The German University in Cairo (GUC). **Discrete Mathematics**, Fall 2022.

SAMPLE ANSWER: **Q #1** Date: 22-Oct.-22 Time: 25 Minutes (max.) Marks: 10

Group / Name / ID:

1. Ashry says: "if `money solves problems`, then `the quiz is easy`", and : "if `money solves problems`, then `the quiz is difficult`". Is it valid to conclude: "`money does not solve problems`" based on both of Ashry's hypotheses? Prove your answer.
   N.B. In this question, we simply use "`the quiz is difficult`" as the negation of "`the quiz is easy`", and "`money does not solve problems`" as the negation of "`money solves problems`".

[5 Marks]

Ans.
- First, translate the given statements into a symbolic, propositional logic form. Say we use p: "money solves problems", and q: "the question is easy".
- Therefore, we want to prove whether $(p \to q) \wedge (p \to \neg q) \to \neg p$ is a valid argument.
- Since the last column in the following truth table is always true, therefore the given argument is valid.

p	q	$p \to q$ (1)	$p \to \neg q$ (2)	1∧2 (3)	$3 \to \neg p$ (4)
F	F	T	T	T	T✓
F	T	T	T	T	T✓
T	F	**F**	T	**F**	**T✓**
T	T	T	**F**	**F**	**T✓**

N.B.: As an alternative answer that uses a proof sequence, Negation Introduction (NI) can be directly applied to derive the conclusion (but after the translation step).

1. $p \to q$ hyp.
2. $p \to \neg q$ hyp.
3. $\therefore \neg p$ 1, 2, NI

Therefore the given argument is valid.

2. Complete the following sequence of derivations to prove that the given argument is valid

$$\neg(\boldsymbol{R} \to \boldsymbol{Q}) \wedge (\neg \boldsymbol{S} \to \boldsymbol{P}) \wedge (\boldsymbol{S} \to \boldsymbol{Q}) \wedge (\boldsymbol{U} \vee \neg \boldsymbol{P}) \longrightarrow \boldsymbol{U}$$

[5 Marks]

#:	expression:	Justification:
1.	$\neg(R \to Q)$	hypothesis
2.	$\neg S \to P$	hypothesis
3.	$\vdots$	$\vdots$
	$\therefore U$	

Ans. •

#:	expression:	Justification:
1.	$\neg(R \to Q)$	hypothesis
2.	$\neg S \to P$	hypothesis
3.	$S \to Q$	hypothesis
4.	$U \vee \neg P$	hypothesis
5.	$P \to U$	4, IMP
6.	$\neg(\neg R \vee Q)$	1, IMP
7.	$R \wedge \neg Q$	6, De Morgan
8.	$\neg Q$	7, SIMP (simplification)
9.	$\neg S$	8, 3, MT (Modus Tollens)
10.	P	9, 2, MP (Modus Ponens)
11.	U	10, 5, MP (Modus Ponens)

Rules of inference: Other rules are missing (e.g., commutativity, associativity, double negation, De Morgan's, etc.)

Name (abbreviation) of the rule:		**The rule:**
Implication	(IMP)	$P \to Q \equiv \neg P \vee Q$.
Contrapositive	(CONT)	$P \to Q \equiv \neg Q \to \neg P$.
Modus Ponens	(MP)	from P and $P \to Q$, deduce Q.
Modus Tollens	(MT)	from $P \to Q$ and $\neg Q$, deduce $\neg P$.
Simplification	(SIMP)	from $P \wedge Q$, deduce P (or deduce Q).
Conjunction	(CONJ)	from P and Q, deduce $P \wedge Q$.
Hypothetical Syllogism	(HS)	from $P \to Q$ and $Q \to R$, deduce $P \to R$.
Disjunctive Syllogism	(DS)	from $P \vee Q$ and $\neg P$, deduce Q.
Addition	(ADD)	from P, deduce $P \vee Q$.
Negation Introduction	(NI)	from $P \to Q$ and $P \to \neg Q$, deduce $\neg P$.
Negation Elimination	(NE)	from P and $\neg P$, deduce Q.