Fundamentels of Robotics

Assignment 3 - Jacobian. Innopolis University, Fall 2020

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GitHub Repository: here (It is public after the submission deadline)

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Section 1: Introduction

The robot is a manipulator from KUKA (KR 10 R1100-2). This robot has 6 joints and is 6 DoF manipulator, moreover, it has a spherical writs.

Data-sheet of the manipulator can be found here

Two methods have been chosen to compute the Jacobian:

- Skew Theory
- Numerical Derivatives

Note: a symbolic solution based on differentiation for the symbolic transformation matrix has been made in order to get the first 3 elements in the jacobian $(\dot{x}, \dot{y}, \dot{z})$ *to debug because there was a bug and did not know which method provide the correct results.*

Section2: Computing Jacobian using Skew Theory

First we define the transformation matrix from world to the end effector as following:

$$T = A_0 A_1 A_2 A_3 A_4 A_5 A_6$$

Such that:

$A_0 = T_{base} = T_0^w$: transformation from the world to the place of the first joint's frame

$A_1 = R_z(q_0)T_z(l_0)T_x(l_1) = T_1^0$: transformation from the first joint's frame to the second joint frame

$A_2 = R_y(q_1)T_x(l_2) = T_2^1$

$A_3 = R_y(q_2)T_x(l_3) = T_3^2$

$A_4 = R_x(q_3)T_x(l_4) = T_4^3$

$A_5 = R_y(q_4) = T_5^4$

$A_6 = R_x(q_5)T_x(l_5)T_{tool} = T_6^5$: transformation from the last joint's frame (6^{th}) to the end effector frame

Note: $T_j^i = T_k^i T_j^k$, *which means for example that* $A_3 A_4 = T_3^2 T_4^3 = T_4^2$

The jacobian is in the following form:

$\mathcal{J} = [\vec{\mathcal{J}}_0 \ \vec{\mathcal{J}}_1 \ \vec{\mathcal{J}}_2 \ \vec{\mathcal{J}}_3 \ \vec{\mathcal{J}}_4 \ \vec{\mathcal{J}}_5]$

As all the joints are revolute joints, we will use the following in order to compute each column $\vec{\mathcal{J}}_i$ in the jacobian matrix:

$$\vec{\mathcal{J}}_i = \begin{bmatrix} \vec{U}_i \times (\vec{O}_n - \vec{O}_i) \\ \vec{U}_i \end{bmatrix}$$

such that: n=6 and i=0,1,2,3,4,5

Then we need to get the $\vec{O}$ $\vec{U}$ vectors:

- $\vec{O}_i$ is the positional vector from the transformation matrix from the world to i^th frame (T_i^w)
- $\vec{U}_i$ is the column vector corresponds to the axis of the rotation of the joint from the rotation matrix that is composed into the transformation matrix from the world to i^th frame (T_i^w) (e.g. rotation axis is z-axis, take 3rd column

Section 3: Computing Jacobian using Numerical Derivatives

First we compute the forward kinematics as from the previous assignment from the following formula:

$$T = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, R_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}$$

Second, get the rotation matrix from the previous homogeneous matrix = R, get the inverse of it and compose it into a homogeneous matrix with zero position vector:

$$T_o = \begin{bmatrix} R^{-1} & \vec{0} \\ \vec{0} & 1 \end{bmatrix}$$

Then, with the following formulas we calculate the columns of the jacobian:

The general:

$$\dot{T}_i = T_{base}\; T_{left}\; \dot{H}_i\; T_{right}\; T_{tool}\; T_o$$

such that $\dot{T}_i$ is the derivative of the transformation with respect to the i^{th} joint generalized coordinate (q_i), T_{left} and T_{right} are the right and left transformations from H_i in the original transformation equation T. $\dot{H}_i$ is the derivative of the transformation that depends on the i^{th} joint generalized coordinates (q_i)

Note: it is zero indexed

$$\mathcal{J}_i = \begin{bmatrix} \dot{T}_i[0,3] \\ \dot{T}_i[1,3] \\ \dot{T}_i[2,3] \\ \dot{T}_i[2,1] \\ \dot{T}_i[0,2] \\ \dot{T}_i[1,0] \end{bmatrix}$$

In details:

$\dot{T}_0 = T_{base}\, \dot{R}_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, R_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

And get $\mathcal{J}_0$ as in the general formulas

$\dot{T}_1 = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, \dot{R}_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, R_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

get $\mathcal{J}_1$

$\dot{T}_2 = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, \dot{R}_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, R_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

get $\mathcal{J}_2$

$\dot{T}_3 = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, \dot{R}_x(q_3)\, T_x(l_4)\, R_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

get $\mathcal{J}_3$

$\dot{T}_4 = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, \dot{R}_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

get $\mathcal{J}_4$

$\dot{T}_5 = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, R_y(q_4)\, \dot{R}_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

get $\mathcal{J}_5$

Such that the derivatives of the rotation matrices are from the lecture 6, slide: 26.

Section 4: Singularities

The singularity happens when the robot lose one or more degree of freedom. The singularity of the robot can be determined from the jacobian matrix by 3 methods:

- The jacobian matrix determinant = 0
- The jacobian matrix lose the original rank
- The jacobian matrix's S diagonal matrix from SVD has a minimum value for the absolute of the elements that is very close or equal to zero.

Here, it is showed 3 singularities: wrist, shoulder, elbow

1. **Wrist singularity:**

Axis of joints 4 and 6 are collinear. Any rotation of one of the joints is the same of the rotation of the other joint, hence, one DoF has been lost.

It happens with the following configuration: $\vec{q} = \begin{bmatrix} 0 \\ \pi/4 \\ \pi/3 \\ \theta_1 \\ 0 \\ \theta_2 \end{bmatrix}$ θ_1 θ_2 are pointing to different rotations for joints but when they are applied they are giving the same rotation motion (the same direction / the same axis of), originally, they should have different rotation axis to get different rotation motion.

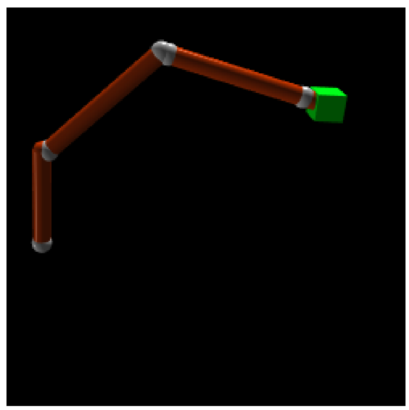

Figure 1. Configuration for wrist singularity

2. **Shoulder & Alignment singularity:**

   It happens when the center of the wrist position (The intersection point of the three wrist's axis) is on the axis of the first joint, thus, any rotation of the first joint, the position of the tool is not changing, hence the position of the spherical wrist manipulator is not changing by rotating the first joint. (Any rotation in the first joint does not affect the end effector position, hence, losing of one DoF. Also, 4th and 1st joints' axes are parallel and 1st joint affects the end effector in the same manner as 4th joint). Moreover, it will cause the 1st and 4th joints to rotate 180 degrees instantaneously when approaching this singularity case.

   Alignment singularity is a subset of shoulder singularity, where 1st and 6th joint axes are colinear, both rotation in these joints will affect the end effector orientation in the same manner, and 1st joint will not affect the position.

   It happens with the following configuration:

Configuration for Shoulder singularity (1st and 4th joint are parallel and position of the center of the spherical wrist in the axis of 1st joint) $\vec{q} = \begin{bmatrix} 0 \\ -1.6154540260077814 \\ +0.04465769921288487 \\ 0 \\ \pi/6 \\ 0.0 \end{bmatrix}$

Configuration for Alignment singularity (1st and 6th joint are parallel and position of the center of the spherical wrist in the axis of 1st joint)

$$\vec{q} = \begin{bmatrix} 0 \\ -\pi/6 \\ -2.2829525304720546 \\ 0 \\ \pi/6 \\ 0.0 \end{bmatrix}$$

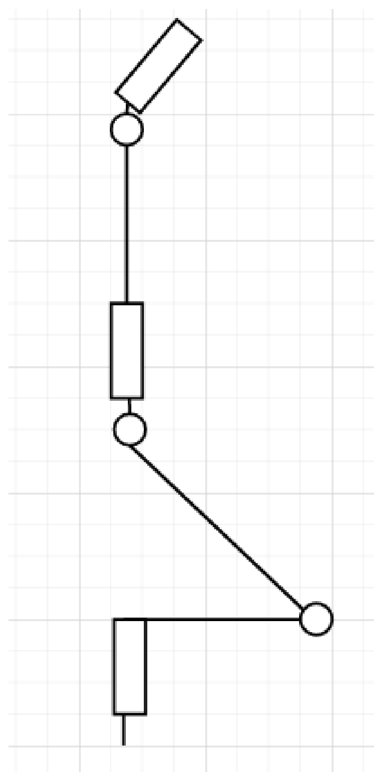

Figure 2. Scheme for Shoulder singularity

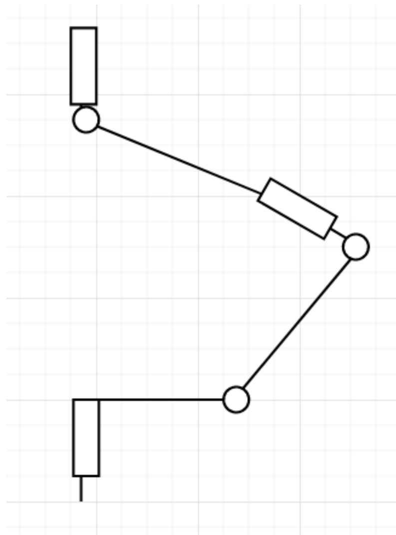

Figure 3. Scheme for Alignment singularity

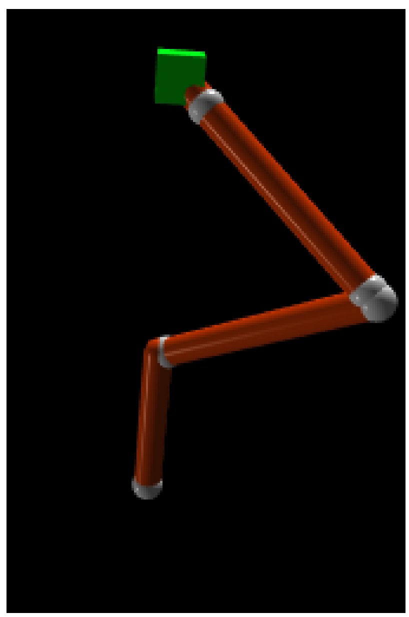

Figure 4. Front perspective for the configuration for the Shoulder singularity

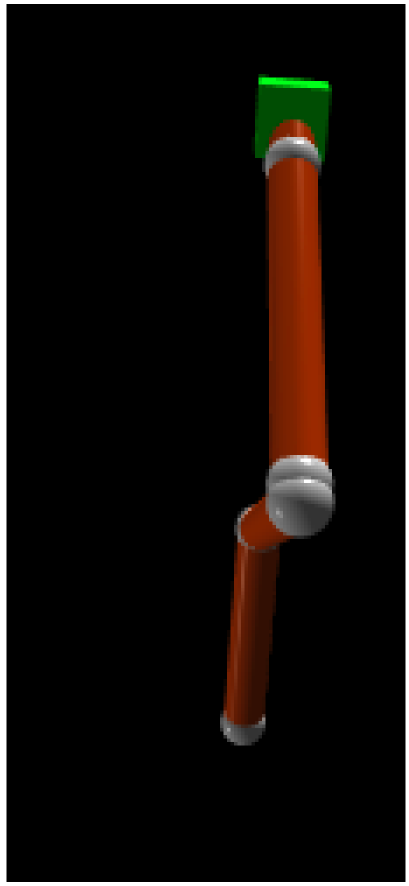

Figure 5. Side perspective for the configuration for the Shoulder singularity

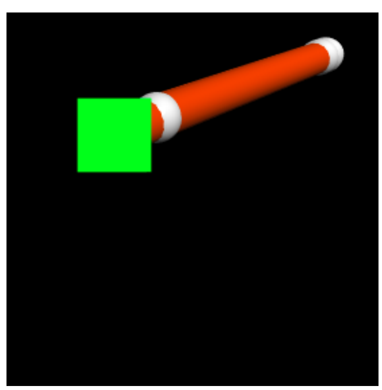

Figure 6. Top perspective for the configuration for the Shoulder singularity

3. **Elbow singularity:**

   It happens when the manipulator reaches the maximum working space, when the whole manipulator is extended (flexed), this happens in the zero configuration according to the developed kinematics model (Wrist and Elbow singularities together). The end effector is blocked to move instantly in z-direction independently without changing the other positions as it can only move in the circular trajectory that bounds the working space, hence, losing of 1 DoF. Generally, Elbow singularity happens when the position of the spherical wrist (position of the intersection of 4th, 5th and 6th axis) lies on the plane that constructed by the axis of 2nd and 3rd joints.

   It happens with the following configurations:

   $\vec{q} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ (Elbow and wrist singularities)

   and

   $\vec{q} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\pi/12 \\ 0 \end{bmatrix}$ (only elbow singularity)

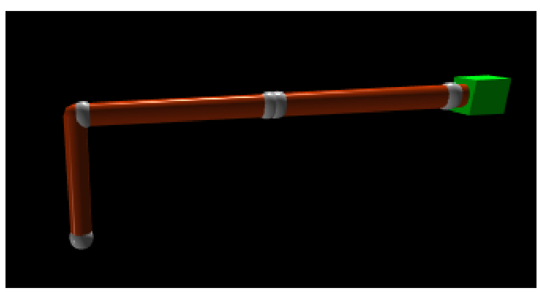

Figure 7. Configuration for the elbow singularity

Note: Green box is the tool on the end effector

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