



COURSE : Mathematical Structures

COURSE CODE : 21MAT41A

MODULE – 3: Functions

Question Bank

Questions

Q.No	Questions
1.	<p>(a) Let $A = \{1,2,3\}$ and $B = \{-1,0\}$ and R be a relation from A to B defined by $R = \{(1,-1), (1,0), (2,-1), (3,0)\}$. Is R a function from A to B?</p> <p>(b) Let $A = \{1,2,3\}$ and $B = \{-1,0\}$ and S be a relation from A to B defined by $S = \{(1,-1), (2,-1), (3,0)\}$. Is S a function?</p>
2.	<p>a) Let $A = \{0, \pm 1, \pm 2, 3\}$. Consider the function $f: A \rightarrow B$ defined by $f(x) = 3x^3 - 6x^2 + 10x + 29$, for x belonging to A. Find the range of f.</p> <p>b) Let $A = \{0, \pm 1, \pm 2, 3\}$. Consider the function $f: A \rightarrow R$ (where R is the set of all real numbers) defined by $f(x) = Ax^3 - 2x^2 + 3x + 1$ for $x \in A$. Find the range of f.</p>
3.	<p>a) Explain the types of Functions with example</p> <p>b) Define a function with an example and Cartesian product .</p>
4.	<p>Let $A = \{1,2,3\}$ and $B = \{1,2,3,4,5\}$. Find whether the following functions from A to B are</p> <p>a) One-to-one (b) Onto</p> <p>i) $f = \{(1,1), (2,3), (3,4)\}$ ii) $g = \{(1,1), (2,3), (3,3)\}$</p>
5.	<p>(a). For $A = \{1,2,3,4,5\}$ and $B = \{w, x, y, z\}$ a function $f: A \rightarrow B$ is defined by $f = \{(1,w), (2,x), (3,x), (4,y), (5,y)\}$. Find the image of the following subsets of A, under f $A_1 = \{1\}, A_2 = \{1,2\}, A_3 = \{1,2,3\}, A_4 = \{2,3\}, A_5 = \{2,3,4,5\}$</p> <p>(b). Let $f: R \rightarrow R$ be defined by</p> $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$ <p>Determine $f(0), f(-1), f(5/3), f(-5/3)$</p>
6.	<p>a) Let z denote the set of all integers. A function $h: z \times z \rightarrow z$ is defined by $h(x, y) = 2x + 3y$. Find $h(0,0), h(-3,7)h(2, -1)$ and $h(A)$, where $A = \{(0, n) n \in \mathbb{Z}^+\}$</p> <p>b) Let $f: z \rightarrow z$ be defined by $f(a) = a + 1$ for $a \in z$. Find whether f is one-to-one or onto (or both or neither)</p>
7.	<p>(a) If $f: A \rightarrow B$, then determine if f is one to one or onto</p> <p>(i) $A = R, B = \{x x \text{ is real number and } x \geq 0\}; f(a) = a$</p> <p>(b) $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}, f = \{(1,a), (2,a), (3,d), (4,c)\}$</p>



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a) Let A and B be finite sets and f is a function defined from A to B , then if f is one to one, then show that $|A| \leq |B|$

b) If there are 120 one-to-one functions from A to B and $|A| = 6$, what is $|B|$?

9. a) Let A and B be finite sets with $|A| = m$ & $|B| = n$. Find how many one-to-one functions are possible from A to B

b) If there are 60 one-to-one functions from A to B and $|A| = 3$, what is $|B|$?

10. a) Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{w, x, y, z\}$. Find the number of onto functions from A to B

b) For $A = \{1, 2, 3\}$ and $B = \{w, x, y, z\}$, let $R = \{(1,w), (2,x), (3,x)\}$ and $R' = \{(1,w), (2,x)\}$. Which is these relations from A to B are functions from A to B ?

11. a) Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. Find how many functions are there from A to B . How many of these are one-to-one? How many are onto?

b) Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. Find how many functions are there from B to A . Find how many of these are onto? How many are one-to-one?

12. a) Evaluate $S(10,6)$, using $S(8,4) = 1701$, $S(8,5) = 1050$ and $S(8,6) = 266$

b) Determine the minimum number of elements to be selected from the set $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ such that the sum of two of them is 10.

13. a) Evaluate $S(5,4)$ and $S(8,6)$.

b) Evaluate $S(7,2)$

14. a) There are six programmers who can assist eight executives. In how many ways can the executives be assisted so that each programmer assists at least one executive?

b) Show that among 13 persons there are at least two persons who are born in the same month.

15. a) Find the number of ways distributing four distinct objects among three identical containers, with some container(s) possibly empty.

(b) Let A and B be finite sets with $|A| = 10$ and $|B| = 15$. Find how many functions are possible from A to B ?

16. a) Find the number of ways of distributing 6 objects among 4 identical containers, with some container(s) possibly empty

b) What is the minimum number of students required in a class to be sure that at least five will receive the same grade if there are four possible grades say A, B, C and D?



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| 1. | <p>(a). Seven members of a family have total of Rs.2886 in their pockets. Show that at least one of them must have at least Rs.416 in his pocket.</p> <p>(b). If 9 books are to be kept in 4 shelves, show that there must be at least one shelf which contains at least 3 books.</p> |
| 2. | <p>(a). How many people must you have to guarantee that at least 9 of them will have birthdays in the same day of the week.</p> <p>(b). Show that if 30 dictionaries in a library contain a total of 61327 pages, then one of the dictionaries must have at least 2045 pages.</p> |
| 3. | <p>a) Consider the functions f and g defined by $f(x) = x^3$ and $g(x) = x^2 + 1, x \in R$ find gof, fog, f^2 and g^2</p> <p>b) Show that among any group of five integers (not necessarily consecutive), there are 2 integers with the same remainder when divided by 4.</p> |
| 4. | <p>i) Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be any two functions. Then show that the following are true.</p> <ul style="list-style-type: none">i) If f and g are one-to-one, so is gof.ii) If gof is one-to-one, then f is one-to-one.iii) If f and g are onto, so is gof.iv) If gof is onto, then g is onto. <p>ii) If 20 candidates appear in a competitive exam, then show that there exists at least two among them, who roll number differ by a multiple of 19.</p> |