Online Appendix to "Optimal Trade Policy with International Technology Diffusion"

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A Central Planner Problem

This section studies a central planner's problem. We first lay out the recursive formulation, we then prove the tax neutrality of Proposition 1 and central planner's optimal policies in Proposition 2 and 3. Lastly, we present a simple example with two countries.

A.1 Recursive Formulation

At period 0, the central planner chooses the sequence of $\left\{\tau^m_{ni,t}, \tau^x_{in,t}, \kappa_{nt}\right\}$ to solve the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t \sum_{n=1}^{N} \lambda_n u\left(\frac{x_{nt}}{P_{nt}}\right)$$

subject to the world market equilibrium, for any t,

$$T_{n,t+1} = (1-\delta)T_{nt} + \alpha_n \sum_{i=1}^{N} \left[\pi_{ni,t}\left(\frac{T_{it}}{\pi_{ni,t}}\right)^{\rho}\right], \quad (\gamma_{Tn}, \quad N) \tag{A.1}$$

$$x_{1t} = w_{1t}L_{1t} + \sum_{i\neq 1}^{N} \frac{\tau^m_{1i,t}}{1+\tau^m_{1i,t}}\pi_{1i,t}x_{1t} + \sum_{i\neq 1}^{N} \frac{\tau^x_{i1,t}}{1+\tau^x_{i1,t}}\frac{1}{1+\tau^m_{i1,t}}\pi_{i1,t}x_{it} - \sum_{i\neq 1}^{N}\kappa_{it}, \quad (\gamma_{x1}) \tag{A.2}$$

$$x_{nt} = w_{nt}L_{nt} + \sum_{i\neq n}^{N} \frac{\tau^m_{ni,t}}{1+\tau^m_{ni,t}}\pi_{ni,t}x_{nt} + \sum_{i\neq n}^{N} \frac{\tau^x_{in,t}}{1+\tau^x_{in,t}}\frac{1}{1+\tau^m_{in,t}}\pi_{in,t}x_{it} + \kappa_{nt}, \quad (\gamma_{xn}, n>1, \quad N-1) \tag{A.3}$$

$$w_{nt}L_{nt} = \sum_{i\neq n}^{N} \frac{1}{1+\tau^x_{in,t}}\frac{1}{1+\tau^m_{in,t}}\pi_{in,t}x_{it} + \pi_{nn,t}x_{nt}, \quad (\gamma_n, n>1, \quad N-1) \tag{A.4}$$

where

$$P_{nt} = \left[T_{nt}w_{nt}^{-\theta} + \sum_{i\neq n} T_{it}(w_{it}(1+\tau^x_{ni,t})(1+\tau^m_{ni,t})d_{ni,t})^{-\theta}\right]^{-\frac{1}{\theta}}$$

$$\pi_{nn,t} = \frac{T_{nt}w_{nt}^{-\theta}}{T_{nt}w_{nt}^{-\theta} + \sum_{i\neq n} T_{it}(w_{it}(1+\tau^x_{ni,t})(1+\tau^m_{ni,t})d_{ni,t})^{-\theta}}$$

$$\pi_{ni,t} = \frac{T_{it}(w_{it}(1+\tau^x_{ni,t})(1+\tau^m_{ni,t})d_{ni,t})^{-\theta}}{T_{nt}w_{nt}^{-\theta} + \sum_{i\neq n} T_{it}(w_{it}(1+\tau^x_{ni,t})(1+\tau^m_{ni,t})d_{ni,t})^{-\theta}}$$

Note that $\sum_i \pi_{ni,t} = 1$ for any n in period t.

Equation (A.2)-(A.3) are the definitions of income. Equation (A.4) are the market clearing conditions; note that due to the Walras law, one of them is redundant, so we have $N-1$ market clearing conditions.

The problem can be written recursively as

$$V\left(\{T_n\}\right) = \max_{\left\{T'_n, w_n, x_n, \kappa_n, \tau^m_{ni}, \tau^x_{in}\right\}} \sum_{n}^{N} \lambda_n u(x_n/P_n) + \beta V\left(\{T'_n\}\right)$$

subject to

$$
\begin{aligned}
T_n' &= (1-\delta)T_n + \alpha_n \sum_{i=1}^{N} \left[\pi_{ni} \left(\frac{T_i}{\pi_{ni}} \right)^{\rho} \right], \quad (\gamma_{Tn}, \quad N) \\
x_1 &= w_1 L_1 + \sum_{i \neq 1}^{N} \frac{\tau_{1i}^m}{1+\tau_{1i}^m} \pi_{1i} x_1 + \sum_{i \neq 1}^{N} \frac{\tau_{i1}^x}{1+\tau_{i1}^x} \frac{1}{1+\tau_{i1}^m} \pi_{i1} x_i - \sum_{i \neq 1}^{N} \kappa_i, \quad (\gamma_{x1}) \\
x_n &= w_n L_n + \sum_{i \neq n}^{N} \frac{\tau_{ni}^m}{1+\tau_{ni}^m} \pi_{ni} x_n + \sum_{i \neq n}^{N} \frac{\tau_{in}^x}{1+\tau_{in}^x} \frac{1}{1+\tau_{in}^m} \pi_{in} x_i + \kappa_n, \quad (\gamma_{xn}, n > 1, \quad N-1) \\
w_n L_n &= \sum_{i \neq n}^{N} \frac{1}{1+\tau_{in}^x} \frac{1}{1+\tau_{in}^m} \pi_{in} x_i + \pi_{nn} x_n, \quad (\gamma_n, n > 1, \quad N-1)
\end{aligned}
$$

A.2 Tax Neutrality: Proof of Proposition 1

We give the central planner all trade policy instruments, including $N(N-1)$ tariff and $N(N-1)$ export tax between each pair of countries and $N-1$ transfers, the purpose of this section is to show that some of these taxes are redundant. First, we show for the central government, the tariff of n on a country i's export is equivalent to the export tax of i to n, with the transfer. This is because both affect the price from i to n. Hence, all export taxes can be normalized to 0. Second, we show with $N(N-1)$ tariffs between countries and $N-1$ aggregate transfers, $N-1$ of them are redundant. Intuitively, there are $N(N-1)$ trade flows to manipulate.

Given $\{\tau_{ni,t}^x + 1, \tau_{ni,t}^m + 1, \kappa_{nt}\}$ where $i \neq n$, let $\mathcal{E}(\tau_{ni,t}^x + 1, \tau_{ni,t}^m + 1, \kappa_{nt})$ denote the set of $\{T_{nt}, \pi_{ni,t}, \frac{w_{nt}}{P_{nt}}, \frac{x_{nt}}{P_{nt}}\}$ that form an equilibrium. We say that from $\{\tau_{ni,t}^x + 1, \tau_{ni,t}^m + 1, \kappa_{nt}\}$ to $\{\check{\tau}_{ni,t}^x + 1, \check{\tau}_{ni,t}^m + 1, \check{\kappa}_{nt}\}$ is neutral if $\mathcal{E}(\tau_{ni,t}^x + 1, \tau_{ni,t}^m + 1, \kappa_{nt})$=$\mathcal{E}(\check{\tau}_{ni,t}^x + 1, \check{\tau}_{ni,t}^m + 1, \check{\kappa}_{nt})$. This captures neutrality because the equilibrium allocations and welfare obtainable with the two sets of policies are the same.

Assume $1 + \check{\tau}_{ni,t}^m = \frac{\xi_{it}}{\xi_{nt}} \mu_{ni,t} (1 + \tau_{ni,t}^m)$ and $1 + \check{\tau}_{ni,t}^x = \frac{1+\tau_{ni,t}^x}{\mu_{ni,t}}$, where $i \neq n$, for any constants $\mu_{ni,t} > 0$ for any $i \neq n$ and $\xi_{nt} > 0$ for any n. We guess the allocations $\{\check{T}_{n,t+1}, \check{T}_{nt}, \check{\pi}_{in,t}, \frac{\check{w}_{nt}}{\check{P}_{nt}}, \frac{\check{x}_{nt}}{\check{P}_{nt}}\}$ in the new equilibrium are the same as allocations in the old equilibrium with $\check{T}_{n,t+1} = T_{n,t+1}, \check{T}_{nt} = T_{nt}$, $\check{\pi}_{ni,t} = \pi_{ni,t}$, $\check{P}_{nt} = \frac{P_{nt}}{\xi_{nt}}$, $\check{w}_{nt} = \frac{w_{nt}}{\xi_{nt}}$, $\check{x}_{nt} = \frac{x_{nt}}{\xi_{nt}}$, $\check{\kappa}_{nt} = \frac{1}{\xi_{nt}} \left[\kappa_{nt} + \sum_{i \neq n}^{N} \frac{\xi_{nt}/\xi_{it} - \mu_{ni,t}}{\mu_{ni,t}(1+\tau_{ni,t}^m)} \pi_{ni,t} x_{nt} + \sum_{i \neq n}^{N} \frac{\mu_{in,t} - 1}{\mu_{in,t}(1+\tau_{in,t}^m)} \pi_{in,t} x_{it} \right]$. We then verify all equilibrium conditions hold.

Evolution of technology

$$
\check{T}_{n,t+1} = (1-\delta)T_{nt} + \alpha_n \sum_{i=1}^{N} \left[\pi_{ni,t} \left(\frac{T_{it}}{\pi_{ni,t}} \right)^{\rho} \right] = T_{n,t+1}.
$$

Price index

$$
\check{P}_{nt} = \left[T_{nt} \left(\frac{w_{nt}}{\xi_{nt}}\right)^{-\theta} + \sum_{i \neq n} T_{it} \left(\frac{w_{it}}{\xi_{it}} \frac{1}{\mu_{ni,t}} (1+\tau_{ni,t}^x) \frac{\xi_{it}}{\xi_{nt}} \mu_{ni,t} (1+\tau_{ni,t}^m) d_{ni,t} \right)^{-\theta} \right]^{-\frac{1}{\theta}} = \frac{P_{nt}}{\xi_{nt}}.
$$

Market clearing conditions ($n > 1$)

$$\check{w}_{nt} L_{nt} = \sum_{i \neq n}^{N} \frac{\mu_{in,t}}{1+\tau^x_{in,t}} \frac{\xi_{it}}{\xi_{nt}} \frac{1}{\mu_{in,t}(1+\tau^m_{in,t})} \pi_{in,t} \frac{x_{it}}{\xi_{it}} + \pi_{nn,t} \frac{x_{nt}}{\xi_{nt}} = \frac{w_{nt}}{\xi_{nt}} L_{nt}.$$

Trade share

$$\check{\pi}_{nn,t} = \frac{T_{nt}(\frac{w_{nt}}{\xi_{nt}})^{-\theta}}{T_{nt}(\frac{w_{nt}}{\xi_{nt}})^{-\theta} + \sum_{i \neq n} T_{it}(\frac{w_{it}}{\xi_{it}} \frac{1}{\mu_{ni,t}}(1+\tau^x_{ni,t}) \frac{\xi_{it}}{\xi_{nt}} \mu_{ni,t}(1+\tau^m_{ni,t}) d_{ni,t})^{-\theta}} = \pi_{nn,t}$$

$$\check{\pi}_{ni,t} = \frac{T_{it}(\frac{w_{it}}{\xi_{it}} \frac{1}{\mu_{ni,t}}(1+\tau^x_{ni,t}) \frac{\xi_{it}}{\xi_{nt}} \mu_{ni,t}(1+\tau^m_{ni,t}) d_{ni,t})^{-\theta}}{T_{nt}(\frac{w_{nt}}{\xi_{nt}})^{-\theta} + \sum_{i \neq n} T_{it}(\frac{w_{it}}{\xi_{it}} \frac{1}{\mu_{ni,t}}(1+\tau^x_{ni,t}) \frac{\xi_{it}}{\xi_{nt}} \mu_{ni,t}(1+\tau^m_{ni,t}) d_{ni,t})^{-\theta}} = \pi_{ni,t}.$$

Country n's expenditure

$$\begin{aligned}\check{x}_{nt} = \frac{w_{nt}}{\xi_{nt}} L_n + \sum_{i \neq n}^{N} \pi_{ni,t} \frac{x_{nt}}{\xi_{nt}} - \sum_{i \neq n}^{N} \frac{\xi_{nt}}{\xi_{it}} \frac{1}{\mu_{ni,t}(1+\tau^m_{ni,t})} \pi_{ni,t} \frac{x_{nt}}{\xi_{nt}} + \sum_{i \neq n}^{N} \frac{\xi_{it}}{\xi_{nt}} \frac{1}{\mu_{in,t}(1+\tau^m_{in,t})} \pi_{in,t} \frac{x_{it}}{\xi_{it}} \\ - \sum_{i \neq n}^{N} \frac{\xi_{it}}{\xi_{nt}} \frac{1}{1+\tau^x_{in,t}} \frac{1}{1+\tau^m_{in,t}} \pi_{in,t} \frac{x_{it}}{\xi_{it}} + \frac{1}{\xi_{nt}} \left[\kappa_{nt} + \sum_{i \neq n}^{N} \frac{\xi_{nt}/\xi_{it} - \mu_{ni,t}}{\mu_{ni,t}(1+\tau^m_{ni,t})} \pi_{ni,t} x_{nt} + \sum_{i \neq n}^{N} \frac{\mu_{in,t} - 1}{\mu_{in,t}(1+\tau^m_{in,t})} \pi_{in,t} x_{it} \right] = \frac{x_{nt}}{\xi_{nt}}.\end{aligned}$$

Transfer

$$\begin{aligned}\sum_{n=1}^{N} \check{\kappa}_{nt} &= \sum_{n=1}^{N} \{ \frac{1}{\xi_{nt}} \left[\kappa_{nt} + \sum_{i \neq n}^{N} \frac{\xi_{nt}/\xi_{it} - \mu_{ni,t}}{\mu_{ni,t}(1+\tau^m_{ni,t})} \pi_{ni,t} x_{nt} + \sum_{i \neq n}^{N} \frac{\mu_{in,t} - 1}{\mu_{in,t}(1+\tau^m_{in,t})} \pi_{in,t} x_{it} \right] \} \\ &= \sum_{n=1}^{N} \{ \frac{1}{\xi_{nt}} \kappa_{nt} + \sum_{i \neq n}^{N} \frac{1/\xi_{it} - \mu_{ni,t}/\xi_{nt}}{\mu_{ni,t}(1+\tau^m_{ni,t})} \pi_{ni,t} x_{nt} + \sum_{i \neq n}^{N} \frac{\mu_{in,t}/\xi_{nt} - 1/\xi_{nt}}{\mu_{in,t}(1+\tau^m_{in,t})} \pi_{in,t} x_{it} \} \\ &= \sum_{n=1}^{N} \{ \frac{1}{\xi_{nt}} \kappa_{nt} + \sum_{i \neq n}^{N} \frac{-1/\xi_{nt}}{(1+\tau^m_{ni,t})} \pi_{ni,t} x_{nt} + \sum_{i \neq n}^{N} \frac{1/\xi_{nt}}{(1+\tau^m_{in,t})} \pi_{in,t} x_{it} \} \\ &= \sum_{n=1}^{N} \{ \frac{1}{\xi_{nt}} [\kappa_{nt} - \sum_{i \neq n}^{N} \frac{1}{(1+\tau^m_{ni,t})} \pi_{ni,t} x_{nt} + \sum_{i \neq n}^{N} \frac{1}{(1+\tau^m_{in,t})} \pi_{in,t} x_{it}] \} \\ &= 0.\end{aligned}$$

The same allocations satisfy the equilibrium conditions under $\{\check{\tau}^x_{ni,t} + 1, \check{\tau}^m_{ni,t} + 1, \check{\kappa}_{nt}\}$, hence we proved the allocations and welfare are the same under $\{\tau^x_{ni,t} + 1, \tau^m_{ni,t} + 1, \kappa_{nt}\}$ and $\{\check{\tau}^x_{ni,t} + 1, \check{\tau}^m_{ni,t} + 1, \check{\kappa}_{nt}\}$. For any policy Γ, we can set $\mu_{ni,t} = 1 + \tau^x_{ni,t}$ and obtain $\check{\tau}^x_{ni,t} = 0$. Second, by further adjusting $\{\frac{\xi_{it}}{\xi_{nt}}\}$ we can set the tariffs of country n on other countries' goods to zero, or normalize the tariffs imposed on country n's goods to zero, or make all transfers $\{\kappa_{nt}\}$ zero. In all the cases, we further remove $N-1$ policy instruments.

A.3 Central Planner Policies: Proof of Proposition 2-3

After normalizing the export taxes between each pair of countries to zero, the world social planner can use international transfers and the import tariffs between each pair of countries, where $\tau^m_{ni,t}$ is the import tariff that country n has on imports from country i. At period 0, the central planner chooses the sequence of $\left\{\tau^m_{ni,t}, \kappa_{nt}\right\}$ to solve the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t \sum_{n}^{N} \lambda_n u(\frac{x_{nt}}{P_{nt}})$$

subject to for any period:

$$\begin{aligned}
T_{n,t+1} &= (1-\delta)T_{nt} + \alpha_n \sum_{i=1}^{N} \left[\pi_{ni,t}\left(\frac{T_{it}}{\pi_{ni,t}}\right)^{\rho}\right], \quad (\gamma_{Tn}, \quad N) \\
x_{1t} &= w_{1t}L_{1t} + \sum_{i\neq 1}^{N} \frac{\tau^m_{1i,t}}{1+\tau^m_{1i,t}}\pi_{1i,t}x_{1t} - \sum_{i\neq 1}^{N}\kappa_{it}, \quad (\gamma_{x1}) \\
x_{nt} &= w_{nt}L_{nt} + \sum_{i\neq n}^{N} \frac{\tau^m_{ni,t}}{1+\tau^m_{ni,t}}\pi_{ni,t}x_{nt} + \kappa_{nt}, \quad (\gamma_{xn}, \quad (N-1), n>1) \\
w_{nt}L_{nt} &= \sum_{i\neq n}^{N} \frac{1}{1+\tau^m_{in,t}}\pi_{in,t}x_{it} + \pi_{nn,t}x_{nt}, \quad (\gamma_n, n>1, \quad N-1)
\end{aligned}$$

where

$$\begin{aligned}
P_{nt} &= \left[T_{nt}w_{nt}^{-\theta} + \sum_{i\neq n} T_{it}(w_{it}(1+\tau^m_{ni,t})d_{ni,t})^{-\theta}\right]^{-\frac{1}{\theta}} \\
\pi_{nn,t} &= \frac{T_{nt}w_{nt}^{-\theta}}{T_{nt}w_{nt}^{-\theta} + \sum_{i\neq n} T_{it}(w_{it}(1+\tau^m_{ni,t})d_{ni,t})^{-\theta}} \\
\pi_{ni,t} &= \frac{T_{it}(w_{it}(1+\tau^m_{ni,t})d_{ni,t})^{-\theta}}{T_{nt}w_{nt}^{-\theta} + \sum_{i\neq n} T_{it}(w_{it}(1+\tau^m_{ni,t})d_{ni,t})^{-\theta}}
\end{aligned}$$

Note that $\sum_i \pi_{ni,t} = 1$ for any n in period t.

Recursively problem can be written as

$$V(\{T_n\}) = \max_{\left\{T'_n, w_n, x_n, \kappa_n, \tau^m_{ni}\right\}} \sum_{n}^{N} \lambda_n u(x_n/P_n) + \beta V\left(\{T'_n\}\right)$$

subject to

$$T_n' = (1-\delta)T_n + \alpha_n \sum_{i=1}^{N}\left[\pi_{ni}\left(\frac{T_i}{\pi_{ni}}\right)^{\rho}\right], \quad (\gamma_{Tn}, \quad N) \tag{A.5}$$

$$x_1 = w_1 L_1 + \sum_{i\neq 1}^{N} \frac{\tau_{1i}^m}{1+\tau_{1i}^m}\pi_{1i}x_1 - \sum_{i\neq 1}^{N}\kappa_i, \quad (\gamma_{x1}) \tag{A.6}$$

$$x_n = w_n L_n + \sum_{i\neq n}^{N} \frac{\tau_{ni}^m}{1+\tau_{ni}^m}\pi_{ni}x_n + \kappa_n, \quad (\gamma_{xn}, \quad (N-1), n>1) \tag{A.7}$$

$$w_n L_n = \sum_{i\neq n}^{N} \frac{1}{1+\tau_{in}^m}\pi_{in}x_i + \pi_{nn}x_n, \quad (\gamma_n, \quad (N-1), n>1) \tag{A.8}$$

Optimal conditions For ease of notation, we define the weighted average of quality of insights in country n as

$$I_n = \sum_{i=1}^{N}\left[\pi_{ni}\left(\frac{T_i}{\pi_{ni}}\right)^{\rho}\right]$$

FOC over x_n

$$\lambda_n \frac{u_{cn}}{P_n} + \sum_{i\neq n}^{N}\gamma_i \frac{1}{1+\tau_{ni}^m}\pi_{ni} + \gamma_n\pi_{nn} - \gamma_{xn}\left(1 - \sum_{i\neq n}^{N}\frac{\tau_{ni}^m}{1+\tau_{ni}^m}\pi_{ni}\right) = 0$$

FOC over τ_{ni}^m

$$\begin{aligned}&\gamma_{Tn}\alpha_n(1-\rho)\theta\left[I_n - \left(\frac{T_i}{\pi_{ni}}\right)^{\rho}\right] - \lambda_n\frac{u_{cn}}{P_n}x_n - \gamma_i\frac{1}{1+\tau_{ni}^m}x_n - \gamma_i x_n\theta\frac{1}{1+\tau_{ni}^m}(1-\pi_{ni}) + \gamma_n\theta\pi_{nn}x_n \\ &+ \sum_{s\neq\{i,n\}}^{N}\gamma_s\frac{1}{1+\tau_{ns}^m}\theta\pi_{ns}x_n + \gamma_{xn}\frac{1}{1+\tau_{ni}^m}x_n - \gamma_{xn}\frac{\tau_{ni}^m}{1+\tau_{ni}^m}\theta(1-\pi_{ni})x_n + \gamma_{xn}\sum_{s\neq\{i,n\}}^{N}\frac{\tau_{ns}^m}{1+\tau_{ns}^m}x_n\theta\pi_{ns} = 0\end{aligned}$$

Combine FOC over τ_{ni}^m and FOC over x_n:

$$((\gamma_{xn}-\gamma_i)\frac{1}{1+\tau_{ni}^m} - \lambda_n\frac{u_{cn}}{P_n})(1+\theta)x_n = \gamma_{Tn}(1-\rho)\theta\alpha_n\left[\left(\frac{T_i}{\pi_{ni}}\right)^{\rho} - I_n\right] \tag{A.9}$$

For country n, it has

$$((\gamma_{xn}-\gamma_n) - \lambda_n\frac{u_{cn}}{P_n})(1+\theta)x_n = \gamma_{Tn}(1-\rho)\theta\alpha_n\left[\left(\frac{T_n}{\pi_{nn}}\right)^{\rho} - I_n\right] \tag{A.10}$$

Combining equation (A.9) and (A.10), the optimal import tariffs satisfy

$$\frac{1}{1+\tau_{ni}^m} = \frac{\gamma_{xn}-\gamma_n}{\gamma_{xn}-\gamma_i} + \gamma_{Tn}\frac{(1-\rho)\theta\alpha_n}{(\gamma_{xn}-\gamma_i)(1+\theta)x_n}\left[\left(\frac{T_i}{\pi_{ni}}\right)^{\rho} - \left(\frac{T_n}{\pi_{nn}}\right)^{\rho}\right] \tag{A.11}$$

Without diffusion, $\rho = 0$,

$$\frac{1}{1+\tau_{ni}^m} = \frac{\gamma_{xn} - \gamma_n}{\gamma_{xn} - \gamma_i} \tag{A.12}$$

For country 1 ($i = 1$)'s exports face tariffs

$$\frac{1}{1+\tau_{n1}^m} = \frac{\gamma_{xn} - \gamma_n}{\gamma_{xn}}. \tag{A.13}$$

Here due to the Walras law, we drop the market clearing condition for country 1 and set $\gamma_1 = 0$. Thus, there are $N-1$ tariffs imposed on country 1, and we can set all of them to zero using $N-1$ transfers. This implies that $\gamma_n = 0$ for all $n > 1$ according to equation (A.13). This further implies that $\frac{1}{1+\tau_{ni}^m} = 1$ for any pair of n and i from equation (A.12). Thus, the central planner sets all import tariffs to 0 when no diffusions. This proves the Proposition 2 and establishes that the world market equilibrium is efficient as it consists of the same allocations as the central planner's problem under some Pareto weights.

Since the tariff of n on a country i's export is equivalent to the export tax of i to n, with the transfer. Equivalently, the central planner can use $N(N-1)/2$ import tariffs and $N(N-1)/2$ export taxes satisfy:

$$\begin{pmatrix} 0 & \tau_{12}^m & \tau_{13}^m & ... & \tau_{1N}^m \\ \tau_{21}^x & 0 & \tau_{23}^m & ... & \tau_{2N}^m \\ \tau_{31}^x & \tau_{32}^x & 0 & ... & \tau_{3N}^m \\ ... & ... & ... & ... & ... \\ \tau_{N1}^x & \tau_{N2}^x & \tau_{N3}^x & ... & 0 \end{pmatrix}$$

$$\frac{1}{1+\tau_{in}^m} = \frac{\gamma_{xi} - \gamma_i}{\gamma_{xi} - \gamma_n} + \gamma_{Ti}\frac{(1-\rho)\theta\alpha_i}{(\gamma_{xi} - \gamma_n)(1+\theta)x_i}[(\frac{T_n}{\pi_{in}})^\rho - (\frac{T_i}{\pi_{ii}})^\rho] \tag{A.14}$$

$$\frac{1}{1+\tau_{ni}^x} = \frac{\gamma_{xn} - \gamma_n}{\gamma_{xn} - \gamma_i} + \gamma_{Tn}\frac{(1-\rho)\theta\alpha_n}{(\gamma_{xn} - \gamma_i)(1+\theta)x_n}[(\frac{T_i}{\pi_{ni}})^\rho - (\frac{T_n}{\pi_{nn}})^\rho] \quad \text{for } i < n \tag{A.15}$$

With transfers, γ_n can be normalized to zero. This finishes the proof of Proposition 3.

Furthermore, the following FOCs jointly pin down the multipliers γ:

FOC over w_n

$$\sum_i^N \gamma_{Ti}\alpha_i(1-\rho)\pi_{in}\left[I_i - (\pi_{in})^{-\rho}\, T_i^\rho\right] + \sum_i^N \gamma_{xi}(\sum_{s\neq i}^N \theta\pi_{is}\pi_{in}\frac{\tau_{is}^m}{1+\tau_{is}^m}x_i - \theta\pi_{in}\frac{\tau_{in}^m}{1+\tau_{in}^m}x_i)$$
$$+ \gamma_{xn}w_nL_n + \sum_{i\neq n}^N \gamma_i(\sum_{s\neq i}^N \theta\pi_{si}\pi_{sn}\frac{1}{1+\tau_{si}^m}x_s + \theta\pi_{ii}\pi_{in}x_i)$$
$$- \gamma_n(\sum_{s\neq n}^N \theta\pi_{sn}(1-\pi_{sn})\frac{1}{1+\tau_{sn}^m}x_s + \theta\pi_{nn}(1-\pi_{nn})x_n) - \gamma_n w_n L_n - \sum_i^N \lambda_i\frac{u_{ci}}{P_i}\pi_{in}x_i = 0$$

FOC over T_n

$$
\begin{aligned}
&-\gamma_{Tn,-1}T_n+\beta\Big\{(1-\delta)\gamma_{Tn}T_n+\sum_{i=1}^{N}\gamma_{Ti}\alpha_i(1-\rho)\pi_{in}\left[-I_i+(\pi_{in})^{-\rho}T_i^{\rho}\right]+\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\rho\theta\pi_{in}^{1-\rho}T_i^{\rho}\\
&+\sum_{i}^{N}\lambda_i\frac{u_{ci}}{P_i}\pi_{in}x_i\frac{1}{\theta}-\sum_{i}^{N}\gamma_{xi}(-\sum_{s\neq i}^{N}\theta\pi_{is}\pi_{in}\frac{\tau_{is}^m}{1+\tau_{is}^m}x_i+\theta\pi_{in}\frac{\tau_{in}^m}{1+\tau_{in}^m}x_i)\\
&-\sum_{i\neq n}^{N}\gamma_i(\sum_{s\neq i}^{N}\theta\pi_{si}\pi_{sn}\frac{1}{1+\tau_{si}^m}x_s+\theta\pi_{ii}\pi_{in}x_i)+\gamma_n(\sum_{s\neq n}^{N}\theta\pi_{sn}(1-\pi_{sn})\frac{1}{1+\tau_{sn}^m}x_s+\theta\pi_{nn}(1-\pi_{nn})x_n)=0
\end{aligned}
$$

FOC over κ_n, where $n>1$

$$\gamma_{xn}-\gamma_{x1}=0.$$

A.4 Simple Example with Two Countries

To illustrate the central planner's incentives for manipulating trade flows and global technology diffusions, we examine a case with two countries. In this case, with transfers, the optimal import tariffs in Proposition 3 can be written as

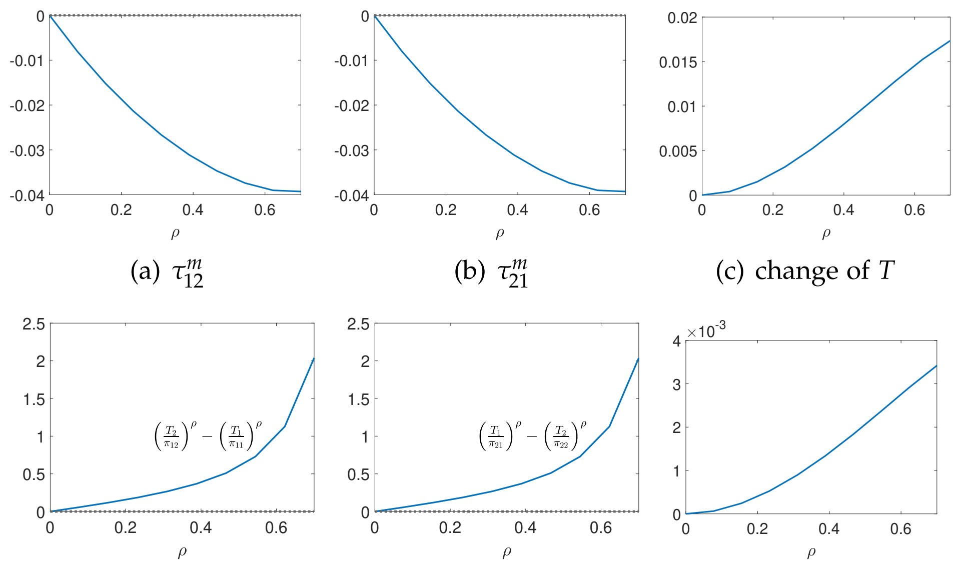
$$\frac{1}{1+\tau_{12}^m}=1+\gamma_{T1}\frac{(1-\rho)\theta\alpha_1}{\gamma_{x1}(1+\theta)x_1}[(\frac{T_2}{\pi_{12}})^{\rho}-(\frac{T_1}{\pi_{11}})^{\rho}],\ \frac{1}{1+\tau_{21}^m}=1+\gamma_{T2}\frac{(1-\rho)\theta\alpha_2}{\gamma_{x2}(1+\theta)x_2}[(\frac{T_1}{\pi_{21}})^{\rho}-(\frac{T_2}{\pi_{22}})^{\rho}]. \tag{A.16}$$

Compared to the no diffusion case where the planner chooses $\tau_{21}^m=\tau_{12}^m=0$, the optimal tariff under technology diffusion takes into account the incentive to alter trade so as to manipulate global technologies. As we discussed above, $(\frac{T_2}{\pi_{12}})^{\rho}-(\frac{T_1}{\pi_{11}})^{\rho}$ is country 1's relative import quality. When $\gamma_{T1}\geq 0$ and the relative import quality for country 1 is positive, it benefits country 1's learning by increasing imports through subsidies ($\tau_{12}^m<0$), as shown in equation (A.16).

When symmetric, the two countries have identical allocations and prices. Let $T_1=T_2\equiv T$, then the planner imposes identical import tariffs on both countries, $\tau_{12}^m=\tau_{21}^m\equiv\tau^m$, and $\frac{1}{1+\tau^m}=1+\gamma_T\frac{(1-\rho)\theta\alpha}{\gamma_x(1+\theta)x}[(\frac{T}{\pi_{12}})^{\rho}-(\frac{T}{\pi_{11}})^{\rho}]$. Furthermore, the multiplier on technology accumulation $\gamma_{T1}=\gamma_{T2}\equiv\gamma_T$ and is always positive as higher technology improves the central planner's welfare. Hence, as long as the import share is smaller than the domestic absorption share, i.e. $\pi_{12}\leq\pi_{11}$, the optimal import tariff is negative $\tau^m\leq 0$. The condition $\pi_{12}\leq\pi_{11}$ holds when there exist some iceberg trade costs in this case.

Intuitively, without iceberg trade costs and home bias, the import share equals domestic share ($\pi_{12}=\pi_{11}$), which means that importers and domestic producers bear identical insights. As a result, importers do not bring new insights, and there is no incentive for the central planner to impose import tariffs (hence $\tau_{12}^m=\tau_{21}^m=0$). In contrast, with trade costs, the import share is smaller. By selection, importers are more productive and carry better insight than domestic firms, which motivates the central planner to lower import tariffs to allow domestic firms to learn from importers. When countries are asymmetric, for example, $T_1>T_2$, country 2 would learn from country 1. The planner would lower country 2's import tariffs by more.

Figure A-1: Central Planner's Policies (Different Diffusion ρ)



(a) τ_{12}^m (b) τ_{21}^m (c) change of T

(d) Country 1's relative import quality (e) Country 2's relative import quality (f) change of c

Note: This figure plots the central planner's optimal policies and the associated changes of T and real consumption c at the steady state when we vary diffusion parameter ρ. Country 1's relative import quality is defined as $(T_2/\pi_{12})^{\rho} - (T_1/\pi_{11})^{\rho}$, and Country 2's relative import quality is $(T_1/\pi_{21})^{\rho} - (T_2/\pi_{22})^{\rho}$. τ_{12}^m and τ_{21}^m are the tariffs.

We further consider a numerical example that varies the diffusion parameter ρ from 0 to 0.7. In this numerical example, $\theta = 4$, $\sigma = 2$, $\beta = 0.94$, $\delta = 0.2$, $d_{12} = d_{21} = 1.2$, $L_1 = L_2 = 1$, $\alpha_1 = \alpha_2 = 0.2$, $\lambda_1 = \lambda_2 = 0.5$. Figure A-1 presents the steady-state optimal policies, relative import qualities, the resultant level of technology, and real consumption as ρ rises. The central government imposes import subsidies compared to without diffusion as the relative import quality from each other are positive (Panel (d) and (e)). Both countries' technology and welfare increased.

B Unilateral Markov Policy

B.1 Characterization of Markov Policy

We normalize the price of final goods as 1 and denote countries by i, n. The evolution of technology is given by

$$T_{nt+1} = (1-\delta)T_{nt} + \alpha_n \sum_{i=1}^{N} (\pi_{ni,t})^{1-\rho} (T_{it})^{\rho}. \tag{A.17}$$

Note that Ramsey and Markov problems are the same, given that there are no forward-looking constraints like a worker-researcher choice constraint under endogenous technology accumulation.

At period 0, the Home country government chooses the sequence of $\{\tau_{nt}^x, \tau_{nt}^m\}$ to maximize the present value of utility subject to the private equilibrium:

$$\max \sum_{t=0}^{\infty} \beta^t u(x_{1t}/P_{1t}) = \sum_{t=0}^{\infty} \beta^t \frac{(x_{1t}/P_{1t})^{1-\sigma}}{1-\sigma}$$

subject to for any period:

$$T_{n,t+1} = (1-\delta)T_{nt} + \alpha_n \sum_{i=1}^{N} (\pi_{ni,t})^{1-\rho} (T_{it})^{\rho}, \quad (\gamma_{Tn}, \quad N)$$

$$x_{1t} = w_{1t}L_{1t} + \sum_{i\neq 1}^{N} \frac{\tau_{it}^x}{1+\tau_{it}^x} \pi_{i1,t} x_{it} + \sum_{i\neq 1}^{N} \frac{\tau_{it}^m}{1+\tau_{it}^m} \pi_{1i,t} x_{1t}, \quad (\gamma_x)$$

$$w_{nt}L_{nt} = \sum_{m\neq 1}^{N} \pi_{mn,t} x_{mt} + \frac{1}{1+\tau_{nt}^m} \pi_{1n,t} x_{1t}, \quad (\gamma_n, \quad (N-1), n>1)$$

where

$$x_{it} = w_{it}L_{it}$$

$$P_{1t} = \left[T_{1t} w_{1t}^{-\theta} + \sum_{n\neq 1} T_{nt}(w_{nt}(1+\tau_{nt}^m)d_{1n,t})^{-\theta} \right]^{-\frac{1}{\theta}} = 1$$

$$P_{it} = \left[T_{1t}(w_{1t}(1+\tau_{it}^x)d_{i1,t})^{-\theta} + \sum_{n\neq 1} T_{nt}(w_{nt}d_{in,t})^{-\theta} \right]^{-\frac{1}{\theta}}$$

$$\pi_{11,t} = \frac{T_1 w_1^{-\theta}}{T_{1t} w_{1t}^{-\theta} + \sum_{n\neq 1} T_{nt}(w_{nt}(1+\tau_{nt}^m)d_{1n,t})^{-\theta}}$$

$$\pi_{i1,t} = \frac{T_{1t}(w_{1t}(1+\tau_{it}^x)d_{i1,t})^{-\theta}}{T_{1t}(w_{1t}(1+\tau_{it}^x)d_{i1,t})^{-\theta} + \sum_{n\neq 1} T_{nt}(w_{nt}d_{in,t})^{-\theta}}$$

$$\pi_{in,t} = \frac{T_{nt}(w_{nt}d_{in,t})^{-\theta}}{T_{1t}(w_{1t}(1+\tau_{it}^x)d_{i1,t})^{-\theta} + \sum_{m\neq 1} T_{mt}(w_{mt}d_{im,t})^{-\theta}}$$

$$\pi_{1i,t} = \frac{T_{it}(w_{it}(1+\tau_{it}^m)d_{1i,t})^{-\theta}}{T_{1t} w_{1t}^{-\theta} + \sum_{n\neq 1} T_{nt}(w_{nt}(1+\tau_{nt}^m)d_{1n,t})^{-\theta}}$$

Note that $\sum_n \pi_{in,t} = 1$ for any i in period t.

Recursively problem can be written as

$$V(\{T_n\}) = \max_{\{T_n', w_n, x_n, \tau_{nt}^x, \tau_{nt}^m\}} u(x_1/P_1) + \beta V(\{T_n'\})$$

subject to

$$T_n' = (1-\delta)T_n + \alpha_n \sum_{i=1}^{N} (\pi_{ni})^{1-\rho} (T_i)^{\rho}, \quad (\gamma_{Tn}, \quad N) \tag{A.18}$$

$$x_1 = w_1 L_1 + \sum_{i\neq 1}^{N} \frac{\tau_i^x}{1+\tau_i^x} \pi_{i1} (w_i L_i) + \sum_{i\neq 1}^{N} \frac{\tau_i^m}{1+\tau_i^m} \pi_{1i} x_1, \quad (\gamma_x) \tag{A.19}$$

$$w_n L_n = \sum_{m\neq 1}^{N} \pi_{mn} x_m + \frac{1}{1+\tau_n^m} \pi_{1n} x_1, \quad (\gamma_n, \quad (N-1), n>1) \tag{A.20}$$

B.2 Tax Neutrality: Proof of Proposition 4

This section provides the proof of tax neutrality in our unilateral problem. Given $\Gamma = \{(\tau_{it}^m + 1, \tau_{it}^x + 1) : \forall i \neq 1, t\}$ and $\check{\Gamma} = \{(\check{\tau}_{it}^m + 1, \check{\tau}_{it}^x + 1) : \forall i \neq 1, t\}$, let $\mathcal{E}(\Gamma)$ denote the set of $\{T_{nt+1}$, T_{nt}, $\pi_{1n,t}$, $\pi_{mn,t}$, $\frac{w_{nt}}{P_{nt}}$, $\frac{x_{nt}}{P_{nt}}\}$ that form an equilibrium. We say that from Γ to $\check{\Gamma}$ is neutral if $\mathcal{E}(\Gamma)$=$\mathcal{E}(\check{\Gamma})$. This captures neutrality because the equilibrium allocations and welfare under Γ and $\check{\Gamma}$ are the same.

We assume $\check{\tau}_{it}^m + 1 = \mu_t(\tau_{it}^m + 1)$ and $\check{\tau}_{it}^x + 1 = (\tau_{it}^x + 1)/\mu_t$. We guess the allocations $\{\check{T}_{nt+1}$, $\check{T}_{nt}$, $\check{\pi}_{1n,t}$, $\check{\pi}_{in,t}$, $\frac{\check{w}_{nt}}{\check{P}_{nt}}$, $\frac{\check{x}_{nt}}{\check{P}_{nt}}\}$ in the new equilibrium have the following relations with allocations in the old equilibrium: $\check{T}_{nt+1} = T_{nt+1}, \check{T}_{nt} = T_{nt}, \check{\pi}_{1n,t} = \pi_{1n,t}, \check{\pi}_{in,t} = \pi_{in,t}, \check{P}_{1t} = P_{1t} = 1, \check{P}_{it} = P_{it}/\mu_t, \check{w}_{1t} = w_{1t}, \check{w}_{it} = w_{it}/\mu_t, \check{x}_{1t} = x_{1t}, \check{x}_{it} = x_{it}/\mu_t$. We then verify the following equations can be satisfied under new equilibrium allocations.

Evolution of technology

$$\check{T}_{nt+1} = (1-\delta)T_{nt} + \alpha_n \sum_{i=1}^{N} (\pi_{ni,t})^{1-\rho} (T_{it})^{\rho} = T_{nt+1}$$

Price index

$$\check{P}_{1t} = \left[T_{1t} w_{1t}^{-\theta} + \sum_{n\neq 1} T_{nt} (\frac{w_{nt}}{\mu_t} \mu_t (1+\tau_{nt}^m) d_{1nt})^{-\theta} \right]^{-\frac{1}{\theta}} = P_{1t} = 1$$

$$\check{P}_{it} = \left[T_{1t} (w_{1t} \frac{1+\tau_{it}^x}{\mu_t} d_{i1t})^{-\theta} + \sum_{n\neq 1} T_{nt} (\frac{w_{nt}}{\mu_t} d_{int})^{-\theta} \right]^{-\frac{1}{\theta}} = \frac{P_{it}}{\mu_t}$$

Market clearing conditions ($n \neq 1$)

$$\check{w}_{nt} L_{nt} = \sum_{i\neq 1}^{N} \pi_{in,t} \frac{x_{it}}{\mu_t} + \frac{1}{\mu_t(1+\tau_{nt}^m)} \pi_{1nt} x_{1t} = \frac{w_{nt}}{\mu_t} L_{nt}$$

Trade shares

$$\check{\pi}_{11t} = \frac{T_{1t}w_{1t}^{-\theta}}{T_{1t}w_{1t}^{-\theta} + \sum_{n\neq 1} T_{nt}(\frac{w_{nt}}{\mu_t}\mu_t(1+\tau_{nt}^m)d_{1nt})^{-\theta}} = \pi_{11t}$$

$$\check{\pi}_{i1t} = \frac{T_{1t}(w_{1t}\frac{(1+\tau_{it}^x)}{\mu_t}d_{i1t})^{-\theta}}{T_{1t}(w_{1t}\frac{(1+\tau_{it}^x)}{\mu_t}d_{i1t})^{-\theta} + \sum_{n\neq 1} T_{nt}(\frac{w_{nt}}{\mu_t}d_{int})^{-\theta}} = \pi_{i1t}$$

$$\check{\pi}_{int} = \frac{T_{nt}(\frac{w_{nt}}{\mu_t}d_{int})^{-\theta}}{T_{1t}(w_{1t}\frac{(1+\tau_{it}^x)}{\mu_t}d_{i1t})^{-\theta} + T_{nt}(\frac{w_{nt}}{\mu_t}d_{int})^{-\theta} + \sum_{m\neq\{1,n\}} T_{mt}(\frac{w_{mt}}{\mu_t}d_{imt})^{-\theta}} = \pi_{int}$$

$$\check{\pi}_{1it} = \frac{T_{it}(\frac{w_{it}}{\mu_t}\mu_t(1+\tau_{it}^m)d_{1it})^{-\theta}}{T_{1t}w_{1t}^{-\theta} + T_{it}(\frac{w_{it}}{\mu_t}\mu_t(1+\tau_{it}^m)d_{1it})^{-\theta} + \sum_{n\neq\{1,i\}} T_{nt}(\frac{w_{nt}}{\mu_t}\mu_t(1+\tau_{nt}^m)d_{1nt})^{-\theta}} = \pi_{1it}$$

Foreign's expenditure

$$\check{x}_{it} = \frac{w_{it}}{\mu_t}L_{it} = \frac{x_{it}}{\mu_t}$$

Home's expenditure

$$\begin{aligned}
\check{x}_{1t} &= w_{1t}L_{1t} + \sum_{i\neq 1}^{N}(1 - \frac{\mu_t}{1+\tau_{it}^x})\pi_{i1t}\left(\frac{w_{it}}{\mu_t}L_{it}\right) + \sum_{i\neq 1}^{N}(1 - \frac{1}{\mu_t(1+\tau_{it}^m)})\pi_{1it}x_{1t} \\
&= w_{1t}L_{1t} + \sum_{i\neq 1}^{N}\frac{1+\tau_{it}^x - \mu_t}{\mu_t(1+\tau_{it}^x)}\pi_{i1t}\left(w_{it}L_{it}\right) + \sum_{i\neq 1}^{N}\frac{\mu_t(1+\tau_{it}^m)-1}{\mu_t(1+\tau_{it}^m)}\pi_{1it}x_{1t} \\
&= w_{1t}L_{1t} + \sum_{i\neq 1}^{N}(\frac{\tau_{it}^x}{1+\tau_{it}^x} + \frac{1-\mu_t}{\mu_t})\pi_{i1t}\left(w_{it}L_{it}\right) + \sum_{i\neq 1}^{N}(\frac{\tau_{it}^m}{1+\tau_{it}^m} + \frac{\mu_t - 1}{\mu_t(1+\tau_{it}^m)})\pi_{1it}x_{1t} \\
&= w_{1t}L_{1t} + \sum_{i\neq 1}^{N}\frac{\tau_{it}^x}{1+\tau_{it}^x}\pi_{i1t}\left(w_{it}L_{it}\right) + \sum_{i\neq 1}^{N}\frac{\tau_{it}^m}{1+\tau_{it}^m}\pi_{1it}x_{1t} = x_{1t}
\end{aligned}$$

where the fourth equation used the balanced trade condition for country 1

$$\sum_{i\neq 1}^{N}\pi_{i1t}w_{it}L_{it} = \sum_{i\neq 1}^{N}\frac{1}{1+\tau_{it}^m}\pi_{1it}x_{1t}.$$

Hence, the home expenditure equation can be satisfied under the guessed equilibrium allocations.

We confirm that the guessed allocations and prices satisfy all the equilibrium conditions. This completes the proof of tax neutrality in this model.

B.3 Home's Unilateral Policy with Exogenous Technology: Proof of Proposition 5

We normalize the price of Home's final goods as 1 and denote countries by i, n. The technology of all countries are exogenously given. The Home country government chooses $\{\tau_n^x, \tau_n^m\}$ to maximize the value of utility subject to the private market equilibrium conditions:

$$\max\ u(x_1/P_1) = \frac{(x_1/P_1)^{1-\sigma}}{1-\sigma}$$

Subject to for any period:

$$x_1 = w_1 L_1 + \sum_{i\neq 1}^{N} \frac{\tau_i^x}{1+\tau_i^x}\pi_{i1}x_i + \sum_{i\neq 1}^{N} \frac{\tau_i^m}{1+\tau_i^m}\pi_{1i}x_1, \quad (\gamma_x)$$

$$w_n L_n = \sum_{i\neq 1}^{N} \pi_{in}x_m + \frac{1}{1+\tau_n^m}\pi_{1n}x_1, \quad (\gamma_n, \quad (N-1), n>1)$$

where

$$x_i = w_i L_i$$

$$P_1 = \left[T_1 w_1^{-\theta} + \sum_{n\neq 1} T_n (w_n(1+\tau_n^m)d_{1n})^{-\theta}\right]^{-\frac{1}{\theta}} = 1$$

$$P_i = \left[T_1 (w_1(1+\tau_i^x)d_{i1})^{-\theta} + \sum_{n\neq 1} T_n (w_n d_{in})^{-\theta}\right]^{-\frac{1}{\theta}}$$

$$\pi_{11} = \frac{T_1 w_1^{-\theta}}{T_1 w_1^{-\theta} + \sum_{n\neq 1} T_n (w_n(1+\tau_n^m)d_{1n})^{-\theta}}$$

$$\pi_{i1} = \frac{T_1 (w_1(1+\tau_i^x)d_{i1})^{-\theta}}{T_1 (w_1(1+\tau_i^x)d)^{-\theta} + \sum_{n\neq 1} T_n (w_n d_{in})^{-\theta}}$$

$$\pi_{in} = \frac{T_n (w_n d_{in})^{-\theta}}{T_1 (w_1(1+\tau_i^x)d_{i1})^{-\theta} + T_n (w_n d_{in})^{-\theta} + \sum_{m\neq\{i,n\}} T_m (w_m d_{im})^{-\theta}}$$

$$\pi_{1i} = \frac{T_i (w_i(1+\tau_i^m)d_{1i})^{-\theta}}{T_1 w_1^{-\theta} + T_i (w_i(1+\tau_i^m)d_{1i})^{-\theta} + \sum_{n\neq\{1,i\}} T_n (w_n(1+\tau_n^m)d_{1n})^{-\theta}}$$

Note that $\sum_n \pi_{i,n} = 1$ for any i.

Optimal conditions

FOC over x_1

$$u_c + \sum_{n\neq 1}^{N} \gamma_n \frac{1}{1+\tau_n^m}\pi_{1n} - \gamma_x \left(1 - \sum_{n\neq 1}^{N} \frac{\tau_n^m}{1+\tau_n^m}\pi_{1n}\right) = 0$$

Further

$$\sum_{n\neq 1}^{N}\frac{\gamma_x-\gamma_n}{1+\tau_n^m}\pi_{1n}=u_c-\gamma_x\pi_{11} \tag{A.21}$$

FOC over τ_n^x

$$\sum_{i\neq 1}^{N}\gamma_i\left[\sum_{m\neq 1}\frac{\partial\pi_{m,i}}{\partial\tau_n^x}x_m\right]+\gamma_x\frac{1}{(1+\tau_n^x)^2}\pi_{n1}x_n+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\frac{\partial\pi_{i1}}{\partial\tau_n^x}x_i=0$$

plugging into derivatives

$$\sum_{i\neq 1}^{N}\gamma_i\left[\theta\pi_{n,i}\pi_{n1}x_n\right]+\gamma_x\frac{1}{(1+\tau_n^x)}\pi_{n1}x_n-\gamma_x\frac{\tau_n^x}{1+\tau_n^x}\theta\pi_{n1}(1-\pi_{n1})x_n=0$$

Hence the optimal export tax

$$1+\tau_n^x=\frac{[1+\theta(1-\pi_{n1})]}{\theta\sum_{i\neq 1}(1-\gamma_i/\gamma_x)\pi_{ni}}$$

FOC over τ_n^m

$$\begin{aligned}&-u_c x_1\frac{\pi_{1n}}{1+\tau_n^m}-\gamma_n\frac{\pi_{1n}}{(1+\tau_n^m)^2}x_1\\&+\gamma_n\frac{1}{1+\tau_n^m}\frac{\partial\pi_{1n}}{\partial\tau_n^m}x_1+\sum_{i\neq\{1,n\}}\gamma_i\frac{1}{1+\tau_i^m}\frac{\partial\pi_{1i}}{\partial\tau_n^m}x_1+\gamma_x\frac{1}{(1+\tau_n^m)^2}\pi_{1n}x_1+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\frac{\partial\pi_{1i}}{\partial\tau_n^m}x_1=0\end{aligned}$$

plugging in derivatives

$$\begin{aligned}&-u_c x_1\frac{\pi_{1n}}{1+\tau_n^m}-\gamma_n\frac{\pi_{1n}}{(1+\tau_n^m)^2}x_1-\gamma_n\frac{1}{1+\tau_n^m}x_1\theta\frac{1}{1+\tau_n^m}\pi_{1n}\left(1-\pi_{1n}\right)+\sum_{i\neq\{1,n\}}\gamma_i\frac{1}{1+\tau_i^m}x_1\theta\frac{1}{1+\tau_n^m}\pi_{1i}\pi_{1n}\\&+\gamma_x\frac{1}{(1+\tau_n^m)^2}\pi_{1n}x_1-\gamma_x\frac{\tau_n^m}{1+\tau_n^m}\theta\frac{1}{1+\tau_n^m}\pi_{1n}\left(1-\pi_{1n}\right)x_1+\gamma_x\sum_{i\neq\{1,n\}}^{N}\frac{\tau_i^m}{1+\tau_i^m}x_1\theta\frac{1}{1+\tau_n^m}\pi_{1i}\pi_{1n}=0\end{aligned}$$

Further

$$-u_c+\sum_{i\neq\{1,n\}}\gamma_i\theta\pi_{1i}+\left(\gamma_x-\gamma_n\right)\frac{1}{(1+\tau_n^m)}\left[1+\theta\left(1-\pi_{1n}\right)\right]-\gamma_x\theta\left(1-\pi_{1n}\right)+\sum_{i\neq\{1,n\}}^{N}\left(\gamma_x-\gamma_i\right)\frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}=0$$

Hence the optimal import tariff

$$\frac{1}{1+\tau_n^m}=\frac{u_c+\gamma_x\left[\theta\pi_{11}+\sum_{i\neq\{1,n\}}^{N}\left(\frac{1-\gamma_i/\gamma_x}{1+\tau_i^m}\right)\theta\pi_{1i}\right]}{\left(\gamma_x-\gamma_n\right)\left[1+\theta\left(1-\pi_{1n}\right)\right]} \tag{A.22}$$

Combine the FOCs of tariffs (A.22) and of x_1 (A.21), we get

$$\tau_n^m = -\frac{\gamma_n}{u_c},\ \gamma_x = u_c \tag{A.23}$$

and

$$1+\tau_n^x = \frac{[1+\theta(1-\pi_{n1})]}{\theta\sum_{i\neq 1}(1+\tau_i^m)\pi_{ni}} \tag{A.24}$$

These imply the optimal trade policies under exogenous T feature country-specific tariffs and country-specific export taxes, which finishes the proof of optimal import tariff and export tax in Proposition 5.

FOC over w_1

$$\begin{aligned}
&-u_c x_1 \frac{\pi_{11}}{w_1} + \sum_{n\neq 1}^{N}\gamma_n \frac{1}{1+\tau_n^m}\frac{\partial \pi_{1n}}{\partial w_1}x_1 + \sum_{n\neq 1}^{N}\gamma_n\sum_{m\neq 1}^{N}\frac{\partial \pi_{mn}}{\partial w_1}x_m \\
&+\gamma_x L_1 + \gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\frac{\partial \pi_{i1}}{\partial w_1}x_i + \gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\frac{\partial \pi_{1i}}{\partial w_1}x_1 = 0
\end{aligned}$$

Plugging in derivatives

$$\begin{aligned}
&-u_c x_1\pi_{11} + \sum_{n\neq 1}^{N}\gamma_n\frac{1}{1+\tau_n^m}\theta\pi_{1n}\pi_{11}x_1 + \sum_{n\neq 1}^{N}\gamma_n\sum_{m\neq 1}^{N}\theta\pi_{mn}\pi_{m1}x_m + \gamma_x w_1 L_1 \\
&-\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\theta\pi_{i1}(1-\pi_{i1})x_i + \gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}\pi_{11}x_1 = 0
\end{aligned}$$

Final on w_1:

$$\begin{aligned}
&\sum_{n\neq 1}^{N}\gamma_n\left[\left(\frac{1}{1+\tau_n^m}\theta\pi_{1n}\pi_{11}x_1 + \sum_{m\neq 1}^{N}\theta\pi_{mn}\pi_{m1}x_m\right)\right] \\
&+\gamma_x\left[w_1L_1 + \left(-\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\theta\pi_{i1}(1-\pi_{i1})x_i + \sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}\pi_{11}x_1\right)\right] = u_c x_1\pi_{11}
\end{aligned}$$

FOC over w_n for $n \neq 1$

$$\begin{aligned}
&-u_c x_1\frac{\pi_{1n}}{w_n} + \gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\frac{\partial\pi_{i1}}{\partial w_n}x_i + \gamma_x\frac{\tau_n^x}{1+\tau_n^x}\pi_{n1}\frac{x_n}{w_n} \\
&+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\frac{\partial\pi_{1i}}{\partial w_n}x_1 + \sum_{i\neq 1}\sum_{m\neq 1}^{N}\gamma_i\frac{\partial\pi_{m,i}}{\partial w_n}x_m + \sum_{i\neq 1}\gamma_i\pi_{n,i}\frac{x_n}{w_n} + \sum_{i\neq 1}\gamma_i\frac{1}{1+\tau_i^m}\frac{\partial\pi_{1,i}}{\partial w_n}x_1 - \gamma_n\frac{x_n}{w_n} = 0
\end{aligned}$$

Plugging in derivatives

$$
\begin{aligned}
&-u_c x_1 \pi_{1n} + \gamma_x \sum_{i\neq 1}^{N} \frac{\tau_i^x}{1+\tau_i^x}\theta\pi_{i1}\pi_{in}x_i + \gamma_x \frac{\tau_n^x}{1+\tau_n^x}\pi_{n1}x_n + \gamma_x \sum_{i\neq\{1,n\}}^{N} \frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}\pi_{1n}x_1 \\
&-\gamma_x \frac{\tau_n^m}{1+\tau_n^m}\theta\pi_{1n}(1-\pi_{1n})x_1 + \sum_{i\neq\{1,n\}}\sum_{m\neq 1}^{N}\gamma_i\theta\pi_{mi}\pi_{mn}x_m - \sum_{m\neq 1}^{N}\gamma_n\theta\pi_{mn}(1-\pi_{mn})x_m + \sum_{i\neq 1}\gamma_i\pi_{n,i}x_n \\
&+\sum_{i\neq\{1,n\}}\gamma_i\frac{1}{1+\tau_i^m}\theta\pi_{1i}\pi_{1n}x_1 - \gamma_n\frac{1}{1+\tau_n^m}\theta\pi_{1n}(1-\pi_{1n})x_1 - \gamma_n x_n = 0
\end{aligned}
$$

Final on w_n:

$$
\begin{aligned}
&-u_c x_1 \pi_{1n} + \gamma_x \sum_{i\neq 1}^{N} \frac{\tau_i^x}{1+\tau_i^x}\theta\pi_{i1}\pi_{in}x_i + \gamma_x \frac{\tau_n^x}{1+\tau_n^x}\pi_{n1}x_n + \gamma_x \sum_{i\neq 1}^{N} \frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}\pi_{1n}x_1 - \gamma_x\frac{\tau_n^m}{1+\tau_n^m}\theta\pi_{1n}x_1 \\
&+\sum_{i\neq 1}\sum_{m\neq 1}^{N}\gamma_i\theta\pi_{mi}\pi_{mn}x_m - \sum_{m\neq 1}^{N}\gamma_n\theta\pi_{mn}x_m + \sum_{i\neq 1}\gamma_i\pi_{n,i}x_n + \sum_{i\neq 1}\gamma_i\frac{1}{1+\tau_i^m}\theta\pi_{1i}\pi_{1n}x_1 - \gamma_n\frac{1}{1+\tau_n^m}\theta\pi_{1n}x_1 - \gamma_n x_n = 0
\end{aligned}
\tag{A.25}
$$

Using equations (A.23) and (A.24), the above equations are system equations of γ_n. Q.E.D.

B.4 Optimal Conditions and Proof of Proposition 6

Recall we define the weighted average of quality of insights in country n as $I_n = \sum_{i=1}^{N} (\pi_{ni})^{1-\rho} (T_i)^{\rho}$.

FOC over x_1

$$
u_c + \sum_{n\neq 1}^{N}\gamma_n\frac{1}{1+\tau_n^m}\pi_{1n} - \gamma_x\left(1 - \sum_{n\neq 1}^{N}\frac{\tau_n^m}{1+\tau_n^m}\pi_{1n}\right) = 0 \tag{A.26}
$$

FOC over τ_n^x

$$
\begin{aligned}
&\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\sum_{m=1}^{N}(1-\rho)(\pi_{im})^{-\rho}(T_m)^{\rho}\frac{\partial\pi_{im}}{\partial\tau_n^x} + \sum_{i\neq 1}^{N}\gamma_i\left[\sum_{m\neq 1}\frac{\partial\pi_{mi}}{\partial\tau_n^x}x_m\right] \\
&+\gamma_x\frac{1}{(1+\tau_n^x)^2}\pi_{n1}x_n + \gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\frac{\partial\pi_{i1}}{\partial\tau_n^x}x_i = 0
\end{aligned}
$$

plugging into derivatives

$$
\begin{aligned}
&\gamma_{Tn}\alpha_n(1-\rho)\theta\left[\sum_{m\neq 1}^{N}(\pi_{nm})^{-\rho}(T_m)^{\rho}\pi_{nm}\pi_{n1} - (\pi_{n1})^{-\rho}(T_1)^{\rho}\pi_{n1}(1-\pi_{n1})\right] \\
&+\sum_{i\neq 1}^{N}\gamma_i\left[\theta\pi_{ni}\pi_{n1}x_n\right] + \gamma_x\frac{1}{(1+\tau_n^x)}\pi_{n1}x_n - \gamma_x\frac{\tau_n^x}{1+\tau_n^x}\theta\pi_{n1}(1-\pi_{n1})x_n = 0
\end{aligned}
$$

Hence the optimal export taxes satisfy

$$1+\tau_n^x = \frac{\gamma_x\left[1+\theta(1-\pi_{n1})\right]}{\gamma_x\theta\sum_{m\neq 1}(1-\gamma_m/\gamma_x)\pi_{nm}+\gamma_{Tn}(1-\rho)\theta\frac{1}{x_n}\alpha_n\left[\left(\frac{T_1}{\pi_{n1}}\right)^{\rho}-I_n\right]}, \tag{A.27}$$

this finishes the proof of optimal export tax in Proposition 6.

FOC over τ_n^m

$$\begin{aligned}
&\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\sum_{m=1}^{N}(1-\rho)\left(\pi_{im}\right)^{-\rho}\left(T_m\right)^{\rho}\frac{\partial\pi_{im}}{\partial\tau_n^m}-u_c x_1\frac{\pi_{1n}}{1+\tau_n^m}-\gamma_n\frac{\pi_{1n}}{(1+\tau_n^m)^2}x_1\\
&+\gamma_n\frac{1}{1+\tau_n^m}\frac{\partial\pi_{1n}}{\partial\tau_n^m}x_1+\sum_{i\neq\{1,n\}}\gamma_i\frac{1}{1+\tau_i^m}\frac{\partial\pi_{1i}}{\partial\tau_n^m}x_1+\gamma_x\frac{1}{(1+\tau_n^m)^2}\pi_{1n}x_1+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\frac{\partial\pi_{1i}}{\partial\tau_n^m}x_1=0
\end{aligned}$$

plugging in derivatives

$$\begin{aligned}
&\gamma_{T1}\alpha_1(1-\rho)\left(1+\tau_n^m\right)^{-1}\left[\sum_{m\neq n}^{N}\left(\pi_{1m}\right)^{-\rho}\left(T_m\right)^{\rho}\theta\pi_{1m}\pi_{1n}-\left(\pi_{1n}\right)^{-\rho}\left(T_n\right)^{\rho}\theta\pi_{1n}(1-\pi_{1n})\right]-u_c x_1\frac{\pi_{1n}}{1+\tau_n^m}\\
&-\gamma_n\frac{\pi_{1n}}{(1+\tau_n^m)^2}x_1-\gamma_n\frac{1}{1+\tau_n^m}x_1\theta\frac{1}{1+\tau_n^m}\pi_{1n}\left(1-\pi_{1n}\right)+\sum_{i\neq\{1,n\}}\gamma_i\frac{1}{1+\tau_i^m}x_1\theta\frac{1}{1+\tau_n^m}\pi_{1i}\pi_{1n}\\
&+\gamma_x\frac{1}{(1+\tau_n^m)^2}\pi_{1n}x_1-\gamma_x\frac{\tau_n^m}{1+\tau_n^m}\theta\frac{1}{1+\tau_n^m}\pi_{1n}\left(1-\pi_{1n}\right)x_1+\gamma_x\sum_{i\neq\{1,n\}}^{N}\frac{\tau_i^m}{1+\tau_i^m}x_1\theta\frac{1}{1+\tau_n^m}\pi_{1i}\pi_{1n}=0
\end{aligned}$$

Further

$$\begin{aligned}
&\gamma_{T1}(1-\rho)\frac{1}{x_1}\theta\alpha_1\left[I_1-\left(\pi_{1n}\right)^{-\rho}\left(T_n\right)^{\rho}\right]-u_c+\sum_{m\neq\{1,n\}}\gamma_m\theta\pi_{1m}\\
&+\left(\gamma_x-\gamma_n\right)\frac{1}{(1+\tau_n^m)}\left[1+\theta\left(1-\pi_{1n}\right)\right]-\gamma_x\theta\left(1-\pi_{1n}\right)+\sum_{i\neq\{1,n\}}^{N}\left(\gamma_x-\gamma_i\right)\frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}=0
\end{aligned}$$

Hence the optimal import tariffs satisfy:

$$\frac{1}{1+\tau_n^m}=\frac{u_c+\gamma_x\left[\theta\pi_{11}+\sum_{i\neq\{1,n\}}^{N}(\frac{1-\gamma_i/\gamma_x}{1+\tau_i^m})\theta\pi_{1i}\right]+\gamma_{T1}(1-\rho)\frac{1}{x_1}\theta\alpha_1\left[\left(\frac{T_n}{\pi_{1n}}\right)^{\rho}-I_1\right]}{\left(\gamma_x-\gamma_n\right)\left[1+\theta\left(1-\pi_{1n}\right)\right]} \tag{A.28}$$

Combine FOC over τ_n^m (A.28) and FOC over x_1 (A.26):

$$((\gamma_x-\gamma_n)\frac{1}{1+\tau_n^m}-u_c)(1+\theta)x_1=\gamma_{T1}(1-\rho)\theta\alpha_1\left[\left(\frac{T_n}{\pi_{1n}}\right)^{\rho}-I_1\right] \tag{A.29}$$

For country 1, it has

$$(\gamma_x-u_c)(1+\theta)x_1=\gamma_{T1}(1-\rho)\theta\alpha_1\left[\left(\frac{T_1}{\pi_{11}}\right)^{\rho}-I_1\right] \tag{A.30}$$

Combining equation (A.29) and (A.30), the optimal import tariffs satisfy

$$\frac{1}{1+\tau_n^m} = \frac{\gamma_x}{\gamma_x - \gamma_n} + \gamma_{T1}\frac{(1-\rho)\theta\alpha_1}{(\gamma_x-\gamma_n)(1+\theta)x_1}\left[\left(\frac{T_n}{\pi_{1n}}\right)^\rho - \left(\frac{T_1}{\pi_{11}}\right)^\rho\right] \tag{A.31}$$

this finishes the proof of optimal tariff in Proposition 6.

To sum up, unilateral optimal policy include $N-1$ tariffs, $N-1$ export taxes, and one of them are redundant,

$$\begin{pmatrix} 0 & \tau_{12}^m & \tau_{13}^m & ... & \tau_{1N}^m \\ \tau_{21}^x & 0 & 0 & ... & 0 \\ \tau_{31}^x & 0 & 0 & ... & 0 \\ ... & ... & ... & ... & ... \\ \tau_{N1}^x & 0 & 0 & ... & 0 \end{pmatrix}$$

$$\frac{1}{1+\tau_n^m} = \frac{\gamma_x}{\gamma_x - \gamma_n} + \gamma_{T1}\frac{(1-\rho)\theta\alpha_1}{(\gamma_x-\gamma_n)(1+\theta)x_1}\left[\left(\frac{T_n}{\pi_{1n}}\right)^\rho - \left(\frac{T_1}{\pi_{11}}\right)^\rho\right] \tag{A.32}$$

$$\frac{1}{1+\tau_n^x} = \frac{\theta\sum_{i\neq 1}(1-\gamma_i/\gamma_x)\pi_{ni} + \frac{(1-\rho)\theta}{\gamma_x x_n}\alpha_n\gamma_{Tn}\left[\left(\frac{T_1}{\pi_{n1}}\right)^\rho - I_n\right]}{1+\theta(1-\pi_{n1})} \tag{A.33}$$

Furthermore, all the multipliers are jointly pinned down by the following additional FOCs:

FOC over w_1

$$\sum_{i=1}^N \gamma_{Ti}\alpha_i \sum_{m=1}^N (1-\rho)\left(\pi_{im}\right)^{-\rho}\left(T_m\right)^\rho \frac{\partial \pi_{im}}{\partial w_1} - u_c x_1 \frac{\pi_{11}}{w_1} + \sum_{n\neq 1}^N \gamma_n \frac{1}{1+\tau_n^m}\frac{\partial \pi_{1n}}{\partial w_1}x_1$$
$$+\sum_{n\neq 1}^N \gamma_n \sum_{m\neq 1}^N \frac{\partial \pi_{mn}}{\partial w_1}x_m + \gamma_x L_1 + \gamma_x \sum_{i\neq 1}^N \frac{\tau_i^x}{1+\tau_i^x}\frac{\partial \pi_{i1}}{\partial w_1}x_i + \gamma_x\sum_{i\neq 1}^N \frac{\tau_i^m}{1+\tau_i^m}\frac{\partial \pi_{1i}}{\partial w_1}x_1 = 0$$

Plugging in derivatives

$$\sum_{i=1}^N \gamma_{Ti}\alpha_i(1-\rho)\left[\sum_{m\neq 1}^N \left(\pi_{im}\right)^{-\rho}\left(T_m\right)^\rho \theta\pi_{im}\pi_{i1} - \left(\pi_{i1}\right)^{-\rho}\left(T_1\right)^\rho\theta\pi_{i1}(1-\pi_{i1})\right] - u_c x_1\pi_{11} + \sum_{n\neq 1}^N \gamma_n \frac{1}{1+\tau_n^m}\theta\pi_{1n}\pi_{11}x_1$$
$$+\sum_{n\neq 1}^N \gamma_n \sum_{m\neq 1}^N \theta\pi_{mn}\pi_{m1}x_m + \gamma_x w_1 L_1 - \gamma_x\sum_{i\neq 1}^N \frac{\tau_i^x}{1+\tau_i^x}\theta\pi_{i1}(1-\pi_{i1})x_i + \gamma_x\sum_{i\neq 1}^N \frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}\pi_{11}x_1 = 0$$

Final on w_1:

$$\sum_{n=1}^{N}\gamma_{Tn}(1-\rho)\theta\pi_{n1}\alpha_n\left[I_n-(\pi_{n1})^{-\rho}(T_1)^{\rho}\right]+\sum_{n\neq 1}^{N}\gamma_n\left[\left(\frac{1}{1+\tau_n^m}\theta\pi_{1n}\pi_{11}x_1+\sum_{m\neq 1}^{N}\theta\pi_{mn}\pi_{m1}x_m\right)\right]$$
$$+\gamma_x\left[w_1L_1+\left(-\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\theta\pi_{i1}(1-\pi_{i1})x_i+\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}\pi_{11}x_1\right)\right]=u_cx_1\pi_{11}$$

FOC over w_n for $n\neq 1$

$$\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\sum_{m=1}^{N}(1-\rho)\left(\pi_{im}\right)^{-\rho}\left(T_m\right)^{\rho}\frac{\partial\pi_{im}}{\partial w_n}-u_cx_1\frac{\pi_{1n}}{w_n}+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\frac{\partial\pi_{i1}}{\partial w_n}x_i+\gamma_x\frac{\tau_n^x}{1+\tau_n^x}\pi_{n1}\frac{x_n}{w_n}$$
$$+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\frac{\partial\pi_{1i}}{\partial w_n}x_1+\sum_{i\neq 1}\sum_{m\neq 1}^{N}\gamma_i\frac{\partial\pi_{mi}}{\partial w_n}x_m+\sum_{i\neq 1}\gamma_i\pi_{n,i}\frac{x_n}{w_n}+\sum_{i\neq 1}\gamma_i\frac{1}{1+\tau_i^m}\frac{\partial\pi_{1i}}{\partial w_n}x_1-\gamma_n\frac{x_n}{w_n}=0$$

Plugging in derivatives

$$\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\sum_{m\neq n}^{N}(1-\rho)\left(\pi_{im}\right)^{-\rho}\left(T_m\right)^{\rho}\theta\pi_{im}\pi_{in}-\sum_{i=1}^{N}\gamma_{Ti}\alpha_i(1-\rho)\left(\pi_{in}\right)^{-\rho}\left(T_n\right)^{\rho}\theta\pi_{in}(1-\pi_{in})$$
$$-u_cx_1\pi_{1n}+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\theta\pi_{i1}\pi_{in}x_i+\gamma_x\frac{\tau_n^x}{1+\tau_n^x}\pi_{n1}x_n+\gamma_x\sum_{i\neq\{1,n\}}^{N}\frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}\pi_{1n}x_1$$
$$-\gamma_x\frac{\tau_n^m}{1+\tau_n^m}\theta\pi_{1n}(1-\pi_{1n})x_1+\sum_{i\neq\{1,n\}}\sum_{m\neq 1}^{N}\gamma_i\theta\pi_{mi}\pi_{mn}x_m-\sum_{m\neq 1}^{N}\gamma_n\theta\pi_{mn}(1-\pi_{mn})x_m+\sum_{i\neq 1}\gamma_i\pi_{ni}x_n$$
$$+\sum_{i\neq\{1,n\}}\gamma_i\frac{1}{1+\tau_i^m}\theta\pi_{1i}\pi_{1n}x_1-\gamma_i\frac{1}{1+\tau_n^m}\theta\pi_{1n}(1-\pi_{1n})x_1-\gamma_nx_n=0$$

Final on w_n:

$$\sum_{i=1}^{N}\gamma_{Ti}(1-\rho)\pi_{in}\theta\alpha_i\left[I_i-(\pi_{in})^{-\rho}(T_n)^{\rho}\right]-u_cx_1\pi_{1n}+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\theta\pi_{i1}\pi_{in}x_i+\gamma_x\frac{\tau_n^x}{1+\tau_n^x}\pi_{n1}x_n$$
$$+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}\pi_{1n}x_1-\gamma_x\frac{\tau_n^m}{1+\tau_n^m}\theta\pi_{1n}x_1+\sum_{i\neq 1}\sum_{m\neq 1}^{N}\gamma_i\theta\pi_{mi}\pi_{mn}x_m-\sum_{m\neq 1}^{N}\gamma_n\theta\pi_{mn}x_m+\sum_{i\neq 1}\gamma_i\pi_{ni}x_n$$
$$+\sum_{i\neq 1}\gamma_i\frac{1}{1+\tau_i^m}\theta\pi_{1i}\pi_{1n}x_1-\gamma_n\frac{1}{1+\tau_n^m}\theta\pi_{1n}x_1-\gamma_nx_n=0$$

FOC over T_1

$$
\begin{aligned}
&-\gamma_{T1,-1}+\beta\Big\{(1-\delta)\gamma_{T1}+\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\rho\left(\pi_{i1}\right)^{1-\rho}\left(T_1\right)^{\rho-1}+\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\sum_{m=1}^{N}(1-\rho)\left(\pi_{im}\right)^{-\rho}\left(T_m\right)^{\rho}\frac{\partial\pi_{im}}{\partial T_1}\\
&+u_c x_1\frac{1}{\theta}\frac{\pi_{11}}{T_1}+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\frac{\partial\pi_{i1}}{\partial T_1}x_i+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\frac{\partial\pi_{1i}}{\partial T_1}x_1+\sum_{i\neq 1}\gamma_i\sum_{m\neq 1}^{N}\frac{\partial\pi_{mi}}{\partial T_1}x_m+\sum_{i\neq 1}\gamma_i\frac{1}{1+\tau_i^m}\frac{\partial\pi_{1i}}{\partial T_1}x_1\Big\}=0
\end{aligned}
$$

Plugging in derivatives

$$
\begin{aligned}
&-\gamma_{T1,-1}T_1+\beta\Big\{(1-\delta)\gamma_{T1}T_1+\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\rho\left(\pi_{i1}\right)^{1-\rho}\left(T_1\right)^{\rho}-\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\sum_{m=1}^{N}(1-\rho)\left(\pi_{im}\right)^{-\rho}\left(T_m\right)^{\rho}\pi_{i1}\pi_{im}\\
&+\sum_{i=1}^{N}\gamma_{Ti}\alpha_i(1-\rho)\left(\pi_{i1}\right)^{-\rho}\left(T_1\right)^{\rho}\pi_{i1}+u_c x_1\frac{1}{\theta}\pi_{11}+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\pi_{i1}(1-\pi_{i1})x_i-\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\pi_{11}\pi_{1i}x_1\\
&-\sum_{i\neq 1}\gamma_i\sum_{m\neq 1}^{N}\pi_{m1}\pi_{mi}x_m-\sum_{i\neq 1}\gamma_i\frac{1}{1+\tau_i^m}\pi_{11}\pi_{1i}x_1\Big\}=0
\end{aligned}
$$

Final on T_1:

$$
\begin{aligned}
&-\gamma_{T1,-1}T_1+\beta\Big\{(1-\delta)\gamma_{T1}T_1+\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\left(\pi_{i1}\right)^{1-\rho}\left(T_1\right)^{\rho}-\sum_{i=1}^{N}\gamma_{Ti}\pi_{i1}(1-\rho)\alpha_i I_i+u_c x_1\frac{1}{\theta}\pi_{11}\\
&+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\pi_{i1}(1-\pi_{i1})x_i-\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\pi_{11}\pi_{1i}x_1-\sum_{i\neq 1}\gamma_i\sum_{m\neq 1}^{N}\pi_{m1}\pi_{mi}x_m-\sum_{i\neq 1}\gamma_i\frac{1}{1+\tau_i^m}\pi_{11}\pi_{1i}x_1\Big\}=0
\end{aligned}
$$

FOC over T_n

$$
\begin{aligned}
&-\gamma_{Tn,-1}+\beta\Big\{(1-\delta)\gamma_{Tn}+\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\rho\left(\pi_{in}\right)^{1-\rho}\left(T_n\right)^{\rho-1}+\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\sum_{m=1}^{N}(1-\rho)\left(\pi_{im}\right)^{-\rho}\left(T_m\right)^{\rho}\frac{\partial\pi_{im}}{\partial T_n}\\
&+u_c x_1\frac{1}{\theta}\frac{\pi_{1n}}{T_n}+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\frac{\partial\pi_{i1}}{\partial T_n}x_i+\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\frac{\partial\pi_{1i}}{\partial T_n}x_1+\sum_{i\neq 1}\gamma_i\sum_{m\neq 1}^{N}\frac{\partial\pi_{mi}}{\partial T_n}x_m+\sum_{i\neq 1}\gamma_i\frac{1}{1+\tau_i^m}\frac{\partial\pi_{1i}}{\partial T_n}x_1\Big\}=0
\end{aligned}
$$

Plugging in derivatives

$$
\begin{aligned}
&-\gamma_{Tn,-1}T_n+\beta\Big\{(1-\delta)\gamma_{Tn}T_n+\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\rho\left(\pi_{i,n}\right)^{1-\rho}\left(T_n\right)^{\rho}-\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\sum_{m=1}^{N}(1-\rho)\left(\pi_{im}\right)^{-\rho}\left(T_m\right)^{\rho}\pi_{im}\pi_{in}\\
&+\sum_{i=1}\gamma_{Ti}\alpha_i(1-\rho)\left(\pi_{in}\right)^{-\rho}\left(T_n\right)^{\rho}\pi_{i,n}+u_c x_1\frac{1}{\theta}\pi_{1n}-\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\pi_{i1}\pi_{in}x_i-\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\pi_{1i}\pi_{1n}x_1\\
&+\gamma_x\frac{\tau_n^m}{1+\tau_n^m}\pi_{1n}x_1-\sum_{i\neq 1}\gamma_i\sum_{m\neq 1}^{N}\pi_{mi}\pi_{mn}x_m+\gamma_n\sum_{m\neq 1}^{N}\pi_{mn}x_m-\sum_{i\neq 1}\gamma_i\frac{1}{1+\tau_i^m}\pi_{1i}\pi_{1n}x_1+\gamma_n\frac{1}{1+\tau_n^m}\pi_{1n}x_1\Big\}=0
\end{aligned}
$$

Final on T_n:

$$
\begin{aligned}
&-\gamma_{Tn,-1}T_n+\beta\Big\{(1-\delta)\gamma_{Tn}T_n+\sum_{i=1}^{N}\gamma_{Ti}\alpha_i\left(\pi_{i,n}\right)^{1-\rho}\left(T_n\right)^{\rho}-\sum_{i=1}^{N}\gamma_{Ti}\pi_{i,n}(1-\rho)\alpha_i I_i \\
&+u_c x_1\frac{1}{\theta}\pi_{1n}-\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\pi_{i1}\pi_{in}x_i-\gamma_x\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\pi_{1i}\pi_{1n}x_1+\gamma_x\frac{\tau_n^m}{1+\tau_n^m}\pi_{1n}x_1 \\
&-\sum_{i\neq 1}\gamma_i\sum_{m\neq 1}^{N}\pi_{mi}\pi_{mn}x_m+\gamma_n\sum_{m\neq 1}^{N}\pi_{mn}x_m-\sum_{i\neq 1}\gamma_i\frac{1}{1+\tau_i^m}\pi_{1i}\pi_{1n}x_1+\gamma_n\frac{1}{1+\tau_n^m}\pi_{1n}x_1\Big\}=0.
\end{aligned}
$$

B.5 Proof of Corollary 2

For the case of two countries, the optimal export tax is derived from the FOC over τ_2^x:

$$\frac{\gamma_x x_2}{1+\tau_2^x}-\gamma_x\frac{\tau_2^x}{1+\tau_2^x}\theta\pi_{22}x_2+\gamma_2\theta\pi_{22}x_2-\gamma_{T_2}(1-\rho)\theta\alpha_2\pi_{22}((\frac{T_1}{\pi_{21}})^{\rho}-(\frac{T_2}{\pi_{22}})^{\rho})=0$$

Optimal export tax:

$$\frac{1}{1+\tau_2^x}=\frac{(\gamma_x-\gamma_2)\theta\pi_{22}x_2+\gamma_{T_2}(1-\rho)\theta\alpha_2\pi_{22}\left[(\frac{T_1}{\pi_{21}})^{\rho}-(\frac{T_2}{\pi_{22}})^{\rho}\right]}{\gamma_x(1+\theta\pi_{22})x_2}$$

To derive the optimal import tariff, we combine the FOC over τ_2^m and FOC over x_1 FOC over τ_2^m:

$$\frac{1}{1+\tau_2^m}=\frac{(u_c+\gamma_x\theta\pi_{11})x_1+\gamma_{T_1}(1-\rho)\theta\pi_{11}\alpha_1((\frac{T_2}{\pi_{12}})^{\rho}-(\frac{T_1}{\pi_{11}})^{\rho})}{(\gamma_x-\gamma_2)(1+\theta\pi_{11})x_1}$$

FOC over x_1:

$$u_c=\gamma_x(1-\frac{\tau_2^m}{1+\tau_2^m}\pi_{12})-\gamma_2\frac{1}{1+\tau_2^m}\pi_{12}$$

Optimal import tariff:

$$\frac{1}{1+\tau_2^m}=\frac{\gamma_x}{\gamma_x-\gamma_2}+\frac{\gamma_{T_1}(1-\rho)\theta\alpha_1\pi_{11}\left[(\frac{T_2}{\pi_{12}})^{\rho}-(\frac{T_1}{\pi_{11}})^{\rho}\right]}{(\gamma_x-\gamma_2)(1+\theta)\pi_{11}x_1}$$

According to the tax neutrality in Proposition 4, we can normalize one variable without affecting real allocations.

1) By normalizing $\tau_2^x=0$, we get

$$\gamma_2=\frac{-\gamma_x x_2+\gamma_{T_2}(1-\rho)\theta\alpha_2\pi_{22}\left[(\frac{T_1}{\pi_{21}})^{\rho}-(\frac{T_2}{\pi_{22}})^{\rho}\right]}{\theta\pi_{22}x_2}$$

$$\frac{1}{1+\tau_2^m} = \frac{1}{\gamma_x - \gamma_2}(\gamma_x + \frac{\gamma_{T_1}(1-\rho)\theta\alpha_1\pi_{11}\left[(\frac{T_2}{\pi_{12}})^\rho - (\frac{T_1}{\pi_{11}})^\rho\right]}{(1+\theta)\pi_{11}x_1})$$

2) By normalizing $\tau_2^m = 0$, we get

$$\gamma_2 = -\frac{\gamma_{T_1}(1-\rho)\theta\alpha_1\pi_{11}\left[(\frac{T_2}{\pi_{12}})^\rho - (\frac{T_1}{\pi_{11}})^\rho\right]}{(1+\theta)\pi_{11}x_1}$$

$$\frac{1}{1+\tau_2^x} = \frac{(\gamma_x-\gamma_2)\theta\pi_{22}x_2 + \gamma_{T_2}(1-\rho)\theta\alpha_2\pi_{22}\left[(\frac{T_1}{\pi_{21}})^\rho - (\frac{T_2}{\pi_{22}})^\rho\right]}{\gamma_x(1+\theta\pi_{22})x_2}$$

3) By normalizing $\gamma_2 = -\frac{\gamma_x}{\theta\pi_{22}}$, we get equation (14) and equation (13) in the paper:

$$\frac{1}{1+\tau_2^x} = 1 + \frac{\gamma_{T_2}(1-\rho)\theta\frac{1}{\gamma_x x_2}\alpha_2\pi_{22}\left[(\frac{T_1}{\pi_{21}})^\rho - (\frac{T_2}{\pi_{22}})^\rho\right]}{1+\theta\pi_{22}}$$

$$\frac{1}{1+\tau_2^m} = \frac{\theta\pi_{22}}{1+\theta\pi_{22}}(1 + \frac{\gamma_{T_1}(1-\rho)\theta\alpha_1\pi_{11}\left[(\frac{T_2}{\pi_{12}})^\rho - (\frac{T_1}{\pi_{11}})^\rho\right]}{\gamma_x(1+\theta)\pi_{11}x_1})$$

To back out multipliers γ_x, γ_{T_1} and γ_{T_2}, we use the following three equations.

1. Combining the FOC on T_1 and T_2:

$$\sum_{n=1}^{2}\gamma_{T_n}T_n\theta(1-\beta+\beta\delta(1-\rho)) = \beta u_c x_1$$

2. Combining the FOC on w_2 and T_2:

$$-\gamma_2\pi_{21}x_2 - \frac{\gamma_{T_2}T_2\theta}{\beta} + \gamma_x\frac{\tau_x}{1+\tau_x}\pi_{21}x_2 + (1-\delta)\gamma_{T_2}T_2\theta + \sum_{n=1}^{2}\gamma_{T_n}\alpha_n\pi_{n2}^{1-\rho}\rho T_2^\rho\theta = 0$$

3. Combining the FOC on x_1 and τ_2^m:

$$(u_c-\gamma_x)(1+\theta)x_1 = \gamma_{T_1}(1-\rho)\theta\pi_{12}\alpha_1((\frac{T_2}{\pi_{12}})^\rho - (\frac{T_1}{\pi_{11}})^\rho)$$

C Computing Optimal Policies at Steady State

1. We guess $\{w_n, T_n, \tau_n^x, \tau_n^m\}$ and assume $\tau_2^x = 0$. The total number of unknow is $N+N+2(N-1)-1$.

2. We find $\{\pi_{ni}, x_n, P_n\}$

$$P_1 = \left[T_1(w_1 d)^{-\theta} + \sum_{n\neq 1}T_n(w_n(1+\tau_n^m)d)^{-\theta}\right]^{-\frac{1}{\theta}}$$

$$x_1 = w_1 L_1 + \sum_{i \neq 1}^{N} \frac{\tau_i^x}{1+\tau_i^x} \pi_{i1} (w_i L_i) + \sum_{i \neq 1}^{N} \frac{\tau_i^m}{1+\tau_i^m} \pi_{1i} x_1, \quad (\gamma_x)$$

$$x_i = w_i L_i$$

3. We find multipliers $1 + (N-1) + N : \{\gamma_x, \gamma_n, \gamma_{Tn}\}$, by setting $\gamma_N = 0$

   FOC over T_1

$$\gamma_x \sum_{i \neq 1}^{N} \left(\frac{\tau_i^x}{1+\tau_i^x} \pi_{i1}(1-\pi_{i1}) x_i - \frac{\tau_i^m}{1+\tau_i^m} \pi_{11} \pi_{1i} x_1 \right) + \sum_{i \neq 1} \gamma_i \left[\frac{\tau_i^m}{1+\tau_i^m} \pi_{11} \pi_{1i} x_1 - \sum_{m=1}^{N} \pi_{m1} \pi_{mi} x_m \right]$$
$$- \left(\frac{1}{\beta} - 1 + \delta \right) \gamma_{T1} T_1 + \sum_{i=1}^{N} \gamma_{Ti} \left[\alpha_i \left(\pi_{i,1}\right)^{1-\rho} \left(T_1\right)^{\rho} - (1-\rho) \delta \pi_{i1} T_i \right] = -u_c x_1 \frac{1}{\theta} \pi_{11}$$

   FOC over T_n

$$- \left(\frac{1}{\beta} - 1 + \delta \right) \gamma_{Tn} T_n + \sum_{i=1}^{N} \gamma_{Ti} \left[\alpha_i \left(\pi_{i,n}\right)^{1-\rho} \left(T_n\right)^{\rho} - (1-\rho) \delta \pi_{i,n} T_i \right]$$
$$+ \gamma_x \left[\sum_{i \neq 1}^{N} \left(- \frac{\tau_i^x}{1+\tau_i^x} \pi_{i1} \pi_{in} x_i - \frac{\tau_i^m}{1+\tau_i^m} \pi_{1i} \pi_{1n} x_1 \right) + \frac{\tau_n^m}{1+\tau_n^m} \pi_{1n} x_1 \right]$$
$$+ \gamma_n \left(\sum_{m=1}^{N} \pi_{mn} x_m - \frac{\tau_n^m}{1+\tau_n^m} \pi_{1n} x_1 \right) + \sum_{i \neq 1} \gamma_i \left(\frac{\tau_i^m}{1+\tau_i^m} \pi_{1i} \pi_{1n} x_1 - \sum_{m=1}^{N} \pi_{mi} \pi_{m,n} x_m \right) = -u_c x_1 \frac{1}{\theta} \pi_{1n}$$

   FOC over x_1

$$u_c + \sum_{n \neq 1}^{N} \gamma_n \frac{1}{1+\tau_n^m} \pi_{1n} - \gamma_x \left(1 - \sum_{n \neq 1}^{N} \frac{\tau_n^m}{1+\tau_n^m} \pi_{1n} \right) = 0$$

   We have a condition for the tariff for $n = 2$, the following holds:

$$u_c - \sum_{i \neq \{1,n\}}^{N} (\gamma_x - \gamma_i) \frac{\tau_i^m}{1+\tau_i^m} \theta \pi_{1i} + \gamma_x \theta (1 - \pi_{1n}) - \sum_{m \neq \{1,n\}} \gamma_m \theta \pi_{1m}$$
$$- \gamma_{T1} (1-\rho) \frac{1}{x_1} \theta \alpha_1 \left[I_1 - \left(\pi_{1n}\right)^{-\rho} \left(T_n\right)^{\rho} \right] = \frac{1}{1+\tau_n^m} \left(\gamma_x - \gamma_n\right) \left[1 + \theta(1-\pi_{1n})\right]$$

   Reorganize

$$\gamma_{T1} (1-\rho) \frac{1}{x_1} \theta \alpha_1 \left[I_1 - \left(\pi_{1,n}\right)^{-\rho} \left(T_n\right)^{\rho} \right] + \gamma_x \left(\sum_{i \neq \{1,n\}}^{N} \frac{\tau_i^m}{1+\tau_i^m} \theta \pi_{1i} - \theta(1-\pi_{1n}) + \frac{1}{1+\tau_n^m} \left[1 + \theta(1-\pi_{1n})\right] \right)$$
$$+ \sum_{i \neq \{1,n\}}^{N} \gamma_i \left(\frac{1}{1+\tau_i^m} \theta \pi_{1i} \right) - \frac{1}{1+\tau_n^m} \gamma_n \left[1 + \theta(1-\pi_{1n})\right] = u_c$$

or

$$\gamma_{T1}(1-\rho)\frac{1}{x_1}\theta\alpha_1\left[I_1-(\pi_{1,n})^{-\rho}(T_n)^{\rho}\right]+\gamma_x\left(\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}-\theta+\frac{1+\theta}{1+\tau_n^m}\right)$$
$$+\sum_{i\neq 1}^{N}\gamma_i\left(\frac{1}{1+\tau_i^m}\theta\pi_{1i}\right)-\frac{1+\theta}{1+\tau_n^m}\gamma_n=u_c$$

FOC over w_n for $n>3$

$$\sum_{i=1}^{N}\gamma_{Ti}(1-\rho)\pi_{in}\theta\alpha_i\left[I_i-(\pi_{in})^{-\rho}(T_n)^{\rho}\right]+\sum_{i\neq 1}\gamma_i\left[\left(\sum_{m\neq 1}^{N}\theta\pi_{mi}\pi_{mn}x_m+\frac{1}{1+\tau_i^m}\theta\pi_{1i}\pi_{1n}x_1+\pi_{ni}x_n\right)\right]$$
$$+\gamma_x\left[\sum_{i\neq 1}^{N}\frac{\tau_i^x}{1+\tau_i^x}\theta\pi_{i1}\pi_{in}x_i+\sum_{i\neq 1}^{N}\frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i}\pi_{1n}x_1+\frac{\tau_n^x}{1+\tau_n^x}\pi_{n1}x_n-\frac{\tau_n^m}{1+\tau_n^m}\theta\pi_{1n}x_1\right]$$
$$-\gamma_n\left[\left(\sum_{m\neq 1}^{N}\theta\pi_{mn}x_m+\frac{1}{1+\tau_n^m}\theta\pi_{1n}x_1\right)+x_n\right]=u_c x_1\pi_{1n}$$

4. We check whether the following equations hold. The total number of equations is $N+N+2(N-1)-1$.

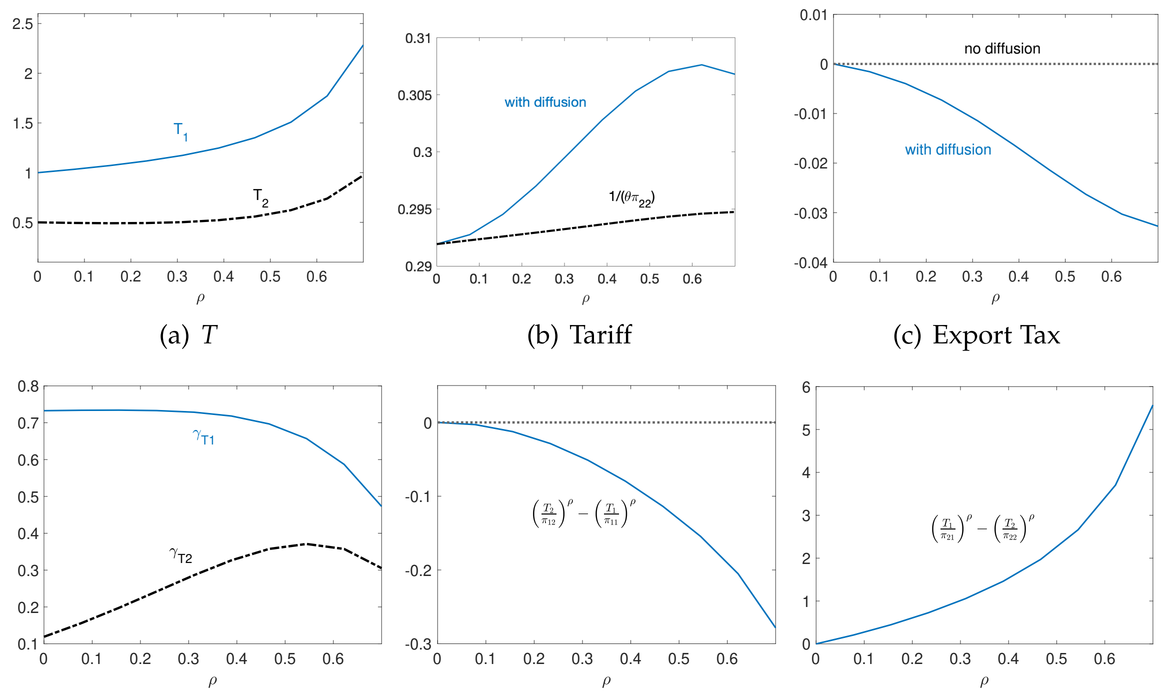
$$P_1=1$$
$$\sum_{m=1}^{N}\pi_{nm}x_n=\sum_{m\neq 1}^{N}\pi_{mn}x_m+\frac{1}{1+\tau_n^m}\pi_{1,n}x_1,\quad(\gamma_n,\quad(N-1),n>1)$$
$$\delta T_n=\alpha_n\sum_{i=1}^{N}(\pi_{ni})^{1-\rho}(T_i)^{\rho},\quad(\gamma_{Tn},\quad N)$$
$$1+\tau_n^x=\frac{1+\theta(1-\pi_{n1})}{\theta\sum_{i\neq 1}(1-\gamma_i/\gamma_x)\pi_{ni}+\frac{(1-\rho)\theta}{\gamma_x x_n}\alpha_n\gamma_{Tn}\left[\left(\frac{T_1}{\pi_{n1}}\right)^{\rho}-I_n\right]}$$
$$\frac{1}{1+\tau_n^m}=\frac{\gamma_x}{\gamma_x-\gamma_n}+\gamma_{T1}\frac{(1-\rho)\theta\alpha_1}{(\gamma_x-\gamma_n)(1+\theta)x_1}\left[\left(\frac{T_n}{\pi_{1n}}\right)^{\rho}-\left(\frac{T_1}{\pi_{11}}\right)^{\rho}\right],n>3$$

D Example of Varying Diffusion Parameter ρ

Here, we consider a case in which the diffusion parameter ρ varies from 0 to 0.7, while retaining the assumption that $\alpha_1=0.2$, $\alpha_2=0.1$. In this numerical example, $\theta=4$, $\sigma=2$, $\beta=0.94$, $\delta=0.2$, $d_{12}=d_{21}=1.2$, $L_1=1$, $L_2=3$. Figure A-2 presents the optimal policies at the steady state, and the resultant level of technology, multipliers, and foreign import qualities at the steady state as ρ rises in value. As in the previous example, the U.S. imports from China because of low import prices, and thus imposes a tariff higher compared to without diffusion as its relative import quality from China is negative (Panel (e)). At the same time, China's relative import quality is positive (Panel (f)), which leads to a lower export tax of the U.S. relative to the case without diffusion.

Although China's technology is improving, its relative import quality is still increasing to a certain degree because the U.S. is becoming more productive. When $\rho = 0$, diffusion doesn't depend on trade, and so optimal policies are identical to those without diffusion, namely a tariff of $\frac{1}{\theta \pi_{22}}$ and zero export tax (Panel (b) and (c)). Diffusion rises along with the value of ρ, allowing both T_1 and T_2 to increase. In this case, U.S. technology T_1 increases faster with ρ, forcing its relative import quality to fall, and China's relative import quality to rise. The impact of diffusion through trade, however, is not monotone in ρ. When ρ is closer to 1, $\alpha_n \sum_{i=1}^{N} \left[\pi_{ni,t} \left(\frac{T_{it}}{\pi_{ni,t}} \right)^{\rho} \right] \approx \alpha_n \sum_{i=1}^{N} T_{it}$, and diffusion depends less on trade. The incentive to use tariffs thus starts to fall, as shown in Panel (b).

Figure A-2: Optimal Policies with Increasing Diffusion



(a) T (b) Tariff (c) Export Tax

(d) γ_T (e) Home's relative import quality (f) Foreign's relative import quality

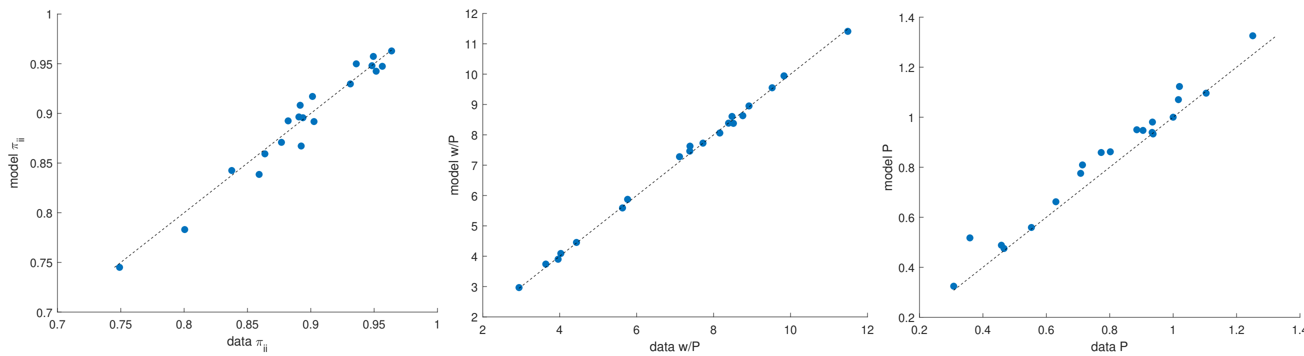
Note: This figure plots the Home optimal policies and the associated equilibrium at the steady state when we vary the diffusion parameter ρ. Home's relative import quality is defined as $(T_2/\pi_{12})^{\rho} - (T_1/\pi_{11})^{\rho}$, and Foreign's relative import quality is $(T_1/\pi_{21})^{\rho} - (T_2/\pi_{22})^{\rho}$. γ_{T_1} and γ_{T_2} are the multipliers on technology accumulations $T_{n,t+1} = (1-\delta) T_{nt} + \alpha_n \sum_{i=1}^{N} \left[\pi_{ni,t} \left(\frac{T_{it}}{\pi_{ni,t}} \right)^{\rho} \right]$.

E Goodness of Fit of the Calibrated Model

First, we compare the model implications with the data in 2016, the last year in our data sample. The exogenous model variables in 2016 consist of arrival rates $\{\alpha_i\}$ for each country and the cross-country trade costs $\{d_{ni}\}$. The endogenous variables include expenditure shares $\{\pi_{ni}\}$, wages $\{w_i\}$, and consumer price indices $\{P_i\}$. Mechanically, the calibration of $\{\alpha_i, d_{ni}\}$ helps match $N + N(N-1)$ endogenous variables. Hence, among the $2N + N(N-1)$ endogenous variables $\{\pi_{ni}, w_i/P_i, P_i\}$, there are still N of them can be used for out-of-sample examination.

Figure A-3 contrasts the model with data in three endogenous variables: domestic share of expenditure π_{ii}, real GDP per input (w_i/P_i),[1] and consumer price index P_i. Each dot is a country in 2016. The model matches the data well; all the dots align around the 45-degree line, with real GDP per input fitting most tightly. Our model also matches successfully π_{ni} for each country with those in the data.

Figure A-3: Model and Data Comparison in 2016



(a) π_{ii} (b) Real Real GDP per input (c) Consumer price index

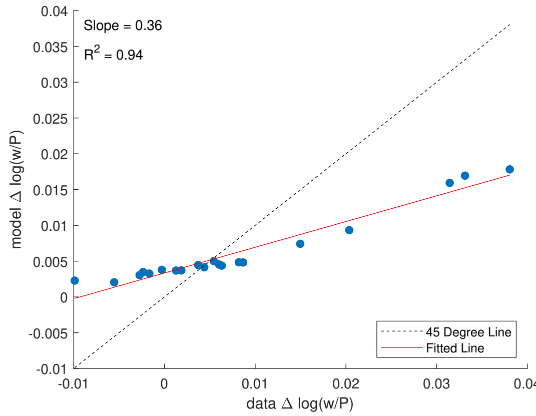
Note: This figure shows the cross-section relationship between the data and the model's expenditure shares on domestic goods, real GDP per input, and the consumer price index (CPI) for 20 countries in the year 2016. All price indices are relative to the U.S. The black dashed line represents the 45-degree line.

Second, although the estimated ρ is significant at 1% level, the model cannot explain most of the fluctuation of annual growth with constant $\{\alpha_i\}$. Nonetheless, the model captures key features of the average growth of real GDP per input between 2000 and 2016. See Figure A-4. The average growth of real GDP per input in the model is 0.62%, similar to that in the data 0.81%. Furthermore, the model and data are highly correlated, with a correlation of 0.97, meaning that high-growth countries in the data also have high growth in the model. Lastly, the model accounts for about 37% of the observed growth variation across countries: the cross-country standard deviation of average growth rates is 0.49% in the model and 1.32% in the data.

It is worth discussing the model's lower cross-country dispersion of growth. In our model, the only exogenous variation across countries and time is the trade costs. However, there are other factors influencing technology growth. For example, the arrival rates $\{\alpha_i\}$ could be time-varying. However, with country-specific time-varying α, additional assumptions or data are needed to calibrate ρ. For example, Buera and Oberfield (2020) adopts an alternative calibration strategy and only uses two years' cross-section observations to calibrate ρ, then it can back out time-varying α_{nt}. While this alternative approach has its limitations, it offers a way to separate ρ and time-varying α, helping match the annual

[1] In the model, real wage $\{w_i/P_i\}$ equals to real GDP per input, which we constructed using real GDP over composite input in the data, i.e., $GDP/(K^{\zeta} emp^{1-\zeta})$ where real GDP, capital, and employment are from PWT10.01 and $\zeta = 0.36$.

Figure A-4: Average Growth in Real GDP per Input, Model and Data



Note: This figure shows the average growth of real GDP per input between 2000 and 2016 for both model and data. The black dashed line represents the 45-degree line, and the red line is the fitted line.

growth of countries. They find $\rho = 0.6$.

Note that in our quantitative analysis, we study the optimal policies after the economy reaches a balanced growth path. To quantify the optimal policies, we only need $\{\alpha_i\}$ in the year 2016 (balanced growth). With $\rho = 0.6$, we can recalculate α_i in 2016 from equation (17) in the paper. The optimal policies under $\rho = 0.6$ are shown in Section F of this online appendix.

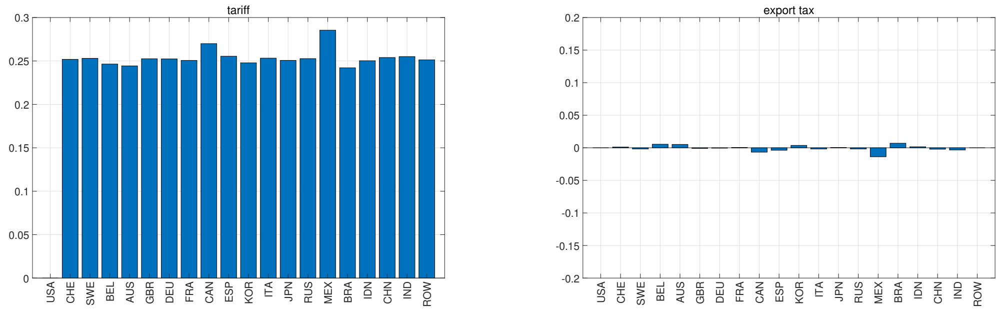
We want to make it clear that Buera and Oberfield (2020) and our paper use two alternative calibration strategies. Neither of them can identify the diffusion mechanism. A better method is to rely on some exogenous events like exogenous changes in trade costs or micro-level data directly related to diffusion. We leave that to future works.

F Transition Paths and Robustnss over ρ

This section includes three parts. We first present the optimal policies without diffusion. We then show the transition paths after the optimal policies of the U.S. and China. Lastly, we present the results under the alternative value of diffusion parameter ρ.

Optimal policies without diffusion Here we consider optimal policies under no diffusions by setting the diffusion parameter to zero, $\rho = 0$. Figure A-5 presents the corresponding U.S. optimal tariffs and export taxes, and Figure A-6 shows China's optimal policies.

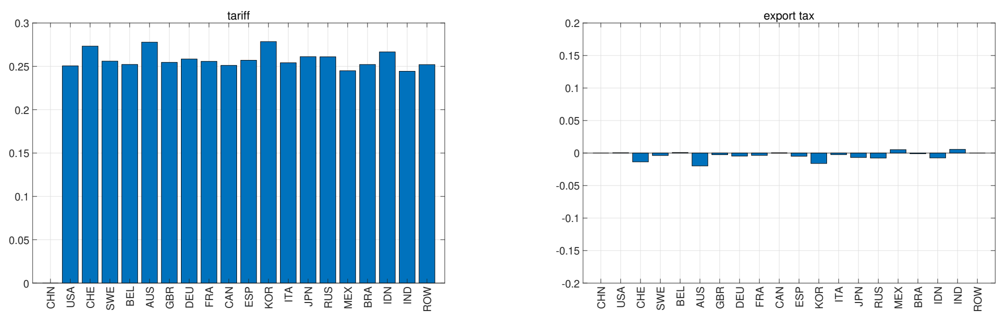
Figure A-5: U.S. Optimal Policies without Diffusion ($\rho = 0$), 20 Countries, Steady State



(a) Tariff (b) Export tax

Note: This figure plots U.S. optimal trade policies at the steady state with 20 countries but no diffusion.

Figure A-6: China Optimal Policies without Diffusion ($\rho = 0$), 20 Countries, Steady State



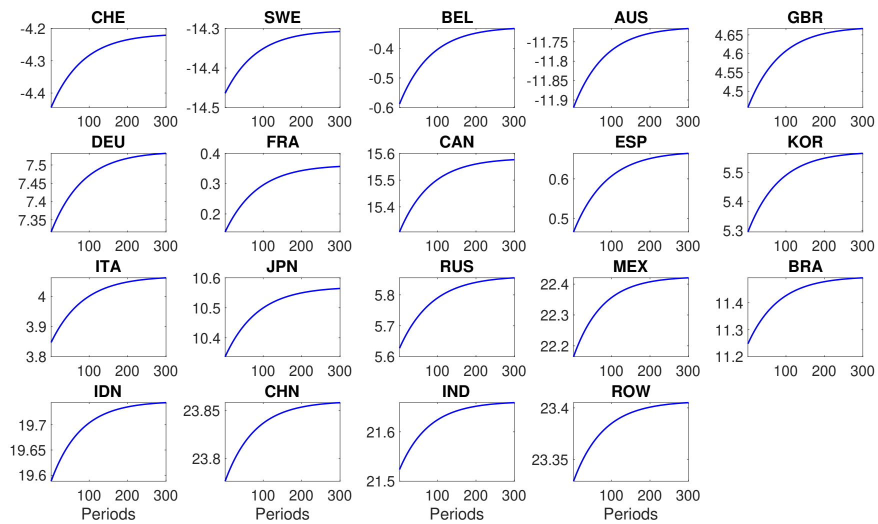
(a) Tariff (b) Export tax

Note: This figure plots China's optimal trade policies at the steady state in the model with 20 countries but no technology diffusion.

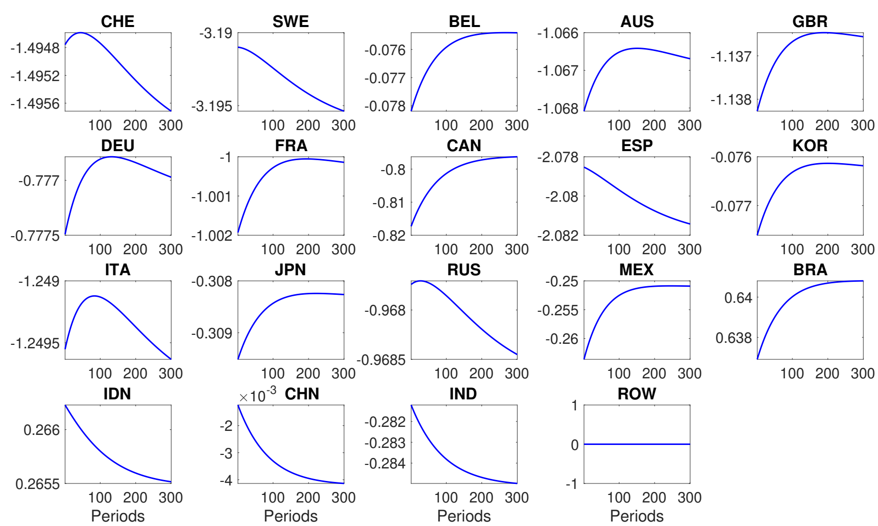
Transition paths under optimal policies During the transitional period, technology levels go from their 2016 levels to the new balanced growth path. Figure A-7, A-8, A-9, and A-10 presents the path of tariffs and export taxes, relative wages, import shares, and consumptions for 20 countries after U.S. optimal policies. Figure A-11, A-12, A-13, and A-14 shows the path of tariffs and export taxes, relative wages, import shares, and consumption for 20 countries under China's optimal policies.

Alternative ρ Table A-1 compares U.S. optimal export taxes and import tariffs and the associated change in technology and welfare cross countries for $\rho = 0.4$, 0.5, and 0.6. Table A-2 presents a similar comparison under China's optimal policies.

Figure A-7: U.S. Optimal Trade Policies during Transition



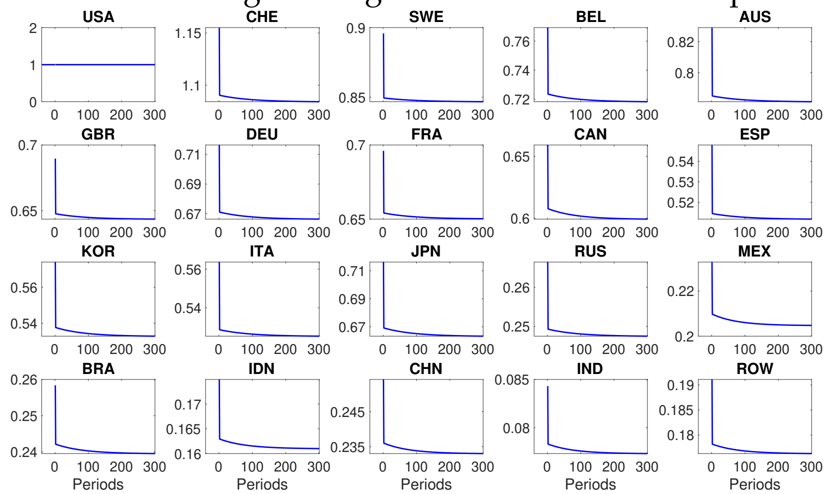
(a) Tariff



(b) Export tax

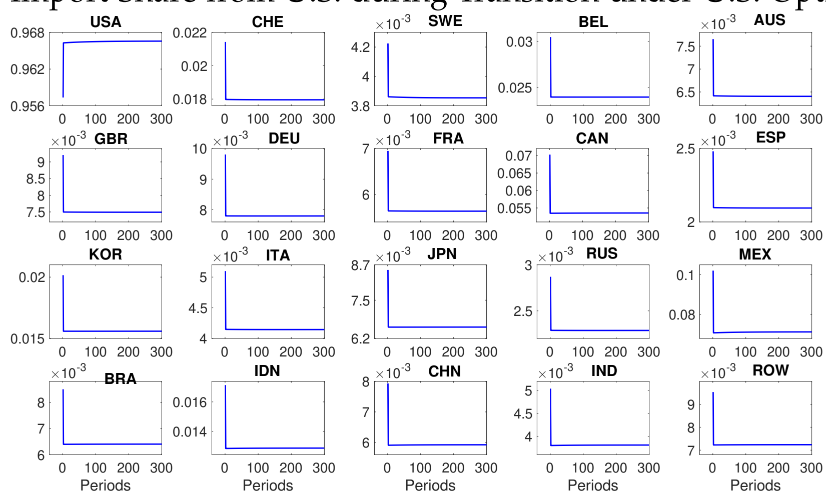
Note: This figure plots the transition paths of U.S. optimal import tariffs (τ^m) and export taxes (τ^x) with technology diffusions, $\rho = 0.5$. The export tax on ROW is normalized to be zero.

Figure A-8: Relative Wage during Transition under U.S. Optimal Policies



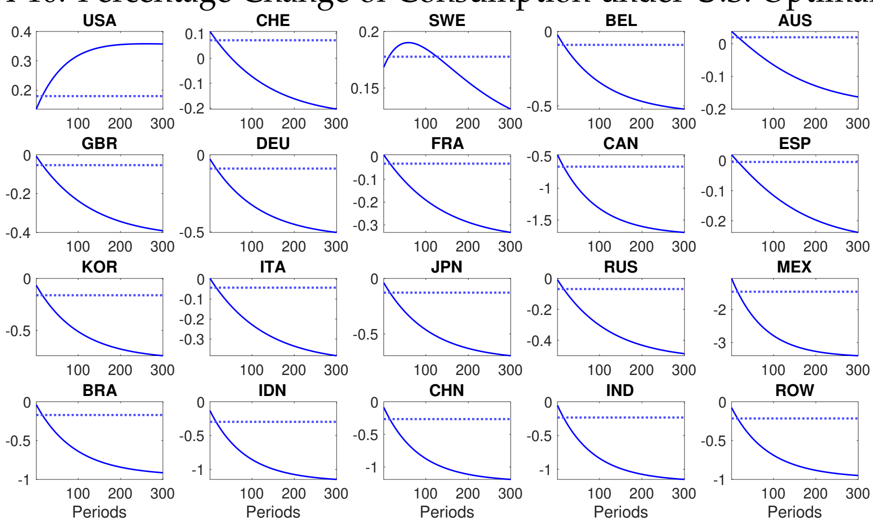
Note: This figure plots relative wage to the U.S. under U.S. optimal trade policies with technology diffusion.

Figure A-9: Import Share from U.S. during Transition under U.S. Optimal Policies



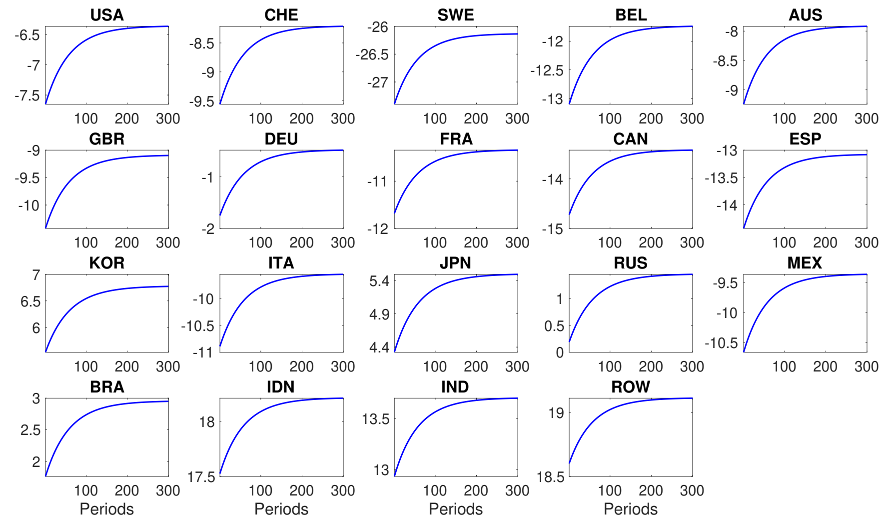
Note: This figure plots the evolution of import shares from U.S. $\{\pi_{i,US}\}$ for $i = 1, 2, .., 20$ overtime under the U.S. optimal trade policies with technology diffusion.

Figure A-10: Percentage Change of Consumption under U.S. Optimal Policies

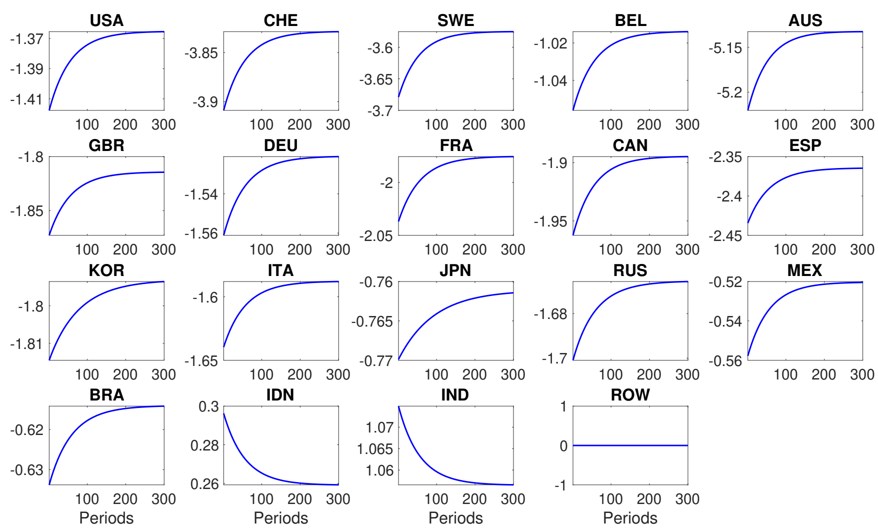


Note: This figure plots the transition path for the percentage change in consumption and consumption equivalence in our benchmark— relative to consumption under the private equilibrium with diffusion. In each subfigure, the solid blue line is the percentage change in consumption, and the dotted blue line is the percentage change in consumption equivalences.

Figure A-11: China's Optimal Trade Policies during Transition



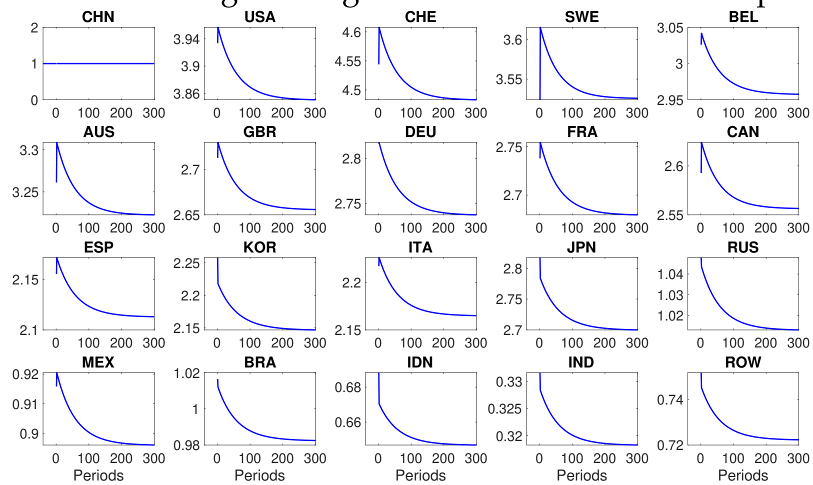
(a) Tariff



(b) Export tax

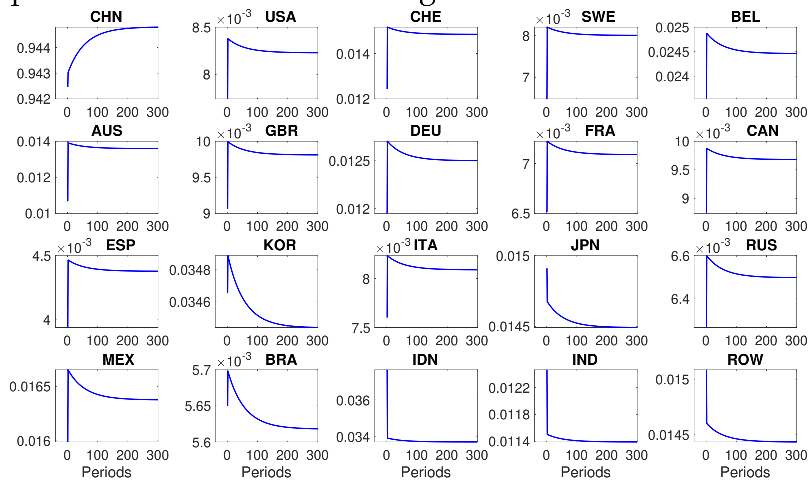
Note: This figure plots the transition paths of China's optimal import tariffs (τ^m) and export taxes (τ^x) under technology diffusion. The export tax on ROW is normalized to be zero.

Figure A-12: Relative Wage during Transition under China's Optimal Policies



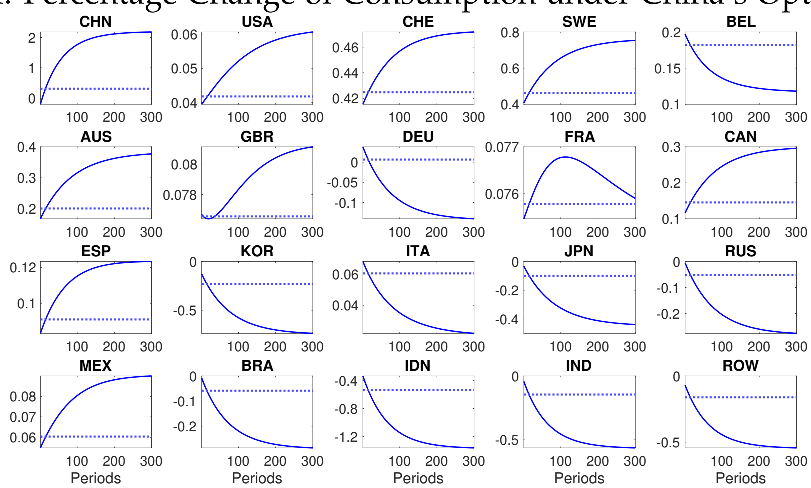
Note: This figure plots the transition path of relative wage to China in each period under China's optimal trade policies with technology diffusion.

Figure A-13: Import Share from China during Transition under China's Optimal Policies



Note: This figure plots the evolution of import shares from China $\{\pi_{i,CN}\}$ for $i = 1, 2, .., 20$ over time under the China's optimal trade policies with technology diffusion.

Figure A-14: Percentage Change of Consumption under China's Optimal Policies



Note: This figure plots the percentage change of consumption and the percentage change of consumption equivalences under China's optimal trade policies with technology diffusion, from the consumption from the private equilibrium without China's trade taxes but under technology diffusion. In each subfigure, the solid blue line is the percentage change of consumption, and the dotted blue line is the percentage change of consumption equivalences.

Table A-1: U.S. Optimal Policies under Alternative ρ

	$\rho = 0.4$				$\rho = 0.5$				$\rho = 0.6$			
Country	τ^x	τ^m	$\%\triangle T$	$\%\triangle CE$	τ^x	τ^m	$\%\triangle T$	$\%\triangle CE$	τ^x	τ^m	$\%\triangle T$	$\%\triangle CE$
USA			-1.05	0.17			0.87	0.18			4.86	0.25
CHE	-0.66	4.21	-1.81	-0.08	-1.50	-4.22	-1.34	0.07	-3.05	-11.59	0.61	0.31
SWE	-1.36	-3.54	-0.84	0.04	-3.20	-14.31	-0.21	0.18	-7.31	-23.23	2.28	0.43
BEL	0.24	6.83	-2.33	-0.17	-0.08	-0.33	-2.13	-0.09	-0.63	-6.71	-0.59	0.03
AUS	-0.18	-1.83	-1.36	-0.04	-1.07	-11.71	-0.91	0.02	-2.78	-20.05	1.29	0.15
GBR	-0.66	10.59	-1.72	-0.08	-1.14	4.67	-1.67	-0.05	-1.98	-0.78	-0.26	0.01
DEU	-0.43	12.62	-1.92	-0.11	-0.78	7.54	-2.04	-0.09	-1.39	2.87	-0.84	-0.05
FRA	-0.49	7.46	-1.49	-0.06	-1.00	0.36	-1.49	-0.03	-1.96	-6.00	-0.16	0.02
CAN	-0.85	18.83	-5.34	-0.73	-0.80	15.58	-5.08	-0.66	-0.74	12.52	-3.11	-0.57
ESP	-1.15	7.84	-1.15	-0.03	-2.08	0.67	-1.16	-0.00	-4.11	-5.75	0.21	0.06
KOR	0.15	11.13	-2.89	-0.19	-0.08	5.57	-2.87	-0.16	-0.50	0.53	-1.32	-0.10
ITA	-0.72	10.22	-1.60	-0.06	-1.25	4.07	-1.68	-0.04	-2.28	-1.53	-0.39	0.00
JPN	-0.15	14.71	-2.46	-0.12	-0.31	10.57	-2.81	-0.13	-0.62	6.76	-1.60	-0.12
RUS	-0.56	11.48	-1.76	-0.06	-0.97	5.86	-2.07	-0.07	-1.78	0.76	-0.92	-0.05
MEX	-0.73	24.08	-10.06	-1.43	-0.25	22.42	-9.71	-1.46	0.30	20.93	-7.19	-1.45
BRA	0.69	15.01	-3.54	-0.15	0.64	11.50	-3.68	-0.17	0.54	8.25	-1.98	-0.16
IDN	0.22	21.05	-4.27	-0.28	0.27	19.75	-4.25	-0.30	0.32	18.66	-2.61	-0.29
CHN	-0.09	24.26	-4.33	-0.23	-0.00	23.86	-4.61	-0.27	0.10	23.55	-3.07	-0.28
IND	-0.31	22.58	-4.33	-0.19	-0.29	21.66	-4.57	-0.23	-0.25	20.92	-2.94	-0.24
ROW	0.00	23.78	-3.57	-0.19	0.00	23.41	-3.67	-0.22	0.00	23.12	-2.18	-0.21

Note: This table summarizes U.S. optimal export taxes and tariffs on the other 19 countries and the associated equilibrium technology and the change in consumption equivalence considering the whole transition path and the steady state, for different ρ. $\%\triangle$ denotes percentage change, CE denotes consumption equivalence.

Table A-2: China's Optimal Policies for Alternative ρ

	$\rho = 0.4$				$\rho = 0.5$				$\rho = 0.6$			
Country	τ^x	τ^m	$\%\triangle T$	$\%\triangle CE$	τ^x	τ^m	$\%\triangle T$	$\%\triangle CE$	τ^x	τ^m	$\%\triangle T$	$\%\triangle CE$
CHN			4.59	0.18			9.46	0.31			10.77	0.43
USA	-1.07	-2.68	0.11	0.02	-1.37	-6.36	0.11	0.04	-1.52	-7.49	0.00	0.05
CHE	-3.32	-3.58	0.13	0.24	-3.83	-8.20	0.39	0.42	-4.03	-10.08	0.46	0.51
SWE	-2.60	-19.21	0.76	0.27	-3.58	-26.13	1.55	0.46	-4.27	-29.78	2.15	0.62
BEL	-0.77	-7.10	-0.09	0.10	-1.01	-11.73	-0.24	0.18	-1.13	-13.59	-0.36	0.21
AUS	-4.56	-3.22	0.39	0.11	-5.13	-7.91	0.93	0.20	-5.28	-9.95	1.20	0.26
GBR	-1.47	-4.87	0.07	0.04	-1.81	-9.09	0.05	0.08	-2.00	-10.69	-0.04	0.09
DEU	-1.32	2.31	-0.44	-0.01	-1.52	-0.47	-0.70	0.01	-1.60	-0.95	-0.89	-0.00
FRA	-1.60	-5.84	0.03	0.04	-1.98	-10.33	0.03	0.08	-2.18	-12.12	-0.05	0.09
CAN	-1.46	-8.50	0.54	0.09	-1.89	-13.41	0.78	0.15	-2.13	-15.50	0.77	0.18
ESP	-1.90	-8.11	0.11	0.05	-2.36	-13.07	0.20	0.09	-2.63	-15.18	0.15	0.11
KOR	-1.93	8.64	-1.70	-0.23	-1.79	6.78	-2.45	-0.23	-1.57	7.01	-2.94	-0.29
ITA	-1.28	-5.20	-0.07	0.03	-1.59	-9.54	-0.16	0.06	-1.75	-11.18	-0.30	0.07
JPN	-0.86	7.26	-0.95	-0.08	-0.76	5.50	-1.66	-0.10	-0.60	5.78	-2.22	-0.15
RUS	-1.53	3.97	-0.69	-0.04	-1.67	1.45	-1.10	-0.05	-1.67	1.21	-1.41	-0.08
MEX	-0.31	-5.29	0.26	0.03	-0.52	-9.35	0.18	0.06	-0.63	-10.79	-0.03	0.07
BRA	-0.57	4.94	-0.67	-0.04	-0.61	2.96	-1.13	-0.06	-0.59	3.07	-1.44	-0.09
IDN	0.01	17.96	-3.39	-0.47	0.26	18.21	-4.21	-0.53	0.42	19.38	-4.26	-0.59
IND	0.93	13.68	-1.41	-0.10	1.06	13.71	-2.11	-0.15	1.13	14.94	-2.31	-0.18
ROW	0.00	18.53	-1.42	-0.13	0.00	19.11	-1.93	-0.16	0.00	20.35	-2.05	-0.19

Note: This table summarizes China's optimal export taxes and tariffs on the other 19 countries and the associated equilibrium technology and the change in consumption equivalence considering the whole transition path and the steady state, for different ρ. $\%\triangle$ denotes percentage change, CE denotes consumption equivalence.

G Heterogeneous ρ Across Countries

Diffusion effects may vary among different countries. In this section, we first derive the technology distributions when there are heterogenous diffusion ρ_{ij}. Then we prove the optimal unilateral policy with heterogenous ρ.

Proposition 1. *With heterogenous diffusion ρ_{ij} across countries, the frontier of knowledge follows a Frechet distribution, with parameter T_{it} and θ, and the evolution of the scale of the Frechet, that is, the stock of knowledge, evolves according to*

$$\dot{T}_{it} = \alpha_{it} \sum_j \Gamma(1-\rho_{ij}) \pi_{ij,t} (\frac{T_{jt}}{\pi_{ij,t}})^{\rho_{ij}}$$

Proof. *Step 1.* As in BO, diffusion is a process involving the random interaction among producers of different countries. New ideas arrive to potential producers of each good stochastically and exogenously. Each idea is a technology to produce a particular good with productivity q. when a new idea arrives, the productivity of the idea in country i is $q = zq_j^{\rho_{ij}}$, which has two random components. There is an insight drawn from another producer, q_j, which is drawn from the source distribution $H_{ij,t}(q_j)$. There is also an original component, z, drawn from an exogenous distribution of country i. We assume that the arrival rate of ideas of country i with an original component greater than z is $A_{it}(z)$.

The economy's productivity depends on the frontier of knowledge. The frontier of knowledge of country i is characterized by the function $F_{it}(q)$ which denotes the fraction of goods for which no producer's productivity exceeds q. Given the frontier of knowledge at time $t, F_{it}(q)$, the source distribution, $H_{ij,t}\left(q_j\right)$, and the exogenous arrival rates of ideas, $A_{it}(z)$, the frontier of knowledge at time $t+\Delta$ satisfies

$$1 - F_{i,t+\Delta}(q) = [1 - F_{it}(q)] + F_{it}(q) \int_t^{t+\Delta} \sum_j \int A_{i\tau} \left(\frac{q}{q_j^{\rho_{ij}}} \right) dH_{ij,\tau}\left(q_j\right) d\tau$$

Rearranging and taking the limit as $\Delta \to 0$, we obtain an expression characterizing the evolution of the frontier of knowledge:

$$\frac{d}{dt} \ln F_{it}(q) = \lim_{\Delta \to 0} \frac{F_{i,t+\Delta t}(q) - F_{it}(q)}{\Delta F_{it}(q)} = -\sum_j \int_0^\infty A_{it} \left(\frac{q}{q_j^{\rho_{ij}}} \right) dH_{ij,t}\left(q_j\right)$$

We assume $A_{it}(z) = \alpha_{it} z^{-\theta}$. Then the frontier of knowledge evolves as

$$\frac{d \ln F_{it}(q)}{dt} = -\alpha_{it} q^{-\theta} \sum_j \int_0^\infty x_j^{\rho_{ij}\theta} dH_{ij,t}\left(x_j\right)$$

Define $T_{it} = \int_{-\infty}^t \alpha_{i\tau} \Sigma_j \int_0^\infty x_j^{\rho_{ij}\theta} dH_{ij,\tau}\left(x_j\right) d\tau$. Then $F_{it}(q) = e^{-T_{it} q^{-\theta}}$ and $\dot{T}_{it} = \alpha_{it} \sum_j \int_0^\infty x_j^{\rho_{ij}\theta} dH_{ij,t}\left(x_j\right)$

Step 2. Prove that the source distribution $H_{ij}(q) = \pi_{ij} e^{-\frac{T_j}{\pi_{ij}} q^{-\theta}}$.

For a producer with productivity q in country j, the cost of providing one unit of the good in country i is $\frac{w_j d_{ij}}{q_j}$. We study an environment in which the producers engage in perfect competition. The country that can provide goods to i at the lowest cost is given by

$$\arg\min_j \frac{w_j d_{ij}}{q_j}$$

The lowest-cost-provider of good for i determines the price of good in i

$$p_i = \min_j \{ \frac{w_j d_{ij}}{q_j} \}$$

For any two countries k and j, a producer in country j has lower costs than a producer in country k if

$$\frac{w_k d_{ik}}{q_k} > \frac{w_j d_{ij}}{q_j} \Rightarrow q_k < \frac{w_k d_{ik}}{w_j d_{ij}} q_j$$

Then the source distribution can be derived by

$$H_{ij}(q_j) = \int_0^{q_j} e^{-\sum_{k\neq j} T_k(\frac{w_k d_{ik}}{w_j d_{ij}} q)^{-\theta}} d(e^{-T_j q^{-\theta}}) = \int_0^{q_j} e^{-\frac{T_j}{\pi_{ij}} q^{-\theta}} d(e^{-T_j q^{-\theta}}) = \pi_{ij} e^{-\frac{T_j}{\pi_{ij}} q_j^{-\theta}}$$

Step 3. Thus,

$$\int_0^\infty x_j^{\rho_{ij}\theta} dH_{ij,t}(x_j) = \pi_{ij,t} \int_0^\infty x_j^{\rho_{ij}\theta} d(e^{-\frac{T_{jt}}{\pi_{ij,t}} x_j^{-\theta}})$$

Let $m = \frac{T_{jt}}{\pi_{ij,t}} x_j^{-\theta}$ and plug it back

$$\int_0^\infty x_j^{\rho_{ij}\theta} dH_{ij,t}(x_j) = \pi_{ij,t} \int_0^\infty (\frac{T_{jt}}{\pi_{ij,t} m})^{\rho_{ij}} d(e^{-m}) = \Gamma(1-\rho_{ij}) \pi_{ij,t} (\frac{T_{jt}}{\pi_{ij,t}})^{\rho_{ij}}$$

Finally,

$$\dot{T}_{it} = \alpha_{it} \sum_j \Gamma(1-\rho_{ij}) \pi_{ij,t} (\frac{T_{jt}}{\pi_{ij,t}})^{\rho_{ij}} \tag{A.34}$$

If ρ is homogenous across all countries, we go back to our baseline $\dot{T}_{it} = \Gamma(1-\rho)\alpha_{it} \sum_j \pi_{ij,t}(\frac{T_{jt}}{\pi_{ij,t}})^\rho$.

We want to point out that this setup with heterogenous ρ and constant exogenous growth of α doesn't have a balanced growth path. To see this point, consider a two-country case. We divide T_{1t} from both sides of (A.34) for $i = 1$ and obtain the growth rate of T_1 as

$$\frac{\dot{T}_{1t}}{T_{1t}} = \frac{\alpha_{1t}}{T_{1t}^{1-\rho_1}} \left[\Gamma(1-\rho_1) \pi_{11,t}^{1-\rho_1} + \Gamma(1-\rho_2) \pi_{12,t}^{1-\rho_2} \left(\frac{T_{2t}}{T_{1t}} \right)^{\rho_2} T_{1t}^{\rho_2-\rho_1} \right]. \tag{A.35}$$

When $\rho_1 = \rho_2 \equiv \rho$, there is a balanced growth path. In this case, $T_{1t}^{\rho_2-\rho_1}$ on the right-hand side (RHS) of (A.35) becomes 1. Suppose the arrival rates α_{1t} and α_{2t} grow exogenously at γ, then both T_{1t} and T_{2t} grow at at the same rates $\gamma/(1-\rho)$. Thus, the world has a balanced growth path under homogeneous ρ.

Now suppose $\rho_1 \neq \rho_2$. Then, $T_{1t}^{\rho_2-\rho_1}$ will not drop from RHS of (A.35), and there would be no balanced growth. Furthermore, even when α_{1t} and α_{2t} grow at the same rate γ, technologies will not grow at the same rates. If $\rho_1 > \rho_2$, eventually, the impact of country 2 technology on country 1 disappears.

Hence, under heterogeneous ρ, there is no balanced growth path. To have an equilibrium under technology accumulation like (A.34), we either assume no growth in the arrival rates α or assume homogeneous ρ. In other words, to have balanced growth in T, we have to assume ρ is homogeneous across countries. Or, to have heterogeneous ρ, we need to assume no growth in the model, which seems to be counterfactual.

G.1 Optimal Unilateral Policies with Heterogenous ρ_{ij}

If we assume there is no exogenous growth but with a steady state, we can still derive optimal policy with heterogenous ρ_{ij},

$$T_n' = (1-\delta)T_n + \alpha_n \sum_{i=1}^{N} \Gamma(1-\rho_{ni})\pi_{ni}(\frac{T_i}{\pi_{ni}})^{\rho_{ni}}, \quad (\gamma_{Tn}, \quad N)$$

while other conditions are the same as those in section B.1.

Proposition 2. *With heterogenous diffusion ρ_{ij} across countries, the optimal import tariff and export tax satisfy*

$$\frac{1}{1+\tau_n^m} = \frac{\gamma_x}{\gamma_x - \gamma_n} + \gamma_{T1}\frac{\theta\alpha_1}{(\gamma_x-\gamma_n)(1+\theta)x_1}\left[\Gamma(1-\rho_{1n})(1-\rho_{1n})\left(\frac{T_n}{\pi_{1n}}\right)^{\rho_{1n}} - \Gamma(1-\rho_{11})(1-\rho_{11})\left(\frac{T_1}{\pi_{11}}\right)^{\rho_{11}}\right] \tag{A.36}$$

$$\begin{aligned}\frac{1}{1+\tau_n^x} &= \frac{\gamma_x\theta\sum_{m\neq 1}(1-\gamma_m/\gamma_x)\pi_{nm}}{\gamma_x\left[1+\theta(1-\pi_{n1})\right]} \\ &+ \frac{\gamma_{Tn}\theta\frac{1}{x_n}\alpha_n\left[\Gamma(1-\rho_{n1})(1-\rho_{n1})\left(\frac{T_1}{\pi_{n1}}\right)^{\rho_{n1}} - \sum_m^N \Gamma(1-\rho_{nm})(1-\rho_{nm})\left(\pi_{nm}\right)^{1-\rho_{nm}}\left(T_m\right)^{\rho_{nm}}\right]}{\gamma_x\left[1+\theta(1-\pi_{n1})\right]}\end{aligned}$$

Proof. To derive the optimal import tariff, we first use the FOC over τ_n^m

$$\begin{aligned}&\gamma_{T1}\frac{1}{x_1}\theta\alpha_1\left[\sum_i \Gamma(1-\rho_{1i})(1-\rho_{1i})\left(\pi_{1i}\right)^{1-\rho_{1i}}\left(T_i\right)^{\rho_{1i}} - \Gamma(1-\rho_{1n})(1-\rho_{1n})\left(\pi_{1n}\right)^{-\rho_{1n}}\left(T_n\right)^{\rho_{1n}}\right] - u_c + \sum_{m\neq\{1,n\}}\gamma_m\theta\pi_{1m} \\ &+ (\gamma_x-\gamma_n)\frac{1}{(1+\tau_n^m)}\left[1+\theta\left(1-\pi_{1n}\right)\right] - \gamma_x\theta\left(1-\pi_{1n}\right) + \sum_{i\neq\{1,n\}}^{N}\left(\gamma_x-\gamma_i\right)\frac{\tau_i^m}{1+\tau_i^m}\theta\pi_{1i} = 0\end{aligned}$$

Hence the optimal import tariffs satisfy

$$\frac{1}{1+\tau_n^m} = \frac{u_c + \gamma_x\left[\theta\pi_{11} + \sum_{i\neq\{1,n\}}^{N}(\frac{1-\gamma_i/\gamma_x}{1+\tau_i^m})\theta\pi_{1i}\right] + \gamma_{T1}\frac{1}{x_1}\theta\alpha_1\left[\Gamma(1-\rho_{1n})(1-\rho_{1n})\left(\frac{T_n}{\pi_{1n}}\right)^{\rho_{1n}} - \sum_i \Gamma(1-\rho_{1i})(1-\rho_{1i})\left(\pi_{1i}\right)^{1-\rho_{1i}}\left(T_i\right)^{\rho_{1i}}\right]}{\left(\gamma_x-\gamma_n\right)\left[1+\theta\left(1-\pi_{1n}\right)\right]} \tag{A.37}$$

Combine FOC over τ_n^m (A.37) and FOC over x_1 (A.26):

$$((\gamma_x-\gamma_n)\frac{1}{1+\tau_n^m} - u_c)(1+\theta)x_1 = \gamma_{T1}\theta\alpha_1\left[\Gamma(1-\rho_{1n})(1-\rho_{1n})\left(\frac{T_n}{\pi_{1n}}\right)^{\rho_{1n}} - \sum_i \Gamma(1-\rho_{1i})(1-\rho_{1i})\left(\pi_{1i}\right)^{1-\rho_{1i}}\left(T_i\right)^{\rho_{1i}}\right] \tag{A.38}$$

For country 1, it has

$$(\gamma_x - u_c)(1+\theta)x_1 = \gamma_{T1}\theta\alpha_1\left[\Gamma(1-\rho_{11})(1-\rho_{11})\left(\frac{T_1}{\pi_{11}}\right)^{\rho_{11}} - \sum_i \Gamma(1-\rho_{1i})(1-\rho_{1i})\left(\pi_{1i}\right)^{1-\rho_{1i}}\left(T_i\right)^{\rho_{1i}}\right] \tag{A.39}$$

Combining equation (A.38) and (A.39), the optimal import tariffs satisfy

$$\frac{1}{1+\tau_n^m} = \frac{\gamma_x}{\gamma_x - \gamma_n} + \gamma_{T1}\frac{\theta\alpha_1}{(\gamma_x-\gamma_n)(1+\theta)x_1}\left[\Gamma(1-\rho_{1n})(1-\rho_{1n})\left(\frac{T_n}{\pi_{1n}}\right)^{\rho_{1n}} - \Gamma(1-\rho_{11})(1-\rho_{11})\left(\frac{T_1}{\pi_{11}}\right)^{\rho_{11}}\right]$$

The optimal export tax is derived from the FOC over τ_n^x:

$$\gamma_{Tn}\alpha_n\theta\left[\sum_{m\neq 1}^{N}\Gamma(1-\rho_{nm})(1-\rho_{nm})\left(\pi_{nm}\right)^{1-\rho_{nm}}\left(T_m\right)^{\rho_{nm}} - \Gamma(1-\rho_{n1})(1-\rho_{n1})\left(\pi_{n1}\right)^{-\rho_{n1}}\left(T_1\right)^{\rho_{n1}}(1-\pi_{n1})\right]$$
$$+\sum_{i\neq 1}^{N}\gamma_i\left[\theta\pi_{ni}x_n\right] + \gamma_x\frac{1}{(1+\tau_n^x)}\pi_{n1}x_n - \gamma_x\frac{\tau_n^x}{1+\tau_n^x}\theta(1-\pi_{n1})x_n = 0$$

Hence the optimal export taxes satisfy

$$\frac{1}{1+\tau_n^x} = \frac{\gamma_x\theta\sum_{m\neq 1}(1-\gamma_m/\gamma_x)\pi_{nm}}{\gamma_x\left[1+\theta(1-\pi_{n1})\right]} + \frac{\gamma_{Tn}\theta\frac{1}{x_n}\alpha_n\left[\Gamma(1-\rho_{n1})(1-\rho_{n1})\left(\frac{T_1}{\pi_{n1}}\right)^{\rho_{n1}} - \sum_m^N\Gamma(1-\rho_{nm})(1-\rho_{nm})\left(\pi_{nm}\right)^{1-\rho_{nm}}\left(T_m\right)^{\rho_{nm}}\right]}{\gamma_x\left[1+\theta(1-\pi_{n1})\right]}$$

When ρ is homogenous across all countries, and we scale down all α by a constant $\Gamma(1-\rho)$, the optimal trade policies are the same as equation (11) and (12) in the paper.

Two-Country Case For the case of two countries, the optimal export tax is derived from the FOC over τ_2^x:

$$\frac{\gamma_x x_2}{1+\tau_2^x} - \gamma_x\frac{\tau_2^x}{1+\tau_2^x}\theta\pi_{22}x_2 + \gamma_2\theta\pi_{22}x_2 - \gamma_{T_2}\theta\alpha_2\pi_{22}[\Gamma(1-\rho_{21})(1-\rho_{21})(\frac{T_1}{\pi_{21}})^{\rho_{21}} - \Gamma(1-\rho_{22})(1-\rho_{22})(\frac{T_2}{\pi_{22}})^{\rho_{22}}] = 0$$

Optimal export tax:

$$\frac{1}{1+\tau_2^x} = \frac{(\gamma_x-\gamma_2)\theta\pi_{22}x_2 + \gamma_{T_2}\theta\alpha_2\pi_{22}[\Gamma(1-\rho_{21})(1-\rho_{21})(\frac{T_1}{\pi_{21}})^{\rho_{21}} - \Gamma(1-\rho_{22})(1-\rho_{22})(\frac{T_2}{\pi_{22}})^{\rho_{22}}]}{\gamma_x(1+\theta\pi_{22})x_2}$$

To derive the optimal import tariff, we combine the FOC over τ_2^m and FOC over x_1 FOC over τ_2^m:

$$\frac{1}{1+\tau_2^m} = \frac{(u_c+\gamma_x\theta\pi_{11})x_1 - \gamma_{T_1}\theta\pi_{11}\alpha_1[\Gamma(1-\rho_{11})(1-\rho_{11})(\frac{T_1}{\pi_{11}})^{\rho_{11}} - \Gamma(1-\rho_{12})(1-\rho_{12})(\frac{T_2}{\pi_{12}})^{\rho_{12}}]}{(\gamma_x-\gamma_2)(1+\theta\pi_{11})x_1}$$

FOC over x_1:

$$u_c = \gamma_x(1-\frac{\tau_2^m}{1+\tau_2^m}\pi_{12}) - \gamma_2\frac{1}{1+\tau_2^m}\pi_{12}$$

Optimal import tariff:

$$\frac{1}{1+\tau_2^m}=\frac{\gamma_x}{\gamma_x-\gamma_2}+\frac{\gamma_{T_1}\theta\alpha_1[\Gamma(1-\rho_{12})(1-\rho_{12})(\frac{T_2}{\pi_{12}})^{\rho_{12}}-\Gamma(1-\rho_{11})(1-\rho_{11})(\frac{T_1}{\pi_{11}})^{\rho_{11}}]}{(\gamma_x-\gamma_2)(1+\theta)x_1}$$

By normalizing $\gamma_2=-\frac{\gamma_x}{\theta\pi_{22}}$, we get:

$$\frac{1}{1+\tau_2^m}=\frac{\theta\pi_{22}}{1+\theta\pi_{22}}\{1+\frac{\theta\alpha_1}{(1+\theta)\gamma_x x_1}\gamma_{T_1}[\Gamma(1-\rho_{12})(1-\rho_{12})(\frac{T_2}{\pi_{12}})^{\rho_{12}}-\Gamma(1-\rho_{11})(1-\rho_{11})(\frac{T_1}{\pi_{11}})^{\rho_{11}}]\}$$

$$\frac{1}{1+\tau_2^x}=1+\frac{\theta\pi_{22}}{1+\theta\pi_{22}}\frac{\alpha_2}{\gamma_x x_2}\gamma_{T_2}\{\Gamma(1-\rho_{21})(1-\rho_{21})(\frac{T_1}{\pi_{21}})^{\rho_{21}}-\Gamma(1-\rho_{22})(1-\rho_{22})(\frac{T_2}{\pi_{22}})^{\rho_{22}}\}$$

When ρ is homogenous across all countries, and we scale down all α by a constant $\Gamma(1-\rho)$, the optimal trade policies are the same as equation (13) and (14) in the paper.

The formula shows very similar insights as our baseline. We also conducted some numerical examples. We find that (1) other things equal, the smaller ρ_{1j}, the less tariff subsidy on country j. (2) The difference from the homogenous ρ case is that Home country is possible to subsidize imports from a low-α-low-T but high-ρ country if Home country learns more from this high ρ country, other things equal (like trade costs and labor endowment).

G.2 Multiple Sector

Now consider the final goods is a Cobb-Douglas function across the consumption of different sector $j\in J$ goods $C_n=\Pi_{j\in J}\left(C_{n,j}\right)^{\beta_j}$, where β_j is constant and reflects the share of sector j. Within a sector, there is a continuum of varieties of consumption goods, which are aggregated with a CobbDouglas function $C_{n,j}=\exp\int_0^1\ln c_{nj}(\omega)d\omega$. All goods are tradable with an iceberg trade cost d_{ni}.

The consumer price of Home is given by

$$P_{1t}=\Pi_j\left[T_{1,j,t}w_{1t}^{-\theta}+\sum_{n\neq 1}T_{n,j,t}\left(w_{nt}\left(1+\tau_{n,j,t}^m\right)d_{1n,t}\right)^{-\theta}\right]^{-\beta_j/\theta} \tag{A.40}$$

If ρ is heterogeneous across sectors, the technology evolution in the semi-endogenous growth model in Buera and Oberfield (2020) is

$$\tilde{T}_{i,j,t+1}=\tilde{T}_{i,j,t}+\tilde{\alpha}_{it}\left[\pi_{ii,j,t}^{1-\rho_j}\tilde{T}_{i,j,t}^{\rho_j}+\sum_{n\neq i}\pi_{in,j,t}^{1-\rho_j}\tilde{T}_{n,j,t}^{\rho_j}\right]. \tag{A.41}$$

where the growth rate of innovation efficiencies is exogenously driven by the population growth γ, hence the scale of the Frechet distribution T_t grows asymptotically at $\frac{\gamma}{1-\rho_j}$.

We normalize $w_{1t}=1$. According to equation (A.40), this normalization impacts the growth rate of Home price, g_{P_1}, which can be expressed as $g_{P_1}=-\sum_j\frac{\beta_j}{\theta}\frac{\gamma}{1-\rho_j}$.

The total expenditure, denoted by $x=wL$, implies that the growth rate of expenditure g_x is the sum of the growth rates of wages and labor, i.e., $g_x = g_w + g_L = \gamma$. Consequently, real consumption grows at a rate $g_C = g_x - g_P = \gamma + \sum_j \frac{\beta_j}{\theta} \frac{\gamma}{1-\rho_j}$.

These relationships confirm the existence of a balanced growth path where: technology in each sector T_{nj} grows at $\frac{\gamma}{1-\rho_j}$, expenditure x_n grows at γ, real consumption C_n grows at $\gamma + \sum_j \frac{\beta_j}{\theta} \frac{\gamma}{1-\rho_j}$, trade share $\pi_{ni,j}$ remains constant.

Using the definition of detrended technology and innovation efficiency as $T_{i,j,t} \equiv \tilde{T}_{i,j,t} e^{-\frac{\gamma}{1-\rho_j}(t-2016)}$ and $\alpha_{ij} \equiv \left(1 - \frac{\gamma}{1-\rho_j}\right) \tilde{\alpha}_{it} e^{-\gamma(t-2016)}$, we can show that equation (A.41) becomes

$$T_{n,j,t+1} = (1-\delta_j) T_{n,j,t} + \alpha_{nj} \sum_{i=1}^{N} \left[\pi_{ni,j,t} \left(\frac{T_{i,j,t}}{\pi_{ni,j,t}} \right)^{\rho_j} \right], \tag{A.42}$$

where $\delta_j = \gamma/(1-\rho_j)$.

H Suggestive Evidence: Learning Through Imports

In this section, we show some patterns in our data sample that motivate our paper. We first show technology convergence across countries. The dispersion of technology has been falling. We further propose one potential explanation for convergence: technology diffusion through trade. Using panel data with trade and patent citations, we find that higher citations of a country's patent are associated with larger imports from that country in the past. This finding shows the explanation of technology diffusion through trade is promising.

Data Source Our empirical analysis uses data on trade, national accounts, and patents.

To examine technology convergence, we use data on trade and national accounts. Trade data is sourced from the BACI database, based on COMTRADE, which provides a harmonized world trade matrix with values at the HS 6-digit level of the 1992 Harmonized System for 253 countries. Real national account data is obtained from the Penn World Table 10.01 (PWT 10.01). We begin with a sample of 169 countries, whose total import value accounts for 98% of global imports in 2016. From this sample, we select the 19 largest countries based on GDP in 2016, grouping all other countries into a "rest of the world" category. The import value of these top 19 countries represents approximately 80% of the total imports among the 169 countries in 2016. For these countries, we use real GDP, physical capital (K), and employment (emp) data from PWT 10.01. We focus on the years between 2000 and 2016.

To provide evidence of trade and technology diffusion, we use data on trade and patents. Our analysis begins with 253 countries from the BACI database and focus solely on cases where the importer is China. We group exporting countries into different categories based on China's imports in 2000. "Top 50" refers to the top 50 countries from which China imported in 2000, with similar definitions for the "Top 40" and "Top 30" groups. Additionally, we include countries classified as IMF advanced economies, OECD high-income countries, and G7 member countries. Industry codes are obtained from the BACI database, which provides the ISIC Rev.3 2-digit codes and product names for each HS 6-digit product. We limit our analysis to manufacturing sectors by including only products with ISIC Rev.3 2-digit codes ranging from 15 to 36. For each sector, we use the trade value for every country pair in each year.

Patent citations come from the European Patent Office (EPO) data. We only include patents from China that belong to the manufacturing sectors, identified by ISIC Rev.3 2-digit codes ranging from 15 to 36. Additionally, we focus on the years between 2000 and 2010, which are defined as the year when the patent application was filed. In the case of a patent from China with multiple inventors, we distribute the patent and citations equally among the inventors, considering their respective contributions. We then associate each inventor with their corresponding country. We aggregate the number of citations within a specific country-IPC-year combination. To map the country-IPC-year measures to the country-sector-year level, we utilize the IPC-sector mapping provided in Lybbert and Zolas (2014).

Technology Convergence In an Eaton-Kortum trade model, an underlying technology is associated with the observed domestic expenditure share π_{ii} and real income per capita w_i/P_i of a country. Specifically, for each country i, the technology is given by

$$T_{it} = \pi_{ii,t} \left(\frac{w_{it}}{P_{it}} \right)^{\theta}. \tag{A.43}$$

In the EK model, real income per capita equals real GDP per input in consumer price.

Following Buera and Oberfield (2020), we measure the composite input with $K^{\zeta} emp^{1-\zeta}$ where K is capital, emp employment, and ζ capital share. The real GDP, physical capital (K), and employment (emp) are from the PWT 10.01. We choose the capital share ζ to be 0.36 to match the corporate labor share in the U.S. calculated by Karabarbounis and Neiman (2014). The Domestic expenditure share π_{ii} is constructed using the bilateral trade data. The Frechet parameter θ is chosen to be 4, consistent with the trade elasticity estimated from Simonovska and Waugh (2014).

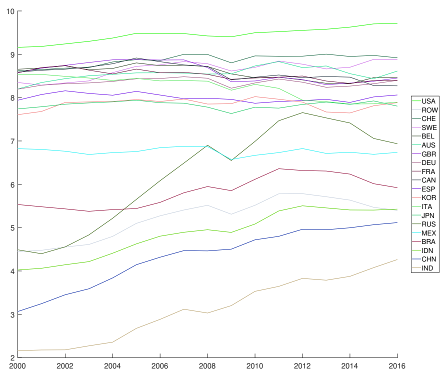
Panel (a) of Figure A-15 plots the logged technology across our 20 countries from 2000 to 2016. On average, developed countries have higher technology development than developing countries. However, developing countries experience faster technology growth. Moreover, the dispersion of technology across countries falls over time, and the standard deviation decreases from around 2.2 to 1.5, as Panel (b) shows. Hence, Figure A-15 indicates a technology convergence.

There are various reasons for technology convergence in Panel (b). One potential one is learning through trade. After 2000, many countries adopted trade liberalization, which not only brought more trade but also technology diffusions through trade across countries. As a result, technology development has become less dispersed across countries. Given that it is challenging to measure technology diffusion, below, we provide suggestive evidence for this channel by documenting a positive relationship between imports and patent citations.

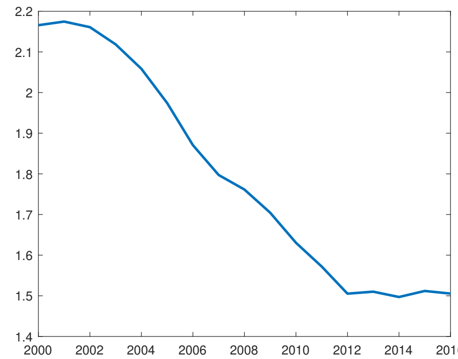
Trade and Technology Diffusion Here, we explore the relationship between trade and technology diffusion. Some literature papers utilize patent citation data to indicate knowledge diffusion (Cai and Li (2019), Acemoglu, Akcigit, and Kerr (2016)). The hypothesis is that when firms in China import from country c, they uncover the underlying technology embedded in the imported goods. Firms further improve upon the technology from country c and cite more the patents from country c. To see the potential of such an explanation, we consider the following panel regression:

$$\log(citation_{cjt}) = \beta_1 \log(import_{cjt-1}) + \beta_2 \log(export_{cjt-1}) + \delta_c + \delta_j + \delta_t + \varepsilon_{cjt}$$

Figure A-15: Technology Covergence



(a) log(T)



(b) Standard Deviation of log(T)

Note: The left panel plots the logged technology backed out using equation (A.43) for each of the 20 countries in our sample. The right panel plots the standard deviation of cross-country technologies in each period from 2000 to 2016.

where $citation_{cjt}$ is the number of patent citations that China cites from country c in sector j in year t, $import_{cjt-1}$ is the value of China's import from country c in sector j in year $t-1$, and $export_{cjt-1}$ is the value of China's export to country c in sector j in year $t-1$. We control the country, sector, and year fixed effects. The sample covers the period from 2000 to 2010.

Table A-3 presents the regression results based on different groups of countries that trade with China. Column (1) includes the top 50 countries from which China imported in 2000, while Columns (2) and (3) focus on the top 40 and top 30 trading partners, respectively. Additionally, we analyze groups of advanced economies defined by the IMF, OECD high-income countries, and G7 member countries.

Table A-3: Regression: Trade on Patent Citations

	(1)	(2)	(3)	(4)	(5)	(6)
	Top50	Top40	Top30	IMF advanced	OECD high income	G7
$\log(import_{cjt-1})$	0.052***	0.058***	0.061***	0.061***	0.053***	0.059***
	(0.007)	(0.010)	(0.012)	(0.011)	(0.011)	(0.028)
$\log(export_{cjt-1})$	-0.041***	-0.045***	-0.046***	-0.046***	-0.082***	-0.011
	(0.008)	(0.011)	(0.013)	(0.016)	(0.014)	(0.034)
FE				country, sector, year		
Obs	11000	8800	6600	9020	7480	1540

Note: Cluster in country-sector. "Top50" refers to retaining only the top 50 countries from which China imported in 2000, with similar definitions applying to the "Top40" and "Top30" groups. The last three columns represent samples restricted to IMF advanced economies, OECD high-income countries, and G7 member countries, respectively. We add 1 to each variable before taking logarithms to handle zero values.

All results demonstrate a positive relationship between patent citation and import: when China imports more from a country c in the past, it also cites more patents from country c. Exports and patent citations, however, exhibit a negative relationship. Thus, a mechanism of learning from imports is consistent with our empirical findings of positively correlated imports and patent citations.

In summary, we find technological convergence across countries. Such convergency could arise from technology diffusion through imports. Our second empirical finding suggests this explanation is promising. Moreover, many trade disputes are on technology and spillover, it is a natural question to ask the policy implication when a Home country considers technology diffusion and would like to maximize its own utility. In the paper, we derive the dynamic optimal policies with international technology diffusion.

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