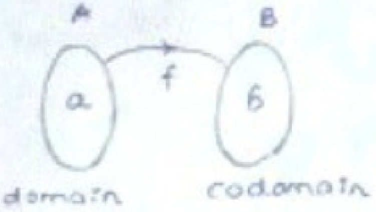
Module - 03

Functions

* Let A and B be the non-empty sets. then A fu^n as a mapping f from A to B is a relation from A to B such that every element of A has unique element in B i.e, $b = f(a)$

* Here 'b' is called the image of 'a' and 'a' is called pre image of 'b'

* A fu^n from A to B is denoted by $f : A \to B$, the pictorial representation of f is



$b = f(a) \longrightarrow$ range of f

Note.

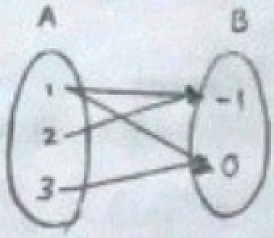
* Every fu^n is a relation but every relation need not to be a fu^n

* A contains m elements. B contains n elements then a fu^n 'f' from $A \to B$ contains n^m elements

① Determine the following are fu^n (or) not ?

i) $A = \{1, 2, 3\}$ $B = \{-1, 0\}$

$R = \{(1,-1), (1,0), (2,-1), (3,0)\}$

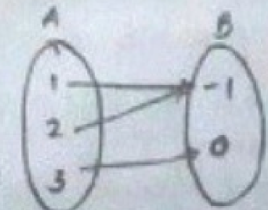


Given $R = \{(1,-1), (1,0), (2,-1), (3,0)\}$

$\therefore$ It is not a fu^n $\because$ 1 of A is related to -1 and 0 of B

ii) $A = \{1, 2, 3\}$ $B = \{-1, 0\}$

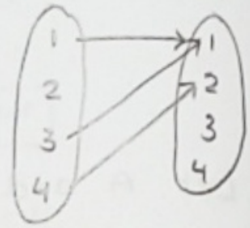
$S = \{(1,-1), (2,-1), (3,0)\}$



Sol^n It is a fu^n $\because$ every eleme-nt of A has unique element in B

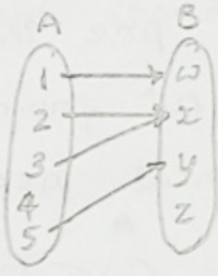
iii) $A = \{1, 2, 3, 4\}$ $g = \{(3,1), (4,2), (1,1)\}$

It is not a fu^n $\because$ every element of A does not have a image in B.



② For $A = \{1, 2, 3, 4, 5\}$ and $B = \{\omega, x, y, z\}$ fu^n f from A to B is defined by $f = \{(1, \omega), (2, x), (5, y), (3, x)\}$. Find the images of the following

i) $A_1 = \{1, 2, 3\}$ ii) $A_2 = \{2, 3, 4, 5\}$



$A_1 = \{1, 2, 3\}$

The images of $f(A_1) = \{\omega, x\}$

$A_2 = \{2, 3, 4, 5\}$

The image $f(A_2) = \{x, y\}$

(2) Let f from $R \to R$ defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$$

i) Determine the following:
$f(0)$, $f(-1)$, $f(5/3)$, $f(-5/3)$

ii) Find $f^{-1}(0)$, $f^{-1}(-1)$, $f^{-1}(3)$, $f^{-1}(6)$

i) $f(0) = 3(0) - 5 = \underline{-5}$

$f(-1) = -3(-1) + 1 = 3 + 1 = \underline{4}$

$f(5/3) = \cancel{3}\left(\frac{5}{\cancel{3}}\right) - 5 = \underline{0}$

$f(-5/3) = -\cancel{3}\left(\frac{-5}{\cancel{3}}\right) + 1 = 5 + 1 = \underline{6}$

ii) $f(a) = b \Rightarrow a = f^{-1}(b)$

be $f^{-1}(0) = x$

$f(x) = 0$

$3x - 5 = 0$ $\qquad$ $-3x + 1 = 0$

$3x = 5$ $\qquad$ $+3x = +1$

$x = \frac{5}{3} > 0$ $\qquad$ $x = \frac{1}{3} > 0$

$3(x) - 5 = 0$ $\qquad$ $3x - 5 = 0$

substitute $x = \frac{5}{3}$ $\qquad$ substitute $x = \frac{1}{3}$

$\cancel{3}\frac{5}{\cancel{3}} - 5 = 0$ $\qquad$ $\cancel{3}\frac{1}{\cancel{3}} - 5 = 0$

$\underline{0 = 0}$ $\qquad$ $\underline{-4 \neq 0}$

$\therefore f^{-1}(0) = \{5/3\}$

$f^{-1}(1) = x$

$f(x) = 1$

$3x - 5 = 1$ $\qquad$ $-3x + 1 = 1$

$3x = 6$ $\qquad$ $-3x = 0$

$x = 2$

$2 > 0$

$3(x) - 5 = 1$

$3(2) - 5 = 1$

$6 - 5 = 1$

$1 = 1$

$x = 0$

$0 \leq 0$

$-3x + 1 = 1$

$-3(0) + 1 = 1$

$0 + 1 = 1$

$1 = 1$

$f^{-1}(1) = \{2, 0\}$

$f^{-1}(-1) = x$

$f(x) = -1$

$3x - 5 = -1$

$3x = 4$

$x = 4/3 > 0$

$3x - 5 = -1$

$\cancel{3}\frac{4}{\cancel{3}} - 5 = -1$

$-1 = -1$

$-3x + 1 = -1$

$-3x = -2$

$x = 2/3 > 0$

$3x - 5 = -1$

$\cancel{3}\left(\frac{2}{\cancel{3}}\right) - 5 = -1$

$2 - 5 = -1$

$-3 \neq -1$

$\therefore f^{-1}(-1) = \{4/3\}$

$f^{-1}(3) = x$

$f(x) = 3$

$3x - 5 = 3$

$3x = 8$

$x = 8/3 > 0$

$3x - 5 = 3$

$\cancel{3}\frac{8}{\cancel{3}} - 5 = 3$

$3 = 3$

$-3x + 1 = 3$

$-3x = 2$

$x = -2/3 \leq 0$

$-3x + 1 = 3$

$-\cancel{3}\left(\frac{-2}{\cancel{3}}\right) + 1 = 3$

$2 + 1 = 3$

$3 = 3$

$\therefore f^{-1}(3) = \{8/3, -2/3\}$

$f^{-1}(6) = x$

$f(x) = 6$

$3x - 5 = 6$ $\qquad$ $-3x + 1 = 6$

$3x = 11$ $\qquad$ $-3x = 5$

$x = 11/3 > 0$ $\qquad$ $x = -5/3 \leq 0$

$3x - 5 = 6$ $\qquad$ $-3x + 1 = 6$

$\cancel{3}\frac{11}{\cancel{3}} - 5 = 6$ $\qquad$ $-\cancel{3}\left(\frac{-5}{\cancel{3}}\right) + 1 = 6$

$6 = 6$ $\qquad$ $5 + 1 = 6$

$6 = 6$

$\therefore f^{-1}(6) = \{11/3, -5/3\}$

iii> $f^{-1}[[-5, 5]] = x$

$f(x) = [-5, 5]$

$-5 \leq 3x - 5 \leq 5$

$-5 + 5 \leq 3x \leq 5 + 5$

$0 \leq 3x \leq 10$

$\frac{0}{3} \leq x \leq \frac{10}{3}$

$0 \leq x \leq 10/3$

$-5 \leq -3x + 1 \leq 5$

$-5 - 1 \leq -3x \leq 5 - 1$

$-6 \leq -3x \leq 4$

$6 \geq 3x \geq -4$

$\frac{6}{3} \geq x \geq \frac{-4}{3}$

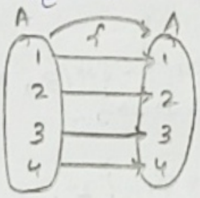
$2 \geq x \geq -4/3$

Types of functions:

① **Identity fun:**

A fun $f: A \to A$ is said to be identity fun if the image of every element of A is A itself, i.e $f(a) = a$ where $a \in A$ & $a \in A$.

Ex: $A = \{1, 2, 3, 4\}$

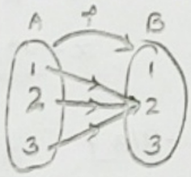


② **Constant fun:**

A fun $f: A \to B$ is said to be an constant fun if all element of A have same image in B

Ex: $A = \{1, 2, 3\}$

$B = \{1, 2, 3\}$

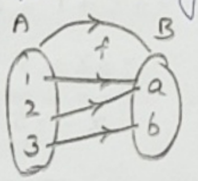


③ **Onto fun / Subjective fun:**

A fun $f: A \to B$ is said to be onto fun if every element in B has a pre image in A.

Ex: $A = \{1, 2, 3\}$

$B = \{a, b\}$



Note:

★ If $|A| \propto |B|$, then it is not onto.

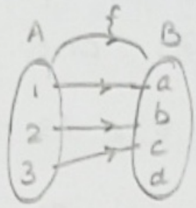
④ **One to one fun / Injective fun:**

A fun $f: A \to B$ is said to be one-to-one if every element of A has a unique image in B and

and every element of $f(a)$ has a unique pre-image in A.

Ex: $A = \{1, 2, 3\}$

$B = \{a, b, c, d\}$

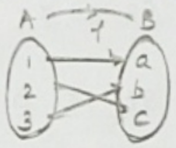


⑤ One-to-one Correspondance / Bijective funⁿ

A funⁿ $f: A \to B$ is said to be bijective funⁿ if it is both one-to-one and onto funⁿ.

Ex: $A = \{1, 2, 3\}$

$B = \{a, b, c\}$

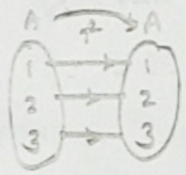


Note:

★ If $|A| = |B|$, then only it is a bijective funⁿ.

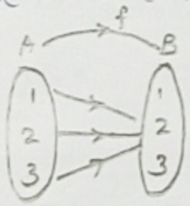
Ⓠ Find the nature of the following funⁿ: $A = \{1, 2, 3\}$

① $f = \{(1,1), (2,2), (3,3)\}$



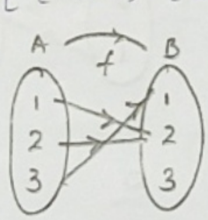
→ f is an identity funⁿ

② $g = \{(1,2), (2,2), (3,2)\}$



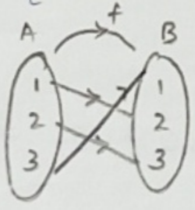
→ g is a constant funⁿ

③ $h = \{(1,2), (2,2), (3,1)\}$



→ h is neither one-to-one nor onto funⁿ

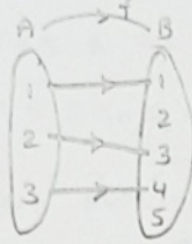
④ $p = \{(1,2), (2,3), (3,1)\}$



→ p is a bijective funⁿ

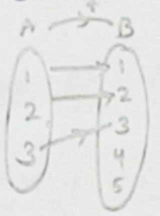
(2) $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$. Find whether it is one-to-one or onto.

(1) $f = \{(1,1), (2,3), (3,4)\}$



$\Rightarrow$ It is one-to-one fun but not onto

(2) $g = \{(1,1), (2,2), (3,3)\}$



$\Rightarrow$ It is a one-to-one fun but not onto.

(3) The fun $f : R \to R$ and $g : R \to R$ are defined by $f(x) = 3x + 7$ for all $x \in R$ and $g(x) = x(x^3 - 1)$ $\forall$ $x \in R$. Verify that f is one-one but g is not.

$f(x) = 3x + 7$ $\forall$ $x \in R$

Let us take x_1 & $x_2 \in R$

Consider $f(x_1) = f(x_2)$

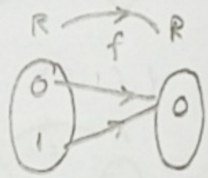
$3x_1 + \cancel{7} = 3x_2 + \cancel{7}$

$\cancel{3}x_1 = \cancel{3}x_2$

$x_1 = x_2$

$\therefore$ f is one-one fun

$g(x) = x(x^3 - 1)$ $\forall$ $x \in R$.



Consider $g(x) = g(y)$

$x(x^3 - 1) = y(y^3 - 1)$

Let $x = 1 \Rightarrow g(x) = g(1) = 1(1^3 - 1) = 0$

$y = 0 \Rightarrow g(y) = g(0) = 0(0 - 1) = 0$

$\therefore$ g is not one-one fun

(4) Let $A = R$. $B = \{x / x$ is real and $x \geq 0\}$. If the fun $f : A \to B$ defined by $f(a) = a^2$

$f(a) = a^2$

$a_1, a_2 \in R$

Consider $f(a_1) = f(a_2)$

$a_1^2 = a_2^2$

$\therefore a_1 = \pm\sqrt{a_2^2}$

$a_1 = \pm a_2$

i.e $a_1 = a_2$ or $a_1 = -a_2$

$\therefore$ f is <u>not one-one</u>

Now take $b \in B$, $f(a) = b$

$a^2 = b$

$a = \pm\sqrt{b}$

$\therefore$ f is <u>onto</u> $(\pm\sqrt{b} \in A \because A \in R)$

<u>Note:</u>

* Let $|A| = m$ and $|B| = n$ then

→ No/. of fu^n from A to B $= n^m$

→ No/. of onto fu^n from A to B $= \sum_{k=0}^{n} (-1)^k \, {}^nC_{n-k} (n-k)^m$

→ If $m<n$, no/. of one-one fu^n from A to B $= \dfrac{n!}{(n-m)!}$

→ If $m=n$, no/. of one-one fu^n from A to B $= m!$

→ If $m>n$, no/. of one-one fu^n from A to B $= 0$

⑤ There are 60 one-one fu^n from A to B and $|A| = 3$ what is $|B| = ?$

$P(n,m) = \dfrac{n!}{(n-m)!}$

$60 = \dfrac{n!}{(n-3)!}$

$60 = \dfrac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}}$

$60 = n(n-1)(n-2)$

$\therefore \underline{n = 5}$

$\therefore \underline{|B| = 5}$

⑥ Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{w, x, y, z\}$ find the no/ of onto fun from A to B

$|A| = 7 = m$ $|B| = 4 = n$

$$P(n,m) = \sum_{k=0}^{n} (-1)^k \, {}^nC_{n-k} \, (n-k)^m$$

$$P(4,7) = \sum_{k=0}^{4} (-1)^k \, {}^4C_{4-k} \, (4-k)^7$$

$k = 0 \Rightarrow P(4,7) = (-1)^0 \, {}^4C_4 \, (4)^7 = 16384$

$k = 1 \Rightarrow P(4,7) = (-1)^1 \, {}^4C_3 \, (3)^7 = -8748$

$k = 2 \Rightarrow P(4,7) = (-1)^2 \, {}^4C_2 \, (2)^7 = 768$

$k = 3 \Rightarrow P(4,7) = (-1)^3 \, {}^4C_1 \, (1)^7 = -4$

$k = 4 \Rightarrow P(4,7) = (-1)^4 \, {}^4C_0 \, (0)^7 = 0$

$P(4,7) = 16384 - 8748 + 768 - 4 + 0 = \underline{8400}$

⑦ If there are 720 one-one fun from A to B and $|A| = 6$ then what is $|B| = ?$

$$P(m,n) = \frac{n!}{(n-m)!}$$

$$720 = \frac{n!}{(n-6)!}$$

$$720 = \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)\cancel{(n-6)!}}{\cancel{(n-6)!}}$$

$720 = n(n-1)(n-2)(n-3)(n-4)(n-5)$

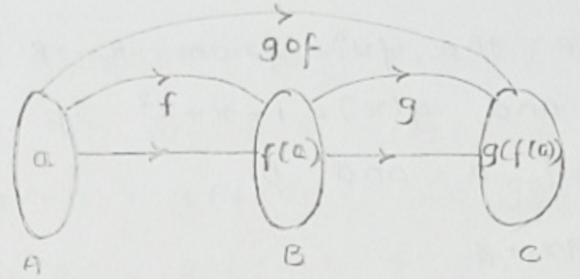
$720 = 6 \times 5 \times 4 \times 3 \times 2 \times 1$ (i.e $6!$)

$\therefore n = 6$

$\therefore |B| = 6$

Composition of functions:

Consider 3 non-empty sets A, B, C and the fun $f : A \to B$ and $g : B \to C$. The composition of these two fun is defined as the fun of $g \circ f : A \to C$ with $g(a) = g(f(a))$ where $a \in A$



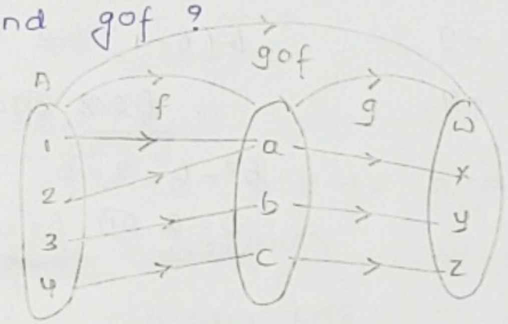
$gof = A \to C$

<u>Note:</u>

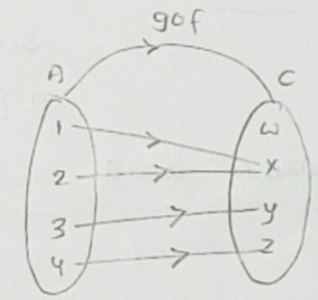
* $f : A \to A$ then $fof = f^2$, $fof^2 = f^3$
* $gof(x) = g(f(x))$
* $fog(x) = f(g(x))$

① Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$ and $C = \{\omega, x, y, z\}$ with $f : A \to B$ and $g : B \to C$ given by $f = \{(1, a), (2, a), (3, b), (4, c)\}$ and $g = \{(a, x), (b, y), (c, z)\}$.

Find gof ?



$\Rightarrow$



② Consider the funⁿ f and g obtained by $f(x) = x^3$ and $g(x) = x^2 + 1$ $\forall x \in R$. Find gof, fog, f^2 and g^2. and also S.T $fog \neq gof$

W.K.T $gof(x) = g(f(x))$

To find gof

$$gof(x) = g(f(x))$$
$$= g(x^3)$$
$$= (x^3)^3 + 1$$
$$= \underline{x^6 + 1}$$

To find fog,

$$fog(x) = f(g(x))$$
$$= f(x^2 + 1)$$
$$= \underline{(x^2 + 1)^3}$$

$\therefore gof(x) \neq fog(x)$

To find f^2

$$fof(x) = f(f(x))$$
$$= f(x^3)$$
$$= (x^3)^3$$
$$= \underline{x^9}$$

To find g^2

$$gog(x) = g(g(x))$$
$$= g(x^2 + 1)$$
$$= \underline{(x^2 + 1)^2 + 1}$$

③ Let f and g be the fun from $R \to R$ defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$ if $gof(x)$. $9x^2 - 9x + 3$. Find a and b

$gof(x) = 9x^2 - 9x + 3$

$g(f(x)) = 9x^2 - 9x + 3$

$g(ax+b) = 9x^2 - 9x + 3$

$1 - (ax+b) + (ax+b)^2 = 9x^2 - 9x + 3$

$1 - ax - b + \underline{a^2x^2} + b^2 + 2axb = 9\underline{x^2} - 9x + 3$

Equating the coefficients of x^2, x and constants we get

$\underline{a^2 = 9}$

$\underline{a = \pm 3}$

$-ax + 2axb = -9x$

$x[-a + 2ab] = -9x$

$2ab - a = -9$

$\underline{a[2b-1] = -9}$

$1 - b + b^2 = 3$

$b^2 - b = 2$

$\cancel{b(b-1) = 2}$

$b^2 - b - 2 = 0$

$\underline{b = 2}$ (or) $\underline{b = -1}$

<u>Theorem 1:</u>

Let $f: A \to B$ and $g: B \to C$ be any two funs. Then the following are true

a) If f and g is one-one then gof is also one-one

c) If f and g are onto. so gof is also onto.

d) If gof is onto then g is onto.

b) If gof is one-one, then f is one-one.

<u>Proof</u>:

a) If $f: A \to B$ and $g: B \to C$ then $gof: A \to C$

To prove gof is one-one

Consider $a_1, a_2 \in A$.

$gof(a_1) = gof(a_2)$

$g(f(a_1)) = g(f(a_2))$ ($\because$ g is one-one)

$f(a_1) = f(a_2)$ ($\because$ f is one-one)

$\therefore$ $\underline{a_1 = a_2}$

$\therefore$ gof is $\underline{\text{one-one}}$

b) Take $a_1, a_2 \in A$ then $f(a_1), f(a_2) \in B$ and $f(a_1) = f(a_2)$

Take 'g' on b.s

$g(f(a_1)) = g(f(a_2))$ ($\because$ gof is one-one)

$gof(a_1) = gof(a_2) \Rightarrow a_1 = a_2$

$\therefore$ f is one-one

c) Take $a \in A$, Since f is onto $\Rightarrow f(a) = b$ and

Since g is onto $\Rightarrow c = g(b)$.

Consequently $gof(a) = g(f(a)) = g(b) = c$

$\therefore$ gof is onto

d) Since gof is onto $\Rightarrow gof(a) = g(f(a)) = g(b) = c$

$\Rightarrow c \in C$

$\therefore$ g is onto

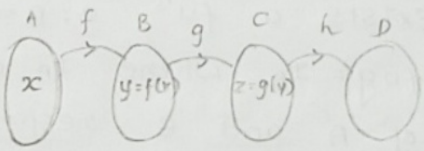
Theorem : 2

Let $f: A \to B$, $g: B \to C$ and $h: C \to D$ be fu^{ns} then P.T

$(hog)of = ho(gof)$

Proof: Consider LHS

$\{(hog)of\}x$; $x \in A$



$(hog)of(x) = ((hog)(f(x)))$

$= hog(y)$ $[\because y = f(x)]$

$= h(g(y))$

$= h(z)$

Consider RHS

$[ho(gof)](x) = h(gof(x))$

$= h(g(f(x)))$

$= h(g(y))$

$= h(z)$

$\therefore$ LHS = RHS

Hence Prooved

(4) Let f, g, h be the fu^{ns} from $Z \to Z$ defined by $f(x) = x - 1$, $g(x) = 3x$, $h(x) = 0$, if x is even and $h(x) = 1$, if x is odd. Determine $[fo(goh)](x)$

and $[(f \circ g) \circ h](x)$ and verify that $f \circ (g \circ h) = (f \circ g) \circ h$

$$[f \circ (g \circ h)](x) = f(g \circ h(x))$$
$$= f(g(h(x)))$$
$$= f(3h(x))$$
$$= 3h(x) - 1$$

$\Rightarrow$ $3(0) - 1 = -1$, if x is even

$3(1) - 1 = 2$, if x is odd

$$[(f \circ g) \circ h](x) = ((f \circ g)(h(x)))$$
$$= f(g(h(x)))$$
$$= f(3(h(x)))$$
$$= 3h(x) - 1$$

$\Rightarrow$ $3(0) - 1 = -1$, if x is even

$3(1) - 1 = 2$, if x is odd

$\therefore$ $f \circ (g \circ h) = (f \circ g) \circ h$

Invertible Functions:

A fu^n $f: A \to B$ is said to be invertible if there exists a fu^n $g: B \to A$ such that $g \circ f = I_A$ and $f \circ g = I_B$ where I_A and I_B are the identity fu^{ns} of A and B respectively then g is called inverse of f i.e $g = f^{-1}$

① Let $A = \{1, 2, 3, 4\}$, f and g be the fu^{ns} from A to A, Given by $f = \{(1,4), (2,1), (3,2), (4,3)\}$ and $g = \{(1,2), (2,3), (3,4), (4,1)\}$. P.T f and g are inverse of each other.

$g \circ f(1) = g(f(1)) = g(4) = 1 = I_1$

$g \circ f(2) = g(f(2)) = g(1) = 2 = I_2$

$g \circ f(3) = g(f(3)) = g(2) = 3 = I_3$

$g \circ f(4) = g(f(4)) = g(3) = 4 = I_4$

$f \circ g(1) = f(g(1)) = f(2) = 1 = I_1$

$f \circ g(2) = f(g(2)) = f(3) = 2 = I_2$

$f \circ g(3) = f(g(3)) = f(4) = 3 = I_3$

$f \circ g(4) = f(g(4)) = f(1) = 4 = I_4$

$\therefore$ f and g are inverse of each other

③ Consider the fun $f: R \to R$ defined by $f(x) = 2x+5$ and a fun $g: R \to R$ defined by $g(x) = \frac{1}{2}(x-5)$. P.T g is inverse of f.

$$gof(x) = g(f(x)) = g(2x+5) = \frac{1}{2}\left[(2x+5)-5\right]$$

$$= \frac{1}{2}[2x] = \underline{\underline{x}}$$

$$fog(x) = f(g(x)) = f\left(\frac{1}{2}(x-5)\right)$$

$$= 2\left[\frac{1}{2}(x-5)\right]+5 = x - \cancel{5} + \cancel{5} = \underline{\underline{x}}$$

$\therefore$ g is inverse of f

Theorem : 03

A fun $f: A \to B$ is invertible if and only if it is one-one and onto.

Proof: If $f: A \to B$ is invertible, then there exists a unique fun $g: B \to A$ such that $gof = I_A$ and $fog = I_B$

Take $a_1, a_2 \in A$.

Consider $f(a_1) = f(a_2)$

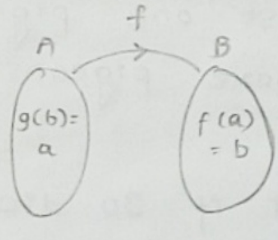
$$g(f(a_1)) = g(f(a_2))$$

$$gof(a_1) = gof(a_2)$$

$$I_A(a_1) = I_A(a_2)$$

$$a_1 = a_2$$

$\therefore$ f is one-one



Take $a \in A$, $f(a) = b = I_B(b)$

$$f(a) = fog(b)$$

$$= f(g(b))$$

$$= f(a)$$

$\therefore$ f is onto

① Conversely is f is one-one and onto P.T F is invertible

$gof = I_A$ and $fog = I_B$

$$gof(a) = g(f(a)) = g(b) = \underline{\underline{a}} = I_A$$

$fog(b) = f(g(b)) = f(a) = b = I_B$

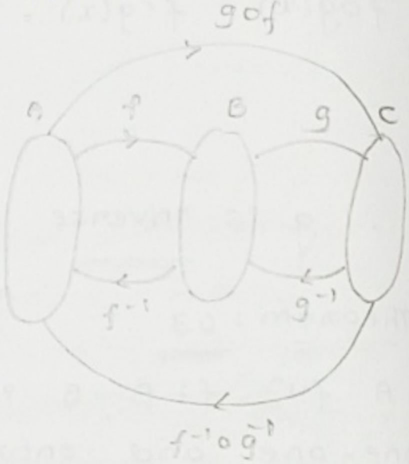
$\therefore$ f is invertible

Theorem : 04

If $f: A \to B$ and $g: B \to C$ are invertible fu^{ns} then $gof: A \to C$ is invertible then S.T $(gof)^{-1} = (f^{-1}og^{-1})$

Proof: Given $f^{-1}: B \to A$ and $g^{-1}: C \to B$

Then consider $h = f^{-1} o g^{-1} : C \to A$



$ho(gof) = (f^{-1} o g^{-1}) o (gof)$
$= f^{-1} o (g^{-1} o g) o f$
$= f^{-1} o I_B o f$
$= f^{-1} o f$
$= \underline{I_A}$

$(gof)oh = (gof) o (f^{-1} o g^{-1})$
$= go(fof^{-1}) o g^{-1}$
$= goI_A o g^{-1}$
$= gog^{-1}$
$= \underline{I_B}$

The Pigeonhole Principle:

If 'm' pigeons occupy 'n' pigeon holes and if $m > n$, then atleast one pigeon hole must contain 2 or more pigeons in it $\boxed{m > n}$

Generalisation:

If 'm' pigeons occupy 'n' pigeons holes then atleast one pigeon hole must contain $(p+1)$ or more pigeons where p = flooring of $\frac{m-1}{n} = \left\lfloor \frac{m-1}{n} \right\rfloor$

① P.T if 30 dictionaries in a library contains a total 61,327 pages, then atleast one of the dictionaries must have atleast 2045 pages.

(pigeon) $m = 61,327$ P.h $(n) = 30$

$p = \left\lfloor \frac{m-1}{n} \right\rfloor = \left\lfloor \frac{61327-1}{30} \right\rfloor = \lfloor 2044.2 \rfloor = \underline{2044}$

Generalisation = $p+1$

$= 2044+1 = \underline{2045}$

Hence atleast one of the dictionaries must have atleast 2045 pages

② 5 colours are used to paint 26 doors. P.T atleast 6 doors will have same colour.

$m = 26 \quad n = 5$

$p = \left\lfloor \frac{m-1}{n} \right\rfloor = \left\lfloor \frac{26-1}{5} \right\rfloor = \left\lfloor \frac{25}{5} \right\rfloor = \underline{5}$

Generalisation : $p+1 = 5+1 = \underline{6}$

Hence proved

③ P.T in any set of 29 persons atleast 5 persons must have born on the same day of the week.

$m = 29 \quad n = 7$

$p = \left\lfloor \frac{m-1}{n} \right\rfloor = \left\lfloor \frac{29-1}{7} \right\rfloor = \left\lfloor \frac{28}{7} \right\rfloor = \underline{4}$

Generalisation: $p+1 = 4+1 = \underline{5}$

Hence proved

④ How many persons must be chosen that atleast 5 of them will have birthdays in same calendar month.

$m = ? \quad n = 12 \quad p+1 = 5 \Rightarrow p = \underline{4}$

$p = \left\lfloor \frac{m-1}{n} \right\rfloor$

$4 = \left\lfloor \frac{m-1}{12} \right\rfloor$

$48 = (m-1)$

$m = 48+1 = \underline{49}$

⑤ P.T if any no/-s from 1 to 8 are choosen then 2 of them will have their sum = 9.

Sum of 9 = {(1, 8), (2, 7), (3, 6), (4, 5)} are the 4 sets whose sum is 9 from 1 to 8.

$m = 8 \qquad n = 4$

$$P = \left\lfloor \frac{m-1}{n} \right\rfloor = \left\lfloor \frac{8-1}{4} \right\rfloor = \left\lfloor \frac{7}{4} \right\rfloor = \lfloor 1.7 \rfloor = \underline{1}$$

$$p+1 = 1+1 = \underline{2}$$

Hence proved.

⑥ Shirts numbered consequently from 1 to 20 are worn by 20 students of a class when any 3 of these students are chosen to be in a debating team from the class, the sum of their shirt no/- is used as the code number of the team. S.T any 8 of the 20 students are selected then from there 8 we may form atleast 2 different teams have same code.

From the 8 of the 20 students are selected to make the team of 3 students in $^8C_3$ ways i.e, 56 ways.

The possibility of the smallest code no/- of the team = $1 + 2 + 3 = \underline{6}$

The possibility of the largest code no/- of the team = $18 + 19 + 20 = \underline{57}$

Code number from 6 to 57 (inclusive) = 52 code no/

$m = 56 \qquad n = 52$

$$P = \left\lfloor \frac{m-1}{n} \right\rfloor = \left\lfloor \frac{56-1}{52} \right\rfloor = \left\lfloor \frac{55}{52} \right\rfloor = \lfloor 1.057 \rfloor = \underline{1}$$

$$p+1 = 1+1 = \underline{2}$$