Self-Enforced Job Matching

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Job Matching

Two-sided many-to-one matching markets with wages

- Labor markets, multi-unit auctions, housing markets ...

Kelso & Crawford (1982): stable matching exists if

- Firms treat workers as substitutable inputs (no complementarity)
- Workers' preferences have no peer effects

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Can we accommodate arbitrary market sizes, firm technologies, and worker preferences?

Matching as a Process

Matching is often an ongoing process.

- E.g., seller-buyer relationships, entry-level hiring, and securities auctions

Long-lived firms + short-lived workers.

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Are dynamic incentives powerful enough to maintain stability?

This Paper

We can always construct a dynamically stable matching process when firms are sufficiently patient.

Key feature: firms maintain dynamic stability through a form of no-poaching agreement.

Related Literature

Existence of stable matching

- Alternative technology or preference assumptions: Hatfield and Milgrom (2005); Sun and Yang (2006); Hatfield and Kojima (2008); Rostek and Yoder (2020); Kojima, Sun, and Yu (2020, 2023); Pycia and Yenmez (2023)
- Large market: Kojima, Pathak and Roth (2013); Azevedo and Hatfield (2018); Che, Kim and Kojima (2019)
- Minimally adjusting quotas: Nguyen and Vohra (2018)

Dynamic stability in matching

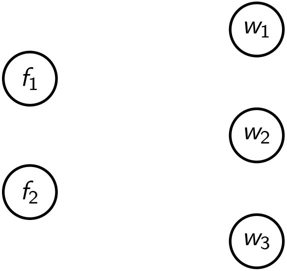
- Damiano and Lam (2005), Du and Livne (2016), Kadam and Kotowski (2018a,b), Altinok (2020), Kotowski (2020), Kurino (2020), Doval (2022), Liu (2023)

Repeated cooperative games

- Liu (2023), Ali and Liu (2020→2024)

Example: Matching with Transfers

No static stable matching



Two firms f_1, f_2, each with 2 hiring slots per year.

Each year, three workers w_1, w_2, w_3 look for jobs.

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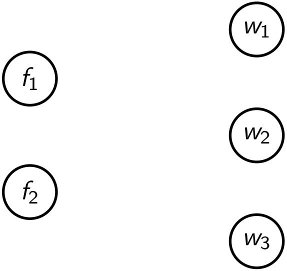

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Workers' payoffs are equal to their wages.

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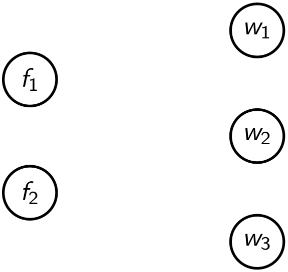

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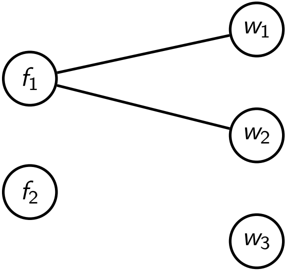

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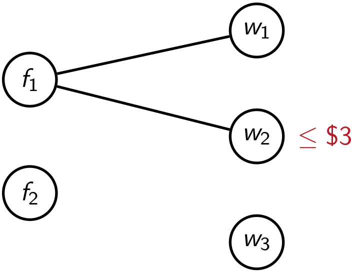

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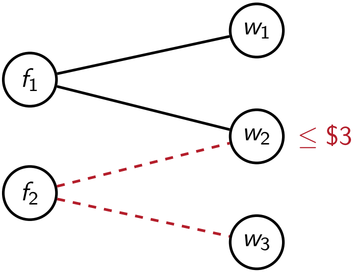

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... But the Market Is More Than One-Shot

Firms may care about the future impacts of today's poaching.

A dynamically stable matching process in the repeated (cooperative) game?

Matching Process

4 states: 2 collusion + 2 punishment

No-Poaching Agreements

Hiring right decided by a biased coin flip:

- Winner: hire 2 workers at 0 wage
- Loser: stay out, no poaching

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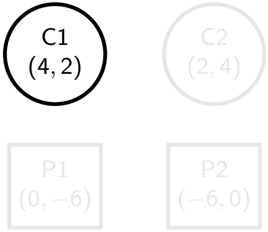
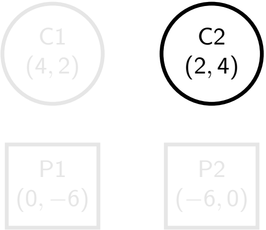


No-Poaching Agreements

Hiring right decided by a biased coin flip:

- Winner: hire 2 workers at 0 wage
- Loser: stay out, no poaching

C2: f_2 wins with prob. $2/3$



Punishments

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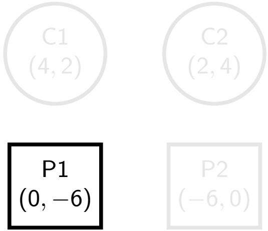
To punish f_1:

- f_2 hires two workers, each at \$6;
- f_1 shuts down.



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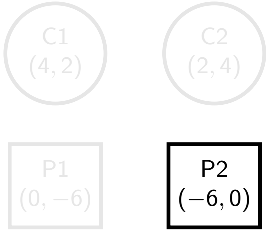
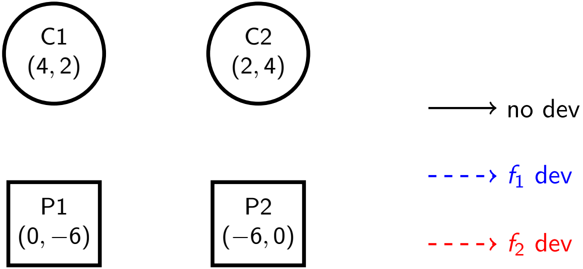


To punish f_2:

- f_1 hires two workers, each at \$6;
- f_2 shuts down.

A Matching Process



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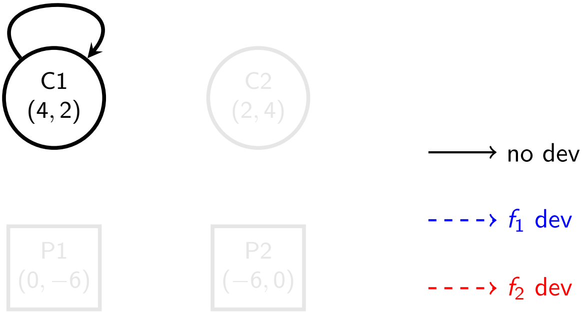

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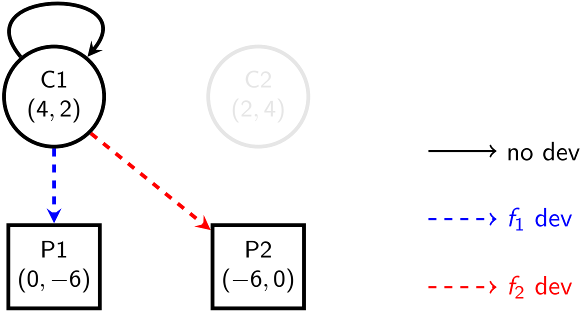

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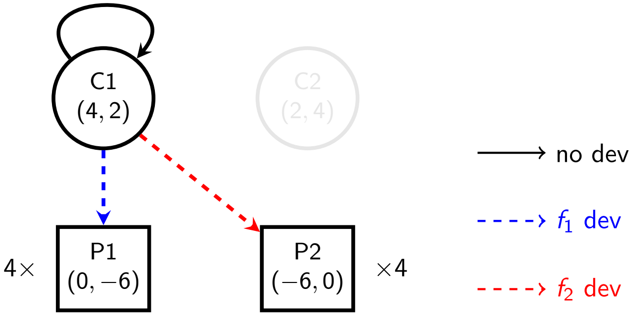

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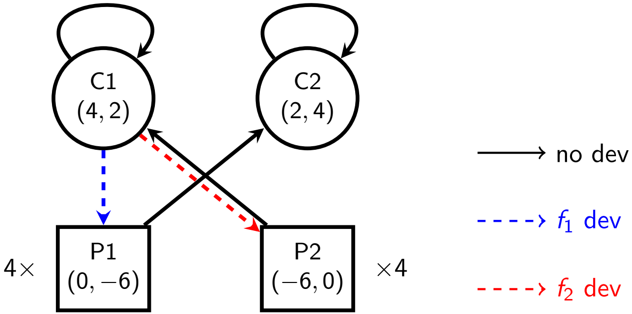

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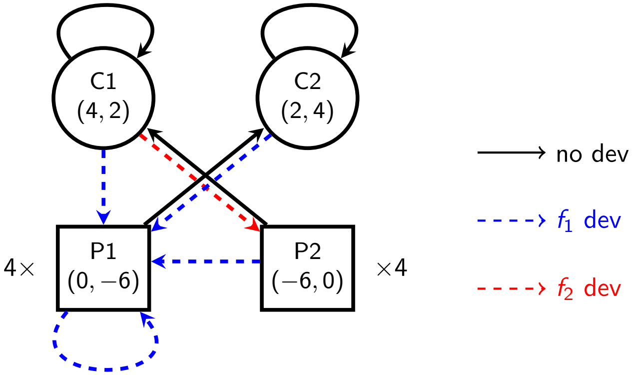

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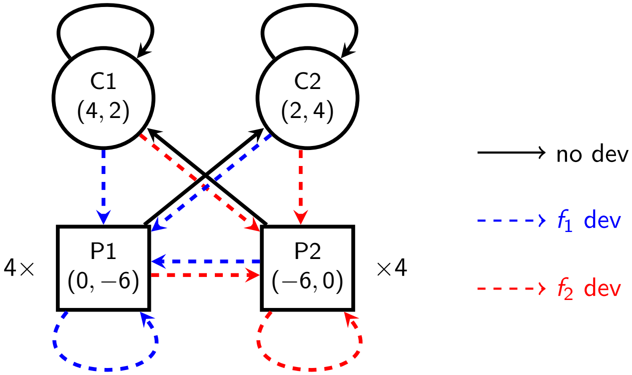
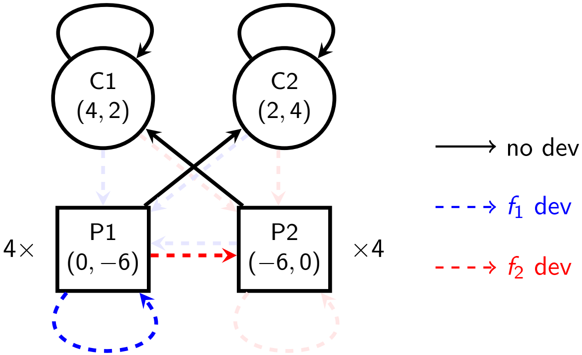


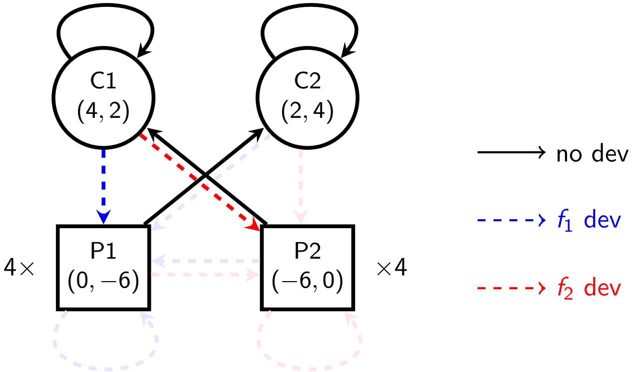
Dynamic Stability When $\delta \to 1$



P1: f_2 hires two workers each at \$6; f_1 shuts down.

- f_1 cannot profitably deviate in the stage game.
- f_2 prefers \$4 (in C2) over \$2 (in C1) in the long run.

Dynamic Stability When $\delta \to 1$



C1: toss a $(\frac{2}{3}, \frac{1}{3})$ coin, winner hires 2 workers at \$0, loser does not poach.

- In the long run, f_1 prefers \$4 over \$2.
- f_2 cannot change the long run, and $\$6 + 4 \times \$0 < \$0 + 4 \times \$2$.

Two Comments

1. Public randomization is not essential; we can instead achieve the same average payoffs using sequence of plays.

2. Wage flexibility (availability of monetary transfers) is essential: later an example to show the nonexistence of static or dynamic stable matching with peer effects in the absence of transfers.

Setup

Long-lived firms, $\mathcal{F}$

- Each firm $f \in \mathcal{F}$ has q_f positions to fill in every period.

Short-lived workers, $\mathcal{W}$, enter the market in every period

- Each worker w is in a *work environment* $\Phi_w = \left(\mathcal{F} \times 2^{\mathcal{W}_t \setminus \{w\}}\right) \cup \{(\emptyset, \emptyset)\}$.

States of the world Θ redrawn every period: $\pi \in \Delta(\Theta)$

- can be quite general, e.g., $\Theta = \Theta_0 \times \prod_{f \in \mathcal{F}} \Theta_f \times \prod_{w \in \mathcal{W}} \Theta_w$
- can be degenerate, i.e., Θ is a singleton

Payoffs

- Firm f's payoff in a period: $\widetilde{u}_f(W, \theta)$
- Worker w's utility in a period: $\widetilde{v}_w(f, W, \theta)$
- They share a common discount factor δ

Main Result

Theorem

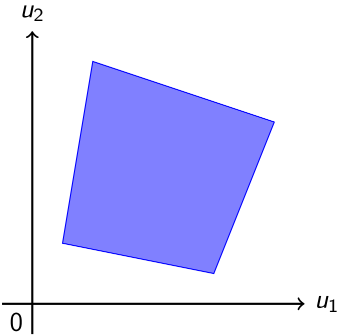
A self-enforced stable matching process exists when $\delta \to 1$.

No restrictions on firm technology, worker preference, and market size.

On the path of play, firms suppress wages and refrain from poaching.

Repeated Matching

Plot the firms' feasible payoff profiles.



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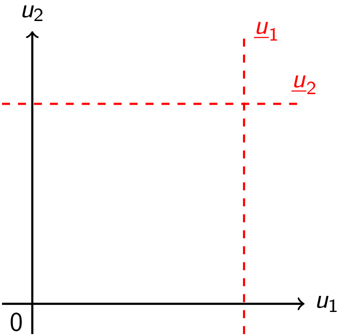
We can also define firms' "minmax" payoffs.



Repeated Matching

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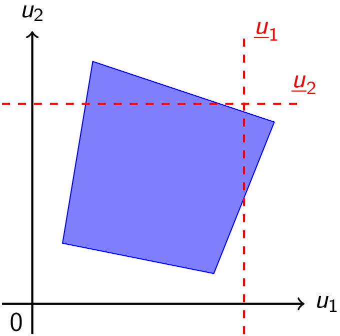
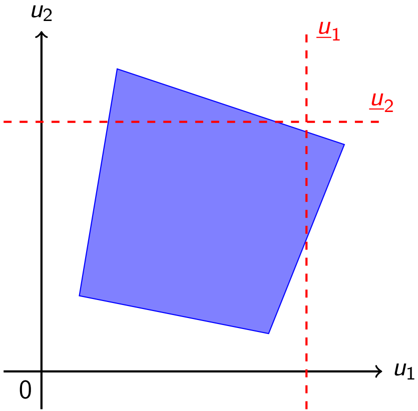
We can also define firms' "minmax" payoffs.

There may not be any payoff profile that is

- Feasible, and
- Higher than players' minmaxes.



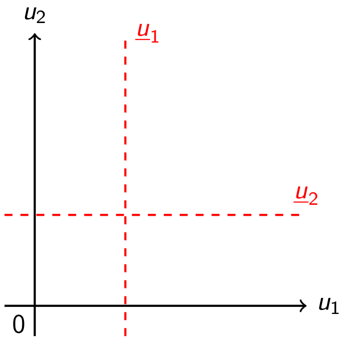
How to Prove Dynamic Stability?



We want to show that this is NOT the case.

Proof Idea

Step 1. Characterize firms' minmaxes.



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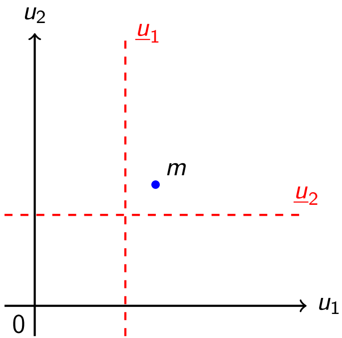

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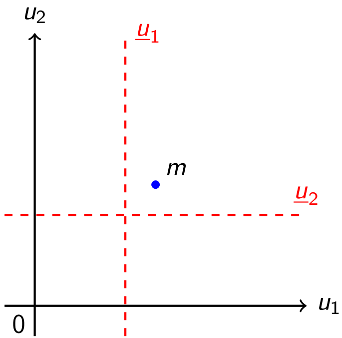


Stage-Game Matching

A *stage-game matching outcome* $m = (\phi, p)$ consists of

- *assignment* ϕ, a mapping defined on $\mathcal{F} \cup \mathcal{W}$ such that
    - $\phi(w) \in \Phi_w$ for every $w \in \mathcal{W}$
    - $\phi(f) \subseteq \mathcal{W}$ and $|\phi(f)| \leq q_f$ for every $f \in \mathcal{F}$
    - $w \in \phi(f)$ if and only if $\phi(w) = (f, W')$ for some $W' \subseteq \mathcal{W} \backslash \{w\}$
- *wage vector* p
    - nonzero transfer only between firms and their own employees: $p_{fw} = 0$ for every $w \notin \phi(f)$.

Quasilinear utilities of firms and workers

$$u_f(m, \theta) \equiv \widetilde{u}_f(\phi(f), \theta) - \sum_{w' \in \mathcal{W}} p_{fw'}$$

$$v_w(m, \theta) \equiv \widetilde{v}_w(\phi(w), \theta) + \sum_{f' \in \mathcal{F}} p_{f'w}$$

Deviation

- In static settings, there is no need to specify how other players will be matched after a coalitional deviation.
- In our dynamic model, players' future behavior is influenced by past histories.
- To study the stability of matching processes, we need to specify the realized stage-game outcome after a deviation. We adopt the following assumption.

Assumption 1

After coalitional deviation (f, W, p_f) from matching $m = (\phi, p)$, let matching $m' = [m, (f, W, p_f)] \in M$ denote the resulting stage-game matching and let ϕ' denote the assignment in m'. Assume $\phi'(f) = W$ (deviators are matched together) and $\phi'(f') = \phi(f') \backslash W$ for every $f' \neq f$ (those untouched by deviators remain intact, and partners abandoned by deviators do not rematch).

"Perfect monitoring": When a matching m is blocked by a coalition, the firm in the deviating coalition is identifiable.

Repeated Matching

A new cohort of workers arrives.

A state of the world θ is drawn and a public randomization $\gamma \in \Gamma$ is realized.

A matching (ϕ, p) is recommended based on realized (θ, γ).

Players decide whether to deviate from the recommended matching.

Histories

A t-period *ex ante history* $\overline{h}_t = (\theta_\tau, \gamma_\tau, m_\tau)_{\tau=0}^{t-1} \in \overline{\mathcal{H}}_t$.

$\overline{\mathcal{H}} \equiv \bigcup_{t=0}^{\infty} \overline{\mathcal{H}}_t$ set of all ex ante histories.

$\mathcal{H} \equiv \overline{\mathcal{H}} \times \Theta \times \Gamma$ set of ex post histories.

A matching process $\mu : \mathcal{H} \to M$ specifies a stage-game matching m for every ex post history $h \in \mathcal{H}$.

Firm f's continuation payoff from matching process μ after history $\widehat{h} \in \overline{\mathcal{H}} \cup \mathcal{H}$

$$U_f(\widehat{h} \,|\, \mu) \equiv (1-\delta)\mathbb{E}_\mu \left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} u_f(m_\tau(h_\infty), \theta_\tau) \,\middle|\, \widehat{h} \right]$$

Deviation Plan

A deviation plan $(d : \mathcal{H} \to 2^{\mathcal{W}}, \eta : \mathcal{H} \to \mathbb{R}^{|\mathcal{W}|})$ for firm f is a complete contingent plan that specifies, at every ex post history, a set of workers to recruit and their wage offers. [$|d(h)| \leq q_f$ for any h and $\eta_w(h) \neq 0$ only if $w \in d(h)$.]

Given μ and deviation plan (d, η), the manipulated matching process is

$$\Big[\mu, (f, d, \eta)\Big](h) \equiv \Big[\mu(h), \ \Big(f, d(h), \eta(h)\Big)\Big] \quad \forall h \in \mathcal{H}.$$

Firm f's deviation plan (d, η) from μ is feasible if at every ex post history h,

$$v_w\Big(\Big[\mu, (f, d, \eta)\Big](h), \ \theta\Big) > v_w\Big(\mu(h), \theta\Big) \quad \forall w \in d(h).$$

Deviation plan (d, η) is profitable if there exists an ex post history h such that

$$U_f\big(h \,\big|\, [\mu, (f, d, \eta)]\big) > U_f(h|\mu).$$

Self-Enforced Matching Process

Definition 1

Matching process μ is self-enforcing if

1. $v_w(\mu(h), \theta) \geq 0$ for every $w \in \mathcal{W}$ at every ex post history $h \in \mathcal{H}$ and
2. no firm has a feasible and profitable deviation plan.

Remarks

- It requires no deviation at every ex post history: a form of sequential rationality similar to subgame perfection.
- Coalitions of a firm and multiple workers: stronger than pairwise stability but weaker than group stability.
- It coincides with Kelso and Crawford's notion of stability, hence a dynamic generalization of it.

Main Result

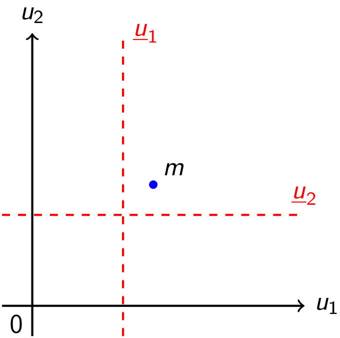
When firms are sufficiently patient, there exists a self-enforcing matching process in which players match according to the outcome of a random serial dictatorship in every period on path.

Proof Idea

Step 1. Characterize firms' minmaxes.

Step 2. Payoffs above minmaxes can be sustained dynamically.

Step 3. There is a feasible matching where payoffs dominate the minmaxes (random serial dictatorship).



Step 1. Minmax Payoffs

$M^\circ(\theta)$: set of stage-game matchings that are individually rational for workers.

$D_f(m,\theta)$: set of feasible stage-game deviations for f at state θ.

Firm f's minmax payoff at state θ is

$$\underline{u}_f(\theta) \equiv \inf_{m \in M^\circ(\theta)} \sup_{(W',p'_f) \in D_f(m,\theta)} u_f([m,(f,W',p'_f)],\theta).$$

Lemma 1. Let $Q \equiv \sum_{f' \in \mathcal{F}} q_{f'}$ represent the sum of all firms' hiring quotas. For every firm f and state θ, f's minmax payoff is

$$\underline{u}_f(\theta) = \min_{W' \subseteq \mathcal{W}, |W'| \leq Q} \max_{W \subseteq \mathcal{W} \setminus W', |W| \leq q_f} s(f,W,\theta),$$

where

$$s(f,W,\theta) \equiv \widetilde{u}_f(W,\theta) + \sum_{w \in W} \widetilde{v}_w(f, W \setminus \{w\}, \theta)$$

A firm's minmax payoff equals the maximum surplus it can generate after Q workers have been removed in an adversarial manner.

Example: Minmax Payoff $\neq$ Payoff from Minmax Matching

- A single firm f with capacity $q_f = 2$
- Three workers w_1, w_2, w_3, each worker can generate a revenue of \$1
- Minmax matching $\underline{m}_f$: Two workers (say, w_1 and w_2) match with f and each gets \$1, so f gets \$0 in the minmax matching.
- f's best response is to abandon $\{w_1, w_2\}$ and hire w_3 at a wage of \$0: $\underline{u}_f = \$1$.
- In standard repeated games, the mimmaxed player gets the minmax payoff from the minmaxing action profile.

Step 2. Characterization

$u(m,\theta) \equiv (u_f(m,\theta))_{f\in\mathcal{F}}$: firms' payoff profile under matching $m \in M^\circ(\theta)$

$\mathcal{U}^* \equiv \Big\{ \sum_{\theta\in\Theta} \pi(\theta)u(\theta) : u(\theta) \in \mathcal{U}(\theta) \forall\theta \in \Theta \Big\}$: convex hull of these payoffs

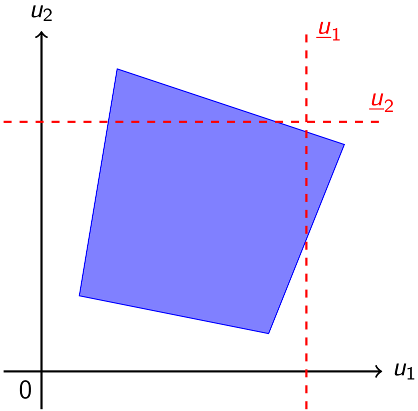
$\underline{u}_f^* \equiv \mathbb{E}_\pi[\underline{u}_f(\theta)]$: expected minmax payoff over states of the world

Lemma 2.

- If $u \in \mathcal{U}^*$ satisfies $u_f > \underline{u}_f^*$ for all $f \in \mathcal{F}$, then there is a $\underline{\delta} \in (0,1)$ such that for every $\delta \in (\underline{\delta}, 1)$, there exists a self-enforcing matching process with firms' continuation payoffs u at the beginning of period 0.
- Suppose μ is a self-enforcing matching process for a given $\delta \in (0,1)$. For every ex ante history $\overline{h} \in \overline{\mathcal{H}}$, firms' continuation payoff profile satisfies $\big(U_f(\overline{h} \,|\, \mu)\big)_{f\in\mathcal{F}} \in \mathcal{U}^*$ and $U_f(\overline{h} \,|\, \mu) \geq \underline{u}_f^*$ for every $f \in \mathcal{F}$.

A payoff profile is supported by a self-enforcing matching process if and only if it is strictly higher than minmax payoffs.

Where are we?



We want to show that this is NOT the case.

Step 3. Random Serial Dictatorship

Serial dictatorship according to ordering $o : \mathcal{F} \to \{1, \ldots, |\mathcal{F}|\}$: firms in order o hire workers at wage 0.

Firm f's maximum feasible payoff

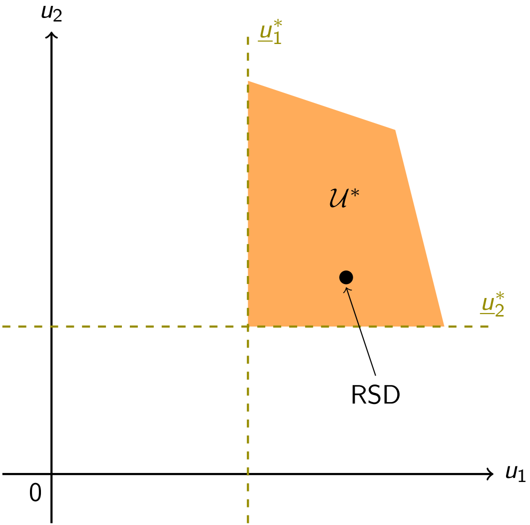
$$\overline{u}_f(\theta) \equiv \max_{W \subseteq \mathcal{W}, |W| \leq q_f} s(f, W, \theta)$$

Lemma 3. For every firm f,

$$\frac{1}{|\mathcal{O}|} \sum_{o \in \mathcal{O}} \mathbb{E}_\pi \Big[u_f \big(\widehat{m}(\theta, o), \theta \big) \Big] > \underline{u}_f^*,$$

where $\mathcal{O}$ is all orderings o.

Random Serial Dictatorship



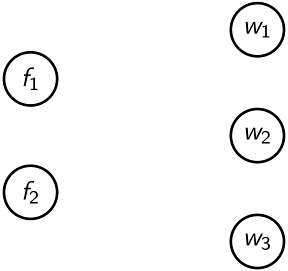
Random Serial Dictatorship may not be Pareto optimal

	w_1	w_2	w_3	w_4
f_1	10	1	ϵ	ϵ
f_2	1	10	ϵ	ϵ

- Each firm can hire two workers, $\epsilon \in [0, 1)$.
- Uniform RSD:
  - One firm picks w_1 and w_2 to get 11.
  - The other firm gets 2ϵ.
  - Expected payoff from uniform RSD: $5.5 + \epsilon$.
- Alternative allocation:
  - Firm f_1 hires w_1 and w_3 (or w_4) to get $10 + \epsilon$.
  - Firm f_2 hires w_2 and w_4 (or w_3) to get $10 + \epsilon$.

Counterexample 1: Matching without Transfers

Peer effects (not considered in Liu 2023): No static or dynamic stable matching



Two firms f_1, f_2, each with 2 hiring slots per year.

Each year, three workers w_1, w_2, w_3 look for jobs.

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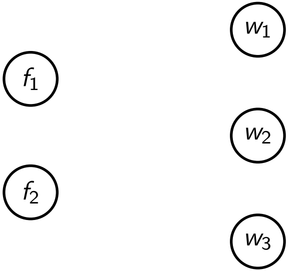

Firm's payoff is # of workers.
Worker prefers working together to working alone
w_1 prefers w_2 to w_3, w_2 prefers w_3 to w_1, w_3 prefers w_1 to w_2

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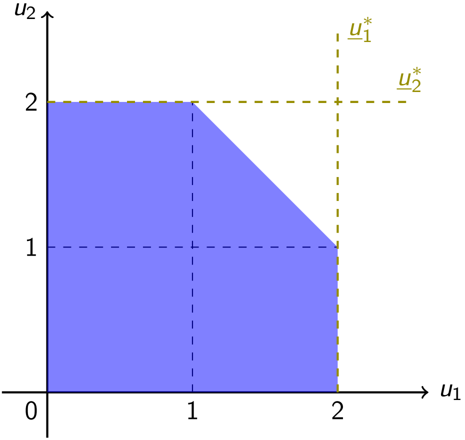
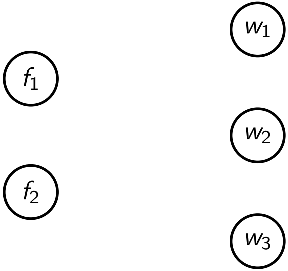


Counterexample 2: Matching with Externalities

Firms care about each other: No static or dynamic stable matching



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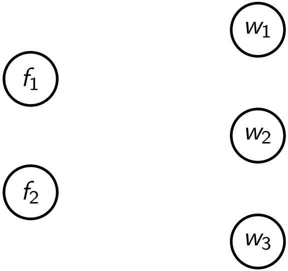
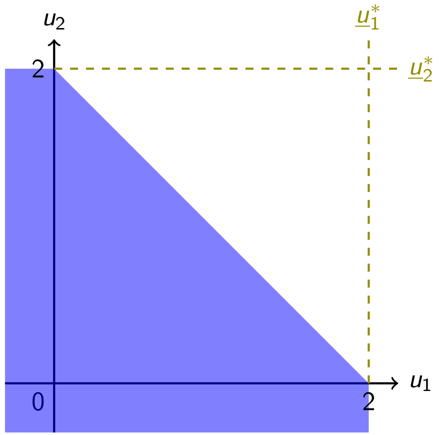


Firm 1 gets 2 if it hires the same number of workers as firm 2.
Firm 2 gets 2 if it hires a different number of workers as firm 1.
Workers are indifferent between being unemployed and working for free.

Counterexample 2: Matching with Externalities



Key: RSD no longer works in this example with externalities.

Takeaway: Existence

Received wisdom: market disrupted unless stable outcome is implemented.

With realistic preferences and technologies, stable matching is unlikely to exist.

But we don't see complete chaos in many matching markets.

Stability is the result of a dynamic process, self-fulfilled by expectations.

- Expectation should themselves be consistent with stability.

Takeaway: No-Poaching Agreement

No-poaching agreements are found in many matching markets

- Informal agreements among firms (*US v. Adobe Systems Inc., et al.*)

Controversial: subject of ongoing anti-trust litigations

- E.g., University financial aid (*Henry, et al. v. Brown University, et al.*)

This paper: informal NPAs maintain stability in matching markets.

- Crucial if complementarities + peer effects destabilize static matchings.
- Prohibiting such agreements could lead to market disruption.

THANK YOU!