DMS – Module – II – Fundamental of Logic

Fundamental logics

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~~Predicate calculus~~ Propositional calculus

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- Logical operators
- Truth tables.
- Tautology, Contradiction, Contingency.
- Laws of logics
- Connectives
- Rules of Inferences.

Predicate Calculus

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- Quantifies
- Rules of Inference.

Propositional Calculus:

It is a statement which gives an output true or false but not both.

Note: The lower case letters represents the statements p, q, r.

Examples:-

Sentence	proposition	Reason.
1. Karnataka is in India.	Yes, It is!	We can declare Yes/NO
2. 1+1 = 2	Yes, It is!	We can declare Yes/NO
3. x+1 = 2	No, It is not!	We cannot declare Yes/No
4. Do your Home work	NO	We cannot declare Yes/No.
5. What are you doing?	NO	We cannot declare Yes/No.
6. It will rain tomorrow	~~Yes~~ NO	We cannot declare Yes/NO

Logical operator:-

Operator	Truth Table	Meaning.
1. Negation ($\sim$)	<table><tr><th>p</th><th>$\sim p$</th></tr><tr><td>T</td><td>F</td></tr><tr><td>F</td><td>T</td></tr></table>	Truth value of $\sim p$ is opposite to p. [$\sim p$ - not p]
2. Conjunction ($\wedge$) [AND]	<table><tr><th>p</th><th>q</th><th>$p \wedge q$</th></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>F</td></tr><tr><td>F</td><td>F</td><td>F</td></tr></table>	The conjuction p + q ($p \wedge q$) is true only when both p and q are true. Otherwise it is false.
3. Disjunctive ($\vee$) [OR]	<table><tr><th>p</th><th>q</th><th>$p \vee q$</th></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>T</td></tr><tr><td>F</td><td>T</td><td>T</td></tr><tr><td>F</td><td>F</td><td>F</td></tr></table>	$p \vee q$ is false when both p and q are false otherwise it is true.
4. Conditional ($\rightarrow$) (If ... Then)	<table><tr><th>p</th><th>q</th><th>$p \rightarrow q$</th></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>T</td></tr><tr><td>F</td><td>F</td><td>T</td></tr></table>	If p is true and q is false then $p \rightarrow q$ is false, otherwise it is true.
5. Bi-conditional ($\leftrightarrow$) (If and only if)	<table><tr><th>p</th><th>q</th><th>$p \leftrightarrow q$</th></tr><tr><td>T</td><td>T</td><td>T</td></tr><tr><td>T</td><td>F</td><td>F</td></tr><tr><td>F</td><td>T</td><td>F</td></tr><tr><td>F</td><td>F</td><td>T</td></tr></table>	If p iff q is true when the statements p and q have the same truth value.
6. Exclusive OR. ($\underline{\vee}$, $\oplus$)	<table><tr><th>p</th><th>q</th><th>$p \oplus q$</th></tr><tr><td>T</td><td>T</td><td>F</td></tr><tr><td>T</td><td>F</td><td>T</td></tr><tr><td>F</td><td>T</td><td>T</td></tr><tr><td>F</td><td>F</td><td>F</td></tr></table>	$p \oplus q$ is false when both p and q are same otherwise it is true.

1. Write down in symbolic form.

p : Ramu is rich

q : Ramu is handsome.

a) Ramu is poor but handsome

b) Ramu is rich and handsome

c) Ramu is either rich or handsome

d) It is not true that ramu is rich or handsome.

e) Ramu is neither rich nor handsome

Ans:- a) $\sim p \wedge q$

b) $p \wedge q$

c) $p \oplus q$

d) $\sim (p \vee q)$

e) $\sim (p \wedge q)$ (or) $\sim p \vee \sim q$

2. write down in symbolic form.

p : you have the flu

q : you missed the final exam

r : you passed the course.

a) you have the flu or you missed the exam or you pass $p \vee q \vee r$.

b) If you have flu then you will not pass the course or if you miss final exam, you will not pass the course.

c) you have flu and you missed exam or you will not miss the exam and you passed the course

Ans:- (a) $p \vee q \vee r$

(b) $(p \rightarrow \sim r) \vee (q \rightarrow \sim r)$

(c) $(p \wedge q) \vee (\sim q \wedge r)$.

3. write down in symbolic form.

1. Jack and jill went up the hill.
2. If either jerry takes calculus or john takes algebra then alex takes engish.
3. The growth of Crop will be good if there is a rain.

Ans: i) $p \wedge q$, ii) $(p \vee q) \rightarrow r$, iii) $q \rightarrow p$.

4. Truth Table:

4. Construct the truth table for i) $\sim p \wedge \sim q$. ii) $(p \vee q) \leftrightarrow (q \vee p)$

i) $\sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

ii) $(p \vee q) \leftrightarrow (q \vee p)$

p	q	$p \vee q$	$q \vee p$	$(p \vee q) \leftrightarrow (q \vee p)$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

(5). Construct a truth table for $\sim(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \vee r))$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$\sim(p \vee (q \wedge r))$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$\sim(p \vee (q \wedge r)) \leftrightarrow (p \vee q) \wedge (p \vee r)$
T	T	T	T	T	F	T	T	T	F
T	T	F	F	T	F	T	T	T	F
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	F	T	T	T	F
F	T	T	T	T	F	T	T	T	F
F	T	F	F	F	T	T	F	F	F
F	F	T	F	F	T	F	T	F	F
F	F	F	F	F	T	F	F	F	F

6. ~~write~~ Construct the truth table for $p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p)$

p	q	r	$q \leftrightarrow r$	$p \wedge (q \leftrightarrow r)$	$r \leftrightarrow p$	$p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p)$
T	T	T	T	T	T	T
T	T	F	F	F	F	F
T	F	T	F	F	T	T
T	F	F	T	T	F	T
F	T	T	T	F	F	F
F	T	F	F	F	T	T
F	F	T	F	F	F	F
F	F	F	T	F	T	T

Converse, Contrapositive & Inverse.

Converse:

Def: The proposition $q \rightarrow p$ is called the Converse of $p \rightarrow q$.

Contrapositive:

Def: The proposition $\sim q \rightarrow \sim p$ is called the Contrapositive of $p \rightarrow q$.

Inverse:-

Def: The Proposition $\sim p \rightarrow \sim q$ is called inverse of $p \rightarrow q$.

7. obtain Converse, Contrapositive and inverse of "Team India wins whenever Dhoni is captain"

Soln:

Let p : Dhoni is Captain

q : Team India wins

Converse ($q \rightarrow p$) : If team India wins, then Dhoni is captain.

Contrapositive ($\sim q \rightarrow \sim p$) : If Team India does not win then Dhoni is not Captain.

Inverse ($\sim p \rightarrow \sim q$) : If Dhoni is not captain, then Team india doesnot win.

Tautology:

Def: A Compound propositions which is true always irrespective of the truth values of the individual variables is called a Tautology.

Eg: $p \vee \sim p$

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

$\therefore$ $p \vee \sim p$ is Tautology.

Contradiction:

Def: A Compound propositions which is false always is called a Contradiction.

Eg: $p \wedge \sim p$.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

$\therefore p \wedge \sim p$ is Contradiction

Contigency:

Def: A compound proposition which is neither Tautology nor Contradiction is called Contigency.

Eg: $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$\therefore p \leftrightarrow q$ is contigency.

Eg:- a) Find the following statements are Tautology.

i) $p \rightarrow (p \vee Q)$.

ii) $((p \vee Q) \wedge (\sim (P \vee R)) \rightarrow (Q \vee R)$

b) Find the following statements are Contradiction.

i) $\sim (Q \rightarrow R) \wedge R \wedge (P \rightarrow Q)$

ii) $(P \wedge Q) \wedge \sim (P \vee Q)$.

8. Find the following statement is Tautology.

$((P \vee Q) \wedge (\sim(P \vee R)) \rightarrow (Q \vee R)$.

P	Q	R	$P \vee Q$	$\sim P$	$\sim(P \vee R)$	$Q \vee R$	$((P \vee Q) \wedge \sim(P \vee R)$	$((P \vee Q) \wedge \sim(P \vee R)) \rightarrow (Q \vee R)$
T	T	T	T	F	T	T	T	T
T	T	F	T	F	F	T	F	T
T	F	T	T	F	T	T	T	T
T	F	F	T	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	F	T	T	T	F	T
F	F	F	F	T	T	F	F	T

∴ It is a Tautology.

9. Construct Truth trable for the following statements and find the statements are contradiction.

i) $\sim(Q \rightarrow R) \wedge R \wedge (P \rightarrow Q)$ ii) $(P \wedge Q) \wedge \sim(P \vee Q)$.

Ans : (i)

P	Q	R	$Q \rightarrow R$	$\sim(Q \rightarrow R)$	$P \rightarrow Q$	$\sim(Q \rightarrow R) \wedge R \wedge (P \rightarrow Q)$.
T	T	T	T	F	T	F
T	T	F	F	T	T	F
T	F	T	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	F
F	T	F	F	T	T	F
F	F	T	T	F	T	F
F	F	F	T	F	T	F.

∴ It is a contradiction.

10. Construct the truth table for the following statement.

$\sim(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \to r))$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$\sim(p \vee (q \wedge r))$	$p \vee q$	$p \to r$	$(p \vee q) \wedge (p \to r)$	$\sim(p \vee (q \wedge r)) \leftrightarrow (p \vee q) \wedge (p \to r)$
T	T	T	T	T	F	T	T	T	F
T	T	F	F	T	F	T	F	F	T
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	F	T	F	F	T
F	T	T	T	T	F	T	T	T	F
F	T	F	F	F	T	T	T	T	T
F	F	T	F	F	T	F	T	F	F
F	F	F	F	F	T	F	T	F	F

It is contingency.

11. Construct the truth table for the following statement.

$(p \to (q \to r)) \wedge ((\sim r \vee p) \wedge q)$.

p	q	r	$q \to r$	$p \to (q \to r)$	$\sim r$	$\sim r \vee p$	$((\sim r \vee p) \wedge q)$	$(p \to (q \to r)) \wedge ((\sim r \vee p) \wedge q)$
T	T	T	T	T	F	T	T	T
T	T	F	F	F	T	T	T	F
T	F	T	T	T	F	T	F	F
T	F	F	T	T	T	T	F	F
F	T	T	T	T	F	F	F	F
F	T	F	F	T	T	T	T	T
F	F	T	T	T	F	F	F	F
F	F	F	T	T	T	T	F	F

It is a contigency.

12. Construct the truth table for the following statement

$$((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r.$$

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$(p \vee q) \wedge (p \rightarrow r)$	$((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r))$	$((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	T
F	F	T	F	T	T	F	F	T
F	F	F	F	T	T	F	F	T

It is a Tautology.

Logical equivalence and Implication formula.

Def of Equivalence:

Let S be the set of statements and P, Q be the statements generated from S then P and Q are equivalent or logically equivalent of $P \leftrightarrow Q$ is a Tautology.

ie) $P \Leftrightarrow Q$ iff $P \leftrightarrow Q$ is a Tautology.

Def of Implication:-

Let S be the set of statements and P, Q are statements generated from S, then $P \Rightarrow Q$ if $P \to Q$ is a Tautology.

Equivalance Formula:

[Note: $P \vee Q \Leftrightarrow \sim P \to Q$
$P \to Q \Leftrightarrow \sim P \vee Q$
$P \wedge Q \Leftrightarrow \sim (P \to \sim Q)$]

E_1	Double Negation	$\sim\sim P \Leftrightarrow P$.
E_2	Idempotent law	$P \vee P \Leftrightarrow P$ $P \wedge P \Leftrightarrow P$
E_3	Complement law	$P \vee \sim P \Leftrightarrow T$ $P \wedge \sim P \Leftrightarrow F$
E_4	Dominant law	$P \vee T \Leftrightarrow T$ | $P \wedge T \Leftrightarrow P$ $P \vee F \Leftrightarrow P$ | $P \wedge F \Leftrightarrow F$
E_5	Commutative law	$P \wedge Q \Leftrightarrow Q \wedge P$ $P \vee Q \Leftrightarrow Q \vee P$.
E_6	Associative law	$P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$ $P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$.
E_7	Distributive law	$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$ $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$
E_8	De Morgan's law	$\sim (P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$ $\sim (P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$
E_9	Absorption law	$P \vee (P \wedge Q) \Leftrightarrow P$ $P \wedge (P \vee Q) \Leftrightarrow P$
E_{10}	Contrapositive law	$P \to Q \Leftrightarrow \sim Q \to \sim P$ $P \to Q \Leftrightarrow \sim P \vee Q$.

13. prove that $P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R$.

$P \to (Q \to R)$	Reason.
$\Leftrightarrow \sim P \vee (Q \to R)$	Contrapositive law
$\Leftrightarrow \sim P \vee (\sim Q \vee R)$	Contrapositive law
$\Leftrightarrow (\sim P \vee \sim Q) \vee R$	Associative law
$\Leftrightarrow \sim (P \wedge Q) \vee R$	De-Morgan's law
$\Leftrightarrow (P \wedge Q) \to R$	Contrapositive law

Hence proved.

(14). prove that $(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

$(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$	Reason.
$\Leftrightarrow (\sim P \wedge (\sim Q \wedge R)) \vee P \wedge (Q \vee R)$ $((P \vee Q) \wedge R)$	Distributive law
$\Leftrightarrow (\sim (P \wedge Q) \wedge R) \vee ((P \vee Q) \wedge R)$	De-Morgan's law
$\Leftrightarrow [\sim (P \vee Q) \vee (P \vee Q)] \wedge R$	Distributive law
$\Leftrightarrow T \wedge R$	$\sim P \wedge P \Leftrightarrow T$ (Compliment law)
$\Leftrightarrow R$	Dominant law.

(15). Prove that $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ is Tautology by Equivalence formula.

$Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$	Reason.
$\Leftrightarrow Q \vee (P \wedge \sim Q) \vee \sim (P \vee Q)$	De-Morgan's law
$\Leftrightarrow Q \vee (\sim Q \wedge P) \vee \sim (P \vee Q)$	Commutative law
$\Leftrightarrow (Q \vee \sim Q) \wedge P \vee \sim (P \vee Q)$	Associative law
$\Leftrightarrow T \wedge P \vee \sim (P \vee Q)$.	Compliment law
$\Leftrightarrow P \vee \sim (P \vee Q)$	Dominant law
$\Leftrightarrow (P \vee \sim P) \vee \sim Q$	Associative law
$\Leftrightarrow T \vee \sim Q$	compliment law
$\Leftrightarrow T$	Dominant law.

Connectives:-

Def: Connective is an operation that is used to Connect two or more than two statements, simply it is called sentencial connectives. It is also called as logical connectives.

Compound proposition:

Def: The statement contain one or more primary statements and some connectives are called compound proposition or modular statements or composite statements.

statement formula:

Def: It is an expression which is a string consisting of variables (with or without subscripts). paranthesis and connecting symbols.

Well-formed formula:

Def: It can be generated by the following rules.

1. A statement variable standing alone is a well-formed formula.
2. If A is well-formed formula, then $\neg A$ is also well-formed formula.
3. If A and B are well-formed formulae, then $(A \wedge B)$, $(A \vee B)$, $A \rightarrow B$, $A \leftrightarrow B$ are also well-formed formula.

Other Connectives :-

1. EX - OR [$\bar{\vee}$, $\underline{\vee}$, $\oplus$]

Def: Let P & Q be any two statements formula then $P \oplus Q$ is called exclusive - OR and it is defind as true whenever either P or Q is true but not both.

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

2. NAND ($\uparrow$) : The connective NAND is denoted by $\uparrow$ and it is defind as $P \uparrow Q \Leftrightarrow \sim(P \wedge Q)$.

P	Q	$P \wedge Q$	$P \uparrow Q \Leftrightarrow \sim(P \wedge Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

3. NOR ($\downarrow$) : The connective NOR is denoted by $\downarrow$ and it is defind as $P \downarrow Q \Leftrightarrow \sim(P \vee Q)$.

P	Q	$P \vee Q$	$P \downarrow Q \Leftrightarrow \sim(P \vee Q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

5). prove that $P \uparrow Q \Leftrightarrow (P \downarrow Q)$! soln: - $\sim(P \wedge Q) \Leftrightarrow \sim(P \vee Q)$

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$	$P \vee Q$	$\sim(P \vee Q)$	$\sim(P \wedge Q) \Leftrightarrow \sim(P \vee Q)$
T	T	T	F	T	F	T
T	F	F	T	T	F	F
F	T	F	T	T	F	F
F	F	F	T	F	T	T

Theory of Inferences.

Premises:

A premises is the statement which is assumed to be true.

Types of premises: Direct Method & Indirect Method.

Direct Method:

Rule of Inferences, A set of premises $H_1, H_2, \cdots H_n$ and a conclusion 'c' given and assume that set of premises $H_1, H_2 \cdots H_n$ are true. we want to conclude that C is a true.

Indirect Method:

In order to show that conclusion C follows logically from the premises $H_1, H_2, \cdots H_m$. we assume C is "false" and consider $\sim C$ as an additional premises.

Rule P:

A premises may be introduced at any point in the direction (Given Premises)

Rule T:

A formula S may be introduced in a derivative of S is tautologically implied by any one or more preceding formula in the derivation

	Rule in Tautological form	Rule name.
1.	$p \to q$ $q \to r$ $\therefore p \to r$	Transitive Rule.
2.	$\sim q$ $p \to q$ $\therefore \sim p$	Modus Tollens.
3.	$p \vee q$ $\sim p$ $\therefore q$	Disjuctive syllogism.
4.	p $p \to q$ $\therefore q$	Modus Phonus
5.	$(p \wedge q) \to p$; $p \wedge q \Rightarrow p$ $(p \wedge q) \to q$; $p \wedge q \Rightarrow q$	Simplification.
6.	$p \to (p \vee q)$ $q \to (p \vee q)$	Addition
7.	$(p \wedge q) \to (p \wedge q)$	Conjuction.
8.	$(p \wedge (p \to q)) \to q$	Modus Ponens.
9.	$(\sim q \wedge (p \to q)) \to \sim p$	Modus Tollens.
10.	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical Syllogism. (Si-luh-ji-zm)
11.	$[(p \vee q) \wedge (\sim p \vee r)] \to (q \vee r)$	~~Disjuctive syllogism.~~ Resolution.
12.	$((p \vee q) \wedge \sim p) \to q$	Disjuctive syllogism
13.	$[(p \vee q) \wedge (p \to r) \wedge (q \to r)]$ $\to r$.	Dilemma.

17. Show that $\sim P$ follows from the premises $P \to Q$, $\sim Q$.

Step	premises	Reason.
1.	$P \to Q$	Rule P.
2.	$\sim Q$	Rule P.
3.	$\sim P$.	Rule T (Modus Tollens) 1 & 2.

✓

18. Show that $S \vee R$ are Tautologically implied by $P \vee Q$, $P \to R$, $Q \to S$.

Step	premises	Reason.
1.	$P \vee Q$	Rule P
2.	$P \to R$	Rule P.
3.	$Q \to S$	Rule P.
4.	$\sim P \to Q$	Rule T (from 1) Contrapositive law.
5.	$\sim S \to \sim Q$	Rule T (from 3) Contrapositive law
6.	$\sim Q \to P$	Rule T (from 4) Contrapositive law.
7.	$\sim S \to P$.	Rule T (from 5 & 6) Transitive law
8.	$\sim S \to R$	Rule T (frm 2 & 7) Transitive law
9.	$S \vee R$	Rule T (from 8) Contrapositive law
	$\therefore$ $S \vee R$ proved.	

(19). Show that $P \to (Q \to S)$ from the premises $P \to (Q \to R)$, $Q \to (R \to S)$.

Step.	premises	Reason:
1.	$P \to (Q \to R)$	Rule P.
2.	$Q \to (R \to S)$	Rule P.
3.	$\sim P \vee (Q \to R)(S \to R)$	Rule T (1)
4.	$\sim P$	Rule T (3)
5.	$(\sim Q \vee S) \vee \sim R$	Rule T (2)
6.	$(Q \to S) \vee \sim R$	Rule T (5)
7.	$Q \to S$	Rule T (6)
8.	$\sim P \vee (Q \to S)$	Rule T (4, 7)
9.	$P \to (Q \to S)$	Rule T (8).

Quantifiers

Universal Quantifiers

Existential quantifiers.

Universal Quantifier:- Let $P(x)$ be the statement function, then the notation (for all x) $\forall x\, p(x)$ (or) $x\, p(x)$. denotes the Universal quantifiers.

Eg: Let $M(x)$ be statement where x is a man.

Let $C(x)$ be Statement where x is clever.

The symbolic form of statement "All men are clever" is $\forall x\, (M(x) \to C(x))$

Existential Quantifier:-

Let $P(x)$ be ~~exist~~ statement function then the notation $\exists x\, P(x)$ denotes the existential quantifier.

Eg: " some men are clever ":

$\exists x. [M(x) \wedge C(x)]$

Universe of discourse:

It is defined as a set of all values taken by a variable. the universe of discourse specific the possible values of variable x.

Therefore, it is defined as the domain of a variable in a propositional function.

Q. Write each of the following in symbolic form

i) All men are clever.

ii) No men are clever.

iii) Some men are clever

iv) Some men are not clever.

Soln: Let $M(x)$ be the statement where x is a men
$C(x)$ be the statement where x is clever.

i) All men are clever : $\forall x [M(x) \rightarrow C(x)]$

ii) No men are clever : $\forall x [M(x) \rightarrow \sim C(x)]$

iii) Some men are clever : $\exists x [M(x) \wedge C(x)]$

iv) Some men are not clever : $\exists x [M(x) \wedge \sim C(x)]$

Rule of Inference.

There are four rules of Inference.

- US — Universal of specification
- UG — Universal of Generalisation
- ES — Existensial Specification
- EG — Existential Generalisation

Note:

US — $\forall x\, p(x) \Rightarrow p(y)$.

UG — $py \Rightarrow \forall x\, p(x)$.

ES — $\exists x\, p(x) \Rightarrow p(y)$.

EG — $p(y) \Rightarrow \exists x\, p(x)$.

20). Show that $\forall x[p(x) \rightarrow Q(x)] \wedge (\forall x)[Q(x) \rightarrow R(x)] \rightarrow \forall x\, [p(x) \rightarrow R(x)]$.

Step.	Premises	Reason.
1.	$\forall x[p(x) \rightarrow Q(x)]$	Rule P.
2.	$\forall x\, [Q(x) \rightarrow R(x)]$	Rule P.
3.	$p(y) \rightarrow Q(y)$	Rule T (US). from 1
4.	$Q(y) \rightarrow R(y)$	Rule T (US) from 2
5.	$p(y) \rightarrow R(y)$	Rule T (frm 3 & 4) Transitive law
6.	$\forall x[p(x) \rightarrow R(x)]$	Rule G (from 5).

21. Prove that $\exists x\, P(x) \rightarrow \forall x\, Q(x) \Rightarrow \forall x [P(x) \rightarrow Q(x)]$

~~by indirect method~~

Step.	Premises	Reason.
1.	$\sim [\forall x\, P(x) \rightarrow Q(x)]$	Assumed premises.
2.	$\exists x \sim [P(x) \rightarrow Q(x)]$	Rule T
3.	$\exists x\, P(x) \rightarrow \forall x\, Q(x)$	Rule P.
4.	$P(y) \rightarrow \forall x\, Q(x)$	Rule ES [from 3]
5.	$P(y) \rightarrow Q(y)$	Rule US (4)
6.	$\forall (x)\ [P(x) \rightarrow Q(x)]$	Rule UG (5)

22. Show that $\forall x [P(x) \vee Q(x)] \Rightarrow (\forall x) P(x) \vee (\exists x) Q(x)$ by Indirect Method.

Step	Premises	Reason.
1.	$\sim [(\forall x) P(x) \vee (\exists x) Q(x)]$	Assumed Premises.
2.	$(\exists x) \sim P(x) \wedge (\forall x) \sim Q(x)$	Rule T (1)
3.	$\forall x [P(x) \vee Q(x)]$	Rule P.
4.	$(\exists x) \sim P(x)$	Rule T (2)
5.	$(\forall x) \sim Q(x)$	Rule T (2)
6.	$P(y) \vee Q(y)$	Rule US (from 3)
7.	$\sim P(y) \rightarrow Q(y)$	Rule T (6) Contrapositive law
8.	$\sim P(y)$	Rule ES (4)
9.	$\sim Q(y)$	Rule US (5)
10.	$Q(y)$	Rule T (7, 8)
11.	$Q(y) \wedge \sim Q(y)$	Rule T (9, 10)
12	F	proved.

23. Show that the argument is valid

i) Every micro computer has a serial interface port.

ii) Some microcomputers have a parallel port.

iii) Therefore, some micro computer have both serial and parallel port.

Soln:- Let $M(x)$: x is a micro computer.

$S(x)$: x is a serial port.

$P(x)$: x is a parallel port.

The given argument, $(\forall x)[M(x) \rightarrow S(x)]$,

$(\exists x)[M(x) \wedge P(x)]$.

Conclusion : $\exists x[M(x) \wedge S(x) \wedge P(x)]$.

Step.	Premises	Reason.
1.	$\forall x[M(x) \rightarrow S(x)]$	Rule P.
2.	$\exists x[M(x) \wedge P(x)]$	Rule P.
3.	$M(y) \rightarrow S(y)$.	Rule US (1)
4.	$M(y) \wedge P(y)$	Rule ES (2)
5.	$M(y)$	Rule T (fm 4)
6.	$P(y)$	Rule T (4)
7.	$S(y)$	Rule T (3,5) Transitive law
8.	$M(y) \wedge P(y) \wedge S(y)$	Rule T (5,6,7).
9.	$\exists x[M(x) \wedge S(x) \wedge P(x)]$	Rule EG (8)

Hence proved.

HW

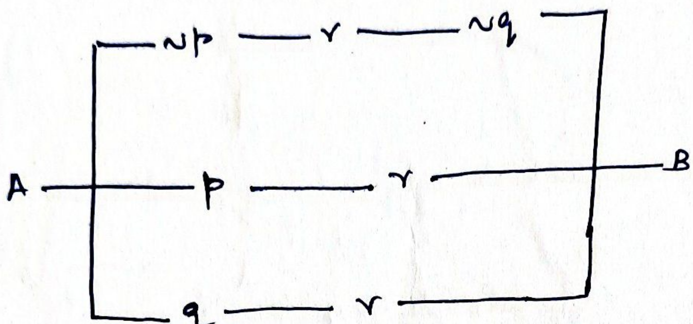
24. Show that the argument is valid:

i) All maths professors have studied calculus.

ii) Raja is a maths professor.

iii) Therefore, Raja has studied calculus.

25. Explain switching network, simplify the switching network using laws of logic.



Soln:- The given network can be represented as

$(\sim p \wedge r \wedge \sim q) \vee (p \wedge r) \vee (q \wedge r)$.

Step	Premises	Rule (Reason)
1	$(\sim p \wedge r \wedge \sim q) \vee (p \wedge r) \vee (q \wedge r)$	Given.
1.	$(\sim p \wedge r \wedge \sim q) \vee ((p \vee q) \wedge r)$	Distributive law
2.	$(\sim (p \vee q) \wedge r)((p \vee q) \wedge r)$	Demorgan's law
3.	$((p \vee q) \vee \sim (p \vee q) \wedge r)$	Associative law
4.	$((p \vee q) \vee \sim (p \vee q)) \wedge r$	Distributive law & Inverse law
5.	$(T \wedge r)$	Identity law.
6.	r	

∴ The network can be represented as 

14) a) Test the validity of the following argument:

- I will become famous or I will not become a musician
- I will become a musician.

$\therefore$ I will become famous

Sln:- Let p : I will become famous

q : I will become musician.

Then, the given argument reads,

$$\begin{array}{r} p \vee \sim q \\ q \\ \hline \therefore \quad p \\ \hline \end{array}$$

This argument is logically equivalent to

$$\begin{array}{l} q \rightarrow p \\ q \\ \hline \therefore p \\ \hline \end{array}$$

(because $p \vee \sim q \Leftrightarrow q \vee p \Leftrightarrow q \rightarrow p$)

$\therefore$ By the Modus Pones Rules, this argument is valid

b) Test whether the following is a valid argument.

If I study, then I do not fail in the examination

If I do not fail in the examination, my father gifts a two-wheeler to me.

$\therefore$ If I study then my father gifts a two-wheeler to me.

Sln:- p : I study   q : I do not fail in the examination

r : My father gifts a two-wheeler to me.

Then the given arguments reads,

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

By rule of syllogism, this is a valid argument.

15) a) Test the validity of the following arguments:

If I study, I will not fail in the exam
If I do not watch TV in the evenings, I will study
I failed in the exam.

$\therefore$ I must have watched TV in the evenings.

Sln:- Let p: I study q: I fail in the exam
r: I watch TV in the evenings.

Then, the given argument reads
$$\begin{array}{l} p \to \sim q \\ \sim r \to p \\ q \\ \hline \therefore r \end{array}$$

This argument is logically equivalent to,
$$\begin{array}{ll} q \to \sim p & (\text{because } (p \to \sim q) \Leftrightarrow (\sim\sim q \to \sim p)) \\ \sim p \to r & (\text{because } (\sim r \to p) \Leftrightarrow (\sim p \to r)) \\ q & \\ \hline \therefore r & \end{array}$$

This is equivalent to,
$$\begin{array}{ll} q \to r & (\text{using Rule of syllogism} \\ q & \\ \hline \therefore r & \end{array}$$

This argument is valid by the Modus Pones Rule.

15) b) Consider the following argument;
I will get grade A in this course or I will not graduate
If I do not graduate, I will join the army.
I got grade A.

$\therefore$ I will not join the army.

Is this a valid argument?

S/n:- Let p: I got grade A in this course
q: I do not graduate. r: I join the army.

Then the given argument reads
$$\begin{array}{l} p \vee q \\ q \rightarrow r \\ p \\ \hline \therefore \sim r \end{array}$$

This argument is logically equivalent to
$\sim q \rightarrow p$ (because $p \vee q \equiv q \vee p \Leftrightarrow \sim q \rightarrow p$)
$\sim r \rightarrow \sim q$ (using contrapositive)
$\therefore \sim r$

This is logically equivalent to
$\sim r \rightarrow p$ (using Rule of Syllogism)
p
$\therefore \sim r$

$\therefore$ This is not a valid argument ($\because$ the truth table not a tautology)

16) a) Test the validity of the following.

(i)
$$\begin{array}{l} p \\ p \rightarrow \sim q \\ \sim q \rightarrow \sim r \\ \hline \therefore \sim r \end{array}$$

S/n The permises $p \rightarrow \sim q$ and $\sim q \rightarrow \sim r$ together yield the permise $p \rightarrow \sim r$. Since p is true, this premise $(p \rightarrow \sim r)$ establishes that $\sim r$ is true. Hence the given argument is valid.

(ii)
$$\begin{array}{l} p \rightarrow r \\ q \rightarrow r \\ \hline \therefore (p \vee q) \rightarrow r \end{array}$$

S/n:- We not that
$(p \rightarrow r) \wedge (q \rightarrow r)$
$\Leftrightarrow (\sim p \vee r) \wedge (-q \vee r)$

$\Longleftrightarrow (r \vee \sim p) \wedge (r \vee \sim q)$, by commutative law

$\Longleftrightarrow r \vee (\sim p \wedge \sim q)$, by distributive law

$\Longleftrightarrow \sim (p \vee q) \vee r$, by commutative & DeMorgan laws

$\Longleftrightarrow (p \vee q) \rightarrow r$

This logical equivalance shows that the given argument is valid.

(iii) $p \rightarrow q$
$r \rightarrow s$
$p \vee r$
$\therefore q \vee s$

Sln: we not that

$(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)$

$\Longleftrightarrow (p \rightarrow q) \wedge (r \rightarrow s) \wedge (\sim p \rightarrow r)$

$\Longrightarrow (p \rightarrow q) \wedge (\sim p \rightarrow s)$ (by Rule of syllogism)

$\Longleftrightarrow (\sim q \rightarrow \sim p) \wedge (\sim p \rightarrow s)$ (by contrapositive)

$\Rightarrow \sim q \rightarrow s$, (by Rule of Syllogism)

$\Longleftrightarrow q \vee s$

$\therefore$ This shows that the given argument is valid.

(iv) $p \rightarrow q$
$r \rightarrow s$
$\sim q \vee \sim s$
$\therefore \sim (p \wedge r)$

Sln: we note that

$(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\sim q \vee \sim s)$

$\Longleftrightarrow (p \rightarrow q) \wedge (r \rightarrow s) \wedge (q \rightarrow \sim s)$

$\Rightarrow (p \rightarrow \sim s) \wedge (r \rightarrow s)$

(by commutative law & rule of syllogism)

$\Longleftrightarrow (p \rightarrow \sim s) \wedge (\sim s \rightarrow \sim r)$ (by contrapositive

$\Rightarrow p \rightarrow \sim r$, (by Rule of syllogism)

$\Longleftrightarrow \sim p \vee \sim r \Longleftrightarrow \sim (p \wedge r)$

This shows that the given argument is valid.

18) a) Consider the following open statements with the set of all real numbers as the universe $p(x): x \geq 0$, $q(x): x^2 > 0$ $r(x): x^2 - 3x - 4 = 0$, $s(x): x^2 - 3 > 0$. Determine the truth values of the following statements.

(i) $\forall x, r(x) \vee s(x)$ (ii) $\exists x, p(x) \wedge r(s)$ (iii) $\forall x, r(x) \rightarrow p(x)$

Sln:- i) we have $x^2 - 3x - 4 = (x-4)(x+1)$.

Hence, $r(x)$ is true only for $x = 4$ (or) $x = -1$.

As such, $r(x)$ and $s(x)$ are false for $x = 1$

Thus $r(x) \vee s(x)$ is not always true.

$\therefore \forall x, r(x) \vee s(x)$ is false, its truth value is 0.

(ii)

$x^2 - 3x - 4 = (x-4)(x+1)$.

for $(x-4)$, both $p(x)$ and $r(x)$ are true.

Therefore, $\exists x, p(x) \wedge r(x)$ is true, its truth value is 1.

(iii)

It is observed that $p(x)$ is false and $r(x)$ is true for $x = -1$.

Hence $r(x) \rightarrow p(x)$ is false for $x = -1$.

$\therefore r(x) \rightarrow p(x)$ is not true always.

Accordingly, $\forall x, r(x) \rightarrow p(x)$ is false, its truth value is 0

18) b) Write down the following proposition in symbolic form and find its negation: "All integers are rational numbers and some rational numbers are not integers".

Sln:- Let $p(x)$: x is a rational number

$q(x)$: x is an integer

and Z: Set of all integers, Q: Set of all rational numbers.

Then, in symbolic form, the given proposition reads,

$$\{\forall x \in Z, p(x)\} \wedge \{\exists x \in Q, \sim q(x)\}.$$

The negation of this is

$$\sim\{\forall x \in Z, p(x)\} \vee \sim\{\exists x \in Q, \sim q(x)\}$$

$$\equiv \{\exists x \in Z, \sim p(x)\} \vee \{\forall x \in Q, q(x)\}$$

In words, this reads:

"Some integers are not rational numbers or every rational number is an integer".

(Here the given proposition is true and the negation is false).

19)a) Write down the following proposition in symbolic form and find its negation, "If all triangles are right-angled then no triangle is equiangular".

Sln: Let T denote the set of all triangles,

Also let, $p(x)$: x is right angled

$q(x)$: x is equiangular

Then, in symbolic form, the given proposition reads

$$\{\forall x \in T, p(x)\} \longrightarrow \{\forall x \in T, \sim q(x)\}$$

The negation of this is

$$\{\forall x \in T, p(x)\} \wedge \{\exists x \in T, q(x)\}$$

In words, this reads "All triangles are right-angled and some triangles are equiangular".

19) b) Write down the negation of each of the following statements. (2)

(i) For all integers n, if n is not divisible by 2, then n is odd

(ii) If k, m, n are any integers where $(k-m)$ and $(m-n)$ are odd, then $(k-n)$ is even.

Sln: Let Z denote the set of all integers and R denote the set of all real numbers. Then:

(i) The given statement is

$$\forall n \in Z, \sim p(n) \rightarrow q(n).$$

where $p(n)$: n is divisible by 2, $q(n)$: n is odd

Therefore, the negation of the given statement is

$$\exists n \in Z, \sim p(n) \wedge \sim q(n).$$

In words, this negation reads:

For some integer n, n is not divisible by 2 and n is not odd.

(ii) The given statement is

$$\forall k, m, n \in Z, [p(x) \wedge q(x)] \rightarrow r(x),$$

where $p(x)$: $k-m$ is odd, $q(x)$: $m-n$ is odd, $r(x)$: $k-n$ is even.

The negation of this is,

$$\exists k, m, n \in Z, [p(x) \wedge q(x)] \wedge \sim r(x).$$

In words, the negation is

For some integer n, n is not divisible by 2 and n is not odd.