

CSEN 703 Analysis and Design of Algorithms, Winter Term 2022  
Practice Assignment 3

**Exercise 3-1** From CLRS (©MIT Press 2001)

Insertion sort can be expressed as a recursive procedure as follows:

In order to sort  $A[1..n]$ , we recursively sort  $A[1..n-1]$  then insert  $A[n]$  into the sorted array  $A[1..n-1]$ . Write a recurrence for the running time of this recursive version of insertion sort.

**Exercise 3-2** From CLRS (©MIT Press 2001)

Use a recursion tree to determine a good asymptotic upper bound on the recurrence

$$T(n) = 3T(\lfloor n/2 \rfloor) + n.$$

**Exercise 3-3** From CLRS (©MIT Press 2001)

Solve the following recurrence using the recursion tree method.  $T(n) = 7T(n/2) + n^2$

**Exercise 3-4** From CLRS (©MIT Press 2001)

Use the divide-and-conquer approach to write an algorithm that finds the largest item in a list of  $n$  items. Analyze your algorithm and get its worst-case time complexity.

**Exercise 3-5**

Write a divide-and-conquer algorithm for the **Towers of Hanoi** problem. The Towers of Hanoi problem consists of three pegs and  $n$  disks of different sizes. The objective is to move the disks that are stacked on one of the three pegs (in decreasing order of their size) to a new peg using the third one as a temporary peg. The problem should be solved according to the following rules:

- i when a disk is moved, it must be placed on one of the three pegs;
- ii only one disk may be moved at a time, and it must be the top disk on one of the pegs; and
- iii a larger disk may never be placed on top of a smaller disk.

What is the worst-case time complexity of your algorithm?

```
public class Hanoi {
    enum Tower { A, B, C };

    static void moveDisk( Tower from, Tower to ) {
        System.out.println(
            "Move disk from " + from + " to " + to
        );
    }

    public static void towers( int n, Tower x, Tower y, Tower z ) {
        if ( n > 0 ) {
            towers( n-1, x, z, y );
        }
    }
}
```

```
        moveDisk( x, y );
        towers( n-1, z, y, x );
    }
}
```