MATH 135 HOMEWORK 5

A. HENING

Do problems 12, 13, 14, 15, 16, 22, 23 from Rudin Chapter 2 and the following problems.

1. Let A_i, $i = 1, 2, 3 \ldots$, be a countable collection of nonempty compact sets with $A_i \subset A_{i-1}$. Prove that
$$\bigcap_{i=1}^{\infty} A_i \neq \emptyset.$$
2. Let (X, d) be a metric space. Let $A \subset X$. We say $x \in X$ is a *condensation point* of A if every neighborhood of x contains an **uncountable** number of elements of A. Prove every uncountable subset of $\mathbb{R}$ has a condensation point.

   Hint: Use the following: Every open cover of a subset of $\mathbb{R}$ has an *at most countable* subcover.