Measuring assortativeness in marriage

Axiomatic and structural approaches

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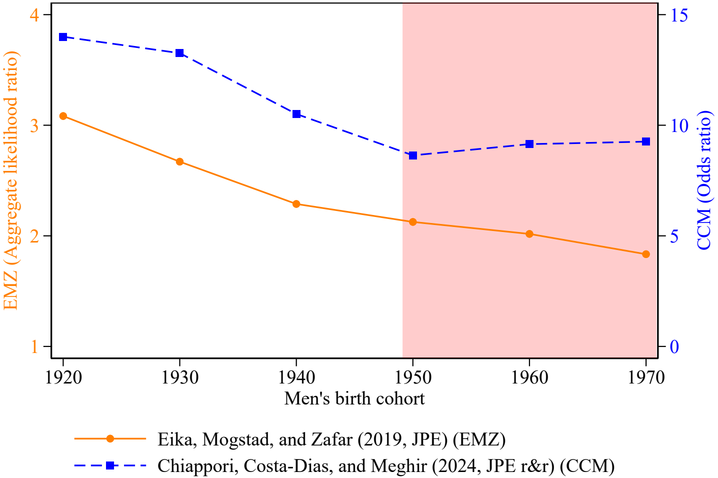
This paper subsumes

- "Measuring assortativeness in marriage" (Chiappori, Costa-Dias, and Meghir) and
- "Axiomatic measures of assortative matching" (Zhang)

A specific empirical debate



IPUMS USA: 40- to 50-year-olds and their heterosexual partners

(Start with) matching markets with binary types

$M = (a, b, c, d)$

	college women $a + c$	noncollege women $b + d$
college men $a + b$	a	b
noncollege men $c + d$	c	d

Each element denotes the # of pairs (also fine to normalize to %).

A general theoretical question

How do we compare

	θ_1 600	θ_2 400
θ_1 600	500	100
θ_2 400	100	300

and

	θ_1 450	θ_2 550
θ_1 500	400	100
θ_1 500	50	450

In general, how do we rank any two markets with different distributions of college and noncollege men and women?

Existing Measures

Perfect Positive Assortative Matching (Perfect PAM).

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & 0 \\ \hline \theta_2 & 0 & d \end{array} \sim_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a' & b' \\ \hline \theta_2 & 0 & d' \end{array} \sim_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a'' & 0 \\ \hline \theta_2 & c'' & d'' \end{array}$$

Perfect Negative Assortative Matching (Perfect NAM).

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & 0 & b \\ \hline \theta_2 & c & 0 \end{array} \sim_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & 0 & b' \\ \hline \theta_2 & c' & d' \end{array} \sim_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a'' & b'' \\ \hline \theta_2 & c'' & 0 \end{array}$$

Random Matching (RM). ($|M| \equiv a + b + c + d$)

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & \frac{a+b}{|M|}\frac{a+c}{|M|}|M| & \frac{a+b}{|M|}\frac{b+d}{|M|}|M| \\ \hline \theta_2 & \frac{a+c}{|M|}\frac{c+d}{|M|}|M| & \frac{c+d}{|M|}\frac{b+d}{|M|}|M| \end{array} = \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & \frac{(a+b)(a+c)}{a+b+c+d} & \frac{(a+b)(b+d)}{a+b+c+d} \\ \hline \theta_2 & \frac{(a+c)(c+d)}{a+b+c+d} & \frac{(c+d)(b+d)}{a+b+c+d} \end{array}$$

Positive Assortative Matching (PAM).

$$\begin{array}{c} \text{observed } \#(\theta_1\theta_1) > \text{random baseline} \\ \Updownarrow \\ a(a+b+c+d) > (a+b)(a+c) \\ \Updownarrow \\ a(c+d) > (a+b)c \\ \Updownarrow \\ ad > bc \end{array}$$

$$\begin{array}{c} \text{observed } \#(\theta_2\theta_2) > \text{random baseline} \\ \Updownarrow \\ d(a+b+c+d) > (d+b)(d+c) \\ \Updownarrow \\ d(a+b) > (d+c)b \\ \Updownarrow \\ ad > bc \end{array}$$

Negative Assortative Matching (NAM). $ad < bc$.

EMZ: Likelihood ratio

Likelihood ratio for each type

$$LR_1(M) = \frac{\text{observed } \#\theta_1\theta_1}{\text{random baseline}} = \frac{a}{\frac{a+b}{|M|}\frac{a+c}{|M|}|M|} = \frac{a(a+b+c+d)}{(a+b)(a+c)}.$$

$$LR_2(M) = \frac{\text{observed } \#\theta_2\theta_2}{\text{random baseline}} = \frac{d}{\frac{d+b}{|M|}\frac{d+c}{|M|}|M|} = \frac{d(a+b+c+d)}{(d+b)(d+c)}.$$

Aggregate likelihood ratio (Eika, Mogstad and Zafar, 2019, JPE) (EMZ)

$$\begin{aligned} LR(M) &= \frac{(a+b)(a+c)LR_1(M) + (d+b)(d+c)LR_2(M)}{(a+b)(a+c) + (d+b)(d+c)} \\ &= \frac{a+d}{\frac{a+b}{|M|}\frac{a+c}{|M|}|M| + \frac{d+b}{|M|}\frac{d+c}{|M|}|M|} = \frac{\text{observed } \#(\theta_1\theta_1 + \theta_2\theta_2)}{\text{random baseline}} \end{aligned}$$

CCM: Odds ratio

(OR) odds ratio; cross-ratio (Chiappori, Costa-Dias and Meghir, 2020, 2022)

$$I_O(a,b,c,d) = \frac{a}{b} / \frac{c}{d} = \frac{ad}{bc}.$$

(Q) Yule's Q; Coefficient of association (Yule, 1900)

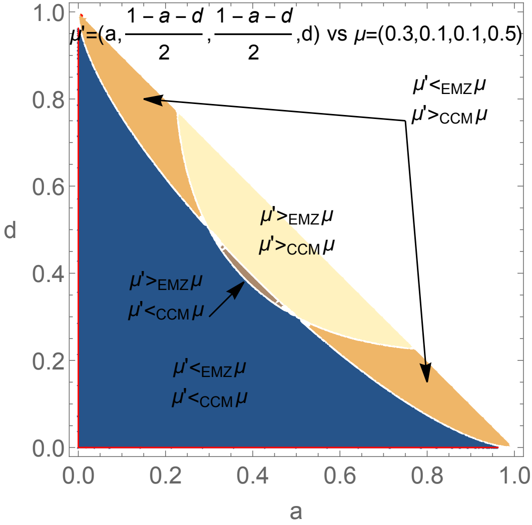
$$I_Q(a,b,c,d) = \frac{ad - bc}{ad + bc} = \frac{1 - \frac{bc}{ad}}{1 + \frac{bc}{ad}} = \frac{\frac{ad}{bc} - 1}{\frac{ad}{bc} + 1}.$$

(Y) Yule's Y; Coefficient of colligation (Yule, 1912)

$$I_Y(a,b,c,d) = \frac{\sqrt{ad} - \sqrt{bc}}{\sqrt{ad} + \sqrt{bc}} = \frac{\sqrt{\frac{ad}{bc}} - 1}{\sqrt{\frac{ad}{bc}} + 1}.$$

Both return $+1$ when PAM and -1 when NAM.

Conflicting conclusion: CCM vs EMZ



Other measures

(PR) Pure-random normalization (minimum distance)
Fernández and Rogerson (2001, QJE), Liu and Lu (2006, EL), Greenwood, Guner, Kocharkov and Santos (2014, AER), Shen (2020, PhD thesis):

$$I_{PR}(a,b,c,d) = \frac{ad - bc}{(\max\{b,c\}+d)(a+\max\{b,c\})}.$$

(Corr) Correlation

$$I_{Corr}(a,b,c,d) = \frac{ad - bc}{\sqrt{(a+b)(c+d)(a+c)(b+d)}}.$$

(Chi) Spearman's rank correlation (degree away from random matching)

$$I_{\chi}(a,b,c,d) = [I_{Corr}(a,b,c,d)]^2 = \frac{(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)}.$$

Hou et al. (2022, PNAS) use all aforementioned measures for robustness checks.

Axioms

[SI] Scale Invariance. The market exhibits the same assortativity when all entries scale by the same constant. For all $\lambda > 0$,

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array} \sim_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & \lambda a & \lambda b \\ \hline \theta_2 & \lambda c & \lambda d \end{array}$$

[TSym] Type Symmetry. The market exhibits the same assortativity when types are relabeled.

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array} \sim_A \begin{array}{c|c|c} & \theta_2 & \theta_1 \\ \hline \theta_2 & d & c \\ \hline \theta_1 & b & a \end{array}$$

[SSym] Side Symmetry. The market exhibits the same assortativity when sides are relabeled.

$$\begin{array}{c|c|c} m\backslash w & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array} \sim_A \begin{array}{c|c|c} w\backslash m & \theta_1 & \theta_2 \\ \hline \theta_1 & a & c \\ \hline \theta_2 & b & d \end{array}$$

[DMon] Diagonal Monotonicity. For all $\epsilon > 0$,

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a+\epsilon & b \\ \hline \theta_2 & c & d \end{array} \succeq_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array}$$

and

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d+\epsilon \end{array} \succeq_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array}$$

where the equalities hold if and only if $bc = 0$.

$\Longrightarrow$ **[wPAM] Weak Positive Assortative Matching.** (all symbols are positive)

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & 0 \\ \hline \theta_2 & 0 & d \end{array} \sim_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a' & b' \\ \hline \theta_2 & 0 & d' \end{array} \sim_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a'' & 0 \\ \hline \theta_2 & c'' & d'' \end{array} \succeq_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a''' & b''' \\ \hline \theta_2 & c''' & d''' \end{array}$$

Do the measures satisfy the axioms?

	equivalence conditions		
	SI	TSym	SSym
LR_i (EMZ)	✓	X	✓
LR (EMZ)	✓	✓	✓
OR (CCM)	✓	✓	✓

[DMon'] Strict Diagonal Monotonicity. For all $\epsilon > 0$,

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a+\epsilon & b \\ \hline \theta_2 & c & d \end{array} \succeq_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array}$$

and

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d+\epsilon \end{array} \succeq_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array}$$

where the equalities hold if and only if $b = c = 0$.

$\Longrightarrow$ **[sPAM] Strict Positive Assortative Matching.** (all symbols are positive)

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & 0 \\ \hline \theta_2 & 0 & d \end{array} \succ_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a' & b' \\ \hline \theta_2 & 0 & d' \end{array}, \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a'' & 0 \\ \hline \theta_2 & c'' & d'' \end{array}, \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a''' & b''' \\ \hline \theta_2 & c''' & d''' \end{array}$$

[ODMon] Off-Diagonal Monotonicity. For all $\epsilon > 0$,

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array} \succeq_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b+\epsilon \\ \hline \theta_2 & c & d \end{array}$$

and

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array} \succeq_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c+\epsilon & d \end{array}$$

where the equalities hold if and only if $ad = 0$.

$\Longrightarrow$ **[wNAM] Weak Negative Assortative Matching.** (all symbols are positive)

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & 0 & b \\ \hline \theta_2 & c & 0 \end{array} \sim_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & 0 & b' \\ \hline \theta_2 & c' & d' \end{array} \sim_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a'' & b'' \\ \hline \theta_2 & c'' & 0 \end{array} \prec_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a''' & b''' \\ \hline \theta_2 & c''' & d''' \end{array}$$

[ODMon'] Strict Off-Diagonal Monotonicity. For all $\epsilon > 0$,

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array} \succeq_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b+\epsilon \\ \hline \theta_2 & c & d \end{array}$$

and

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array} \succeq_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c+\epsilon & d \end{array}$$

where the equalities hold if and only if $a = d = 0$.

$\Longrightarrow$ **[sNAM] Strict Negative Assortative Matching.** (all symbols are positive)

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & 0 & b \\ \hline \theta_2 & c & 0 \end{array} \prec_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & 0 & b' \\ \hline \theta_2 & c' & d' \end{array}, \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a'' & b'' \\ \hline \theta_2 & c'' & 0 \end{array}, \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a''' & b''' \\ \hline \theta_2 & c''' & d''' \end{array}$$

[MMon] Marginal Monotonicity. Suppose $M = (a, b, c, d) \gg 0$ and $M' = (a', b', c', d') \gg 0$ have the same marginals: $a + b = a' + b'$, $a + c = a' + c'$, $b + d = b' + d'$, $c + d = c' + d'$.

$$M \succ_A M' \Leftrightarrow a > a' \Leftrightarrow b < b' \Leftrightarrow c < c' \Leftrightarrow d > d'$$

Equivalently, for all $M = (a, b, c, d) \gg 0$ and $\epsilon \in (0, \min\{a, d\})$,

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array} \succ_A \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a-\epsilon & b+\epsilon \\ \hline \theta_2 & c+\epsilon & d-\epsilon \end{array}$$

- DMon and ODMon imply MMon.
  Proof:

$$\begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a & b \\ \hline \theta_2 & c & d \end{array} \underset{\text{by DMon}}{\succ_A} \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a-\epsilon & b \\ \hline \theta_2 & c & d-\epsilon \end{array} \underset{\text{by ODMon}}{\succ_A} \begin{array}{c|c|c} & \theta_1 & \theta_2 \\ \hline \theta_1 & a-\epsilon & b+\epsilon \\ \hline \theta_2 & c+\epsilon & d-\epsilon \end{array}$$

Do the measures satisfy the axioms?

	equivalence conditions			monotonicity conditions		
	SI	TSym	SSym	MMon	DMon	ODMon
LR_i (EMZ)	✓	X	✓	✓	✓	✓
LR (EMZ)	✓	✓	✓	✓	X	X
OR (CCM)	✓	✓	✓	✓	✓	✓

[MI] Marginal Independence (Edwards, 1963, JRSSA).
For all $\lambda > 0$,

$$
\begin{array}{c|c|c}
 & \theta_1 & \theta_2 \\ \hline
\theta_1 & a & b \\ \hline
\theta_2 & c & d
\end{array}
\sim_A
\begin{array}{c|c|c}
 & \theta_1 & \theta_2 \\ \hline
\theta_1 & \lambda a & b \\ \hline
\theta_2 & \lambda c & d
\end{array}
\sim_A
\begin{array}{c|c|c}
 & \theta_1 & \theta_2 \\ \hline
\theta_1 & a & \lambda b \\ \hline
\theta_2 & c & \lambda d
\end{array}
$$

$$
\sim_A
\begin{array}{c|c|c}
 & \theta_1 & \theta_2 \\ \hline
\theta_1 & \lambda a & \lambda b \\ \hline
\theta_2 & c & d
\end{array}
\sim_A
\begin{array}{c|c|c}
 & \theta_1 & \theta_2 \\ \hline
\theta_1 & a & b \\ \hline
\theta_2 & \lambda c & \lambda d
\end{array}
$$

- MI implies SI, TSym, SSym.
- MMon and MI together imply DMon and ODMon.

Odds ratio: unique total order

Proposition

The unique total order that satisfies MI (which implies SI, TSym, SSym) and MMon (which together with MI implies DMon and ODMon) is the order induced by the odds ratio $(ad)/(bc)$.
In other words, the unique index, up to monotonic transformation, that satisfies MI and MMon is the odds ratio.

Structural interpretation of the odds ratio

- Consider an underlying transferable-utility matching model of men $X \ni x$ and women $Y \ni y$.
- Suppose the surplus generated by a match between man x of type θ_i and woman y of type θ_j takes the separable form

$$s_{xy} = Z^{\theta_i \theta_j} + \epsilon_x^{\theta_j} + \epsilon_y^{\theta_i},$$

  where $Z^{\theta_i \theta_j}$ is a deterministic component depending on types and ϵ's are random shocks reflecting unobserved heterogeneity among individuals.
- If ϵ's follow T1EV (Choo and Siow, 2006), then the supermodular core equals twice the odds ratio:

$$Z^{\theta_i \theta_i} + Z^{\theta_j \theta_j} - Z^{\theta_i \theta_j} - Z^{\theta_j \theta_i} = 2\frac{ad}{bc}.$$

- The odds ratio directly reflects changes in surplus (irrespective of changes in marginal distribution).

Call $M = (a, b, c, d) \gg 0$ a full-support market. Call M and M' a full-support decomposition of a full-support market $M + M'$ if $M \gg 0$ and $M' \gg 0$.

[Dec] Decomposability. For any full-support decomposition of any full-support market, the assortativity of the market is the population-weighted average of the assortativity of the two markets decomposed from the market. For $M = (a, b, c, d) \gg 0$ and $M' = (a', b', c', d') \gg 0$,

$$I(M + M') = \frac{|M|}{|M + M'|} I(M) + \frac{|M'|}{|M + M'|} I(M'),$$

where $|M| = a + b + c + d$ and $|M'| = a' + b' + c' + d'$.

- Dec implies SI.
- Dec, SI, TSym, and MMon imply DMon and ODMon.

Normalized trace: unique cardinal measure

Proposition

The unique index, up to linear transformation, that satisfies SI, TSym, SSym, DMon, ODMon, and Dec is **normalized trace (proportion of like pairs) with boundary adjustment**

$$I_{tr}(a,b,c,d) = \begin{cases} 1 & \text{if } bc = 0 \\ \frac{a+d}{a+b+c+d} \in (0,1) & \text{if } abcd \neq 0 \\ 0 & \text{if } ad = 0 \end{cases}$$

Normalized trace: unique cardinal measure

Proposition

The unique index, up to linear transformation, that satisfies SI, TSym, SSym, MMon, and Dec is **normalized trace (proportion of like pairs) with boundary adjustment**

$$I_{tr}(a,b,c,d) = \begin{cases} 1 & \text{if } bc = 0 \\ \frac{a+d}{a+b+c+d} \in (0,1) & \text{if } abcd \neq 0 \\ 0 & \text{if } ad = 0 \end{cases}$$

[Dec’] Generalized Decomposability.
For any $M = (a, b, c, d)$ and $M' = (a', b', c', d')$,

$$I(M + M') = \frac{|M|}{|M + M'|} I(M) + \frac{|M'|}{|M + M'|} I(M').$$

Proposition

The unique index, up to linear transformation, that satisfies SI, TSym, SSym, DMon’, ODMon’, and Dec’ is **normalized trace (proportion of like pairs)**

$$I_{tr}(a, b, c, d) = \frac{a + d}{a + b + c + d}.$$

Call $M = (a, b, c, d) \gg 0$ a full-support market. Call M and M' a full-support decomposition of a full-support market $M + M'$ if $M \gg 0$ and $M' \gg 0$.

[RDec] Random Decomposability. For any full-support decomposition of any full-support market, the assortativity of the market is a weighted average of the assortativity of the two markets decomposed from the market, where the weight is the expected number of assortative pairs:

$$r(M) \equiv \frac{a+b}{|M|}\frac{a+c}{|M|}|M| + \frac{d+b}{|M|}\frac{d+c}{|M|}|M|.$$

For $M = (a, b, c, d) \gg 0$ and $M' = (a', b', c', d') \gg 0$,

$$I(M + M') = \frac{r(M)}{r(M + M')}I(M) + \frac{r(M')}{r(M + M')}I(M'),$$

where $|M| = a + b + c + d$ and $|M'| = a' + b' + c' + d'$.

EMZ's likelihood ratio

Proposition

An index satisfies SI, TSym, SSym, MMon, and RDec if and only if it is proportional to likelihood ratio

$$
\begin{aligned}
LR(M) &= \frac{(a+b)(a+c)LR_1(M) + (d+b)(d+c)LR_2(M)}{(a+b)(a+c) + (d+b)(d+c)} \\
&= \frac{a+d}{\frac{a+b}{|M|}\frac{a+c}{|M|}|M| + \frac{d+b}{|M|}\frac{d+c}{|M|}|M|} = \frac{\text{observed } \#(\theta_1\theta_1 + \theta_2\theta_2)}{\text{random baseline}}.
\end{aligned}
$$

Axioms for binary types

	equivalence conditions			monotonicity conditions			
	SI	TSym	SSym	MMon	DMon	ODMon	unique
LR_i (EMZ)	✓	X	✓	✓	✓	✓	
LR (EMZ)	✓	✓	✓	✓	X	X	RDec
OR (CCM)	✓	✓	✓	✓	✓	✓	MI
trace	✓	✓	✓	✓	✓	✓	Dec

Singles and same-sex couples

Singles

Consider the markets with singles. Expand the table without singles by adding a row and a column to indicate the singles.

$$\widetilde{M} = \begin{array}{cccc} m \backslash w & \theta_1 & \theta_2 & \emptyset \\ \theta_1 & M_{11} & M_{12} & M_{10} \\ \theta_2 & M_{21} & M_{22} & M_{20} \\ \emptyset & M_{01} & M_{02} & \end{array}$$

Singles examples

If we do not consider singles, the following three tables give us the same assortativity: (p=pairs)

$\widetilde{M}_1$

$m\backslash w$	θ_1	θ_2	$\emptyset$
θ_1	$50p$	0	25
θ_2	0	$50p$	0
$\emptyset$	25	0	

$\widetilde{M}_2$

$m\backslash w$	θ_1	θ_2	$\emptyset$
θ_1	$50p$	0	0
θ_2	0	$50p$	0
$\emptyset$	0	0	

$\widetilde{M}_3$

$m\backslash w$	θ_1	θ_2	$\emptyset$
θ_1	$75p$	0	0
θ_2	0	$50p$	0
$\emptyset$	0	0	

If we consider singles, arguably,

- $\widetilde{M}_2$ is more assortative than $\widetilde{M}_1$ because there are no singles who could have matched with each other;
- $\widetilde{M}_3$ is more assortative than $\widetilde{M}_1$ because unmatched individuals in $\widetilde{M}_1$ are assortatively matched in $\widetilde{M}_3$.

Normalized trace with singles

[SMon] Singles Monotonicity.
Consider $\widetilde{M} = (M_{ij})_{i,j\in\{0,1,2\}}$ *and* $\widetilde{M}' = (M'_{ij})_{i,j\in\{0,1,2\}}$. *When* $M_{i0} > M'_{i0}$ *for an* i *and* $M_{jk} = M'_{jk}$ *for any other combination of* j *and* k, $\widetilde{M} \succ_A \widetilde{M}'$.

Proposition

Normalized trace with singles is the unique index (up to linear transformation) that satisfies SI, TSym, and SSym, DMon0, ODMon0, Dec0, and SMon.

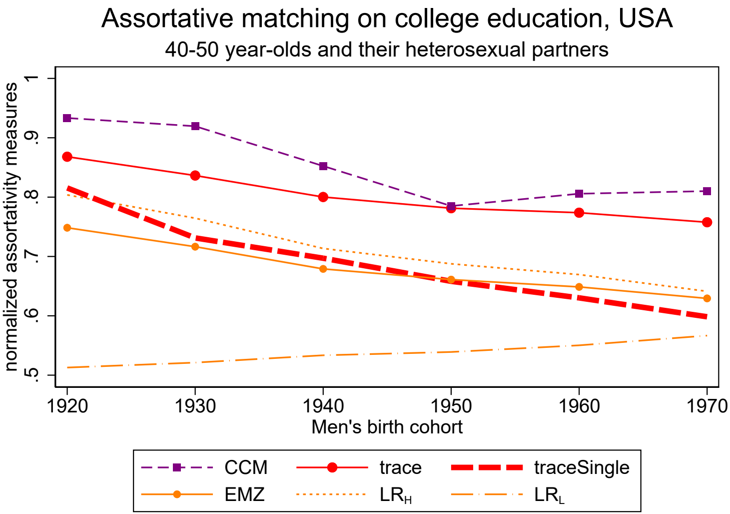
$$\widetilde{I}_{tr}(\widetilde{M}) = \frac{\mathrm{tr}(\widetilde{M})}{|\widetilde{M}|}.$$

In this case, $\widetilde{I}_{tr}(\widetilde{M}_1) = 200/250 = 4/5$ and $\widetilde{I}_{tr}(\widetilde{M}_2) = \widetilde{I}_{tr}(\widetilde{M}_3) = 1$.

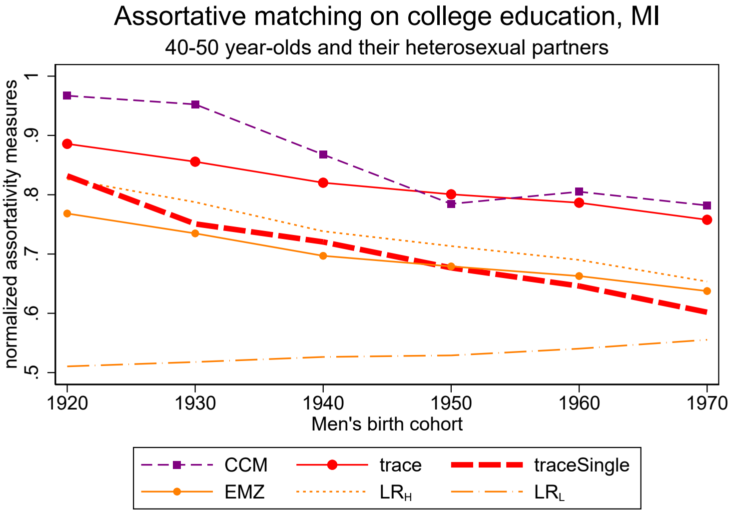
Axioms beyond binary types

	equivalence conditions	monotonicity conditions	singles	same-sex	multiple types
LR_i (EMZ)	X	✓	✓	✓	✓
LR (EMZ)	✓	X	✓	✓	✓
OR (CCM)	✓	✓	X	X	X
trace	✓	✓	✓	✓	✓

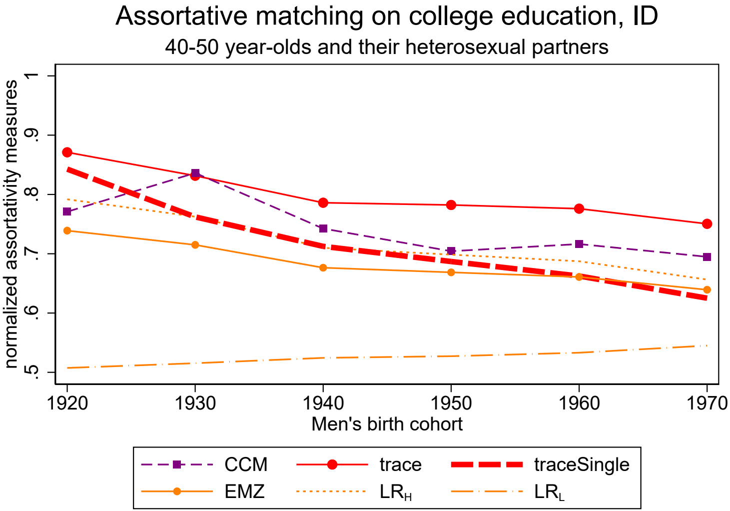
Evidence from US



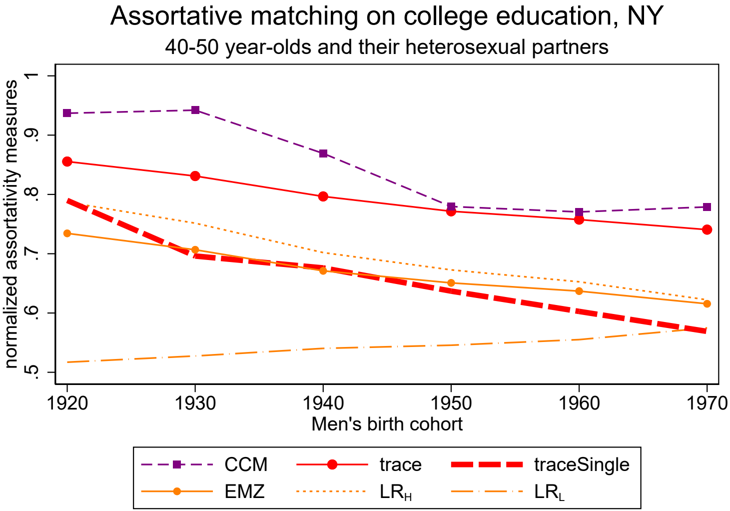
Evidence from MI



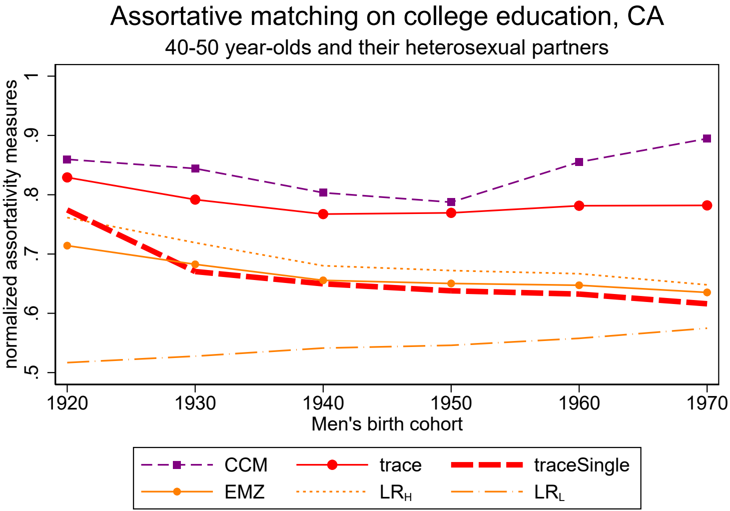
Evidence from ID



Evidence from NY



Evidence from CA



What is a marriage market in practice?

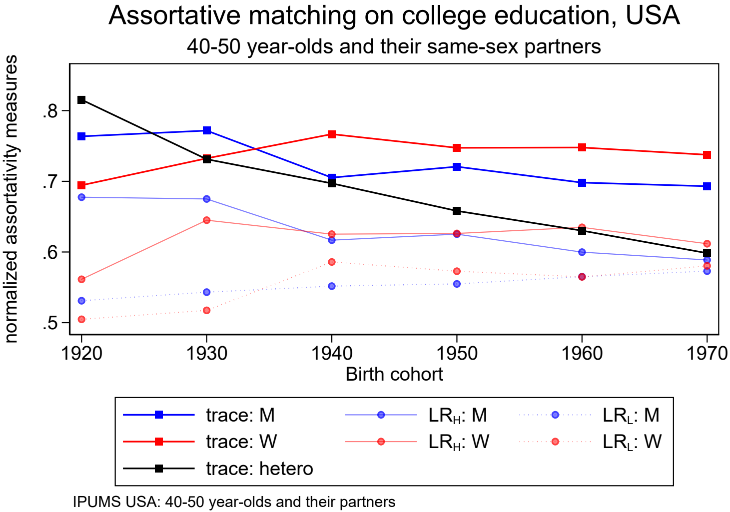
- 40-50 year-olds and their spouses
- 40-50 year-old men and their wives
- 40-50 year-old women and their husbands
- all those of various birth cohorts who marry in the same year/decade
- cohabitation versus marriage

Normalized trace for same-sex couples

Proposition

Consider same-sex matching of binary types. The unique index that satisfies SI, TSym, SSym, DMon, ODMon, and Dec is the normalized trace, up to linear transformation.

Evidence for same-sex couples



Multiple discrete types

Multiple discrete types

educd	
00	N/A or no schooling
01	Nursery school to grade 4
02	Grade 5, 6, 7, or 8
03	Grade 9
04	Grade 10
05	Grade 11
06	Grade 12
07	1 year of college
08	2 years of college
09	3 years of college
10	4 years of college
11	5+ years of college

Normalized trace in multiple types

Proposition

Suppose there are N types: $\theta_1, \theta_2, \ldots, \theta_N$. The unique index that satisfies SI, TSym, SSym, DMon, ODMon, and Dec is the normalized trace, up to linear transformation.

	θ_1	θ_2	θ_3
θ_1	M_{11}	M_{12}	M_{13}
θ_2	M_{21}	M_{22}	M_{23}
θ_3	M_{31}	M_{32}	M_{33}

Robustness to categorization

[RC] Robustness to Categorization.
Let $M|_C$ denote the market given categorization C. $M \succeq_A M'$ if and only if $M|_C \succeq_A M'|_C$ for any categorization C, and $M \succ_A M'$ if and only if $M|_C \succ_A M'|_C$ for any categorization C.

	θ_1	θ_2	θ_3
θ_1	M_{11}	M_{12}	M_{13}
θ_2	M_{21}	M_{22}	M_{23}
θ_3	M_{31}	M_{32}	M_{33}

	θ_1	θ_2	θ_3
θ_1	M_{11}	M_{12}	M_{13}
θ_2	M_{21}	M_{22}	M_{23}
θ_3	M_{31}	M_{32}	M_{33}

No complete assortativity order on multi-type M

Proposition
No total order satisfies MMon and RC.

Proof by counterexample. Consider markets

$$M = \begin{array}{c|c|c} 1/9 & 1/9 & 1/9 \\ \hline 1/9 & 1/9 & 1/9 \\ \hline 1/9 & 1/9 & 1/9 \end{array} \text{ and } M' = \begin{array}{c|c|c} 1/9-\epsilon & 1/9+\epsilon & 1/9 \\ \hline 1/9+\epsilon & 1/9 & 1/9-\epsilon \\ \hline 1/9 & 1/9-\epsilon & 1/9+\epsilon \end{array}$$

When we group θ_1 and θ_2,

$$M|_{(\{1,2\}\{3\})} = \begin{array}{c|c} 4/9 & 2/9 \\ \hline 2/9 & 1/9 \end{array} \prec_A M'|_{(\{1,2\}\{3\})} = \begin{array}{c|c} 4/9+\epsilon & 2/9-\epsilon \\ \hline 2/9-\epsilon & 1/9+\epsilon \end{array}$$

When we group θ_2 and θ_3,

$$M|_{(\{1\}\{2,3\})} = \begin{array}{c|c} 1/9 & 2/9 \\ \hline 2/9 & 4/9 \end{array} \succ_A M'|_{(\{1\}\{2,3\})} = \begin{array}{c|c} 1/9-\epsilon & 2/9+\epsilon \\ \hline 2/9+\epsilon & 4/9-\epsilon \end{array}$$

No complete assortativity order on multi-type M

Proposition

No total order satisfies DMon+ODMon and RC.

Proof by counterexample. Consider markets

$$M = \begin{array}{c|c|c} 1/9 & 1/9 & 1/9 \\ \hline 1/9 & 1/9 & 1/9 \\ \hline 1/9 & 1/9 & 1/9 \end{array} \text{ and } M' = \begin{array}{c|c|c} 1/9-\epsilon & 1/9+\epsilon & 1/9 \\ \hline 1/9+\epsilon & 1/9 & 1/9-\epsilon \\ \hline 1/9 & 1/9-\epsilon & 1/9+\epsilon \end{array}$$

When we group θ_1 and θ_2,

$$M|_{(\{1,2\}\{3\})} = \begin{array}{c|c} 4/9 & 2/9 \\ \hline 2/9 & 1/9 \end{array} \prec_A M'|_{(\{1,2\}\{3\})} = \begin{array}{c|c} 4/9+\epsilon & 2/9-\epsilon \\ \hline 2/9-\epsilon & 1/9+\epsilon \end{array}$$

When we group θ_2 and θ_3,

$$M|_{(\{1\}\{2,3\})} = \begin{array}{c|c} 1/9 & 2/9 \\ \hline 2/9 & 4/9 \end{array} \succ_A M'|_{(\{1\}\{2,3\})} = \begin{array}{c|c} 1/9-\epsilon & 2/9+\epsilon \\ \hline 2/9+\epsilon & 4/9-\epsilon \end{array}$$

Summary

- Likelihood ratio is the unique index (up to linear transformation) that satisfies SI, TSym, SSym, MMon and Random Decomposability.
  - fails DMon and ODMon
- Odds ratio is the unique total order on binary-types markets that satisfies MMon and Marginal Independence (implies SI, TSym, SSym).
  - no analogous measure on multi-type markets; a local measure of assortativity
- Normalized trace is the unique index (up to linear transformation) that satisfies SI, TSym, SSym, MMon, and Decomposability.
  - naturally extends to multi-type markets, markets with singles, and one-sided markets.
- No total order satisfies MMon and Robustness to Categorization.

THANK YOU!

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