DMET 502 - Computer Graphics
Graphics Primitives Solutions

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Q1: Given a 2D polygon specified by the vertices $[-3,0]^T$, $[3,-1]^T$, $[1,0]^T$, and $[4,2]^T$, test whether it is convex or concave.

Solution: The cross product of two vectors results in a norm which is perpendicular to these two vectors, and which has a positive or negative direction. In a convex polygon, the norms of all vectors have the same direction, which means they all have the same sign. On the other hand, in a concave polygon, at least one norm goes in a direction opposite to that of the others; this norm also has a different sign.

Since the polygon has 4 points, then there are 4 vectors that could be calculated, which are:

- $v_0 = p_1 - p_0 = [3,-1]^T - [-3,0]^T = [6,-1]^T$
- $v_1 = p_2 - p_1 = [1,0]^T - [3,-1]^T = [-2,1]^T$
- $v_2 = p_3 - p_2 = [4,2]^T - [1,0]^T = [3,2]^T$
- $v_3 = p_0 - p_3 = [-3,0]^T - [4,2]^T = [-7,-2]^T$

The angles between the vectors are not known, which leads to the calculation of the cross product of these vectors using determinant multiplication.

$$v_0 \times v_1 = [6,-1]^T \times [-2,1]^T = \begin{vmatrix} 6 \\ -1 \end{vmatrix} \times \begin{vmatrix} -2 \\ 1 \end{vmatrix} = 6 \times 1 - (-1 \times -2) = 6 - 2 = 4$$

$$v_1 \times v_2 = [-2,1]^T \times [3,2]^T = \begin{vmatrix} -2 \\ 1 \end{vmatrix} \times \begin{vmatrix} 3 \\ 2 \end{vmatrix} = -2 \times 2 - (1 \times 3) = -4 - 3 = -7$$

$$v_2 \times v_3 = [3,2]^T \times [-7,-2]^T = \begin{vmatrix} 3 \\ 2 \end{vmatrix} \times \begin{vmatrix} -7 \\ -2 \end{vmatrix} = 3 \times -2 - (2 \times -7) = -6 + 14 = 8$$

$$v_3 \times v_0 = [-7,-2]^T \times [6,-1]^T = \begin{vmatrix} -7 \\ -2 \end{vmatrix} \times \begin{vmatrix} 6 \\ -1 \end{vmatrix} = -7 \times -1 - (-2 \times 6) = 7 + 12 = 19$$

One of the four values obtained has a different sign than the other three, which means that this polygon is a **concave** one. As can be noticed, after the second cross product there were two values with different signs; this means that it is not necessary to calculate all cross products to determine that the type of the polygon is concave, one only needs to calculate until different signs are encountered.

Q2: A circle having a radius of 5 pixels and centered at $[3{,}7]^T$ is to be drawn on a computer screen. Use the 8-way symmetry algorithm to determine what pixels should constitute the circle.

Solution: The first step would be to get all the points of this circle around the origin, which is at the point $[0{,}0]^T$. The point $[3{,}7]^T$ is then added to all resulting points in order to get the final pixel values that constitute the circle.

Using the formula $\cos\theta = \frac{x}{r}$, it could be deduced that $x = r\cos\theta$.
Since $r = 5$ and $\theta = 45°$, $x = 5\cos 45° = 3.53$, which means that x will range from 0 to 3.

For each value of x, a value of y is computed using the following formula:
$y = \sqrt{r^2 - x^2}$, where a ceiling (rounding up) is applied to the y values that contain fractions.

x	y
0	$\sqrt{25-0}=5$
1	$\sqrt{25-1}=5$
2	$\sqrt{25-4}=5$
3	$\sqrt{25-9}=4$

The calculated x and y values are reflected accordingly in order to get the remaining colored pixels of the circle while the center is located at the origin. The resulting points are displayed in the following table:

(x,y)	(y,x)	(y,-x)	(x,-y)	(-x,-y)	(-y,-x)	(-y,x)	(-x,y)
(0,5)	(5,0)	(5,0)	(0,-5)	(0,-5)	(-5,0)	(-5,0)	(0,5)
(1,5)	(5,1)	(5,-1)	(1,-5)	(-1,-5)	(-5,-1)	(-5,1)	(-1,5)
(2,5)	(5,2)	(5,-2)	(2,-5)	(-2,-5)	(-5,-2)	(-5,2)	(-2,5)
(3,4)	(4,3)	(4,-3)	(3,-4)	(-3,-4)	(-4,-3)	(-4,3)	(-3,4)

In order to get the points of the circle centered at $[3{,}7]^T$, all points calculated in the previous table are translated by the vector $[3{,}7]^T$; this means that each point has vector $[3{,}7]^T$ added to it.

(x,y)	(y,x)	(y,-x)	(x,-y)	(-x,-y)	(-y,-x)	(-y,x)	(-x,y)
(3,12)	(8,7)	(8,7)	(3,2)	(3,2)	(-2,7)	(-2,7)	(3,12)
(4,12)	(8,8)	(8,6)	(4,2)	(2,2)	(-2,6)	(-2,8)	(2,12)
(5,12)	(8,9)	(8,5)	(5,2)	(1,2)	(-2,5)	(-2,9)	(1,12)
(6,11)	(7,10)	(7,4)	(6,3)	(0,3)	(-1,4)	(-1,10)	(0,11)

It should be noted that, since there are negative values in the 6th and the 7th columns, the points in these columns are located outside the screen.

Q3: Shown in Figure 1 is the left upper corner of a computer screen. The horizontal and vertical axes are shown with values representing pixel locations.

Suppose that a curve spanning from $[5,15]^T$ to $[15,5]^T$ is drawn as two circle quadrants. The centers of the circles are shown as black dots. Use the 8-way symmetry algorithm to determine what pixels should constitute the curve.

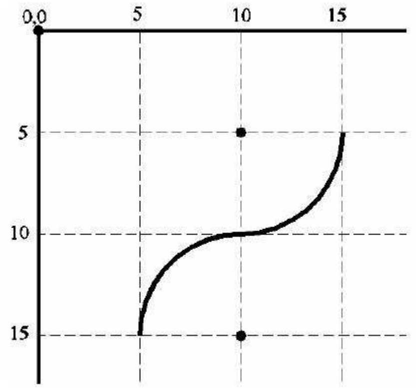

Figure 1: Curve Graph

Solution: The first step in solving this problem is to get the radius, which as shown in the graph is equal to 5.

$x = r \cos \theta = 5 \cos 45° = 3.53$

For each value of x, a value of y is computed using the following formula:
$y = \sqrt{r^2 - x^2}$, where a ceiling is applied to the y values that contain fractions.

x	y
0	$\sqrt{25-0}$ =5
1	$\sqrt{25-1}$ =5
2	$\sqrt{25-4} = 5$
3	$\sqrt{25-9} = 4$

The first circle, whose center lies at $[10,5]^T$, has only the pixels (x,y) and (y,x) drawn. These points, while still around the origin, are:

(x,y)	(y,x)
(0,5)	(5,0)
(1,5)	(5,1)
(2,5)	(5,2)
(3,4)	(4,3)

The calculated points are then transformed by $[10,5]^T$ to obtain the final pixels of the first circle, which are:

(x,y)	(y,x)
(10,10)	(15,5)
(11,10)	(15,6)
(12,10)	(15,7)
(13,9)	(14,8)

Meanwhile, the second circle, whose center lies at $[10,15]^T$, has only the pixels (-x,-y) and (-y,-x) drawn. These points, while still around the origin, are:

(-x,-y)	(-y,-x)
(0,-5)	(-5,0)
(-1,-5)	(-5,-1)
(-2,-5)	(-5,-2)
(-3,-4)	(-4,-3)

The calculated points are then transformed by $[10,15]^T$ to obtain the final pixels of the second circle, which are:

(-x,-y)	(-y,-x)
(10,10)	(5,15)
(9,10)	(5,14)
(8,10)	(5,13)
(7,11)	(6,12)

Q4: Shown in Figure 2 is the upper left corner of a computer screen. The horizontal and vertical axes are shown with values representing pixel locations.

Suppose that the curve shown consists of 3 segments, two of them are line segments, and the third is one-eighth of a circle whose center is shown as a black dot. Use the 8-way symmetry and Bresenham's line drawing algorithms to determine what pixels should constitute the curve.

Solution: Before being able to draw the first line segment, the end point of this segment needs to be determined by first drawing the circle segment. Similar to the previous questions, the radius of the circle is 5.

$x = r \cos\theta = 5 \cos 45\ ^\circ = 3.53$

For each value of x, a value of y is computed using the following formula:
$y = \sqrt{r^2 - x^2}$, where a ceiling is applied to the y values that contain fractions.

The resulting points are $[0,5]^T$, $[1,5]^T$, $[2,5]^T$, and $[3,4]^T$. The pixels drawn at the origin for this circle are those with the coordinates (-x,-y), which are $[0,-5]^T$, $[-1,-5]^T$, $[-2,-5]^T$, and $[-3,-4]^T$.

Since the circle is position around $[10,10]^T$, all points are translated by that vector. The final pixels constituting the circle are $[10,5]^T$, $[9,5]^T$, $[8,5]^T$, and $[7,6]^T$ which is the end point of the first line segment.

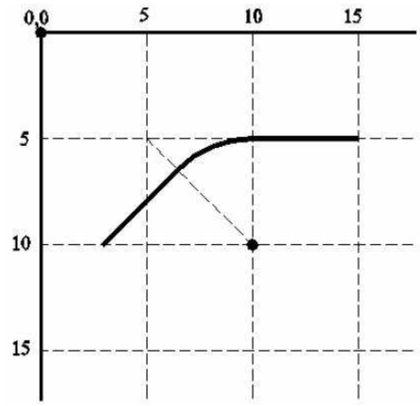

Figure 2: Curve Graph

The first line segment extends between points $[3,10]^T$ and $[7,6]^T$. The slope, also known as $\triangle$ error, of this line is:

slope $= \frac{y_1-y_0}{x_1-x_0} = \frac{|6-10|}{|7-3|} = \frac{4}{4} = 1$

The value of the error always starts with 0, and has the value of the slope added to it. If the resulting error is ≥ 0.5, then y is incremented by 1, and the error is decremented by 1. This is repeated until the final point in the line is drawn.

Since y_0 is greater than y_1, in Bresenham's line drawing algorithm this is the case where instead of incrementing the value of y by 1, it is decremented by 1.

x	**error**	y	**new error**
3	0	10	0
4	$0 + 1 = 1$	9	1 - 1 = 0
5	$0 + 1 = 1$	8	1 - 1 = 0
6	$0 + 1 = 1$	7	1 - 1 = 0
7	$0 + 1 = 1$	6	1 - 1 = 0

The points constituting the first line segment are $[3,10]^T$, $[4,9]^T$, $[5,8]^T$, $[6,7]^T$, and $[7,6]^T$.

Finally, the second line segment extending from $[10,5]^T$ to $[15,5]^T$ is drawn. The slope of this line is equal to 0, ensuring that the value of y is never incremented. The points constituting this line are $[10,5]^T$, $[11,5]^T$, $[12,5]^T$, $[13,5]^T$, $[14,5]^T$, and $[15,5]^T$.

Q5: You are asked to draw a line segment between the points $[1,1]^T$ and $[4,3]^T$. Use Bresenham's line drawing algorithm to specify the locations of the pixels that should approximate the line.

Solution: The first step in solving this problem is to calculate the slope of the line.

slope $= \frac{y_1 - y_0}{x_1 - x_0} = \frac{3-1}{4-1} = \frac{2}{3}$

This question represents the general case of Bresenham's line drawing algorithm, where x_1 is greater than x_0, y_1 is greater than y_0, and the slope is less than 1.

According to the table below, each value for y is determined using the value of the slope, which is also defined as $\triangle$ error. Every time the value of the incremented error is greater than or equal to 0.5, the value of y is increased by 1, while the value of the error is decreased by 1.

x	**error**	y	**new error**
1	0	1	0
2	0 + 0.66 = 0.66	2	0.66 - 1 = -0.34
3	-0.34 + 0.66 = 0.32	2	0.32
4	0.32 + 0.66 = 0.98	3	0.98 - 1 = -0.02

The points constituting the line are $[1,1]^T$, $[2,2]^T$, $[3,2]^T$, and $[4,3]^T$.

Q6: Suppose that a clipping window is indicated by its two upper-left and lower-right corners $[100,50]^T$ and $[300,200]^T$ as illustrated in Figure 3. Test whether each of the following line segments can be trivially accepted in the window, trivially rejected from the window, or would need further processing:

- A line extending from $[171,88]^T$ to $[233,171]^T$
- A line extending from $[150,101]^T$ to $[233,39]^T$
- A line extending from $[52,15]^T$ to $[98,45]^T$

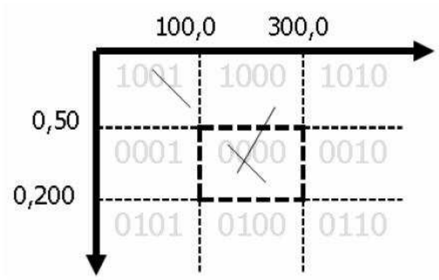

Figure 3: Clipping Window

Solution: The **first line**, which extends from $[171,88]^T$ to $[233,171]^T$, has both points lying inside the clipping window. This means that each of the starting point and the ending point have the value 0000. The operation 0000 *or* 0000 results in 0, which means that this line is *trivially accepted*.

The **second line**, which extends from $[150,101]^T$ to $[233,39]^T$, has one point inside the clipping window, and one outside. According to the graph given in Figure 3, the starting point has the value 0000, while the ending point has the value 1000. The operation 0000 *or* 1000 does not yield 0, which is why this line is not trivially accepted. In addition, 0000 *and* 1000 yields 0, which is why this line is not trivially rejected. Therefore, $further\ calculations$ are needed.

There are two ways to get the intersecting point between the line segment and the clipping window, one could either use parametric line equations, or the equation of a line could be applied.

Using parametric line equations:
The first line extends from $[150,101]^T$ to $[233,39]^T$, while the second extends from $[100,50]^T$ to $[300,50]^T$.

derivation vector$_{line1}$ = p_1 - p_0 = $[233 - 150, 39 - 101]^T = [83,-62]^T$
$x = x_0 + t(x_1 - x_0) = 150 + t(233 - 150) = 150 + 83t$
$y = y_0 + t(y_1 - y_0) = 101 + t(39 - 101) = 101 - 62t$

derivation vector$_{line2}$ = p_1 - p_0 = $[300 - 100, 50 - 50]^T = [200,50]^T$
$x = x_0 + t(x_1 - x_0) = 100 + t(300 - 100) = 100 + 200t$
$y = y_0 + t(y_1 - y_0) = 50 + t(50 - 50) = 50$

By equating both values of y:
$101 - 62t = 50 \rightarrow t = \frac{50-101}{-62} = \frac{-51}{-62} = \frac{51}{62}$

By replacing the value of t in the first equation for x:
$x = 150 + \frac{83 \times 51}{62} \approx 218.2$

The intersecting point is at $[218.2,50]^T$.

Using the equation of a line:
$y - y_0 = \frac{y_1 - y_0}{x_1 - x_0} (x - x_0)$

The line extending between $[150,101]^T$ and $[233,39]^T$ results in the following line equation:
$y - 101 = \frac{39-101}{233-150} (x - 150)$

Since the intersecting line is $[100,50]^T$ to $[300,50]^T$, the intersecting point is at $y = 50$, which yields:
$50 - 101 = \frac{-62}{83} (x - 150) \rightarrow x = \frac{-51 \times 83}{-62} + 150 \approx 218.2$

The intersecting point is at $[218.2,50]^T$.

The **third line**, which extends from $[52,15]^T$ to $[98,45]^T$, has both points outside the clipping window. The first point and second point both have the value 1001. The operation 1001 *or* 1001 does not yield 0, which means this line is not trivially accepted, yet 1001 *and* 1001 yields a non-zero value, which means this line is *trivially rejected*.