3D Reconstruction with Fast Dipole Sums

MS Thesis Presentation

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Introduction

3D Reconstruction

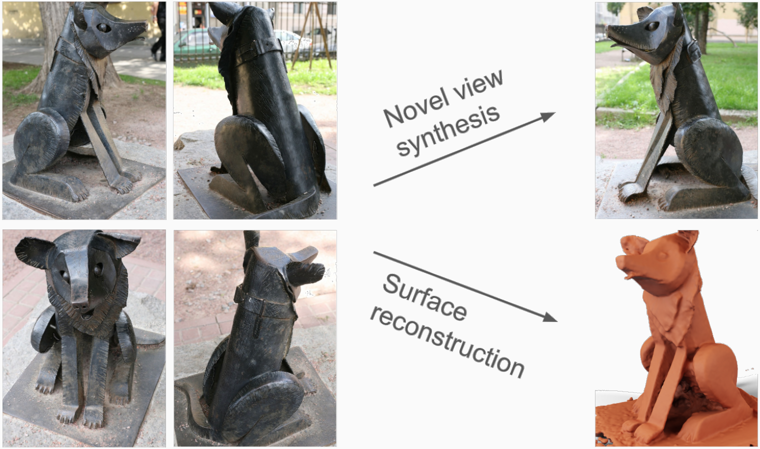
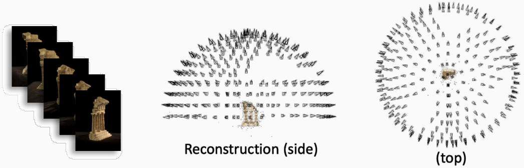


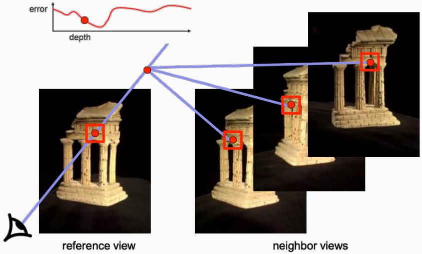
Figure 1: Dog scene from the Blended MVS dataset

Traditional Methods

- Structure from motion



- Multi-view stereo



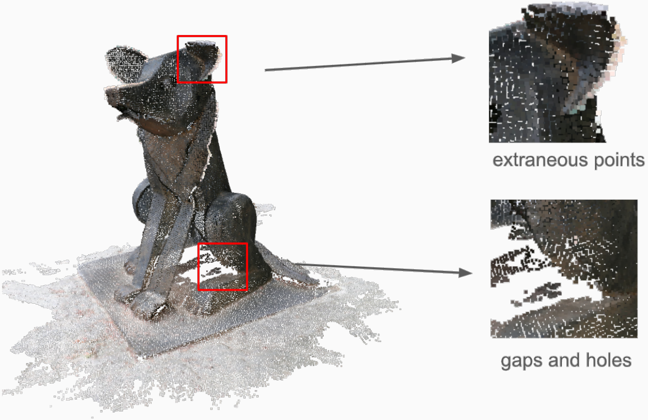
Kanazawa and Efros [2021]

Traditional Methods

What do the outputs look like?

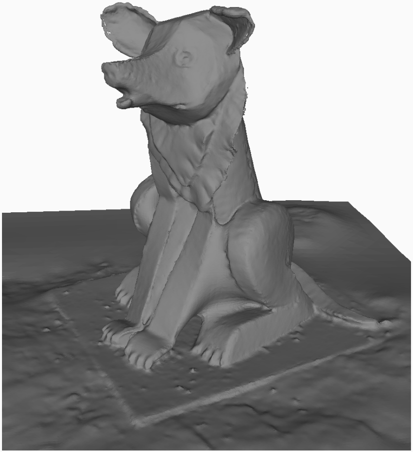
Traditional Methods

Looks pretty good, except...



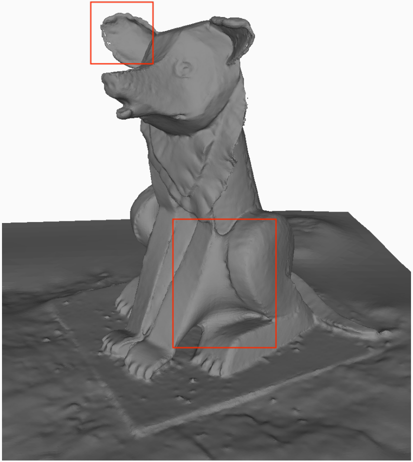
Traditional Methods

Typically, Poisson surface reconstruction [Kazhdan et al., 2006] is used to extract surfaces.



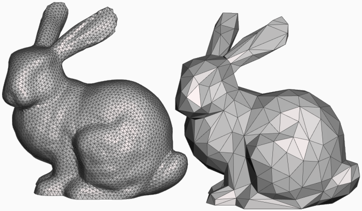
Traditional Methods

… and we see the same issues

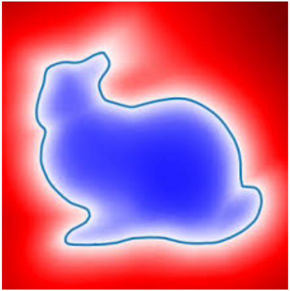


Volumetric Neural Rendering

Surface Rendering



(a) Triangle mesh



(b) Signed-distance function

Schmidt [2017], Park et al. [2019]

Volume Rendering

$$
\begin{aligned}
dL(\mathbf{x},\vec{\omega}) = &-\sigma_a(\mathbf{x})L(\mathbf{x},\vec{\omega})dz \quad -\sigma_s(\mathbf{x})L(\mathbf{x},\vec{\omega})dz \\
&+\sigma_a(\mathbf{x})L_e(\mathbf{x},\vec{\omega})dz + \sigma_s(\mathbf{x})L_s(\mathbf{x},\vec{\omega})dz
\end{aligned}
$$

Bitterli et al. [2018], Gkioulekas [2024]

Rendering Equation

Assuming an *emissive* volume with no scattering, we can derive the rendering equation as

$$L(\mathbf{x}, \vec{\omega}) = \int_0^z \mathrm{Tr}(\mathbf{x}, \mathbf{x}_t) \sigma(\mathbf{x}_t, \vec{\omega}) L_e(\mathbf{x}_t, \vec{\omega}) \, \mathrm{d}t,$$

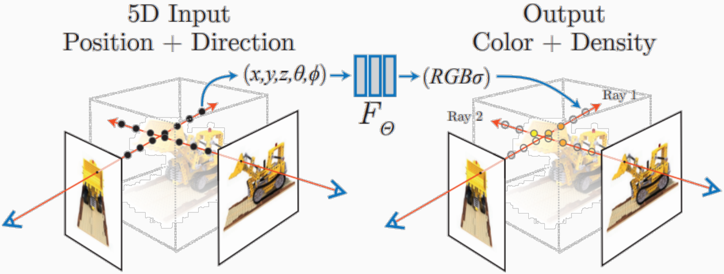
where $\mathrm{Tr}(\mathbf{x}, \mathbf{x}_t)$ is the transmittance

$$\mathrm{Tr}(\mathbf{x}, \mathbf{x}_t) = \exp\left(-\int_0^t \sigma(\mathbf{x}_s, \vec{\omega}) \, \mathrm{d}s\right),$$

and $\sigma(\mathbf{x}_s, \vec{\omega})$ is the spatially varying attenuation coefficient.

Neural Radiance Fields (NeRF)

Representing objects as volumes (instead of surfaces).



Color $L_e(\mathbf{x}, \vec{\omega})$ and density $\sigma(\mathbf{x}, \vec{\omega})$ modeled by a neural network.

[Mildenhall et al., 2020]

Neural Radiance Fields (NeRF)

NeRFs work particularly well for novel-view synthesis tasks
… but not so much for surface extraction.

[Mildenhall et al., 2020]

Neural Surface Representations

Can we explicitly model the geometry of the scene?

Yes, there are many works that already do this:

- Occupancy $\leftrightarrow$ density (NeuS [Wang et al., 2021])
- Signed-distance function $\leftrightarrow$ density (VolSDF [Yariv et al., 2021])

Follow-up works improve reconstruction quality and efficiency:

- Multi-resolution hash grids (Neuralangelo [Li et al., 2023], NeuS2 [Wang et al., 2023])
- Photo-consistency constraints (Neuralwarp [Darmon et al., 2022], Geo-NeuS [Fu et al., 2022])
- Sparse voxel representation (Voxurf [Wu et al., 2023])

Neural Surface Representations

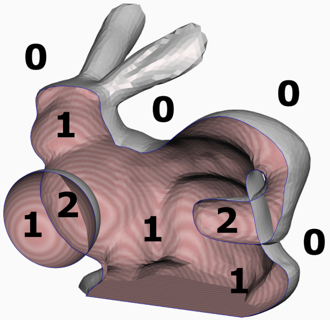
Limitations of existing methods:

- Trade-off between speed and reconstruction quality
- Cannot effectively leverage known information

Our solution: Directly build off of the output of traditional methods (i.e. dense point clouds)

Winding Number & Dipole Sums

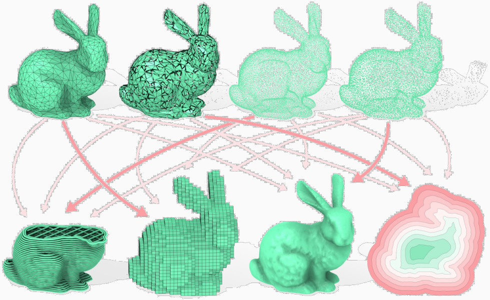
The Winding Number



$$w_S(\mathbf{q}) = \frac{1}{4\pi}\int_S \mathrm{d}\Omega(\mathbf{q}) = \int_S \frac{(\mathbf{p}-\mathbf{q})\cdot\hat{\mathbf{n}}}{4\pi\|\mathbf{p}-\mathbf{q}\|^3}\,\mathrm{d}\mathbf{p}$$

Schmidt [2018], Barill et al. [2018]

The Generalized Winding Number



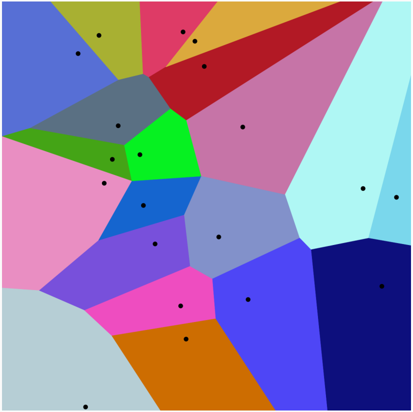
For point clouds,

$$w_S(\mathbf{q}) \approx \sum_{m=1}^{N} a_m \frac{(\mathbf{p}_m - \mathbf{q}) \cdot \hat{\mathbf{n}}_m}{4\pi \|\mathbf{p}_m - \mathbf{q}\|^3}$$

where a_m are “area-weights” (e.g. geodesic Voronoi area).

Barill et al. [2018]

The Generalized Winding Number



Aside: Laplace BVP

The winding numbers are also the solutions to the Laplace boundary value problem (BVP)

$$\begin{cases} \Delta w(\mathbf{p}) = 0, & \mathbf{p} \in \mathbb{R}^3 \backslash \partial\Omega \\ w^+(\mathbf{p}) - w^-(\mathbf{p}) = f(\mathbf{p}), & \mathbf{p} \in \partial\Omega \\ \frac{\partial w^+}{\partial \widehat{\mathbf{n}}}(\mathbf{p}) - \frac{\partial w^-}{\partial \widehat{\mathbf{n}}}(\mathbf{p}) = 0, & \mathbf{p} \in \partial\Omega, \end{cases}$$

for the case where $f(\mathbf{p}) \equiv 1$ on the boundary.

The Generalized Winding Number

Naively, we can directly extract the 0.5-isosurface of the winding number field. However, this often fails in practice...

What if we further generalize the winding number

… and allow $f(\mathbf{p}) \not\equiv 1$?

$$w_S(\mathbf{q}) = \int_S \frac{(\mathbf{p}-\mathbf{q}) \cdot \hat{\mathbf{n}}}{4\pi\|\mathbf{p}-\mathbf{q}\|^3} \cdot 1 \,\mathrm{d}\mathbf{p}$$

$$\Downarrow$$

$$u_S^f(\mathbf{q}) = \int_S \frac{(\mathbf{p}-\mathbf{q}) \cdot \hat{\mathbf{n}}}{4\pi\|\mathbf{p}-\mathbf{q}\|^3} \cdot f(\mathbf{p}) \,\mathrm{d}\mathbf{p}$$

The Dipole Sum

This is equivalent to having non-unit per-point attributes f_m:

$$w_{\mathrm{pc}}(\mathbf{q}) = \sum_{m=1}^{N} a_m \frac{(\mathbf{p}_m - \mathbf{q}) \cdot \hat{\mathbf{n}}_m}{4\pi \|\mathbf{p}_m - \mathbf{q}\|^3} \cdot 1$$

$$\Downarrow$$

$$u^f_{\mathrm{pc}}(\mathbf{q}) = \sum_{m=1}^{N} a_m \frac{(\mathbf{p}_m - \mathbf{q}) \cdot \hat{\mathbf{n}}_m}{4\pi \|\mathbf{p}_m - \mathbf{q}\|^3} \cdot f_m$$

In a differentiable rendering pipeline, we can make them learnable!

The Fast Dipole Sum

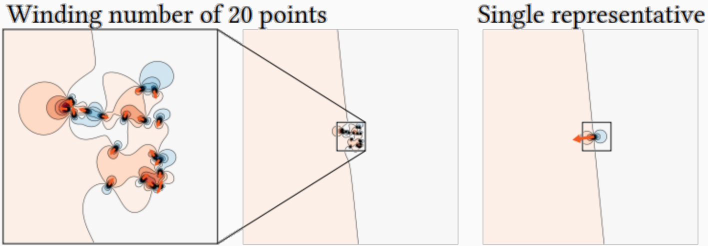
A dense point cloud can have tens of thousands of points

… and to render a single image, we need to potentially query the dipole sum at billions of distinct locations in the scene.

Obviously, we cannot compute the entire sum for every query…

The Barne-Hut Approximation

As proposed by Barill et al. [2018], we can consider a cluster of points far away from the query point as a single “dipole” with attributes computed using the Barnes-Hut approximation.



The Barne-Hut Approximation

We construct an octree from the point cloud, and assign each tree node t a centroid, a weighted normal, and a radius, computed from its leaves $\mathcal{L}(t)$:

$$\tilde{\mathbf{p}}_t \equiv \frac{\sum_{m \in \mathcal{L}(t)} a_m \mathbf{p}_m}{\sum_{m \in \mathcal{L}(t)} a_m}$$

$$\tilde{\mathbf{n}}_t^f \equiv \sum_{m \in \mathcal{L}(t)} a_m \hat{\mathbf{n}}_m \cdot f_m$$

$$\tilde{r}_t \equiv \max_{m \in \mathcal{L}(t)} \|\tilde{\mathbf{p}}_t - \mathbf{p}_m\|$$

To query the fast dipole sum, we recurse the octree with aggressive pruning in $O(\log N)$ time.

Fast Backpropogation

Naively backpropagating through the dipole sum takes $O(N \cdot M)$ time for M queries.

We propose a two-stage backprop scheme:

1. We detach the per-node parameters from the per-point parameters and cache gradients at nodes.
2. We use the chain rule to propogate gradients from the nodes to all leaves (points).

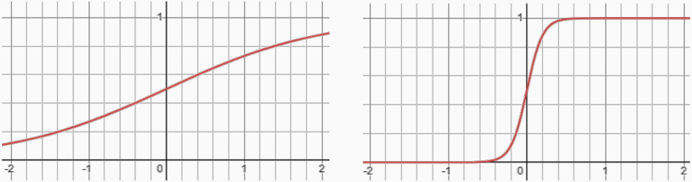
Each iteration of backprop only takes $O((M + N) \log N)$ time.

Rendering

To convert the dipole sum[†] into occupancy values between 0 and 1, we apply a logistic sigmoid

$$O(\mathbf{q}) = \frac{1}{1 + \exp\left(-s \cdot (u^f_{\mathrm{pc}}(\mathbf{q}) - 0.5)\right)}$$

with a learnable scale parameter s.



† We actually use a regularized version of the dipole sum to avoid singularities

Occupancy $\rightarrow$ Density

To convert the occupancy into density (attenuation coefficient) values, we follow Miller et al. [2023]:

$$\sigma(\mathbf{q}, \vec{\omega}) = \frac{|\nabla O(\mathbf{q}) \cdot \vec{\omega}|}{1 - O(\mathbf{q})}$$

Intuitively, we have

$\sigma(\mathbf{q}, \vec{\omega})$	High	Low
$O(\mathbf{q})$	High	Low
$\nabla O(\mathbf{q})$	Large magnitude	Small magnitude
$\vec{\omega}$	Normal incidence	Grazing angle

Neural Features

We can also use the dipole sum as an interpolation scheme to interpolate per-point neural features $\mathbf{l}_m \in \mathbb{R}^d$:

$$u_{\mathrm{pc}}^{\mathbf{l}}(\mathbf{q}) = \sum_{m=1}^{N} a_m \frac{(\mathbf{p}_m - \mathbf{q}) \cdot \hat{\mathbf{n}}_m}{4\pi \|\mathbf{p}_m - \mathbf{q}\|^3} \cdot \mathbf{l}_m$$

Unlike for geometry, we do not want sharp discontinuities in the interpolated features, so in practice, we instead use

$$u_{\mathrm{pc}}^{\mathbf{l}}(\mathbf{q}) = \sum_{m=1}^{N} a_m \frac{1}{4\pi \|\mathbf{p}_m - \mathbf{q}\|^2} \cdot \mathbf{l}_m$$

Neural Features

We predict colors using a simple neural network

$$L_e(\mathbf{q}, \vec{\omega}) \approx \mathrm{NN}(u^{\mathbf{l}}_{\mathrm{pc}}(\mathbf{q}), \mathrm{Enc}(\vec{\omega}))$$

with a directional encoding function that encodes the viewing direction using spherical harmonics, following Verbin et al. [2022].

Results

Setup

We implement our method in PyTorch based on the NeuS codebase [Wang et al., 2021]. Dipole sum and octree operations are implemented in C++ and CUDA with PyTorch bindings.

We evaluate our method against Neuralangelo [Li et al., 2023] on the DTU dataset [Jensen et al., 2014] and BlendedMVS dataset [Yao et al., 2020] on a single NVIDIA RTX4090 GPU. Each scene takes around 3-4 hours to fully train.

Results: DTU

We quantitatively evaluate our method on the DTU dataset using the Chamfer distance metric (↓).

Scan	Ours	Neuralangelo
24	0.46	**0.37**
37	**0.65**	0.72
40	**0.33**	0.35
55	**0.33**	0.35
63	0.95	**0.87**
65	0.78	**0.54**
69	**0.53**	**0.53**
83	**1.23**	1.29
97	**0.84**	0.97
105	**0.70**	0.73
106	**0.46**	0.47
110	**0.55**	0.74
114	0.33	**0.32**
118	**0.37**	0.41
122	**0.36**	0.43
Mean†	**0.50**	0.52

† We exclude scans 63, 83, 105 from the mean calculation due to inaccurate ground truths.

Results: BlendedMVS

Qualitative equal-time comparisons on the BlendedMVS dataset:

Results: BlendedMVS

Neuralangelo completely fails when there are few views available:

Visualizations: Occupancy & Color

We visualize the occupancy and color on slices of the scene:

Conclusion

Summary

We propose a novel method for 3D reconstruction that combines the efficiency of point clouds (via the Barnes-Hut approximation) with the expressiveness of neural rendering (via learnable boundary conditions and neural features).

Our method achieves results comparable to the state-of-the-art in less than a quarter of the training time.

Our representation is extremely compact: the entire model consists of an oriented point cloud with a single additional scalar attribute at each point (< 50 MB).

Limitations

Our reconstruction quality is highly dependent on the quality of the input point cloud, as we do not optimize point locations.

For larger scale scenes, estimating a dense point cloud with Colmap can be a bottleneck for performance.

Our method inherits certain limitations from Colmap, such as the difficulty in handling textureless surfaces and highly specular surfaces, where points are noisy or completely missing.

Questions?

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Thank you!