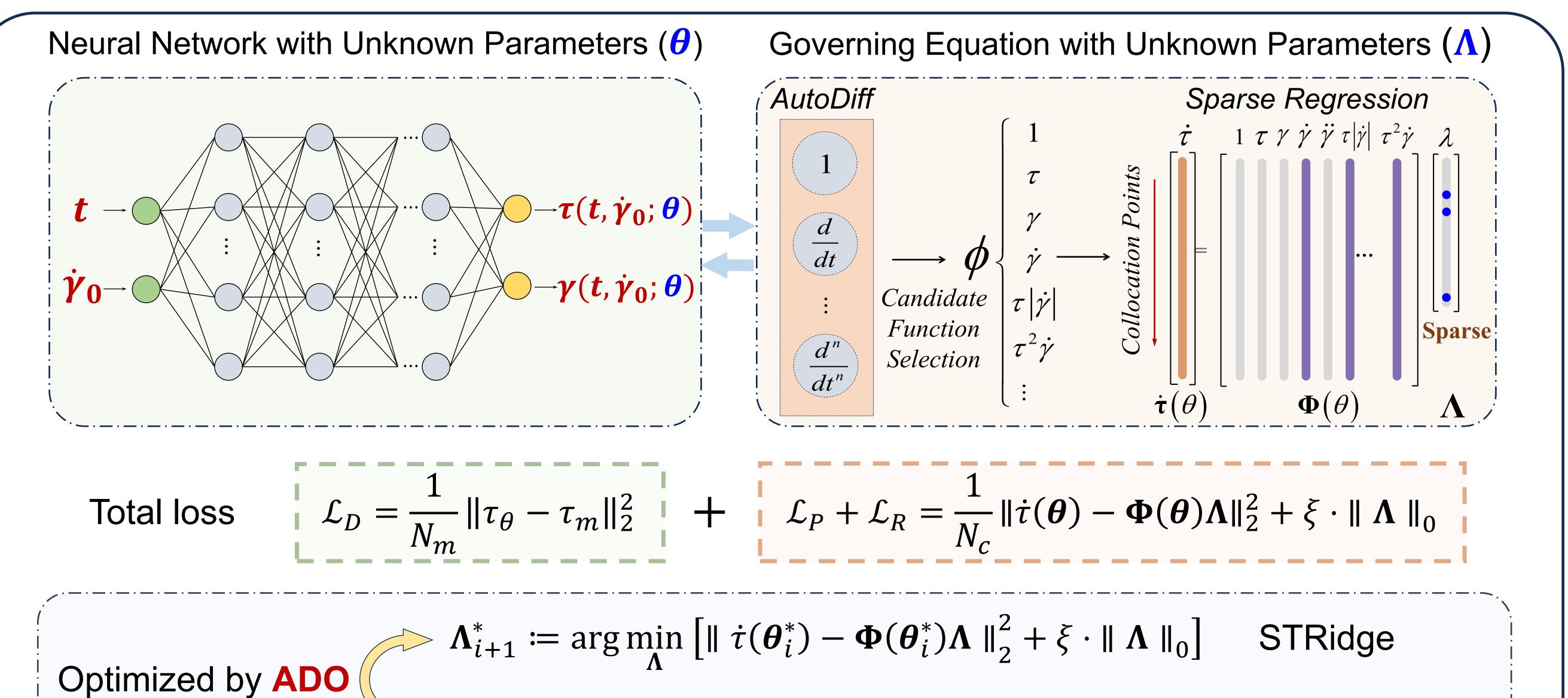
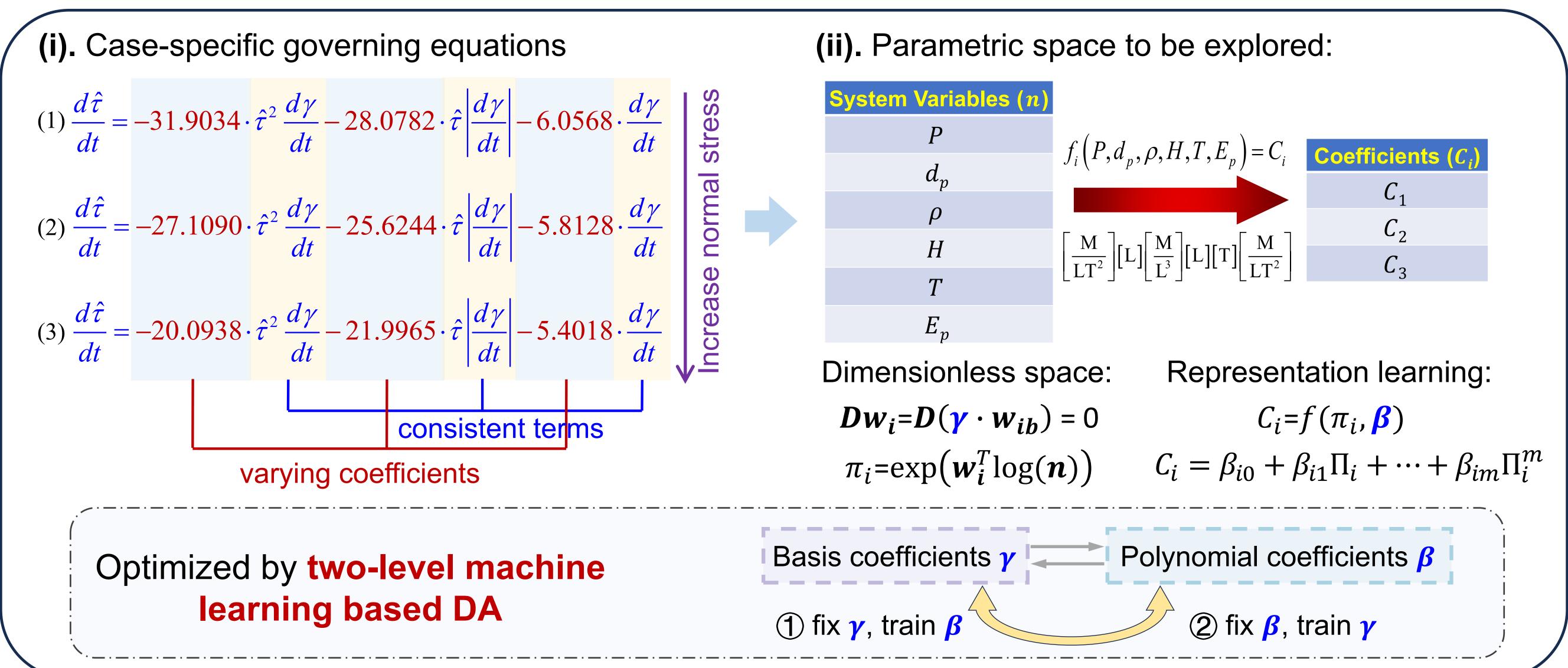
a. PINNSR for granular noisy data



 $\Rightarrow \boldsymbol{\theta}_{i+1}^* \coloneqq \arg\min_{\boldsymbol{\rho}} \left[\mathcal{L}_d(\boldsymbol{\theta}; \mathcal{D}_m) + \lambda \cdot \mathcal{L}_p(\boldsymbol{\theta}, \boldsymbol{\Lambda}_{i+1}^*; \mathcal{D}_c) \right]$

PINNs training

b. Dimensional analysis for learning coefficients



Granular Rheology

General governing equation for sheared granular materials:

$$\frac{d\mu}{dt} = f_1(\chi) \cdot \frac{d\gamma}{dt} + f_2(\chi) \cdot \mu \left| \frac{d\gamma}{dt} \right| + f_3(\chi) \cdot \mu^2 \frac{d\gamma}{dt}$$

Dimensionless number:

$$\chi = n_H \cdot \kappa$$
 $n_H = H/d_p$ $\kappa = E_p/P$