



# DAYANANDA SAGAR COLLEGE OF ENGINEERING

(An Autonomous Institute Affiliated to VTU, Belagavi)

Shavige Malleshwara Hills, Kumaraswamy Layout, Bengaluru-560078

## DEPARTMENT OF MATHEMATICS

### MATHEMATICAL STRUCTURES

COURSE CODE : 21MAT41A

#### Module-5: Graph Theory and Its Applications

#### Question Bank

Q.No	Questions
1.	<p>a) Define with an example</p> <ol style="list-style-type: none"> <li>Graph</li> <li>Multi graph</li> <li>Simple Graph</li> </ol> <p>b) Define the following terms with a suitable example:</p> <ol style="list-style-type: none"> <li>Directed graph.</li> <li>Undirected graph.</li> </ol>
2.	<p>a) Define with an example</p> <ol style="list-style-type: none"> <li>Degree of vertex</li> <li>Isolated vertex</li> </ol> <p>b) Define with an example</p> <ol style="list-style-type: none"> <li>Bipartite graph</li> <li>Complete bipartite Graph</li> </ol>
3.	<p>a) Define with an Example</p> <ol style="list-style-type: none"> <li>Pseudographs</li> <li>Cycle Graph</li> <li>Wheel Graph</li> </ol> <p>b) Verify <math>\sum_{i=1}^n \deg^+(v_i) = \sum_{i=1}^n \deg^-(v_i) =  E  = e</math> in the following graph.</p> <p> </p>
4.	<p>a) Draw the Graphs</p> <ol style="list-style-type: none"> <li><math>K_n</math> for <math>1 \leq n \leq 4</math></li> <li>a) <math>C_3</math> b) <math>C_4</math> c) <math>C_5</math> d) <math>C_6</math> e) <math>C_8</math></li> </ol> <p>b) Is <math>C_3</math> and <math>C_6</math> bipartite? Explain</p>



# DAYANANDA SAGAR COLLEGE OF ENGINEERING

(An Autonomous Institute Affiliated to VTU, Belagavi)

Shavige Malleshwara Hills, Kumaraswamy Layout, Bengaluru-560078

## DEPARTMENT OF MATHEMATICS

5.	<p>a) Draw the complete bipartite graphs <math>K_{2,3}</math>, <math>K_{3,3}</math>, <math>K_{3,5}</math> and <math>K_{2,6}</math></p> <p>b) Draw the Graphs of a) <math>W_3</math> b) <math>W_4</math> c) <math>W_5</math> d) <math>W_6</math> e) <math>W_7</math></p>
6.	<p>a) Define Complement Graph and if the simple graph <math>G</math> has <math>v</math> vertices and edges, how many edges does <math>\bar{G}</math> have?</p> <p>b) State Handshaking property. Show that every simple graph has two vertices of the same degree</p>
7.	<p>a) Define Adjacency matrix and incidence matrix</p> <p>b) Write the Adjacency Matrix of <math>C_4</math> and <math>W_4</math></p>
8.	<p>a) Draw a graph of the given adjacency matrix</p> <p>i) <math>\begin{bmatrix} 0 &amp; 1 &amp; 0 \\ 1 &amp; 0 &amp; 1 \\ 0 &amp; 1 &amp; 0 \end{bmatrix}</math> ii) <math>\begin{bmatrix} 1 &amp; 1 &amp; 1 &amp; 0 \\ 0 &amp; 0 &amp; 1 &amp; 0 \\ 1 &amp; 0 &amp; 1 &amp; 0 \\ 1 &amp; 1 &amp; 1 &amp; 0 \end{bmatrix}</math></p> <p>b) Find the adjacency matrix of the given directed multigraph</p>
9.	<p>a) Draw Petersen graph.</p> <p>b) Is there a simple graph with 1,1,3,3,3,4,6,7 as the degree of its vertices?</p>

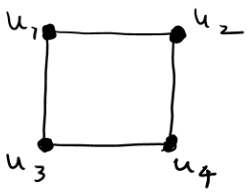
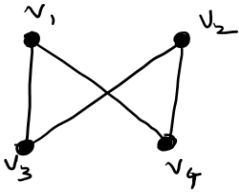
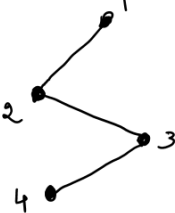
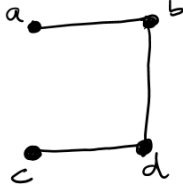
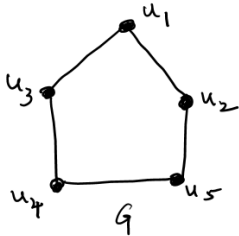
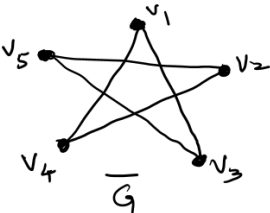
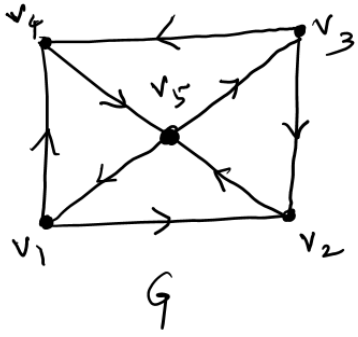
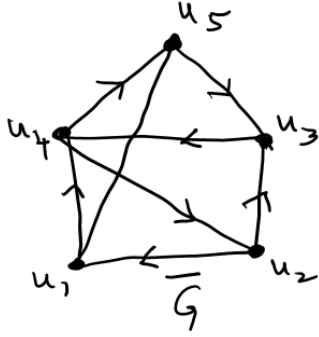


# DAYANANDA SAGAR COLLEGE OF ENGINEERING

(An Autonomous Institute Affiliated to VTU, Belagavi)

Shavige Malleshwara Hills, Kumaraswamy Layout, Bengaluru-560078

## DEPARTMENT OF MATHEMATICS

10.	<p>a) Explain a Regular graph with example</p> <p>b) How many vertices does a Regular graph of degree 4 with 10 edges have?</p>
11.	<p>a) Define isomorphism of graphs. Show that the graphs <math>G=(V,E)</math> and <math>H=(W,F)</math> shown in the figures are isomorphic.</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p>b) Show that the two graphs shown in the figure are isomorphic.</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>
12.	<p>a) Show that the graphs <math>G</math> and <math>\bar{G}</math> shown in the figure are isomorphic.</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p>b) Show that the Digraphs are isomorphic.</p> <div style="display: flex; justify-content: space-around; align-items: center;">   </div>

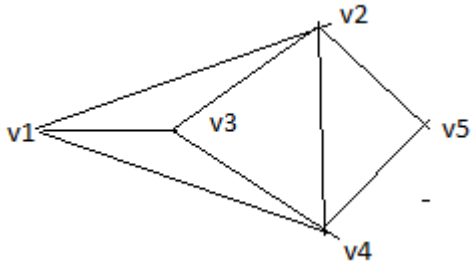


## DAYANANDA SAGAR COLLEGE OF ENGINEERING

(An Autonomous Institute Affiliated to VTU, Belagavi)

Shavige Malleshwara Hills, Kumaraswamy Layout, Bengaluru-560078

### DEPARTMENT OF MATHEMATICS

13.	<p>a) Define with an example each</p> <ul style="list-style-type: none"> <li>(i) Euler Circuit</li> <li>(ii) Euler path</li> <li>(iii) Euler Trail</li> </ul> <p>b) Define with an example each</p> <ul style="list-style-type: none"> <li>(i) A Hamiltonian path</li> <li>(ii) A Hamiltonian Circuit</li> </ul>
14.	<p>a) Define with an example : (a) Subgraph of a graph (b) spanning sub graph</p> <p>b) Show that the complement of a bipartite graph need not be a bipartite graph.</p>
15.	<p>Which of the following graphs are Eulerian?</p> <ul style="list-style-type: none"> <li>i) The complete graph <math>K_5</math></li> <li>ii) The complete bipartite graph <math>K_{2,3}</math></li> <li>iii) The graph of the Octahedron</li> <li>iv) The Petersen graph</li> </ul>
16.	<p>Prove that a simple graph with <math>n</math> vertices and <math>k</math> components can have at most <math>(n-k)(n-k+1)/2</math> edges.</p>
17.	<p>a) i) Give an example of a graph which has a Hamiltonian circuit but not an Euler circuit.  ii) Give an example of a graph which has an Euler circuit but not a Hamiltonian circuit.</p> <p>b) Show that <math>K_n</math> has a Hamilton circuit whenever <math>n \geq 3</math></p>
18.	<p>a) Define Graph coloring of a graph with an example.</p> <p>b) Define chromatic number of a graph with an example.</p>
19.	<p>a) Find the chromatic number of the graph shown below</p> 



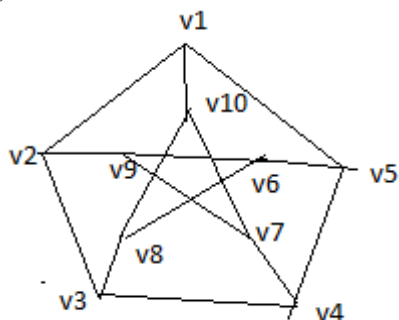
## DAYANANDA SAGAR COLLEGE OF ENGINEERING

(An Autonomous Institute Affiliated to VTU, Belagavi)

Shavige Malleshwara Hills, Kumaraswamy Layout, Bengaluru-560078

### DEPARTMENT OF MATHEMATICS

b) Find the chromatic number of the graph shown below



20.

- Prove that a graph of order ( $n \geq 2$ ) consisting of a single cycle is 2- chromatic if  $n$  is even, and 3-chromatic if  $n$  is odd.
- Prove that a graph  $G$  is 2-chromatic if and only if it is a non-null bipartite graph.