Math 135

A. Hening

Midterm Practice

Full name: ______________________________

- Outside scratch paper, formula sheets, and calculators are not allowed.
- Write your answers in the spaces following each question. If you need more room, use the reverse side of the page and check the box indicating I should look at the back.
- Circle or box important parts of your work and cross out portions you do not want evaluated.
- You may ask me questions, but unless there is an error on the exam I will probably refuse to answer.
- Unless told otherwise, **show all your work**.

Question:	1	2	3	4	5	Total
Points:	5	5	5	5	5	25
Score:						

Work on reverse ☐

1. (5 points) True/false, multiple choice, and short answer. No justification required. *Read the statements carefully.* Each is worth $\frac{1}{2}$ point; the total will be rounded (up) to an integer.

   (a) True or false: The subset $A = \{(x, y) \in \mathbb{R}^2 : |xy| \leq 1\}$ of $\mathbb{R}^2$ with the standard metric is compact.

   (b) True or false: If $E \subset K \subset X$ is an infinite subset of the compact set K then E has at least one limit point in K.

   (c) True of false: Every metric space (X, d) has at least two clopen (closed and open) sets, namely $X, \emptyset$.

   (d) True of false: If a closed subset K of a metric space (X, d) is covered by finitely many open sets, then K is compact.

   (e) True of false: If K is a closed subset of $(\mathbb{R}^k, d)$, where d is the standard metric, such that the set
   $$\{d(y, 0) : y \in K\}$$
   is bounded, then K is compact.

   (f) True or false: There is a countable subset of $\mathbb{R}^2$ that is dense in $\mathbb{R}^2$.

   (g) True or false: Given a subset E of a metric space (X, d), $\overline{E}$ is the intersection of all closed sets that contain E.

   (h) True or false: The set of complex numbers of the form $\{z = bi : 0 < b < 1\}$ is an open subset of $\mathbb{C}$. Note: You can identify $\mathbb{C}$ with $\mathbb{R}^2$ and the metric on $\mathbb{C}$ will be given by the standard metric in $\mathbb{R}^2$.

   (i) True or false: If K is a compact subset of a metric space (X, d), then for any $\epsilon > 0$ there are finitely many neighborhoods of radius ϵ whose union contains K.

   (j) True or false: Any subset of the rational numbers is at most countable.

Work on reverse ☐

2. (5 points) Calculate the supremum and infimum of the following subsets of $\mathbb{R}$:

   (a) $A = \{1/n : n \in \mathbb{N}\}$

   (b) $B = \bigcap_{x \in (0,1)} [-1, x)$

Work on reverse $\square$

3. (5 points) Suppose A is a non-empty set of positive real numbers that is bounded away from 0. That means, there exists $\delta > 0$ such that $a \geq \delta$ for all $a \in A$. Let $B = \{1/a \mid a \in A\}$. Show that $\sup B = 1/\inf A$

Work on reverse □

4. (5 points) Let (X, d) be a metric space.

   (a) Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

   (b) Show that $\overline{A \cap B} \subset \overline{A} \cap \overline{B}$. Do we have equality? Prove or give a counterexample.

Work on reverse □

5. (5 points) Let (X, d) be a metric space. A family $(E_\lambda)_{\lambda\in\Lambda}$ of subsets of X is said to have the finite intersection property (FIP) if

$$\bigcap_{\lambda\in F} E_\lambda \neq \emptyset \quad \text{for all finite } F \subset \Lambda$$

Prove that X is compact if and only if every family $(E_\lambda)_{\lambda\in\Lambda}$ of closed subsets with the finite intersection propert satisfies

$$\bigcap_{\lambda\in\Lambda} E_\lambda \neq \emptyset$$

Extra sheet