The German University in Cairo (GUC). **Discrete Mathematics**, Fall 2022.
SAMPLE ANSWER: **Q #1** Date: 22-Oct.-22 Time: 25 Minutes (max.) Marks: 10

Group / Name / ID:

1. Write a propositional logic form of the following statement, then write a plausible negation both in propositional logic and in English. (Don't use "`It is not true that` ....")

   "`the statement is not satisfiable` only if `the statement is a contradiction`."

   [2 Marks]

Ans. (a) Form: $p \to q$, where p: "`the statement is NOT satisfiable`" and q: "`the statement is a contradiction`".
Negation: $\neg(p \to q) \equiv \neg(\neg p \vee q) \equiv p \wedge \neg q$: "`the statement is NOT satisfiable` AND `the statement is NOT a contradiction`".

N.B. It's also okay if p were "`the statement is satisfiable`". However, in such a case, the form must be $\neg p \to q$, and the negation $\neg p \wedge \neg q$, for example.

2. Prove that the Negation Introduction (NI) $(\boldsymbol{P \to Q}) \wedge (\boldsymbol{P \to \neg Q}) \to \neg \boldsymbol{P}$ is a valid argument.

   [3 Marks]

   Remember: This argument means that if one statement leads to another one as well as to is negation (of that other one), then the first statement must be false.

Ans.
- Since the last column in the following truth table is always true, therefore the given argument is valid.

P	Q	$P \to Q$ (1)	$P \to \neg Q$ (2)	$1 \wedge 2$ (3)	$3 \to \neg P$ (4)
F	F	T	T	T	T✓
F	T	T	T	T	T✓
T	F	**F**	T	**F**	**T**✓
T	T	T	**F**	**F**	**T**✓

- A proof sequence using would also work fine (yet seemingly less obvious):

1.	$P \to Q$	hyp. (given)
2.	$P \to \neg Q$	hyp. (given)
3.	$Q \to \neg P$	2, CONT ($\equiv$)
4.	$P \to \neg P$	1, 3, HS ($\to$)
5.	$\neg P \vee \neg P$	4, CONT ($\equiv$)
6.	$\therefore \neg P$	5, SIMP ($\equiv$)

3. Prove that $((\neg \boldsymbol{A} \to \boldsymbol{B}) \to (\boldsymbol{C} \to \boldsymbol{D})) \wedge ((\neg \boldsymbol{B} \to \boldsymbol{A}) \to (\boldsymbol{C} \wedge \neg \boldsymbol{D})) \to (\neg \boldsymbol{A} \wedge \neg \boldsymbol{B})$ is a valid argument.

   Hints: (1) Remember that $\neg\left(Ⓧ \to Ⓨ\right) \equiv Ⓧ \wedge \neg Ⓨ$, and apply the valid argument of question 2 and/or other rules from the "formula sheet". (2) If you're puzzled, answer using a truth table... Tick Tock!

   [5 Marks]

#:	expression:	Justification:
1.	$(\neg A \to B) \to (C \to D)$	hypothesis
2.	$(\neg B \to A) \to (C \land \neg D)$	hypothesis
3.	$(\neg A \to B) \to (C \land \neg D)$	$\equiv$2, CONT (of antecedent)
4.	$(\neg A \to B) \to \neg(\neg C \lor D)$	$\equiv$3, De Morgan's (of consequent)
5.	$(\neg A \to B) \to \neg(C \to D)$	$\equiv$4, CONT (of consequent)
6.	$\neg(\neg A \to B)$	5, 1, NI (question 2)
7.	$\neg(\neg\neg A \lor B)$	6, CONT
8.	$\therefore \neg A \land \neg B$	7, De Morgan's

Ans. • 4.

N.B. If this question is answered using a truth table, the answer must also state the concluding statement: "therefore, the given argument is valid because it's a tautology.".

Rules of inference: Other rules are missing (e.g., commutativity, associativity, double negation, De Morgan's, etc.)

Name (abbreviation) of the rule:		The rule:
Implication	(IMP)	$P \to Q \equiv \neg P \lor Q$.
Contrapositive	(CONT)	$P \to Q \equiv \neg Q \to \neg P$.
Modus Ponens	(MP)	from P and $P \to Q$, deduce Q.
Modus Tollens	(MT)	from $P \to Q$ and $\neg Q$, deduce $\neg P$.
Simplification	(SIMP)	from $P \land Q$, deduce P (or deduce Q).
Conjunction	(CONJ)	from P and Q, deduce $P \land Q$.
Hypothetical Syllogism	(HS)	from $P \to Q$ and $Q \to R$, deduce $P \to R$.
Disjunctive Syllogism	(DS)	from $P \lor Q$ and $\neg P$, deduce Q.
Addition	(ADD)	from P, deduce $P \lor Q$.
Negation Introduction	(NI)	from $P \to Q$ and $P \to \neg Q$, deduce $\neg P$.
Negation Elimination	(NE)	from P and $\neg P$, deduce Q.