MATH 135 HOMEWORK 3

A. HENING

Do problems 13, 15, 16, 18 from Rudin Chapter 1, problems 1, 2, 3, 4 from Rudin Chapter 2 and the following problems.

1. a.) Let $A = \{f : \mathbb{Z} \to \{0,1\} |$ if $n \geq m$ then $f(n) \geq f(m)\}$ (the set of non-decreasing functions from $\mathbb{Z}$ to $\{0,1\}$). Prove A is countable.
   b.) Is this still true for $B = \{f : \mathbb{Q} \to \{0,1\} |$ if $p > q$ then $f(p) \geq f(q)\}$?
2. Let M be an uncountable subset of the real positive numbers, $M \subset \mathbb{R}^+$. Show that for every $r \in \mathbb{R}$ there is a finite number of different elements $a_1, a_2, \dots, a_n \in M$ such that
$$\sum_{k=1}^{n} a_k \geq r$$
   Hint: Look at the sets $M_N = \{a \in M | a \geq \frac{1}{N}\}$, $N \in \mathbb{N}$.