

# Fundamentels of Robotics

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## Task1

The complete model is as following (in the notations of the lecture):

$$T = [T_x \ T_y \ T_z \ R_x \ R_y \ R_z]_b R_x(q_1 + \Delta q_1) [T_y \ T_z \ R_y \ R_z]_1 T_x(q_2 + \Delta q_2) [R_y \ R_z]_2 T_z(q_3 + \Delta q_3) [R_x \ R_y]_3 [T_x \ T_y \ T_z \ R_x \ R_y \ R_z]_t$$

Note:  $[R_e]_i = R_{ei}$

We will remove:

- (a)  $R_x(\Delta q_1)$  as it will be added to the base
- (b)  $R_{z2}$  as two consecutive orthogonal Prismatic joints
- (c)  $T_z(\Delta q_3)$  as it will be added to the tool
- (d)  $R_{z2}$  as it can swapped with  $T_z$  of the prismatic joint as both of them on the same axis and then can be added to the tool
- (e)  $R_{x3}, R_{y3}$  as they will be added to the tool
- (f)  $R_{xt}, R_{yt}, R_{zt}$  as it will be removed as we are not measuring orientation information for the end effector but position for the references

Note: I mean by "added to" that it is swapped and can be identified with that parameter.

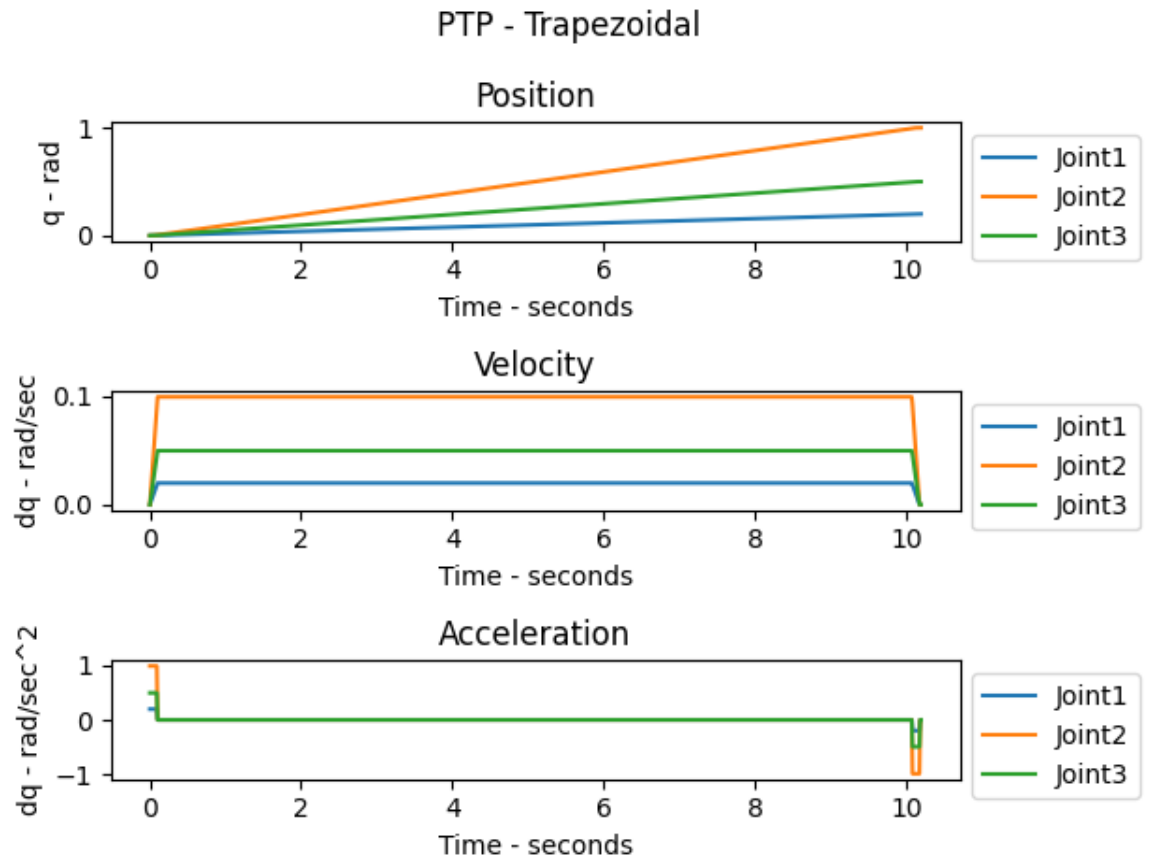
The irreducible model after eliminations:

$$T = [T_x \ T_y \ T_z \ R_x \ R_y \ R_z]_b R_x(q_1) [T_y \ T_z \ R_y \ R_z]_1 T_x(q_2 + \Delta q_2) [R_y]_2 T_z(q_3) [T_x \ T_y \ T_z]_t$$

Then, we have six parameters for  $T_{robot}$ , six parameters for  $T_{base}$ , and 3 parameters for  $T_{tool}$

## Task2

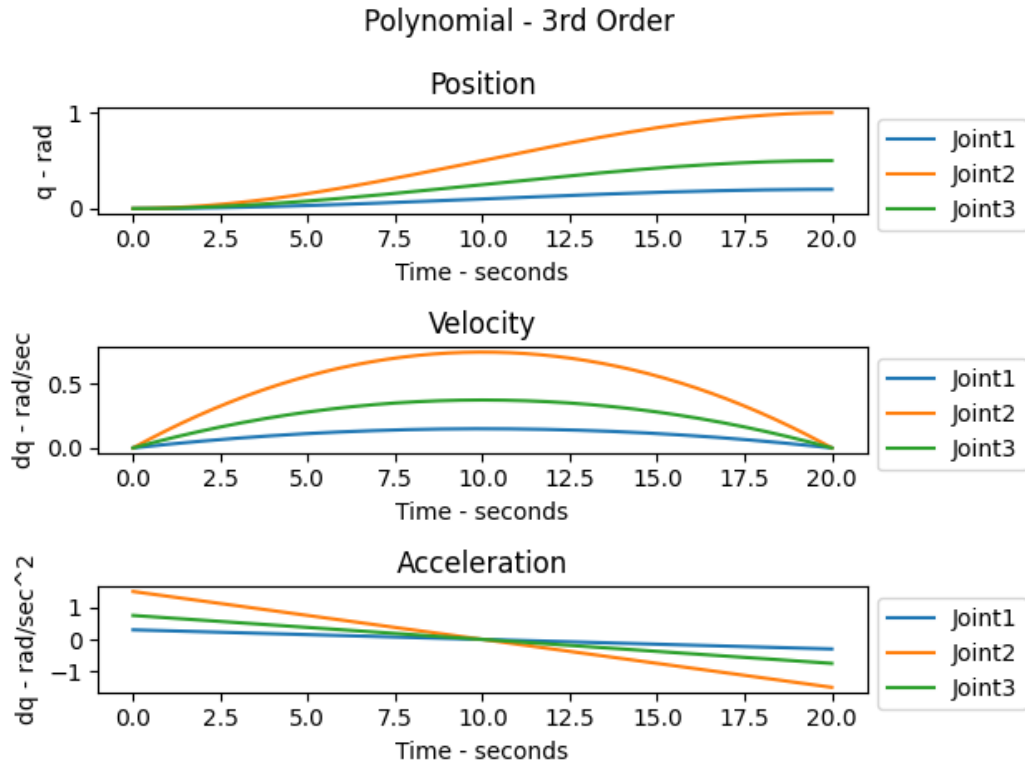
For PTP motion, I have used Trapezoidal profile, and the following figures determine the joint positions, velocities and accelerations.



PTP trajectories

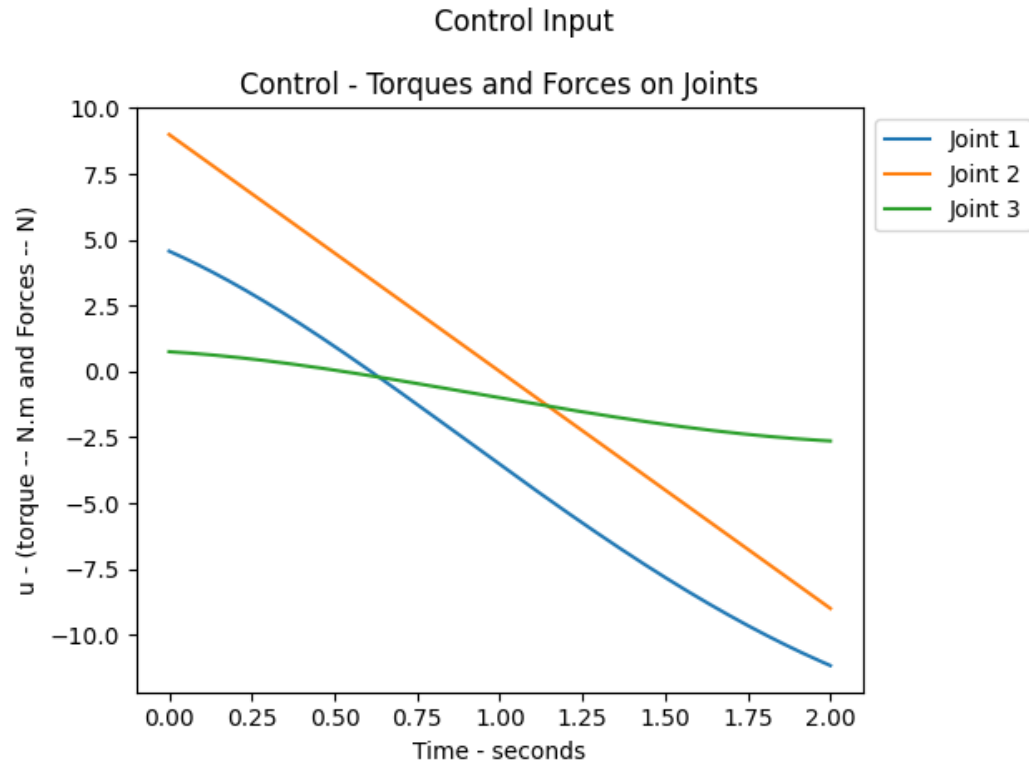
### Task3

First, I have used 3rd order polynomial in order to satisfy the 4 constraints and generate a polynomial trajectory, and it is as the following:



### Trajectory 3rd degree polynomial

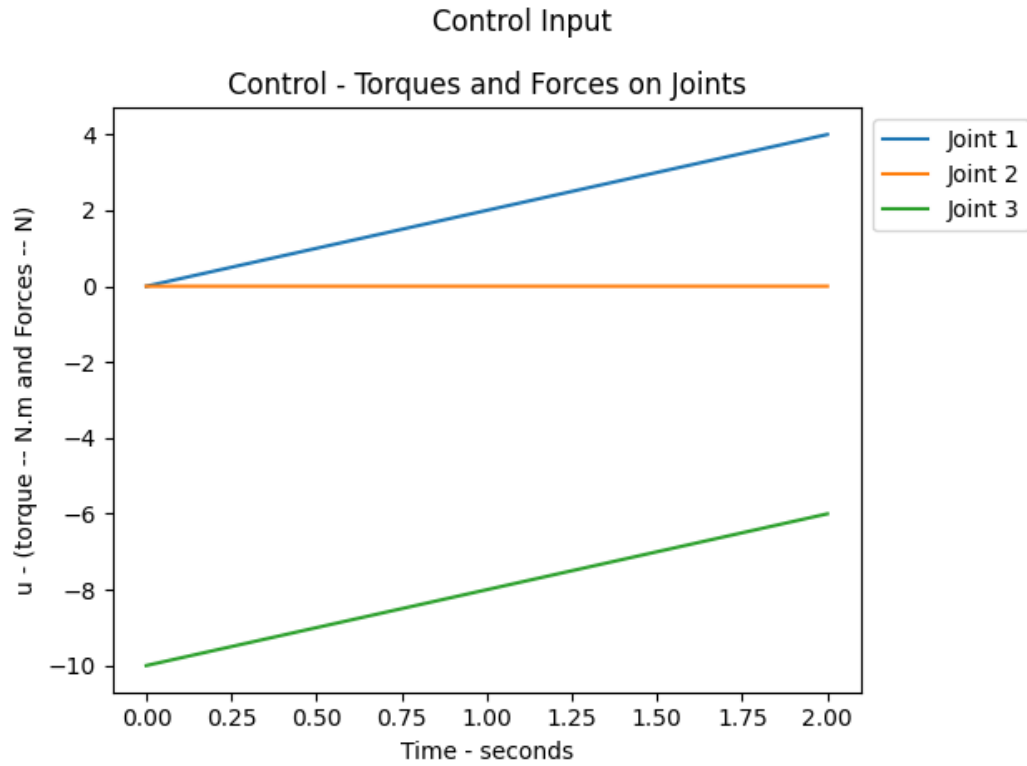
Then, using the inverse dynamics for Euler-Lagrange in order to get the torques and forces corresponds to this trajectory.



**Torques and Forces plots**

## Task4

First, I have used 1st order polynomial in order to satisfy the 2 constraints on the force/torque equation and generate a polynomial trajectory (function depends on  $t$ ), and it is as the following:



**Force/Torque polynomial function with time**

Then, using the direct dynamics for Euler-Lagrange in order to get the trajectory this function of time for forces and torques.

It was as following (Reference: [here](#)):

- numerical integration, at **current** state  $(q, \dot{q})$ , of  

$$\ddot{q} = M^{-1}(q)[u - (c(q, \dot{q}) + g(q))] = M^{-1}(q)[u - n(q, \dot{q})]$$
- Coriolis, centrifugal, and gravity terms  

$$n = NE_g(q, \dot{q}, 0) \quad \text{complexity } O(N)$$
- $i$ -th column of the inertia matrix, for  $i = 1, \dots, N$   

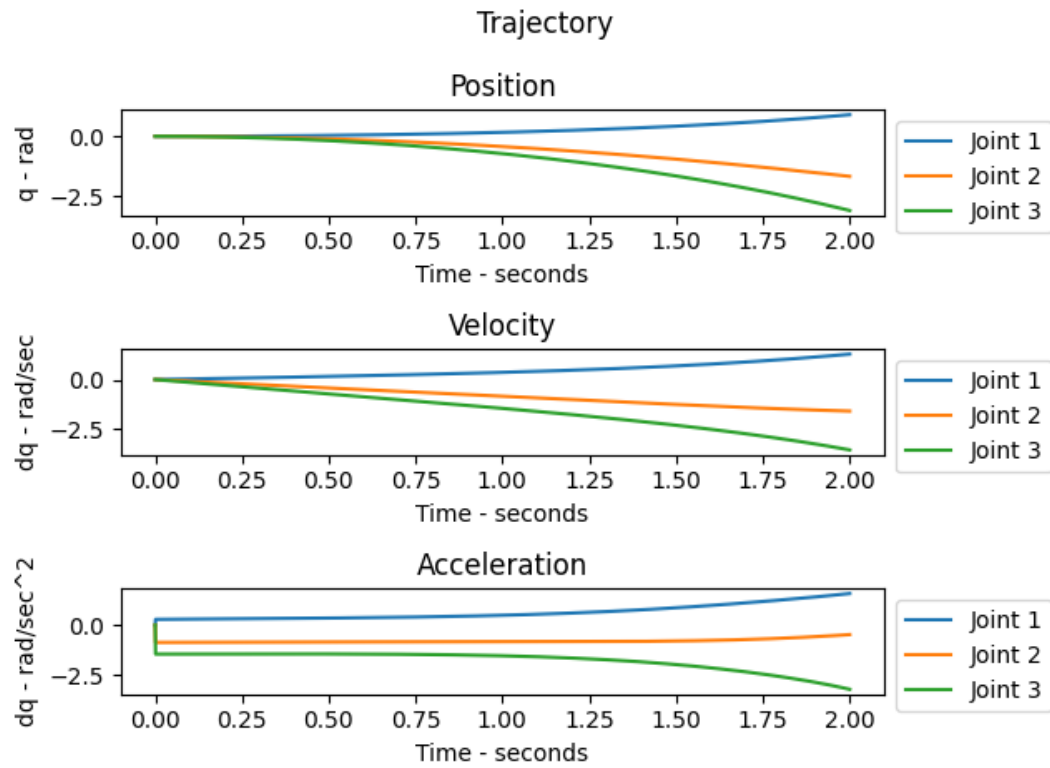
$$M_i = NE_0(q, 0, e_i) \quad O(N^2)$$
- numerical inversion of inertia matrix  

$$InvM = \text{inv}(M) \quad O(N^3) \text{ but with small coefficient}$$
- given  $u$ , integrate acceleration computed as  

$$\ddot{q} = InvM * [u - n] \quad \rightarrow \quad \text{new state } (q, \dot{q}) \text{ and repeat over time ...}$$

Method for Direct Dynamics based on the recursive solution of the inverse dynamics

The plots:



Torques and Forces plots