Module 2:-

Set Theory and Number Theory

→ Sets and Subsets
→ Important Identities
→ Set operations and Law's of Set Theory
→ Venn diagram
→ Addition principal.

Definition:-

A set is determined or defined as a collection of well defined ~~as~~ objects.

(with this their won't be any ambiguity & doubt in the selection of set)

The each object in the set is called members & elements. Ex: Books. cities. numbers.

If the number of elements in the set are finite we say that it is Finite set.

If the set having infinite number of elements then Infinite set.

The set having only on element is called 'Singleton set'

~~Power set~~ :-

Set representation:-

The set can be represented in two ways

* The tabulation method $S = \{1, 3, 5, 7 ..\}$
* The rule method $S = \{x \mid x \text{ is a +ve odd integer}\}$

Power set :- P(A) - Set of all subsets of a set A.
It has 2^n elements in n - no/ of element of A

The null set :- The set not containing any element is called '**Empty set**' or '**Null set**'. $\{\}$. ϕ. Φ

Equal set :-
Two sets A and B are said to be equal if they have precisely the same elements the we write $A = B$

Ex:- $A = \{1, 2, 3, 4\}$
$B = \{x / x \text{ is a positive integer with } x^2 < 20\}$

$\Rightarrow A = B$

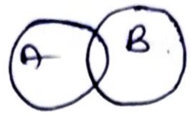
Subsets :-
Given two sets A and B, we say that A is a subset of B, or that A is contained in B if every element of A is an element of B as well.
$A \subseteq B$ [subset, contained in]

Ex:- $A = \{1, 2, 3.4\}$ $B = \{1, 2.3.4.5.6\}$ $C = \{2, 3.5.6\}$
$A \subseteq B$ but $A \nsubseteq C$

* **proper subset:** If B contain atleast on element not in A & contain all elements of A then A is called **Proper subset**

~~Universal set~~ : **Venn diagram :-**
The relationship between one & more set through diagrams is called '**Venn diagram**'

Ex:- $A \subset B$ $\qquad$ $A \not\subset B$



Some consequences:-

(1) Every set is a subset of itself
(2) The two set are equal if $A \subseteq B$ & $B \subseteq A$
(3) The null set ϕ is a subset of every set A
(4) For any set A, B and C, if $A \subseteq B$, $B \subseteq C \Rightarrow A \subseteq C$
(5) For any set $A = B$ & $B = C$ then $A = C$

Universal Set :-

All the possible ~~subsets~~ sets we consider are subset of certain set U. This set U is called 'Universal set'

Ex:- Study concern ~~on~~ with integers & integer set

* The universal set is not unique

power set Ex:-

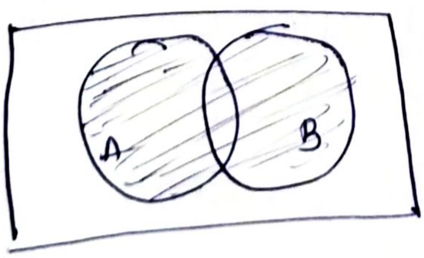
$A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\} \{2,3\} \{1,3\}, A\}$

operations on sets:-

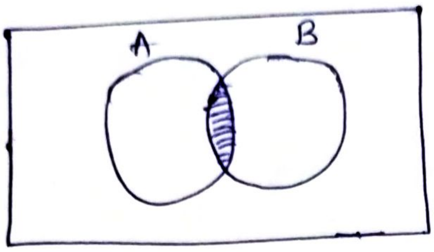
1. Union of ~~two~~ sets:-

$A \cup B = \{x \mid x \in A$ or $x \in B\}$



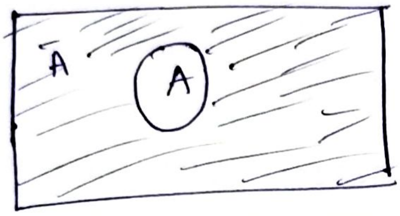
2. <u>Intersection of Sets</u> :-

$A \cap B = \{x / x \in A \text{ and } x \in B\}$



3. <u>complement of a set</u> :-

$\bar{A} = \{x / x \in U \text{ and } x \notin A\}$

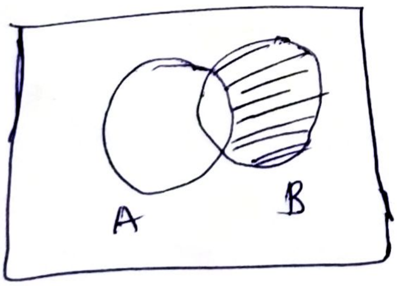


<u>Relatively complement</u> :-

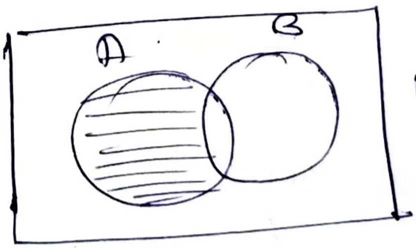
Given two sets A and B, the set of all elements that belong to B but not to A is called the complement of A relative to B denoted by. [relative complement of A in B]

$B - A = \{x / x \in B \text{ and } x \notin A\}$

$A - B = \{x / x \in A \text{ and } x \notin B\}$



not same



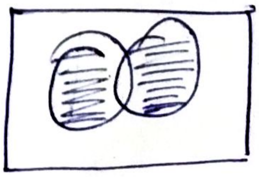
A-B

<u>Symmetric Difference</u> :-

The two sets A and B the relative complement of $A \cap B$ in $A \cup B$ is called Symmetric difference of A and B.

$A \Delta B = (A \cup B) - (A \cap B)$

$= \{x / x \in A \cup B \text{ and } x \notin A \cap B\}$



The Laws of Set Theory :-

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1st June
138 → yesterday/

1. Commutative Law :-

$A \cup B = B \cup A$, $A \cap B = B \cap B$

2. Associative law :-

$A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$

3. Distributive law :-

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4. Idempotent Laws :-

$A \cup A = A$
$A \cap A = A$

5. Identity Law

$A \cup \Phi = A$ $A \cap U = A$

6. Law of double complement :

$\bar{\bar{A}} = A$

7. Inverse Laws :-

$A \cup \bar{A} = U$ $A \cap \bar{A} = \Phi$

8. Demorgan Laws :-

$\overline{A \cup B} = \bar{A} \cap \bar{B}$ $\overline{A \cap B} = \bar{A} \cup \bar{B}$

9. Domination Law

$\cancel{A \cup B \neq}$ $A \cup U = U$ $A \cap \Phi = \Phi$

10. Absorption Laws :-

$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$

1. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Duality :- It's a principle.

Union is replaced by intersection and vice versa ~~and~~ also universal set by empty and vice versa

i.e $\quad \cup$ by $\cap \Leftrightarrow \cap$ by $\cup \qquad p^d$.

$\qquad U$ by $\Phi \Leftrightarrow \Phi$ by U

Ex:- $\quad P = \{(A \cup B) \cap \phi\} \cup \{(C \cap D) \cup U\}$

$\qquad p^d = \{(A \cap B) \cup U\} \cap \{(C \cup D) \cap \Phi\}$

Problems:- 1. For any three sets A, B, C prove that

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\Rightarrow$ let $x \in A \cup (B \cap C)$

$\cup$ &

$\cap$ and

$x \in A$ or $x \in B \cap C$

$x \in A$ or $x \in B$ and $x \in C$

$(x \in A$ or $x \in B)$ and $(x \in A$ & $x \in C)$

$\Rightarrow x \in (A \cup B) \cap . x \in (A \cup C)$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \rightarrow ①$

now $x \in (A \cup B) \cap (A \cup C)$

$(x \in A$ & $x \in B)$ and $(x \in A$ & $x \in C)$

i.e $x \in A$ & $x \in B$ and $x \in C$

i.e $x \in A \cup (B \cap C)$ $\quad \therefore (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \rightarrow ②$

from ① & ② $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\Rightarrow \quad x \in [A \cap (B \cup C)]$

$\Rightarrow \quad x \in A$ and $x \in B \cup C$

$\Rightarrow \quad x \in A$ and $x \in B$ or $x \in C$

$\Rightarrow \quad (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$\Rightarrow \quad (x \in A \cap B)$ or $(x \in A \cap C)$

$\Rightarrow \quad x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow \therefore \quad A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \rightarrow (1)$

now in other hand

$(A \cap B) \cup (A \cap C)$

$\Rightarrow x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow (x \in A$ and $x \in B)$ or $(x \in A$ and $x \in C)$

$\Rightarrow x \in A$ and $x \in B$ or $x \in C$

$\Rightarrow x \in A \cap (B \cup C)$

$\Rightarrow (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \rightarrow (2)$

from (1) & (2)

$\underline{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

consequences

1. For any two sets A & B $A-B$ & $B-C$ are disjoint
2. If A & B are disjoint then $A-B = A$
   $B-A = B.$
3. If U is the universal set & $A \subseteq B$ then
   $U - A = \overline{A}$
4. $A = (A \cup B) - (B - A)$
   $B = (A \cup B) - (A - B)$
5. $A = (A \cap B) \cup (A - B)$
   $B = (A \cap B) \cup (B - A)$

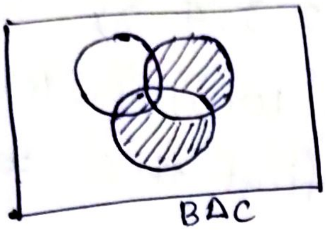
(2) ~~$\overline{A \cap B \cup C} = \overline{A}$~~

$\overline{(A \cup B) \cap C} = (\overline{A} \cap \overline{B}) \cup \overline{C}$

(3) using Venn Diagram prove that for any three sets A.B.C

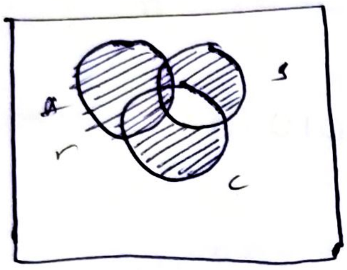
$A \Delta (B \Delta C) = (A \Delta B) \Delta C$

$B \Delta C = (B \cup C) - (B \cap C)$

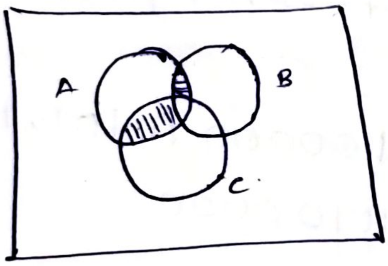


$B \Delta C$

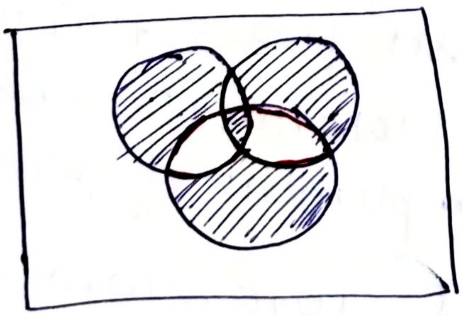
$A \cup (B \Delta C) =$



$A \cap (B \Delta C)$

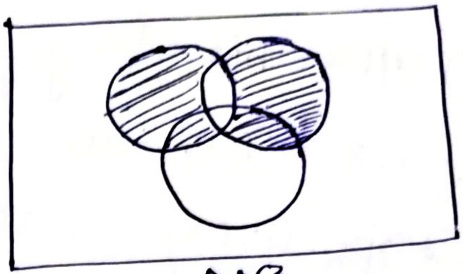


$A \Delta (B \Delta C) = (A \cup (B \Delta C)) - (A \cap (B \Delta C))$



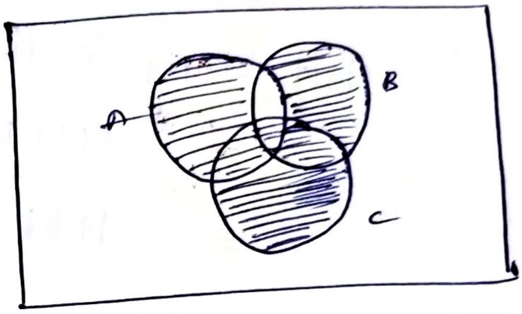
RHS

$A \Delta B = (A \cup B) - (A \cap B)$

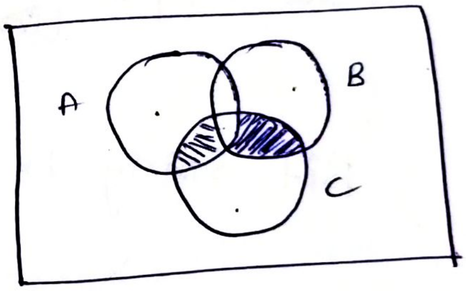


$A \Delta B$

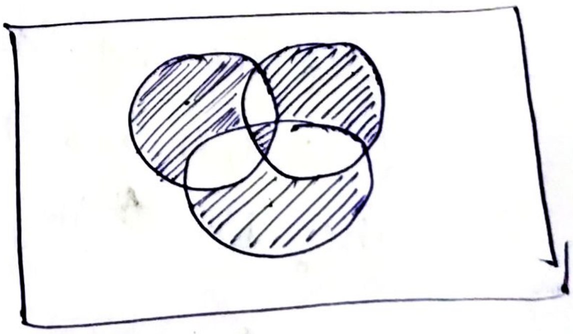
$(A \Delta B) \cup C$



$(A \Delta B) \cap C$



$(A \Delta B) \Delta C = [(A \Delta B) \cup C] - [(A \Delta B) \cap C]$



(b) If for any sets A.B.C, $A \Delta C = B \Delta C$ then $A = B$

Next

4 (a) The bit string for the set $\{1,2,3,4.5\}$ & $\{1,3,5,7,9\}$ are 1111100000 and 1010101010 respectively. Use bit string to find the union and intersection of these sets.

Soln: Bit String for the union.

$= 1111100000 \cup 1010101010$

$= 1111101010$ ✓

Set component of the union $= \{1,2,3,4,5,7,9\}$

Bit String Intersection $= 1111100000 \cap 1010101010$

$= 1010100000$

Set corresponding intersection $= \{1,3,5\}$

4 (b) If the bit string for the set $\{1,3,5,7.9\}$ in U $\{1,2,3,4.5,6.7.8,9.10\}$ is 1010101010 what is the bit string for the complement of this set

Soln: Bit string complement of 1010101010 is 0101010101

Set corresponding to the complement is $= \{2,4,6,8,10\}$

Countable and uncountable sets:-

Consider two non-empty sets A and B. we say that A and B have same size or cardinality if there exist one-to-one correspondence between their elements

Countable set:-

A set A is said to be countable (or denumerable) if ① A is finite set
② $A \sim Z^+$ where Z^+ is set of positive integers

Ex:- set $A = \{1, 3, 6, 7\}$ is countable. because the set is finite. Also $A \sim Z^+$

A set which is not countable called an uncountable set

(1) Every subset of a countable set is a countable set

(2) The union of two countable set is countable

(3) The intersection of two uncountable sets need not be uncountable.

Addition Principle:-

The number of finite elements in a set is called the order, size, or the cardinality of the set and is denoted as $o(S)$, $n(S)$, $|S|$.

If we consider the union two finite sets A and B and wish to determine the order of $A \cup B$ which is obviously a finite set. and can be obtained as.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

This result through which we can count the number of elements in the union of two finite sets is known as the 'principal of inclusion - exclusion for two sets. also called 'Addition principal'.

In the particular case where A and B are disjoint sets so that $A \cap B = \phi$ the addition principal stated above becomes,

$$\boxed{|A \cup B| = |A| + |B| - |\phi|}$$

This is known as the Principal of disjunctive counting for two sets.

* $|\bar{A}| = |U| - |A|$

$|A - B| = |A| - |A \cap B|$

$|\bar{A} \cap \bar{B}| = |U| - |A| - |B| + |A \cap B|$.

1. A computer company requires 30 programmers to handle systems programming jobs and 40 programmes for applications programming. If the company appoints 55 programmers to carry out these jobs.

How many of these perform jobs of both types?
How many handle only system programming jobs?
How many handle only applications programming?

Soln:

A denote S.P.
B denote A.P

$A \cup P \rightarrow$ for both handle.

$|A| = 30, \quad |B| = 40, \quad |A \cup B| = 55$

$\therefore \quad |A \cup B| = |A| + |B| - |A \cap B|$

$\therefore |A \cap B| = |A| + |B| - |A \cup B| = 30 + 40 - 55 = 15$

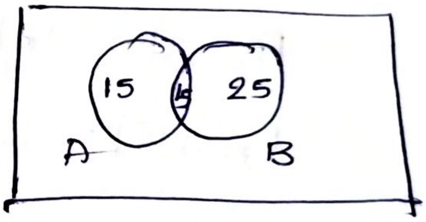
This means that 15 programmers perform both types of jobs

$\therefore$ Therefore the numbers of programmers who handle only ~~applications~~ System prog jobs is

$|A - B| = |A| - |A \cap B| = 30 - 15 = 15$

The number programmers handle only application progr

$|B - A| = |B| - |A \cap B| = 40 - 15 = 25$



② A professor has two dozen textbooks on computer Science and is concerned about their coverage of topics viz

ⓐ compilers ⓑ Data Structures ⓒ operating systems.

The following are the numbers of books that contain material on these topics

$|A| = 8, \quad |B| = 13 = |C|, \quad |A \cap B| = 5. \quad |A \cap C| = 3.$

$|B \cap C| = 6. \quad |A \cap B \cap C| = 2.$ ① How many have no material on Compiler

Q2:- How many of the books include material on exactly one of these topics? $|A \cup B \cup C|$

Q3: How many do not deal with any of the topic?

~~ABC~~ A . B . C

(a) How many have no material on compilers.

let U be the universal set $|U| = 24$.

then no material with compiler is

$|\bar{A}| = U - |A| = 24 - 8 = 16$

(b) The number of books which cover at least one of the indicated topic is

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$= 8 + 13 + 13 - 5 - 6 - 3 + 2$$

$$= 22$$

∴ The number of books which do not deal with any of the indicated topics is,

$\overline{|A \cup B \cup C|} = |U| - |A \cup B \cup C| = 24 - 22$

$= 2$

(c) Let A_1 denote the set of books which covers only compilers, B_1 denotes only data Structure. & C_1 denotes only operating system. then.

$A_1 = A - B - C$, $B_1 = B - A - C$ $C_1 = C - A - B$

Accordingly, the number of books which covers only compiler. is

$$|A_1| = |A - B - C| = |A| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$$

$$= 8 - 5 - 3 + 2$$

$$= 2$$

Illy

$$|B_1| = |B - A - C| = |B| - |B \cap A| - |B \cap C| + |B \cap C \cap A|$$

$$= 13 - 5 - 6 + 2$$

$$= 4$$

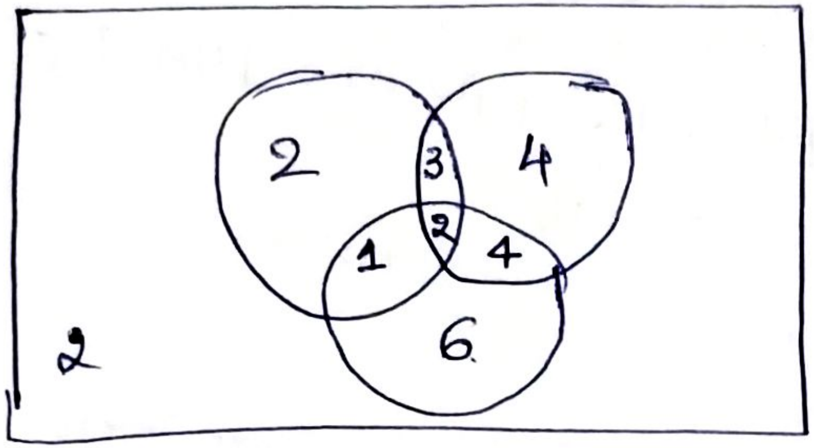
6 (a) and the number of books which covers only O.S

$$|C_1| = |C| - |C \wedge A| - |C \wedge B| + |C \wedge B \wedge A|$$

$$= 13 - 3 - 6 + 2 = 6$$

∴ The no. of books which includes material on exactly one of the topic is.

$$|A_1| + |B_1| + |C_1| = 2 + 4 + 6 = 12$$



5 (a) A certain computer center employs 100 programmers. Of these 47 can program in Java, 35 in Python, 20 in C++, 23 in Java and Python, 12 in C++ and Java, 11 in Python and C++ and 5 in all three of these languages. How many can program in none of these languages?

Ans:- Let A, B, C represent the programmers who program in Java, Python, and C++ resp.

given $|A| = 47$. $|B| = 35$. $|C| = 20$.

$|A \cap B| = 23$. $|(A \cap C)| = 12$. $|B \cap C| = 11$.

$|A \cap B \cap C| = 5$

then We need to find no. of programmers who can program in none of the language. (i.e complement)

$= |\overline{A \cup B \cup C}|$

$= 100 - |A \cup B \cup C|$

$= 100 - [|A| + |B| + |C| - |A \cap B| - |(A \cap C)| - |B \cap C| + |A \cap B \cap C|]$

$= 100 - [47 + 35 + 20 - 23 - 12 - 11 + 5]$

$= 100 - [107 - 46]$

$= 39$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

Extended additional principal.

6(a). How many integers are between 1 and 200 which are divisible by any one of the integers 2, 3 and 5?

$U = 200$

A = Set of all even integers.

B = Set of all integers multiples of 3.

C = set of all integers multiple of 5

$|A| = \frac{200}{2} = 100$, $|B| = \frac{200}{3} = 66$ $|C| = 40$

$|B \cap C| = \frac{200}{\cancel{64 \times 40}\ 3 \times 5} = 13$, $|A \cap B| = \frac{200}{2 \times 3} = 33$, $|A \cap C| = \frac{200}{2 \times 5} = 20$

$|A \cap B \cap C| = \frac{200}{2 \times 3 \times 5} = 6$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$= 100 + 66 + 40 - 33 - 13 - 20 + 6$

$= 146$ //

(b) How many integers are between 1 and 250 which are divisible by any one of the integer 3, 5 and 7.

Classwork.

7 (a) Cartesian product :-

Given two sets A and B, the cartesian product $A \times B$ is defined by $A \times B = \{ (a, b) \mid a \in A, b \in B \}$

i.e $A \times B$ is a set of ordered pairs.

Example :- $A = \{1, 2, 3\}$ $B = \{a, b, c\}$

$A \times B = \{ (1, a), (1, b), (1, c), (2, a) (2, b) (2, c), (3, a), (3, b), (3, c) \}$

$A \times B \neq B \times A$

$A \times A \Rightarrow$ cartesian product of A itself.

Ex :- $A \times A = \{ (1, 1) (1, 2) (1, 3) (2, 1) (2, 2) (2, 3) (3, 1) (3, 2) (3, 3) \}$

* For any two finite, non-empty sets A & B
$|A \times B| = |A| \, |B|$

Proof:- let $|A| = m$. $|B| = n$.

The elements of $A \times B$ are ordered pairs in which first element belongs to the A and Second element belongs to B..

i.e. $(x.y) \in A \times B$ where $x \in A$, $y \in B$

There are n ways to choose $x \in A$ for the first position in ordered pair and 'm' ways to choose $y \in B$ for second position in the ordered pair.

So. by multiplication principle there $m \times n$ ways to form an order pair (a,b)

i.e. $\boxed{|A \times B| = mn = |A|\,|B|}$

8(b). For any three sets A, B and C prove that

① $A \times (B \cup C) = (A \times B) \cup (A \times C)$

② $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution:- Consider $A \times (B \cup C) = \{(x.y) \mid x \in A \ \& \ y \in (B \cup C)\}$

$= \{(x.y) \mid x \in A \text{ and } (y \in B \text{ or } y \in C)\}$

$= \{(x.y) \mid (x \in A \text{ and } y \in B) \text{ or } (x \in A \text{ and } y \in C)\}$

$= (A \times B) \cup (A \times C)$.

(ii) $A \times (B \cap C) = \{(x.y) / x \in A \text{ and } y \in (B \cap C)\}$

$= \{(x.y) \mid x \in A \text{ and } (y \in B \text{ and } y \in C)\}$

$= \{(x.y) / [(x \in A) \text{ and } (y \in B)] \text{ and } [x \in A \text{ and } y \in C]\}$

$= (A \times B) \cap (A \times C)$

$\therefore \quad A \times (B \cap C) = (A \times B) \cap (A \times C)$

7 (a) Let $A = \{1, 3, 5\}$ $B = \{2, 3\}$, $C = \{4, 6\}$.

Find $(A \cup B) \times C$

$(A \times B) \cap (B \times A)$

$(A \times B) \cup (B \times C)$

(1) $A \cup B = \{1, 2, 3, 5\}$. $C = \{2, 3\}$

$(A \cup B) \times C = \{(1,2)(1,3)(2,2)(2,3)(3,2)(3,3)(5,2)(5,3)\}$

(2) $(A \times B) \cap (B \times A)$

$(A \times B) = \{(1,2)(1,3)(3,2)(3.3)(5,2)(5,3)\}$

$(B \times A) = \{(2,1)(2,3)(2.5)(3,1)(3,3)(3,5)\}$

$(A \times B) \cap (B \times A) = \{(3,3)\}$

$(A \times B) \cup (B \times C) \rightarrow$ class work.

7(b): Verify $A \times (B-C) = (A \times B) - (A \times C)$

$B - C = \{2,3\} - \{4,6\} = \{2,3\}$

$\therefore A \times B = \{(1,2)(1,3)(3,2)(3,3)(5,2)(5,3)\}$ ✓

$(A \times C) = \{(1,4)(1,6)(3,4)(3,6)(5,4)(5,6)\}$

$(A \times B) - (A \times C) = \{(1,2)(1,3)(3,2)(3,3)(5,2)(5,3)\}$ ✓

(ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

→ class work.

9(a). Prove the De Morgan's laws also prove using Venn diagram.

(i) $\overline{A \cap B} = \bar{A} \cup \bar{B}$

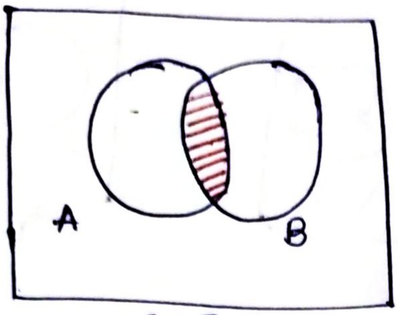
$\overline{A \cap B} = \{x / x \notin A \cap B\}$

$= \{x / x \notin A \text{ or } x \notin B\}$

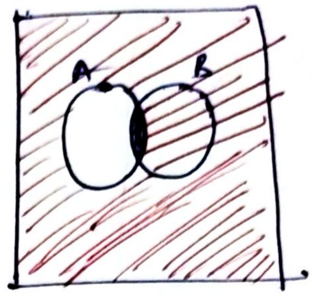
$= \{x / x \in \bar{A} \text{ or } x \in \bar{B}\}$

$= \{x / x \in \bar{A} \cup \bar{B}\}$

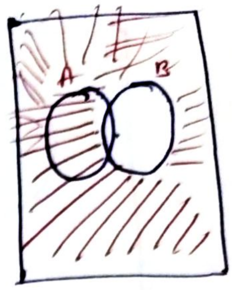
$\overline{A \cap B} = \bar{A} \cup \bar{B}$



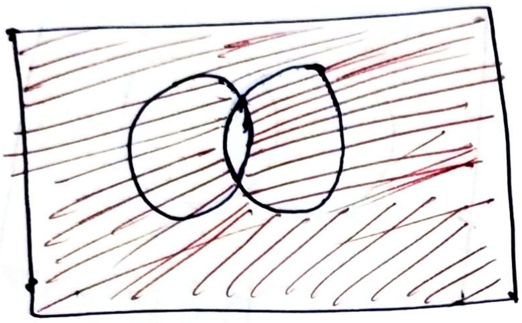
$A \cap B$



$\overline{A}$

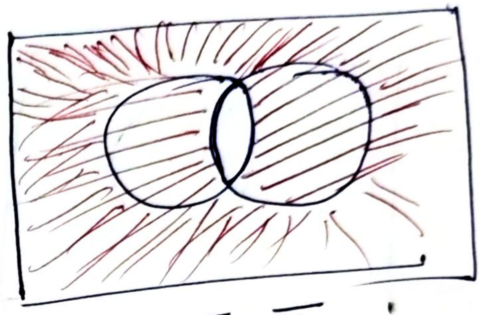


$\overline{B}$



$\overline{A \cap B}$

$\Leftrightarrow$



$\overline{A} \cup \overline{B}$

(ii)

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A \cup B} = \{x / x \in \overline{A \cup B}\}$

$= \{x / x \notin A \cup B\}$

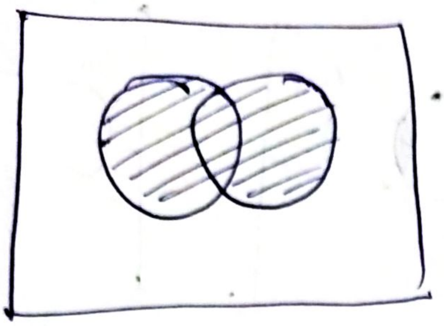
$= \{x / x \notin A$ and $x \notin B\}$

$= \{x / x \in \overline{A}$ and $x \in \overline{B}\}$

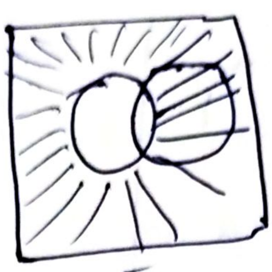
$= \{x / x \in \overline{A}$ and $x \in \overline{B}\}$

$= \{x / x \in \overline{A} \cap \overline{B}\}$

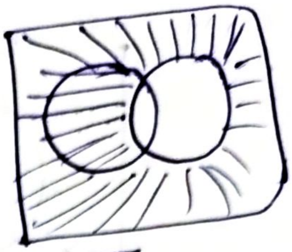
$\overline{A \cup B} = \overline{A} \cap \overline{B}$



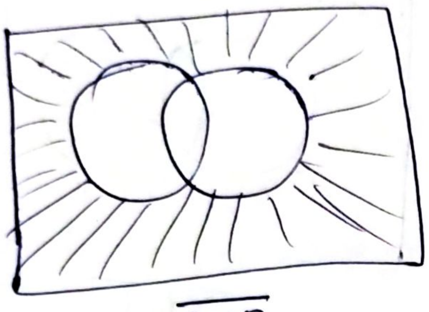
$A \cup B$



$\overline{A}$

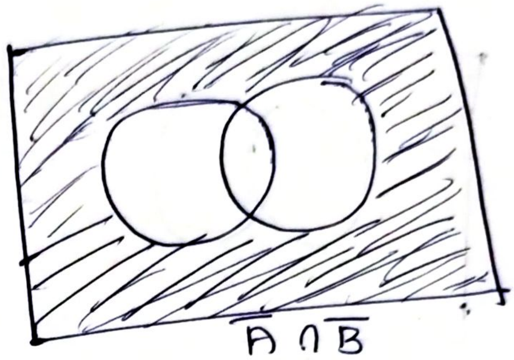


$\overline{B}$



$\overline{A \cup B}$

$\Leftrightarrow$



$\overline{A} \cap \overline{B}$

9(b.) $A = \{x / 3x^2 - 7x - 6 = 0\}$ $B = \{x / 6x^2 - 5x - 6 = 0\}$

find $A \cap B$

$3x^2 - 7x - 6 = 0$

$x = \frac{7 \pm \sqrt{49 - (-72)}}{6} = \frac{7 \pm \sqrt{121}}{6} = \frac{7 \pm 11}{6} = \frac{18}{6}$ or $\frac{-4}{6}$

i.e $x = 3$ or $-2/3$

$6x^2 - 5x - 6 = 0$ if solve.

$x = 3/2$ or $-2/3$

$\therefore A = \{3, -2/3\}$. $B = \{3/2, -2/3\}$

$\therefore A \cap B = \{-2/3\}$

$A \cup B = \{3, 3/2, -2/3\}$

10 (a) Two finite sets have m & n elements. The total number of subsets of the first set is 48 more than the total number of subsets of the second set. The values of m and n are?

<u>Soln:-</u> let A and B represent the sets with m & n elements respectively.

$|P(A)| = 2^m$. $\quad |P(B)| = 2^n$. (using powerset definition)

given that $|P(A)| = |P(B)| + 48$

$$2^m = 2^n + 48$$

$$2^m - 2^n = 48$$

$$2^n[2^{m-n} - 1] = 2^4 \times 3$$

$$= 2^4[2^2 - 1]$$

$$\begin{array}{l|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$2^4 \times 3$

$\therefore n = 4$. $\quad m - n = 2$

$$m = 2 + n$$

$$m = 6$$

$\therefore$ $m = 6$, $n = 4$

10 (b) In an examination 70% of the candidates pass in English, 65% in Mathematics, 27% failed in both subjects and 248 passed in both the subjects. Find the total number of candidates.

Soln:- let A and B be the sets of candidates who passed in Eng and Maths.

$|A| = 70\%$ $|B| = 65\%$

given. no/. of candidates failed in A and failed in B = 27%

$\Rightarrow |\bar{A} \cap \bar{B}| = 27\%$

$\Rightarrow |\overline{A \cup B}| = 27\%$

$\Rightarrow |A \cup B| = 100 - 27 = 73\%$

Also given no/ of candidates passed in A & B = 248

$\Rightarrow |A \cap B| = 248$

$\Rightarrow |A| + |B| - |A \cup B| = 248$

$\Rightarrow (70 + 65 - 73)\% = 248$

$\Rightarrow$ 62% of the total = 248

i.e $\frac{62}{100} \times N = 248$

$N = \frac{248 \times 100}{62}$

$N = 400$

11 (a) :- In a Survey of 100 students it was found that 50 used the college library, 40 had their own library and 30 borrowed books.
of these 20 used both college library and their own, 15 used their own library and borrowed books and 10 used college library & barrowed books.

How many students used all the three sources of books?

Soln:- Let A, B, C represents the set of students who used college library, own library, barrowed books respectively

$|A| = 50$, $|B| = 40$, $|C| = 30$

$|A \cap C| = 10$. $|A \cup B \cup C| = 100$

$|A \cap B| = 20$, $|B \cap C| = 15$.

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

$$|A \cap B \cap C| = 100 - 50 - 40 - 30 + 20 + 15 + 10$$

$$= 25 //.$$

11 (b) :- A firm has 40 workers working in the factory premises, 30 working in it's office & 20 working in the both factory & office.
How many workers are there in the firm? How many are working in

(i) In factory alone

(ii) office alone.

Soln:- $|A| = 40$. $|B| = 30$. $|A \cap B| = 20$

Total no/ of workers $= |A \cup B|$

$= |A| + |B| - |A \cap B|$

$= 40 + 30 - 20$

$= 50$

no/ of workers only in factor $= |A| - |A \cap B|$

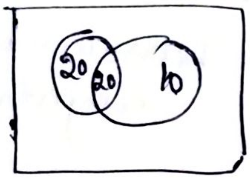
$= 40 - 20$

$= 20$

no/ of workers in the office $= |B| - |A \cap B|$

$= 30 - 20$

$= 10$



12 (a). In a group of 20 adults there are 8 females, 9 literate and 6 female literate. Find the no/. of male illiterates in the group.

Soln:- Let A and B be the set of males & illerates respectively

$|A| = 12$ (males) $|B| = 9$ (literates)

$|\bar{A}| = 8$ (Females) $|\bar{B}| = 11$ (Illiterates)

Also given. $|\bar{A} \cap B| = 6$ [no of female literates]

no/ of male illiterates $= |A \cap \bar{B}|$

$= |\overline{\bar{A} \cup B}|$

$= 20 - |\bar{A} \cup B|$

$= 20 - [|\bar{A}| + |B| - |\bar{A} \cap B|]$

$= 20 - [8 + 9 - 6]$

$= 9$

12 (b):- Set A has 4 elements and set B has 7 element. What can be the minimum number of elements in $A \cup B$?

Soln:- given. $|A| = 4$, $|B| = 7$.

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 4 + 7 - |A \cap B|$

$|A \cup B|$ is mimumb when. $|A \cap B|$ is maximum

i.e when A is subset of B.

i.e when $|A \cap B| = 4$.

$\therefore$ minum $|A \cup B| = 4 + 7 - 4$

$= 7$