SET-THEORY

(12-06-23)

Symmetric Difference

$A \Delta B = (A \cup B) - (A \cap B)$

$= \{x \mid x \in A \cup B \text{ \& } x \notin A \cap B\}$

$B - A = \{x \in B \mid x \notin A\}$

Determine the sets A and B, given that

$A - B = \{1, 3, 7, 11\}$

$B - A = \{2, 6, 8\}$

$A \cap B = \{4, 9\}$

Sol $A = \{1, 3, 7, 11, 4, 9\}$

$B = \{2, 6, 8, 4, 9\}$

The laws of Set Theory:

For any sets A, B, C belonging to U where U is the universal set, the following laws hold good.

1. Commutative Law : $A \cup B = B \cup A$, $A \cap B = B \cap A$
2. Associative law : $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$
3. Distributive law : $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4. Idempotent law : $A \cup A = A$

   $A \cap A = A$

5. Identity Laws : $A \cup \phi = A$, $A \cap U = A$

   $\cancel{A \cap \phi = \phi}$

6. Law of double compliment : $(A^c)^c = A$ or $\bar{\bar{A}} = A$
7. Inverse Laws, $A \cup \bar{A} = U$, $A \cap \bar{A} = \phi$
8. Demorgan's law, $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

   $\overline{A \cap B} = \bar{A} \cup \bar{B}$

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   ~~$A \cap \phi = \phi$~~
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8. Demorgan's Law, $(\overline{A \cup B}) = \bar{A} \cap \bar{B}$
   $\overline{A \cap B} = \bar{A} \cup \bar{B}$

9) Domination Laws : $A \cap \phi = \phi$

$A \cup U = U$

10) Absorption Laws : $A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

Q. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof - from LHS, let $x \in A \cup (B \cap C)$

let $B \cap C = D$

$\therefore$ ① $\Rightarrow x \in A \cup D$

$\Rightarrow x \in A$ or $x \in D$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A$ or $[x \in B$ & $x \in C]$

$\Rightarrow (x \in A$ or $x \in B)$ & $(x \in A$ or $x \in C)$

$\Rightarrow (x \in A$ or $B)$ & $(x \in A$ or $C)$

$\Rightarrow (x \in A \cup B)$ & $(x \in A \cup C)$

$\Rightarrow (x \in A \cup B) \cap (x \in A \cup C)$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

Q. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

from LHS, let $x \in A \cap (B \cup C)$

let $B \cup C = D$

$\therefore$ ① $\Rightarrow x \in A \cap D$

$\Rightarrow x \in A$ & $x \in D$

$\Rightarrow x \in A$ & $x \in (B \cup C)$

$\Rightarrow x \in A$ & $[x \in B$ or $x \in C]$

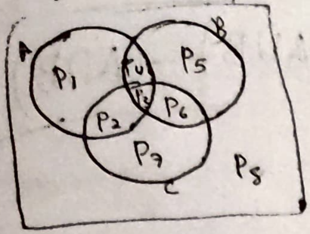
$\Rightarrow (x \in A$ & $x \in B)$ or $(x \in A$ & $x \in C)$

$\Rightarrow (x \in A$ & $B)$ or $(x \in A$ & $C)$

$\Rightarrow (x \in A \cap B)$ or $(x \in A \cap C)$

$\Rightarrow (x \in A \cap B) \cup (x \in A \cap C) \Rightarrow x \in (A \cap B) \cup (A \cap C)$

Q. Using Venn Diagram, prove that $\overline{A\cap(B\cup C)} = (\bar{A}\cup\bar{B})\cap\bar{C}$



$A = P_1\cup P_2\cup P_3\cup P_4$, $\bar{A} = P_5\cup P_6\cup P_7\cup P_8$

$B = P_3\cup P_4\cup P_5\cup P_6$ $\quad \bar{B} = P_1\cup P_2\cup P_7\cup P_8$

$C = P_2\cup P_3\cup P_6\cup P_7$ $\quad \bar{C} = P_1\cup P_4\cup P_5\cup P_8$

$(B\cup C) = P_2\cup P_3\cup P_4\cup P_5\cup P_6\cup P_7$

$A\cap(B\cup C) = P_2\cup P_3\cup P_4$

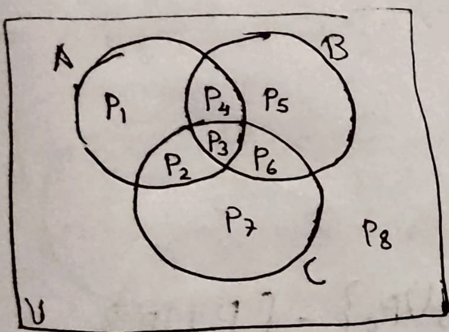
$\overline{A\cap(B\cup C)} = U - \{P_2\cup P_3\cup P_4\}$

$= P_1\cup P_5\cup P_6\cup P_7\cup P_8$ — LHS

RHS $(\bar{A}\cup\bar{B})\cap C$) . $(\bar{A}\cup B) = P_1\cup P_2\cup P_5\cup P_6\cup P_7\cup P_8$

$(\bar{A}\cup\bar{B})\cap\bar{C} = \cancel{P_2\cup P_6\cup P_7}\ P_1\cup P_5\cup P_8$

Q Prove $\overline{(A\cup B)\cap C} = (\bar{A}\cap\bar{B})\cup\bar{C}$



LHS

$A = P_1\cup P_2\cup P_3\cup P_4$

$B = P_3\cup P_4\cup P_5\cup P_6$

$C = P_2\cup P_3\cup P_6\cup P_7$

$(A\cup B) = P_1\cup P_2\cup P_3\cup P_3\cup P_4\cup P_4\cup P_5\cup P_6$

$(A\cup B)\cap C = P_2\cup P_3\cup P_6$

$\overline{[(A\cup B)\cap C]} = U - \{P_2\cup P_3\cup P_6\} = P_1\cup P_4\cup P_5\cup P_7\cup P_8$ — (1)

$\bar{A} = P_5\cup P_6\cup P_7\cup P_8$

$\bar{B} = P_1\cup P_2\cup P_7\cup P_8$

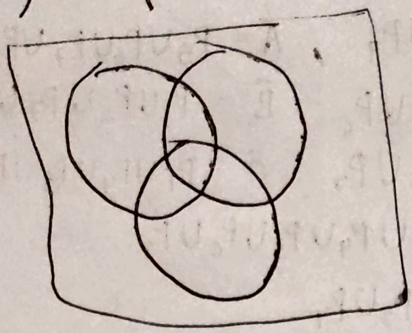
$\bar{C} = P_1\cup P_4\cup P_5\cup P_8$

$\bar{A}\cap\bar{B} = P_7\cup P_8$

$(\bar{A}\cap\bar{B})\cup\bar{C} = \{P_1\cup P_4\cup P_5\cup P_7\cup P_8\}$ — (2)

Since (1) = (2) eqⁿ holds true

3) $A\Delta(B\Delta C) = (A\Delta B)\Delta C$



$$\boxed{A \Delta B = (A\cup B) - (A\cap B)}$$

for $B\Delta C$

$B\cup C = \{P_2 \cup P_3 \cup P_4 \cup P_5 \cup P_6 \cup P_7\}$

$B\cap C = \{P_3 \cup P_6\}$

$(B\cup C) - (B\cap C) = \{P_2 \cup P_4 \cup P_5 \cup P_7\}$

$A\Delta(B\Delta C) = \{A\cup(B\Delta C) - A\cap(B\Delta C)\}$

$= \{P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5 \cup P_7\} - \{P_2 \cup P_4\}$

$= P_1 \cup P_3 \cup P_5 \cup P_7$

<u>RHS</u> $(A\Delta B)\Delta C$

$A\Delta B = (A\cup B) - (A\cap B)$

$= \{P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5 \cup P_6\} - \{P_3 \cup P_4\}$

$= \{P_1 \cup P_2 \cup P_5 \cup P_6\}$

$(A\Delta B)\Delta C = ((A\Delta B)\cup C) - (A\Delta B)\cap C$

$= \{P_1 \cup P_2 \cup P_3 \cup P_5 \cup P_6 \cup P_7\} - \{P_2 \cup P_6\}$

$= \{P_1 \cup P_3 \cup P_5 \cup P_7\}$

LHS = RHS

Hence <u>Proved</u>

Q. If $A \Delta C = B \Delta C$, then $A = B$

$\Rightarrow (A \cup C) - (A \cap C) = (B \cup C) - (B \cap C)$

~~Dividing both sides by ΔC~~

$$\cancel{\frac{A \Delta C}{\Delta C} = \frac{B \Delta C}{\Delta C}}$$

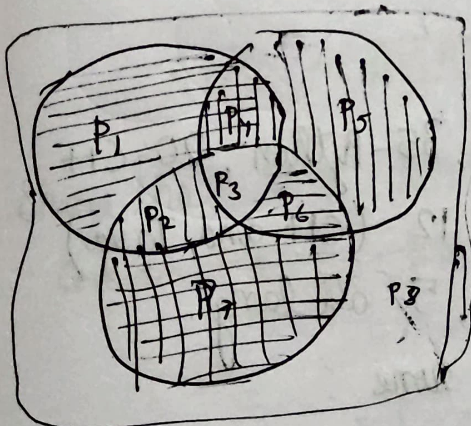
$A = C$

$A = P_1 \cup P_2 \cup P_3 \cup P_4$, $B = P_3 \cup P_4 \cup P_5 \cup P_6$, $C = P_2 \cup P_3 \cup P_6 \cup P_7$.

$\{P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_6 \cup P_7\} - \{P_2 \cup P_3\}$

$= \{P_2 \cup P_3 \cup P_4 \cup P_5 \cup P_6 \cup P_7\} - \{P_3 \cup P_6\}$

$= P_1 \cup P_4 \cup P_6 \cup P_7 = P_2 \cup P_4 \cup P_5 \cup P_7$



4a) The bit string for the set {1,2,3,4,5} and {1,3,5,7,9} are 11 1110 0000 and 10 1010 1010 respectively. Use bit strings to find union and intersection of these sets.

$\bar{A}$ = 0 000000011

$\bar{B}$ = 0101000000

A∪B = 1111110011

A∩B = 1010100000

4b) If the bit string for the set {1,3,5,7,9} in U {1,2,3,4,5,6,7,8,9,10}. is 1010101010
What is the bit string for the complement of this set.

Ans 0101010101

5a) 100 - programmer, 47 - Java, 35 - Python, 20 - C++
23 - (Java and Python), 12 - (C++ and Java)
11 - (Python and C++), 5 - all lang.
How many can program in none.

Sol $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= 47 + 35 + 20 + 5 - 23 - 12 - 11$

$= 61$

$|\overline{A \cup B \cup C}| = 100 - 61 = 39$

25
46 35
60

107
46

Q State and prove Demorgan's Law

① $\overline{A \cup B} = \bar{A} \cap \bar{B}$ ——— ①

consider, $\bar{A} \cap \bar{B} = \{x \mid x \in \bar{A} \ \& \ x \in \bar{B}\}$

$= \{x \mid x \notin A \ \& \ x \notin B\}$

$= \{x \mid x \notin A \cup B\}$

$= x \in \overline{A \cup B}$

$\Rightarrow \overline{A \cup B}$

② $\overline{A \cap B} = \bar{A} \cup \bar{B}$ ——— ②

consider $\bar{A} \cup \bar{B} = \{x \in \bar{A} \text{ or } x \in \bar{B}\}$

$= \{x \notin A \text{ or } x \notin B\}$

$= \{x \notin (A \cap B)\}$

$= x \in \overline{A \cap B}$

$\bar{A} \cup \bar{B} \Rightarrow \overline{A \cap B}$

Q. Show that $A - B = A \cap \bar{B}$ ——— ③

Consider $A - B = \{x \in A \ \& \ x \notin B\}$

$= \{x \in U \mid x \in A \ \& \ x \notin B\}$

$= \{x \in U \mid x \in A \ \& \ x \in \bar{B}\}$

$= x \in A \cap \bar{B}$

Q. Show that $\overline{A - B} = \bar{A} \cup B$ ——— ④

from ③ ~~wkt~~ $\cancel{\overline{A \cup B} = \bar{A} \cap \bar{B}}$

$\cancel{\overline{A - B} = \overline{A \cup B}}$ $\cap$ $\quad \overline{A - B} = \overline{A \cap \bar{B}}$

$= \cancel{\bar{\bar{A}} \cup \bar{B}}$ $\quad = \bar{A} \cup \bar{\bar{B}}$

$= \bar{A} \cup B$

5b) ~~C~~ - Total - 24

$|A| = 8$, $|B| = 13$, $|C| = 13$

$|A \cap B| = 5$, $|A \cap C| = 3$, $|B \cap C| = 6$, $|A \cap B \cap C| = 2$

(ii)

(i) ~~$P(A) - P(A \cup B) - P(A \cup C) + P(A \cap B \cap C)$~~

$P(A) = |A| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$

$= 8 - 5 - 3 + 2$

$= 2$

$P(B) = |B| - |A \cap B| - |B \cap C| + |A \cap B \cap C|$

$= 13 - 5 - 6 + 2$

$= 4$

$P(C) = |C| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= 13 - 3 - 6 + 2$

$= 6$

7 a) $A = \{1, 3, 5\}$, $B = \{2, 3\}$ and $C = \{4, 6\}$

(i) $(A \cup B) \times C = \{1, 2, 3, 5\} \times \{4, 6\}$

$= (1,4)\ (1,6)\ \cancel{(2,4)}\ (2,6)$

$(3,4)\ (3,6)\ (5,4)\ (5,6)$

(ii) $(A \times B) \cap (B \times A)$, $A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$

$B \times A = \{(2,1), (2,3), (2,5), (3,1)\ (3,3)\ (3,5)\}$

$(A \times B) \cap (B \times A) = (3,3)$

=

$(B\times C) = (2,4)\ (2,6)\ (3,4)\ (3,6)$

$(A\times B)\cup(B\times C) = \{(1,2)\ (1,3)\ (3,2)\ (3,3)\ (5,2)\ (5,3)$
$(2,4)\ (2,6)\ (3,4)\ (3,6)\}$

~~#1 verify that~~ (i) $A\times(B-C) = (A\times B)-(A\times C)$

Q Show that $A-(\cancel{A\cap B}\ B) = A\cap(\overline{A\cap B})$

RHS $= A\cap(\bar{A}\cup\bar{B})$ $\overline{A\cap B} = \bar{A}\cup\bar{B}$ DM Law

$= (A\cap\bar{A})\cup(A\cap\bar{B})$ Dist. law

$= \phi\cup A\cap\bar{B}$

$= A\cap\bar{B}$

$= A-B$

Q. $\overline{A-B} = \bar{A}\cup(A\cap B)$

LHS $=$ ~~A~~ from ④ $\overline{A-B} = \cancel{\bar{A}\cup B}$ $\overline{A\cap(\overline{A\cap B})}$

$= \bar{A}\cup(A\cap B)$

Q. $A-(A-B) = A\cap B$

$\Rightarrow$ LHS $\Rightarrow A\cap(\overline{A-B})$

$\Rightarrow A\cap(\overline{A\cap\bar{B}}) = A\cap(\bar{A}\cup B)$

$\Rightarrow (A\cap\bar{A})\cup(A\cap B)$

$\Rightarrow \phi\cup A\cap B$

$\Rightarrow A\cap B$

Q. For any 3 sets, show that $A \cap (B - C) = (A \cap B) - C$

$$B - C = B \cap \bar{C}$$

$$A \cap (B \cap \bar{C}) = (A \cap B) \cap (A \cap \bar{C})$$

$$= (A \cap B) \cap (A - C)$$

$$= (A \cap B) - C$$

Q. $(A-B) \cap (A-C) = A - (B \cup C)$

LHS $\Rightarrow$ $(A \cap \bar{B}) \cap (A \cap \bar{C})$

$\Rightarrow A \cap (\bar{B} \cap \bar{C})$

$\Rightarrow A \cap (\overline{B \cup C})$

$\Rightarrow A - (B \cup C)$

RHS $= A \cap (\overline{B \cup C})$

$\Rightarrow A \cap (\bar{B} \cap \bar{C})$

$\Rightarrow (A \cap \bar{B}) \cap (A \cap \bar{C})$

$\Rightarrow (A - B) \cap (A - C)$

$\Rightarrow$ LHS

Q. For any 3 sets, show that $(A-B) - C = A - (B \cup C)$

RHS $\Rightarrow$ $A \cap (\overline{B \cup C}) = A \cap (\bar{B} \cap \bar{C})$

$$= (A \cap \bar{B}) \cap (A \cap \bar{C})$$

$$= (A - B) \cap (A - C)$$

$(A-B) - C = (A \cap \bar{B}) - C$

$\Rightarrow (A \cap \bar{B}) \cap \bar{C})$

$\Rightarrow A \cap (\bar{B} \cap \bar{C}) = A \cap (\overline{B \cup C})$

$\Rightarrow A - (B \cup C)$

Q. $(A-C)-(B-C) = A-(B\cup C)$

$\Rightarrow (A\cap \bar{C}) - (B\cap \bar{C})$

$\Rightarrow (A\cap \bar{C}) \cap (\overline{B\cap \bar{C}})$

$\Rightarrow (A\cap \bar{C}) \cap (\bar{B}\cup C) = ((A\cap\bar{C})\cap\bar{B}) \cup ((A\cap\bar{C})\cap C)$

$\Rightarrow \cancel{[A\cap(\overline{B\cap \bar{C}})] \cup [A\cap(\bar{C}\cup \bar{C})]}$ $\quad A\cap(\bar{C}\cap\bar{B})\cup(A\cap\phi)$

$\Rightarrow A\cap(\overline{B\cup C}) \cup [A\cap \bar{C}]$ $\quad A\cap(\overline{C\cup B})$

$A-(B\cup C)$

Q. For any 3 sets prove that $(A-B)-C = (A-C)-(B-C)$

Sol from last question $(A-B)-C = A-(B\cup C) = (A-C)-(B-C)$

Q. ~~$(A-C)-(B-C)$~~ If S,T is $\subseteq U$ prove that S and T are disjoint if and only if $S\cup T$ is equal to $S\Delta T$

Sol We know that $S\Delta T = (S\cup T)-(S\cap T)$ — {By definition

$= (S\cup T)\cap(\overline{S\cap T})$ — ①

Now suppose S & T are disjoint, i.e. $S\cap T = \phi$

$\therefore \overline{S\cap T} = \bar{\phi} = U$

from ① $S\Delta T = (S\cup T)\cap\bar{\phi}$

$= (S\cup T)\cap U$

$= S\cup T$

Conversely, suppose $S\Delta T = (S\cup T)$, then

eqⁿ ① $\Rightarrow S\cup T = (S\cup T)\cap(\overline{S\cap T})$

This shows that ~~$S\cup T$~~ $\overline{S\cap T} = U$

$S\cap T = \bar{U} = \phi$

$\therefore$ S & T are disjoint

3(b) for any 3 sets A, B, C, show that if $A \Delta C = B \Delta C$ then $A = B$

Sol: for any sets A, B, C if $A \cap C = B \cap C$ and $A \cup C = B \cup C$.

$\Rightarrow A = B$

Sol Take any x belonging to A then $x \in A \cup C$ because A is contained in $A \cup C$ ($A \subseteq A \cup C$)

Since, $A \cup C = B \cup C$

$\therefore x \in B \cup C$

$\therefore x \in B$ or $x \in C$

if $x \in B$, then $A \subseteq B$

if $x \in C$, then $x \in A \cap C = B \cap C$

So that $x \in B$, thus $A \subseteq B$ —— ①

Let any $y \in B$, then $y \in B \cup C = A \cup C$

$\therefore y \in A$ or $y \in C$

If $y \in A$, then $B \subseteq A$

if $y \in C$, then $y \in B \cap C = A \cap C$

so that $y \in A$, thus $B \subseteq A$

$\therefore$ from ① & ② we can say that

$A \subseteq B$ & $B \subseteq A$

$\Rightarrow A = B$

Q. How many no.'s from 1 to 1000 are not divisible by 2, 3, 5, 7

$A \to 2 \Rightarrow \frac{1000}{2} = 500$

$B \to 3 \Rightarrow \frac{1000}{3} = 333$

$C \to 5 \Rightarrow \frac{1000}{5} = 200$

$D \to 7 \Rightarrow \frac{1000}{7} = 142$

~~$n(A\cap B\cap C\cap D)$~~ =

$n(A\cap B)$	$n(A\cap C)$	$n(A\cap D)$	$n(B\cap C)$	$n(B\cap D)$	$n(C\cap D)$
$\frac{1000}{2\times3}$	$\frac{1000}{5\times2}$	$\frac{1000}{7\times2}$	$\frac{1000}{15}$	$\frac{1000}{3\times7}$	$\frac{1000}{7\times5}$
$=166$	$=100$	$=71$	$=66$	$=47$	$=28$

$n(A\cap B\cap C)$	$n(A\cap C\cap D)$	$n(B\cap C\cap D)$	$n(A\cap B\cap D)$
$\frac{1000}{2\times3\times5}$	$\frac{1000}{2\times3\times7}$	$\frac{1000}{3\times5\times7}$	$\frac{1000}{2\times3\times7}$
$=33$	$=23$	$=9$	$=14$

$n(A\cap B\cap C\cap D)$

$$\frac{1000}{2\times3\times5\times7} = 4$$

~~$A\cup B\cup C\cup D$~~ $+ (A+B+C+D) - (A\cap B + A\cap C + A\cap D + B\cap C + C\cap D) + (A\cap B\cap C + A\cap C\cap D + B\cap C\cap D + A\cap B\cap D) - (A\cap B\cap C\cap D) = 772$

Total number of numbers which aren't divisible by 2,3,5,7 ~~are~~ from 1 to 1000 are

$1000 - 772 = 228$

Q. How many integers b/w 1 and 250 which are divisible by one of the integers 3,5,7

1 to 200
2,3,5

Sol

$A \to 3 \Rightarrow \frac{250}{3} = 83$

$B \to 5 \Rightarrow \frac{250}{5} = 50$

A+B+C-

$C \to 7 \Rightarrow \frac{250}{7} = 35$

168

$\frac{21}{5}$
105

$83+50+35 - \frac{250}{15} - \frac{250}{35} - \frac{250}{21} + \frac{250}{105}$

$-16 - 7 - 11 + 2 = 136$

$\underline{2,3,5}$ $\quad 50+33+20-16-6-10+3$

74

9(b) $A = \{x \mid 3x^2-7x-6=0\}$ $\quad B = \{x \mid 6x^2-5x-6=0\}$

$(x-9)(x+2)$ $\quad (x-9)(x+4)$

$\boxed{A \cap B = 9}$

8(b) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Consider the LHS for $A \times (B \cup C)$

Let $(x,y) \in A \times (B \cup C)$

$x \in A$ & $y \in B \cup C$

$(x \in A$ & $y \in B)$ or $(x \in A$ & $y \in C)$

$\Rightarrow (x,y) \in (A \times B)$ or $(x,y) \in A \times C)$

$\Rightarrow (x,y) \in (A \times B) \cup (A \times C)$ $\quad \}$ $(x,y) \in A \times (B \cap C)$ & $(x,y) \in (A \times B) \cap (A \times C)$

$\Rightarrow A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$

$\underline{\text{from RHS}}$, Let $(x,y) \in (A \times B) \cup (A \times C)$

$(x,y) \in (A \times B)$ or $(x,y) \in (A \times C)$

$(x \in A$ & $y \in B)$ or $(x \in A$ & $y \in C)$

$\Rightarrow (x \in A)$ & $(y \in B$ or $y \in C)$

$\Rightarrow (x \in A)$ & $(y \in B \cup C)$

$\therefore (A \times B) \cup (A \times C) \subseteq A \times B \cup C)$

from (1) & (2)

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

8(b) II $A \times (B \cap C) = (A \times B) \cap (A \times C)$

from LHS Let $(x,y) \in A \times (B \cap C)$

$\Rightarrow x \in A$ & $y \in (B \cap C)$

$\Rightarrow (x \in A$ & $y \in B)$ & $(x \in A$ & $y \in C)$

$\Rightarrow (x,y) \in (A \times B)$ & $(x \in (A \times C))$

$\Rightarrow (x,y) \in (A \times B) \cap (A \times C)$

$\Rightarrow A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$

from RHS Let $(x,y) \in (A \times B) \cap (A \times C)$

$\Rightarrow (x,y) \in (A \times B)$ & $(x,y) \in (A \times C)$

$\Rightarrow (x \in A$ & $y \in B)$ & $(x \in A$ & $y \in C)$

$\Rightarrow (x \in A)$ & $(y \in B$ & $y \in C)$

$\Rightarrow (x \in A)$ & $(y \in B \cap C)$

$\Rightarrow A \times (B \cap C)$

$\Rightarrow$ ~~$A \times (B \cap C)$~~ $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$

from ① & ② $A \times (B \cap C) = (A \times B) \cap (A \times C)$

10(a) Two finite sets have ~~man~~ m and n elements. The total no. of subsets of set A is 48 more than total no of subsets in B. The values of m & n are.

$|A| = m$, $|B| = n$ $\Rightarrow$ subsets of $A = 2^m$

$B = 2^n$

$2^m = 48 + 2^n$

$2^m - 2^n = 48 = 64 - 16$

$= 2^6 - 2^4$

$\boxed{m = 6, n = 4}$

(b) In an exam, passed in English - 70%
passed in Maths - 65%
failed in English & Math - 27%
passed in both - 248
Find the total no. of candidates

Sol. failed in Eng. - 30%, failed in Maths - 35%
failed in Math & Eng = 27%

failed in Eng only = 30% - 27% = 3%
Maths only = 35% - 27% = 8%
Total failed = 27 + 3 + 8 = 38%
pass = 100 - 38 = 62%

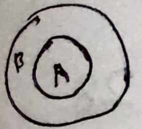
$$\frac{62}{100} \times x = 248$$

$$x = \frac{248 \times 100}{62} = 400$$

13. $A = \{\alpha, \beta\}$, $B = \{1, 2, 3\}$
(i) $A \times B = \{(\alpha, 1), (\alpha, 2), (\alpha, 3), (\beta, 1), (\beta, 2), (\beta, 3)\}$
(ii) $B \times A = \{(1, \alpha), (2, \alpha), (3, \alpha), (1, \beta), (2, \beta), (3, \beta)\}$
(iii) $A \times A = \{(\alpha, \alpha)\ (\alpha, \beta)\}$
(iv) $B \times B = \{(\beta, \alpha)\ (\beta, \beta)\}$
(v) $(A \times B) \cup A = \{((\alpha, 1), \alpha)\ (\alpha, 2, \alpha)\ (\alpha, 3, \alpha)$ ~~$(\alpha, 1, \alpha)$~~
$(\beta, 1, \alpha)\ (\beta, 2, \alpha)\ (\beta, 3, \alpha)$
$(\alpha, 1, \beta)\ (\alpha, 2, \beta)\ (\alpha, 3, \beta)$
$(\beta, 1, \beta)\ (\beta, 2, \beta)\ (\beta, 3, \beta)\}$

(vi) $(A \times B) \cap B$

12(a) A - 4 elements , B - 7 elements



Min = 7
Max = 11

11(a) Total - 100 students , 50 - college lib. , 40 - own lib.
30 - borrowed , 20 - both college and own
15 - own and borrow
10 - college and borrowed

$100 - (50+40+30) + (20+15+10) = x$

$\boxed{25 = x}$

11 (b) $n(f) = 40$, $n(0) = 30$, $n(f \cap 0) = 20$

$n(f \cup 0) = n(f) + n(0) - n(f \cap 0)$
$= 70 - 20 = 50$

Factory alone $= n(f) - n(f \cap 0)$
$= 40 - 20 = 20$

Office alone $= n(0) - n(f \cap 0)$
$= 30 - 20 = 10$

12) ~~Male~~ Adults - 20, ~~Ma~~ Female - 8 , Male - 12
Literate - ~~12~~9

	Male	Female	Total
Literate	8	1	9
Illiterate	4	7	3
Total	~~9~~12	8	~~1~~20

14 (a) $n \leq 3000$ divisible by 3, 5, 7

$3 \Rightarrow 1000$ $\quad$ $3 \cap 5 \Rightarrow 200$ $\quad$ $3 \cap 5 \cap 7 \Rightarrow 28$

$5 \Rightarrow 600$ $\quad$ $3 \cap 7 \Rightarrow 142$

$7 \Rightarrow 428$ $\quad$ $5 \cap 7 \Rightarrow 85$

$\frac{142}{\cancel{3000}}$ 21

$$n(3 \cup 5 \cup 7) = n(3) + n(5) + n(7) - (n(3 \cap 5) + n(3 \cap 7) + n(5 \cap 7)) + n(3 \cap 5 \cap 7)$$

$$= 1629$$

(b) $n \leq 2076$ divisible by neither 4 ~~no~~ or 5

$$\Rightarrow 2076 - (\cancel{519}\ 519 + 415 - 103)$$

$$= 1245$$

18 (a) Prove every integer ≥ 2 has a prime factor

A) We shall prove this result by using the concept of strong induction which says that we assume the statement holds for all values preceding k

By strong induction, the given statement is clearly true if $n = 2$.
Now assume that it is true for every positive integer $n \leq k$ where $k \geq 2$.
Consider the integer $k+1$

Case 1: If $k+1$ is a prime no., then $k+1$ is a prime factor of itself

Case 2: If $k+1$ is not a prime, then $k+1$ is a composite no. so it must have a factor $d \leq k$. ~~so~~
Then by inductive hypothesis, d has a prime factor p so p is a factor of $k+1$

Thus, by strong induction, the statement is true for every integer ≥ 2 i.e. every integer ≥ 2 has a prime factor

15(a) Express 10110_2 in base ten

$\Rightarrow (2^4 + 2^2 + 2^1) = 22$

(b) $3ABC_{16}$

As the bases go higher than 10, we assign values for A as 10, B - 11, C - 12 and so on

$\Rightarrow (3\times 16^3) + (10\times 16^2) + (11\times 16^1) + (12\times 16^0) =$

$\Rightarrow (58\times 16^2) + 2176 + 12 = 15036$

16 (a) Express 15036 in hexadecimal system

$$\begin{array}{r|l} 16 & 15036 \\ 16 & 939 \\ & 3 \end{array}$$

$$\begin{array}{r} 15036 \,(939 \\ 15024 \\ \hline 12 \end{array}$$

$\Rightarrow 15036 = 16.939 + 12$

$$\begin{array}{r} 16 \,\big)\, 939 \,\big(\, 58 \\ 928 \\ \hline 11 \end{array}$$

$939 = 16\times 58 + 11$

$58 = 16\times 3 + 10$

$3 = 16\times 0 + 3$

$= 3ABC$

(b) 3014

$$\begin{array}{r|l} 8 & 3014 \\ \hline 8 & 376 \rightarrow 6 \\ \hline 8 & 47 \rightarrow 0 \\ \hline 8 & 5 \rightarrow 7 \\ \hline & 0 \rightarrow 5 \end{array}$$

$\Rightarrow (5706)_8$

The largest power of 8 contained in 3014 is 512, apply with 3014 as dividend and 512 as a divisor we get $3014 = 512\times 4 + 454$

Now 454 lies b/w 64 & 512

The largest power of 8 we can use is 64, hence $454 = 7\times 64 + 6$

continuing the process until remainder becomes < 8, $6 = 6\times 1 + 0$

3014 =

$\therefore 3014 = 5(512) + 7(64) + 6(1)$

$= 5(8^3) + 7(8^2) + 0(8) + 6(8^0)$

$= 5706_8$

Example Prove that there are atleast $3\lfloor n/2 \rfloor$ primes in the range n through $n!$, where $n \geq 4$.

Proof Assume $n \geq 10$

Suppose n is even, say $n = 2k$, where $k \geq 5$ then

$n! = 1.2.3 \cdots (2k-2)(2k-1)n$

$= 2^k[1.2.3 \cdots (k-1)][1.3.5 \cdots (2k-1)]n$

$> 2^{k-1}(k-1)!\,2^{k+2}n$

$\geq 2^{k-1}.2^{k-1}.2^{k+2}n$, $k \geq 5$

$= 2^{3k}n$

A repeated application of Bertrand's conjecture shows there are atleast $3k = 3(n/2) = 3\lfloor n/2 \rfloor$ primes in the range n through $2^{3k}n$, that is, between n & $n!$

Case 2 : Suppose n is odd, say, $n = 2k+1$, where $k \geq 5$. Then

$n! = 1 \times 2 \times 3 \cdots (2k-1)(2k)n$

$= 2^k\, k!\,[1.3.5 \cdots (2k-1)]n$

$= 2^k.2^k.2^{k+2}n$, since $k \geq 5$

$> 2^{3k}n$

Thus, as before, there are at least $3k = 3\lfloor (n-1)/2 \rfloor$

$= 3\lfloor n/2 \rfloor$ primes in the range n through $2^{3k}n$, that is, b/w n & $n!$

Thus, in both cases result is true.