

Fundamentels of Robotics

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Task1

The complete model is as following (in the notations of the lecture):

$$T = [T_x \ T_y \ T_z \ R_x \ R_y \ R_z]_b R_x(q_1 + \Delta q_1) [T_y \ T_z \ R_y \ R_z]_1 T_x(q_2 + \Delta q_2) [R_y \ R_z]_2 T_z(q_3 + \Delta q_3) [R_x \ R_y]_3 [T_x \ T_y \ T_z \ R_x \ R_y \ R_z]_t$$

Note: $[R_e]_i = R_{ei}$

We will remove:

- (a) $R_x(\Delta q_1)$ as it will be added to the base
- (b) R_{z2} as two consecutive orthogonal Prismatic joints
- (c) $T_z(\Delta q_3)$ as it will be added to the tool
- (d) R_{z2} as it can swapped with T_z of the prismatic joint as both of them on the same axis and then can be added to the tool
- (e) R_{x3}, R_{y3} as they will be added to the tool
- (f) R_{xt}, R_{yt}, R_{zt} as it will be removed as we are not measuring orientation information for the end effector but position for the references

Note: I mean by "added to" that it is swapped and can be identified with that parameter.

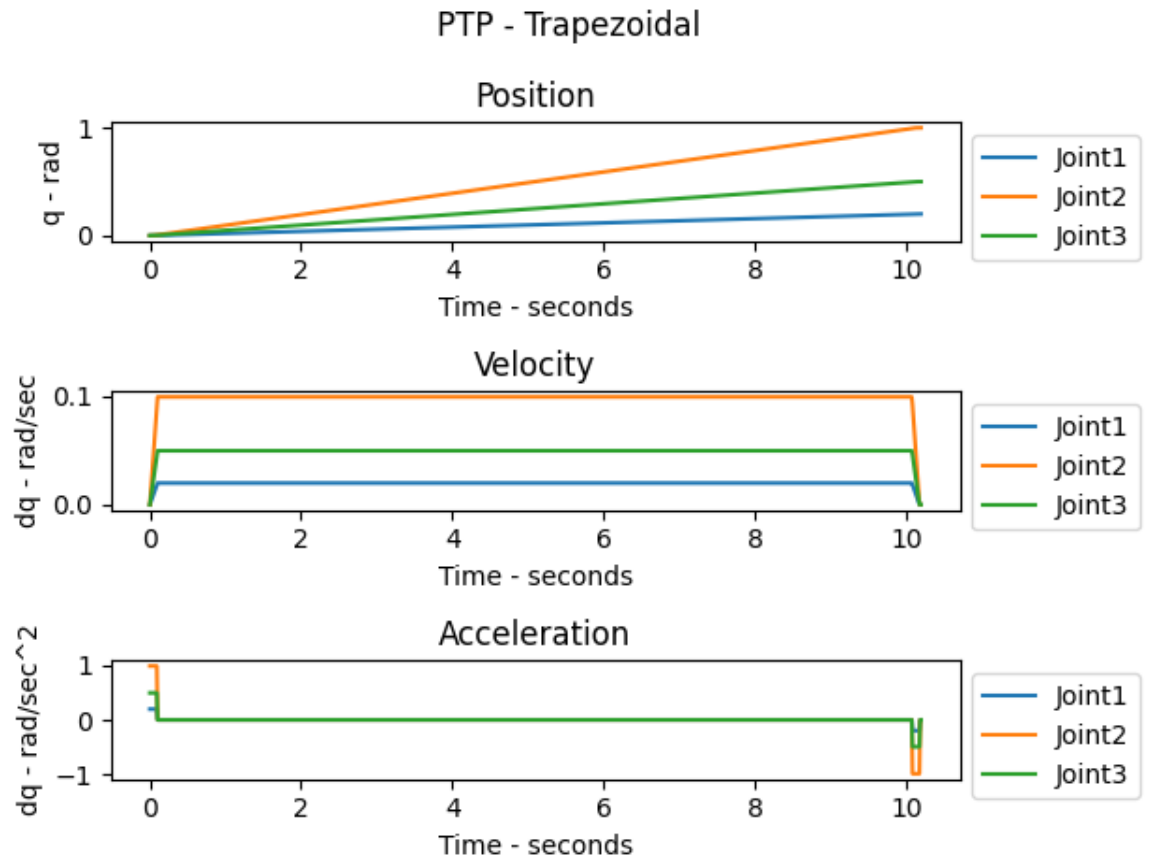
The irreducible model after eliminations:

$$T = [T_x \ T_y \ T_z \ R_x \ R_y \ R_z]_b R_x(q_1) [T_y \ T_z \ R_y \ R_z]_1 T_x(q_2 + \Delta q_2) [R_y]_2 T_z(q_3) [T_x \ T_y \ T_z]_t$$

Then, we have six parameters for T_{robot} , six parameters for T_{base} , and 3 parameters for T_{tool}

Task2

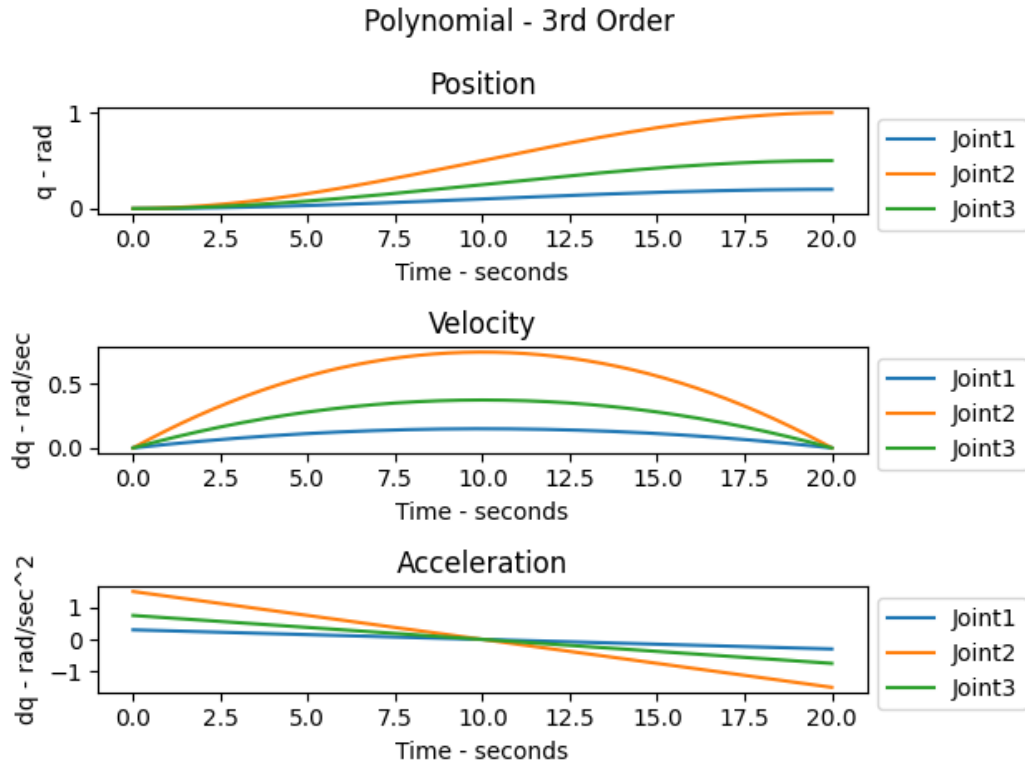
For PTP motion, I have used Trapezoidal profile, and the following figures determine the joint positions, velocities and accelerations.



PTP trajectories

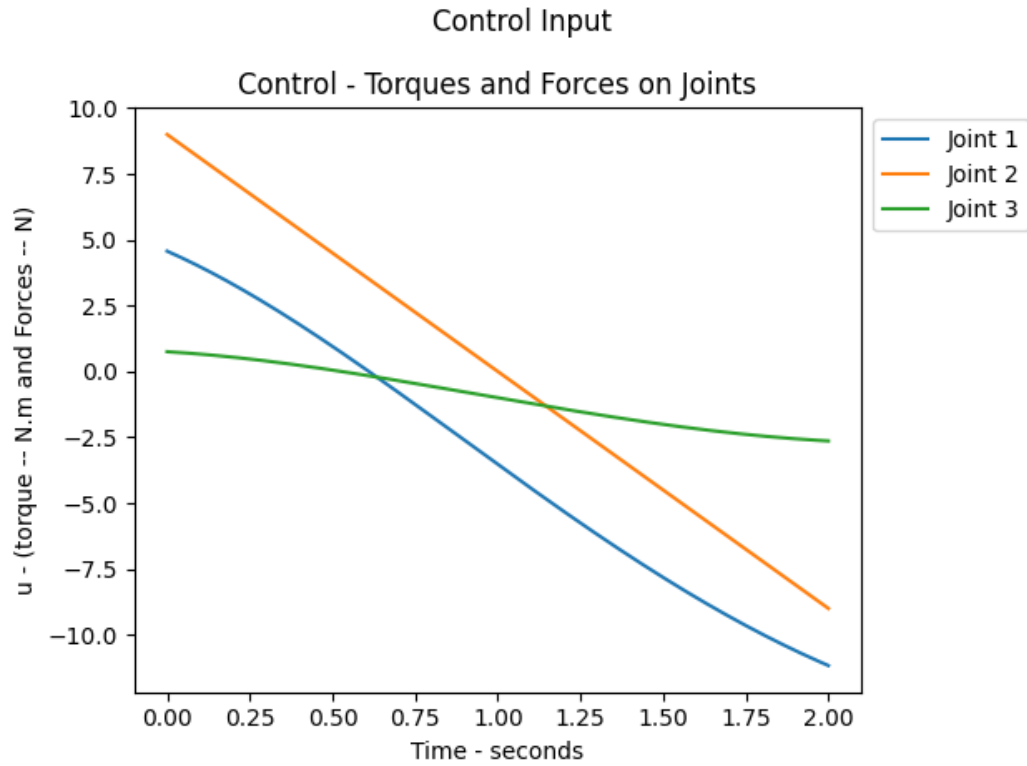
Task3

First, I have used 3rd order polynomial in order to satisfy the 4 constraints and generate a polynomial trajectory, and it is as the following:



Trajectory 3rd degree polynomial

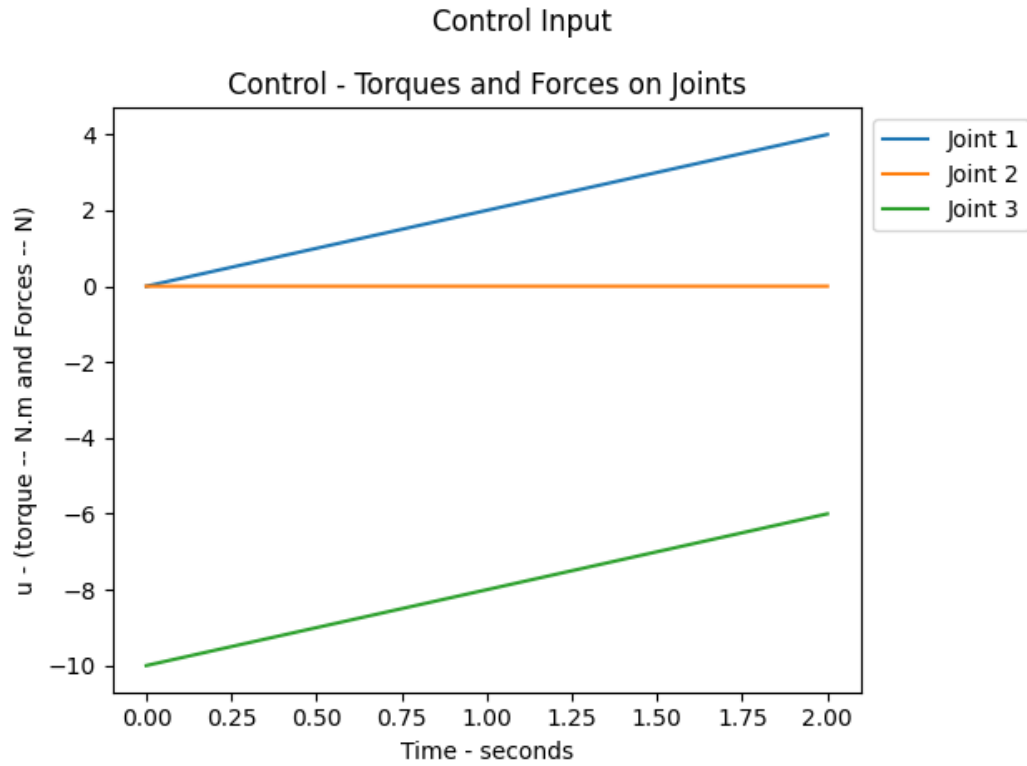
Then, using the inverse dynamics for Euler-Lagrange in order to get the torques and forces corresponds to this trajectory.



Torques and Forces plots

Task4

First, I have used 1st order polynomial in order to satisfy the 2 constraints on the force/torque equation and generate a polynomial trajectory (function depends on t), and it is as the following:



Force/Torque polynomial function with time

Then, using the direct dynamics for Euler-Lagrange in order to get the trajectory this function of time for forces and torques.

It was as following (Reference: [here](#)):

- numerical integration, at **current** state (q, \dot{q}) , of

$$\ddot{q} = M^{-1}(q)[u - (c(q, \dot{q}) + g(q))] = M^{-1}(q)[u - n(q, \dot{q})]$$
- Coriolis, centrifugal, and gravity terms

$$n = NE_g(q, \dot{q}, 0) \quad \text{complexity } O(N)$$
- i -th column of the inertia matrix, for $i = 1, \dots, N$

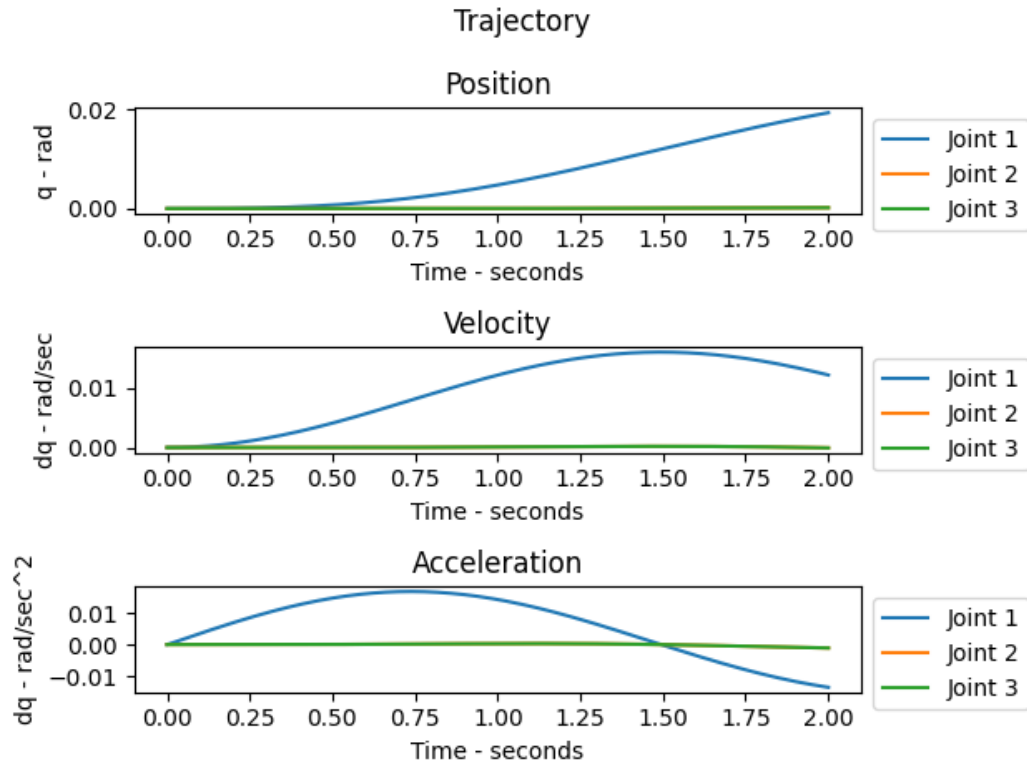
$$M_i = NE_0(q, 0, e_i) \quad O(N^2)$$
- numerical inversion of inertia matrix

$$InvM = \text{inv}(M) \quad O(N^3) \text{ but with small coefficient}$$
- given u , integrate acceleration computed as

$$\ddot{q} = InvM * [u - n] \quad \rightarrow \quad \text{new state } (q, \dot{q}) \text{ and repeat over time ...}$$

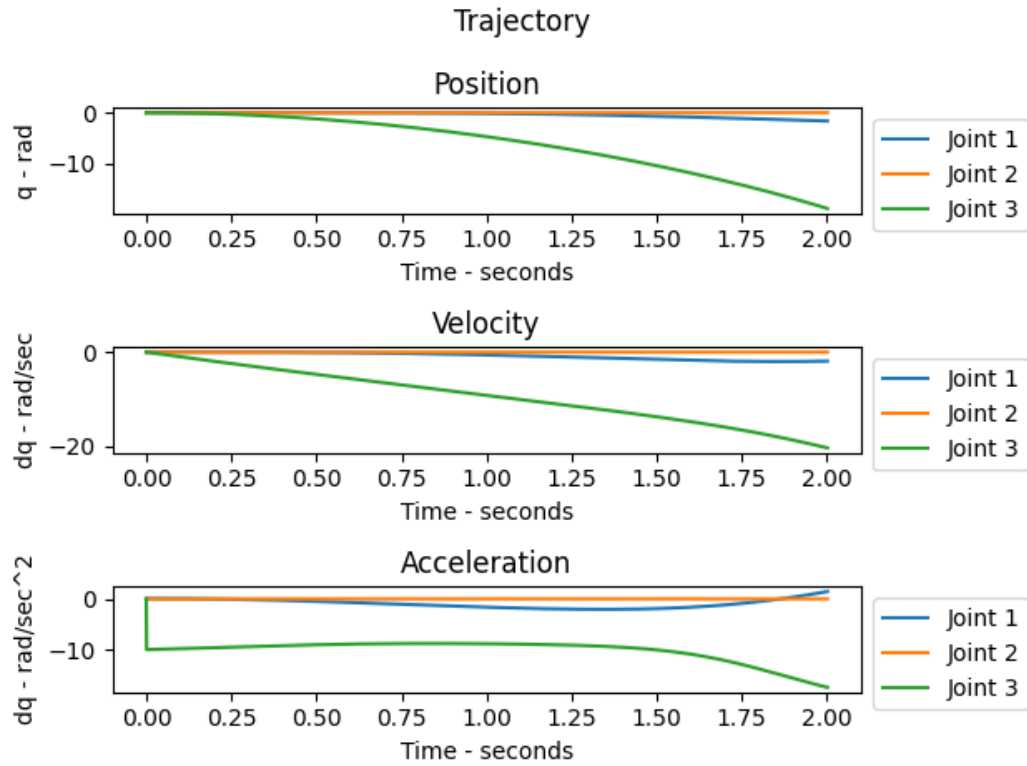
Method for Direct Dynamics based on the recursive solution of the inverse dynamics

The plots:



Torques and Forces plots

Moreover, I implemented the direct dynamics for Euler-Lagrange in order to get the trajectory this function of time for forces and torques.



Torques and Forces plots

Note: They should be the same or one of them had a problem but did not have time to find the error (most probably in Newton euler)