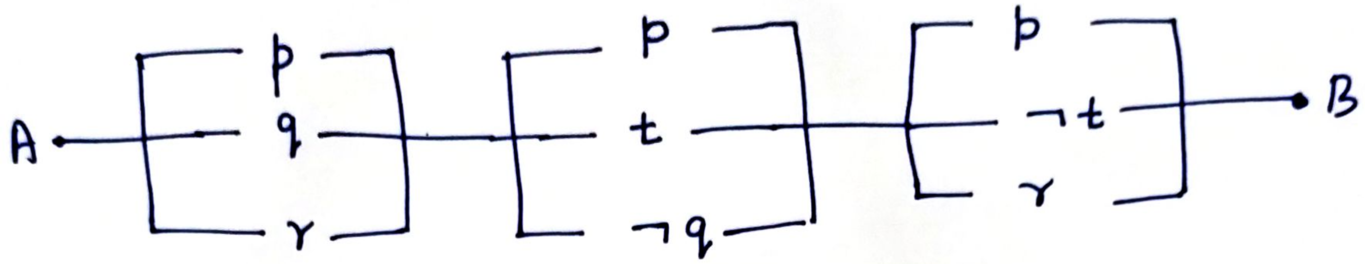
⑤ ⓐ Apply laws of logic and simplify the following switching networks.



Soln: The given network is represented by

$u \equiv \{p \vee q \vee r\} \wedge \{p \vee t \vee \neg q\} \wedge \{p \vee \neg t \vee r\}$

$\therefore \quad u \equiv [\{p \vee (q \vee r)\} \wedge \{p \vee (t \vee \neg q)\}] \wedge \{p \vee (\neg t \vee r)\}$ by associative law.

$\equiv p \vee \{(q \vee r) \wedge (t \vee \neg q)\} \wedge \{p \vee (\neg t \vee r)\}$

$\equiv p \vee [\{(q \vee r) \wedge (t \vee \neg q)\} \wedge (\neg t \vee r)]$, by distributive law

$\equiv p \vee [(t \vee \neg q) \wedge \{(r \vee q) \wedge (r \vee \neg t)\}]$, using commutative, and associative laws.

$\equiv p \vee [(t \vee \neg q) \wedge \{r \vee (q \wedge \neg t)\}]$ by distribution law.

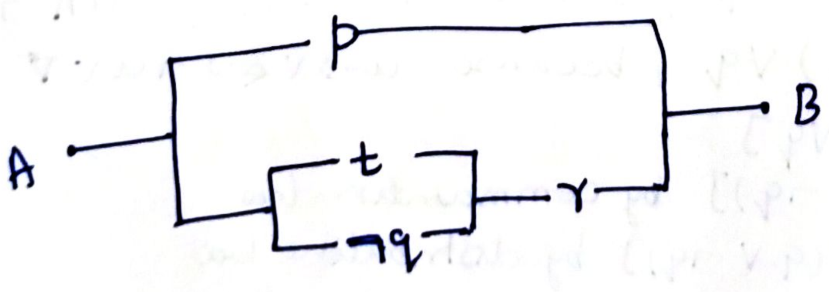
$\equiv p \vee [(t \vee \neg q) \wedge \{r \vee \neg (t \vee \neg q)\}]$ using DeMorgan's law and commutative law.

$\equiv p \vee [\{(t \vee \neg q) \wedge r\} \vee \{(t \vee \neg q) \wedge \neg (t \vee \neg q)\}]$, using distribution law

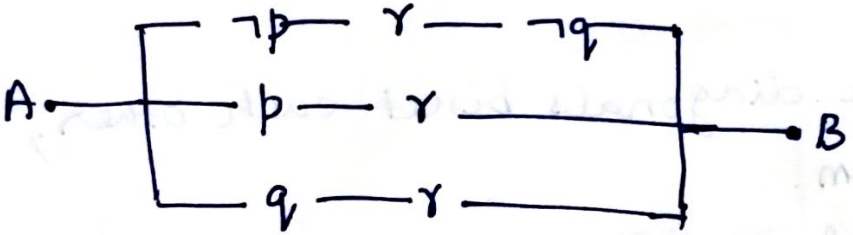
$\equiv p \vee [(t \vee \neg q) \wedge r] \vee F$ by inverse law.

$\equiv p \vee [(t \vee \neg q) \wedge r]$ by identity law.

This shows that the given network which has 9 switches is equivalent to the network $p \vee [(t \vee \neg q) \wedge r]$ which has four switches. The simplified network is shown below



(5)(b) Explain switching network. Simplify the switching network using laws of logic



Soln: $(\neg p \wedge r \wedge \neg q) \vee (p \wedge r) \vee (q \wedge r)$

$r \wedge (\neg p \vee p) \vee (\neg q \vee q)$ (distributive law)

$r \wedge (T \vee \neg q \vee q)$

$r \wedge (T \vee T)$

$r \wedge T$

$= r$

$\therefore$

4. Simplify the following compound propositions using the laws of logic :

(i) $[(p \vee q) \wedge (p \vee \neg q)] \vee q \equiv p \vee q$

Soln: $(p \vee q) \wedge (p \vee \neg q) \equiv p \vee (q \wedge \neg q)$ by Distributive law

$\equiv p \vee F$

$\equiv p$ by Identity law.

$\therefore [(p \vee q) \wedge (p \vee \neg q)] \vee q \equiv p \vee q$

(ii) $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$

Soln: $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \equiv (p \to q) \wedge [\neg q \wedge (\neg q \vee r)]$ By Commutative law

$\equiv (p \to q) \wedge \neg q$ by Absorption law.

$\equiv \neg [(p \to q) \to q$, because $\neg (u \to v) \Leftrightarrow u \wedge \neg v$

$\equiv \neg [\neg (p \to q) \vee q$, because $u \to v \Leftrightarrow \neg u \vee v$

$\equiv \neg [(p \wedge \neg q) \vee q]$

$\equiv \neg [q \vee (p \wedge \neg q)]$ by commutative law

$\equiv \neg [(q \vee p) \wedge (q \vee \neg q)]$ by distributive law

$\equiv \neg [(q \vee p) \wedge T]$

$\equiv \neg (q \vee p)$ by Identity law.

⑥ State the converse, inverse and contrapositive of

(i) If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Soln : Converse: If the diagonals bisect each other, then it is a parallelogram.

Inverse: If a quadrilateral is not a parallelogram, the the diagonals do not bisect each other.

Contrapositive: If the diagonals of a quadrilateral do not bisect each other, then it is not a parallelogram.

(ii) If a real number x^2 is greater than zero, then x is not equal to zero.

Soln : Converse : If a real number x is not equal to zero, then x^2 is greater than zero.

Inverse : If a real number x^2 is not greater than zero, then x is equal to zero.

Contrapositive : If a real number x is equal to zero, then x^2 is not greater than zero.

⑦ Test the validity of the following arguments:

(i) If Ravi goes out with friends, he will not study.
If Ravi does not study, his father becomes angry.
His father is not angry

∴ Ravi has not gone out with friends.

Soln: Let p: Ravi goes out with friends.
q: Ravi does not study.
r: His father gets angry

Then the given argument reads:

$$\begin{array}{l} p \to q \\ q \to r \\ \neg r \\ \hline \therefore \neg p \end{array}$$