Optimal Trade Policies and Market Power in General Equilibrium Trade Models*

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Abstract

We derive optimal trade policies in a multi-country, multi-sector general-equilibrium model that unifies a wide range of supply-side assumptions. Under our CES supply system, which nests most existing specifications, two-country optimal tariffs and export taxes across sectors depend only on elasticity parameters and relative market shares. With multiple countries, country and sector interdependencies—due to the cross-country trade network—make optimal tariffs imposed by the home country differ across countries and sectors while converging within sectors. Only when foreign countries do not trade with each other do the home country's optimal policies depend solely on bilateral trade. Using trade data, we quantify optimal policies and find that ignoring interdependencies can lead to substantially lower welfare gains for the home country.

Keywords: Optimal Policies, Tariffs, Export Taxes, General Equilibrium, CES supply system, Imperfectly Substitutable Labor, Economies of Scales, Endogenous Self- and Cross-Export Supply Elasticities

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1 Introduction

A large body of research establishes that multi-country, multi-sector general equilibrium (GE) models are successful in accounting for global trade patterns. The literature has emphasized how general equilibrium effects play an important role in evaluating trade policy impact. Yet characterizing optimal trade policies within such frameworks remains challenging due to inherent complex interdependencies. When a tariff is imposed in one sector, it creates cascading effects: it changes demand and supply in both domestic and foreign markets, reallocates factors across sectors, and changes global prices, which in turn reshape trade flows and determine optimal trade policy incentives.

Traditional optimal trade policy literature abstracts away from these interdependencies by using simple general equilibrium or partial equilibrium models, which reduces the problem to a single-foreign-country single-good monopolist or monopsonist. This simplification leads to a straightforward formula: optimal tariffs should equal the inverse of the self-elasticity of foreign export supply curves. This relationship captures the home country's ability to influence foreign prices by suppressing foreign demand—essentially measuring how much market power the home country can exercise.[1]

Recent studies have advanced the analysis of optimal trade policies in modern trade models. Costinot, Donaldson, Vogel, and Werning (2015) studies optimal trade policies for a two-country, multi-good model with neoclassical production functions, characterizing the structure of optimal trade taxes across goods and explaining the economic rationale behind import tariffs and export taxes. Subsequent research further quantifies optimal trade policies in multi-country and multi-sector settings with differential economies of scale across sectors.[2] We contribute to this literature by developing a unified framework that nests existing supply-side assumptions while deriving simple, explicit formulas for optimal policies. Our approach enhances theoretical clarity and streamlines computation.

We revisit optimal trade policies in multi-country, multi-sector GE models with a generic

[1] See Bagwell and Staiger (1999), Broda, Limao, and Weinstein (2008) and Feenstra (2015).

[2] Costinot et al. (2015) explains why optimal import tariffs should be uniform across goods using the canonical Ricardian model Dornbusch, Fischer, and Samuelson (1977). Lashkaripour and Lugovskyy (2023) emphasizes that trade policy measures are ineffective in correcting misallocation using the generalized Krugman et al. (1980) model. Bartelme, Costinot, Donaldson, and Rodriguez-Clare (2025) quantifies the gains from optimal industrial policy using modern trade models with economies of scale.

supply system that nests many theories such as extended multi-sector Armingtion models, Eaton and Kortum (2002), and Krugman et al. (1980) models, allowing for different economies of scale across sectors and their combination with different labor market specifications, including perfectly mobile labor, fixed labor, and imperfectly substitutable labor across sectors as in the Ricardo-Roy model. The home government chooses domestic and trade policies to maximize welfare, taking into account how these policies affect consumption, production, trade flows, and prices. The economic rationale for these policies is classic: exploiting terms-of-trade benefits and correcting domestic market distortions. We use the Lagrange multiplier method to characterize Home's unilateral optimal policies.

We have three findings. First, we derive exact, tractable formulas for optimal policies in two-country models featuring a CES import demand system across countries and our defined *CES supply system* across sectors—a structure that characterizes many modern quantitative trade models. Although there are interlinkages across sectors and countries, optimal policies follow simple formulas that link sectoral tariffs to structural supply elasticity parameters and Home's net import share as a fraction of foreign income in each sector. To our knowledge, this is the first exact formulas applicable to a broad class of quantitative trade models. Second, we derive exact formulas for the multiple-country case and find that Home's optimal tariffs exhibit two opposing features: they are *heterogeneous* across sectors and countries, yet general equilibrium effects from the trade network alter the home country's market power, leading to *convergence* of tariffs within a sector across countries. Lastly, we quantify optimal unilateral and Nash trade policies using cross-country trade data and our exact formulas. We find that ignoring interdependencies can lead to lower welfare gains for the home country.

Specifically, for models belonging to the CES supply system, in the two-country case, optimal sectoral policies of Home (country 1) satisfy

$$
\begin{aligned}
\text{Domestic tax:} \quad 1+\tau_j^d &= (1+\bar{\tau}^d)\frac{\eta_j}{\eta_j-1}, \\
\text{Import tariff:} \quad \tau_j^m - \tau_i^m &= \frac{1}{\eta_j}\frac{\beta_{1,j}x_1 - (1+\tau_j^d)Y_{1,j}}{Y_{2,j}} - \frac{1}{\eta_i}\frac{\beta_{1,i}x_1 - (1+\tau_i^d)Y_{1,i}}{Y_{2,i}}, \\
\text{Export tax:} \quad 1+\tau_j^x &= \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\epsilon_j \pi_{22,j}}\right),
\end{aligned}
$$

where ϵ_j is the trade elasticity, $\pi_{22,j}$ is the foreign expenditure shares on its own goods, Y_{nj} is the income of country n and sector j, $\beta_{1,j}x_1 - (1+\tau_j^d)Y_{1,j}$ captures the net import of the home country in sector j.

The parameter η_j represents a sector-specific constant that relates to supply elasticity parameters. For example, in a model with imperfectly substitutable labor (with elasticity κ) and production economies of scale (with parameter ψ_j for sector j), the parameter η_j is given by $\frac{1}{\eta_j} = \frac{1}{\kappa} - \psi_j \frac{\kappa-1}{\kappa}$. This relationship covers a wide range of quantitative trade models. In a multi-sector Armington model, $\kappa = \infty$ and $\psi_j = 0$. A Ricardo-Roy trade model features finite κ and $\psi_j = 0$. In a multi-sector model with sectoral economies of scale ψ_j and perfectly mobile labor $\kappa = \infty$, such as Lashkaripour and Lugovskyy (2023) and Bartelme et al. (2025), $\frac{1}{\eta_j} = -\psi_j$.[3]

Three insights arise from the optimal policy formulas. First, relative tariffs across any two sectors depend on the relative magnitude of Home's net imports over foreign income scaled by sectoral supply elasticity parameters, reflecting Home's relative import market power across the two sectors. Under positive supply elasticity, Home imposes higher tariffs in sectors where it is a larger buyer of foreign goods, and the tariff decreases with the sector's supply elasticity. Second, given other taxes, a sector's export tax depends on the inverse of trade elasticity and Home's market shares in foreign consumption in that sector, together reflecting Home's export market power. Home uses high export taxes on sectors where it is a large seller to the foreign country. Third, Home uses Pigouvian domestic taxes to fix domestic inefficiency.

Our formulas generalize the existing findings of uniform import tariffs under perfectly mobile labor across sectors. This model corresponds to infinite κ and $\psi_j = 0$, hence, the optimal tariff differences across sectors go to zero. This is because, with labor freely mobile across sectors, wages are equalized and technology is exogenously fixed, Home cannot change the relative price of any two imports and therefore uses uniform or zero tariffs. In contrast, our general model allows for general supply-side assumptions. For example,

[3]The economies of scale parameter ψ_j takes different forms across model specifications: in a model of multi-sector Krugman, $\psi_j = 1/(\sigma_j - 1)$, where σ_j is the elasticity of substitution across varieties in sector j; in multi-sector Eaton and Kortum (2001) with endogenous technology, $\psi_j = 1/\theta$, where θ is the Fréchet shape parameter; and in a model of multi-sector Armington with external economies of scale, ψ_j is differential sectoral economies of scale.

with imperfectly substitutable labor across sectors ($\kappa < \infty$), Home imposes relatively higher tariffs on sectors with net imports. This occurs because decreased demand reduces foreign sectoral wages and hence supply prices in those sectors, thereby improving Home's terms of trade.

With alternative supply-side assumptions, such as Marshallian economies of scale, or endogenous technology, or those in multi-sector Krugman models with identical scale parameters across sectors, Home imposes relatively lower tariffs on sectors with net imports.[4] When Home policies can affect foreign production technology, Home has incentives to use differential tariffs to influence relative prices. In this case, lower tariffs increase demand for foreign goods, which encourages foreign technological advancements. These technological advances reduce foreign production costs, lowering supply prices and improving Home's terms of trade. While the same optimal-tariff formula applies, the elasticity constant $1/\eta$ now equals the negative of the scale elasticity.

With different scale elasticities across sectors, production allocation is inefficient. Home uses Pigouvian domestic taxes to fix domestic inefficiency, reflecting the Bhagwati-Johnson principle of targeting. Differential tariffs across sectors still depend on the relative magnitude of Home's net imports over foreign income scaled by the sectoral supply elasticity parameter, reflecting Home's import market power and how effectively Home can affect foreign prices across sectors.

To explain how these models incorporate general equilibrium effects while yielding tractable formulas, we define a *CES supply system*. Models within this system share a common feature: cross-elasticities of sectoral supply with respect to any two other sectors' prices are symmetric when scaled by each sector's size and supply parameters. This symmetry holds even when labor is imperfectly substitutable across sectors or when sectors have different returns to scale. These cross-elasticity symmetries lead to our tractable formulas. Because each sector's supply responds to price changes in any two other sectors with the same magnitude, the effects from all other sectors cancel out in equilibrium. Fur-

[4] Bai, Jin, and Lu (2025) extend the endogenous technology model in Eaton and Kortum (2001) to multiple sectors and study optimal dynamic trade policies with endogenous technology accumulation. The multi-sector versions of Krugman with identical scale and trade elasticity across sectors are isomorphic to Bai, Jin, and Lu (2025) at the steady state.

thermore, the difference between a sector's own-price supply elasticity (scaled by its size and supply parameter) and its cross-price elasticity with respect to any other sector (scaled by that sector's size and supply parameter) equals one. As a result, optimal relative import tariffs between any two sectors depend only on those sectors' supply elasticity parameters and the home country's net import shares relative to foreign income.

Note that the CES supply system does not imply that each sector's partial supply elasticity is constant, nor that the self-elasticity of each sector's export supply to its price is fixed at η_j. This distinguishes our framework from partial equilibrium models that assume exogenous, constant self-elasticities of foreign export supply. Moreover, under general equilibrium, the CES supply system generates non-zero cross-elasticities between sectors, which depend on the sector's (the price changing sector) size and supply elasticity. The defining feature of a CES supply system is its symmetry property in cross-elasticities, which simplifies the formulas of optimal trade policies despite cross-sector interdependencies.

An example of a non-CES supply system helps to show how asymmetric interdependencies across sectors matter for optimal policies. Consider a multi-factor Ricardo-Roy model, where workers with different skills select into different sectors based on comparative advantage. Price hence wage changes in any two sectors can asymmetrically affect supply in a third sector, depending on the types of workers employed in the first two sectors and their likelihood of migrating to the third. This multi-factor reallocation creates asymmetric interdependencies across sectors, requiring the optimal tariff formula to incorporate all self- and cross-sector supply elasticities explicitly.

The multi-country framework differs from the two-country case in two key respects. First, even if the supply system within individual countries satisfies the CES supply system requirement, this property may not extend across foreign countries. For example, labor is more substitutable within domestic markets than across countries. This asymmetry in factor mobility breaks the symmetry of cross-elasticities across sectors and countries even under CES supply systems. Second, the home government lacks sufficient instruments to directly influence trade flows between any two foreign countries. In contrast, in the two-country case, the home government can use sectoral-level import tariffs and export taxes to directly influence bilateral trade flows between Home and Foreign. In fact, when foreign countries do not trade with each other, or when they are symmetric, Home's optimal import tariffs and export taxes would again follow the same formulas based on bilateral trade flows between Home and each foreign country.

Since trade between foreign countries cannot be directly affected, price changes in any two country-sectors affect third-country exports to Home asymmetrically, as they depend on the changes in all foreign demand for third-country goods. This yields endogenous self-elasticities and asymmetric endogenous cross-elasticities that depend on the trade network structure. As a result, optimal tariff formulas must depend on all these endogenous cross-country-sector and self-elasticities explicitly.

To isolate these trade network effects, we consider constant returns to scale production and imperfectly substitutable labor, which generate a single supply elasticity parameter κ for all sectors. If the home country imposes a higher tariff on a heavily imported country-sector, this may lead to a reduction in wages in that country-sector. As a result, other countries increase their imports from that country-sector, further decreasing its exports to the home country, resulting in a larger endogenous own-elasticity. Moreover, an increase in wages in other countries can create an income effect that boosts demand for goods from this sector, generating higher negative cross-elasticities. These factors lead to a relative reduction in the optimal tariff for that specific heavily imported country-sector, ultimately fostering convergence of optimal tariffs within the sector across countries.

Ignoring cross-country interdependency and simply applying bilateral trade policy formulas would reduce potential welfare gains. Moreover, since supply elasticities and imports are endogenous variables that respond to changes in Home's policies, using pre-tax trade shares while ignoring how trade policies themselves alter these shares can generate welfare losses rather than gains. Through numerical examples, we demonstrate that these welfare costs become larger when foreign countries are more asymmetric and engage in more extensive trade with each other.

We quantify optimal policies for the U.S. and China with 20 countries and 13 sectors, using sectoral trade and production data. By employing explicit first-order conditions for optimal policies, our method achieves notable gains in accuracy and computational efficiency. Quantitatively, we find that optimal tariffs differ across countries and sectors, reflecting Home's varying buyer power across different markets. Meanwhile, tariffs within a given sector tend to converge across countries. To illustrate this convergence, we compute the slope of sectoral optimal tariffs with respect to Home's scaled net import shares across sectors for each country. In a two-country or symmetric foreign countries model with CES supply systems, this slope should be equal to one. However, our quantitative analysis reveals that the slopes differ from 1, indicating that optimal tariffs are not solely determined by each country's bilateral relationship with Home, but are also influenced by their trade network. Thus, optimal tariffs derived from observed trade patterns exhibit small heterogeneity across countries.

We find that two factors drive convergence in optimal policies across sectors. First, the high trade costs or small technology variations implied by observed trade data reduce dispersion in optimal policies across sectors. For two-digit sectors across countries, both the level and standard deviation of the U.S. or Chinese import expenditure shares are very small, indicating high trade costs and limited heterogeneity in technology across sectors and foreign countries.

Second, as we discussed above, the model has an endogenous convergence mechanism operating through the general equilibrium effect. The trade network increases the elasticity of foreign countries' export supply, and both supply elasticities and trade flows respond endogenously to Home's policies. When higher tariffs are imposed on large import sectors, self-supply elasticity rises while import shares fall, leading to tariff convergence. Our welfare analysis confirms the importance of accounting for these endogenous effects. Policies using our formula, but based on *bilateral*, endogenous trade shares, yield smaller welfare gains than our optimal approach. Furthermore, policies using our formula, but with *bilateral observed* trade shares, yield only 80% of the optimal welfare gains for the U.S. and 77% for China, as these policies overlook endogenous changes in trade patterns and export supplies.

Our paper is mostly related to the literature on optimal trade policies in GE models.[5] Our approach of providing a general formula for optimal trade policies can be traced back to the classic work of Dixit (1985), which sets up the general problem of optimal taxes in

[5]See Caliendo and Parro (2022) for a review of optimal trade policies in general equilibrium models.

an open economy as a fictitious planning problem and derives the associated first-order conditions under a given foreign offer curve. However, Dixit's framework does not provide directly applicable formulas for contemporary trade models. The endogenous foreign offer curves and self- and cross-elasticities of foreign export supply under various specifications were not well understood in these workhorse GE trade models. We use the Lagrange multiplier method and derive optimal policies, trade shares, and prices from the home government's first-order conditions and equilibrium conditions. This method allows us to derive optimal policies explicitly, ensuring accurate theoretical explanations and efficient computation.

Our model abstracts from cross-country technology diffusion. Technology diffusion and innovation are important considerations for industry and trade policies. Bai, Jin, Lu, and Wang (2025) studies optimal dynamic trade policies when there is technology diffusion through imports. Hémous, Lepot, Sampson, and Schärer (2023) quantitatively analyzes optimal patent policies for open economies. Santacreu (2025) presents a quantitative theory of bilateral trade agreements that include intellectual property (IP) provisions. We abstract from incomplete markets, as emphasized in Waugh (2023) and Donald, Fukui, and Miyauchi (2023), where first-best allocations are difficult to achieve.

Our paper makes three contributions. First, we derive exact optimal policy formulas within a unified framework with general supply-side specifications, strengthening both theoretical clarity and computational efficiency. Second, we bridge the gap between traditional literature that relies on simplified general equilibrium or partial equilibrium models and contemporary trade models that emphasize sectoral and cross-country interdependencies. Third, while quantitative models have recognized the importance of imperfectly mobile labor and extensively analyzed their trade implications, optimal trade policies within these frameworks remain largely unexplored. Our paper fills the gaps by providing a theoretical optimal taxation formula for both general and specific cases of labor markets commonly used in quantitative trade models, and by quantifying their welfare impacts.

The paper proceeds as follows: Section 2 extends the multi-country, multi-sector trade framework to incorporate general supply-side specifications. Section 3 derives the optimal policies; in particular, Section 3.1 works with a two-country case to focus on cross-sector interdependencies, Section 3.2 examines how trade networks and interdependencies in multi-country models affect the endogenous foreign export supply elasticities and, consequently, the optimal policies. Section 4 quantifies the optimal unilateral policy for the US and China across 20 countries, along with the Nash optimal policies and the associated welfare changes. Section 5 concludes.

2 Theoretical Framework

We study optimal trade and domestic policies in a multi-country, multi-sector general equilibrium model with a generic supply system that encompasses various supply-side specifications from existing models.

The world consists of N countries and J sectors. Each country n has a measure of $\bar{L}_n$ labor, with Country 1 designated as Home. All sectors utilize labor as the sole input. Consumers in all countries have the same utility function on final goods C given by $u(C) = C$. The final goods is a Cobb-Douglas function over the consumptions of different sector $j \in J$ goods, $C_n = \prod_{j \in J} \left(C_{n,j}\right)^{\beta_{n,j}}$, where the constant $\beta_{n,j}$ captures the consumption share of sector j in country n. All goods are tradable under an iceberg trade cost d_{ni} between country n and i.

Within each sector, preference and technology assumptions can be those in Eaton and Kortum (2002) (EK) or Armington model, or a multi-sector model with external economies of scale, which isomorphic to a generalized multi-sector Krugman et al. (1980) model of internal economies of scale. As shown in Kucheryavyy, Lyn, and Rodríguez-Clare (2023), the equilibrium conditions across all these models can be characterized by a unified system of general equilibrium conditions. We denote ϵ_j as the elasticity of bilateral trade flows with respect to bilateral trade costs in sector j. The set $\mathbb{S}$ represents general supply-side assumptions and is defined as $\mathbb{S} = \{(p_{n,j}, w_{n,j}, L_{n,j})\}$. For example, in a generalized multi-sector Krugman model, $\mathbb{S} = \left\{(p_{n,j}, w_{n,j}, L_{n,j}) : p_{n,j} = \bar{T}_{n,j}^{-1} w_{n,j} L_{n,j}^{1/(1-\sigma_j)}\right\}$ with $\bar{T}_{n,j}$ denoting the exogenous technology level beyond the economies of scale in sector j of country n.

In addition to these supply-side assumptions, we incorporate general labor market specifications that encompass those extensively studied in the trade and labor literature. The labor market specification determines wages and labor in equilibrium. Let $w_{n,j}$ and $L_{n,j}$ be the wage and labor in sector j of country n, and let Ω represent the equilibrium labor conditions. For example, when labor is perfectly mobile across sectors, we can define $\Omega = \{(w_{n,j}, L_{n,j}) : \sum_{j=1}^{J} L_{n,j} = \bar{L}_n,\ w_{n,j} = w_n\}$. Section 3.1 provides specific examples of Ω and the associated optimal policies, including extensions to multi-factor g with endowment $\bar{L}_{n,g}$.

Suppose that Home imposes unilateral trade and domestic policies, but foreign countries are passive with zero taxes. Specifically, the home government imposes country-sector-specific import tariffs $\tau^m_{n,j}$, export taxes $\tau^x_{n,j}$, and sector-specific domestic sales tax τ^d_j on its imports, exports, and domestic sales, respectively. Let P_n and x_n be the country n's final goods price and expenditures, respectively, with $x_n = P_n C_n$.[6] Below, we define the world market equilibrium under these Home's policies.

Definition 1 (World Market Equilibrium)**.** *The world market equilibrium consists of an allocation of labor $\{L_{n,j}\}$, consumption $\{C_n\}$, expenditures $\{x_n\}$, consumer price indexes $\{P_n\}$, prices $\{p_{n,j}\}$ and wages $\{w_{n,j}\}$ such that consumers maximize utility, firms maximize profits, and the market clearing conditions hold, taking as given the home government's policies $\left\{\tau^m_{n,j}, \tau^x_{n,j}, \tau^d_j\right\}$:*

1. *The labor market specifications $\Omega(\{w_{n,j}, L_{n,j}\})$ hold.*
2. *The supply-side assumptions $\mathbb{S}(\{p_{n,j}, w_{n,j}, L_{n,j}\})$ hold.*
3. *Trade shares satisfy, for each sector j*

$$\pi_{11,j} = \frac{(p_{1,j}(1+\tau^d_j))^{-\epsilon_j}}{(p_{1,j}(1+\tau^d_j))^{-\epsilon_j} + \sum_{i\neq 1}^{N}(p_{i,j}(1+\tau^m_{i,j})d_{1i})^{-\epsilon_j}}, \tag{1}$$

$$\pi_{1n,j} = \frac{(p_{n,j}(1+\tau^m_{n,j})d_{1n})^{-\epsilon_j}}{(p_{1,j}(1+\tau^d_j))^{-\epsilon_j} + \sum_{i\neq 1}^{N}(p_{i,j}(1+\tau^m_{i,j})d_{1i})^{-\epsilon_j}}, \quad n \neq 1, \tag{2}$$

$$\pi_{n1,j} = \frac{(p_{1,j}(1+\tau^x_{n,j})d_{n1})^{-\epsilon_j}}{(p_{1,j}(1+\tau^x_{n,j})d_{n1})^{-\epsilon_j} + \sum_{i\neq 1}^{N}(p_{i,j}d_{ni})^{-\epsilon_j}}, \quad n \neq 1, \tag{3}$$

$$\pi_{nm,j} = \frac{(p_{m,j}d_{nm})^{-\epsilon_j}}{(p_{1,j}(1+\tau^x_{n,j})d_{n1})^{-\epsilon_j} + \sum_{i\neq 1}^{N}(p_{i,j}d_{ni})^{-\epsilon_j}}, \quad n \neq 1. \tag{4}$$

[6] We assume perfect domestic financial market and C_n is the household's real consumption in country n.

4. *Expenditures are given by*

$$x_1 = \sum_{j=1}^{J} Y_{1,j} + \sum_{j=1}^{J}\sum_{i\neq 1}^{N} \beta_{i,j} \frac{\tau^x_{i,j}}{1+\tau^x_{i,j}} \pi_{i1,j} x_i + \sum_{j=1}^{J} \beta_{1,j} \sum_{i\neq 1}^{N} \frac{\tau^m_{i,j}}{1+\tau^m_{i,j}} \pi_{1i,j} x_1 + \sum_{j=1}^{J} \beta_{1,j} \frac{\tau^d_j}{1+\tau^d_j} \pi_{11,j} x_1, (\gamma_x) \tag{5}$$

$$x_n = \sum_{j=1}^{J} Y_{n,j}, \quad n \neq 1, \tag{6}$$

where $Y_{n,j} = w_{n,j} L_{n,j}$ is the income in sector j of country n.

5. *Consumer price indexes are given by*

$$P_1 = \prod_{j=1}^{J} \left[(p_{1,j}(1+\tau^d_j))^{-\epsilon_j} + \sum_{i\neq 1}^{N} (p_{i,j}(1+\tau^m_{i,j}) d_{1i})^{-\epsilon_j} \right]^{-\frac{\beta_{1,j}}{\epsilon_j}}, \tag{7}$$

$$P_n = \prod_{j=1}^{J} \left[(p_{1,j}(1+\tau^x_{n,j}) d_{n1})^{-\epsilon_j} + \sum_{i\neq 1}^{N} (p_{i,j} d_{ni})^{-\epsilon_j} \right]^{-\frac{\beta_{n,j}}{\epsilon_j}}, \quad n \neq 1. \tag{8}$$

6. *Goods market clearing conditions, for each sector j*

$$Y_{1,j} - \sum_{i\neq 1}^{N} \beta_{i,j} \frac{1}{1+\tau^x_{i,j}} \pi_{i1,j} x_i = \beta_{1,j} \frac{1}{1+\tau^d_j} \pi_{11,j} x_1, \quad (\gamma_{1,j}) \tag{9}$$

$$Y_{n,j} - \sum_{i\neq 1}^{N} \beta_{i,j} \pi_{in,j} x_i = \beta_{1,j} \frac{1}{1+\tau^m_{n,j}} \pi_{1n,j} x_1, \quad (\gamma_{n,j}), \quad n \neq 1. \tag{10}$$

Without loss of generality, we normalize the price of Home's final goods, $P_1 = 1$. Note that in the world market equilibrium, the market-clearing conditions and the definition of expenditures imply that the balanced trade conditions hold for each country.

3 Optimal Unilateral Policies

This section examines the unilateral policies of the home country, assuming that foreign governments remain passive. We consider country-sector-specific export taxes, import tariffs, and sector-specific domestic taxes to address the externalities arising from terms of trade and potential domestic market frictions.

Definition 2. *Home government's problem is to choose country-sector-specific trade policies* $\{\tau^x_{n,j}, \tau^m_{n,j}\}$ *and sector-specific domestic policies* $\{\tau^d_j\}$*, for* $\forall j, n \neq 1$*, to maximize domestic consumers' consumption,* $\max x_1/P_1$*, subject to the world market equilibrium given by Definition 1.*

Section 3.1 characterizes optimal policies for the problem defined in Definition 2 within a two-country setting, highlighting the impact of cross-sector interdependencies due to factor allocation and returns to scale. We show that for models featuring symmetric cross-elasticities in foreign supply curves, the relative import tariffs only depend on sectoral parameters and Home's relative net import shares across sectors. Section 3.2 extends the analysis to multiple countries, introducing both cross-sector and cross-country interdependencies. We establish that import tariffs generally have two key features: heterogeneity across countries and sectors and convergence within each sector. We illustrate how the underlying trade network—shaped by technology and trade barriers—determines the relative strength of these heterogeneity and convergence forces.

Below, we first provide the property of tax neutrality, which establishes equivalency between different policies and allows for the normalization of certain domestic taxes, export taxes, and tariffs, then proceed to present the optimal policies in the following propositions.

Proposition 1 (Tax Neutrality). *Consider two sets of Home government's trade policies* $\Gamma = \left\{\left(1+\tau^m_{n,j}, 1+\tau^x_{n,j}, 1+\tau^d_j\right) : \forall n \neq 1, j\right\}$ *and* $\check{\Gamma} = \left\{\left(\lambda(1+\tau^m_{n,j}), \frac{1+\tau^x_{n,j}}{\mu}, \lambda\frac{1+\tau^d_j}{\mu}\right) : \forall n \neq 1, j\right\}$ *for any constants* $\mu > 0$ *and* $\lambda > 0$*. When the labor market allocation in* Ω *is homogeneous of degree zero on wages, and prices in* $\mathbb{S}$ *are homogeneous of degree one on wages, the welfare and allocations of the world market equilibrium under* Γ *and* $\check{\Gamma}$ *are the same.*

Proof: Appendix A. This proposition shows that tax neutrality holds in the unilateral problem if the supply-side specifications have homogeneity properties. The home government can manipulate $2J(N-1)+J$ prices. According to Proposition 1, we can first normalize one domestic tax to zero, for example, $\check{\tau}^d_1 = 0$ for sector 1. For any τ^d_1, we can always choose λ and μ, such that $\mu/\lambda = 1+\tau^d_1$ to get $\check{\tau}^d_1 = \lambda(1+\tau^d_1)/\mu - 1 = 0$ and scale other taxes accordingly. The welfare and allocation are equivalent. Second, we can set an import tariff from one specific country and sector to zero, for example, $\check{\tau}^m_{2,1} = 0$

by setting $\lambda = 1/(\tau^m_{2,1} + 1)$ for any $\tau^m_{2,1}$ and adjusting other taxes accordingly to achieve the same allocation. Alternatively, one of the export taxes, $\check{\tau}^x_{2,1}$, can be normalized to zero by setting $\mu = 1 + \tau^x_{2,1}$ for any $\tau^x_{2,1}$. In either case, it suffices for the government to use only $2J(N-1) + J - 2$ instruments to implement the same allocation. In our optimization framework, we normalize one domestic tax and one trade policy instrument to uniquely determine the level of all remaining taxes.

3.1 Cross-Sector Interdependencies and Optimal Policies

Here, we isolate the impact of cross-sector interdependencies on optimal policies by analyzing the case with two countries, Home and Foreign.

We solve the home government's optimization problem using the Lagrange multiplier method through a two-step approach. In the first step, we derive each country's supply curves $Y_{n,j}\left(\{p_{n,k}\}\right)$ where $Y_{n,j}$ represents the sectoral income as $Y_{n,j} = w_{n,j}L_{n,j}$. This derivation combines the supply-side assumption $\mathbb{S}\left(\{p_{n,j}, w_{n,j}, L_{n,j}\}\right)$ with the labor market specification $\Omega\left(\{w_{n,j}, L_{n,j}\}\right)$. We can then express the equilibrium variables, including $\{w_{n,j}, L_{n,j}\}$, as functions of sectoral prices $\{p_{n,j}\}$ and policy instruments. In the second step, we take first-order conditions with respect to expenditures, sectoral prices, and policy instruments. We impose multipliers on the goods-market clearing conditions while treating income $Y_{n,j}$ as functions of sectoral prices $\{p_{n,k}\}$ through the derived supply curves. Additional details are provided in Appendix B.

This approach provides a unified framework for characterizing optimal policies by first deriving supply curves that encapsulate the underlying supply structure. A key advantage is its generality: One can readily apply the resulting general optimal formulas to any specifications of $\mathbb{S}$ and Ω. The policy formulas obtained through this approach also offer clear economic intuition, with interpretations built on supply elasticities and trade shares that reflect the home country's market power.[7]

While the supply-curve approach shares similarity with the classic literature, our en-

[7] An alternative approach involves directly taking first-order conditions with respect to $\{w_{n,j}, L_{n,j}\}$ and imposing multipliers on the labor market specifications Ω. We can determine $\{p_{n,j}\}$ using the supply-side assumptions $\mathbb{S}(\{p_{n,j}, w_{n,j}, L_{n,j}\})$. Both approaches yield identical optimal policies and equilibrium outcomes. We demonstrate the equivalence of these two approaches across three examples in Appendix C.

dogenous supply curves capture important cross-sector linkages, with sectoral income $Y_{n,j}$ depending not only on own-sector prices $p_{n,j}$ but also on prices across all other sectors $p_{n,k}$. These interdependencies arise from the general equilibrium nature of factor allocation and production technologies with returns to scale. As a result, policy interventions in one sector generate spillover effects that propagate throughout the economy.

According to Walras' law, one equilibrium condition is redundant; thus, we drop Home's expenditure constraint from the optimization problem and only consider the goods market clearing conditions and the supply curves derived from $\mathbb{S}$ and Ω. Let $\gamma_{1,j}$ and $\gamma_{2,j}$ be the multipliers on the goods market clearing conditions (9) and (10), respectively. [8]

Lemma 1 (**Two-country Optimal Policies and Multipliers**). *Irrespective of supply-side specifications, optimal policies in sector j take the form of*

$$1+\tau_j^d = -\gamma_{1,j}, \quad 1+\tau_j^m = -\gamma_{2,j}, \quad 1+\tau_j^x = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\epsilon_j \pi_{22,j}}\right). \tag{11}$$

Proof. Appendix B.

These formulas reflect the two reasons why the home government utilizes taxes: to manipulate terms of trade and to address domestic distortions. Terms-of-trade gains come from Home's ability to change import and export prices by affecting the demand that Foreign faces and the supply of Home goods to Foreign. In other words, these gains depend on the Home country's buyer power in the foreign goods market and its monopoly power as an exporter.

The multiplier $\gamma_{2,j}$ captures how changing foreign goods demand affects Home utility, thus reflecting the home government's consideration of exploiting its buyer power. When $\gamma_{2,j}$ is positive, the home government would like to relatively subsidize imports of sector j goods, yielding the relationship $1+\tau_j^m = -\gamma_{2,j}$. Regarding domestic distortions, the home government uses domestic policies to correct market inefficiencies. Domestic sales taxes τ_j^d are determined by the multiplier $\gamma_{1,j}$, which measures how changes in domestic

[8] Equivalently, if we use Home's budget constraint, one of the market-clearing conditions becomes redundant. By utilizing Home's budget constraint and γ_x, we can eliminate one of the goods market-clearing conditions by setting $\gamma_{2,J}=0$. The same optimal policy formulas can be attained. We provide an example of the proof of Home's optimal policies in the two-country case in Appendix I.

goods demand impact Home utility. When $\gamma_{1,j}$ is positive, the home government relatively subsidizes the j sector. Export taxes depend on a combination of both multipliers, trade elasticity, and foreign expenditure shares on its own goods.

We further unpack these multipliers and show that the home government's market power depends on foreign supply curves $Y_{2,j}(\{p_{2,k}\})$ and the foreign country's domestic demand $\beta_{2,j}\pi_{22,j}x_2$, which is affected by Home's exports to Foreign. We define Foreign's supply elasticity and introduce a term representing Foreign's domestic demand effects. Together, these two components characterize optimal tariffs in the two-country case.

Definition 3. *Matrix Λ_n consists of scaled partial supply elasticities in country n, with the jth-row-ith-column entry as $\frac{\partial \ln Y_{n,i}}{\partial \ln p_{n,j}} \frac{Y_{n,i}}{Y_{n,j}}$. Vector Ψ_1 contains Home's sectoral net import spending share in foreign income, with the jth entry defined as $\frac{\beta_{1,j}x_1 - (1+\tau_j^d)Y_{1,j}}{Y_{2,j}}$.*

The following proposition prescribes the optimal policies for the general two-country case.

Proposition 2 (Two-Country General Optimal Policy Formula). *Home government's unilateral optimal policy satisfies,*

$$\begin{aligned}
&\textit{Domestic tax vector:} && (\Lambda_1 - I)(1+\boldsymbol{\tau^d}) && = 0, && (12)\\
&\textit{Import tariff vector:} && \boldsymbol{\tau^m} && = \Lambda_2^{-1}\Psi_1, && (13)\\
&\textit{Export tax:} && 1+\tau_j^x && = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\epsilon_j \pi_{22,j}}\right), \quad \forall j, && (14)
\end{aligned}$$

and the equilibrium conditions (1)-(10), where $\boldsymbol{\tau^d}$ and $\boldsymbol{\tau^m}$ are vectors of domestic taxes and import tariffs across sectors, respectively. I is an identity matrix.

Proof. Appendix B. First, domestic taxes are directly related to Home's scaled partial supply elasticities Λ_1, which depend on Home's own supply-side and labor market specifications. Second, with two countries, import tariffs directly depend on Foreign's scaled partial supply elasticity matrix Λ_2 and how important Home purchase is in Foreign, Ψ_1. Lastly, the export taxes depend on trade elasticity and the share of foreign spending on Home exports, given other taxes. All taxes are jointly determined with equilibrium allocations and prices satisfying conditions (1)-(10).

To demonstrate how various forces influence endogenous supply and demand and optimal policies, we focus below on a class of models featuring a CES supply system with symmetric cross-elasticities effects.

3.1.1 Optimal Policies under CES Supply System

Here, we study a class of models with *CES supply system*, as defined in Definition 4. This system offers two key advantages. First, many standard trade models fall within this system. Second, this system yields analytical policy formulas, even considering general equilibrium effects and interlinkages across sectors.

Definition 4 (CES Supply System). *A CES supply system satisfies*

$$\frac{\partial \ln Y_s}{\partial \ln p_j}\frac{Y_s}{Y_j}\frac{1}{\eta_j} - \frac{\partial \ln Y_s}{\partial \ln p_i}\frac{Y_s}{Y_i}\frac{1}{\eta_i} = \begin{cases} 0 & \textit{for } s \neq i, j \\ 1 & \textit{for } s = j, \end{cases} \tag{15}$$

for any sector s, i, j, where η_j is sector specific constant related to supply elasticity.

Under a CES supply system, the partial elasticities of a sector's income with respect to prices in any other two sectors (scaled by the size and elasticity parameters of the two sectors) are the same. Additionally, the difference between a sector's own-price supply elasticity (scaled by its size and supply parameter) and its cross-price elasticity relative to any other sector (scaled by that sector's size and supply parameter) equals one, i.e., $\frac{\partial \ln Y_j}{\partial \ln p_j}\frac{1}{\eta_j} - \frac{\partial \ln Y_j}{\partial \ln p_i}\frac{Y_j}{Y_i}\frac{1}{\eta_i} = 1$.

The CES supply system encompasses most quantitative trade models, including extended multi-sector Armington, Eaton-Kortum with exogenous technology, and different economies of scale across sectors like Armington with external economies of scale or the generalized Krugman model. It also accommodates various labor market specifications: perfectly mobile labor, fixed labor, and imperfectly substitutable labor across sectors as in the one-factor Ricardo-Roy model.

Proposition 3 (Two-country Optimal Policies under CES Supply System). *Under a CES supply system, the home government's unilateral optimal policies satisfy the following conditions* *for any two sectors j and i,*

$$\text{Domestic tax:} \quad 1+\tau_j^d = (1+\bar{\tau}^d)\frac{\eta_j}{\eta_j - 1}, \tag{16}$$

$$\text{Import tariff:} \quad \tau_j^m - \tau_i^m = \frac{1}{\eta_j}\frac{\beta_{1,j}x_1 - (1+\tau_j^d)Y_{1,j}}{Y_{2,j}} - \frac{1}{\eta_i}\frac{\beta_{1,i}x_1 - (1+\tau_i^d)Y_{1,i}}{Y_{2,i}}, \tag{17}$$

$$\text{Export tax:} \quad 1+\tau_j^x = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\epsilon_j \pi_{22,j}}\right). \tag{18}$$

and the equilibrium conditions (1)-(10).

Proof. Appendix B. Given the tax neutrality established in Proposition 1, one domestic tax level and one trade tax level remain indeterminate. Thus, the uniform shifter $\bar{\tau}^d$ in equation (16) can take any arbitrary value, and one trade tax can be normalized without changing the real allocations and welfare impacts of optimal policies.

This proposition demonstrates that under a CES supply system, Home imposes differential domestic taxes for all sectors if supply elasticities are heterogeneous. Note that if η_j are identical across sectors, Home imposes uniform domestic taxes, or equivalently zero domestic taxes by choosing $\bar{\tau}^d$. Given other taxes, sector j's export tax is higher if the demand elasticity ϵ_j is smaller, i.e., the demand is less elastic, and if $\pi_{22,j}$ is lower, which implies a greater foreign demand for Home goods. The schedule of export taxes is consistent with Costinot et al. (2015), which shows that export taxes reflect the home government's incentive to manipulate relative prices in its favor by exploiting its export market power.

The relative import tariffs between any two sectors depend only on the relative Home net import over foreign sectoral income and these two sectors' elasticity parameters η_j and η_i, which can be positive or negative. A larger Home net import spending relative to Foreign income, $\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}$, reflects greater Home buyer power and thus justifies a higher import tariff when $\eta_j > 0$ or a higher tariff subsidy when $\eta_j < 0$. Importantly, the relative import tariffs formula $\tau_j^m - \tau_i^m$ does not explicitly depend on Home's import share in other sectors. Furthermore, instead of depending on the whole supply elasticity matrix, where each sector's self-elasticity is not a constant and cross-elasticities are not zero, the relative import tariffs only depend on the parameters η_j and η_i.

In the following, we present three examples of the CES supply system and explain the mechanisms. The first focuses on labor supply and allocation across sectors. The second examines models with different returns to scale across sectors. The third explores different returns to scale across sectors combined with labor market specifications.

Example 1: One-Factor Ricardo-Roy Model (imperfectly substitutable inputs) We adopt the framework in Galle, Rodríguez-Clare, and Yi (2023), which combines the multi-sector EK model in Costinot, Donaldson, and Komunjer (2012) and the Roy model as in Lagakos and Waugh (2013) and Hsieh, Hurst, Jones, and Klenow (2019). We set up the general multi-factor setup and show that when there is only one factor, the model belongs to a CES supply system. See Appendix C.1 for proof.

There are G types of workers in each country. The total number of type-g workers in country n is fixed at $\bar{L}_{n,g}$. A type-g worker in country n draws an efficiency unit z_j in sector j from a Fréchet distribution $F_{n,j,g}$ with a shape parameter κ and a scale parameter $A_{n,j,g}$. A worker with the vector $\mathbf{z} = (z_1, z_2, \dots, z_J)$ chooses to work in a sector that gives the highest income based on her productivity and wage in that sector. Let $\Xi_{n,j}$ be the set of workers choosing sector j, $\Xi_{n,j} \equiv \{\mathbf{z}\colon w_{n,j}z_j \geq w_{n,k}z_k \text{ for all } k\}$ and $F_{n,g}(\mathbf{z})$ be the joint probability distribution of $\mathbf{z}$ for type-g workers across sectors in country n.

In equilibrium, the share of type-g workers that enter sector j in country n is given by

$$\lambda_{n,j,g} = \frac{A_{n,j,g}(w_{n,j})^{\kappa}}{\sum_{s=1}^{J} A_{n,s,g}(w_{n,s})^{\kappa}}.$$

The sum of type-g efficiency units supplied to sector j is $L_{n,j,g} = \frac{[\sum_{s=1}^{J} A_{n,s,g}(w_{n,s})^{\kappa}]^{\frac{1}{\kappa}}}{w_{n,j}} \lambda_{n,j,g}\bar{L}_{n,g}$, and the total efficiency labor working in sector j in country n is given by $L_{n,j} = \sum_{g=1}^{G} L_{n,j,g}$.[9] Thus, the labor market specification Ω satisfies

$$\Omega = \left\{ (w_{n,j}, L_{n,j}) : w_{n,j}L_{n,j} = \sum_{g=1}^{G} \left[\frac{A_{n,j,g}(w_{n,j})^{\kappa}}{W_{n,g}^{\kappa}} W_{n,g}\bar{L}_{n,g} \right], \; W_{n,g} = \left[\sum_{s=1}^{J} A_{n,s,g}(w_{n,s})^{\kappa} \right]^{\frac{1}{\kappa}} \right\}. \tag{19}$$

[9] The sum of type-g efficiency units supplied to sector j in country n is $L_{n,j,g} \equiv L_{n,g} \int_{\Xi_{n,j}} z_j dF_{n,g}(\mathbf{z}) = \frac{\xi_n [\sum_{s=1}^{J} A_{n,s,g}(w_{n,s})^{\kappa}]^{\frac{1}{\kappa}}}{w_{n,j}} \lambda_{n,j,g}\bar{L}_{n,g}$, where $\xi_n \equiv \Gamma(1 - 1/\kappa)$. For notation simplification, we scale $A_{n,s,g}$ by ξ_n^{κ}, i.e., setting $A_{n,s,g}$ to $\xi_n^{\kappa} A_{n,s,g}$.

With perfect competition in the goods market, price equals marginal cost in each sector. The supply-side assumption $\mathbb{S}$ therefore specifies that for any country n and sector j:

$$p_{n,j} = \frac{w_{n,j}}{\bar{T}_{n,j}}, \tag{20}$$

Consider the case with only one type of worker ($G = 1$).[10] We can derive supply curves by combining the labor market specification Ω (Eq.19) with the supply-side assumption $\mathbb{S}$ (Eq.20) and the definition of income $Y_{n,j} = w_{n,j}L_{n,j}$. Eliminating wages and labor $\{w_{n,j}, L_{n,j}\}$ yields the following supply curves:

$$Y_{n,j} = A_{n,j}\left(p_{n,j}\bar{T}_{n,j}\right)^{\kappa}\left[\sum_{s=1}^{J} A_{n,s}\left(p_{n,s}\bar{T}_{n,s}\right)^{\kappa}\right]^{\frac{1-\kappa}{\kappa}} \bar{L}_n.$$

The corresponding element of the scaled partial supply elasticity matrix Λ_n is given by

$$\frac{\partial \ln(Y_{n,s})}{\partial \ln(p_{n,j})}\frac{Y_{n,s}}{Y_{n,j}} = I_{s=j}(\kappa) - (\kappa - 1)\lambda_{n,s}, \tag{21}$$

where $I_{s=j}(\kappa)$ is an indicator function that equals κ when $s = j$ and 0 otherwise. $\lambda_{n,j} = Y_{n,j}/Y_n$ is the share of income in sector j produced in country n.

We can derive the optimal tariff formula in two equivalent ways. In the first way, we substitute the scaled partial elasticities in equation (21) into the general formula in Proposition 2 directly, i.e.,

$$\sum_{s=1}^{J} \tau_s^m \frac{\partial \ln(Y_{2,s})}{\partial \ln(p_{2,j})}\frac{Y_{2,s}}{Y_{2,j}} = \frac{\beta_{1,j}x_1 - (1+\tau_j^d)Y_{1,j}}{Y_{2,j}}, \quad \forall j, \tag{22}$$

Using the scaled partial supply elasticities from equation (21), then divide each sector j equation (22) by κ and taking differences between sectors i and j, we obtain the optimal tariff formula (17) in Proposition 3 under the CES supply system:

$$\tau_j^m - \tau_i^m = \frac{1}{\kappa}\frac{\beta_{1,j}x_1 - (1+\tau_j^d)Y_{1,j}}{Y_{2,j}} - \frac{1}{\kappa}\frac{\beta_{1,i}x_1 - (1+\tau_i^d)Y_{1,i}}{Y_{2,i}}.$$

[10]The one-factor Ricardo-Roy model is equivalent to a CES labor market characterized by constant elasticity of substitution of labor across sectors. See Appendix C.4 for details.

Alternatively, we can demonstrate that this model's supply system—featuring one-factor Ricardo-Roy structure with perfect goods market competition—satisfies the CES supply system requirements. Hence, the optimal policies satisfy the formula in Proposition 3. Using the elements from equation (21), the matrix Λ_n takes the form

$$\Lambda_n = \begin{pmatrix} \kappa - (\kappa-1)\lambda_{n,1} & \dots & -(\kappa-1)\lambda_{n,j} & \dots & -(\kappa-1)\lambda_{n,J} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa-1)\lambda_{n,1} & \dots & \kappa - (\kappa-1)\lambda_{n,j} & \dots & -(\kappa-1)\lambda_{n,J} \\ \dots & \dots & \dots & \dots & \dots \\ -(\kappa-1)\lambda_{n,1} & \dots & -(\kappa-1)\lambda_{n,j} & \dots & \kappa - (\kappa-1)\lambda_{n,J} \end{pmatrix}. \tag{23}$$

When we take the difference between any two rows of Λ_n (say i and j), all off-diagonal elements cancel out, yielding a diagonal matrix with κ on the diagonals. The CES supply system, as defined in Definition 4, is clearly satisfied with the constant parameter $\eta_i = \eta_j = \kappa$. This enables direct application of Proposition 3 to determine optimal policies. Given that η_j is constant across sectors, optimal domestic taxes are uniform, reflecting an efficient domestic market.

The salient symmetry feature of the partial supply elasticity matrix Λ_n produces the powerful tariff formula in (17). The cross-elasticities are symmetric, meaning any two sectors' prices have symmetric impacts on a third sector. This symmetry makes the third-sector supply responses do not show up explicitly in the optimal relative tariffs between any two sectors, even when cross-elasticities are non-zero. In this model, relative foreign supply curves are upward-sloping with an elasticity of $\kappa > 0$ (the labor substitutability parameter). To induce relatively lower foreign prices in a given sector, the home government must raise tariffs in that sector, which reduces domestic demand for foreign goods. Since labor is not perfectly substitutable across sectors, this demand reduction lowers wages and thus prices in the corresponding foreign sector, improving Home's terms of trade. Furthermore, Proposition 3 demonstrates that the home government should increase tariffs more in sectors where Home has larger import shares in foreign income, as Home has greater buyer power in these sectors.

Two important special cases merit attention. When $\kappa = \infty$, the Ricardo-Roy model reduces to the standard specification of perfectly mobile labor across sectors. Proposition 3 then implies that $\tau_j^m - \tau_i^m = 0$ for any two sectors i and j, generating uniform optimal tariffs across all sectors. When $\kappa = 1$, the model becomes equivalent to the sector-specific labor case, and optimal relative tariffs equal the relative net import shares of Home across any two sectors.

Example 2: Generalized Multi-Sector Krugman (different scale elasticity across sectors) We introduce increasing returns to scale through a generalized multi-sector Krugman with different scale elasticities across sectors. Appendix C.2 presents this model, proves it belongs to the CES supply system, and also provides a model-specific proof that yields the same optimal policy formula as in Proposition 3.

In this case, we assume perfectly mobile labor across sectors. The labor market specification Ω is characterized by equal wages across sectors, $w_{n,j} = w_n$ for any j, and labor market clearing $\sum_j L_{n,j} = \bar{L}_n$. The supply-side assumption $\mathbb{S}$ incorporates increasing return to scale with

$$p_{n,j} = \bar{T}_{n,j}^{-1} L_{n,j}^{-\psi_j} w_n, \tag{24}$$

for each country n and sector j. Here, the scale elasticity ψ_j relates to σ_j, the elasticity of substitution across varieties within a sector j through the relationship $\psi_j = 1/(\sigma_j - 1)$. Combining Ω and $\mathbb{S}$, the supply curves in country n are given by,

$$Y_{n,j} = w_n^{(1+\psi_j)/\psi_j} \left(p_{n,j}\bar{T}_{n,j}\right)^{-1/\psi_j}, \quad \text{where } w_n \text{ satisfies } \sum_{j=1}^{J} \left(p_{n,j}\bar{T}_{n,j}\right)^{-\frac{1}{\psi_j}} w_n^{\frac{1}{\psi_j}} = \bar{L}_n. \tag{25}$$

Using the implicit function theorem, we obtain $\frac{\partial \ln w_n}{\partial \ln p_{n,j}} = \frac{Y_{n,j}/\psi_j}{\sum_{s=1}^{J} Y_{n,s}/\psi_s}$. Incorporating these derivatives, we derive the scaled partial supply elasticities:

$$\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}} \frac{Y_{n,s}}{Y_{n,j}} = I_{s=j}\left(-\frac{1}{\psi_j}\right) + \frac{1}{\psi_j} \frac{(1+\frac{1}{\psi_s})Y_{n,s}}{\sum_{k=1}^{J} \frac{1}{\psi_k} Y_{n,k}}.$$

This model satisfies the requirements of the CES supply system with $\frac{1}{\eta_j} = -\psi_j$. In particular, the scaled partial supply elasticities of a sector's income with respect to price (scaled by the supply elasticity parameters) satisfy $\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}} \frac{Y_{n,s}}{Y_{n,j}}(-\psi_j) - \frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,i}} \frac{Y_{n,s}}{Y_{n,i}}(-\psi_i) = 0$ for $s \neq i$ or j, and $\frac{\partial \ln Y_{n,j}}{\partial \ln p_{n,j}} \frac{Y_{n,j}}{Y_{n,j}}(-\psi_j) - \frac{\partial \ln Y_{n,j}}{\partial \ln p_{n,i}} \frac{Y_{n,j}}{Y_{n,i}}(-\psi_i) = 1$.

We can then apply Proposition 3 to get optimal policies. Since the scale elasticity ψ_j is sector-specific and introduces inefficiency in the product market, optimal domestic taxes are no longer uniform across sectors. Meanwhile, optimal tariffs satisfy

$$\tau_j^m - \tau_k^m = -\psi_j \frac{\beta_{1,j} x_1 - (1+\tau_j^d) Y_{1,j}}{Y_{2,j}} + \psi_k \frac{\beta_{1,k} x_1 - (1+\tau_k^d) Y_{1,k}}{Y_{2,k}}.$$

Increasing return to scale generates a downward-sloping relative supply curve. In this scenario, to achieve lower prices in a foreign sector, the home government reduces import tariffs in that sector, thereby increasing domestic demand for foreign goods. This higher demand incentivizes foreign producers to allocate more labor to that sector. Due to the increasing returns to scale, Home consumers benefit from lower import prices. As a result, the home government should reduce tariffs more in sectors where Home has larger net import shares in foreign income.

Note that this generalized multi-sector Krugman framework is isomorphic to multi-sector Eaton-Kortum and Armington models with Marshallian economies of scale or endogenous technology but different returns to scale across sectors.[11] Our optimal policy formulas therefore apply to these models.

Example 3: Differential Scale Elasticities and Imperfectly Mobile Labor We now extend the generalized multi-sector Krugman model with imperfectly mobile labor. This example combines features of the previous two examples: imperfectly mobile labor across sectors, as in Example 1, and the increasing returns to scale, as in Example 2. Appendix C.3 proves that this model belongs to the CES supply system and provides a model-specific

[11]See Bartelme et al. (2025) for a trade framework with Marshallian externalities. Kucheryavyy, Lyn, and Rodríguez-Clare (2023) show the isomorphic gravity equations for models with varying returns to scale, including the generalized multi-sector Krugman model, multi-sector Eaton-Kortum and Armington models with external economies of scale. Eaton and Kortum (2001) allow for endogenous technology accumulation, where total labor is divided into two components: production labor and researchers. Labor moves freely across production and research, and technology endogenously depends on the number of researchers. Bai, Jin, and Lu (2025) extend this framework to a multi-sector setting and study the optimal dynamic trade policies. In steady-state equilibrium, the ratio of production labor to researchers remains constant, allowing technology to be expressed as a function of the sector's labor as in equation (A.76).

proof that derives the same optimal policy formulas stated in Proposition 3.

Specifically, the labor market specification Ω follows equation (19), and the supply-side assumption follows equation (24). Using these two specifications, we can derive the supply curves as

$$Y_{n,j} = A_{n,j} \left(p_{n,j} \bar{T}_{n,j} \right)^{\frac{\kappa}{1-\psi_j(\kappa-1)}} W_n^{1-\frac{\kappa}{1-\psi_j(\kappa-1)}} \bar{L}_n^{\frac{\psi_j \kappa}{1-\psi_j(\kappa-1)}+1}, \tag{26}$$

with $(W_n)^\kappa = \sum_{j=1}^J A_{n,j} \left(p_{n,j} \bar{T}_{n,j} W_n^{-\psi_j(\kappa-1)} \bar{L}_n^{\psi_j} \right)^{\frac{\kappa}{1-\psi_j(\kappa-1)}}$. Using implicit function theorem, we obtain scaled partial supply elasticities[12]

$$\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}} \frac{Y_{n,s}}{Y_{n,j}} = I_{s=j} \left(\frac{\kappa}{1-\psi_j(\kappa-1)} \right) + \frac{\kappa}{1-\psi_j(\kappa-1)} \frac{\left(1 - \frac{\kappa}{1-\psi_s(\kappa-1)}\right) Y_{n,s}}{\sum_{k=1}^J \frac{\kappa}{1-\psi_k(\kappa-1)} Y_{n,k}}.$$

It is straightforward to show that these elasticities satisfy the requirement of CES supply systems, where parameters η_j incorporate both the scale elasticity and labor market parameter:

$$\frac{1}{\eta_j} = \frac{1}{\kappa} - \psi_j \frac{\kappa - 1}{\kappa}. \tag{27}$$

We apply Proposition 3 to determine the optimal policies. Since the scale elasticity ψ_j is sector-specific and introduces inefficiency in the product market, optimal domestic taxes are no longer uniform across sectors. Whether the foreign relative supply curve in sector j is upward- or downward-sloping depends on the sign of the parameter η_j, which influences whether the home government imposes a relatively higher or lower tariff on this sector.

In sum, the two-country case with a CES supply system yields a clear formula showing that Home's relative optimal tariffs depend on its buyer power across foreign sectors. By emphasizing buyer power, this formula echoes traditional optimal tariff theory while differing significantly from those partial equilibrium setups that use exogenous, constant foreign export elasticities or overlook sectoral linkages. The key distinction lies in how our formula captures interdependencies across sectors through general equilibrium effects. Although

[12]When deriving the scaled partial supply elasticities, we have incorporated the derivative $\partial \ln W_n / \partial \ln p_{n,j} = \frac{1}{1-\psi_j(\kappa-1)} Y_{n,j} / \sum_{s=1}^J \frac{1}{1-\psi_s(\kappa-1)} Y_{n,s}$.

the formula appears sector-independent at first glance, all underlying components—trade shares, prices, and trade policies—are jointly determined. Traditional formulations abstract away from these connections, treating foreign sectors in isolation. In the following subsection, we highlight the importance of *interdependence across sectors* by studying the two-country case with a general supply system.

3.1.2 Optimal Policies under General Supply System

When a model does not belong to CES supply systems, interdependence across sectors shows up explicitly: the cross-elasticities of foreign scaled supply curves are not symmetric, and the relative import tariffs across any two sectors depend not only on Home's relative net import shares of these two sectors but also on those of other sectors. Thus, the import tariff formula depends on a matrix of foreign partial supply elasticities and a vector of Home net import shares, as shown in Proposition 2. In other words, Home's buyer power across sectors is determined jointly.

One example of a non-CES supply system is the multi-factor Ricardo-Roy model, as laid out in Example 1. In this case, the supply-side assumption $\mathbb{S}$ is specified by equation (20) and Ω is defined as equation (19). Combining these two yields the following supply curves:

$$Y_{n,j} = \left(p_{n,j}\bar{T}_{n,j}\right)^{\kappa} \sum_{g=1}^{G} A_{n,j,g} \left[\sum_{s=1}^{J} A_{n,s,g} \left(p_{n,s}\bar{T}_{n,s}\right)^{\kappa}\right]^{\frac{1-\kappa}{\kappa}} \bar{L}_{n,g}$$

The corresponding element of the scaled partial supply elasticity matrix Λ_n is given by

$$\frac{\partial \ln(Y_{n,s})}{\partial \ln(p_{n,j})} \frac{Y_{n,s}}{Y_{n,j}} = I_{s=j}(\kappa) - (\kappa - 1) \frac{\sum_g \lambda_{n,s,g} \lambda_{n,j,g} W_{n,g} \bar{L}_{n,g}}{Y_{n,j}}$$

where $\lambda_{n,j,g} = Y_{n,j,g}/Y_{n,g}$ is the share of type-g workers that enter sector j in country n. In the matrix format, Λ_n takes the form of

$$\Lambda_2 = \begin{pmatrix} \kappa - (\kappa-1)\frac{\sum_g \lambda_{2,1,g}\lambda_{2,1,g}W_{2,g}\bar{L}_{2,g}}{Y_{2,1}} & ... & -(\kappa-1)\frac{\sum_g \lambda_{2,1,g}\lambda_{2,j,g}W_{2,g}\bar{L}_{2,g}}{Y_{2,1}} & ... & -(\kappa-1)\frac{\sum_g \lambda_{2,1,g}\lambda_{2,J,g}W_{2,g}\bar{L}_{2,g}}{Y_{2,1}} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\frac{\sum_g \lambda_{2,j,g}\lambda_{2,1,g}W_{2,g}\bar{L}_{2,g}}{Y_{2,j}} & ... & \kappa-(\kappa-1)\frac{\sum_g \lambda_{2,j,g}\lambda_{2,j,g}W_{2,g}\bar{L}_{2,g}}{Y_{2,j}} & ... & -(\kappa-1)\frac{\sum_g \lambda_{2,j,g}\lambda_{2,J,g}W_{2,g}\bar{L}_{2,g}}{Y_{2,j}} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\frac{\sum_g \lambda_{2,J,g}\lambda_{2,1,g}W_{2,g}\bar{L}_{2,g}}{Y_{2,J}} & ... & -(\kappa-1)\frac{\sum_g \lambda_{2,J,g}\lambda_{2,j,g}W_{2,g}\bar{L}_{2,g}}{Y_{2,J}} & ... & \kappa-(\kappa-1)\frac{\sum_g \lambda_{2,J,g}\lambda_{2,J,g}W_{2,g}\bar{L}_{2,g}}{Y_{2,J}} \end{pmatrix}.$$

It is easy to demonstrate that this general Ricardo-Roy model fails to meet the CES supply requirements. Foreign sector i's supply, represented by $Y_{2,i}$, is affected by the sector j's price $p_{2,j}$ through the expression $-(\kappa-1)\sum_g \frac{\lambda_{2,j,g}Y_{2,g}}{Y_{2,j}}\lambda_{2,i,g}$, and affected by sector J's price $p_{2,J}$ through $-(\kappa-1)\sum_g \frac{\lambda_{2,J,g}Y_{2,g}}{Y_{2,J}}\lambda_{2,i,g}$. In general, these cross-elasticities are asymmetrically influenced by endogenous variables and cannot be canceled out. Thus, the multi-factor Ricardo-Roy model is not a CES supply system.

The Home government internalizes the impact of its import tariffs on sectoral wages, which in turn influence worker allocation across sectors. Whether an increase in sector j's wage in Foreign reduces labor and supply in the foreign sector i depends on what type of workers in sector j, $\frac{\lambda_{2,j,g}Y_{2,g}}{Y_{2,j}}$, and how likely they enter sector i, i.e., $\lambda_{2,i,g}$. The covariance between $\frac{\lambda_{2,j,g}Y_{2,g}}{Y_{2,j}}$ and $\lambda_{2,i,g}$ indicates how likely workers in type g within sector j apply to sector i, and a higher covariance results in greater elasticity, meaning that changes in sector j's wage will have a more pronounced effect on sector i's income due to the increased likelihood of worker mobility between the two sectors. The covariance between sector j and i can differ from that between sector J and i. Thus, the impact of price changes of sector j and sector J on the third sector i is different. Home government takes these interdependencies into consideration and chooses trade and domestic policies all together.

We can directly apply Proposition 2 for optimal policies. In addition, even if the condition for the CES supply system is not met, we can still prove that the home government does not use any domestic taxes, i.e., $\tau_j^d = 0$ for any sector j. The reason is that the Ricardo-Roy model is efficient and has no market frictions. See Appendix D for details.

3.2 Cross-Country Interdependencies and Optimal Policies

This section analyzes the optimal policies under the multi-country setting. Beyond the sectoral interdependencies examined in the two-country case, the multi-country context introduces an additional layer of complexity through interdependencies across countries within the trade network. We first explain how optimal policies depend on trade networks, and conclude by exploring the heterogeneity and convergence inherent in optimal trade policies by comparing various scenarios. We demonstrate how comparative advantage, trade costs, and endogenous trade shares shape interdependencies, and thus trade policies and their associated welfare implications.

A multi-country framework differs from the two-country case in two key respects. First, even if the supply system within individual countries satisfies the CES supply system requirement, this property may not extend across foreign countries. For example, labor is more substitutable within domestic markets than across countries. This asymmetry breaks the symmetry of cross-elasticities across sectors and countries even under CES supply systems. Second, in the two-country case, the home government can use sectoral-level import tariffs and export taxes to directly influence bilateral trade flows between Home and Foreign. With multiple countries, however, the home government lacks sufficient instruments to directly influence trade flows between any two foreign countries. In fact, when foreign countries do not trade with each other or when they are symmetric, Home's optimal import tariffs and export taxes would again follow the same simple formulas based on bilateral trade flows between Home and any foreign country.

Therefore, with multiple countries, Home's optimal tariff formula for imports from a particular country depends not only on that country's supply curves but also on other countries' demand for that country's goods. Now, the key concept related to tariffs is the foreign export supply curve, which incorporates both foreign supply and demand systems. The export supply curve $E_{1n,j}$ of country n to Home in sector j is defined as

$$E_{1n,j} \equiv Y_{n,j} - \sum_{i\neq 1}^{N} \beta_{i,j}\pi_{in,j}x_i. \tag{28}$$

This expression corresponds to the left-hand side of goods market clearing conditions (10).

The following proposition characterizes Home's general optimal unilateral policies with multiple countries.

Proposition 4 (Optimal Unilateral Policies). *Irrespective of supply-side specifications, the home government's unilateral optimal policy on domestic taxes, import tariffs, and export taxes takes the following form:*

Domestic tax:

$$1+\tau_j^d = -\gamma_{1,j}, \tag{29}$$

where $\gamma_{1,j}$ is the multiplier on Home's goods market clearing condition (Eq.9). Furthermore,

$$\sum_{s=1}^{J}(1+\tau_s^d)\frac{\partial \ln Y_{1,s}}{\partial \ln p_{1,j}}\frac{Y_{1,s}}{Y_{1,j}} = 1+\tau_j^d.$$

Import tariff:

$$1+\tau_{n,j}^m = -\gamma_{n,j}, \tag{30}$$

where $\gamma_{n,j}$ is the multiplier on the goods market clearing condition of country n, sector j (Eq.10). Furthermore,

$$\begin{pmatrix} \frac{\partial \boldsymbol{E_{12}}}{\partial \ln \boldsymbol{p_2}} \circ \frac{1}{\boldsymbol{Y_2}} & \frac{\partial \boldsymbol{E_{13}}}{\partial \ln \boldsymbol{p_2}} \circ \frac{1}{\boldsymbol{Y_2}} & \dots & \frac{\partial \boldsymbol{E_{1N}}}{\partial \ln \boldsymbol{p_2}} \circ \frac{1}{\boldsymbol{Y_2}} \\ \frac{\partial \boldsymbol{E_{12}}}{\partial \ln \boldsymbol{p_3}} \circ \frac{1}{\boldsymbol{Y_3}} & \frac{\partial \boldsymbol{E_{13}}}{\partial \ln \boldsymbol{p_3}} \circ \frac{1}{\boldsymbol{Y_3}} & \dots & \frac{\partial \boldsymbol{E_{1N}}}{\partial \ln \boldsymbol{p_3}} \circ \frac{1}{\boldsymbol{Y_3}} \\ \dots & \dots & & \dots \\ \frac{\partial \boldsymbol{E_{12}}}{\partial \ln \boldsymbol{p_N}} \circ \frac{1}{\boldsymbol{Y_N}} & \frac{\partial \boldsymbol{E_{13}}}{\partial \ln \boldsymbol{p_N}} \circ \frac{1}{\boldsymbol{Y_N}} & \dots & \frac{\partial \boldsymbol{E_{1N}}}{\partial \ln \boldsymbol{p_N}} \circ \frac{1}{\boldsymbol{Y_N}} \end{pmatrix} \begin{pmatrix} 1+\boldsymbol{\tau_2^m} \\ 1+\boldsymbol{\tau_3^m} \\ \dots \\ 1+\boldsymbol{\tau_N^m} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\beta_1} x_1 \circ \boldsymbol{\pi_{12}} \circ \frac{1}{\boldsymbol{Y_2}} - \sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial E_{11,k}}{\partial \ln \boldsymbol{p_2}} \circ \frac{1}{\boldsymbol{Y_2}} \\ \boldsymbol{\beta_1} x_1 \circ \boldsymbol{\pi_{13}} \circ \frac{1}{\boldsymbol{Y_3}} - \sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial E_{11,k}}{\partial \ln \boldsymbol{p_3}} \circ \frac{1}{\boldsymbol{Y_3}} \\ \dots \\ \boldsymbol{\beta_1} x_1 \circ \boldsymbol{\pi_{1N}} \circ \frac{1}{\boldsymbol{Y_N}} - \sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial E_{11,k}}{\partial \ln \boldsymbol{p_N}} \circ \frac{1}{\boldsymbol{Y_N}} \end{pmatrix}, \tag{31}$$

where $\boldsymbol{E_{1n}} = [E_{1n,1}, ..., E_{1n,J}]'$, $\boldsymbol{p_n} = [p_{n,1}, ..., p_{n,J}]'$, $\boldsymbol{\tau_n^m} = [\tau_{n,1}^m, ..., \tau_{n,J}^m]'$, and $\boldsymbol{\pi_{1n}} = [\pi_{1n,1}, ..., \pi_{1n,J}]'$ are vectors of export supplies, prices, import tariffs, and trade shares across sectors from country n, respectively. Each element of the left-hand-side matrix is a $J * J$ matrix, with $(\frac{\partial \boldsymbol{E_{1n}}}{\partial \ln \boldsymbol{p_i}})_{k,j} = \frac{\partial E_{1n,k}}{\partial \ln p_{i,j}}$ representing the partial derivative of the k-th element of $\boldsymbol{E_{1n}}$ with respect to the j-th element of $\ln \boldsymbol{p_i}$. The $J * 1$ vector $\frac{\partial E_{11,k}}{\partial \ln \boldsymbol{p_n}}$ represents the partial derivative of $E_{11,k}$ with respect to each element of $\ln \boldsymbol{p_n}$, where the j-th entry corresponds to the derivative with respect to the j-th element of $\ln \boldsymbol{p_n}$. $\boldsymbol{\beta_1} = [\beta_{1,1}, ..., \beta_{1,J}]'$ is the vector of expenditure share of country 1. $\boldsymbol{Y_n} = [Y_{n,1}, ..., Y_{n,J}]'$ is the vector of income of country n. The operator $\circ$ denotes element-wise multiplication.

Export tax:

$$1+\tau^x_{n,j} = \frac{\gamma_{1,j}(1+\epsilon_j(1-\pi_{n1,j}))}{\sum_{i\neq 1}^N \gamma_{i,j}\epsilon_j\pi_{ni,j}} = (1+\tau^d_j)\frac{1+\epsilon_j(1-\pi_{n1,j})}{\sum_{i\neq 1}^N (1+\tau^m_{i,j})\epsilon_j\pi_{ni,j}}. \tag{32}$$

All policies are jointly determined with equilibrium conditions (1)-(10)*.*

Proof. Appendix E derives the optimality conditions and solves the optimal policies. According to the Proposition, domestic taxes are directly associated with Home's own partial supply elasticities, reflecting the Bhagwati-Johnson principle of targeting that instruments should be matched to the source of distortion. Import tariffs depend on the inverse of the matrix of foreign *export* supply elasticities. Given other taxes, export taxes depend on trade elasticities and foreign demand for Home goods.

These formulas reflect the underlying cross-sector interdependencies due to factor allocation and cross-country interdependencies arising through trade networks. Changes in one sector's policies alter factor prices and resource allocation across all sectors, which in turn affects prices and trade flows throughout the economy. Similarly, policy changes in one country propagate through trade linkages to influence foreign demand and supply, export supply responses, and ultimately the elasticities that govern optimal policy choices, creating complex feedback effects. Thus, all these elasticities and trade shares are jointly determined in general equilibrium and are affected by optimal policies themselves.

3.2.1 Export taxes

With multiple countries, the optimal export taxes exploit Home's monopoly power while accounting for trade interdependency. To see the monopoly power incentives, consider a simplified scenario in which all foreign countries are symmetric, resulting in uniform import tariffs across countries. Due to tax neutrality, one tariff can be normalized to zero, implying that all tariffs become zero. Under such conditions, the optimal export tax (32) simplifies to $1+\tau^x_{n,j} = \frac{1+\epsilon_j(1-\pi_{n1,j})}{\epsilon_j(1-\pi_{n1,j})}$. This indicates that the export tax for a specific country-sector rises with the market power (expenditure share $\pi_{n1,j}$) of the Home's goods in that country.

In the general setting with multiple asymmetric countries, Home imposes country-sector-specific export taxes, taking into account the import tariffs and trade shares of other countries, as in equation (32), to exploit Home's seller power.

3.2.2 Import tariffs

We now analyze the general formula of optimal import tariff (31) in Proposition 4. With multiple countries, the Home's optimal tariffs target the relative prices of foreign producers across countries and sectors, thereby relating to the multipliers $\gamma_{n,j}$ on goods market conditions. Furthermore, using the first-order conditions with respect to country-sector prices, Proposition 4 shows that the optimal tariffs can be expressed in equation (31) that depends on a vector related to Home's imports and a matrix of foreign export supply elasticities.

For convenience, we name the foreign-export-supply-elasticity matrix $d\boldsymbol{E}$:

$$d\boldsymbol{E} \equiv \begin{pmatrix} \frac{\partial E_{12}}{\partial \ln p_2} \circ \frac{1}{Y_2} & \frac{\partial E_{13}}{\partial \ln p_2} \circ \frac{1}{Y_2} & \cdots & \frac{\partial E_{1N}}{\partial \ln p_2} \circ \frac{1}{Y_2} \\ \frac{\partial E_{12}}{\partial \ln p_3} \circ \frac{1}{Y_3} & \frac{\partial E_{13}}{\partial \ln p_3} \circ \frac{1}{Y_3} & \cdots & \frac{\partial E_{1N}}{\partial \ln p_3} \circ \frac{1}{Y_3} \\ \cdots & \cdots & & \cdots \\ \frac{\partial E_{12}}{\partial \ln p_N} \circ \frac{1}{Y_N} & \frac{\partial E_{13}}{\partial \ln p_N} \circ \frac{1}{Y_N} & \cdots & \frac{\partial E_{1N}}{\partial \ln p_N} \circ \frac{1}{Y_N} \end{pmatrix}.$$

This matrix reflects the comprehensive effects of import tariffs on equilibrium prices and export supplies from foreign countries. In the following, we first examine the structure of the $d\boldsymbol{E}$ matrix, emphasizing how it reflects the interdependence across countries and sectors. We then present a key feature of optimal tariffs under multilateral trade: tariffs on imports from a specific country-sector pair may be loosely related to Home's net import from that sector. We demonstrate this feature through both theoretical analysis and numerical examples.

Decoding $d\boldsymbol{E}$ Matrix This matrix captures the overall network effect through trade, including both direct and indirect effects. Directly, imposing a tariff on a country n and sector j affects the price and the supply in this sector through the supply system. As a result, the matrix first reflects the direct impact of price $p_{n,j}$ on country n-sector j's own supply $Y_{n,j}$. Moreover, the demand for goods produced in the country-sector nj from other destination markets, excluding Home, will adjust in response to price changes, thereby affecting the supply of nj to Home. These direct effects of Home tariffs—on country n's supply and its foreign demands—are reflected in the diagonal elasticities $\frac{\partial \boldsymbol{E_{1n}}}{\partial \ln \boldsymbol{p_n}} \circ \frac{1}{\boldsymbol{Y_n}}$. In addition to these direct effects, such changes will influence prices globally and, in turn, affect global supply and demand, captured by the off-diagonal matrices of $d\boldsymbol{E}$. Ultimately, all these price changes will be transmitted into export supplies to the Home.

To understand these trade network effects, we consider constant returns to scale production and imperfectly substitutable labor, which generate a single supply elasticity parameter κ for all sectors. In this case, the diagonal blocks of $d\boldsymbol{E}$ become

$$\frac{\partial \boldsymbol{E_{1n}}}{\partial \ln \boldsymbol{p_n}} \circ \frac{1}{\boldsymbol{Y_n}} = \begin{pmatrix} \frac{\partial E_{1n,1}}{\partial \ln p_{n,1}} \frac{1}{Y_{n,1}} & \cdots & \frac{\partial E_{1n,J}}{\partial \ln p_{n,1}} \frac{1}{Y_{n,1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial E_{1n,1}}{\partial \ln p_{n,J}} \frac{1}{Y_{n,J}} & \cdots & \frac{\partial E_{1n,J}}{\partial \ln p_{n,J}} \frac{1}{Y_{n,J}} \end{pmatrix}$$

$$= \begin{pmatrix} \kappa - (\kappa-1)\lambda_{n,1} + \epsilon_1 \sum_{h\neq 1}^{N} (1-\pi_{hn,1}) s_{hn,1} - \beta_{n,1}\pi_{nn,1} & \cdots & -(\kappa-1)\lambda_{n,J} - \beta_{n,J}\pi_{nn,J} \\ \vdots & & \vdots \\ -(\kappa-1)\lambda_{n,1} - \beta_{n,1}\pi_{nn,1} & \cdots & \kappa - (\kappa-1)\lambda_{n,J} + \epsilon_J \sum_{h\neq 1}^{N} (1-\pi_{hn,J}) s_{hn,J} - \beta_{n,J}\pi_{nn,J} \end{pmatrix},$$

where $s_{hn,j} \equiv \frac{\pi_{hn,j}\beta_{h,j}x_h}{Y_{n,j}}$ is exporter nj's income share from an importing country h.

On the supply side, the term $[\kappa - (\kappa-1)\lambda_{n,j}]$ represents the partial supply elasticity $\frac{\partial \ln(Y_{n,j})}{\partial \ln(p_{n,j})}$, which reflects how price changes in a sector alter the sector's supply through production reallocation across sectors. On the demand side, two components measure how price changes affect foreign demand for country n's goods in sector j. The first component, $\epsilon_j \sum_{h\neq 1}^{N} (1-\pi_{hn,j}) s_{hn,j}$, captures the price effect on trade shares $\pi_{hn,j}$ across countries: as $p_{n,j}$ increases, country h reduces its demand for goods from country-sector nj, which increases nj's export supply to Home. The second component, $-\beta_{n,j}\pi_{nn,j}$, reflects the income effect: price changes alter country n's total spending, which affects demand for its own goods. The off-diagonal elements of the matrix $\frac{\partial \boldsymbol{E_{1n}}}{\partial \ln \boldsymbol{p_n}} \circ \frac{1}{\boldsymbol{Y_n}}$ are simpler. Because prices in sector j do not directly influence trade shares in other sectors $k \neq j$, the off-diagonal terms contain only two components: the supply effect of labor reallocation and the income effect.

In a two-country model, price changes affect trade only through the direct channels, and thus the $d\boldsymbol{E}$ matrix includes only matrix $\frac{\partial \boldsymbol{E_{12}}}{\partial \ln \boldsymbol{p_2}} \circ \frac{1}{\boldsymbol{Y_2}}$. However, when there are multiple countries, a price change in one country generates additional spillover effects by altering other countries' export supply to Home. These indirect effects appear in the off-diagonal blocks of $d\boldsymbol{E}$:

$$\frac{\partial \boldsymbol{E_{1i}}}{\partial \ln \boldsymbol{p_n}} \circ \frac{1}{\boldsymbol{Y_n}} = \begin{pmatrix} \frac{\partial E_{1i,1}}{\partial \ln p_{n,1}} \frac{1}{Y_{n,1}} & \cdots & \frac{\partial E_{1i,J}}{\partial \ln p_{n,1}} \frac{1}{Y_{n,1}} \\ \cdots & & \cdots \\ \frac{\partial E_{1i,1}}{\partial \ln p_{n,J}} \frac{1}{Y_{n,J}} & \cdots & \frac{\partial E_{1i,J}}{\partial \ln p_{n,J}} \frac{1}{Y_{n,J}} \end{pmatrix}$$

$$= \begin{pmatrix} -\epsilon_1 \sum_{h \neq 1}^{N} \pi_{hi,1} s_{hn,1} - \beta_{n,1} \pi_{ni,1} & \cdots & -\beta_{n,J} \pi_{ni,J} \\ \vdots & & \vdots \\ -\beta_{n,1} \pi_{ni,1} & \cdots & -\epsilon_J \sum_{h \neq 1}^{N} \pi_{hi,J} s_{hn,J} - \beta_{n,J} \pi_{ni,J} \end{pmatrix}.$$

The matrix $\frac{\partial \boldsymbol{E_{1i}}}{\partial \ln \boldsymbol{p_n}} \circ \frac{1}{\boldsymbol{Y_n}}$ demonstrates how changes in $p_{n,j}$ impact the export supply of sector k's good from country i to Home, by influencing global demand for country i's goods. The first term in the diagonal $-\epsilon_j \sum_{h \neq 1}^{N} \pi_{hi,j} s_{hn,j}$ captures the cross-country interdependency within sector j. This term depends on the covariance between $\pi_{hi,j}$ (country h's import share from country i in sector j) and $s_{hn,j}$ (how important country h is in buying goods from country n). The economic mechanism operates as follows: when country n's prices rise, other countries reduce their imports from n and increase their purchases from alternative suppliers, such as country i. This substitution effect reduces country i's supply for export to Home. The magnitude of this indirect effect depends on how likely countries importing from n also source goods from country i. Higher covariance amplifies the elasticity, meaning that country i's export to Home is more sensitive to price changes elsewhere. Ultimately, this covariance is shaped by the correlation between productivity and trade costs, and trade networks across countries. The second term, $-\beta_{n,k} \pi_{ni,k}$, captures the income effect. It describes how a change in $p_{n,j}$ affects country n's demand for country i's goods in sector k, which in turn affects country i's supply to Home.

In summary, the inverse of the foreign export supply matrix $d\boldsymbol{E}$ captures both direct and indirect effects of price changes due to tariffs. When a country mainly engages in trade with the home country, the off-diagonal elasticities are small, and the self-elasticity dominates. However, when a country trades with multiple countries, these multilateral connections influence the cross-elasticities of their exports to the Home country. Home's optimal tariffs across countries and sectors are jointly determined, considering how price adjustments in one country affect the export supply responses across all other countries and sectors. This interdependence tends to generate a convergence of tariffs across countries within the same sector. We establish this feature in the following analysis.

Optimal Tariffs: Convergence

Lemma 2. *Under a CES supply system with parameters κ and multiple countries,*

1. *the home government's unilateral optimal tariffs satisfy, for any sector j,*

$$\sum_{n\neq 1}^{N}(\delta_n + \tau^m_{n,j})Y_{n,j} = \frac{1}{\kappa}\left[\beta_{1,j}x_1 - Y_{1,j}\right], \tag{33}$$

   where δ_n is endogenous but independent of sectors;[13]

2. *when foreign countries are symmetric or when they do not trade with each other, optimal policies only depend on bilateral trade shares: for any sectors j and i,*

$$\tau^m_{n,j} - \tau^m_{n,i} = \frac{1}{\kappa}\left[\frac{\pi_{1n,j}\beta_{1,j}x_1 - \frac{1}{1+\tau^x_{n,j}}\pi_{n1,j}\beta_{n,j}x_n}{Y_{n,j}} - \frac{\pi_{1n,i}\beta_{1,i}x_1 - \frac{1}{1+\tau^x_{n,i}}\pi_{n1,i}\beta_{n,i}x_n}{Y_{n,i}}\right], \tag{34}$$

$$1+\tau^x_{n,j} = \frac{1}{1+\tau^m_{n,j}}\left(1+\frac{1}{\epsilon_j(1-\pi_{n1,j})}\right). \tag{35}$$

and equilibrium conditions (1)-(10) *are satisfied.*

Proof. Appendix F.

Equation (33) demonstrates that the average tariff on a sector is strongly related to Home's overall net import $(\beta_{1,j}x_1 - Y_{1,j})$ in that sector. When $\kappa > 0$, Home applies higher average tariffs to sectors where it has larger net imports. This tariff pattern reflects Home's incentive to reduce foreign wages in sectors where Home has greater buyer power. By

[13] See Appendix F for the δ_n expression.

depressing foreign wages, Home reduces its import prices and improves its overall terms of trade.

Importantly, this tariff pattern suggests that Home's tariffs on sector j imports from country n do not necessarily correlate with the magnitude of Home's bilateral imports from country n in that particular sector. The interconnected nature of multilateral trade networks creates situations where Home may impose substantial tariffs on imports from a specific country and sector, even when those bilateral flows are relatively small compared to other sectors.

The second point in Lemma 2 demonstrates that when foreign countries are symmetric or do not trade with each other, optimal policies again depend on bilateral trade shares, consistent with formulas (17) and (18) in the two-country case.[14] The sum of Home's net imports in sector j across all foreign countries, $\sum_{n\neq 1}^{N}[\pi_{1n,j}\beta_{1,j}x_1 - \frac{1}{1+\tau_{n,j}^x}\pi_{n1,j}\beta_{n,j}x_n]$, gives $\beta_{1,j}x_1 - Y_{1,j}$ and formula (33) still applies here.

Similar to the case with two countries and CES supply system, when foreign countries are symmetric, the symmetric cross-elasticities cancel out, and optimal differential tariffs between two sectors of a country depend solely on the bilateral trade shares of these two sectors. However, when foreign countries are asymmetric, optimal tariffs for each country-sector are jointly determined considering all self- and cross-elasticities across countries and sectors. In the numerical examples below, we use two asymmetric foreign countries to highlight the impact of cross-country interdependencies on optimal policies.

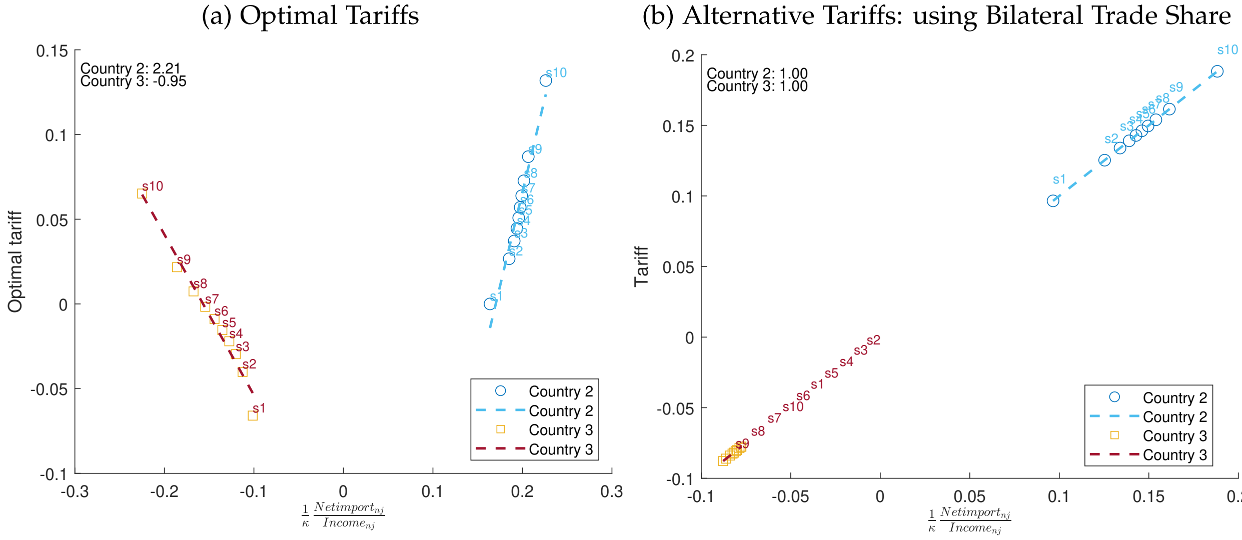
Consider an example with three countries and ten sectors.[15] Country 2 has a comparative advantage in sector 10, relative to Home (country 1), while country 3 has a comparative advantage in sector 1. The technology parameters are as follows: $T_1 = [50, ..., 1]$, $T_2 = [1, ..., 50]$, and $T_3 = T_1 \circ r$, where r represents values uniformly ranging from 2 to 0.5 with 10 elements. Due to asymmetric trade costs (listed in Case 4 of Table 1), Home imports significantly less from country 3 than country 2, while also exporting less to country 2.

Figure 1 plots tariffs against after-tax Home net import shares from each country-sector

[14]Note that in a two-country case, our defined net import in sector j, $\beta_{1,j}x_1 - Y_{1,j} = \pi_{12,j}\beta_{1,j}x_1 - \frac{1}{1+\tau_j^x}\pi_{21,j}\beta_{2,j}x_2$. Export taxes τ_j^x appear in the expression because they affect Home's export to foreign countries, hence foreign demand for its own goods.

[15]In all numerical examples, $\epsilon = 4, \kappa = 1.1, \bar{L}_n = 1, \forall n$. $A_{n,j} = 1, \forall n, j$.

Figure 1: Effects of Trade Network Between Foreign Countries



Note: These figures plot the relationship between Home's optimal import tariffs and Home net import share(defined as after-tax Home net import from a certain country-sector over the foreign country-sector income), normalized by κ, under two scenarios: the optimal tariffs (Panel a) and the tariffs Home would use if only considering bilateral trade share (Panel b).

relative to foreign country-sector income, normalized by κ. We refer to the slope of this relationship as the *tariff-import slope*. Panel (a) presents the optimal tariffs calculated using the optimal policies from Proposition 4 and the equilibrium conditions. A key takeaway is that although country 3 lacks a comparative advantage in sector 10, Home nevertheless imposes a higher tariff on imports of sector 10 from country 3.

The underlying mechanism operates as follows: Given Home's large imports from sector 10 of country 2, Home possesses large buyer power and would like to impose high tariffs on these goods. The high tariff reduces sector 10 wages in country 2, making country 2's exports more attractive to country 3. Hence, country 3 increases its imports from country 2, which further reduces country 2's export supply to Home. This increases the self-elasticity of sector 10 in country 2. The cross-elasticity becomes more negative for sector 10 because the income effects—which leads to increased imports by country 3 from country 2—is larger in this high-trade sector. Home thus exploits its monopsony power as a large buyer in sector 10, using tariffs to depress foreign wages and improve its terms of trade across all suppliers in that sector. As a result, Home's optimal tariffs on sector 10 converge across both foreign countries.

In Panel (b), we isolate the impact of trade network effects by analyzing an alternative policy where Home disregards the interdependence across countries and instead imposes import tariffs based solely on bilateral relationships, as in a two-country case, i.e., Home treats each foreign country as independent when determining policies. Specifically, the import tariffs and export taxes are given by equations (34) and (35), even though the foreign countries in this numerical example are asymmetric and trade with each other.[16]

According to equation (34), the tariff-import slope should be equal to one, as shown in Panel (b). Within each foreign country, the ranking of sectoral tariffs aligns with Home's after-tax net import share in that sector. Home imposes higher tariffs on sectors where its net imports are a higher fraction of foreign sectoral incomes. Given the opposing comparative advantages of countries 2 and 3 relative to Home, the sectoral tariff rankings are largely reversed across the two countries when only bilateral relationships are considered. However, the optimal ones align with Home's overall sectoral net imports. Ignoring trade networks and interdependence leads to a smaller welfare gain: 2.51%, only 76% compared to the gain (3.31%) achieved with optimal policies (Table 1, Case 4, Column 2).

Note that while the formulas (34) and (35) may not be optimal in this numerical example, the resulting trade shares, prices, and expenditures are still from the equilibrium induced by these formulas. The tariffs and export tax are consistent with the equilibrium following the alternative formulas. Thus, the policies largely capture the endogenous direct effect $\frac{\partial E_{1n}}{\partial \ln p_n}$, the self-elasticities, and imports changes. When prices change, they trigger sectoral reallocations and alter other foreign countries' imports from a specific country-sector, which in turn affects its export supply to the Home country. All these are incorporated into the equilibrium induced by the policies prescribed by (34) and (35). As a result, this alternative policy still captures 76% of welfare gains.

In contrast, policies simply putting *pre-tax* trade shares in (34) and (35) would overlook

[16] Due to tax neutrality in Proposition 1, only one of the country-sector tariffs can be normalized to zero when Home imposes optimal policies on multiple countries. However, using the alternative policies, Home treats each country independently, but the relative levels of taxes across countries matter and will lead to different results. Hence, the particular alternative policies we consider takes the formula: $\tau^m_{n,j} = \frac{1}{\kappa}\frac{\pi_{1n,j}\beta_{1,j}x_1 - \frac{1}{1+\tau^x_{n,j}}\pi_{n1,j}\beta_{n,j}x_n}{Y_{n,j}}$, where trade shares $\pi_{in,j}$, expenditures x_n, and wages are the equilibrium outcome under these policy formulas. If we take the difference of this formula between two sectors j and i, Home's relative tariffs on country n are consistent with (34).

endogenous elasticities and trade shares, leading to smaller welfare gains or even losses. To illustrate this point, we use the example above, setting policies according to (34) and (35) based on the pre-tax equilibrium trade shares. Now, the home country would experience a welfare loss of 5.09%, rather than the welfare gain achieved with our equilibrium-based policies.

3.2.3 Numerical Examples: Heterogeneity and Convergence in Optimal Policies

Our analysis has shown that optimal tariffs can vary between sectors and countries. Meanwhile, interdependence works as a counterforce that moves tariffs closer. This section presents numerical examples to highlight how technologies and trade costs affect tariff heterogeneity and convergence across countries and sectors. The welfare implications of these examples are summarized in Table 1.

Tariff heterogeneity We begin with the simplest case with two countries and two sectors, with a focus on sector heterogeneity. Table 1, Case 1 presents the technologies and trade costs for this case with $T_1 = [50, 1]$ and $T_2 = [1, 50]$. Tariffs across the two sectors can be very different: we normalize the tariff in sector 1 to zero and find that the optimal tariff on sector 2 is 51.1%. In this case, formulas (34) and (35), which incorporate endogenous bilateral trade shares, exactly capture the home government's incentives to exploit its monopsony power. In contrast, substituting pre-tax bilateral trade shares for endogenous bilateral trade shares would greatly reduce the welfare gain from trade policies, from 4.85% (Column 1) to 0.52% (Column 3). This occurs because pre-tax trade shares fail to account for the endogenous elasticities and import patterns across sectors that are influenced by trade policies.

The variation in optimal tariffs across sectors depends on the heterogeneity of Home's buyer power, which arises from comparative advantages between countries. To illustrate this point, we expand the model from two to ten sectors while maintaining the same technology ranges for both countries (Case 1a of Table 1). This expansion reduces sectoral differences as foreign goods become more similar across sectors, comparative advantages weaken, and the home country's buyer powers become more uniform across sectors. Al-

Table 1: Welfare Implications: Optimal versus Alternative Policies

	Parameters	Optimal (1)	Endogenous bilateral share (2)	Pre-tax bilateral share (3)
Case 1	$N=2,\ J=2,\ d=\begin{pmatrix}1 & 1.2\\ 1.2 & 1\end{pmatrix}$	4.85	4.85	0.52
Case 1a	$N=2,\ J=10,\ d=\begin{pmatrix}1 & 1.2\\ 1.2 & 1\end{pmatrix}$	2.89	2.89	1.54
Case 2	$N=3,\ J=10,\ d=\begin{pmatrix}1 & 1.2 & 1.2\\ 1.2 & 1 & 1.2\\ 1.2 & 1.2 & 1\end{pmatrix}$	2.67	2.61	0.96
Case 3	$N=3,\ J=10,\ d=\begin{pmatrix}1 & 1.2 & 1.2\\ 1.2 & 1 & 10\\ 1.2 & 10 & 1\end{pmatrix}$	4.84	4.84	−1.61
Case 4	$N=3,\ J=10,\ d=\begin{pmatrix}1 & 1.01 & 2\\ 2 & 1 & 1.01\\ 1.01 & 1.01 & 1\end{pmatrix}$	3.31	2.51	−5.09

Note: This table summarizes welfare gains for the Home country following policy implementation, measured as the percentage change of the real consumption (x_1/P_1) relative to real consumption before policies. Column (1) reports results under optimal policies, Column (2) applies policies from equations (34) and (35) using endogenous bilateral shares, and Column (3) uses the same policy formulas but uses pre-tax bilateral shares. All values are expressed in percentages. $\epsilon = 4$, $\kappa = 1.1$, $\bar{L}_n = 1, \forall n$, $A_{n,j} = 1, \forall n, j$. $T_1 = [50, ..., 1]$, $T_2 = [1, ..., 50]$. If there are three countries, $T_3 = T_1 \circ r$, where r represents values ranging from 2 to 0.5, comprising 10 elements.

though optimal tariffs still range from 0 (sector 1) to 51.1% (sector 10) as in the two-sector case, the share of each sector declines, resulting in lower cross-sectoral variation in tariffs. This small variation is associated with a reduction in welfare gains from optimal policies by 40%, from 4.85% to 2.89%.

Interdependency When the home country trades with more than one partner, its optimal tariffs are also shaped by the interdependency across foreign countries. Case 2 of Table 1 illustrates this with a three-country example using the technologies from Figure 1, where country 3 has a comparative advantage in sector 1, relative to Home, however, the trade cost between countries is symmetric in this case. The optimal tariffs in this multi-country setting have two distinct features. First, the bilateral trade share formulas (34) and (35) break down with more than two countries. Ignoring cross-country interdependency and applying only bilateral trade formulas would reduce welfare gains from the optimal 2.67% to 2.61%. Nevertheless, using the endogenous trade shares and prices still outperforms using pre-tax trade shares, which would further reduce the welfare gain to 0.96%.

Second, expanding from two to more countries weakens Home's market power by providing alternative trading partners for foreign countries. When country 2 can redirect trade toward country 3, its self-elasticity becomes larger, reducing Home's ability to use tariffs to exploit its market power. As a result, welfare gains decline when an additional country is added, with underlying parameter patterns remaining comparable across cases. The lower welfare outcome in Case 2 (three countries) compared to Case 1a (two countries) illustrates this interdependency effect.

When foreign countries have a weaker network, the home country can impose higher and more differential tariffs on the foreign country-sector and achieve greater welfare gains. Case 3 provides an example when the interdependence between countries 2 and 3 is weaker than Case 2, allowing Home to extract higher welfare from its policies. Specifically, we shut down trade flows between country 3 and country 2 by setting an extremely high trade cost ($d_{23} = d_{32} = 10$). This restriction forces countries 2 and 3 to export exclusively to country 1 (Home), reducing their export supply elasticity in response to Home's tariffs. As a result, Home gains greater monopoly power, leading to welfare gains of 4.84%, much higher than those in Case 2. In addition, using formulas (34) and (35) with endogenous bilateral trade shares generates welfare gains of 4.84%, identical to the optimal one, as proved in Lemma 2. In contrast, using only the pre-tax trade shares fails to capture the general equilibrium effects that endogenously change countries' self- and cross-elasticities. This results in welfare losses of -1.61% for the home country.

Case 4 illustrates how country asymmetries and trade network matters for Home's optimal policies. According to Lemma 2, when foreign countries are symmetric or do not trade with each other, optimal policies depend solely on bilateral trade shares. Hence, if foreign countries are asymmetric and engage in large bilateral trade with each other, Home's policies based purely on bilateral trade shares tend to deviate significantly from optimal levels, resulting in lower welfare gains or even losses.

Case 4 presents such a scenario. We construct an asymmetric trade network when country 1 imports almost exclusively from country 2 by imposing high export costs from country 3 to country 1 ($d_{13} = 2$), while country 1 exports exclusively to country 3 due to high export costs from country 1 to country 2 ($d_{21} = 2$). This structure creates highly asymmetric trading relationships between Home and the two foreign countries. Moreover, countries 2 and 3 face low bilateral trade costs ($d_{23} = d_{32} = 1.01$) and thus engage in extensive trade with each other.

The trade network creates important policy spillovers that bilateral formulas fail to capture. Without accounting for these trade network effects, Home would impose high tariffs on imports from country 2 due to its large imports from that country. However, such high tariffs would reduce country 2's prices and, given the low bilateral trade costs between countries 2 and 3, encourage country 3 to increase its imports from country 2. This increased demand from country 3 reduces country 2's export supply to Home.

Policies that overlook these network effects can generate substantial welfare costs. Simply applying bilateral share formulas generates the outcome in Figure 1b, where Home imposes relatively low tariffs on sector 10 of country 3. In contrast, the optimal policies internalize the large trade between countries 2 and 3, leading to policy convergence as shown in Figure 1a. The difference leads to large welfare impacts. Using endogenous bilateral trade shares yields 25% lower welfare gain compared to the optimal policy (2.51% versus 3.31%). The large general equilibrium effect further implies that using pre-tax trade shares actually causes welfare losses rather than gains.

4 Quantitative Analysis

In this section, we quantify the optimal policies and calculate the welfare changes associated with these policies in the multi-country case satisfying the CES supply system with constant returns to scale, where $\eta_j = \kappa$. We study both unilateral and Nash optimal trade policies. We begin by extending the exact hat method, using optimal policy formulas and equilibrium conditions, to compute the counterfactual equilibrium under these optimal policies.

4.1 Computing Welfare with Exact Hat Method

To compute the welfare changes after trade policies, we employ the exact hat method. Let variables without 'prime' denote the observed variables, which include the trade matrix $\{\pi_{ni,j}\}$ and sectoral income $\{w_{n,j}L_{n,j}\}$. Variables denoted with 'prime' represent counterfactuals after implementing the optimal policies, and variables with 'hat' denote the ratios of prime variables to the observed ones. The following two propositions describe the welfare changes under the unilateral optimal policy and the Nash optimal policy.

Proposition 5 (Welfare Changes under Optimal Policy). *Given data on trade shares and sectoral income, $\{\pi_{ni,j}, w_{n,j}L_{n,j}\}$, along with parameter values $\{\epsilon_j, \kappa, \beta_{n,j}\}$, we can evaluate the welfare implications of Home country's optimal policy using the exact hat method. Specifically, this involves solving for the variables $\{\hat{w}_{n,j}, \hat{L}_{n,j}, \hat{P}_n, x'_n, \pi'_{ni,j}, 1+\tau^{m\prime}_{n,j}, 1+\tau^{x\prime}_{n,j}\}$, which constitute the solution to the system of equations detailed in Appendix H.1, which uses optimal policies in Proposition 4 and equilibrium conditions.*[17] *The welfare effects are $\hat{C}_n = \hat{x}_n/\hat{P}_n$.*

We also consider Nash optimal policies where all countries are able to retaliate against the policies of other countries. Country i's policy is defined as $\{\tau^x_{ni,j}, \tau^m_{in,j}\}$, where $\tau^x_{ni,j}$ denotes the export tax imposed by country i on country n in sector j, and $\tau^m_{in,j}$ represents the import tariff imposed by country i on country n in sector j. Given the policies of other countries, country i's government chooses $\{\tau^x_{ni,j}, \tau^m_{in,j}\}$ to maximize domestic consumers' consumption, $\max x_i/P_i$, subject to world market equilibrium and the policies of other countries. Appendix G derives the Nash optimal policies. Appendix H.2 lists the counterfactual equilibrium conditions under Nash optimal policies using the exact hat method and the formulas for optimal policies.

Proposition 6 (Welfare Changes under Nash Optimal Policy). *Given observable data on trade shares and sectoral income, $\{\pi_{ni,j}, w_{n,j}L_{n,j}\}$, along with parameter values $\{\epsilon_j, \kappa, \beta_{n,j}\}$, we can evaluate the economic impact of Nash optimal policy using the exact hat method. Specifically, this involves solving for the variables $\{\hat{w}_{n,j}, \hat{L}_{n,j}, \hat{P}_n, x'_n, \pi'_{ni,j}, 1+\tau^{m\prime}_{ni,j}, 1+\tau^{x\prime}_{in,j}\}$, which constitute the solution to the system of equations detailed in Appendix H.2. The welfare effects of the Nash optimal policies for each country are fully characterized by $\hat{C}_n = \hat{x}_n/\hat{P}_n$.*

[17] As we have proven in Appendix F, under the multi-country case we consider, a country implementing optimal policies will not impose any domestic taxes because the domestic market is efficient. Therefore, domestic taxes will not be included from now on.

In summary, the method outlined in Propositions 5 and 6 enables us to calculate optimal policies and equilibrium changes using bilateral trade and sector-level income data, without the need to explicitly recover technology $\bar{T}_{n,j}$, fundamental productivity $A_{n,j}$, or trade costs d_{ni}. We adapt the standard exact hat method but incorporate labor market specifications with imperfect labor substitution as in Ricardo-Roy, and our first-order conditions (FOCs) for optimal policies. With explicit FOCs for optimal policies, our method provides notable gains in accuracy and computational efficiency.[18]

4.2 Data and Measurement

To conduct the analysis, we need sectoral-level data on gross production and bilateral trade for each country. Data on domestic and international production and expenditure are from the World Input-Output Database (WIOD) 2016 Release, which covers 56 sectors, 43 countries, and an aggregate for the rest of the world for the period 2000-2014. The 56 sectors are classified according to International Standard Industrial Classification Revision 4 (ISIC Rev. 4). We utilize data from 2014. Given the absence of certain sector data for specific countries, we aggregate some of the 56 sectors and focus on $J = 13$ manufacturing sectors, similar to Lashkaripour and Lugovskyy (2023). See Table 2 for the list of aggregated sectors. We retain data for the top 19 countries by GDP in 2014.[19] The 19 largest countries include the U.S., China, Japan, Germany, the United Kingdom, France, Brazil, Italy, the Russian Federation, India, Canada, the Republic of Korea, Australia, Spain, Mexico, Turkey, Netherland, Indonesia, and Switzerland. All other countries are aggregated into the rest of the world (ROW).

The model parameters include those that we assume to be common across countries and sectors—such as the trade elasticity parameter ϵ and CES supply system elasticity κ. We set $\epsilon = 4$, consistent with the trade elasticity estimated by Simonovska and Waugh (2014), and $\kappa = 1.5$ following Galle, Rodríguez-Clare, and Yi (2023). The sector-level expenditure

[18] Noted by Lashkaripour and Lugovskyy (2023), without FOCs, we would have to rely on numerical optimization, which can become increasingly difficult and unstable to implement when dealing with many policy variables with multiple sectors and countries.

[19] Nominal GDP are from the World Bank's World Development Indicators (WDI) database. Saudi Arabia is excluded due to the lack of data in the WIOD database.

Table 2: List of Manufacturing Sectors

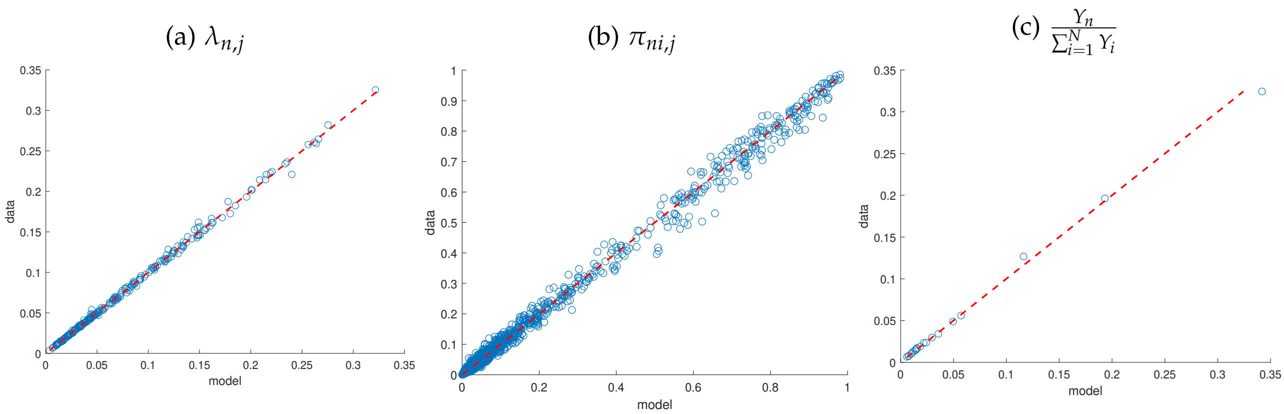
Sector Code	Short Name	Description
1	Food	Food, Beverages and Tobacco
2	Textiles	Textiles, wearing apparel and leather products
3	Wood	Wood and of products of wood and cork; articles of straw and plaiting materials
4	Paper	Paper and paper products, Printing and reproduction of recorded media
5	Oil	Coke and refined petroleum products
6	Chemicals	Chemicals and chemical products
7	Rubber	Rubber and plastics
8	Mineral	Other Non-metallic mineral
9	Metals	Basic metals and fabricated metal
10	Computer	Computer, electronic, optical products and electrical equipment
11	Machinery	Machinery and equipment n.e.c.
12	Vehicles	Vehicles and other transport equipment
13	Furniture	Furniture and other manufacturing

shares for each country n are based on the following calculations:

$$\beta_{n,j} = \frac{\sum_{i=1}^{N} X_{ni,j}}{\sum_{k=1}^{J} \sum_{i=1}^{N} X_{ni,k}}$$

where $X_{ni,j}$ is the trade flow from country i to country n in sector j.

Figure 2: Model and Data Comparison



Note: This figure shows the relationship between the data and the model's income shares $\lambda_{n,j}$, trade shares $\pi_{ni,j}$, and the income of a country in global income $\frac{Y_n}{\sum_{i=1}^{N} Y_i}$. The red dashed line represents the 45-degree line.

To ensure that the data satisfy the world market equilibrium conditions described in equations (1)-(10), we first apply the hat method under the assumption of zero export taxes and import tariffs, and balanced trade[20] then update the trade shares and sectoral income

[20]Our model features an equilibrium with balanced trade. In the data, trade is not balanced. This implies

values using $\{\pi'_{ni,j}, w'_{n,j}L'_{n,j}\}$, using the system of equations detailed in the Appendix H.1. Figure 2 compares the model with data in three endogenous variables: the share of sector j income in country n, denoted by $\lambda_{n,j}$, the trade share from country i to country n in sector j, $\pi_{ni,j}$, and the share of country n's income in global income $\frac{Y_n}{\sum_{i=1}^{N} Y_i}$. The model captures the data well, with all the dots aligning around the 45-degree line.

4.3 Unilateral Optimal Policies of the U.S.

Figure 3 plots the U.S. optimal unilateral policies.[21] We rank sectors in ascending order based on the U.S. sectoral net imports relative to world sectoral income before policy implementation. For example, relative to other sectors, the U.S. oil sector has the highest share of net exports, whereas the U.S. textiles sector is imported on net. Following the tax neutrality principle established in Proposition 1, we set the import tariff on oil from ROW to zero.

The first feature is that export taxes and import tariffs are heterogeneous across sectors and countries. Second, tariffs imposed by the U.S. on other countries are generally higher in its importing sectors, while export taxes are higher in U.S. exporting sectors. As shown in columns (1) and (3) of Table 3, on average, U.S. optimal import tariffs are lowest in the oil sector, at just 0.03%, and highest in the textiles sector, at 4.17%, with an average rate of 1.54%. The oil sector faces the highest export tax at 26.05%, while the textiles sector has the lowest export tax at 20.62%. The average export tax rate across all sectors is 24.15%.

that the welfare gains computed include both the welfare changes from eliminating trade imbalances and those from implementing optimal policies. To separate them, we first eliminate the imbalances. We then take the new equilibrium to be our private equilibrium observations, which are used to calculate the optimal policies and welfare changes. The results show additional changes brought about by optimal policies.

[21] Optimal tariffs and export taxes on Canada and Mexico are separately shown because their means and standard deviations are much higher than those of other countries.

Table 3: U.S. Policies Across Sectors: Optimal, Alternative, and Nash

	Optimal				Endogenous Bilateral Share				Nash			
	Tariff (%)		Export tax(%)		Tariff (%)		Export tax(%)		Tariff (%)		Export tax(%)	
Sectors	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Oil	0.03	1.67	26.05	2.77	-0.86	2.04	27.24	3.37	0.07	1.49	25.92	2.73
Paper	0.51	1.14	25.11	2.27	-0.41	1.52	26.33	2.74	0.43	1.30	25.05	2.43
Chemicals	0.68	1.15	25.61	2.28	-0.52	1.48	27.17	2.86	0.42	1.15	25.71	2.25
Machinery	1.72	3.59	25.71	3.68	0.08	2.28	27.05	3.31	1.64	4.50	26.13	4.58
Food	0.75	0.42	24.46	0.98	-0.14	0.56	25.59	1.14	0.59	0.53	24.53	0.83
Vehicles	1.54	2.01	24.79	1.96	0.21	3.11	26.51	3.46	1.14	1.64	24.77	2.11
Rubber	1.35	1.49	24.25	1.26	0.28	1.80	25.58	1.71	0.88	1.27	24.51	1.49
Mineral	1.28	0.57	23.75	0.55	0.28	1.00	25.02	1.14	0.88	0.54	24.05	0.68
Wood	1.22	1.15	23.82	1.13	0.27	1.91	25.10	1.81	0.98	1.46	23.85	1.92
Metals	1.50	0.85	23.83	0.74	0.60	1.36	24.94	0.95	1.16	1.03	23.85	1.10
Computer	2.77	2.82	23.23	1.38	0.96	3.79	25.24	2.72	2.01	2.52	23.78	1.12
Furniture	2.49	1.54	22.71	0.92	1.16	2.85	24.53	3.00	1.89	1.50	23.03	1.74
Textiles	4.17	2.58	20.62	1.00	2.47	3.27	22.43	3.16	3.13	1.93	21.19	1.73
Avg.	1.54	1.61	24.15	1.61	0.34	2.07	25.60	2.41	1.17	1.60	24.34	1.90

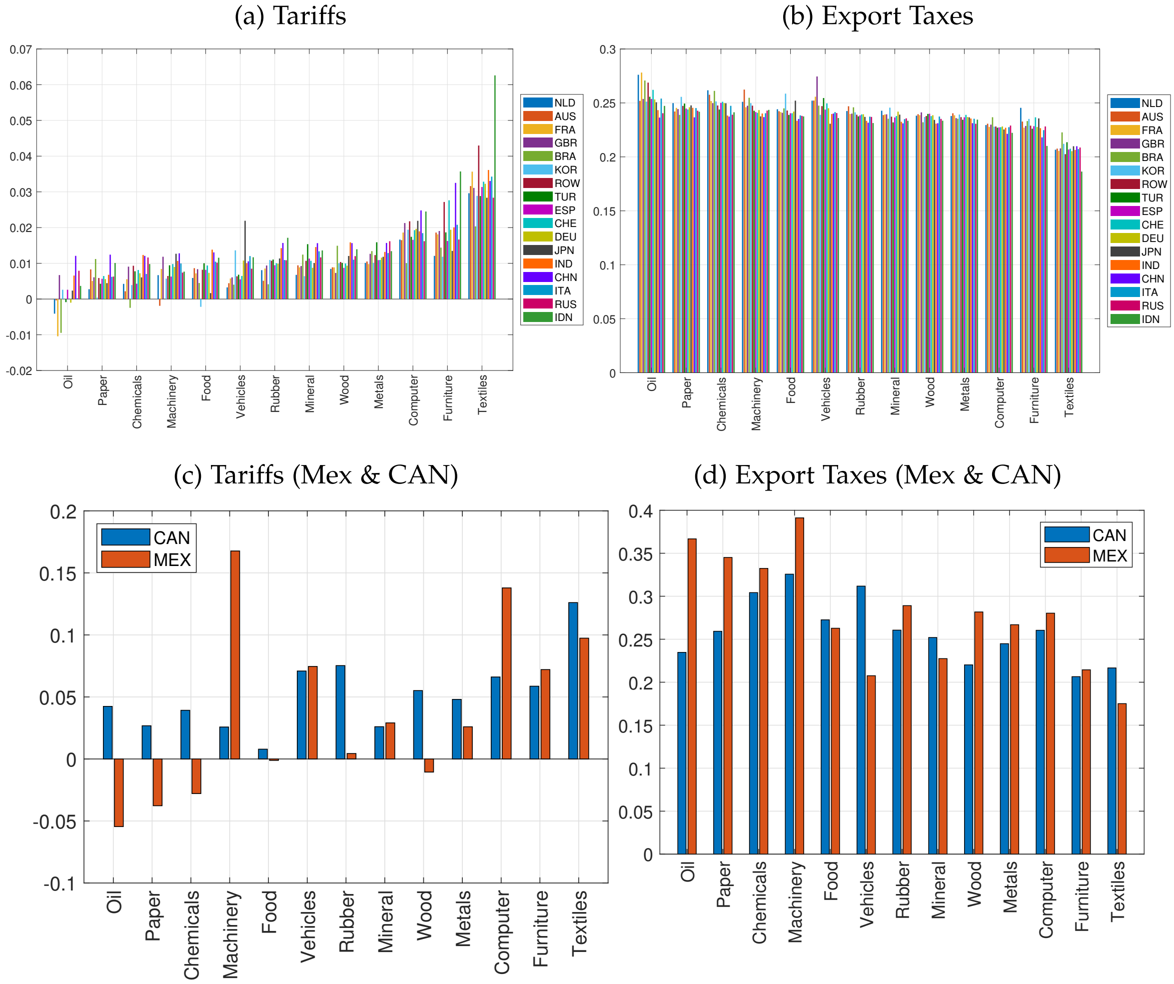
Note: This table summarizes the U.S. policies (mean and standard deviation) for 13 sectors under three scenarios: unilateral optimal policies, alternative policies using formulas (34) and (35) with endogenous bilateral trade share, and Nash optimal policies. The sectors are ranked in ascending order based on the U.S. sectoral net imports relative to world sectoral income before the policy. The last row represents the average of the entire column.

Table 4: U.S. Policies Across Countries: Optimal, Alternative, and Nash

	Optimal					Endogenous Bilateral Share					Nash				
	Tariff (%)			Extax(%)		Tariff (%)			Extax(%)		Tariff (%)			Extax(%)	
Cty	Mean	Std	*Slope*	Mean	Std	Mean	Std	*Slope*	Mean	Std	Mean	Std	*Slope*	Mean	Std
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
NLD	0.85	0.77	0.18	24.45	1.56	-1.13	1.10	1	27.34	2.11	0.30	0.65	0.23	25.09	1.47
AUS	0.94	0.85	0.16	24.19	1.33	-0.89	1.39	1	26.79	2.24	0.34	0.66	0.19	24.81	1.28
FRA	1.00	1.00	0.60	24.12	1.62	-0.18	1.31	1	25.83	2.18	0.60	1.02	0.50	24.64	1.45
GBR	1.22	0.71	0.22	24.11	1.47	0.18	0.86	1	25.50	1.82	0.73	0.51	0.28	24.43	1.25
BRA	0.75	0.79	0.61	24.30	1.26	0.02	1.04	1	25.38	1.64	0.25	0.63	0.60	24.75	1.19
KOR	0.94	0.79	0.42	24.18	1.21	-0.60	1.09	1	26.31	1.56	0.49	0.65	0.43	24.68	1.18
ROW	1.32	1.09	0.70	23.94	1.45	0.15	1.45	1	25.44	2.05	1.00	0.81	0.59	24.09	1.32
TUR	1.19	0.72	0.53	23.81	1.14	-0.32	0.95	1	25.82	1.50	1.01	0.59	0.37	24.09	1.15
ESP	1.05	0.72	0.44	23.76	1.13	0.07	0.42	1	25.15	0.63	0.74	0.49	0.45	24.13	0.87
CHE	1.24	0.86	0.25	23.95	1.22	-0.17	1.41	1	25.73	2.00	0.89	0.78	0.20	24.42	1.15
DEU	1.15	0.79	0.84	23.83	1.18	0.09	0.48	1	25.28	0.61	0.82	0.61	0.65	24.20	0.92
JPN	1.22	0.77	0.42	23.74	1.11	-0.03	1.04	1	25.43	1.33	0.93	0.58	0.42	24.00	1.01
IND	1.49	0.72	0.91	23.33	0.95	0.20	0.65	1	24.92	0.89	1.43	0.59	0.74	23.36	0.95
CHN	1.74	0.74	0.69	23.05	0.85	0.40	0.97	1	24.66	1.14	1.50	0.49	0.60	23.25	0.72
ITA	1.29	0.79	0.87	23.63	1.12	0.30	0.73	1	24.86	1.00	0.99	0.58	0.62	23.93	0.97
CAN	5.14	2.89	0.32	25.92	3.55	3.94	3.08	1	26.16	5.40	4.19	1.94	0.23	24.09	3.83
RUS	1.26	0.56	-0.16	23.45	0.88	0.07	0.61	1	25.02	0.80	1.17	0.32	0.08	23.58	0.62
IDN	1.81	1.50	0.54	23.07	1.58	1.16	2.51	1	23.80	3.01	1.87	1.51	0.45	22.83	2.06
MEX	3.67	6.60	0.68	28.01	6.25	3.13	7.10	1	26.93	8.10	2.97	7.03	0.65	27.99	7.64
Avg.	1.54	1.25	0.49	24.15	1.62	0.34	1.48	1	25.60	2.11	1.17	1.08	0.44	24.34	1.63

Note: This table summarizes the U.S. import tariffs (mean, standard deviation, and tariff-import slope) and export taxes (mean and standard deviation) for the other 19 countries under three scenarios: unilateral optimal policies, alternative policies using formulas (34) and (35) with endogenous bilateral trade share, and Nash optimal policies. The column "*Slope*" represents tariff-import slope, which is the regression coefficients of U.S. optimal tariffs on each country's sectors with respect to the after-tax U.S. net import share relative to that country-sector income in different sectors, normalized by κ. Countries are ranked in ascending order based on the U.S. total net import share relative to each country's total income before the policy. The last row represents the average of the entire column.

Figure 3: U.S. Unilateral Optimal Policies

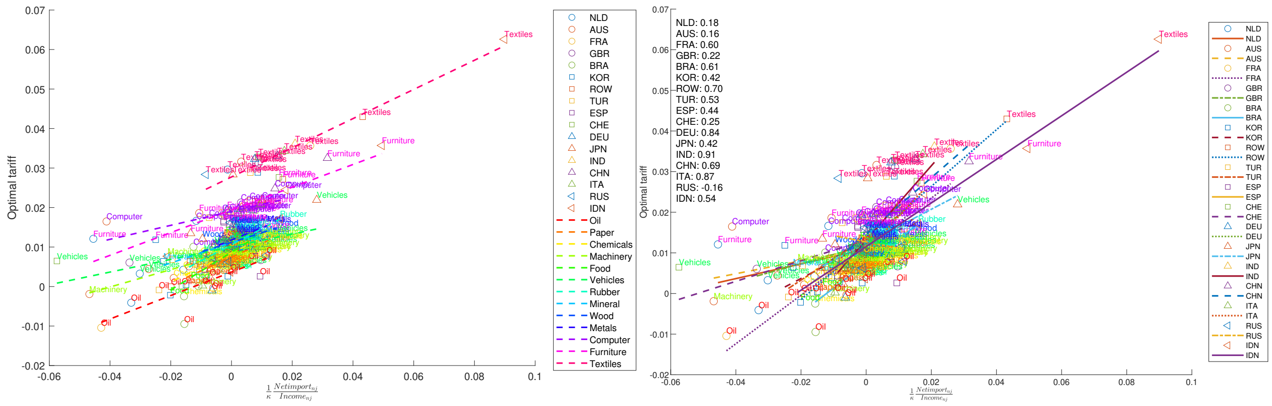


Note: This figure plots U.S. unilateral optimal policies. The sectors are ranked in ascending order based on the U.S. sectoral net imports relative to world sectoral income before the policy. Countries are ranked in ascending order based on the U.S. total net import share relative to each country's total income before the policy. Optimal tariffs and export taxes on Canada and Mexico are separately shown because their means and standard deviations are much higher than those of other countries.

Third, the average import tariffs vary significantly across countries, as demonstrated in column (1) of Table 4, where countries are ranked in ascending order based on the U.S. total net import share relative to each country's total income before the policy. The U.S. imposes the highest import tariffs on Canada (5.14%) and Mexico (3.67%), while the lowest tariffs are levied on Brazil (0.75%) and the Netherlands (0.85%), with an average rate of 1.54%, as shown in column (1). The export taxes across countries have a mean of 24.15% and mostly range from 23% to 25%, with the exceptions of Mexico (28.01%) and Canada (25.92%), as shown in column (4).

Figure 4: U.S. Unilateral Optimal Tariffs and Net Import Share

(a) Fitted Lines within Sectors (b) Fitted Lines within Countries



Note: This figure shows the relationship between U.S. unilateral optimal tariffs and U.S. net import share, normalized by κ. The net import share is defined as after-tax U.S. net imports from a certain country-sector over the foreign country-sector income. Panel (a) plots separate fitted regression lines for each sector, and panel (b) presents separate fitted regression lines for each country.

Fourth, optimal tariffs depend not only on U.S. market power over individual country-sector pairs but also on overall U.S. market power within a sector, reflecting the interdependency among countries. Within sectors, there is dispersion in tariffs across countries, as illustrated by the scatter points in Figure 4.[22] Columns (2) and (4) of Table 3 quantify this variation by reporting the standard deviations of the trade policies within each sector. Consider the textiles sector, where import tariffs range from 2.5% for Russia and 6% for Indonesia, yielding a standard deviation of 2.58%. The U.S. tends to impose higher tariffs on countries where it has a larger net import share relative to their income. This relationship explains why optimal U.S. tariffs are higher on Indonesia in the textiles sector and on Japan in vehicles, compared to other countries within those respective sectors.

However, in addition to U.S. market power over individual country sectors, interdependence among countries also shapes the tariffs imposed. Figure 4(a) plots the fitted regression line of tariffs against bilateral after-tax U.S. net import shares relative to each country's income for each sector. The textiles line has the highest intercept, reflecting that import tariffs on textiles exceed those of other sectors across most countries. Furthermore, column (3)

[22]Figure 4 excludes Canada and Mexico because optimal tariff values on these two countries are substantially larger than those for other countries, as shown in Figure 3(c) and (d). This reflects their large trade volumes and correspondingly low trade costs with the U.S., which gives the U.S. significantly greater market power in these two countries. Including Canada and Mexico in Figure 4 would compress the figure's scale and obscure tariff patterns in other trading partners.

of Table 4 presents, for each country, the regression coefficients of optimal sectoral tariffs with respect to the bilateral after-tax U.S. net import share relative to that country's income. These coefficients are consistently below 1, averaging 0.49. Figure 4(b) illustrates how these regression slopes differ across countries while remaining below 1.

As explained in the theoretical results in Section 3.2.2, the tariff-import slope in optimal policies deviates from 1 due to cross-country interdependency. Even though the U.S. does not import textiles from Russia, it imposes higher tariffs because the cross-country trade network means that relative wage changes affect demand and supply. Without tariffs on Russia's textiles, the U.S. would import relatively more from Russia, and relative wage change would increase Russia's imports from Indonesia and China, further reducing their export supplies to the U.S. Therefore, optimal tariffs within a sector tend to converge across countries due to these interdependencies.

Alternative policies. We now consider a scenario where the U.S. does not account for interdependence across countries. The import tariff and export tax are determined by equations (34) and (35), but the trade shares and prices are endogenously determined in equilibrium given these trade policy formulas. Columns (5)-(8) of Table 3 show the cross-sector differences under U.S. policies under "Endogenous Bilateral Share". Compared to the "Optimal" case, both tariffs and export taxes are more dispersed, with the standard deviation within each sector significantly higher than that of the "Optimal" case.

Columns (6)-(10) of Table 4 illustrate the cross-country differences under these alternative U.S. policies. In this case, the regression coefficients of U.S. tariffs on each country's sectors with respect to the U.S. after-tax net import share relative to that country-sector income (normalized by κ) in different sectors are 1, which is shown in column (8). These columns exhibit significantly different patterns compared to the "Optimal": optimal policies depend little on bilateral imports and have smaller standard deviations, again highlighting the impact of the trade network among countries on optimal policies.

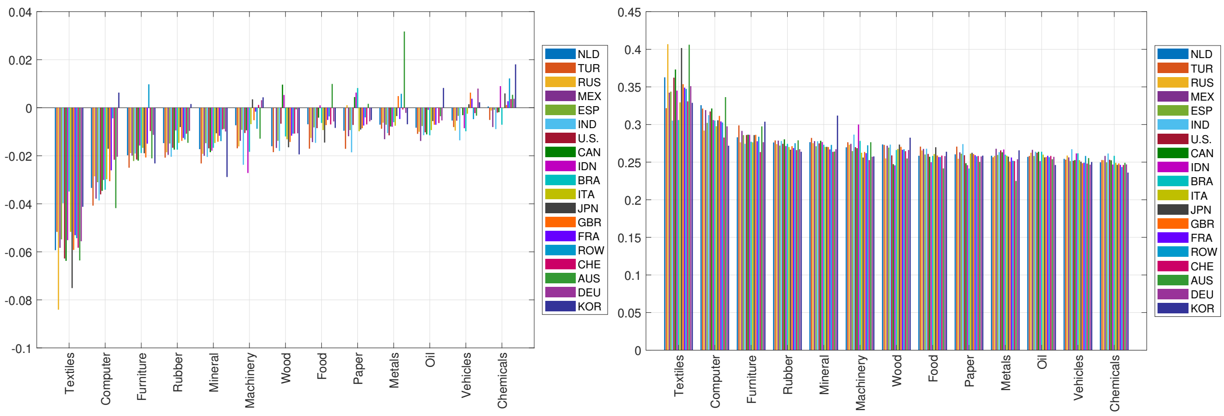
Welfare. The first column of Table 7 presents the welfare changes resulting from the unilateral optimal policies implemented by the US. The results indicate that the U.S. experiences welfare gains of approximately 1.3%, while other countries suffer welfare losses, as the U.S.'s unilateral policies improve its own terms of trade while worsening those of other countries. The U.S. welfare gains under optimal policies are larger than those under the "Endogenous Bilateral Share" and "Observed Bilateral Share" scenarios, as shown in columns (2) and (3).

4.4 Unilateral Optimal Policies of China

Now we turn to the case when China imposes unilateral policies. Figure 5 shows China's optimal policies across countries and sectors, with sectors ranked in ascending order based on China's sectoral net imports over world sectoral income. The import tariff on the ROW's oil sector is normalized to zero. As with our U.S. analysis, we summarize these results in two ways: Table 5 reports average trade policies by sector, while Table 6 shows average policies by country, along with the tariff-import slopes. Figure 6 illustrates these tariff-import relationships and interdependency through scatter plots of optimal tariffs against China's net import shares.[23] Panel (a) displays sector-specific fitted lines, and panel (b) shows country-specific fitted lines.

Figure 5: China's Unilateral Optimal Policies

(a) Tariff (b) Export Tax



Note: This figure plots China's unilateral optimal policies. The sectors are ranked in ascending order based on China's sectoral net imports relative to world sectoral income before the policy. Countries are ranked in ascending order based on China's total net import share relative to each country's total income before the policy.

China's optimal trade policies follow similar principles to those in the U.S. First, both optimal export taxes and import tariffs are heterogeneous across sectors and countries.

[23]Russia is excluded from Figure 6 because China's net import share from it in the textiles sector (−0.61) is an extreme outlier compared to other countries. See Figure 5 for tariffs on Russia.

Second, China tends to impose higher tariffs on sectors where it is a net importer and higher export taxes on sectors where it is a net exporter, as shown in columns (1) and (3) of Table 5. Across sectors, China's optimal import tariffs range from -5.67% in textiles (where China is a major exporter) to 0.14% in chemicals (where China is a net importer). Export taxes show the reverse pattern, reaching 35.09% in textiles while falling to 24.99% in chemicals.

Third, cross-country interdependencies shape these policies greatly. Column (3) of Table 6 and Figure 6(b) show that tariff-import slopes are consistently smaller than 1, indicating that optimal trade policies toward individual countries reflect broader sectoral considerations rather than purely bilateral trade flows. For example, China imposes relatively high tariffs on chemical imports from Mexico despite its small imports from Mexico in this sector. China also imposes lower tariffs on textiles for most countries.

Alternative policies. Similarly, we consider a scenario where China does not account for interdependence across countries. The import tariff and export tax are determined by equations (34) and (35). Columns (5)-(8) of Table 5 illustrate the cross-sector differences under the policies implemented by China, "Endogenous Bilateral Share". Compared to the "Optimal", most sectors exhibit a significantly higher standard deviation, reflecting less similarity in tariffs and export taxes across countries. Columns (6)-(10) of Table 6 illustrate the cross-country differences under the policies implemented by China, "Endogenous Bilateral Share". In this case, the regression coefficients of China's optimal tariffs on each country's sectors with respect to China's after-tax net import share relative to that country-sector income in different sectors (normalized by κ) are 1, which is shown in column (8). These columns exhibit significantly different patterns compared to the optimal case. The standard deviations of import tariffs and export taxes in most countries are larger than those in the optimal case, highlighting the impact of trade networks among countries on optimal policies.

Column (4) of Table 7 illustrates the welfare changes resulting from China's unilateral optimal policies. China benefits from welfare gains, while other countries experience welfare losses. China's welfare gains are greater than those under the "Endogenous Bilateral Share" and "Observed Bilateral Share" scenarios, as shown in columns (5) and (6).

Table 5: China's Policies: Optimal, Alternative, and Nash

	Optimal				Endogenous Bilateral Share				Nash			
	Tariff (%)		Export tax(%)		Tariff (%)		Export tax(%)		Tariff (%)		Export tax(%)	
Sectors	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Textiles	-5.67	1.10	35.09	2.90	-6.92	6.27	37.64	11.30	-4.95	1.34	34.91	3.80
Computer	-2.79	1.20	30.70	1.49	-3.52	3.52	32.42	5.23	-2.35	1.36	30.35	2.32
Furniture	-1.74	0.74	28.41	0.93	-0.95	1.97	27.75	2.74	-1.22	0.73	27.60	1.09
Rubber	-1.40	0.51	27.29	0.48	-0.19	0.82	25.77	1.11	-1.07	0.58	26.71	0.65
Mineral	-1.61	0.50	27.48	1.00	-0.47	1.27	26.07	2.24	-1.29	0.70	27.02	1.47
Machinery	-0.93	0.87	26.96	1.11	-0.30	2.67	26.47	3.91	-0.59	1.07	26.39	1.72
Wood	-1.02	0.80	26.56	0.92	0.14	1.88	25.22	2.46	-0.65	1.16	25.86	1.70
Food	-0.75	0.60	26.01	0.67	0.55	0.93	24.42	1.16	-0.62	0.85	25.68	1.13
Paper	-0.57	0.71	25.82	0.75	0.73	1.21	24.25	1.46	-0.40	0.95	25.34	1.29
Metals	-0.36	0.96	25.74	0.91	0.69	1.93	24.57	2.25	-0.23	1.41	25.25	1.86
Oil	-0.69	0.50	25.85	0.47	0.36	0.77	24.62	0.89	-0.66	0.55	25.70	0.57
Vehicles	-0.30	0.55	25.42	0.52	0.68	1.27	24.40	1.60	-0.17	0.73	25.04	0.85
Chemicals	0.14	0.67	24.99	0.56	0.87	1.12	24.28	1.33	-0.02	0.76	24.94	0.84
Avg.	-1.36	0.75	27.41	0.98	-0.64	1.97	26.76	2.90	-1.09	0.94	26.98	1.48

Note: This table summarizes China's policies (mean and standard deviation) for 13 sectors under three scenarios: unilateral optimal policies, alternative policies using formulas (34) and (35) with endogenous bilateral trade share, and Nash optimal policies. The sectors are ranked in ascending order based on China's sectoral net imports relative to world sectoral income before the policy. The last row represents the average of the entire column.

Table 6: China's Policies: Optimal, Alternative, and Nash

	Optimal					Endogenous Bilateral Share					Nash				
	Tariff (%)			Extax(%)		Tariff (%)			Extax(%)		Tariff (%)			Extax(%)	
Cty	Mean	Std	*Slope*	Mean	Std	Mean	Std	*Slope*	Mean	Std	Mean	Std	*Slope*	Mean	Std
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
NLD	-1.57	1.50	0.31	27.74	3.09	-2.03	3.48	1	28.89	6.01	-1.53	1.52	0.24	27.68	3.26
TUR	-2.03	1.26	0.20	27.80	2.23	-1.31	1.99	1	27.58	3.63	-2.10	1.07	0.20	27.80	2.11
RUS	-1.72	2.10	0.12	27.73	3.89	-1.67	6.88	1	28.58	12.46	-1.42	2.06	0.12	27.21	4.64
MEX	-1.95	1.35	0.74	27.96	2.40	-0.93	1.16	1	26.99	2.40	-1.78	1.13	0.09	27.85	2.24
ESP	-1.52	1.38	0.64	27.30	2.45	-0.98	1.68	1	26.89	2.93	-1.34	1.17	0.59	27.08	2.37
IND	-1.98	0.93	0.44	27.73	1.51	-0.66	1.11	1	26.28	1.95	-2.09	0.65	0.43	27.84	1.32
U.S.	-1.62	1.61	0.48	27.67	2.97	-0.85	2.82	1	26.90	4.77	-1.33	1.37	0.43	27.22	3.08
CAN	-1.38	1.82	0.28	27.64	3.40	-0.91	4.52	1	27.30	7.34	-0.86	2.00	0.24	26.86	4.14
IDN	-1.16	1.72	0.25	27.33	2.85	-0.44	3.04	1	26.73	4.82	-1.06	1.56	0.26	26.97	2.99
BRA	-1.42	1.11	0.61	27.10	1.75	-0.25	1.47	1	25.74	2.26	-0.95	0.88	0.60	26.40	1.62
ITA	-1.43	1.29	0.87	27.05	2.06	-0.16	0.94	1	25.63	1.59	-1.23	1.04	0.70	26.75	1.91
JPN	-1.29	1.95	0.41	27.71	3.90	-0.62	3.60	1	27.12	6.36	-0.98	1.83	0.40	27.15	4.30
GBR	-1.30	1.65	0.48	27.31	2.85	-0.57	2.93	1	26.60	4.75	-0.91	1.48	0.43	26.77	2.98
FRA	-1.15	1.42	0.56	27.09	2.65	-0.42	2.12	1	26.28	3.63	-0.89	1.43	0.37	26.81	2.88
ROW	-0.69	1.58	0.64	27.28	2.61	0.90	1.89	1	25.10	2.61	-0.56	1.23	0.57	26.53	2.25
CHE	-1.07	1.51	0.73	26.50	2.10	0.34	1.69	1	24.84	2.33	-0.59	1.34	0.47	26.01	2.03
AUS	-0.94	2.27	0.25	27.61	4.62	-1.48	5.79	1	28.73	9.74	-0.27	2.67	0.28	26.73	5.80
DEU	-0.94	1.52	0.48	26.94	2.68	-0.10	2.69	1	25.93	4.29	-0.59	1.48	0.32	26.46	2.91
KOR	-0.71	1.62	0.39	27.24	2.62	-0.01	2.88	1	26.35	4.25	-0.32	1.47	0.42	26.56	2.87
Avg.	-1.36	1.56	0.47	27.41	2.77	-0.64	2.77	1	26.76	4.64	-1.09	1.44	0.38	26.98	2.93

Note: This table summarizes China's tariffs (mean, standard deviation, and tariff-import slope) and export taxes (mean and standard deviation) for the other 19 countries under three scenarios: unilateral optimal policies, alternative policies using formulas (34) and (35) with endogenous bilateral trade share, and Nash optimal policies. The column "*Slope*" represents tariff-import slope, which is the regression coefficients of China's optimal tariffs on each country's sectors with respect to the after-tax China's net import share relative to that country-sector income in different sectors, normalized by κ. Countries are ranked in ascending order based on China's total net import share relative to each country's total income before the policy. The last row represents the average of the entire column.

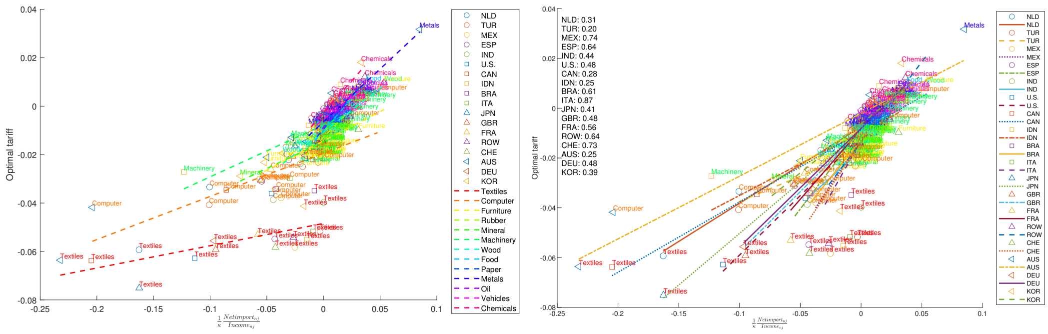
Table 7: Welfare Change

	U.S. Unilateral Policy (%)			China's Unilateral Policy (%)			Nash Policy (%)
	Optimal	Endogenous Bilateral Share	Observed Bilateral Share	Optimal	Endogenous Bilateral Share	Observed Bilateral Share	Optimal
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
U.S.	**1.307**	**1.286**	**1.052**	-0.386	-0.418	-0.521	-0.888
NLD	-0.373	-0.411	-0.443	-0.724	-0.877	-1.186	-7.985
AUS	-0.284	-0.321	-0.376	-1.399	-1.712	-2.089	-4.150
FRA	-0.263	-0.276	-0.285	-0.331	-0.350	-0.409	-4.610
GBR	-0.517	-0.536	-0.574	-0.498	-0.555	-0.704	-3.266
BRA	-0.214	-0.219	-0.222	-0.196	-0.204	-0.219	-0.965
KOR	-0.252	-0.262	-0.263	-0.839	-0.866	-0.924	-1.924
ROW	-0.287	-0.295	-0.305	-0.824	-0.804	-0.798	-1.483
TUR	-0.164	-0.170	-0.178	0.061	0.059	0.123	-3.742
ESP	-0.048	-0.050	-0.031	-0.201	-0.213	-0.249	-2.193
CHE	-0.269	-0.280	-0.281	-0.139	-0.141	-0.153	-3.554
DEU	-0.210	-0.211	-0.189	-0.435	-0.458	-0.530	-3.220
JPN	-0.235	-0.244	-0.246	-0.672	-0.710	-0.847	-1.519
IND	-0.076	-0.075	-0.073	-0.113	-0.111	-0.084	-0.686
CHN	-0.076	-0.076	-0.075	**0.571**	**0.558**	**0.438**	-0.297
ITA	-0.125	-0.126	-0.117	-0.063	-0.042	0.008	-1.705
CAN	-4.857	-4.787	-5.262	-0.555	-0.642	-0.845	-5.563
RUS	-0.014	-0.018	-0.021	-0.838	-1.190	-1.576	-2.914
IDN	-0.165	-0.191	-0.221	-0.352	-0.382	-0.388	-2.352
MEX	-4.829	-4.657	-5.279	-0.145	-0.140	-0.102	-5.424

Note: The welfare change is calculated as the percentage change relative to real consumption (x_n/P_n) before the policy. Countries except the U.S. are ranked in ascending order based on the U.S. total net import share relative to each country's total income before the policy. The welfare change for each country under the unilateral policies of the U.S. and China is evaluated across three scenarios: optimal policies, alternative policies using formulas (34) and (35) with endogenous bilateral trade shares, and alternative policies using formulas (34) and (35) with pre-tax observed bilateral trade shares. The last column presents each country's welfare change under the optimal Nash policy.

Figure 6: China's Unilateral Optimal Tariffs and Net Import Share

(a) Fitted Line within Sectors (b) Fitted Line within Countries



Note: This figure shows the relationship between China's unilateral optimal tariffs and China's net import share, normalized by κ. The net import share is defined as after-tax China's net import from a certain country-sector over the foreign country-sector income. Panel (a) plots separate fitted regression lines for each sector, and panel (b) presents separate fitted regression lines for each country.

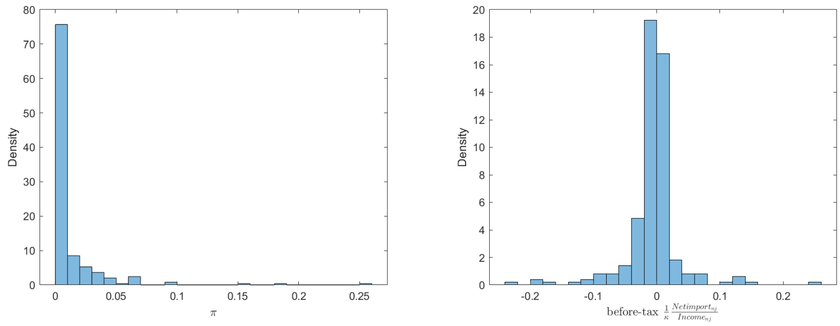
4.5 Discussions and Nash Policies

Welfare discussion The quantitative exercise illustrates that, given the underlying fundamentals, optimal policies are positively related to Home's overall net imports, with convergence across countries. The welfare change is not substantial, which is typical, as the data suggests high trade costs. Additionally, the welfare differences arising from using the bilateral (endogenous) trade share are small. In our theoretical and numerical examples, we demonstrate that these welfare differences can vary significantly across different scenarios. Using our formula, but pre-tax observed trade shares result in a much larger gap.

For the two-digit sectors and the 20 countries we analyzed, the following observations contribute to the small differences: The level and standard deviation of the U.S. or China's import expenditure share across country-sector are very small (see Figure 7(a) and 8(a)), and the pre-tax net import share also exhibits very small standard deviations (see Figure 7(b) and 8(b)). This indicates high trade costs and limited heterogeneity across two-digit sectors and foreign countries. Although the optimal tariff-import slope is not equal to 1 for all countries, the overall optimal tariffs remain close to the 45-degree line. This suggests that foreign countries have similar comparative advantages relative to the U.S., indicating limited heterogeneity across foreign countries for the U.S. Hence, using our formula and bilateral endogenous trade shares, which partly account for the endogenous self-elasticities, largely captured the gains. Using our formula with pre-tax observed trade shares result in a larger gap: the welfare change is about 80% of the optimal for the U.S. and 77% for China, as these policies overlook the endogenous changes in export supplies in response to the policies.

Figure 7: Distribution of U.S. Trade and Bilateral Net Import Share

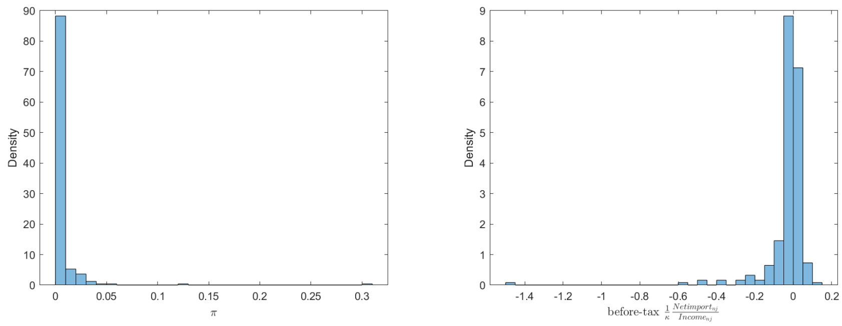
(a) π (b) Net Import Share



Note: Panel (a) shows the distribution of U.S. before-tax import share for all country-sector. Panel (b) shows the distribution of U.S before-tax net import share in foreign income, normalized by κ, for all country-sector.

Figure 8: Distribution of China's Trade and Bilateral Net Import Share

(a) π (b) Net Import Share



Note: Panel (a) shows the distribution of China's before-tax import share for all country-sector. Panel (b) shows the distribution of China's before-tax net import share in foreign income, normalized by κ, for all country-sector.

Nash Optimal Policies We analyze the scenario in which the governments of all countries implement policies that result in a Nash equilibrium. Table 3-6 shows the U.S. and China's optimal policies across countries and sectors under the Nash equilibrium.[24]

[24]To compare with the unilateral optimal policies, we use the same normalizations as unilateral when the U.S. and China implement optimal policies in the Nash equilibrium.

Similarly to the unilateral policy case, export taxes and import tariffs under the Nash equilibrium are heterogeneous across sectors and countries. For example, the U.S. imposes higher tariffs on Indonesia in the textiles sector and on Japan in the vehicles sector, compared to other countries. U.S. optimal import tariffs are, on average, lowest in the oil sector and highest in the textiles sector, which reflects the same pattern observed under unilateral policies. However, the magnitude of these policies differs under the Nash equilibrium.

In general, Nash policies are much more complicated due to foreign endogenous market power and the interdependencies in their foreign trading partners. In a two-country world with a sectoral comparative advantage, trade will be significantly reduced by Nash policies. As differential market power across sectors diminishes, the standard deviation of Home's tariffs across sectors would also decrease, compared with unilateral policies. However, with multiple countries, the third-country policies would affect trade between Home and a foreign country. This may lead to an increase in the dispersion of imports across foreign sectors, meaning that the standard deviation of tariffs across sectors for a foreign country may not necessarily decrease, compared with unilateral policies.

The welfare outcomes under Nash optimal policies, presented in the last column of Table 7, while unilateral policies yield higher welfare gains for Home, the countries in Nash equilibrium face retaliation, resulting in lower welfare for all countries. Countries that face significant additional reductions in welfare, for example, European countries, are those countries that trade a lot with other countries than the U.S. and China.

5 Conclusion

In this paper, we provide a comprehensive understanding of optimal trade policies within a general equilibrium context. We derive explicit formulas for a unified setup incorporating general supply-side specifications that not only enhance theoretical clarity but also streamline the calculation of optimal taxes. The explicit formula for a two-country model with the CES supply system establishes a direct link to market power, offering novel insights into optimal policies in general equilibrium.

We also examine specifications beyond the CES framework. Our analysis reveals how sectoral interdependencies and trade network effects across countries influence endogenous foreign (self- and cross-) export supply elasticities and shape optimal policies. We quantify these impacts for unilateral policies by the U.S. and China within a 20-country framework, along with the derivation of Nash optimal policies.

In addition to assumptions on production technology, other factors can also influence trade and optimal trade policies. These factors include preferences, market structure, and political and strategic considerations. While extensive literature has examined some of these factors, our paper specifically concentrates on the implications of supply-side assumptions and the general equilibrium interdependencies across sectors and countries for trade policies. Using a unified framework, we aim to provide a clear and explicit understanding of how different model specifications matter in optimal taxation.

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Online Appendix to "Optimal Trade Policies and Market Power in General Equilibrium Trade Models"

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This appendix is organized as follows.

A. Proof for Proposition 1: Tax Neutrality

B. Proof for Lemma 1, Proposition 2 and 3: General Formula for Two-Country Optimal Policies and Optimal Policies under CES Supply System

C. Optimal Policies under CES Supply System: Examples

  C.1. One-factor Ricardo-Roy Model

  C.2. Generalized Multi-sector Krugman Model

  C.3. Differential Scale Elasticities and Imperfectly Mobile Labor

  C.4. CES Labor Market

D. Non-CES Supply System: Multi-factor Ricardo-Roy Model

E. Proof for Proposition 4: Optimal Unilateral Policies with Multiple Countries

F. Proof for Lemma 2

G. Nash Optimal Policies

H. Quantifying the Consequences of Optimal Policies

I. Alternative Proof for Proposition 2 and 3

A Proof for Proposition 1: Tax Neutrality

This section presents the Home government's problem and proves tax neutrality.

Home's problem Let $\tau^x_{n,j}$ denote the export tax imposed by country 1 (Home) on country n in sector j. Similarly, $\tau^m_{n,j}$ represents the import tariff imposed by Home on country n in sector j, and τ^d_j represents the domestic tax imposed by Home in sector j. Home government chooses $\{\tau^x_{n,j}, \tau^m_{n,j}, \tau^d_j\}$ for $n \neq 1$ to solve the following problem:

$$\max \frac{x_1}{P_1},$$

subject to world market equilibrium characterized by the following constraints:

The labor market specifications $\Omega(\{w_{n,j}, L_{n,j}\})$ hold.

The supply-side assumptions $\mathbb{S}(\{p_{n,j}, w_{n,j}, L_{n,j}\})$ hold.

Expenditures are given by

$$x_1 = \sum_{j=1}^{J} Y_{1,j} + \sum_{j=1}^{J}\sum_{i\neq 1}^{N} \beta_{i,j} \frac{\tau^x_{i,j}}{1+\tau^x_{i,j}} \pi_{i1,j} x_i + \sum_{j=1}^{J} \beta_{1,j} \sum_{i\neq 1}^{N} \frac{\tau^m_{i,j}}{1+\tau^m_{i,j}} \pi_{1i,j} x_1 + \sum_{j=1}^{J} \beta_{1,j} \frac{\tau^d_j}{1+\tau^d_j} \pi_{11,j} x_1, \quad (\gamma_x)$$
$$x_n = \sum_{j=1}^{J} Y_{n,j}, \quad n \neq 1,$$

where $Y_{n,j} = w_{n,j} L_{n,j}$ is the income in sector j of country n.

Goods market clearing conditions, for each j

$$E_{11,j} \equiv Y_{1,j} - \sum_{i\neq 1}^{N} \beta_{i,j} \frac{1}{1+\tau^x_{i,j}} \pi_{i1,j} x_i = \beta_{1,j} \frac{1}{1+\tau^d_j} \pi_{11,j} x_1, \quad (\gamma_{1,j})$$
$$E_{1n,j} \equiv Y_{n,j} - \sum_{i\neq 1}^{N} \beta_{i,j} \pi_{in,j} x_i = \beta_{1,j} \frac{1}{1+\tau^m_{n,j}} \pi_{1n,j} x_1, \quad (\gamma_{n,j}), \quad n \neq 1$$

where we define $E_{11,j}$ as the supply of Home to its domestic market in sector j, and $E_{1n,j}$ as the export supply of country n to Home in sector j.

Consumer prices indexes are given by

$$P_1 = \prod_{j=1}^{J} \left[(p_{1,j}(1+\tau^d_j))^{-\epsilon_j} + \sum_{i\neq 1}^{N} (p_{i,j}(1+\tau^m_{i,j}) d_{1i})^{-\epsilon_j} \right]^{-\frac{\beta_{1,j}}{\epsilon_j}},$$
$$P_n = \prod_{j=1}^{J} \left[(p_{1,j}(1+\tau^x_{n,j}) d_{n1})^{-\epsilon_j} + \sum_{i\neq 1}^{N} (p_{i,j} d_{ni})^{-\epsilon_j} \right]^{-\frac{\beta_{n,j}}{\epsilon_j}}, \quad n \neq 1.$$

Trade shares for each sector j are given by:

$$\pi_{11,j} = \frac{(p_{1,j}(1+\tau_j^d))^{-\epsilon_j}}{(p_{1,j}(1+\tau_j^d))^{-\epsilon_j} + \sum_{i\neq 1}^N (p_{i,j}(1+\tau_{i,j}^m)d_{1i})^{-\epsilon_j}},$$
$$\pi_{1n,j} = \frac{(p_{n,j}(1+\tau_{n,j}^m)d_{1n})^{-\epsilon_j}}{(p_{1,j}(1+\tau_j^d))^{-\epsilon_j} + \sum_{i\neq 1}^N (p_{i,j}(1+\tau_{i,j}^m)d_{1i})^{-\epsilon_j}}, \quad n \neq 1,$$
$$\pi_{n1,j} = \frac{(p_{1,j}(1+\tau_{n,j}^x)d_{n1})^{-\epsilon_j}}{(p_{1,j}(1+\tau_{n,j}^x)d_{n1})^{-\epsilon_j} + \sum_{i\neq 1}^N (p_{i,j}d_{ni})^{-\epsilon_j}}, \quad n \neq 1,$$
$$\pi_{nm,j} = \frac{(p_{mj}d_{nm})^{-\epsilon_j}}{(p_{1,j}(1+\tau_{n,j}^x)d_{n1})^{-\epsilon_j} + \sum_{i\neq 1}^N (p_{i,j}d_{ni})^{-\epsilon_j}}, \quad n \neq 1.$$

Tax neutrality Given $\Gamma = \{(\tau_{n,j}^m + 1, \tau_{n,j}^x + 1, \tau_j^d + 1) : \forall j, n \neq 1\}$ and $\check{\Gamma} = \{(\check{\tau}_{n,j}^m + 1, \check{\tau}_{n,j}^x + 1, \check{\tau}_j^d + 1) : \forall j, n \neq 1\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{\pi_{1m,j}, \pi_{nm,j}, \frac{w_{1,j}}{P_1}, \frac{w_{n,j}}{P_n}, \frac{x_1}{P_1}, \frac{x_n}{P_n}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

Assume $1 + \check{\tau}_{n,j}^m = \lambda(1+\tau_{n,j}^m)$, $1 + \check{\tau}_{n,j}^x = \frac{1+\tau_{n,j}^x}{\mu}$, $1 + \check{\tau}_j^d = \lambda\frac{1+\tau_j^d}{\mu}$ for any constant $\mu > 0$ and $\lambda > 0$. We guess the allocations $\{\check{\pi}_{1m,j}, \check{\pi}_{nm,j}, \frac{\check{w}_{1,j}}{\check{P}_1}, \frac{\check{w}_{n,j}}{\check{P}_n}, \frac{\check{x}_1}{\check{P}_1}, \frac{\check{x}_n}{\check{P}_n}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{\pi}_{1m,j} = \pi_{1m,j}$, $\check{\pi}_{nm,j} = \pi_{nm,j}$, $\check{P}_1 = P_1$, $\check{P}_n = \frac{P_n}{\lambda}$, $\check{w}_{1,j} = \mu\frac{w_{1,j}}{\lambda}$, $\check{w}_{n,j} = \frac{w_{n,j}}{\lambda}$, $\check{x}_1 = x_1$, $\check{x}_n = \frac{x_n}{\lambda}$ for $n \neq 1$. When the labor market specification in Ω is homogeneous of degree zero on wages, $\check{L}_{n,j} = L_{n,j}$, and prices $\mathbb{S}$ are homogeneous of degree one on wages, $\frac{\check{p}_{n,j}}{p_{n,j}} = \frac{\check{w}_{n,j}}{w_{n,j}}$. We then verify that all equilibrium conditions hold under the new equilibrium.

$$\begin{aligned}
\check{x}_1 &= \frac{\mu}{\lambda}\sum_{j=1}^J Y_{1,j} + \sum_{i\neq 1}^N\sum_{j=1}^J (1 - \frac{\mu}{1+\tau_{i,j}^x})\beta_{i,j}\pi_{i1,j}x_i\frac{1}{\lambda} + \sum_{j=1}^J \beta_{1,j}\sum_{i\neq 1}^N (1 - \frac{1}{\lambda(1+\tau_{i,j}^m)})\pi_{1i,j}x_1 \\
&+ \sum_{j=1}^J \beta_{1,j}(1 - \frac{\mu}{\lambda(1+\tau_j^d)})\pi_{11,j}x_1 \\
&= \frac{\mu}{\lambda}\sum_{j=1}^J \beta_{1,j}\frac{1}{1+\tau_j^d}\pi_{11,j}x_1 + \frac{\mu}{\lambda}\sum_{i\neq 1}^N \beta_{i,j}\frac{1}{1+\tau_{i,j}^x}\pi_{i1,j}x_i + \sum_{i\neq 1}^N\sum_{j=1}^J \beta_{i,j}(1 - \frac{\mu}{1+\tau_{i,j}^x})\pi_{i1,j}\frac{x_i}{\lambda} \\
&+ \sum_{j=1}^J \beta_{1,j}\sum_{i\neq 1}^N (1 - \frac{1}{\lambda(1+\tau_{i,j}^m)})\pi_{1i,j}x_1 + \sum_{j=1}^J \beta_{1,j}(1 - \frac{\mu}{\lambda(1+\tau_j^d)})\pi_{11,j}x_1 \\
&= \sum_{i\neq 1}^N\sum_{j=1}^J \beta_{i,j}\pi_{i1,j}\frac{x_i}{\lambda} + \sum_{j=1}^J \beta_{1,j}\sum_{i\neq 1}^N (1 - \frac{1}{\lambda(1+\tau_{i,j}^m)})\pi_{1i,j}x_1 + \sum_{j=1}^J \beta_{1,j}\pi_{11,j}x_1 \\
&= \sum_{i\neq 1}^N\sum_{j=1}^J \beta_{1,j}\pi_{1i,j}x_1 + \sum_{j=1}^J \beta_{1,j}\pi_{11,j}x_1 = x_1,
\end{aligned}$$

$$\check{x}_n = \sum_{j=1}^{J} \frac{1}{\lambda} Y_{n,j} = \frac{x_n}{\lambda}, \quad n \neq 1$$

$$\check{Y}_{1,j} = \check{w}_{1,j} L_{1,j} = \frac{\mu}{\lambda} w_{1,j} L_{1,j} = \frac{\mu}{\lambda} Y_{1,j},$$

$$\check{Y}_{n,j} = \check{w}_{n,j} L_{n,j} = \frac{\mu}{\lambda} w_{n,j} L_{n,j} = \frac{1}{\lambda} Y_{n,j}, \quad n \neq 1$$

$$\check{Y}_{1,j} = \frac{\mu}{\lambda(1+\tau_j^d)} \beta_{1,j} \pi_{11,j} x_1 + \sum_{i \neq 1}^{N} \beta_{i,j} \frac{\mu}{1+\tau_{i,j}^x} \pi_{i1,j} \frac{x_i}{\lambda} = \frac{\mu}{\lambda} Y_{1,j},$$

$$\check{Y}_{n,j} = \beta_{1,j} \frac{1}{\lambda(1+\tau_{n,j}^m)} \pi_{1n,j} x_1 + \sum_{i \neq 1}^{N} \beta_{i,j} \pi_{in,j} \frac{x_i}{\lambda} = \frac{1}{\lambda} Y_{n,j}, \quad n \neq 1$$

$$\check{P}_1 = \prod_{j=1}^{J} \left[\left(\frac{\mu}{\lambda} p_{1,j} \frac{\lambda}{\mu} (1+\tau_j^d)\right)^{-\epsilon_j} + \sum_{i \neq 1}^{N} \left(\frac{p_{i,j}}{\lambda} \lambda (1+\tau_{i,j}^m) d_{1i}\right)^{-\epsilon_j} \right]^{-\frac{\beta_{1,j}}{\epsilon_j}} = P_1 = 1,$$

$$\check{P}_n = \prod_{j=1}^{J} \left[\left(\frac{\mu}{\lambda} p_{1,j} \frac{1+\tau_{n,j}^x}{\mu} d_{n1}\right)^{-\epsilon_j} + \sum_{i \neq 1}^{N} \left(\frac{p_{i,j}}{\lambda} d_{ni}\right)^{-\epsilon_j} \right]^{-\frac{\beta_{n,j}}{\epsilon_j}} = \frac{P_n}{\lambda}, \quad n \neq 1$$

$$\check{\pi}_{11,j} = \frac{\left(\frac{\mu}{\lambda} p_{1,j} \frac{\lambda}{\mu} (1+\tau_j^d)\right)^{-\epsilon_j}}{\left(\frac{\mu}{\lambda} p_{1,j} \frac{\lambda}{\mu} (1+\tau_j^d)\right)^{-\epsilon_j} + \sum_{i \neq 1}^{N} \left(\frac{p_{i,j}}{\lambda} \lambda (1+\tau_{i,j}^m) d_{1i}\right)^{-\epsilon_j}} = \pi_{11,j},$$

$$\check{\pi}_{n1,j} = \frac{\left(\frac{\mu}{\lambda} p_{1,j} \frac{1+\tau_j^x}{\mu} d_{n1}\right)^{-\epsilon_j}}{\left(\frac{\mu}{\lambda} p_{1,j} \frac{1+\tau_j^x}{\mu} d_{n1}\right)^{-\epsilon_j} + \sum_{i \neq 1}^{N} \left(\frac{p_{i,j}}{\lambda} d_{ni}\right)^{-\epsilon_j}} = \pi_{n1,j}. \quad n \neq 1$$

The same allocations satisfy the equilibrium conditions under $\{\check{\tau}_{n,j}^m + 1, \check{\tau}_{n,j}^x + 1, \check{\tau}_j^d + 1\}$. Hence, we proved that the allocations and welfare are the same under $\{\tau_{n,j}^m + 1, \tau_{n,j}^x + 1, \tau_j^d + 1\}$ and $\{\check{\tau}_{n,j}^m + 1, \check{\tau}_{n,j}^x + 1, \check{\tau}_j^d + 1\}$.

The Home government can manipulate $2J(N-1)+J$ prices. We can first normalize one domestic tax to zero, for example, $\check{\tau}_1^d = 0$ for sector 1. For any τ_1^d, we can always choose λ and μ, such that $\mu/\lambda = 1+\tau_1^d$ to get $\check{\tau}_1^d = \lambda(1+\tau_1^d)/\mu - 1 = 0$ and scale other taxes accordingly. The welfare and allocation are equivalent. Second, we can set an import tariff from one specific country and sector to zero, for example, $\check{\tau}_{2,1}^m = 0$ by setting $\lambda = 1/(\tau_{2,1}^m + 1)$ for any $\tau_{2,1}^m$ and adjusting other taxes accordingly to achieve the same allocation. Alternatively, one of the export taxes, $\check{\tau}_{2,1}^x$, can be normalized to zero by setting $\mu = 1 + \tau_{2,1}^x$ for any $\tau_{2,1}^x$. In either case, it suffices for the government to use only the $2J(N-1)+J-2$ instruments to implement the same allocation. In our optimization framework, we normalize one domestic tax and one trade tax to uniquely determine the level of all remaining taxes. □

B Proof for Lemma 1, Proposition 2 and 3: General Formula for Two-Country Optimal Policies and Optimal Policies under CES Supply System

This section presents the Home government's problem in the two-country case. We first present the maximization problem, then prove the optimal policies under our defined CES supply system and the general framework, as shown in Propositions 2 and 3.

Home government chooses policies $\{\tau_j^d, \tau_j^x, \tau_j^m\}$ to maximize domestic households' consumption x_1/P_1 subject to the world market equilibrium defined in Definition 1. We solve this optimization problem with the Lagrangian multiplier through a two-step approach.

In the first step, we derive supply curves $Y_{n,j}(\{p_{n,k}\})$ for each country-sector pair using the supply-side assumptions $\mathbb{S}$ and labor market specifications Ω. We can then express the equilibrium variables, including $\{w_{n,j}, L_{n,j}\}$, as functions of sectoral prices $\{p_{n,j}\}$ and policy instruments. In the second step, we take first-order conditions with respect to expenditures x_1, sectoral prices $\{p_{1,j}, p_{2,j}\}$, and policy instruments, imposing multipliers on the goods-market clearing conditions while treating income $Y_{n,j}$ as endogenous functions of $\{p_{n,k}\}$ through the derived supply curves.

An alternative approach involves taking first-order conditions directly with respect to $\{w_{n,j}, L_{n,j}, p_{n,j}\}$ while imposing additional multipliers on the supply-side assumptions $\mathbb{S}$ and labor market specifications Ω. Both approaches yield identical optimal policies and equilibrium outcomes. However, our two-step approach provides a unified framework for characterizing optimal policies by first deriving supply curves that encapsulate the underlying supply structure. The policy formulas obtained through this approach also offer a clear economic intuition and interpretation. We demonstrate the equivalence of these two approaches across three examples in Appendix C.

Step 1: Derive supply curves $Y_{n,j}(\{p_{n,k}\})$ Income $Y_{n,j}$ in each sector is defined as $Y_{n,j} = w_{n,j}L_{n,j}$. We derive the supply curves $Y_{n,j}(\{p_{n,k}\})$ for sector j in country n by combining the supply-side assumptions $\mathbb{S}$ and the labor market specifications Ω. In each country, the set $\mathbb{S}$ specifies J equations that links sectoral prices, wages, and labor $\{p_{n,j}, w_{n,j}, L_{n,j}\}$ both within and across sectors, while the set Ω provides an additional J equations governing labor market conditions.

For each country n, we have $4J$ variables $\{Y_{n,j}, p_{n,j}, w_{n,j}, L_{n,j}\}$ that must satisfy the following $3J$

conditions,

$$\begin{aligned} &[\text{income definition}] && Y_{n,j} = w_{n,j} L_{n,j}, \quad \text{for all } j \\ &[\text{supply-side assumption}] && \mathbb{S}(\{p_{n,j}, w_{n,j}, L_{n,j}\}), \\ &[\text{labor market specification}] && \Omega(\{w_{n,j}, L_{n,j}\}). \end{aligned}$$

This system of $3J$ equations in $4J$ unknowns yields J degrees of freedom, which generate the supply curves or linking sectoral income $Y_{n,j}$ with prices across all sectors $\{p_{n,k}\}$. From these supply curves, We can construct a matrix of partial supply elasticities Λ_n, where each element represents the weighted elasticity $\frac{\partial \ln Y_{n,k}}{\partial \ln p_{n,j}} \frac{Y_{n,k}}{Y_{n,j}}$.

The derived supply curves have incorporated both the supply-side assumptions $\mathbb{S}$ and the labor market specifications Ω. This allows us to simplify our optimization problem by eliminating the explicit consideration of conditions $\mathbb{S}$ and Ω along with wages and labor $\{w_{n,j}, L_{n,j}\}$. Instead, we can focus on the reduced system involving only $\{Y_{n,j}, p_{n,k}\}$ and the derived supply relationships.

Example: Consider a generalized Krugman model where each sector has identical scale elasticity. The supply-side assumption $\mathbb{S}$ specifies $p_{n,j} = \bar{T}_{n,j}^{-1} w_{n,j} L_{n,j}^{1/(1-\sigma)}$. The labor market specification Ω assumes perfectly mobile labor across sectors, which generates one labor constraint $\sum_{j=1}^{J} L_{n,j} = \bar{L}_n$ and wage equalization conditions $w_{n,j} = w_n$ for all sectors. The complete system of $3J$ conditions for country n consists of

$$\begin{aligned} &[\text{definition of income}] && Y_{n,j} = w_{n,j} L_{n,j}, \\ &[\text{supply-side assumption}] && p_{n,j} = \bar{T}_{n,j}^{-1} w_{n,j} L_{n,j}^{1/(1-\sigma)}, \\ &[\text{labor market specification}] && w_{n,j} = w_n, \\ &&& \sum_{j=1}^{J} L_{n,j} = \bar{L}_n. \end{aligned}$$

To derive the supply curves, we eliminate the wage and labor variables $w_{n,j}$ and $L_{n,j}$ from this system, and obtain the following supply curves

$$Y_{n,j} = \left(\frac{1}{\bar{L}_n} \sum_{k=1}^{J} p_{n,k}^{1-\sigma} \bar{T}_{n,k}^{1-\sigma} \right)^{\frac{\sigma}{1-\sigma}} p_{n,j}^{1-\sigma} \bar{T}_{n,j}^{1-\sigma}.$$

This expression demonstrates that sectoral income $Y_{n,j}$ depends not only on its own prices $p_{n,j}$ but also on prices across all other sectors $p_{n,k}$.

Step 2: Derive optimal policies We now replace the conditions $\mathbb{S}$ and Ω with the derived supply curves $Y_{n,j}(\{p_{n,k}\})$. Note that foreign prices do not affect home supply curves directly.

In the two-country case, the Home government choose $\{x_1, p_{1,j}, p_{2,j}, \tau_j^d, \tau_j^x, \tau_j^m\}$ to solve the following problem:

$$\max_{\{\tau_j^d,\tau_j^x,\tau_j^m,x_1,p_{1,j},p_{2,j}\}} \frac{x_1}{P_1},$$

subject to world market equilibrium characterized by the following constraints:

Home expenditures

$$x_1 = \sum_{j=1}^{J} Y_{1,j}(\{p_{1,k}\}) + \sum_{j=1}^{J} \beta_{2,j} \frac{\tau_j^x}{1+\tau_j^x} \pi_{21,j} x_2 + \sum_{j=1}^{J} \beta_{1,j} \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \sum_{j=1}^{J} \beta_{1,j} \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1, \quad (\gamma_x) \tag{A.1}$$

Goods market clearing conditions

$$Y_{1,j}(\{p_{1,k}\}) - \beta_{2,j} \frac{1}{1+\tau_j^x} \pi_{21,j} x_2 = \beta_{1,j} \frac{1}{1+\tau_j^d} \pi_{11,j} x_1, \quad (\gamma_{1,j}, \quad J) \tag{A.2}$$

$$Y_{2,j}(\{p_{2,k}\}) - \beta_{2,j} \pi_{22,j} x_2 = \beta_{1,j} \frac{1}{1+\tau_j^m} \pi_{12,j} x_1, \quad (\gamma_{2,j}, \quad J) \tag{A.3}$$

where $\{x_2, P_1, P_2\}$ and trade shares are given by

$$x_2 = \sum_{j=1}^{J} Y_{2,j}(\{p_{2,k}\}), \tag{A.4}$$

$$\pi_{11,j} = \frac{(p_{1,j}(1+\tau_j^d))^{-\epsilon_j}}{(p_{1,j}(1+\tau_j^d))^{-\epsilon_j} + (p_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j}}, \tag{A.5}$$

$$\pi_{12,j} = \frac{(p_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j}}{(p_{1,j}(1+\tau_j^d))^{-\epsilon_j} + (p_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j}}, \tag{A.6}$$

$$\pi_{21,j} = \frac{(p_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j}}{(p_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j} + (p_{2,j})^{-\epsilon_j}}, \tag{A.7}$$

$$\pi_{22,j} = \frac{(p_{2,j})^{-\epsilon_j}}{(p_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j} + (p_{2,j})^{-\epsilon_j}}, \tag{A.8}$$

$$P_1 = \prod_{j=1}^{J} \left[(p_{1,j}(1+\tau_j^d))^{-\epsilon_j} + (p_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j} \right]^{-\frac{\beta_{1,j}}{\epsilon_j}} = 1, \tag{A.9}$$

$$P_2 = \prod_{j=1}^{J} \left[(p_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j} + p_{2,j}^{-\epsilon_j} \right]^{-\frac{\beta_{2,j}}{\epsilon_j}}. \tag{A.10}$$

One of the equilibrium conditions in (A.1), (A.2) and (A.3) is redundant, according to Walras' law, thus, we drop the expenditure constraint in optimization and set $\gamma_x = 0$. Or, equivalently, if we use Home's income equation, one of the market-clearing conditions is redundant.

FOC over x_1

$$1 + \sum_{j=1}^{J} \gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\pi_{11,j} + \sum_{j=1}^{J} \gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\pi_{12,j} = 0.$$

FOC over export tax τ_j^x

$$-\gamma_{1,j}\beta_{2,j}\frac{\pi_{21,j}x_2}{(1+\tau_j^x)^2} + \gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\frac{\partial \pi_{21,j}}{\partial \tau_j^x}x_2 + \gamma_{2,j}\beta_{2,j}\frac{\partial \pi_{22,j}}{\partial \tau_j^x}x_2 = 0.$$

After plugging in derivatives and simplifying, the FOC over τ_j^x becomes

$$1 + \tau_j^x = \frac{\gamma_{1,j}(\epsilon_j \pi_{22,j} + 1)}{\gamma_{2,j}\epsilon_j \pi_{22,j}}. \tag{A.11}$$

FOC over import tariff τ_j^m

$$-\beta_{1,j}x_1\frac{\pi_{12,j}}{1+\tau_j^m} + \gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial \tau_j^m}x_1 - \gamma_{2,j}\beta_{1,j}\frac{\pi_{12,j}}{(1+\tau_j^m)^2}x_1 + \gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial \tau_{n,j}^m}x_1 = 0.$$

Plugging in derivatives and simplifying, the FOC over τ_j^m becomes

$$1 + \tau_j^m = \frac{-\gamma_{2,j}\left(1 + \epsilon_j \pi_{11,j}\right)}{-\gamma_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j \pi_{11,j} + 1}. \tag{A.12}$$

FOC over domestic tax τ_j^d

$$-\beta_{1,j}x_1\frac{\pi_{11,j}}{1+\tau_j^d} - \gamma_{1,j}\frac{1}{(1+\tau_j^d)^2}\beta_{1,j}\pi_{11,j}x_1 + \gamma_{1,j}\frac{1}{1+\tau_j^d}\beta_{1,j}\frac{\partial \pi_{11,j}}{\partial \tau_j^d}x_1 + \gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\frac{\partial \pi_{12,j}}{\partial \tau_j^d}x_1 = 0.$$

Plugging in derivatives and simplifying, the FOC over τ_j^d becomes

$$1 + \tau_j^d = \frac{-\gamma_{1,j}\left(1 + \epsilon_j \pi_{12,j}\right)}{-\gamma_{2,j}\frac{1}{1+\tau_j^m}\epsilon_j \pi_{12,j} + 1}. \tag{A.13}$$

Combining the optimal tariff (A.12) and domestic tax (A.13), we can get

$$1+\tau_j^m = -\gamma_{2,j}, \qquad 1+\tau_j^d = -\gamma_{1,j}. \tag{A.14}$$

Combining the above two equations with equation (A.11), we get

$$1+\tau_j^x = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\epsilon_j \pi_{22,j}}\right). \tag{A.15}$$

FOC over $p_{1,j}$

$$\begin{aligned}
&-\beta_{1,j}x_1\pi_{11,j} - \gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}\pi_{12,j}x_1 - \gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2 \\
&+\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1 + \gamma_{2,j}\beta_{2,j}\epsilon_j\pi_{22,j}\pi_{21,j}x_2 + \sum_{k=1}^{J}(-\gamma_{1,k})\frac{\partial Y_{1,k}}{\partial p_{1,j}}p_{1,j} = 0.
\end{aligned} \tag{A.16}$$

FOC over $p_{2,j}$

$$\begin{aligned}
&-\beta_{1,j}x_1\pi_{12,j} + \gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}\pi_{12,j}x_1 + \gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2 \\
&-\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1 - \gamma_{2,j}\beta_{2,j}\epsilon_j\pi_{22,j}\pi_{21,j}x_2 + \sum_{k=1}^{J}(cons_1-\gamma_{2,k})\frac{\partial Y_{2,k}}{\partial p_{2,j}}p_{2,j} = 0,
\end{aligned} \tag{A.17}$$

where $cons_1 = \sum_{s=1}^{J}(\gamma_{1,s}\beta_{2,s}\frac{1}{1+\tau_s^x}\pi_{21,s} + \gamma_{2,s}\beta_{2,s}\pi_{22,s})$.

B.1 Proof of Optimal Domestic Tax Formula

Combining FOCs over $p_{1,j}$ (A.16), τ_j^x(A.15), and τ_j^d, τ_j^m (A.14), we get

$$\begin{aligned}
&-\beta_{1,j}x_1\pi_{11,j} + \beta_{2,j}\left(\frac{1+\tau_j^d}{1+\tau_j^x} - (1+\tau_j^m)\right)\epsilon_j\pi_{21,j}\pi_{22,j}x_2 + \sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial Y_{1,k}}{\partial p_{1,j}}p_{1,j} = 0 \\
&\Rightarrow -\beta_{1,j}x_1\pi_{11,j} - \beta_{2,j}\frac{1+\tau_j^d}{1+\tau_j^x}\pi_{21,j}x_2 + \sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial Y_{1,k}}{\partial p_{1,j}}p_{1,j} = 0 \\
&\Rightarrow \sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial Y_{1,k}}{\partial p_{1,j}}p_{1,j} - (1+\tau_j^d)Y_{1,j} = 0 \\
&\Rightarrow \sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial \ln(Y_{1,k})}{\partial \ln(p_{1,j})}\frac{Y_{1,k}}{Y_{1,j}} - (1+\tau_j^d) = 0.
\end{aligned} \tag{A.18}$$

Hence, domestic taxes $\boldsymbol{\tau^d} = [\tau_1^d, ..., \tau_j^d, ..., \tau_J^d]'$ for J sectors satisfy

$$(\Lambda_1 - I)(1 + \boldsymbol{\tau^d}) = 0,$$

where element of matrix Λ_n at row j and column k is given by $\frac{\partial \ln(Y_{n,k})}{\partial \ln(p_{n,j})} \frac{Y_{n,k}}{Y_{n,j}}$, and

$$\Lambda_1 - I = \begin{pmatrix} \frac{\partial ln(Y_{1,1})}{\partial ln(p_{1,1})} \frac{Y_{1,1}}{Y_{1,1}} - 1 & ... & \frac{\partial ln(Y_{1,j})}{\partial ln(p_{1,1})} \frac{Y_{1,j}}{Y_{1,1}} & ... & \frac{\partial ln(Y_{1,J})}{\partial ln(p_{1,1})} \frac{Y_{1,J}}{Y_{1,1}} \\ ... & ... & ... & ... & ... \\ \frac{\partial ln(Y_{1,1})}{\partial ln(p_{1,j})} \frac{Y_{1,1}}{Y_{1,j}} & ... & \frac{\partial ln(Y_{1,j})}{\partial ln(p_{1,j})} \frac{Y_{1,j}}{Y_{1,j}} - 1 & ... & \frac{\partial ln(Y_{1,J})}{\partial ln(p_{1,j})} \frac{Y_{1,J}}{Y_{1,j}} \\ ... & ... & ... & ... & ... \\ \frac{\partial ln(Y_{1,1})}{\partial ln(p_{1,J})} \frac{Y_{1,1}}{Y_{1,J}} & ... & \frac{\partial ln(Y_{1,j})}{\partial ln(p_{1,J})} \frac{Y_{1,j}}{Y_{1,J}} & ... & \frac{\partial ln(Y_{1,J})}{\partial ln(p_{1,J})} \frac{Y_{1,J}}{Y_{1,J}} - 1 \end{pmatrix}.$$

Divide equation (A.18) by η_j. For any sector k and j, optimal domestic taxes satisfy

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{1,s}}{\partial \ln p_{1,j}} \frac{Y_{1,s}}{Y_{1,j}} (1 + \tau_s^d) \frac{1}{\eta_j} = (1 + \tau_j^d) \frac{1}{\eta_j}, \tag{A.19}$$

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{1,s}}{\partial \ln p_{1,k}} \frac{Y_{1,s}}{Y_{1,k}} (1 + \tau_s^d) \frac{1}{\eta_k} = (1 + \tau_k^d) \frac{1}{\eta_k}, \tag{A.20}$$

where η_j is the partial supply elasticity parameter in sector j. Using the definition of CES supply system in Definition 4, and subtracting (A.20) from (A.19), we have

$$(1 + \tau_j^d) - (1 + \tau_k^d) = (1 + \tau_j^d) \frac{1}{\eta_j} - (1 + \tau_k^d) \frac{1}{\eta_k} \quad \Rightarrow \quad \frac{1 + \tau_j^d}{1 + \tau_k^d} = \frac{\frac{\eta_j}{\eta_j - 1}}{\frac{\eta_k}{\eta_k - 1}}.$$

According to the tax neutrality shown in Proposition 1, we can set $1 + \tau_j^d = (1 + \bar{\tau}^d) \frac{\eta_j}{\eta_j - 1}, \forall j$. The uniform shifter $\bar{\tau}^d$ can be assigned any arbitrary value, and one of the trade policies can be normalized, without changing the real allocations and welfare impacts of optimal policies.

B.2 Proof of Optimal Import Tariff Formula

Using FOC over $p_{2,j}$ (A.17), τ_j^x(A.15), and τ_j^d, τ_j^m (A.14), we get for each sector j

$$\begin{aligned}
&\sum_{k=1}^{J}(cons_1+1+\tau_k^m)\left[\frac{\partial\ln Y_{2,k}}{\partial\ln p_{2,j}}Y_{2,k}\right]=\beta_{1,j}\pi_{12,j}x_1+\left[\frac{1+\tau_j^d}{1+\tau_j^x}-(1+\tau_j^m)\right]\beta_{2,j}\epsilon_j\pi_{21,j}\pi_{22,j}x_2\\
\Rightarrow&\sum_{k=1}^{J}(cons_1+1+\tau_k^m)\left[\frac{\partial\ln Y_{2,k}}{\partial\ln p_{2,j}}\frac{Y_{2,k}}{Y_{2,j}}\right]=\frac{\beta_{1,j}\pi_{12,j}x_1}{Y_{2,j}}-\frac{1+\tau_j^d}{1+\tau_j^x}\beta_{2,j}\pi_{21,j}\frac{x_2}{Y_{2,j}} \qquad \text{(A.21)}\\
\Rightarrow&\sum_{k=1}^{J}(cons_1+1+\tau_k^m)\left[\frac{\partial\ln Y_{2,k}}{\partial\ln p_{2,j}}\frac{Y_{2,k}}{Y_{2,j}}\right]=\frac{\beta_{1,j}\pi_{12,j}x_1}{Y_{2,j}}-(1+\tau_j^d)\frac{Y_{1,j}}{Y_{2,j}}+\frac{\beta_{1,j}\pi_{11,j}x_1}{Y_{2,j}},
\end{aligned}$$

where $cons_1$ is common across all sectors. using the tax neutrality property established in Proposition 1, we normalize $cons_1+1=0$ and the above equation becomes

$$\sum_{k=1}^{J}\tau_k^m\left[\frac{\partial\ln Y_{2,k}}{\partial\ln p_{2,j}}\frac{Y_{2,k}}{Y_{2,j}}\right]=\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}. \qquad \text{(A.22)}$$

Let $\boldsymbol{\tau^m}=[\tau_1^m,...,\tau_j^m,...,\tau_J^m]'$ denote the import tariff vector. We can write equation (A.22) in matrix form as $\Lambda_2\boldsymbol{\tau^m}=\Psi_1$, where Λ_2 and Ψ_1 are given by

$$\Lambda_2=\begin{pmatrix}
\frac{\partial ln(Y_{2,1})}{\partial ln(p_{2,1})}\frac{Y_{2,1}}{Y_{2,1}} & ... & \frac{\partial ln(Y_{2,j})}{\partial ln(p_{2,1})}\frac{Y_{2,j}}{Y_{2,1}} & ... & \frac{\partial ln(Y_{2,J})}{\partial ln(p_{2,1})}\frac{Y_{2,J}}{Y_{2,1}}\\
... & ... & ... & ... & ...\\
\frac{\partial ln(Y_{2,1})}{\partial ln(p_{2,j})}\frac{Y_{2,1}}{Y_{2,j}} & ... & \frac{\partial ln(Y_{2,j})}{\partial ln(p_{2,j})}\frac{Y_{2,j}}{Y_{2,j}} & ... & \frac{\partial ln(Y_{2,J})}{\partial ln(p_{2,j})}\frac{Y_{2,J}}{Y_{2,j}}\\
... & ... & ... & ... & ...\\
\frac{\partial ln(Y_{2,1})}{\partial ln(p_{2,J})}\frac{Y_{2,1}}{Y_{2,J}} & ... & \frac{\partial ln(Y_{2,j})}{\partial ln(p_{2,J})}\frac{Y_{2,j}}{Y_{2,J}} & ... & \frac{\partial ln(Y_{2,J})}{\partial ln(p_{2,J})}\frac{Y_{2,J}}{Y_{2,J}}
\end{pmatrix},\quad
\Psi_1=\begin{pmatrix}
\frac{\beta_{1,1}x_1-(1+\tau_1^d)Y_{1,1}}{Y_{2,1}}\\
...\\
\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}\\
...\\
\frac{\beta_{1,J}x_1-(1+\tau_J^d)Y_{1,J}}{Y_{2,J}}
\end{pmatrix}. \qquad \text{(A.23)}$$

Divide equation (A.22) by η_j. For any sector k and j, optimal import tariffs satisfy

$$\sum_{s=1}^{J}\frac{\partial\ln Y_{2,s}}{\partial\ln p_{2,j}}\frac{Y_{2,s}}{Y_{2,j}}\tau_s^m\frac{1}{\eta_j}=\frac{1}{\eta_j}\Psi_{1,j}, \qquad \text{(A.24)}$$

$$\sum_{s=1}^{J}\frac{\partial\ln Y_{2,s}}{\partial\ln p_{2,k}}\frac{Y_{2,s}}{Y_{2,k}}\tau_s^m\frac{1}{\eta_k}=\frac{1}{\eta_k}\Psi_{1,k}. \qquad \text{(A.25)}$$

Using the definition of CES supply system in Definition 4, and subtracting (A.25) from (A.24), we have

$$
\begin{aligned}
&\left(\frac{\partial \ln Y_{2,j}}{\partial \ln p_{2,j}}\frac{1}{\eta_j}-\frac{\partial \ln Y_{2,j}}{\partial \ln p_{2,k}}\frac{Y_{2,j}}{Y_{2,k}}\frac{1}{\eta_k}\right)\tau_j^m+\left(\frac{\partial \ln Y_{2,k}}{\partial \ln p_{2,j}}\frac{Y_{2,k}}{Y_{2,j}}\frac{1}{\eta_j}-\frac{\partial \ln Y_{2,k}}{\partial \ln p_{2,k}}\frac{1}{\eta_k}\right)\tau_k^m=\frac{1}{\eta_j}\Psi_{1,j}-\frac{1}{\eta_k}\Psi_{1,k}\\
\Rightarrow&\tau_j^m-\tau_k^m=\frac{1}{\eta_j}\Psi_{1,j}-\frac{1}{\eta_k}\Psi_{1,k},
\end{aligned}
$$

where $\Psi_{1,j}$ and $\Psi_{1,k}$ are the jth and kth row of vector Ψ_1. □

C Optimal Policies under CES Supply System: Examples

This section presents three examples of CES supply system and derives their optimal policies. The three models include a one-factor Ricardo-Roy model (Section C.1), a generalized multi-sector Krugman model (Section C.2), and a model with sector-specific scale elasticities and imperfectly mobile labor (Section C.3). In addition, Section C.4 provides a proof that the labor market specification in a CES labor market is equivalent to that in the one-factor Ricardo-Roy model, and examines two special cases: perfectly mobile labor across sectors and immobile/sector-specific labor.

We use two methods to characterize optimal policies and demonstrate that both yield the same tax formulas.

In the first method, we formulate the Home government's problem and the world market equilibrium based on the original equilibrium conditions of each model. In each case, we use the explicit functional forms of $\mathbb{S}$ and Ω specific to each case and solve $\{p_{n,j}\}$ as functions of wages and labor $\{w_{n,j}, L_{n,j}\}$ using the model-specific supply function $\mathbb{S}(\{p_{n,j}, w_{n,j}, L_{n,j}\})$. We then take first-order conditions with respect to $\{w_{n,j}, L_{n,j}\}$ and impose multipliers on each model's labor market conditions Ω. This method differs from our general approach in two ways: first, it relies on specific functional forms of $\mathbb{S}$ and Ω rather than treating them generically, and second, it takes derivatives directly with respect to $\{w_{n,j}, L_{n,j}\}$ rather than with respect to derived supply curves while treating wages and labor as functions of sectoral prices $\{p_{n,j}\}$.

In the second method, we prove that each model satisfies the CES supply system requirement and apply the formulas from Proposition 3 directly. Both methods yield identical tax formulas, confirming the validity of our general approach.

C.1 One-factor Ricardo-Roy Model

In Section C.1.1, we formulate the Home government's problem and the world market equilibrium based on the original equilibrium conditions of the one-factor Ricardo-Roy model. We solve $\{p_{n,j}\}$ as functions of wages and labor $\{w_{n,j}, L_{n,j}\}$ using the model-specific supply function $\mathbb{S}(\{p_{n,j}, w_{n,j}, L_{n,j}\})$. We then directly take first-order conditions with respect to $\{w_{n,j}, L_{n,j}\}$ to derive optimal policies in this model. In Section C.1.2, we show that this model is a CES supply system and employ matrix operations to establish a connection with Proposition 3.

C.1.1 Method 1: Model Specific Proof

In the one-factor Ricardo-Roy model, due to the constant returns to scale, prices are proportional to wages. The supply-side assumption $\mathbb{S}$ satisfies

$$p_{n,j} = \frac{w_{n,j}}{\bar{T}_{n,j}}. \tag{A.26}$$

where $\bar{T}_{n,j}$ denotes the exogenous technology level beyond the economies of scale in the sector j of country n. In the following market equilibrium constraints, we substitute $p_{n,j}$ with $w_{n,j}$.

Applying the labor market specificaiton Ω (Eq.19) with one-factor $G = 1$, we obtain the one-factor Ricardo-Roy model Ω satisfying

$$w_{n,j}L_{n,j} = A_{n,j}(\frac{w_{n,j}}{W_n})^{\kappa} W_n \bar{L}_n, \quad W_n = [\sum_{j=1}^{J} A_{n,j} {w_{n,j}}^{\kappa}]^{\frac{1}{\kappa}}. \tag{A.27}$$

Using this condition, we can eliminate $L_{n,j}$ from the optimization system and substitute

$$L_{n,j} = A_{n,j}(w_{n,j})^{\kappa-1} \left[\sum_{j=1}^{J} A_{n,j} {w_{n,j}}^{\kappa}\right]^{\frac{1-\kappa}{\kappa}} \bar{L}_n.$$

Now Home government chooses $\{\tau_j^d, \tau_j^x, \tau_j^m, x_1, w_{1,j}, w_{2,j}\}$ to solve the following problem:

$$\max \ \frac{x_1}{P_1},$$

subject to world market equilibrium conditions

$$x_1 = \sum_{j=1}^{J} A_{1,j}(w_{1,j})^{\kappa} \left[\sum_{j=1}^{J} A_{1,j}{w_{1,j}}^{\kappa}\right]^{\frac{1-\kappa}{\kappa}} \bar{L}_1 + \sum_{j=1}^{J} \beta_{2,j}\frac{\tau_j^x}{1+\tau_j^x}\pi_{21,j}x_2 + \sum_{j=1}^{J}\beta_{1,j}\frac{\tau_j^m}{1+\tau_j^m}\pi_{12,j}x_1$$
$$+ \sum_{j=1}^{J}\beta_{1,j}\frac{\tau_j^d}{1+\tau_j^d}\pi_{11,j}x_1, \quad (\gamma_x) \tag{A.28}$$

$$A_{1,j}(w_{1,j})^{\kappa}\left[\sum_{j=1}^{J} A_{1,j}{w_{1,j}}^{\kappa}\right]^{\frac{1-\kappa}{\kappa}} \bar{L}_1 - \beta_{2,j}\frac{1}{1+\tau_j^x}\pi_{21,j}x_2 = \beta_{1,j}\frac{1}{1+\tau_j^d}\pi_{11,j}x_1, \quad (\gamma_{1,j}, \quad J) \tag{A.29}$$

$$A_{2,j}(w_{2,j})^{\kappa}\left[\sum_{j=1}^{J} A_{2,j}{w_{2,j}}^{\kappa}\right]^{\frac{1-\kappa}{\kappa}} \bar{L}_2 - \beta_{2,j}\pi_{22,j}x_2 = \beta_{1,j}\frac{1}{1+\tau_j^m}\pi_{12,j}x_1, \quad (\gamma_{2,j}, \quad J) \tag{A.30}$$

where $\{x_2, P_1, P_2\}$ and trade shares are given by

$$x_2 = \sum_{j=1}^{J} A_{2,j}(w_{2,j})^{\kappa}\left[\sum_{j=1}^{J} A_{2,j}{w_{2,j}}^{\kappa}\right]^{\frac{1-\kappa}{\kappa}} \bar{L}_n, \tag{A.31}$$

$$\pi_{11,j} = \frac{\bar{T}_{1,j}^{\epsilon_j}(w_{1,j}(1+\tau_j^d))^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j}(w_{1,j}(1+\tau_j^d))^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j}(w_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j}}, \tag{A.32}$$

$$\pi_{12,j} = \frac{\bar{T}_{2,j}^{\epsilon_j}(w_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j}(w_{1,j}(1+\tau_j^d))^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j}(w_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j}}, \tag{A.33}$$

$$\pi_{21,j} = \frac{\bar{T}_{1,j}^{\epsilon_j}(w_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j}(w_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j}(w_{2,j})^{-\epsilon_j}}, \tag{A.34}$$

$$\pi_{22,j} = \frac{\bar{T}_{2,j}^{\epsilon_j}(w_{2,j})^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j}(w_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j}(w_{2,j})^{-\epsilon_j}}, \tag{A.35}$$

$$P_1 = \prod_{j=1}^{J}\left[\bar{T}_{1,j}^{\epsilon_j}(w_{1,j}(1+\tau_j^d))^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j}(w_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j}\right]^{-\frac{\beta_{1,j}}{\epsilon_j}} = 1, \tag{A.36}$$

$$P_2 = \prod_{j=1}^{J}\left[\bar{T}_{1,j}^{\epsilon_j}(w_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j}w_{2,j}^{-\epsilon_j}\right]^{-\frac{\beta_{2,j}}{\epsilon_j}}. \tag{A.37}$$

One of the equilibrium conditions in (A.28), (A.29) and (A.30) is redundant, according to Walras' law, thus, we drop the expenditure constraint in optimization and set $\gamma_x = 0$. Or, equivalently, if we use Home's budget constraint, one of the market-clearing conditions is redundant.

The FOC over $\tau_j^d, \tau_j^m, \tau_j^x$ are the same as in Appendix B. We can get the optimal policies:

$$1+\tau_j^m = -\gamma_{2,j}, \qquad 1+\tau_j^d = -\gamma_{1,j}. \tag{A.38}$$

$$1+\tau_j^x = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\epsilon_j \pi_{22,j}}\right). \tag{A.39}$$

FOC over x_1

$$1+\sum_{j=1}^{J}\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\pi_{11,j}+\sum_{j=1}^{J}\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\pi_{12,j}=0.$$

FOC over $w_{1,j}$

$$\begin{aligned}
&-\beta_{1,j}x_1\pi_{11,j}-\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}\pi_{12,j}x_1-\gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2+\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1\\
&+\gamma_{2,j}\beta_{2,j}\epsilon_j\pi_{22,j}\pi_{21,j}x_2-\kappa\gamma_{1,j}w_{1,j}L_{1,j}+\sum_{s=1}^{J}\gamma_{1,s}(\kappa-1)\lambda_{1,s}w_{1,j}L_{1,j}=0,
\end{aligned} \tag{A.40}$$

where the share of sector j income in country n as $\lambda_{n,j}=Y_{n,j}/Y_n$ and $Y_{n,j}=w_{n,j}L_{n,j}$, $Y_n=\sum_{j=1}^{J}Y_{n,j}$.

FOC over $w_{2,j}$

$$\begin{aligned}
&-\beta_{1,j}x_1\pi_{12,j}+\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}\pi_{12,j}x_1+\gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2-\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1\\
&-\gamma_{2,j}\beta_{2,j}\epsilon_j\pi_{22,j}\pi_{21,j}x_2+\kappa(cons_1-\gamma_{2,j})w_{2,j}L_{2,j}-\sum_{s=1}^{J}(cons_1-\gamma_{2,s})(\kappa-1)\lambda_{2,s}w_{2,j}L_{2,j}=0,
\end{aligned} \tag{A.41}$$

where $cons_1=\sum_{s=1}^{J}(\gamma_{1,s}\beta_{2,s}\frac{1}{1+\tau_s^x}\pi_{21,s}+\gamma_{2,s}\beta_{2,s}\pi_{22,s})$.

Combining FOCs over $w_{1,j}$ (A.40), τ_j^x(A.39), and τ_j^d, τ_j^m (A.38), we get

$$\begin{aligned}
&-\beta_{1,j}x_1\pi_{11,j}+\beta_{2,j}(\frac{1+\tau_j^d}{1+\tau_j^x}-(1+\tau_j^m))\epsilon_j\pi_{21,j}\pi_{22,j}x_2-\kappa\gamma_{1,j}w_{1,j}L_{1,j}+\sum_{s=1}^{J}\gamma_{1,s}(\kappa-1)\lambda_{1,s}w_{1,j}L_{1,j}=0\\
\Rightarrow&-\beta_{1,j}x_1\pi_{11,j}-\beta_{2,j}\frac{1+\tau_j^d}{1+\tau_j^x}\pi_{21,j}x_2-\kappa\gamma_{1,j}w_{1,j}L_{1,j}+\sum_{s=1}^{J}\gamma_{1,s}(\kappa-1)\lambda_{1,s}w_{1,j}L_{1,j}=0\\
\Rightarrow&(\kappa-1)(1+\tau_j^d)w_{1,j}L_{1,j}-\sum_{s=1}^{J}(1+\tau_s^d)(\kappa-1)\lambda_{1,s}w_{1,j}L_{1,j}=0\\
\Rightarrow&1+\tau_j^d=\sum_{s=1}^{J}(1+\tau_s^d)\lambda_{1,s}.
\end{aligned}$$

Thus, the optimal domestic tax is uniform across sectors.

Combining FOCs over $w_{2,j}$ (A.41), τ_j^x(A.39), and τ_j^d, τ_j^m (A.38), we get

$$\kappa(cons_1 - \gamma_{2,j})Y_{2,j} - \sum_{s=1}^{J}(cons_1 - \gamma_{2,s})(\kappa-1)\lambda_{2,s}Y_{2,j} = \beta_{1,j}\pi_{12,j}x_1 + \left[\frac{1+\tau_j^d}{1+\tau_j^x} - (1+\tau_j^m)\right]\beta_{2,j}\epsilon_j\pi_{21,j}\pi_{22,j}x_2,$$

$$\Rightarrow \kappa(cons_1 + 1 + \tau_j^m) - \sum_{s=1}^{J}(cons_1 + 1 + \tau_s^m)(\kappa-1)\lambda_{2,s} = \frac{\beta_{1,j}\pi_{12,j}x_1}{Y_{2,j}} - \frac{1+\tau_j^d}{1+\tau_j^x}\beta_{2,j}\pi_{21,j}\frac{x_2}{Y_{2,j}}$$

$$\Rightarrow \kappa(cons_1 + 1 + \tau_j^m) - \sum_{s=1}^{J}(cons_1 + 1 + \tau_s^m)(\kappa-1)\lambda_{2,s} = \frac{\beta_{1,j}\pi_{12,j}x_1}{Y_{2,j}} - (1+\tau_j^d)\frac{Y_{1,j}}{Y_{2,j}} + \frac{\beta_{1,j}\pi_{11,j}x_1}{Y_{2,j}}$$

$$\Rightarrow (cons_1 + 1 + \tau_j^m) - \sum_{s=1}^{J}(cons_1 + 1 + \tau_s^m)\frac{\kappa-1}{\kappa}\lambda_{2,s} = \frac{1}{\kappa}\frac{\beta_{1,j}x_1 - (1+\tau_j^d)Y_{1,j}}{Y_{2,j}}. \tag{A.42}$$

Finally, we take the difference of (A.42) between sectors j and k and derive the formula for optimal tariffs

$$\tau_j^m - \tau_k^m = \frac{1}{\kappa}\left[\frac{\beta_{1,j}x_1 - (1+\tau_j^d)Y_{1,j}}{Y_{2,j}} - \frac{\beta_{1,k}x_1 - (1+\tau_k^d)Y_{1,k}}{Y_{2,k}}\right]. \tag{A.43}$$

C.1.2 Method 2: Applying Proposition 3

We first prove that the one-factor Ricardo-Roy is a CES supply system, then apply the optimal policy formulas from Proposition 3 using the structural parameters from this model.

We can derive supply curves by combining the labor market specification Ω (Eq.A.27) with the supply-side assumption $\mathbb{S}$ (Eq.A.26) and the definition of income $Y_{n,j} = w_{n,j}L_{n,j}$. Eliminating the wage and labor $\{w_{n,j}, L_{n,j}\}$ yields the following supply curves:

$$Y_{n,j} = A_{n,j}\left(p_{n,j}\bar{T}_{n,j}\right)^{\kappa}\left[\sum_{s=1}^{J}A_{n,s}\left(p_{n,s}\bar{T}_{n,s}\right)^{\kappa}\right]^{\frac{1-\kappa}{\kappa}}\bar{L}_n.$$

The corresponding element of the scaled partial supply elasticity matrix Λ_n is given by

$$\frac{\partial\ln(Y_{n,s})}{\partial\ln(p_{n,j})}\frac{Y_{n,s}}{Y_{n,j}} = I_{s=j}(\kappa) - (\kappa-1)\lambda_{n,s},$$

where $I_{s=j}(\kappa)$ is an indicator function that equals κ when $s=j$ and 0 otherwise, $\lambda_{n,j} = Y_{n,j}/Y_n$ is the share of income in sector j produced in country n.

The scaled partial supply elasticity matrix Λ_n of country n takes the form of

$$\Lambda_n = \begin{pmatrix} \kappa-(\kappa-1)\lambda_{n,1} & ... & -(\kappa-1)\lambda_{n,j} & ... & -(\kappa-1)\lambda_{n,J} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\lambda_{n,1} & ... & \kappa-(\kappa-1)\lambda_{n,j} & ... & -(\kappa-1)\lambda_{n,J} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\lambda_{n,1} & ... & -(\kappa-1)\lambda_{n,j} & ... & \kappa-(\kappa-1)\lambda_{n,J} \end{pmatrix}. \tag{A.44}$$

Obviously, this model satisfies the requirement of the CES supply system. The partial elasticities of a sector's income with respect to price (scaled by the two sectors' size and elasticity parameter) satisfy $\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}} \frac{Y_{n,s}}{Y_{n,j}} \frac{1}{\eta_j} - \frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,i}} \frac{Y_{n,s}}{Y_{n,i}} \frac{1}{\eta_i} = 0$ for $s \neq i$ or j, and $\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}} \frac{Y_{n,s}}{Y_{n,j}} \frac{1}{\eta_j} - \frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,i}} \frac{Y_{n,s}}{Y_{n,i}} \frac{1}{\eta_i} = 1$ for $s = j$, where the parameter $\eta_j = \kappa$.

Proof of optimal domestic taxes According to Proposition 2, optimal domestic taxes satisfy

$$(\Lambda_1 - I)(1 + \boldsymbol{\tau^d}) = 0.$$

Consider two rows, jth and kth of (A.44),

$$-(\kappa-1)\sum_{s=1}^{J} \lambda_{2,s}(1+\tau_s^d) + (\kappa-1)(1+\tau_j^d) = 0,$$
$$-(\kappa-1)\sum_{s=1}^{J} \lambda_{2,s}(1+\tau_s^d) + (\kappa-1)(1+\tau_k^d) = 0.$$

The difference of the two implies

$$(\kappa-1)(\tau_j^d - \tau_k^d) = 0.$$

This indicates $\tau_j^d = \tau_k^d$ for any k, j. Thus, the domestic tax is uniform across all sectors.

Proof of optimal import tariffs According to Proposition 2, the optimal tariffs satisfy

$$\sum_{k=1}^{J} \tau_k^m \frac{\partial \ln(Y_{2,k})}{\partial \ln(p_{2,j})} \frac{Y_{2,k}}{Y_{2,j}} = \frac{\beta_{1,j}x_1 - (1+\tau_j^d)Y_{1,j}}{Y_{2,j}}, \forall j$$

Consider two rows, jth and kth of (A.44)

$$
\begin{aligned}
-(\kappa-1)\sum_{s=1}^{J}\lambda_{2,s}\tau_s^m+\kappa\tau_j^m &= \frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}},\\
-(\kappa-1)\sum_{s=1}^{J}\lambda_{2,s}\tau_s^m+\kappa\tau_k^m &= \frac{\beta_{1,k}x_1-(1+\tau_k^d)Y_{1,k}}{Y_{2,k}}.
\end{aligned}
$$

The difference of the two implies

$$
\begin{aligned}
\kappa(\tau_j^m-\tau_k^m) &= \frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}-\frac{\beta_{1,k}x_1-(1+\tau_k^d)Y_{1,k}}{Y_{2,k}}.\\
\Rightarrow \tau_j^m-\tau_k^m &= \frac{1}{\kappa}\left[\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}-\frac{\beta_{1,k}x_1-(1+\tau_k^d)Y_{1,k}}{Y_{2,k}}\right].
\end{aligned}
$$

The optimal policies align with Proposition 3 under the CES supply system, where the parameter $\eta_j=\kappa$. Furthermore, this formula is the same as (A.43) that we derived directly using the Lagrange method. $\square$

C.2 Generalized Multi-sector Krugman Model

In Section C.2.1, we formulate the Home government's problem and the world market equilibrium based on the original equilibrium conditions of the generalized multi-sector Krugman model. We solve $\{p_{n,j}\}$ as functions of wages and labor $\{w_{n,j}, L_{n,j}\}$ using the model-specific supply function $\mathbb{S}(\{p_{n,j}, w_{n,j}, L_{n,j}\})$. We then directly take first-order conditions with respect to $\{w_{n,j}, L_{n,j}\}$ and impose multipliers on the labor market specifications Ω to derive optimal policies in this model. In Section C.2.2, we show that this model is a CES supply system and employ matrix operations to establish a connection with Proposition 3.

Furthermore, in Section C.2.3, we demonstrate that the multi-sector Armington and Eaton and Kortum (2002) model with external economies of scale, and model with endogenous technology but different returns to scale across sectors are isomorphic to this model.

C.2.1 Method 1: Model Specific Proof

In the generalized multi-sector Krugman model, labor is perfectly mobile, which indicates $w_{n,j}=w_n$ for any j. Due to the increasing returns to scale, prices are affected by both wages and labor. The supply-side assumption $\mathbb{S}$ satisfies

$$p_{n,j} = \frac{w_n}{\bar{T}_{n,j} L_{n,j}^{\psi_j}}, \tag{A.45}$$

where $\bar{T}_{n,j}$ denotes the exogenous technology level beyond the economies of scale in sector j of country n and $\psi_j = 1/(\sigma_j - 1)$ is the scale elasticity in sector j. In the following market equilibrium constraints, we substitute $p_{n,j}$ with w_n and $L_{n,j}$.

Home government solves the following problem:

$$\max_{\{\tau_j^d, \tau_j^x, \tau_j^m, x_1, w_1, w_2, L_{1,j}, L_{2,j}\}} \frac{x_1}{P_1},$$

subject to world market equilibrium characterized by the following constraints:

$$x_1 = \sum_{j=1}^{J} w_1 L_{1,j} + \sum_{j=1}^{J} \beta_{2,j} \frac{\tau_j^x}{1+\tau_j^x} \pi_{21,j} x_2 + \sum_{j=1}^{J} \beta_{1,j} \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \sum_{j=1}^{J} \beta_{1,j} \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1, \quad (\gamma_x) \tag{A.46}$$

$$w_1 L_{1,j} - \beta_{2,j} \frac{1}{1+\tau_j^x} \pi_{21,j} x_2 = \beta_{1,j} \frac{1}{1+\tau_j^d} \pi_{11,j} x_1, \quad (\gamma_{1,j}, \quad J) \tag{A.47}$$

$$w_2 L_{2,j} - \beta_{2,j} \pi_{22,j} x_2 = \beta_{1,j} \frac{1}{1+\tau_j^m} \pi_{12,j} x_1, \quad (\gamma_{2,j}, \quad J) \tag{A.48}$$

The labor market specifications are

$$\sum_{j=1}^{J} L_{1,j} = \bar{L}_1, \quad (\gamma_{L1}) \tag{A.49}$$

$$\sum_{j=1}^{J} L_{2,j} = \bar{L}_2, \quad (\gamma_{L2}) \tag{A.50}$$

$\{x_2, P_1, P_2\}$ and trade shares are given by

$$x_2 = \sum_{j=1}^{J} w_2 L_{2,j}, \tag{A.51}$$

$$\pi_{11,j} = \frac{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j \psi_j} (w_1(1+\tau_j^d))^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j \psi_j} (w_1(1+\tau_j^d))^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j \psi_j} (w_2(1+\tau_j^m) d_{12})^{-\epsilon_j}}, \tag{A.52}$$

$$\pi_{12,j} = \frac{\bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j \psi_j} (w_2(1+\tau_j^m) d_{12})^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j \psi_j} (w_1(1+\tau_j^d))^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j \psi_j} (w_2(1+\tau_j^m) d_{12})^{-\epsilon_j}}, \tag{A.53}$$

$$\pi_{21,j} = \frac{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j \psi_j} (w_1(1+\tau_j^x) d_{21})^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j \psi_j} (w_1(1+\tau_j^x) d_{21})^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j \psi_j} (w_2)^{-\epsilon_j}}, \tag{A.54}$$

$$\pi_{22,j} = \frac{\bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j \psi_j} (w_2)^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j \psi_j} (w_1(1+\tau_j^x) d_{21})^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j \psi_j} (w_2)^{-\epsilon_j}}, \tag{A.55}$$

$$P_1 = \prod_{j=1}^{J} \left[\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j \psi_j} (w_1(1+\tau_j^d))^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j \psi_j} (w_2(1+\tau_j^m) d_{12})^{-\epsilon_j} \right]^{-\frac{\beta_{1,j}}{\epsilon_j}} = 1, \tag{A.56}$$

$$P_2 = \prod_{j=1}^{J} \left[\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j \psi_j} (w_1(1+\tau_j^x) d_{21})^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j \psi_j} w_2^{-\epsilon_j} \right]^{-\frac{\beta_{2,j}}{\epsilon_j}}. \tag{A.57}$$

One of the equilibrium conditions in (A.46), (A.47) and (A.48) is redundant, according to Walras' law, thus, we drop the expenditure constraint in optimization and set $\gamma_x = 0$. Or, equivalently, if we use Home's budget constraint, one of the market-clearing conditions is redundant. The derivation of the market equilibrium constraints is provided in Appendix C.2.3.

The FOC over $\tau_j^d, \tau_j^m, \tau_j^x$ are the same as in Appendix B. We can get the optimal policies:

$$1+\tau_j^m = -\gamma_{2,j}, \qquad 1+\tau_j^d = -\gamma_{1,j}. \tag{A.58}$$

$$1+\tau_j^x = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\epsilon_j \pi_{22,j}}\right). \tag{A.59}$$

FOC over x_1

$$1 + \sum_{j=1}^{J} \gamma_{1,j} \beta_{1,j} \frac{1}{1+\tau_j^d} \pi_{11,j} + \sum_{j=1}^{J} \gamma_{2,j} \beta_{1,j} \frac{1}{1+\tau_j^m} \pi_{12,j} = 0.$$

FOC over w_1

$$\begin{aligned}
&-\sum_{j=1}^{J}\beta_{1,j}x_1\pi_{11,j}-\sum_{j=1}^{J}\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}\pi_{12,j}x_1-\sum_{j=1}^{J}\gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2\\
&+\sum_{j=1}^{J}\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1+\sum_{j=1}^{J}\gamma_{2,j}\beta_{2,j}\epsilon_j\pi_{22,j}\pi_{21,j}x_2-\sum_{j=1}^{J}\gamma_{1,j}w_1L_{1,j}=0.
\end{aligned}\tag{A.60}$$

FOC over w_2

$$\begin{aligned}
&-\sum_{j=1}^{J}\beta_{1,j}x_1\pi_{12,j}+\sum_{j=1}^{J}\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}\pi_{12,j}x_1+\sum_{j=1}^{J}\gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2\\
&-\sum_{j=1}^{J}\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1-\sum_{j=1}^{J}\gamma_{2,j}\beta_{2,j}\epsilon_j\pi_{22,j}\pi_{21,j}x_2+\sum_{j=1}^{J}(cons_1-\gamma_{2,j})w_2L_{2,j}=0,
\end{aligned}\tag{A.61}$$

where $cons_1=\sum_{s=1}^{J}(\gamma_{1,s}\beta_{2,s}\frac{1}{1+\tau_s^x}\pi_{21,s}+\gamma_{2,s}\beta_{2,s}\pi_{22,s})$.

FOC over $L_{1,j}$

$$\begin{aligned}
&\psi_j\beta_{1,j}x_1\pi_{11,j}+\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\psi_j\pi_{11,j}\pi_{12,j}x_1+\gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\psi_j\pi_{21,j}\pi_{22,j}x_2\\
&-\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\psi_j\pi_{12,j}\pi_{11,j}x_1-\gamma_{2,j}\beta_{2,j}\epsilon_j\psi_j\pi_{22,j}\pi_{21,j}x_2-\gamma_{1,j}w_1L_{1,j}-\gamma_{L1}L_{1,j}=0.
\end{aligned}\tag{A.62}$$

FOC over $L_{2,j}$

$$\begin{aligned}
&\psi_j\beta_{1,j}x_1\pi_{12,j}-\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\psi_j\pi_{11,j}\pi_{12,j}x_1-\gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\psi_j\pi_{21,j}\pi_{22,j}x_2\\
&+\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\psi_j\pi_{12,j}\pi_{11,j}x_1+\gamma_{2,j}\beta_{2,j}\epsilon_j\psi_j\pi_{22,j}\pi_{21,j}x_2+(cons_1-\gamma_{2,j})w_2L_{2,j}-\gamma_{L2}L_{2,j}=0.
\end{aligned}\tag{A.63}$$

Combining FOCs over $L_{1,j}$ (A.62), τ_j^x(A.59), and τ_j^d, τ_j^m (A.58), we get

$$\begin{aligned}
&\psi_j\beta_{1,j}x_1\pi_{11,j}-\beta_{2,j}(\frac{1+\tau_j^d}{1+\tau_j^x}-(1+\tau_j^m))\epsilon_j\psi_j\pi_{21,j}\pi_{22,j}x_2-\gamma_{1,j}w_1L_{1,j}-\gamma_{L1}L_{1,j}=0\\
\Rightarrow&\psi_j\beta_{1,j}x_1\pi_{11,j}+\beta_{2,j}\frac{1+\tau_j^d}{1+\tau_j^x}\psi_j\pi_{21,j}x_2-\gamma_{1,j}w_1L_{1,j}-\gamma_{L1}L_{1,j}=0\\
\Rightarrow&-\gamma_{1,j}w_1L_{1,j}-\gamma_{L1}L_{1,j}+\psi_j(1+\tau_j^d)w_1L_{1,j}=0\\
\Rightarrow&(\psi_j+1)(1+\tau_j^d)=\gamma_{L1}\frac{1}{w_1}.
\end{aligned}$$

Hence, for any sectors j and k, the domestic taxes satisfy

$$\frac{1+\tau_j^d}{1+\tau_k^d} = \frac{1+\psi_k}{1+\psi_j}.$$

Combining FOCs over $L_{2,j}$ (A.63), τ_j^x(A.59), and τ_j^d, τ_j^m (A.58), we get

$$\begin{aligned}
&-(cons_1-\gamma_{2,j})w_2L_{2,j}+\gamma_{L2}L_{2,j}=\psi_j\beta_{1,j}\pi_{12,j}x_1+\left[\frac{1+\tau_j^d}{1+\tau_j^x}-(1+\tau_j^m)\right]\beta_{2,j}\epsilon_j\psi_j\pi_{21,j}\pi_{22,j}x_2\\
&\Rightarrow -(cons_1+1+\tau_j^m)+\gamma_{L2}\frac{1}{w_2}=\psi_j\frac{\beta_{1,j}\pi_{12,j}x_1}{Y_{2,j}}-\frac{1+\tau_j^d}{1+\tau_j^x}\beta_{2,j}\psi_j\pi_{21,j}\frac{x_2}{Y_{2,j}}\\
&\Rightarrow -(cons_1+1+\tau_j^m)+\gamma_{L2}\frac{1}{w_2}=\psi_j\frac{\beta_{1,j}\pi_{12,j}x_1}{Y_{2,j}}-\psi_j(1+\tau_j^d)\frac{Y_{1,j}}{Y_{2,j}}+\psi_j\frac{\beta_{1,j}\pi_{11,j}x_1}{Y_{2,j}}\\
&\Rightarrow -(cons_1+1+\tau_j^m)+\gamma_{L2}\frac{1}{w_2}=\psi_j\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}. \qquad \text{(A.64)}
\end{aligned}$$

Finally, we take the difference of (A.64) between sectors j and k and derive the formula for optimal tariffs

$$\tau_j^m-\tau_k^m=-\psi_j\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}+\psi_k\frac{\beta_{1,k}x_1-(1+\tau_k^d)Y_{1,k}}{Y_{2,k}}, \qquad \text{(A.65)}$$

where $Y_{n,j}=w_nL_{n,j}$ and $\psi_j=1/(\sigma_j-1)$.

C.2.2 Method 2: Applying Proposition 3

We first prove that the generalized multi-sector Krugman is a CES supply system, then apply the optimal policy formulas from Proposition 3 using the structural parameters from this model.

We begin by deriving $L_{n,j}$ as a function of $p_{n,j}$ and w_n, from equation (A.45), then summing them up to derive w_n as a function of $\bar{L}_n$, $p_{n,j}$ and other sectors' prices. Then, we use the implicit function theorem to obtain $\frac{\partial \ln w_n}{\partial \ln p_{n,j}}$.

$$
\begin{aligned}
& p_{n,j} = \bar{T}_{n,j}^{-1} L_{n,j}^{-\psi_j} w_n \\
\Rightarrow & L_{n,j} = \left(p_{n,j} \bar{T}_{n,j} w_n^{-1} \right)^{-\frac{1}{\psi_j}} \\
\Rightarrow & \bar{L}_n = \sum_{j=1}^{J} \left(p_{n,j} \bar{T}_{n,j} \right)^{-\frac{1}{\psi_j}} w_n^{\frac{1}{\psi_j}} \\
\Rightarrow & \sum_{j=1}^{J} \frac{1}{\psi_j} \left(p_{n,j} \bar{T}_{n,j} \right)^{-\frac{1}{\psi_j}} w_n^{\frac{1}{\psi_j}} \frac{\partial \ln w_n}{\partial \ln p_{n,j}} = \frac{1}{\psi_j} \left(p_{n,j} \bar{T}_{n,j} \right)^{-\frac{1}{\psi_j}} w_n^{\frac{1}{\psi_j}} \\
\Rightarrow & \frac{\partial \ln w_n}{\partial \ln p_{n,j}} = \frac{\frac{1}{\psi_j} \left(p_{n,j} \bar{T}_{n,j} \right)^{-\frac{1}{\psi_j}} w_n^{\frac{1}{\psi_j}}}{\sum_{s=1}^{J} \frac{1}{\psi_s} \left(p_{n,s} \bar{T}_{n,s} \right)^{-\frac{1}{\psi_s}} w_n^{\frac{1}{\psi_s}}} = \frac{\frac{1}{\psi_j} Y_{n,j}}{\sum_{s=1}^{J} \frac{1}{\psi_s} Y_{n,s}}. \qquad \text{(A.66)}
\end{aligned}
$$

Since the sectoral income is $Y_{n,j} = w_n L_{n,j}$, the supply curves in country n are given by,

$$
Y_{n,j} = w_n^{\frac{1+\psi_j}{\psi_j}} \left(p_{n,j} \bar{T}_{n,j} \right)^{-\frac{1}{\psi_j}}, \quad \text{where } w_n \text{ satisfies } \sum_{j=1}^{J} \left(p_{n,j} \bar{T}_{n,j} \right)^{-\frac{1}{\psi_j}} w_n^{\frac{1}{\psi_j}} = \bar{L}_n. \qquad \text{(A.67)}
$$

We then derive scaled partial supply elasticities $\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}} \frac{Y_{n,s}}{Y_{n,j}}$ in Λ_n by incorporating (A.66) and (A.67).

$$
\begin{aligned}
\frac{\partial \ln Y_{n,k}}{\partial \ln p_{n,j}} \frac{Y_{n,k}}{Y_{n,j}} &= I_{k=j}(-\frac{1}{\psi_j}) + (1 + \frac{1}{\psi_k}) \frac{Y_{n,k}}{Y_{n,j}} \frac{\partial \ln w_n}{\partial \ln p_{n,j}} \\
&= I_{k=j}(-\frac{1}{\psi_j}) + (1 + \frac{1}{\psi_k}) \frac{\frac{1}{\psi_j} Y_{n,k}}{\sum_{s=1}^{J} \frac{1}{\psi_s} Y_{n,s}} \\
&= I_{k=j}(-\frac{1}{\psi_j}) + \frac{1}{\psi_j} \frac{(1 + \frac{1}{\psi_k}) Y_{n,k}}{\sum_{s=1}^{J} \frac{1}{\psi_s} Y_{n,s}}, \qquad \text{(A.68)}
\end{aligned}
$$

where $I_{s=j}(-\frac{1}{\psi_j})$ is an indicator function that equals $-\frac{1}{\psi_j}$ when $s = j$, and 0 otherwise.

The partial supply elasticity matrix Λ_n of country n takes the form of

$$\Lambda_n = \begin{pmatrix} -\frac{1}{\psi_1}(1-\frac{(1+\frac{1}{\psi_1})Y_{n,1}}{\sum_{s=1}^J \frac{1}{\psi_s}Y_{n,s}}) & \dots & \frac{1}{\psi_1}\frac{(1+\frac{1}{\psi_j})Y_{n,j}}{\sum_{s=1}^J \frac{1}{\psi_s}Y_{n,s}} & \dots & \frac{1}{\psi_1}\frac{(1+\frac{1}{\psi_J})Y_{n,J}}{\sum_{s=1}^J \frac{1}{\psi_s}Y_{n,s}} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{1}{\psi_j}\frac{(1+\frac{1}{\psi_1})Y_{n,1}}{\sum_{s=1}^J \frac{1}{\psi_s}Y_{n,s}} & \dots & -\frac{1}{\psi_j}(1-\frac{(1+\frac{1}{\psi_j})Y_{n,j}}{\sum_{s=1}^J \frac{1}{\psi_s}Y_{n,s}}) & \dots & \frac{1}{\psi_j}\frac{(1+\frac{1}{\psi_J})Y_{n,J}}{\sum_{s=1}^J \frac{1}{\psi_s}Y_{n,s}} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{1}{\psi_J}\frac{(1+\frac{1}{\psi_1})Y_{n,1}}{\sum_{s=1}^J \frac{1}{\psi_s}Y_{n,s}} & \dots & \frac{1}{\psi_J}\frac{(1+\frac{1}{\psi_j})Y_{n,j}}{\sum_{s=1}^J \frac{1}{\psi_s}Y_{n,s}} & \dots & -\frac{1}{\psi_J}(1-\frac{(1+\frac{1}{\psi_J})Y_{n,J}}{\sum_{s=1}^J \frac{1}{\psi_s}Y_{n,s}}) \end{pmatrix}. \tag{A.69}$$

The formula for Λ_n shows that this model satisfies the CES supply system in Definition 4. The partial elasticities of a sector's income with respect to price (scaled by the two sectors' size and elasticity parameter) satisfy $\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}}\frac{Y_{n,s}}{Y_{n,j}}\frac{1}{\eta_j} - \frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,i}}\frac{Y_{n,s}}{Y_{n,i}}\frac{1}{\eta_i} = 0$ for $s \neq i$ or j, and $\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}}\frac{Y_{n,s}}{Y_{n,j}}\frac{1}{\eta_j} - \frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,i}}\frac{Y_{n,s}}{Y_{n,i}}\frac{1}{\eta_i} = 1$ for $s = j$, where the parameter

$$\frac{1}{\eta_j} = -\psi_j = \frac{1}{1-\sigma_j}.$$

Proof of optimal domestic taxes According to Proposition 2, optimal domestic taxes satisfy

$$(\Lambda_1 - I)(1+\boldsymbol{\tau}^{\boldsymbol{d}}) = 0.$$

Consider two rows, jth and kth of (A.69)

$$-\frac{1}{\psi_j}(1+\tau_j^d) + \frac{1}{\psi_j}\sum_{s=1}^J \frac{(1+\frac{1}{\psi_s})Y_{n,s}}{\sum_{i=1}^J \frac{1}{\psi_i}Y_{n,i}}(1+\tau_s^d) = (1+\tau_j^d), \tag{A.70}$$

$$-\frac{1}{\psi_k}(1+\tau_k^d) + \frac{1}{\psi_k}\sum_{s=1}^J \frac{(1+\frac{1}{\psi_s})Y_{n,s}}{\sum_{i=1}^J \frac{1}{\psi_i}Y_{n,i}}(1+\tau_s^d) = (1+\tau_k^d). \tag{A.71}$$

Multiplying (A.70) and (A.71) by $-\psi_j$ and $-\psi_k$ respectively, and subtracting (A.71) from (A.70), we have

$$\begin{aligned} &(1+\tau_j^d) - (1+\tau_k^d) = -(1+\tau_j^d)\psi_j + (1+\tau_k^d)\psi_k \\ \Rightarrow &\frac{1+\tau_j^d}{1+\tau_k^d} = \frac{1+\psi_k}{1+\psi_j} = \frac{\frac{\eta_j}{\eta_j - 1}}{\frac{\eta_k}{\eta_k - 1}}, \end{aligned}$$

where $\frac{1}{\eta_j} = -\psi_j = \frac{1}{1-\sigma_j}$ and ψ_j is the scale elasticity in sector j. When ψ_j is identical across sectors, we get the uniform domestic tax across sectors.

Proof of optimal import tariffs According to Proposition 2, the optimal tariffs satisfy

$$\sum_{k=1}^{J} \tau_k^m \frac{\partial \ln(Y_{2,k})}{\partial \ln(p_{2,j})} \frac{Y_{2,k}}{Y_{2,j}} = \frac{\beta_{1,j} x_1 - (1+\tau_j^d) Y_{1,j}}{Y_{2,j}}, \forall j$$

Consider two rows, jth and kth of (A.69),

$$-\frac{1}{\psi_j}\tau_j^m + \frac{1}{\psi_j}\sum_{s=1}^{J} \frac{(1+\frac{1}{\psi_s})Y_{n,s}}{\sum_{i=1}^{J}\frac{1}{\psi_i}Y_{n,i}}\tau_s^m = \Psi_{1,j}, \tag{A.72}$$

$$-\frac{1}{\psi_k}\tau_k^m + \frac{1}{\psi_k}\sum_{s=1}^{J} \frac{(1+\frac{1}{\psi_s})Y_{n,s}}{\sum_{i=1}^{J}\frac{1}{\psi_i}Y_{n,i}}\tau_s^m = \Psi_{1,k}, \tag{A.73}$$

where $\Psi_{1,j}$ and $\Psi_{1,k}$ are the jth and kth row of vector Ψ_1. Multiplying (A.72) and (A.73) by $-\psi_j$ and $-\psi_k$ respectively, and subtracting (A.73) from (A.72), we have

$$\begin{aligned}
\tau_j^m - \tau_k^m &= -\psi_j \frac{\beta_{1,j} x_1 - (1+\tau_j^d)Y_{1,j}}{Y_{2,j}} + \psi_k \frac{\beta_{1,k} x_1 - (1+\tau_k^d)Y_{1,k}}{Y_{2,k}} \\
\Rightarrow \tau_j^m - \tau_k^m &= \frac{1}{1-\sigma_j}\frac{\beta_{1,j} x_1 - (1+\tau_j^d)Y_{1,j}}{Y_{2,j}} - \frac{1}{1-\sigma_k}\frac{\beta_{1,k} x_1 - (1+\tau_k^d)Y_{1,k}}{Y_{2,k}}.
\end{aligned}$$

The optimal policies align with Proposition 3 under the CES supply system, where the parameter $1/\eta_j = -\psi_j$. Furthermore, this formula is the same as (A.65) that we derived directly using the Lagrange method.

C.2.3 Isomorphism Between the Generalized Multi-Sector Krugman Model, Multi-Sector Armington, and Eaton-Kortum model with EES and the Endogenous Technology Model

Kucheryavyy, Lyn, and Rodríguez-Clare (2023) proves the isomorphic gravity equations for models with varying returns to scale, including the generalized multi-sector Krugman model and multi-sector Armington and EK model with external economies of scale (EES). We also revisit Bai, Jin, and Lu (2025), who extend Eaton and Kortum (2001) with endogenous technology to a multi-sector setting. We provide proofs demonstrating that these models are isomorphic to the generalized multi-sector Krugman model. Consequently, the optimal policies align with those derived in Appendix C.2.2.

Generalized Multi-sector Krugman model Each sector contains a continuum of differentiated varieties. Preferences are structured as follows: Cobb-Douglas across sectors in country n with weights $\beta_{n,j}$, CES across country bundles within a sector j with an elasticity of substitution ϕ_j, and CES across varieties within a country bundle with elasticity σ_j. The corresponding demand function for a representative variety in sector j produced in country i and consumed in country n is:

$$q_{ni,j} = p_{ni,j}^{-\sigma_j} P_{ni,j}^{\sigma_j - 1} (\frac{P_{ni,j}}{P_{n,j}})^{1-\phi_j} \beta_{n,j} x_n,$$

where $p_{ni,j}$ is the price of the variety. $P_{ni,j} = M_{i,j}^{1/\left(1-\sigma_j\right)} p_{ni,j}$ is the price index in country n of country i varieties of sector j. $M_{i,j}$ is the measure of firms or the measure of varieties produced in country i sector j. $P_{n,j} = (\sum_i P_{ni,j}^{1-\phi_j})^{1/(1-\phi_j)}$ is the sector price index in country n.

Let $A_{i,j}$ represent the exogenous productivity in sector j of country i, which is common across all firms in that sector. Similarly, let $F_{i,j}$ denote the fixed cost (in terms of labor) associated with producing any variety in sector j of country i. Under monopolistic competition, the price index in country n for varieties from sector j in country i can be expressed as:

$$P_{ni,j} = M_{i,j}^{1/\left(1-\sigma_j\right)} \left(\bar{\sigma}_j w_i (1+\tau_{ni,j}) d_{ni,j} / A_{i,j}\right),$$

where $\bar{\sigma}_j \equiv \sigma_j / \left(\sigma_j - 1\right)$ is the markup. When $n \neq i$, $\tau_{ni,j}$ denotes the import tariff imposed by country n on sector j imports from country i. When $n = i$, $\tau_{nn,j}$ represents the domestic tax imposed by country n on sector j. Denoting the revenue of the representative firm from sector j in country i by $R_{i,j} \equiv \sum_n \frac{1}{1+\tau_{ni,j}} p_{ni,j} q_{ni,j}$, the corresponding profit is then given by $R_{i,j}/\sigma_j$.

$$\begin{aligned} \frac{R_{i,j}}{\sigma_j} &= \frac{1}{\sigma_j} \sum_n \frac{1}{1+\tau_{ni,j}} (\frac{p_{ni,j}}{P_{ni,j}})^{1-\sigma_j} (\frac{P_{ni,j}}{P_{n,j}})^{1-\phi_j} \beta_{n,j} x_n = \frac{M_{i,j}^{-1}}{\sigma_j} \sum_n \frac{1}{1+\tau_{ni,j}} (\frac{P_{ni,j}}{P_{n,j}})^{1-\phi_j} \beta_{n,j} x_n = \frac{M_{i,j}^{-1}}{\sigma_j} \sum_n \frac{\pi_{ni,j}}{1+\tau_{ni,j}} \beta_{n,j} x_n \\ &= \frac{M_{i,j}^{-1}}{\sigma_j} w_i L_{i,j}. \end{aligned}$$

where sector-level trade shares are

$$\pi_{ni,j} = \left(\frac{P_{ni,j}}{P_{n,j}}\right)^{1-\phi_j} = \frac{A_{i,j}^{\phi_j - 1} M_{i,j}^{(\phi_j-1)/(\sigma_j-1)} \left(w_i (1+\tau_{ni,j}) d_{ni,j}\right)^{1-\phi_j}}{\sum_l A_{l,j}^{\phi_j-1} M_{l,j}^{(\phi_j-1)/(\sigma_j-1)} \left(w_l (1+\tau_{nl,j}) d_{nl,j}\right)^{1-\phi_j}}.$$

Free entry then implies,

$$\frac{R_{i,j}}{\sigma_j} - F_{i,j}w_i = 0 \quad \Rightarrow \quad M_{i,j} = \frac{L_{i,j}}{\sigma_j F_{i,j}}.$$

Total labor $L_{i,j}$ must be consistent with the amounts produced for each market $L^p_{i,j}$, plus the fixed cost of entry, and labor markets must clear, $\sum_{j=1}^{J} L_{i,j} = \bar{L}_i$.

$$\begin{aligned}
L_{i,j} &= M_{i,j}\sum_n q_{ni,j}d_{ni,j}/A_{i,j} + M_{i,j}F_{i,j} = M_{i,j}\sum_n p_{ni,j}^{-\sigma_j}P_{ni,j}^{\sigma_j-1}(\frac{P_{ni,j}}{P_{n,j}})^{1-\phi_j}d_{ni,j}\beta_{n,j}x_n/A_{i,j} + \frac{1}{\sigma_j}\sum_n \frac{1}{1+\tau_{ni,j}}\pi_{ni,j}\beta_{n,j}x_n/w_i \\
&= \frac{1}{\bar{\sigma}_j}\sum_n \frac{1}{1+\tau_{ni,j}}(\frac{P_{ni,j}}{P_{n,j}})^{1-\phi_j}\beta_{n,j}x_n/w_i + \frac{1}{\sigma_j}\sum_n \frac{1}{1+\tau_{ni,j}}\pi_{ni,j}\beta_{n,j}x_n/w_i \\
&= \frac{1}{\bar{\sigma}_j}\sum_n \frac{1}{1+\tau_{ni,j}}\pi_{ni,j}\beta_{n,j}x_n/w_i + \frac{1}{\sigma_j}\sum_n \frac{1}{1+\tau_{ni,j}}\pi_{ni,j}\beta_{n,j}x_n/w_i = \sum_n \frac{1}{1+\tau_{ni,j}}\pi_{ni,j}\beta_{n,j}x_n/w_i = L_{i,j}. \\
L^p_{i,j} &= \frac{\sigma_j - 1}{\sigma_j}L_{i,j}. \qquad \text{(A.74)}
\end{aligned}$$

(A.74) implies that production labor maintains a constant share of total labor.

Trade shares are then given by

$$\pi_{ni,j} = \left(\frac{P_{ni,j}}{P_{n,j}}\right)^{1-\phi_j} = \frac{A_{i,j}^{\phi_j-1}F_{i,j}^{-(\phi_j-1)/(\sigma_j-1)}L_{i,j}^{(\phi_j-1)/(\sigma_j-1)}\left(w_i(1+\tau_{ni,j})d_{ni,j}\right)^{1-\phi_j}}{\sum_l A_{l,j}^{\phi_j-1}F_{l,j}^{-(\phi_j-1)/(\sigma_j-1)}L_{l,j}^{(\phi_j-1)/(\sigma_j-1)}\left(w_l(1+\tau_{nl,j})d_{nl,j}\right)^{1-\phi_j}},$$

with the price index given by

$$P_{n,j} = \sigma^{\frac{1}{\sigma_j-1}}\bar{\sigma}_j\left[\sum_l A_{l,j}^{\phi_j-1}F_{l,j}^{-(\phi_j-1)/(\sigma_j-1)}L_{l,j}^{(\phi_j-1)/(\sigma_j-1)}\left(w_l(1+\tau_{nl,j})d_{nl,j}\right)^{1-\phi_j}\right]^{\frac{1}{1-\phi_j}}.$$

In the generalized Krugman model, the trade and scale elasticities are $\epsilon_j = \phi_j - 1$ and $\psi_j = 1/(\sigma_j - 1)$. If we set $\phi_j = \sigma_j$ for all j, the model is just the standard Krugman model, where preferences are CES across country bundles within a sector with an elasticity of substitution σ_j, and CES across varieties within a country bundle with the same elasticity σ_j.

Multi-sector Armington and EK model with External Economies of Scale Each sector contains a continuum of differentiated varieties. Preferences are structured as follows: Cobb-Douglas across sectors in country n with weights $\beta_{n,j}$, CES across country bundles within sector j with an elasticity of substitution σ_j. The corresponding demand function in sector j produced in country i

and consumed in country n is:

$$q_{ni,j} = p_{ni,j}^{-\sigma_j} P_{n,j}^{\sigma_j - 1} \beta_{n,j} x_n,$$

where $P_{n,j} = (\sum_i P_{ni,j}^{1-\sigma_j})^{1/(1-\sigma_j)}$ is sector j's price index in country n.

In multi-sector Armington model, labor productivity in sector j country i is given by $A_{i,j} L_{i,j}^{\psi_j}$, where $A_{i,j}$ is an exogenous productivity parameter in sector j in country i, and $L_{i,j}^{\psi_j}$ captures EES in sector j.

Thus, the unit cost of good in sector j country i for delivery to market n is

$$c_{ni,j} = \frac{w_i (1 + \tau_{ni,j}) d_{ni,j}}{A_{i,j} L_{i,j}^{\psi_j}}.$$

When $n \neq i$, $\tau_{ni,j}$ denotes the import tariff imposed by country n on sector j imports from country i. When $n = i$, $\tau_{nn,j}$ represents the domestic tax imposed by country n on sector j. All markets are perfectly competitive, indicating $p_{ni,j} = c_{ni,j}$. Total labor $L_{i,j}$ must be consistent with the amounts produced for each market, and labor markets must clear, $\sum_{j=1}^{J} L_{i,j} = \bar{L}_i$.

$$\begin{aligned} L_{i,j} &= \sum_n q_{ni,j} d_{ni,j} \frac{1}{A_{i,j} L_{i,j}^{\psi_j}} = \sum_n p_{ni,j}^{-\sigma_j} P_{n,j}^{\sigma_j - 1} d_{ni,j} \beta_{n,j} x_n \frac{1}{A_{i,j} L_{i,j}^{\psi_j}} = \sum_n \frac{1}{1 + \tau_{ni,j}} p_{ni,j}^{1-\sigma_j} P_{n,j}^{\sigma_j - 1} \beta_{n,j} x_n \frac{1}{w_i} \\ &= \sum_n \frac{\pi_{ni,j} \beta_{n,j} x_n}{1 + \tau_{ni,j}} \frac{1}{w_i}, \end{aligned} \tag{A.75}$$

where the sector-level trade shares are given by

$$\pi_{ni,j} = \left(\frac{p_{ni,j}}{P_{n,j}} \right)^{1-\sigma_j} = \frac{A_{i,j}^{\sigma_j - 1} L_{i,j}^{\psi_j (\sigma_j - 1)} \left(w_i (1 + \tau_{ni,j}) d_{ni,j} \right)^{1-\sigma_j}}{\sum_l A_{l,j}^{\sigma_j - 1} L_{l,j}^{\psi_j (\sigma_j - 1)} \left(w_l (1 + \tau_{nl,j}) d_{nl,j} \right)^{1-\sigma_j}}.$$

We can demonstrate that the multi-sector Armington model with EES is isomorphic to the generalized Krugman model, where the trade elasticity is given by $\epsilon_j = \sigma_j - 1$.

In the multi-sector EK model with EES, labor productivity in sector j of country i is given by $A_{i,j} L_{i,j}^{\psi_j}$, where the exogenous productivity parameter $A_{i,j}$ are independently drawn from a Fréchet distribution with shape parameter θ_j and scale parameter $\bar{T}_{i,j}$.

Similarly, the goods market clearing condition satisfies

$$w_i L_{i,j} = \sum_n \frac{\pi_{ni,j} \beta_{n,j} x_n}{1 + \tau_{ni,j}},$$

where the sector-level trade shares are

$$\pi_{ni,j} = \left(\frac{p_{ni,j}}{P_{n,j}}\right)^{-\theta_j} = \frac{\bar{T}_{i,j} L_{i,j}^{\psi_j \theta_j} \left(w_i(1+\tau_{ni,j})d_{ni,j}\right)^{-\theta_j}}{\sum_l \bar{T}_{l,j} L_{l,j}^{\psi_j \theta_j} \left(w_l(1+\tau_{nl,j})d_{nl,j}\right)^{-\theta_j}}.$$

We can demonstrate that the multi-sector EK model with EES is isomorphic to the generalized Krugman model, where the trade elasticity is given by $\epsilon_j = \theta_j$. Therefore, the optimal policies in these two models are identical to those in the generalized Krugman model.

Endogenous Technolgoy We revisit the steady-state optimal policies in Bai, Jin, and Lu (2025) (BJL hereafter), an example of optimal trade policies in a multi-sector, multi-country GE model under productivity-driven supply curves. This framework extends Eaton and Kortum (2001) with endogenous technology to a multi-sector setting.

In BJL, total labor in each sector is divided into two components: production labor and researchers. Labor moves freely across sectors, and technology endogenously depends on the number of researchers. In steady-state equilibrium, the ratio of production labor to researchers in each sector remains constant, implying that sectoral technology can be expressed as a function of sectoral labor. Let r_j denote the fraction of researchers in each sector. Let $T_{n,j}$ denote the technology level of sector j in country n, which depends on an efficiency parameter $\rho_{n,j}$ and the endogenous researchers in the sector. Specifically, $T_{n,j} = \rho_{n,j} L_{n,j}^r = \rho_{n,j} r_j L_{n,j}$.

More generally, there may be constant, decreasing, or increasing returns to scale in innovation. The change in innovation effort further influences innovation efficiency. Specifically, the technology of the sector j in country n is a function of the sector's researchers, i.e.,

$$T_{n,j} = \rho_{n,j}(r_j L_{n,j})^{\nu_j}, \tag{A.76}$$

where ν_j governs the strength of economies of scale. The baseline BJL represents a special case of constant returns to scale where $\nu_j = 1$.

Therefore, the price in sector j of country n satisfies

$$p_{n,j} = T_{n,j}^{-\frac{1}{\theta_j}} w_n = w_n / (\bar{T}_{n,j} L_{n,j}^{\frac{\nu_j}{\theta_j}}),$$

where $\bar{T}_{n,j} = (\rho_{n,j} r^{\nu_j})^{-1/\theta_j}$ and θ_j is the trade elasticity in sector j.

Firms engage in Bertrand competition, where the lowest-cost producer of each good in each market captures the entire market by charging a markup just enough to keep the second-lowest-cost producer out of the market. In equilibrium, markups follow a Pareto distribution with parameter θ_j. Since all firms selling in the market charge a markup drawn from the same distribution, the total profits of firms earned by firms in the market are a constant share of that market's total sales. Specifically, firms (both domestic and foreign) that sell in the market earn profits equal to $1/(1+\theta_j)$ of total sales. The remaining $\theta_j/(1+\theta_j)$ share of total sales goes to production labor. Therefore, we can express the total sales of the sector j in country n as $\frac{1+\theta_j}{\theta_j} w_n L^p_{n,j}$, where $L^p_{n,j}$ can be replaced by $(1-r_j)L_{n,j}$.

Goods market clearing conditions are

$$Y_{1,j} - \beta_{2,j}\frac{1}{1+\tau^x_j}\pi_{21,j}x_2 = \beta_{1,j}\frac{1}{1+\tau^d_j}\pi_{11,j}x_1, \tag{A.77}$$

$$Y_{2,j} - \beta_{2,j}\pi_{22,j}x_2 = \beta_{1,j}\frac{1}{1+\tau^m_j}\pi_{12,j}x_1, \tag{A.78}$$

where $x_2 = \sum_{j=1}^{J} Y_{2,j}$, and $Y_{n,j} = \frac{1+\theta_j}{\theta_j} w_n(1-r_j)L_{n,j}$, $(1+\theta_j)/\theta_j$ captures firms' markup.

Home's expenditure is

$$x_1 = \sum_{j=1}^{J} Y_{1,j} + \sum_{j=1}^{J}\beta_{2,j}\frac{\tau^x_j}{1+\tau^x_j}\pi_{21,j}x_2 + \sum_{j=1}^{J}\beta_{1,j}\frac{\tau^m_j}{1+\tau^m_j}\pi_{12,j}x_1 + \sum_{j=1}^{J}\beta_{1,j}\frac{\tau^d_j}{1+\tau^d_j}\pi_{11,j}x_1. \tag{A.79}$$

Trade shares and prices satisfy equations (A.5) - (A.10), where the trade and scale elasticities are $\epsilon_j = \theta_j$ and $\psi_j = \nu_j/\theta_j$. Thus, the expressions for trade shares and sector price indexes in this endogenous technology model are the same as in generalized Krugman models. The optimal policies in this model are identical to those in generalized Krugman model. When $\nu = 1$, the model reverts to the BJL framework, where the trade elasticity and scale elasticity are given by $\epsilon = \theta$ and $\psi = 1/\theta$, respectively, and are constant across sectors. □

C.3 Differential Scale Elasticities and Imperfectly Mobile Labor

In this section, we extend the generalized multi-sector Krugman model with imperfectly mobile labor to analyze alternative supply-side specifications in a unified framework. This example combines features of the previous two examples: imperfectly mobile labor across sectors, as in Example 1, and the increasing returns to scale, as in Example 2.

In Section C.3.1, we formulate the Home government's problem and the world market equilibrium based on the original equilibrium conditions of this model. We solve $\{p_{n,j}\}$ as functions of wages and labor $\{w_{n,j}, L_{n,j}\}$ using the model-specific supply function $\mathbb{S}(\{p_{n,j}, w_{n,j}, L_{n,j}\})$. We then directly take first-order conditions with respect to $\{w_{n,j}, L_{n,j}\}$ and impose multipliers on the labor market specifications Ω to derive optimal policies in this model. In Section C.3.2, we show that this model is a CES supply system and employ matrix operations to establish a connection with Proposition 3.

C.3.1 Method 1: Model Specific Proof

In the model with differential scale elasticities and imperfectly mobile labor, due to the increasing returns to scale, prices are affected by both wages and labor. The supply-side assumption $\mathbb{S}$ satisfies

$$p_{n,j} = \frac{w_{n,j}}{\bar{T}_{n,j} L_{n,j}^{\psi_j}}, \tag{A.80}$$

where $\bar{T}_{n,j}$ denotes the exogenous technology level beyond the economies of scale in sector j of country n and ψ_j is the scale elasticity in sector j. In the following market equilibrium constraints, we substitute $p_{n,j}$ with $w_{n,j}$ and $L_{n,j}$.

Home government solves the following problem:

$$\max_{\{\tau_j^d, \tau_j^x, \tau_j^m, x_1, w_{1,j}, w_{2,j}, L_{1,j}, L_{2,j}\}} \frac{x_1}{P_1},$$

subject to world market equilibrium characterized by the following constraints:

$$x_1 = \sum_{j=1}^{J} w_{1,j} L_{1,j} + \sum_{j=1}^{J} \beta_{2,j} \frac{\tau_j^x}{1+\tau_j^x} \pi_{21,j} x_2 + \sum_{j=1}^{J} \beta_{1,j} \frac{\tau_j^m}{1+\tau_j^m} \pi_{12,j} x_1 + \sum_{j=1}^{J} \beta_{1,j} \frac{\tau_j^d}{1+\tau_j^d} \pi_{11,j} x_1, \quad (\gamma_x) \tag{A.81}$$

$$w_{1,j} L_{1,j} - \beta_{2,j} \frac{1}{1+\tau_j^x} \pi_{21,j} x_2 = \beta_{1,j} \frac{1}{1+\tau_j^d} \pi_{11,j} x_1, \quad (\gamma_{1,j}, \quad J) \tag{A.82}$$

$$w_{2,j} L_{2,j} - \beta_{2,j} \pi_{22,j} x_2 = \beta_{1,j} \frac{1}{1+\tau_j^m} \pi_{12,j} x_1, \quad (\gamma_{2,j}, \quad J) \tag{A.83}$$

The labor market specifications are

$$L_{1,j} = A_{1,j}(\frac{w_{1,j}}{W_1})^{\kappa-1}\bar{L}_1, \quad (\gamma_{L1,j}, \quad J) \tag{A.84}$$

$$L_{2,j} = A_{2,j}(\frac{w_{2,j}}{W_2})^{\kappa-1}\bar{L}_2, \quad (\gamma_{L2,j}, \quad J) \tag{A.85}$$

where $W_n = [\sum_{j=1}^J A_{n,j} w_{n,j}{}^{\kappa}]^{\frac{1}{\kappa}}$ and $\{x_2, P_1, P_2\}$ and trade shares are given by

$$x_2 = \sum_{j=1}^J w_{2,j} L_{2,j}, \tag{A.86}$$

$$\pi_{11,j} = \frac{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j\psi_j}(w_{1,j}(1+\tau_j^d))^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j\psi_j}(w_{1,j}(1+\tau_j^d))^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j\psi_j}(w_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j}}, \tag{A.87}$$

$$\pi_{12,j} = \frac{\bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j\psi_j}(w_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j\psi_j}(w_{1,j}(1+\tau_j^d))^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j\psi_j}(w_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j}}, \tag{A.88}$$

$$\pi_{21,j} = \frac{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j\psi_j}(w_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j\psi_j}(w_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j\psi_j}(w_{2,j})^{-\epsilon_j}}, \tag{A.89}$$

$$\pi_{22,j} = \frac{\bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j\psi_j}(w_{2,j})^{-\epsilon_j}}{\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j\psi_j}(w_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j\psi_j}(w_{2,j})^{-\epsilon_j}}, \tag{A.90}$$

$$P_1 = \prod_{j=1}^J \left[\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j\psi_j}(w_{1,j}(1+\tau_j^d))^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j\psi_j}(w_{2,j}(1+\tau_j^m)d_{12})^{-\epsilon_j}\right]^{-\frac{\beta_{1,j}}{\epsilon_j}} = 1, \tag{A.91}$$

$$P_2 = \prod_{j=1}^J \left[\bar{T}_{1,j}^{\epsilon_j} L_{1,j}^{\epsilon_j\psi_j}(w_{1,j}(1+\tau_j^x)d_{21})^{-\epsilon_j} + \bar{T}_{2,j}^{\epsilon_j} L_{2,j}^{\epsilon_j\psi_j} w_{2,j}^{-\epsilon_j}\right]^{-\frac{\beta_{2,j}}{\epsilon_j}}. \tag{A.92}$$

One of the equilibrium conditions in (A.81), (A.82) and (A.83) is redundant, according to Walras' law, thus, we drop the expenditure constraint in optimization and set $\gamma_x = 0$. Or, equivalently, if we use Home's budget constraint, one of the market-clearing conditions is redundant.

The FOC over $\tau_j^d, \tau_j^m, \tau_j^x$ are the same as in Appendix B. We can get the optimal policies:

$$1+\tau_j^m = -\gamma_{2,j}, \qquad 1+\tau_j^d = -\gamma_{1,j}. \tag{A.93}$$

$$1+\tau_j^x = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\epsilon_j \pi_{22,j}}\right). \tag{A.94}$$

FOC over x_1

$$1 + \sum_{j=1}^J \gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\pi_{11,j} + \sum_{j=1}^J \gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\pi_{12,j} = 0.$$

FOC over $w_{1,j}$

$$-\beta_{1,j}x_1\pi_{11,j}-\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}\pi_{12,j}x_1-\gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2+\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1$$
$$+\gamma_{2,j}\beta_{2,j}\epsilon_j\pi_{22,j}\pi_{21,j}x_2-\gamma_{1,j}w_{1,j}L_{1,j}+\gamma_{L1,j}(\kappa-1)L_{1,j}-\sum_{s=1}^{J}\gamma_{L1,s}(\kappa-1)L_{1,s}\frac{w_{1,j}L_{1,j}}{W_1\bar{L}_1}=0. \tag{A.95}$$

FOC over $L_{1,j}$

$$\psi_j\beta_{1,j}x_1\pi_{11,j}+\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\psi_j\pi_{11,j}\pi_{12,j}x_1+\gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\psi_j\pi_{21,j}\pi_{22,j}x_2$$
$$-\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\psi_j\pi_{12,j}\pi_{11,j}x_1-\gamma_{2,j}\beta_{2,j}\epsilon_j\psi_j\pi_{22,j}\pi_{21,j}x_2-\gamma_{1,j}w_{1,j}L_{1,j}-\gamma_{L1,j}L_{1,j}=0. \tag{A.96}$$

FOC over $w_{2,j}$

$$-\beta_{1,j}x_1\pi_{12,j}+\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}\pi_{12,j}x_1+\gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2-\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1$$
$$-\gamma_{2,j}\beta_{2,j}\epsilon_j\pi_{22,j}\pi_{21,j}x_2+(cons_1-\gamma_{2,j})w_{2,j}L_{2,j}+\gamma_{L2,j}(\kappa-1)L_{2,j}-\sum_{s=1}^{J}\gamma_{L2,s}(\kappa-1)L_{2,s}\frac{w_{2,j}L_{2,j}}{W_2\bar{L}_2}=0, \tag{A.97}$$

where $cons_1=\sum_{s=1}^{J}(\gamma_{1,s}\beta_{2,s}\frac{1}{1+\tau_s^x}\pi_{21,s}+\gamma_{2,s}\beta_{2,s}\pi_{22,s})$.

FOC over $L_{2,j}$

$$\psi_j\beta_{1,j}x_1\pi_{12,j}-\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\psi_j\pi_{11,j}\pi_{12,j}x_1-\gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\psi_j\pi_{21,j}\pi_{22,j}x_2$$
$$+\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\psi_j\pi_{12,j}\pi_{11,j}x_1+\gamma_{2,j}\beta_{2,j}\epsilon_j\psi_j\pi_{22,j}\pi_{21,j}x_2+(cons_1-\gamma_{2,j})w_{2,j}L_{2,j}-\gamma_{L2,j}L_{2,j}=0. \tag{A.98}$$

Combining FOCs over $w_{1,j}$ (A.95) and $L_{1,j}$ (A.96), we get

$$-(\psi_j+1)\gamma_{1,j}w_{1,j}L_{1,j}+\gamma_{L1,j}(\psi_j(\kappa-1)-1)L_{1,j}-\psi_j\sum_{s=1}^{J}\gamma_{L1,s}(\kappa-1)L_{1,s}\frac{w_{1,j}L_{1,j}}{W_1\bar{L}_1}=0.$$
$$\Rightarrow-(\psi_j+1)\gamma_{1,j}+\gamma_{L1,j}(\psi_j(\kappa-1)-1)\frac{1}{w_{1,j}}-\psi_j\sum_{s=1}^{J}\gamma_{L1,s}(\kappa-1)L_{1,s}\frac{1}{W_1\bar{L}_1}=0. \tag{A.99}$$

Combining FOCs over $L_{1,j}$ (A.96), τ_j^x(A.94), and τ_j^d, τ_j^m (A.93), we get

$$
\begin{aligned}
&\psi_j\beta_{1,j}x_1\pi_{11,j}-\beta_{2,j}(\frac{1+\tau_j^d}{1+\tau_j^x}-(1+\tau_j^m))\epsilon_j\psi_j\pi_{21,j}\pi_{22,j}x_2-\gamma_{1,j}w_{1,j}L_{1,j}-\gamma_{L1,j}L_{1,j}=0\\
\Rightarrow&\psi_j\beta_{1,j}x_1\pi_{11,j}+\beta_{2,j}\frac{1+\tau_j^d}{1+\tau_j^x}\psi_j\pi_{21,j}x_2-\gamma_{1,j}w_{1,j}L_{1,j}-\gamma_{L1,j}L_{1,j}=0\\
\Rightarrow&-\gamma_{1,j}w_{1,j}L_{1,j}-\gamma_{L1,j}L_{1,j}+\psi_j(1+\tau_j^d)w_{1,j}L_{1,j}=0\\
\Rightarrow&(\psi_j+1)(1+\tau_j^d)=\gamma_{L1,j}\frac{1}{w_{1,j}}.
\end{aligned}
$$

Plugging back to (A.99),

$$
\begin{aligned}
&(\psi_j+1)(1+\tau_j^d)+(\psi_j(\kappa-1)-1)(\psi_j+1)(1+\tau_j^d)-\psi_j\sum_{s=1}^{J}\gamma_{L1,s}(\kappa-1)L_{1,s}\frac{1}{W_1\bar{L}_1}=0\\
\Rightarrow&\psi_j(\kappa-1)(\psi_j+1)(1+\tau_j^d)-\psi_j\sum_{s=1}^{J}\gamma_{L1,s}(\kappa-1)L_{1,s}\frac{1}{W_1\bar{L}_1}=0\\
\Rightarrow&(\psi_j+1)(1+\tau_j^d)=\sum_{s=1}^{J}\gamma_{L1,s}L_{1,s}\frac{1}{W_1\bar{L}_1}.
\end{aligned}
$$

Hence, for any sectors j and k, the optimal domestic taxes satisfy

$$
\frac{1+\tau_j^d}{1+\tau_k^d}=\frac{1+\psi_k}{1+\psi_j}.
$$

Combining FOCs over $L_{2,j}$ (A.98), τ_j^x(A.94), and τ_j^d, τ_j^m (A.93), we get

$$
\begin{aligned}
&-(cons_1-\gamma_{2,j})w_{2,j}L_{2,j}+\gamma_{L2,j}L_{2,j}=\psi_j\beta_{1,j}\pi_{12,j}x_1+\left[\frac{1+\tau_j^d}{1+\tau_j^x}-(1+\tau_j^m)\right]\beta_{2,j}\epsilon_j\psi_j\pi_{21,j}\pi_{22,j}x_2\\
\Rightarrow&-(cons_1+1+\tau_j^m)+\gamma_{L2,j}\frac{1}{w_{2,j}}=\psi_j\frac{\beta_{1,j}\pi_{12,j}x_1}{Y_{2,j}}-\frac{1+\tau_j^d}{1+\tau_j^x}\beta_{2,j}\psi_j\pi_{21,j}\frac{x_2}{Y_{2,j}}\\
\Rightarrow&-(cons_1+1+\tau_j^m)+\gamma_{L2,j}\frac{1}{w_{2,j}}=\psi_j\frac{\beta_{1,j}\pi_{12,j}x_1}{Y_{2,j}}-\psi_j(1+\tau_j^d)\frac{Y_{1,j}}{Y_{2,j}}+\psi_j\frac{\beta_{1,j}\pi_{11,j}x_1}{Y_{2,j}}\\
\Rightarrow&-(cons_1+1+\tau_j^m)+\gamma_{L2,j}\frac{1}{w_{2,j}}=\psi_j\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}.
\end{aligned}
$$

Combining FOCs over $w_{2,j}$ (A.97) and $L_{2,j}$ (A.98), we get

$$
\begin{aligned}
&(\psi_j+1)(cons_1-\gamma_{2,j})w_{2,j}L_{2,j}+\gamma_{L2,j}(\psi_j(\kappa-1)-1)L_{2,j}-\psi_j\sum_{s=1}^{J}\gamma_{L2,s}(\kappa-1)L_{2,s}\frac{w_{2,j}L_{2,j}}{W_2\bar{L}_2}=0.\\
\Rightarrow&(\psi_j+1)(cons_1+1+\tau_j^m)+\gamma_{L2,j}(\psi_j(\kappa-1)-1)\frac{1}{w_{2,j}}-\psi_j\sum_{s=1}^{J}\gamma_{L2,s}(\kappa-1)L_{2,s}\frac{1}{W_2\bar{L}_2}=0\\
\Rightarrow&\psi_j\kappa(cons_1+1+\tau_j^m)+(\psi_j(\kappa-1)-1)(\psi_j\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}})-\psi_j\sum_{s=1}^{J}\gamma_{L2,s}(\kappa-1)L_{2,s}\frac{1}{W_2\bar{L}_2}=0\\
\Rightarrow&cons_1+1+\tau_j^m+(\psi_j\frac{\kappa-1}{\kappa}-\frac{1}{\kappa})\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}-\sum_{s=1}^{J}\gamma_{L2,s}\frac{\kappa-1}{\kappa}L_{2,s}\frac{1}{W_2\bar{L}_2}=0. \qquad \text{(A.100)}
\end{aligned}
$$

Finally, we take the difference of (A.100) between sectors j and k and derive the formula for optimal tariffs

$$
\tau_j^m-\tau_k^m=(\frac{1}{\kappa}-\psi_j\frac{\kappa-1}{\kappa})\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}-(\frac{1}{\kappa}-\psi_k\frac{\kappa-1}{\kappa})\frac{\beta_{1,k}x_1-(1+\tau_k^d)Y_{1,k}}{Y_{2,k}}, \qquad \text{(A.101)}
$$

where $Y_{n,j}=w_{n,j}L_{n,j}$.

C.3.2 Method 2: Applying Proposition 3

We first prove that the model with differential scale elasticities and imperfectly mobile labor satisfies CES supply system, then apply the optimal policy formulas from Proposition 3 using the structural parameters from this model.

In this extended model, goods market clearing conditions, expenditures, trade shares and prices satisfy equations (A.2) - (A.10). The labor market specifications are given by equations (A.84) and (A.85). As $\kappa\to\infty$, the model converges to one with increasing returns to scale and perfect labor mobility, similar to the generalized Krugman model.

We begin by deriving $w_{n,j}$ as a function of $p_{n,j}$ and W_n, from the supply-side assumption $\mathbb{S}$ (A.80).

$$
\begin{aligned}
p_{n,j}&=\bar{T}_{n,j}^{-1}L_{n,j}^{-\psi_j}w_{n,j}=\bar{T}_{n,j}^{-1}w_{n,j}^{1-\psi_j(\kappa-1)}W_n^{\psi_j(\kappa-1)}\bar{L}_n^{-\psi_j}\\
\Rightarrow w_{n,j}&=\left(p_{n,j}\bar{T}_{n,j}W_n^{-\psi_j(\kappa-1)}\bar{L}_n^{\psi_j}\right)^{\frac{1}{1-\psi_j(\kappa-1)}}. \qquad \text{(A.102)}
\end{aligned}
$$

Since the sectoral income is $Y_{n,j}=w_{n,j}L_{n,j}$, the supply curves in country n are given by

$$
Y_{n,j}=A_{n,j}w_{n,j}^{\kappa}W_n^{1-\kappa}\bar{L}_n=A_{n,j}\left(p_{n,j}\bar{T}_{n,j}\right)^{\frac{\kappa}{1-\psi_j(\kappa-1)}}W_n^{1-\frac{\kappa}{1-\psi_j(\kappa-1)}}\bar{L}_n^{\frac{\psi_j\kappa}{1-\psi_j(\kappa-1)}+1}. \qquad \text{(A.103)}
$$

Plugging (A.102) into $W_n = [\sum_{j=1}^{J} A_{n,j} w_{n,j}{}^{\kappa}]^{\frac{1}{\kappa}}$, we derive W_n as a function of $p_{n,j}$.

$$W_n^{\kappa} = \sum_{j=1}^{J} A_{n,j} \left(p_{n,j} \bar{T}_{n,j} W_n^{-\psi_j(\kappa-1)} \bar{L}_n^{\psi_j} \right)^{\frac{\kappa}{1-\psi_j(\kappa-1)}}. \tag{A.104}$$

Using the implicit function theorem, we obtain $\frac{\partial \ln W_n}{\partial \ln p_{n,j}}$.

$$\begin{aligned}
\kappa \frac{\partial \ln W_n}{\partial \ln p_{n,j}} W_n^{\kappa} &= \frac{\kappa}{1-\psi_j(\kappa-1)} A_{n,j} \left(p_{n,j} \bar{T}_{n,j} W_n^{-\psi_j(\kappa-1)} \bar{L}_n^{\psi_j} \right)^{\frac{\kappa}{1-\psi_j(\kappa-1)}} \\
&\quad - \sum_{s=1}^{J} \psi_s(\kappa-1) \frac{\kappa}{1-\psi_s(\kappa-1)} A_{n,s} \left(p_{n,s} \bar{T}_{n,s} W_n^{-\psi_s(\kappa-1)} \bar{L}_n^{\psi_s} \right)^{\frac{\kappa}{1-\psi_s(\kappa-1)}} \frac{\partial \ln W_n}{\partial \ln p_{n,j}} \\
\Rightarrow \frac{\partial \ln W_n}{\partial \ln p_{n,j}} &= \frac{\frac{1}{1-\psi_j(\kappa-1)} A_{n,j} \left(p_{n,j} \bar{T}_{n,j} W_n^{-\psi_j(\kappa-1)} \bar{L}_n^{\psi_j} \right)^{\frac{\kappa}{1-\psi_j(\kappa-1)}}}{\sum_{s=1}^{J} \frac{1}{1-\psi_s(\kappa-1)} A_{n,s} \left(p_{n,s} \bar{T}_{n,s} W_n^{-\psi_s(\kappa-1)} \bar{L}_n^{\psi_s} \right)^{\frac{\kappa}{1-\psi_s(\kappa-1)}}} \\
&= \frac{\frac{1}{1-\psi_j(\kappa-1)} Y_{n,j} W_n^{\kappa-1} / \bar{L}_n}{\sum_{s=1}^{J} \frac{1}{1-\psi_s(\kappa-1)} Y_{n,s} W_n^{\kappa-1} / \bar{L}_n} = \frac{\frac{1}{1-\psi_j(\kappa-1)} Y_{n,j}}{\sum_{s=1}^{J} \frac{1}{1-\psi_s(\kappa-1)} Y_{n,s}}.
\end{aligned} \tag{A.105}$$

We then derive scaled partial supply elasticities $\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}} \frac{Y_{n,s}}{Y_{n,j}}$ in Λ_n by incorporating (A.105) in (A.103).

$$\begin{aligned}
\frac{\partial \ln Y_{n,k}}{\partial \ln p_{n,j}} \frac{Y_{n,k}}{Y_{n,j}} &= I_{k=j}\left(\frac{\kappa}{1-\psi_j(\kappa-1)}\right) + \left(1 - \frac{\kappa}{1-\psi_k(\kappa-1)}\right) \frac{Y_{n,k}}{Y_{n,j}} \frac{\partial \ln W_n}{\partial \ln p_{n,j}} \\
&= I_{k=j}\left(\frac{\kappa}{1-\psi_j(\kappa-1)}\right) + \left(1 - \frac{\kappa}{1-\psi_k(\kappa-1)}\right) \frac{\frac{1}{1-\psi_j(\kappa-1)} Y_{n,k}}{\sum_{s=1}^{J} \frac{1}{1-\psi_s(\kappa-1)} Y_{n,s}} \\
&= I_{k=j}\left(\frac{\kappa}{1-\psi_j(\kappa-1)}\right) + \frac{\kappa}{1-\psi_j(\kappa-1)} \frac{1 - \frac{\kappa}{1-\psi_k(\kappa-1)} Y_{n,k}}{\sum_{s=1}^{J} \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}},
\end{aligned}$$

where $I_{s=j}(\frac{\kappa}{1-\psi_j(\kappa-1)})$ is an indicator function that equals $\frac{\kappa}{1-\psi_j(\kappa-1)}$ when $s=j$, and 0 otherwise.

The scaled partial supply elasticity matrix Λ_n of country n takes the form of

$$\Lambda_n = \begin{pmatrix}
\frac{\kappa}{1-\psi_1(\kappa-1)}\left(1 + \frac{(1-\frac{\kappa}{1-\psi_1(\kappa-1)})Y_{n,1}}{\sum_{s=1}^{J} \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}}\right) & \cdots & \frac{\kappa}{1-\psi_1(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_j(\kappa-1)})Y_{n,j}}{\sum_{s=1}^{J} \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} & \cdots & \frac{\kappa}{1-\psi_1(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_J(\kappa-1)})Y_{n,J}}{\sum_{s=1}^{J} \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\frac{\kappa}{1-\psi_j(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_1(\kappa-1)})Y_{n,1}}{\sum_{s=1}^{J} \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} & \cdots & \frac{\kappa}{1-\psi_j(\kappa-1)}\left(1 + \frac{(1-\frac{\kappa}{1-\psi_j(\kappa-1)})Y_{n,j}}{\sum_{s=1}^{J} \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}}\right) & \cdots & \frac{\kappa}{1-\psi_j(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_J(\kappa-1)})Y_{n,J}}{\sum_{s=1}^{J} \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\frac{\kappa}{1-\psi_J(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_1(\kappa-1)})Y_{n,1}}{\sum_{s=1}^{J} \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} & \cdots & \frac{\kappa}{1-\psi_J(\kappa-1)} \frac{(1-\frac{\kappa}{1-\psi_j(\kappa-1)})Y_{n,j}}{\sum_{s=1}^{J} \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}} & \cdots & \frac{\kappa}{1-\psi_J(\kappa-1)}\left(1 + \frac{(1-\frac{\kappa}{1-\psi_J(\kappa-1)})Y_{n,J}}{\sum_{s=1}^{J} \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}}\right)
\end{pmatrix}. \tag{A.106}$$

The formula for Λ_n shows that this model satisfies the CES supply system in Definition 4. The scaled partial elasticities of a sector's income with respect to price (scaled by the two sectors' size and elasticity parameter) satisfy $\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}} \frac{Y_{n,s}}{Y_{n,j}} \frac{1}{\eta_j} - \frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,i}} \frac{Y_{n,s}}{Y_{n,i}} \frac{1}{\eta_i} = 0$ for $s \neq i$ or j, and $\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}} \frac{Y_{n,s}}{Y_{n,j}} \frac{1}{\eta_j} - \frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,i}} \frac{Y_{n,s}}{Y_{n,i}} \frac{1}{\eta_i} = 1$ for $s = j$, where

$$\frac{\partial \ln Y_{n,k}}{\partial \ln p_{n,j}} \frac{Y_{n,k}}{Y_{n,j}} \frac{1}{\eta_j} = I_{k=j}(1) + \frac{(1 - \frac{\kappa}{1-\psi_k(\kappa-1)})Y_{n,k}}{\sum_{s=1}^{J} \frac{\kappa}{1-\psi_s(\kappa-1)} Y_{n,s}},$$

and

$$\frac{1}{\eta_j} = \frac{1}{\kappa} - \psi_j \frac{\kappa - 1}{\kappa}.$$

When $\kappa \to \infty$, $\frac{1}{\eta_j} \to -\psi_j$, the model goes back to generalized Krugman model, and the scaled partial supply elasticity matrix Λ_n of country n simplifies to (A.69).

Proof of optimal domestic taxes According to Proposition 2, optimal domestic taxes satisfy

$$(\Lambda_1 - I)(1 + \boldsymbol{\tau}^d) = 0.$$

Consider two rows, jth and kth of (A.106)

$$\frac{\kappa}{1 - \psi_j(\kappa - 1)}(1 + \tau_j^d) + \frac{\kappa}{1 - \psi_j(\kappa - 1)} \sum_{s=1}^{J} \frac{(1 - \frac{\kappa}{1-\psi_s(\kappa-1)})Y_{n,s}}{\sum_{i=1}^{J} \frac{\kappa}{1-\psi_i(\kappa-1)} Y_{n,i}}(1 + \tau_s^d) = (1 + \tau_j^d), \tag{A.107}$$

$$\frac{\kappa}{1 - \psi_k(\kappa - 1)}(1 + \tau_k^d) + \frac{\kappa}{1 - \psi_k(\kappa - 1)} \sum_{s=1}^{J} \frac{(1 - \frac{\kappa}{1-\psi_s(\kappa-1)})Y_{n,s}}{\sum_{i=1}^{J} \frac{\kappa}{1-\psi_i(\kappa-1)} Y_{n,i}}(1 + \tau_s^d) = (1 + \tau_k^d). \tag{A.108}$$

Multiplying (A.107) and (A.108) by $(\frac{1}{\kappa} - \psi_j \frac{\kappa-1}{\kappa})$ and $(\frac{1}{\kappa} - \psi_k \frac{\kappa-1}{\kappa})$ respectively, and subtracting (A.108) from (A.107), we have

$$\begin{aligned}
&(1 + \tau_j^d) - (1 + \tau_k^d) = (1 + \tau_j^d)(\frac{1}{\kappa} - \psi_j \frac{\kappa - 1}{\kappa}) - (1 + \tau_k^d)(\frac{1}{\kappa} - \psi_k \frac{\kappa - 1}{\kappa}) \\
\Rightarrow &\frac{1 + \tau_j^d}{1 + \tau_k^d} = \frac{1 + \psi_k}{1 + \psi_j} = \frac{\frac{\eta_j}{\eta_j - 1}}{\frac{\eta_k}{\eta_k - 1}},
\end{aligned}$$

where ψ_j is the scale elasticity in sector j.

Proof of optimal import tariffs According to Proposition 2, the optimal tariffs satisfy

$$\sum_{k=1}^{J} \tau_k^m \frac{\partial \ln(Y_{2,k})}{\partial \ln(p_{2,j})} \frac{Y_{2,k}}{Y_{2,j}} = \frac{\beta_{1,j} x_1 - (1+\tau_j^d) Y_{1,j}}{Y_{2,j}}, \forall j$$

Consider two rows, jth and kth of (A.106),

$$\frac{\kappa}{1-\psi_j(\kappa-1)}\tau_j^m + \frac{\kappa}{1-\psi_j(\kappa-1)} \sum_{s=1}^{J} \frac{(1-\frac{\kappa}{1-\psi_s(\kappa-1)})Y_{n,s}}{\sum_{i=1}^{J} \frac{\kappa}{1-\psi_i(\kappa-1)} Y_{n,i}} \tau_s^m = \Psi_{1,j}, \tag{A.109}$$

$$\frac{\kappa}{1-\psi_k(\kappa-1)}\tau_k^m + \frac{\kappa}{1-\psi_k(\kappa-1)} \sum_{s=1}^{J} \frac{(1-\frac{\kappa}{1-\psi_s(\kappa-1)})Y_{n,s}}{\sum_{i=1}^{J} \frac{\kappa}{1-\psi_i(\kappa-1)} Y_{n,i}} \tau_s^m = \Psi_{1,k}, \tag{A.110}$$

where $\Psi_{1,j}$ and $\Psi_{1,k}$ are the jth and kth row of vector Ψ_1. Multiplying (A.109) and (A.110) by $(\frac{1}{\kappa} - \psi_j \frac{\kappa-1}{\kappa})$ and $(\frac{1}{\kappa} - \psi_k \frac{\kappa-1}{\kappa})$ respectively, and subtracting (A.110) from (A.109), we have

$$\tau_j^m - \tau_k^m = (\frac{1}{\kappa} - \psi_j \frac{\kappa-1}{\kappa}) \frac{\beta_{1,j} x_1 - (1+\tau_j^d) Y_{1,j}}{Y_{2,j}} - (\frac{1}{\kappa} - \psi_k \frac{\kappa-1}{\kappa}) \frac{\beta_{1,k} x_1 - (1+\tau_k^d) Y_{1,k}}{Y_{2,k}}.$$

The optimal policies align with Proposition 3 under the CES supply system, where the parameter $\frac{1}{\eta_j} = \frac{1}{\kappa} - \psi_j \frac{\kappa-1}{\kappa}$. Furthermore, this formula is the same as (A.101) that we derived directly using the Lagrange method. □

C.4 CES Labor Market

In this section, we first demonstrate that the labor market specification in a CES labor market is equivalent to that in one-factor Ricardo-Roy model. Next, we prove that it satisfies the CES supply system and derive the formula for Home's optimal policies. In Sections C.4.1 and C.4.2, we discuss two special cases: perfectly mobile labor across sectors and immobile/sector-specific labor.

Here, total labor is a CES aggregator over labor in different sectors, $\bar{L}_n = [\sum_{j=1}^{J} \alpha_j^{-\frac{1}{\kappa-1}} L_{n,j}^{\frac{\kappa}{\kappa-1}}]^{\frac{\kappa-1}{\kappa}}$, where $\{\alpha_j\}$ are constants and $\kappa - 1 \geq 0$ captures the elasticity of substitution across labor in different sectors. This implies a labor supply curve as

$$L_{n,j} = \alpha_j \left(\frac{w_{n,j}}{W_n}\right)^{\kappa-1} \bar{L}_n, \quad \text{with } W_n = \left[\sum_{j=1}^{J} \alpha_j w_{n,j}{}^{\kappa}\right]^{\frac{1}{\kappa}}. \tag{A.111}$$

When $\kappa \to \infty$, $\bar{L}_n = \sum_{j=1}^{J} L_{n,j}$ and labor is perfectly substitutable across sectors, which is widely adopted in the literature. For example, Costinot, Donaldson, and Komunjer (2012) extends Eaton and Kortum (2002) to multi-sector. In this case, the equilibrium wages are equalized across sectors, leading to $\Omega = \{(w_{n,j}, L_{n,j}) : \sum_{j=1}^{J} L_{n,j}(\boldsymbol{w}) = \bar{L}_n,\ w_{n,j} = w_n\}$. When $\kappa = 1$, the model is the same as the specific-factor model with fixed labor in each sector, and thus $\Omega = \{(w_{n,j}, L_{n,j}) : L_{n,j} = \alpha_j \bar{L}_n,\ w_{n,j} = w_{n,j}(\boldsymbol{L})\}$. The optimal policies are in Sections C.4.1 and C.4.2 for details.

In general, we can rewrite the labor supply curve (A.111) and obtain the equilibrium labor market specification as

$$\Omega = \left\{(w_{n,j}, L_{n,j}) : \frac{w_{n,j}L_{n,j}}{W_n \bar{L}_n} = \alpha_j \left(\frac{w_{n,j}}{W_n}\right)^{\kappa},\ W_n = [\sum_{j=1}^{J} \alpha_j {w_{n,j}}^{\kappa}]^{\frac{1}{\kappa}}\right\}. \tag{A.112}$$

The labor market specification (A.112) is equivalent to the one-factor Ricardo-Roy model, and it further implies a supply system,

$$Y_{n,j} = \alpha_j \left(\frac{w_{n,j}}{W_n}\right)^{\kappa} Y_n,$$

where the income in sector j, $Y_{n,j} = w_{n,j}L_{n,j}$. Let the share of sector j income in country n as $m_{n,j} = Y_{n,j}/Y_n$. Due to the constant returns to scale, prices are proportional to wages, which satisfy $p_{n,j} = \frac{w_{n,j}}{\bar{T}_{n,j}}$. The supply curves are

$$Y_{n,j} = \alpha_j \left(p_{n,j}\bar{T}_{n,j}\right)^{\kappa} \left[\sum_{s=1}^{J} \alpha_s \left(p_{n,s}\bar{T}_{n,s}\right)^{\kappa}\right]^{\frac{1-\kappa}{\kappa}} \bar{L}_n.$$

The corresponding element of the scaled partial supply elasticity matrix Λ_n is given by

$$\frac{\partial \ln(Y_{n,s})}{\partial \ln(p_{n,j})} \frac{Y_{n,s}}{Y_{n,j}} = I_{s=j}(\kappa) - (\kappa - 1)m_{n,s}.$$

The scaled partial supply elasticity matrix Λ_n takes the form of

$$\Lambda_n = \begin{pmatrix} \kappa - (\kappa-1)m_{n,1} & \ldots & -(\kappa-1)m_{n,j} & \ldots & -(\kappa-1)m_{n,J} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -(\kappa-1)m_{n,1} & \ldots & \kappa - (\kappa-1)m_{n,j} & \ldots & -(\kappa-1)m_{n,J} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -(\kappa-1)m_{n,1} & \ldots & -(\kappa-1)m_{n,j} & \ldots & \kappa - (\kappa-1)m_{n,J} \end{pmatrix}. \tag{A.113}$$

Therefore, this model satisfies the requirement of the CES supply system. The partial elasticities of a sector's income with respect to price (scaled by the two sectors' size and elasticity parameter) satisfy $\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}} \frac{Y_{n,s}}{Y_{n,j}} \frac{1}{\eta_j} - \frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,i}} \frac{Y_{n,s}}{Y_{n,i}} \frac{1}{\eta_i} = 0$ for $s \neq i$ or j, and $\frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,j}} \frac{Y_{n,s}}{Y_{n,j}} \frac{1}{\eta_j} - \frac{\partial \ln Y_{n,s}}{\partial \ln p_{n,i}} \frac{Y_{n,s}}{Y_{n,i}} \frac{1}{\eta_i} = 1$ for $s = j$, where the parameter $\eta_j = \kappa$.

Consider the jth and kth rows of (12) in Proposition 2.

$$
\begin{aligned}
-(\kappa-1)\sum_{s=1}^{J} m_{2,s}\tau_s^d + (\kappa-1)\tau_j^d &= 0,\\
-(\kappa-1)\sum_{s=1}^{J} m_{2,s}\tau_s^d + (\kappa-1)\tau_k^d &= 0.
\end{aligned}
$$

The difference of the two implies $\tau_k^d = \tau_j^d$. Intuitively, without domestic frictions in the model, the home government has no incentives to use differential domestic taxes.

Consider the jth and kth rows of (13) in Proposition 2.

$$
\begin{aligned}
-(\kappa-1)\sum_{s=1}^{J} m_{2,s}\tau_s^m + \kappa\tau_j^m &= \Psi_{1,j},\\
-(\kappa-1)\sum_{s=1}^{J} m_{2,s}\tau_s^m + \kappa\tau_k^m &= \Psi_{1,k}.
\end{aligned}
$$

The difference of the two implies,

$$
\begin{aligned}
\kappa(\tau_j^m - \tau_k^m) &= \Psi_{1,j} - \Psi_{1,k}\\
\Rightarrow \tau_j^m - \tau_k^m &= \frac{1}{\kappa}\left[\frac{\beta_{1,j}x_1 - Y_{1,j}}{Y_{2,j}} - \frac{\beta_{1,k}x_1 - Y_{1,k}}{Y_{2,k}}\right].
\end{aligned}
$$

This optimal tariff formula equation nests the case with perfectly mobile labor across sectors ($\kappa \to \infty$). In such a case, the optimal import tariffs can be set to zero, $\tau_j^m = 0$ for any j. This is because under a completely elastic relative Foreign supply, the Home has no market power to change relative foreign supply prices across sectors and, thus, no incentives to use heterogeneous tariffs across sectors. In contrast, with imperfect substitute labor, including the sector-specific labor model, where $\kappa < \infty$, imposing a tariff decreases the demand that Foreign sector faces and depresses the sector wage and price. The larger Home's net import share in the Foreign sectoral income, the greater the impact of tariffs on Foreign prices, and the higher the tariff.

To sum up, the optimal policies align with Proposition 3 under CES supply system, where the parameter $\eta_j = \kappa$. Next, we discuss two special cases: perfectly mobile labor across sectors and immobile/sector-specific labor.

C.4.1 Perfectly Mobile Labor Across Sectors

With perfectly mobile labor across sectors ($\kappa \to \infty$), the labor market specifications are

$$\sum_{j=1}^{J} L_{1,j} = \bar{L}_1,$$
$$\sum_{j=1}^{J} L_{2,j} = \bar{L}_2.$$

In this case, we can sum over goods markets across sectors. Now, the goods market clearing conditions become:

$$w_1\bar{L}_1 - \sum_{j=1}^{J} \beta_{2,j}\frac{1}{1+\tau_j^x}\pi_{21,j}x_2 = \sum_{j=1}^{J}\beta_{1,j}\frac{1}{1+\tau_j^d}\pi_{11,j}x_1, \quad (\gamma_1)$$
$$w_2\bar{L}_2 - \sum_{j=1}^{J} \beta_{2,j}\pi_{22,j}x_2 = \sum_{j=1}^{J}\beta_{1,j}\frac{1}{1+\tau_j^m}\pi_{12,j}x_1. \quad (\gamma_2)$$

It is important to note that when technology $T_{n,j}$ is endogenous and depends on $L_{n,j}$, as demonstrated in Appendix C.2.3, we cannot simply aggregate the goods market conditions.

Since the multipliers γ_1 and γ_2 are not sector-specific, the optimal policy following equations (A.14) becomes $1+\tau_j^m = -\gamma_2 = 1+\tau^m$ and $1+\tau_j^d = -\gamma_1 = 1+\tau^d$. Thus, domestic taxes or tariffs are also not independent of sectors. Furthermore, since the labor market specification is homogeneous of degree zero on wages, tax neutrality holds. Both the uniform tariff and domestic tax can be normalized to zero.

C.4.2 Immobile and Sector-Specific Labor

In this case, labor is fixed in each sector, $L_{n,j} = \bar{L}_{n,j}$, which is (A.111) with $\kappa = 1$. The goods market clearing conditions are sector-specific:

$$w_{1,j}\bar{L}_{1,j} - \beta_{2,j}\frac{1}{1+\tau_j^x}\pi_{21,j}x_2 = \beta_{1,j}\frac{1}{1+\tau_j^d}\pi_{11,j}x_1, \quad (\gamma_{1,j}, \quad J)$$
$$w_{2,j}\bar{L}_{2,j} - \beta_{2,j}\pi_{22,j}x_2 = \beta_{1,j}\frac{1}{1+\tau_j^m}\pi_{12,j}x_1. \quad (\gamma_{2,j}, \quad J)$$

In this case, the element of matrix Λ_n at row j and column k is given by $\frac{\partial \ln(Y_{n,k})}{\partial \ln(p_{n,j})}\frac{Y_{n,k}}{Y_{n,j}}$, which is 1 if $j = k$ and 0 if $j \neq k$. Therefore, $\Lambda_n = I$), which satisfies the definition of CES supply system. We can then apply the Proposition 3 for optimal policies, which shows the optimal domestic tax is uniform across sectors.

Proof of optimal import tariffs Since $\Lambda_2 = I$, consider the jth and kth rows of (13) in Proposition 2.

$$\tau_j^m = \Psi_{1,j},$$
$$\tau_k^m = \Psi_{1,k}.$$

The difference of the two implies

$$\tau_j^m - \tau_k^m = \frac{\beta_{1,j}x_1 - (1+\tau_j^d)Y_{1,j}}{Y_{2,j}} - \frac{\beta_{1,k}x_1 - (1+\tau_k^d)Y_{1,k}}{Y_{2,k}}.$$

Thus, Home's import tariff imposed on sector j increases with the share of Home's net imports in Foreign's income, relative to sector k.

D Non-CES Supply System: Multi-factor Ricardo-Roy Model

In this section, we explain why multi-factor Ricardo-Roy model does not satisfy CES supply system. In this model, Home government's problem and world market equilibrium are the same as those in one-factor Ricardo-Roy model shown in Section C.1, except for the labor market specifications, which satisfy

$$w_{n,j}L_{n,j} = \sum_g \frac{A_{n,j,g}w_{n,j}^\kappa}{W_{n,g}^\kappa} W_{n,g}\bar{L}_{n,g}, \quad W_{n,g} = \left[\sum_{s=1}^J A_{n,s,g}(w_{n,s})^\kappa\right]^{\frac{1}{\kappa}}. \tag{A.114}$$

Since there are constant returns to scale, prices are proportional to wages, which satisfy $p_{n,j} = \frac{w_{n,j}}{\bar{T}_{n,j}}$. We can derive supply curves by combining the labor market specification Ω (Eq. A.114) with the supply-side assumption $\mathbb{S}$ (Eq. A.26) and the definition of income $Y_{n,j} = w_{n,j}L_{n,j}$. Eliminating the wage and labor $\{w_{n,j}, L_{n,j}\}$ yields the following supply curves:

$$Y_{n,j} = \left(p_{n,j}\bar{T}_{n,j}\right)^\kappa \sum_g A_{n,j,g}\left[\sum_{s=1}^J A_{n,s,g}\left(p_{n,s}\bar{T}_{n,s}\right)^\kappa\right]^{\frac{1-\kappa}{\kappa}} \bar{L}_{n,g}.$$

The corresponding element of the scaled partial supply elasticity matrix Λ_n is given by

$$\frac{\partial \ln(Y_{n,s})}{\partial \ln(p_{n,j})}\frac{Y_{n,s}}{Y_{n,j}} = I_{s=j}(\kappa) - (\kappa - 1)\frac{\sum_g \lambda_{n,s,g}\lambda_{n,j,g}W_{n,g}\bar{L}_{n,g}}{Y_{n,j}},$$

where $I_{s=j}(\kappa)$ is an indicator function that equals κ when $s = j$ and 0 otherwise. $\lambda_{n,j,g} = Y_{n,j,g}/Y_{n,g}$ is the share of type-g workers that enter sector j.

The scaled partial supply elasticity matrix Λ_n of country n, with its jth-row-ith-column entry as $\frac{\partial \ln Y_{n,i}}{\partial \ln p_{n,j}}\frac{Y_{n,i}}{Y_{n,j}}$, takes the form of

$$\Lambda_n = \begin{pmatrix} \kappa - (\kappa-1)\frac{\sum_g \lambda_{n,1,g}\lambda_{n,1,g}W_{n,g}\bar{L}_{n,g}}{Y_{n,1}} & ... & -(\kappa-1)\frac{\sum_g \lambda_{n,1,g}\lambda_{n,j,g}W_{n,g}\bar{L}_{n,g}}{Y_{n,1}} & ... & -(\kappa-1)\frac{\sum_g \lambda_{n,1,g}\lambda_{n,J,g}W_{n,g}\bar{L}_{n,g}}{Y_{n,1}} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\frac{\sum_g \lambda_{n,j,g}\lambda_{n,1,g}W_{n,g}\bar{L}_{n,g}}{Y_{n,j}} & ... & \kappa - (\kappa-1)\frac{\sum_g \lambda_{n,j,g}\lambda_{n,j,g}W_{n,g}\bar{L}_{n,g}}{Y_{n,j}} & ... & -(\kappa-1)\frac{\sum_g \lambda_{n,j,g}\lambda_{n,J,g}W_{n,g}\bar{L}_{n,g}}{Y_{n,j}} \\ ... & ... & ... & ... & ... \\ -(\kappa-1)\frac{\sum_g \lambda_{n,J,g}\lambda_{n,1,g}W_{n,g}\bar{L}_{n,g}}{Y_{n,J}} & ... & -(\kappa-1)\frac{\sum_g \lambda_{n,J,g}\lambda_{n,j,g}W_{n,g}\bar{L}_{n,g}}{Y_{n,J}} & ... & \kappa - (\kappa-1)\frac{\sum_g \lambda_{n,J,g}\lambda_{n,J,g}W_{n,g}\bar{L}_{n,g}}{Y_{n,J}} \end{pmatrix}. \tag{A.115}$$

Foreign sector i's supply, represented by $Y_{2,i}$, is affected by the sector j's price $p_{2,j}$ through the expression $-(\kappa-1)\sum_g \frac{\lambda_{2,j,g}Y_{2,g}}{Y_{2,j}}\lambda_{2,i,g}$, and affected by sector J's price $p_{2,J}$, $-(\kappa-1)\sum_g \frac{\lambda_{2,J,g}Y_{2,g}}{Y_{2,J}}\lambda_{2,i,g}$. In general, these cross-elasticities are influenced by asymmetric endogenous variables and cannot be canceled out. Hence, this model does not satisfy the CES supply system as defined in Definition 4. The relative optimal tariffs between two sectors are also influenced by other sectors. Optimal policies still satisfy the general formula as in Proposition 2.

Proof of optimal domestic taxes According to Proposition 2, optimal domestic taxes satisfy

$$(\Lambda_1 - I)(1 + \boldsymbol{\tau}^{\boldsymbol{d}}) = 0,$$

where

$$\Lambda_1 - I = (\kappa-1)\begin{pmatrix} 1-\frac{\sum_g \lambda_{1,1,g}\lambda_{1,1,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,1}} & \dots & -\frac{\sum_g \lambda_{1,1,g}\lambda_{1,j,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,1}} & \dots & -\frac{\sum_g \lambda_{1,1,g}\lambda_{1,J,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,1}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g \lambda_{1,j,g}\lambda_{1,1,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,j}} & \dots & 1-\frac{\sum_g \lambda_{1,j,g}\lambda_{1,j,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,j}} & \dots & -\frac{\sum_g \lambda_{1,j,g}\lambda_{1,J,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,j}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g \lambda_{1,J,g}\lambda_{1,1,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,J}} & \dots & -\frac{\sum_g \lambda_{1,J,g}\lambda_{1,j,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,J}} & \dots & 1-\frac{\sum_g \lambda_{1,J,g}\lambda_{1,J,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,J}} \end{pmatrix}. \tag{A.116}$$

Due to tax neutrality, the domestic tax can only be determined at a relative level. We define a $(J-1)*(J-1)$ matrix M that satisfies

$$M = (\kappa-1)\begin{pmatrix} 1-\frac{\sum_g \lambda_{1,1,g}\lambda_{1,1,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,1}} & \dots & -\frac{\sum_g \lambda_{1,1,g}\lambda_{1,j,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,1}} & \dots & -\frac{\sum_g \lambda_{1,1,g}\lambda_{1,J-1,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,1}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g \lambda_{1,j,g}\lambda_{1,1,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,j}} & \dots & 1-\frac{\sum_g \lambda_{1,j,g}\lambda_{1,j,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,j}} & \dots & -\frac{\sum_g \lambda_{1,j,g}\lambda_{1,J-1,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,j}} \\ \dots & \dots & \dots & \dots & \dots \\ -\frac{\sum_g \lambda_{1,J-1,g}\lambda_{1,1,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,J-1}} & \dots & -\frac{\sum_g \lambda_{1,J-1,g}\lambda_{1,j,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,J-1}} & \dots & 1-\frac{\sum_g \lambda_{1,J-1,g}\lambda_{1,J-1,g}W_{1,g}\bar{L}_{1,g}}{Y_{1,J-1}} \end{pmatrix}.$$

Using this matrix, we can rewrite the system of equations (A.116) as

$$M \cdot \begin{pmatrix} \tau_1^d - \tau_J^d \\ \dots \\ \tau_j^d - \tau_J^d \\ \dots \\ \tau_{J-1}^d - \tau_J^d \end{pmatrix} = \begin{pmatrix} 0 \\ \dots \\ 0 \\ \dots \\ 0 \end{pmatrix}.$$

As $|M| \neq 0$, $\tau_j^d - \tau_J^d = 0$, and thus domestic taxes are uniform across all sectors. □

E Proof for Proposition 4: Optimal Unilateral Policies under Multi-country Case

In this section, we use the Lagrange multiplier method to study the optimization problem constrained to world equilibrium conditions shown in Appendix A, where γ_x is the multiplier on Home's expenditure, and $\gamma_{1,j}$ and $\gamma_{n,j}$ are the multipliers on the goods market clearing conditions that determine prices. According to the Walras' law, one of the equilibrium conditions is redundant; thus, we drop the expenditure constraint in optimization and set $\gamma_x = 0$. Or, equivalently, if we use Home's budget constraint, one of the market-clearing conditions is redundant. In addition, we normalize the price index of Home as the numeraire.

FOC over x_1

$$1 + \sum_{j=1}^{J} \gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\pi_{11,j} + \sum_{i\neq 1}^{N}\sum_{j=1}^{J}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau_{i,j}^m}\pi_{1i,j} = 0.$$

FOC over import tariff $\tau_{n,j}^m$, $n \neq 1$

$$-\beta_{1,j}x_1\frac{\pi_{1n,j}}{1+\tau_{n,j}^m} + \gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial \tau_{n,j}^m}x_1 - \gamma_{n,j}\beta_{1,j}\frac{\pi_{1n,j}}{(1+\tau_{n,j}^m)^2}x_1 + \sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau_{i,j}^m}\frac{\partial \pi_{1i,j}}{\partial \tau_{n,j}^m}x_1 = 0.$$

Plugging into derivatives

$$-\beta_{1,j}x_1\pi_{1n,j} + \gamma_{1,j}\frac{\beta_{1,j}\epsilon_j\pi_{11,j}\pi_{1n,j}x_1}{1+\tau_j^d} - \gamma_{n,j}\frac{\beta_{1,j}\pi_{1n,j}x_1}{1+\tau_{n,j}^m} + \sum_{i\neq 1}^{N}\gamma_{i,j}\frac{\beta_{1,j}\epsilon_j\pi_{1i,j}\pi_{1n,j}x_1}{1+\tau_{i,j}^m} - \gamma_{n,j}\frac{\beta_{1,j}\epsilon_j\pi_{1n,j}x_1}{1+\tau_{n,j}^m} = 0 \tag{A.117}$$

$$\Rightarrow -1 + \gamma_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j} - \gamma_{n,j}\frac{1}{1+\tau_{n,j}^m} + \sum_{i\neq 1}^{N}\gamma_{i,j}\frac{1}{1+\tau_{i,j}^m}\epsilon_j\pi_{1i,j} - \gamma_{n,j}\frac{1}{1+\tau_{n,j}^m}\epsilon_j = 0$$

$$\Rightarrow -1 + \gamma_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j} - \gamma_{n,j}\frac{1+\epsilon_j}{1+\tau_{n,j}^m} + \sum_{i\neq 1}^{N}\gamma_{i,j}\frac{1}{1+\tau_{i,j}^m}\epsilon_j\pi_{1i,j} = 0. \tag{A.118}$$

Thus, $\gamma_{n,j}\frac{1}{1+\tau_{n,j}^m}$ is the same across all countries in sector j.

FOC over domestic tax τ_j^d

$$-\beta_{1,j}x_1\frac{\pi_{11,j}}{1+\tau_j^d} + \gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\frac{\partial \pi_{11,j}}{\partial \tau_j^d}x_1 - \gamma_{1,j}\beta_{1,j}\frac{\pi_{11,j}}{(1+\tau_j^d)^2}x_1 + \sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau_{i,j}^m}\frac{\partial \pi_{1i,j}}{\partial \tau_j^d}x_1 = 0.$$

Plugging into derivatives

$$-\beta_{1,j}x_1\pi_{11,j}-\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}(1-\pi_{11,j})x_1-\gamma_{1,j}\beta_{1,j}\frac{\pi_{11,j}}{1+\tau_j^d}x_1+\sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau_{i,j}^m}\epsilon_j\pi_{1i,j}\pi_{11,j}x_1=0 \tag{A.119}$$

$$\Rightarrow -1-\gamma_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j(1-\pi_{11,j})-\gamma_{1,j}\frac{1}{1+\tau_j^d}+\sum_{i\neq 1}^{N}\gamma_{i,j}\frac{1}{1+\tau_{i,j}^m}\epsilon_j\pi_{1i,j}=0.$$

Combining with (A.118),

$$\begin{aligned}\gamma_{1,j}\frac{1}{1+\tau_j^d}&=\gamma_{n,j}\frac{1}{1+\tau_{n,j}^m}\\ \Rightarrow 1+\tau_j^d&=-\gamma_{1,j},\quad 1+\tau_{n,j}^m=-\gamma_{n,j}.\end{aligned} \tag{A.120}$$

FOC over export tax $\tau_{n,j}^x$, $n\neq 1$

$$-\gamma_{1,j}\beta_{n,j}\frac{1}{(1+\tau_{n,j}^x)^2}\pi_{n1,j}x_n+\gamma_{1,j}\beta_{n,j}\frac{1}{1+\tau_{n,j}^x}\frac{\partial\pi_{n1,j}}{\partial\tau_{n,j}^x}x_n+\sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{n,j}\frac{\partial\pi_{ni,j}}{\partial\tau_{n,j}^x}x_n=0.$$

Plugging into derivatives

$$-\gamma_{1,j}\beta_{n,j}\frac{1}{1+\tau_{n,j}^x}\pi_{n1,j}x_n-\gamma_{1,j}\beta_{n,j}\frac{1}{1+\tau_{n,j}^x}\epsilon_j\pi_{n1,j}(1-\pi_{n1,j})x_n+\sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{n,j}\epsilon_j\pi_{ni,j}\pi_{n1,j}x_n=0 \tag{A.121}$$

$$\begin{aligned}&\Rightarrow -\gamma_{1,j}\frac{1}{1+\tau_{n,j}^x}-\gamma_{1,j}\frac{1}{1+\tau_{n,j}^x}\epsilon_j(1-\pi_{n1,j})+\sum_{i\neq 1}^{N}\gamma_{i,j}\epsilon_j\pi_{ni,j}=0\\ &\Rightarrow \frac{1}{1+\tau_{n,j}^x}=\frac{\sum_{i\neq 1}^{N}\gamma_{i,j}\epsilon_j\pi_{ni,j}}{\gamma_{1,j}(1+\epsilon_j(1-\pi_{n1,j}))}=\frac{1}{1+\tau_j^d}\frac{\sum_{i\neq 1}^{N}(1+\tau_{i,j}^m)\epsilon_j\pi_{ni,j}}{1+\epsilon_j(1-\pi_{n1,j})}.\end{aligned} \tag{A.122}$$

FOC over $p_{1,j}$

$$\begin{aligned}&-x_1\beta_{1,j}\frac{\pi_{11,j}}{p_{1,j}}+\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\frac{\partial\pi_{11,j}}{\partial p_{1,j}}x_1-\sum_{k=1}^{J}\gamma_{1,k}\frac{\partial E_{11,k}}{\partial p_{1,j}}+\sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau_{i,j}^m}\frac{\partial\pi_{1i,j}}{\partial p_{1,j}}x_1-\sum_{k=1}^{J}\sum_{i\neq 1}^{N}\gamma_{i,k}\frac{\partial E_{1i,k}}{\partial p_{1,j}}=0\\ &\Rightarrow -x_1\beta_{1,j}\pi_{11,j}-\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}(1-\pi_{11,j})x_1-\sum_{k=1}^{J}\gamma_{1,k}\frac{\partial Y_{1,k}}{\partial\ln p_{1,j}}-\gamma_{1,j}\sum_{i\neq 1}^{N}\beta_{i,j}\frac{1}{1+\tau_{i,j}^x}\epsilon_j\pi_{i1,j}(1-\pi_{i1,j})x_i\\ &+\sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau_{i,j}^m}\epsilon_j\pi_{1i,j}\pi_{11,j}x_1+\sum_{i\neq 1}^{N}\gamma_{i,j}\sum_{n\neq 1}^{N}\beta_{n,j}\epsilon_j\pi_{ni,j}\pi_{n1,j}x_n=0,\end{aligned} \tag{A.123}$$

where $\frac{\partial E_{11,k}}{\partial \ln p_{1,j}} = \frac{\partial Y_{1,k}}{\partial \ln p_{1,j}} + I_{k=j}\left[\sum_{i\neq 1}^{N} \beta_{i,k} \frac{x_i}{1+\tau_{i,k}^x} \epsilon_k \pi_{i1,k}(1-\pi_{i1,k})\right] - \beta_{1,k} \frac{\pi_{11,k}}{1+\tau_{1,k}^x} \frac{\partial \sum_{s=1}^{J} Y_{1,s}}{\partial \ln p_{1,j}}$.

Combining (A.123) with (A.120) and (A.121), we derive the formula for domestic taxes

$$\begin{aligned}
& - x_1 \beta_{1,j} \pi_{11,j} - \sum_{k=1}^{J} \gamma_{1,k} \frac{\partial Y_{1,k}}{\partial \ln p_{1,j}} + \gamma_{1,j} \sum_{n\neq 1}^{N} \beta_{n,j} \frac{1}{1+\tau_{n,j}^x} \pi_{n1,j} x_n = 0 \\
\Rightarrow & - x_1 \beta_{1,j} \pi_{11,j} - \sum_{k=1}^{J} \gamma_{1,k} \frac{\partial Y_{1,k}}{\partial \ln p_{1,j}} + \gamma_{1,j} Y_{1,j} - \gamma_{1,j} \beta_{1,j} \frac{1}{1+\tau_j^d} \pi_{11,j} x_1 = 0 \\
\Rightarrow & \sum_{k=1}^{J} \gamma_{1,k} \frac{\partial Y_{1,k}}{\partial \ln p_{1,j}} = \gamma_{1,j} Y_{1,j} \\
\Rightarrow & \sum_{k=1}^{J} (1+\tau_k^d) \frac{\partial \ln Y_{1,k}}{\partial \ln p_{1,j}} \frac{Y_{1,k}}{Y_{1,j}} = 1 + \tau_j^d.
\end{aligned} \tag{A.124}$$

FOC over $p_{n,j}$, $n \neq 1$

$$\begin{aligned}
& - x_1 \beta_{1,j} \frac{\pi_{1n,j}}{p_{n,j}} + \gamma_{1,j} \beta_{1,j} \frac{1}{1+\tau_j^d} \frac{\partial \pi_{11,j}}{\partial p_{n,j}} x_1 - \sum_{k=1}^{J} \gamma_{1,k} \frac{\partial E_{11,k}}{\partial p_{n,j}} + \sum_{i\neq 1}^{N} \gamma_i \beta_{1,j} \frac{1}{1+\tau_{i,j}^m} \frac{\partial \pi_{1i,j}}{\partial p_{n,j}} x_1 - \sum_{k=1}^{J} \sum_{i\neq 1}^{N} \gamma_{i,k} \frac{\partial E_{1i,k}}{\partial p_{n,j}} = 0 \\
\Rightarrow & - \sum_{k=1}^{J} \sum_{i\neq 1}^{N} \gamma_{i,k} \frac{\partial E_{1i,k}}{\partial p_{n,j}} p_{n,j} = x_1 \beta_{1,j} \pi_{1n,j} + \sum_{k=1}^{J} \gamma_{1,k} \frac{\partial E_{11,k}}{\partial p_{n,j}} p_{n,j} \\
\Rightarrow & \sum_{k=1}^{J} \sum_{i\neq 1}^{N} (1+\tau_{i,k}^m) \frac{\partial E_{1i,k}}{\partial \ln p_{n,j}} = x_1 \beta_{1,j} \pi_{1n,j} - \sum_{k=1}^{J} (1+\tau_k^d) \frac{\partial E_{11,k}}{\partial \ln p_{n,j}}.
\end{aligned} \tag{A.125}$$

We can rewrite (A.125) into matrix and determine the $(N-1) \times J$ tariffs

$$\left(\begin{array}{ccc:ccc}
\frac{\partial E_{12,1}}{\partial \ln p_{2,1}} \frac{1}{Y_{2,1}} & \cdots & \frac{\partial E_{12,J}}{\partial \ln p_{2,1}} \frac{1}{Y_{2,1}} & \frac{\partial E_{13,1}}{\partial \ln p_{2,1}} \frac{1}{Y_{2,1}} & \cdots & \frac{\partial E_{1N,J}}{\partial \ln p_{2,1}} \frac{1}{Y_{2,1}} \\
\cdots & & \cdots & \cdots & & \cdots \\
\frac{\partial E_{12,1}}{\partial \ln p_{2,J}} \frac{1}{Y_{2,J}} & \cdots & \frac{\partial E_{12,J}}{\partial \ln p_{2,J}} \frac{1}{Y_{2,J}} & \frac{\partial E_{13,1}}{\partial \ln p_{2,J}} \frac{1}{Y_{2,J}} & \cdots & \frac{\partial E_{1N,J}}{\partial \ln p_{2,J}} \frac{1}{Y_{2,J}} \\
\hdashline
\frac{\partial E_{12,1}}{\partial \ln p_{3,1}} \frac{1}{Y_{3,1}} & \cdots & \frac{\partial E_{12,J}}{\partial \ln p_{3,1}} \frac{1}{Y_{3,1}} & \frac{\partial E_{13,1}}{\partial \ln p_{3,1}} \frac{1}{Y_{3,1}} & \cdots & \frac{\partial E_{1N,J}}{\partial \ln p_{3,1}} \frac{1}{Y_{3,1}} \\
\cdots & & \cdots & \cdots & & \cdots \\
\frac{\partial E_{12,1}}{\partial \ln p_{N,J}} \frac{1}{Y_{N,J}} & \cdots & \frac{\partial E_{12,J}}{\partial \ln p_{N,J}} \frac{1}{Y_{N,J}} & \frac{\partial E_{13,1}}{\partial \ln p_{N,J}} \frac{1}{Y_{N,J}} & \cdots & \frac{\partial E_{1N,J}}{\partial \ln p_{N,J}} \frac{1}{Y_{N,J}}
\end{array}\right)
\begin{pmatrix} 1+\tau_{2,1}^m \\ \cdots \\ 1+\tau_{2,J}^m \\ 1+\tau_{3,1}^m \\ \cdots \\ 1+\tau_{N,J}^m \end{pmatrix}
=
\begin{pmatrix}
\beta_{1,1} \pi_{12,1} x_1 \frac{1}{Y_{2,1}} - \sum_{k=1}^{J} (1+\tau_k^d) \frac{\partial E_{11,k}}{\partial \ln p_{2,1}} \frac{1}{Y_{2,1}} \\
\cdots \\
\beta_{1,J} \pi_{12,J} x_1 \frac{1}{Y_{2,J}} - \sum_{k=1}^{J} (1+\tau_k^d) \frac{\partial E_{11,k}}{\partial \ln p_{2,J}} \frac{1}{Y_{2,J}} \\
\beta_{1,1} \pi_{13,1} x_1 \frac{1}{Y_{3,1}} - \sum_{k=1}^{J} (1+\tau_k^d) \frac{\partial E_{11,k}}{\partial \ln p_{3,1}} \frac{1}{Y_{3,1}} \\
\cdots \\
\beta_{1,J} \pi_{1N,J} x_1 \frac{1}{Y_{N,J}} - \sum_{k=1}^{J} (1+\tau_k^d) \frac{\partial E_{11,k}}{\partial \ln p_{N,J}} \frac{1}{Y_{N,J}}
\end{pmatrix}, \tag{A.126}$$

where the semi-export-supply-elasticities are

$$\frac{\partial E_{11,k}}{\partial \ln p_{n,j}} = -I_{k=j}\left[\sum_{i\neq 1}^{N}\beta_{i,k}\frac{x_i}{1+\tau^x_{i,k}}\epsilon_k\pi_{i1,k}\pi_{in,k}\right] - \beta_{n,k}\frac{\pi_{n1,k}}{1+\tau^x_{n,k}}\frac{\partial\sum_{s=1}^{J}Y_{n,s}}{\partial \ln p_{n,j}}, \quad (n\neq 1)$$

$$\frac{\partial E_{1n,k}}{\partial \ln p_{n,j}} = \frac{\partial Y_{n,k}}{\partial \ln p_{n,j}} + I_{k=j}\left[\sum_{s\neq 1}^{N}\beta_{s,k}x_s\epsilon_k(1-\pi_{sn,k})\pi_{sn,k}\right] - \beta_{n,k}\pi_{nn,k}\frac{\partial\sum_{s=1}^{J}Y_{n,s}}{\partial \ln p_{n,j}}, \quad (n\neq 1)$$

$$\frac{\partial E_{1i,k}}{\partial \ln p_{n,j}} = -I_{k=j}\left[\sum_{s\neq 1}^{N}\beta_{s,k}x_s\epsilon_k\pi_{si,k}\pi_{sn,k}\right] - \beta_{n,k}\pi_{ni,k}\frac{\partial\sum_{s=1}^{J}Y_{n,s}}{\partial \ln p_{n,j}}, \quad (i\neq n, i\neq 1).$$

□

F Proof for Lemma 2

In this section, we first present the formula for the optimal domestic taxes and tariffs within a CES supply system with constant returns to scale. Then, we prove Lemma 2.

Uniform optimal domestic taxes In a multi-country case, consider a CES supply system exhibiting constant returns to scale with $\eta_j = \kappa$ for any j (see proof in Section C.1). We integrate Definition 4 with Equation (A.124):

$$\sum_{k=1}^{J}(1+\tau^d_k)\frac{\partial \ln Y_{1,k}}{\partial \ln p_{1,j}}\frac{Y_{1,k}}{Y_{1,j}} = 1+\tau^d_j \quad\Rightarrow\quad \tau^d_j - \tau^d_i = 0.$$

Thus, the optimal domestic tax is uniform across sectors. Due to the tax neutrality in Proposition 1, one domestic tax can be normalized to zero. Therefore, all domestic taxes are zero.

Optimal tariffs Returning to equation (A.126) in Proposition 4, within a CES supply system with constant returns to scale, the semi-export-supply-elasticities are

$$\begin{aligned}
\frac{\partial E_{11,k}}{\partial \ln p_{n,j}} &= -I_{k=j}\left[\sum_{i\neq 1}^{N}\beta_{i,k}\frac{x_i}{1+\tau^x_{i,k}}\epsilon_k\pi_{i1,k}\pi_{in,k}\right] - \beta_{n,k}\frac{\pi_{n1,k}}{1+\tau^x_{n,k}}\frac{\partial\sum_{s=1}^{J}Y_{n,s}}{\partial \ln p_{n,j}}, \quad (n\neq 1)\\
&= -I_{k=j}\left[\sum_{i\neq 1}^{N}\beta_{i,k}\frac{x_i}{1+\tau^x_{i,k}}\epsilon_k\pi_{i1,k}\pi_{in,k}\right] - \beta_{n,k}\frac{\pi_{n1,k}}{1+\tau^x_{n,k}}Y_{n,j},\\
\frac{\partial E_{1n,k}}{\partial \ln p_{n,j}} &= \frac{\partial Y_{n,k}}{\partial \ln p_{n,j}} + I_{k=j}\left[\sum_{s\neq 1}^{N}\beta_{s,k}x_s\epsilon_k(1-\pi_{sn,k})\pi_{sn,k}\right] - \beta_{n,k}\pi_{nn,k}\frac{\partial\sum_{s=1}^{J}Y_{n,s}}{\partial \ln p_{n,j}}, \quad (n\neq 1)\\
&= I_{k=j}\left[\kappa Y_{n,k}\right] - (\kappa-1)\lambda_{n,k}Y_{n,j} + I_{k=j}\left[\sum_{s\neq 1}^{N}\beta_{s,k}x_s\epsilon_k(1-\pi_{sn,k})\pi_{sn,k}\right] - \beta_{n,k}\pi_{nn,k}Y_{n,j},\\
\frac{\partial E_{1i,k}}{\partial \ln p_{n,j}} &= -I_{k=j}\left[\sum_{s\neq 1}^{N}\beta_{s,k}x_s\epsilon_k\pi_{si,k}\pi_{sn,k}\right] - \beta_{n,k}\pi_{ni,k}\frac{\partial\sum_{s=1}^{J}Y_{n,s}}{\partial \ln p_{n,j}}, \quad (i\neq n\neq 1)\\
&= -I_{k=j}\left[\sum_{s\neq 1}^{N}\beta_{s,k}x_s\epsilon_k\pi_{si,k}\pi_{sn,k}\right] - \beta_{n,k}\pi_{ni,k}Y_{n,j}.
\end{aligned}$$

The optimal tariffs depend on the LHS matrix, $d\boldsymbol{E}$. The diagonal blocks of $d\boldsymbol{E}$ are

$$\frac{\partial \boldsymbol{E_{1n}}}{\partial \ln \boldsymbol{p_n}}\circ\frac{1}{\boldsymbol{Y_n}} = \begin{pmatrix} \frac{\partial E_{1n,1}}{\partial \ln p_{n,1}}\frac{1}{Y_{n,1}} & \cdots & \frac{\partial E_{1n,J}}{\partial \ln p_{n,1}}\frac{1}{Y_{n,1}}\\ \vdots & \ddots & \vdots\\ \frac{\partial E_{1n,1}}{\partial \ln p_{n,J}}\frac{1}{Y_{n,J}} & \cdots & \frac{\partial E_{1n,J}}{\partial \ln p_{n,J}}\frac{1}{Y_{n,J}}\end{pmatrix}$$

$$= \begin{pmatrix} \kappa-(\kappa-1)\lambda_{n,1}+\epsilon_1\sum_{h\neq 1}^{N}(1-\pi_{hn,1})s_{hn,1}-\beta_{n,1}\pi_{nn,1} & \cdots & -(\kappa-1)\lambda_{n,J}-\beta_{n,J}\pi_{nn,J}\\ \vdots & & \vdots\\ -(\kappa-1)\lambda_{n,1}-\beta_{n,1}\pi_{nn,1} & \cdots & \kappa-(\kappa-1)\lambda_{n,J}+\epsilon_J\sum_{h\neq 1}^{N}(1-\pi_{hn,J})s_{hn,J}-\beta_{n,J}\pi_{nn,J}\end{pmatrix},$$

where we define $s_{hn,j}=\frac{\pi_{hn,j}\beta_{h,j}x_h}{Y_{n,j}}$ as the share of the exporter nj's income from an importer h.

The off-diagonal blocks of $d\boldsymbol{E}$ are

$$\frac{\partial \boldsymbol{E_{1i}}}{\partial \ln \boldsymbol{p_n}} \circ \frac{1}{\boldsymbol{Y_n}} = \begin{pmatrix} \frac{\partial E_{1i,1}}{\partial \ln p_{n,1}} \frac{1}{Y_{n,1}} & \cdots & \frac{\partial E_{1i,J}}{\partial \ln p_{n,1}} \frac{1}{Y_{n,1}} \\ \cdots & & \cdots \\ \frac{\partial E_{1i,1}}{\partial \ln p_{n,J}} \frac{1}{Y_{n,J}} & \cdots & \frac{\partial E_{1i,J}}{\partial \ln p_{n,J}} \frac{1}{Y_{n,J}} \end{pmatrix}$$

$$= \begin{pmatrix} -\epsilon_1 \sum_{h\neq 1}^{N} \pi_{hi,1} s_{hn,1} - \beta_{n,1}\pi_{ni,1} & \cdots & -\beta_{n,J}\pi_{ni,J} \\ \vdots & & \vdots \\ -\beta_{n,1}\pi_{ni,1} & \cdots & -\epsilon_J \sum_{h\neq 1}^{N} \pi_{hi,J} s_{hn,J} - \beta_{n,J}\pi_{ni,J} \end{pmatrix}.$$

F.1 Optimal tariffs and average net import share

Next, we prove the first point in Lemma 2. By summing up the FOCs over $p_{1,j}$ in (A.123) and $p_{n,j}$ in (A.125), for $n \neq 1$, we derive:

$$\begin{aligned}
&\sum_{h=1}^{N}\sum_{k=1}^{J}\sum_{i\neq 1}^{N}(1+\tau_{i,k}^{m})\frac{\partial E_{1i,k}}{\partial \ln p_{h,j}} = x_1\beta_{1,j} - \sum_{h=1}^{N}\sum_{k=1}^{J}\frac{\partial E_{11,k}}{\partial \ln p_{h,j}} \\
\Rightarrow &\sum_{n\neq 1}^{N}\sum_{k=1}^{J}(1+\tau_{n,k}^{m})\frac{\partial Y_{n,k}}{\partial \ln p_{n,j}} + \sum_{n\neq 1}^{N} cons_n \frac{\partial \sum_{k=1}^{J} Y_{n,k}}{\partial \ln p_{n,j}} = x_1\beta_{1,j} - \sum_{k=1}^{J}\frac{\partial Y_{1,k}}{\partial \ln p_{1,j}} \\
\Rightarrow &\sum_{n\neq 1}^{N}\sum_{k=1}^{J}(cons_n + 1 + \tau_{n,k}^{m})\frac{\partial Y_{n,k}}{\partial \ln p_{n,j}} = x_1\beta_{1,j} - Y_{1,j} \\
\Rightarrow &\sum_{n\neq 1}^{N}(cons_n + 1 + \tau_{n,j}^{m})Y_{n,j} + \sum_{n\neq 1}^{N}\sum_{k=1}^{J}(cons_n + 1 + \tau_{n,k}^{m})\frac{\partial L_{n,k}}{\partial \ln p_{n,j}} = x_1\beta_{1,j} - Y_{1,j} \\
\Rightarrow &\kappa\sum_{n\neq 1}^{N}(cons_n + 1 + \tau_{n,j}^{m})Y_{n,j} - (\kappa-1)\sum_{n\neq 1}^{N}\sum_{k=1}^{J}(cons_n + 1 + \tau_{n,k}^{m})\lambda_{n,k}Y_{n,j} = x_1\beta_{1,j} - Y_{1,j} \\
\Rightarrow &\sum_{n\neq 1}^{N}(\delta_n + \tau_{n,j}^{m})Y_{n,j} = \frac{1}{\kappa}\left[x_1\beta_{1,j} - Y_{1,j}\right], \qquad \text{(A.127)}
\end{aligned}$$

where

$$\begin{aligned}
cons_n &= -\sum_{i\neq 1}^{N}\sum_{k=1}^{J}\beta_{n,k}(1+\tau_{i,k}^{m})\pi_{ni,k} - \sum_{k=1}^{J}\beta_{n,k}\frac{\pi_{n1,k}}{1+\tau_{n,k}^{x}}, \\
\delta_n &= -\frac{\kappa-1}{\kappa}\sum_{k=1}^{J}(cons_n + 1 + \tau_{n,k}^{m})\lambda_{n,k} + cons_n + 1.
\end{aligned}$$

F.2 Symmetric foreign countries

We then prove the second point in Lemma 2. When foreign countries are symmetric, the optimal import tariffs and export taxes are identical across all countries within a sector j, i.e., $\tau^m_{n,j} = \tau^m_{i,j}$ and $\tau^x_{n,j} = \tau^x_{i,j}, \forall n \neq i > 1, \forall j$. Trade shares between any two foreign countries within a sector are equal. Furthermore, $\beta_{n,j} = \beta_{i,j}$ and $x_n = x_i, \forall n \neq i > 1, \forall j$. The optimal export tax in (A.122) can be expressed as

$$
\begin{aligned}
\frac{1}{1+\tau^x_{n,j}} &= (1+\tau^m_{n,j})\frac{\sum_{i\neq 1}^N \epsilon_j \pi_{ni,j}}{1+\epsilon_j(1-\pi_{n1,j})} = (1+\tau^m_{n,j})\frac{\epsilon_j(1-\pi_{n1,j})}{1+\epsilon_j(1-\pi_{n1,j})} \\
\Rightarrow 1+\tau^x_{n,j} &= \frac{1}{1+\tau^m_{n,j}}\left(1+\frac{1}{\epsilon_j(1-\pi_{n1,j})}\right).
\end{aligned}
\tag{A.128}
$$

We can derive the optimal tariff in country n sector j from FOC over $p_{n,j}$ (A.125)

$$
\begin{aligned}
&\sum_{k=1}^J (1+\tau^m_{n,k}) \sum_{i\neq 1}^N \frac{\partial E_{1i,k}}{\partial \ln p_{n,j}} = x_1\beta_{1,j}\pi_{1n,j} - \sum_{k=1}^J \frac{\partial E_{11,k}}{\partial \ln p_{n,j}} \\
\Rightarrow &\sum_{k=1}^J (1+\tau^m_{n,k}) \left[I_{k=j}\left[\kappa Y_{n,k}\right] - (\kappa-1)\lambda_{n,k}Y_{n,j} - \beta_{n,k}(1-\pi_{n1,k})Y_{n,j} + I_{k=j}\left[\sum_{i\neq 1}^N \beta_{i,k}x_i\epsilon_k\pi_{i1,k}\pi_{in,k}\right]\right] \\
&= x_1\beta_{1,j}\pi_{1n,j} + \sum_{k=1}^J \left[I_{k=j}\left[\sum_{i\neq 1}^N \beta_{i,k}\frac{x_i}{1+\tau^x_{i,k}}\epsilon_k\pi_{i1,k}\pi_{in,k}\right] + \beta_{n,k}\frac{\pi_{n1,k}}{1+\tau^x_{n,k}}Y_{n,j}\right] \\
\Rightarrow &(1+\tau^m_{n,j})\kappa Y_{n,j} - \sum_{k=1}^J (1+\tau^m_{n,k})(\kappa-1)\lambda_{n,k}Y_{n,j} + (1+\tau^m_{n,j})\sum_{i\neq 1}^N \beta_{i,j}x_i\epsilon_j\pi_{i1,j}\pi_{in,j} - \sum_{k=1}^J (1+\tau^m_{n,k})\beta_{n,k}(1-\pi_{n1,k})Y_{n,j} \\
&= x_1\beta_{1,j}\pi_{1n,j} + \sum_{i\neq 1}^N \beta_{i,j}\frac{x_i}{1+\tau^x_{i,j}}\epsilon_j\pi_{i1,j}\pi_{in,j} + \sum_{k=1}^J \beta_{n,k}\frac{\pi_{n1,k}}{1+\tau^x_{n,k}}Y_{n,j} \\
\Rightarrow &(1+\tau^m_{n,j})\kappa - (\kappa-1)\sum_{k=1}^J (1+\tau^m_{n,k})\lambda_{n,k} + cons_n = \frac{\pi_{1n,j}\beta_{1,j}x_1}{Y_{n,j}} + \left(\frac{1}{1+\tau^x_{n,j}} - (1+\tau^m_{n,j})\right)\sum_{i\neq 1}^N \beta_{i,j}\epsilon_j\pi_{i1,j}\pi_{in,j}\frac{x_i}{Y_{n,j}} \\
\Rightarrow &(1+\tau^m_{n,j})\kappa - (\kappa-1)\sum_{k=1}^J (1+\tau^m_{n,k})\lambda_{n,k} + cons_n = \frac{\pi_{1n,j}\beta_{1,j}x_1}{Y_{n,j}} + \frac{1}{1+\tau^x_{n,j}}\frac{1}{1-\pi_{n1,j}}\sum_{i\neq 1}^N \beta_{i,j}\pi_{i1,j}\pi_{in,j}\frac{x_i}{Y_{n,j}} \\
\Rightarrow &(1+\tau^m_{n,j})\kappa - (\kappa-1)\sum_{k=1}^J (1+\tau^m_{n,k})\lambda_{n,k} + cons_n = \frac{\pi_{1n,j}\beta_{1,j}x_1}{Y_{n,j}} - \frac{1}{1+\tau^x_{n,j}}\frac{1}{1-\pi_{n1,j}}\pi_{n1,j}\beta_{n,j}\frac{x_n}{Y_{n,j}}(1-\pi_{n1,j}) \\
\Rightarrow &(1+\tau^m_{n,j})\kappa - (\kappa-1)\sum_{k=1}^J (1+\tau^m_{n,k})\lambda_{n,k} + cons_n = \frac{\pi_{1n,j}\beta_{1,j}x_1 - \frac{1}{1+\tau^x_{n,j}}\pi_{n1,j}\beta_{n,j}x_n}{Y_{n,j}},
\end{aligned}
\tag{A.129}
$$

$$
\text{or } \sum_{k=1}^J (cons_n + 1 + \tau^m_{n,j})\frac{\partial \ln Y_{n,k}}{\partial \ln p_{n,j}}\frac{Y_{n,k}}{Y_{n,j}} = \frac{\pi_{1n,j}\beta_{1,j}x_1 - \frac{1}{1+\tau^x_{n,j}}\pi_{n1,j}\beta_{n,j}x_n}{Y_{n,j}},
\tag{A.130}
$$

where $cons_n = -\sum_{k=1}^J (1+\tau^m_{n,k})\beta_{n,k}(1-\pi_{n1,k}) - \sum_{k=1}^J \beta_{n,k}\frac{\pi_{n1,k}}{1+\tau^x_{n,k}}$. Equation (A.130) is identical to

(A.21) in the two-country case. Thus, we take the difference of (A.130) between sectors j and k and derive the formula for optimal tariffs

$$\tau^m_{n,j} - \tau^m_{n,k} = \frac{1}{\kappa}\left[\frac{\pi_{1n,j}\beta_{1,j}x_1 - \frac{1}{1+\tau^x_{n,j}}\pi_{n1,j}\beta_{n,j}x_n}{Y_{n,j}} - \frac{\pi_{1n,k}\beta_{1,k}x_1 - \frac{1}{1+\tau^x_{n,k}}\pi_{n1,k}\beta_{n,k}x_n}{Y_{n,k}}\right]. \tag{A.131}$$

F.3 No trade between foreign countries

When foreign countries do not trade with each other, the trade shares between any two foreign countries are zero, i.e., $\pi_{ni,j} = 0$ and $\pi_{nn,j} = 1 - \pi_{n1,j}, \forall n, i \neq 1, \forall j$. The optimal export tax in (A.122) can be written as

$$\begin{aligned}
\frac{1}{1+\tau^x_{n,j}} &= \frac{(1+\tau^m_{n,j})\epsilon_j\pi_{nn,j}}{1+\epsilon_j(1-\pi_{n1,j})} \\
\Rightarrow 1+\tau^x_{n,j} &= \frac{1}{1+\tau^m_{n,j}}\left(1+\frac{1}{\epsilon_j(1-\pi_{n1,j})}\right).
\end{aligned} \tag{A.132}$$

The diagonal blocks of $d\boldsymbol{E}$ are

$$\frac{\partial \boldsymbol{E_{1n}}}{\partial \ln \boldsymbol{p_n}} \circ \frac{1}{\boldsymbol{Y_n}} = \begin{pmatrix} \kappa-(\kappa-1)\lambda_{n,1}+\epsilon_1\pi_{n1,1}s_{nn,1}-\beta_{n,1}\pi_{nn,1} & \dots & -(\kappa-1)\lambda_{n,J}-\beta_{n,J}\pi_{nn,J} \\ \vdots & & \vdots \\ -(\kappa-1)\lambda_{n,1}-\beta_{n,1}\pi_{nn,1} & \dots & \kappa-(\kappa-1)\lambda_{n,J}+\epsilon_J\pi_{n1,J}s_{nn,J}-\beta_{n,J}\pi_{nn,J} \end{pmatrix}.$$

The off-diagonal blocks of $d\boldsymbol{E}$ are

$$\frac{\partial \boldsymbol{E_{1i}}}{\partial \ln \boldsymbol{p_n}} \circ \frac{1}{\boldsymbol{Y_n}} = \begin{pmatrix} 0 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \end{pmatrix}.$$

The RHS of equation (A.126) for country n is

$$\boldsymbol{\beta_1} x_1 \circ \boldsymbol{\pi_{1n}} \circ \frac{1}{\boldsymbol{Y_n}} - \sum_{k=1}^{J}\frac{\partial E_{11,k}}{\partial \ln \boldsymbol{p_n}} \circ \frac{1}{\boldsymbol{Y_n}} = \begin{pmatrix} \beta_{1,1}x_1\pi_{1n,1}\frac{1}{Y_{n,1}} + \beta_{n,1}\frac{x_n}{1+\tau^x_{n,1}}\epsilon_1\pi_{n1,1}\pi_{nn,1}\frac{1}{Y_{n,1}} + \sum_{k=1}^{J}\beta_{n,k}\frac{\pi_{n1,k}}{1+\tau^x_{n,k}} \\ \dots \\ \beta_{1,J}x_1\pi_{1n,J}\frac{1}{Y_{n,J}} + \beta_{n,J}\frac{x_n}{1+\tau^x_{n,J}}\epsilon_J\pi_{n1,J}\pi_{nn,J}\frac{1}{Y_{n,J}} + \sum_{k=1}^{J}\beta_{n,k}\frac{\pi_{n1,k}}{1+\tau^x_{n,k}} \end{pmatrix}.$$

Thus, we can derive the optimal tariff in country n sector j

$$
\begin{aligned}
&\sum_{k=1}^{J}(1+\tau^m_{n,k})\frac{\partial E_{1n,k}}{\partial \ln p_{n,j}} = x_1\beta_{1,j}\pi_{1n,j} - \sum_{k=1}^{J}\frac{\partial E_{11,k}}{\partial \ln p_{n,j}} \\
\Rightarrow &(1+\tau^m_{n,j})\kappa Y_{n,j} - (\kappa-1)\sum_{k=1}^{J}(1+\tau^m_{n,k})\lambda_{n,k}Y_{n,j} - \sum_{k=1}^{J}(1+\tau^m_{n,k})\beta_{n,k}\pi_{nn,k}Y_{n,j} + (1+\tau^m_{n,j})\beta_{n,j}x_n\epsilon_j\pi_{n1,j}\pi_{nn,j} \\
&= x_1\beta_{1,j}\pi_{1n,j} + \beta_{n,j}\frac{x_n}{1+\tau^x_{n,j}}\epsilon_j\pi_{n1,j}\pi_{nn,j} + \sum_{k=1}^{J}\beta_{n,k}\frac{\pi_{n1,k}}{1+\tau^x_{n,k}}Y_{n,j} \\
\Rightarrow &(1+\tau^m_{n,j})\kappa - (\kappa-1)\sum_{k=1}^{J}(1+\tau^m_{n,k})\lambda_{n,k} + cons_n = \frac{\pi_{1n,j}\beta_{1,j}x_1}{Y_{n,j}} + \beta_{n,j}\left(\frac{1}{1+\tau^x_{n,j}} - (1+\tau^m_{n,j})\right)\epsilon_j\pi_{n1,j}\pi_{nn,j}\frac{x_n}{Y_{n,j}} \\
\Rightarrow &(1+\tau^m_{n,j})\kappa - (\kappa-1)\sum_{k=1}^{J}(1+\tau^m_{n,k})\lambda_{n,k} + cons_n = \frac{\pi_{1n,j}\beta_{1,j}x_1 - \frac{1}{1+\tau^x_{n,j}}\pi_{n1,j}\beta_{n,j}x_n}{Y_{n,j}}, \qquad \text{(A.133)}
\end{aligned}
$$

$$
\text{or } \sum_{k=1}^{J}(cons_n + 1 + \tau^m_{n,j})\frac{\partial \ln Y_{n,k}}{\partial \ln p_{n,j}}\frac{Y_{n,k}}{Y_{n,j}} = \frac{\pi_{1n,j}\beta_{1,j}x_1 - \frac{1}{1+\tau^x_{n,j}}\pi_{n1,j}\beta_{n,j}x_n}{Y_{n,j}}, \qquad \text{(A.134)}
$$

where $cons_n = -\sum_{k=1}^{J}(1+\tau^m_{n,k})\beta_{n,k}(1-\pi_{n1,k}) - \sum_{k=1}^{J}\beta_{n,k}\frac{\pi_{n1,k}}{1+\tau^x_{n,k}}$. Once again, equation (A.134) is identical to (A.21) in the two-country case. Thus, we take the difference of (A.134) between sectors j and k and derive the formula for optimal tariffs

$$
\tau^m_{n,j} - \tau^m_{n,k} = \frac{1}{\kappa}\left[\frac{\pi_{1n,j}\beta_{1,j}x_1 - \frac{1}{1+\tau^x_{n,j}}\pi_{n1,j}\beta_{n,j}x_n}{Y_{n,j}} - \frac{\pi_{1n,k}\beta_{1,k}x_1 - \frac{1}{1+\tau^x_{n,k}}\pi_{n1,k}\beta_{n,k}x_n}{Y_{n,k}}\right]. \qquad \text{(A.135)}
$$

□

G Nash Optimal Policies

In this section, all countries are allowed to impose policies on other countries. We derive a country's optimal policies given the policies of other countries.

As proven in Appendix F, under the multi-country setup that satisfies the CES supply system with constant returns to scale, a country implementing optimal policies will not impose any domestic taxes. Therefore, domestic taxes will not be included in the setup in this section.

We consider Nash equilibrium where all countries implement optimal policies given other countries policies. Country i's policy is defined as $\Gamma = \{1+\tau^x_{ni,j}, 1+\tau^m_{in,j}\}$, where $\tau^x_{ni,j}$ denotes the export tax imposed by country i on country n in sector j, and $\tau^m_{in,j}$ represents the import tariff imposed by country i on country n in sector j. Given the policies of other countries, country i's government chooses $\{\tau^x_{ni,j}, \tau^m_{in,j}\}$ to maximize domestic consumers' consumption, $\max x_i/P_i$, subject to world market equilibrium and the policies of other countries. The world converges to a Nash equilibrium, where each country adopts its optimal policy given the policies of other countries.

Take country 1 as an example. Country 1's government solves the following problem:

$$\max_{\{\tau^x_{n1,j},\tau^m_{1n,j},x_1,x_n,p_{1,j},p_{n,j}\}} \frac{x_1}{P_1},$$

subject to world market equilibrium characterized by the following constraints:

$$Y_{1,j} - \sum_{i\neq 1}^{N} \beta_{i,j} \frac{1}{1+\tau^x_{i1,j}} \frac{1}{1+\tau^m_{i1,j}} \pi_{i1,j} x_i = \beta_{1,j} \pi_{11,j} x_1, \quad (\gamma_{1,j}) \tag{A.136}$$

$$Y_{n,j} - \sum_{i\neq 1}^{N} \beta_{i,j} \frac{1}{1+\tau^m_{in,j}} \frac{1}{1+\tau^x_{in,j}} \pi_{in,j} x_i = \beta_{1,j} \frac{1}{1+\tau^m_{1n,j}} \frac{1}{1+\tau^x_{1n,j}} \pi_{1n,j} x_1, \quad (\gamma_{n,j}) \tag{A.137}$$

$$x_1 = \sum_{j=1}^{J} Y_{1,j} + \sum_{i\neq 1}^{N} \sum_{j=1}^{J} \left[\beta_{i,j} \frac{\tau^x_{i1,j}}{1+\tau^x_{i1,j}} \frac{1}{1+\tau^m_{i1,j}} \pi_{i1,j} x_i + \beta_{1,j} \frac{\tau^m_{1i,j}}{1+\tau^m_{1i,j}} \pi_{1i,j} x_1 \right], \quad (\gamma_{x1}) \tag{A.138}$$

$$x_n = \sum_{j=1}^{J} Y_{n,j} + \sum_{i\neq n}^{N} \sum_{j=1}^{J} \left[\beta_{i,j} \frac{\tau^x_{in,j}}{1+\tau^x_{in,j}} \frac{1}{1+\tau^m_{in,j}} \pi_{in,j} x_i + \beta_{n,j} \frac{\tau^m_{ni,j}}{1+\tau^m_{ni,j}} \pi_{ni,j} x_n \right], \quad (\gamma_{xn}) \tag{A.139}$$

and the labor market specifications

$$L_{1,j} = A_{1,j} \left(\frac{w_{1,j}}{W_1}\right)^{\kappa-1} \bar{L}_1,$$
$$L_{n,j} = A_{n,j} \left(\frac{w_{n,j}}{W_n}\right)^{\kappa-1} \bar{L}_n,$$

where $\{P_1, P_n\}$ and trade shares are given by

$$P_1 = \prod_{j=1}^{J}\left[p_{1,j}^{-\epsilon_j} + \sum_{i\neq 1}^{N}(p_{i,j}(1+\tau^x_{1i,j})(1+\tau^m_{1i,j})d_{1i})^{-\epsilon_j}\right]^{-\frac{\beta_{1,j}}{\epsilon_j}},$$

$$P_n = \prod_{j=1}^{J}\left[(p_{1,j}(1+\tau^x_{n1,j})(1+\tau^m_{n1,j})d_{n1})^{-\epsilon_j} + \sum_{i\neq 1}^{N}(p_{i,j}(1+\tau^x_{ni,j})(1+\tau^m_{ni,j})d_{ni})^{-\epsilon_j}\right]^{-\frac{\beta_{n,j}}{\epsilon_j}},$$

$$\pi_{11,j} = \frac{(p_{1,j})^{-\epsilon_j}}{(p_{1,j})^{-\epsilon_j} + \sum_{i\neq 1}^{N}(p_{i,j}(1+\tau^m_{1i,j})(1+\tau^x_{1i,j})d_{1i})^{-\epsilon_j}},$$

$$\pi_{1n,j} = \frac{(p_{n,j}(1+\tau^m_{1n,j})(1+\tau^x_{1n,j})d_{1n})^{-\epsilon_j}}{(p_{1,j})^{-\epsilon_j} + \sum_{i\neq 1}^{N}(p_{i,j}(1+\tau^m_{1i,j})(1+\tau^x_{1i,j})d_{1i})^{-\epsilon_j}},$$

$$\pi_{n1,j} = \frac{(p_{1,j}(1+\tau^x_{n1,j})(1+\tau^m_{n1,j})d_{n1})^{-\epsilon_j}}{(p_{1,j}(1+\tau^x_{n1,j})(1+\tau^m_{n1,j})d_{n1})^{-\epsilon_j} + \sum_{i\neq 1}^{N}(p_{i,j}(1+\tau^x_{ni,j})(1+\tau^m_{ni,j})d_{ni})^{-\epsilon_j}},$$

$$\pi_{nm,j} = \frac{(p_{mj}(1+\tau^x_{nm,j})(1+\tau^m_{nm,j})d_{nm})^{-\epsilon_j}}{(p_{1,j}(1+\tau^x_{n1,j})(1+\tau^m_{n1,j})d_{n1})^{-\epsilon_j} + \sum_{i\neq 1}^{N}(p_{i,j}(1+\tau^x_{ni,j})(1+\tau^m_{ni,j})d_{ni})^{-\epsilon_j}}.$$

According to the Walras' law, one of the equilibrium conditions in equations (A.136)-(A.139) is redundant; thus, we drop country 1's expenditure constraint in optimization and set $\gamma_{x1} = 0$.

Without loss of generality, we normalize country 1's final goods price $P_1 = 1$. We define $E_{11,j} = Y_{1,j} - \sum_{i\neq 1}^{N}\beta_{i,j}\frac{1}{1+\tau^x_{i1,j}}\frac{1}{1+\tau^m_{i1,j}}\pi_{i1,j}x_i$ as the supply of country 1 on its domestic market, and $E_{1n,j} = Y_{n,j} - \sum_{i\neq 1}^{N}\beta_{i,j}\frac{1}{1+\tau^x_{in,j}}\frac{1}{1+\tau^m_{in,j}}\pi_{in,j}x_i$ as the export supply of country i on country 1's market.

We apply the Lagrange multiplier method to derive the formula for each country's optimal policies. The following proposition presents the optimal policies for country 1, given the policies of other countries.

Proposition 7 (Nash Optimal Policies and Multipliers). *Given the policies of countries $n \neq 1$, the optimal policy of country 1's government satisfies, for any country $n \neq 1$ and sector $\forall j$,*

$$\textit{Import tariff: } 1+\tau^m_{1n,j} = \frac{(\gamma_{xn}-\gamma_{n,j})\frac{1}{1+\tau^x_{1n,j}} - \gamma_{xn}}{1/P_1}, \tag{A.140}$$

Export tax:

$$\frac{1}{1+\tau^x_{n1,j}} = \frac{\sum_{i\neq 1,n}^{N}\frac{\gamma_{xi}-\gamma_{i,j}}{1+\tau^x_{ni,j}}\frac{1}{1+\tau^m_{ni,j}}\epsilon_j\pi_{ni,j} + (\gamma_{xn}-\gamma_{n,j})\epsilon_j\pi_{nn,j} + \sum_{i\neq 1,n}^{N}\frac{\gamma_{xn}-\gamma_{xi}}{1+\tau^m_{ni,j}}\epsilon_j\pi_{ni,j} - \gamma_{xn}\frac{1}{1+\tau^m_{n1,j}}\epsilon_j(1-\pi_{n1,j})}{\frac{1}{P_1}\frac{1}{1+\tau^m_{n1,j}}(1+\epsilon_j(1-\pi_{n1,j}))}, \tag{A.141}$$

where $\gamma_{n,j}$ is the multiplier on the goods market clearing condition of country n, and γ_{xn} is the multiplier on *country n's expenditure. These multipliers satisfy the FOCs with respect to prices and FOCs with respect to expenditures.*

Tax neutrality The proof of tax neutrality for the country i follows a similar approach to that presented in Proposition 1. When a country implements its optimal policies, it can normalize a tariff it imposes on a specific country and sector to zero, given the policies of other countries. Since there are N countries in total and each country can set one tariff to zero while implementing its optimal policies, a total of N tariffs can be normalized to zero.

Country 1's optimal conditions

FOC over x_1

$$\frac{1}{P_1}+\sum_{j=1}^{J}\gamma_{1,j}\beta_{1,j}\pi_{11,j}+\sum_{i\neq 1}^{N}\sum_{j=1}^{J}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau^m_{1i,j}}\frac{1}{1+\tau^x_{1i,j}}\pi_{1i,j}+\sum_{i\neq 1}^{N}\gamma_{xi}\sum_{j=1}^{J}\beta_{1,j}\frac{\tau^x_{1i,j}}{1+\tau^x_{1i,j}}\frac{1}{1+\tau^m_{1i,j}}\pi_{1i,j}=0. \tag{A.142}$$

FOC over x_n, $n \neq 1$

$$-\gamma_{xn}+\sum_{j=1}^{J}\gamma_{n,j}\beta_{n,j}\pi_{nn,j}+\sum_{i\neq n}^{N}\sum_{j=1}^{J}\gamma_{i,j}\beta_{n,j}\frac{1}{1+\tau^m_{ni,j}}\frac{1}{1+\tau^x_{ni,j}}\pi_{ni,j}+\sum_{i\neq n}^{N}\gamma_{xi}\sum_{j=1}^{J}\beta_{n,j}\frac{\tau^x_{ni,j}}{1+\tau^x_{ni,j}}\frac{1}{1+\tau^m_{ni,j}}\pi_{ni,j}=0. \tag{A.143}$$

FOC over import tariff $\tau^m_{1n,j}$, $n \neq 1$

$$\begin{aligned}
&-\frac{1}{P_1}\beta_{1,j}x_1\frac{\pi_{1n,j}}{1+\tau^m_{1n,j}}+\gamma_{1,j}\beta_{1,j}\frac{\partial\pi_{11,j}}{\partial\tau^m_{1n,j}}x_1-\gamma_{n,j}\beta_{1,j}\frac{\pi_{1n,j}}{(1+\tau^m_{1n,j})^2}\frac{1}{1+\tau^x_{1n,j}}x_1+\sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau^m_{1i,j}}\frac{1}{1+\tau^x_{1i,j}}\frac{\partial\pi_{1i,j}}{\partial\tau^m_{1n,j}}x_1\\
&-\gamma_{xn}\beta_{1,j}\frac{\tau^x_{1n,j}}{1+\tau^x_{1n,j}}\frac{1}{(1+\tau^m_{1n,j})^2}\pi_{1n,j}x_1+\sum_{i\neq 1}\gamma_{xi}\beta_{1,j}\frac{\tau^x_{1i,j}}{1+\tau^x_{1i,j}}\frac{1}{1+\tau^m_{1i,j}}\frac{\partial\pi_{1i,j}}{\partial\tau^m_{1n,j}}x_1=0.
\end{aligned}$$

Plugging into derivatives

$$
\begin{aligned}
&-\frac{1}{P_1}\beta_{1,j}x_1\pi_{1n,j}+\gamma_{1,j}\beta_{1,j}\epsilon_j\pi_{11,j}\pi_{1n,j}x_1-(\gamma_{n,j}-\gamma_{xn})\beta_{1,j}\frac{\pi_{1n,j}}{1+\tau^m_{1n,j}}\frac{1+\epsilon_j}{1+\tau^x_{1n,j}}x_1+\sum_{i\neq 1}^{N}(\gamma_{i,j}-\gamma_{xi})\frac{\beta_{1,j}\pi_{1n,j}}{1+\tau^m_{1i,j}}\frac{\epsilon_j\pi_{1i,j}x_1}{1+\tau^x_{1i,j}}\\
&-\gamma_{xn}\beta_{1,j}\frac{1}{1+\tau^m_{1n,j}}\pi_{1n,j}x_1+\sum_{i\neq 1}\gamma_{xi}\beta_{1,j}\frac{1}{1+\tau^m_{1i,j}}\epsilon_j\pi_{1i,j}\pi_{1n,j}x_1-\gamma_{xn}\beta_{1,j}\frac{1}{1+\tau^m_{1n,j}}\epsilon_j\pi_{1n,j}x_1=0 \quad \text{(A.144)}\\
\Rightarrow &-\frac{1}{P_1}+\gamma_{1,j}\epsilon_j\pi_{11,j}-\frac{\gamma_{n,j}-\gamma_{xn}}{1+\tau^m_{1n,j}}\frac{1+\epsilon_j}{1+\tau^x_{1n,j}}+\sum_{i\neq 1}^{N}\frac{\gamma_{i,j}-\gamma_{xi}}{1+\tau^m_{1i,j}}\frac{\epsilon_j\pi_{1i,j}}{1+\tau^x_{1i,j}}-\gamma_{xn}\frac{1+\epsilon_j}{1+\tau^m_{1n,j}}+\sum_{i\neq 1}\gamma_{xi}\frac{\pi_{1i,j}}{1+\tau^m_{1i,j}}\epsilon_j=0.
\end{aligned}
$$

Thus, $(\gamma_{n,j}-\gamma_{xn})\frac{1}{1+\tau^m_{1n,j}}\frac{1}{1+\tau^x_{1n,j}}+\gamma_{xn}\frac{1}{1+\tau^m_{1n,j}}$ is the same across all countries in sector j. Replacing each $(\gamma_{i,j}-\gamma_{xi})\frac{1}{1+\tau^m_{1i,j}}\frac{1}{1+\tau^x_{1i,j}}+\gamma_{xi}\frac{1}{1+\tau^m_{1i,j}}$, we derive

$$
\begin{aligned}
&-\frac{1}{P_1}+\gamma_{1,j}\epsilon_j\pi_{11,j}-\frac{\gamma_{n,j}-\gamma_{xn}}{1+\tau^m_{1n,j}}\frac{1+\epsilon_j}{1+\tau^x_{1n,j}}-\gamma_{xn}\frac{1+\epsilon_j}{1+\tau^m_{1n,j}}+\frac{\gamma_{n,j}-\gamma_{xn}}{1+\tau^m_{1n,j}}\frac{\epsilon_j(1-\pi_{11,j})}{1+\tau^x_{1n,j}}+\gamma_{xn}\frac{\epsilon_j}{1+\tau^m_{1n,j}}(1-\pi_{11,j})=0\\
\Rightarrow &-\frac{1}{P_1}+\gamma_{1,j}\epsilon_j\pi_{11,j}-(\gamma_{n,j}-\gamma_{xn})\frac{1+\epsilon_j\pi_{11,j}}{1+\tau^m_{1n,j}}\frac{1}{1+\tau^x_{1n,j}}-\gamma_{xn}\frac{1+\epsilon_j\pi_{11,j}}{1+\tau^m_{1n,j}}=0.
\end{aligned}
$$

Combining FOCs over x_1 and $\tau^m_{1n,j}$, we derive

$$
\frac{1}{P_1}+\sum_{j=1}^{J}((\gamma_{n,j}-\gamma_{xn})\frac{1}{1+\tau^x_{1n,j}}+\gamma_{xn})\beta_{1,j}\frac{1}{1+\tau^m_{1n,j}}=0,
$$

or

$$
\sum_{j=1}^{J}\beta_{1,j}\frac{(\gamma_{1,j}+\frac{1}{P_1})\epsilon_j\pi_{11,j}}{1+\epsilon_j\pi_{11,j}}=0. \quad \text{(A.145)}
$$

FOC over export tax $\tau^x_{n1,j}$, $n\neq 1$

$$
\begin{aligned}
&-\gamma_{1,j}\beta_{n,j}\frac{1}{(1+\tau^x_{n1,j})^2}\frac{1}{1+\tau^m_{n1,j}}\pi_{n1,j}x_n+\gamma_{1,j}\beta_{n,j}\frac{1}{1+\tau^x_{n1,j}}\frac{1}{1+\tau^m_{n1,j}}\frac{\partial\pi_{n1,j}}{\partial\tau^x_{n1,j}}x_n+\sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{n,j}\frac{1}{1+\tau^x_{ni,j}}\frac{1}{1+\tau^m_{ni,j}}\frac{\partial\pi_{ni,j}}{\partial\tau^x_{n1,j}}x_n\\
&+\sum_{i\neq 1,n}^{N}\gamma_{xi}\beta_{n,j}\frac{\tau^x_{ni,j}}{1+\tau^x_{ni,j}}\frac{1}{1+\tau^m_{ni,j}}\frac{\partial\pi_{ni,j}}{\partial\tau^x_{n1,j}}x_n+\gamma_{xn}\sum_{i\neq n}^{N}\beta_{n,j}\frac{\tau^m_{ni,j}}{1+\tau^m_{ni,j}}\frac{\partial\pi_{ni,j}}{\partial\tau^x_{n1,j}}x_n=0.
\end{aligned}
$$

Plugging into derivatives

$$
\begin{aligned}
&-\gamma_{1,j}\beta_{n,j}\frac{1}{1+\tau^x_{n1,j}}\frac{1}{1+\tau^m_{n1,j}}\pi_{n1,j}(1+\epsilon_j(1-\pi_{n1,j}))x_n+\sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{n,j}\frac{1}{1+\tau^x_{ni,j}}\frac{1}{1+\tau^m_{ni,j}}\epsilon_j\pi_{ni,j}\pi_{n1,j}x_n\\
&+\sum_{i\neq 1,n}^{N}\gamma_{xi}\beta_{n,j}\frac{\tau^x_{ni,j}}{1+\tau^x_{ni,j}}\frac{1}{1+\tau^m_{ni,j}}\epsilon_j\pi_{ni,j}\pi_{n1,j}x_n+\gamma_{xn}\sum_{i\neq n}^{N}\beta_{n,j}\frac{\tau^m_{ni,j}}{1+\tau^m_{ni,j}}\epsilon_j\pi_{ni,j}\pi_{n1,j}x_n-\gamma_{xn}\beta_{n,j}\frac{\tau^m_{n1,j}}{1+\tau^m_{n1,j}}\epsilon_j\pi_{n1,j}x_n=0
\end{aligned}
\tag{A.146}
$$

$$
\begin{aligned}
\Rightarrow &-\gamma_{1,j}\frac{1}{1+\tau^x_{n1,j}}\frac{1}{1+\tau^m_{n1,j}}(1+\epsilon_j(1-\pi_{n1,j}))+\sum_{i\neq 1,n}^{N}(\gamma_{i,j}-\gamma_{xi})\frac{1}{1+\tau^x_{ni,j}}\frac{1}{1+\tau^m_{ni,j}}\epsilon_j\pi_{ni,j}+(\gamma_{n,j}-\gamma_{xn})\epsilon_j\pi_{nn,j}\\
&+\sum_{i\neq 1,n}^{N}(\gamma_{xi}-\gamma_{xn})\frac{1}{1+\tau^m_{ni,j}}\epsilon_j\pi_{ni,j}+\gamma_{xn}\frac{1}{1+\tau^m_{n1,j}}\epsilon_j(1-\pi_{n1,j})=0.
\end{aligned}
$$

FOC over $p_{1,j}$

$$
\begin{aligned}
&-\frac{1}{P_1}x_1\beta_{1,j}\frac{\pi_{11,j}}{p_{1,j}}+\gamma_{1,j}\beta_{1,j}\frac{\partial\pi_{11,j}}{\partial p_{1,j}}x_1-\sum_{k=1}^{J}\gamma_{1,k}\frac{\partial E_{11,k}}{\partial p_{1,j}}+\sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau^x_{1i,j}}\frac{1}{1+\tau^m_{1i,j}}\frac{\partial\pi_{1i,j}}{\partial p_{1,j}}x_1-\sum_{k=1}^{J}\sum_{i\neq 1}^{N}\gamma_{i,k}\frac{\partial E_{1i,k}}{\partial p_{1,j}}\\
&+\sum_{n\neq 1}^{N}\gamma_{xn}\sum_{i\neq n}^{N}\left[\beta_{i,j}\frac{\tau^x_{in,j}}{1+\tau^x_{in,j}}\frac{1}{1+\tau^m_{in,j}}\frac{\partial\pi_{in,j}}{\partial p_{1,j}}x_i+\beta_{n,j}\frac{\tau^m_{ni,j}}{1+\tau^m_{ni,j}}\frac{\partial\pi_{ni,j}}{\partial p_{1,j}}x_n\right]=0\\
\Rightarrow &-\frac{1}{P_1}x_1\beta_{1,j}\pi_{11,j}-\gamma_{1,j}\beta_{1,j}\epsilon_j\pi_{11,j}(1-\pi_{11,j})x_1-\sum_{k=1}^{J}\gamma_{1,k}\frac{\partial Y_{1,k}}{\partial p_{1,j}}p_{1,j}-\gamma_{1,j}\sum_{i\neq 1}^{N}\beta_{i,j}\frac{1}{1+\tau^x_{i1,j}}\frac{1}{1+\tau^m_{i1,j}}\epsilon_j\pi_{i1,j}(1-\pi_{i1,j})x_i\\
&+\sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau^m_{1i,j}}\frac{1}{1+\tau^x_{1i,j}}\epsilon_j\pi_{1i,j}\pi_{11,j}x_1+\sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{n,j}\sum_{n\neq 1}^{N}\frac{1}{1+\tau^m_{ni,j}}\frac{1}{1+\tau^x_{ni,j}}\epsilon_j\pi_{ni,j}\pi_{n1,j}x_n\\
&+\sum_{n\neq 1}^{N}\gamma_{xn}\sum_{i\neq n}^{N}\left[\beta_{i,j}\frac{\tau^x_{in,j}}{1+\tau^x_{in,j}}\frac{1}{1+\tau^m_{in,j}}\epsilon_j\pi_{in,j}\pi_{i1,j}x_i+\beta_{n,j}\frac{\tau^m_{ni,j}}{1+\tau^m_{ni,j}}\epsilon_j\pi_{ni,j}\pi_{n1,j}x_n\right]-\sum_{n\neq 1}^{N}\gamma_{xn}\beta_{n,j}\frac{\tau^m_{n1,j}}{1+\tau^m_{n1,j}}\epsilon_j\pi_{n1,j}x_n=0.
\end{aligned}
\tag{A.147}
$$

Combining (A.147) with (A.144) and (A.146),

$$
\begin{aligned}
&-\frac{1}{P_1}x_1\beta_{1,j}-\sum_{k=1}^{J}\gamma_{1,k}\frac{\partial Y_{1,k}}{\partial p_{1,j}}p_{1,j}+\gamma_{1,j}\sum_{n\neq 1}^{N}\frac{\beta_{n,j}}{1+\tau^{x}_{n1,j}}\frac{\pi_{n1,j}x_n}{1+\tau^{m}_{n1,j}}-\sum_{n\neq 1}^{N}\frac{\gamma_{n,j}\beta_{1,j}}{1+\tau^{m}_{1n,j}}\frac{\pi_{1n,j}x_1}{1+\tau^{x}_{1n,j}}\gamma_{xn}\beta_{1,j}\frac{\tau^{x}_{1n,j}}{1+\tau^{x}_{1n,j}}\frac{\pi_{1n,j}x_1}{1+\tau^{m}_{1n,j}}=0\\
\Rightarrow&-\frac{1}{P_1}x_1\beta_{1,j}-\sum_{k=1}^{J}\gamma_{1,k}\frac{\partial Y_{1,k}}{\partial p_{1,j}}p_{1,j}-\gamma_{1,j}\beta_{1,j}\pi_{11,j}x_1+\gamma_{1,j}Y_{1,j}-\left(\frac{\gamma_{n,j}}{1+\tau^{x}_{1n,j}}+\gamma_{xn}\frac{\tau^{x}_{1n,j}}{1+\tau^{x}_{1n,j}}\right)\frac{\beta_{1,j}}{1+\tau^{m}_{n,j}}(1-\pi_{11,j})x_1=0\\
\Rightarrow&-\frac{1}{P_1}x_1\beta_{1,j}-\sum_{k=1}^{J}\gamma_{1,k}\frac{\partial L_{1,k}}{\partial \ln p_{1,j}}-\gamma_{1,j}\beta_{1,j}\pi_{11,j}x_1-\beta_{1,j}\frac{-\frac{1}{P_1}+\gamma_{1,j}\epsilon_j\pi_{11,j}}{1+\epsilon_j\pi_{11,j}}(1-\pi_{11,j})x_1=0\\
\Rightarrow&-\sum_{k=1}^{J}\gamma_{1,k}\frac{\partial L_{1,k}}{\partial \ln p_{1,j}}-\beta_{1,j}\frac{\frac{1}{P_1}+\gamma_{1,j}}{1+\epsilon_j\pi_{11,j}}(1+\epsilon_j)x_1=0\\
\Rightarrow&-\gamma_{1,j}(\kappa-1)Y_{1,j}+\sum_{k=1}^{J}\gamma_{1,k}(\kappa-1)\frac{Y_{1,k}}{W_1\bar{L}_1}Y_{1,j}-\beta_{1,j}\frac{\frac{1}{P_1}+\gamma_{1,j}}{1+\epsilon_j\pi_{11,j}}(1+\epsilon_j)x_1=0\\
\Rightarrow&\sum_{j=1}^{J}\beta_{1,j}\frac{\frac{1}{P_1}+\gamma_{1,j}}{1+\epsilon_j\pi_{11,j}}(1+\epsilon_j)x_1=0. \qquad \text{(A.148)}
\end{aligned}
$$

Using (A.145), we solve for $\gamma_{1,j}=-\frac{1}{P_1}$. Thus, the optimal import tariff is

$$
\begin{aligned}
&-\frac{1}{P_1}-(\gamma_{n,j}-\gamma_{xn})\frac{1}{1+\tau^{m}_{1n,j}}\frac{1}{1+\tau^{x}_{1n,j}}-\gamma_{xn}\frac{1}{1+\tau^{m}_{1n,j}}=0\\
\Rightarrow&1+\tau^{m}_{1n,j}=\frac{(\gamma_{xn}-\gamma_{n,j})\frac{1}{1+\tau^{x}_{1n,j}}-\gamma_{xn}}{1/P_1}. \qquad \text{(A.149)}
\end{aligned}
$$

Plugging (A.149) into (A.146), the optimal export tax is

$$
\frac{1}{1+\tau^{x}_{n1,j}}=\frac{\sum_{i\neq 1,n}^{N}\frac{\gamma_{xi}-\gamma_{i,j}}{1+\tau^{x}_{ni,j}}\frac{1}{1+\tau^{m}_{ni,j}}\epsilon_j\pi_{ni,j}+(\gamma_{xn}-\gamma_{n,j})\epsilon_j\pi_{nn,j}+\sum_{i\neq 1,n}^{N}\frac{\gamma_{xn}-\gamma_{xi}}{1+\tau^{m}_{ni,j}}\epsilon_j\pi_{ni,j}-\gamma_{xn}\frac{1}{1+\tau^{m}_{n1,j}}\epsilon_j(1-\pi_{n1,j})}{\frac{1}{P_1}\frac{1}{1+\tau^{m}_{n1,j}}(1+\epsilon_j(1-\pi_{n1,j}))}. \qquad \text{(A.150)}
$$

where $\gamma_{n,j}$ reflects how changes in country n's goods demand impact country 1's utility and γ_{xn} reflects how changes in country n's expenditure affect country 1's utility. When $\gamma_{xn}=0$ for $n\neq 1$, and other countries do not impose policies, (A.149) and (A.150) are equivalent to (A.120) and (A.122) under country 1's unilateral policies.

FOC over $p_{n,j}$, $n \neq 1$

$$
\begin{aligned}
& -\frac{1}{P_1}x_1\beta_{1,j}\frac{\pi_{1n,j}}{p_{n,j}} - \frac{1}{P_1}\beta_{1,j}\frac{\partial \pi_{11,j}}{\partial p_{n,j}}x_1 + \sum_{k=1}^{J}\frac{\partial E_{11,k}}{\partial p_{n,j}} + \sum_{i\neq 1}^{N}\gamma_i\beta_{1,j}\frac{1}{1+\tau^m_{1i,j}}\frac{1}{1+\tau^x_{1i,j}}\frac{\partial \pi_{1i,j}}{\partial p_{n,j}}x_1 - \sum_{k=1}^{J}\sum_{i\neq 1}^{N}\gamma_{i,k}\frac{\partial E_{1i,k}}{\partial p_{n,j}} \\
& + \gamma_{xn}\sum_{k=1}^{J}\frac{\partial Y_{n,k}}{\partial p_{n,j}}p_{n,j} + \sum_{h\neq 1}^{N}\gamma_{xh}\sum_{i\neq h}^{N}\left[\beta_{i,j}\frac{\tau^x_{ih,j}}{1+\tau^x_{ih,j}}\frac{1}{1+\tau^m_{ih,j}}\frac{\partial \pi_{ih,j}}{\partial p_{n,j}}x_i + \beta_{h,j}\frac{\tau^m_{hi,j}}{1+\tau^m_{hi,j}}\frac{\partial \pi_{hi,j}}{\partial p_{n,j}}x_h\right] = 0 \\
\Rightarrow & -\frac{1}{P_1}x_1\beta_{1,j}\pi_{1n,j} - \frac{1}{P_1}\beta_{1,j}\epsilon_j\pi_{11,j}\pi_{1n,j}x_1 + \sum_{k=1}^{J}(\gamma_{xn}-\gamma_{n,k})\frac{\partial Y_{n,k}}{\partial p_{n,j}}p_{n,j} + \gamma_{1,j}\sum_{i\neq 1}^{N}\beta_{i,j}\frac{1}{1+\tau^x_{i1,j}}\frac{1}{1+\tau^m_{i1,j}}\epsilon_j\pi_{i1,j}\pi_{in,j}x_i \\
& + \sum_{i\neq 1}^{N}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau^m_{1i,j}}\frac{1}{1+\tau^x_{1i,j}}\epsilon_j\pi_{1i,j}\pi_{1n,j}x_1 - \gamma_{n,j}\beta_{1,j}\frac{1}{1+\tau^m_{1n,j}}\frac{1}{1+\tau^x_{1n,j}}\epsilon_j\pi_{1n,j}x_1 + \sum_{i\neq 1}^{N}\gamma_{i,j}\sum_{h\neq 1}^{N}\frac{\beta_{h,j}x_h}{1+\tau^m_{hi,j}}\frac{\epsilon_j\pi_{hi,j}\pi_{hn,j}}{1+\tau^x_{hi,j}} \\
& - \gamma_{n,j}\sum_{h\neq 1}^{N}\beta_{h,j}\frac{1}{1+\tau^m_{hn,j}}\frac{1}{1+\tau^x_{hn,j}}\epsilon_j\pi_{hn,j}x_h + \sum_{h\neq 1}^{N}\gamma_{xh}\sum_{i\neq h}^{N}\beta_{i,j}\frac{\tau^x_{ih,j}}{1+\tau^x_{ih,j}}\frac{1}{1+\tau^m_{ih,j}}\epsilon_j\pi_{ih,j}\pi_{in,j}x_i \\
& + \sum_{h\neq 1}^{N}\gamma_{xh}\sum_{i\neq h}^{N}\beta_{h,j}\frac{\tau^m_{hi,j}}{1+\tau^m_{hi,j}}\epsilon_j\pi_{hi,j}\pi_{hn,j}x_h - \gamma_{xn}\sum_{i\neq n}^{N}\beta_{i,j}\frac{\tau^x_{in,j}}{1+\tau^x_{in,j}}\frac{1}{1+\tau^m_{in,j}}\epsilon_j\pi_{in,j}x_i - \sum_{h\neq 1}^{N}\gamma_{xh}\beta_{h,j}\frac{\tau^m_{hn,j}}{1+\tau^m_{hn,j}}\epsilon_j\pi_{hn,j}x_h = 0 \\
\Rightarrow & -\frac{1}{P_1}x_1\beta_{1,j}\pi_{1n,j} + \sum_{k=1}^{J}(\gamma_{xn}-\gamma_{n,k})\frac{\partial Y_{n,k}}{\partial \ln p_{n,j}} + \gamma_{1,j}\sum_{i\neq 1}^{N}\frac{\beta_{i,j}}{1+\tau^x_{i1,j}}\frac{\epsilon_j\pi_{i1,j}\pi_{in,j}x_i}{1+\tau^m_{i1,j}} + \sum_{i\neq 1}^{N}\gamma_{i,j}\sum_{h\neq 1}^{N}\frac{\beta_{h,j}}{1+\tau^m_{hi,j}}\frac{\epsilon_j\pi_{hi,j}\pi_{hn,j}x_h}{1+\tau^x_{hi,j}} \\
& - \gamma_{n,j}\sum_{h\neq 1}^{N}\beta_{h,j}\frac{1}{1+\tau^m_{hn,j}}\frac{1}{1+\tau^x_{hn,j}}\epsilon_j\pi_{hn,j}x_h + \sum_{h\neq 1}^{N}\gamma_{xh}\sum_{i\neq\{h,n\}}^{N}\beta_{i,j}\frac{\tau^x_{ih,j}}{1+\tau^x_{ih,j}}\frac{1}{1+\tau^m_{ih,j}}\epsilon_j\pi_{ih,j}\pi_{in,j}x_i \\
& - \gamma_{xn}\sum_{i\neq n}^{N}\beta_{i,j}\frac{\tau^x_{in,j}}{1+\tau^x_{in,j}}\frac{1}{1+\tau^m_{in,j}}\epsilon_j\pi_{in,j}x_i + \sum_{h\neq 1}^{N}\gamma_{xh}\sum_{i\neq h}^{N}\beta_{h,j}\frac{\tau^m_{hi,j}}{1+\tau^m_{hi,j}}\epsilon_j\pi_{hi,j}\pi_{hn,j}x_h - \sum_{h\neq 1}^{N}\gamma_{xh}\beta_{h,j}\frac{\tau^m_{hn,j}}{1+\tau^m_{hn,j}}\epsilon_j\pi_{hn,j}x_h = 0.
\end{aligned}
\tag{A.151}
$$

H Quantifying the Consequences of Optimal Policies

In this section, we compute the counterfactual equilibrium under unilateral and Nash optimal policies using the exact hat method and the formula for the optimal policies in the multi-country case satisfying CES supply system with constant returns to scale, where $\eta_j = \kappa$. Variables without 'prime' are observed variables from data (trade matrix $\pi_{ni,j}$; sectoral income $w_{n,j}L_{n,j}$; sectoral income share $\lambda_{n,j}$), and variables with 'prime' are counterfactuals after implementing optimal policies. Variables with 'hat' denote the ratios of prime variables to the observed ones.

H.1 Unilateral optimal policies

Given Home's policies, The counterfactual equilibrium satisfies[25]

$$\hat{p}_{1,j} = \hat{w}_{1,j}, \tag{A.152}$$

$$\hat{p}_{n,j} = \hat{w}_{n,j}, \quad n \neq 1 \tag{A.153}$$

$$Y'_{1,j} = w'_{1,j} L'_{1,j} = \hat{w}_{1,j} \hat{L}_{1,j} w_{1,j} L_{1,j}, \tag{A.154}$$

$$Y'_{n,j} = w'_{n,j} L'_{n,j} = \hat{w}_{n,j} \hat{L}_{n,j} w_{n,j} L_{n,j}, \quad n \neq 1 \tag{A.155}$$

$$\hat{P}_n = \prod_{j=1}^{J} \left[\pi_{n1,j} \left(\hat{p}_{1,j}(1+\tau^{x\prime}_{n,j}) \right)^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{ni,j} \hat{p}_{i,j}^{-\epsilon_j} \right]^{-\beta_{n,j}/\epsilon_j}, \quad n \neq 1 \tag{A.156}$$

$$\hat{P}_1 = \prod_{j=1}^{J} \left[\pi_{11,j} \hat{p}_{1,j}^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{1i,j} (\hat{p}_{i,j}(1+\tau^{m\prime}_{i,j}))^{-\epsilon_j} \right]^{-\beta_{1,j}/\epsilon_j}, \tag{A.157}$$

$$\pi'_{11,j} = \frac{\pi_{11,j} \hat{p}_{1,j}^{-\epsilon_j}}{\pi_{11,j} \hat{p}_{1,j}^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{1i,j} \left(\hat{p}_{i,j}(1+\tau^{m\prime}_{i,j}) \right)^{-\epsilon_j}}, \tag{A.158}$$

$$\pi'_{1n,j} = \frac{\pi_{1n,j} \left(\hat{p}_{n,j}(1+\tau^{m\prime}_{n,j}) \right)^{-\epsilon_j}}{\pi_{11,j} \hat{p}_{1,j}^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{1i,j} \left(\hat{p}_{i,j}(1+\tau^{m\prime}_{i,j}) \right)^{-\epsilon_j}}, \quad n \neq 1 \tag{A.159}$$

$$\pi'_{n1,j} = \frac{\pi_{n1,j} \left(\hat{p}_{1,j}(1+\tau^{x\prime}_{n,j}) \right)^{-\epsilon_j}}{\pi_{n1,j} \left(\hat{p}_{1,j}(1+\tau^{x\prime}_{n,j}) \right)^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{ni,j} \hat{p}_{i,j}^{-\epsilon_j}}, \quad n \neq 1 \tag{A.160}$$

$$\pi'_{nm,j} = \frac{\pi_{nm,j} \hat{p}_{mj}^{-\epsilon_j}}{\pi_{n1,j} \left(\hat{p}_{1,j}(1+\tau^{x\prime}_{n,j}) \right)^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{ni,j} \hat{p}_{i,j}^{-\epsilon_j}}, \quad n \neq 1, m \neq 1 \tag{A.161}$$

$$x'_1 = \sum_{j=1}^{J} Y'_{1,j} + \sum_{i \neq 1}^{N} \sum_{j=1}^{J} \frac{\tau^{x\prime}_{i,j}}{1+\tau^{x\prime}_{i,j}} \pi'_{i1,j} \beta_{i,j} x'_i + \sum_{i \neq 1}^{N} \sum_{j=1}^{J} \frac{\tau^{m\prime}_{i,j}}{1+\tau^{m\prime}_{i,j}} \pi'_{1i,j} \beta_{1,j} x'_1, \tag{A.162}$$

$$x'_n = \sum_{j=1}^{J} Y'_{n,j}, \quad n \neq 1 \tag{A.163}$$

$$Y'_{1,j} - \sum_{i \neq 1}^{N} \frac{1}{1+\tau^{x\prime}_{i,j}} \pi'_{i1,j} \beta_{i,j} x'_i = \pi'_{11,j} \beta_{1,j} x'_1, \tag{A.164}$$

$$Y'_{n,j} - \sum_{i \neq 1}^{N} \pi'_{in,j} \beta_{i,j} x'_i = \frac{1}{1+\tau^{m\prime}_{n,j}} \pi'_{1n,j} \beta_{1,j} x'_1, \quad n \neq 1. \tag{A.165}$$

[25] As we have proven in Appendix F, in the setup we use, a country implementing optimal policies will not impose any domestic taxes. Therefore, domestic taxes will not be included in the setup in this section.

The labor market specifications become

$$\lambda'_{n,j} = \frac{\lambda_{n,j}\hat{w}^{\kappa}_{n,j}}{\sum_{k=1}\lambda_{n,k}\hat{w}^{\kappa}_{n,k}}, \quad W'_n = W_n(\sum_{k=1}\lambda_{n,k}\hat{w}^{\kappa}_{n,k})^{\frac{1}{\kappa}}.$$

Thus,

$$W'_n\bar{L}_n = W_n\bar{L}_n(\sum_{k=1}\lambda_{n,k}\hat{w}^{\kappa}_{n,k})^{\frac{1}{\kappa}} \quad \Rightarrow \quad \frac{w'_{n,j}L'_{n,j}}{\lambda'_{n,j}} = \frac{w_{n,j}L_{n,j}}{\lambda_{n,j}}(\sum_{k=1}\lambda_{n,k}\hat{w}^{\kappa}_{n,k})^{\frac{1}{\kappa}}$$

$$\Rightarrow \hat{L}_{n,j} = \frac{\hat{w}^{\kappa-1}_{n,j}}{(\sum_{k=1}\lambda_{n,k}\hat{w}^{\kappa}_{n,k})^{\frac{\kappa-1}{\kappa}}}. \tag{A.166}$$

The equations for optimal export taxes and import tariffs are derived from (A.122) and (A.125),

$$\frac{1}{1+\tau^{x\prime}_{n,j}} = \frac{\sum_{i\neq 1}^{N}(1+\tau^{m\prime}_{i,j})\epsilon_j\pi'_{ni,j}}{1+\epsilon_j(1-\pi'_{n1,j})}, n \neq 1, \forall j \tag{A.167}$$

$$\sum_{k=1}^{J}\sum_{i\neq 1}^{N}(1+\tau^{m\prime}_{i,k})\frac{\partial E'_{1i,k}}{\partial \ln p'_{n,j}} = x'_1\beta_{1,j}\pi'_{1n,j} - \sum_{k=1}^{J}\frac{\partial E'_{11,k}}{\partial \ln p'_{n,j}}, n \neq 1, \forall j \tag{A.168}$$

where

$$\frac{\partial E'_{11,k}}{\partial \ln p'_{n,j}} = -I_{k=j}\left[\sum_{i\neq 1}^{N}\beta_{i,k}\frac{x'_i}{1+\tau^{x\prime}_{i,k}}\epsilon_k\pi'_{i1,k}\pi'_{in,k}\right] - \beta_{n,k}\frac{\pi'_{n1,k}}{1+\tau^{x\prime}_{n,k}}Y'_{n,j}, \quad (n \neq 1)$$

$$\frac{\partial E'_{1n,k}}{\partial \ln p'_{n,j}} = I_{k=j}\left[\kappa Y'_{n,k}\right] - (\kappa-1)\lambda'_{n,k}Y'_{n,j} + I_{k=j}\left[\sum_{s\neq 1}^{N}\beta_{s,k}x'_s\epsilon_k(1-\pi'_{sn,k})\pi'_{sn,k}\right] - \beta_{n,k}\pi'_{nn,k}Y'_{n,j},$$

$$\frac{\partial E'_{1i,k}}{\partial \ln p'_{n,j}} = -I_{k=j}\left[\sum_{s\neq 1}^{N}\beta_{s,k}x'_s\epsilon_k\pi'_{si,k}\pi'_{sn,k}\right] - \beta_{n,k}\pi'_{ni,k}Y'_{n,j}. \quad (i \neq n)$$

We use equations (A.152)-(A.168) to determine variables $\{\hat{w}_{n,j}, \hat{L}_{n,j}, \hat{p}_{n,j}, \hat{P}_n, x'_n, \pi'_{ni,j}, 1+\tau^{m\prime}_{n,j}, 1+\tau^{x\prime}_{n,j}\}$. Due to tax neutrality, we normalize $\tau^{m\prime}_{2,1} = 0$. We can then calculate the change in welfare for each country ($\hat{C}_n = \frac{\hat{x}_n}{\hat{P}_n}$).

H.2 Nash optimal policies

Given policies of all countries, the counterfactual equilibrium satisfies

$$\hat{p}_{1,j} = \hat{w}_{1,j}, \tag{A.169}$$

$$\hat{p}_{n,j} = \hat{w}_{n,j}, \quad n \neq 1 \tag{A.170}$$

$$Y'_{1,j} = w'_{1,j} L'_{1,j} = \hat{w}_{1,j} \hat{L}_{1,j} w_{1,j} L_{1,j}, \tag{A.171}$$

$$Y'_{n,j} = w'_{n,j} L'_{n,j} = \hat{w}_{n,j} \hat{L}_{n,j} w_{n,j} L_{n,j}, \quad n \neq 1 \tag{A.172}$$

$$\hat{P}_n = \prod_{j=1}^{J} \left[\pi_{n1,j} \left(\hat{p}_{1,j} (1 + \tau^{x\prime}_{n1,j})(1 + \tau^{m\prime}_{n1,j}) \right)^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{ni,j} \left(\hat{p}_{i,j} (1 + \tau^{x\prime}_{ni,j})(1 + \tau^{m\prime}_{ni,j}) \right)^{-\epsilon_j} \right]^{-\beta_{n,j}/\epsilon_j}, \quad n \neq 1 \tag{A.173}$$

$$\hat{P}_1 = \prod_{j=1}^{J} \left[\pi_{11,j} \hat{p}_{1,j}^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{1i,j} (\hat{p}_{i,j} (1 + \tau^{m\prime}_{1i,j})(1 + \tau^{x\prime}_{1i,j}))^{-\epsilon_j} \right]^{-\beta_{1,j}/\epsilon_j}, \tag{A.174}$$

$$\pi'_{11,j} = \frac{\pi_{11,j} \hat{p}_{1,j}^{-\epsilon_j}}{\pi_{11,j} \hat{p}_{1,j}^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{1i,j} \left(\hat{p}_{i,j} (1 + \tau^{m\prime}_{1i,j})(1 + \tau^{x\prime}_{1i,j}) \right)^{-\epsilon_j}}, \tag{A.175}$$

$$\pi'_{1n,j} = \frac{\pi_{1n,j} \left(\hat{p}_{n,j} (1 + \tau^{m\prime}_{1n,j})(1 + \tau^{x\prime}_{1n,j}) \right)^{-\epsilon_j}}{\pi_{11,j} \hat{p}_{1,j}^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{1i,j} \left(\hat{p}_{i,j} (1 + \tau^{m\prime}_{1i,j})(1 + \tau^{x\prime}_{1i,j}) \right)^{-\epsilon_j}}, \quad n \neq 1 \tag{A.176}$$

$$\pi'_{n1,j} = \frac{\pi_{n1,j} \left(\hat{p}_{1,j} (1 + \tau^{x\prime}_{n1,j})(1 + \tau^{m\prime}_{n1,j}) \right)^{-\epsilon_j}}{\pi_{n1,j} \left(\hat{p}_{1,j} (1 + \tau^{x\prime}_{n1,j})(1 + \tau^{m\prime}_{n1,j}) \right)^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{ni,j} \left(\hat{p}_{i,j} (1 + \tau^{x\prime}_{ni,j})(1 + \tau^{m\prime}_{ni,j}) \right)^{-\epsilon_j}}, \quad n \neq 1 \tag{A.177}$$

$$\pi'_{nm,j} = \frac{\pi_{nm,j} \left(\hat{p}_{mj} (1 + \tau^{x\prime}_{nm,j})(1 + \tau^{m\prime}_{nm,j}) \right)^{-\epsilon_j}}{\pi_{n1,j} \left(\hat{p}_{1,j} (1 + \tau^{x\prime}_{n,j}) \right)^{-\epsilon_j} + \sum_{i \neq 1}^{N} \pi_{ni,j} \left(\hat{p}_{i,j} (1 + \tau^{x\prime}_{ni,j})(1 + \tau^{m\prime}_{ni,j}) \right)^{-\epsilon_j}}, \quad n \neq 1 \tag{A.178}$$

$$x'_1 = \sum_{j=1}^{J} Y'_{1,j} + \sum_{i \neq 1}^{N} \sum_{j=1}^{J} \frac{\tau^{x\prime}_{i1,j}}{1 + \tau^{x\prime}_{i1,j}} \frac{1}{1 + \tau^{m\prime}_{i1,j}} \pi'_{i1,j} \beta_{i,j} x'_i + \sum_{i \neq 1}^{N} \sum_{j=1}^{J} \frac{\tau^{m\prime}_{1i,j}}{1 + \tau^{m\prime}_{1i,j}} \pi'_{1i,j} \beta_{1,j} x'_1, \tag{A.179}$$

$$x'_n = \sum_{j=1}^{J} Y'_{n,j} + \sum_{i \neq n}^{N} \sum_{j=1}^{J} \frac{\tau^{x\prime}_{in,j}}{1 + \tau^{x\prime}_{in,j}} \frac{1}{1 + \tau^{m\prime}_{in,j}} \pi'_{in,j} \beta_{i,j} x'_i + \sum_{i \neq n}^{N} \sum_{j=1}^{J} \frac{\tau^{m\prime}_{ni,j}}{1 + \tau^{m\prime}_{ni,j}} \pi'_{ni,j} \beta_{n,j} x'_n, \quad n \neq 1 \tag{A.180}$$

$$Y'_{1,j} = \pi'_{11,j} \beta_{1,j} x'_1 + \sum_{i \neq 1}^{N} \frac{1}{1 + \tau^{x\prime}_{i1,j}} \frac{1}{1 + \tau^{m\prime}_{i1,j}} \pi'_{i1,j} \beta_{i,j} x'_i, \tag{A.181}$$

$$Y'_{n,j} = \frac{1}{1 + \tau^{m\prime}_{1n,j}} \frac{1}{1 + \tau^{x\prime}_{1n,j}} \pi'_{1n,j} \beta_{1,j} x'_1 + \sum_{i \neq 1}^{N} \frac{1}{1 + \tau^{m\prime}_{in,j}} \frac{1}{1 + \tau^{x\prime}_{in,j}} \pi'_{in,j} \beta_{i,j} x'_i, \quad n \neq 1. \tag{A.182}$$

The labor market specifications become

$$\lambda'_{n,j} = \frac{\lambda_{n,j} \hat{w}^{\kappa}_{n,j}}{\sum_{k=1} \lambda_{n,k} \hat{w}^{\kappa}_{n,k}}, \quad W'_n = W_n \left(\sum_{k=1}^{J} \lambda_{n,k} \hat{w}^{\kappa}_{n,k} \right)^{\frac{1}{\kappa}}.$$

Thus,

$$W_n'\bar{L}_n = W_n\bar{L}_n(\sum_{k=1}\lambda_{n,k}\hat{w}_{n,k}^{\kappa})^{\frac{1}{\kappa}} \quad\Rightarrow\quad \frac{w_{n,j}'L_{n,j}'}{\lambda_{n,j}'} = \frac{w_{n,j}L_{n,j}}{\lambda_{n,j}}(\sum_{k=1}\lambda_{n,k}\hat{w}_{n,k}^{\kappa})^{\frac{1}{\kappa}}$$

$$\Rightarrow\hat{L}_{n,j} = \frac{\hat{w}_{n,j}^{\kappa-1}}{(\sum_{k=1}\lambda_{n,k}\hat{w}_{n,k}^{\kappa})^{\frac{\kappa-1}{\kappa}}}. \tag{A.183}$$

Given the policies of other countries, country 1's unilateral optimal policies are determined by the FOC equations (A.142), (A.143), (A.149), (A.150), and (A.151). Similarly, in this counterfactual equilibrium, we solve the following FOC equations:

FOC over x_1'

$$\frac{1}{P_1'} + \sum_{j=1}^{J}\gamma_{1,j}\beta_{1,j}\pi_{11,j}' + \sum_{i\neq 1}^{N}\sum_{j=1}^{J}\gamma_{i,j}\beta_{1,j}\frac{1}{1+\tau_{1i,j}^{m\prime}}\frac{1}{1+\tau_{1i,j}^{x\prime}}\pi_{1i,j}' + \sum_{i\neq 1}^{N}\gamma_{xi}\sum_{j=1}^{J}\beta_{1,j}\frac{\tau_{1i,j}^{x\prime}}{1+\tau_{1i,j}^{x\prime}}\frac{1}{1+\tau_{1i,j}^{m\prime}}\pi_{1i,j}' = 0. \tag{A.184}$$

FOC over x_n', $n\neq 1$

$$-\gamma_{xn} + \sum_{j=1}^{J}\gamma_{n,j}\beta_{n,j}\pi_{nn,j}' + \sum_{i\neq n}^{N}\sum_{j=1}^{J}\gamma_{i,j}\beta_{n,j}\frac{1}{1+\tau_{ni,j}^{m\prime}}\frac{1}{1+\tau_{ni,j}^{x\prime}}\pi_{ni,j}' + \sum_{i\neq n}^{N}\gamma_{xi}\sum_{j=1}^{J}\beta_{n,j}\frac{\tau_{ni,j}^{x\prime}}{1+\tau_{ni,j}^{x\prime}}\frac{1}{1+\tau_{ni,j}^{m\prime}}\pi_{ni,j}' = 0. \tag{A.185}$$

FOC over $\tau_{1n,j}^{m\prime}$, $n\neq 1$

$$1+\tau_{1n,j}^{m\prime} = \frac{(\gamma_{xn}-\gamma_{n,j})\frac{1}{1+\tau_{1n,j}^{x\prime}} - \gamma_{xn}}{1/P_1'}. \tag{A.186}$$

FOC over $\tau_{n1,j}^{x\prime}$, $n\neq 1$

$$\frac{1}{1+\tau_{n1,j}^{x\prime}} = \frac{\sum_{i\neq 1,n}^{N}\frac{\gamma_{xi}-\gamma_{i,j}}{1+\tau_{ni,j}^{x\prime}}\frac{1}{1+\tau_{ni,j}^{m\prime}}\epsilon_j\pi_{ni,j}' + (\gamma_{xn}-\gamma_{n,j})\epsilon_j\pi_{nn,j}' + \sum_{i\neq 1,n}^{N}\frac{\gamma_{xn}-\gamma_{xi}}{1+\tau_{ni,j}^{m\prime}}\epsilon_j\pi_{ni,j}' - \gamma_{xn}\frac{1}{1+\tau_{n1,j}^{m\prime}}\epsilon_j(1-\pi_{n1,j}')}{\frac{1}{P_1'}\frac{1}{1+\tau_{n1,j}^{m\prime}}(1+\epsilon_j(1-\pi_{n1,j}'))}. \tag{A.187}$$

FOC over $p'_{n,j}$, $n \neq 1$

$$
\begin{aligned}
&-\frac{1}{P'_1} x'_1 \beta_{1,j} \pi'_{1n,j} + \sum_{k=1}^{J} (\gamma_{xn} - \gamma_{n,k}) \frac{\partial Y'_{n,k}}{\partial \ln p'_{n,j}} + \gamma_{1,j} \sum_{i \neq 1}^{N} \frac{\beta_{i,j}}{1+\tau^{x\prime}_{i1,j}} \frac{\epsilon_j \pi'_{i1,j} \pi'_{in,j} x'_i}{1+\tau^{m\prime}_{i1,j}} + \sum_{i \neq 1}^{N} \gamma_{i,j} \sum_{h \neq 1}^{N} \frac{\beta_{h,j}}{1+\tau^{m\prime}_{hi,j}} \frac{\epsilon_j \pi'_{hi,j} \pi'_{hn,j} x'_h}{1+\tau^{x\prime}_{hi,j}} \\
&- \gamma_{n,j} \sum_{h \neq 1}^{N} \beta_{h,j} \frac{1}{1+\tau^{m\prime}_{hn,j}} \frac{1}{1+\tau^{x\prime}_{hn,j}} \epsilon_j \pi'_{hn,j} x'_h + \sum_{h \neq 1}^{N} \gamma_{xh} \sum_{i \neq \{h,n\}}^{N} \beta_{i,j} \frac{\tau^{x\prime}_{ih,j}}{1+\tau^{x\prime}_{ih,j}} \frac{1}{1+\tau^{m\prime}_{ih,j}} \epsilon_j \pi'_{ih,j} \pi'_{in,j} x'_i \\
&- \gamma_{xn} \sum_{i \neq n}^{N} \beta_{i,j} \frac{\tau^{x}_{in,j}}{1+\tau^{x\prime}_{in,j}} \frac{1}{1+\tau^{m\prime}_{in,j}} \epsilon_j \pi'_{in,j} x'_i + \sum_{h \neq 1}^{N} \gamma_{xh} \sum_{i \neq h}^{N} \beta_{h,j} \frac{\tau^{m\prime}_{hi,j}}{1+\tau^{m\prime}_{hi,j}} \epsilon_j \pi'_{hi,j} \pi'_{hn,j} x'_h - \sum_{h \neq 1}^{N} \gamma_{xh} \beta_{h,j} \frac{\tau^{m\prime}_{hn,j}}{1+\tau^{m\prime}_{hn,j}} \epsilon_j \pi'_{hn,j} x'_h = 0,
\end{aligned}
\tag{A.188}
$$

where $\frac{\partial Y'_{n,k}}{\partial \ln p'_{n,j}} = I_{k=j}\left[\kappa Y'_{n,j}\right] - (\kappa - 1)\lambda'_{n,k} Y'_{n,j}$. Without loss of generality, we normalize country 1's final goods price $P'_1 = 1$.

Due to tax neutrality, when a country implements its optimal policies, it can normalize a tariff it imposes on a specific country and sector to zero, given the policies of other countries. Since there are N countries in total and each country can set one tariff to zero while implementing its optimal policies, a total of N tariffs can be normalized to zero. To derive the equilibrium variables, we follow several steps. First, we let country 1 impose unilateral optimal policies and use equations (A.169)–(A.188) to determine the variables $\{\hat{w}_{n,j}, \hat{L}_{n,j}, \hat{p}_{n,j}, \hat{P}_n, x'_n, \pi'_{nm,j}, 1+\tau^{m\prime}_{1i,j}, 1+\tau^{x\prime}_{i1,j}\}$ and multipliers $\{\gamma_{xi}, \gamma_{n,j}\}$, $\forall i \neq 1, \forall n, m, j$, given that the policies of other countries are initially set to zero. Next, we update country 1's policies. Similarly, country 2 imposes unilateral optimal policies, given the policies of all other countries. We repeat these steps until each variable remains unchanged when any country imposes optimal policies. Finally we calculate the change in welfare for each country ($\hat{C}_n = \frac{\hat{x}_n}{\hat{P}_n}$).

I Alternative Proof for Proposition 2 and 3

As shown in Appendix B, one of the equilibrium conditions in (A.1), (A.2) and (A.3) is redundant according to Walras' law. In this section, we use Home's budget constraint and eliminate one of the goods market clearing conditions by setting $\gamma_{2,J} = 0$. We then prove the optimal policies under the general framework and our defined CES supply system, as shown in Propositions 2 and 3. The Home government's maximization problem and the world market equilibrium constraints in the two-country case are detailed in Appendix B.

FOC over x_1

$$1+\sum_{j=1}^{J}\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\pi_{11,j}+\sum_{j=1}^{J}\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\pi_{12,j}-\gamma_x\left(1-\sum_{j=1}^{J}\beta_{1,j}\frac{\tau_j^m}{1+\tau_j^m}\pi_{12,j}-\sum_{j=1}^{J}\beta_{1,j}\frac{\tau_j^d}{1+\tau_j^d}\pi_{11,j}\right)=0.$$

FOC over export tax τ_j^x

$$-\gamma_{1,j}\beta_{2,j}\frac{\pi_{21,j}x_2}{(1+\tau_j^x)^2}+\gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\frac{\partial\pi_{21,j}}{\partial\tau_j^x}x_2+\gamma_{2,j}\beta_{2,j}\frac{\partial\pi_{22,j}}{\partial\tau_j^x}x_2+\gamma_x\beta_{2,j}\frac{\pi_{21,j}x_2}{(1+\tau_j^x)^2}+\gamma_x\beta_{2,j}\frac{\tau_j^x}{1+\tau_j^x}\frac{\partial\pi_{21,j}}{\partial\tau_j^x}x_2=0.$$

After plugging in derivatives and simplifying, the FOC over τ_j^x becomes

$$1+\tau_j^x=\frac{\left(\gamma_x-\gamma_{1,j}\right)\left(\epsilon_j\pi_{22,j}+1\right)}{\left(\gamma_x-\gamma_{2,j}\right)\epsilon_j\pi_{22,j}}. \tag{A.189}$$

FOC over import tariff τ_j^m

$$\begin{aligned}&-\beta_{1,j}x_1\frac{\pi_{12,j}}{1+\tau_j^m}+\gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\frac{\partial\pi_{11,j}}{\partial\tau_j^m}x_1-\gamma_{2,j}\beta_{1,j}\frac{\pi_{12,j}}{(1+\tau_j^m)^2}x_1+\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\frac{\partial\pi_{12,j}}{\partial\tau_{n,j}^m}x_1\\&+\gamma_x\beta_{1,j}\frac{1}{(1+\tau_j^m)^2}\pi_{12,j}x_1+\gamma_x\beta_{1,j}\frac{\tau_j^m}{1+\tau_j^m}\frac{\partial\pi_{12,j}}{\partial\tau_j^m}x_1+\gamma_x\beta_{1,j}\frac{\tau_j^d}{1+\tau_j^d}\frac{\partial\pi_{11,j}}{\partial\tau_j^m}x_1=0.\end{aligned}$$

Plugging in derivatives and simplifying, the FOC over τ_j^m becomes

$$1+\tau_j^m=\frac{\left(\gamma_x-\gamma_{2,j}\right)\left(1+\epsilon_j\pi_{11,j}\right)}{\left(\gamma_x-\gamma_{1,j}\right)\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}+1}. \tag{A.190}$$

FOC over domestic tax τ_j^d

$$\begin{aligned}&-\beta_{1,j}x_1\frac{\pi_{11,j}}{1+\tau_j^d}-\gamma_{1,j}\frac{1}{(1+\tau_j^d)^2}\beta_{1,j}\pi_{11,j}x_1+\gamma_{1,j}\frac{1}{1+\tau_j^d}\beta_{1,j}\frac{\partial\pi_{11,j}}{\partial\tau_j^d}x_1+\gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\frac{\partial\pi_{12,j}}{\partial\tau_j^d}x_1\\&+\gamma_x\beta_{1,j}\frac{\tau_j^m}{1+\tau_j^m}\frac{\partial\pi_{12,j}}{\partial\tau_j^d}x_1+\gamma_x\beta_{1,j}\frac{1}{(1+\tau_j^d)^2}\pi_{11,j}x_1+\gamma_x\beta_{1,j}\frac{\tau_j^d}{1+\tau_j^d}\frac{\partial\pi_{11,j}}{\partial\tau_j^d}x_1=0.\end{aligned}$$

Plugging in derivatives and simplifying, the FOC over τ_j^d becomes

$$1+\tau_j^d=\frac{\left(\gamma_x-\gamma_{1,j}\right)\left(1+\epsilon_j\pi_{12,j}\right)}{\left(\gamma_x-\gamma_{2,j}\right)\frac{1}{1+\tau_j^m}\epsilon_j\pi_{12,j}+1}. \tag{A.191}$$

Combining the optimal tariff (A.190) and domestic tax (A.191), we can get

$$1+\tau_j^m = \gamma_x - \gamma_{2,j}, \qquad 1+\tau_j^d = \gamma_x - \gamma_{1,j}. \tag{A.192}$$

Combining the above two equations with equation (A.189), we get

$$1+\tau_j^x = \frac{1+\tau_j^d}{1+\tau_j^m}\left(1+\frac{1}{\epsilon_j \pi_{22,j}}\right). \tag{A.193}$$

When $\gamma_x = 0$, the optimal policy formulas are identical to those in Appendix B.

FOC over $p_{1,j}$

$$\begin{aligned}
&-\beta_{1,j}x_1\pi_{11,j} - \gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}\pi_{12,j}x_1 - \gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2 + \gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1 \\
&+\gamma_{2,j}\beta_{2,j}\epsilon_j\pi_{22,j}\pi_{21,j}x_2 - \gamma_x\beta_{2,j}\frac{\tau_j^x}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2 + \gamma_x\beta_{1,j}\frac{\tau_j^m}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1 - \gamma_x\beta_{1,j}\frac{\tau_j^d}{1+\tau_j^d}\epsilon_j\pi_{11,j}\pi_{12,j}x_1 \\
&+\sum_{k=1}^{J}(\gamma_x-\gamma_{1,k})\frac{\partial Y_{1,k}}{\partial p_{1,j}}p_{1,j} = 0.
\end{aligned} \tag{A.194}$$

FOC over $p_{2,j}$

$$\begin{aligned}
&-\beta_{1,j}x_1\pi_{12,j} + \gamma_{1,j}\beta_{1,j}\frac{1}{1+\tau_j^d}\epsilon_j\pi_{11,j}\pi_{12,j}x_1 + \gamma_{1,j}\beta_{2,j}\frac{1}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2 - \gamma_{2,j}\beta_{1,j}\frac{1}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1 \\
&-\gamma_{2,j}\beta_{2,j}\epsilon_j\pi_{22,j}\pi_{21,j}x_2 + \gamma_x\beta_{2,j}\frac{\tau_j^x}{1+\tau_j^x}\epsilon_j\pi_{21,j}\pi_{22,j}x_2 - \gamma_x\beta_{1,j}\frac{\tau_j^m}{1+\tau_j^m}\epsilon_j\pi_{12,j}\pi_{11,j}x_1 + \gamma_x\beta_{1,j}\frac{\tau_j^d}{1+\tau_j^d}\epsilon_j\pi_{12,j}\pi_{11,j}x_1 \\
&+\sum_{k=1}^{J}(cons_1-\gamma_{2,k})\frac{\partial Y_{2,k}}{\partial p_{2,j}}p_{2,j} = 0,
\end{aligned} \tag{A.195}$$

where $cons_1 = \sum_{s=1}^{J}(\gamma_x\beta_{2,s}\frac{\tau_k^x}{1+\tau_s^x}\pi_{21,s} + \gamma_{1,s}\beta_{2,s}\frac{1}{1+\tau_s^x}\pi_{21,s} + \gamma_{2,s}\beta_{2,s}\pi_{22,s})$.

I.1 Proof of optimal domestic taxes

Combining FOCs over $p_{1,j}$ (A.194), τ_j^x(A.193), and τ_j^d, τ_j^m (A.192), we get

$$
\begin{aligned}
&-\beta_{1,j}x_1\pi_{11,j}+\beta_{2,j}\Big(\frac{1+\tau_j^d}{1+\tau_j^x}-(1+\tau_j^m)\Big)\epsilon_j\pi_{21,j}\pi_{22,j}x_2+\sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial Y_{1,k}}{\partial p_{1,j}}p_{1,j}=0\\
\Rightarrow &-\beta_{1,j}x_1\pi_{11,j}-\beta_{2,j}\frac{1+\tau_j^d}{1+\tau_j^x}\pi_{21,j}x_2+\sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial Y_{1,k}}{\partial p_{1,j}}p_{1,j}=0\\
\Rightarrow &\sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial Y_{1,k}}{\partial p_{1,j}}p_{1,j}-(1+\tau_j^d)Y_{1,j}=0\\
\Rightarrow &\sum_{k=1}^{J}(1+\tau_k^d)\frac{\partial\ln(Y_{1,k})}{\partial\ln(p_{1,j})}\frac{Y_{1,k}}{Y_{1,j}}\frac{1}{\eta_j}-(1+\tau_j^d)\frac{1}{\eta_j}=0.
\end{aligned}
\tag{A.196}
$$

Hence, domestic taxes $\boldsymbol{\tau^d}=[\tau_1^d,...,\tau_j^d,...,\tau_J^d]'$ for J sectors satisfy

$$(\Lambda_1-I)(1+\boldsymbol{\tau^d})=0,$$

where element of matrix Λ_n at row j and column k is given by $\frac{\partial\ln(Y_{n,k})}{\partial\ln(p_{n,j})}\frac{Y_{n,k}}{Y_{n,j}}$, and

$$
\Lambda_1-I=\begin{pmatrix}
\frac{\partial ln(Y_{1,1})}{\partial ln(p_{1,1})}\frac{Y_{1,1}}{Y_{1,1}}-1 & ... & \frac{\partial ln(Y_{1,j})}{\partial ln(p_{1,1})}\frac{Y_{1,j}}{Y_{1,1}} & ... & \frac{\partial ln(Y_{1,J})}{\partial ln(p_{1,1})}\frac{Y_{1,J}}{Y_{1,1}}\\
... & ... & ... & ... & ...\\
\frac{\partial ln(Y_{1,1})}{\partial ln(p_{1,j})}\frac{Y_{1,1}}{Y_{1,j}} & ... & \frac{\partial ln(Y_{1,j})}{\partial ln(p_{1,j})}\frac{Y_{1,j}}{Y_{1,j}}-1 & ... & \frac{\partial ln(Y_{1,J})}{\partial ln(p_{1,j})}\frac{Y_{1,J}}{Y_{1,j}}\\
... & ... & ... & ... & ...\\
\frac{\partial ln(Y_{1,1})}{\partial ln(p_{1,J})}\frac{Y_{1,1}}{Y_{1,J}} & ... & \frac{\partial ln(Y_{1,j})}{\partial ln(p_{1,J})}\frac{Y_{1,j}}{Y_{1,J}} & ... & \frac{\partial ln(Y_{1,J})}{\partial ln(p_{1,J})}\frac{Y_{1,J}}{Y_{1,J}}-1
\end{pmatrix}.
$$

Divide equation (A.196) by η_j. For any sector k and j, optimal domestic taxes satisfy

$$\sum_{s=1}^{J}\frac{\partial\ln Y_{1,s}}{\partial\ln p_{1,j}}\frac{Y_{1,s}}{Y_{1,j}}(1+\tau_s^d)\frac{1}{\eta_j}=(1+\tau_j^d)\frac{1}{\eta_j},\tag{A.197}$$

$$\sum_{s=1}^{J}\frac{\partial\ln Y_{1,s}}{\partial\ln p_{1,k}}\frac{Y_{1,s}}{Y_{1,k}}(1+\tau_s^d)\frac{1}{\eta_k}=(1+\tau_k^d)\frac{1}{\eta_k}.\tag{A.198}$$

where η_j is the partial supply elasticity parameter in sector j. Using the definition of CES supply system in Definition 4, and subtracting (A.198) from (A.197), we have

$$
\begin{aligned}
&(1+\tau_j^d)-(1+\tau_k^d)=(1+\tau_j^d)\frac{1}{\eta_j}-(1+\tau_k^d)\frac{1}{\eta_k}\\
\Rightarrow &\frac{1+\tau_j^d}{1+\tau_k^d}=\frac{\frac{\eta_j}{\eta_j-1}}{\frac{\eta_k}{\eta_k-1}}.
\end{aligned}
$$

Due to the tax neutrality shown in Proposition 1, we can set $1+\tau_j^d=(1+\bar{\tau}^d)\frac{\eta_j}{\eta_j-1},\forall j$. The uniform shifter $\bar{\tau}^d$ can be assigned any arbitrary value, and one of the trade policies can be normalized, without changing the real allocations and welfare impacts of optimal policies.

I.2 Proof of optimal import tariffs

Combining FOC over $p_{2,j}$ (A.195), τ_j^x(A.193), and τ_j^d, τ_j^m (A.192), we get

$$
\begin{aligned}
&\sum_{k=1}^{J}(cons_1-\gamma_x+1+\tau_k^m)\left[\frac{\partial\ln Y_{2,k}}{\partial\ln p_{2,j}}Y_{2,k}\right]=\beta_{1,j}\pi_{12,j}x_1+\left[\frac{1+\tau_j^d}{1+\tau_j^x}-(1+\tau_j^m)\right]\beta_{2,j}\epsilon_j\pi_{21,j}\pi_{22,j}x_2\\
\Rightarrow &\sum_{k=1}^{J}(cons_1-\gamma_x+1+\tau_k^m)\left[\frac{\partial\ln Y_{2,k}}{\partial\ln p_{2,j}}\frac{Y_{2,k}}{Y_{2,j}}\right]=\frac{\beta_{1,j}\pi_{12,j}x_1}{Y_{2,j}}-\frac{1+\tau_j^d}{1+\tau_j^x}\beta_{2,j}\pi_{21,j}\frac{x_2}{Y_{2,j}} \qquad \text{(A.199)}\\
\Rightarrow &\sum_{k=1}^{J}(cons_1-\gamma_x+1+\tau_k^m)\left[\frac{\partial\ln Y_{2,k}}{\partial\ln p_{2,j}}\frac{Y_{2,k}}{Y_{2,j}}\right]=\frac{\beta_{1,j}\pi_{12,j}x_1}{Y_{2,j}}-(1+\tau_j^d)\frac{Y_{1,j}}{Y_{2,j}}+\frac{\beta_{1,j}\pi_{11,j}x_1}{Y_{2,j}},
\end{aligned}
$$

where $cons_1$ is common across all sectors. using the tax neutrality property established in Proposition 1, we normalize $cons_1-\gamma_x+1=0$ and the above equation becomes

$$
\sum_{k=1}^{J}\tau_k^m\left[\frac{\partial\ln Y_{2,k}}{\partial\ln p_{2,j}}\frac{Y_{2,k}}{Y_{2,j}}\right]=\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}. \qquad \text{(A.200)}
$$

Let $\boldsymbol{\tau^m}=[\tau_1^m,...,\tau_j^m,...,\tau_J^m]'$ denote the import tariff vector. We can write equation (A.200) in matrix form as $\Lambda_2\boldsymbol{\tau^m}=\Psi_1$, where Λ_2 and Ψ_1 are given by

$$
\Lambda_2=\begin{pmatrix}
\frac{\partial ln(Y_{2,1})}{\partial ln(p_{2,1})}\frac{Y_{2,1}}{Y_{2,1}} & ... & \frac{\partial ln(Y_{2,j})}{\partial ln(p_{2,1})}\frac{Y_{2,j}}{Y_{2,1}} & ... & \frac{\partial ln(Y_{2,J})}{\partial ln(p_{2,1})}\frac{Y_{2,J}}{Y_{2,1}}\\
... & ... & ... & ... & ...\\
\frac{\partial ln(Y_{2,1})}{\partial ln(p_{2,j})}\frac{Y_{2,1}}{Y_{2,j}} & ... & \frac{\partial ln(Y_{2,j})}{\partial ln(p_{2,j})}\frac{Y_{2,j}}{Y_{2,j}} & ... & \frac{\partial ln(Y_{2,J})}{\partial ln(p_{2,j})}\frac{Y_{2,J}}{Y_{2,j}}\\
... & ... & ... & ... & ...\\
\frac{\partial ln(Y_{2,1})}{\partial ln(p_{2,J})}\frac{Y_{2,1}}{Y_{2,J}} & ... & \frac{\partial ln(Y_{2,j})}{\partial ln(p_{2,J})}\frac{Y_{2,j}}{Y_{2,J}} & ... & \frac{\partial ln(Y_{2,J})}{\partial ln(p_{2,J})}\frac{Y_{2,J}}{Y_{2,J}}
\end{pmatrix},\quad
\Psi_1=\begin{pmatrix}
\frac{\beta_{1,1}x_1-(1+\tau_1^d)Y_{1,1}}{Y_{2,1}}\\
...\\
\frac{\beta_{1,j}x_1-(1+\tau_j^d)Y_{1,j}}{Y_{2,j}}\\
...\\
\frac{\beta_{1,J}x_1-(1+\tau_J^d)Y_{1,J}}{Y_{2,J}}
\end{pmatrix}. \qquad \text{(A.201)}
$$

Divide equation (A.200) by η_j. For any sector k and j, optimal import tariffs satisfy

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{2,s}}{\partial \ln p_{2,j}} \frac{Y_{2,s}}{Y_{2,j}} \tau_s^m \frac{1}{\eta_j} = \frac{1}{\eta_j} \Psi_{1,j}, \tag{A.202}$$

$$\sum_{s=1}^{J} \frac{\partial \ln Y_{2,s}}{\partial \ln p_{2,k}} \frac{Y_{2,s}}{Y_{2,k}} \tau_s^m \frac{1}{\eta_k} = \frac{1}{\eta_k} \Psi_{1,k}. \tag{A.203}$$

Using the definition of CES supply system in Definition 4, and subtracting (A.203) from (A.202), we have

$$\left(\frac{\partial \ln Y_{2,j}}{\partial \ln p_{2,j}} \frac{1}{\eta_j} - \frac{\partial \ln Y_{2,j}}{\partial \ln p_{2,k}} \frac{Y_{2,j}}{Y_{2,k}} \frac{1}{\eta_k} \right) \tau_j^m + \left(\frac{\partial \ln Y_{2,k}}{\partial \ln p_{2,j}} \frac{Y_{2,k}}{Y_{2,j}} \frac{1}{\eta_j} - \frac{\partial \ln Y_{2,k}}{\partial \ln p_{2,k}} \frac{1}{\eta_k} \right) \tau_k^m = \frac{1}{\eta_j} \Psi_{1,j} - \frac{1}{\eta_k} \Psi_{1,k}$$
$$\Rightarrow \tau_j^m - \tau_k^m = \frac{1}{\eta_j} \Psi_{1,j} - \frac{1}{\eta_k} \Psi_{1,k},$$

where $\Psi_{1,j}$ and $\Psi_{1,k}$ are the jth and kth row of vector Ψ_1.