MATH 135 HOMEWORK 2

A. HENING

Do problems 7, 8, 9, 10, 11, 12, 17 from Rudin Chapter 1 and the following two problems. Problem 7 f)g) is a BONUS problem. (you don't have to do it but if you do, you get extra credit).

(1) Let $z = \frac{1+\sqrt{3}i}{2}$.
   a) Determine $|z|, z^{-1}, \bar{z}, z^n$ for $n \in \mathbb{N}$.
   b) Let $T \subset \mathbb{C}$ be the triangle with vertices $a = 5, b = 6 + i, c = 7$. What geometric shape do $z^n a, z^n b, z^n c$ form for any $n \in \mathbb{N}$? Draw them into the complex plane together with T and z.
   c) Calculate $(1 + i)^n + (1 - i)^n$ for any $n \in \mathbb{Z}$. Why is the result always a real number?

(2) If F is a field we say map $\varphi : F \to F$ is a *field automorphism* if:
   (i) $\varphi(1) = 1$.
   (ii) $\varphi(a + b) = \varphi(a) + \varphi(b)$ for all $a, b \in F$.
   (iii) $\varphi(ab) = \varphi(a)\varphi(b)$ for all $a, b \in F$.
   (iv) φ is one to one and onto. (In fact (i) guarentees you φ is one to one)

   Let $\varphi : \mathbb{R} \to \mathbb{R}$ be a field automorphism.
   (a) Show that if $r \in \mathbb{Q}$ then $\varphi(r) = r$.
   (b) Show that if $x \in \mathbb{R}$ and $x > 0$ then $\varphi(x) > 0$.
   (c) Deduce using part (b) that if $a < b$ then $\varphi(a) < \varphi(b)$.
   (d) Use parts (a) and (c) to prove that for all $x \in \mathbb{R}$ $\varphi(x) = x$ (thus proving that φ is the identity and the only automorphism of $\mathbb{R}$ is the identity).
   (e) For comparison give an automorphism of $\mathbb{C}$ that is not the identity.