The Optimal Sequence of Prices and Auctions

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Auction Costs and Dynamic Buyer Arrival

- In theory, Myerson (1981) shows a second price auction with carefully chosen reserve price is optimal (expected profit maximizing).
- In practice,
  1. Costs: auctions are usually more costly and complicated than simple prices
     - More time needed to organize, higher display cost, more attention required from both buyers and sellers, sophistication of buyers, buyers' distaste for auctions.
  2. Dynamics: Buyers arrive over time.

This Paper

- Takes auction costs to sellers and buyers as given and looks at the optimal sequence of mechanism choices between prices and auctions when buyers arrive over time.
- Main result: For a wide range of auction costs and under various settings,
  - Prices-then-auctions mechanism sequence is optimal.
  - Any other mechanism sequence, e.g. auctions-prices, although feasible, is never optimal.
- Implication: The prices-then-auctions mechanism sequence resembles eBay's buy-it-now.

Related Literature

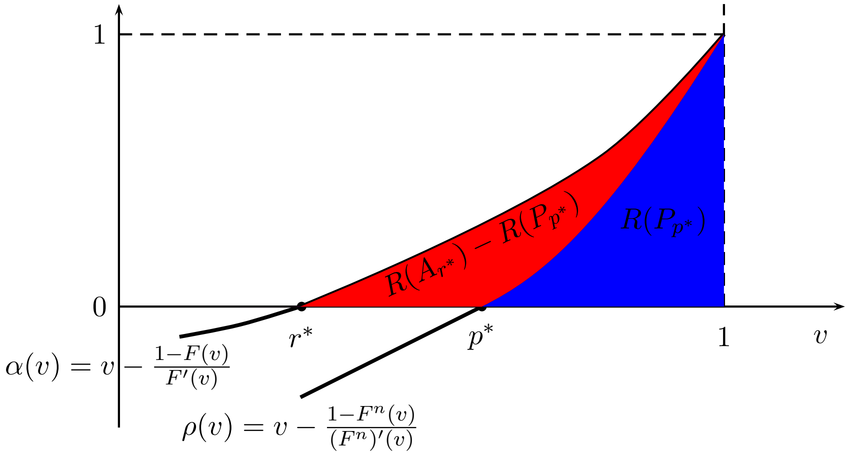
1. First to explicitly model intermingled choices between auctions and prices when buyers arrive over time
   - Wang (1993, AER), Dilme and Li (2014, REStud), Board and Skrzypacz (2016, JPE)

2. Provides a new justification of the use of buy-it-now option
   - Budish and Takeyama (2001, EL), Mathews (2004, JE), Anwar and Zheng (2015, GEB)

Basic Setup

- A monopolist of an indivisible good
- Lives for $T \in \{1, 2, \cdots, \infty\}$ periods
- Discounts each period by $\delta \in [0, 1]$
- In each period the seller chooses between
  - a reserve price auction A_r with cost c
  - a posted price P_p without cost
- In each period n buyers with independent private values $v \sim F$ are in the market
  - F satisfies monotone hazard rate $(1 - F)/f$
  - Buyers are short-lived

Static Optimal Prices and Revenues - Graph



Static Optimal Mechanism

Proposition 1

Suppose $T = 1$. Let r^* and p^* be the unique solutions to

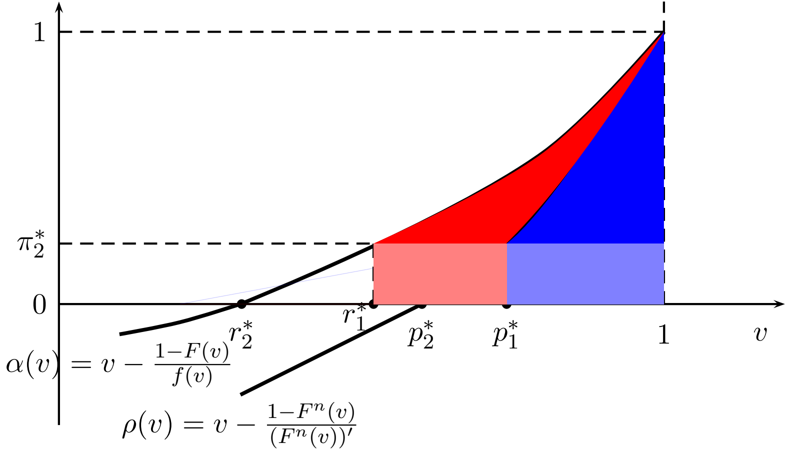
$$\alpha(r^*) \equiv r^* - \frac{1 - F(r^*)}{(F(r^*))'} = \rho(p^*) \equiv p^* - \frac{1 - F^n(p^*)}{(F^n(p^*))'} = 0.$$

and

$$c^* = R(A_{r^*}) - R(P_{p^*}) = \int_{r^*}^{1} \alpha(v) dF^n(v) - \int_{p^*}^{1} \rho(v) dF^n(v).$$

The seller's optimal mechanism is A_{r^*} if $c < c^*$, and is P_{p^*} if $c > c^*$. A cost c^* seller is indifferent between A_{r^*} and P_{p^*}.

Two-Period's Optimal Prices and Revenues - Graph



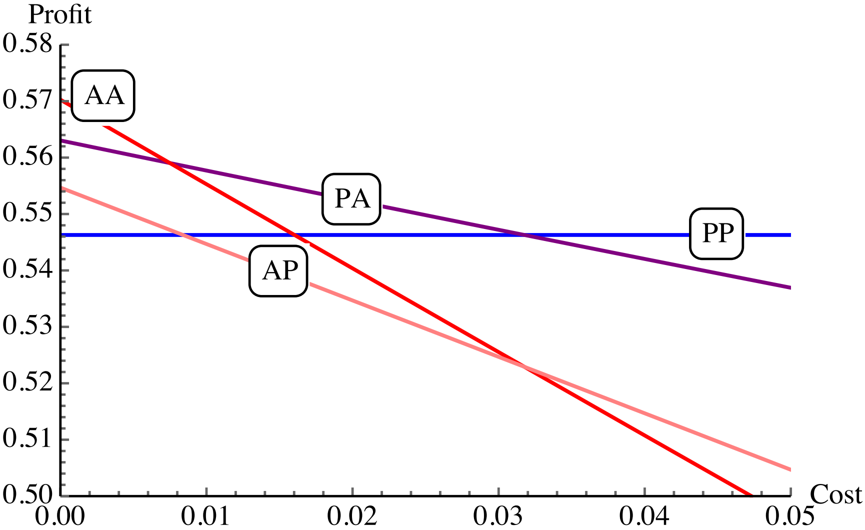
Benefit-Cost Analysis

- One benefit of an auction
  - Revenue advantage in the current period
    - smaller in earlier periods
- Two costs of an auction
  - Auction cost
    - constant
  - **Endogenous opportunity cost**: retention value of the good
    - bigger in earlier periods
    - a higher chance of selling from an auction than from a posted price because the optimal reserve price r_t^* is smaller than the optimal posted price p_t^*
- Conclusion: better off running an auction later than earlier.

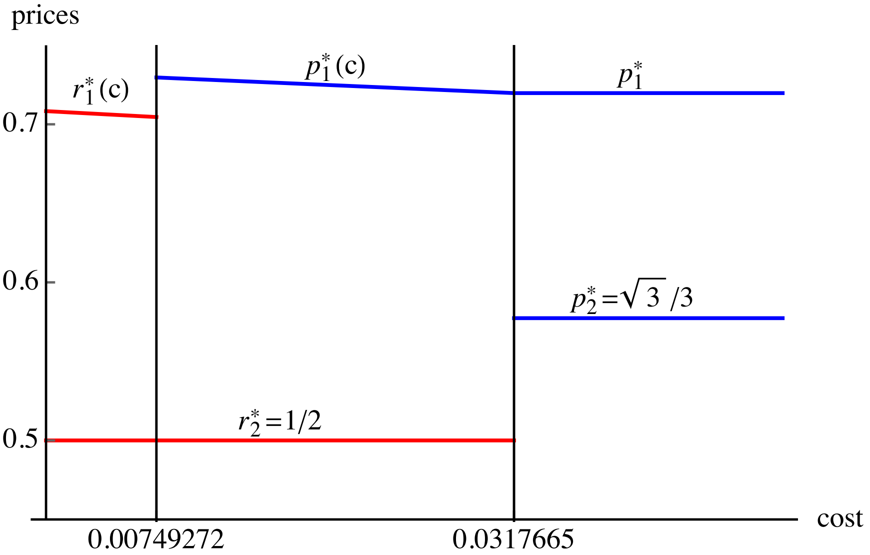
A Two-Period Example

- Two periods: $T = 2$
- No discounting: $\delta = 1$
- Two buyers in each period: $n = 2$
- Each buyer has uniform distribution: $F(v) = v$

Optimal Two-Period Mechanism Sequence



Optimal Prices



Finite-Horizon Optimal Mechanism Sequence

- Period T: cutoff cost c_T^*,

$$c_T^* = R(A_{r_T^*}) - R(P_{p_T^*}) = \int_0^1 x d[F^n(\alpha^{-1}(x)) - F^n(\rho^{-1}(x))].$$

- Period $t < T$: cutoff cost c_t^*

$$c_t^* = \int_{\delta\pi_{t+1}^*(c_t^*)}^1 [x - \delta\pi_{t+1}^*(c_t^*)] d\left[F^n(\alpha^{-1}(x)) - F^n(\rho^{-1}(x))\right].$$

Proposition 2

Suppose T is finite. Let $\alpha(r_t^*(c)) = \rho(p_t^*(c)) = \delta\pi_{t+1}^*(c)$. A cost c seller's optimal mechanism in period t is $A_{r_t^*(c)}$ if $c < c_t^*(c)$, and is $P_{p_t^*(c)}$ if $c > c_t^*(c)$.

Optimality of Prices-Then-Auctions Sequence

Proposition 3

For an intermediate level of auction cost, the seller posts prices until some period and runs auctions afterwards.

Proposition 4

(Corollary of Proposition 3) A mechanism sequence with auctions before prices is never optimal. (An Alternative Proof useful for Propositions 5 and 6)

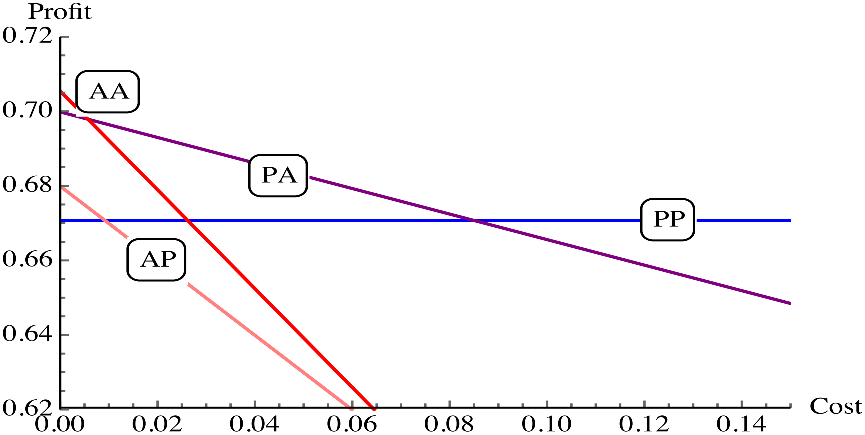
Extensions with Short-Lived Buyers

1. Seller has a stochastic sale deadline.
2. Seller becomes increasingly impatient.
3. Seller incurs decreasing auction cost.
4. Buyers arrive stochastically.
5. Buyers have outside options.
6. Buyers incur bidding costs.
7. Separate auctions and prices markets.
8. Procurement contracts.
9. Sequentially selling multiple objects.

Long-Lived Myopic Buyers

Proposition 5

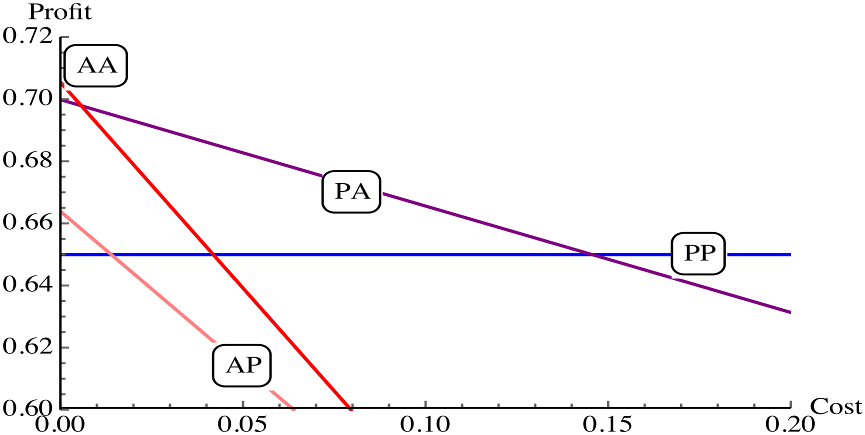
Suppose there are two periods. When buyers are long-lived and forward-looking, the auction-price sequence is never optimal. (Proof)



Long-Lived Forward-Looking Buyers

Proposition 6

Suppose there are two periods. When buyers are long-lived and forward-looking, the auction-price sequence is never optimal. (Proof)



Seller's Infinite-Horizon Problem

- The problem is stationary.
- The optimal mechanism is a constant price or auctions with constant reserve price.
- A comparative statics result: as the seller becomes more patient (δ increases), an auction's revenue advantage decreases.
  - Intuition: If infinitely patient, just post a price arbitrarily close to 1.
  - Implication: As the market becomes thicker (as eBay expands), more people will post price (Einav et al., 2013).

Summary

- A monopolist sells an item with prices and auctions in a dynamic environment with buyers arriving over time.

- Optimal finite-period mechanism sequence: prices then auctions, resembling a buy-it-now.

Thanks!

Proof of Proposition 4

- Suffices to show an auction-price combination is never optimal.
- Proof by contradiction.
- Suppose $(A_{r_1}, P_{p_2}, \mathbf{m})$ is optimal.
- If $r_1 > p_2$: It cannot dominate both $(A_{r_1}, A_{p_2}, \mathbf{m})$ and $(P_{r_1}, P_{p_2}, \mathbf{m})$.
  - If it dominates both, $R(A_{r_1}) - R(P_{r_1}) \leq c \leq R(A_{p_2}) - R(P_{p_2})$.
- If $r_1 \leq p_2$: It cannot dominate both $(P_{p_2}, P_{p_2}, \mathbf{m})$ and $(A_{r_1}, A_{r_2}, \mathbf{m})$ where $r_2 = \alpha^{-1}(\delta\pi(\mathbf{m}))$.
- QED.

Proof Sketch of Proposition 5

- Proof by contradiction.
- Suppose (A_{r_1}, P_{p_2}) is optimal.
- If $r_1 > p_2$: It cannot dominate both (A_{r_1}, A_{p_2}) and (P_{r_1}, P_{p_2}).
- If $r_1 \leq p_2$: It cannot dominate both (P_{p_2}, P_{p_2}) and (A_{r_1}, A_{r_2}) where $r_2 = \alpha^{-1}(\delta\pi(\mathbf{m}))$, as in the proof of Proposition 4.
- QED.

Proof Sketch of Proposition 6

- Proof by contradiction.
- Suppose (A_{r_1}, P_{p_2}) is optimal.
- If $r_1 > p_2$: It cannot dominate both (A_{r_1}, A_{r_2}) and (P_{p_1}, P_{p_2}) where in all three mechanisms value $\widetilde{v}$ buyer is indifferent between buying in the current period and in the second period.
- If $r_1 \leq p_2$: The proof follows the proof of Proposition 5.
- QED.

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