14(a) $n \leq 3000$ divisible by 3, 5, 7

$3 \Rightarrow 1000$ $\quad$ $3 \cap 5 \Rightarrow 200$ $\quad$ $3 \cap 5 \cap 7 \Rightarrow 28$

$5 \Rightarrow 600$ $\quad$ $3 \cap 7 \Rightarrow 142$

$7 \Rightarrow 428$ $\quad$ $5 \cap 7 \Rightarrow 85$

142
~~2000~~
24

$$n(3 \cup 5 \cup 7) = n(3) + n(5) + n(7) - (n(3 \cap 5) + n(3 \cap 7) + n(5 \cap 7)) + n(3 \cap 5 \cap 7)$$

$$= 1629$$

(b) $n \leq 2076$ divisible by neither 4 nor 5

$$\Rightarrow 2076 - (519 + 415 - 103)$$

$$= 1245$$

18(a) Prove every integer ≥ 2 has a prime factor

A) We shall prove this result by using the concept of strong induction which says that we assume the statement holds for all values preceding k

By strong induction, the given statement is clearly true if $n = 2$.

Now assume that it is true for every positive integer $n \leq k$ where $k \geq 2$.

Consider the integer $k+1$

Case 1: If $k+1$ is a prime no., then $k+1$ is a prime factor of itself

Case 2: If $k+1$ is not a prime, then $k+1$ is a composite no. so it must have a factor $d \leq k$.

Then by inductive hypothesis, d has a prime factor p so p is a factor of $k+1$

Thus, by strong induction, the statement is true for every integer ≥ 2 i.e. every integer ≥ 2 has a prime factor

15(a) Express 10110_2 in base ten

$\Rightarrow (2^4 + 2^2 + 2^1) = 22$

(b) $3ABC_{16}$

As the bases go higher than 10, we assign values for A as 10, B - 11, C - 12 and so on

$\Rightarrow (3 \times 16^3) + (10 \times 16^2) + (11 \times 16^1) + (12 \times 16^0) =$

$\Rightarrow (58 \times 16^2) + 2176 + 12 = 15036$

16 (a) Express 15036 in hexadecimal system

$$\begin{array}{r|l} 16 & 15036 \\ 16 & 939 \quad 12 \\ & 58 \quad 11 \\ & 3 \end{array}$$

$15036 \,(939$
$\underline{15024}$
12

$\Rightarrow 15036 = 16 \cdot 939 + 12$

$16 \overline{)939} \, 58$
$\underline{928}$
11

$939 = 16 \times 58 + 11$

$58 = 16 \times 3 + 10$

$3 = 16 \times 0 + 3$

$= 3ABC$

(b) 3014

The largest power of 8 contained in 3014 is 512, apply ... with 3014 as dividend

$$\begin{array}{r|l} 8 & 3014 \\ \hline 8 & 376 \rightarrow 6 \\ \hline 8 & 47 \rightarrow 0 \\ \hline 8 & 5 \rightarrow 7 \\ \hline & 0 \rightarrow 5 \end{array}$$

$\Rightarrow (5706)_8$

and 512 as a divisor we get $3014 = 512 \times 4 + 454$

Now 454 lies b/w 64 & 512

The largest power of 8 we can use is 64, hence $454 = 7 \times 64 + 6$ continuing this process until remainder becomes < 8, $6 = 6 \times 1 + 0$

3014 =

$\therefore 3014 = 5(512) + 7(64) + 6(1)$

$= 5(8^3) + 7(8^2) + 0(8) + 6(8^0)$

$= 5706_8$

Example Prove that there are atleast $3\lfloor n/2 \rfloor$ primes in the range n through $n!$, where $n \geq 4$.

Proof Assume $n \geq 10$

Suppose n is even, say $n = 2k$, where $k \geq 5$ then

$n! = 1.2.3 \ldots (2k-2)(2k-1)n$

$= 2^k[1.2.3 \ldots (k-1)][1.3.5 \ldots (2k-1)]n$

$> 2^{k-1}(k-1)!\, 2^{k+2} n$

$\geq 2^{k-1}.2^{k-1}.2^{k+2} n$, $k \geq 5$

$= 2^{3k} n$

A repeated application of Bertrand's conjecture shows there are atleast $3k = 3(n/2) = 3\lfloor n/2 \rfloor$ primes in the range n through $2^{3k} n$, that is, between n & $n!$

Case 2: Suppose n is odd, say, $n = 2k+1$, where $k \geq 5$. Then

$n! = 1 \times 2 \times 3 \ldots (2k-1)(2k)n$

$= 2^k k! [1.3.5 \ldots (2k-1)]n$

$= 2^k.2^k.2^{k+2} n$, since $k \geq 5$

$> 2^{3k} n$

Thus, as before, there are at least $3k = 3\lfloor (n-1)/2 \rfloor$

$= 3\lfloor n/2 \rfloor$ primes in the range n through $2^{3k} n$, that is, b/w n & $n!$

Thus, in both cases result is true.

Let A & B be two non-empty sets, then the realation from A to B is a subset of $A \times B$ i.e. $R \subseteq (A \times B)$

If $R \subseteq (A \times B)$ is a relation from $A \times B$ and if $(a,b) \in R$ then we say that a R-related to b i.e aRb

Domain & Range of a relation

If $R \subseteq (A \times B)$ then the domain (R) $= \{a : (a,b) \in R\}$

Range (R) $= \{b : (a,b) \in R\}$

Inverse Relation

If $R \subseteq (A \times B)$ be a relation from A to B then the inverse of R denoted by R^{-1} is a relation from B to A which is defined as $R^{-1} = \{(b,a) \mid \begin{matrix} b \in B \\ a \in A \end{matrix}\}$

or $\mid (a,b) \in R$

Note : The domain and range of R^{-1} is the range and the domain of R respectively.

Identity Relation

A relation R in a set A is said to be an identity relation denoted by I_A if $I_A = \{[x,y] / \begin{matrix} x = y \\ x,y \in I_A \end{matrix}\}$

Let $A = \{1,2,3\} \Rightarrow I_A = \{(1,1)\ (2,2)\ (3,3)\}$

To represent a relation as a tabular notation, digraph representation, matrix representation

1a) Let $A = \{1,2\}$ and $B = \{p,q,r,s\}$ write down

- $R : A \to B = \{(1,q), (1,r), (2,p), (2,s)\}$

- matrix of R

$$M_R = \begin{array}{c|cccc} & p & q & r & s \\ \hline 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{array}$$

b) Consider $R: X \to Y$; $X = \{1,2,3\}$, $Y = \{8,9\}$

$R = \{(1,8)(2,8)(1,9)(3,9)\}$

find the complement of R

$X \times Y = \{(1,8)(1,9)\ (2,8)(2,9)\ (3,8)\ (3,9)\}$

$R^c = \{(2,9), (3,8)\}$

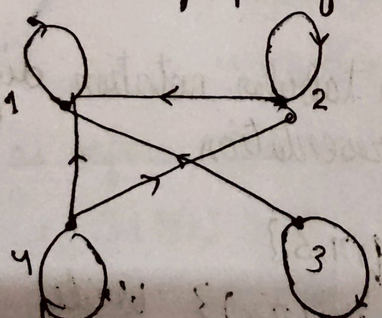
2) Let $A = \{1,2,3,4\}$ and let R be the relation on A defined by $x R y$ if and only if "x divides y" written $x | y$

a) Write down R as a set of ordered pairs

$A \times A = \{(1,1)(1,2)(1,3)(1,4)\ (2,1)(2,2)(2,3)(2,4)$
$(3,1)(3,2)(3,3)(3,4)\ (4,1)(4,2)(4,3)(4,4)\}$

$R = \{(1,1)(2,1)(2,2)(3,1)(3,3)(4,1)(4,2)(4,4)\}$

b) Draw the digraph of R



c) Determine indegrees & outdegrees

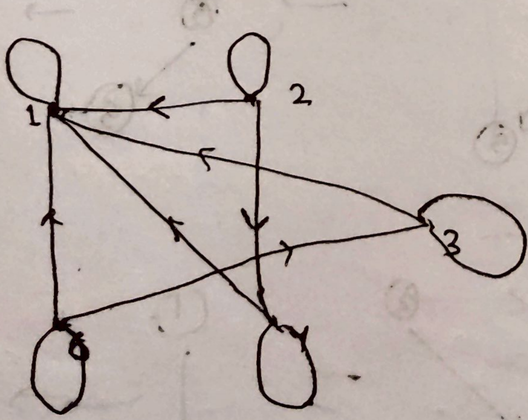
	$\deg v^+$	$\deg v^-$
1	4	1
2	2	2
3	1	2
4	1	3

3(a) Let A = {1, 2, 3, 4, 6} and R be a relⁿ on A defined by aRb iff a is a multiple of b. Represent ~~rela~~ relⁿ R as a matrix and draw digraph

(i) A×A = {(1,1) (1,2) (1,3) (1,4) ~~(1,5)~~ (1,6)
(2,1) (2,2) (2,3) (2,4) ~~(2,5)~~ (2,6)
(3,1) (3,2) (3,3) (3,4) ~~(3,5)~~ (3,6)
(4,1) (4,2) (4,3) (4,4) ~~(4,5)~~ (4,6)
~~(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)~~
(6,1) (6,2) (6,3) (6,4) ~~(6,5)~~ (6,6) }

R = { (1,1) (~~1,2~~ 2,1) (2,2) (3,1) (3,3) (4,1) (4,2) (4,4)
~~(5,1) (5,5)~~ (6,1) (6,2) (6,3) (6,6) }

$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & \cancel{5} & 6 \\ \hline 1 & 1 & 0 & 0 & 0 & \cancel{0} & 0 \\ 2 & 1 & 1 & 0 & 0 & \cancel{0} & 0 \\ 3 & 1 & 0 & 1 & 0 & \cancel{0} & 0 \\ 4 & 1 & 1 & 0 & 1 & \cancel{0} & 0 \\ \cancel{5} & \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{1} & \cancel{0} \\ 6 & 1 & 1 & 1 & 0 & \cancel{0} & 1 \end{array}$$



b) Show that I_A is eq.

I_A = {(1,1) (2,2) (3,3) (4,4) (6,6)}

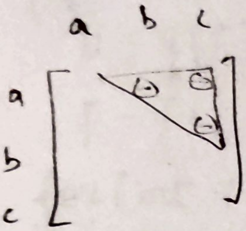
4. $M_R = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$

$A = \{a, b, c, d\}$

$B = \{1, 2, 3\}$

$A \times B = \{ (a,1) (a,2) (a,3)$
$(b,1) (b,2) (b,3)$
$(c,1) (c,2) (c,3)$
$(d,1) (d,2) (d,3) \}$

$R = \{ (a,1), (a,3) (b,1) (b,2)$
$(c,3) (d,1) \}$

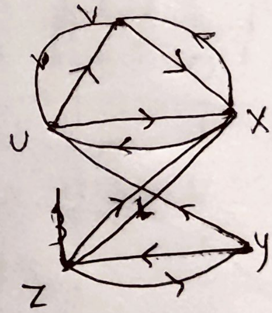


4(b) Let $A = \{u, v, x, y, z\}$ and R be a relation on A whose matrix is a given below. Determine R and the digraph of matrix

All the matrix entities do not classify to be reflexive, symmetric and transitive

$\begin{array}{c|c} & \begin{array}{ccccc} u & v & x & y & z \end{array} \\ \begin{array}{c} u \\ v \\ x \\ y \\ z \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$

$R = (u,2) (u,3) (v,1)$



Diagonal entries are zero, therefore not reflexive

Path in a relation

Let R be a relation on a set A, a path of length n in ~~are~~ R from ~~A~~ a to b is a finite sequence given by $a, x_1, x_2 \ldots x_{n-1}, b$ which starts the initial vertex a and ends with the terminal vertex b s.t. $aRx_1, x_1Rx_2, \ldots x_{n-2}Rx_{n-1}, x_{n-1}Rb$.

· A path of length n has n+1 elements which are distinct

1) R^n is the relation that is defined on the set A which is of

length n, between the vertices a to b and is denoted by $aR^n b$.

2) R^∞ on a set A is called a connectivity relation for R and it is defined on some path from a to b in R.
The length of the path depends on a and b

3) R^* on a set A is called the reachability relation in R and R^* is defined as aR^*b iff $a=b$ or $aR^\infty b$, i.e. b is reachable from a if either $b=a$ or there exists some path from a to b.

Computation of the matrix (R^n, R^∞ & R^*)

Let R be a relation on a finite set A then the matrix of R^n is defined as $M_{R^n} = M_R \times M_R \times \text{---} M_R$ n times only for $n \geq 2$

② $M_{R^\infty} = M_R \vee M_{R^2} \vee M_{R^3} \vee \text{---}$

③ The matrix of R^*, $M_{R^*} = I_n \vee M_{R^\infty}$ where I_n is the $n \times n$ is the identity matrix

$\cup \rightarrow$ Union, $M_R \vee M_S$ or

$\cap$ • Int, $M_R \wedge M_S$ and

Q. from the rel^n $R = \{(1,2)(2,4)(3,2)(3,5)(5,4)\}$. find M_R

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Relations

5. Let $A = \{1,2,3,4\}$, $B = \{w,x,y,z\}$, $C = \{5,6,7\}$

A to B, $R_1 = \{(1,x)\ (2,x)\ (3,y)\ (3,z)\}$

B to C, $R_2 = \{(w,5)\ (x,6)\}$

~~R_3~~ $R_3 = \{(w,5)\ (w,6)\}$

$$M_{R_1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_{R_2} = \begin{array}{c} \\ w \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccc} 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_{R_3} \quad \begin{array}{c} \\ w \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccc} 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$R_1 \circ R_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{4\times4} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{4\times3}$$

$A \times C$

$$M_{R_1 \circ R_2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{4\times3} \end{array}$$

c) $R(R_p \circ R_2) = \{(1,6),(2,6)\}$.

$R_1 \circ R_3$

$$M_{R_1 \circ R_3} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_{R_1 \circ R} = \{\phi\}$

6. a) For relation R_1 and R_2, $R_1 = \{(1,x)(2,x)(3,y)(3,z)\}$

$R_2 = \{(w,5),(x,6)\}$

$M(R_1)$, $M(R_2)$, $M(R_1 \circ R_2)$

$$M(R_1) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad 4 \times 5$$

$$M(R_2) = \begin{array}{c} \\ w \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$R_1 o R_2 = \{ (1,6)\ (2,6) \}$

$R_1 o R_2 = \{ x_{ik}$ if $(x_{ij} \in R_1, x_{jk} \in R_2) \}$.

$$M_{R_1 o R_2} = \begin{array}{c|c} & \begin{array}{ccc} 5 & 6 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

6 b) $A = \{1,2,3,4\}$, $R = \{(1,1)(1,3)(3,2)(3,4)(4,2)\}$

$S = \{(2,1),(3,2),(3,4)(4,1)\}$

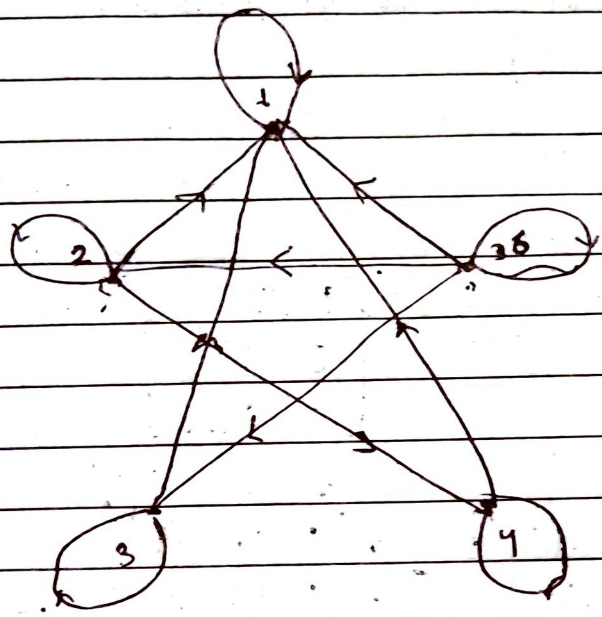
$$RoS = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$SoR = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

7(b) $A = \{1,2,3,4,6\}$ R be a rel on A, aRb iff a is a multiple of b.

$A \times A = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ 5\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ 5\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ 5\ (4,6)$
~~5,1 5,2 5 3 4 5 6~~
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ 5\ (6,6) \}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (6,6)$
$(2,1)\ (3,1)\ (4,1)\ (6,1)\ (4,2)$
$(6,2)\ (6,3)\ \}$



$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Let R and S be any two relations defined from $R: A \to B$ & $S: B \to C$, where A, B, C be any non-empty set then the composition of the relations R and S denoted by $R \circ S$ is defined as the set of all elements containing A, C s.t. $A, C \in R$ & $(b,c) \in S$.)

$R \circ S = \{(a,c) \mid (a,b) \in R$ & $(b,c) \in S$

The composition of R with itself is denoted as the power of R i.e $R \circ R = R^2$

Similarly $R \circ R \circ R = R^3$ & $R \circ R \circ R \circ \ldots \circ R$ (n times) $= R^n$

1) $R \circ S \neq S \circ R$
2) $(R^{-1})^{-1} = R$
3) $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$
4) The compos^n of relations is an associate
$(R \circ S) \circ T = R \circ (S \circ T)$

8. a) $A = \{a, b, c\}$

$$M_R = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$R \circ S \Rightarrow \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} = \begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 0 & 1 \\ b & 1 & 1 & 1 \\ c & 0 & 1 & 1 \end{array}$$

$R \circ S = \{(a,a), (a,c), (b,a), (b,b), (b,c), (c,b), (c,c)\}$

7 a) If $A = \{1, 2, 3, 4\}$, $R = \{(1,2), (1,3), (2,4), (4,4)\}$
$S = \{(1,1), (1,2), (1,3), (1,4), (2,3), (2,4)\}$

~~$M_{R \circ S}$~~ $R \circ S = \{(1,3), (1,4)\}$

$S \circ R = \{(1,2) (1,3) (1,4) (2,4)\}$

$R^2 = \{(1,4), (2,4), (4,4)\}$

$S^2 = \{(1,1) (1,3) (1,4), (1,2)\}$

9(a) $A = \{1,2,3,4\}$, $R = \{(1,2)(1,3)(2,4)(3,2)(3,3)(3,4)\}$

$R^2 = \{(1,4)(1,2)(1,3)(1,4)(3,3)(3,4)\}$

$R^3 =$

8(a) $R = \{(a,a),(a,c)(b,a)(b,b)(b,c)(c,b)\}$
$S = \{(a,a),(b,b)(b,c),(c,a)(c,c)\}$

$S \circ R = \{(a,a)(a,c)(b,a)(b,b)(c,c)(c,b),(c,a)\}$

$R \circ R = \{(a,a)(a,b)(a,c)(b,a)(b,b)(b,c)(c,a)(c,b)(c,c)\}$

$S \circ S = \{(a,a)(b,a)(b,b)(b,c)(c,a)(c,c)\}$ Matrix must

(b) R: aRb iff $ab > 0$

$\Rightarrow Z = \{-a \ldots 0, 1, 2 \ldots\}$

(i) for reflexive : for any $a \in Z - \{0\}$ $(a,a) \in Z$
s.t. $aRa \Rightarrow a.a > 0$
hence reflexive

(ii) for symmetric : for any $a,b \in Z$ $(a,b) \in Z$
$\Rightarrow aRb$ iff a&b are both positive
or a&b are both negative
$\Rightarrow ab > 0$

(iii) for any $(b,a) \in Z$
bRa iff b&a are both positive $\in Z$
or b&a are both negative $\in Z$
$ba > 0$ or $ab > 0$

Hence symmetric

(iii) for transitive
for any (a,b) & $(b,c) \in Z$

case I $\begin{cases} \Rightarrow aRb \text{ iff } a \& b \geq 0 \in Z \\ bRc \text{ iff } b \& c \geq 0 \in Z \end{cases}$ aRc iff $a \& c > 0 \in Z$

case II $\begin{cases} aRb \text{ iff } a \& b < 0 \in Z \\ bRc \text{ iff } b \& c < 0 \in Z \end{cases}$ aRc iff $a \& c < 0 \in Z$

$\therefore$ from ③ & ④ $\Rightarrow aRc$ iff $ac > 0$, Hence transitive

Hence the given set of Z, under aRb iff $ab > 0$ is an equivalence rel^n.

Q. On the set of positive integers Z^+, $R = aRb$ iff a/b.

antisymmetric rel^n and a poset

reflexive, antisymmetric & transitive

42
20

Theorem

If R is a set relation on a set A, then R is transitive iff $R^2 \in R$

Let R be a transitive relation and let $(x,z) \in R^2$. then by the definition of R^2, there exists an element y belonging to A st. $(x,y) \in R$ and $(y,z) \in R$
Since R is transitive, it implies $(x,z) \in R$
This implies $R^2 \subseteq R$
Conversely,
If $R^2 \subseteq R$, then we need to show that R is transitive.
Let (x,y) and (y,z) belong to R, then by definition of R^2, we get $(x,z) \in R^2$.
$R^2 \subseteq R \Rightarrow (x,z) \in R$
$\Rightarrow$ R is transitive

Note : In terms of matrices, we note that the relation R is transitive if $M_{R^2} + M_R = M_R$
where M_R is the matrix of the relation R

1) $M_{R \cup S} = M_R \vee M_S$ { Join ($\vee$ = or) }
2) $M_{R \cap S} = M_R \wedge M_S$ } Intersection ($\wedge$ = and).
(Meet)

If $A = \{1,2,3,4\}$ and $R = \{(1,1)(1,2)(2,3)(3,4)\}$
$S = \{(3,1),(2,4),(4,4),(1,4)\}$
$\therefore R \cup S = \{(1,1)(1,2)(2,3)(3,4)(3,1)(2,4)(4,4)(1,4)\}$

$$M_{R \cup S} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_{R \cap S} = \{\emptyset\} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

not reflexive, antisymmetrical

THEOREM

Let R and S be any two relations on a set A, then

~~(i) If R is reflexive iff $\bar{R}$ is irreflexive~~

~~(i) If R is reflexive, so is R^c~~

(i) If R and S are reflexive, so are $R \cap S$ and $R \cup S$

(ii) symmetric, so are $R \cap S$ and $R \cup S$

(iii) antisymmetric, so is $R \cap S$

(iv) transitive, so is $R \cap S$

(i) Given that R and S are two relations defined on the set A and let R and S be reflexive.

$\forall a \in A \Rightarrow (a,a) \in R$ & $(a,a) \in S$

$\Rightarrow (a,a) \in R \cup S$

$\Rightarrow (a,a) \in R \cap S$

Hence $R \cup S$ and $R \cap S$ are reflexive

~~Let $(a,b) \in R \Rightarrow$~~

(ii) Let $(a,b) \in R \Rightarrow (b,a) \in R$

also, let $(a,b) \in S \Rightarrow (b,a) \in S$

$\Rightarrow (a,b) \in R \cup S \Rightarrow (b,a) \in R \cup S \Rightarrow R \cup S$ is symmetric

$(a,b) \in R \cap S \Rightarrow (b,a) \in R \cap S \Rightarrow R \cap S$ is symmetric

(iii) If R and S are antisymmetric i.e. if $(a,b) \in R$ & $(b,a) \in R$

$\Rightarrow a = b$

if $(a,b) \in S$ & $(b,a) \in S \Rightarrow a = b$

this implies, if (a,b) & $(b,a) \in R \cap S \Rightarrow a = b$

if both R & S are anti-symm. then $R \cup S$ need not be anti symmetric

(iv) If R and S are transitive, i.e.

(a,b) & $(b,c) \in R \Rightarrow (a,c) \in R$

(a,b) & $(b,c) \in S \Rightarrow (a,c) \in S$

if (a,b) & $(b,c) \in R \cap S \Rightarrow (a,c) \in R \cap S$

$\therefore$ $R \cap S$ is transitive

while $R \cup S$ is not transitive

For eg. for $A = \{1,2,3,4\}$, $R = \{(1,1)(1,2)(2,3)(1,3)\}$

$S = \{(2,2)(2,3)(3,1)(2,1)\}$

$R \cup S = \{(1,1)(1,2)(2,3)(1,3)(2,2)(3,1)(2,1)\}$ Not transitive

$R \cap S = \{(2,3)\}$

12. Let $A = \{1,2,3,4\}$ and $R = \{(1,1)(1,2)(2,1)(2,2)$
$(3,4)(4,3)(3,3)(4,4)\}$ be a relation on A

Verify that R is an equivalence rel^n.

$R \subseteq A \times A$

$A \times A = \{(1,1),(1,2)(1,3)(1,4)(2,1)(2,2)(2,3)$
$(2,4)(3,1)(3,2)(3,3)(3,4),(4,1)(4,2)$
$(4,3)(4,4)\}$

$\cancel{(1,3)}\ \cancel{(3,1)} \rightarrow$ ~~Not Symm.~~

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Finding the type of relation using matrix entries

(i) If R is defined on the set itself i.e. A : B then the matrix of the relation M_R is a square matrix

(ii) A relation R is said to be reflexive if all of its principal diagonal entries are

(iii) A relation is symmetric if its matrix entries are symmetric about the principal diagonal.

(iv) A rel^n is anti-symmetric whenever $M_{ij} \neq M_{ji}$ when $i \neq j$

(v) A rel^n is transitive if $M_{ij} = 1$ and $M_{jk} = 1$ $\Rightarrow M_{ik} = 1$, for all $i, j, k = 1, 2, 3, 4$ so on

Eg. $M_{R_1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{array}{c} \quad a_{12} \quad a_{13} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} a_{23} \end{array}$

Reflexive ✓ (all diagonal elements

Non-symmetric

Non-antisymmetric

Transitive

Eg $\begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 1 & 1 & 1 & 1 \\ 21 & 0 & 0 & 0 & 0 \\ 31 & 1 & 1 & 1 & 1 \\ 41 & 0 & 1 & 0 & 0 \end{array}$

Non-reflexive

Non-symmetric

Non-antisymmetric

$a_{13}\ a_{31}\ a_{11}$

Transitive

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Non-reflexive.
Non-symmetric
Non-transitive
Non-anti symmetric

Digraph represent

On a set B called the set of vertices, the set of all ordered pairs of elements of V called as arcs or edges, is called the directed graph (digraph) which is usually represented by

$$D(G) = (V(G), E(G))$$

The nature of a rel^n can be determined by using the direction given to the ordered ~~p~~pair i.e. if $(a,b) \in R$ then in the digraph $represent^n$, we denote (a,b) as an arc or an edge having dir^n from a to b where a is the initial node and b is the terminal node —

if ~~of~~ the presence of the tuple (a,a) represents a self loop about the vertex specified.

RULES FOR CHECKING THE NATURE OF REL^n

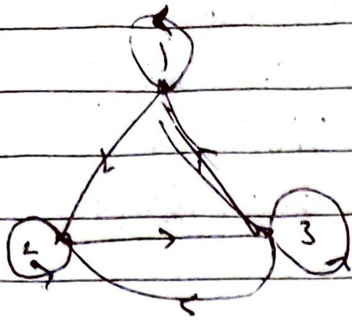
(i) R is said to be reflexive if we have a self loop around each node.

(ii) R is symmetric, if for all the edges we find bidirectional arrows.

(iii) R is antisymmetric, if b/w any pair of distinct nodes, we do not find any bidirectional arc in the digraph.

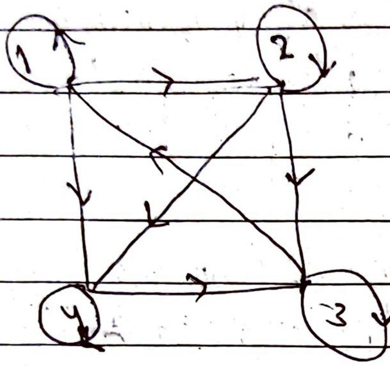
(iv) R is transitive, if a directed edge exists b/w $V_1 \rightarrow V_3$, whenever there ~~is an edge~~ exists a vertex V_2, s.t. there's an edge $V_1 \rightarrow V_2$ and $V_2 \rightarrow V_3$

(1)



Reflexive
Not Symmetric
Not Antisymmetric
Not Transitive

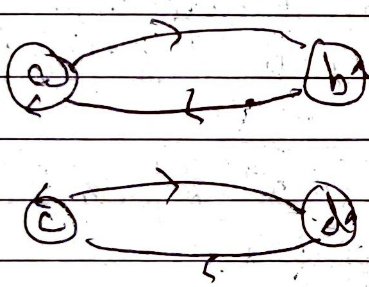
(2)



Reflexive
Non-symmetric
Anti-symmetric
~~Not~~ Transitive

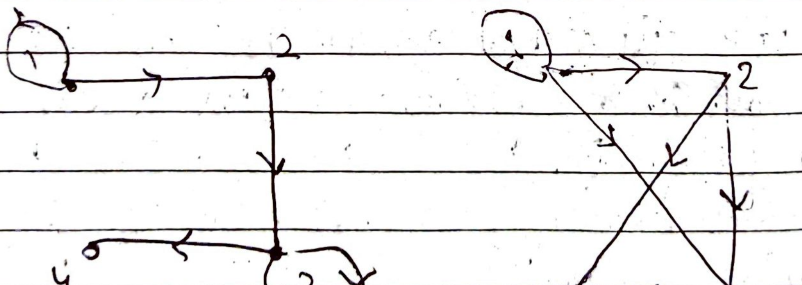
$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 1 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 & 1 \end{array}$$

(3)



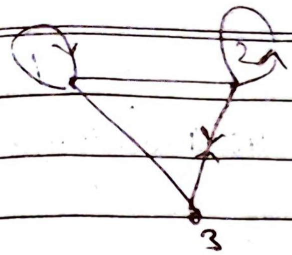
Reflexive
Symmetric

(4)

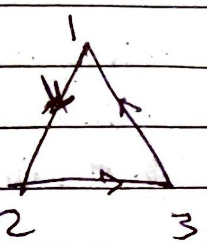


(5)

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad M_R^2 = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(5) 

Non reflexive

(6.) 

Non reflexive

Given a set A ~~whose~~ where cardinality is 'M' ; $|A| = n$ and the relation R on A. Let M denote the relation matrix of R, then the following conditions hold:

(i) R is reflexive iff $I_n \leq M$, where I_n is the identity matrix of order n.
(ii) R is transitive iff $M^2 \leq M$
(iii) R is antisymmetric iff $M \cap M^T \leq I_n$ where M^T is the transpose of the relⁿ matrix M

> If R is a relation on A and ~~cardinality~~ $|A| = n$ then $R^\infty = R \cup R^2 \cup R^3 \cup$ ___ R^n

> If R & S are equivalence relⁿ of A, then $(R \cup S)^\infty$ is the smallest equivalence relⁿ on A containing both R & S. $\in$

ANTISYMMETRIC REL^n

A relation R on a set A is said to be antisymmetric if $\underbrace{aRb}_{\substack{(a,b)\\ a \text{ is related to } b}}$ & $\underbrace{bRa}_{\substack{(b,a)\\ b \text{ is related to } a}} \Rightarrow a=b$

Eg. Let Z denote the set of integers and R be a rel defined on Z as a/b which is antisymmetric because if a divides b and b divides a then $a=b$.

Eg. Let A be a set and R be ~~the~~ a rel^n defined by $a<b$ then aRb and bRa $\Rightarrow a=b$ and hence R is antisymmetric.

POSET

A partially ordered set or a POSET in short, consists of a set A and a rel^n R. For a given set to be a poset, it must satisfy the property of being reflexive antisymmetric and transitive. Usually a poset is denoted by $(A, \leq)$ or $<A, \leq>$

induced subgraph

For eg. the set ~~{1,3,5}~~ $\{<1,2,3,4,5 \leq >\}$ forms a poset, ~~also~~

$\Rightarrow$ Divisors of 30 under division forms a poset along with set of all naturals under the rule $\leq$,

Set of all +ve integers under the rule division,
" " " " " $\leq$,

are all examples of posets

16 a) If R is a relation on the set A = {1,2,3,4} defined by xRx if x/y, prove that (A,R) is a poset. Draw a Hasse diagram — x divides y

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 1 & 0 \\ 4 & 1 & 1 & 0 & 1 \end{array}$$

(3,2) = (2,3)

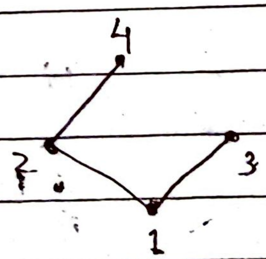
$M_{ij} \neq M_{ji}$
i / j

$A \times A$ = {(1,1) (2,2) (3,3) (4,4) (1,2) (1,3) (1,4) (2,4)}.

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 0 & 1 \\ 3 & 0 & 0 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

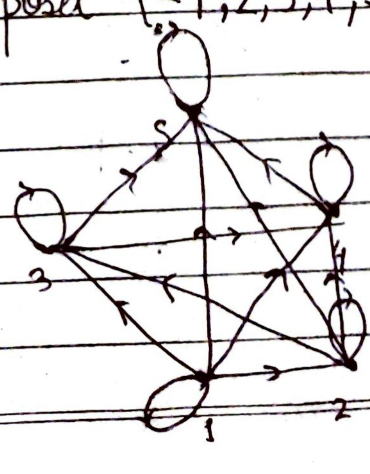
(3,2) = (2,3) { not in the set

$M_{ij} \neq M_{ji}$ when $i \neq j$ for elements in (A×A)



Hasse graph
(all self-loops and transitivity eg edges are removed)
(~~not~~ directions aren't specified)

Q The poset {<1,2,3,4,5>, ≤}



R = {(1,1) (2,2) (3,3) (4,4) (5,5),
(1,2) (1,3) (1,4) (1,5) (2,3)
(2,4) (2,5) (3,4) (3,5) (4,5)}

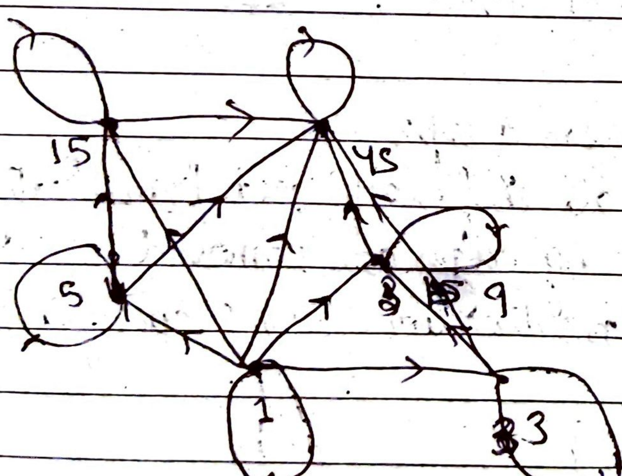
Hasse diagram

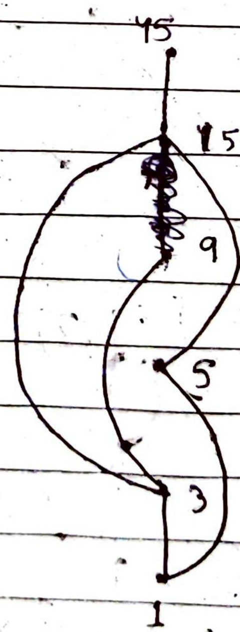
Q. Divisors of 45 , $\{\langle D_{45} \rangle, /\}$

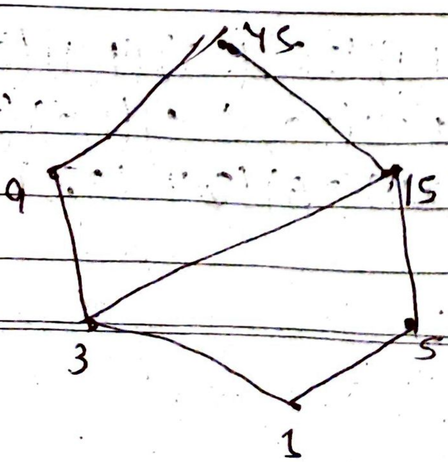
B = {1, 3, 5, 9, 15, 45}, / }

R = { ~~(1,45), (3,15), (5,9)~~

{ (1,45) (3,45) (5,45) (9,45) (15,45) (45,45)
(1,3) (1,5) (1,9) (1,45) (3,9) (5,15)
(1,1) (3,3) (5,5) (9,9) (15,15) (45,45) }





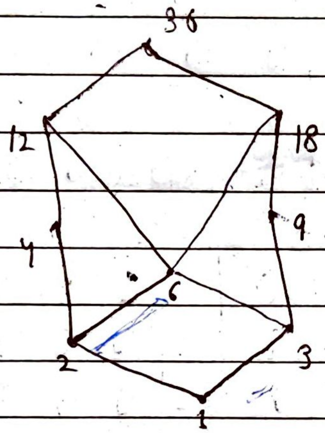


Number theory, rel^n, func^n (Q.15 tak)

36 : 1, 2, 3, 4, 6, 9, 12, 18, 36

{(1,1) (2,2) (3,3) (4,4) (6,6) (9,9) (12,12) (18,18) (36,36)
(1,2) (1,3) (1,4) (1,6) (1,9) (1,12) (1,18) (1,36)
(2,4) (2,6) (2,12) (2,18) (2,36)
(3,6) (3,9) (3,12) (3,18) (3,36)
(4,12) (4,36) (6,12) (6,18) (6,36) (9,18) (9,36)
(12,36) & (18,36)

(T, 17)

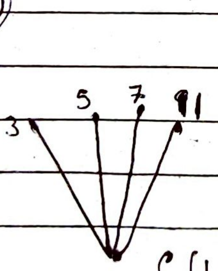


a,b b,c a,c
(1,2) (2,6) (1,6)

2,18 18,36

(2,36)

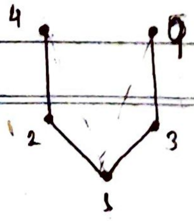
15 b)



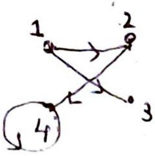
R = {(1,3) (1,5) (1,7) (1,11)}

Since no (a,b) (b,c) tuples present, it is not transitive (even if one tuple is present, it is transitive)

The given diagram is a Hasse diagram



R = {(1,2) (1,3) (2,4) (3,9)}



(19, 4)

18 b) If $A = [1,2,3,4]$ and R & S are relations on a defined $R = \{(1,2),(1,3),(2,4),(4,4)\}$
$S = \{(1,1),(1,2),(1,3),(1,4),(2,3),(2,4)\}$

(i) Verify $M(R^\circ S) = M(R)M(S)$

$R^\circ S = \{(1,3)(1,4)\}$

$$M(R^\circ S) = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$$M(R)\,M(S) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ LHS = RHS

(ii) $S^\circ R = \{(1,2)(1,3)(1,4)(1,4)(2,4)\}$

(iii) $R^2 = \{(1,4),(2,4),(4,4)\}$

(iv) $S^2 = \{(1,1)(1,2)(1,3)(\cancel{13})(1,4)\}$

19 (i) $(2x, x+y) = (6,1)$

$2x = 6 \Rightarrow x = 3$

$x+y = 1 \Rightarrow y = 1-3 = -2$

(ii) $(y-2, 2x+1) = (x-1, y+2)$.

$\Rightarrow y-2 = x-1$ | $2x+1 = y+2$

$x-y = -1$ | $2x-y = 1$

$\boxed{x=2}$ $\boxed{y=3}$

20) a) (ii) b) (iii) $(x,y) = (x^2, y^2)$

$x = x^2 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0$

$y = y^2 \Rightarrow y^2 - y = 0 \Rightarrow y(y-1) = 0$

$x = 0,1$, $y = 0,1$

(iv) $(x,y) = (y^2, x^2)$

$x = y^2 \Rightarrow x = x^4$

$y = x^2$

$x(x^3-1) = 0$

$x(x-1)(x^2+x+1) = 0$

$x = 0, 1, \{$ $\}$

13 (a) $A = A_1 \cup A_2 \cup A_3$ where $A_1 = \{1,2\}$,
$A_2 = \{2,3,4\}$, $A_3 = \{5\}$
xRy iff x and y are in the same set
A_i, $i = 1,2,3$. Is R an equivalence relⁿ?

$A =$ ~~$\{1,2,3,4,5\}$~~
$\{ (1,1)\ (1,2)\ (2,1)\ \cancel{(2,2)}$
$(2,2)\ (2,3)\ (2,4)\ (3,2)\ (3,3)\ (3,4)\ (4,2)\ (4,3)\ (4,4)$
$(5,5)\ \}$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 1 & 1 & 0 \\ 3 & 0 & 1 & 1 & 1 & 0 \\ 4 & 0 & 1 & 1 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 \end{array}$$

Diagonal elements present → reflexive $\{(1,1)(2,2)(3,3)(4,4)$ $(5,5)\}$ present
Symmetric $\{(1,2)(2,1)\ (2,3)(3,2)\ (2,4)\ (4,2)$
Transitive $(3,4)(4,3)\}$
$(2,4)(4,2) \Rightarrow (2,2)$
Hence, it is an equivalence rel

26-07-23

Functions

Let X and Y be any two non-empty sets a rule or a correspondance which associates to each element $x \in X$ to a unique element $y \in Y$ is called a function or a mapping from X to Y and is written as $f: x \to y$, the element y is called the image of x under f and is denoted by f(x) ie $y = f(x)$ and x is called the pre-image of y.
The set X is called the domain of the function F and the set Y is called the codomain of the funcⁿ F

Note: Every funcⁿ can be a relⁿ but every relⁿ is not a funcⁿ

$f : \{1, 2, 3\} \to \{a, b\}$
$\quad\quad X \quad \to \quad Y$

$R' = \{(1, a)(1, b)(2, a)\}$
$R' \subseteq R \quad , R = \{X \times Y\}$

The above set R' is not a funct since the first element 1 appears in two tuples

Equality

Two functions f & g of $A \to B$ are said to be equal iff, $f(x) = g(x)\ \forall x \in A$ and hence we write $f = g$

{ if atleast any one element x exists s.t $f(x) \neq g(x)$ then the functions are unequal }

1 (a) $A = \{1, 2, 3\}$, $B = \{-1, 0\}$
$R = \{(1,-1), (1,0), (2,-1), (3,0)\}$

1 is associated with -1 as well as 0,
$\therefore$ it is not a func^n. Domain element 1 repeated in two functions

(b) $A = \{1, 2, 3\}$ $B = \{-1, 0\}$
$S = \{(1,-1), (2,-1), (3,0)\}$

2(a) $A = \{0, \pm 1, \pm 2, 3\}$
$f(x) = 3x^3 - 6x^2 + 10x + 29 \quad \forall x \in f$
Range of f

$\cancel{f'(x) = 9x^2 - 12x + 10}$

$f(0) = 29$ $(0, 29)$
$f(1) = 36$ $(1, 36)$
$f(-1) = -3 - 6 - 10 + 29 = 10$ $(-1, 10)$
$f(2) = 24 - 24 + 20 + 29 = 49$ $(+2, 49)$
$f(-2) = -24 - 24 - 20 + 29 = -39$ $(-2, -39)$
$f(3) = 81 - 54 + 30 + 29 = 86$ $(3, 86)$

Range space $(-39, 10, 29, 36, 49, 86)$

(b) $A = \{0, \pm 1, \pm 2, 3\}$
$f(x) = Ax^3 - 2x^2 + 3x + 1 \quad \forall x \in A$

$f(0) = 1$
$f(1) = A + 2$

$\lambda = 0 \quad f(x) = -2x^2 + 3x + 1$

$-1 \qquad -x^3 + 2x^2 + 3x + 1$

$1 \qquad x^3 + 2x^2 + 3x + 1$

$2 \qquad 2x^3 - 2x^2 + 3x + 1$

$-2 \qquad -2x^3 - 2x^2 + 3x + 1$

$3 \qquad 3x^3 - 3x^2 + 3x + 1$

Range $= \{R - \{\phi\}\}$ ϕ

Into mapping / Into func^n

→ If there is ~~a mapping~~ mapping $f: A \to B$ s.t. there is atleast one element in B which is not the f-image of any element in A, then we say that f is a mapping of A into B.

In this case, the range of f is a proper subset of the co-domain of f i.e. $f(A) \subseteq f(B)$

Onto mapping

→ If the mapping $f: A \to B$ is s.t. each element in B is an f-image of atleast one element in A.
In this case, the range of f is equal to the co-domain of f i.e. $f(A) = B$.
An onto mapping is also called a surjective mapping or a surjection.

One-one mapping

→ $f: A \to B$ is said to be one-one or one to one if different elements in A have different f-images in B
i.e. if $f(x) = f(x_1) \Rightarrow x = x_1$

Also called as injective mapping

Many-one

⇒ A mapping is said to be many-one mapping if two distinct elements in A have the same f-image in B

i.e. $f(x) = f(x_1) \Rightarrow x \neq x_1$

Bijection

A mapping $f: A \to B$ is said to be bijective if it is both one-one and onto. (one-one onto)

Note: Two sets A and B are said to have the same no of elements iff one to one mapping of A onto B exists.

Such sets are said to be cardinally equivalent and is written as $A \sim B$

Identity mapping / functn / transfer reln

Let A be ~~an the~~ any set. Let $F: A \to A$, then $f(x) = x$ $\forall x \in A$, i.e. each element of A is mapped onto itself

We denote this by I_A s.t. $I_A(x) = x$

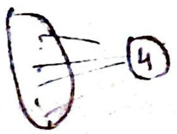
Let $A = \{1, 2, 3, 4\}$, then $f = \{(1,1)(2,2)(3,3)(4,4)\}$ is an eg. of an identity mapping on A.

f is also an equivalence reln.

Constant mapping

A funcn $f: A \to B$ is called a constant mapping if there exists an element $c \in B$ if its assigned to every element $\forall A$ i.e. $f(x) = c$ where $x \in A$.

eg. $f(x) = 4$, $x \in R$



Inverse image of an element

Let $f: A \to B$ and let $b \in B$, then the ~~ine~~ under f denoted by $f^{-1}(b)$ consists of elements in A which has B as their f-image

$$f^{-1}(x) = \left\{ x \;\middle|\; \begin{array}{l} x \in A \\ f(x) = b \end{array} \right\}$$

$f^{-1}(b)$ is always a subset of A

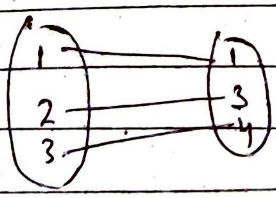
Cartesian product

Let $A \times B$ be finite set then $A \times B = \{(x,y) \mid x \in A, y \in B\}$

4a) $A = \{1,2,3\}$, $B = \{1,2,3,4,5\}$
Find whether $f: A \to B$ is

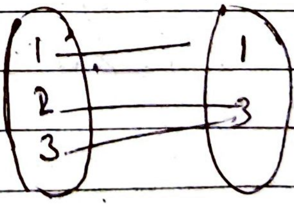
(a) one-to-one (b) onto

(i) $f = \{(1,1)\ (2,3)\ (3,4)\}$



~~Bijective~~ one to one
not onto
(some elements in B not mapped in A)

(ii) $f = \{(1,1)\ (2,3)\ (3,3)\}$



~~one~~ many-one
not onto

into

5 (a) $A = \{1,2,3,4,5\}$ $B = \{w,x,y,z\}$

$f = \{(1,w),(2,x),(3,x),(4,y),(5,y)\}$

Find the image of the following subsets of A under f.

~~$A(1) = f\{1\}$~~

$A_1 = \{1\}$, $A_2 = \{1,2\}$, $A_3 = \{1,2,3\}$, $A_4 = \{2,3\}$

$A_5 = \{2,3,4,5\}$

Ans $\{w\}$, $\{w,x\}$, $\{w,x\}$, $\{x\}$,

$\{x,y\}$

(b) $f: R \to R$, be defined by

$$f(x) = \begin{cases} 3x-5 & \text{for } x>0 \\ -3x+1 & \text{for } x \leq 0 \end{cases}$$

Determine $f(0), f(-1), f\left(\frac{5}{3}\right), f\left(\frac{-5}{3}\right)$

Ans $f(0) = 1$

$f(-1) = 4$

$f\left(\frac{5}{3}\right) = 0$

$f\left(\frac{-5}{3}\right) = 6$

6 (a) Let z denote the set of all integers. A function $h: z \times z \to z$ is defined by $h(x,y) = 2x+3y$

Find $h(0,0)$, $h(-3,7)$ $h(2,-1)$ and $h(A)$

$A = \{(0,n) \mid n \in Z+\}$

Ans, $h(0,0) = 0$, $h(-3,7) = 15$, $h(2,-1) = -1$

$h(A) = 3n$, $n \in Z^+$, $A = \{(0,n) \mid n \in z^+\} \Rightarrow \{3,6,9,12 \ldots\}$

6b) Let $f: Z \to Z$ be defined by $f(a) = a+1 \ \forall a \in Z$

one-one, onto

$f(3) = 3+1 = 4$

$f(-3) = -3+1 = -2$

One-one because every element has unique image in B

onto because every in B is an image of some element in A

7a) $f: A \to B$ determine f is one one or onto

(i) $A = R$, $B = \{x \mid x$ is real no. and $x \geq 0\}$;

$f(a) = |a|$

It is not one-one because two, negative and non-neg. counterparts will have same image in B, which is non-unique

$f(-1) = 1$, $f(1) = 1$

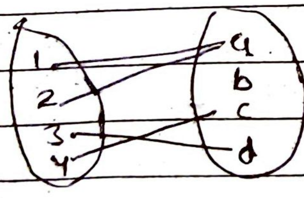
onto - Every element in B is an image of some element in A.

~~$f(1) = 1$~~

~~$f(-1) = 1$~~

(b) $A = \{1,2,3,4\}$, $B = \{a,b,c,d\}$

$f = \{(1,a), (2,a), (3,d), (4,c)\}$



→ Since 1, 2 have same image (a) in B, not one-one

→ (b) has no pre image, not onto

8(b) 120 one-one func^n A to B

$|A| = 6$

$|B| = ?$

$f: \overset{m}{A} \to \overset{n}{B} \to \cancel{m^n}\ n^m$

$\boxed{{}^mP_n}$

$\frac{6!}{(6-n)!} = 120$

$6 = (6-n)!$

$\Rightarrow n^6 = 120$

$6 \log n = \log 120$

$n = 2.22096155$

3y

$|x\vec{c}|$

60 one-on

$^6P_n = 120$

$\frac{6!}{(6-n)!} = 120$

$\frac{\cancel{5!}\times 6}{(6-n)!} = \cancel{120}$

$6 = (6-n)!$

$\boxed{n=3}$

9(b) $|A| = 3$ $|B|$

$^3P_n = 60$

$\frac{3!}{(3-n)!} = 60$

$10(3-n)! = 1$

not possible

10(a) $A = \{1,2,3,4,5,6,7\}$, $B = \{w,x,y,z\}$

Find no. of onto from A to B

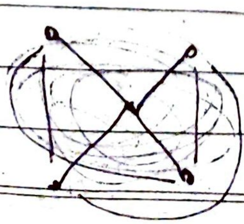
Ans To find no. of onto func^n, we use Sterling no. of 2nd kind

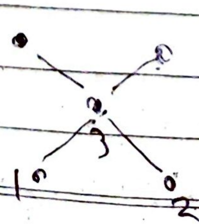
$S(m,n) = \frac{p(m,n)}{n!} = \frac{1}{n!}\sum_{k=0}^{n}(-1)^k\left({}^nC_{n-k}\right)(n-k)^m$

$S(7,4) = \frac{1}{4!}\sum_{k=0}^{4}(-1)^k\left({}^4C_{4-k}\right)(4-k)^7$

$= \frac{1}{4!}\left({}^4C_4(4^7) - {}^4C_3\, 3^7 + {}^4C_2\, 2^7 - {}^4C_1(1)^7\right)$

$= 240$





Hasse

$1 \to 3 \to 2$

Q. Let A and B be finite sets with ~~cardinality of~~ $|B| = 3$ if there are 4096 rel^n from A to B, what is $|A|$?

$|A| = m$, $|B| = n$ $\Rightarrow$

$R = A \times B = 2^{mn}$
$m \times n$

$\rightarrow 2^{mn} = 4096$

$2^{3n} = 4096$

$2^{3n} = 2^{12}$

$3n = 12$

$\boxed{n = 4}$

64^2
8^4
2^{34}

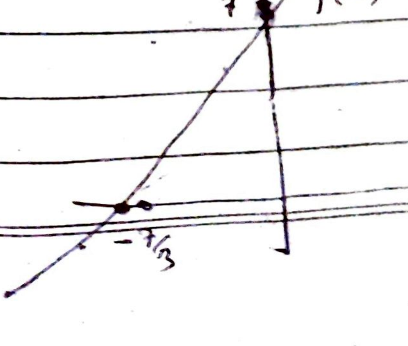
Binary relations $R: A \to A \rightsquigarrow 2^{m \cdot m} = 2^{m^2}$
$|m| \; |m|$

$B \times A = 2^{mn}$
$B \to B = 2^{n^2}$

Q $f: R \to R$ and $g: R \to R$ defined by $f(x) = 3x + 7 \ \forall x \in R$ and $g(x) = x(x^3 - 1) \ \forall x \in R$
Verify that f and g are one-one, if not provide reas

~~$f(0)$~~ $= f(1) = 10$
$f($

one-one



$g(0) = 0$
$g(1) = 0$

not one one

~~Onto for~~ sterling no. use $s(m,n) = \frac{1}{n!}$

Onto func^n = $n! \times$ sterling no.

11 a) $A = \{1,2,3,4,\cancel{5,6}\}$, $B = \{1,2,3,4,5,6\}$
Find how many func^n from A to B
How many one-one?

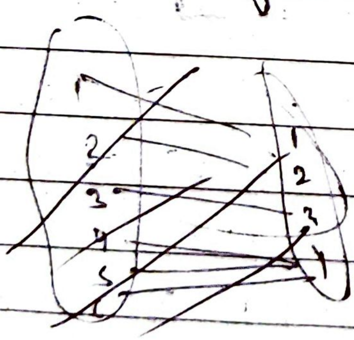
$|A| = 4 = m$
$|B| = 6 = n$ $\quad {}^n\!\cancel{P_m}$

Total no. of func^n from A to B = $n^m = 6^4 = 1296$
B to A = $m^n = 4^6 = 4096$

No. of one-one func^n from A to B = $\cancel{{}^mP_n = \frac{m!}{(m-n)!} = \frac{4!}{(4}}$
$= {}^nP_m = \frac{n!}{(n-m)!} = \frac{6!}{(6-4)!} = 360$

→ There exists no one-one func^n from B to A
no onto from B to A

Onto func^n from A to B



$S(n,m)$ = from $A \to B$

$$S(6,4) = \sum_{k=0}^{4} (-1)^k \left({}^4C_{4-k}\right)(4-k)^6$$

$= 1560$

$S(5,4) = \frac{240}{4!} = 10$

$= \frac{1}{4!} \sum_{k=0}^{n} (-1)^k ({}^4C_{4-k})(4-k)^5 = \frac{240}{4!} = 10$

$S(8,8) = 266$

$S(7,2) = 63$

Q.a) $S(10,6)$, $S(8,4) = 1701$, $S(8,5) = 1050$, $S(8,6) = 266$

$\boxed{S(m+1,n) = S(m,n-1) + n\,S(m,n)}$

$S(10,6) = S(9,5) + 6S(9,6)$

$S(9,5) = S(8,4) + 5\,S(8,5)$
$= 1701 + 5(1050)$
$= 6951$

$S(9,6) = S(8+1,6) = S(8,5) + 6S(8,6)$
$= 1050 + 6(266) = 2646$

$S(10,6) = 6951 + 6(2646)$
$= 22827$

Rel^n → 30-28

Func^n → 10-12 → 10

Num T → 10