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NUMBER PATTERN.

$= 9\times10^n - [1\cdot10^{n-1} + 1\cdot10^{n-2} + \dots + 1\cdot10^1 + 1\cdot10^0] - 1$

$= 9\times10^n - (\underbrace{1111\dots1}_{n \text{ times}}) - 1$

$\because 9\times10^n = 900000\dots$ n times

$= 9\times10^n - (1111\dots1) - 1$

$=$ 8888 ... 9 (n+1) times -1.

$=$ 888 .... (n+1) times

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MODULE-3

RELATIONS

Syllabus -

- Relations, types of relations.
- Verification of equivalence relation.
- Understanding the partial order.
- Hasse diagram.
- Matrix & diagraph.
- Poset definition.
- Problems.

RELATIONS.

Let A and B be two sets. The set of all ordered pairs A, B where $a \in A$ and $b \in B$. The cartesian product or the cross product of A & B is denoted as $A \times B = \{(a, b) / a \in A \ \& \ b \in B\}$.

Any subset of $A \times B$ is called a relation from A to B denoted by $R: A \rightarrow B$, aRb (element wise) or a related b by R which consisting in the form of ordered pair (a,b).

Note -

(*) i. If A is a set with m elements and B is the set with n elements, then the total number of relations from A to B, $R = 2^{mn}$.

ii. If R is a relation from A to A, we say that R is a binary relation on A.

TYPES OF RELATIONS

1. Reflexive relation - A relation on a set A is said to be reflexive if the element $(a, a) \in R$ for all $a \in A$. otherwise if $(a, a) \notin R$ it is an anti-reflexive or irreflexive relation.

Eg - $A = \{1, 2, 3, 4\}$

$R_1 : \{(1, 2), (1, 3), (1, 4), (2, 4), (2, 3)\}$

$R_2 : \{(1, 2), (2, 3), (3, 3), (1, 4), (1, 1), (2, 2), (4, 4)\}$

$R_3 : \{(1, 2), (1, 4), (2, 2), (2, 4)\}$

R_2 is reflexive.

2. Symmetric relation - A relation R on a set is said to be symmetric if $(b, a) \in R$ whenever $(a, b) \in R$ for all

$a, b \in R$.
otherwise asymmetric relation.

Eg - $A = \{1, 2, 3\}$
$R_1 : \{(1,1), (1,2), (2,1), (1,3), (3,1)\}$
$R_2 : \{(1,2), (1,3), (2,3), (3,2)\}$

R_1 is symmetric & R_2 is asymmetric.

3. Transitive relation - A relation R on a set A is said to be transitive if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ should also exist for all $a, b, c \in A$.

Eg - $A = \{1, 2, 3\}$
$R = \{(1,1), (1,2), (2,3), (1,3), (3,3)\}$.

4. Equivalence relation - A relation R on a set A is said to an equivalence relation on A if R is reflexive, symmetric and transitive.

Eg - $A = \{1, 2, 3\}$
$R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (1,3), (3,1), (2,3), (3,2)\}$

P.T.O

5. Anti Symmetric - A Relation R is said to be anti symmetric if whenever $(a,b) \in R$ and $(b,a) \in R$ then $a=b$.

PARTIAL ORDER / POSET.

Let A be a non empty set. Suppose there exists non empty sub sets $A_1, A_2, A_3, \dots A_k$ such that the following two conditions satisfied by the set A -

i. A must be union of $A_1, A_2, A_3, \dots, A_k$, ie $A = A_1 \cup A_2 \cup \dots \cup A_k$.

ii. Any two subsets $A_1, A_2, \dots, A_k$ are disjoint $\Rightarrow$ $A_i \cap A_j = \phi$.

Then the set $P = \{A_1, A_2, \dots, A_k\}$ is called partition of A.

The elements $A_1, A_2, A_3 \dots A_k$ are called blocks or cells of the partition.

A relation R on a set A is said to be partial ordering relation or a partial order on A if -

i. It should be reflexive

ii. It should be anti symmetric.

iii. It should be transitive.

A set A with a partial order ~~are~~ defined on it is called a partially ordered set or ordered set or poset, and is represented as (A, R).

Eg - $A = \{1, 2, 3, 4, 6, 12\}$

$R = \{(1,2), (1,3), (1,4), (1,6), (1,12), (2,4), (2,6), (2,12), (3,6), (3,12), (4,12), (6,12), (1,1), (2,2), (3,3), (4,4), (6,6), (12,12)\}$.

→ R is reflexive.

→ R is antisymmetric.

→ R is transitive.

Eg - Is the relation division of | a partial order on the set of all integers?

No, it is not antisymmetric.

Eg - Is the set of integers of elements $(Z, \leq)$ & $(Z, \geq)$ is poset or not?

P.T.O

MATRIX

Let R be a relation from A to B so that R is a subset of A X B.

Let $m_{ij} = (a_i, b_j)$ be assigned 1 if $(a_i, b_j \in R)$ and 0 otherwise.

$$m_{ij} \begin{cases} 1, & (a_i, b_j) \in R \\ 0, & \text{otherwise} \end{cases}$$

A contains 'm' elements & B contains 'n' elements, then the relation matrix R is of $m \times n$ order is denoted as M_R or $M(R)$.

Eg - Consider a set $A = \{1, 2, 3, 4\}$ and relation R defined on A is $R = \{(1,2), (1,3), (2,4), (3,2)\}$. Construct M_R.

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & (1,1) & (1,2) & (1,3) & (1,4) \\ 2 & (2,1) & (2,2) & (2,3) & (2,4) \\ 3 & (3,1) & (3,2) & (3,3) & (3,4) \\ 4 & (4,1) & (4,2) & (4,3) & (4,4) \end{array} \qquad M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$$M_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Note -

i. For any relation R from set A to B, $M_R = [0]$ / zero matrix when $R = \phi$.

ii. $[M(R)]^n = M(R^n)$.

iii. $M[R_1 \circ R_2] = M(R_1) \cdot M(R_2)$.

DIGRAPH.

→ The elements are represented by circles are called vertices.
→ The ordered element $(x, y) \in R$ is represented as edges.
→ if $\exists'(x, x) \in R$ it is a self loop.
→ The vertex from where an edge emerging is called initial/origin.
→ The edge where it ends is called terminal vertex.
→ The vertex which has no connections is called an isolated vertex.

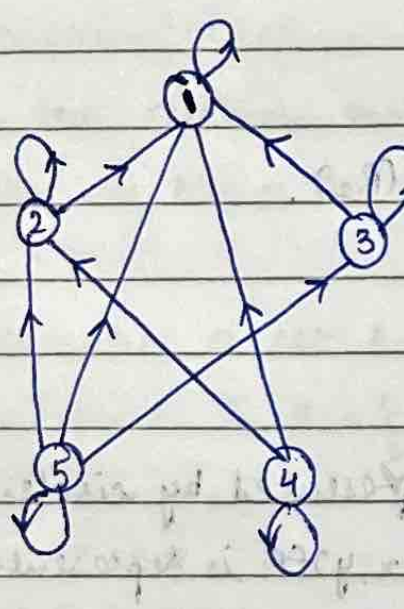
Eg - $A = \{1, 2, 3, 4, 5\}$

$R = \{(1,1), (2,1), (2,2), (3,1), (3,3), (4,1), (4,2), (4,$

$(5,1), (5,2), (5,3), (5,5)\}$

Construct M_R and digraph.

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$



OPERATIONS ON RELATIONS :-

1. $R_1 \cup R_2 = \{(a,b) \in R_1$ or $(a,b) \in R_2 \Rightarrow (a,b) \in R_1 \cup R_2\}$

2. $R_1 \cap R_2 = \{(a,b) \in R_1 \cap R_2$ iff $(a,b) \in R_1$ and $(a,b) \in R_2\}$

3. Given a relation R from $A \to B$, the complement of R is defined as -

$\overline{R} = \{ (a,b) \in \overline{R},$ iff $(a,b) \notin R \}$

4. Note - If M_R is a relation matrix then the transpose $(M_R)^T$ then matrix of R^C is $(M_R)^T$.

- $(R^C)^C = R$.

4. Given a relation R from set $A \to B$, the converse of R is defined as $R^C = \{ (a,b) \in R^C$ iff $(b,a) \in R \}$

5. Consider a Relation R from set $A \to B$ and a relation S from set $B \to C$. Then the product of or composition of R & S from set $A \to C$ is defined as

$$R \cdot S = \left\{ \begin{array}{l} (a,c) \in R \cdot S, \ a \in A, \ c \in C \text{ and } \exists' b \in B \text{ with} \\ (a,b) \in R(A), \ (b,c) \in S(B) \end{array} \right\}$$

Note -

1. If there is a matrix for the relation R, M(R) & for S M(S) $\Rightarrow$ $M(R) \times M(S) = M(R \cdot S)$.
2. $R \cdot (S \cdot T) = (R \cdot S) \cdot T$

P.T.O

Eg - Consider a set $A = \{a, b, c\}$, $B = \{1, 2, 3\}$ and the relations ~~ate~~ $R = \{(a,1), (b,1), (c,2), (c,3)\}$ & $S = \{(a,1), (a,2), (b,1), (b,2)\}$ from $A \to B$.

i. Compute $\bar{R}$, $\bar{S}$, $R \cup S$, $R \cap S$, R^c, S^c.

$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1), (c,2), (c,3)\}$

$\bar{R} = \{(a,2), (a,3), (b,2), (b,3), (c,1)\}$

$\bar{S} = \{(a,3), (b,3), (c,1), (c,2), (c,3)\}$

$R \cup S = \{(a,1), (a,2), (b,1), (b,2), (c,2), (c,3)\}$

$R \cap S = \{(a,1), (b,1)\}$

~~$R^c = \{(1,a)$~~, $R^c = \{(1,a), (1,b), (2,c), (3,c)\}$

$S^c = \{(1,a), (2,a), (1,b), (2,b)\}$

Q. Let $A = \{1, 2, 3, 4\}$, $B = \{w, x, y, z\}$, $C = \{5, 6, 7\}$

R_1 = from $A \to B$; $R_1 = \{(1,x), (2,x), (3,y), (3,z)\}$

R_2 & R_3 are the relations from B to C

$R_2 = \{(w,5), (x,6)\}$ $\quad R_3 = \{(w,5), (w,6)\}$

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find $R_1 \cdot R_2$ & $R_1 \cdot R_3$.

$R_1 \cdot R_2 = \{(1,6), (2,6)\}$

~~$R_1 \cdot R_3 = \{(1,6), (2,6)\}$~~

$R_1 \cdot R_3 = \{\phi\}$.

Q. Let $A = \{1,2,3\}$ and $B = \{1,2,3,4\}$. The relations R & S represented by the following matrices.

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Determine $\bar{R}$, $R \cup S$, $R \cap S$, S^c & $R \cdot S$

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 1 & 0 \\ 4 & & & & \end{array}$$

$R = \{(1,1), (1,3), (2,4), (3,1), (3,2), (3,3)\}$

$$M_S = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 1 \end{array}$$

$S = \{(1,1), (1,2), (1,3), (1,4), (2,4), (3,2), (3,4)\}$

$A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4\}$

$A \times B = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4)$
$(3,1), (3,2), (3,3), (3,4)\}$

$R = \{(1,1), (1,3), (2,4), (3,1), (3,2), (3,3)\}$

$S = \{(1,1), (1,2), (1,3), (1,4), (2,4), (3,2), (3,4)\}$

$\overline{R} = \{(1,2), (1,4), (2,1), (2,2), (2,3), (3,4)\}$

$R \cup S = \{(1,1), (1,2), (1,3), (2,4), (3,1), (3,2), (3,3), (3,4)\}$

$R \cap S = \{(1,1), (1,3), (2,4), (3,2)\}$

$\cancel{R^c = \{(1,1), (3,1), (2,4)}$

$S^c = \{(1,1), (2,1), (3,1), (4,1), (4,2), (2,3), (4,3)\}$

$R \cdot S = \{(1,1), (1,2), (1,3), (1,4), (3,1), (3,2), (3,3),$
$(3,4)\}$

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Q. $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,3), (3,2), (3,4), (4,2)\}$

$S = \{(2,1), (3,3), (3,4), (4,1)\}$

Find $R \cdot S$, $S \cdot R$, $R \cdot R$, $S \cdot S$.

HASSE DIAGRAM

The digraph of partial order drawn by adapting the following conventions is called a Poset Diagram / Hasse Diagram.

1. Since a partial order is reflexive, at every vertex in the digraph there would be a cycle of length 1 and self loop need not be exhibited explicitly.

2. If vertex A connected to vertex B and B connected to C. Then A connected to C since it is transitive. Hence there is no need to exhibit / connect an edge from A to C explicitly.

Note -

1. The matrix of a reflexive reflexive must have '1' on its main diagonal.
2. If principle diagonal elements are zero, then it is

irreflexive.

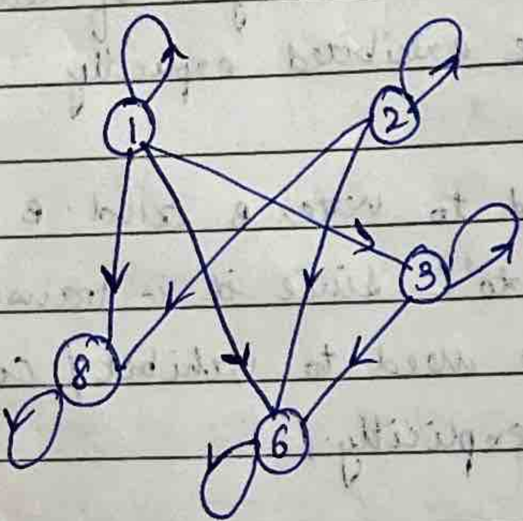
3. If the matrix is a symmetric matrix, then the relation is also symmetric.

4.

Eg-

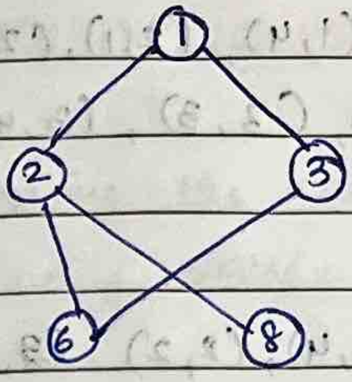
Q. Consider a set $A = \{1, 2, 3, 6, 8\}$. Write the digraph and also verify ~~that~~ (A, R) is a Poset and write the Hasse diagram.

$R = \{(1,1), (1,3), (1,6), (1,8), (2,2), (2,6), (2,8), (3,3), (3,6), (6,6), (8,8)\}$



1 $\forall a \in A \ (a, a) \in R$

- reflexive
- Anti symmetric
- Transitive



PROBLEMS.

Q1. Let $A = \{1, 2\}$ and $B = \{p, q, r, s\}$ and let the relⁿ R from $A \to B$ be $R = \{(1, q), (1, r), (2, p), (2, s)\}$.
Write M_R.

A.

	p	q	r	s
1	0	1	1	0
2	1	0	0	1

$$M_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Q2. Consider the relation R from $X \to Y$ where $X = \{1, 2, 3\}$ $Y = \{8, 9\}$. $R = \{(1, 8), (2, 8), (1, 9), (3, 9)\}$.
Find $\overline{R}$.

A. $X \times Y = \{(1, 8), (1, 9), (2, 8), (2, 9), (3, 8), (3, 9)\}$

$\overline{R} = \{(2, 9), (3, 8)\}$.

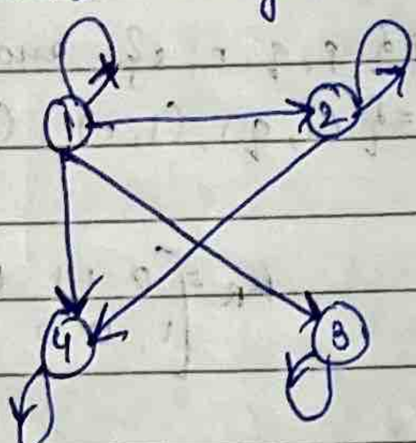
Q3. Let $A = \{1, 2, 3, 4\}$. Let R be the relation xRy iff x divides y.

$A \times A = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

(i) Draw digraph.

(ii) Determine indegree & outdegree.



id(1) = 1   id(3) = 2   od(1) = 4   od(3) = 1

id(2) = 2   id(4) = 3   od(2) = 2   od(4) = 1

Q4. Let $A = \{1, 2, 3, 4, 6\}$

R : {if a is a multiple of b}

① write R

② write M_R

③ draw digraph.

$R = \{(1,1), (2,1), (3,1), (4,1), (6,1), (4,2), (6,2), (2,2)$

(3,3), (4,4), (6,6), (6,3)}

Q5. Determine the relation $R: A \to B$ described by the following matrix. $M_R =$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

$R = \{(a,a), (a,c), (b,a), (b,b), (c,c), (d,d)\}$

Q6. $A = \{u, v, x, y, z\}$ & R be a relation on A

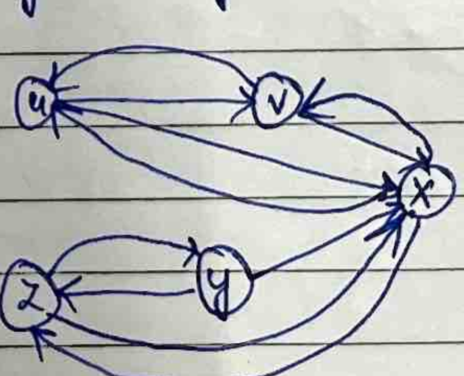
$M_R =$

$$\begin{array}{c} \\ u \\ v \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccccc} u & v & x & y & z \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

(i) determine R
(ii) Draw digraph

(i) $R = \{(u,v), (u,x), (v,u), (v,x), (x,u), (x,v), (x,z), (y,x), (y,z), (z,x), (z,y)\}$

(ii)



Q7. For the relations R_1 & R_2 where $R_1 = \{(1,x), (2,x), (3,y), (3,z)\}$ and $R_2 = \{(w,5), (x,6)\}$.
Find (i) $M(R_1)$ (ii) M_{R_2} (iii) $[M(R_1 \cdot R_2)]$.
Verify if $M(R_1 \cdot R_2) = M(R_1) \cdot M(R_2)$.

A.
$$\begin{array}{c|ccc} & x & y & z \\ \hline 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 0 & 1 & 1 \end{array} \qquad M(R_1) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{3\times 3}$$

$$\begin{array}{c|cc} & 5 & 6 \\ \hline w & 1 & 0 \\ x & 0 & 1 \end{array} \qquad M(R_2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

$R_1 \cdot R_2 = \{(1,6), (2,6)\}$

$$M(R_1 \cdot R_2) = \begin{array}{c|cc} & 5 & 6 \\ \hline 1 & 0 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 0 \end{array} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

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Q8. Let $A = \{1, 2, 3, 4\}$ and $B = \{w, x, y, z\}$. Let $R_1 = \{(1, x), (2, x), (3, y), (3, z)\}$ & $R_2 = \{(w, 1), (x, 2)\}$.

Find :- $M(R_1)$, $M(R_2)$ and $M(R_1 \cdot R_2)$. Also verify $M(R_1 \cdot R_2) = M(R_1) \cdot M(R_2)$.

A. $M(R_1) = \begin{array}{c|cccc} & w & x & y & z \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$ $\quad M(R_1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$M(R_2) = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline w & 1 & 0 & 0 & 0 \\ x & 0 & 1 & 0 & 0 \\ y & 0 & 0 & 0 & 0 \\ z & 0 & 0 & 0 & 0 \end{array}$ $\quad M(R_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_1 \cdot R_2 = \{(1, 2), (2, 2)\}$

$M(R_1 \cdot R_2) = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 \end{array}$ $\quad M(R_1 \cdot R_2) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$M(R_1) \times M(R_2) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = M(R_1 \cdot R_2)$$

Q9. Let $A = \{a, b, c\}$ and R & S be relations on A whose matrices are given below

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Find $R \cdot S$, $R \cdot R$, $S \cdot S$, $S \cdot R$

$R = \{(a,a), (a,c), (b,a), (b,b), (b,c), (c,b)\}$

$S = \{(a,a), (b,b), (b,c), (c,a), (c,c)\}$

$R \cdot S = \{(b,a), (b,b), (b,c), (c,b), (a,a), (a,c), (c,c)\}$

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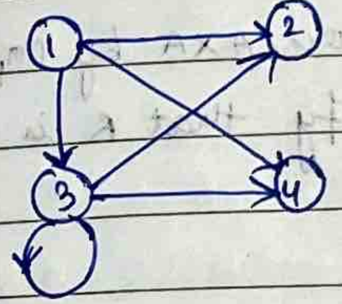
Q10. $A = \{1, 2, 3, 4\}$ and R is a relation on A defined by $R = \{(1,2), (1,3), (2,4), (3,2), (3,3), (3,4)\}$. Find R^2 and R^3 also write down their diagraphs.

A. $R^2 = \{(1,4), (1,2), (1,3), (3,4), (3,2), (3,3)\}$

$R^3 = \{(1,4), (1,2), (1,3), (3,4), (3,3), (3,2)\}$



same digraph for R^2 & R^3.

Q11. $A = \{1, 2, 3, 4\}$ & $R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)\}$.

Verify if R is an equivalent relation.

A. By default all R are transitive.

It is - reflexive.
- symmetric.
- transitive.

Q12. Let $A = \{u, v, x, y, z\}$ be a relation on A.

P.T.O

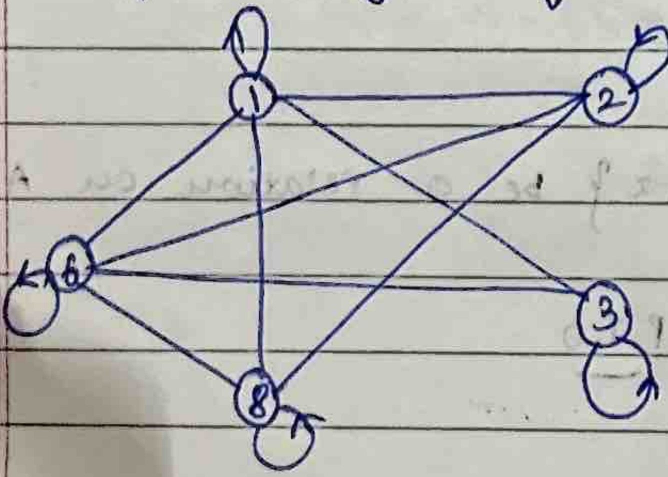
$$M_R = \begin{array}{c|ccccc} & u & v & x & y & z \\ \hline u & 0 & 1 & 1 & 0 & 0 \\ v & 1 & 0 & 1 & 0 & 0 \\ x & 1 & 1 & 0 & 0 & 1 \\ y & 1 & 0 & 0 & 0 & 1 \\ z & 0 & 0 & 1 & 1 & 0 \end{array}$$

$R = \{(u,v), (u,x), (v,u), (v,x), (x,u), (x,v), (x,z), (y,u), (y,z), (z,y), (z,x)\}$

Q12. $A = \{1, 2, 3, 4, 5\}$ defined R on $A \times A$ by $(x_1, y_1) R (x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$. Verify that R is an equivalence relation.

1. Reflexive $(x_1, y_1) \in A \times A \Rightarrow x_1 + y_1 = x_1 + y_1$
2. Symmetric (x_1, y_1) & $(x_2, y_2) \in A \times A \Rightarrow x_1 + y_1 = x_2 + y_2$
3. Transitive $(x_1, y_1), (x_2, y_2), (x_3, y_3)$

Q13. Diagraph of the relation $A = \{1, 2, 3, 4\}$ is given below. Verify A, R is a poset & then draw its poset diagram for it.



Date 27/7/23.

Q14. For a fixed integer $n > 1$, prove that congruent modulo n is an equivalence relation on the set of all integers $\mathbb{Z}$!

A.

Eg - $5 \equiv 17 \pmod{3} \Rightarrow 5 \div 3 = 2(R) \quad Q = 1$

$17 \div 3 = 2(R) \quad Q = 5$

$17 \equiv 5 \pmod{3} \Rightarrow$ also congruent.

$\Rightarrow$ Symmetric.

$a \equiv b \pmod{n}$	$5 \equiv 17 \pmod{3}$
$b \equiv c \pmod{n}$	$17 \equiv 26 \pmod{3}$
$a \equiv c \pmod{n} \rightarrow$ Transitive	$5 \equiv 26 \pmod{3}$.

For all (a, b) in $\mathbb{Z}$, we say that $a \equiv b \pmod{n}$ if $a - b$ is a multiple of n or n/a or n/b leaves the same remainder or $n / a-b$ i.e. $(a-b) = nk$ for $k \in \mathbb{Z}$.

First, we note that for every a in $\mathbb{Z}$ $(a-a) = 0$ i.e, ~~a ≡ mod~~ $a \equiv a \pmod{n}$.

$\therefore R$ is reflexive.

For all (a, b) in $\mathbb{Z}$ if $a \equiv b \pmod{n}$ if $aRb \Rightarrow$ $a \equiv b \pmod{n} \Rightarrow (a-b)$ is a multiple of $n \Rightarrow$ $(b-a)$ is also a multiple of n. $\Rightarrow bRa$

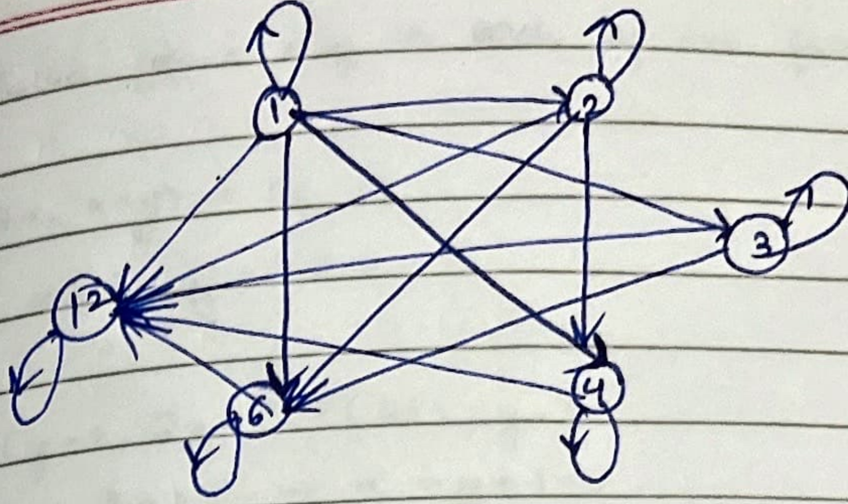
$\therefore$ R is symmetric.

For all $(a,b), c) \in \mathbb{Z}$, aRb & bRc $\Rightarrow a \equiv b \pmod n$
$b \equiv c \pmod n$ $\Rightarrow$ $(a-b)$ & $(b-c)$ is a multiple of n
$\Rightarrow (a-b)+(b-c)$ is a multiple of n.
$\Rightarrow$ $(a-c)$ is a multiple of n.
$\Rightarrow$ $a \equiv c \pmod n$
$\Rightarrow$ aRc
$\therefore$ R is transitive.

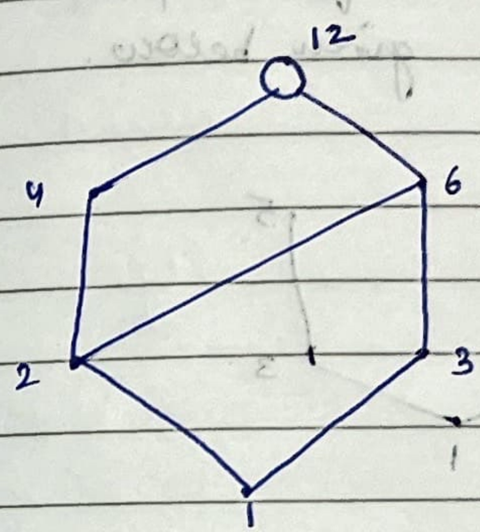
Q15. Let R be a relation on the set $A = \{1, 2, 3, 4, 6, 12\}$.
Define the relation R by aRb iff <u>a divides b</u>.
Prove that R is a partial order on A and also draw poset diagram.

A. $R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,12), (2,4), (2,6),$
$(2,2), (2,12), (3,3), (3,6), (3,12), (4,4), (4,12),$
$(6,6), (6,12), (12,12)\}$

R is reflexive. $\rightarrow (a,a) \in R$
R is transitive. $\rightarrow (a,b), (b,c), (a,c) \in R$
R is antisymmetric. $\rightarrow (a,b) \in R, (b,a) \in R$ & $a=b$.



← Digraph



← Poset diagram

Q16. Draw the poset diagram representing the positive divisors of 36.

A. $S = \{1, 2, 3, 4, 6, 12, 9, 18, 36\}$

~~R = {(1,2), (1,1), (1,3), (1,4)~~

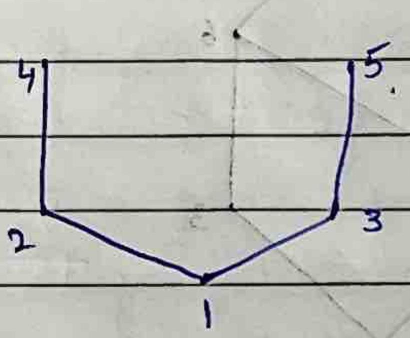
The relation R of divisibility (i.e, aRb iff a divides b).

We need to write a partial order set.

$R = \{($

Q. Determine the matrix of the partial order whose poset diagram is given below.

~~i. Construct the~~



$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,2), (1,3), (1,4), (1,5), (2,4), (3,5)\}$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 & 0 & 1 \\ 4 & 0 & 0 & 0 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 \end{array}$$

Date 31/7/23

Q. Solve for x & y in each of the following-

i. $(2x, x+y) = (6, 1)$.

$x = 3,\ y = -2$

ii. $(y-2, 2x+1) = (x-1, y+2)$

$y-2 = x-1 \Rightarrow x - y + 1 = 0$.

$2x+1 = y+2 \Rightarrow 2x - y - 1 = 0$.

MODULE 4

FUNCTIONS

Syllabus -

→ Cartesian product.
→ Definition + example.
→ Types of functions.
→ Properties of functions.
→ Stirling numbers of the second kind.
→ The pigeon hole principle.
→ Function composition - Case study.
→ Number theory
→ ~~Relations [full]~~

VALID FUNCTIONS

Let A and B be two non empty sets. Then a function A → B is a relation from A → B such that for each element of A, there is a unique image in set B such that $(a, b) \in f$ where $a \in A$, $b \in B$.

Here 'b' is called image and a is called preimage.
$b = f(a)$.
'a' is called domain & 'b' is called codomain / range.

Every function is a relation but not every relation is a function.

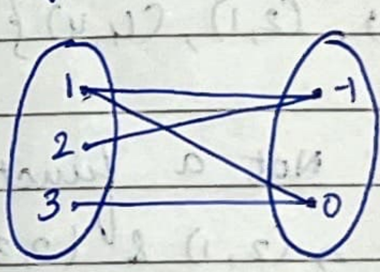
Note -

i. Every $a \in A$ belongs to some pair $(a,b) \in f$ and if $(a_1, b_1) \in f$ and $(a_1, b_2) \in f$ then $b_1 = b_2$.

ii. An element $b \in B$ need not have a preimage in A under f.

iii. If an element $b \in B$ can be a/have a preimage in $a \in A$ to two different elements. Two diff elements of A can have the same image in B.

Eg -



→ not a function

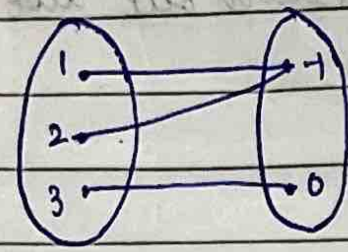
Q. $A = \{1, 2, 3\}$, $B = \{-1, 0\}$, $S : A \to B$, $S = \{(1,-1), (2,-1), (3,0)\}$. Is S a function?

A. (Above diagram):

1 is mapped to two distinct images which means it is not a function.

P.T.O

(This Q)



It is a function.

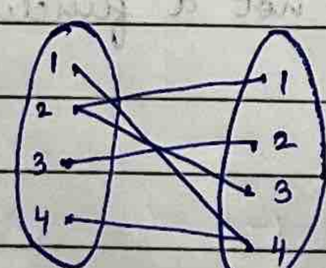
Q. $A = \{1, 2, 3, 4\}$. Determine whether or not the following relations of A are functions?

(i) $f : \{(2,3), (1,4), (2,1), (3,2), (4,4)\}$

(ii) $g : \{(3,1), (4,2), (1,1)\}$

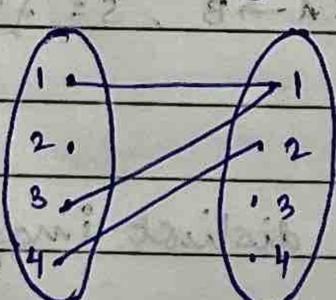
(iii) $h : \{(2,1), (3,4), (1,4), (2,1), (4,4)\}$

A-(i)



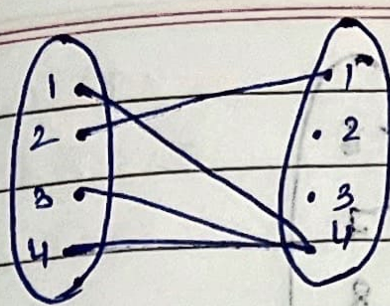
Not a function
$\Rightarrow (2,1)$ & $(2,3)$

(ii)



It is not a function
$\Rightarrow$ 2 does not an image in B.

(iii)



It is a function.

Q. Let $A = \{0, \pm 1, \pm 2, 3\}$. Consider the function $f : A \to R$ [where R is the set of all real numbers] defined by $f(x) = x^3 - 2x^2 + 3x + 1$ for all $x \in A$. Find the range of f.

A. $f(0) = 1$ $\quad f(-1) = -\cancel{1} - 2 - 3 + \cancel{1} = -5$

$f(1) = \cancel{1} - \cancel{2} + 3 + \cancel{1} = 3$ $\quad f(2) = 8 - 2(4) + 3(2) + 1 = 7$

$f(-2) = -8 - 2(4) - 6 + 1 = -21$ $\quad f(3) = 27 - 2(9) + 3(2) + 1$

$= 19.$

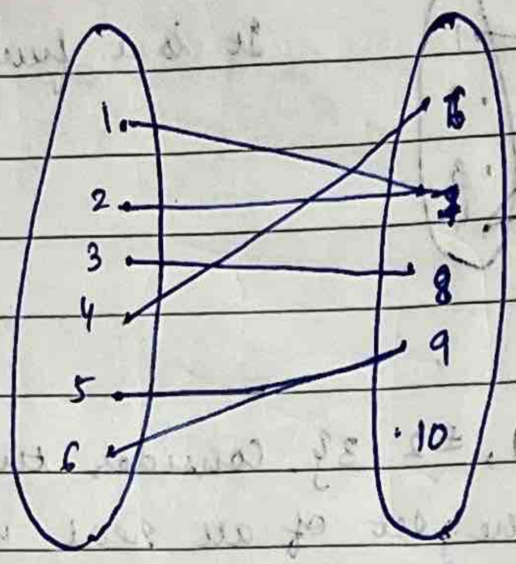
Range $= \{-21, -5, 1, 3, 7, 19\} = B.$

Q. $A = \{1, 2, 3, 4, 5, 6\}$ & $B = \{6, 7, 8, 9, 10\}$. If a function $f : A \to B$ is defined by $f : \{(1,7), (2,7), (3,8), (4,6), (5,9), (6,9)\}$. Determine $f^{-1}(6)$ & $f^{-1}(9)$. Also if $B_1 = \{7, 8\}$, $B_2 = \{8, 9, 10\}$, Find $f^{-1}(B_1)$ and $f^{-1}(B_2)$.

A. $f^{-1}(6) = \{4\}$ $\quad f^{-1}(B_1) = \{2, 3\}$

$f^{-1}(9) = \{5, 6\}$ $\quad f^{-1}(B_2) = \{3, 5, 6\}$

P.T.O



Q. For $A = \{1, 2, 3, 4, 5\}$, $B = \{w, x, y, z\}$ a function $f: A \to B$ defined by $f = \{(1, w), (2, x), (3, x), (4, y), (5, y)\}$. Find the image of the following subset of A under $A_1 = \{1\}$; $A_2 = \{1, 2\}$, $A_3 = \{1, 2, 3\}$, $A_4 = \{2, 3\}$, $A_5 = \{2, 3, 4, 5\}$.

1/8/23 Q. Let $F: R \to R$ be defined by $f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$

Determine $f(0)$, $f(-1)$, $f(5/3)$, $f(-5/3)$.

A. $f(0) = 1$

$f(-1) = -3(-1) + 5 = -8$

$f(5/3) = 3 \cdot \frac{5}{3} - 5 = 0$

$f(-5/3) = -3 \cdot \left(\frac{-5}{3}\right) + 1 = 6$

Q. Let Z denote the set of all integers. A function $h: (Z \times Z) \to Z$ is defined by $h(x, y) = 2x + 3y$. Find $h(0, 0)$, $h(-3, 7)$, $h(2, -1)$.

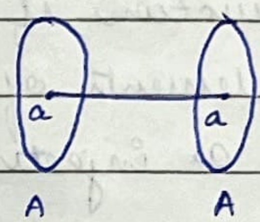
$h(0,0)=0$

$h(-3,7)=15$

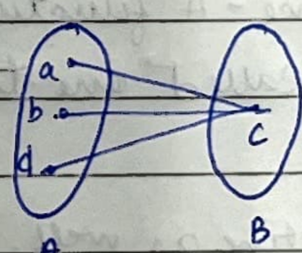
$h(2,-1)=1.$

TYPES OF FUNCTION.

1. Identity function : $(a,a) \in F$. A function $f: A \to A$ s.t. $f(a)=a$ for every $a \in A$ is called identity function & it is denoted as I_A.

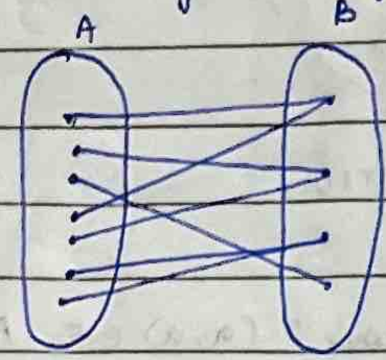


2. Constant function : A function $f: A \to B$ such that $f(a)=c$ for every $a \in A$, where c is a fixed element of B.

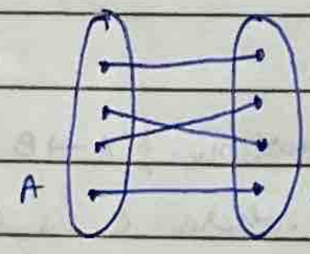


3. Onto function : A function $f: A \to B$ is said to be an onto function if for every element $b \in B$ there is an element $a \in A$ such that $f(a)=b$ [every element of b

must be having a preimage in A]. This function also called as surjective function.



4. One to one function: A function $f: A \to B$ is said to be a one to one function if different elements of A have different elements of B as ~~img~~ images under f. Also called as injective function.



5. One to one correspondence - A function which is both one to one & onto is called one-to-one correspondence or bijective function.

Above example is bijective as well.

Q. Find the nature of the function defined on A, $A = \{1, 2, 3\}$ and the functions are -

$f = \{(1,1), (2,2), (3,3)\}$

$g = \{(1,2), (2,2), (3,2)\}$

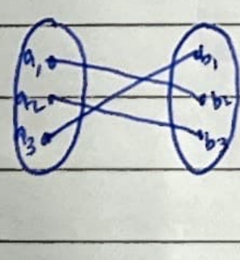
$h = \{(1,2), (2,2), (3,1)\}$

$p = \{(1,2), (2,3), (3,1)\}$.

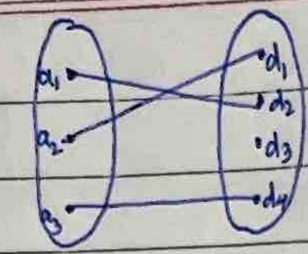
A. $f \rightarrow$ Identity func^n.
$g \rightarrow$ constant func^n.
$h \rightarrow$ none of the 5 types - many to one.
$p \rightarrow$ onto func^n, one-one func^n $\longrightarrow$ bijective func^n.

Q. Let $A = \{a_1, a_2, a_3\}$ & $B = \{b_1, b_2, b_3\}$ & $C = \{c_1, c_2\}$ & $D = \{d_1, d_2, d_3, d_4\}$. Let $f_1 : A \rightarrow B$, $f_2 : A \rightarrow D$, $f_3 : B \rightarrow C$ and $f_4 : D \rightarrow B$. The functions are defined as follows
$f_1 : \{(a_1, b_2), (a_2, b_3), (a_3, b_1)\}$
$f_2 : \{(a_1, d_2), (a_2, d_1), (a_3, d_4)\}$
$f_3 : \{(b_1, c_2), (b_2, c_2), (b_3, c_1)\}$ Surjective
$f_4 : \{(d_1, b_1), (d_2, b_2), (d_3, b_1), (d_4, b_2)\}$ none.
Find nature of each function.

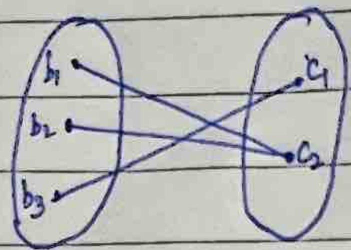
A. i.



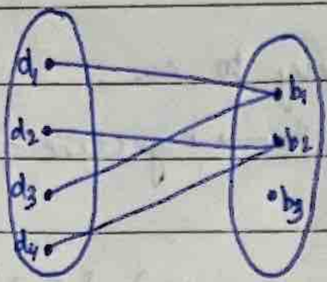
Bijective.

ii. 

One-one func^n.

iii. 

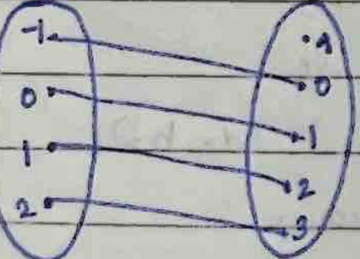
onto func^n.

iv. 

None.

Q. Let $f: Z \to Z$ defined by $f(a) = a+1 \ \forall a \in Z$. Find whether f is one-one or onto or both or neither.

A.



→ Onto.
→ One-one.
→ Bijective.