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Fundamentals of Logic:

1. (b). (i) $p \wedge q$

(ii) $\sim p \wedge q$

(iii) $\sim q \wedge p$

(iv) $\sim p \wedge \sim q$

(v) $p \rightarrow q$

2. (a) (a). $\sim p \wedge q$

(b) $p \wedge q$

(c) $p \vee q$

(d) $\sim (p \vee q)$

(e) $\sim p \wedge \sim q$

(b). (i) $p \vee q \vee r$

(ii) $(p \rightarrow \sim r) \wedge (q \rightarrow \sim r)$

(iii) $(p \wedge q) \vee (\sim q \wedge r)$

3. (a) 1. Jack went up the Hill $\rightarrow p$

Jill went up the Hill $\rightarrow q$

$\therefore p \wedge q$

2. Jerry takes calculus $\rightarrow p$

John takes algebra $\rightarrow q$

Alex takes English $\rightarrow r$

$(p \vee q) \rightarrow r$

(b). $\sim (p \wedge (q \vee r))$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$\sim(p \wedge (q \vee r))$
0	0	0	0	0	1
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	1	0	1
1	0	0	0	0	1
1	0	1	1	1	0
1	1	0	1	1	0
1	1	1	1	1	0

4.

(a). $\sim(p \vee (q \wedge r)) \longleftrightarrow ((p \vee q) \wedge (p \vee r))$

p	q	r	$q \wedge r$ (I)	$p \vee \text{I}$ (II)	$\sim\text{II}$	$p \vee q$	$p \vee r$	$((p \vee q) \wedge (p \vee r))$ (III)	$\sim\text{II} \leftrightarrow \text{III}$
0	0	0	0	0	1	0	0	0	0
0	0	1	0	0	1	0	1	0	0
0	1	0	0	0	1	1	0	0	0
0	1	1	1	1	0	1	0	0	0
1	0	0	0	1	0	1	1	1	0
1	0	1	0	1	0	1	1	1	0
1	1	0	0	1	0	1	1	1	0
1	1	1	1	1	0	1	1	1	0

(b).

p	q	r	$q \leftrightarrow r$ (I)	$r \leftrightarrow p$ (II)	$\text{I} \vee \text{II}$	$p \wedge (\text{I} \vee \text{II})$
0	0	0	1	1	1	0
0	0	1	0	0	0	0
0	1	0	0	1	1	0
0	1	1	1	0	1	0
1	0	0	1	0	1	1
1	0	1	0	1	1	1
1	1	0	0	0	0	0
1	1	1	1	1	1	1

5.

(b) Team India wins whenever Dhoni is Captain.

p: Team India wins $\qquad$ q: Dhoni is Captain

Converse: Whenever Dhoni is Captain, Team India wins.

$q \rightarrow p$

Contrapositive: Whenever Dhoni is not the captain, Team India does not win.

$\sim q \rightarrow \sim p$.

Inverse: Team India does not win whenever Dhoni is Captain.

$\sim p \rightarrow \sim q$

6.(a) Home Team wins whenever it rains

$\hookrightarrow p$ $\qquad$ $\hookrightarrow q$

(i) Converse: Whenever it rains, the Home Team wins.

Contrapositive: Whenever it does not rain, the Home Team does not win.

Inverse: The Home Team does not win whenever it rains.

7.(a) $((p \vee q) \wedge (\sim(p \vee r)) \rightarrow (q \vee r)$

p	q	r	I $p \vee q$	$(p \vee r)$	II $\sim(p \vee r)$	III I$\wedge$II	$q \vee r$	III $\rightarrow (q \vee r)$
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	1	0	1	1	1	1
0	1	1	1	1	0	0	1	1
1	0	0	1	1	0	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	0	0	1	1
1	1	1	1	1	0	0	1	1

$\therefore$ Tautology

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(b) $\sim(q \to r) \wedge r \wedge (p \to q)$

p	q	r	$q \to r$	$\overset{\mathrm{I}}{\sim(q \to r)}$	$\overset{\mathrm{II}}{\mathrm{I} \wedge r}$	$\overset{\mathrm{III}}{p \to q}$	$\mathrm{II} \wedge \boxed{\mathrm{III}}$
0	0	0	1	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	1	0	1	0
0	1	1	1	0	0	1	0
1	0	0	1	0	0	0	0
1	0	1	1	0	0	0	0
1	1	0	0	1	0	1	0
1	1	1	1	0	0	1	0

∴ contradiction //

8(a). $\sim(p\lor(q\land r))\leftrightarrow((p\lor q)\land(p\to r))$

				I		II	III	IV	
p	q	r	$q\land r$	$p\lor(q\land r)$	$\sim$I	$p\lor q$	$p\to r$	II$\land$III	$\sim$I$\leftrightarrow$IV
0	0	0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1	0	0
0	1	0	0	0	1	1	1	1	1
0	1	1	1	1	0	1	1	1	0
1	0	0	0	1	0	1	0	0	1
1	0	1	0	1	0	1	1	1	0
1	1	0	0	1	0	1	0	0	1
1	1	1	1	1	0	1	1	1	0

$\therefore$ Contingency //

8.

(b). $(p\to(q\to r))\land((\sim r\lor p)\land q)$

				I	II	III	
p	q	r	$q\to r$	$p\to(q\to r)$	$\sim r\lor p$	II$\land q$	I$\land$III
0	0	0	1	1	1	0	0
0	0	1	1	1	0	0	0
0	1	0	0	1	1	1	1
0	1	1	1	1	0	0	0
1	0	0	1	1	1	0	0
1	0	1	1	1	1	0	0
1	1	0	0	0	1	1	0
1	1	1	1	1	1	1	1

$\therefore$ Contingency //

9.

(a). $((p \to q) \wedge (q \to r)) \to (p \to r)$

p	q	r	I $p \to q$	II $q \to r$	III $\mathrm{I} \wedge \mathrm{II}$	IV $p \to r$	$\mathrm{III} \to \mathrm{IV}$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

9.(b). $((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r$

p	q	r	I $p \vee q$	II $p \to r$	$\mathrm{I} \wedge \mathrm{II}$	III $q \to r$	IV $\mathrm{I} \wedge \mathrm{II} \wedge \mathrm{III}$	$\mathrm{IV} \to r$
0	0	0	0	1	0	1	0	1
0	0	1	0	1	0	1	0	1
0	1	0	1	1	1	0	0	1
0	1	1	1	1	1	1	1	1
1	0	0	1	0	0	10	0	1
1	0	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

10.

(b). $p \to (q \to r) \iff (p \wedge q) \to r$

p	q	r	$q \to r$	I $p \to (q \to r)$	$p \wedge q$	II $(p \wedge q) \to r$
0	0	0	1	1	0	1
0	0	1	1	1	0	1
0	1	0	0	1	0	1
0	1	1	1	1	0	1
1	0	0	1	1	0	1
1	0	1	1	1	0	1
1	1	0	0	0	1	0
1	1	1	1	1	1	1

I = II $\therefore$ logical equivalence holds.

11.(a). $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \iff r$

p	q	r	I $\sim q \wedge r$	IV $\sim p \wedge$ I	II $q \wedge r$	III $p \wedge r$	V II $\vee$ III	IV $\vee$ V
0	0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0	1
0	1	0	0	0	0	0	0	0
0	1	1	0	0	1	0	1	1
1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	1	1	1
1	1	0	0	0	0	0	0	0
1	1	1	0	0	1	1	1	1

IV $\vee$ V $\iff r$

Hence, holds.

11.

(b). $q \vee (p \wedge \sim q) \vee (\sim p \wedge q)$

p	q	$p \wedge \sim q$	$\sim p \wedge q$	$q \vee (p \wedge \sim q) \vee (\sim p \wedge q)$
0	0	0	0	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

Hence tautology //.

10(b). $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$

$$p \rightarrow (q \rightarrow r) \Leftrightarrow p \rightarrow (\sim q \vee r)$$

$\sim p \vee (\sim q \vee r)$ [laws for conditionals]

$(\sim p \vee \sim q) \vee r$ [association]

$\sim (p \wedge q) \vee r$ [De morgan law]

$(p \wedge q) \rightarrow r$ [law of conditional]

11.

(a) $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$

$\Leftrightarrow [(\sim p \wedge \sim q) \wedge r] \vee (q \wedge r) \vee (p \wedge r)$ [associative]

$\Leftrightarrow [\sim (p \vee q) \wedge r] \vee (q \wedge r) \vee (p \wedge r)$ [De morgan]

$\Leftrightarrow [\sim (p \vee q) \wedge r] \vee [(q \vee p) \wedge r]$

$\Leftrightarrow [\sim (p \vee q) \vee (q \vee p)] \wedge r$

$\Leftrightarrow [\sim (p \vee q) \vee (p \vee q)] \wedge r$

$\Leftrightarrow [T_0] \wedge r$

$\Leftrightarrow r$

Hence proved.//