COURSE : Mathematical Structures

COURSE CODE : 21MAT41A

MODULE – 4: Relations

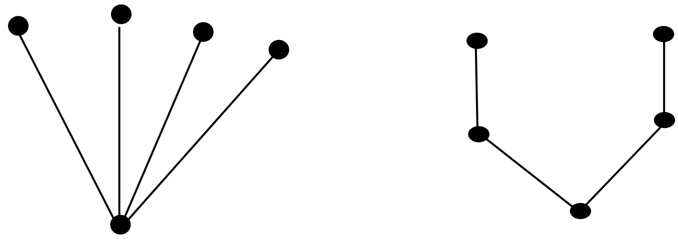
Question Bank

Q.No	Questions
1.	a) Let $A = \{1,2\}$ and $B = \{p, q, r, s,\}$ and let the relation R from A to B be defined by $R = \{(1, q), (1, r), (2, p), (2, s)\}$. Write down the matrix of R. b) Consider the relation R from X to Y , X = {1, 2, 3}, Y = {8, 9} and R = {(1, 8) (2, 8) (1, 9) (3, 9)} Find the complement relation of R.
2.	Let $A = \{1,2,3,4\}$ and let R be the relation on A defined by xRy if and only if "x divides y", written x/y. a) Write down R as a set of ordered pairs b) Draw the digraph of R c) Determine the in-degrees and out-degrees of the vertices in the diagraph
3.	a) Let $A = \{1,2,3,4,6\}$ and R be a relation on A defined by aRb if and only if a is a multiple of b. Represent the relation R as a matrix and draw its diagraph. b) Show that the identity relation on a set A is an equivalence relation
4.	a)Determine the relation R from a set B as described by the following matrix $$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$ b)Let A = {u, v, x, y, z} and R be a relation on A whose matrix is as given below. Determine R and the digraph of the matrix. $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$.
5.	Let $A = \{1,2,3,4\}, B = \{w, x, y, z\}$ and $C = \{5,6,7\}$. Also let R_1 be a relation from A to B defined by $R_1 = \{(1, x), (2, x), (3, y), (3, z)\}$ and R_2 and R_3 be relations from B to C defined by $R_2 = \{(w, 5), (x, 6)\}, R_3 = \{(w, 5), (w, 6)\}$. Find $R_1 o R_2$ and $R_1 o R_3$
6.	a) For the relation $R_1$and R_2, where $R_1 = \{(1, x)(2, x), (3, y)(3, z)\}$, $R_2 = \{(w, 5), (x, 6)\}$, Find $M(R_1), M(R_2)$ and $M(R_1 o R_2)$. Also verify that $M(R_1 o R_2) = M(R_1).M(R_2)$ b) Let A = { 1, 2, 3, 4} with R = {(1, 1), (1, 3), (3, 2), (3, 4), (4, 2)} and S = {(2, 1), (3, 3), (3, 4), (4, 1)} as relations on A. Find $R°S, S°R, R°R, S°S$.
7.	a) If $A = \{1,2,3,4\}$ and R, S are relations on A defined by $R = \{(1,2), (1,3), (2,4), (4,4)\}$, $S = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4)\}$. Find RoS, SoR, R^2, S^2, write down their matrices.

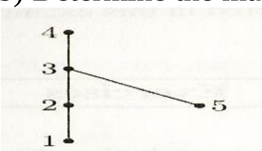
	b) Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by $_{a}R_{b}$ iff a is a multiple of b. Represent the relation R as a matrix and draw its digraph.
8.	a) Let $A = \{a, b, c\}$ and R and S be relations on A whose matrices are as given below. $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$; $M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ Find the composite relations RoS, SoR, RoR, SoS and their matrices. b) On the set of all integers Z defined by the relation R by aRb iff ab > 0, Show that R is an equivalence relation.
9.	(a). If $A = \{1,2,3,4\}$ and R is a relation on A defined by $R = \{(1,2), (1,3), (2,4), (3,2), (3,3), (3,4)\}$, find R^2 and R^3. Write down their diagraphs. (b). Let $R = \{(1,2), (3,4), (2,2)\}$ and $S = \{(4,2), (2,5), (3,1), (1,3)\}$ be relations on the set $A = \{1,2,3,4,5\}$. Find $Ro(RoS), Ro(SoR), So(RoS), So(SoR)$.
10.	Let R be a relation on a set A. Prove that i) R is reflexive iff $\bar{R}$ is irreflexive ii) If R is reflexive, so is R^C iii) If R is symmetric, so are R^C& $\bar{R}$ iv) If R is transitive, so is R^C
11.	Let R and S be relations on a set A. Prove that i) If R and S are reflexive, so are $R \cap S$ and $R \cup S$ ii) If R and S are symmetric, so are $R \cap S$ and $R \cup S$ iii) If R and S are antisymmetric, so is $R \cap S$ iv) If R and S are transitive, so is $R \cap S$
12.	Let $A = \{1,2,3,4\}$ and $R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,4)\ (4,3)\ (3,3)\ (4,4)\}$ be a relation on A. Verify that R is an equivalence relation.
13.	a) If $A = A_1 \cup A_2 \cup A_3$, where $A_1 = \{1,2\}, A_2 = \{2,3,4\}\ and\ A_3 = \{5\}$, define the relation R on by xRy iff x and y are in the same set A_i, $i = 1,2,3$. Is R an equivalence relation? b) Let A = {u, v, x, y, z} and R be a relation on A whose matrix is as below. Determine R and also draw the associated digraph. $M(R) = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$
14.	a) For a fixed integer $n > 1$, prove that the relation "Congruent modulo n" is an equivalence relation on the set of all integers, z. b) For the relations R_1 and R_2 defined on the sets A={1, 2, 3, 4}, B={w,x,y,z} and C={5, 6, 7} as R_1 = {(1,x), (2,x), (3,y), (3,z)} and R_2 = {(w,5), (x,6)} verify that $M(R_1 \circ R_2) = M(R_1)M(R_2)$
15.	a) Let $A = \{1,2,3,4,5\}$. Define a relation R on AXA by $((x_1, y_1)\ R\ (x_2, y_2)$ if and only if

$x_1 + y_1 = x_2 + y_2$. Verify that R is an equivalence relation on AXA.

b) Find the matrix of the partial order relation whose Hasse diagram is given by :



16. a) If R is a relation on the set $A = \{1,2,3,4\}$ defined by xRx if x/y, prove that (A, R) is a poset. Draw a Hasse diagram.
b) Determine the matrix of the partial order whose Hasse diagram is given below.



17. a) Let $A = \{1,2,3,4,6,12\}$. On A, define the relation R by aRb iff a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation
b) Draw the Hasse diagram of the relation R on A = {1,2,3,4,5} whose matrix is

$$M(R) = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

18. a) Draw the Hasse diagram representing the positive divisors of 36.

b) If A = {1, 2, 3, 4} and R and S are relations on a defined by R = {(1,2), (1,3) (2,4), (4,4)}, S = {(1,1), (1,2), (1,3), (1,4), (2,3), (2,4)}. Verify the following

$$(i) M(R \circ S) = M(R)M(S) \quad (ii) M(S \circ R) = M(S)M(R)$$
$$(iii) M(R^2) = [M(R)]^2 \qquad (iv) M(S^2) = [M(S)]^2$$

19. Find x and y in each of the following
i) $(2x, x + y) = (6, 1)$
ii) $(y - 2, 2x + 1) = (x - 1, y + 2)$

20. a) Find x and y in each of the following
i) $(2x - 3, 3y + 1) = (5,7)$
ii) $(x + 2,4) = (5,2x + y)$
b) Find x and y in each of the following
iii) $(x, y) = (x^2, y^2)$
iv) $(x, y) = (y^2, x^2)$