* \beta_j \frac{1}{1+\tau_j^x} \frac{1}{1+\tau_j^{m*}} \theta \pi_{21,j} \pi_{22,j} x_2 \\
&+ \gamma_{2j}^* \beta_j \frac{1}{1+\tau_j^m} \frac{1}{1+\tau_j^{x*}} \theta \pi_{12,j} \pi_{11,j} x_1 + \gamma_{2j}^* \beta_j \frac{1}{1+\tau_j^{d*}} \theta \pi_{22,j} \pi_{21,j} x_2 + \gamma_{x1}^* w_{1j} L_{1j} \\
&- \gamma_{x1}^* \beta_j \frac{\tau_j^x}{1+\tau_j^x} \frac{1}{1+\tau_j^{m*}} \theta \pi_{21,j} \pi_{22,j} x_2 + \gamma_{x1}^* \beta_j \frac{\tau_j^m}{1+\tau_j^m} \theta \pi_{12,j} \pi_{11,j} x_1 - \gamma_{x1}^* \beta_j \frac{\tau_j^d}{1+\tau_j^d} \theta \pi_{11,j} \pi_{12,j} x_1 \\
&+ \gamma_{x2}^* \beta_j \frac{\tau_j^{x*}}{1+\tau_j^{x*}} \frac{1}{1+\tau_j^m} \theta \pi_{12,j} \pi_{11,j} x_1 - \gamma_{x2}^* \beta_j \frac{\tau_j^{m*}}{1+\tau_j^{m*}} \theta \pi_{21,j} \pi_{22,j} x_2 + \gamma_{x2}^* \beta_j \frac{\tau_j^{d*}}{1+\tau_j^{d*}} \theta \pi_{22,j} \pi_{21,j} x_2 + A_j^* = 0
\end{aligned}
$$

Where $A_j^* = \sum_{k=1}^J (\gamma_{x1}^* - \gamma_{1k}^*) w_{1k} \frac{\partial L_{1k}}{\partial w_{1j}} w_{1j} = \sum_{k=1}^J (\gamma_{x1}^* - \gamma_{1k}^*) \frac{\partial w_{1k} L_{1k}}{\partial w_{1j}} w_{1j} - (\gamma_{x1}^* - \gamma_{1j}^*) w_{1j} L_{1j}$.

FOC over w_{2j}

$$
\begin{aligned}
&-\frac{u_{c2}}{P_2}\beta_j x_2\frac{\pi_{22,j}}{w_{2j}}-\gamma^*_{2j}L_{2j}+\gamma^*_{1j}\beta_j\frac{1}{1+\tau^d_j}\frac{\partial\pi_{11,j}}{\partial w_{2j}}x_1+\gamma^*_{1j}\beta_j\frac{1}{1+\tau^x_j}\frac{1}{1+\tau^{m*}_j}\frac{\partial\pi_{21,j}}{\partial w_{2j}}x_2\\
&+\gamma^*_{2j}\beta_j\frac{1}{1+\tau^m_j}\frac{1}{1+\tau^{x*}_j}\frac{\partial\pi_{12,j}}{\partial w_{2j}}x_1+\gamma^*_{2j}\beta_j\frac{1}{1+\tau^{d*}_j}\frac{\partial\pi_{22,j}}{\partial w_{2j}}x_2+\gamma^*_{x2}L_{2j}+\gamma^*_{x1}\beta_j\frac{\tau^x_j}{1+\tau^x_j}\frac{1}{1+\tau^{m*}_j}\frac{\partial\pi_{21,j}}{\partial w_{2j}}x_2\\
&+\gamma^*_{x1}\beta_j\frac{\tau^m_j}{1+\tau^m_j}\frac{\partial\pi_{12,j}}{\partial w_{2j}}x_1+\gamma^*_{x1}\beta_j\frac{\tau^d_j}{1+\tau^d_j}\frac{\partial\pi_{11,j}}{\partial w_{2j}}x_1+\gamma^*_{x2}\beta_j\frac{\tau^{x*}_j}{1+\tau^{x*}_j}\frac{1}{1+\tau^m_j}\frac{\partial\pi_{12,j}}{\partial w_{2j}}x_1\\
&+\gamma^*_{x2}\beta_j\frac{\tau^{m*}_j}{1+\tau^{m*}_j}\frac{\partial\pi_{21,j}}{\partial w_{2j}}x_2+\gamma^*_{x2}\beta_j\frac{\tau^{d*}_j}{1+\tau^{d*}_j}\frac{\partial\pi_{22,j}}{\partial w_{2j}}x_2+B^*_j/w_{2j}=0 \qquad \text{(A.62)}
\end{aligned}
$$

Further

$$
\begin{aligned}
&-\frac{u_{c2}}{P_2}\beta_j x_2\pi_{22,j}-\gamma^*_{2j}w_{2j}L_{2j}+\gamma^*_{1j}\beta_j\frac{1}{1+\tau^d_j}\theta\pi_{11,j}\pi_{12,j}x_1+\gamma^*_{1j}\beta_j\frac{1}{1+\tau^x_j}\frac{1}{1+\tau^{m*}_j}\theta\pi_{21,j}\pi_{22,j}x_2\\
&-\gamma^*_{2j}\beta_j\frac{1}{1+\tau^m_j}\frac{1}{1+\tau^{x*}_j}\theta\pi_{12,j}\pi_{11,j}x_1-\gamma^*_{2j}\beta_j\frac{1}{1+\tau^{d*}_j}\theta\pi_{22,j}\pi_{21,j}x_2+\gamma^*_{x2}w_{2j}L_{2j}\\
&+\gamma^*_{x1}\beta_j\frac{\tau^x_j}{1+\tau^x_j}\frac{1}{1+\tau^{m*}_j}\theta\pi_{21,j}\pi_{22,j}x_2-\gamma^*_{x1}\beta_j\frac{\tau^m_j}{1+\tau^m_j}\theta\pi_{12,j}\pi_{11,j}x_1+\gamma^*_{x1}\beta_j\frac{\tau^d_j}{1+\tau^d_j}\theta\pi_{11,j}\pi_{12,j}x_1\\
&-\gamma^*_{x2}\beta_j\frac{\tau^{x*}_j}{1+\tau^{x*}_j}\frac{1}{1+\tau^m_j}\theta\pi_{12,j}\pi_{11,j}x_1+\gamma^*_{x2}\beta_j\frac{\tau^{m*}_j}{1+\tau^{m*}_j}\theta\pi_{21,j}\pi_{22,j}x_2-\gamma^*_{x2}\beta_j\frac{\tau^{d*}_j}{1+\tau^{d*}_j}\theta\pi_{22,j}\pi_{21,j}x_2+B^*_j=0
\end{aligned}
$$

Where $B^*_j=\sum_{k=1}^J(\gamma^*_{x2}-\gamma^*_{2k})w_{2k}\frac{\partial L_{2k}}{\partial w_{2j}}w_{2j}=\sum_{k=1}^J(\gamma^*_{x2}-\gamma^*_{2k})\frac{\partial w_{2k}L_{2k}}{\partial w_{2j}}w_{2j}-(\gamma^*_{x2}-\gamma^*_{2j})w_{2j}L_{2j}$.

K.3 Proof of $A_j=0$, $B^*_j=0$ and optimal domestic taxes

For Home's optimal conditions, we combine FOCs over w_{1j} (A.55), τ^x_j(A.51), and τ^m_j (A.53):

$$
\begin{aligned}
&-u_{c1}\beta_j x_1+(\gamma_{x1}-\gamma_{1j})w_{1j}L_{1j}+(\gamma_{x2}-\gamma_{2j})\beta_j\frac{1}{1+\tau^m_j}\frac{1}{1+\tau^{x*}_j}\pi_{12,j}x_1\\
&-(\gamma_{x1}-\gamma_{1j})\beta_j\frac{1}{1+\tau^x_j}\frac{1}{1+\tau^{m*}_j}\pi^j_{21}x_2+(\gamma_{x1}-\gamma_{x2})\beta_j\frac{1}{1+\tau^m_j}\pi^j_{12}x_1+A_j=0
\end{aligned}
$$

hence,

$$
\begin{aligned}
&-u_{c1}\beta_j x_1+(\gamma_{x2}-\gamma_{2j})\beta_j\frac{1}{1+\tau^m_j}\frac{1}{1+\tau^{x*}_j}\pi_{12,j}x_1\\
&+(\gamma_{x1}-\gamma_{1j})\beta_j\frac{1}{1+\tau^d_j}\pi_{11,j}x_1+(\gamma_{x1}-\gamma_{x2})\beta_j\frac{1}{1+\tau^m_j}\pi^j_{12}x_1+A_j=0.
\end{aligned}
$$

Using the optimal policies in (A.54), we arrive at

$$-u_{c1}\beta_j x_1 \pi_{12,j} + u_{c1}\beta_j x_1 \pi_{12,j} + A_j = 0.$$

which implies $A_j = 0$. Now let us revisit the definition of A_j:

$$\sum_{k=1}^{J}(\gamma_{x1} - \gamma_{1k})\frac{\partial w_{1k} L_{1k}}{\partial w_{1j}} w_{1j} - (\gamma_{x1} - \gamma_{1j}) w_{1j} L_{1j} = 0.$$

Plugging the optimal formula for τ_j^d in (A.54) to the above equation, we can further show that

$$\sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial \ln(w_{1k}L_{1k})}{\partial \ln(w_{1j})}\frac{w_{1k}L_{1k}}{w_{1j}L_{1j}} - (1+\tau_j^d) = 0.$$

Hence, domestic taxes of Home country $\boldsymbol{\tau^d} = [\tau_1^d, ..., \tau_j^d, ..., \tau_J^d]'$ for J sectors satisfy

$$(\Lambda_1 - I)(1 + \boldsymbol{\tau^d}) = 0,$$

where the element of matrix Λ_1 at row j and column i is given by $\frac{\partial \ln(w_{1i}L_{1i})}{\partial \ln(w_{1j})}\frac{w_{1i}L_{1i}}{w_{1j}L_{1j}}$, and

$$\Lambda_1 - I = \begin{pmatrix} \frac{\partial ln(w_{11}L_{11})}{\partial ln(w_{11})}\frac{w_{11}L_{11}}{w_{11}L_{11}} - 1 & ... & \frac{\partial ln(w_{1j}L_{1j})}{\partial ln(w_{11})}\frac{w_{1j}L_{1j}}{w_{11}L_{11}} & ... & \frac{\partial ln(w_{1J}L_{1J})}{\partial ln(w_{11})}\frac{w_{1J}L_{1J}}{w_{11}L_{11}} \\ ... & ... & ... & ... & ... \\ \frac{\partial ln(w_{11}L_{11})}{\partial ln(w_{1j})}\frac{w_{11}L_{11}}{w_{1j}L_{1j}} & ... & \frac{\partial ln(w_{1j}L_{1j})}{\partial ln(w_{1j})}\frac{w_{1j}L_{1j}}{w_{1j}L_{1j}} - 1 & ... & \frac{\partial ln(w_{1J}L_{1J})}{\partial ln(w_{1j})}\frac{w_{1J}L_{1J}}{w_{1j}L_{1j}} \\ ... & ... & ... & ... & ... \\ \frac{\partial ln(w_{11}L_{11})}{\partial ln(w_{1J})}\frac{w_{11}L_{11}}{w_{1J}L_{1J}} & ... & \frac{\partial ln(w_{1j}L_{1j})}{\partial ln(w_{1J})}\frac{w_{1j}L_{1j}}{w_{1J}L_{1J}} & ... & \frac{\partial ln(w_{1J}L_{1J})}{\partial ln(w_{1J})}\frac{w_{1J}L_{1J}}{w_{1J}L_{1J}} - 1 \end{pmatrix}.$$

For Foreign's optimal conditions, we combine FOCs over w_{2j} (A.62), τ_j^{x*}(A.57), and τ_j^{m*} (A.59):

$$\begin{aligned} &-\frac{u_{c2}}{P_2}\beta_j x_2 + (\gamma_{x2}^* - \gamma_{2j}^*) w_{2j} L_{2j} + (\gamma_{x1}^* - \gamma_{1j}^*)\beta_j \frac{1}{1+\tau_j^{m*}}\frac{1}{1+\tau_j^x}\pi_{21,j} x_2 \\ &-(\gamma_{x2}^* - \gamma_{2j}^*)\beta_j \frac{1}{1+\tau_j^{x*}}\frac{1}{1+\tau_j^m}\pi_{12}^j x_1 + (\gamma_{x2}^* - \gamma_{x1}^*)\beta_j \frac{1}{1+\tau_j^{m*}}\pi_{21}^j x_2 + B_j^* = 0 \end{aligned}$$

hence,

$$
\begin{aligned}
&-\frac{u_{c2}}{P_2}\beta_j x_2+(\gamma^*_{x1}-\gamma^*_{1j})\beta_j\frac{1}{1+\tau^{m*}_j}\frac{1}{1+\tau^x_j}\pi_{21,j}x_2\\
&+(\gamma^*_{x2}-\gamma^*_{2j})\beta_j\frac{1}{1+\tau^{d*}_j}\pi_{22,j}x_2+(\gamma^*_{x2}-\gamma^*_{x1})\beta_j\frac{1}{1+\tau^{m*}_j}\pi^j_{21}x_2+B^*_j=0.
\end{aligned}
$$

Using the optimal policies in (A.60), we arrive at

$$
-\frac{u_{c2}}{P_2}\beta_j x_2\pi_{21,j}+\frac{u_{c2}}{P_2}\beta_j x_2\pi_{21,j}+B^*_j=0.
$$

which implies $B^*_j=0$. Now let us revisit the definition of B^*_j:

$$
\sum_{k=1}^{J}(\gamma^*_{x2}-\gamma^*_{2k})\frac{\partial w_{2k}L_{2k}}{\partial w_{2j}}w_{2j}-(\gamma^*_{x2}-\gamma^*_{2j})w_{2j}L_{2j}=0.
$$

Plugging the optimal formula for τ^{d*}_j in (A.60) to the above equation, we can further show that

$$
\sum_{k=1}^{J}(1+\tau^{d*}_k)\frac{\partial\ln(w_{2k}L_{2k})}{\partial\ln(w_{2j})}\frac{w_{2k}L_{2k}}{w_{2j}L_{2j}}-(1+\tau^{d*}_j)=0.
$$

Hence, domestic taxes of Foreign country $\boldsymbol{\tau^{d*}}=[\tau^{d*}_1,...,\tau^{d*}_j,...,\tau^{d*}_J]'$ for J sectors satisfy

$$
(\Lambda_2-I)(1+\boldsymbol{\tau^{d*}})=0,
$$

where the element of matrix Λ_2 at row j and column i is given by $\frac{\partial\ln(w_{2i}L_{2i})}{\partial\ln(w_{2j})}\frac{w_{2i}L_{2i}}{w_{2j}L_{2j}}$.

Under the CES supply system, following an approach similar to the proof in section C, the Home country does not implement differential domestic taxes across sectors. Given tax neutrality, we can normalize domestic taxes to zero in both Home and Foreign countries.

K.4 Proof of optimal import tariffs

For Home's optimal conditions, we combine FOCs over w_{1j} (A.55) and w_{2j} (A.56) and the formula of B_j:

$$
\begin{aligned}
&- u_{c1}\beta_j x_1 + (\gamma_{x1} - \gamma_{1j}) w_{1j} L_{1j} + (\gamma_{x2} - \gamma_{2j}) w_{2j} L_{2j} + B_j = 0 \\
(\Rightarrow) &- u_{c1}\beta_j x_1 + (\gamma_{x1} - \gamma_{1j}) w_{1j} L_{1j} + (\gamma_{x2} - \gamma_{2j}) w_{2j} L_{2j} \\
&+ \sum_{k=1}^{J} (\gamma_{x2} - \gamma_{2k}) \frac{\partial w_{2k} L_{2k}}{\partial w_{2j}} w_{2j} - (\gamma_{x2} - \gamma_{2j}) w_{2j} L_{2j} = 0 \\
(\Rightarrow) &\sum_{k=1}^{J} (\gamma_{x2} - \gamma_{2k}) \frac{\partial w_{2k} L_{2k}}{\partial w_{2j}} w_{2j} = u_{c1}\beta_j x_1 - (\gamma_{x1} - \gamma_{1j}) w_{1j} L_{1j}
\end{aligned}
$$

Finally, we get

$$
\begin{aligned}
&\sum_{k=1}^{J} \frac{\gamma_{x2} - \gamma_{2k}}{u_{c1}} \frac{\partial \ln(w_{2k} L_{2k})}{\partial \ln(w_{2j})} \frac{w_{2k} L_{2k}}{w_{2j} L_{2j}} = \frac{\beta_j x_1 - (1 + \tau_j^d) w_{1j} L_{1j}}{w_{2j} L_{2j}} \\
(\Rightarrow) &\sum_{k=1}^{J} (cons + \tau_k^m)(1 + \tau_k^{x*}) \frac{\partial \ln(w_{2k} L_{2k})}{\partial \ln(w_{2j})} \frac{w_{2k} L_{2k}}{w_{2j} L_{2j}} = \frac{\beta_j x_1 - (1 + \tau_j^d) w_{1j} L_{1j}}{w_{2j} L_{2j}}
\end{aligned}
$$

where $cons = 1 - \frac{\gamma_{x1} - \gamma_{x2}}{u_{c1}}$ is common across all the sectors. Due to the tax neutrality, we can normalize it to zero, thus, $\gamma_{x1} - \gamma_{x2} = u_{c1}$.

Hence, given export tax $\boldsymbol{\tau}^{\boldsymbol{x}*} = [\tau_1^{x*}, ..., \tau_j^{x*}, ..., \tau_J^{x*}]'$ imposed by Foreign country, Home's import tariffs $\boldsymbol{\tau}^{\boldsymbol{m}} = [\tau_1^m, ..., \tau_j^m, ..., \tau_J^m]'$ for J sectors satisfy

$$
\Lambda_2[\boldsymbol{\tau}^{\boldsymbol{m}} \cdot (1 + \boldsymbol{\tau}^{\boldsymbol{x}*})] = \begin{pmatrix} \frac{\beta_1 x_1 - (1+\tau_1^d) w_{11} L_{11}}{w_{21} L_{21}} \\ \ldots \\ \frac{\beta_j x_1 - (1+\tau_j^d) w_{1j} L_{1j}}{w_{2j} L_{2j}} \\ \ldots \\ \frac{\beta_J x_1 - (1+\tau_J^d) w_{1J} L_{1J}}{w_{2J} L_{2J}} \end{pmatrix} = \Psi_1, \tag{A.63}
$$

Under a CES supply system with foreign elasticity κ_2, the optimal import tariffs of sector j and k satisfy

$$
\tau_j^m (1 + \tau_j^{x*}) - \tau_k^m (1 + \tau_k^{x*}) = \frac{1}{\kappa_2} \left[\frac{\beta_j x_1 - w_{1j} L_{1j}}{w_{2j} L_{2j}} - \frac{\beta_k x_1 - w_{1k} L_{1k}}{w_{2k} L_{2k}} \right]
$$

Revisiting the formula for Home's import tariff (A.54) and export tax (A.52), we can simplify them to:

$$1+\tau_j^m = 1+\frac{(\gamma_{x2}-\gamma_{2j})\frac{1}{1+\tau_j^{x*}}}{u_{c1}} \tag{A.64}$$

$$1+\tau_j^x = \frac{u_{c1}}{(\gamma_{x2}-\gamma_{2j})\,(1+\tau_j^{m*})+u_{c1}}\frac{\theta\pi_{22,j}+1}{\theta\pi_{22,j}}. \tag{A.65}$$

Combining (A.64) and (A.65),

$$(1+\tau_j^x)(1+\tau_j^{x*})\tau_j^m(1+\tau_j^{m*})+(1+\tau_j^x) = 1+\frac{1}{\theta\pi_{22,j}} \tag{A.66}$$

Similarly, for Foreign's optimal conditions, we combine FOCs over w_{1j} (A.61) and w_{2j} (A.62) and the formula of A_j^*:

$$\begin{aligned}
&-\frac{u_{c2}}{P_2}\beta_j x_2+(\gamma_{x1}^*-\gamma_{1j}^*)w_{1j}L_{1j}+(\gamma_{x2}^*-\gamma_{2j}^*)w_{2j}L_{2j}+A_j^*=0\\
(\Rightarrow)&-\frac{u_{c2}}{P_2}\beta_j x_2+(\gamma_{x2}^*-\gamma_{2j}^*)w_{2j}L_{2j}+(\gamma_{x1}^*-\gamma_{1j}^*)w_{1j}L_{1j}\\
&+\sum_{k=1}^{J}(\gamma_{x1}^*-\gamma_{1k}^*)\frac{\partial w_{1k}L_{1k}}{\partial w_{1j}}w_{1j}-(\gamma_{x1}^*-\gamma_{1j}^*)w_{1j}L_{1j}=0\\
(\Rightarrow)&\sum_{k=1}^{J}(\gamma_{x1}^*-\gamma_{1k}^*)\frac{\partial w_{1k}L_{1k}}{\partial w_{1j}}w_{1j}=\frac{u_{c2}}{P_2}\beta_j x_2-(\gamma_{x2}^*-\gamma_{2j}^*)w_{2j}L_{2j}
\end{aligned}$$

Finally, we get

$$\begin{aligned}
&\sum_{k=1}^{J}\frac{\gamma_{x1}^*-\gamma_{1k}^*}{u_{c2}/P_2}\frac{\partial\ln(w_{1k}L_{1k})}{\partial\ln(w_{1j})}\frac{w_{1k}L_{1k}}{w_{1j}L_{1j}}=\frac{\beta_j x_2-(1+\tau_j^{d*})w_{2j}L_{2j}}{w_{1j}L_{1j}}\\
(\Rightarrow)&\sum_{k=1}^{J}(cons^*+\tau_k^{m*})(1+\tau_k^x)\frac{\partial\ln(w_{1k}L_{1k})}{\partial\ln(w_{1j})}\frac{w_{1k}L_{1k}}{w_{1j}L_{1j}}=\frac{\beta_j x_2-(1+\tau_j^{d*})w_{2j}L_{2j}}{w_{1j}L_{1j}}
\end{aligned}$$

where $cons^* = 1-\frac{\gamma_{x2}^*-\gamma_{x1}^*}{u_{c2}/P_2}$ is common across all the sectors. Due to the tax neutrality, we can normalize it to zero, thus, $\gamma_{x2}^*-\gamma_{x1}^* = u_{c2}/P_2$.

Given export tax $\boldsymbol{\tau}^x = [\tau_1^x, ..., \tau_j^x, ..., \tau_J^x]'$ imposed by Home country, Foreign's import tariffs $\boldsymbol{\tau}^{m*} =$

$[\tau_1^{m*}, ..., \tau_j^{m*}, ..., \tau_J^{m*}]'$ for J sectors satisfy

$$\Lambda_1[\boldsymbol{\tau}^{m*} \cdot (1+\boldsymbol{\tau}^x)] = \begin{pmatrix} \frac{\beta_1 x_2 - (1+\tau_1^{d*}) w_{21} L_{21}}{w_{11} L_{11}} \\ ... \\ \frac{\beta_j x_2 - (1+\tau_j^{d*}) w_{2j} L_{2j}}{w_{1j} L_{1j}} \\ ... \\ \frac{\beta_J x_2 - (1+\tau_J^{d*}) w_{2J} L_{2J}}{w_{1J} L_{1J}} \end{pmatrix} = \Psi_2. \tag{A.67}$$

Under a CES supply system with Home elasticity κ_1, the optimal import tariffs of sector j and k satisfy

$$\tau_j^{m*}(1+\tau_j^x) - \tau_k^{m*}(1+\tau_k^x) = \frac{1}{\kappa_1}\left[\frac{\beta_j x_2 - w_{2j}L_{2j}}{w_{1j}L_{1j}} - \frac{\beta_k x_2 - w_{2k}L_{2k}}{w_{1k}L_{1k}}\right]$$

Revisiting the formula for Foreign's import tariff (A.60) and export tax (A.58), we can simplify them to:

$$1+\tau_j^{m*} = 1 + \frac{(\gamma_{x1}^* - \gamma_{1j}^*)\frac{1}{1+\tau_j^x}}{u_{c2}/P_2} \tag{A.68}$$

$$1+\tau_j^{x*} = \frac{u_{c2}/P_2}{\left(\gamma_{x1}^* - \gamma_{1j}^*\right)(1+\tau_j^m) + u_{c2}/P_2}\,\frac{\theta\pi_{11,j}+1}{\theta\pi_{11,j}}. \tag{A.69}$$

Combining (A.68) and (A.69),

$$(1+\tau_j^x)(1+\tau_j^{x*})\tau_j^{m*}(1+\tau_j^m) + (1+\tau_j^{x*}) = 1 + \frac{1}{\theta\pi_{11,j}} \tag{A.70}$$

K.5 Proof of tax neutrality

Given the policies of the other country, the proof of tax neutrality for country n follows a similar approach to that presented in section F. For any policy vector Γ, we can normalize one domestic tax to zero in each country (e.g., setting $\check{\tau}_1^d = 0$ and $\check{\tau}_1^{d*} = 0$). Additionally, one tariff in each country can be normalized to zero. Alternatively, instead of normalizing one tariff, we can normalize *cons* and *cons** to zero.

L Quantifying the consequences of optimal policies

L.1 Counterfactual equilibrium from unilateral optimal policies

In this section, we compute the counterfactual equilibrium under unilateral optimal policies using the exact hat method and the formula for the optimal policies in Equation (20)-(22). Variables without prime are originals from data (trade matrix $\pi_{ni,j}$; sectoral output $w_{nj}L_{nj}$; sectoral income share m_{nj}), and variables with prime are counterfactuals after implementing optimal policies. Variables with hat are the ratio of prime and original.

$$
\begin{aligned}
Y'_{nj} &= w'_{nj}L'_{nj} = \hat{w}_{nj}\hat{L}_{nj}w_{nj}L_{nj} \\
\hat{P}_2 &= \Pi_j\left[\pi_{21,j}\left(\hat{w}_{1j}(1+\tau_j^{x\prime})\right)^{-\theta} + \pi_{22,j}\hat{w}_{2j}^{-\theta}\right]^{-\beta_j/\theta} \\
\hat{P}_1 &= \Pi_j\left[\pi_{11,j}\left(\hat{w}_{1j}(1+\tau_j^{d\prime})\right)^{-\theta} + \pi_{12,j}(\hat{w}_{2j}(1+\tau_j^{m\prime}))^{-\theta}\right]^{-\beta_j/\theta} \\
\pi'_{11,j} &= \frac{\pi_{11,j}\left(\hat{w}_{1j}(1+\tau_j^{d\prime})\right)^{-\theta}}{\pi_{11,j}\left(\hat{w}_{1j}(1+\tau_j^{d\prime})\right)^{-\theta} + \pi_{12,j}\left(\hat{w}_{2j}(1+\tau_j^{m\prime})\right)^{-\theta}} \\
\pi'_{12,j} &= \frac{\pi_{12,j}\left(\hat{w}_{2j}(1+\tau_j^{m\prime})\right)^{-\theta}}{\pi_{11,j}\left(\hat{w}_{1j}(1+\tau_j^{d\prime})\right)^{-\theta} + \pi_{12,j}\left(\hat{w}_{2j}(1+\tau_j^{m\prime})\right)^{-\theta}} \\
\pi'_{21,j} &= \frac{\pi_{21,j}\left(\hat{w}_{1j}(1+\tau_j^{x\prime})\right)^{-\theta}}{\pi_{21,j}\left(\hat{w}_{1j}(1+\tau_j^{x\prime})\right)^{-\theta} + \pi_{22,j}\hat{w}_{2j}^{-\theta}} \\
\pi'_{22,j} &= \frac{\pi_{22,j}\hat{w}_{2j}^{-\theta}}{\pi_{21,j}\left(\hat{w}_{1j}(1+\tau_j^{x\prime})\right)^{-\theta} + \pi_{22,j}\hat{w}_{2j}^{-\theta}} \\
x'_1 &= \sum_j \hat{w}_{1j}\hat{L}_{1j}w_{1j}L_{1j} + \sum_j \frac{\tau_j^{x\prime}}{1+\tau_j^{x\prime}}\pi'_{21,j}\beta_j x'_2 + \sum_j \frac{\tau_j^{m\prime}}{1+\tau_j^{m\prime}}\pi'_{12,j}\beta_j x'_1 + \sum_j \frac{\tau_j^{d\prime}}{1+\tau_j^{d\prime}}\pi'_{11,j}\beta_j x'_1 \\
x'_2 &= \sum_j \hat{w}_{2j}\hat{L}_{2j}w_{2j}L_{2j} \\
\hat{w_{1j}}\hat{L}_{1j}w_1L_{1j} &= \pi'_{11,j}\frac{1}{1+\tau_j^{d\prime}}\beta_j x'_1 + \frac{1}{1+\tau_j^{x\prime}}\pi'_{21,j}\beta_j x'_2 \\
\hat{w_{2j}}\hat{L}_{2j}w_2L_{2j} &= \frac{1}{1+\tau_j^{m\prime}}\pi'_{12,j}\beta_j x'_1 + \pi'_{22,j}\beta_j x'_2
\end{aligned}
$$

The labor market clearing conditions become

$$m'_{nj} = \frac{m_{nj}\hat{w}_{nj}^{\kappa_n}}{\sum_{k=1} m_{nk}\hat{w}_{nk}^{\kappa_n}}$$
$$W'_n = W_n(\sum_{k=1} m_{nk}\hat{w}_{nk}^{\kappa_n})^{\frac{1}{\kappa_n}}$$

Thus,

$$W'_n\bar{L}_n = W_n\bar{L}_n(\sum_{k=1} m_{nk}\hat{w}_{nk}^{\kappa_n})^{\frac{1}{\kappa_n}}$$
$$\Rightarrow \frac{w'_{nj}L'_{nj}}{m'_{nj}} = \frac{w_{nj}L_{nj}}{m_{nj}}(\sum_{k=1} m_{nk}\hat{w}_{nk}^{\kappa_n})^{\frac{1}{\kappa_n}}$$
$$\Rightarrow \hat{L}_{nj} = \frac{\hat{w}_{nj}^{\kappa_n-1}}{(\sum_{k=1} m_{nk}\hat{w}_{nk}^{\kappa_n})^{\frac{\kappa_n-1}{\kappa_n}}}$$

The optimal trade policies are

$$1+\tau_j^{x\prime} = \frac{1+\tau_j^{d\prime}}{1+\tau_j^{m\prime}}(1+\frac{1}{\theta\pi'_{22,j}})$$

$$\tau_j^{m\prime} - \tau_k^{m\prime} = \frac{1}{\kappa_2}\left[\frac{\beta_j x'_1 - (1+\tau_j^{d\prime})Y'_{1j}}{Y'_{2j}} - \frac{\beta_k x'_1 - (1+\tau_k^{d\prime})Y'_{1k}}{Y'_{2k}}\right]$$

$$\tau_j^{d\prime} = \bar{\tau}^{d\prime}$$

Due to tax neutrality, we normalize $\tau_1^{m\prime} = 0$ and $\bar{\tau}^{d\prime} = 0$. We can then solve for $(\hat{w}_{nj}, \hat{L}_{nj}, \tau_j^{x\prime}, \tau_j^{m\prime}, \tau_j^{d\prime})$ and calculate the change in welfare for each country ($\hat{c}_n = \frac{\hat{x}_n}{\hat{P}_n}$).

L.2 Counterfactual equilibrium from Nash optimal policies

In this section, we compute the counterfactual equilibrium under Nash optimal policies using the exact hat method and the formula for the optimal policies in (A.63), (A.67), (A.66) and (A.70). Variables without prime are originals from data (trade matrix $\pi_{ni,j}$; sectoral output $w_{nj}L_{nj}$; sectoral income share m_{nj}), and variables with prime are counterfactuals after implementing optimal policies. Variables with hat are the ratio of prime and original.

$$Y'_{nj} = w'_{nj}L'_{nj} = \hat{w}_{nj}\hat{L}_{nj}w_{nj}L_{nj}$$

$$\hat{P}_2 = \Pi_j \left[\pi_{21,j}\left(\hat{w}_{1j}(1+\tau_j^{x\prime})(1+\tau_j^{m*\prime})\right)^{-\theta} + \pi_{22,j}\left(\hat{w}_{2j}(1+\tau_j^{d*\prime})\right)^{-\theta}\right]^{-\beta_j/\theta}$$

$$\hat{P}_1 = \Pi_j \left[\pi_{11,j}\left(\hat{w}_{1j}(1+\tau_j^{d\prime})\right)^{-\theta} + \pi_{12,j}\left(\hat{w}_{2j}(1+\tau_j^{m\prime})(1+\tau_j^{x*\prime})\right)^{-\theta}\right]^{-\beta_j/\theta}$$

$$\pi'_{11,j} = \frac{\pi_{11,j}\left(\hat{w}_{1j}(1+\tau_j^{d\prime})\right)^{-\theta}}{\pi_{11,j}\left(\hat{w}_{1j}(1+\tau_j^{d\prime})\right)^{-\theta} + \pi_{12,j}\left(\hat{w}_{2j}(1+\tau_j^{m\prime})(1+\tau_j^{x*\prime})\right)^{-\theta}}$$

$$\pi'_{12,j} = \frac{\pi_{12,j}\left(\hat{w}_{2j}(1+\tau_j^{m\prime})(1+\tau_j^{x*\prime})\right)^{-\theta}}{\pi_{11,j}\left(\hat{w}_{1j}(1+\tau_j^{d\prime})\right)^{-\theta} + \pi_{12,j}\left(\hat{w}_{2j}(1+\tau_j^{m\prime})(1+\tau_j^{x*\prime})\right)^{-\theta}}$$

$$\pi'_{21,j} = \frac{\pi_{21,j}\left(\hat{w}_{1j}(1+\tau_j^{x\prime})(1+\tau_j^{m*\prime})\right)^{-\theta}}{\pi_{21,j}\left(\hat{w}_{1j}(1+\tau_j^{x\prime})(1+\tau_j^{m*\prime})\right)^{-\theta} + \pi_{22,j}\left(\hat{w}_{2j}(1+\tau_j^{d*\prime})\right)^{-\theta}}$$

$$\pi'_{22,j} = \frac{\pi_{22,j}\left(\hat{w}_{2j}(1+\tau_j^{d*\prime})\right)^{-\theta}}{\pi_{21,j}\left(\hat{w}_{1j}(1+\tau_j^{x\prime})(1+\tau_j^{m*\prime})\right)^{-\theta} + \pi_{22,j}\left(\hat{w}_{2j}(1+\tau_j^{d*\prime})\right)^{-\theta}}$$

$$x'_1 = \sum_j \hat{w}_{1j}\hat{L}_{1j}w_{1j}L_{1j} + \sum_j \frac{\tau_j^{x\prime}}{1+\tau_j^{x\prime}}\frac{1}{1+\tau_j^{m*\prime}}\pi'_{21,j}\beta_j x'_2 + \sum_j \frac{\tau_j^{m\prime}}{1+\tau_j^{m\prime}}\pi'_{12,j}\beta_j x'_1 + \sum_j \frac{\tau_j^{d\prime}}{1+\tau_j^{d\prime}}\pi'_{11,j}\beta_j x'_1$$

$$x'_2 = \sum_j \hat{w}_{2j}\hat{L}_{2j}w_{2j}L_{2j} + \sum_j \frac{\tau_j^{x*\prime}}{1+\tau_j^{x*\prime}}\frac{1}{1+\tau_j^{m\prime}}\pi'_{12,j}\beta_j x'_1 + \sum_j \frac{\tau_j^{m*\prime}}{1+\tau_j^{m*\prime}}\pi'_{21,j}\beta_j x'_2 + \sum_j \frac{\tau_j^{d*\prime}}{1+\tau_j^{d*\prime}}\pi'_{22,j}\beta_j x'_2$$

$$\hat{w_{1j}}\hat{L}_{1j}w_1L_{1j} = \frac{1}{1+\tau_j^{d\prime}}\pi'_{11,j}\beta_j x'_1 + \frac{1}{1+\tau_j^{x\prime}}\frac{1}{1+\tau_j^{m*\prime}}\pi'_{21,j}\beta_j x'_2$$

$$\hat{w_{2j}}\hat{L}_{2j}w_2L_{2j} = \frac{1}{1+\tau_j^{m\prime}}\frac{1}{1+\tau_j^{x*\prime}}\pi'_{12,j}\beta_j x'_1 + \frac{1}{1+\tau_j^{d*\prime}}\pi'_{22,j}\beta_j x'_2$$

The labor market clearing conditions become

$$m'_{nj} = \frac{m_{nj}\hat{w}_{nj}^{\kappa_n}}{\sum_{k=1} m_{nk}\hat{w}_{nk}^{\kappa_n}}$$

$$W'_n = W_n(\sum_{k=1} m_{nk}\hat{w}_{nk}^{\kappa_n})^{\frac{1}{\kappa_n}}$$

Thus,

$$
\begin{aligned}
W_n'\bar{L}_n &= W_n\bar{L}_n(\sum_{k=1} m_{nk}\hat{w}_{nk}^{\kappa_n})^{\frac{1}{\kappa_n}} \\
\Rightarrow \frac{w_{nj}'L_{nj}'}{m_{nj}'} &= \frac{w_{nj}L_{nj}}{m_{nj}}(\sum_{k=1} m_{nk}\hat{w}_{nk}^{\kappa_n})^{\frac{1}{\kappa_n}} \\
\Rightarrow \hat{L}_{nj} &= \frac{\hat{w}_{nj}^{\kappa_n-1}}{(\sum_{k=1} m_{nk}\hat{w}_{nk}^{\kappa_n})^{\frac{\kappa_n-1}{\kappa_n}}}
\end{aligned}
$$

Recalling the tax neutrality discussed in section K.5, we normalize $cons = 0$, $cons^* = 0$ and $\bar{\tau}^{d\prime} = 0$, $\bar{\tau}^{d*\prime} = 0$. The equations of the optimal trade policies are

$$
(1+\tau_j^{x\prime})(1+\tau_j^{x*\prime})\tau_j^{m\prime}(1+\tau_j^{m*\prime}) + (1+\tau_j^{x\prime}) = 1 + \frac{1}{\theta\pi_{22,j}'}
$$

$$
(1+\tau_j^{x\prime})(1+\tau_j^{x*\prime})\tau_j^{m*\prime}(1+\tau_j^{m\prime}) + (1+\tau_j^{x*\prime}) = 1 + \frac{1}{\theta\pi_{11,j}'}
$$

$$
\kappa_2\tau_j^{m\prime}(1+\tau_j^{x*\prime}) - (\kappa_2-1)\sum_{s=1}\frac{Y_{2s}'}{W_2'\bar{L}_2}\tau_s^{m\prime}(1+\tau_s^{x*\prime}) = \frac{\beta_j x_1' - Y_{1j}'}{Y_{2j}'}
$$

$$
\kappa_1\tau_j^{m*\prime}(1+\tau_j^{x\prime}) - (\kappa_1-1)\sum_{s=1}\frac{Y_{1s}'}{W_1'\bar{L}_1}\tau_s^{m*\prime}(1+\tau_s^{x\prime}) = \frac{\beta_j x_2' - Y_{2j}'}{Y_{1j}'}
$$

We can then solve for $(\hat{w}_{nj}, \hat{L}_{nj}, \tau_j^{x\prime}, \tau_j^{m\prime}, \tau_j^{d\prime}, \tau_j^{x*\prime}, \tau_j^{m*\prime}, \tau_j^{d*\prime})$ and calculate the change in welfare for each country ($\hat{c}_n = \frac{\hat{x}_n}{\hat{P}_n}$).