

Solution

$$y' = y^2 \cdot e^x - 2 \cdot y: \quad y = \frac{1}{e^x(1 + c_1 e^x)}$$

Steps

$$y' = y^2 \cdot e^x - 2 \cdot y$$

First order Bernoulli Ordinary Differential Equation

A first order Bernoulli ODE has the form of $y' + p(x)y = q(x)y^n$

Rewrite in the form of a first order Bernoulli ODE

[Show Steps](#)

$$y' + 2y = e^x y^2$$

$$y' + p(x)y = q(x)y^n$$

$$p(x) = 2, \quad q(x) = e^x, \quad n = 2$$

The general solution is obtained by substituting $v = y^{1-n}$ and solving $\frac{1}{1-n}v' + p(x)v = q(x)$

$$\text{Transform to } \frac{1}{1-n}v' + p(x)v = q(x): \quad -v' + 2v = e^x$$

[Show Steps](#)

$$\text{Solve } -v' + 2v = e^x: \quad v = e^{2x}(e^{-x} + c_1)$$

[Show Steps](#)

$$\text{Substitute back } v = y^{-1}: \quad y^{-1} = e^{2x}(e^{-x} + c_1)$$

[Show Steps](#)

$$\text{Isolate } y: \quad y = \frac{1}{e^x(1 + c_1 e^x)}$$

[Show Steps](#)

$$y = \frac{1}{e^x(1 + c_1 e^x)}$$