Overcoming Borrowing Stigma: The Design of Lending-of-Last-Resort Policies

Yunzhi Hu* Hanzhe Zhang†

July 25, 2019‡

Abstract

How should the government effectively provide liquidity to banks during periods of financial distress? During the most recent financial crisis, banks avoided borrowing from the Fed's Discount Window (DW) but actively participated in its Term Auction Facility (TAF), although both programs shared the same borrowing requirements. Moreover, banks bid and paid higher interest rates in the TAF than the concurrent discount rate in the DW. Using a model with endogenous borrowing stigmas, we explain how the combination of the DW and the TAF increased banks' borrowings and willingnesses to pay for loans from the Fed. Using micro-level data on DW borrowing and TAF bidding from 2007 to 2010, we confirm our theoretical predictions about the pre-borrowing and post-borrowing conditions of banks in different facilities. Finally, we discuss the effects of the design of lending-of-last-resort policies on liquidity provision.

Keywords: discount window stigma, auction, adverse selection, lending of last resort

JEL: G01, D44, E58

*Kenan-Flagler Business School, University of North Carolina, yunzhi_hu@kenan-flagler.unc.edu.

†Department of Economics, Michigan State University, hanzhe@msu.edu.

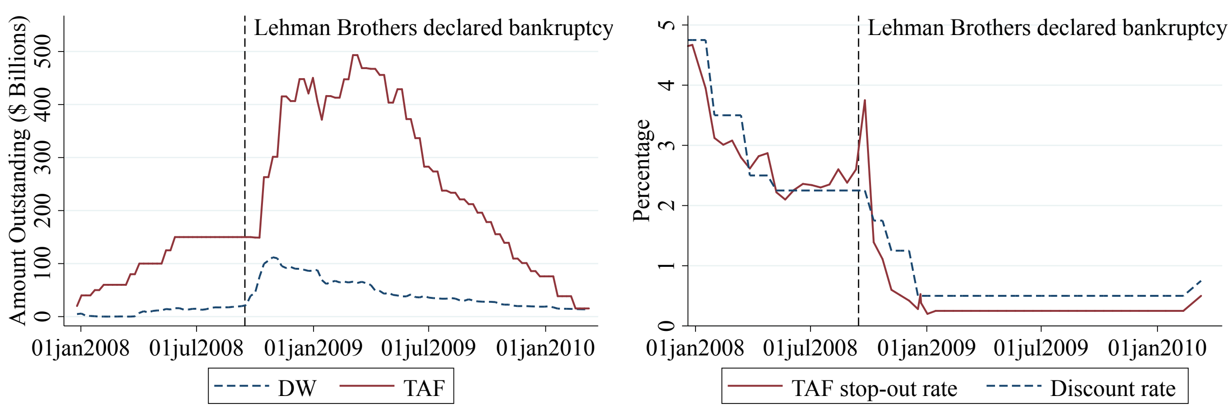
‡We are grateful to comments and suggestions from Sriya Anbil, Olivier Armantier, Yasser Boualam, Chen Cheng, Huberto Ennis, Paolo Fulghieri, Robin Greenwood, Aeimit Lakdawala, Raoul Minetti, Anil Kashyap, Jacob Sagi, Larry Wall, Yiqing Xing, and participants at UNC Kenan-Flagler, JHU Carey, MSU economics, Doug Diamond's 65th birthday conference, EFA, Northeastern Finance Conference, FIRS, and Stony Brook Game Theory Festival. We also thank the research assistance of Sunwoo Hwang and Dongming Yu.

1 Introduction

Financial crises are typically accompanied by liquidity shortage in the entire banking sector. How should the central bank lend to depository institutions during such episodes? The answer is not obvious. The discount window (DW) has been the primary lending facility used by the Federal Reserve, but it was severely under-used when the interbank market froze at the beginning of the financial crisis in late 2007. A main reason for such under use is believed to be the stigma associated with DW borrowing: tapping the discount window conveys a negative signal about the borrowers' financial conditions to their counterparties, competitors, regulators, and the public.[1] As suggestive evidence, banks have regularly paid more for loans from the interbank market than they could readily get from the DW (Peristiani, 1998; Furfine, 2001, 2003, 2005).

Figure 1: Borrowing amounts and rates in DW and TAF from 2008 to 2010

(a) Borrowing amount (b) Borrowing rates



In response to the credit crunch and banks' reluctance to borrow from the DW, the Fed created a temporary program, the Term Auction Facility (TAF), in December 2007. The TAF held an auction every other week, providing a pre-announced amount of loans with *identical* loan maturity, collateral margins, and eligibility criteria as the DW.

Surprisingly, the TAF provided much more liquidity than the DW: Figure 1a shows that the outstanding balance in the TAF far exceeded that in the DW during 2007-2010.[2] Moreover, interestingly, banks sometimes paid a higher interest rate to obtain liquidity through the auction: Figure 1b shows that the stop-out rate – the rate that cleared the auction – was higher than the concurrent

[1] Although the Fed does not disclose publicly which institutions have received loans from the DW, the Board of Governors publishes weekly the total amount of DW lending by each of the twelve Federal Reserve Districts. Therefore, a surge in total DW borrowing could send the market scrambling to identify the loan recipients. Because of the interconnectedness of the interbank lending market, it is not impossible for other banks to infer which institutions went to the discount window. Market participants and social media could also infer from other activities.

[2] Outstanding balance in the DW made up at most 33.4 percent of the total outstanding balance between 2007 and 2010. See Figure A1a in the appendix for the DW balance as a percent of the total balance week by week between 2007 and 2010.

discount rate – the rate readily available in the DW – in 21 out of the 60 auctions, especially from March to September 2008, the peak of the financial crisis.[3]

This episode suggests the importance of the design of emergency lending programs to effectively cope with liquidity shortage. More specifically, it raises a series of questions about the lending-of-last-resort policies. Why could the TAF overcome the stigma and generate more borrowing than the DW? Shouldn't the similar stigma also prevent banks from participating in the TAF? How did banks decide to borrow from the DW and/or the TAF? Was there any systematic difference between the banks that borrowed from the two facilities? How to further improve the program? The answers to these questions remain unclear, even to policy makers involved (Armantier and Sporn, 2013; Bernanke, 2015).

This paper provides a comprehensive analysis of lending of last resort in the presence of borrowing stigma. We introduce a dynamic model in which banks have private information about their financial conditions. Weaker banks have more urgent liquidity need and enjoy higher borrowing benefits. Two lending facilities are available. An auction is held once to allocate a set amount of liquidity, and the DW is always available – before, during, and after the auction. Borrowing from each facility incurs a stigma cost, which is endogenously determined by the financial conditions of participating banks.

In equilibrium, banks self select into different programs. Since the DW always guarantees lending, the weakest banks borrow from it immediately, because they are desperate for liquidity and cannot afford to wait. Stronger banks, in contrast, are lured to participate in the auction because the potential of borrowing cheap renders the auction more attractive than the DW. Their liquidity needs are not that imperative and they value lower expected price in the auction more than their weaker counterparts. Among the banks who participate in the TAF, some may bid higher than the discount rate because they would like to avoid the discount window stigma brought by the association with the weakest banks. As a result, the clearing price in the auction may exceed the discount rate. Among the banks who have lost in the TAF, relatively weaker ones might still borrow from the DW.[4]

Our model demonstrates that the introduction of the TAF in addition to the DW could increase liquidity provision through three channels. First, by setting a low reserve price in the auction, the TAF attracted relatively strong banks to participate and take their chances of borrowing cheap. Second, participating banks can internalize any stigma cost associated with the TAF by adjusting

[3]The stop-out rate ranged from 1.5 percentage points above (September 25, 2008) to 0.83 percentage points below (December 4, 2008) the concurrent discount rate. The stop-out rate was above the concurrent discount rate for almost all auctions between March 2008 (when Bear Sterns filled for bankruptcy) and September 2008 (when Lehman Brothers filed for bankruptcy). See Figure A1b in the appendix for the difference between the stop-out rate and the concurrent discount rate auction by auction.

[4]Finally, the strongest banks do not borrow at all.

their bids, which endogenously leads to a positive payoff if they win. Third and finally, due to selection, the auction stigma is endogenously lower than the discount window stigma. Hence, it is the combination of the TAF and the DW that expands the set of the banks who try to and may obtain liquidity, thus increasing the overall supply of short-term credit to the economy.

We use micro-level data on DW borrowing and TAF bidding to verify the model's predictions. We obtain two sets of empirical results consistent with our predictions. The first set of results confirms our prediction that banks opt into different borrowing programs. We find that (i) weaker banks – measured by tier-1 capital ratios – were more likely to tap the DW relative to the TAF and (ii) among the banks who participated in the TAF, those who submitted higher bids (and thus were more likely to be winners) pledged collaterals of lower quality and were more likely to bid again in subsequent auctions (a sign of weakness). The second set of results confirms our prediction that the stigmas are different for different programs. Since the TAF schedule was announced weeks before the auction date, tapping the discount window right before the auction signals a bank's urgency for liquidity. Using an event-study approach, we confirm that the banks who borrowed from the DW within three days before an auction were associated with more negative subsequent abnormal returns in their stock prices.

First, our paper improves the understanding of interventions during the financial crisis, and more specifically, contributes to the literature that studies government intervention in markets plagued by adverse selection (Philippon and Skreta, 2012; Tirole, 2012; Ennis and Weinberg, 2013; La'O, 2014; Lowery, 2014; Fuchs and Skrzypacz, 2015; Gauthier et al., 2015; Li et al., 2016; Ennis, 2017; Che et al., 2018). In these studies, either there is no explicit stigma cost in government-sponsored facility, or stigmas are implicitly assumed to be uniform across all programs. Our paper endogenizes the different stigma costs associated with DW and TAF as well as banks' heterogeneous decisions on which facility to use.

Second, this paper, to the best of our knowledge, is the first to combine micro-level data on DW borrowing and TAF bidding and link them to information on banks' fundamentals and subsequent stock market performances. Existing papers largely focus on empirical estimates of either the DW stigma or subsequent economic effects of TAF borrowing. Peristiani (1998); Furfine (2001, 2003, 2005) offer evidence that banks prefer the Federal Funds Market to the DW, suggesting the existence of the DW stigma. More recently, Armantier et al. (2015) show that more than half of the TAF participants submitted bids above the discount rate during the 2007-2008 financial crisis. McAndrews et al. (2017) and Wu (2011) study the effect of the TAF and conclude that it was effective in lowering Libor and reducing liquidity concern in the interbank lending market. Moore (2017) finds that the TAF had a benefit on the real economy. Cassola et al. (2013) study the financial crisis from the bidding data in the European central bank from January to December 2007 to confirm that the banks were strategic in their bidding.

The rest of the paper is organized as follows. Section 2 describes the lending-of-last-resort facilities during the 2007-2008 financial crisis. Section 3 sets up the model. Section 4 characterizes the equilibrium of the model. Section 5 presents empirical evidence consistent with the predictions of the model. Section 6 discusses the effects of the design of lending-of-last-resort policies on liquidity provision. Section 7 concludes. The appendix contains omitted figures and proofs.

2 Background

The stress in the interbank lending market began to loom in the summer of 2007. In June, two of Bear Sterns' mortgage-heavy hedge funds reported large losses. On July 31, they declared bankruptcy. On August 9, BNP Paribas, France's largest bank, barred investors from withdrawing money from its investments backed by U.S. subprime mortgages, citing evaporated liquidity as the main reason. Subsequently, many other banks and financial institutions experienced dry-ups in wholesale funding (in the form of asset-based commercial paper or repurchase agreements).

With the growing scarcity of short-term funding, banks were supposed to borrow from the lender of last resort (LOLR). In the United States, the role of LOLR has been largely fulfilled by the discount window, which allows eligible institutions, mostly commercial banks, to borrow money from the Federal Reserve on a short-term basis to meet temporary shortages of liquidity caused by internal and external disruptions.[5] Discount window loans were extended to sound institutions with good collateral. Since its funding a century ago, the Fed has never lost a penny on a discount window loan. However, banks were reluctant to use the discount window, due to the widely held perception that a stigma was associated with borrowing from the Fed. As advised by Bagehot (1873), a penalty – one percentage point above the target federal funds rate – was charged on discount window loans, with the goal to encourage banks to look first to private markets for funding. However, this penalty generated a side effect on banks – banks would look weak if it became known that they had borrowed from the Fed.

Discount window borrowing was strictly kept confidential.[6] However, banks were nervous that investors, in particular money market participants, could guess when they had come to the window by observing banks' behavior or through careful analysis of the Fed's balance sheet figures, because the Fed has to disclose the level of discount window borrowing at both the aggregate and district level, another potential source of detection.[7],[8]

[5] In history, the discount window was once literally a teller window manned by a lending officer.

[6] The Dodd-Frank act required the disclosure of details of discount window loans after July 2010 on a two-year lag from the date on which the loan is made.

[7] According to Bernanke (2015), Ron Logue, CEO of State Street, approached the Boston Fed and checked whether the weekly district-by-district reporting of loan totals could be eliminated. The request was turned down due to legal feasibility reasons and concerns for market-wide confidence.

[8] The stigma associated with borrowing from the government is also significant in the United Kingdom. Shin (2009)

The Fed subsequently made a few changes to the discount window policies. In particular, on August 16, 2007, it halved the interest rate penalty on discount window loans.[9] The maturity of loans was also extended for up to thirty days with an implicit promise for further renewal. Moreover, the Fed tried to persuade some leading banks to borrow at the window, thereby suggesting that borrowing did not equal weakness. On August 17, Timothy Geithner and Donald Kohn hosted a conference call with the Clearing House Association, claiming that the Fed would consider borrowing at the discount window "a sign of strength." Following the call, on August 22, Citi announced it was borrowing $500 million for thirty days. JPMorgan Chase, Bank of America, and Wachovia subsequently made similar announcements that they had borrowed the same amount, increasing the total discount window borrowing amount to $2 billion. However, the four big banks – with the borrowing stigma in mind – made it very clear in their announcements that they did not need the money. Thirty days later, the discount window borrowing fell back to $207 million.[10]

To further relieve the stress in the short-term lending market, the Fed implemented the term auction facility in December 2007. The first auction held on December 17 released $20 billion in the form of 28-day loans. The participation requirement was the same for the auction as for the DW.[11] The Fed received over $63 billion in bids and released the full $20 billion to 93 different institutions. In February 2008, Dick Fuld, CEO of Lehman Brothers, urged the Board to include Wall Street investment banks in the regular TAF auctions, which would require invoking Section 13(3) to allow the Fed to have authority to lend to non-bank institutions. The final auction was

described the storyline of the Northern Rock bank run in the United Kingdom. In the U.K, there was no government deposit insurance. Banks relied on an industry-funded program that only partially protected depositors. On September 13, 2007, BBC evening television news broadcast first broke the news that Northern Rock had sought the Bank of England's support. The next morning, the Bank of England announced that it would provide emergency liquidity support. It was only *after* that announcement, that is, after the central bank had announced its intervention to support the bank, that retail depositors started queuing outside the branch offices. Another story is in August 2007. Barclays tapped twice the emergency lending facility offered by the Bank of England. News first came out on Thursday, August 30, when the Bank of England said it had supplied almost 1.6 billion pounds as lender of last resort, without naming the borrower(s). Journalists and the market scrambled to find out. Barclays first declined to confirm that it had used the central bank's standing borrowing facility. Later on, it cited technical breakdown in the UK clearing system as the reasons for the large pile of cash. In its statement, Barclays said: "The Bank of England sterling standby facility is there to facilitate market operations in such circumstances. Had there not been a technical breakdown, this situation would not have occurred." Its share fell 2.5 pounds immediately after the statement, which casted doubt on its 45 billion pounds bid to take over the Dutch bank ABN Amro.

[9] On December 11, 2007, the Fed lowered its discount rate to 4.75%.

[10] Records released later show that JPMorgan and Wachovia returned most of the money the next day, whereas Bank of America and Citi, already showing signs of problems, kept the money for a month.

[11] The rule of the auction was as follows. On Monday, banks phoned their local Fed regional banks to submit their bids specifying their interest rate (and loan amount) and to post collaterals. On Tuesday, the Fed secretly informed the winners and publicly announced the stop-out rate (as well as the number of banks receiving loans), determined by the highest losing bid (or the minimum reserve price if the auction was under-subscribed). On Thursday, the Fed released the loans to the banks. Throughout the whole auction process, banks were free to borrow from the DW. The following Monday, each regional Fed published total lending from last week; banks may be inferred from these summaries or other channels.

held on March 8, 2010.

As shown in Figure 1, the TAF was clearly more successful than the DW in providing liquidity. Banks were also willing to pay a higher interest rate in the TAF than the concurrent discount rate in the DW.[12] As acknowledged in Bernanke (2015), before implementing TAF, the policy makers were also concerned that the stigma that had kept banks away from the discount window would attach to the auctions. The program was implemented as "give it a try and see what happens."

3 The Model

We introduce a two-period model with n banks in the economy. A period corresponds to a week in real time. The timeline of the model is as follows. Each bank is endowed with an illiquid asset that pays off after the second week. Before the asset pays off, a liquidity shock may hit a bank with a probability that is privately known by the bank. Before the shock, each bank can borrow from two facilities: discount window (DW) and term auction facility (TAF). Borrowing banks may incur a penalty if detected of borrowing. Figure 2 sketches the timing and sequence of events, which we will describe in detail next.

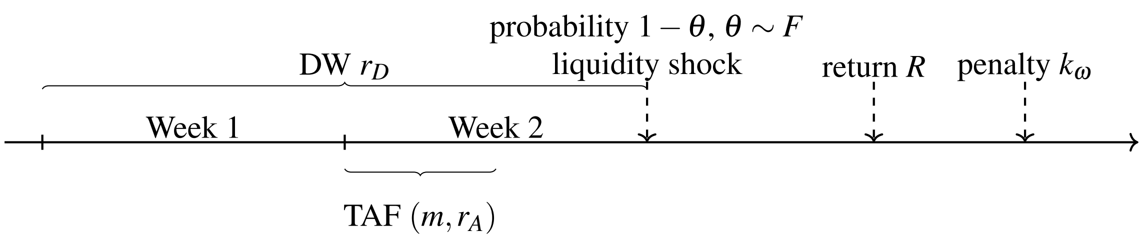


Figure 2: Timeline of the model

3.1 Preferences, Technology, and Shocks

All parties are risk neutral and do not discount future cash flows.

At the beginning of the first week, each bank has one unit of long-term, illiquid assets that will mature at the end of the second week. The asset generates cash flows R upon maturity but nothing if liquidated early. Shortly before the end of the second week, each bank may be hit with a liquidity shock à la Holmström and Tirole (1988).[13] The size of the shock is normalized as one unit. Let $1-\theta_i \in [0,1]$ be the probability that the liquidity shock hits bank i, where θ_i follows the

[12]To this date, the specific reason for why TAF was more successful is still unclear. According to Bernanke (2015), the auction *might possibly* reduce the stigma because interest rates would be set through a competitive auction rather than fixed. In that case, borrowers could claim they were paying a market rate, not a penalty rate.

[13]In reality, a liquidity shock can occur any time. In the model, to avoid the non-stationarity of distribution of banks when banks may be hit with a liquidity shock any time during the two weeks, for tractability, we assume that all banks realize their shocks at the end of the second week.

independent and identically distributed cdf F and associated pdf f on the support $[0,1]$. Throughout the paper, we assume that θ_i is private information and only known by the bank itself. We drop subscript i whenever no confusion arises. Type θ is also referred to as a bank's financial strength. We will sometimes refer to a type-θ bank simply as bank θ.[14]

Before the liquidity shock hits, each bank has opportunities to borrow. We will describe the choices of borrowing. For now, let r be the gross interest rate of a received loan. A loan will help the bank defray the liquidity shock and therefore brings net benefits $(1-\theta)R$ at the cost of interest rate r. Finally, to capture the idea that earlier liquidity is more valuable, we assume the net benefits are discounted by a common factor δ if borrowing is accomplished in week 2. The factor δ can be interpreted as the cost incurred when banks sell illiquid assets at fire-sale prices in order to satisfy immediate liquidity needs.[15] To summarize, a type-θ bank's payoff is $\pi_1(\theta,r) = (1-\theta)R - r$ if it borrows in week 1, and is $\pi_2(\theta,r) = \delta(1-\theta)R - r$ if it borrows in week 2. As it becomes clear later on, the specific functional form of the borrowing benefit does not matter. What matters is that the benefit is lower if the bank is stronger or if the interest rate is higher.[16]

We describe the two lending facilities in the next subsection.

3.2 Borrowing Facilities

We will describe in the appendix an extension in which the interbank market is well-functioned. In the basic model, any bank is only able to borrow from either the discount window or the term auction facility.[17]

3.2.1 Discount Window

The discount window is a facility that offers loans at a fixed interest rate r_D, which is commonly referred to as the discount rate and is exogenously set by the Federal Reserve. Since a bank can always borrow from the discount window with certainty, the net borrowing benefit is $\pi_t(\theta, r_D)$ when the loan is taken out in period $t = 1,2$.

[14] In reality, one can proxy a bank's strength θ by either its reserve of liquid assets or the level of its demandable liabilities that can evaporate in a flash.

[15] The discount factor δ can also be microfounded by a liquidity shock in the first week.

[16] According to Bernanke (2015), one reason to implement the term auction facility was it takes time to conduct an auction and determine the winning bids so that borrowers would receive funds with a delay, making clear that they were not desperate for cash.

[17] The interbank market essentially froze during the 2007-2008 financial crisis.

3.2.2 Term Auction Facility

The term auction facility allocates pre-announced m units of liquidity through an auction. In the auction, banks who decide to participate simultaneously submit their sealed bids.[18] Bid β_i specifies the maximum interest rate bank i is willing to pay. The bid needs to be higher than the reserve interest rate r_A.

After receiving all the bids, the auctioneer ranks them from the highest to the lowest. The auction takes a uniform-price format: all winners pay for the same interest rate while losers do not pay anything. If there are fewer bids than the units of liquidity provided, each bidder receives a loan and pays r_A. If there are more bidders than the total offering liquidity, each of the m highest bidders receives one unit of liquidity by paying the highest *losing* bid. In this case, the highest losing bid is also called the stop-out rate s, which is the clearing price at which aggregate demand in the auction matches the aggregate supply. Formally, suppose there are l bidders in total. If $l \leq m$, bidding banks each receive a loan by paying $s = r_A$. If $l > m$, the m highest bidding banks each receive one unit of liquidity by paying the $m+1^{\text{st}}$ highest bid. The remaining $l - m$ banks do not pay anything and, of course, do not receive any liquidity either.

Let $w(\theta, \beta)$ denote the (equilibrium) probability that bank θ can win the auction by bidding β. We will focus on symmetric strategies in bidding and therefore can write $w(\theta, \beta(\theta))$ as $w(\theta)$ without loss of generality. Let $b(\theta)$ be the expected payment that bank θ pays conditional on winning the auction. The expected net borrowing benefit is $w(\theta)\pi_2(\theta, b(\theta))$.

We have essentially modeled the TAF auction as an extended second-price auction: all winning parties pay the highest losing bid. In reality, TAF is closer to an extended first-price auction: all winning banks pay the lowest winning bid. The two auctions generate the same expected payoffs, by the Revenue Equivalence Theorem (Myerson, 1981), and consequently the same borrowing decisions. We present the analysis with the extended second-price auction, because it is a weakly dominant strategy for each bank to simply bid the maximum interest rate it is willing to pay.[19]

3.3 Stigmas

A key reason that banks were reluctant to borrow from the lender of last resort is stigma. Detected borrowing may signal financial weakness to counter-parties, investors, and regulators. Although θ is private information, the public can still make inference based on whether the bank has borrowed or which facility the bank has used if it has borrowed. We assume that upon detection,

[18] We assume that each bank is restricted to bid for only one unit of liquidity so that no secondary market exists for the auction-allocated liquidity.

[19] In contrast, in the first-price auction, banks shade their bids that depend on the liquidity supply and other participating banks. The fact that their actual bids (that were supposedly results of shading from their maximum willingnesses to pay) were above the concurrent discount rate further proves their high willingnesses to avoid the discount window.

the public can perfectly tell whether the borrowing has been achieved through the discount window or the auction. In the basic model, we assume the public cannot tell *when* the bank has borrowed from the discount window. Later on, we will show that none of our results is driven by the specific assumptions on detection.

Let G_D, G_A, and G_N be the type distributions of the banks that have borrowed from the DW, from the TAF, and have not borrowed, respectively. We capture the notion of stigma in a parsimonious way. Specifically, we assume that after all the borrowings are accomplished, the banks that have successfully borrowed may be detected independently with probability p, after which a penalty will be imposed. This penalty can be understood as a cost in bank's deteriorated reputation, a cost in a reduced chance to find counterparties, or a cost from a heightened chance of runs and increasing withdrawals by creditors. Let the stigma cost be $k(\theta, G_\omega)$, where $\omega \in \{D, A, N\}$. The stigma cost is naturally assumed to be higher when the borrowing banks are worse: formally, $k(\theta, G) > k(\theta, G')$ if G is strictly first-order stochastically dominated by G'. In the baseline model, we eliminate the dependence of stigma cost on a bank's private type and instead assume that it only depends on $\omega \in \{D, A, N\}$. In other words, $k(\theta, G_\omega) = k(G_\omega) \equiv k_\omega$.

3.4 Equilibrium

In summary, the setting is summarized by the return R, type distribution F of banks, discount rate r_D in the DW, number m of units of liquidity auctioned and minimum bid r_A in the TAF, and the penalty function $k(G)$ attached to different belief distributions of bank's type.

A type-θ bank's strategy can be succinctly described by $\sigma(\theta) = (\sigma_D(\theta), (\sigma_A(\theta), \beta(\theta)))$, where $\sigma_\omega(\theta)$ is the probability of borrowing from $\omega \in \{D, A\}$ and $\beta(\theta)$ is its bid if it participates in the auction. Given strategies $\sigma(\cdot)$, beliefs about the financial situation can be inferred by the Bayes' Rule. In this case, we say aggregate strategies $\sigma(\cdot)$ generate posterior belief system $G = (G_A, G_D, G_N)$. Note that we have restricted each bank's strategy to be symmetric so that $\sigma(\cdot)$ only depends on θ.

Definition 1. $(\sigma^*(\cdot), G^*)$ *form an equilibrium if*

1. *each type-θ bank's strategy $\sigma^*(\theta)$ maximizes its expected payoff given belief system G^*, and*
2. *the belief system G^* is consistent with banks' aggregate strategies $\sigma^*(\cdot)$.*[20]

Clearly, the best (i.e., type-1) bank has no intention to borrow at all, because it would only pay a price and stigma cost but has no benefit from borrowing. We assume that the borrowing benefit of the worst (i.e., type-0) bank is sufficiently high so that it has a strict incentive to borrow even given

[20] If one of the two facilities is not used on the equilibrium path, the beliefs of the banks must satisfy the intuitive criterion (Cho and Kreps, 1987).

the most pessimistic belief about the banks who borrow: $\delta R - r_D - k(G) > 0$, where $G(\theta) = 1$ for all $\theta > 0$.[21]

4 Characterization of the Equilibrium

4.1 Special Cases: Equilibrium with only DW or only TAF

We present the solutions of two special cases (with only DW or TAF) before presenting the solution of the generic case of the model (with both DW and TAF facilities).

4.1.1 Equilibrium with only DW

We start by examining the equilibrium when the government only sets up the discount window (finite r_D and $m = 0$). Clearly, no bank would ever want to delay borrowing from the discount window. We show that any equilibrium borrowing decision can be characterized by one threshold: weaker banks borrow from the discount window, and stronger banks do not borrow at all.[22]

Proposition 1. *If r_D is finite and $m = 0$, any equilibrium is characterizable by a threshold θ^{DW}.*

1. *Banks $\theta \in \left[0, \theta^{DW}\right]$ borrow from the DW.*
2. *Banks $\theta \in \left(\theta^{DW}, 1\right]$ do not borrow at all.*

Note (again) that the best bank never borrows: it knows that a liquidity shock could never occur and therefore never need the liquidity, but borrowing incurs an interest rate cost as well as a stigma cost. The worst bank, as we have assumed, has a strict incentive to borrow from the discount window. By continuity of the benefit function, the banks that will be hit by a liquidity shock with a sufficiently large probability will have a strict incentive to borrow from the discount window.

4.1.2 Equilibrium with only TAF

Next, we examine the equilibrium when the government only sets up the auction (infinite r_D and $m > 0$). The equilibrium can also be characterized by one threshold: weaker banks bid in the auction and stronger banks do not borrow at all.

Proposition 2. *If r_D is infinite and $m > 0$, any equilibrium is characterizable by a threshold θ^{TAF}.*

[21] Given this assumption about the sure participation of the worst bank, the refinement with the intuitive criterion in the previous footnote will be effectively not needed at all. We include it for the sake of completeness.

[22] It is possible to have multiple equilibria, but the decisions in all equilibria can be characterized by one threshold. A condition similar to the one used in Akerlof (1970) guarantees the existence of a unique equilibrium.

1. *Banks* $\theta \in \left[0, \theta^{TAF}\right]$ *bid in the TAF.*

2. *Banks* $\theta \in \left(\theta^{TAF}, 1\right]$ *do not bid at all.*

4.2 Generic Case: Equilibrium with both DW and TAF

We now solve for the equilibrium when both discount window and term auction facility are available. We will first describe a bank's bidding strategy in TAF, followed by its incentives in choosing between DW and TAF. Our result shows that relatively stronger banks have more incentives to bid in TAF rather than borrow immediately from DW, which is the key force behind the separation of types in equilibrium.

Let's start by describing a bank's bid in the auction. In general, a bank's bidding strategy depends on its plan after losing in the auction: it can either borrow from the DW in the second period or not to borrow at all. Clearly in this case, the incentive to borrow declines with a bank's financial strength.

Lemma 1. *In any equilibrium, banks* $\theta \leq \theta_2$ *will borrow from the discount window in the second week if they have not borrowed.*

Let $\beta^D(\theta)$ be a type-θ bank's bid if it plans to borrow from discount window after losing the auction. Let $\beta^N(\theta)$ be its bid if it doesn't plan to borrow after losing the auction. Given that a bank's bid does not (directly) affect its payment conditional on winning the auction, a bank bid its own willingness to pay (WTP), as follows.

Lemma 2. *Bank* θ *who borrows from the discount window after losing in the auction bids*

$$\beta^D(\theta) \quad = \quad r_D + p(k_D - k_A). \tag{1}$$

Bank θ *who does not borrow from the discount window after losing in the auction bids*

$$\beta^N(\theta) \quad = \quad \delta(1-\theta)R - pk_A. \tag{2}$$

Note that $\beta^D(\theta)$ does not depend on θ. In other words, any bank who plans to go to the discount window bids up to the same amount, which equals the sum of r_D, the discount rate, and $(k_D - k_A)$, the net stigma cost of discount window relative to TAF. Intuitively, these banks will always borrow in equilibrium, from either the DW or the TAF. Therefore, since the discount window charges the same rate to all borrowers and the stigma cost is also homogeneous across all borrowers from the same facility, their WTPs are also the same. In the most general case where k_ω depends both on the borrowing decision $\omega \in \{D, A\}$ and a bank's own financial strength θ, $\beta^D(\theta)$

will decrease in θ as long as $k(\theta, G_D) - k(\theta, G_A) > 0$ for any θ. On the other hand, $\beta^N(\theta)$, however, does depend on θ. Among these banks, weaker ones have higher WTPs because they have stronger demand for liquidity but will not borrow if they lose in TAF.

Proposition 3 is a main result of this paper. It describes the incentive to borrow from DW1 against participating in the auction. In particular, it shows the skimming property that stronger banks are more willing to wait for the TAF.

Proposition 3 (Skimming property)**.** *Let $u_1(\theta)$ be bank θ's expected equilibrium payoff if it borrows from the discount window in period 1, and $u_A(\theta)$ be its expected payoff if it bids in the auction. In any equilibrium, $u_1(\theta) - u_A(\theta)$ is strictly decreasing in θ.*

Intuitively, auction introduces uncertainty in terms of whether a bidding bank is able to borrow and if so at what price. Specifically, it introduces one mechanism that enables a bank to borrow at a low rate, lower than its own willingness to pay, at the cost of potentially failing to borrow (for banks $\theta \in [\theta_2, 1]$) or delaying to borrow (for banks $\theta \in [0, \theta_2]$). This cost of not borrowing (or delayed borrowing) is lower for stronger banks because their borrowing benefits are lower. Therefore, they are more inclined to participate in the auction and take advantage of the opportunity to borrow when rates are sufficiently low. In this case, auction is able to separate borrowers into two groups, the so-called "single-crossing" condition. Mathematically, a bank $\theta \in [0, \theta_2]$ will always borrow even if it chooses to participate in the TAF: it will turn to the discount window in week 2 in the event of losing in the TAF, in which case the cost of delay is $(1-\delta)(1-\theta)R$, decreasing in θ. Bank $\theta \in [\theta_2, 1]$ no longer borrows if it loses in the auction, with the cost of failing to borrowing being $(1-\theta)R$.

It is worthwhile to point out that our result on separation does not depend on the assumption that delaying cost is bigger for weaker banks. In the appendix, we present another version of the model in which cost of delay is homogeneous across all banks and show all results carry through. Moreover, we would like to emphasize that not any mechanism that offers a tradeoff between probability of winning and price paid can always separate borrower. To see this, note that a bank's overall payoff has three components that vary with θ. First, a stronger bank has lower borrowing benefits: $\frac{d(1-\theta)R}{d\theta} = -R < 0$. Second, in equilibrium, a stronger bank is less likely to win in the auction. However, conditional on winning in the auction, however, it pays less in expectation. When a bank bids optimally, it is indifferent between raising the bid to increase the winning probability and paying more conditional on winning. Therefore, the last two effects exactly cancel out on each other As a result, the overall effect is simply the decreasing benefits of borrowing times the probability of winning in the auction: $-R[1-H(\theta)]$. Next, let us consider a mechanism $(w(\theta), b(\theta))$ where $w(\theta)$ is the probability of receiving one unit of liquidity and

$b(\theta)$ is the price paid. Let a bank's payoff in participating this mechanism be $u_M(\theta)$.

$$u_1(\theta) - u_M(\theta) = w(\theta)\left[b(\theta) + k_M + \delta - r_D - k_D\right] + \left[1 - w(\theta)\right]\left[(1-\theta)R - r_D - k_D\right].$$

By taking derivatives with respect to θ, we can see clearly that the overall effect is ambiguous.

Given Proposition 3, in any equilibrium, weaker banks choose to borrow from the discount window in week 1, and stronger banks bid in the auction. Among the banks who lose in the auction, relatively stronger ones (if any) will still go to the auction.

Theorem 1. *Any equilibrium can be characterized by three thresholds, θ_D, θ_2, and θ_A.*

1. *Banks $\theta \in [0, \theta_D]$ borrow directly from week 1's DW.*

2. *Banks $\theta \in (\theta_D, \theta_A]$ participate in the auction.*

   (a) *If $\Delta(\theta_2|H(\cdot|\theta_2)) \leq 0$, then $\theta_D < \theta_2$. Banks $\theta \in (\theta_D, \theta_2]$ bid $\beta(\theta) = r_A + p(k_D - k_A)$ in the auction, and borrow from week 2's DW if they lose in the auction. Banks $\theta \in (\theta_2, \theta_A]$ bid $\beta(\theta) = \delta b(\theta) - pk_A$ in the auction, and choose not to borrow from week 2's DW if they lose in the auction.*

   (b) *If $\Delta(\theta_2|H(\cdot|\theta_2)) > 0$, then $\theta_D \geq \theta_2$. Banks $\theta \in (\theta_D, \theta_A]$ bid $\beta(\theta) = \delta b(\theta) - pk_A$ in the auction, and choose not to borrow if they lose in the auction.*

3. *Banks $\theta \in (\theta_A, 1]$ do not borrow at all.*

Proposition 3 immediately implies that the stigma associated with discount window borrowing exceeds that with TAF borrowing.

Corollary 1. *In equilibrium, $k_D^* > k_A^*$. In words, the stigma attached to the discount window is endogenously higher than the stigma attached to the term auction facility.*

5 Empirical Analysis

In this section, we offer some empirical evidence that is largely consistent with the prediction of our model. The central prediction of our model is banks borrowing from DW are fundamentally weaker than banks borrowing from TAF, which in turn are weaker than banks who borrow from neither facilities. This will be the main hypothesis throughout the empirical analysis. Specifically, we conduct two tests. First, we study the correlation between banks' fundamentals and their borrowing decisions. Second, we apply an event-study approach and examine how the market reacted to these borrowing decisions.

5.1 Data and Summary

Throughout the empirical exercise, we combine several datasets. The first one is obtained through Bloomberg and includes 407 institutions that borrowed from the Federal Reserve between August 1, 2007 and April 30, 2010. These data were released by the Fed on March 31, 2011, under a court order, after Bloomberg filed a lawsuit against Fed board for information disclosure.[23] The data contain information on each institution's daily outstanding balance of its borrowing from the discount window, the Term Auction Facility as well as five other related programs.

Table 1 provides summary statistics. The borrowing institutions are mostly banks ($\frac{313}{407} \approx 73\%$), together with diversified financial services (mostly asset management firms), insurance companies, savings and loans, and other financial service firms. Foreign banks who borrowed through their U.S. subsidiaries were also included. Among them, 92 borrowers were foreign banks who borrowed mainly through their U.S. subsidiaries. Banks' choices of borrowing facilities were quite heterogeneous. While a majority (260 out of 407) tapped both facilities, some only used one throughout the period. The total borrowing events also exhibit sharp heterogeneity: some banks never tapped the discount window, whereas one bank (Alaska USA Federal Credit Union) used it a total of 242 times. Among the 60 TAF auctions, Mitsubishi UFJ Financial Group borrowed a total of 28 times. On average, TAF offered more liquidity (3174 million) than DW (1529 million), consistent with the evidence in Figure 1a. However, the Dexia Group, the bank that borrowed the most from DW took out a total of approximately \$190 billion loans over the three-year period, exceeding its counterpart in TAF ($\approx$ \$100 billion by Bank of America Corp). This evidence also suggests that DW banks were in more need of liquidity than TAF banks.

Our second dataset provides details on all 60 TAFs, including names of bidders (both winners and losers), their bidding rates, the amount awarded, as well as the collaterals pledged to back these loans. We obtain this data by filing a Freedom of Information Act (FOIA) request to the Federal Reserve. Table 2 describes the summary statistics. A total of 434 banks have submitted their bids in TAF. Among them, 22 were classified as Global systemically important (G-SIBs), and 82 were foreign. Indeed, G-SIBs and foreign banks made on average more bids than the rest of the sample, consistent with the existing evidence that that their liquidity positions could be in bigger troubles (Benmelech, 2012). Similar to DW borrowing, banks bidding decisions are also highly skewed: while the median bank submitted a total of eight bids, the most aggressive bank – Mitsubishi UFJ

[23]For details, see https://www.bloomberg.com/news/articles/2011-03-31/federal-reserve-releases-discount-window-loan-records-under-court-order. In May 2008, Bloomberg News' reporter Mark Pittman filed a FOIA request with the Fed, requesting data about details of discount window lending and collateral. Unsurprisingly, it was stonewalled by the central bank. In Nov 2008, Bloomberg LP's Bloomberg News filed a lawsuit challenging the Fed, with Fox News Network later filing a similar lawsuits. Other news organization also showed support by filing legal briefs. In March 2011, the U.S. Supreme Court ruled that the Fed to release the discount loans in response to the lawsuits. Later that month, the Fed released the data, in the form of 894 PDF files with more than 29,000 pages on two CD-ROMS. Bloomberg News later published an exhaustive analysis that included the detailed data.

Table 1: Summary Statistics of Bloomberg

	N	Mean	Max	Min	SD	10^{th}	50^{th}	90^{th}
No. of Borrowers	407							
Banks	313							
Diversified Financial Services	24							
Insurance Companies	12							
Savings and Loans	30							
Market Cap on Aug 1, 2007 (MM)		28525	399089	11	49876.8	107	7331	81813
Foreign Banks	92							
DW-only banks	18							
TAF-only banks	86							
borrow both	260							
Total DW events		12	242	0	28.7	0	2	35
Total TAF events		5	28	0	5.1	0	3	13
Total DW amount (MM)		1529	190155	0	10393.8	0	20	1809
Total TAF amount (MM)		3174	100167	0	10727.5	0	58	7250
Number of days in debt to Fed		323	814	28	196.8	85	306	606

Financial Group – submitted a total of 95 bids through its New York Branch.

Table 2: Summary Statistics of TAF

	N	Mean	Max	Min	SD	10^{th}	50^{th}	90^{th}
No. of Banks	434							
No. of G-SIBs	22							
No. of Foreign Banks	82							
All Banks: no. of bids		13	95	1	13.9	1	8	35
G-SIBs		27	95	1	24.5	1	25	57
Foreign Banks		25	95	1	18.5	4	23	50
All: high-haircut collaterals		0.19	1.00	0.00	0.3	0.00	0.00	0.79
All: low-rating collaterals		0.19	1.00	0.00	0.3	0.00	0.00	0.72

Figure 3 and 4 offer a snapshot of in 60 TAF auctions. All the auctions before Lehman Brothers filed for bankruptcy (23 out of 60) were over-subscribed, meaning that the total bidding amount exceeding the total amount of liquidity provided in the auction. After that, the Federal Reserve increased the offering amount from $20bn to $150bn and subsequently, all the auctions were under-subscribed. Banks' bidding amount also vary across all the 60 auctions. In all the 60 auctions, there were banks that submitted the minimum bidding amount ($5 million) whereas across most auctions until May 4, 2009, some bank submitted the maximum bidding amount (15% of the total auction amount). This observation is consistent with the general impression that the stress in interbank lending market gets much more relieved after mid 2009. The stress in the interbank market is also revealed prominently in their bidding rates. Figure 4 shows the highest bidding rate surged up to 10% when Lehman Brother declared bankruptcy and subsequently gets releaved.

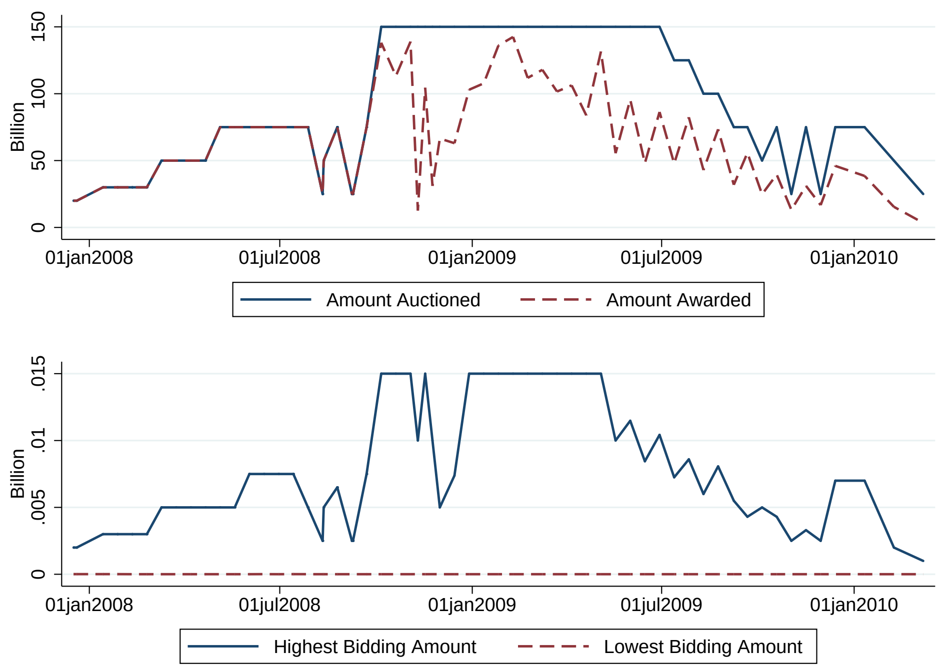


Figure 3: TAF Auction Amount

Finally, we merged both the Bloomberg and the TAF data with each borrower's daily stock market returns as well as various proxies for banks' health (tier-1 capital and liquid assets) obtained from the Bank Regulatory database (Y-9C) at the Band Holding Companies level.

5.2 Bank Fundamentals

Were banks borrowing from the discount window fundamentally different from banks borrowing from TAF? To answer this question, we conduct the following econometric analysis

$$\frac{\mathrm{DW}_{it}}{\mathrm{DW}_{it}+\mathrm{TAF}_{it}} = a_i + b \times \mathrm{T1CAR}_{it} + c \times S_{it} + Q_t + \varepsilon_i, \tag{3}$$

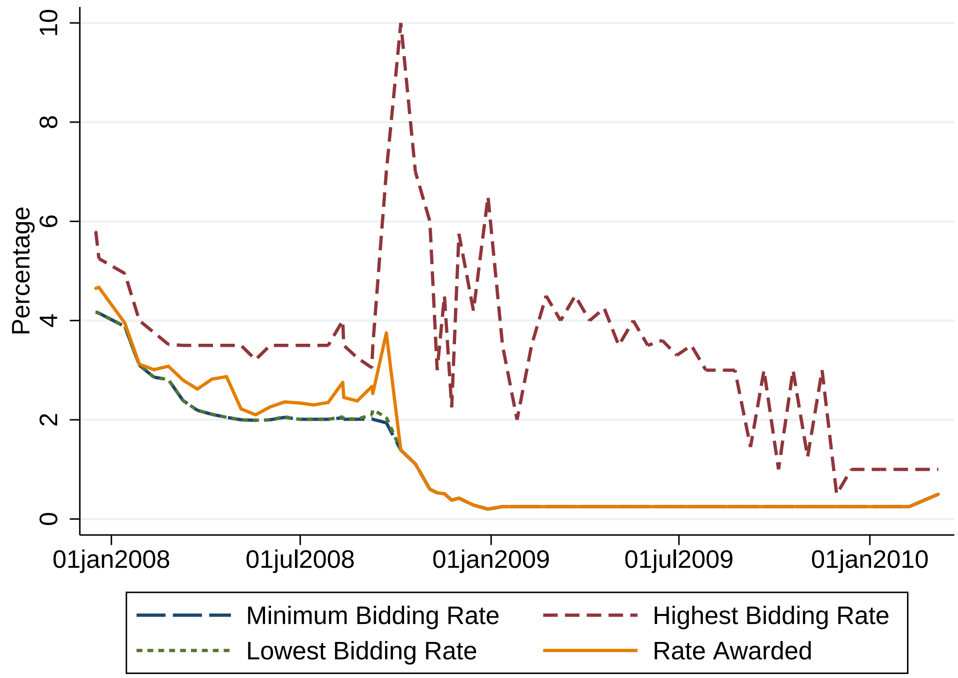


Figure 4: TAF Auction Rate

where the left-hand-side variable, $\frac{\mathrm{DW}_{it}}{\mathrm{DW}_{it}+\mathrm{TAF}_{it}}$, is the share of bank i's total discount window borrowing in quarter t, divided by the sum of total discount window and TAF borrowing within the same period. On the right-hand-side, we control for banks' size S_{it} and quarter fixed effects Q_t. Our main interested variable is tier-1 capital ratio, defined as a bank's tier 1 capital divided by its total risk weighted assets. Tier-1 capital ratio is the most commonly-used measurement for bank's financial health. Our identification relies on the assumption that a bank's tier-1 capital ratio is closely related to its fundamental health. Moreover, consistent with the model that θ captures the information on banks' health that is unobservable to the public, we use the *contemporaneous* measurement of tier-1 capital ratio, as opposed to one that lagged by one period.

Table 3 report the baseline regression results. Column (1) - (4) differ in the type of fixed-effects controlled in the regression. On average, a 1% increase in an average bank's tier-1 capital ratio reduces the total amount of DW borrowing by 3%. The within-quarter effect gets mitigated to 2.2% (Column (2)), whereas the within-bank effect is reduced to 2.1% (Column (3)). Once both fixed effects are taken into account, the effect is effectively unidentifiable, potentially due to the fact that financial strength is only partially captured by the a bank's tier-1 capital ratio. Across all columns, it is clear that larger banks borrower more from TAF than from DW, consistent with the widely held perception that they were more concerned with the discount window stigma.

Table 4 further explores the effect in subsamples. We separate all banks into two groups,

Table 3: Borrowing Decision and Tier-1 Capital Ratio

	(1) OLS	(2) Time FE	(3) Bank FE	(4) Both FE
T1CAR	-2.976***	-2.233***	-2.105*	-0.250
	(0.771)	(0.793)	(1.170)	(1.292)
log (total asset)	-0.051***	-0.059***	-0.747***	-0.578***
	(0.009)	(0.009)	(0.194)	(0.193)
Constant	1.618***	2.274***	10.819***	8.953***
	(0.174)	(0.226)	(2.604)	(2.590)
N	561	561	561	561
R^2	0.070	0.137	0.558	0.599

depending on their market capitalization on August 1, 2007. Column (1) and (2) show that the effect is stronger and more significant among small banks. Comparing the results in Column (3) and (4), we can see the effect is also stronger and more significant in the period before the second period of 2009, arguably due to the fact that concerns for stigma is more prominent during that period.

Table 4: Borrowing Decision in Subsamples

	(1) Large	(2) Small	(3) Pre 2009Q2	(4) Post 2009Q2
T1CAR	-1.655	-2.865**	-4.622***	-0.275
	(1.710)	(1.440)	(0.934)	(1.486)
log (total asset)	0.018	-0.037*	-0.064***	-0.011
	(0.033)	(0.023)	(0.010)	(0.023)
Constant	0.121	1.356***	1.982***	0.710*
	(0.657)	(0.397)	(0.199)	(0.376)
N	107	283	435	126
R^2	0.011	0.021	0.116	0.002

5.3 Market Responses

How did market respond to borrowings from different programs? In this subsection, we conduct an event-study analysis following each borrowing event and study how the stock price changes. Specifically, the estimation window is set as the period before the interbank market froze up: January 3, 2005 to August 1, 2007. Predicted returns are estimated using the market model. We choose the length of the event window as five days after the borrowing event, based on the assumption that one source of detection is the weekly public report of aggregate DW borrowings. Table 5 reports the five-day cumulative abnormal returns (CAR) following each borrowing event.

Table 5: CAR following Borrowing Events

	(1) DW	(2) DW/TAF	(3) TAF/DW	(4) TAF
CAR	-0.009***	-0.015**	0.004	-0.005
	(0.002)	(0.008)	(0.007)	(0.004)
N	2948	209	257	720

Column (1) shows for an average bank, its stock price declines by abot 0.9% in the subsequent five days following a DW borrowing. In contrast, Column (4) shows the CAR is only 0.5% (also not significant) following a TAF borrowing. In Column (2) and (3), we classify the DW borrowing event into two groups. The first group borrowed from the discount window within the three-day window before a TAF auction. Presumably, this group of banks were desperate for liquidity and if detected, the cost of stigma should be the highest. The second group is comprised of banks who tapped discount window shortly after the auction. According to our model, their liquidity conditions should be further stronger. Column (2) shows that the CAR is even more negative for banks who borrowed DW shortly before TAF, whereas according to Column (3), the CAR is no longer significant for banks who borrowed DW shortly after TAF. These results are consistent with our model's prediction that on average, DW borrowing carries a higher stigma cost, and such cost is even higher if a bank tapped DW shortly before a TAF were to be held. One caveat though, is that the real world is dynamic, whereas our model only spans two periods. Consequently, banks who borrowed from the discount window shortly after TAF might also be those who were newly hit with liquidity shocks. If we take this effect into account, the comparison between the second and the last group will be ambiguous.

6 Liquidity Provision

6.1 Only DW versus Only TAF

First, it is not the case that the TAF is always more effective than the DW to provide liquidity. If the facilities are used alone, it is unclear which one will provide more liquidity.

It is relatively straightforward to show that more patience, higher discount rate, more liquidity provision in the auction, less returns, and lower reserve price in the auction will make TAF borrowing more attractive, relative to DW borrowing.

Claim 1. *The increase in the participation in the TAF relative to the DW, $nF(\theta^{TAF}) - nF(\theta^{DW})$, increases in δ, r_D, and m, and decreases in r_A and R.*

6.2 Both DW and TAF

We study how the introduction of TAF affects total liquidity provision in equilibrium. We will focus on the case that $\theta_2 > \theta_A$ so that losers in TAF will not borrow from the discount window. The other case can be analyzed in a similar vein.

The following proposition shows that TAF may or may not increase total liquidity provision.

Claim 2. *In equilibrium, $\theta_A > \theta^{DW} > \theta_D$.*

The result $\theta_A > \theta^{DW}$ clearly implies that the introduction of TAF expands the set of banks that *may* receive liquidity. However, $\theta^{DW} > \theta_D$ so that the set of banks will guarantee to receive liquidity actually gets smaller. Intuitively, the chances of borrowing at low rate at TAF induce some banks that would borrow from DW to wait for the auction.

Let us illustrate the effect introduced by TAF through the comparative static analysis with respect to *m*, the total amount of liquidity auctioned in TAF. One can think of the introduction of TAF as *m* increasing from 0 to 1. We will separate between the direct effect and the indirect effect. To focus on the direct effect, let us first take k_A and k_D as given and study how an increase in *m* changes θ_D and θ_A. Intuitively, an increase in the TAF amount will lure more banks switching from the discount window to the auction. Moreover, if we treat θ_A as an implicit function of θ_D, it is easily shown that $\theta_A'(\theta_D) = \frac{\beta(\theta_D)g(\theta_D)-(1-\delta)R}{[\beta(\theta_A)-r_A]g(\theta_A)}$. If $1-\delta$ is small enough such that the effect of discounting is limited, then θ_A will also increase with *m*. Otherwise, θ_A may actually decrease with *m*. There are two effects at work here. Intuitively, an increase in the TAF auction amount offers more possibilities for banks to borrow cheap so that it should attract more bids and participation. However, since TAF is only held with a delay, the decision to wait for TAF may also reduce participation due to the cost incurred in delay.

Next, let us turn to the indirect effect. An increase in m will decrease θ_D, deteriorating the pool of banks who actually borrowed from discount window. As a result, k_D the stigma carried with discount window borrowing, goes up. By contrast, the change to stigma incurred by TAF borrowing, k_A, remains unclear. Although more banks switch from DW to TAF, an increase in m may also attract more banks who are financially healthier. The overall effect depends on the relative magnitude of both effects. Note that changes to k_D and k_A will feedback into the marginal bank θ_D and θ_A, further obfuscating the overall effects on liquidity provision.

A Numerical Example. We pick the following parameter values: $p = 0.8$, $n = 20$, $r_D = 1.3$, $R = 5$, $m = 1$, $r_A = 1$, and $\delta = 0.45$. Moreover, we assume uniform distribution so that $f(\cdot) \equiv 1$ and $k(\theta) = p(1-\theta)$. In this case, the equilibrium thresholds are $\theta_D = 0.62$ and $\theta_A = 0.68$. In other words, banks whose types are in $[0, 0.62]$ borrow from discount window directly, whereas banks whose types in $[0.62, 0.68]$ bid in the auction for the unit of liquidity. Indeed, $\theta_A > \theta^{DW}$ so that TAF expands the set of banks that may receive liquidity.

If TAF were not available, the threshold $\theta^{DW} = 0.66$, which gives rise to total expected liquidity provision of 13.2. With TAF, the total expected liquidity actually gets reduced to 12.41. The reason is $\theta^D < \theta^{DW}$ so that the set of banks that will borrow from discount window in week 1 drop significantly. After TAF, discount window only lends to 12.40 unit of liquidity, compared to 13.20 without TAF. However, the set of banks who bid in TAF is still limited and therefore the expected liquidity provided in TAF does not make up for the shortfalls in discount window liquidity.

7 Conclusion

In this paper, we investigated how the Term Auction Facility mitigates the stigma associated with borrowing from the Discount Window. We constructed an auction model with endogenous participation and showed optimal auction bidding strategies that internalized any stigma associated with the auction naturally increased participate and consequently mitigated the borrowing stigma.

We showed the following results consistent with the empirical observations. First, banks with strong financial health were reluctant to borrow from the Discount Window due to the standard adverse selection logic à la Akerlof (1970). Second, when both DW and TAF are available, the weakest banks borrowed from the DW, and relatively strong ones participated in TAF. Among those who lost in the auction, relatively weak ones moved on to borrow from the DW. Third, we show the introduction of TAF may or may not expand the set of banks who obtained liquidity. Lastly, our model suggests the stop-out rate of TAF may be higher or lower than the primary discount rate.

References

Akerlof, George A, “The Market for "Lemons": Quality Uncertainty and the Market Mechanism,” *The Quarterly Journal of Economics*, August 1970, *84* (3), 488–500.

Armantier, Olivier and John Sporn, “Auctions Implemented by the Federal Reserve Bank of New York during the Great Recession,” Staff Reports 635, Federal Reserve Bank of New York September 2013.

_ , Eric Ghysels, Asani Sarkar, and Jeffrey Shrader, “Discount Window Stigma during the 2007-2008 Financial Crisis,” *Journal of Financial Economics*, August 2015, *118* (483), 317–335.

Bagehot, Walter, *Lombard Street: A description of the money market*, London: HS King, 1873.

Benmelech, Efraim, “An Empirical Analysis of the Fed’s Term Auction Facility,” *CATO Papers on Public Policy*, August 2012, *2*, 1–42.

Bernanke, Ben S, *The Courage to Act: A Memoir of a Crisis and Its Aftermath*, WW Norton & Company, 2015.

Cassola, Nuno, Ali Hortaçsu, and Jakub Kastl, “The 2007 Subprime Market Crisis through the Lens of European Central Bank Auctions for Short-Term Funds,” *Econometrica*, July 2013, *81* (4), 1309–1345.

Che, Yeon-Koo, Chongwoo Choe, and Keeyoung Rhee, “Bailout Stigma,” February 2018. Working Paper, Department of Economics, Columbia University.

Cho, In-Koo and David M Kreps, “Signaling games and stable equilibria,” *The Quarterly Journal of Economics*, 1987, *102* (2), 179–221.

Ennis, Huberto M., “Interventions in Markets with Adverse selection: Implications for Discount Window Stigma,” October 2017. Federal Reserve Bank of Richmond Working Paper No. 17-01R.

_ and John A. Weinberg, “Over-the-counter loans, adverse selection, and stigma in the interbank market,” *Review of Economic Dynamics*, October 2013, *16* (4), 601–616.

Fuchs, William and Andrzej Skrzypacz, “Government Interventions in a Dynamic Market with Adverse Selection,” *Journal of Economic Theory*, 2015, *158*, 371–406.

Furfine, Craig, "The Reluctance to Borrow from the Fed," *Economics Letters*, August 2001, *72* (2), 209–213.

—, "Standing Facilities and Interbank Borrowing: Evidence from the Federal Reserve's New Discount Window," *International Finance*, November 2003, *6* (3), 329–347.

—, "Discount Window Borrowing: Understanding Recent Experience," *Chicago Fed Letter*, March 2005, (212).

Gauthier, Céline, Alfred Lehar, Héctor Pérez Saiz, and Moez Souissi, "Emergency Liquidity Facilities, Signalling and Funding Costs," December 2015. Bank of Canada Staff Working Paper 2015-44.

Holmström, Bengt and Jean Tirole, "Private and Public Supply of Liquidity," *Journal of Political Economy*, 1988, *106* (1), 1–40.

La'O, Jennifer, "Predatory Trading, Stigma and the Fed's Term Auction Facility," *Journal of Monetary Economics*, July 2014, *65*, 57–75.

Li, Mei, Frank Milne, and Junfeng Qiu, "The Signaling Effect and Optimal LOLR Policy," January 2016. Queen's Economics Department Working Paper No. 1353.

Lowery, Richard, "Comment on: "Predatory Trading, Stigma, and the Fed's Term Auction Facility" by Jennifer La'o," *Journal of Monetary Economics*, July 2014, *65*, 76–79.

McAndrews, James, Asani Sarkar, and Zhenyu Wang, "The Effect of the Term Auction Facility on the London Interbank Offered Rate," *Journal of Banking & Finance*, 2017, *83*, 135–152.

Moore, Eric, "Auction-Based Liquidity of Last Resort," June 2017. Working Paper.

Myerson, Roger, "Optimal Auction Design," *Mathematics of Operations Research*, 1981, *6* (1), 58–73.

Peristiani, Stavros, "The Growing Reluctance to Borrow at the Discount Window: An Empirical Investigation," *Review of Economics and Statistics*, November 1998, *80* (4), 611–620.

Philippon, Thomas and Vasiliki Skreta, "Optimal Interventions in Markets with Adverse Selection," *American Economic Review*, February 2012, *102* (1), 1–28.

Shin, Hyun Song, "Reflections on Northern Rock: The bank run that heralded the global financial crisis," *Journal of economic perspectives*, 2009, *23* (1), 101–19.

Tirole, Jean, "Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning," *American Economic Review*, February 2012, *102* (1), 29–59.

Wu, Tao, "The U.S. Money Market and the Term Auction Facility in the Financial Crisis of 2007-2009," *Review of Economics and Statistics*, May 2011, *93* (2), 617–631.

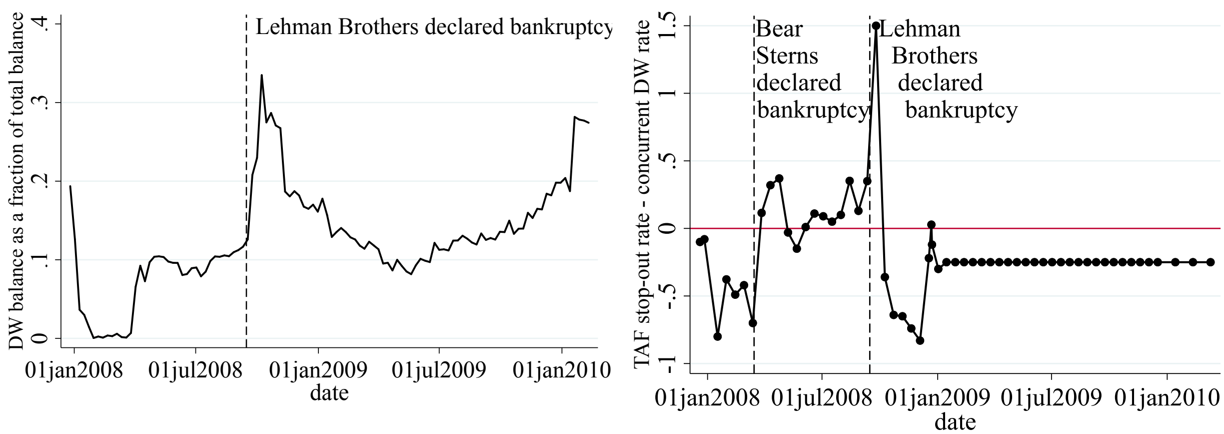
A Appendix

A.1 Additional Figures

Figure A1: Borrowing amounts and rates in DW versus TAF from 2008 to 2010

(a) DW balance as a fraction of total balance

(b) TAF stop-out rate minus DW rate



A.2 Omitted Proofs

A.2.1 Proof of Proposition 1

Proof. A type-θ bank borrows from discount window against not borrowing at all if and only if

$$u_D = (1-\theta)R - r_D - k_D \geq 0.$$

Clearly, the incentive to borrow from discount window decreases with θ. Therefore, for any given k_D, a bank borrows from discount window if and only if it type satisfies

$$\theta \leq \frac{R - r_D - k_D}{R}.$$

Bank θ^{DW} is indifferent between borrowing, implying that

$$\left(1-\theta^{DW}\right)R - r_D = k_D.$$

Specifically, θ^{DW} is determined by

$$\left(1-\theta^{DW}\right)R - r_D = \frac{\int_0^{\theta^{DW}} k(\theta)\, f(\theta)\, d\theta}{F\left(\theta^{DW}\right)}.$$

□

A.2.2 Proof of Proposition 2

Proof. In this case, all banks bid their WTP, which equal to

$$\beta(\theta) = (1-\theta)R - k_A - \delta.$$

Bank θ_{TAF} bids exactly up to reserve rate r_A:

$$\theta_{TAF} = 1 - \frac{k_A + \delta + r_A}{R}.$$

The unique solution is

$$\left(1-\theta^{TAF}\right)R - r_A - \delta = \int_0^{\theta^{TAF}} \frac{k(\theta)f(\theta)d\theta}{F(\theta^{TAF})}.$$

□

A.2.3 Proof of Lemma 1

Proof. A bank borrows from discount window during period 2 if and only if

$$\begin{aligned} u_2(\theta) = (1-\theta)R - r_D - k_D - \delta &\geq u_N = -k_N \\ \theta &\leq \theta_2(k_D, k_N) \equiv 1 - \frac{r_D + k_D - k_N + \delta}{R}. \end{aligned}$$

□

A.2.4 Proof of Lemma 2

Proof. In the auction, the winning bank pays the highest loser's bid. Therefore, its own bid does not affect its equilibrium payments, only its chances of winning the auction. Therefore, it is its dominant strategy to bid its own willingness to pay.

Bank θ's willingness to pay satisfies

$$(1-\theta)R - \beta(\theta) - k_A - \delta = \max\{(1-\theta)R - r_D - k_D - \delta, -k_N\}.$$

If $(1-\theta)R - r_D - k_D - \delta \geq -k_N$ so that the losing bank will go to the discount window, then

$$\beta(\theta) = \beta^D(\theta) = r_D + (k_D - k_A).$$

Otherwise,

$$\beta(\theta) = \beta^N(\theta) = (1-\theta)R - (k_A - k_N) - \delta.$$

□

A.2.5 Proof of Proposition 3

Proof. Clearly,

$$u_1(\theta) = (1-\theta)R - r_D - k_D.$$

Let $\tau \in [0,1]$ be the highest losing bank and $H(\tau)$ be its distribution. Let us first consider $u_A(\theta)$ for $\theta < \theta_2$. If $\tau < \theta_2$, bank θ's payoff from winning the auction is $(1-\theta)R - \beta^D(\theta) - k_A - \delta$, which simplifies to $(1-\theta)R - r_D - k_D - \delta$. If it loses, it turns to discount window again and receives the same payoff $(1-\theta)R - r_D - k_D - \delta$ as well. If $\tau \geq \theta_2$, a bank $\theta < \theta_2$ wins the auction for sure and receives payoff $(1-\theta)R - \beta^N(\tau) - k_A - \delta$, which simplifies to $(1-\theta)R - (1-\tau)R - k_N$. Therefore,

$$u_A(\theta) = [(1-\theta)R - r_D - k_D - \delta]H(\theta_2) + \int_{\theta_2}^{1} [(1-\theta)R - (1-\tau)R - k_N]\, dH(\tau) \quad \text{if } \theta < \theta_2.$$

Next, we consider $u_A(\theta)$ for $\theta > \theta_2$. In this case, a bank θ receives $(1-\theta)R - (1-\tau)R - k_N$ if it wins in the auction ($\tau > \theta$). If it loses, it receives $-k_N$. Therefore,

$$u_A(\theta) = \int_0^{\theta} (-k_N)\, dH(\tau) + \int_{\theta}^{1} [(1-\theta)R - (1-\tau)R - k_N]\, dH(\tau) \quad \text{if } \theta \geq \theta_2.$$

Taking the difference, we have

$$u_1(\theta) - u_A(\theta) = \begin{cases} \delta + \int_{\theta_2}^{1} [(\theta_2 - \tau)R]\, dH(\tau) & \text{if } \theta < \theta_2 \\ [(\theta_2 - \theta)R + \delta] + \int_{\theta}^{1} (\theta - \tau)R dH(\tau) & \text{if } \theta \geq \theta_2. \end{cases}$$

Clearly, $u_1(\theta) - u_A(\theta)$ is continuous and stays at a positive constant when $\theta < \theta_2$. When $\theta > \theta_2$, it is easily checked that

$$\frac{d(u_1(\theta) - u_A(\theta))}{d\theta} = -H(\theta)R < 0.$$

□

A.2.6 Proof of Theorem 1

Proof. The three equilibrium thresholds $\{\theta_D, \theta_2, \theta_A\}$ are determined by

$$
\begin{aligned}
k_A + \int_{\theta_D}^{\theta_A} \beta(\tau) g(\tau) d\tau + [1 - G(\theta_A)] r_A &= r_D + k_D + (1-\delta)(1-\theta_D) R \quad (4)\\
\delta(1-\theta_2) R - r_D - k_D &= 0 \quad (5)\\
\delta(1-\theta_A) R - k_A &= r_A \quad (6)
\end{aligned}
$$

Let $h_m^n(x) \equiv \binom{n}{m} x^m (1-x)^{n-m}$. Define three correspondences:

$$\phi_1(\theta_1, \theta_2, \theta_A) = \left\{ \theta : u_1(\theta|\theta_1, \theta_2, \theta_A) - \max\{u_A(\theta|\theta_1, \theta_2, \theta_A), u_N(\theta|\theta_1, \theta_2, \theta_A)\} \geq 0 \right\} \cup \{0\},$$

$$\phi_2(\theta_1, \theta_2, \theta_A) = \left\{ \theta : u_2(\theta|\theta_1, \theta_2, \theta_A) - u_N(\theta|\theta_1, \theta_2, \theta_A) \geq 0 \right\} \cup \{0\},$$

and

$$\phi_A(\theta_1, \theta_2, \theta_A) = \left\{ \theta : u_A(\theta|\theta_1, \theta_2, \theta_A) - u_N(\theta|\theta_1, \theta_2, \theta_A) \geq 0 \right\} \cup \{0\},$$

where

$$u_1(\theta|\theta_1, \theta_2, \theta_A) = (1-\theta)R - r_D - k_D(\theta_1, \theta_2, \theta_A),$$

$$u_2(\theta|\theta_1, \theta_2, \theta_A) = (1-\theta)R - r_D - k_D(\theta_1, \theta_2, \theta_A) - \delta,$$

$$
u_A(\theta|\theta_1, \theta_2, \theta_A) = \\
\begin{cases}
(1-\theta)R - \int_{\theta_1}^{1} [\max(\beta(\tau), r_A) - k_A(\theta_1, \theta_2, \theta_A)]\, dh_m^{n-1}[F(\tau) - F(\theta_1)] - \delta & 0 \leq \theta \leq \theta_1 \\
\int_{\theta}^{1} [(1-\theta)R - \max(\beta(\tau), r_A) - k_A(\theta_1, \theta_2, \theta_A)]\, dh_m^{n-1}[F(\tau) - F(\theta_1)] - \delta & \\
+ \int_{\theta_1}^{\theta} [-k_N(\theta_1, \theta_2, \theta_A)]\, dh_m^{n-1}[F(\tau) - F(\theta_1)] & \theta_1 \leq \theta \leq \theta_A
\end{cases},
$$

and

$$u_N(\theta|\theta_1, \theta_2, \theta_A) = -k_N(\theta_1, \theta_2, \theta_A).$$

Economically, if it is believed that (i) $[0, \theta_1]$ is the set of banks willing to borrow from discount window 1, (ii) $[0, \theta_A]$ is the set of banks willing to bid if it has not borrowed from discount window 1, and (iii) $[0, \theta_2]$ is the set of banks willing to borrow from discount window 2 if it has not borrowed after auction, then optimally, (i) $\phi_1(\theta_1, \theta_2, \theta_A)$ is the set of banks willing to borrow from discount window 1, (ii) $\phi_A(\theta_1, \theta_2, \theta_A)$ is the set of banks willing to bid in the auction if it has not borrowed from discount window 1, and (iii) $\phi_A(\theta_1, \theta_2, \theta_A)$ is the set of banks willing to borrow from discount window 2 if it has not borrowed after auction. We have an equilibrium if the belief is consistent with the optimal action: $[0,\theta_1] = \phi_1(\theta_1,\theta_2,\theta_A)$, $[0,\theta_2] = \phi_2(\theta_1,\theta_2,\theta_A)$, and $[0,\theta_A] = \phi_A(\theta_1,\theta_2,\theta_A)$; or more simply, if $(\theta_1,\theta_2,\theta_A) \in \phi(\theta_1,\theta_2,\theta_A) \equiv (\phi_1(\theta_1,\theta_2,\theta_A), \phi_2(\theta_1,\theta_2,\theta_A), \phi_A(\theta_1,\theta_2,\theta_A))$. Hence, to prove the existence of an equilibrium, it suffices to show that the correspondence $\phi \equiv (\phi_1,\phi_2,\phi_A)$ has a fixed point.

Mathematically, each of the three correspondences is well-defined on $X \equiv [0,1]^3 \cap \{(\theta_1,\theta_2,\theta_A) : \theta_1 \leq \theta_A\}$, a non-empty, compact, and convex subset of the Euclidean space $\mathbb{R}^3$, and is upperhemi-continuous with the property that $\phi_\omega(x)$ for each $\omega \in \{1,2,A\}$ is non-empty, closed, and convex for all $x \in X$. By Kakutani's fixed point theorem, $\phi : X \to 2^X$ has a fixed point $x \in X$. $\square$