FUNCTIONS

1 (a) R is not a function from A to B as 1 maps to both ~~-5~~ -1 and 0.

(b) Yes S is a function

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2 (a) $A = \{0, \pm 1, \pm 2, 3\}$ $f(x) = 3x^3 - 6x^2 + 10x + 29$

Ans) $f(0) = 29$ $\quad f(2) = \cancel{7}\ 49$ $\quad$ Range $= \{\cancel{-21, 5, 1, 3, 7, 19}\}$

$f(1) = 36$ $\quad f(-2) = \cancel{21} -39$

$f(-1) = -\cancel{5}10$ $\quad f(3) = \cancel{19}\ 86$ $\quad \{-39, 10, 29, 36, 49, 86\}$

2 (b) $A = \{0, \pm 1, \pm 2, 3\}$ $f(x) = Ax^3 - 2x^2 + 3x + 1.$

$x^4 - 2x^2 + 3x + 1$

Find range

Ans) $A = 0$ $\quad f(0) = 1$

$A = -1$ $\quad f(-1) = \cancel{1} - \cancel{2} \cancel{4} - 3 \cancel{+1} \Rightarrow -3$

$A = 1$ $\quad f(1) = 3$

$A = 2$ $\quad f(2) \Rightarrow 16 - 8 + 6 + 1 \Rightarrow 15$

$A = -2$ $\quad f(-2) \Rightarrow 16 - 8 - 6 + 1 \Rightarrow 2$

$A = 3$ $\quad f(3) \Rightarrow 81 - 18 + 9 + 1 \Rightarrow 73$

3(a, b) check from pdf

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4 (a) $A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4, 5\}$ One one, Onto tell?

(i) $f = \{(1,1), (2,3), (3,4)\}$ (ii) $g = \{(1,1)(2,3)(3,3)\}$

(i) One one $\quad$ Not one one

Not onto

#5

5 (a) $A = \{1, 2, 3, 4, 5\}$ $B = \{w, x, y, z\}$

$f = \{(1, w)\ (2, x)\ (3, x)\ (4, y)\ (5, y)\}$.. Find image of the following subsets of A under f $A\_1 = \{1\}$, $A\_2 = \{1, 2\}$, $A\_3 = \{1, 2, 3\}$, $A\_4 = \{2, 3\}$, $A\_5 = \{2, 3, 4, 5\}$

Ans] $f(A\_1) = w$, $f(A\_2) = \{w, x\}$, $f(A\_3) = \{w, x\}$, $f(A\_4) = \{x\}$, $A_5 = \{x, y\}$

(b) $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$. Determine $f(0)$, $f(-1)$, $f(5/3)$, $f(-5/3)$

Ans] $f(0) \Rightarrow 1$ $f(5/3) \Rightarrow -2$

$f(-1) \Rightarrow 4$ $f(-5/3) = 6$

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6 (a) Let Z denote the set of all integers. A function $h: Z \times Z \to Z$ is defined by $h(x, y) = 2x + 3y$. Find $h(0, 0)$, $h(-3, 7)$ $h(2, -1)$ and $h(A)$, where $A = \{(0, n) \mid n \in Z^+$

Ans) $h(0, 0) = 0$, $h(-3, 7) = 15$, $h(2, -1) = -1$, $h(A) = 3n, n \in$

$A = \{(0, n) \mid n \in Z^+)$

6 (b) $f(a) = a + 1$. Find whether f is one-one or onto

(Ans) $f(-3) = -2$; $f(2) = 3$.

$f(-2) = -1$

$\vdots$

$f(0) = 1$

so one to one as every element in a has distinct image in B.

Onto as some element of B is image of some element in A.

7 (a) ~~(ii)~~ If $f: A \to B$, then determine if f is one to one or onto

(i) $A = \mathbb{R}$, $B = \{x \mid x$ is real no and $x \geq 0\}$; $f(a) = |a|$

Ans) It is onto because $f(\mathbb{R}) = |\mathbb{R}|$, $B \geq 0$.

Not one one because for every $A \gtrless 0$ ~~$f(A)$~~ $f(-A) = f(A) = A$

(b) $A = \{1, 2, 3, 4\}$ $B = \{a, b, c, d\}$ $f = \{(1, a)(2, a)(3, d)(4, c)\}$

Ans) Not one one as 1 and 2 both have image as a in B.

Not onto as not all element of B are image of some element in A.

If f is one one from $A \to B$, then show that $|A| \leq |B|$.

8 (a) If f is one one and B has m elements.

Ans) lets assume ~~a~~ A has n element and B has m elements.

f is one-one so each element in A has unique element b in B such that $f(a) = b$. Since f is one-to-one, every element of A maps to distinct element in B.

If $|B| < |A|$. This would mean there are more element in A than B and since f is one-one, it is not possible to map all elements of $A \to B$.

Therefore for f to be one-one $|A| \leq |B|$.

8 (b) 120 one to one function. $|A| = 6$, what is $|B|$.

Ans) ${}^{m}P_{n} \Rightarrow {}^{6}P_{n} = 120 \Rightarrow \frac{6!}{(6-n)!} = 120 \Rightarrow \frac{\cancel{720}^{6}}{\cancel{120}} = (6-n)!$

$3 = n$

9 (a) $|A| = m$, $|B| = n$.

For 1st ele → n choices

For 2nd ele → $(n-1)$ choices

⋮

For last ele → $n-(m-1) = n-m+1$ choice

No. of ~~fun~~ one-to-one func → $n \times (n-1) \times (n-2) \cdots \times (n-m+1)$

→ ${}^{n}P_{m}$

9 (b) ~~If~~ 60 one to one func^n. $|A| = 3$, $|B| = ?$

${}^{3}P_{n} = 60$. Not possible.

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10 (a) $A = \{1,2,3,4,5,6,7\}$, $B = \{w,x,y,z\}$

Ans) $S(m,n) = \sum_{k=0}^{n} (-1)^k ({}^{n}C_{n-k}) (n-k)^m$

$m > n$

$S(7,4) \rightarrow [1(4)^7 - 4\times3^7 + 6\times2^7 - 4\times1^7 + 0]$

→ 8400

10 (b) $A = \{1,2,3\}$ $B = \{w,x,y,z\}$: $R = \{(1,w)(2,x)(3,x)\}$

$R' = \{(1,w)(2,x)\}$. Which is these relations from A to B are functions from A to B }

Ans) R is a function. R' is not.

ⓐ $A = \{1,2,3,4\}$ $B = \{1,2,3,4,5,6\}$

No of functions from $A \to B$ = $6^4 = 1296$

No of functions from $B \to A$ ⇒ $4^6 = 4096$

No of 1-1 function from $A \to B$ ⇒ 360

No of 1-1 ~~to~~ function from $B \to A$ ⇒ Not possible.

No of onto f^n from $A \to B$ ⇒ 0

No of onto f^n from $B \to A$ ⇒ $S(6,4) = 1560$

2ⓐ $S(10, \cancel{4}6)$ using $S(8,4) = 1701$, $S(8,5) = 1050$,
$S(8,6) = 266$. $S(m+1,n) = S(m,n-1) + n\,S(m,n)$

Ans) $S(10,6) = S(9+1,6) ⇒ S(9,5) + 6S(9,6)$

$S(9,5) ⇒ S(8+1,5) ⇒ S(8,4) + 5S(8,5)$

$S(9,5) = 1701 + 5 \times 1050 = 6951$

$S(9,6) = S(8+1,6) = S(8,5\cancel{6}) + 6\,(8,6)$

⇒ $1050 + 6 \times 266$

⇒ 2646

$S(10,6) = 6951 + 6(2646) = 22827$

12 6

13 a $S(5,4) = 10$

$S(8,6) = 266$

$S(7,2) = 63$