Measuring assortativeness in marriage*

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January 2, 2024

Abstract

Measuring the extent to which assortative matching differs between two economies is challenging when the marginal distributions of the characteristic along which sorting takes place (e.g. education) also changes for either or both sexes. We show how the use of different indices can generate different conclusions, and discuss their underlying, structural interpretation.

1 Introduction

The study of sorting in the marriage market has recently attracted renewed attention. The degree of homogamy in marriage - defined as people's tendency to 'marry their own' - has important consequences for family inequalities and intergenerational transmission of human capital. It is therefore surprising that the various studies on this topic have not reached a consensus on the evolution of homogamy over the recent years, and even on how to best measure homogamy (e.g. Chiappori et al., 2017; Ciscato and Weber, 2020; Eika et al., 2019;

*We thank Dan Anderberg, Magne Mogstad, Bernard Salanié, three anonymous referees and the editor James Heckman for discussion and comments. All errors are our own.

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Fernández and Rogerson, 2001; Gihleb and Lang, 2020; Greenwood et al., 2014; Mare and Schwartz, 2005; Siow, 2015, to mention but a few).

The goal of this note is to understand why different approaches to the same problem, using similar data, can reach opposite conclusions and, more generally, to clarify the theoretical issues underlying the choice of a particular measure of assortativeness. Our analysis considers two populations, women and men, sorting in marriage according to a one-dimensional characteristic, say education. Whenever the marginal distributions change – e.g., women's average education increases – matching patterns will change. The main problem faced by any measure of assortativeness is to disentangle the mechanical effects of such variations in the marginal distributions from deeper changes in the matching structure itself, for instance originating from changes in the gain generated by assortativeness along that characteristic. The latter represents what one would call 'changes in assortativeness'. Existing studies proposed various indices to measure assortativeness and changes therein. These indices achieve this goal in different ways, and may therefore generate diverging conclusions.

2 Measuring assortative matching and changes in assortativeness

2.1 Defining Assortative Matching

Consider an economy where an equal mass of men and women match by their level of education, which can only take two values - say, High School versus College. We will abstract from singles - everybody gets matched. The matching patterns in this population are summarized by the 2×2 matching Table (a, b, c, d) shown below. In the Table, $a + b$ and $a + c$ are the number of female and male college graduates respectively, while a is the number of couples where both spouses are college graduates; we assume without loss of generality that $b \geq c$ - i.e., there are at least as many educated women as educated men.

Table 1: Table (a, b, c, d)

$_w\backslash^h$	C	HS
C	a	b
HS	c	d

We say that Table (a, b, c, d) exhibits Positive Assortative Matching (PAM) if *the number of couples with equal education* (the 'diagonal' of the Table) *is larger than what would obtain under random matching.* Under random matching the number of couples where both spouses are college graduates will simply be $\frac{(a+b)(a+c)}{a+b+c+d}$. Then we have PAM if and only if $a(a+b+c+d) \geq (a+b)(a+c)$ or equivalently if $ad \geq bc$; this also implies that more High School graduates marry each other than would be implied by random matching. In other words PAM arises when extra forces generate more matches between equally educated people than would happen for random reasons.

2.2 Defining 'increases in Assortative Matching'

We now consider two Tables, $T = (a, b, c, d)$ and $T' = (a', b', c', d')$, and ask under what conditions can it be concluded that T is more assortative than T'. One can think of two types of answers: an ordinal criterion, that would (at least in some cases) allow to claim that T is more assortative than T', and a cardinal one, that would moreover allow to *quantify* the 'degree of assortativeness' of each Table. Each type of answer would result in the definition of a *preorder* $\succeq$ on the set of Tables, where $T \succeq T'$ reads as 'T is at least as assortative as T''. In particular, we require reflexiveness ($T \succeq T$) and transitivity ($T \succeq T'$ and $T' \succeq T'' \Rightarrow T \succeq T''$). The main difference, obviously, is that a cardinal criterion will define a *complete* preorder, in the sense that for any T and T', either $T \succeq T'$ or $T' \succeq T$ (or both). In contrast, an ordinal criterion may fail to rank two given Tables.

In all cases, however, we make the general requirement that an assortativeness criterion should be scale invariant, in the sense that it does not depend on the total size of the population but only on the respective proportions of the population in each cell.[1]

Clearly, there exist many ways of ranking even simple, 2×2 Tables by assortativeness; some will be presented below. In some cases, however, the ranking is straightforward; we start with two such simple examples.

2.2.1 Same marginal probabilities

The first straightforward case obtains when comparing two Tables T and T' that have identical marginal distributions - i.e., the same proportion of educated men and the same proportion of educated women, in which case

$$\frac{a+b}{a+b+c+d} = \frac{a'+b'}{a'+b'+c'+d'} \quad \text{and} \tag{1}$$

$$\frac{a+c}{a+b+c+d} = \frac{a'+c'}{a'+b'+c'+d'} \tag{2}$$

Then T is more assortative than T' if and only if the proportion of couples on the diagonal of the Table is larger in T than in T', i.e. if

$$\frac{a}{a+b+c+d} \geq \frac{a'}{a'+b'+c'+d'}, \text{ or equivalently}$$

$$\frac{d}{a+b+c+d} \geq \frac{d'}{a'+b'+c'+d'}$$

The motivation for this property is clear: with identical marginals, a larger fraction of individuals marry their own type in Table T than in Table T'. This is the exact definition of increased assortative matching.

In other words, ranking Tables with the same marginal distributions is straightforward. Problems arise precisely when marginals differ.

[1] In particular, one can readily normalize the total population $P = a+b+c+d$ to be 1; then the analysis can be rephrased in terms of probability distributions.

2.2.2 Maximum Homogamy

A second polar case is the notion of *Maximum Homogamy* (HMax Robbins, 2009), defined as '*the maximum homogamy possible, given fixed marginal distributions*' (p. 381). A Table displays HMax if *all* individuals on the short side of the educated category (men in our case) marry an educated partner - i.e., when $c = 0$; intuitively, the only reason why we observe 'mixed' couples is the lack of educated men in sufficient number. A natural requirement is that any Table displaying Maximum Homogamy should be a maximal element for the preorder $\succeq$.

Formally, this leads to the following two definitions:

Definition 1 ***(HMax)*** *A preorder $\succeq$ satisfies the Hmax property if, for any two Tables (a, b, c, d) and (a', b', c', d'),*

$$c = 0 \Rightarrow (a, b, 0, d) \succeq (a', b', c', d')$$

In particular, if the preorder $\succeq$ is derived from an index I, then I should be maximal for any Table of the form $(a, b, 0, d)$.

A weaker version requires that the previous property be satisfied only when $b = c = 0$, i.e. only when the entire population is concentrated on the diagonal of the Table. This requires an equal number of educated men and women; and the matching must be perfectly (positive) assortative, in the sense that *all* individuals marry a spouse with the same education (there are no 'mixed marriages'). In statistical terms, the association between spouses' educations is then *absolute* in the sense of Kendall and Stuart (1961). Hence the definition:

Definition 2 ***(Weak HMax)*** *A preorder $\succeq$ satisfies the Weak Hmax property if, for any two Tables (a, b, c, d) and (a', b', c', d'),*

$$b = c = 0 \Rightarrow (a, 0, 0, d) \succeq (a', b', c', d')$$

In particular, the Weak HMax criterion requires that an assortativeness preorder cannot possibly rank a Table of the $(a, 0, 0, d)$ type as less assortative than some other Table (a', b', c', d') with $b' \times c' > 0$. Again, if the preorder $\succeq$ is derived from an index I, then I should be maximal for any Table of the form $(a, 0, 0, d)$. As we shall see, this property is satisfied for most, but not all, indices used in the literature.

3 Properties of some existing indices

We now briefly review some of the most commonly used criteria in the literature. All of them are cardinal;[2] i.e., they rely on the definition of an index of assortativeness $I(a, b, c, d)$ such that the preorder is defined by

$$T \succeq T' \Longleftrightarrow I(a, b, c, d) \geq I(a', b', c', d')$$

3.1 Some widely used criteria

Odds Ratio This is probably the most widely used index:

$$I_O(a, b, c, d) = \ln\left(\frac{ad}{bc}\right)$$

The odds ratio is popular in the demographic literature, as it can be directly derived from the log-linear approach (see for instance Mare (2001), Mare and Schwartz (2005), Bouchet-Valat (2014)); in economics, it was used by Siow (2015) ('local odds ratio') Chiappori et al. (2017), Ciscato and Weber (2020), Chiappori, Costa-Dias, Crossman and Meghir (2020) among many others. One can readily check that the odds ratio satisfies the HMax property, since it becomes infinite when $c = 0$.

[2] For examples of ordinal criteria, see Chiappori, Costa Dias and Meghir (2022).

Minimum Distance In the minimum distance approach of Fernández and Rogerson (2001) and Abbott et al. (2019), one constructs the convex combination of two extreme cases (random and maximum homogamy) that minimizes the distance with the Table under consideration, and defines the weight of the maximum homogamy component as the index. In our case, it is equal to:

$$I_{MD}\left(a,b,c,d\right)=\frac{ad-bc}{A}$$

where $A=\left(c+d\right)\left(a+c\right)$ if $b\geq c$ and $A=\left(b+d\right)\left(a+b\right)$ if $c\geq b$. This coincides in our context, with the 'perfect-random normalization' of Liu and Lu (2006) and Shen (2019).

By construction, we have that $I_{MD}\left(a,b,c,d\right)\leq 1$ and $I_{MD}\left(a,b,0,d\right)=1$. Therefore this index satifies the HMax property.

Correlation Another natural index is the correlation between wife's and husband's educations, each considered as a Bernoulli random variable taking the value C with probability $\frac{a+b}{a+b+c+d}$ (resp. $\frac{a+c}{a+b+c+d}$) and HS with probability $\frac{c+d}{a+b+c+d}$ (resp. $\frac{b+d}{a+b+c+d}$). This has been used in various contributions (for instance Greenwood et al., 2003, 2014), either explicitly or through a linear regression framework.

Here:

$$I_{Corr}\left(a,b,c,d\right)=\frac{ad-bc}{\sqrt{\left(a+b\right)\left(c+d\right)\left(a+c\right)\left(b+d\right)}}$$

One can readily check that, in this 2×2 case, the correlation index also coincides with Spearman's rank correlation, which exploits the natural ranking of education levels (C > HS). Equivalently, one can consider the χ^2 index, which is $\chi^2\left(a,b,c,d\right)=\left[I_{Corr}\left(a,b,c,d\right)\right]^2$

The correlation index obviously satisfies the Weak HMax property (since the correlation is then equal to 1). However, it does not satisfy HMax. For instance:

$$I_{Corr}\left(.45,.05,.05,.45\right)=.8>.577=I_{Corr}\left(.25,.25,0,.5\right)$$

Likelihood Ratio This index, used for instance by Eika et al. (2019) among several criteria, measures marital sorting between men of education level I and women of education level J by the ratio of the actual probability of matching relative to what would occur at random:

$$I_L(a,b,c,d) = \frac{a(a+b+c+d)}{(a+b)(a+c)}$$

Unlike the previous indices, the Likelihood ratio does not satisfy either HMax or Weak HMax. This point can be illustrated by the following example. Compare Tables A and B in Table 2 corresponding, for instance, to two different cohorts in the same economy (population sizes normalized to 1):

Table 2: Changes in Assortative Matching - An Example

Cohort A

$_w\backslash^h$	C	HS
C	.03	.07
HS	.07	.83

Cohort B

$_w\backslash^h$	C	HS
C	.5	0
HS	0	.5

The distribution of education is independent of gender in both A and B, but the number of educated people has increased from 10% to 50% between A an B. Cohort A exhibits PAM in the usual sense (more people on the diagonal than would obtain under random matching); yet, 70% of educated people marry an uneducated spouse. Cohort B displays perfect sorting, with all college educated individuals marrying together. The first three indices discussed above, all of which satisfy Weak HMax, indeed conclude that B displays more assortativeness than A. However, the likelihood ratio yields $I_L(A) = 3$ and $I_L(B) = 2$, suggesting that assortativeness has *decreased* from A to B.

Measures of association To explain this paradox, it is interesting to refer to an older statistical literature that discusses the properties of measures of association in the case of paired attributes (i.e., in our case, husband's and wife's education). The Marginal Independence requirement posed by Edwards (1963) states that the association should not be '*influenced by the relative sizes of the marginal totals*' (p. 110). That is, the measure should not change if one starts from a Table $T(a, b, c, d)$ and doubles the number of couples where the man is educated (while keeping unchanged the ratio of educated versus uneducated wives of educated men). Formally, for any non negative (a, b, c, d) and any positive λ, it should hold that:

$$I(\lambda a, \lambda b, c, d) = I(\lambda a, b, \lambda c, d) = I(a, \lambda b, c, \lambda d) = I(a, b, \lambda c, \lambda d) = I(a, b, c, d)$$

Edwards (1963) justifies this property by posing that the measure must only be a function of *the proportion of educated women whose husband is educated and the proportion of uneducated women whose husband is educated* (and conversely), so that any population change that keeps these proportions constant should not affect the index. This imposes, in particular, that global changes in the marginal distributions, for instance a global increase in the number of educated women, should not systematically impact the index; only changes in the odds of marrying different types of spouses should matter.[3]

Among the indices just reviewed, only one - the odds ratio - satisfies Edwards's marginal invariance. It is interesting to consider *how* the other indices violate this requirement. Consider Table $T_\lambda = (\lambda a, \lambda b, c, d)$ with $ad > bc$ (PAM) and $\lambda \geq 1$. Suppose λ increases. Then:

[3] The condition was later generalized by Altham (1970) to the $n \times n$ case.

- The minimum distance index *increases* since $\partial I_{MD}/\partial\lambda > 0$;
- The correlation and Spearman correlation may increase or decrease depending on parameters;
- Finally, the likelihood ratio *decreases* since $\partial I_{LR}/\partial\lambda < 0$.

In the previous example, educational attainment increases between cohorts A and B. This mechanically pushes down the likelihood ratio, driving the paradoxical result.

3.2 Structural interpretations

Finally, it is important to note that among the various indices, the odds ratio has the additional advantage of having a known structural interpretation. Specifically, assume that the observed matching behavior constitutes the stable equilibrium of a frictionless matching model under transferable utility. Assume, furthermore, that the surplus generated by a match between woman i belonging to category I and man j belonging to category J takes the separable form:

$$s(i,j) = Z^{IJ} + \alpha_i^J + \beta_j^I \tag{3}$$

where Z is a deterministic component depending only on individual educations and the α, β are random shocks reflecting unobserved heterogeneity among individuals.[4] It is now well known (Graham (2011), Chiappori (2017)) that, keeping constant the distribution of the shocks, assortativeness is related to the supermodularity of the matrix Z^{IJ} (i.e., in the 2×2 case, to the sign of the supermodular core $Z^{HS,HS} + Z^{C,C} - Z^{HS,C} - Z^{C,HS}$). More importantly, if, following the seminal contribution by Choo and Siow (2006), one assumes that the random shocks follow Type 1 extreme value distributions (the so-called Separable Extreme Value or SEV model), then the supermodular core equals twice the odds ratio I_O.

[4] In the literature, Transferable Utility models models satisfying condition 3 are usually referred to as separable models.

This structural interpretation is especially useful for disentangling possible changes in the value of different matches from the mechanical effect of variations in the marginal distributions of education among individuals: 'structural changes', here, can only affect either the matrix Z or the distributions of random shocks. It is also useful for constructing counterfactual simulations, since the same structure can be applied to different distributions of education by genders, using standard techniques to solve for the stable equilibrium of the corresponding matching game.[5]

Two remarks can be made at this point. First, as clearly pointed out by Choo and Siow in their original contribution as well as by the subsequent literature,[6] this structural model can be identified (in the econometric sense) from matching patterns, but only under strong parametric restrictions. For instance, the initial Choo and Siow framework, which generates the odds ratio as an estimator of the supermodularity of the surplus, is *exactly* identified under the assumption that the random shocks α and β follow a type 1 extreme value distribution. Indeed, the consensus among specialists is that, in order to be identified from the sole observation of matching patterns, any structural model would require strong parametric assumption regarding the distribution of random variables. It follows that, in general, the ranking (in terms of assortativeness, i.e. supermodularity of the deterministic surplus) may vary with the specific assumptions made on the distribution of the stochastic factors.

Several routes can be followed to overcome this limitation. One may, following Chiappori et al. (2017), consider repeated cross sections and impose restrictions on how the structural components change over time. Alternatively, Chiappori et al. (2018) argue that direct observation of post-marital behavior provides additional information on the surplus (since the latter, under TU, is simply the sum of individual utilities and can be estimated from the observation of the demand functions); this information can be used in particular

[5]See Chiappori, Costa-Dias, Crossman and Meghir (2020) and Chiappori, Costa-Dias and Meghir (2020) for an application of these ideas.

[6]See for instance Galichon and Salanié (2021) and the survey by Chiappori and Salanié (2016)

to relax parametric assumptions made on the stochastic components of the model. In a recent contribution, Gualdani and Sinha (2022) also show how partial (i.e. set) identification may obtain using general assumptions on the distribution of stochastic shocks (for instance, independence of taste shocks from covariates and quantile or symmetry restrictions).

Second and more importantly, while robust examples can be given where an assortativeness ranking is reversed when the assumptions regarding stochastic distributions are changed, one can nevertheless define conditions under which the ranking will be the same for any separable TU model, *irrespective of the stochastic distribution*, provided the latter satisfy some basic properties (such as independence). Specifically, Chiappori, Costa-Dias and Meghir (2020) show that if two Tables T and T', which both display PAM, are such that:

$$\frac{a}{a+b} \geq \frac{a'}{a'+b'}, \; \frac{a}{a+c} \geq \frac{a'}{a'+c'}, \tag{4}$$

$$\frac{d}{b+d} \geq \frac{d'}{b'+d'} \text{ and } \frac{d}{c+d} \geq \frac{d'}{c'+d'}, \tag{5}$$

then irrespective of the stochastic distributions of α and β (provided they are independent from each other and from the observed characteristics), the deterministic surplus corresponding to T will be more supermodular than T'.[7] As a result, for any stochastic distributions of α and β, while the numerical value of the supermodular cores will depend on the choice of distributions, the structural model *will always rank Table T above Table T' in terms of assortativeness*.

In other words, one can define a preorder $\succeq$ over Tables that is totally robust to changes in distributional assumptions. The price to pay for this generalization, however, is that the preorder is no longer complete: if some inequalities in (4) are satisfied while others are violated, the Tables simply cannot be compared.

[7] This is the 'Generalized Separable' (GS) criterion in Chiappori, Costa-Dias and Meghir (2020).

4 Empirical example: educational homogamy among educated people in the US

As an illustration, consider the evolution of homogamy by education in the US over the recent decades. Our goal is to compare the answer given by various indices to the same basic question, namely: Did educational homogamy increase between different cohorts? Gihleb and Lang (2020) consider this issue from a general perspective, using a variety of indices (correlation, rank correlation, Goodman and Kruskal's γ, Kendall's τ adjusted and not adjusted for ties). A difference from our approach is that they consider changes affecting the entire distribution of education and many classes, whereas we focus on the top of the education distribution and consider two broad education 'classes', thus fitting our 2×2 framework. Taking into account Gihleb and Lang's findings that different groupings of educational categories can change the results, we use three different splits: College and above vs Some College, College and above vs less than College, and Some College and above vs HS and below. We consider various cohorts, from individuals born in the 1930s to those born in the 1970s. In each case, we compute the difference between the values obtained for different cohorts for the indices described above (odds ratio, χ^2, minimum distance and likelihood ratio).

The results are summarized in Table 3. For the oldest cohorts, born in the 50s or earlier, estimates show some mixed patterns. The indices indicate unanimously that homogamy in marriage *increased* at the top of the education distribution over this period (first four columns in Table) but that, at the same time, it *decreased* when broader education groups are considered (middle and final sets of four columns in Table). These results are consistent with the Gihleb and Lang (2020) finding that the level of aggregation matters for what can be said about the direction of change in education homogamy in marriage.

From a theoretical perspective, of particular interest is the difference between the generation born in the 50s and that born in the 70s (second to last row of the Table). It is now

Table 3: Marital assortativeness at the top of the distribution of education – comparing cohorts born between 1930-39 and 1970-75

	College vs Some College				College vs less than College				Some College vs HS and below			
	Odds ratio	χ^2	Min dist	Ll ratio	Odds ratio	χ^2	Min dist	Ll ratio	Odds ratio	χ^2	Min dist	Ll ratio
1940 vs 30	0.189	0.028	0.019	0.023	-0.259	0.032	-0.021	-1.125	-0.132	0.018	-0.017	-0.610
adj p-val	0.000	0.000	0.355	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.005	0.000
1950 vs 30	0.366	0.069	0.020	0.118	-0.521	0.020	-0.106	-1.469	-0.430	-0.026	-0.119	-0.904
adj p-val	0.000	0.000	0.137	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1960 vs 30	0.534	0.098	0.010	0.126	-0.486	0.044	-0.148	-1.619	-0.380	-0.021	-0.116	-0.968
adj p-val	0.000	0.000	0.477	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 30	0.580	0.097	0.022	0.071	-0.459	0.060	-0.080	-1.889	-0.137	0.006	-0.048	-1.030
adj p-val	0.000	0.000	0.123	0.000	0.000	0.000	0.000	0.000	0.000	0.272	0.000	0.000
1950 vs 40	0.178	0.041	0.002	0.094	-0.262	-0.013	-0.085	-0.344	-0.298	-0.044	-0.102	-0.294
adj p-val	0.000	0.000	0.857	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1960 vs 40	0.345	0.071	-0.009	0.102	-0.227	0.011	-0.127	-0.495	-0.248	-0.039	-0.099	-0.358
adj p-val	0.000	0.000	0.337	0.000	0.000	0.009	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 40	0.391	0.070	0.003	0.048	-0.201	0.028	-0.059	-0.765	-0.005	-0.012	-0.031	-0.419
adj p-val	0.000	0.000	0.778	0.000	0.000	0.000	0.000	0.000	0.845	0.014	0.000	0.000
1960 vs 50	0.167	0.029	-0.010	0.008	0.035	0.024	-0.042	-0.150	0.050	0.005	0.004	-0.064
adj p-val	0.000	0.000	0.352	0.108	0.100	0.000	0.000	0.000	0.029	0.182	0.697	0.000
1970 vs 50	0.214	0.029	0.001	-0.046	0.061	0.040	0.026	-0.421	0.293	0.032	0.071	-0.125
adj p-val	0.000	0.000	0.878	0.000	0.040	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 60	0.047	-0.001	0.012	-0.054	0.026	0.016	0.068	-0.270	0.243	0.027	0.068	-0.062
adj p-val	0.403	0.889	0.148	0.000	0.565	0.001	0.000	0.000	0.000	0.000	0.000	0.000

Notes: Columns identify each of the 2×2 sorting matrices. In each panel, row 1 shows estimates of the difference in the respective index between the latest and earliest cohorts; row 2 shows p-values for 2-sided significance testing adjusted for multiple hypothesis using the stepdown method for the three outcomes on the row. (Romano and Wolf (2005), Romano et al. (2008), Romano and Wolf (2016)). *Data source*: March extract of the US Current Population Survey, subsample of married individuals observed when aged 35-44 and born in 1930-59 to 1970-75.

widely agreed that the return to investments in human capital increased substantially during the 80s and later. Educational and marital choices by the older generation were mostly made before that period; on the contrary, individuals born in the 70s, when choosing both their education level and their spouse, were fully aware of the new context. Some authors have argued that, as a result, homogamy should increase between these two cohorts (for instance Chiappori, 2017; Chiappori et al., 2017). This is indeed the conclusion obtained when considering three of the four indices - namely odds ratio, χ^2 and minimum distance

– for all levels of aggregation in education classes considered here. On the contrary, the likelihood ratio concludes that homogamy has decreased over the period (and actually over all periods under consideration for the 'College vs less than College' and 'Some College vs HS and below' splits)). More broadly, all indices but the likelihood ratio consistently show that the direction of change in homogamy by education started to change in the 50s, towards increasing assortativeness. Given the previous analysis, a possible interpretation is that this difference is related to the global increase in the proportion of college graduates over the period, particularly among women; as discussed above, such an increase tends to mechanically reduce the likelihood ratio.

Finally, it is important to note that the Generalized Separable conditions (4) and (5) are never simultaneously satisfied in our data, (although in one case the violations are not significant), as can be seen from Appendix Table 4. This implies that one can always find specific distributions for the random terms such that the ranking would be reversed - a conclusion similar to that of Gualdani and Sinha (2022).

5 Concluding Remarks

It is relatively simple to estimate whether there is positive assortative matching in a stochastic marriage market along the dimensions of a characteristic such as education. However, measuring the extent to which such assortative matching differs between two economies or between two points in time for the same economy is challenging when the marginal distributions of the characteristics also change. We show, in this note, that different indices may generate different conclusions regarding the evolution of homogamy over time. In particular, we conclude that a particular index, the likelihood ratio, should be used with caution, since it violates the maximum homogamy (HMax) condition, and tends therefore to be especially sensitive to changes in the marginal distributions.

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6 Appendix

Table 4: General test of assortativeness at the top of the distribution of education – comparing birth cohorts 1930-59 to 1970-75

	College vs Some College				College vs less than College				Some College vs HS and below			
	$\frac{a}{a+b}$	$\frac{a}{a+c}$	$\frac{d}{d+b}$	$\frac{d}{d+c}$	$\frac{a}{a+b}$	$\frac{a}{a+c}$	$\frac{d}{d+b}$	$\frac{d}{d+c}$	$\frac{a}{a+b}$	$\frac{a}{a+c}$	$\frac{d}{d+b}$	$\frac{d}{d+c}$
1940 vs 30	0.056	-0.007	0.058	-0.011	0.111	0.017	-0.045	-0.036	0.121	0.047	-0.070	-0.066
adj p-val	0.000	0.198	0.000	0.194	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000
1950 vs 30	0.081	-0.062	0.193	0.003	0.176	-0.033	-0.048	-0.080	0.201	0.041	-0.125	-0.177
adj p-val	0.000	0.000	0.000	0.764	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1960 vs 30	0.144	-0.073	0.250	-0.030	0.269	-0.057	-0.027	-0.123	0.278	0.020	-0.089	-0.258
adj p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 30	0.207	-0.075	0.262	-0.106	0.357	-0.050	-0.028	-0.185	0.337	0.046	-0.071	-0.305
adj p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1950 vs 40	0.025	-0.055	0.135	0.014	0.065	-0.050	-0.002	-0.044	0.081	-0.006	-0.055	-0.111
adj p-val	0.000	0.000	0.000	0.039	0.000	0.000	0.200	0.000	0.000	0.046	0.000	0.000
1960 vs 40	0.089	-0.066	0.191	-0.019	0.158	-0.075	0.018	-0.088	0.157	-0.027	-0.018	-0.192
adj p-val	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 40	0.151	-0.068	0.204	-0.095	0.246	-0.068	0.018	-0.149	0.216	-0.001	-0.000	-0.239
adj p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.979	0.945	0.000
1960 vs 50	0.064	-0.011	0.056	-0.033	0.094	-0.024	0.021	-0.043	0.076	-0.021	0.037	-0.081
adj p-val	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 50	0.126	-0.013	0.069	-0.109	0.181	-0.018	0.020	-0.105	0.135	0.005	0.055	-0.128
adj p-val	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.071	0.000	0.000
1970 vs 60	0.063	-0.002	0.013	-0.076	0.088	0.007	-0.001	-0.062	0.059	0.026	0.018	-0.047
adj p-val	0.000	0.890	0.097	0.000	0.000	0.190	0.804	0.000	0.000	0.000	0.000	0.000

Notes: Columns identify each of the 2×2 sorting matrices. In each panel, row 1 shows estimates of the difference in the respective index between the latest and earliest cohorts; row 2 shows p-values for 2-sided significance testing adjusted for multiple hypothesis using the stepdown method for the three outcomes on the row. (Romano and Wolf (2005), Romano et al. (2008), Romano and Wolf (2016)). *Data source*: March extract of the US Current Population Survey, subsample of married individuals observed when aged 35-44 and born in 1930-59 to 1970-75.