Digital Villages

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Abstract

A notable phenomenon in the digital economy is the emergence of *digital villages*: e-commerce platforms support and subsidize suppliers in less developed rural areas to manufacture and sell products online. We argue that, despite platforms' philanthropic claims, these actions are strategically designed to enhance profitability. By providing early subsidies to young sellers, platforms incentivize entry and reduce learning costs, later recouping these investments as sellers gain experience and increase sales. Using a dynamic model of two-sided markets, we analyze the intertemporal and cross-side pricing strategies of a monopoly platform. Our findings indicate that sellers' network externalities and learning-by-doing effects reinforce each other, motivating the platform to subsidize them. This study bridges two typically distinct areas of the economy: global online platforms and less developed rural regions.

Keywords: digital villages, two-sided markets, dynamic pricing, price discrimination, economic development

JEL Codes: D62, O12, L11, L81

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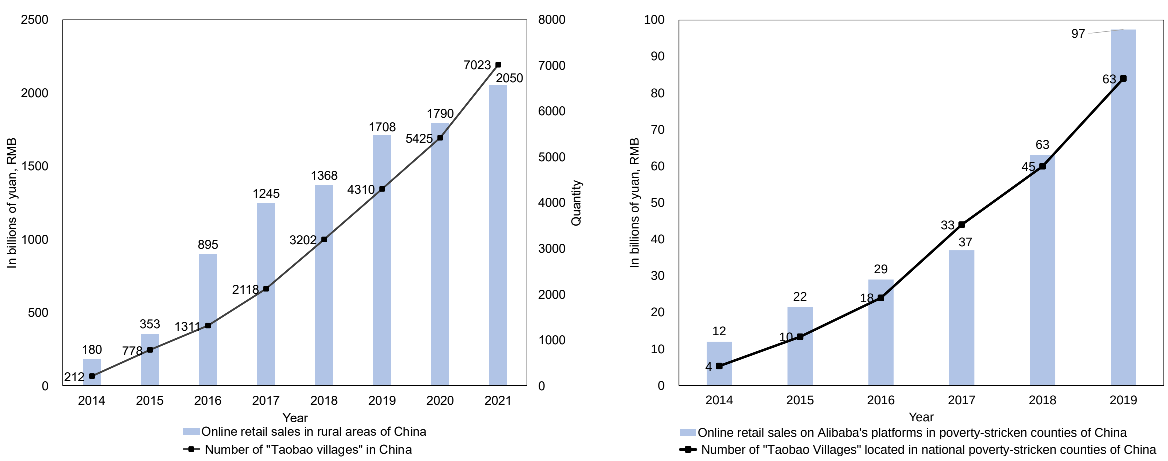
1 Introduction

Digital marketplaces have grown rapidly in recent years. E-commerce giants, such as Amazon, eBay, Mercado Libre, Alibaba, JD, and Temu, link buyers and sellers through their online trading services. A notable phenomenon is the rise of *digital villages*: rural villages with a high volume of transactions and a significant number and proportion of online stores (Luo and Niu, 2019). Defined by AliResearch (2015), a "Taobao village" on Alibaba's platforms (mainly including Taobao and Tmall) is an administrative village in which the number of active online shops exceeds 100 or the ratio of the active online shops to the local households exceeds 10%, and the annual turnover in e-commerce exceeds 10 million yuan (approximately US$1.52 million).[1] In the span of five years from 2014 to 2019, online sales in rural China increased almost ten-fold from $27.27 billion to $258.83 billion, and the number of Taobao villages increased more than twenty-fold from 212 to 4,310 (Figure 1(a)). When considering only less developed areas (LDAs)—832 poverty-stricken counties—in China, online retail sales on Alibaba's platforms increased eight-fold from $1.82 billion in 2014 to $14.76 billion in 2019, and during the same period, the number of Taobao villages increased more than fifteen-fold from 4 to 63 (Figure 1(b)).[2]



(a) All rural counties (b) Poverty-stricken counties

Figure 1: Online retail sales and the number of Taobao Villages in rural China, 2014–2019
Notes: Data sources are provided in Online Appendix A.

What role have e-commerce giants played in driving economic development in LDAs? Take

[1]Dollar to yuan nominal exchange rate used in this paper is 6.6, the average from 2014 to 2019.

[2]In 2014, the State Council Leading Group Office of Poverty Alleviation and Development identified 832 counties in China as poverty-stricken because of their extremely low income per capita. We refer to them as LDAs.

Alibaba as an example. In October 2014, Alibaba launched the "1,000 Counties and 10,000 Villages" program (also known as the rural Taobao model). This program planned to invest 10 billion yuan ($1.52 billion) over three to five years and aimed to build an e-commerce service system with 100,000 administrative villages in 1,000 counties in rural areas of China.[3] In December 2017, Alibaba initiated the Poverty Alleviation Fund, investing 10 billion yuan ($1.52 billion) over five years. This fund aimed at poverty reduction and alleviation in five target areas: e-commerce, ecology, education, health, and women.[4]

In 2018, Alibaba held a 58.2% share of China's e-commerce market, significantly surpassing other platforms.[5] Why would a platform with significant market power be incentivized to engage heavily in seemingly philanthropic public affairs? How does a profit-maximizing company like Alibaba benefit from investing in LDAs? The existing literature lacks a targeted theory to analyze these paramount questions related to two-sided markets and economic development. This paper aims to fill this void.

By incorporating the learning-by-doing of sellers in the theory of two-sided markets with heterogeneous sellers and buyers, we build a dynamic overlapping generations model to explore both intertemporal and cross-side pricing strategies of a monopoly platform (or any platform with market power). Our model proceeds as follows. The profit-maximizing monopoly platform sets membership fees (which may be negative) for buyers and sellers, and then after observing these prices, the buyers and sellers simultaneously make choices between this platform and an outside option in each period. After each period, a new generation of two-period-lived buyers and sellers is born. All agents are rational and can anticipate the responses of other agents in each period. The utility function of buyers and sellers participating in online sales on this platform is the simplified version of Weyl (2010): Buyers and sellers are heterogeneous in membership values, while they obtain

[3] According to AliResearch (2015) and World Bank and Alibaba (2018), the following three outcomes are noteworthy: (1) By the end of 2018, there were 30,000 village-level service stations and a rural service team at the village level of nearly 60,000 people (including part-time and full-time workers). This program incubated 160 regional agricultural brands; (2) By the end of 2017, Taobao University had built 11 e-commerce training bases that conducted 133 e-commerce courses to train entrepreneurs; (3) By the end of 2017, Ant Financial, a subsidiary of Alibaba Group, had provided $1.70 billion in loans to entrepreneurs in poor counties and underdeveloped areas.

[4] World Bank and Alibaba (2018) highlight three notable outcomes of this program. First, in 2019, poverty-stricken counties recorded sales revenues of $14.76 billion on Alibaba's platforms. Second, in 2018, Alibaba trained over 260,000 people, both employed and self-employed in e-commerce and cloud computing, and opened nine e-commerce training bases in impoverished counties. Third, the program trained 18,200 women and helped 10,600 women gain employment in e-commerce.

[5] In 2018, the second and third biggest e-commerce giants, JD and Pinduoduo, owned 16.3% and 5.2% of the retail e-commerce sales shares in China, respectively.

homogeneous interaction values from online trade, respectively, and all buyers and sellers obtain zero utility from offline trade.[6] More specifically, the membership values of buyers are always positive (we call them membership benefits), and the membership values of sellers are negative (we call them membership costs) when they are young. Due to learning-by-doing, sellers who participated in online sales early on have lower membership costs in the next period, while the costs of new entrants remain unchanged. This specification for sellers' membership costs is novel, and the heterogeneity of sellers' membership costs can be interpreted as the difference in online business ability.

In the baseline model, the platform can charge different fees for newly entered sellers and previously entered sellers, which is a form of third-degree price discrimination, while when sellers are young, only a unified fee for all sellers is charged.[7] Meanwhile, the platform can only charge the same fee for all buyers in each period. In equilibrium, the platform has an incentive to subsidize young sellers if the network externality each online seller makes is higher than each online buyer makes or if the growth in the business ability (i.e., learning-by-doing) of each online seller is high enough; the cross-side externality and learning-by-doing of each online seller can reinforce each other in reducing the platform's charges (or increasing its subsidies) to young sellers when the externality of each seller is high enough, and that of each buyer is not too high. The theoretical results also indicate that both the prices for young buyers and old sellers who always enter the platform are higher than the price for young sellers. Intuitively, although sellers must bear membership costs in the initial period, the relatively higher externality and the expected increase in business ability are taken into account when the platform implements cross-side and intertemporal pricing strategies. This subsidization mechanism for sellers explains the monopoly e-commerce platform's considerable investment of resources, such as training unskilled merchants, building village-level service stations, and providing loans in LDAs. Moreover, we show that the numbers of online buyers in the two periods are equal, and the numbers of online sellers in the two periods are equal, because the platform can internalize the cross-side network externalities through its dynamic pricing power.

In the extended model, we consider an environment in which third-degree price discrimination toward sellers is prohibited in two different ways. First, we restrict the platform from third-

[6] One could argue that the specification of unidimensional heterogeneity is similar to that of Armstrong (2006), but our framework allows for the outside option and does not involve the so-called "cost of distance" under the Hotelling-based framework. In this sense, our framework is closer to that of Weyl (2010).

[7] In practice, price discrimination between entrants and existing members, especially on the sellers' side, is usually implemented by the platform; see related discussions in Cabral (2019).

degree price discriminating the sellers when they are more experienced based on their previous homing choice. This does not affect the equilibrium results achieved in the benchmark model. This is because, regardless of third-degree price discrimination based on previous homing choice, the platform prefers to set a high enough price for experienced sellers such that the seller who is on the margin of joining when they are new is also on the margin of joining when they are experienced. Thus, sellers who do not receive the learning-by-doing benefit will never join the platform even when restricting third-degree price discrimination based on sellers' homing choice. We also consider the case in which the platform cannot price discriminate the sellers in any form, i.e. the platform sets a unified price. Because the platform must still commit to its price schedule ex ante, the only difference this makes is that the unified price each seller faces is the average of the two effective prices from the benchmark model. In other words, social welfare remains the same with or without price discrimination.

The rest of this paper is organized as follows. The remainder of this section reviews the related literature. Section 2 presents the baseline model, Section 3 solves the equilibrium, and Section 4 analyzes its equilibrium. Section 5 analyzes the equilibrium when price discrimination is prohibited. Section 6 provides a further discussion of our model, and Section 7 concludes. All proofs are included in the Online Appendix.

Related literature

The most important contribution of our paper is that we bridge the gap between two fields in economics—industrial organization and economic development—by incorporating learning-by-doing among merchants in LDAs into a dynamic two-sided market framework with a monopoly platform. The theory of two-sided markets is proposed in seminal works by Rochet and Tirole (2003, 2006), Caillaud and Jullien (2003), and Armstrong (2006).[8] More recent papers, such as Weyl (2010), White and Weyl (2010, 2016), Jullien and Pavan (2019), Karle et al. (2020), and Tan and Zhou (2021), further develop the literature on two-sided markets. These papers offer general frameworks for exploring conventional topics in industrial organization, such as pricing, competition, and platform entry in the presence of cross-group externalities. Existing theoretical and empirical studies cover a range of industries, such as payment systems (Bedre-Defolie and Calvano, 2013; Dolfen et al., 2019; Li et al., 2020; Rochet and Tirole, 2003; Wright, 2012), informational

[8]Previously, network externalities in the information communication technology industry were studied in pioneer papers like Katz and Shapiro (1985, 1986) and Farrell and Saloner (1985, 1986).

intermediaries via the internet (Caillaud and Jullien, 2003), video games (Hagiu, 2006; Landsman and Stremersch, 2011; Lee, 2013; Zhou, 2017; Zhu and Iansiti, 2012), media markets (Anderson and Coate, 2005; Anderson et al., 2018; Athey et al., 2013; Ferrando et al., 2004), newspapers (Argentesi and Filistrucchi, 2007; Chandra and Collard-Wexler, 2009; Fan, 2013; Seamans and Zhu, 2014), magazines (Kaiser and Wright, 2006), sport card conventions (Jin and Rysman, 2015), labor matching markets (Lee and Schwarz, 2017), and online real estate trade (Karle et al., 2020). However, incentives for a monopoly platform to promote economic development in LDAs have not been the focus of theoretical studies.

In recent years, some empirical papers have examined the development of e-commerce in rural China, for example, Luo and Niu (2019) and Couture et al. (2018, 2021). In particular, the rapid development of digital villages has attracted much academic and policy attention (Ding et al., 2018; Luo and Niu, 2019; Qi et al., 2019). For example, Fan et al. (2018) demonstrate that e-commerce can reduce spatial consumption inequality by lowering fixed market-entry costs and distance-based trade barriers; Luo and Niu (2019) highlight the role of e-commerce in fostering entrepreneurship. Moreover, several studies have explored how learning-by-doing fosters economic development in theory (Arrow, 1962; Lucas, 1988). More broadly, Nunn (2020) reviews the literature on economic development from a historical perspective. However, the formation of sellers' learning-by-doing and the role of monopoly platforms in economic development remain underexplored. In summary, the development of e-commerce in LDAs and two-sided platforms has been treated separately in the literature. Our paper links them together.

Another contribution of our paper is that it enriches the literature on dynamic pricing in two-sided markets. The pricing strategy of a monopolistic two-sided platform has been explored in pioneering studies. For example, Rochet and Tirole (2003) and Armstrong (2006) examine the price structure in the benchmark model with a monopoly platform, and Weyl (2010) develops an analytical framework centered on monopoly pricing in two-sided markets. However, these models are static and cannot be used to analyze the dynamic pricing strategy of the monopoly platform driving e-commerce development in LDAs. Although some theoretical articles have studied dynamic games with network effects, they do not explicitly model the dynamic game among three rational parties—buyers, sellers, and the monopoly platform—in a two-sided market. For example, Doganoglu (2003), Cabral (2011), Radner et al. (2014), Biglaiser and Crémer (2020) and Halaburda et al. (2020) focus solely on one-sided network effects of consumers, rather than cross-side externalities in two-sided markets. Although Chen and Tse (2008) and Cabral (2019) consider dynamic pricing in two-sided markets, the agents in Cabral (2019) only take current payoffs into account, and the numbers of buyers and sellers in Chen and Tse (2008) only depend on the anticipated market segment in the subsequent period.[9] Using a two-period model of two-sided markets, Lam (2017) analyzes the impact of switching costs on price competition between two symmetric platforms.[10] In contrast, our model explores cross-sided and intertemporal pricing strategies implemented by a monopoly platform, characterizing the distinct ability distributions of buyers and sellers and deriving closed-form price solutions in a dynamic framework.[11]

Our paper also contributes to the literature on price regulation and price discrimination in two-sided markets.[12] Price discrimination between groups of agents on the two sides is well documented in early studies, for example, Caillaud and Jullien (2003), Armstrong (2006) and Weyl (2010). However, when the two sides consist of different types of agents (e.g., buyers and sellers), the price difference between the two sides does not qualify as price discrimination.[13] A few papers have recognized price discrimination on different types of agents on each side. For example, using a model of a two-sided monopoly platform, Jeon et al. (2022) document second-degree price discrimination on two types of agents on one side and analyze the impact of price regulation on social welfare.[14] Liu and Serfes (2013) explore perfect price discrimination within each group in two-sided markets using a static model. In contrast, our dynamic model allows for the characterization of third-degree price discrimination across different groups of old sellers, with the grouping of sellers occurring endogenously when they are young.

[9]Doganoglu (2003) and Radner et al. (2014) assume that consumers are myopic. Moreover, Chen and Tse (2008) and Cabral (2019) do not explicitly characterize the cross-side network externalities between buyers and sellers as in Rochet and Tirole (2003), Armstrong (2006), and Weyl (2010).

[10]There are only two types of agents—consumers and platforms—in Lam's paper, although the consumers are distributed on the different sides. To simplify the analysis, Lam also assumes that the interaction benefits each consumer obtains from any consumer on the other side are the same no matter which side this consumer belongs to.

[11]Our research topic and the model setup are quite different from those of Lam (2017). Lam's model focuses on the effects of switching costs on the first-period price competition and social welfare in two-sided markets.

[12]Some papers have examined price discrimination in two-sided markets (Evans, 2003; Gomes and Pavan, 2016; Jeon et al., 2022; Liu and Serfes, 2013; Rysman, 2009; Wang and Wright, 2017; Zhang and Liu, 2016). Weisman and Kulick (2010), referring to the Notice of Proposed Rulemaking issued by the Federal Communications Commission in the United States, provide a detailed discussion of price discrimination and two-sided markets in the regulation circumstance of net neutrality.

[13]The price difference between different sides is called "cross-subsidization" in some papers, for example, Gomes and Pavan (2016), Cabral (2019) and Tan and Zhou (2021).

[14]Choi et al. (2015), Böhme (2016), and Lin (2020) also analyze second-degree price discrimination in two-sided markets.

2 Baseline Model

There is an infinitely lived monopoly platform and overlapping generations of buyers and sellers. The mass of each generation of buyers and sellers is constant and normalized to one, and each agent lives for two periods. Let $N^b_{1,t}$ (resp., $N^s_{1,t}$) and $N^b_{2,t+1}$ (resp., $N^s_{2,t+1}$) denote the mass of buyers (resp., sellers) of generation t who join the platform in the first and second periods of their lives (i.e., periods t and $t+1$), respectively. Let $N^b_t = N^b_{1,t} + N^b_{2,t}$ and $N^s_t = N^s_{1,t} + N^s_{2,t}$ represent the total mass of buyers and sellers present on the platform at time t. We now provide more details for each player.

2.1 The Monopoly Platform

The platform has pricing power over both buyers and sellers in each period and aims to maximize its aggregate profit. The platform charges a uniform fee P^s_1 in each seller's first period of life. In each seller's second period of life, the platform charges P^s_{21} if the seller was not affiliated with the platform in the previous period and P^s_{22} otherwise. Here, a latent assumption is that in period 2, the monopoly platform is allowed to implement third-degree price discrimination on sellers. This is easily implemented in our setting of interest: A platform offers a discount to sellers only when it enters a village. Later, we will adjust this assumption and analyze the economic implications of prohibiting price discrimination in Section 5. For buyers, the platform charges uniform fees P^b_1 and P^b_2 for each generation depending on age. We use $N^s_{22,t+1}$ and $N^s_{21,t+1}$ to denote the mass of generation-t sellers who join the platform if the seller was (resp., was not) affiliated with the platform when they were young.

In other words, when third-degree price discrimination is allowed, the monopoly platform's maximization problem for each generation t is

$$\pi = \max_{\boldsymbol{P}} \left[P^s_1 N^s_{1,t} + P^b_1 N^b_{1,t} + \frac{1}{1+r} \left(P^s_{21} N^s_{21,t+1} + P^s_{22} N^s_{22,t+1} + P^b_2 N^b_{2,t+1} \right) \right], \tag{1}$$

where $\boldsymbol{P} = \left\{ P^s_1, P^b_1, P^s_{21}, P^s_{22}, P^b_2 \right\}$ represents the list of platform prices and r is the interest rate on the platform's profit from period t to period $t+1$. Of course, the number of buyers and sellers affiliated with the platform will be endogenously determined by the list of prices. For the next two sections describing the sellers and buyers, we will omit subscripts t on newly introduced parameters for tractability. This is without loss given our solution concept, which we will discuss in Section 2.4.

2.2 Sellers

The target of each seller is to maximize the aggregate discounted utility of selling products in their two periods of life. For each seller in each period, there are two choices: selling online and offline. Importantly, for each generation t, sellers initially differ in their business ability $B_{i,1}^s$, which obeys a uniform distribution on the interval $[-1, 0]$. If they engage with the platform when they are young, they gain a positive increment $c \in (0, 1)$ in business ability in the second period due to learning-by-doing; otherwise, their business ability remains unchanged across the two periods.

Intuitively, poor sellers face a relatively higher startup cost due to their lower degree of education and the lower internet penetration in rural areas compared to rich sellers.[15] With the deepening of the digital economy in rural regions, poor merchants who are affiliated with the platform when they are young obtain the learning-by-doing benefit that is available in the next period. In this scenario, the law of motion of business ability is

$$B_{i,2}^s = B_{i,1}^s + c, \tag{2}$$

where $B_{i,2}^s$ represents the seller i's business ability when they are old. However, if the merchant i chooses not to sell products on the digital platform in period 1, the law of motion of the business ability becomes

$$B_{i,2}^s = B_{i,1}^s, \tag{3}$$

which means that there is no enhancement in the business ability of the seller i. As documented in Spence (1981), the hypothesis of the learning curve (or learning-by-doing) implies that the unit cost of making a product of a firm decreases with the accumulation of experience. Stokey (1988) studies the role of learning by doing in driving economic growth. Minniti and Bygrave (2001) also argue that the knowledge of entrepreneurs from past experiences determines the sequence of their choices. In contrast, all old (heterogeneous) sellers obtain the same level of cost reduction c from period 1 to period 2 in our model. This can be seen as a simplified version of the learning-by-doing setup in the literature.

For each generation t, seller i's utilities from selling on the e-commerce platform when they

[15] This comparison implies that when a poor seller and a rich seller face the same entrepreneurship program, it is more difficult for the poor one to accomplish it.

are young and old are expressed as

$$U_{i,1}^s = B_{i,1}^s + \alpha N_t^b - P_1^s, \tag{4}$$

$$U_{i,2}^s = \underbrace{B_{i,1}^s + d_1^i c}_{\equiv B_{i,2}^s} + \alpha N_t^b - \underbrace{\left[\left(1-d_1^i\right) P_{21}^s + d_1^i P_{22}^s\right]}_{\equiv P_{i,2}^s}, \tag{5}$$

where d_1^i is a binary variable that indicates whether the seller i sells online when they are young ($d_1^i = 1$ if yes, $d_1^i = 0$ if no); $\alpha \in (0,1)$ denotes the interaction benefit from each buyer on the other side; $\underline{U}^s$ is the utility of offline selling.

2.3 Buyers

Each generation of buyers is characterized as a unit mass of completely heterogeneous buyers in the economy. Each buyer faces the choice between buying goods on the online platform and from offline stores. Meanwhile, buyers of generation t are distinguished by their ability to conduct business online. Specifically, B_j^b is the level of business ability of the buyer j, which follows a uniform distribution on the interval $[0,1]$. There is no enhancement of buyers' online business abilities as there is for sellers; thus, the distribution of buyers does not change over time. We assume that the utility of buying offline is a constant value $\underline{U}^b$. Thus, a buyer j's utility from buying online when they are young and old is formed as

$$U_{j,1}^b = B_j^b + \beta N_t^s - P_1^b, \tag{6}$$

$$U_{j,2}^b = B_j^b + \beta N_t^s - P_2^b, \tag{7}$$

where $\beta \in (0,1)$ denotes the buyer's interaction benefit from each seller.

2.4 Timing and Solution Concept

Given the fees charged by the monopoly platform, each living buyer and seller chooses to join it or take the outside option (e.g., offline) in each period. All agents are rational and possess complete information. Therefore, at the outset of the game, the platform can establish its list of pricing, enabling all buyers and sellers to make participation decisions for each period.

Throughout the rest of the paper, we assume a steady-state exists and solve for the equilibrium. (We do not solve for the transition to the stationary equilibrium.) That is, we solve for the unique Nash equilibrium such that for all (sufficiently high) t, N_t^b and N_t^s are equal which we will now conveniently denote N^b and N^s. Note that this implies that $N_{1,t}^b$, $N_{2,t+1}^b$, $N_{1,t}^s$, and $N_{2,t+1}^s$ are the same across generations in the steady-state equilibrium, because the platform pricing does not depend on t.. Moreover, as each seller and buyer's initial ability in every generation are identically and independently distributed, we can solve the Nash Equilibrium without denoting specific generations. As such, we will now refer to periods 1 and 2 synonymously with the time when an agent is young and old, respectively.

3 Equilibrium Solution

We first characterize the possible homing choices of sellers and buyers to solve for the unique steady-state equilibrium.

3.1 Seller's Homing Choices

Consider a single generation of generation-t sellers and their optimal homing choice in both periods of life (periods 1 and 2). In general, given the prices implemented by the monopoly platform, each seller's homing choices in the two periods are concluded as a two-period game as in Figure 2. $\widehat{U}^s_{i,2}$

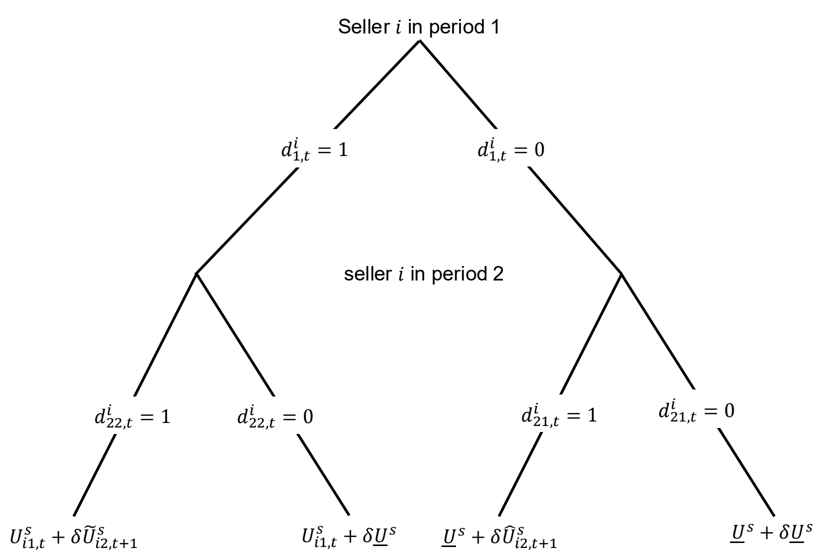


Figure 2: Two-period Homing-choice Game of Each Seller

denotes seller i's utility in period 2 when the seller did not choose to join the platform in period 1, and $\widetilde{U}^s_{i,2}$ denotes seller i's utility in period 2 under the condition that this seller had chosen the platform in period 1. Specifically, according to equation (5), $\widehat{U}^s_{i,2}$ and $\widetilde{U}^s_{i,2}$ can be expressed as

$\hat{U}^s_{i,2} = B^s_{i,1} + \alpha N^b_t - P^s_{21}$ and $\widetilde{U}^s_{i,2} = B^s_{i,1} + c + \alpha N^b_t - P^s_{22}$, respectively. Then, seller i's payoff-maximizing problem can be formulated as

$$\max_{d^i_1 \in \{0,1\}} \left\{ (1-d^i_1) \left[\underline{U}^s + \delta \max_{d^i_{21} \in \{0,1\}} \left\{ (1-d^i_{21})\underline{U}^s + d^i_{21}\widehat{U}^s_{i,2} \right\} \right] \right.$$
$$\left. + d^i_1 \left[U^s_{i,1} + \delta \max_{d^i_{22} \in \{0,1\}} \left\{ (1-d^i_{22})\underline{U}^s + d^i_{22}\widetilde{U}^s_{i,2} \right\} \right] \right\},$$

where d^i_{21} is a binary variable indicating whether seller i, who did not sell on the platform in period 1 (i.e., $d^i_1 = 0$), chooses to sell on the platform in period 2 ($d^i_{21} = 1$ if the seller does, and $d^i_{21} = 0$ otherwise); d^i_{22} is a binary variable representing whether seller i, who did sell on the platform in period 1 (i.e., $d^i_1 = 1$), continues to sell on the platform in period 2 ($d^i_{22} = 1$ if the seller does, and $d^i_{22} = 0$ otherwise); δ denotes the discount factor. For simplicity, we assume that the utility of both buyers and sellers is intertemporally additive with a zero discount rate (i.e., $\delta = 1$).

Next, we describe the homing choice of sellers when third-degree price discrimination toward sellers is allowed in period 2. Sellers are completely heterogeneous because of the difference in their ability to run an online business. The seller's homing choice in the two periods can be characterized as shown in Figure 3. Although the figure only presents a specific scenario of sellers' homing choice, the logic of their best responses can be seen. Here, the seller k is the critical one in period 1, and the

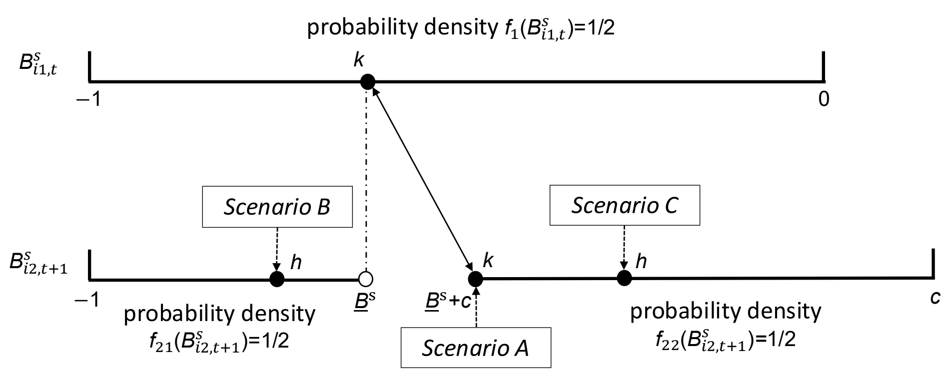


Figure 3: Distribution of Completely Heterogeneous Sellers in Two Periods (for a given generation)

seller h (and the seller $\widetilde{h}$) is (are) at the critical point(s) in period 2. For the sake of analysis, let $\underline{B}^s$ denote the business ability of seller k in period 1. Moreover, $f_1(B^s_{i,1})$ is the probability density of $B^s_{i,1}$ in period 1, and $f_{2,1}(B^s_{i,2})$ and $f_{2,2}(B^s_{i,2})$ denote the probability densities of $B_{i,2}$ on the intervals $[-1, \underline{B}^s)$ and $[\underline{B}^s + c, c]$ in period 2, respectively. All these probability densities equal 1.

Theoretically, given the prices proposed by the monopoly platform, there are four scenarios for the critical seller(s) in the two periods as follows.

Scenario A (the sellers on the platform are the same in both periods). The seller h and the seller k are the same, which implies that three conditions, $U^s_{k,1} = \underline{U}^s$, $\widetilde{U}^s_{k,2} = \underline{U}^s$, and $U^s_{k,1} + \widetilde{U}^s_{k,2} = \underline{U}^s + \widehat{U}^s_{k,2}$, are satisfied. In period 1, given price P^s_1, the critical condition $U^s_{k,1} = \underline{U}^s$ is satisfied. The reason is that in the second (last) period, the platform will charge a high enough fee such that the critical seller's utility is indifferent between online and offline sales. From this, we can obtain the number of sellers choosing the platform in period 1 as

$$\begin{aligned} N^s_{1,t}(P^s_1, N^b) &= \Pr\left(B^s_{i,1} \geq \underline{B}^s\right) \\ &= \int_{\underline{U}^s + P^s_1 - \alpha N^b}^{0} f_1(B^s_{i,1}) dB^s_{i,1} \\ &= \alpha N^b - P^s_1 - \underline{U}^s. \end{aligned} \tag{8}$$

From the last two conditions, we have that the number of sellers affiliated with the platform in period 2 is

$$N^s_{2,t+1} = \alpha N^b + c - P^s_{22} - \underline{U}^s = N^s_{1,t} \tag{9}$$

only if the following two conditions are satisfied:

$$P^s_{22} = -N^s_{1,t} + c + \alpha N^b - \underline{U}^s = c + P^s_1, \tag{10}$$

$$P^s_{21} = P^s_{22} - c = P^s_1. \tag{11}$$

Scenario B (some sellers enter the platform in period 2). There is only one critical seller h in period 2, and this seller has weaker business ability compared with seller k, as shown in Figure 3; that is, the seller h's business ability is on the interval $[-1, \underline{B}^s)$. In this scenario, four conditions, $\widetilde{U}^s_{k,2} = \underline{U}^s$, $\widehat{U}^s_{h,2} = \underline{U}^s$, $U^s_{k,1} + \widetilde{U}^s_{k,2} = \underline{U}^s + \widehat{U}^s_{k,2}$ and $\widehat{U}^s_{h,2} < \widehat{U}^s_{k,2}$, should be satisfied. From the first condition, we can indirectly obtain the number of sellers on the platform in period 1:

$$\begin{aligned} N^s_{1,t}(P^s_{22}, N^b) &= \Pr\left(B^s_{i,1} \geq \underline{B}^s\right) \\ &= \int_{\underline{U}^s + P^s_{22} - \alpha N^b - c}^{0} f_1(B^s_{i,1}) dB^s_{i,1} \\ &= \alpha N^b + c - P^s_{22} - \underline{U}^s. \end{aligned} \tag{12}$$

From the second condition, we can derive the number of sellers on the platform in period 2:

$$\begin{aligned} N^s_{2,t+1}(P^s_{21}, N^b) &= \Pr\left(B^s_{i,2} \geq B^s_{h,2}\right) \\ &= \int_{\underline{U}^s + P^s_{21} - \alpha N^b}^{\underline{B}^s} f_{21}(B^s_{i,2}) dB^s_{i,2} + \int_{\underline{B}^s + c}^{c} f_{22}(B^s_{i,2}) dB^s_{i,2} \\ &= \alpha N^b - P^s_{21} - \underline{U}^s. \end{aligned} \tag{13}$$

Moreover, the first and third conditions imply that $U^s_{k,1} = \widehat{U}^s_{k,2}$; that is,

$$P^s_1 = P^s_{21}. \tag{14}$$

Scenario C (some sellers leave the platform in period 2). There is only one critical seller $\widetilde{h}$ in period 2, and this seller has stronger business ability compared to seller k. The seller $\widetilde{h}$'s business ability is on the interval $(\underline{B}^s + c, c]$. In this scenario, four conditions, $U^s_{k,1} = \underline{U}^s$, $\widetilde{U}^s_{\widetilde{h},2} = \underline{U}^s$, $\widetilde{U}^s_{k,2} < \widetilde{U}^s_{\widetilde{h},2}$ and $\widehat{U}^s_{k,2} \leq \underline{U}^s$, are satisfied. Then, from the first condition, we have the same expression for the number of sellers on the platform in period 1 as in equation (8). Based on the second condition, we obtain the number of sellers on the platform in period 2 as

$$\begin{aligned} N^s_{2,t+1}(P^s_{22}, N^b) &= \Pr\left(B^s_{i,2} \geq B^s_{\widetilde{h},2}\right) \\ &= \int_{\underline{U}^s + P^s_{22} - \alpha N^b}^{c} f_{22}(B^s_{i,2}) dB^s_{i,2} \\ &= \alpha N^b + c - P^s_{22} - \underline{U}^s. \end{aligned} \tag{15}$$

When the last condition binds, we have

$$\begin{aligned} P^s_{21} &= \underline{B}^s + \alpha N^b - \underline{U}^s \\ &= P^s_{22} - c. \end{aligned} \tag{16}$$

Scenario D (some sellers leave the platform, but some sellers enter the platform in period 2). There are two critical sellers, h and $\widetilde{h}$, in period 2. Compared to the critical seller k, the seller h has weaker business ability and the seller $\widetilde{h}$ has stronger business ability. In this scenario, the following five conditions, $\widehat{U}^s_{h,2} = \underline{U}^s$, $\widetilde{U}^s_{\widetilde{h},2} = \underline{U}^s$, $\underline{U}^s + \widehat{U}^s_{k,2} = U^s_{k,1} + \underline{U}^s$, $\widehat{U}^s_{h,2} < \widehat{U}^s_{k,2}$, and $\widetilde{U}^s_{k,2} < \widetilde{U}^s_{\widetilde{h},2}$, are satisfied. From the first condition, we have the number of sellers who have higher business ability compared with seller h:

$$\widetilde{N}^s_{2,t} = \alpha N^b - P^s_{21} - \underline{U}^s. \tag{17}$$

Then, we can derive the number of sellers who are only affiliated with the platform in period 2 as

$$N^s_{21,t+1} = \widetilde{N}^s_{2,t} - N^s_{1,t} = \alpha N^b - P^s_{21} - \underline{U}^s - N^s_{1,t}. \tag{18}$$

From the second condition, we obtain the number of sellers who have higher business ability than seller $\widetilde{h}$

$$N^s_{22,t+1} = \alpha N^b + c - P^s_{22} - \underline{U}^s. \tag{19}$$

We then obtain the number of sellers affiliated with the platform in period 2 from equations (18) and (19); that is,

$$N^s_{2,t+1} = N^s_{21,t+1} + N^s_{22,t+1} = 2\alpha N^b + c - P^s_{21} - P^s_{22} - 2\underline{U}^s - N^s_{1,t}. \tag{20}$$

The third condition implies that

$$P^s_{21} = P^s_1. \tag{21}$$

3.2 Buyers' Homing Choices

Now, we consider generation-t buyers and their optimal homing choice when they are young and old (periods 1 and 2). Theoretically, there are three scenarios for buyers' choices when the aggregate utility of two periods is not less than $2\underline{U}^b$. First, the utility is greater than $\underline{U}^b$ in period 1 but less than $\underline{U}^b$ in period 2. This scenario cannot occur because buyers will anticipate and choose offline buying in period 2. Second, the buyer bears a utility that is less than $\underline{U}^b$ in period 1 and then obtains a higher utility in period 2. However, if the buyer deviates from this strategy by choosing offline buying in period 1, he will achieve a higher utility $\underline{U}^b$ in that period, and the utility he obtains from the platform in period 2 will not be affected.[16] Third, each buyer will choose a level of utility that is not less than $\underline{U}^b$ in each period. This scenario is the best response for each buyer.

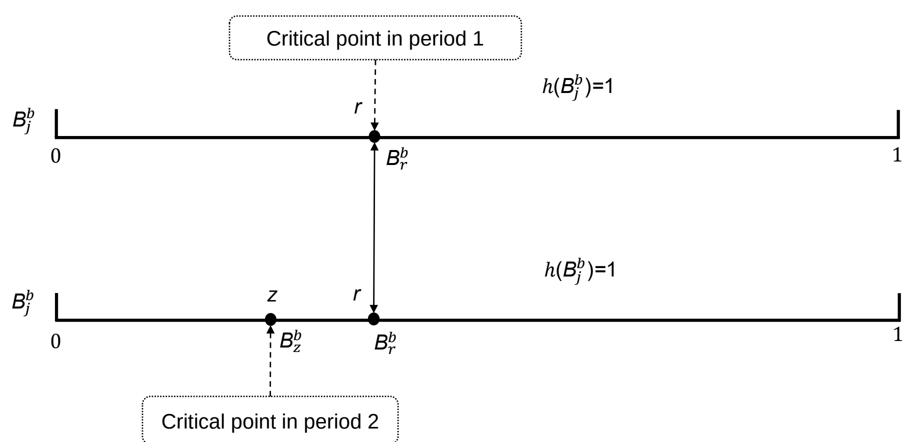


Figure 4: Distribution of Completely Heterogeneous Buyers in Two Periods (for a given generation)

The homing choice of buyers is described in Figure 4. Buyer r is the critical buyer in period

[16]As measure zero of the economy, this buyer's choice does not affect the platform's price or other buyers' choices. This also implies that buyers cannot form a collective action to negotiate with the monopoly platform.

1, and buyer z is that in period 2, which gives us

$$B_r^b = \underline{U}^b + P_1^b - \beta N^s, \tag{22}$$

$$B_z^b = \underline{U}^b + P_2^b - \beta N^s. \tag{23}$$

In each period, if a buyer has greater business ability than the critical buyer, he will affiliate with the online platform. Thus, we have the numbers of buyers affiliated with the online platform in the two periods as

$$N_{1,t}^b(P_1^b, N^s) = \Pr(B_j^b \geq B_r^b) = \int_{\underline{U}^b + P_1^b - \beta N^s}^{1} h(B_j^b) dB_j^b = 1 + \beta N^s - \underline{U}^b - P_1^b, \tag{24}$$

and

$$N_{2,t+1}^b(P_2^b, N^s) = \Pr(B_j^b \geq B_z^b) = \int_{\underline{U}^b + P_2^b - \beta N^s}^{1} h(B_j^b) dB_j^b = 1 + \beta N^s - \underline{U}^b - P_2^b. \tag{25}$$

Combining the best responses of all buyers, we can obtain homing-choice equilibria in the four scenarios discussed above. We can conclude homing-choice equilibria as follows.

Scenario A. The equilibrium of homing choices can be solved by combining buyers' choices (24) and (25) with sellers' choices (8) and (9). Furthermore, the equalities (10) and (11) need to hold in equilibrium.

Scenario B. From four equalities, (24), (25), (12) and (13), we derive the buyers' and sellers' homing choice equilibria. Note that the last condition, $\widehat{U}_{\tilde{h},2}^s < \widehat{U}_{k,2}^s$, and the equality (14) should be satisfied in equilibrium.

Scenario C. Putting equations (24) and (25) together with equations (8) and (15), we have the homing choice equilibria for buyers and sellers. Note that in equilibrium, the two conditions, inequality $\widetilde{U}_{k,2}^s < \widetilde{U}_{\tilde{h},2}^s$ and equality (16), should be satisfied.

Scenario D. Although the number of sellers on the platform in period 1 cannot be expressed directly, it is included in equation (20). Therefore, by combining equations (24) and (25) with equations (20) and (21), the equilibrium homing choices of buyers and sellers are obtained. Moreover, the last two conditions, $\widehat{U}_{\tilde{h},2}^s < \widehat{U}_{k,2}^s$ and $\widetilde{U}_{k,2}^s < \widetilde{U}_{\tilde{h},2}^s$, have not yet been considered and need to be guaranteed in equilibrium.

Note that, up to now, all prices for buyers and sellers in the two periods are given. We present the total number of buyers and sellers on the platform (N^b and N^s) in Online Appendix B.1 which allows us to derive the buyers' and sellers' homing choices in each period.

3.3 Nash Equilibrium

To simplify this dynamic problem, but not to jeopardize the core mechanics, we assume that $\underline{U}^s = 0$, $\underline{U}^b = 0$, and $r = 0$. If the platform is allowed to practice third-degree price discrimination, there are two prices for sellers in period 2 (i.e., P^s_{21} and P^s_{22}). Four possible homing choices in equilibrium are described in Subsection 3.1. The discussion of these four scenarios describes the results of combining the best responses of buyers and sellers under all the possible prices to which the platform has access. The final results after a two-stage game indicate that *Scenario A* is the Nash equilibrium as described in the following lemma.

Lemma 1. *When third-degree price discrimination is allowed, the platform's optimal pricing strategy indicates that Scenario A, characterized in Subsection 3.1, is the unique Nash equilibrium.*

Proof. See Online Appendix C. □

The proof of Lemma 1 shows that Scenarios B, C, and D tend to cross the discontinuity point of the sellers' distribution, and that Scenario A is the only Nash equilibrium after the optimization of the monopoly platform. Two features in this scenario are worth noting. One is that the price for sellers who only want to enter the platform in period 2 (i.e., P^s_{21}) is a commitment or threat fee because no seller chooses this strategy in the final Nash equilibrium. Another feature is that in *Scenario A* , sellers sell online or offline in both periods, which implies that the monopoly platform is more powerful in charging sellers a higher fee in period 2 due to their enhanced business ability.

Next, we present and discuss the homing choices of buyers and sellers, the prices set by the platform, and the two-period profit the platform obtains in Nash equilibrium.

Theorem 1 (Intertemporal Price Structure When Allowing Price Discrimination)**.** *Suppose that third-degree price discrimination is allowed in period 2. In equilibrium:*

1. *The prices on buyers in the two periods are:*
$$P^b_1 = P^b_2 = \frac{2 - 4\alpha(\alpha+\beta) - (\alpha-\beta)c}{4(1-(\alpha+\beta)^2)}. \tag{26}$$
2. *The prices on sellers in the two periods satisfy that $P^s_{21} = P^s_1$ and $P^s_{22} = P^s_1 + c$ where*
$$P^s_1 = P^s_{21} = \frac{2(\alpha-\beta) - [1-2\alpha(\alpha+\beta)]c}{4(1-(\alpha+\beta)^2)}, \tag{27}$$
implying that
$$P^s_{22} = c + \frac{2(\alpha-\beta) - [1-2\alpha(\alpha+\beta)]c}{4(1-(\alpha+\beta)^2)}. \tag{28}$$

Proof. We obtain equilibrium prices by maximizing (1) with respect to (8), (9), (24), (25), (41), and (42). □

Putting the equilibrium prices in Theorem 1 back into equations (8), (9), (24), and (25), we obtain the homing choice equilibria for each generation t when young and old as

$$N^s_{1,t} = N^s_{2,t+1} = \frac{2(\alpha+\beta)+c}{4(1-(\alpha+\beta)^2)}, \tag{29}$$

$$N^b_{1,t} = N^b_{2,t+1} = \frac{2+(\alpha+\beta)c}{4(1-(\alpha+\beta)^2)}. \tag{30}$$

This, of course, implies that the total number of sellers and buyers on the platform per period are

$$N^s = \frac{2(\alpha+\beta)+c}{2(1-(\alpha+\beta)^2)}, \tag{31}$$

$$N^b = \frac{2+(\alpha+\beta)c}{2(1-(\alpha+\beta)^2)} \tag{32}$$

and the two-period profit of the monopoly platform for each generation in equilibrium is

$$\pi = \frac{2+2(\alpha+\beta)c+0.5c^2}{4(1-(\alpha+\beta)^2)}. \tag{33}$$

Theorem 1 states that (i) there is no difference in price for buyers and new sellers in the two periods, and (ii) the price for old sellers in period 2 is c greater than the price in period 1. The distribution of buyer ability and the externality parameters of the network remain unchanged during the two periods. Then, the platform's optimal pricing strategy for buyers is to set the same price in both periods. A further result of this pricing strategy for buyers is that the number of buyers who buy on the platform is the same in the two periods. However, in period 2 the price for sellers who choose to join the online platform in period 1 will be adjusted upward by the business ability increment c. The reason for this upward trend is that this promotion of business skills guarantees that old sellers can afford higher expenses than previously. Overall, the intertemporal price structure will only change as the business ability on the same side changes over time.[17]

The discussion of cross-side and intertemporal price structures has provided some insights into economic development in the digital economy. The next proposition discusses the dynamic change in the cross-side price structure over time.

Proposition 1 (Intertemporal Cross-Side Price Structure When Allowing Price Discrimination)**.** *Suppose that third-degree price discrimination is allowed in period 2. Given the feasible set of parameters in Lemma 2, in equilibrium, the cross-side price difference between buyers and sellers*

[17] If price discrimination on the part of sellers in period 2 is prohibited, the results here will change. The next section provides details on this scenario.

in period 1 is greater than that in period 2 (that is, $P_2^b - P_{22}^s < P_1^b - P_1^s$); the intertemporal difference in cross-side price differences satisfies $(P_2^b - P_{22}^s) - (P_1^b - P_1^s) = -c$.

Proof. Recalling that $P_1^b = P_1^s$, $P_1^s = P_{21}^s$, and $P_{22}^s = P_1^s + c$, we can easily obtain $(P_2^b - P_{22}^s) - (P_1^b - P_1^s) = -c < 0$. □

Proposition 1 indicates that the platform tends to set a higher cross-side price difference between buyers and sellers in period 1 and the change in this price difference completely depends on the increment in online sellers' skills. More specifically, the source of this intertemporal cross-side change is only the increase in online sellers' prices from period 1 to 2, as discussed above. A key reason is that the increment in online sellers' ability can be anticipated at the beginning because of complete information, but it is realized in the next and last period. This theorem also implies that, according to learning-by-doing, the enhancement in the business ability of one type of agent in two-sided markets will affect the cross-side and intertemporal price structures and the platform enterprise's profit. Although these agents only have relatively low initial skills in running a business, the platform includes them in online transactions through intertemporal and/or cross-side price adjustments. The intertemporal and cross-side pricing strategy implemented by the monopoly platform also serves its goal of maximizing aggregate two-period profit, and hence the platform has an incentive to do this.

4 Equilibrium Analysis

In this section, we analyze the equilibria of the baseline model. Before we analyze the equilibria, we need to provide a feasible set of parameters as shown in the following lemma.

Lemma 2. *To establish $0 < N_{1,t}^s, N_{1,t}^b, N_{2,t+1}^s, N_{2,t+1}^b < 1$ and $\pi > 0$, we have a feasible set of parameters as $0 < \alpha + \beta < 1/2$, and $0 < c < 2$.*

Proof. See Online Appendix D. □

Intuitively, if the network externality parameters $\{\alpha, \beta\}$ and the enhancement of business ability c are too large, the outside options for buyers and sellers, such as offline selling and buying, will disappear in the economy. Throughout the following analysis in this subsection, Lemma 2 always holds.[18]

All prices in our model can be interpreted as *membership fees*, as discussed in Rochet and Tirole (2006). These fees measure the allocation of the gross surplus between buyers and sellers. For example, consider a one-shot trade on the monopoly platform. The highest willingness of a buyer to pay for an online transaction is W, and the seller's reserve price is R. The platform charges the fees P^b and P^s to buyers and sellers, respectively. This transaction occurs only when $P^b + P^s \leq W - R$. Then, prices P^b and P^s reflect the (latent) bargaining power of buyers and sellers, respectively. The higher the price or fee charged by the online platform, the weaker the agent's bargaining power. More importantly, the two prices are comparable, which will be useful for analyzing the price structure in equilibrium later.

Subsidization is a common pricing strategy not only in traditional one-sided markets but also in two-sided markets. In static multi-sided markets, cross-subsidization is attributed to cross-side externalities (Tan and Zhou, 2021). Subsidies (that is, negative prices) are also discussed in dynamic models (Cabral, 2011, 2019; Halaburda et al., 2020). Moreover, Cabral (2019) points out that under a dynamic framework, cross-side and intertemporal externalities are addressed when a platform determines the optimal pricing strategy.

In our model, in the beginning, all the sellers are poor with negative business ability distributed on the interval $[-1, 0]$, which reduces the utility they obtain from online transactions. If the platform charges a positive price for participating sellers in the initial period, the only source of positive utility for sellers is the cross-side network externality. However, buyers have positive ability, which is uniformly distributed on the interval $[0, 1]$. A common incentive the platform has is that to maintain transactions in two-sided markets, the platform may subsidize sellers to some extent to ensure that their utility level is at least higher than that of the offline option. This motivation can be interpreted as cross-subsidization, as described in Tan and Zhou (2021). Before analyzing the price structures, we explore the first-period price that the platform charges (poor) sellers.

Theorem 2 (Initial Price for Online Sellers When Allowing Price Discrimination)**.** *Suppose that third-degree price discrimination is allowed for sellers in period 2. Given the feasible set for parameters in Lemma 2, in equilibrium:*

[18] In the literature relevant to two-sided markets, the parameters are usually limited to a certain range. For example, Armstrong (2006) provides an assumption on the relationship between the network externality parameters and the differentiation parameters, to avoid a corner solution. Choi (2010) also imposes some assumptions on parameters to obtain certain homing choices.

1. *If either (i) $\beta > \alpha$, or (ii) $\alpha > \beta$ and $c > \frac{2(\alpha-\beta)}{1-\alpha(\alpha+\beta)}$, the price for sellers in period 1 is negative (that is, $P_1^s < 0$). If $\alpha > \beta$ and $c < \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)}$, the opposite result applies (that is, $P_1^s > 0$).*
2. *P_1^s is decreasing in c and β.*
3. *If $\beta > \frac{1-2\alpha^2-\sqrt{1-4\alpha^2}}{2\alpha}$, we have $\frac{\partial^2 P_1^s}{\partial c \partial \beta} < 0$. Otherwise, we have the opposite result (that is, $\frac{\partial^2 P_1^s}{\partial c \partial \beta} > 0$).*

Proof. See Online Appendix E. □

Item 1 in Theorem 2 shows that the platform has the incentive to subsidize sellers in the initial period when their network externality or growth in business ability is high enough. First, when $\beta > \alpha$, the price P_1^s is negative. Inequality $\beta > \alpha$ implies that sellers have a stronger market power than buyers because, on the platform, a buyer obtains more from each seller than a seller obtains from each buyer. Hence, the platform is more afraid of losing valuable sellers and chooses to subsidize them to attract greater participation in online transactions. The result that price P_1^s is decreasing in β in item 2 also provides evidence of the subsidy decision the platform may make.[19] Second, although the network externality each seller produces is lower than that each buyer produces, if the enhancement of sellers' business ability is high enough (that is, $c > \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)}$), the platform also subsidizes sellers on it in period 1. The reason is that the platform anticipates that in period 2, these old sellers will gain more utility from their business ability than before, and this utility increment is the basis for the platform to charge higher fees. Furthermore, as shown in item 2, an increase in the increment of business ability c will induce a lower price for sellers in period 1. This result is due to the latent intertemporal pricing strategy.[20] Item 3 in the theorem implies that if the externality of each seller (i.e., β) is high enough and the externality of each buyer (i.e., α) is not too high, β and c (i.e., learning-by-doing) are mutually reinforcing in reducing platform's charges (or increasing its subsidies) to sellers. The reason is that in this case, the platform has more substantial incentives to use the learning-by-doing benefit to attract sellers, thus providing more price concessions to sellers in period 1.

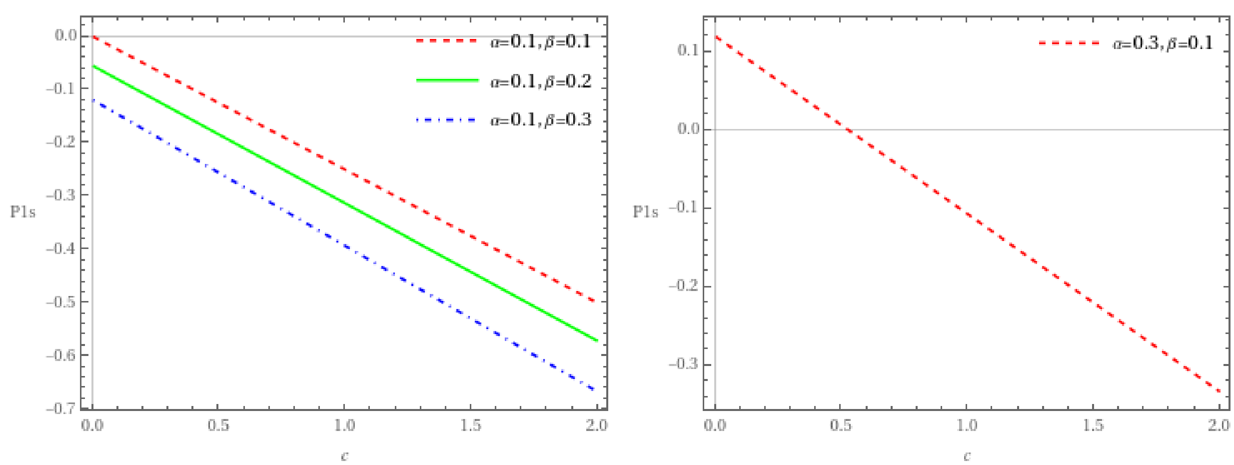
Although Cabral (2019) points out that subsidizing participants at an early stage may be optimal in dynamic pricing theory with learning-by-doing or network effects, the coordinated effect between the learning-by-doing benefit and the cross-sided network externalities has not attracted

[19] A special feature in our model is that the sellers' price charged by the monopoly platform depends on their own externality parameter α, which is different from that in Armstrong (2006).

[20] Later, we will discuss more details of the intertemporal pricing attributions.

much attention. In the model of Cabral (2019), the cross-sided network externalities are not explicitly characterized, and agents are assumed to be myopic. However, in our model, all agents have rational expectations, and the cross-sided network externalities and the learning-by-doing benefit are simultaneously explicitly featured within a unified framework. Our model shows a closed-form solution of prices for buyers and sellers in the two periods and thus provides the concrete conditions under which the platform will subsidize early sellers.[21]

Figure 5 shows several numerical examples to illustrate the conditions of the subsidy to online sellers in period 1 and the impact of changes in c and β on P_1^s. Figure 5(a) presents the situation in which $\beta > \alpha$. The scenario in which $\alpha > \beta$ is presented in Figure 5(b). These numerical examples show that P_1^s is decreasing in β (see Figure 5(a)) and c (see Figures 5(a) and 5(b)).



(a) $\alpha = 0.1$, $\beta = 0.1, 0.2, 0.3$ (b) $\alpha = 0.3$, $\beta = 0.1$

Figure 5: Relationship between the Price for Sellers in Period 1 and the Increment in Online Business Ability c

Notes: Panel (a) plots the relationships between P_1^s and c in three numerical examples, (1) $\alpha = 0.1$ and $\beta = 0.1$; (2) $\alpha = 0.1$ and $\beta = 0.2$; and (3) $\alpha = 0.1$ and $\beta = 0.3$, respectively, with $0 < c < 2$. Panel (b) plots the relationship between P_1^s and c when $\alpha = 0.3$ and $\beta = 0.1$ with $0 < c < 2$.

Next, we analyze the features of cross-side price structures, intertemporal price structures, and the interaction of the two. We also discuss the effects of network externalities and the enhancement of business ability on these price structures.

Theorem 3 (Cross-Side Price Structure When Allowing Price Discrimination)**.** *Suppose that third-degree price discrimination is allowed for sellers in period 2. Given the feasible set for parameters*

[21] Halaburda et al. (2020) present the dynamic competition between the two platforms with one-sided network effects and homogeneous consumers, while they focus on the quality and focal status of the two competing platforms.

in Lemma 2, in equilibrium:

1. *In period 1, buyers are charged a higher fee than sellers; that is,* $P_1^b > P_1^s$.
2. *The cross-side price difference* $P_1^b - P_1^s$ *is increasing in* c *and* β *and decreasing in* α.
3. *If* c *is sufficiently low, the price for buyers is higher than that for sellers who are affiliated with the platform in the two periods (that is,* $P_2^b > P_{22}^s$*).*
4. *The cross-side price difference* $P_2^b - P_{22}^s$ *is increasing in* β *and decreasing in* c *and* α.

Proof. See Online Appendix F. □

Theorem 3 highlights the cross-side price structures in the two periods. In period 1, the conclusion is that the platform always charges a higher fee to buyers than to sellers. This occurs for the same reason as explained earlier—buyers have an absolute advantage in obtaining utility from their own ability compared to sellers. The cross-side price difference depends on two externality parameters (α and β) and improvement in business ability c. The cross-side price difference measures the (latent) relative bargaining power between buyers and sellers when negotiating with the monopoly platform.[22] Consider that α and β denote the values of the network externalities of each buyer and seller, respectively, in two-sided markets. The phenomenon that $P_1^b - P_1^s$ is increasing in β and decreasing in α is obvious. Moreover, when the platform anticipates a higher enhancement of the business ability of sellers, the platform tends to increase the relative price between buyers and sellers in the initial period. As shown in Theorem 2, P_1^s decreases as c increases. However, the change in P_1^b is not clear with an increase in c because it depends on the relative size between α and β.[23] Even if P_1^b may decrease as c increases, the relative price $P_1^b - P_1^s$ will increase with an increase in c, as proven in Online Appendix F, because the increase in c changes the relative market power between buyers and sellers.

In period 2, the cross-side price structure changes. Here, two main features are noteworthy. One is that period 2 is the last period, and the game ends on this date. It also implies that there is no further change in behavior, which the platform needs to consider. Another feature is that sellers who sell online in period 1 have achieved enhanced business ability, which gives them an increase in utility. Then, as described in the theorem, when the buyers' network externality parameter β is high enough or the business ability increment c of sellers is low enough, the platform charges buyers a higher fee than sellers. It is straightforward that $P_2^b - P_{22}^s$ is increasing in β and decreasing

[22] In reality, pricing negotiations may not occur between the monopoly platform and buyers or sellers.

[23] From equality (26), we can derive that the derivative of P_1^b with respect to c is $\frac{\beta-\alpha}{8-2(\alpha+\beta)^2}$. Hence, if $\beta > \alpha$, P_1^b is increasing in c. If $\beta < \alpha$, the opposite result applies.

in α from the explanation of cross-side price structure in period 1.[24] One difference is that the price gap between buyers and sellers in period 2 is decreasing in the business ability increment c. From equation (28), we can derive that $\frac{\partial P_{22}^s}{\partial c} > 0$. Intuitively, the greater is the utility that sellers obtain from improvement in their ability, the higher is the fee the platform charges them. Since $P_1^b = P_2^b$, the analysis of the derivative of P_2^b with respect to c is the same as that for P_1^b. Even if P_2^b is increasing in c when $\beta > \alpha$, the price difference $P_2^b - P_{22}^s$ will decrease with an increase in c due to increased growth in P_{22}^s. This also means that when poor sellers become more capable of running the online business through learning-by-doing, the platform tends to charge them an asymmetrically higher fee than buyers.

5 Prohibiting Price Discrimination

In our original model, the platform possessed the ability to discriminate in third-degree prices on two different fronts: (i) toward sellers given their homing history (P_{21}^s versus P_{22}^s) and (ii) toward sellers according to age. In this section, we model the game when third-degree price discrimination toward sellers in either of these two cases is prohibited. Given the prices for buyers in the two periods, the best responses of buyers are the same as those characterized in Subsection 3.2.

5.1 Third-Degree Price Discrimination by Age

Suppose the platform is prohibited from third-degree price discriminating based on sellers' homing history. That is, all online sellers face a unified price P_2^s proposed by the platform when they become old, which means that P_{21}^s and P_{22}^s above are replaced by P_2^s. As a result, there is only one critical seller in each period because of the unified price and continuous ability distribution. In theory, given the two-period prices for buyers and sellers of each generation proposed by the platform, there are three scenarios for the homing choice of this critical seller (Figure 3 illustrates the critical seller's strategy). For each generation of sellers, the sellers on the platform are the same in both periods, some sellers enter the platform when they age, or some sellers leave when they age. For a full characterization of these scenarios and the derivation of the equilibrium, refer to Appendix H.

In equilibrium, the sellers in each generation on the platform are the same regardless of age. This means that the profit-maximization problem for the monopoly platform mirrors exactly that

[24]These results are consistent with some conclusions in the literature. For example, Hagiu (2009) points out that the platform will extract more profits from agents on the more powerful side compared to those on the other side.

of before in Section 3.3. To see this, observe that when the platform could third-degree price discriminate based on each seller's homing choice when they were young, they would set their prices such that (i) if sellers joined the platform when they were young, they would still join when they are old and (ii) if sellers did not join when they were young, they still would not join. Setting the unified price $P_2^s = P_{22}^s$ achieves this outcome as $P_{22}^s > P_{21}^s$ and sellers who did not join the platform when they were young also did not join when they were old when facing price P_{21}^s. As we solved in Section 3.3, P_{22}^s also maximizes the platform's profit. Thus, none of our equilibrium results are affected by removing the flexibility of the platform's pricing for older sellers.

5.2 Uniform Pricing for Sellers

Now, we restrict the monopoly platform from price discriminating toward the sellers in any form. Specifically, the platform can only set a uniform price P^s for each generation of sellers, regardless of age. However, the platform may still differentiate their pricing among young and old buyers. Given this restriction in seller pricing, it is immediate that the number of sellers that join the platform for each generation is the same regardless of their age. This is because if a seller joins when they are old, they should also join when they are young as their payoffs would be greater by c amount. Additionally, if a seller only joins when they are young, they must also join in the next period as their payoff increases by c. As such, given P^s, a seller i joins when they are young iff

$$\begin{aligned}&\Big(1+\delta\Big)\Big[B_{i,1}^s - P^s + \alpha N^b\Big] + \delta c \geq \Big(1+\delta\Big)\underline{U}^s\\ \iff\ & B_{i,1}^s \geq \underline{U}^s - \frac{\delta c}{1+\delta} + P^s - \alpha N^b.\end{aligned}$$

From this condition, the number of sellers of generation t on the platform, regardless of age, is

$$N_t^s(P^s, N^b) = \alpha N^b + \frac{\delta c}{1+\delta} - P^s - \underline{U}^s \tag{34}$$

and the number of sellers on the platform in a steady-state equilibrium is

$$N^s = 2N_t^s(P^s, N^b) = 2\alpha N^b + \frac{2\delta c}{1+\delta} - 2P^s - 2\underline{U}^s. \tag{35}$$

Without any price discrimination toward sellers, the platform's maximization problem for each generation t is simply

$$\hat{\pi} = \max_{\widehat{\boldsymbol{P}}} \left[\Big(1 + \frac{1}{1+r}\Big)P^s N_t^s + P_1^b N_{1,t}^b + \frac{1}{1+r} P_2^b N_{2,t+1}^b\right] \tag{36}$$

subject to (24), (25), (34), and (35) where the list of the platform prices is $\widehat{\boldsymbol{P}} \equiv \{P^s, P_1^b, P_2^b\}$. Under this economic environment that prohibits any price discrimination toward sellers, we use the "hat" script overhead to distinguish the equilibrium results from before. Assuming $\underline{U}^s = \underline{U}^b = r = 0$ and $\delta = 1$ as before, it is straightforward to show that the profit-maximizing prices in equilibrium are

$$\widehat{P_1^b} = \widehat{P_2^b} = \frac{2 - 4\alpha(\alpha+\beta) - c(\alpha-\beta)}{4(1-(\alpha+\beta)^2)}, \tag{37}$$

$$\widehat{P^s} = \frac{2(\alpha-\beta) + c(1-2\beta(\alpha+\beta))}{4(1-(\alpha+\beta)^2)} \tag{38}$$

and the number of buyers and sellers joining the platform for each generation t are

$$\widehat{N_{1,t}^b} = \widehat{N_{2,t+1}^b} = \frac{2+(\alpha+\beta)c}{4(1-(\alpha+\beta)^2)}, \tag{39}$$

$$\widehat{N_t^s} = \frac{2(\alpha+\beta)+c}{4(1-(\alpha+\beta)^2)}. \tag{40}$$

Notice that, by comparing this with (29) and (30), the number of buyers and sellers that join the platform does not change when restricting the platform to a uniform price for sellers. This holds true for each generation and for the total number of buyers and sellers in each time period. Also, observe that the structure of the buyer's prices remain the same. Each buyer, regardless of age, faces the same price and we have $\widehat{P_1^b} = \widehat{P_2^b} = P_1^b = P_2^b$ from (26). In addition, observe that $\widehat{P^s} = \frac{P_1^s + P_{22}^s}{2}$ from (27) and (28). This is because the platform must commit to the uniform price and the same individual rationality constraint must hold for each seller from before (with price discrimination). Thus, the platform can just average out its pricing across each period to ensure the critical seller when allowing price discrimination still joins. Overall, our results imply that the platform's maximized profits do not change as $\hat{\pi} = \pi$ from (33).

Theorem 4 (Uniform Price for Online Sellers When Prohibiting Price Discrimination)**.** *Suppose that price discrimination is prohibited for sellers. Given the feasible set for parameters in Lemma 2, in equilibrium:*

1. *If $\beta > \alpha$ and $c < \frac{2(\beta-\alpha)}{1-2\beta(\alpha+\beta)}$, the price for sellers is negative (that is, $\widehat{P^s} < 0$). If either (i) $\alpha > \beta$ or (ii) $\beta > \alpha$ and $c > \frac{2(\beta-\alpha)}{1-2\beta(\alpha+\beta)}$, the opposite result applies (that is, $\widehat{P^s} > 0$).*
2. *$\widehat{P^s}$ is increasing in c and decreasing in β.*
3. *If $\beta > \frac{1-2\alpha^2-\sqrt{1-4\alpha^2}}{2\alpha}$, we have $\frac{\partial^2 \widehat{P^s}}{\partial c \partial \beta} < 0$. Otherwise, we have the opposite result (that is, $\frac{\partial^2 \widehat{P^s}}{\partial c \partial \beta} > 0$).*

Proof. See Online Appendix I. □

Item 1 in Theorem 4 is slightly different than when price discrimination was allowed. Now, the platform is incentivized to subsidize sellers when their network externality is sufficiently large and the enhancement of business ability is sufficiently low. Intuitively, if sellers have more market power, but receive minimal benefits from entering the platform early, the platform must subsidize them to ensure that they participate and attract users on the buyer side. This is precisely why $\widehat{P^s}$ is increasing in c. Indeed, the platform anticipates that sellers enjoy a higher utility when they get old, which means the platform can set a higher price for both periods. This also implies that some consumers may face a negative utility when they are young and recoup their losses when they become old, which is not the case as before. However, we would still be in the same scenario where the same sellers are present on the platform for each generation. The rest of items 2 and 3 follow similarly to Theorem 2.

6 Discussions

As mentioned in Section 2, our model focuses on poor sellers in rural areas to analyze the economic phenomenon of emerging "digital villages." Compared to the literature on the learning-by-doing hypothesis that emphasizes productivity improvement or cost reduction due to increased production volume within a firm, our model focuses on the improvement of business ability of more microscopic individuals (sellers) on the digital platform. In this sense, these individuals have the characteristics of entrepreneurs. Thus, the learning-by-doing benefit of old sellers over time is very similar to human capital accumulation, permanently improving business ability. Although there are only two periods in our model, we can expect continued benefits for sellers from learning-by-doing in the future. Including high-skill sellers from metropolitan areas in the model will complicate the analysis, while the core mechanics have not been changed (and thus the main results of this paper may still hold). Indeed, the logic in our model can be widely applied to any dynamic case with cross-side network externalities in imperfectly competitive markets.

Educational investment is widely recognized as a key determinant of economic development. Acquiring knowledge and skills through various channels such as preschool programs, schools, and formal training programs has been shown to improve productivity (Behrman, 2010). In rural areas, education investments are predominantly funded by the government, while enterprise-based skills training rarely reaches impoverished populations. In industries that lack network effects, providing skills training to the poor becomes profitable only when learning-by-doing is sufficiently strong. Hypothetically, if the financial market functioned perfectly, the poor would have incentives to invest in their own education and skills. However, in industries characterized by one-sided network effects, such as the telephone and electronic payment industries, the exchange of physical goods is minimal, limiting the positive network externalities that the rural poor can generate. Digital technology, which facilitates the establishment of two-sided markets, has disrupted this landscape. When cross-side network effects become sufficiently strong, digital platform enterprises are incentivized to provide skills training to the rural poor, particularly during the early stages of economic development. As a result, high-quality agricultural products from rural areas gain access to online markets, significantly broadening consumer choices and enhancing social welfare.

7 Conclusion

By incorporating the learning-by-doing of sellers into the theory of two-sided markets with heterogeneous sellers and buyers and a monopoly platform, we provide a novel dynamic model to analyze a new platform-based phenomenon of economic development—emerging "digital villages" in China. The most important contribution of our paper is that we fill the void in understanding the interaction between two fields, industrial organization and economic development. Our paper also provides a tractable framework to explore the dynamic pricing strategy of a monopoly platform in a situation where price discrimination occurs within one group (on the sellers' side), along with the enhancement of the business ability of online sellers.

The key result is that due to the anticipated growth of the business ability of online sellers and cross-side network externalities, the rational and profit-maximizing platform tends to charge sellers a lower fee in period 1 by manipulating cross-side and intertemporal price structure. In particular, if (i) the network externality that each online seller produces is higher than that which each online buyer produces or (ii) the growth in the business ability of online sellers is high enough, the platform will subsidize sellers in period 1. When the externality of each seller is high enough and the externality of each buyer is not too high, the cross-side externality and learning-by-doing of each online seller can mutually reinforce each other in reducing the platform's charges (or increasing its subsidies) to sellers in period 1. With the regulation on price discrimination, homing choices and social welfare are unaffected. Therefore, we conclude that the usage of third-degree price discrimination for sellers stems from the platform's inability to commit to uniform pricing.

Our dynamic model of two-sided markets can be used to analyze other aspects of economic development. For example, two-sided platforms tend to invest substantial funds or resources during the early stages of development to cultivate users' stickiness, habits, and cross-side networks.

Moreover, our theory can be extended to address at least the following two fundamental theoretical problems: dynamic development on the buyers' side and on both sides of the market. Other important issues, such as endogenous investment decisions (made by platforms, buyers, or sellers) and income (or wealth) inequality caused by heterogeneous abilities in the digital economy, are left for future research.

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Online Appendix

A Data Sources of Online Retail Sales

1. Data on the number of "Taobao villages" in China and the number of "Taobao villages" located in national poverty-stricken counties of China are from the China Taobao Village Research Report (2020) (Chinese title: 中国淘宝村研究报告 2020).
2. Data on online retail sales in rural areas of China come from the E-commerce in China 2019 (Chinese title: 中国电子商务报告 2019).
3. Data sources of online retail sales on Alibaba's platforms in poverty-stricken counties of China:
   (a) The datum for 2014 is from 2014 county and rural e-commerce data (Chinese title: 2014 年县域暨农村电商数据).
   (b) The data for 2015 is from the 2015 China County E-Commerce Report (Chinese title: 2015).
   (c) The datum for 2016 is from E-commerce Development: Experience from China.
   (d) The datum for 2017 is from New Country, New Consumption, New Business: Rural Business Research Report (Chinese title: 新乡村新消费新商业——农村商业研究报告).
   (e) The datum for 2018 is from Alibaba Poverty Alleviation Work Report 2018 (Chinese title: 阿里巴巴脱贫工作报告 2018).
   (f) The datum for 2019 is from Alibaba's 2020 Fiscal Year Public Welfare "Financial Report" (Chinese title: 阿里巴巴 2020 财年公益“财报”).

B Homing Choice Equilibria

When all prices are given by the digital platform, homing choice equilibria are formed after all buyers' and sellers' best responses.

B.1 Allowing Third-Degree Price Discrimination

We solve for the total amount of buyers and sellers in each scenario which can be used to derive agents' optimal homing choice.

Scenario A: By combining buyer choices (24) and (25) with seller choices (8) and (9), we derive the total number of buyers and sellers on the platform:

$$N^b = \frac{2 - 2\underline{U}^b - 4\beta\underline{U}^s - 4\beta P_1^s - (P_1^b + P_2^b)}{1 - 4\alpha\beta} \tag{41}$$

$$N^s = \frac{4\alpha - 4\alpha\underline{U}^b - 2\underline{U}^s - 2P_1^s - 2\alpha(P_1^b + P_2^b)}{1 - 4\alpha\beta}. \tag{42}$$

In addition, we need to maintain the equalities (10) and (11) in equilibrium.

Scenario B: From equalities (24), (25), (12), and (13), we derive the total number of buyers and sellers on the platform:

$$N^b = \frac{2 - 2\underline{U}^b - 4\beta\underline{U}^s - (P_1^b + P_2^b) - 2\beta(P_{22}^s + P_{21}^s) + 2\beta c}{1 - 4\alpha\beta} \tag{43}$$

$$N^s = \frac{4\alpha - 4\alpha\underline{U}^b - 2\underline{U}^s - 2\alpha(P_1^b + P_2^b) - (P_{22}^s + P_{21}^s) + c}{1 - 4\alpha\beta}. \tag{44}$$

Inequality $\widehat{U}_{h,2}^s < \widehat{U}_{k,2}^s$ and equality (14) should also be satisfied in equilibrium.

Scenario C: Putting equations (24) and (25) together with equations (8) and (15), we have the total number of buyers and sellers on the platform are exactly like that in *Scenario B*. That is, N^b and N^s follow (43) and (44). In equilibrium, inequality $\widetilde{U}_{k,2}^s < \widetilde{U}_{\tilde{h},2}^s$ and equality (16) should be satisfied.

Scenario D: By combining equations (24) and (25) with equations (20) and (21), the total number of buyers and sellers on the platform are exactly like that in *Scenario B* again. That is, N^b and N^s follow (43) and (44). Meanwhile, conditions $\widehat{U}_{h,2}^s < \widehat{U}_{k,2}^s$ and $\widetilde{U}_{k,2}^s < \widetilde{U}_{\tilde{h},2}^s$ must be guaranteed in equilibrium.

B.2 Prohibiting Third-Degree Price Discrimination for Old Sellers

Scenario A: From equations (24), (25), (47), and (48), we similarly derive the total number of buyers and sellers from Appendix B.1. Note that when $P_2^s = P_{22}^s$, (47), (48), and (49) are the same as (8) (9), and (11) so N^b and N^s follow from (41) and (42). Also, the relationship between P_1^s and P_2^s, as shown in equation (49), must hold in equilibrium.

Scenario B: From equations (24), (25), (50), and (51), the total number of buyers and sellers in each period are

$$N^b = \frac{2 - 2\underline{U}^b - 4\beta\underline{U}^s - (P_1^b + P_2^b) - 2\beta(P_1^s + P_2^s) + 2\beta c}{1 - 4\alpha\beta}, \tag{45}$$

$$N^s = \frac{4\alpha - 4\alpha\underline{U}^b - 2\underline{U}^s - (P_1^s + P_2^s) - 2\alpha(P_1^b + P_2^b) + c}{1 - 4\alpha\beta}. \tag{46}$$

In addition, the third condition $\widehat{U}_{h,2}^s < \widehat{U}_{k,2}^s$ holds in equilibrium.

Scenario C: By combining equations (24) and (25) with equations (52) and (53), we can solve the total number of buyers and sellers on the platform in each period. Note that (52) and (53) are the same as (50) and (51), so N^b and N^s just follow from (45) and (46). Meanwhile, condition $\widetilde{U}_{k,2}^s < \widetilde{U}_{\tilde{h},2}^s$ should be satisfied in equilibrium.

C Proof of Lemma 1

Proof. All possible scenarios are described above (i.e., *Scenarios A, B, C*, and *D* in subsection 3.1). First, we check whether *Scenario B* is the optimal choice of both buyers and sellers after the monopoly platform sets the optimal prices. It is noteworthy that $N_{22,t+1}^s = N_{1,t}^s$ and $N_{21,t+1}^s = N_{2,t+1}^s - N_{22,t+1}^s$ in this scenario. By making decisions on P_1^b, P_2^b, P_{21}^s, and P_{22}^s, the platform's problem is to maximize profit (1) subject to equalities (12), (13), (14), (24), (25), (43), and (44). The maximization problem is reduced to

$$\max_{\boldsymbol{P}} (P_{21}^s + P_{22}^s)(\alpha N^b + c - P_{22}^s) + P_1^b(1 - P_1^b + \beta N^s) +$$
$$P_2^b(1 - P_2^b + \beta N^s) + P_{21}^s(P_{22}^s - P_{21}^s - c).$$

Equating the F.O.C.s with respect to P_{21}^s and P_{22}^s gives us $P_{21}^s = P_{22}^s - 0.5c$. But, putting this into (12) and (13) implies that $N_{1,t}^s > N_{2,t+1}^s$ which is a contradiction that some sellers enter the platform when they get old. Thus, *Scenario B* does not exist.

Second, we consider *Scenario C*. Under this scenario, $N_{21}^s = 0$ and $N_{22}^s = N_2^s$. By choosing P_1^b, P_2^b, P_1^s and P_{22}^s, the monopoly platform maximizes profit (1) subject to equalities (8), (15), (16), (24), (25), (43), and (44). The profit maximization problem can be reduced to

$$\max_{\boldsymbol{P}} P_1^s(\alpha N^b - P_1^s) + P_1^b(1 - P_1^b + \beta N^s) + P_2^b(1 - P_2^b + \beta N^s) + P_{22}^s(\alpha N^b + c - P_{22}^s).$$

Equating the F.O.C.s with respect to P_1^s and P_{22}^s gives us $P_1^s = P_{22}^s - 0.5c$. But, putting this into (8) and (15), implies that $N_{2,t+1}^s > N_{1,t}^s$ which is a contradiction that some sellers leave the platform when they get old. Thus, *Scenario C* does not exist.

Third, we need to verify whether *Scenario D* is an equilibrium result. We can obtain $N^s_{21,t+1}$ and $N^s_{22,t+1}$ from equations (18) and (19), respectively. The platform's problem is to maximize (1) subject to equalities (21), (24), (25), (43), and (44), according to the platform's decisions on P^b_1, P^b_2, P^s_1, P^s_{21}, and P^s_{22}. Again, we reduce the platform's profit maximization problem to:

$$\max_{\boldsymbol{P}} P^s_1(\alpha N^b - P^s_1) + P^b_1(1 - P^b_1 + \beta N^s) + P^b_2(1 - P^b_2 + \beta N^s) + P^s_{22}(\alpha N^b + c - P^s_{22}).$$

Equating the F.O.C.s with respect to P^s_1 and P^s_{22} gives us $P^s_1 = P^s_{22} - 0.5c$ again. Putting this into (18) and (19), implies that $N^s_{22,t+1} > N^s_{21,t+1} + N^s_{1,t}$ which is a contradiction that some sellers leave the platform when they get old. Thus, *Scenario D* does not exist.

In summary, *Scenarios B, C*, and *D* cannot form a final equilibrium. Thus, we conclude that only *Scenario A* is a final equilibrium result. □

D Proof of Lemma 2

Proof. Consider the final equilibria (29), (30), and (33) with $\alpha \in (0, 1)$, $c \in (0, 1)$, and $\beta \in (0, 1)$. We have that $N^s_{1,t}, N^b_{1,t}, N^s_{2,t+1}, N^b_{2,t+1}, \pi > 0$ if $\alpha + \beta < 1$. It is easy to verify that $N^s_{1,t} = N^s_{2,t+1} < N^b_{1,t} = N^b_{2,t+1}$ for $c < 2$ and $N^s_{1,t} = N^s_{2,t+1} \geq N^b_{1,t} = N^b_{2,t+1}$ for $c \geq 2$. So, for $c < 2$, $N^b_{1,t} < 1$ if $\alpha + \beta < \frac{-c+\sqrt{c^2+32}}{8}$. The right hand side of this expression is decreasing in c, so $\alpha + \beta < 1/2$ is a sufficient bound.

□

E Proof of Theorem 2

Proof. P^s_1 in equality (27) can be rearranged as

$$P^s_1 = \frac{2(\alpha - \beta) - [1 - 2\alpha(\alpha + \beta)]c}{4(1 - (\alpha + \beta)^2)}$$

Given our parameter restrictions in Lemma 2, $P^s_1 < 0 \iff c > \frac{2(\alpha-\beta)}{1-2\alpha(\alpha+\beta)}$ which immediately gives item 1. (Note the denominator is positive.)

The derivatives of P^s_1 with respect to c and β satisfy

$$\frac{\partial P^s_1}{\partial c} = \frac{-[1 - 2\alpha(\alpha + \beta)]c}{4(1 - (\alpha + \beta)^2)} < 0,$$

$$\frac{\partial P^s_1}{\partial \beta} = \frac{-2 + 2\alpha c}{4(1 - (\alpha + \beta)^2)} + \frac{8(\alpha + \beta)}{4(1 - (\alpha + \beta)^2)} P^s_1 < 0 \iff P^s_1 < \frac{1 - \alpha c}{4(\alpha + \beta)}.$$

Note that the last statement holds under our parameter restrictions in Lemma 2, because the numerator of P_1^s is always less than $1-\alpha c$ and the denominator of P_1^s is always larger than $4(\alpha+\beta)$ (both are positive). Meanwhile, we have the cross derivative $\frac{\partial^2 P_1^s}{\partial c \partial \beta}$ as

$$\frac{\partial^2 P_1^s}{\partial c \partial \beta} = \frac{8\alpha - 8\alpha(\alpha+\beta)^2 + 16\alpha(\alpha+\beta)^2 - 8(\alpha+\beta)}{[4(1-(\alpha+\beta)^2)]^2} < 0$$

$$\iff \alpha(\alpha+\beta)^2 < \beta \iff \beta > \frac{1-2\alpha^2-\sqrt{1-4\alpha^2}}{2\alpha}.$$

which gives item 3. □

F Proof of Theorem 3

Proof. From equalities (26) and (27), we have

$$P_1^b - P_1^s = \frac{(2+c)(1-\alpha+\beta-2\alpha(\alpha+\beta))}{4(1-(\alpha+\beta)^2)} > 0.$$

Obviously, given Lemma 2, we have $P_1^b > P_1^s$. Meanwhile, by taking derivatives of $P_1^b - P_1^s$ with respect to c, β, and α, we have the following results:

$$\frac{\partial(P_1^b - P_1^s)}{\partial c} = \frac{1-\alpha+\beta-2\alpha(\alpha+\beta)}{4(1-(\alpha+\beta)^2)} > 0,$$

$$\frac{\partial(P_1^b - P_1^s)}{\partial \beta} = \frac{(c+2)(1-(1+2\alpha)(\alpha+\beta)^2+2\beta+2\beta^2-2\alpha^2)}{4(1-(\alpha+\beta)^2)^2} > 0,$$

$$\frac{\partial(P_1^b - P_1^s)}{\partial \alpha} = -\frac{((c+2)((1+2\beta)(\alpha+\beta)^2+2\beta^2-2\alpha-2\alpha^2-1)}{4(1-(\alpha+\beta)^2)^2} < 0.$$

To see the latter two inequalities, note that both $(1+2\alpha)(\alpha+\beta)^2+2\alpha^2$ and $(1+2\beta)(\alpha+\beta)^2+2\beta^2$ cannot exceed 1 given our parameter restrictions. From inequalities (26) and (28), $P_2^b - P_{22}^s$ can be expressed as

$$P_2^b - P_{22}^s = P_1^b - P_1^s - c = \frac{(2+c)(1-\alpha+\beta-2\alpha(\alpha+\beta))}{4(1-(\alpha+\beta)^2)} - c.$$

Taking derivatives of $P_2^b - P_{22}^s$ with respect to c, α, and β, we have the following results:

$$\frac{\partial(P_2^b - P_{22}^s)}{\partial c} = \frac{(\alpha+\beta)(2\alpha+4\beta)-\alpha+\beta-3}{4(1-(\alpha+\beta)^2)} < 0,$$

$$\frac{\partial(P_2^b - P_{22}^s)}{\partial \alpha} = \frac{\partial(P_1^b - P_1^s)}{\partial \alpha} < 0,$$

$$\frac{\partial(P_2^b - P_{22}^s)}{\partial \beta} = \frac{\partial(P_2^b - P_1^s)}{\partial \beta} > 0.$$

Note that as $c \to 0$, $P_2^b - P_{22}^s \to \frac{1-\alpha+\beta-2\alpha(\alpha+\beta)}{2(1-(\alpha+\beta)^2)} > 0$ and as $c \to 2$, $P_2^b - P_{22}^s \to \frac{2\alpha\beta+2\beta^2+\beta-\alpha-1}{1-(\alpha+\beta)^2} < 0$. Because $\frac{\partial(P_2^b - P_{22}^s)}{\partial c} < 0$, there exists $\bar{c}$, such that $\forall c < \bar{c}$ we have $P_2^b - P_{22}^s > 0$ and $\forall c \geq \bar{c}$ we have

$P_2^b - P_{22}^s \leq 0.$ $\square$

G Proof of Lemma 3

Proof. All possible scenarios are discussed in Subsection 5.1 (*Scenarios A, B*, and *C*). First, we check whether *Scenario B* is the optimal choice after the monopoly platform sets all prices on buyers and sellers in the two periods. By choosing four prices $\{\widetilde{P_1^b}, \widetilde{P_1^s}, \widetilde{P_2^b}, \widetilde{P_2^s}\}$, the platform's problem is to maximize profit (54) subject to equalities (24), (25), (50), (51), (45) and (46). The maximization problem is reduced to

$$\max_{\widetilde{\boldsymbol{P}}} P_1^s(\alpha N^b + c - P_1^s) + P_1^b(1 - P_1^b + \beta N^s) + P_2^b(1 - P_2^b + \beta N^s) + P_2^s(\alpha N^b - P_2^s).$$

Equating the first order conditions with respect to P_1^s and P_2^s gives us the relationship $\widetilde{P_2^s} + .5c = \widetilde{P_1^s}$. This implies that $\widetilde{N_{1,t}^s} > \widetilde{N_{2,t+1}^s}$ which is a contradiction that some sellers enter the platform when they get old. Therefore, *Scenario B* is not a final equilibrium.

Next, we explore *Scenario C*. By taking into account four prices $\{\widetilde{P_1^b}, \widetilde{P_1^s}, \widetilde{P_2^b}, \widetilde{P_2^s}\}$, the platform maximizes profit (54) subject to equalities (24), (25), (52), (53), (45) and (46). The profit maximization problem then becomes:

$$\max_{\widetilde{\boldsymbol{P}}} P_1^s(\alpha N^b - P_1^s) + P_1^b(1 - P_1^b + \beta N^s) + P_2^b(1 - P_2^b + \beta N^s) + P_2^s(\alpha N^b - P_2^s + c).$$

Similar to before, equating the F.O.C.s with respect to P_1^s and P_2^s gives us the relationship $\widetilde{P_2^s} = \widetilde{P_1^s} + 0.5c$. However, given this relationship in equilibrium, $\widetilde{N_{2,t+1}^s} > \widetilde{N_{1,t}^s}$ which is a contradiction as sellers should be leaving the platform. Thus, *Scenario C* is not a final equilibrium result.

In summary, neither *Scenario B* nor *Scenario C* exists. Hence, we conclude that only *Scenario A* forms a final equilibrium. $\square$

H Derivation of Equilibrium with Price Discrimination by Age

Scenario A (the sellers on the platform are the same in both periods). The seller h and the seller k are the same, which implies that three conditions, $U_{k,1}^s = \underline{U}^s$, $\widetilde{U}_{h,2}^s = \underline{U}^s$, and $B_{k,1}^s = B_{h,1}^s$, are satisfied. From these three conditions, we have that the numbers of sellers affiliated with the platform in the two periods are

$$N_{1,t}^s(P_1^s, N^b) = \alpha N^b - P_1^s - \underline{U}^s, \tag{47}$$

and

$$N^s_{2,t+1} = N^s_{1,t} \tag{48}$$

with the following relationship between P_1^s and P_2^s

$$P_2^s = c + P_1^s. \tag{49}$$

Then, we obtain the homing choice equilibria for buyers and sellers from equations (24), (25), (47), and (48) (see Online Appendix B.2). Meanwhile, the relationship between P_1^s and P_2^s, as shown in equation (49), must be guaranteed in equilibrium.[25]

Scenario B (some sellers enter the platform in period 2). The critical seller h in period 2 has weaker business ability than the seller k. Hence, $\widehat{U}^s_{h,2} = \underline{U}^s$, $U^s_{k,1} + \widetilde{U}^s_{k,2} = \underline{U}^s + \widehat{U}^s_{k,2}$ and $\widehat{U}^s_{h,2} < \widehat{U}^s_{k,2}$. Then, from the first two conditions, the numbers of sellers in the two periods can be expressed as

$$N^s_{1,t}(P_1^s, N^b) = \alpha N^b + c - P_1^s - \underline{U}^s, \tag{50}$$

and

$$N^s_{2,t+1}(P_2^s, N^b) = \alpha N^b - P_2^s - \underline{U}^s. \tag{51}$$

From equations (24), (25), (50) and (51), the homing choice equilibria can be obtained (see Online Appendix B.2). In addition, the third condition $\widehat{U}^s_{h,2} < \widehat{U}^s_{k,2}$ must hold in equilibrium.

Scenario C (some sellers leave the platform in period 2). The critical seller $\widetilde{h}$ in period 2 has a stronger business ability than the seller k. In this scenario, $U^s_{k,1} = \underline{U}^s$, $\widetilde{U}^s_{\widetilde{h},2} = \underline{U}^s$ and $\widetilde{U}^s_{k,2} < \widetilde{U}^s_{\widetilde{h},2}$. The first two conditions imply that the numbers of sellers in the two periods are

$$N^s_{1,t}(P_1^s, N^b) = \alpha N^b - P_1^s - \underline{U}^s, \tag{52}$$

and

$$N^s_{2,t+1}(P_2^s, N^b) = \alpha N^b + c - P_2^s - \underline{U}^s. \tag{53}$$

By combining (24) and (25) with equations (52) and (53), we can solve the homing choice problem of all buyers and sellers (see Online Appendix B.2). Meanwhile, the third condition $\widetilde{U}^s_{k,2} < \widetilde{U}^s_{\widetilde{h},2}$ should be satisfied in equilibrium.

[25] It could be argued that an alternative method to solve the homing choice equilibria in *Scenario A* would be to obtain $N^s_{2,t+1}$ from condition $\widetilde{U}^s_{h,2} = \underline{U}^s$ (that is, $N^s_{2,t+1} = c + \alpha N^b - P_2^s - \underline{U}^s$) and then utilize the relation $N^s_{1,t} = N^s_{2,t+1}$ to obtain $N^s_{1,t}$. Therefore, the relationship between the two prices for sellers becomes $P_1^s = P_2^s - c$. This method is wrong because the monopoly platform will apply a backward induction method to develop the pricing strategy. Hence, the game requires that P_2^s is the result of the platform's best response in period 2 when P_1^s is given, and then P_1^s will be chosen by the platform to maximize the two-period profit.

When third-degree price discrimination is prohibited, the aggregate profit of the monopoly platform in the two periods is expressed as

$$\widetilde{\pi} = \max_{\widetilde{\boldsymbol{P}}} \left[P_1^s N_{1,t}^s + P_1^b N_{1,t}^b + \frac{1}{1+r} \left(P_2^s N_{2,t+1}^s + P_2^b N_{2,t+1}^b \right) \right], \tag{54}$$

where the list of the platform prices is

$$\widetilde{\boldsymbol{P}} = \left\{ P_1^s, P_1^b, P_2^s, P_2^b \right\}.$$

All the other parameters are the same as previously described. Here, we also assume $\underline{U}^s = 0$, $\underline{U}^b = 0$, and $r = 0$ for simplicity.

Under the economic environment that prohibits price discrimination based on sellers' homing choice when young, we use the "tilde" script overhead to distinguish the equilibrium results from the results of allowing price discrimination based on homing history. After a two-period game, the final equilibrium results in different scenarios demonstrate the following lemma.

Lemma 3. *When third-degree price discrimination based on sellers' homing history is prohibited, the platform's optimal pricing strategy indicates that Scenario A is a unique Nash equilibrium.*

Proof. First, we check whether *Scenario B* is the optimal choice after the monopoly platform sets all prices on buyers and sellers in the two periods. By choosing four prices $\{\widetilde{P_1^b}, \widetilde{P_1^s}, \widetilde{P_2^b}, \widetilde{P_2^s}\}$, the platform's problem is to maximize profit (54) subject to equalities (24), (25), (50), (51), (45) and (46). The maximization problem is reduced to

$$\max_{\widetilde{\boldsymbol{P}}} P_1^s(\alpha N^b + c - P_1^s) + P_1^b(1 - P_1^b + \beta N^s) + P_2^b(1 - P_2^b + \beta N^s) + P_2^s(\alpha N^b - P_2^s).$$

Equating the first order conditions with respect to P_1^s and P_2^s gives us the relationship $\widetilde{P_2^s} + .5c = \widetilde{P_1^s}$. This implies that $\widetilde{N_{1,t}^s} > \widetilde{N_{2,t+1}^s}$ which is a contradiction that some sellers enter the platform when they get old. Therefore, *Scenario B* is not a final equilibrium.

Next, we explore *Scenario C*. By taking into account four prices $\{\widetilde{P_1^b}, \widetilde{P_1^s}, \widetilde{P_2^b}, \widetilde{P_2^s}\}$, the platform maximizes profit (54) subject to equalities (24), (25), (52), (53), (45) and (46). The profit maximization problem then becomes:

$$\max_{\widetilde{\boldsymbol{P}}} P_1^s(\alpha N^b - P_1^s) + P_1^b(1 - P_1^b + \beta N^s) + P_2^b(1 - P_2^b + \beta N^s) + P_2^s(\alpha N^b - P_2^s + c).$$

Similar to before, equating the F.O.C.s with respect to P_1^s and P_2^s gives us the relationship $\widetilde{P_2^s} = \widetilde{P_1^s} + 0.5c$. However, given this relationship in equilibrium, $\widetilde{N_{2,t+1}^s} > \widetilde{N_{1,t}^s}$ which is a contradiction as sellers should be leaving the platform. Thus, *Scenario C* is not a final equilibrium result.

In summary, neither *Scenario B* nor *Scenario C* exists. Hence, we conclude that only *Scenario A* forms a final equilibrium. □

By Lemma 3, the platform maximizes profit (54) subject to equalities (24), (25), (41), (42), (47), (48), and (49). Because $N^s_{21,t+1} = 0$ for all t in equilibrium with price discrimination, (1) is exactly that of (54) if we let $P^s_{22} = P^s_2$. This also implies (47), (48), and (49) are exactly that of (8) (9), and (11). Thus, these profit maximization problems are identical giving us

$$\widetilde{P^s_2} = P^s_{22} = c + \frac{2(\alpha-\beta) - [1-2\alpha(\alpha+\beta)]c}{4(1-(\alpha+\beta)^2)}$$
$$\widetilde{P^b_1} = \widetilde{P^b_2} = P^b_1 = P^b_2 = \frac{2-4\alpha(\alpha+\beta)-(\alpha-\beta)c}{4(1-(\alpha+\beta)^2)},$$
$$\widetilde{P^s_1} = P^s_1 = \frac{2(\alpha-\beta) - [1-2\alpha(\alpha+\beta)]c}{4(1-(\alpha+\beta)^2)}$$

and the same homing choice equilibria and platform profits from (29), (30), (31), (32), and (33).

I Proof of Theorem 4

Proof. Given our parameter restrictions in Lemma 2, $\widehat{P^s} < 0 \iff c < \frac{2(\beta-\alpha)}{1-2\beta(\alpha+\beta)}$ which immediately gives item 1. (Note the denominator is positive.)

The derivatives of $\widehat{P^s}$ with respect to c and β satisfy

$$\frac{\partial \widehat{P^s}}{\partial c} = \frac{1-2\beta(\alpha+\beta)}{4(1-(\alpha+\beta)^2)} > 0,$$
$$\frac{\partial \widehat{P^s}}{\partial \beta} = \frac{-2-2\alpha c-4\beta c}{4(1-(\alpha+\beta)^2)} + \frac{8(\alpha+\beta)}{4(1-(\alpha+\beta)^2)}\widehat{P^s} < 0 \iff \widehat{P^s} < \frac{2(1+\alpha c+2\beta c)}{8(\alpha+\beta)}.$$

Note that the last statement holds under our parameter restrictions in Lemma 2, because the numerator of $\widehat{P^s}$ is always less than $2+2\alpha c+4\beta c$ and the denominator of $\widehat{P^s}$ is always larger than $8(\alpha+\beta)$ (both are positive). Meanwhile, we have the cross derivative $\frac{\partial^2 P^s_1}{\partial c \partial \beta}$ as

$$\frac{\partial^2 \widehat{P^s}}{\partial c \partial \beta} = \frac{-2\alpha-4\beta}{4(1-(\alpha+\beta)^2)} + \frac{8(\alpha+\beta)}{4(1-(\alpha+\beta)^2)}\frac{\partial \widehat{P^s}}{\partial c} < 0 \iff \frac{\partial \widehat{P^s}}{\partial c} < \frac{\alpha+2\beta}{4(\alpha+\beta)}$$
$$\iff \alpha(\alpha+\beta)^2 < \beta \iff \beta > \frac{1-2\alpha^2-\sqrt{1-4\alpha^2}}{2\alpha}.$$

which gives item 3. □