60 one-on

$^6P_n = 120$

$\frac{6!}{(6-n)!} = 120$

$\frac{\cancel{5!}\times 6}{(6-n)!} = \cancel{120}$

$6 = (6-n)!$

$\boxed{n=3}$

9(b) $|A| = 3$ $|B|$

$^3P_n = 60$

$\frac{3!}{(3-n)!} = 60$

$10(3-n)! = 1$

not possible

10(a) $A = \{1,2,3,4,5,6,7\}$, $B = \{w,x,y,z\}$

Find no. of onto from A to B

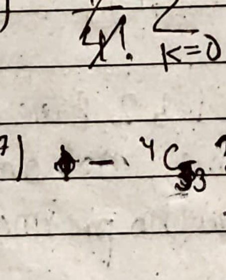
Ans To find no. of onto func^n, we use Sterling no. of 2nd kind

$S(m,n) = \frac{p(m,n)}{n!} = \frac{1}{n!}\sum_{k=0}^{n}(-1)^k\left({}^nC_{n-k}\right)(n-k)^m$

$S(7,4) = \frac{1}{4!}\sum_{k=0}^{4}(-1)^k\left({}^4C_{4-k}\right)(4-k)^7$

$= \frac{1}{4!}\left({}^4C_4(4^7) - {}^4C_3\,3^7 + {}^4C_2\,2^7 - {}^4C_1(1)^7\right)$

$= 240$



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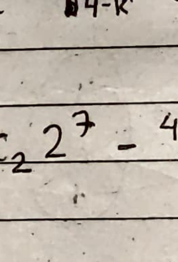


$1 \to 3 \to 2$

$1 \to 2$

Let A and B be finite sets with ~~cardinality of~~ $|B| = 3$ if there are 4096 rel^n from A to B, what is $|A|$?

$|A| = m$, $|B| = n$ $\Rightarrow$

$R = A \times B = 2^{mn}$
$m \times n$

$\Rightarrow 2^{mn} = 4096$

$2^{3n} = 4096$

$2^{3n} = 2^{12}$ $\qquad 64^2$

$3n = 12$ $\qquad 8^4$

$\boxed{n = 4}$ $\qquad 2^{34}$

Binary relations $R: A \to A \rightsquigarrow 2^{m \cdot m} = 2^{m^2}$
$|m| \; |m|$

$B \times A = 2^{mn}$

$B \to B = 2^{n^2}$

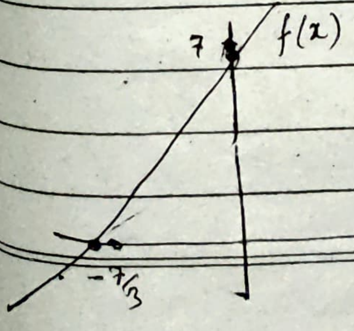
Q. $f: R \to R$ and $g: R \to R$ defined by $f(x) = 3x + 7 \; \forall x \in R$ and $g(x) = x(x^3 - 1) \; \forall x \in R$

Verify that f and g are one-one, if not provide reason

$\cancel{f(0)} = f(1) = 10$ $\qquad$ $g(0) = 0$

$f($ $\qquad$ $g(1) = 0$

One-one $\qquad$ not one one



~~Only for~~ sterling no. use $S(m,n) = \frac{1}{n!}$

Onto func^n = $n! \times$ Sterling no.

11 a) $A = \{1,2,3,4, \cancel{5,6}\}$, $B = \{1,2,3,4,5,6\}$

Find how many func^n from A to B

How many one-one?

$|A| = 4 = m$

$|B| = 6 = n$

Total no. of func^n from A to B $= n^m = 6^4 = 1296$

B to A $= m^n = 4^6 = 4096$

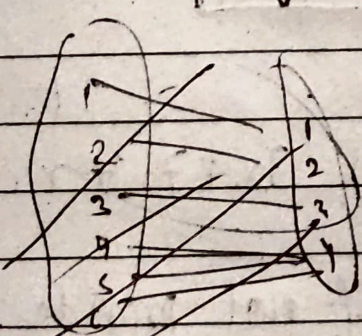
No. of one-one func^n from A to B $= \cancel{{}^mP_n = \frac{m!}{(m-n)!} = \frac{4!}{(4}}$

$= {}^nP_m = \frac{n!}{(n-m)!} = \frac{6!}{(6-4)!} = 360$

→ There exists no one-one func^n from B to A

no onto from B to A

Onto func^n from A to B



$S(n,m)$ = from $A \to B$

$S(6,4) = \sum_{k=0}^{4} (-1)^k \, ({}^4C_{4-k})(4-k)^6$

$= 1560$

$S(5,4) = \frac{240}{4!} = 10$

$= \frac{1}{4!} \sum_{k=0}^{n} (-1)^k ({}^4C_{4-k})(4-k)^5 = \frac{240}{4!} = 10$

$S(8,8) = 266$

$S(7,2) = 63$

12.a) $S(10,6)$, $S(8,4) = 1701$, $S(8,5) = 1050$, $S(8,6) = 266$

$$\boxed{S(m+1,n) = S(m,n-1) + n\,S(m,n)}$$

$S(10,6) = S(9,5) + 6S(9,6)$

$S(9,5) = S(8,4) + 5S(8,5)$
$= 1701 + 5(1050)$
$= 6951$

$S(9,6) = S(8+1,6) = S(8,5) + 6S(8,6)$
$= 1050 + 6(266) = 2646$

$S(10,6) = 6951 + 6(2646)$
$= 22827$

Rel^n → 30-28

Func^n → 10-12 → 10

Num T → 10

$S(5,4) = \frac{240}{4!} = 10$

$= \frac{1}{4!} \sum_{k=0}^{n} (-1)^k ({}^4C_{4-k})(4-k)^5 = \frac{240}{4!} = 10$

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$= 22827$

Rel^n → 30-28

Func^n → 10-12 → 10

Num T → 10

QUIZ APT, Module 1 to 4

OBT, Logic and one of these modules

Q

$S(m,n)$, $m \geq n$

$S(m,1) = 1$

$S(m,m) = m$

$P(8,6) = 6! \times S(8,6)$

$P(m,n) = n! \times S(m,n)$

14. There are ~~6~~ programmers who can assist eight executives. In how many of these are one to one?

How many onto?

$P(8,6) = 6! \times S(8,6)$

$= 720 \times 266$

$= 191520$

$$\begin{array}{r} 266 \\ 72 \\ \hline 532 \\ 1862\times \\ \hline 19152 \end{array}$$

(b)

Djikstra's
Tree

$f(x) = y = x+2$
$x = y-2$

$f^{-1}(x) = x-2$

$f(x) = x+2,\ g(x) = x^2$
$fog \rightarrow f(g(x))$
$gof \rightarrow g(f(x))$

$f(g(x)) = f(x^2) = x^2+2$
$g(f(x)) = g(x+2) = (x+2)^2$

LOGIC : Module new

Introduction - Logic is the science which deals with the principles of reasoning which were founded by George Boom b/w 1815 to 1864

Proposition - A ~~pro~~proposition is a statement, which in a given context is either true or false.

Truth value - The truth or the falsity of a proposition is called the truth value.
The propositions are always denoted by smaller values in the form of p, q, r etc.
If a propos^n is true, we denote it by T or 1
if false, by F or 0

Connectives - The words or phrases likes 'NOT', 'OR', 'and', 'if', 'if then', 'if and only if' are called as connectives

Simple proposition - Propos^n obtained by using only one connective.
Compound - by using one or more connectives
Components - Propos^n used in constructing a compound propos^n.

Negation : If p is true then ~p is false
p is false then ~p is true

p	~p
T	F
F	T

Conjunction - $p \wedge q$ is true only when both p and q are true

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$
T	T	T	T	T *
T	F	F	T	F
F	T	F	T	T *
F	F	F	F	T *

*

$p \vee q$ is false when both p and q are false

Implication : $p \rightarrow q$

Equivalence / biconditional : $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q$
T *
F
F
T *

Inverse - If $p \rightarrow q$ is the implication that is given, then the inverse of it will be $\sim p \rightarrow \sim q$.

T
T
F
T

onverse : $q \to p$ [T, T, F, T]

ontrapositive : $\sim q \to \sim p$ [T, F, T, T]

Tautology / ~~Pautology~~ : A compound compos^n is said to be a tautology if it is always true for all the possible values.

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

logical equivalence

$\sim(p \to q) \equiv p \wedge (\sim q)$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

Contradiction - if it is always false for all the values

Q. $p \vee \sim q$

$\sim($

Methods of Proof

i) Rules of inference / Transformation rules

Inference rule is the act of drawing a conclusion based on the form of premises interpreted as a function which takes premises, analyses their syntax and returns a conclusion.

The standard form is as follows

Premise #1

Premise #2

,

,

Premise #n

$\therefore$ Conclusion

No. of premises = 2^n

Rules of inference	Tautology	Name
① $\cfrac{p}{\therefore p \vee q}$	$p \vee (p \to q)$	Addition
② $\cfrac{p \wedge q}{\therefore p}$	$(p \wedge q) \to p$	Simplification
③ $\cfrac{p \quad q}{\therefore p \wedge q}$	~~$(p \wedge q) \to p$~~ $((p) \wedge (q)) \to p \wedge q$	Conjunction
$\cfrac{p \quad p \to q}{\therefore q}$	$p \wedge (p \to q) \to q$	Modus Ponens
$\cfrac{\sim q \quad p \to q}{\therefore \sim p}$	$(\sim q) \wedge (p \to q) \to \sim p$	Modus Tellons

$p \to q$ $q \to r$ $\therefore p \to r$	$(p \to q) \wedge (q \to r) \to (p \to r)$	Hypothetical Syllogism
$p \vee q$ $\sim p$ $\therefore q$	$((p \vee q) \vee (\sim p)) \to q$	Disjunctive Syllogism
$p \vee q$ $\sim p \vee r$ $\therefore q \vee r$	$[(p \vee q) \vee (\sim p \vee r)] \to (q \vee r)$	~~Low of~~ Resolution

p	q	$p \vee q$	$p \to q$	$p \vee (p \to q)$
T	T	T	T	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

Show that for any 2 propositions $(p \vee q) \vee (p \leftrightarrow q)$ is a tautology

p	q	$p \vee q$	$p \leftrightarrow q$	$(p \vee q) \vee (p \leftrightarrow q)$
T	T	T	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	T

Q $(p \vee q) \wedge (p \leftrightarrow q)$ → ~~no contradiction~~ contigency (T & F)

$(p \vee q) \wedge (p \leftrightarrow q)$
T
F
F
F

(i) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	I $p \to q$	II $q \to r$	I ∧ II III	IV $p \to r$	III → IV V
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

↑ Tautology

$p \to$ IV VI	I → IV VII	VI → VII VIII
T	T	T
F	F	T
T	T	T
F	T	T
T	T	T
T	T	T
T	T	T
T	T	T

↑ Tautology

$$\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$$

$p \vee \sim p$, $\sim(p \wedge q)$, $\leftrightarrow$

Negation, Conjunction, Disjunction, Conditional, Biconditional

Q. If p ~~is~~ : "This book is good"
q : "This book is costly"

Write down the following statements in symbolic form

(i) This book is good and costly : $p \wedge q$
(ii) This book is not good but costly : $\sim p \wedge q$
(iii) This books is cheap but good : $\sim q \wedge p$
(iv) neither good nor costly : ~~$\sim p \wedge \sim$~~ $\sim(p \wedge q)$
(v) If this book is good then it's ~~g~~costly : $p \rightarrow q$ $\downarrow$ $\sim p \vee \sim q$

Q. ~~Ra~~ ~~p~~ Ramu is poor but handsome : $\sim p \wedge q$

p : Ramu is rich
q : Ramu is handsome

⇒ Rich and handsome : $p \wedge q$
⇒ Either rich or handsome : ~~$p \vee q$~~ $(\sim p \wedge q) \vee (\sim q \wedge p)$
⇒ It is not true that Ramu is rich or handsome $\sim(p \vee q)$
⇒ Ramu is neither rich nor handsome $\sim(p \wedge q)$

Q p: You have flu.
q: You missed the final exam.
r: You passed the course

1. You have flu or you missed the exam or you passed

$p \vee q \vee r$

2. If you have flu, then you'll not pass the course or If you miss the final exam, you'll not pass the course

$(p \to \sim r) \vee (q \to \sim r)$

3. You have flu and you missed your exam or you'll not miss the exam and you passed the course

$(p \wedge q) \vee (\sim q \wedge r)$

Symbolic form:

1) Jack and jill went up the hill : $p \wedge q$

2) If either Jerry takes calculus or John takes algebra then Alexa takes English

p: Jerry takes calculus.
q: John takes algebra.
r: Alexa takes English

$(p \vee q) \to r$

3) The growth of the crop will be good if there is rain

p: There is rain
q: Growth of crop is good $(p \to q)$

3) Construct the truth table for $\sim(p \wedge (q \vee r))$.

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$\sim(p \wedge (q \vee r))$
T	T	T	T	T	F
T	T	F	T	T	F
T	F	T	T	T	F
T	F	F	F	F	T
F	T	T	T	F	T
F	T	F	T	F	T
F	F	T	T	F	T
F	F	F	F	F	T

$\sim(p \vee (q \wedge r)) \leftrightarrow (p \vee q) \wedge (p \vee r)$

$\sim(p \vee (q \wedge r))$	$p \vee q$	$p \vee r$	$\wedge$	
~~T~~ F	T	T	T	F
~~T~~ F	T	T	T	F
~~T~~ F	T	T	T	F
~~T~~ F	T	T	T	F
~~T~~ F	T	T	T	F
~~T~~ ~~F~~ T	T	F	F	~~T~~ F
T ~~F~~ T	F	T	F	~~T~~ F
~~F~~ T	F	F	F	F

$p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p)$

p	q	r	$q \leftrightarrow r$	$r \leftrightarrow p$
0	0	0	1	1
0	0	1	0	0
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	0	1
1	1	0	0	0
1	1	1	1	1

5b) Obtain inverse, converse and contrapositive of "Team India wins whenever Dhoni is captain"

Inverse : $\sim p \rightarrow \sim q$

Converse : $q \rightarrow p$

Contrapositive : $\sim q \rightarrow \sim p$

if — then

p : Team India wins
q : Dhoni is the captain

I. If team India does not win, then Dhoni is not the captain

Co. If Dhoni is the captain then Team India wins

C+. If Dhoni is not the captain, then Team India does not win.

6 a) "Home team wins whenever it rains"
p : Home team wins
q : It rains

I. Home team does not win if it does not rain

~~c~~ -- If home team does not win then it does not rain.

CV -- If it rains then it home team wins

Ct -- If it does not rain then home team does not win.

8 a) $\sim(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \to r))$

p	q	r	I $q \wedge r$	$\sim p \vee I$	II $p \vee q$	III $p \to r$	IV $II \wedge III$	$I \leftrightarrow IV$	V $q \to r$	VI $p \to V$	$\sim r$
0	0	0	0	~~0~~ 1	0	1	0	1	1	1	1
0	0	1	0	~~0~~ 1	0	1	0	1	1	1	0
0	1	0	0	~~0~~ 1	1	1	1	0	0	1	1
0	1	1	1 ~~0~~	0	1	1	1	1	1	1	0
1	0	0	0	0	1	0	0	1	1	1	1
1	0	1	0	0	1	1	1	0	1	1	0
1	1	0	0	0	1	0	0	1	0	0	1
1	1	1	1	0	1	1	1	1	1	1	0

b) $(p \to (q \to r)) \wedge ((\sim r \vee p) \wedge q)$

VII $\sim r \vee p$	IX $VII \wedge q$	$VIII \wedge IX$	X $II \wedge III$	XI $X \wedge V$	$XI \to r$
1	0	0	0	0	
0	0	0	0	0	
1	0	0	1	0	
0	0	0	1	1	
1	0	0	0	0	
1	0	0	1	1	
1	1	0	0	0	
1	1	1	1	1	

b) $((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r$

An argument is called valid if whenever all the hypothesis ~~or~~ is true, the conclusion is true. Consequently, showing that q logically follows from the hypothesis form $p_1, p_2 \ldots p_n$ is the same as showing the implication $(p_1, p_2 \ldots p_n) \to q$

However, a valid argument can lead to an incorrect conclusion if one or more false propositions are used within the argument

Eg S1) I play hockey and the next day I'm tired
S2: I use Whirlpool
Argue That I do not use a whirlpool

p: I play hockey
q: The next day I'm tired
r: I use Whirlpool
~r: I do not use a whirlpool.

$$\left.\begin{array}{l} p \to q \\ q \to r \end{array}\right\} \text{Hypothetical syllogism}$$

$\overline{(p \to r}$ } Since this is not true, it is not a valid statement

> p: Every student has an internet account
q: ~~Omar~~ does not have an internet acc.
r: Maggie has an internet acc.

p: If Sachin hits a century then he gets a free car
q: Sachin hits a century

$\therefore$ Sachin gets a free car

p: Sachin hits a century
q: He gets a free car

$$\left.\begin{array}{l} p \to q \\ \underline{\quad p \quad} \\ \therefore q \end{array}\right\} \text{Rule of Modus ponens}$$