Defn :- A graph G consists of 2 sets i.e, $G=(V,E)$ where V- set of ~~matric~~ vertices and E- the two element subsets of V.

Defn :- The elements of the edge set are known as edges of G.

Defn :- A graph is called a finite graph if both V & E are finite, otherwise called an infinite graph.

Defn :- In a finite graph the number of vertices is denoted by p & is known as order of the graph G.

Defn :- In a finite graph the no. of edges is denoted by q & is known as size of G.

Note :- Unless mentioned otherwise all graphs are finite with non-empty vertex set. If V is null set, E must be null set & thus graph is known as the null graph.

Defn :- A graph is said to be labelled if all the p vertices of G are labelled with some labelling say $V_1, V_2, V_3, \ldots, V_p$. Otherwise, it is called as an unlabelled graph.

Defn :- If an edge has both initial & terminal vertex same, then such an edges are called a self loop.

Defn :- The edges between the same initial & terminal vertices are known as multiple edges or parallel edges.

Parallel.

Defn:- A graph is said to be a multiple edge graph or a multigraph, inshort, if it contains multiple edges. (It doesnot have self loop).

Defn:- A graph is said to be a pseudograph if it contains self loop or multiedges.

Defn:- A graph is said to be directed if ordered pairs of elements are considered as edges.

Defn:- A graph is said to be directed graph or digraph, if it contains directed edges. Otherwise, it is called an undirected graph.

Defn:- A graph is said to be a simple graph if it doesnot contain multiple edges or self loops.

Note:- Unless mentioned otherwise by graph we mean a simple & directed graph.

Defn:- Two vertices are said to be adjacent to each other if there exist an edge b/w them. Otherwise, they are said to be non-adjacent.

Defn:- Two edges are said to be incident with each other if they share a common end vertex.

Defn:- The no of vertices, a vertex is adjacent with is known as the degree of that vertex.

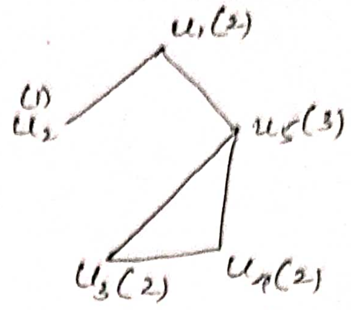
$$\text{degree}, \deg(v_i) = |\{v_j / v_i v_j \in E(G)\}|$$

Defn :- The minimum of the degrees of the vertices of G is denoted by $\delta(G)$ & the maximum is denoted by $\Delta(G)$.

$\delta(G_1) = 1$

$\Delta(G_1) = 3$.

G_1 :



Degree sequence $(1\ 2\ 2\ 2\ 3)$

$(3\ 2\ 2\ 2\ 1)$

$(1\ 2^3\ 3)$

$G_2 : (2^5) \Rightarrow$

$(2^4, 0) \Rightarrow$

2-degree $\Rightarrow$ regular

3 $\Rightarrow$ Cubic

If degree is one then it is called pendant

If degree is 0 then it is called isolated.

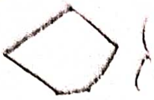
Defn :- A graph G is called K-regular graph if for every vertex of G, $\deg(v) = k$ (same for all vertex)

Defn :- A 3-regular graph is known as a cubic-graph & 2-regular graph is known as a bigraph.

Defn :- A graph G is known as the Pendent vertex if its degree is equal to 1 & the isolated vertex if its degree is zero.

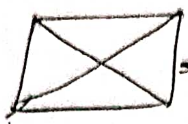
Defn :- The unique edge incident with a pendant vertex is known as a pendent edge.

Defn:- A sequence formed by the degrees of the vertices arranged in non-increasing or non-decreasing order is known as the degree sequence of the graph G.



(2^5)

(5,5) graph

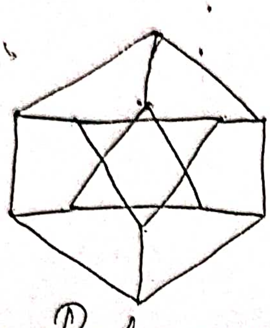


$\Rightarrow 3^4$, no of edges

(4,6) graph

(3 3 3 3)

v→5,

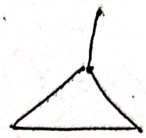


$\Rightarrow 3^{10+2}$

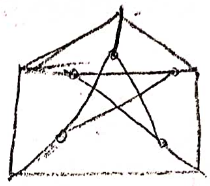
Peterson graph

V E

(4,4) graph (5)



$(1, 2^2, 3)$



$\Rightarrow 3^{10}$

14/11/2014.

The Handshaking Lemma

Peterson graph

If two vertices are not adjacent in Peterson graph then they have exactly one common neighbour.

In a (p, q) graph G,

$$\sum_{i=1}^{p} deg(v_i) = 2q$$

Proof:- When degree sum is taken, every edge is counted twice at both end vertices of the edge and hence $\sum_{i=1}^{p} deg\, v_i = 2 \cdot q$.

Corollary:- In a graph G, the no. of odd degree vertices is even.

Proof:- In a graph G, we know that

$$\sum_{i=1}^{p} deg\, v_i = 2q.$$

If there are k no. of odd degree vertices in the graph then, we take the sum of degree of vertices with odd degree & even degree seperately,

i.e $$\sum_{i=1}^{p} \deg v_i = \sum_{i=1}^{k} \deg v_i + \sum_{i=k+1}^{p} \deg v_i = 2q$$

i.e $$\sum_{i=1}^{k} \deg v_i = 2q - \sum_{i=k+1}^{p} \deg v_i .$$

Since the rhs is an even number,

$$\sum_{i=1}^{k} \deg v_i \text{ is even.}$$

Since each entry in the summation is an odd integer, the summation is an even integ only when k is even.

Hence, the no. of odd degree vertices in a graph is even.

Theorem

In a graph G, its not possible to have all vertices with distinct degrees.

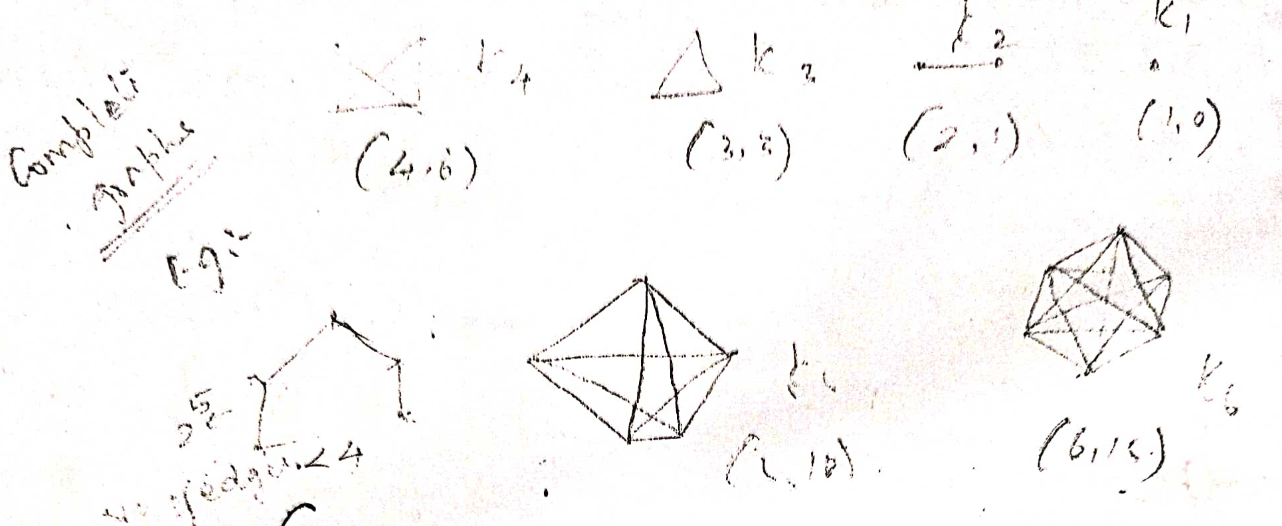
Proof:- Let G be order p:

If suppose all vertex degrees are distinct then they will be 0, 1, 2, ... , p-1.

The vertex of degree (p-1) is a maximum degree vertex and hence it will be adjacent with all other vertices of G, contradicting the fact that there exists an isolated vertex.

Thus, it's not possible to have a simple graph G with all vertex degrees distinct.

Defn:- A vertex of degree $(p-1)$ is known as a full degree vertex.



Defn:- A graph G is called a complete graph on p- no of vertices if each vertex in G is of full degree i.e., all p-vertices have degree $(p-1)$. Hence the number of edges in a complete graph is $\frac{p(p-1)}{2} = C(P,2)$.

Usually, a complete graph on p-vertices is denoted by K_p.

Note:- Trivially; K_1 is considered as a complete graph.

Note: Other than the complete graph on p-vertices, all graphs have their size less than ${}^{P}C_2$.

Vertex set $= \phi \to$ Null graph

Defn:- A graph G on p-vertices is known as a totally disconnected graph if all the p-vertices are isolated vertices.

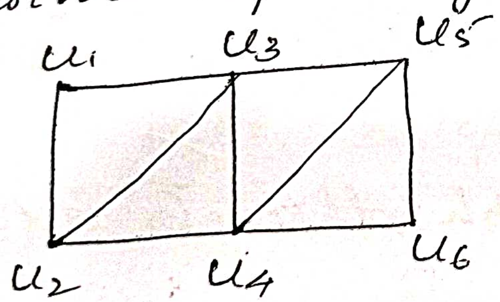
Defn: Given a graph $G=(V,E)$.
If $V_1 \subseteq V$ and $E_1 \subseteq E$, then the graph $G_1=(V_1,E_1)$ is known as a subgraph of the given graph G.

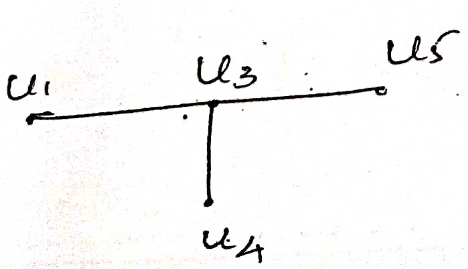
Defn:- A Sub-graph is known as an induced subgraph if it is a maximal subgraph on the considered vertex set.
Usually, it is denoted by $\langle V_1 \rangle$.

Note:- An induced subgraph on a vertex set $V_1 \subseteq V$ contains all the edges that were present in G on the vertices of V_1.

Defn:- A subgraph is called a spanning subgraph if $V_1 = V$.

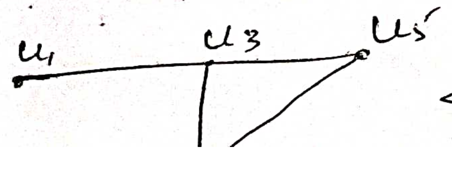
Note:- For a given graph G, the totally dis-connected graph & the graph itself are trivial spanning subgraphs.



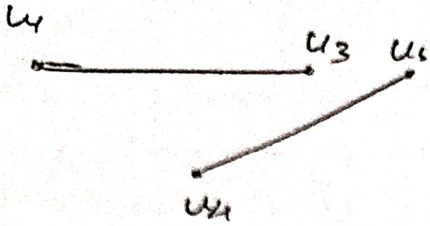


$V_1 = \{u_1, u_3, u_5, u_4\}$.
$E_1 = \{u_1u_3, u_3u_4, u_3u_5\}$

$\Rightarrow$ Proper subgraph
Not induced
Not spanning.



$\langle \{u_1, u_3, u_4, u_5\} \rangle \Rightarrow$ induced.



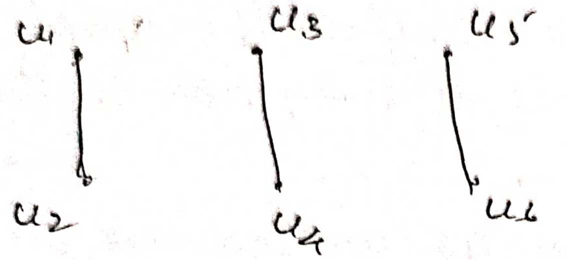
$V_2 = \{u_1, u_3, u_4, u_5\}$
$E_2 = \{u_1u_3, u_4u_5\}$

Induced spanning or V = graph itself

$V_3 = \{u_1, u_2, u_3, u_4, u_5\} = V.$

$E_3 = \{u_1u_2, u_3u_4, u_5u_6\}.$

$\langle V \rangle = G.$



⇒ Spanning
Not induced.

Note :- There exists only one induced spanning subgraph i.e. the graph itself.

Defn :- Complement of a graph G.

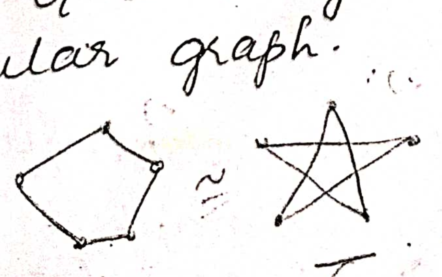
Given a graph G, its complement is denoted by $\bar{G}$ and is defined on the same vertex set as of G and two vertices in $\bar{G}$ are adjacent iff they are non-adjacent in G.

Any other graph other than complete graph, will have atleast one edge in the complement. (i.e. if there is any one non-full degree vertex)

G = Complete graph.
$\bar{G}$ = totally disconnected graph

If G is a regular graph then $\bar{G}$ is also a regular graph.

$G \to k$
$\bar{G} \to (p-1-k)$



$G \cong \bar{G}$

∴ self complementary graph.

Here $P = 5$

Defn:- A sequence of vertices and edges arranged alternatively starting with a verte and ending with a vertex is known as a walk in the graph G.

Eg: $v_1 e_1 v_2 e_2 v_3 e_2 v_3 e_4 v_4$

i.e, if u_1, u_2 have e_1 b/w them, then in the graph G the edge e_1 is forme b/w u_1 and u_2.

Defn:- If the initial & terminal vertices of a walk coincide then such a walk is called a closed walk otherwise, open.

Defn:- A walk is called ~~trial~~ trail if no edge is repeated.

open & close trail.

Defn:- A walk is called a path if there is no repeatation of vertices

Defn:- A closed path is known as a cycle.

$v_6 \; v_7 \; v_8 \; v_9 \; v_6 \Rightarrow$ cycle

Defn:- The number of edges in a walk, trail or path is known as its length.

If the graph itself is a path of p-vertices then is called the path graph (or) in short a path, denoted by 'P_p'.

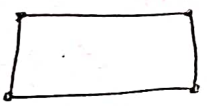
A path of p-vertices has length $(p-1)$ & the degree sequence is ~~ratio~~ written as $(1^2, 2^{p-2})$

Eg:- 4 & 5 edges respectively.

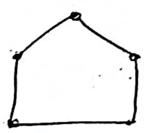
If the graph itself is a cycle on p-vertices then is called a cycle graph or just a cycle.

A cycle on p-vertices has p-edges & every vertex has its degree 2 & hence the degree sequence (2^p)

Eg:-



C_4

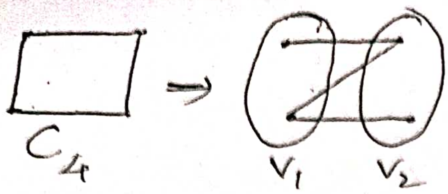


Note:- Cycle graph start only in C_3 but degree sequence is

A graph $G=(V,E)$ is said to be bi-partite graph if the vertex ~~at~~ set 'V' can be partitioned into two subsets V_1 & V_2 i.e, $V = V_1 \cup V_2$ and the edge set consists of the pairs uv such that $u \in V_1$ and $v \in V_2$.

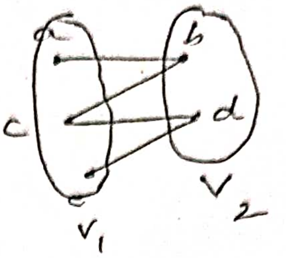
The induced subgraph induced by V_1 & V_2 are null sets.

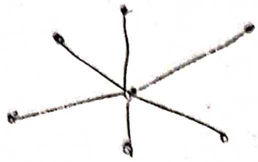
Eg:- C_4, C_6, P_5 are bipartite graphs.



C_4

P_5





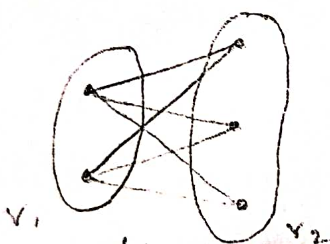
$K_{1,6}$

Star

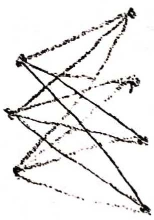
$K_{1,3}$ - claw.

$K_{1,1} = K_2 = P_2$

If Cardinality of $V_1 = m$ & cardinality of $V_2 = n$ then the graph $K_{m,n}$ is a complete bi-partite graph such that each vertex of V_1 has full degree (in $V_2 = n$) and each vertex of V_2 has full degree (in $V_1 =)m$. So the degree sequence of $K_{m,n}$ is (m^n, n^m).



$K_{2,3}$.

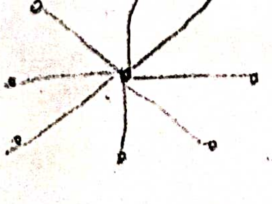


$K_{3,4}$

A graph is bipartite iff it has no odd cycle.

Note:- $K_{m,n}$ has order $= m+n$.
Size $= m \times n$.

defn: If $m=1$ in a $K_{m,n}$, then it is called a star.



$K_{1,8}$

Star

If $m=1$ & $n=3 \Rightarrow$ claw.

Defn :- Two graphs G_1 and G_2 are said to be isomorphic to each other if there exists an adjacency preserving bijection from $V(G_1)$ to $V(G_2)$.

Eg:- 1)

$f(u_1) = a$, $f(u_2) = b$, $f(u_3) = c$, $f(u_4) = d$

G_1: u_1u_2, u_1u_4, u_2u_3, u_3u_4.

$f(u_1)f(u_2) \Rightarrow ab$ ✓ : G_2

$f(u_1)f(u_4) \to ad$ ✓

$f(u_2)f(u_3) \to bc$ ✓

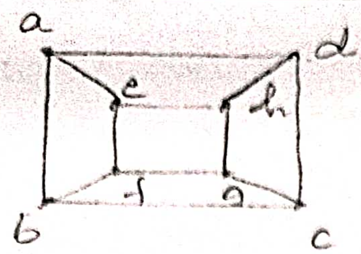
$f(u_3)f(u_4) \to cd$ ✓

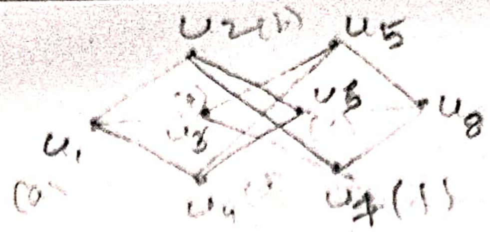
Adjacency

Each property of G_1 then holds true in G_2.

2)

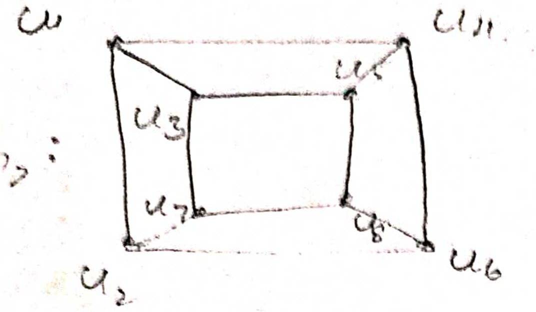
G_1 :





: G_2

G_2 :



$f(a) = u_1$
$f(b) = u_2$
$f(c) = u_6$
$f(d) = u_{11}$
$f(e) = u_3$
$f(f) = u_7$
$f(g) = u_8$
$f(h) = u_5$

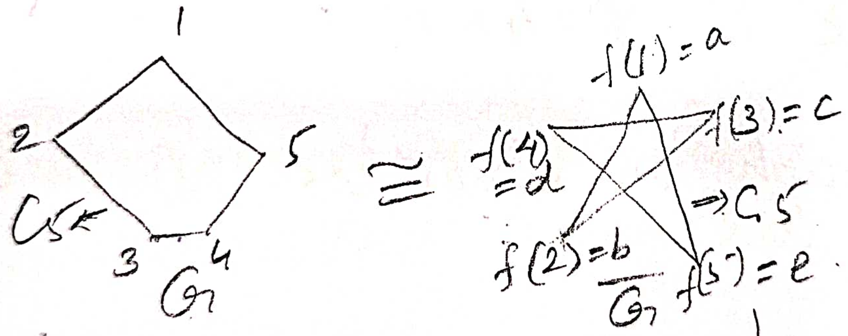
$f(u_1) = a$
$f(u_2) = b$
$f(u_3) = e$
$f(u_{11}) = d$
$f(u_5) = h$
$f(u_6) = c$
$f(u_7) = f$
$f(u_8) = g$

Adjacency list

ab	$f(a)f(b)$	$u_1u_2 \in E(G_2)$
ae	$f(a)f(e)$	$u_1u_3 \in E(G_2)$
ad	$f(a)f(d)$	$u_1u_{11} \in E(G_2)$
eh	$f(e)f(h)$	$u_3u_5 \in E(G_2)$
ef	$f(e)f(f)$	$u_3u_7 \in E(G_2)$
fb	$f(f)f(b)$	$u_2u_7 \in E(G_2)$
gh	$f(g)f(h)$	$u_8u_5 \in E(G_2)$
bc	$f(b)f(c)$	$u_2u_6 \in E(G_2)$
cd	$f(c)f(d)$	$u_6u_{11} \in E(G_2)$
gf	$f(g)f(f)$	$u_7u_8 \in E(G_2)$
gc	$f(g)f(c)$	$u_6u_8 \in E(G_2)$
bd	$f(b)f(d)$	$u_2u_{11} \in E(G_2)$

$\therefore G_1 \cong G_2$

Theorem:- Two graphs G and H are **isomorphic** iff their complements are isomorphic.

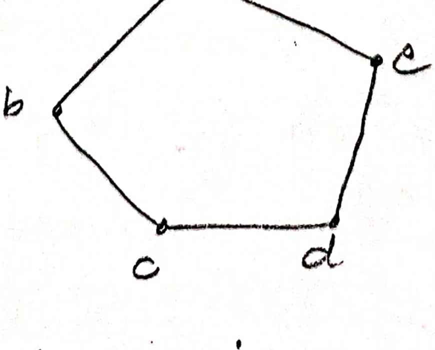


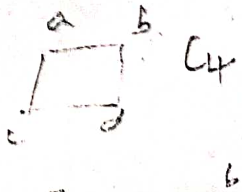
$\Rightarrow$ Self complement[ary]

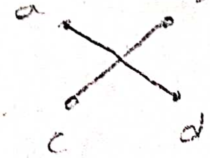
Δ ∴

Defn :- A graph G is said to be self-complementary graph if $G \cong \overline{G}$.

$f(1)=a$
$f(2)=b$
$f(3)=c$
$f(4)=d$
$f(5)=e$



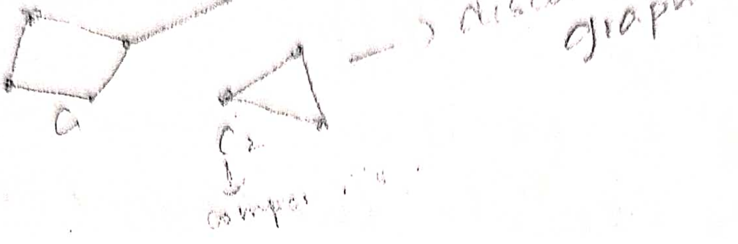




$C_4 \ncong 2K_2$

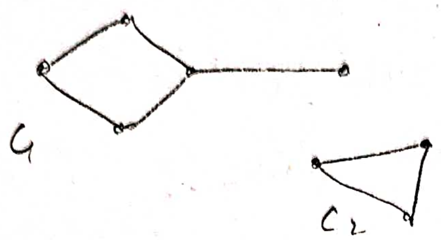
$G = \overline{G}$ Total of

Defn :- A graph G is said to be connected if there exists a path b/w every pair of vertices. Otherwise, it is said to be disconnected.



Defn :- A maximum connected sub-graph of a graph G is known as component of a graph G.

Note :- A connected graph has 1-component & a disconnected graph has atleast two components.



Defn :- A vertex 'v' is said to be deleted from graph G if 'v' is removed from G along with all the adjacencies of 'v'. Usually it is denoted by G−v.

Defn :- An edge 'e' is said to be deleted from graph G if the edge is removed from G without affecting the other adjacencies of the end vertices of 'e'. Usually this is denoted by G−e.

Note that G−v gives a proper subgraph of G & G−e gives a spanning subgraph of G.

Defn :- A vertex 'v' is said to be cut vertex if G−v has more components than G and an edge 'e' is said to be a cut-edge or a bridge if the number of components in G−e is more than the no of components in G.