MATH 135 HOMEWORK 7

A. HENING

Do problems 20, 21, 22, 23, 24 (only parts a), b) and c)), from Rudin Chapter 3 and the following problems.

1. Fix $\alpha > 0$ and let $x_1 > \sqrt{\alpha}$. Let
$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{\alpha}{x_n}\right),\ n \in \mathbb{N}$$
   a) Show that $x_n > \sqrt{\alpha}$ for all $n \in \mathbb{N}$.
   b) Conclude that the sequence is monotonically decreasing.
   c) Show that $(x_n)_{n\in\mathbb{N}}$ converges to $\sqrt{\alpha}$.
2. For any $\alpha \in \mathbb{R}$ we define
$$\lfloor \alpha \rfloor = \max_{n\in\mathbb{Z}}\{n \mid n \leq \alpha\}$$
   and
$$\alpha \mod 1 = \alpha - \lfloor \alpha \rfloor.$$
   Let $\alpha \in \mathbb{R}$ be irrational.
   (a) Given $n \in \mathbb{N}$, show that $\{k\alpha \mod 1 \mid k \in \mathbb{N}\} \cap [0, \frac{1}{n}] \neq \emptyset$.
   (b) Prove that $\{n\alpha \mod 1 \mid n \in \mathbb{N}\}$ is dense in $[0, 1]$.