Question 5

1. The Principle Components are $T = XU$ where X are the data set and

$$U = \begin{bmatrix} \text{-0.7930} & 0.6093 \\ 0.6093 & 0.7930 \end{bmatrix} \tag{1}$$

2. There is no dominant linear direction in the data, since the ratio of two eigenvalues of covariance matrix is about 1.

Question 6

1. The Principle Components are $T = XU$ where X are the data set and

$$U = \begin{bmatrix} \text{-1.0000} & 0.0070 \\ 0.0070 & 1.0000 \end{bmatrix} \tag{2}$$

   You can see clearly the data are stretched most along the first primary component direction.

2. The dominant direction is along $[0.0070, 1.0000]$ with the primary eigenvalue of 0.9672, which is much larger than the second eigenvalue 0.0630.

Code for Questions

```
%% Sample ~1000 points uniformly from a ball

X_Circle=2*rand(floor(1000*(4/pi)),2)-[1,1]; % Generate data uniformly
    at random
X_Circle=X_Circle(sum(X_Circle.^2,2)<1,:); %Use rejection sampling to
    get points to live in ball

%%First Principle component
Cov_Circle = cov(X_Circle);
[V_C,D_C] = eig(Cov_Circle);
%%PCA
PC_1 = X_Circle * V_C(:,1);
PC_2 = X_Circle * V_C(:,2);
length = zeros(size(PC_1,1),1);
close all; % Display data
%scatter(X_Circle(:,1),X_Circle(:,2));
scatter(length,PC_1);
axis equal
title('Data Generated Uniformly From a Ball of Radius
    1','Interpreter','Latex');
```

```
%% Sample ~1000 points uniformly from an ellipse with major axis lengths
    2,.5.


X_Ellipse=4*rand(floor(1000*(16/pi)),2)-[2,2];
X_Ellipse=X_Ellipse(sum([4,.25].*X_Ellipse.^2,2)<1,:);
%%First Principle component
Cov_Ellipse = cov(X_Ellipse);
[V_E,D_E] = eig(Cov_Ellipse);
%%Principle Components
PC_E1 = X_Ellipse * V_E(:,1);
PC_E2 = X_Ellipse * V_E(:,2);
length_E = zeros(size(PC_E1,1),1);

close all;
scatter(X_Ellipse(:,1),X_Ellipse(:,2));


axis equal
title('Data Generated Uniformly From Interior of the Ellipse
    $4x^{2}+\frac{y^{2}}{4}=1$','Interpreter','latex');
```