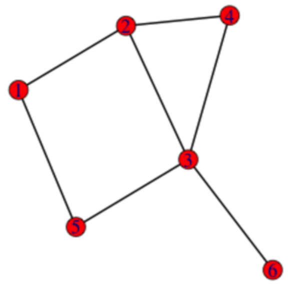
Week 1.1: Introduction to Networks

Content

1.1 Undirected Simple Network



- **Number of Nodes** (n): 6
- **Number of Edges** (m): 7
- **Degree**
    - **Degree of Node** (ki): The number of ends attached to each node

      $$k_i = \sum_{j=1}^{n} A_{ij} \quad k_5 = \sum_{j=1}^{6} A_{5j} = A_{51} + \ldots + A_{56}$$

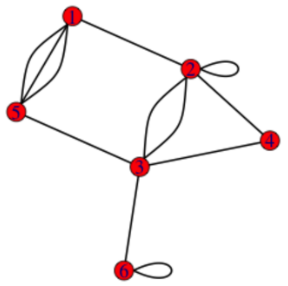
    - **Mean Degree of Node** (<ki>): The mean degree across nodes

      $$< k > = \frac{1}{n} \sum_{i=1}^{n} k_i = \frac{2m}{n}$$

- **Adjacency Matrix**:
    - **Symmetry**: Symmetrical (Aij=Aji)
    - **Representation**: (1) Aij= 1 means a connection exists between the two points; (2) Aij=0 means a connection does not exist between the two points

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

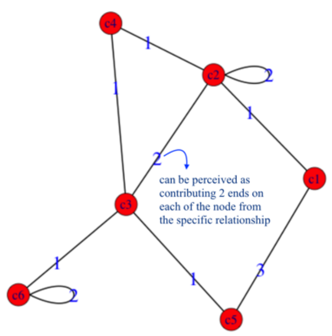
1.2 Undirected Multi-edge Network



- **Number of Nodes** (n): 6
- **Number of Edges** (m): 12. Different links between the same two nodes can represent different relationships between agents (e.g., different modes of transportation)
- **Adjacency Matrix**:
  - **Symmetry**: Symmetrical (Aij=Aji)
  - **Representation**: (1) Aij represents the number of ends (i.e., where nodes and edges are connected) contributed by the link(s) between i and j; (2) Aii=0 or 2, where 2 represents a self-loop

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 & 3 & 0 \\ 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix}$$

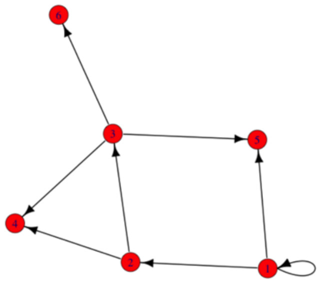
1.3 Weighted Network



- **Number of Nodes** (n): 6
- **Adjacency Matrix**:
  - **Symmetry**: Symmetrical (Aij=Aji)
  - **Representation**: (1) Aij represents the number of ends (i.e., where nodes and edges are connected) contributed by the link(s) between i and j, incorporating the weight in; (2) Aii=0 or 2, where 2 represents a self-loop

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{bmatrix} \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 & 3 & 0 \\ 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix}$$

1.4 Directed Network



- **Number of Nodes** (n): 6
- **Number of Edges** (m): 8

$$m = \sum_{i=1}^{n} k_i^{in} = \sum_{j=1}^{n} k_j^{out} = \sum_{ij} A_{ij}$$

- **Notation**:
  - **Aij =1**: There is a link from j to i
  - **Aij = 0**: There is no link from j to i
- **Degree**:
  - **In-degree**: The number of links pointing to you

  $$k_i^{in} = \sum_{j=1}^{n} A_{ij}$$

  - **Out-degree**: The number of links pointing to others

  $$k_j^{out} = \sum_{i=1}^{n} A_{ij}$$

  - **Mean Degree of Node** (<k>): m/n
- **Adjacency Matrix**:
  - **Symmetry**: No longer symmetrical because the links are not mutually the same
  - **Loop**: Loop has the value of 1 in this case due to the directionality

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} \end{bmatrix} \qquad A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Diameter: The greatest shortest path between two nodes in the network