Self-Enforced Job Matching

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Job Matching

Two-sided many-to-one matching markets with wages

- Labor markets, multi-unit auctions, housing markets . . .

Kelso & Crawford (1982): stable matching exists if

- Firms treat workers as substitutable inputs (no complementarity)
- Workers' preferences have no peer effects

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Many tasks require workers with complementary skills.

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But . . .

Colleagues are important when choosing where to work.

Motivation: Existence (?)

Existing ways to accommodate complementarities or peer effects:

- Large markets
- Alternative assumptions on technologies / preferences

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Can we accommodate arbitrary market sizes, firm technologies, and worker preferences?

Matching as a Process

Matching is often an ongoing process.

- E.g. seller-buyer relationships, entry-level hiring, and securities auctions

Long-lived firms + short-lived workers.

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Are dynamic incentives powerful enough to maintain stability?

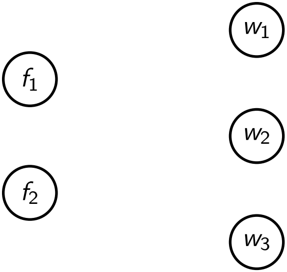
This Paper

We can always construct a dynamically stable matching process when firms are sufficiently patient.

Key feature: firms maintain dynamic stability through a form of no-poaching agreements.

Example: Matching with Transfers

No static stable matching



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Each year, three workers w_1, w_2, w_3 look for jobs.

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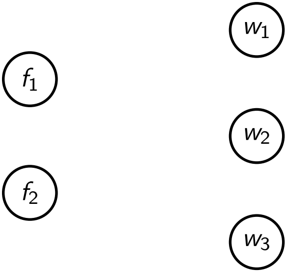

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Workers' payoffs are equal to their wages.

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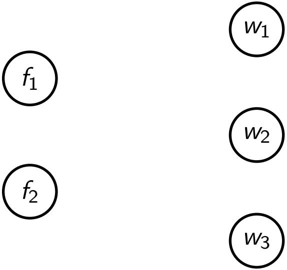

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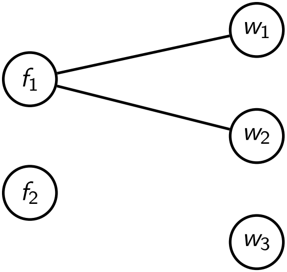

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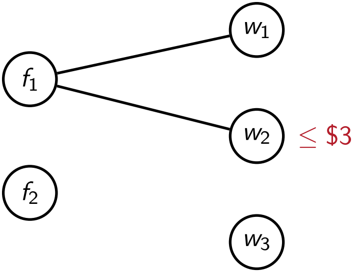

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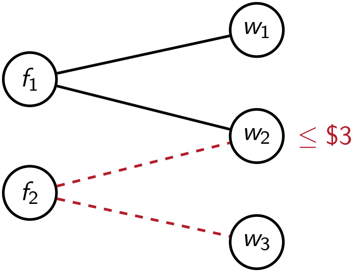

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... But the Market Is More Than One-Shot

Firms may care about the future impacts of today's poaching.

A dynamically stable matching process in the repeated (cooperative) game?

4 states: 2 collusion + 2 punishment

No-Poaching Agreements

Hiring right decided by a biased coin flip:

- Winner: hire 2 workers at 0 wage
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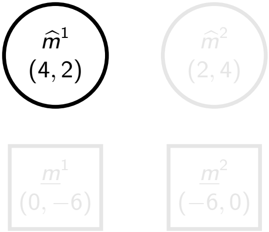

$\widehat{m}^1$: f_1 wins with prob. $2/3$

No-Poaching Agreements

Hiring right decided by a biased coin flip:

- Winner: hire 2 workers at 0 wage
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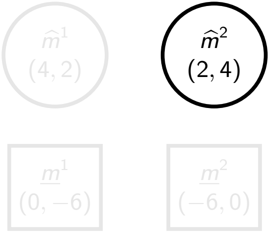
$\widehat{m}^2$: f_2 wins with prob. $2/3$



Punishments

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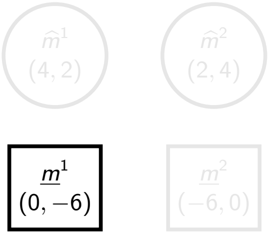
To punish f_1:

- f_2 hires two workers, each at \$6;
- f_1 shuts down.



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To punish f_2:

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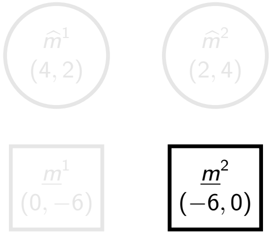
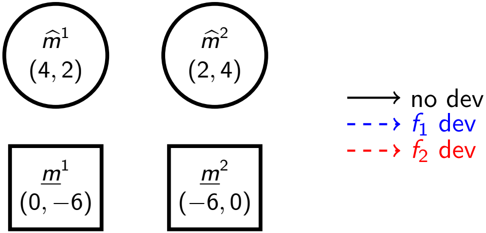


A Matching Process



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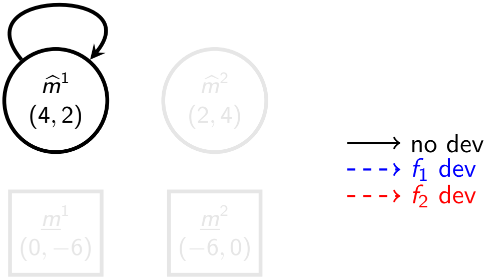

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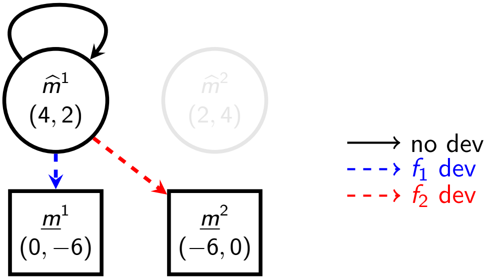

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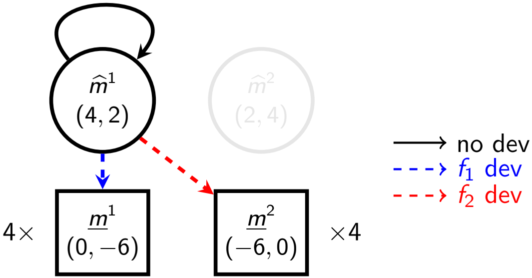

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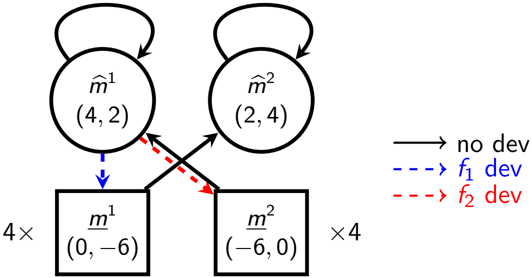

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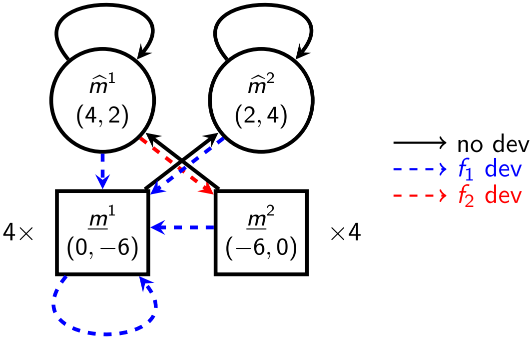

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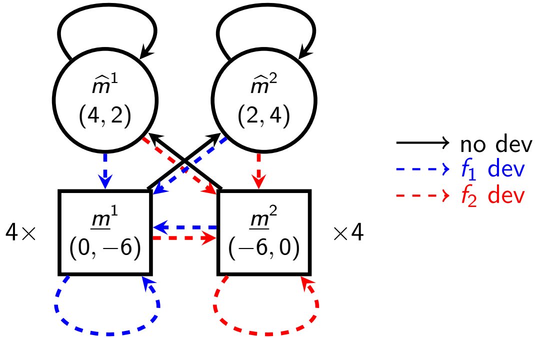
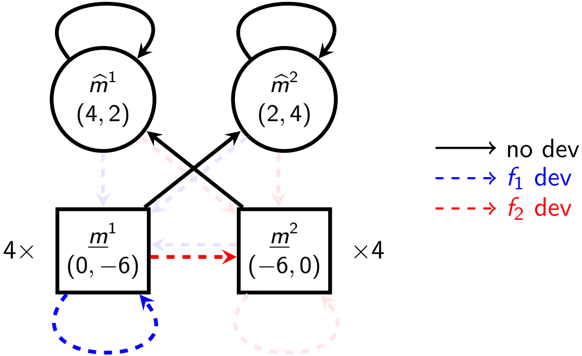


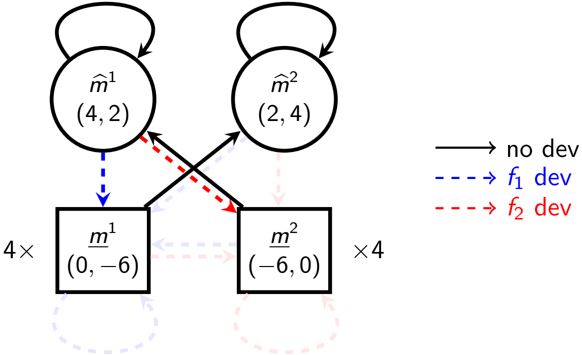
Dynamic Stability When $\delta \to 1$



$\underline{m}^1$: f_2 hires two workers each at \$6; f_1 shuts down.

- f_1 cannot profitably deviate in the stage game.
- f_2 prefers \$4 over \$2 in the long run.

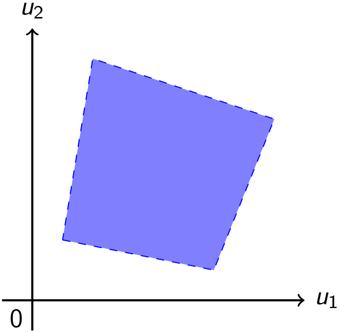
Dynamic Stability When $\delta \to 1$



$\widehat{m}^1$: toss a $(\frac{2}{3}, \frac{1}{3})$ coin, winner hires 2 workers at \$0, loser does not poach.

- In the long run, f_1 prefers \$4 over \$2.
- f_2 cannot change the long run, and $\$6 + 4 \times \$0 < \$0 + 4 \times \$2$.

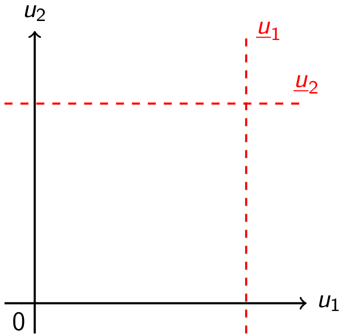
Plot the firms' feasible payoff profiles.



Repeated Matching

Plot the firms' feasible payoff profiles.

We can also define firms' "minmax" payoffs.

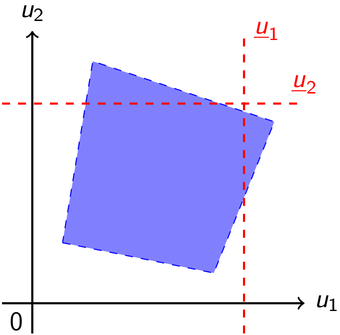


Plot the firms' feasible payoff profiles.

We can also define firms' "minmax" payoffs.

There may not be any payoff profile that is

- Feasible, and
- Higher than players' minmaxes.



Main Finding

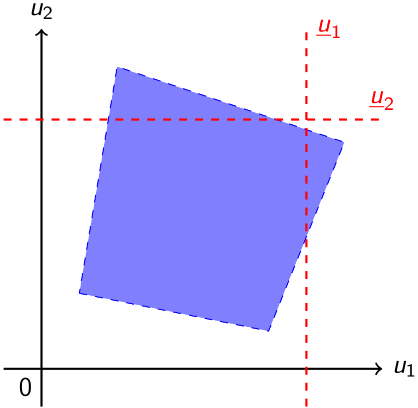
Theorem

A self-enforced stable matching process exists when $\delta \to 1$.

No restrictions on firm technology, worker preference, and market size.

On the path of play, firms suppress wages and refrain from poaching.

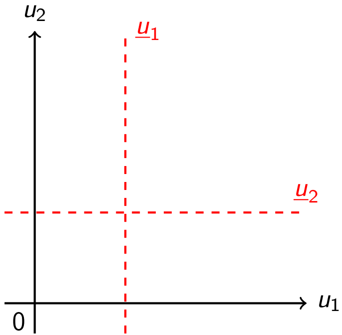
How to Prove Dynamic Stability?



We want to show that this is NOT the case.

Proof Idea

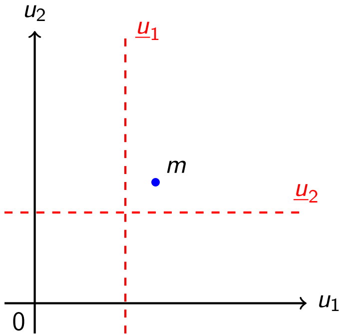
Lemma 1: Characterize firms' minmaxes.



Proof Idea

Lemma 1: Characterize firms' minmaxes.

Lemma 2: There is a feasible matching where payoffs dominate the minmaxes (random serial dictatorship).



Proof Idea

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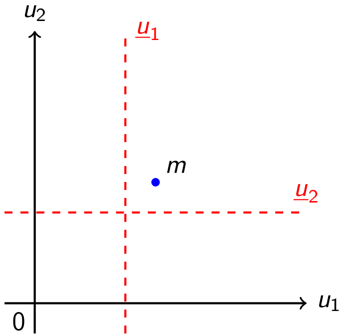
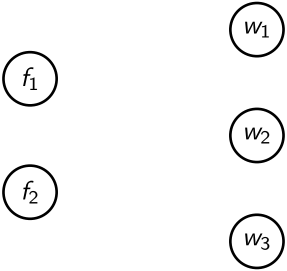
Lemma 3: Payoffs above minmaxes can be sustained dynamically.



Example: Matching without Transfers

Peer effects (not considered in Liu 2023): No static or dynamic stable matching



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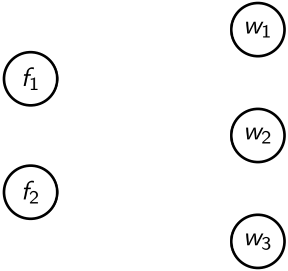

Firm's payoff is # of workers.

Worker prefers working together to working alone

w_1 prefers w_2 to w_3, w_2 prefers w_3 to w_1, w_3 prefers w_1 to w_2

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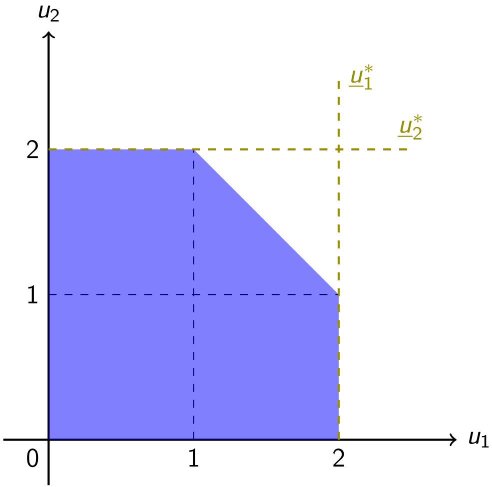

Takeaway: Existence

Received wisdom: market disrupted unless stable outcome is implemented.

With realistic preferences and technologies, stable matching is unlikely to exist.

But we don't see complete chaos in many matching markets.

Stability is the result of a dynamic process, self-fulfilled by expectations.

- Expectation should themselves be consistent with stability.

Takeaway: No-Poaching Agreement

No-poaching agreements are found in many matching markets

- Informal agreements among firms (*US v. Adobe Systems Inc., et al.*)

Controversial: subject of ongoing anti-trust litigations

- E.g., University financial aid (*Henry, et al. v. Brown University, et al.*)

This paper: informal NPAs maintain stability in matching markets.

- Crucial if complementarities + peer effects destabilize static matchings.
- Prohibiting such agreements could lead to market disruption.

THANK YOU!

General Model

Firms: long-lived players

- A finite set of firms, $\mathcal{F}$.
- Each firm $f \in \mathcal{F}$ has q_f positions to fill every period.

Workers: short-lived players

- A new generation of (finite) workers $\mathcal{W}$ enter the market every period.

Each worker $w \in \mathcal{W}$ has a type $\theta_w \in \Theta_w$, re-drawn every period.

- Let $\Theta \equiv \times_{w \in \mathcal{W}} \Theta_w$.

Distribution of type profile $\pi \in \Delta(\Theta)$

- Independent across time
- Not necessarily independent across w
- Can be degenerate

Firm f's payoff function:

$$\widetilde{u}_f : 2^{\mathcal{W}} \times \Theta \to \mathbb{R}$$

Worker w's utility function:

$$\widetilde{v}_w : \left(\left(\mathcal{F} \times 2^{\mathcal{W} \setminus \{w\}} \right) \cup \{(\emptyset, \emptyset)\} \right) \times \Theta \to \mathbb{R}$$