The New York Times

December 31, 2012

The Life of Pi, and Other Infinities

By NATALIE ANGIER

On this day that fetishizes finitude, that reminds us how rapidly our own earthly time share is shrinking, allow me to offer the modest comfort of infinities.

Yes, infinities, plural. The popular notion of infinity may be of a monolithic totality, the ultimate, unbounded big tent that goes on forever and subsumes everything in its path — time, the cosmos, your complete collection of old Playbills. Yet in the ever-evolving view of scientists, philosophers and other scholars, there really is no single, implacable entity called infinity.

Instead, there are infinities, multiplicities of the limit-free that come in a vast variety of shapes, sizes, purposes and charms. Some are tailored for mathematics, some for cosmology, others for theology; some are of such recent vintage their fontanels still feel soft. There are flat infinities, hunchback infinities, bubbling infinities, hyperboloid infinities. There are infinitely large sets of one kind of number, and even bigger, infinitely large sets of another kind of number.

There are the infinities of the everyday, as exemplified by the figure of pi, with its endless post-decimal tail of nonrepeating digits, and how about if we just round it off to 3.14159 and then serve pie on March 14 at 1:59 p.m.? Another stalwart of infinity shows up in the mathematics that gave us modernity: calculus.

“All the key concepts of calculus build on infinite processes of one form or another that take limits out to infinity,” said Steven Strogatz, author of the recent book “The Joy of x: A Guided Tour of Math, From One to Infinity” and a professor of applied mathematics at Cornell. In calculus, he added, “infinity is your friend.”

Yet worthy friends can come in prickly packages, and mathematicians have learned to handle infinity with care.

“Mathematicians find the concept of infinity so useful, but it can be quite subtle and quite dangerous,” said Ian Stewart, a mathematics researcher at the University of Warwick in England and the author of “Visions of Infinity,” the latest of many books. “If you treat infinity like a normal number, you can come up with all sorts of nonsense, like saying, infinity plus one is equal to infinity, and now we subtract infinity from each side and suddenly r one. You can’t be freewheeling in your use of infinity.”

Then again, a very different sort of infinity may well be freewheeling you. Based on recent studies of the cosmic microwave afterglow of the Big Bang, with which our known universe began 13.7 billion years ago, many cosmologists now believe that this observable universe is just a tiny, if relentlessly expanding, patch of space-time embedded in a greater universal fabric that is, in a profound sense, infinite. It may be an infinitely large monoverse, or it may be an infinite bubble bath of infinitely budding and inflating multiverses, but infinite it is, and the implications of that infinity are appropriately huge.

“If you take a finite physical system and a finite set of states, and you have an infinite universe in which to sample them, to randomly explore all the possibilities, you will get duplicates,” said Anthony Aguirre, an associate professor of physics who studies theoretical cosmology at the University of California, Santa Cruz.

Not just rough copies, either. “If the universe is big enough, you can go all the way,” Dr. Aguirre said. “If I ask, will there be a planet like Earth with a person in Santa Cruz sitting at this colored desk, with every atom, every wave function exactly the same, if the universe is infinite the answer has to be yes.”

In short, your doppelgängers may be out there and many variants, too, some with much better hair who can play Bach like Glenn Gould. A far less savory thought: There could be a configuration, Dr. Aguirre said, “where the Nazis won the war.”

Given infinity’s potential for troublemaking, it’s small wonder the ancient Greeks abhorred the very notion of it.

“They viewed it with suspicion and hostility,” said A. W. Moore, professor of philosophy at Oxford University and the author of “The Infinite” (1990). The Greeks wildly favored tidy rational numbers that, by definition, can be defined as a ratio, or fraction — the way 0.75 equals ¾ and you’re done with it — over patternless infinitums like the square root of 2.

On Pythagoras’ Table of Opposites, “the finite” was listed along with masculinity and other good things in life, while “the infinite” topped the column of bad traits like femininity. “They saw it as a cosmic fight,” Dr. Moore said, “with the finite constantly having to subjugate the infinite.”

Aristotle helped put an end to the rampant infiniphobia by drawing a distinction between what he called “actual” infinity, something that would exist all at once, at a given moment — which he declared an impossibility — and “potential” infinity, which would unfold over time and which he deemed perfectly intelligible. As a result, Dr. Moore said, “Aristotle believed in finite space and infinite time,” and his ideas held sway for the next 2,000 years.

Newton and Leibniz began monkeying with notions of infinity when they invented calculus, which solves tricky problems of planetary motions and accelerating bodies by essentially breaking down curved orbits and changing velocities into infinite series of tiny straight lines and tiny uniform motions. “It turns out to be an incredibly powerful tool if you think of the world as being infinitely divisible,” Dr. Strogatz said.

In the late 19th century, the great German mathematician Georg Cantor took on infinity not as a means to an end, but as a subject worthy of rigorous study in itself. He demonstrated that there are many kinds of infinite sets, and some infinities are bigger than others. Hard as it may be to swallow, the set of all the possible decimal numbers between 1 and 2, being unlistable, turns out to be a bigger infinity than the set of all whole numbers from 1 to forever, which in principle can be listed.

In fact, many of Cantor’s contemporaries didn’t swallow, dismissing him as “a scientific charlatan,” “laughable” and “wrong.” Cantor died depressed and impoverished, but today his set theory is a flourishing branch of mathematics relevant to the study of large, chaotic systems like the weather, the economy and human stupidity.

With his majestic theory of relativity, Einstein knitted together time and space, quashing old Aristotelian distinctions between actual and potential infinity and ushering in the contemporary era of infinity seeking. Another advance came in the 1980s, when Alan Guth introduced the idea of cosmic inflation, a kind of vacuum energy that vastly expanded the size of the universe soon after its fiery birth.

New theories suggest that such inflation may not have been a one-shot event, but rather part of a runaway process called eternal inflation, an infinite ballooning and bubbling outward of this and possibly other universes.

Relativity and inflation theory, said Dr. Aguirre, “allow us to conceptualize things that would have seemed impossible before.” Time can be twisted, he said, “so from one point of view the universe is a finite thing that is growing into something infinite if you wait forever, but from another point of view it’s always infinite.”

Or maybe the universe is like Jorge Luis Borges’s fastidiously imagined Library of Babel, composed of interminable numbers of hexagonal galleries with polished surfaces that “feign and promise infinity.”

Or like the multiverse as envisioned in Tibetan Buddhism, “a vast system of 10^59 universes, that together are called a Buddha Field,” said Jonathan C. Gold, who studies Buddhist philosophy at Princeton.

The finite is nested within the infinite, and somewhere across the glittering, howling universal sample space of Buddha Field or Babel, your doppelgänger is hard at the keyboard, playing a Bach toccata.