Detection of Dynamicity of Covariance Matrix

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Introduction

- It is common for researchers to use high dimensional matrices to store large collections of data and extract information from the covariance matrices.
- Researchers from home and abroad have studied a lot and enjoyed several mature results on the estimation of static matrices. For example, see Bickel(2008), Cai(2011), etc.
- However, in some real cases, covariance may vary with other variables.
- Here we consider the case that covariance matrices vary with time U.
- It will be of great value if we can detect the dynamic property of covariance matrices.

Great Value of Dynamic Covariance Model

- Schizophrenia and bipolar disorder are clinically hard to diffentiate, for they both often present with psychotic symptoms. Research shows that some bipolar disorder patients can go years misdiagnosed as much as 45% of the time (Meyer, 2009).
- The consequence of miscategorization is costly both economically and in terms of human suffering (DiLuca, 2014)
- Data clearly show that incorporation of dynamics may provide a more sensitive and specific marker of disease than static connectivity(Calhoun, 2014).

Data Description

- Attention Deficit Hyperactivity Disorder (ADHD) affects at least 5-10% of school-age children and is associated with substantial lifelong impairment.
- The data contain 776 resting-state fMRI and anatomical datasets aggregated across 8 independent imaging sites, 491 of which were obtained from typically developing individuals and 285 in children and adolescents with ADHD (ages: 7-21 years old).
- ADHD-200 Phenotypic Key:
  0: Typically Developing Children
  1: ADHD-Combined
  2: ADHD-Hyperactive/Impulsive
  3: ADHD-Inattentive

Detection of Dynamic Property

- The data of each individual consists 172 scanned data of 190 different regions of interest (ROI).
- We split the 172 observations into two sets, each consisting 86 observations. The heatmaps of the first 20 ROIs for individual 0010002 (DX=3) are shown below

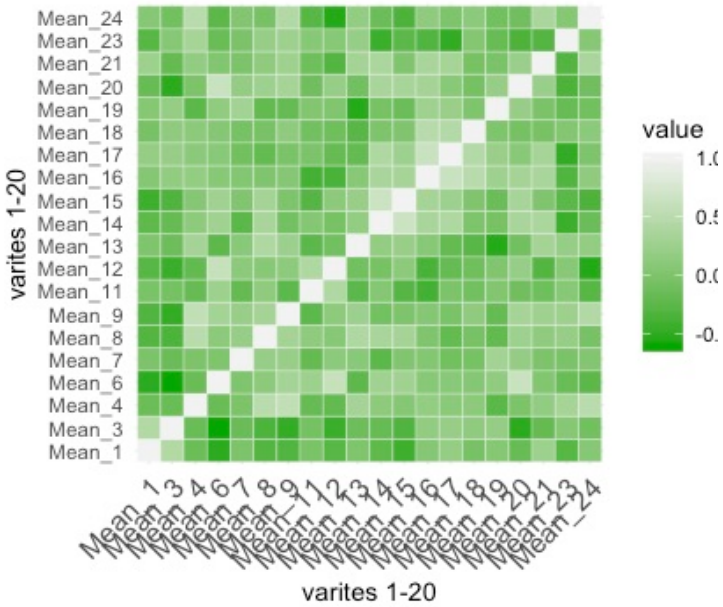


Figure: 1-86 scans

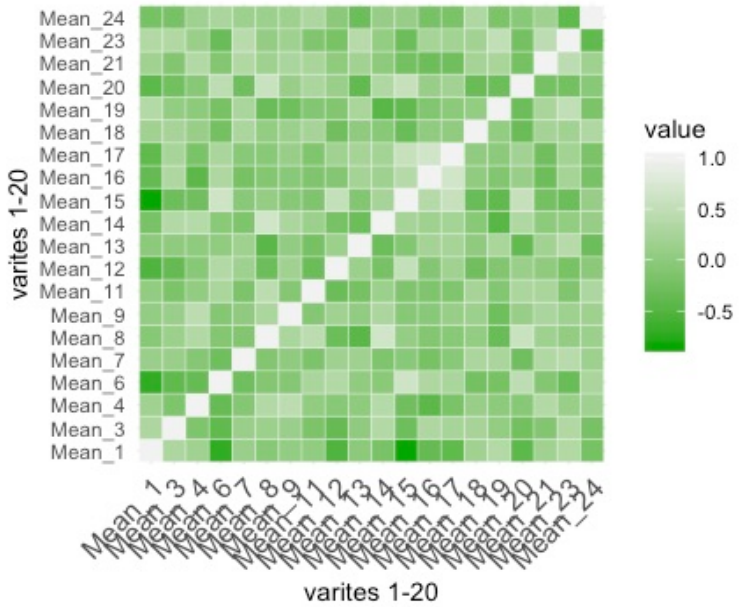


Figure: 87-172 scans

Detection of Dynamic Property

- The heatmaps of the first 20 ROIs for individual 1000804 (DX=0):

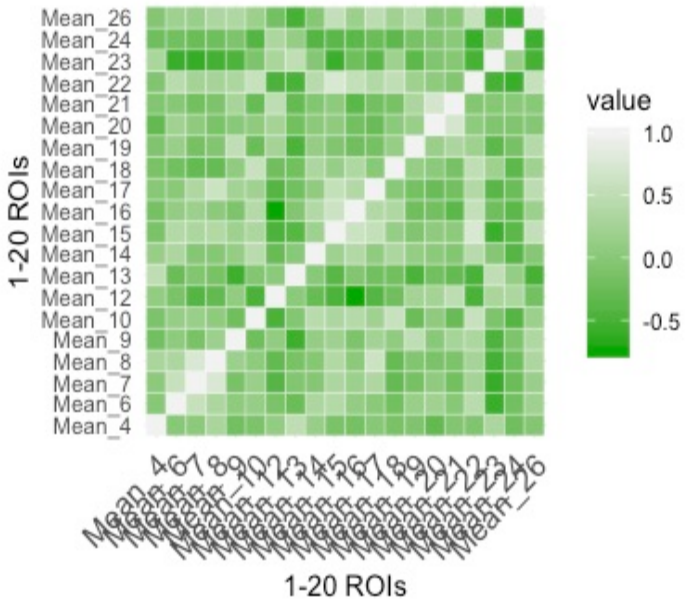


Figure: 1-86 scans

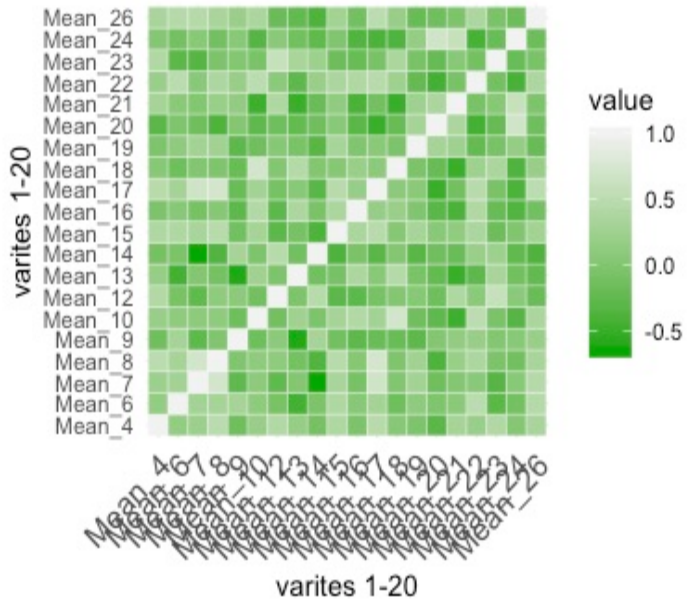


Figure: 87-172 scans

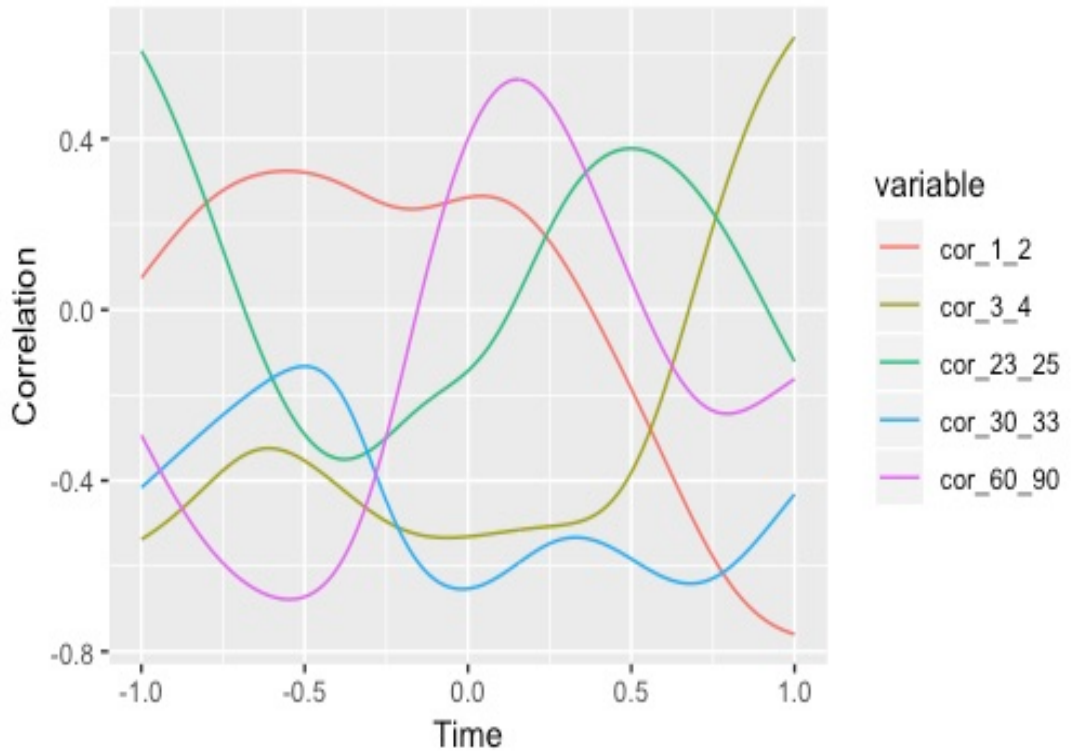
Dynamic Model for Covaricance Matrices

- Chen(2016) has proposed an effective dynamic model to estimation the dynamic covariance matrix.
- Let $Y = (Y_1, \cdots, Y_p)^T$ be a p-dimensional random vector and U denote time. Suppose that $\{Y_i, U_i\}$ with $Y_i = (Y_{i1}, \cdots, Y_{ip})^T$ is a random sample from the population $\{Y, U\}$, for $i = 1, \cdots, n$.
- The empirical sample conditional covariance matrix based on kernel smoothing is

$$\begin{aligned}\hat{\Sigma}(u) := & \left\{\sum_{i=1}^{n} K_h(U_i - u) Y_i Y_i^T\right\} \cdot \left\{\sum_{i=1}^{n} K_h(U_i - u)\right\}^{-1} \\ & - \left\{\sum_{i=1}^{n} K_h(U_i - u) Y_i\right\} \left\{\sum_{i=1}^{n} K_h(U_i - u) Y_i^T\right\} \\ & \times \left\{\sum_{i=1}^{n} K_h(U_i - u)\right\}^{-2}\end{aligned}$$

Dynamic Model for Covaricance Matrices

- Under the model from Chen(2016), we plot some correlations of individual 1023964 (DX=3):



- It is clear that the correlation between each pair changes a lot with the time going.

Future Expectation

- It is expected that voxels of some or all ROIs may vary with DX and have similarity among people with the same sympotom.
- The scatter plots of absolute sums of individuals 1000804 (DX=0) and 0010005 (DX=2) are shown below

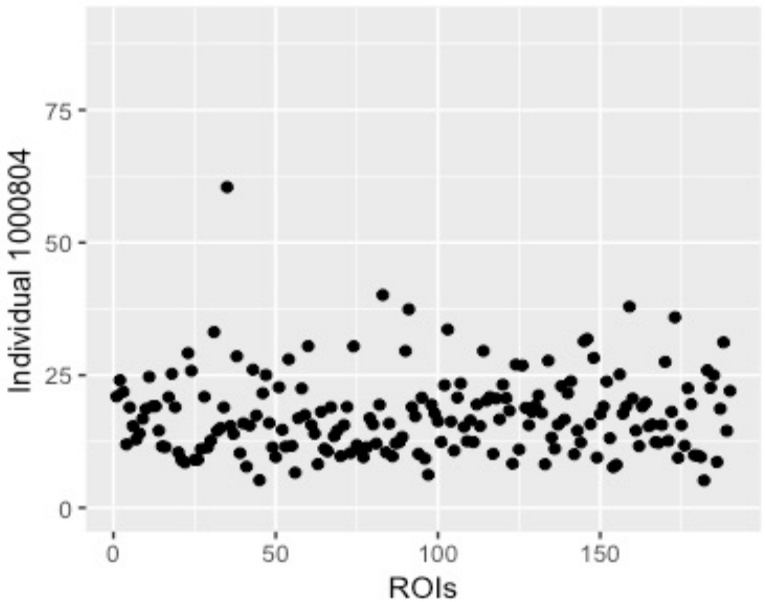


Figure: 1000804 (DX=0)

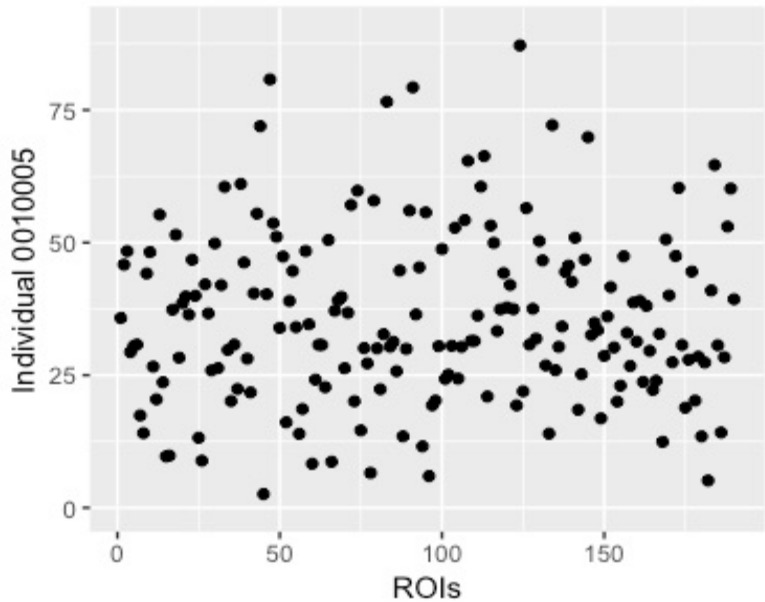
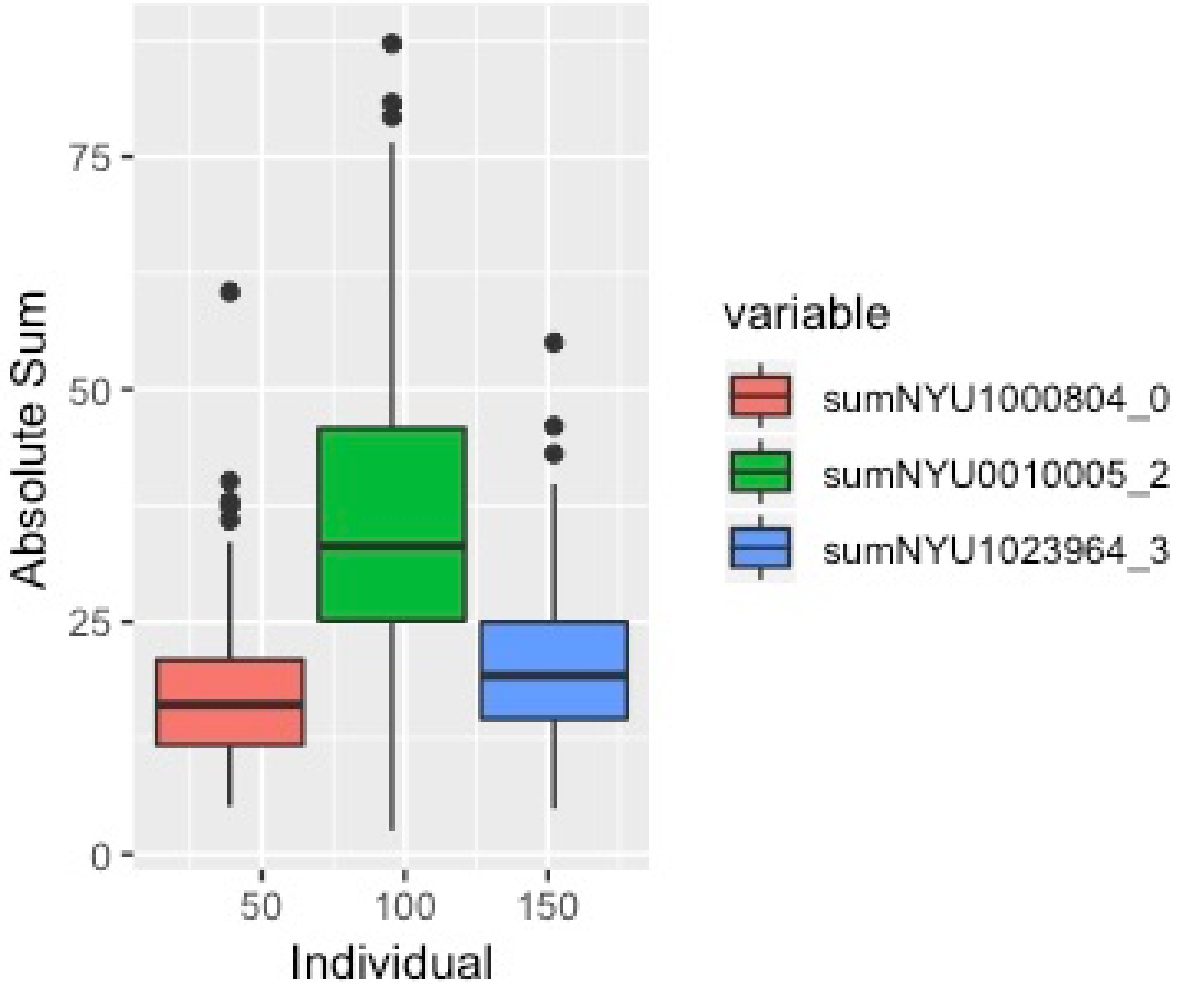


Figure: 0010005 (DX=2)

Future Expectation

- Boxplots for absolute sums of individuals 1000804 (DX=0), 0010005 (DX=2) and 1023964 (DX=3) are shown below



Thank you!

Questions?