Tufts University
Department of Mathematics
Fall 2018

MA 126: Numerical Analysis

Homework 2 (v1.0) [1]

Assigned Friday 14 September 2018
Due Friday 21 September 2018 at 3 pm

1. Suppose that we have a function f with $n+1$ continuous derivatives in open interval I. Use mathematical induction to prove that an *exact* expression for the nth remainder term of the Taylor polynomial for f, centered at $a \in I$, is

$$R_n(x) = \frac{1}{n!} \int_a^x dt \; f^{(n+1)}(t)\,(x-t)^n$$

2. Show that the expression for the Taylor polynomial in Problem 1 implies the form of the Taylor Remainder Theorem that we studied in class, and in Homework 1. Hint: The integral form of the Mean Value Theorem may be helpful for this purpose.

3. In class, we noted that the polynomial

$$p(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n$$

can be recast as

$$p(x) = c_0 + x\,(c_1 + x\,(c_2 + x\,(\cdots + x)))\,.$$

This latter method of evaluating $p(x)$ is sometimes called *Horner's method*. Recall that it takes one clock cycle on a computer's CPU to do any one of (i) addition, (ii) multiplication, or (iii) multiplication followed by addition (a so-called *mult-add*). Assuming that all the c_j are nonzero, how many clock cycles are required for the evaluation of $p(x)$,

(a) in its standard form, supposing that it takes $k-1$ multiplications to compute x^k?

(b) in its standard form, supposing that you compute x^{2^j} for j sufficiently large, and multiply these together to obtain x^k, as described in class?

(c) using Horner's method?

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