Fundamentels of Robotics

Assignment 3 - Jacobian. Innopolis University, Fall 2020

Name: Hany Hamed
Group Number: BS18-Robotics
GitHub Repository: here (It is public after the submission deadline)

Contents

FoR, Innopolis University, Fall 2020
Author: Hany Hamed

Section 1: Introduction

The robot is a manipulator from KUKA (KR 10 R1100-2). This robot has 6 joints and is 6 DoF manipulator, moreover, it has a spherical writs.

Data-sheet of the manipulator can be found here

Two methods have been chosen to compute the Jacobian:

- Skew Theory
- Numerical Derivatives

Note: a symbolic solution based on differentiation for the symbolic transformation matrix has been made in order to get the first 3 elements in the jacobian $(\dot{x}, \dot{y}, \dot{z})$ *to debug because there was a bug and did not know which method provide the correct results.*

Section2: Computing Jacobian using Skew Theory

First we define the transformation matrix from world to the end effector as following:

$$T = A_0 A_1 A_2 A_3 A_4 A_5 A_6$$

Such that:

$A_0 = T_{base} = T_0^w$: transformation from the world to the place of the first joint's frame

$A_1 = R_z(q_0)T_z(l_0)T_x(l_1) = T_1^0$: transformation from the first joint's frame to the second joint frame

$A_2 = R_y(q_1)T_x(l_2) = T_2^1$

$A_3 = R_y(q_2)T_x(l_3) = T_3^2$

$A_4 = R_x(q_3)T_x(l_4) = T_4^3$

$A_5 = R_y(q_4) = T_5^4$

$A_6 = R_x(q_5)T_x(l_5)T_{tool} = T_6^5$: transformation from the last joint's frame (6^{th}) to the end effector frame

Note: $T_j^i = T_k^i T_j^k$*, which means for example that* $A_3 A_4 = T_3^2 T_4^3 = T_4^2$

The jacobian is in the following form:

$\mathcal{J} = [\vec{\mathcal{J}}_0\ \vec{\mathcal{J}}_1\ \vec{\mathcal{J}}_2\ \vec{\mathcal{J}}_3\ \vec{\mathcal{J}}_4\ \vec{\mathcal{J}}_5]$

As all the joints are revolute joints, we will use the following in order to compute each column $\vec{\mathcal{J}}_i$ in the jacobian matrix:

$$\vec{\mathcal{J}}_i = \begin{bmatrix} \vec{U}_i \ \times \ (\vec{O}_n - \vec{O}_i) \\ \vec{U}_i \end{bmatrix}$$

such that: n=6 and i=0,1,2,3,4,5

Then we need to get the $\vec{O}$ $\vec{U}$ vectors:

- $\vec{O}_i$ is the positional vector from the transformation matrix from the world to $i^t h$ frame (T_i^w)
- $\vec{U}_i$ is the column vector corresponds to the axis of the rotation of the joint from the rotation matrix that is composed into the transformation matrix from the world to $i^t h$ frame (T_i^w) (e.g. rotation axis is z-axis, take 3rd column

Section 3: Computing Jacobian using Numerical Derivatives

First we compute the forward kinematics as from the previous assignment from the following formula:

$$T = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, R_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}$$

Second, get the rotation matrix from the previous homogeneous matrix = R, get the inverse of it and compose it into a homogeneous matrix with zero position vector:

$$T_o = \begin{bmatrix} R^{-1} & \vec{0} \\ \vec{0} & 1 \end{bmatrix}$$

Then, with the following formulas we calculate the columns of the jacobian:

The general:

$$\dot{T}_i = T_{base}\ T_{left}\ \dot{H}_i\ T_{right}\ T_{tool}\ T_o$$

such that $\dot{T}_i$ is the derivative of the transformation with respect to the i^{th} joint generalized coordinate (q_i), T_{left} and T_{right} are the right and left transformations from H_i in the original transformation equation T. $\dot{H}_i$ is the derivative of the transformation that depends on the i^{th} joint generalized coordinates (q_i)

Note: it is zero indexed

$$\mathcal{J}_i = \begin{bmatrix} \dot{T}_i[0,3] \\ \dot{T}_i[1,3] \\ \dot{T}_i[2,3] \\ \dot{T}_i[2,1] \\ \dot{T}_i[0,2] \\ \dot{T}_i[1,0] \end{bmatrix}$$

In details:

$\dot{T}_0 = T_{base}\, \dot{R}_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, R_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

And get $\mathcal{J}_0$ as in the general formulas

$\dot{T}_1 = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, \dot{R}_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, R_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

get $\mathcal{J}_1$

$\dot{T}_2 = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, \dot{R}_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, R_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

get $\mathcal{J}_2$

$\dot{T}_3 = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, \dot{R}_x(q_3)\, T_x(l_4)\, R_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

get $\mathcal{J}_3$

$\dot{T}_4 = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, \dot{R}_y(q_4)\, R_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

get $\mathcal{J}_4$

$\dot{T}_5 = T_{base}\, R_z(q_0)\, T_z(l_0)\, T_x(l_1)\, R_y(q_1)\, T_x(l_2)\, R_y(q_2)\, T_x(l_3)\, R_x(q_3)\, T_x(l_4)\, R_y(q_4)\, \dot{R}_x(q_5)\, T_x(l_5)\, T_{tool}\, T_o$

get $\mathcal{J}_5$

Such that the derivatives of the rotation matrices are from the lecture 6, slide: 26.

Section 4: Singularities

The singularity happens when the robot lose one or more degree of freedom. The singularity of the robot can be determined from the jacobian matrix by 3 methods:

- The jacobian matrix determinant = 0
- The jacobian matrix lose the original rank
- The jacobian matrix's S diagonal matrix from SVD has a minimum value for the elements that is very close or equal to zero.

Here, it is showed 3 singularities: wrist, shoulder, elbow

1. **Wrist singularity:**

Axis of joints 4 and 6 are collinear. Any rotation of one of the joints is the same of the rotation of the other joint, hence, one DoF has been lost.

It happens with the following configuration: $\vec{q} = \begin{bmatrix} 0 \\ \pi/4 \\ 0 \\ \theta_1 \\ 0 \\ \theta_2 \end{bmatrix}$ θ_1 θ_2 are pointing to different rotations for joints but when they are applied they are giving the same rotation motion (the same direction), originally, they should have different rotation axis to get different rotation motion.

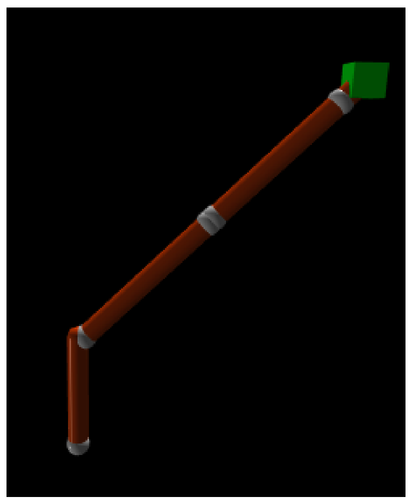

Figure 1. Configuration for wrist singularity

2. **Shoulder singularity:**

   It happens when tool position is on the axis of the first joint, thus, any rotation of the first joint, the position of the tool is not changing, hence the position of the spherical wrist manipulator is not changing by rotating the first joint. (Any rotation in the first joint does not affect the end effector position, hence, losing of one DoF)

   It happens with the following configuration: $\vec{q} = \begin{bmatrix} 0 \\ -\pi/6 \\ -1.990468423823186 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

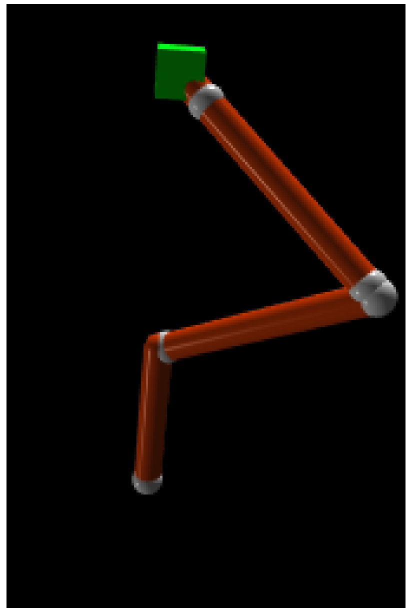

Figure 2. Front perspective for the configuration for the shoulder singularity

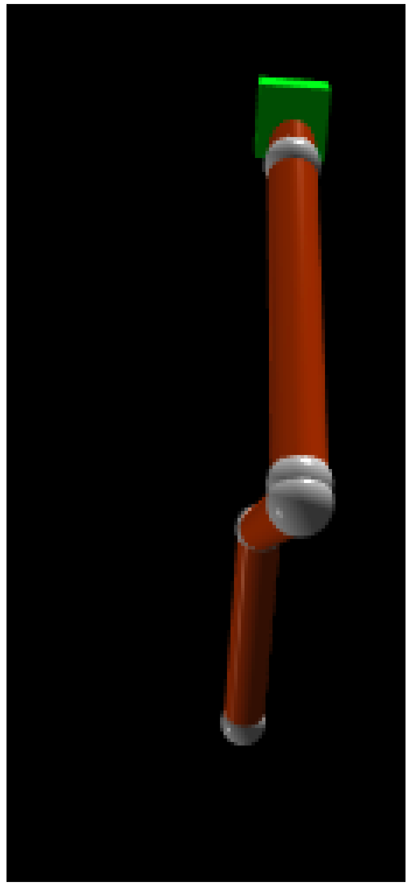

Figure 3. Side perspective for the configuration for the shoulder singularity

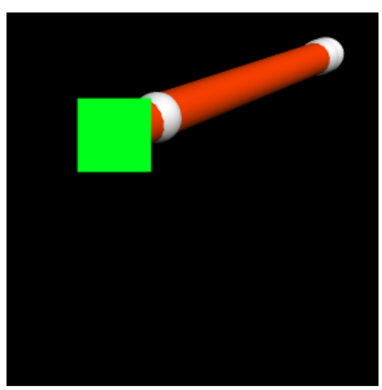

Figure 4. Top perspective for the configuration for the shoulder singularity

3. **Elbow singularity:**

   It happens when the manipulator reaches the maximum working space, when the whole manipulator is extended (flexed), this happens in the zero configuration according to the developed kinematics model (Wrist and Elbow singularities together). The end effector cannot move directly in the z direction without changing the x position (two position directions depends on each other, cannot change one direction (z direction) independently instead it can only move in the circular trajectory that bounds the working space, hence, losing of 1 DoF) . Generally, Elbow singularity happens when the position of the spherical wrist (position of the 4th joint) lies on the plane that constructed by the axis of 2nd and 3rd joints.

   It happens with the following configurations:

   $$\vec{q} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ (Elbow and wrist singularities)}$$

   and

   $$\vec{q} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\pi/12 \\ 0 \end{bmatrix} \text{ (only elbow singularity)}$$

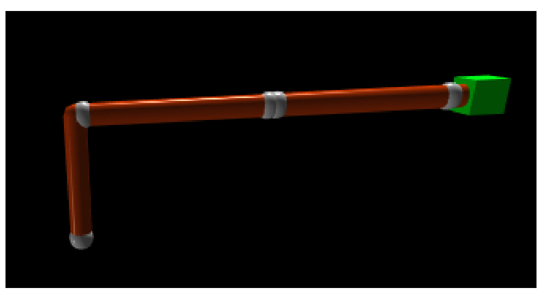

Figure 5. Configuration for the elbow singularity

Note: Green box is the tool on the end effector

References

[1] Kang, Z.H., Cheng, C.A. and Huang, H.P., 2019. A singularity handling algorithm based on operational space control for six-degree-of-freedom anthropomorphic manipulators. International Journal of Advanced Robotic Systems, 16(3), p.1729881419858910.

[2] Chevallereau, C. and Rabah, R., 1996, July. The singular value decomposition and the control of a robot in singular configuration.

[3] Dewolf, T., 2017. Robot control part 6: Handling singularities. studywolf. Available at: https://studywolf.wordpress.com/2013/10/10/robot-control-part-6-handling-singularities/ [Accessed November 10, 2020].

[4] Mecademic.com. (2020). What are singularities in a six-axis robot arm? [online] Available at: https://www.mecademic.com/resources/Singularities/Robot-singularities [Accessed 10 Nov. 2020].

[5] CS 4733 Class Notes: Kinematic Singularities and Jacobians 1 Kinematic Singularities. (n.d.). [online] Available at: http://www.cs.columbia.edu/ allen/F15/NOTES/jacobians.pdf [Accessed 10 Nov. 2020].

[6] Owen-Hill, A. (2016). 3 types of robot singularities and how to avoid them — Robohub. [online] Robohub.org. Available at: https://robohub.org/3-types-of-robot-singularities-and-how-to-avoid-them/ [Accessed 10 Nov. 2020].

[7] Dimeas, F., Moulianitis, V.C., Papakonstantinou, C. and Aspragathos, N. (2016). Manipulator performance constraints in Cartesian admittance control for human-robot cooperation. 2016 IEEE International Conference on Robotics and Automation (ICRA). [online] Available at: https://ieeexplore.ieee.org/abstract/document/7487469. YouTube. Available at: https://www.youtube.com/watch?v=1zTDmiDjDOAab_channel=HEBIRoboticst=99s [Accessed 10 Nov. 2020].