Report of Midterm Project
Introduction to MATLAB
Southern University of Science and Technology
April 10-13, 2020, Shenzhen, Guangdong, CHINA

Introduction to MATLAB-ME112

EFFECTIVE FACTORS IN TRAPPING DROPLET WITH QUADRUPOLE

Xudong Han[1]

[1]Southern University of Science and Technology, Shenzhen, China

ABSTRACT

Electro-dynamic balance (EDB) is widely used in mass spectrometry, control of charged micro droplet and other study. By EDB, an electrodynamic balance levitation system with linear quadrupole is developed. Considering the lifetime of the droplet, the response time in trapping the droplet is important, which could be controlled by AC voltage and frequency. And for evaporation of the droplet, parameters should be controlled to make the droplet trapped successfully. As the effective coverage of the quadrupole is limit, the initial velocity of the droplet should be considered to insure the motion inside the limit coverage.

Keywords: quadrupole, AC voltage, frequency, evaporation

1. INTRODUCTION

Electro-dynamic balance (EDB) is capable of levitating droplet diameters of a few tens or micrometers (Shaw, 2000; Davis, 1980; Agnes, 2002) that is desirable for investigating both diffusion-rate controlled and kinetically controlled evaporation regime (Davis, 1980). EDB is developed based on the quadrupole mass filter that is proposed by Paul and Steinwedel (1953). A charged particle suspended by means of the electrical fields. The device can trap particles with the size range of nanometer to hundreds of micrometers by adjusting the AC frequency and magnitude. The target is to identify the parameters that affect trapping the charged droplet and control the process so that we could get desired results.

2. PRINCIPLE AND METHOD

The linear quadrupole setup simply composes of four metal rods placed parallel with each other as shown in Figure 1. We will discuss the principle of trapping a charged micro droplet with linear quadrupole in 2.1 and adopt numerical approach with MATLAB to develop the method to determine influence of AC voltage and frequency on the response. In discussion, we first considered no evaporation for simple model to describe the influence of AC voltage and frequency. But for evaporation, the droplet's size and mass would change but the charge would keep until Rayleigh limit, before which we would discuss the process.

FIGURE 1: LINEAR QUADRUPOLE SETUP

2.1 Principle

As shown in Figure 1, Two rods opposite each other have the same voltage, which is *Vcos(wt)*, the other two opposing rods have the same magnitude of voltage but differ in polarity *(-Vcos(wt))*. w is the frequency of the AC voltage and V is the magnitude of it. This structure could build up an electric field.

The electric field generated by ac voltage, V_{ac}, applied on the quadrupole is expressed as

$$E_x = -\frac{2x}{r_0^2} V_{ac} cos(wt) \quad (1)$$

$$E_y = \frac{2y}{r_0^2} V_{ac} cos(wt) \quad (2)$$

where r_0 is the distance from center to rod surface (shown in Figure 2).

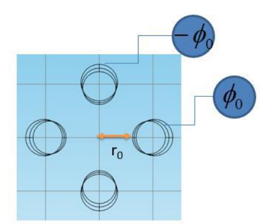

FIGURE 2: SIDE VIEW OF LINEAR QUADRUPOLE

From the expressions of electrical field induced by ac voltage, we can see the charged particle in the space surrounded by the electrodes is subject to periodic forces whose strength are proportional to the distance from the center of the device. The electric field that is produced by a DC voltage, V_{dc} , is

$$E_{dc} = \frac{V_{dc}}{r_0} \tag{3}$$

Suppose a charged particle is injected to a dynamic field. The trajectory of a charged particle with the mass of m and charge of q in the electrical filed are described as

$$F_x = m\frac{d^2x}{dt^2} = -C_d\frac{dx}{dt} + qE_x \tag{4}$$
$$F_y = m\frac{d^2y}{dt^2} = -C_d\frac{dy}{dt} + qE_y - mg + qE_{dc} \tag{5}$$

where C_d is the drag coefficient and for a spherical particle it can be defined by

$$C_d = 3\pi d\mu \tag{6}$$

where d is the particle diameter, and μ is the viscosity of the gas phase.

Substitute equations 1, 2, 3 and 6 into equations 4 and 5, we could have

$$m\frac{d^2x}{dt^2} = -\frac{2x}{r_0^2}V_{ac}cos(wt) - 3\pi d\mu\frac{dx}{dt} \tag{7}$$
$$m\frac{d^2y}{dt^2} = \frac{2y}{r_0^2}V_{ac}cos(wt) - \frac{qV_{dc}}{r_0} - 3\pi d\mu\frac{dy}{dt} - mg \tag{8}$$

In the absence of external forces, the function of dc voltage thus becomes to overbalance the gravitational force.

$$-\frac{qV_{dc}}{r_0} = mg \tag{9}$$

Then Eq.7 and 8 becomes

$$m\frac{d^2x}{dt^2} = -\frac{2x}{r_0^2}V_{ac}cos(wt) - 3\pi d\mu\frac{dx}{dt} \tag{10}$$
$$m\frac{d^2y}{dt^2} = \frac{2y}{r_0^2}V_{ac}cos(wt) - 3\pi d\mu\frac{dy}{dt} \tag{11}$$

Equations 10 and 11 are second order nonlinear differential equations which describe the path of the droplet involved. But the equations are not easy to get analytical solution directly. We could change them into Mathieu equation whose stability region has already known.

$$\frac{d^2u}{d\xi^2} + (a_u - 2q_u cos2\xi)u = 0 \tag{12}$$

where

$$a_x = -a_y = \frac{8qV_{dc}}{mr_0w^2} \tag{13}$$
$$q_x = -q_y = -\frac{4qV_{ac}}{mr_0w^2} \tag{14}$$
$$\xi = \frac{wt}{2} \tag{15}$$

The droplet should be stable on both x and y direction to pass through the quadrupole, which is the principle for selecting charged droplet. Mathieu equation's stability region is shown in Figure 3 in parameter space of a and q.

The parameters involved in determining whether trapped particles can stabilize in the null point finally are V_{ac}, V_{dc}, w and charge mass ratio (q / m). It is easy to find the value of V_{dc} to overbalance gravity. We would discuss the influence of different V_{ac} and w on certain droplet. Also, we are interested in the influence of the velocity on the droplet.

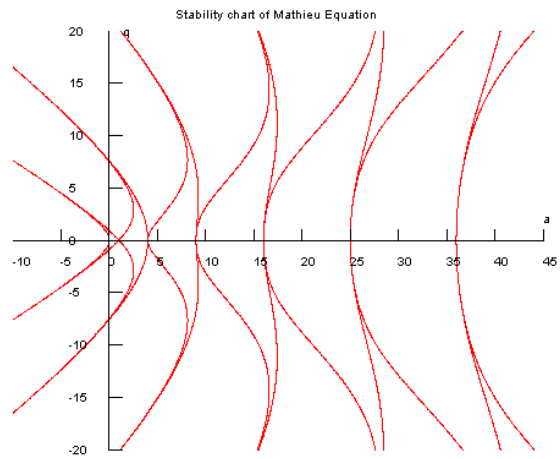

FIGURE 3: STABILITY CHART OF MATHIEU EQUATION

2.2 Method

The response time in trapping the droplet is important when studying evaporation, reaction and combustion process because of the short lifetime of the droplet. Equations 10 and 11 could describe the path of a certain charged droplet but are not easy to get analytical solution directly. We adopted the numerical simulation approach to solve them with proper initial conditions . The droplet with size of 50μm are studied under different velocities and AC voltage and frequency. The charge ratio of all the cases discussed below is 30% Raleigh limit.

The droplet vibrates under the AC electrical field and will finally stabilize in the center point when DC is set to overbalance gravity and AC voltage and frequency was set to satisfy the stability criteria. The distance from center to rod surface r_0 is 12 mm, so the displacement should always be within the -12~12 mm during the vibration process.

We assumed that the environment is at room temperature 25℃. The air dynamic viscosity η is 1.849×10^{-5}kg/m-s. The density of the droplet is $0.9974456 g/cm^3$. The surface tension is 7.28×10^{-2}N/m.

First, we only considered the motion on x direction (y = 0). We injected the droplet with the initial position and velocity of droplet as x = 1mm and v_x = 1mm/s. Then we chose several V and w values under which the droplet can be trapped to plot x versus t. The MATLAB code is provided in Appendix B. By analyzing the plot, we could figure out the affect of the AC voltage and frequency on the response time.

Then, we did the same thing for the combination of both x and y direction motions. We injected the droplet with the initial position and velocity of droplet as x = 1mm, y = 1mm, v_x = 1mm/s, and v_y = 1mm/s. The path of the droplet could be simulated. The MATLAB code is provided in Appendix B.

When we considered the evaporation of the droplet, we could build up the relation between d^2/d_0^2 and t/d_0^2.

$$\frac{d^2}{d_0^2} = k\frac{t}{d_0^2} + 1 \tag{16}$$

where assumed $k = 1.0\times10^{-9}\text{m}^2/\text{s}$, and initial diameter d_0 = 50µm = 5.0×10^{-5}m. Then equation 16 becomes

$$d = \sqrt{d_0^2 - 1.0\times10^{-9}t} \tag{17}$$

Then considering d changing with t, we did the same thing for only x direction motions, only y direction motions and the combination of both x and y direction motions. As the maximum charge a droplet can hold is Rayleigh limit, and if the charge reaches Rayleigh limit, Coulombic fission happens and that is a complex process that we would not discuss. We only considered the process before Rayleigh limit. The MATLAB code is provided in Appendix C.

Different induced velocity could affect the path of the droplet. Although AC voltage could change the velocity, large initial velocity could make the droplet run out of the effective coverage. We could choose several different velocities to show the effect. And the MATLAB code is provided in Appendix D.

3. RESULTS AND DISCUSSION

We assumed no evaporation and considered different V_{ac} and w affecting on the response. First, we chose different w = 900, 1200, and 1900rad/s with stable V_{ac} = 2000V. The MATLAB code is shown in Appendix B. And the results is shown in Figures 4~12.

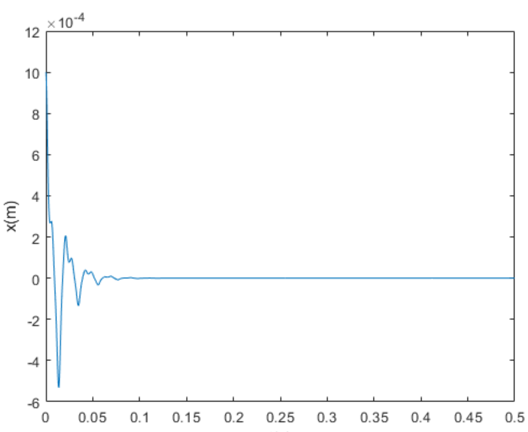

FIGURE 4: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 2000V, w = 900rad/s

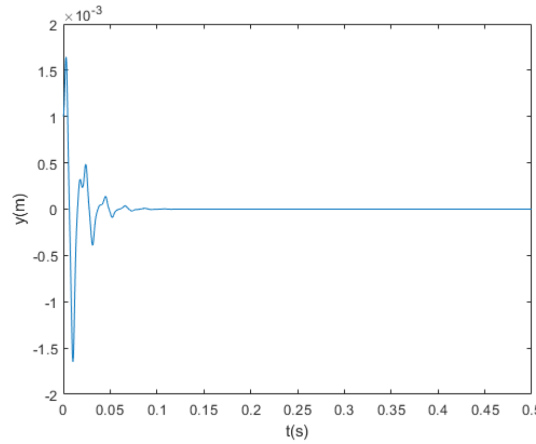

FIGURE 5: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 2000V, w = 900rad/s

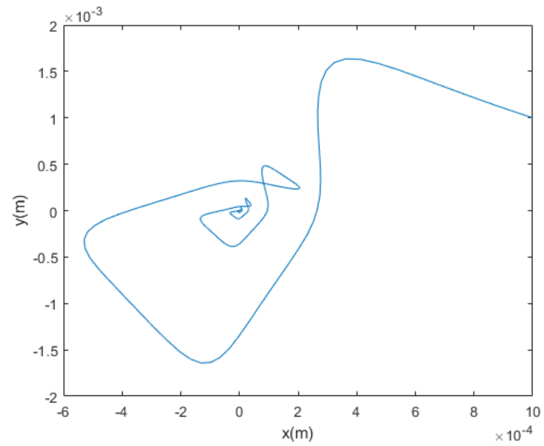

FIGURE 6: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 2000V, w = 900rad/s

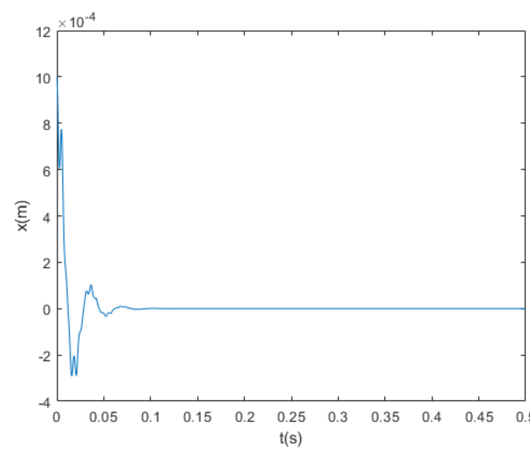

FIGURE 7: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 2000V, w = 1200rad/s

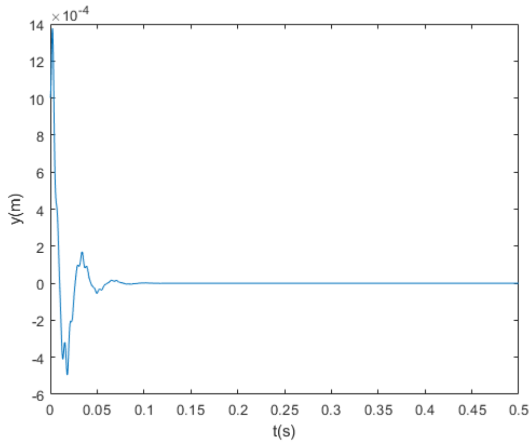

FIGURE 8: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 2000V, w = 1200rad/s

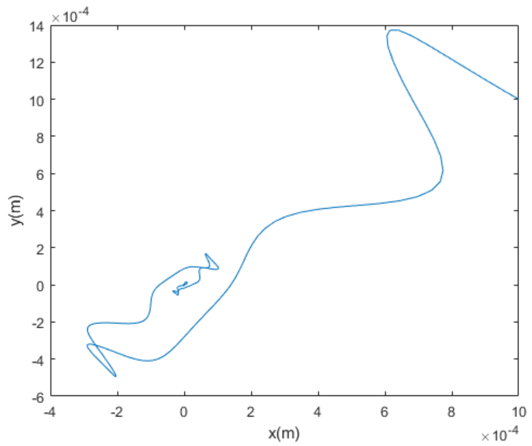

FIGURE 9: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 2000V, w = 1200rad/s

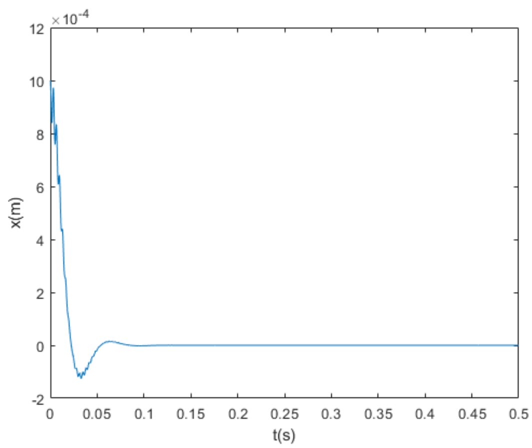

FIGURE 10: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 2000V, w = 1900rad/s

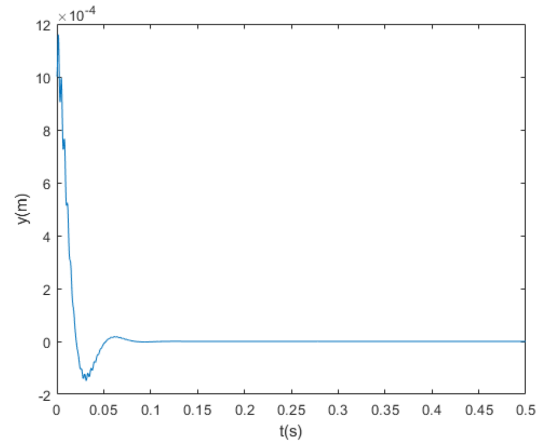

FIGURE 11: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 2000V, w = 1900rad/s

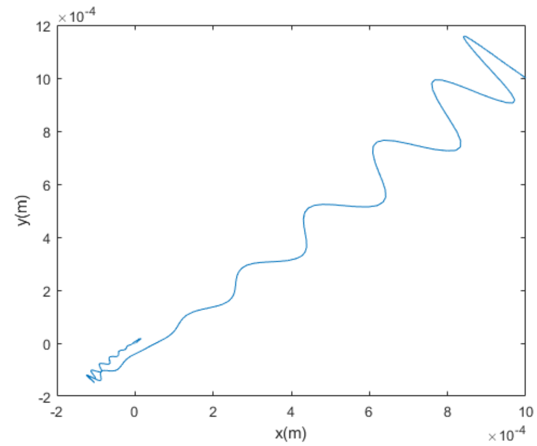

FIGURE 12: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 2000V, w = 1900rad/s

Second, we chose different V_{ac} = 1000, 1500, 2000V with stable w = 1200rad/s. The result is shown in Figure 4~12.

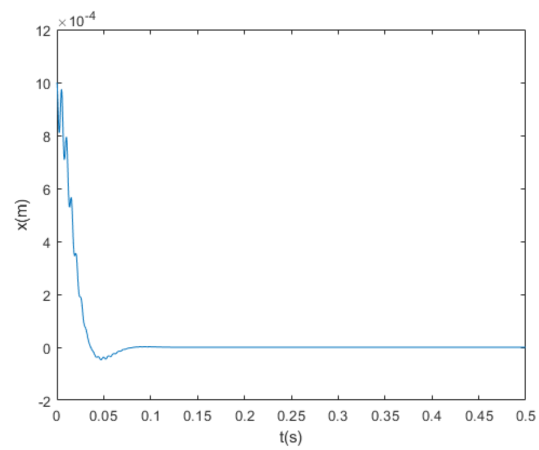

FIGURE 13: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 1000V, w = 1200rad/s

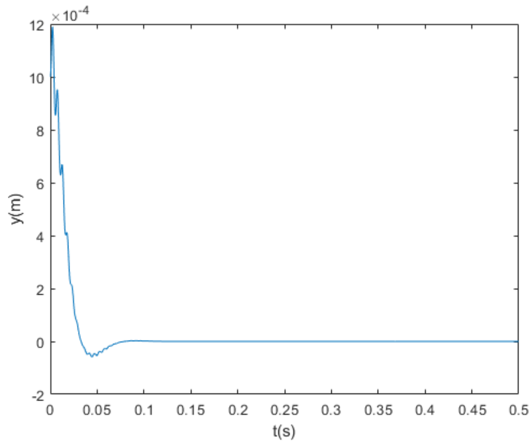

FIGURE 14: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 1000V, w = 1200rad/s

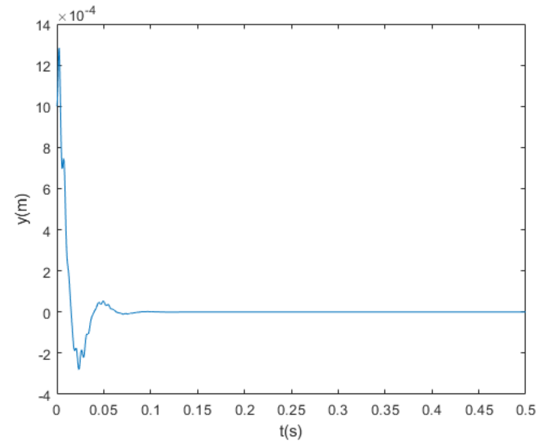

FIGURE 17: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 1500V, w = 1200rad/s

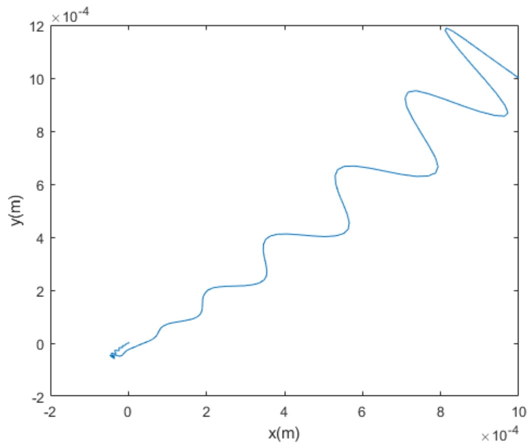

FIGURE 15: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 1000V, w = 1200rad/s

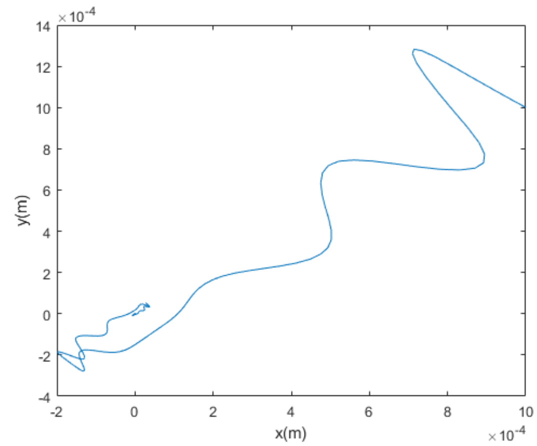

FIGURE 18: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 1500V, w = 1200rad/s

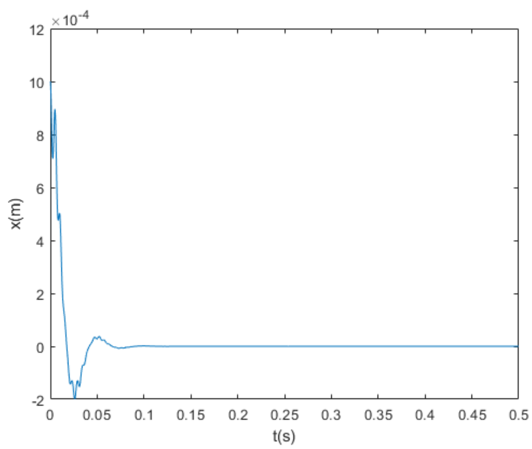

FIGURE 16: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 1500V, w = 1200rad/s

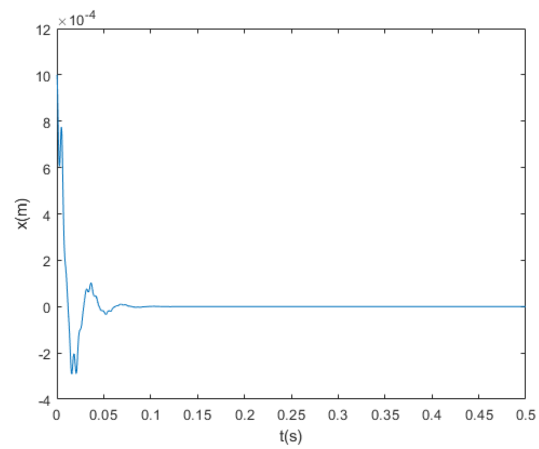

FIGURE 19: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 2000V, w = 1200rad/s

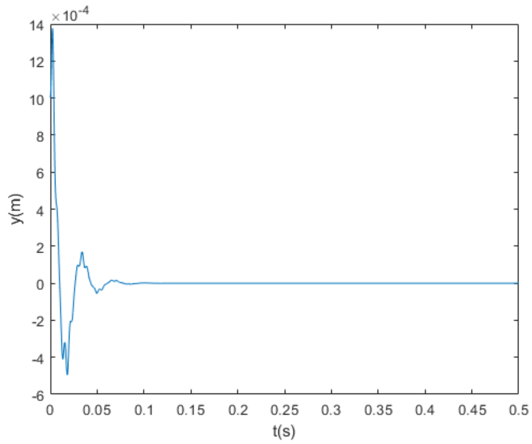

FIGURE 20: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 2000V, w = 1200rad/s

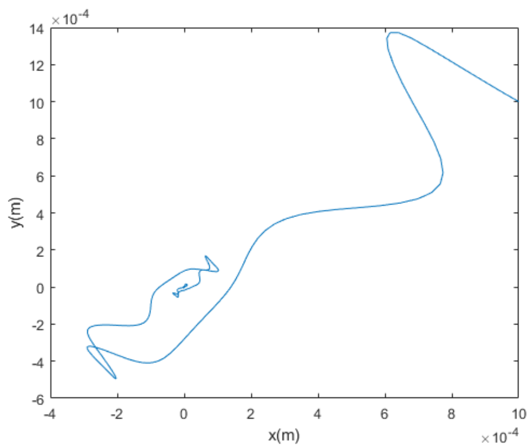

FIGURE 21: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 2000V, w = 1200rad/s

According to the result shown in Figures 4~12, the droplet is trapped successfully but the response time is different with different AC voltage and frequency. The vibration number and amplitude both decrease as w increases. According to the result shown in Figures 13~21, the vibration number increases but the amplitude decreases as V_{ac} increases. That means high w and low V_{ac} in a suitable region could decrease the response time to the stability.

In Figures 4~21, we learned that x and y are both converged to 0. Since

$$\lim_{t\to\infty} x = 0 \tag{18}$$
$$\lim_{t\to\infty} y = 0 \tag{19}$$

If evaporation is considered, the process before Rayleigh limit would be discussed because of the complex process of Coulombic fission when the charge reaches Rayleigh limit. We chose V_{ac} = 300V and w = 1000rad/s, V_{ac} = 400V and w = 1000rad/s, and V_{ac} = 400V and w = 1300rad/s.

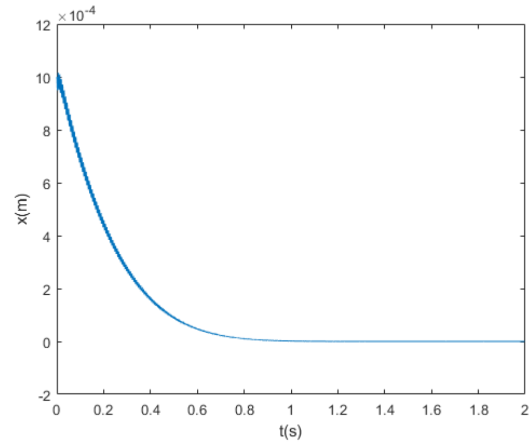

FIGURE 22: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 300V, w = 1000rad/s

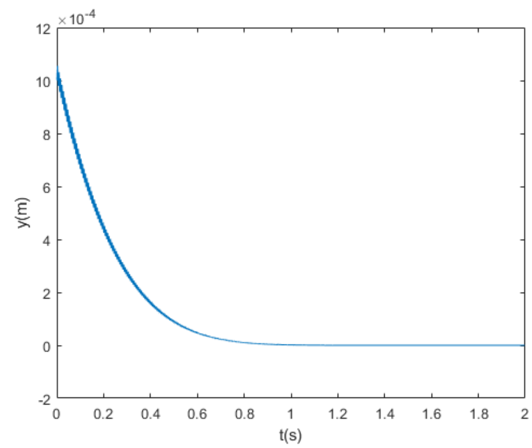

FIGURE 23: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 300V, w = 1000rad/s

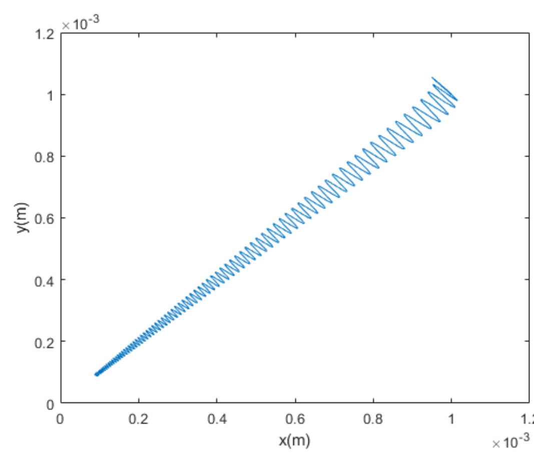

FIGURE 24: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 300V, w = 1000rad/s

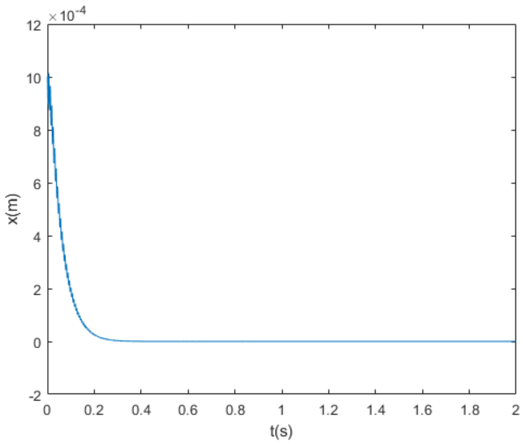

FIGURE 25: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 400V, w = 1000rad/s

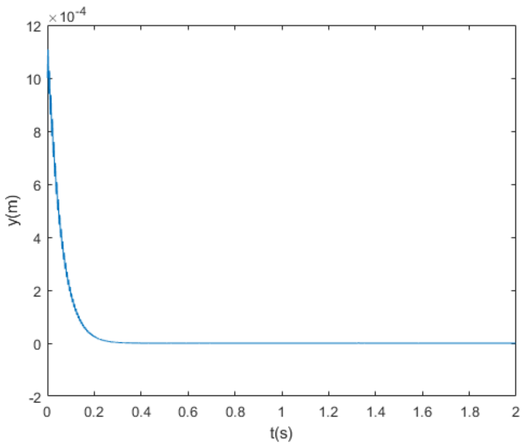

FIGURE 26: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 400V, w = 1000rad/s

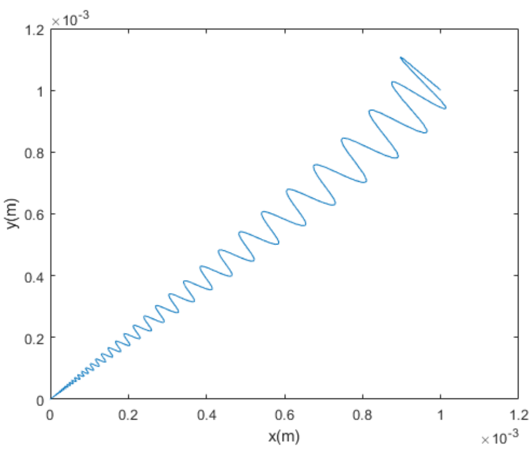

FIGURE 27: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 400V, w = 1000rad/s

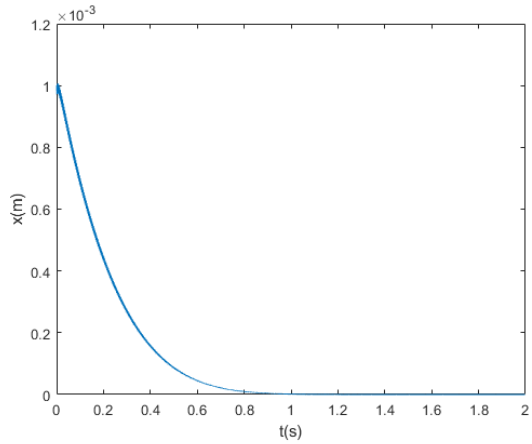

FIGURE 28: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 400V, w = 1300rad/s

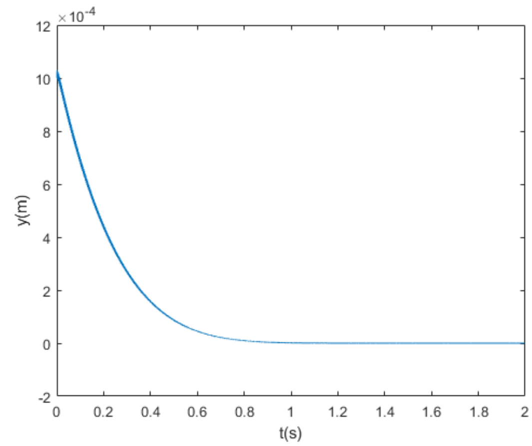

FIGURE 29: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 400V, w = 1300rad/s

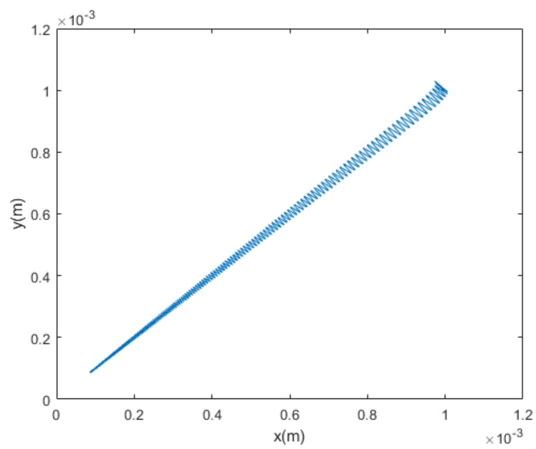

FIGURE 30: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 400V, w = 1300rad/s

In Figures 22~24, we could learn that too small V_{ac} would not make the droplet stable at null point before the charge reached Rayleigh limit. In Figures 25~27, suitable V_{ac} and w

could make the droplet vibrate to stability. In Figures 28~30, we could learn that too large *w* also would not make the droplet stable before Rayleigh limit.

Low AC voltage and high frequency might make the droplet not run to null point for stability. When AC voltage is high, we could learn that the response time is short and the droplet could easily run to the stable position as shown in Figures 31~33. We chose V_{ac} = 2000V, and *w* = 900, 1200, 1900rad/s.

In Figures 31~39, we could learn that although evaporation included, the trajectory of the droplet is similar with that in Figures 4~12. Both chose V_{ac} = 2000V, and *w* = 900, 1200, 1900rad/s. But there is still some little difference. For V_{ac} = 2000V, and *w* = 900rad/s, as shown in Figures 4~6 and Figures 31~33, most of the trajectory are similar, but at the end part, evaporating droplet would vibrate again and the amplitude became larger and larger. We could learn that at low frequency, the droplet would run to the stable position but after some time it would vibrate again and run against the stable position further and further. If we trap the droplet at low frequency, the vertical velocity must be large enough to travel through the quadrupole in time. But at high frequency, there is no worry about second vibration as the droplet keep stable in a long time.

The evaporation process could also show the relation between the size of the droplet and frequency *w*. During the evaporation, the diameter deceases following equation 17. As time increases, the diameter or the size decreases but the charge never change before Coulombic fission. And AC voltage V_{ac} and frequency *w* keep stable during the process. Then we concluded that the certain AC voltage and frequency could only trap the stability droplet with the size in a region. On the other hand, the droplet with certain size could only be trapped by AC voltage and frequency in a region. Such region could be found by Mathieu equation solution as shown in Figure 3. In the region, the droplet with larger size and same charge needs higher frequency to be trapped.

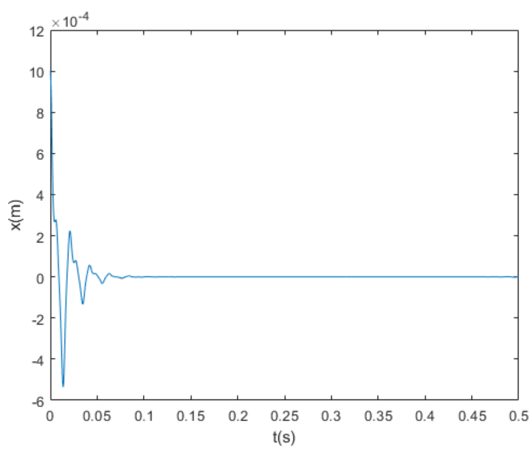

FIGURE 31: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 2000V, *w* = 900rad/s

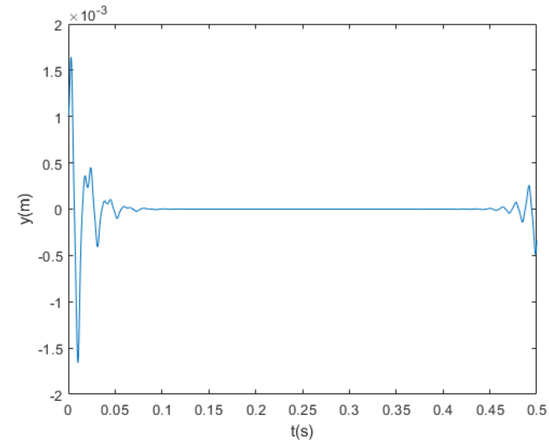

FIGURE 32: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 2000V, *w* = 900rad/s

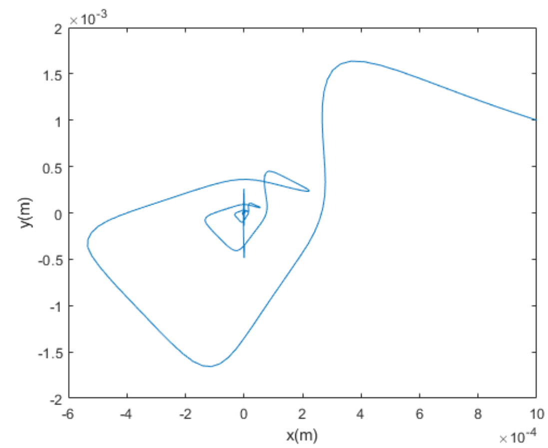

FIGURE 33: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 2000V, *w* = 900rad/s

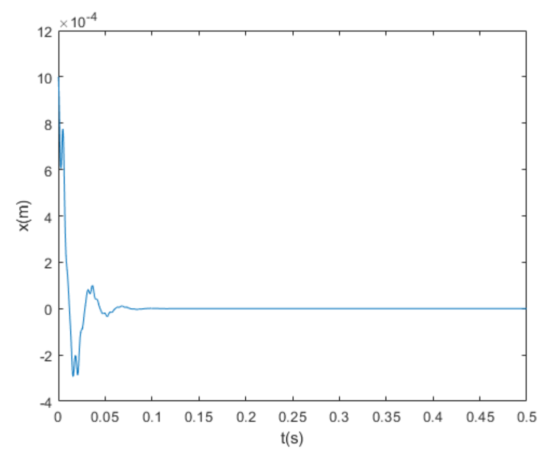

FIGURE 34: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 2000V, *w* = 1200rad/s

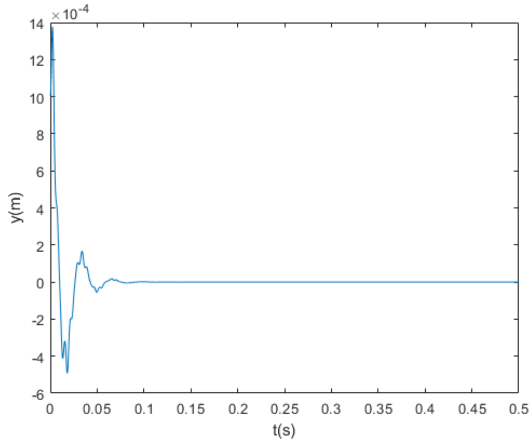

FIGURE 35: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 2000V, w = 1200rad/s

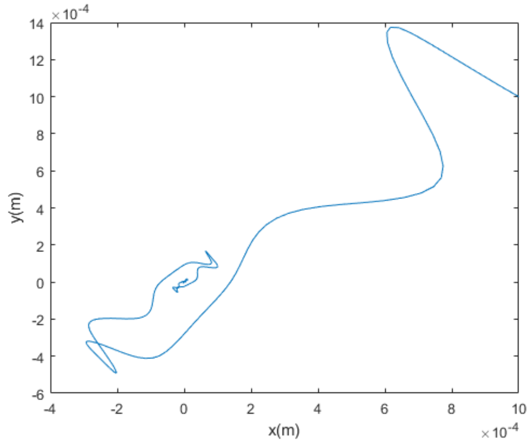

FIGURE 36: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 2000V, w = 1200rad/s

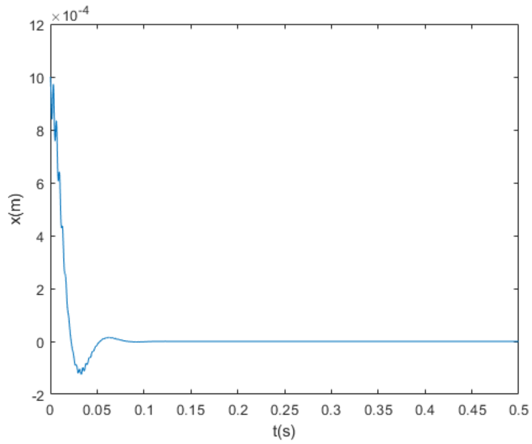

FIGURE 34: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 2000V, w = 1900rad/s

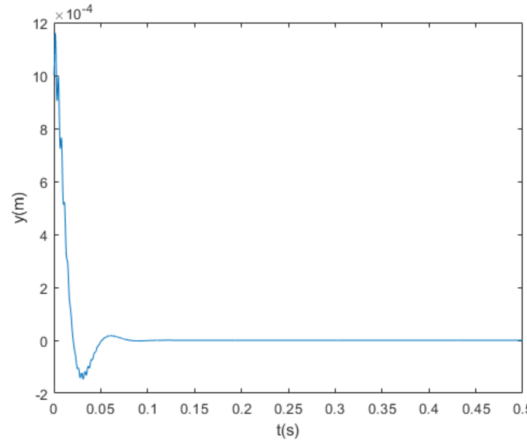

FIGURE 34: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 2000V, w = 1900rad/s

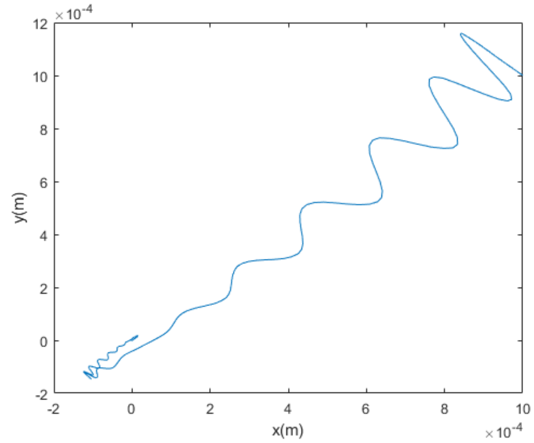

FIGURE 34: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 2000V, w = 1900rad/s

Initial velocity is an important parameter that would affect the motion path of the droplet. We chose several different velocities, v_x = v_y = 0.5, 1, and 2m/s, with V_{ac} = 2000V, w = 1000rad/s and no evaporation.

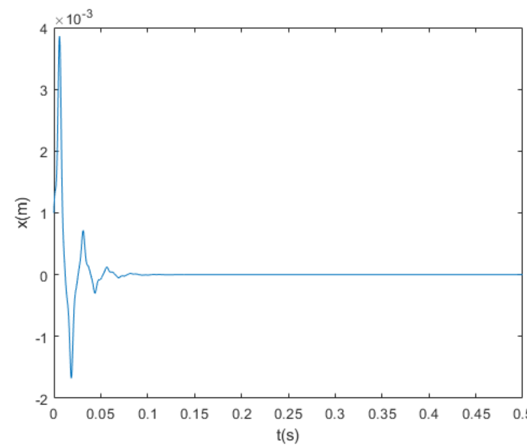

FIGURE 40: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 2000V, w = 1000rad/s, v_x = v_y = 0.5m/s

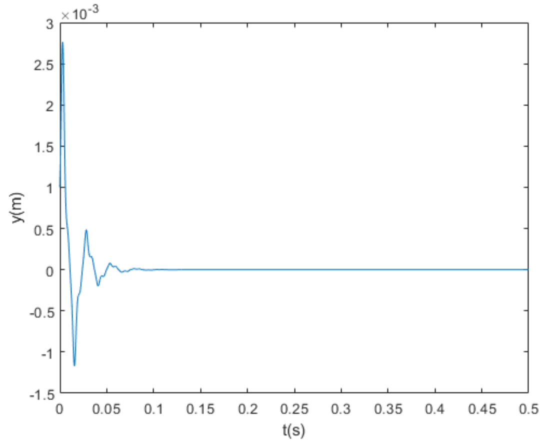

FIGURE 41: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 400V, w = 1000rad/s, v_x = v_y = 0.5m/s

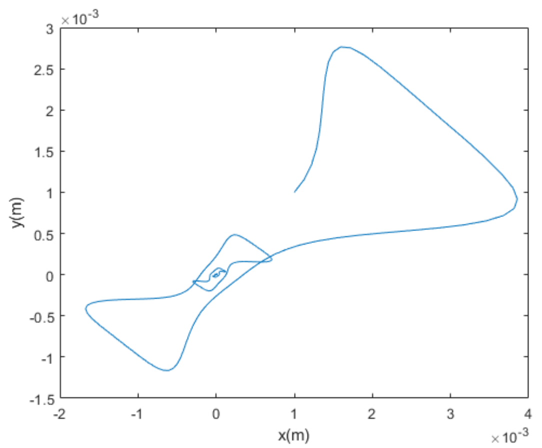

FIGURE 42: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 2000V, w = 1000rad/s, v_x = v_y = 0.5m/s

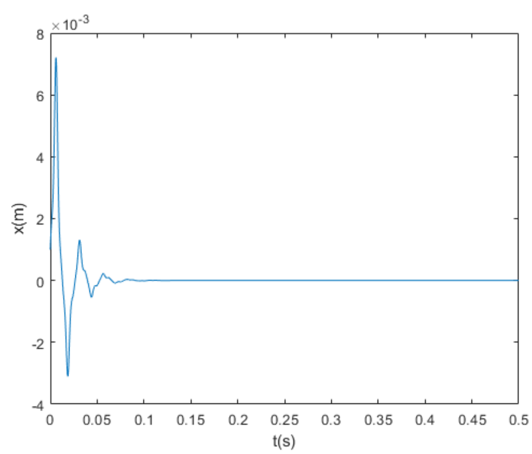

FIGURE 43: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 2000V, w = 1000rad/s, v_x = v_y = 1m/s

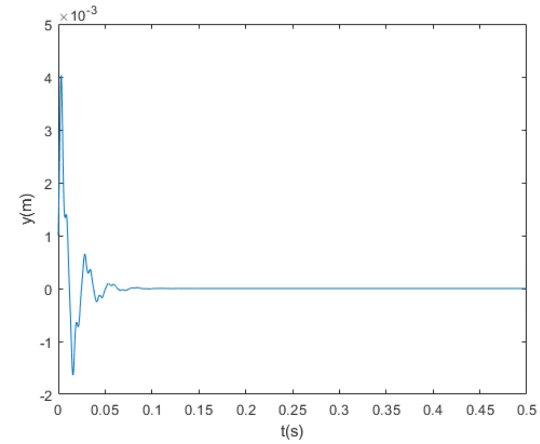

FIGURE 44: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 2000V, w = 1000rad/s, v_x = v_y = 1m/s

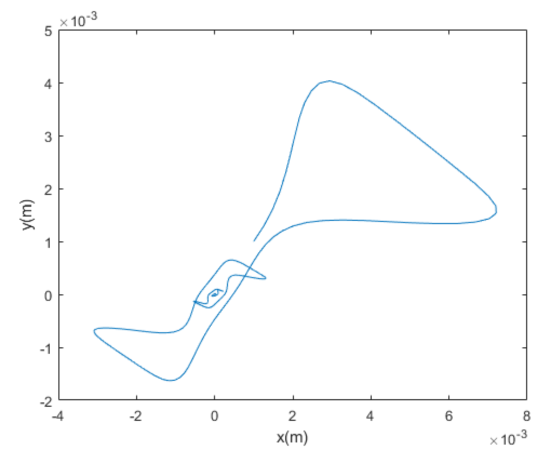

FIGURE 45: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 2000V, w = 1000rad/s, v_x = v_y = 1m/s

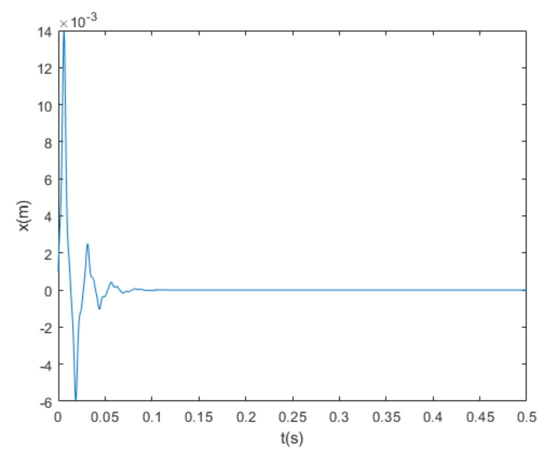

FIGURE 46: TRAJECTORY OF DROPLET IN THE X DIRECTION WITH V_{ac} = 2000V, w = 1000rad/s, v_x = v_y = 2m/s

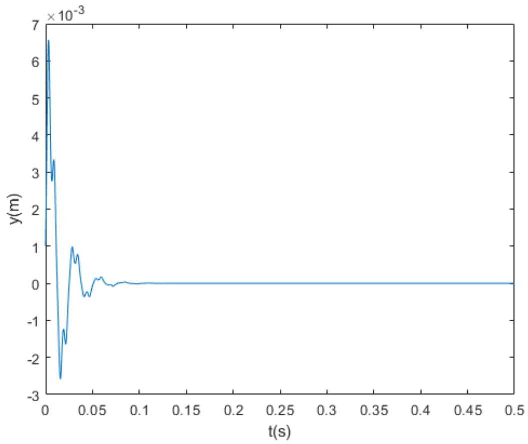

FIGURE 47: TRAJECTORY OF DROPLET IN THE Y DIRECTION WITH V_{ac} = 2000V, w = 1000rad/s, $v_x = v_y$ = 2m/s

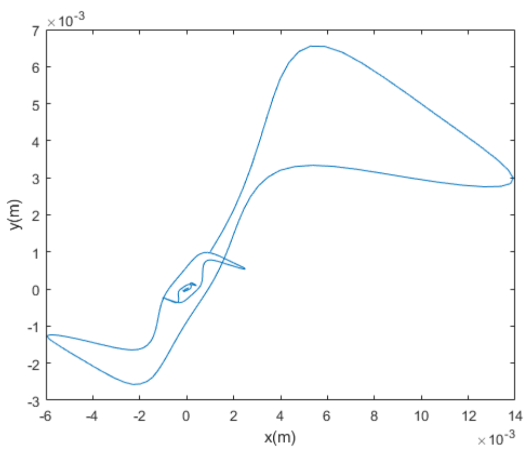

FIGURE 48: CONBIANTION OF X AND Y DIRECTION MOTION WITH V_{ac} = 2000V, w = 1000rad/s, $v_x = v_y$ = 2m/s

Equation 1 and 2 shows the electric field intensity generated by AC voltage. We can assume AC voltage V_{zc} = 2000V and use the effective value of the electric field intensity to express the electric field (shown in Figure 49). The MATLAB code is provided in Appendix E.

The electric field shows that the minimum value of the intensity appears at the center. The effective coverage of the quadrupole is about a circle with radius r_0, the distance from the center to rod surface. And at the boundary, the intensity is almost the maximum value. The whole electric field decreases from periphery to center when considering effective value of intensity.

For the real electric field with *cos(wt)*, the direction of the electric field change and point to opposite direction. Even if the droplet could drive far away to the center by the electrical force pointing outward, next time when the force direction is toward the center, the magnitude is larger than before and the droplet gets closer to the center than initial position, so that the droplet can be trapped.

But there should be a premise that the droplet does never move out of the effective coverage. For certain AC voltage V_{ac} and frequency w, the initial velocity affect the motion as shown in Figures 40~48. As the initial velocity increases, the maximum displacement increases. Figures 40 and 43 shows that the maximum displacement along x direction is 4 and 7mm when the initial velocity is 0.5 and 1m/s. Figure 46, however, shows that the maximum displacement along x direction is 1.4mm when th e initial velocity is 2m/s, which is larger than the effective coverage radius. Therefore, the initial velocity should be controlled when trapping the droplet with such quadrupole electric field in order to stabilize the droplet. Lower initial velocity could also let the vibration decrease faster, which means the response time is reduced and the effect of evaporation could be weakened.

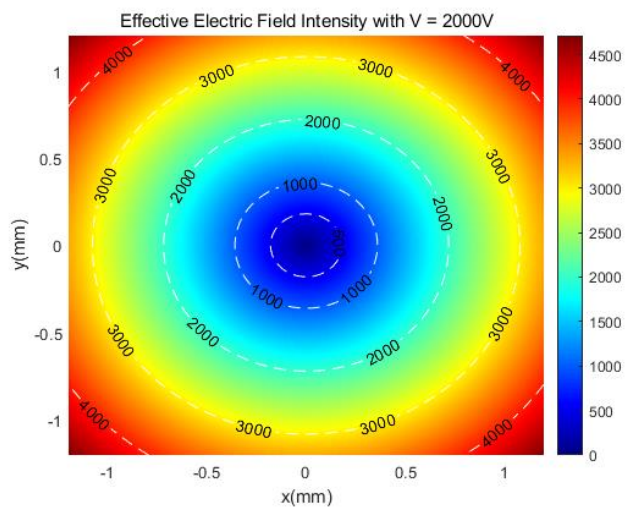

FIGURE 49: EFFECTIVE ELECTRIC FIELD INTENSITY WITH V = 2000V

Further, we could consider the stability region for the droplet. Considering a droplet with certain charge mass ratio *q/m*, we use Mathieu equation to solve the stability region of *Vac* and *w*. The solution could be described by equations 12~15 and Figure 3. As it is complex process, we did not make exact solutions, but characteristic curve could give some information to direct the experiment for trapping the droplet. For example, if we would like to trap a droplet with certain charge mass ratio (*q/m*), according to the stability region shown in Figure 3, appropriate AC voltage V_{ac} and frequency w could be founded.

4. CONCLUSION

Trapping a charged droplet with linear quadrupole is complex process with several parameters, such as AC voltage V_{ac} and frequency w, charge mass ratio *q/m*, initial velocity v. Evaporation is also a considered factor that would affect the size of the droplet. When we need to trap a droplet with certain charge mass ratio, Mathieu equation could help find appropriate AC voltage and frequency. But Mathieu equation does not include evaporation factor. Then the response time becomes important to weaken the evaporation effect. Low initial velocity, high w and low V_{ac} in stability region are available to shorten the response time. But V_{ac} could be too low and w could be too high for vibration amplitude. The model above is a simplified model which has some theoretical significance. When it comes to the real operation process, it needs to be corrected by experience and actual measurements.

ACKNOWLEDGEMENTS

Thanks Dr. Wei for guiding me to clear the target. She helped me to understand the work she did, which is similar to this problem in some aspects. Thanks Erik Liu for sharing some illuminating ideas about solving methods which helped me a lot.

REFERENCES

[1] Wei, Yan. *Effect of Particles on Evaporation of Droplet Containing Particles*, 2015

[2] Xu, Guobin. *Introduction to Quadrupole MS Theory & Technology*, 2010

APPENDIX A

```
clear all;close all; clc
N = 30;                  % Order of the Hill's Determinant
T = cell(1,4);           % Cell array to store Hill determinant matrices
EE = cell(1,4);
epsilon = -20:0.05:20;
for i = 1:length(epsilon)
   e = epsilon(i);
   a = e*ones(N,1);      % Sub diagonal elements
   c = e*ones(N,1);      % Super diagonal elements
   % Hill determinants for odd sine and cosine coefficients
   ndeto = 2*N+1;
   bo = (1:2:ndeto).^2;  % Diagonal elements
   T1 = diag(a,-1)+diag(bo)+diag(c,+1);
   T1(1,1) = 1+e;
   T{1} = T1 ;           % Odd cosine determinant matrix
   T1(1,1) = 1-e;
   T2 = T1;
   T{2} = T2 ;           % Odd sine determinant matrix
   % Hill determinants for even sine and cosine coefficients
   ndete = 2*(N+1);
   be = (2:2:ndete).^2;  % Diagonal elements
   T3 = diag(a,-1)+diag(be)+diag(c,+1);
   T{3} = T3;            % Even cosine determinant matrix
   T4 = zeros(size(T3));
   T4(1,1) = 0 ;T4(1,2) = e ;T4(2,1) = 2*e;
   T4(2:end,2:end)=T3(1:end-1,1:end-1);
   T{4} = T4 ;           % Even sine determinant matrix
   % Calculate the Eigenvalues of the Determinant matrices
   for j = 1:4
      E = eig(T{j});
      EE{j}(:,i) = E;
   end
end
epsilon = epsilon';
E1 = EE{1}';
E2 = EE{2}';
E3 = EE{3}';
E4 = EE{4}';
E = [E1 E2 E3 E4];
figure;
plot(E,epsilon,'r','Linewidth',1);
xlabel('\delta');
ylabel('\epsilon');
hold on;
axis([-10,45,-20,20]);
PlotAxisAtOrigin;

function PlotAxisAtOrigin
%PlotAxisAtOrigin Plot 2D axes through the origin

% GET TICKS
```

```
X=get(gca,'Xtick');
Y=get(gca,'Ytick');

% GET LABELS
XL=get(gca,'XtickLabel');
YL=get(gca,'YtickLabel');

% GET OFFSETS
Xoff=diff(get(gca,'XLim'))./40;
Yoff=diff(get(gca,'YLim'))./40;

% DRAW AXIS LINES
plot(get(gca,'XLim'),[0 0],'k');
plot([0 0],get(gca,'YLim'),'k');

% Plot new ticks
for i=1:length(X)
    plot([X(i) X(i)],[0 Yoff],'-k');
end
for i=1:length(Y)
   plot([Xoff, 0],[Y(i) Y(i)],'-k');
end

% ADD LABELS
t1 = text(X,zeros(size(X))-2.*Yoff,XL);
t2 = text(zeros(size(Y))-3.*Xoff,Y,YL);
set(t1,'FontName','TimesNewRoman','Fontsize',10) ;
set(t2,'FontName','TimesNewRoman','Fontsize',10) ;
% Add Axis Labels
l1 = text(2,max(Y),'a');
set(l1,'FontName','TimesNewRoman','Fontsize',10) ;
l2 = text(max(X),2.,'q');
set(l2,'FontName','TimesNewRoman','Fontsize',10) ;
% Add title
t = title('Stability chart of Mathieu Equation') ;
set(t,'FontName','TimesNewRoman','Fontsize',10) ;
box off;
% axis square;
axis off;
set(gcf,'color','w');
```

Published with MATLAB® R2019b

APPENDIX B

```
clear;close;clc
Varray = [2000, 3000, 4000];
warray = [900, 1200, 1900];
for i = 1:3
    V = Varray(1);                                              %magnitude of AC voltage
    w = warray(i);                                              %frequency of AC voltage
    trange = [0, 0.5];                                          %time range
    x0 = [0.001, 0.001];                                        %initial x position and velocity
    y0 = [0.001, 0.001];                                        %initial y position and velocity
    [t1, x] = ode45(@(t, x) xsimulation(t, x, V, w), trange, x0);  %solve ode equation on x direction
    [t2, y] = ode45(@(t, y) ysimulation(t, y, V, w), trange, y0);  %solve ode equation on y direction
    figure;                                                     %new figure
    plot(t1, x(:, 1));                                          %plot x-t
    xlabel('t(s)');
    ylabel('x(m)');
    figure;                                                     %new figure
    plot(t2, y(:, 1));                                          %plot y-t
    xlabel('t(s)');
    ylabel('y(m)');
    figure;                                                     %new figure
    t = linspace(0, 0.5, size(t1, 1) * 2);                      %new t linspace for interpolation
    xvals = interp1(t1, x(:, 1), t);                            %x table after interpolation
    yvals = interp1(t2, y(:, 1), t);                            %y table after intrepolation
    plot(xvals, yvals);                                         %plot y-x for path of droplet after simulation
    xlabel('x(m)');
    ylabel('y(m)');
end

for i = 1:3
    V = Varray(i);                                              %magnitude of AC voltage
    w = warray(2);                                              %frequency of AC voltage
    trange = [0, 0.5];                                          %time range
    x0 = [0.001, 0.001];                                        %initial x position and velocity
    y0 = [0.001, 0.001];                                        %initial y position and velocity
    [t1, x] = ode45(@(t, x) xsimulation(t, x, V, w), trange, x0);  %solve ode equation on x direction
    [t2, y] = ode45(@(t, y) ysimulation(t, y, V, w), trange, y0);  %solve ode equation on y direction
    figure;                                                     %new figure
    plot(t1, x(:, 1));                                          %plot x-t
    xlabel('t(s)');
    ylabel('x(m)');
    figure;                                                     %new figure
    plot(t2, y(:, 1));                                          %plot y-t
    xlabel('t(s)');
    ylabel('y(m)');
    figure;                                                     %new figure
    t = linspace(0, 0.5, size(t1, 1) * 2);                      %new t linspace for interpolation
    xvals = interp1(t1, x(:, 1), t);                            %x table after interpolation
    yvals = interp1(t2, y(:, 1), t);                            %y table after intrepolation
    plot(xvals, yvals);                                         %plot y-x for path of droplet after simulation
    xlabel('x(m)');
    ylabel('y(m)');
```

```
end

function rk = xsimulation(t, x, V, w)
d = 50 * 10^(-6);                                              %diameter of droplet
n = 1.849 * 10^(-5);                                           %air dynamic viscosity
m = 4 / 3 * pi * (d / 2)^3 * (0.9974456 * 10^3);               %mass of droplet
surface_tension = 7.28 * 10^(-2);                              %surface tension
q = 0.3 * 8 * pi * sqrt(8.85e-12 * surface_tension * (d / 2)^3);   %maximum charge by Rayleigh limit
r0 = 1.2 * 10^(-2);                                            %distance from center to rod surface
E = -2 * x(1) / r0^2 * V * cos(w * t);                         %electric field on x
rk = zeros(2, 1);
rk(1) = x(2);                                                  %1st derivative of x
rk(2) = (E * q - 3 * pi * n * d * x(2)) / m;                   %2nd derivative of x
end

function rk = ysimulation(t, y, V, w)
d = 50 * 10^(-6);                                              %diameter of droplet
n = 1.849 * 10^(-5);                                           %air dynamic viscosity
m = 4 / 3 * pi * (d / 2)^3 * (0.9974456 * 10^3);               %mass of droplet
surface_tension = 7.28 * 10^(-2);                              %surface tension
q = 0.3 * 8 * pi * sqrt(8.85e-12 * surface_tension * (d / 2)^3);   %maximum charge by Rayleigh limit
r0 = 1.2 * 10^(-2);                                            %distance from center to rod surface
E = 2 * y(1) / r0^2 * V * cos(w * t);                          %electric field on y
rk = zeros(2, 1);
rk(1) = y(2);                                                  %1st derivative of y
rk(2) = (E * q - 3 * pi * n * d * y(2)) / m;                   %2nd derivative of y
end
```

Published with MATLAB® R2019b

APPENDIX C

```
clear;close;clc
syms t0;
d0 = 50 * 10^(-6);
qr = 0.3 * 8 * pi * sqrt(8.85e-12 * 7.28 * 10^(-2) * (d0 / 2)^3);
dr = nthroot((qr / 8 / pi)^2 / 8.85e-12 / (7.28 * 10^(-2)), 3) * 2;
tr = (d0^2 - dr^2) / (1.0 * 10^(-9));
Varray = [200, 400, 400];
warray = [1000, 1000, 2000];
for i = 1:3
    V = Varray(i);                                                  %magnitude of AC voltage
    w = warray(i);                                                  %frequency of AC voltage
    trange = [0, tr];                                               %time range
    x0 = [0.001, 0.001];                                            %initial x position and velocity
    y0 = [0.001, 0.001];                                            %initial y position and velocity
    [t1, x] = ode45(@(t, x) xsimulation(t, x, V, w), trange, x0);  %solve ode equation on x direction
    [t2, y] = ode45(@(t, y) ysimulation(t, y, V, w), trange, y0);  %solve ode equation on y direction
    figure;                                                         %new figure
    plot(t1, x(:, 1));                                              %plot x-t
    xlabel('t(s)');
    ylabel('x(m)');
    figure;                                                         %new figure
    plot(t2, y(:, 1));                                              %plot y-t
    xlabel('t(s)');
    ylabel('y(m)');
    figure;                                                         %new figure
    t = linspace(0, 0.5, size(t1, 1) * 2);                          %new t linspace for interpolation
    xvals = interp1(t1, x(:, 1), t);                                %x table after interpolation
    yvals = interp1(t2, y(:, 1), t);                                %y table after intrepolation
    plot(xvals, yvals);                                             %plot y-x for path of droplet after simulation
    xlabel('x(m)');
    ylabel('y(m)');
end

warray = [900, 1200, 1900];
for i = 1:3
    V = 2000;                                                       %magnitude of AC voltage
    w = warray(i);                                                  %frequency of AC voltage
    trange = [0, 0.5];                                              %time range
    x0 = [0.001, 0.001];                                            %initial x position and velocity
    y0 = [0.001, 0.001];                                            %initial y position and velocity
    [t1, x] = ode45(@(t, x) xsimulation(t, x, V, w), trange, x0);  %solve ode equation on x direction
    [t2, y] = ode45(@(t, y) ysimulation(t, y, V, w), trange, y0);  %solve ode equation on y direction
    figure;                                                         %new figure
    plot(t1, x(:, 1));                                              %plot x-t
    xlabel('t(s)');
    ylabel('x(m)');
    figure;                                                         %new figure
    plot(t2, y(:, 1));                                              %plot y-t
    xlabel('t(s)');
    ylabel('y(m)');
    figure;                                                         %new figure
```

```
    t = linspace(0, 0.5, size(t1, 1) * 2);                           %new t linspace for interpolation
    xvals = interp1(t1, x(:, 1), t);                                 %x table after interpolation
    yvals = interp1(t2, y(:, 1), t);                                 %y table after intrepolation
    plot(xvals, yvals);                                              %plot y-x for path of droplet after simulation
    xlabel('x(m)');
    ylabel('y(m)');
end

function rk = xsimulation(t, x, V, w)
d0 = 50 * 10^(-6);
d = sqrt(d0^2 - 10^(-9) * t);                                        %diameter of droplet
n = 1.849 * 10^(-5);                                                 %air dynamic viscosity
m = 4 / 3 * pi * (d / 2)^3 * (0.9974456 * 10^3);                     %mass of droplet
surface_tension = 7.28 * 10^(-2);                                    %surface tension
q = 0.3 * 8 * pi * sqrt(8.85e-12 * surface_tension * (d0 / 2)^3);    %maximum charge by Rayleigh limit
r0 = 1.2 * 10^(-2);                                                  %distance from center to rod surface
E = -2 * x(1) / r0^2 * V * cos(w * t);                               %electric field on x
rk = zeros(2, 1);
rk(1) = x(2);                                                        %1st derivative of x
rk(2) = (E * q - 3 * pi * n * d * x(2)) / m;                         %2nd derivative of x
end

function rk = ysimulation(t, y, V, w)
d0 = 50 * 10^(-6);
d = sqrt(d0^2 - 10^(-9) * t);                                        %diameter of droplet
n = 1.849 * 10^(-5);                                                 %air dynamic viscosity
m = 4 / 3 * pi * (d / 2)^3 * (0.9974456 * 10^3);                     %mass of droplet
surface_tension = 7.28 * 10^(-2);                                    %surface tension
q = 0.3 * 8 * pi * sqrt(8.85e-12 * surface_tension * (d0 / 2)^3);    %maximum charge by Rayleigh limit
r0 = 1.2 * 10^(-2);                                                  %distance from center to rod surface
E = 2 * y(1) / r0^2 * V * cos(w * t);                                %electric field on y
rk = zeros(2, 1);
rk(1) = y(2);                                                        %1st derivative of y
rk(2) = (E * q - 3 * pi * n * d * y(2)) / m;                         %2nd derivative of y
end
```

Published with MATLAB® R2019b

APPENDIX D

```
clear;close;clc
varray = [0.5, 1, 2];
for i = 1:3
   V = 2000;                                                      %magnitude of AC voltage
   w = 1000;                                                      %frequency of AC voltage
   trange = [0, 0.5];                                             %time range
   x0 = [0.001, varray(i)];                                       %initial x position and velocity
   y0 = [0.001, varray(i)];                                       %initial y position and velocity
   [t1, x] = ode45(@(t, x) xsimulation(t, x, V, w), trange, x0);  %solve ode equation on x direction
   [t2, y] = ode45(@(t, y) ysimulation(t, y, V, w), trange, y0);  %solve ode equation on y direction
   figure;                                                        %new figure
   plot(t1, x(:, 1));                                             %plot x-t
   xlabel('t(s)');
   ylabel('x(m)');
   figure;                                                        %new figure
   plot(t2, y(:, 1));                                             %plot y-t
   xlabel('t(s)');
   ylabel('y(m)');
   figure;                                                        %new figure
   t = linspace(0, 0.5, size(t1, 1) * 2);                         %new t linspace for interpolation
   xvals = interp1(t1, x(:, 1), t);                               %x table after interpolation
   yvals = interp1(t2, y(:, 1), t);                               %y table after intrepolation
   plot(xvals, yvals);                                            %plot y-x for path of droplet after simulation
   xlabel('x(m)');
   ylabel('y(m)');
end

function rk = xsimulation(t, x, V, w)
d = 50 * 10^(-6);                                                 %diameter of droplet
n = 1.849 * 10^(-5);                                              %air dynamic viscosity
m = 4 / 3 * pi * (d / 2)^3 * (0.9974456 * 10^3);                  %mass of droplet
surface_tension = 7.28 * 10^(-2);                                 %surface tension
q = 0.3 * 8 * pi * sqrt(8.85e-12 * surface_tension * (d / 2)^3);  %maximum charge by Rayleigh limit
r0 = 1.2 * 10^(-2);                                               %distance from center to rod surface
E = -2 * x(1) / r0^2 * V * cos(w * t);                            %electric field on x
rk = zeros(2, 1);
rk(1) = x(2);                                                     %1st derivative of x
rk(2) = (E * q - 3 * pi * n * d * x(2)) / m;                      %2nd derivative of x
end

function rk = ysimulation(t, y, V, w)
d = 50 * 10^(-6);                                                 %diameter of droplet
n = 1.849 * 10^(-5);                                              %air dynamic viscosity
m = 4 / 3 * pi * (d / 2)^3 * (0.9974456 * 10^3);                  %mass of droplet
surface_tension = 7.28 * 10^(-2);                                 %surface tension
q = 0.3 * 8 * pi * sqrt(8.85e-12 * surface_tension * (d / 2)^3);  %maximum charge by Rayleigh limit
r0 = 1.2 * 10^(-2);                                               %distance from center to rod surface
E = 2 * y(1) / r0^2 * V * cos(w * t);                             %electric field on y
rk = zeros(2, 1);
rk(1) = y(2);                                                     %1st derivative of y
rk(2) = (E * q - 3 * pi * n * d * y(2)) / m;                      %2nd derivative of y
```

```
end
```

Published with MATLAB® R2019b

APPENDIX E

```
[xgrid,ygrid] = meshgrid(-1.2:0.01:1.2);
z=sqrt((-(2.* xgrid)./1.2^2.* 2000).^2+((2.* ygrid)./ 1.2^2.* 2000).^2);
figure;
pcolor(xgrid,ygrid,z);
shading interp;
colorbar;
colormap(jet);
xlabel('x(mm)');
ylabel('y(mm)');
title('Effective Electric Field Intensity with V = 2000V');
hold on;
[cs, h]=contour(xgrid,ygrid,z,[4000 3000 2000 1000 500 0],'w--');
clabel(cs, h,'LabelSpacing',100, 'FontSize', 10, 'Color', 'k');
hold off;
```

Published with MATLAB® R2019b