MATH 135 HOMEWORK 9

A. HENING

Do problems 10, 12, 13, 20, 23 from Rudin Chapter 4 and the following problems.

1. For any point x on the Earth (or any sphere really) the *antipode*, often written as $-x$, is the point exactly on the other side. Let E represent the surface of the earth. Let $T : E \to \mathbb{R}$ be a continuous function where $T(x)$ represents the temperature at the point x. Prove that there exists a point x on the equator such that $T(x) = T(-x)$.
2. Let (X, d) be a metric space. A *contraction* is a continuous function $f : X \to X$ with the property
$$d(f(x), f(y)) \leq cd(x, y)$$
for some $c < 1$. Show that every contraction in a complete metric space has a unique fixed point (a fixed point is an $x \in X$ such that $f(x) = x$).
3. Let f be a continuous real valued function on a metric space X. Let $Z(f)$ be the set of all $p \in X$ such that $f(p) = 0$
   a) Prove that $Z(f)$ is closed.
   b) Recall that for a set $E \subset X$ the distance from a point to this set is defined as
   $$h(x) = \inf_{s \in E} d(x, s)$$
   Prove that h is uniformly continuous.
   c) Use the previous part to show that for any closed set $E \subset X$ there exist a continuous function $f : X \to \mathbb{R}$ that is 0 on E and positive elsewhere.