Decentralized Matching with Transfers: Experimental and Noncooperative Analyses

Simin He[*]   Jiabin Wu[†]   Hanzhe Zhang[‡]

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Abstract

We experimentally and theoretically study decentralized matching markets with transfers of Shapley and Shubik (1972) and Becker (1973). Our contributions are threefold. First, our experiment shows that matching and aggregate surplus achieved are affected by whether efficient matching is assortative and whether the pairwise Nash-Rubinstein bargaining outcome is stable (i.e., in the core), neither of which is predicted to matter in the canonical theory. Second, in markets with equal numbers of participants on the two sides, individual payoffs in our and others' experiments cannot be explained by existing refinements of the core, but are consistent with our noncooperative model's unique prediction. Third and finally, in markets with unequal numbers of participants on the two sides, noncompetitive outcomes—that is, subjects on the long side of the market do not compete fully—are not captured by the canonical theory, but by our noncooperative model. Our investigation suggests that small-size matching markets may be better understood as multilateral bargaining problems.

Keywords: decentralized matching, matching with transfers, assignment games, bargaining
JEL: C71, C72, C78, C90

1 Introduction

The seminal transferable-utilities (TU) two-sided matching model of Shapley and Shubik (1972) and Becker (1973) has been widely applied to study marriage and labor markets, both theoretically and empirically.[1] There is nascent interest in testing the model's predictions on stable/core matching and bargaining outcomes[2] in laboratory experiments.[3] We conduct one of the first comprehensive experiments on the TU matching model, and provide noncooperative analyses of the TU matching market to rationalize experimental findings that are inconsistent with predictions of the canonical cooperative theory.

Our experimental investigation starts with the smallest possible *balanced* markets with nontrivial matching possibilities—that is, both sides have three subjects—and moves on to study *unbalanced* markets with three subjects on one side and four on the other. To mimic the TU matching market, we minimize frictions by allowing the subjects to propose to anyone on the opposite side of the market with any division of the surplus. To avoid market unraveling, we specify that all matches do not become permanent until the end. To avoid congestion, we allow subjects to react if they suddenly become unmatched near the end of the game.

According to the canonical theory, different market structures (i.e., surplus configurations) should not affect people's ability to reach a stable—and equivalently, efficient—matching outcome. However, in practice, several factors could. We first investigate how two features affect matching and bargaining outcomes: (i) whether the efficient matching is assortative and (ii) whether an equal split of each efficiently matched pair's surplus is stable/in the core. To do so, we use a two-by-two comparison of four balanced markets. First, we hypothesize that the configurations that admit an assortative stable matching are arguably more straightforward and intuitive compared with those that do not admit one, especially since sorting has long been observed empirically; it is therefore important to investigate whether subjects in a controlled experiment indeed find it easier to match when assortative matching is theoretically supported. Second, we note that an equal split of every efficiently matched pair's surplus is the pairwise Nash (1950) bargaining outcome, and is also the limit outcome of pairwise Rubinstein (1982) bargaining when subjects are infinitely patient, so subjects may find it easier—and also strategically

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[1]See recent surveys and monographs for a comprehensive overview of the model and its applications: Galichon (2016); Chiappori and Salanié (2016); Chade et al. (2017); and Chiappori (2017). The model has been applied to explain the commonly observed assortative matching in characteristics such as education, height, race, and income (Becker, 1973); cross-country differences in income and growth (Kremer, 1993); increase in CEO pay (Gabaix and Landier, 2008); and college and career choices (Chiappori et al., 2009; Zhang, 2020, 2021; Zhang and Zou, 2020).

[2]A matching and bargaining outcome is stable—equivalently, in the core—if no pair of agents has an incentive to deviate from their respective partners to form a new pair.

[3]A laboratory experiment has the advantage of creating a controlled environment to understand the scope and limitations of the theory, despite the small number of participants and the low incentives provided (Roth, 2015). Related papers are elaborated on in the literature review.

more plausible—to reach and maintain such an outcomes if it is also in the core.[4]

We find that the probability of being matched and the probability of achieving efficient matching are significantly higher in markets with equal splits in the core and, to a lesser extent, in markets with assortative matching. In markets with equal splits in the core, most subjects propose equal division, and most accepted proposals feature equal division. In the other markets, equal division is neither commonly proposed nor commonly accepted. To understand the pattern of proposals in all the cases, we extend the bilateral bargaining model of Rubinstein (1982) to the matching market (related to the setup of Rubinstein and Wolinsky (1990), but allowing for more general surplus configurations), which captures the dynamic bargaining process in our experimental design. We find that the bargaining model features a unique equilibrium when the delay friction is sufficiently small.

Despite a large theoretical core in the assignment game, we find that the agents who efficiently match tend to arrive at certain bargaining outcomes in the core. Average experimental payoffs coincide with the payoffs in the unique equilibrium of our noncooperative model as delay frictions become vanishingly small (Figure 1a). In comparison, existing single-valued and set-valued refinements of the core do not coincide with the experimental payoffs. Whenever equal-splits is in the core, it is our predicted outcome as frictions vanish. It is also the outcome reached in the experiment, because each pair essentially engages in Rubinstein bargaining with their partner, and outside options do not influence their bargaining outcomes in equilibrium. This result helps explain the widespread observation of equal splits without behavioral assumptions.[5] When equal-splits is not in the core, outside threats influence players' bargaining power with their partners, and our noncooperative model incorporates these outside options.

Finally, we investigate unbalanced matching markets by duplicating the agent with the lowest bargaining power in each of the four balanced markets, so that there are three agents on one side and four agents on the other. According to the canonical theory, one would expect competition between the two duplicate agents to drive down their payoffs, even to zero. Interestingly, their payoffs rarely reach zero in the experiment. Their payoffs do not always differ from their payoffs in the balanced markets; in these instances, it is as if there is no competition. Even when their payoffs are lower than their payoffs in the balanced markets, they are significantly above zero. In

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[4]Additional potential reasons include, but are not limited to, complexity, social preferences, and focal points. Moreover, when equal-splits is not in the core, inequality aversion may prohibit people from forming a pair. See Fehr and Schmidt (1999) and Bolton and Ockenfels (2000), who first introduce inequality aversion in the economic literature. Take the marriage market as an example. Suppose a man and a woman tie the knot (and it is a stable matching according to the TU matching model), whereby they have to divide their joint surplus quite unequally. The one who receives a lower share may deem the division unfair and would rather end the relationship, even though he or she cannot do better by matching with someone else. Such a phenomenon can be explained by inequality aversion.
[5]Recent papers by Elliott and Nava (2019) and Talamàs (2020) take a noncooperative approach to matching markets but consider different bargaining protocols and agent replenishment in the market. Both papers obtain similar conclusions regarding the stability of equal splits in markets with equal splits in the core.

Figure 1: Average payoffs in efficient matching
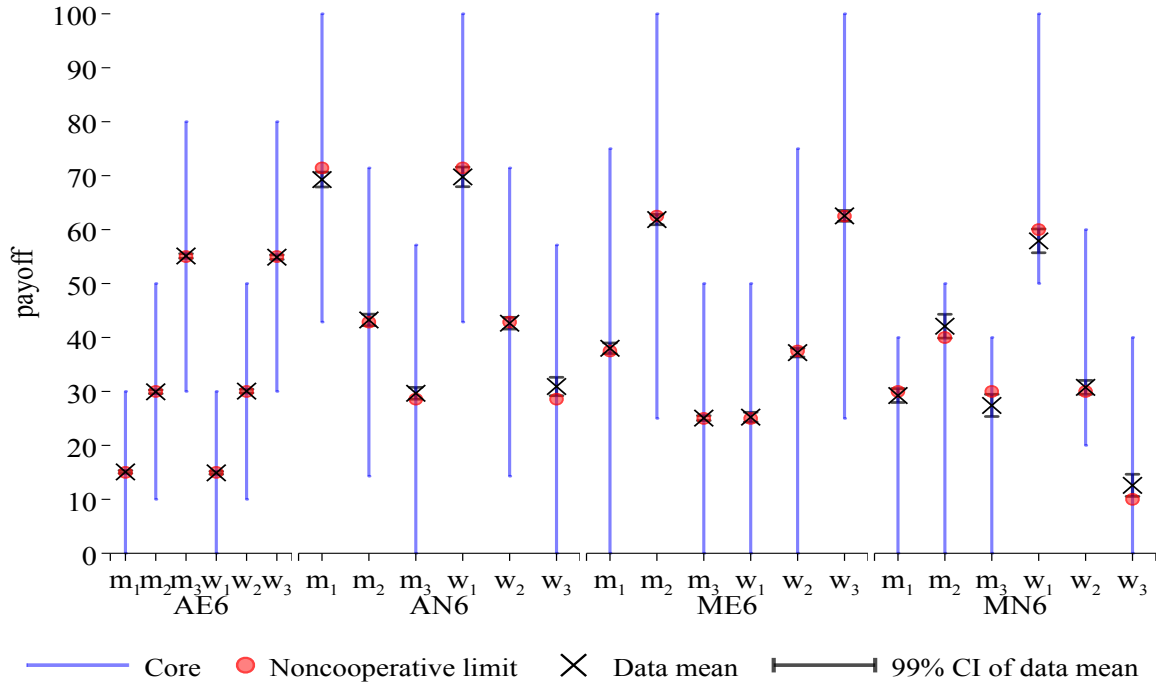
(a) Balanced markets

(b) Unbalanced markets
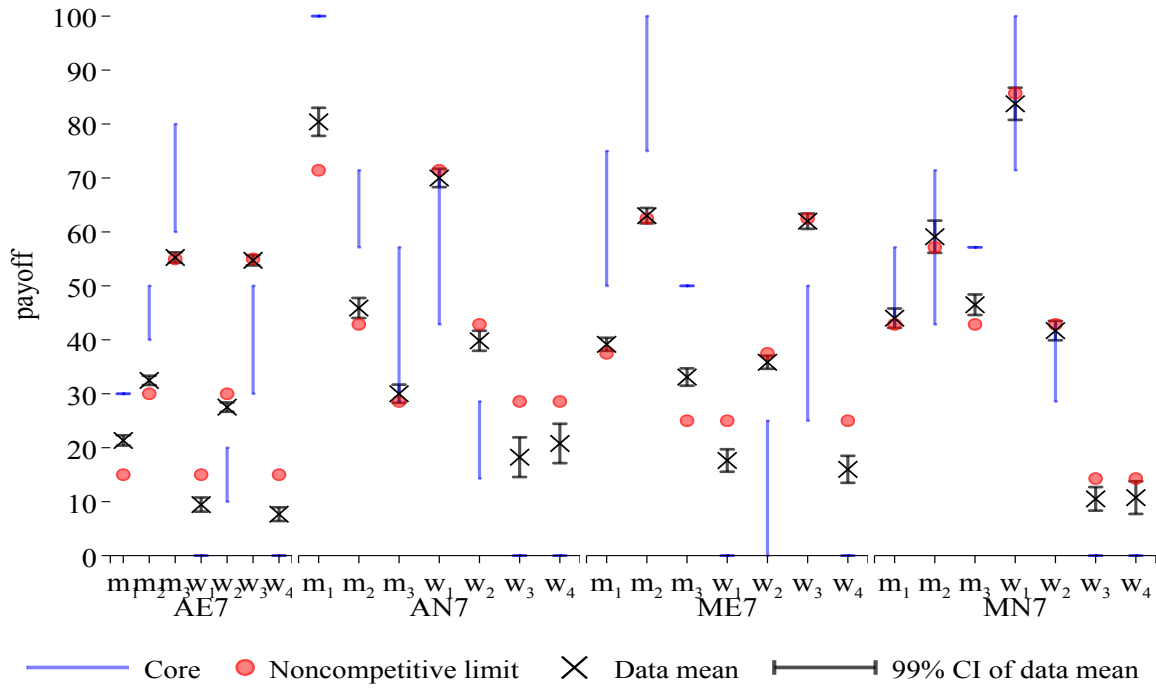
Note. The experimental payoffs in the balanced markets are perfectly predicted by the limit payoffs in our noncooperative model. The experimental payoffs in the unbalanced markets are between the competitive and noncompetitive equilibrium payoffs in our noncooperative model, and are not in the core.

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our noncooperative model, a la folk theorem, there is a continuum of equilibria that speak to these experimental observations: (i) a class of competitive equilibria in which competitors get (near) zero payoffs, (ii) a class of noncompetitive equilibria in which there is essentially no competition between the competitors, and (iii) a class of partially competitive equilibria in which competitors get positive payoffs between the previous two equilibrium payoffs.

Most matching experiments focus on nontransferable-utilities (NTU) matching models following Gale and Shapley (1962), and take a market-design perspective to understand stability, efficiency, and strategyproofness of different algorithms implemented by the centralized clearing house; see Roth (2015) and Hakimov and Kübler (2019) for recent surveys. A few studies consider two-sided decentralized NTU markets in which both sides can make offers: Echenique and Yariv (2013); Chen et al. (2018); and Pais et al. (2020). A few recent experimental studies of trading markets find that in the absence of a competitive equilibrium, markets conform to stable outcomes predicted by matching theory (Hatfield et al., 2012, 2016; Plott et al., 2019).[6]

Several papers can be considered as experimentally testing the TU matching model. Nalbantian and Schotter (1995) set up an experiment to mimic baseball free agency with three "managers" and three "players" phoning one another to negotiate salaries, which is effectively a decentralized TU matching market with incomplete information. In their experiment, subjects do not have complete information on the matching surpluses, and they negotiate by phone to reach permanent agreements. In contrast, in our experiment, subjects have complete information on the matching surpluses and offers are not permanent, which reduces matching frictions. The negotiation process is more structured, which allows us to obtain rich information on the details of the subjects' proposals and their decisions to accept or reject. Otto and Bolle (2011) study the outcome of six different 2-by-2 matching markets with price negotiation and verbal communication. In contrast, our paper focuses on decentralized two-sided matching markets that do not feature verbal negotiation but allow the possibility of negotiation through strategic acceptance/rejection of competing offers from potential matches. Dolgopolov et al. (2020) study a 3-by-3 assignment matching market, and investigate the market outcomes under three institutions different from ours (double auctions, posted prices, and decentralized communication). They find that Nash outcomes are commonly observed under double-auction rules, though efficient outcomes are not always achieved; however, markets with communication achieve higher efficiencies on average. Agranov et al. (2021) compare matching under complete and incomplete information, and they find that incomplete information and submodularity jointly hinder the efficiency and stability of matching. However, their comparison is limited to two assortative markets with and without equal splits in the core. Agranov and Elliott (2021) consider three 2-by-2 markets. However, in their decentralized bargaining process, if a pair is matched, both players leave the market (mim-

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[6]See Kelso and Crawford (1982) and Hatfield et al. (2013) for theoretical studies of decentralized markets.

icking [Elliott and Nava](2019)). Hence, the incentives in their setting differ from ours. [Leng](2020) follows the bargaining protocol of [Perry and Reny](1994) in experiments for 2-by-1 markets and finds that, contrary to the theoretical prediction but similar to the experimental results of our 3-by-4 markets, the core outcome is not achieved, in that agents on the short side of the market do not fully capture all of the surpluses from trade.

In sum, our paper differs from these papers in different aspects, and consequently contributes to the literature in three ways. First, we manipulate the matching configurations to systematically investigate the impact of two features—equal splits in the core and assortativity—on matching and bargaining outcomes. Second, we find that agents tend to achieve certain bargaining outcomes in the core in balanced matching markets. Third and finally, we find that agents can achieve an array of bargaining outcomes outside and inside the core in unbalanced matching markets. Our noncooperative model, which mimics our decentralized bargaining process without verbal communication, can help rationalize both the unique observations in balanced markets and the multiplicities in unbalanced markets.

The remainder of the paper is organized as follows. Section 2 presents definitions and testable implications of the canonical TU matching model. Section 3 introduces the experimental design, procedures, and hypotheses. Section 4 presents experimental results on matching and bargaining outcomes. Section 5 discusses the noncooperative model. Section 6 discusses other experimental results, and Section 7 concludes.

2 Canonical cooperative theory

We briefly go over the canonical cooperative TU matching model based on [Shapley and Shubik](1972) and [Becker](1973) to define notations and terminologies and to introduce its main testable implications. There are two sides consisting of n_M men, $M = \{m_1, \cdots, m_{n_M}\}$, and n_W women, $W = \{w_1, \cdots, w_{n_W}\}$. The entire set of players is denoted by $I = M \cup W$. We say a market is **balanced** if $n_M = n_W$ and **unbalanced** otherwise. For any man $m \in M$ and woman $w \in W$, they produce a total surplus of s_{mw}. The surpluses of all pairs can be summarized by a surplus matrix $s = \{s_{mw}\}_{m \in M, w \in W}$. Each agent gets zero when unmatched, and gets a payoff that depends on the division of the surplus when matched. Note that the surplus matrix s describes the entire market, so we can refer to a matching market simply by s.

Definition 1 (Stable outcome). *A **stable outcome** of market s is described by a **stable matching** $\mu : I \to I \cup \{\emptyset\}$ and vectors of **stable/core payoffs** $u : M \to \mathbb{R}$ and $v : W \to \mathbb{R}$ such that (i) (individual rationality) each person gets at least as much as staying single: $u_m \geqslant 0$ for all $m \in M$ and $v_w \geqslant 0$ for all $w \in W$; (ii) (surplus efficiency) each couple exactly divides up the surplus: $u_m + v_w = s_{mw}$ if $m = \mu(w)$ and $w = \mu(m)$; and (iii) (no blocking pair condition) each couple divides*

the total surplus in such a way that no man and woman pair has an incentive to form a new pair: $u_m + v_w \geqslant s_{mw}$ for any $m \in M$ and $w \in W$.

There always exists a stable outcome, which provides a benchmark theoretical prediction for each matching market. In addition, stable matching and payoffs satisfy some easily testable properties, which we summarize below.

Proposition 1 (Stable matching). *A matching is stable if and only if it is efficient; that is, it maximizes the total surplus. Equivalently, a matching μ is stable if and only if it is the solution to the linear programming problem $\max_{\mu \in \mathcal{M}} \sum_{m \in M} s_{m\mu(m)}$, where $\mathcal{M}$ is the set of feasible matching.*

Corollary 1 (Full matching). *If every element in the surplus matrix is positive, a stable matching is a **full matching**; that is, the number of matched pairs in the stable outcome reaches the maximal possible number.*

Corollary 2 (Efficient matching). *If there is a unique efficient matching, this matching is the unique matching in the stable outcome.*

A key observation of Becker (1973) is that if surplus matrix s (possibly with reordering) satisfies supermodularity, then a stable matching is **assortative**, in that the highest ranked man is matched with the highest ranked woman, the second highest ranked man is matched with the second highest ranked woman, and the nth highest ranked man is matched with the nth highest ranked woman. To slightly abuse terminology for expositional convenience, we say that the surplus matrix is assortative if it can be rearranged to satisfy supermodularity. To formally define an assortative surplus matrix, we need to first define a reordered surplus matrix.

Definition 2 (Reordered surplus matrix). *The surplus matrix $\widetilde{s}$ is a **reordered surplus matrix** of s if there exists a pair of permutations $\pi_M : M \to M$ and $\pi_W : W \to W$ such that $\widetilde{s}_{\pi_M(m)\pi_W(w)} = s_{mw}$ for any $m \in M$ and any $w \in W$.*

Definition 3 (Assortative surplus). *The surplus matrix s is **assortative** if there exists a reordered surplus matrix of s, say $\widetilde{s}$, such that*

$$\widetilde{s}_{mw} + \widetilde{s}_{m'w'} > \widetilde{s}_{m'w} + \widetilde{s}_{m'w} \quad \forall m, m' \in M \text{ and } w, w' \in W \text{ s.t. } m > m' \text{ and } w > w'. \tag{A}$$

*We say the surplus matrix s is **nonassortative** or **mixed** if it is not assortative; that is, there does not exist a reordered surplus matrix that satisfies Condition (A).[7]*

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[7]We say the reordered matrix $\widetilde{s}$ is **positive-assortative** (supermodular in Agranov et al. (2021)) if $\forall m, m' \in M, \forall w, w' \in W : m > m' \Rightarrow \widetilde{s}_{mw} \geq (\leq)\widetilde{s}_{m'w}$ and $w > w' \Rightarrow \widetilde{s}_{mw} \geq (\leq)\widetilde{s}_{mw'}$; or **negative-assortative** (submodular in Agranov et al. (2021)) if $\forall m, m' \in M, w, w' \in W : m > m' \Rightarrow \widetilde{s}_{mw} \geq (\leq)\widetilde{s}_{m'w}$, and $w < w' \Rightarrow \widetilde{s}_{mw} \leq (\geq)\widetilde{s}_{mw'}$. We can always reorder a positive-assortative matrix to be negative-assortative and vice versa (by reversing one side of the market), so positive-assortativeness and negative-assortativeness are defined on reordered matrices rather than on original matrices/markets.

Proposition 2 (Stable/core payoffs). *The set of stable payoffs (Becker, 1973), or equivalently the core (Shapley and Shubik, 1972), is the set of solutions of the following linear programming problem:*

$$\min \sum_{m \in M} u_m + \sum_{w \in W} v_w \quad s.t. \ u_m + v_w \geqslant s_{mw} \ \forall m \in M \ and \ w \in W.$$

With a finite number of agents, there is always a non-singleton set of stable payoffs (given a positive surplus matrix). An equal split of the surplus for each pair in the stable matching is not always in the core (as some surplus matrices chosen in the experiment will show).

Definition 4 (Equal-splits in the core). **Equal-splits is in the core** (**ESIC**) *of game s if the following outcome is stable: (i) efficient matching μ^* and (ii) payoffs $u_m = s_{m\mu^*(m)}/2$ for each matched $m \in M$ and $v_w = s_{\mu^*(w)w}/2$ for each matched $w \in W$. We say that* **equal-splits is <u>not</u> in the core** (**ESNIC**) *of game s otherwise.*

Other solutions. The core in general is a non-singleton set that generates a wide range of predictions. Many solutions—both set-valued and single-valued—attempt to provide more restrictions to refine the core. These solutions may coincide in "simple" symmetric and assortative markets with equal splits in the core, but differ in more "complicated" asymmetric and nonassortative markets without equal splits in the core. Figure 2 provides an illustration of the wide range of refined solutions for such a "complicated" 2-by-2 market. See Núñez and Rafels (2015) for a summary of relevant cooperative solutions.

Figure 2: Cooperative solutions for market $s = (6, 5; 2, 4)$
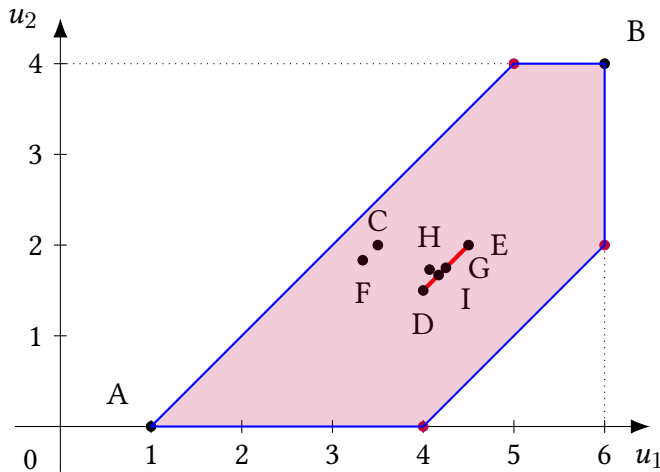
Note. We add cooperative solutions to Figure 1 of Núñez and Rafels (2015). Given that all solutions predict an efficient matching and the efficient matching is unique, row players' payoffs u_1 and u_2 summarize the solutions. The shaded area is the core, and the five points on the boundary are its extreme points (Shapley and Shubik, 1972). A at $(1, 0)$ is row-optimal allocation; B at $(6, 4)$ is column-optimal allocation; C at $(3.5, 2)$ is the fair division point (Thompson, 1980); line segment DE from $(4, 1.5)$ to $(4.5, 2)$ is the kernel (Rochford, 1984); F at $(3, 2)$ is the Shapley (1953) value; G at $(17/4, 7/4)$ is the nucleolus (Schmeidler, 1969); H at $(61/15, 26/15)$ is the centroid of the core; and I at $(25/6, 5/3)$ is median stable matching (Schwarz and Yenmez, 2011).

3 Experiment

3.1 Treatment design

In our treatment design, we use eight surplus configurations, as shown in Table 1. Each surplus configuration represents a different matching market. On the left are four balanced markets, and on the right are four unbalanced markets. The row players are marked by cold color squares and the column players are marked by warm color circles. We also refer the row to men and the column to women. For example, we use m_1 to represent the ■ (blue square) player in the later analysis.

Table 1: Surplus configurations in the experiment

	Balanced markets (6 players)						Unbalanced markets (7 players)							
	ESIC			ESNIC			ESNIC				ESNIC			
	AE6			AN6			AE7				AN7			
	🟣	🔴	🟡	🟣	🔴	🟡	🟣	🔴	🟡	⚪	🟣	🔴	🟡	⚪
■	30	40	50	90	80	70	30	40	50	30	90	80	70	70
■	40	60	80	80	60	40	40	60	80	40	80	60	40	40
■	50	80	110	70	40	10	50	80	110	50	70	40	10	10
	ME6			MN6			ME7				MN7			
	🟣	🔴	🟡	🟣	🔴	🟡	🟣	🔴	🟡	⚪	🟣	🔴	🟡	⚪
■	30	60	80	90	60	30	30	60	80	30	90	60	30	30
■	60	70	100	100	50	30	60	70	100	60	100	50	30	30
■	40	40	60	80	60	40	40	40	60	40	80	60	40	40

(The left two configuration blocks are labeled "Assortative"; the lower two blocks are labeled "Mixed".)

<u>A</u>ssortative: Efficient matching is assortative; <u>M</u>ixed: Efficient matching is not assortative; E<u>S</u>IC: equal split in the core; ES<u>N</u>IC: equal split not in the core

In the balanced markets, the double-underlined surpluses in each configuration show the pairings in the unique efficient matching. We vary the configurations in two dimensions: (i) whether the surplus matrix—and, as a consequence, the efficient matching pattern—is assortative (as defined in Definition 3), and (ii) whether equal-splits is in the core (as defined in Definition 4). Hence, each market (i) is <u>a</u>ssortative (A) or <u>m</u>ixed (M) and (ii) has <u>e</u>qual-splits in the core (<u>E</u>SIC or simply E) or equal-splits <u>n</u>ot in the core (ES<u>N</u>IC or simply N). We refer to the four configurations by AE6, AN6, ME6, and MN6. We also design the surpluses to provide some consistency across markets: The maximum total surplus that all agents can obtain is 200, the average total surplus that all agents can obtain is 180 if they are matched fully and randomly, and the minimum total surplus they can obtain if they are all matched is 160.

The only difference between the unbalanced and the balanced market is that in each market, there is one more warm color (circle) player, yielding a total of seven players in each market. Specifically, each of the four surplus matrices adds a column player by replicating the column

player that yields the lowest surplus in the corresponding balanced market setting. Though equal-splits is no longer in the core (because the duplicate players would get zero in the core), we refer to the four markets by AE7, AN7, ME7, and MN7, to make the connection with their balanced market counterpart clear.

We employ a between-subject design for the balanced and unbalanced markets, and a within-subject design for the four different configurations of each market type. That is, subjects play either the four balanced markets or the four unbalanced markets, but they play the four markets in different orders. Using the Latin square method,[8] for balanced and unbalanced markets, we each have four treatment orders:[9]

	1	2	3	4
Treatment 1	AE	AN	ME	MN
Treatment 2	MN	AE	AN	ME
Treatment 3	ME	MN	AE	AN
Treatment 4	AN	ME	MN	AE

At the beginning of the experiment, subjects are randomly selected to form a group (of six or seven), and this grouping remains fixed throughout the experiment. They stay anonymous, and their roles in the market can change from round to round. Subjects within a group play the four markets in the order corresponding to their assigned treatment. Each market is played for 7 rounds, so they play 28 rounds in total.[10] At the beginning of each round, each subject is randomly assigned a color that represents their role. A cold color (square) can only be matched with a warm color (circle). Each market lasts at least 3 minutes. Within the 3-minute interval, anyone can propose to anyone on the opposite side. To propose, a subject clicks the color they wish to propose to, and decides the division of surplus. The receiver of a proposal has 30 seconds to accept or reject. When the proposer is waiting for the response from the receiver, the proposer cannot make a new proposal to anyone. If a proposal is rejected, both sides are free to make and receive new offers.

If a proposal is accepted, a temporary match is reached; information on the temporary match

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[8]We thank Yan Chen for this suggestion.

[9]One may consider varying the appearance of each surplus matrix by reordering rows and columns to avoid the potential appearance bias that matches on the diagonal are more likely to form. However, each 6-player surplus matrix has $3!\times3! = 36$ ways of appearing, and each 7-player surplus matrix has $3!\times4!/2 = 72$ ways of appearing, so there are $4!\times(72^4+36^4) \approx 6.85\times10^9$ possible order and appearance treatments. It is unclear how to simultaneously vary the appearance of each matrix and the ordering of different matrices using a reasonable number of participants. In addition, Table 2 showing the raw distribution of matching outcomes does not suggest a higher frequency of diagonal pairs. Furthermore, the overall more efficient outcome in ME6 (off-diagonal efficient matching) over AN6 (diagonal efficient matching) suggests that the appearance bias may not have a statistically significant effect on matching and bargaining outcome.

[10]One reason we chose 7 rounds for each market is to ensure an ex ante equal opportunity for subjects in the unbalanced-markets setting, as one of 7 subjects is for sure unmatched and gets zero payoff in each round.

and division of the surplus is shown to everyone in the market. When a temporary match is reached, both subjects can still make and receive proposals. One can always break their current temporary match by reaching a new temporary match (either by proposing to a new person and being accepted, or by accepting another proposal). A market ends at the 3-minute mark and all temporary matches become permanent, unless someone gets released from a temporary match in the last 15 seconds; in that case, they have 15 additional seconds to make a new proposal. If another subject is bumped from their temporary match as a result of the new proposal, the bumped subject gets a chance to make a proposal, and this process continues until no new proposal is accepted. Subjects can see the history of final matches in previous rounds.[11]

3.2 Procedures

The experiment was conducted at the Shanghai University of Finance and Economics. Chinese subjects were recruited from the subject pool of the Economics Lab through Ancademy, a platform for social sciences experiments.[12] In total, 296 subjects participated in our experiment: 156 in the balanced markets and 140 in the unbalanced markets. Each subject participated only once. We ran 8 sessions for the balanced markets and 6 sessions for the unbalanced markets. In each session we ran 3−6 independent markets. For the balanced markets, the number of times each treatment order is used is 7, 7, 6, and 6, respectively, yielding 728 individual rounds of games. For the unbalanced markets, we used each treatment order 5 times, yielding 560 individual rounds of games. Subjects were mostly undergraduate students from various fields of studies.

The experiment was computerized using z-Tree (Fischbacher, 2007) and conducted in Chinese. Upon arrival, each subject was randomly assigned a card with their table number, and seated in the corresponding cubicle. Prior to the start of the experiment, subjects read and signed a consent form agreeing to their participation. All instructions were displayed on their computer screens. Control questions were conducted to check their understanding of the instructions. English translations of the instructions and screenshots are provided in Appendix A.

Subjects were paid the sum of their payoffs in 28 rounds at an exchange rate of 10 units of payoffs to 1 CNY in balanced markets. To keep the average earnings comparable between balanced and unbalanced markets, we lowered the exchange rate of the experimental currency from 12 to 10. Everything else is kept the same as in the balanced market. After finishing the experiment, subjects received their earnings in cash. Average earnings were 85 CNY (equivalent to about 12 USD, or about 20 PPP-adjusted USD) for the balanced markets, and 93 CNY for the

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[11]To be clear, historical information is based on roles (squares and circles) but not on individual experimental subjects, so there is no way to establish a bargaining style or reputation across periods. Subjects may learn better the overall structure of the game over time and consequently perform better (as suggested by experimental results), but they cannot learn about any particular individual over time.

[12]Most subjects installed and used the app on their phones.

unbalanced markets (equivalent to about 14 USD, or about 23 PPP-adjusted USD). Each session lasted around 2 hours.

4 Results

In this section, we focus on the two most important aspects of the model: (i) the aggregate outcomes of matching and surplus and (ii) individual payoffs. We discuss other experimental findings in Section 6.

4.1 Aggregate outcomes: matching and surplus

Table 2 presents a raw distribution of pairs, where each cell indicates the percentage (rounded to the nearest integer) of rounds with such a pair in the final matching outcome. The last row and column of each matrix show the percentage of rounds a player is unmatched.

Table 2: Frequency of pairs formed in the experiment

AE6 (columns: ●pink, ●red, ●yellow, unmatched)

	●	●	●	
■	86%	5%	3%	6%
■	8%	82%	4%	5%
■	3%	7%	86%	4%
	3%	5%	8%	

AN6

	●	●	●	
■	4%	22%	68%	6%
■	26%	51%	8%	15%
■	66%	10%	5%	18%
	3%	17%	19%	

AE7 (columns: ●pink, ●red, ●yellow, ○, unmatched)

	●	●	●	○	
■	53%	1%	1%	44%	1%
■	8%	72%	6%	9%	4%
■	0%	11%	88	0%	1%
	39%	16%	5%	47%	

AN7

	●	●	●	○	
■	1%	9%	46%	42%	1%
■	16%	59%	8%	9%	8%
■	76%	9%	0%	1%	14%
	6%	23%	46%	47%	

ME6

	●	●	●	
■	1%	80%	9%	11%
■	6%	9%	80%	5%
■	87%	3%	2%	8%
	7%	8%	9%	

MN6

	●	●	●	
■	23%	41%	9%	27%
■	69%	6%	8%	17%
■	7%	43%	38%	12%
	2%	10%	45%	

ME7

	●	●	●	○	
■	0%	83%	11%	0%	6%
■	9%	4%	74%	8%	4%
■	51%	1%	0%	44%	3%
	39%	11%	15%	48%	

MN7

	●	●	●	○	
■	23%	56%	6%	4%	11%
■	71%	2%	3%	4%	20%
■	1%	23%	32%	39%	5%
	4%	19%	59%	54%	

Note. Each cell not in the last row or column indicates the percentage of markets in which a pair has formed between the row player and column player. The last row contains the percentage of markets in which each respective column player is unmatched, and the last column contains the percentage for each row player.

The theory predicts that efficient matching is the only outcome in a frictionless setting (Proposition 1). Consequently, we have the following hypotheses: (a) every market sustains the maximum number of matches, (b) all pairs are socially efficiently matched, (c) maximal surplus is achieved, and (d) market features, such as whether matching is assortative and whether equal-splits is in the core, should not affect these outcomes. Subsequently, we derive and test these hypotheses regarding (a) number of matched pairs, (b) number of efficiently matched pairs, (c) total surplus, and (d) determinants of these outcomes.

4.1.1 Full matching

Hypothesis 1a (Full matching). *The number of matched pairs is the maximum feasible number.*

Figure 3: Number of matched pairs

(a) Average number of matched pairs
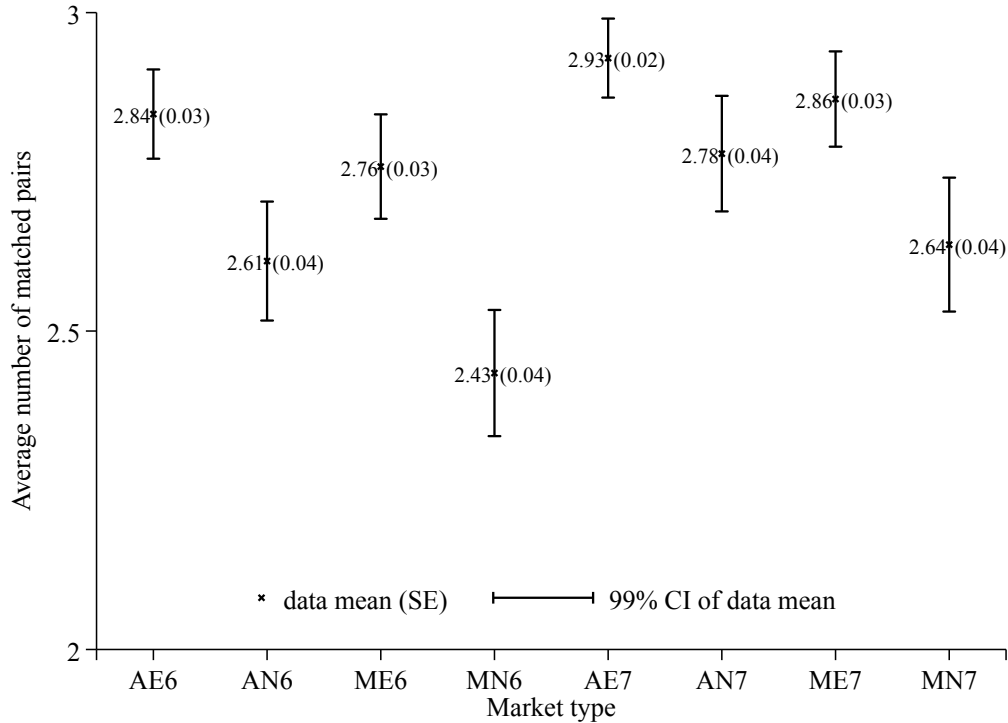
(b) Distribution of number of matched pairs

matched pairs	AE6	AN6	ME6	MN6	AE7	AN7	ME7	MN7	Total
	%	%	%	%	%	%	%	%	%
3 pairs	84.1	61.0	75.8	44.5	93.6	77.9	86.4	63.6	72.4
2 pairs	15.9	39.0	24.2	54.4	5.7	22.1	13.6	36.4	27.3
1 pair	0.0	0.0	0.0	1.1	0.7	0.0	0.0	0.0	0.2

Figure 3a shows the average number of matched pairs and its 99% confidence interval by market type. The average number of matched pairs ranges from 2.43 in MN6 to 2.93 in AE7. We can reject the hypothesis that the maximal number of matched pairs is reached. We observe some similar results in previous experiments. In the market of Nalbantian and Schotter (1995), a 3-by-3 market with nonassortative efficient matching but equal-splits in the core (i.e., a market of type ME6), 9.3% (14 of 150 potential matches) fail to match, which translates to 2.79 pairs, compared with 2.76 (0.04) in our experiment. In the markets of Agranov et al. (2021), who consider two

markets that fit the types of AE6 and AN6, the matching rate is higher than ours, possibly because they do not set a time limit for their matching markets (all matches become permanent after 30 seconds of inactivity).

Nonetheless, the market does not completely break down. Figure 3b shows the distribution of the number of matched pairs by market type. For every market type except MN6 (44.5%), three pairs are matched in over 60% of the games. For almost all games, there are more than two matched pairs. There is one matched pair for only three of the 1,288 games (less than 0.3% of all games): two MN6 games of the 728 balanced market games and one AE7 game of the 560 unbalanced market games. The existence of unmatched individuals suggests that frictions remain in our experimental design to prevent people from being fully matched. In our further discussion section, we take a detailed look at the unmatched individuals' behavior in the experiment and break down the possible reasons that individuals remain unmatched.

4.1.2 Efficient matching and surplus

In a frictionless setting, we should expect that an efficient matching—even if it is not unique—is reached 100% of the time. It goes without saying that this prediction is rejected with the observation that some subjects do not match. We test a more restrictive hypothesis: Some subjects may remain unmatched—and we remain agnostic about the reason—but when the maximum feasible number of matches is reached, the cooperative model predicts efficient matching.

Hypothesis 1b. *Efficient matching is achieved, given that full matching is achieved: The number of efficiently matched pairs is the maximum feasible number.*

Figure 4a provides a breakdown of types of matching with respect to the number of efficiently matched and inefficiently mismatched pairs. In all market types except MN6 (24.7%) and MN7 (46.4%), efficient matching is achieved in the majority of the rounds. We then calculate the conditional probability of efficient matching, given that three pairs are matched; this is shown in Figure 4b. Not all matched pairs in full matching are efficiently matched. In comparison, in Agranov et al. (2021), although subjects are more likely to be matched, they are also not necessarily efficiently matched conditional on being matched. Their AE6 market has a 94% rate of efficient matching, compared with 91% in ours (conditional on being fully matched); their AN6 market has a 73% rate of efficient matching, compared with 68% in ours (conditional on being fully matched).

To evaluate social efficiency, we can also consider the percentage of efficient surplus achieved.

Hypothesis 1c. *Efficient surplus is achieved.*

Figure 5 shows the average percentages of surplus achieved in the eight market types (in all rounds, not restricted to rounds with full matching). They range from 85.05% in MN6 to 95.18%

Figure 4: Efficient matching

(a) Distribution of number of matched and efficiently matched pairs

matching type	market type								
	AE6	AN6	ME6	MN6	AE7	AN7	ME7	MN7	Total
	%	%	%	%	%	%	%	%	%
3 pairs: 3 efficient	76.4	41.2	69.2	24.7	71.4	55.0	69.3	46.4	56.2
3 pairs: 2 efficient	0.0	0.0	0.0	0.0	12.9	7.1	9.3	5.7	3.8
3 pairs: 1 efficient	6.0	17.6	6.6	14.3	8.6	15.7	7.9	7.1	10.6
3 pairs: 0 efficient	1.6	2.2	0.0	5.5	0.7	0.0	0.0	4.3	1.9
2 pairs: 2 efficient	7.1	13.2	11.5	14.3	2.9	8.6	6.4	13.6	9.9
2 pairs: 1 efficient	4.4	17.0	9.3	29.7	2.1	10.7	5.7	13.6	12.0
2 pairs: 0 efficient	4.4	8.8	3.3	10.4	0.7	2.9	1.4	9.3	5.4
1 pair: 1 efficient	0.0	0.0	0.0	1.1	0.0	0.0	0.0	0.0	0.2
1 pair: 0 efficient	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.0	0.1

(b) Percentage of efficient matching given full matching
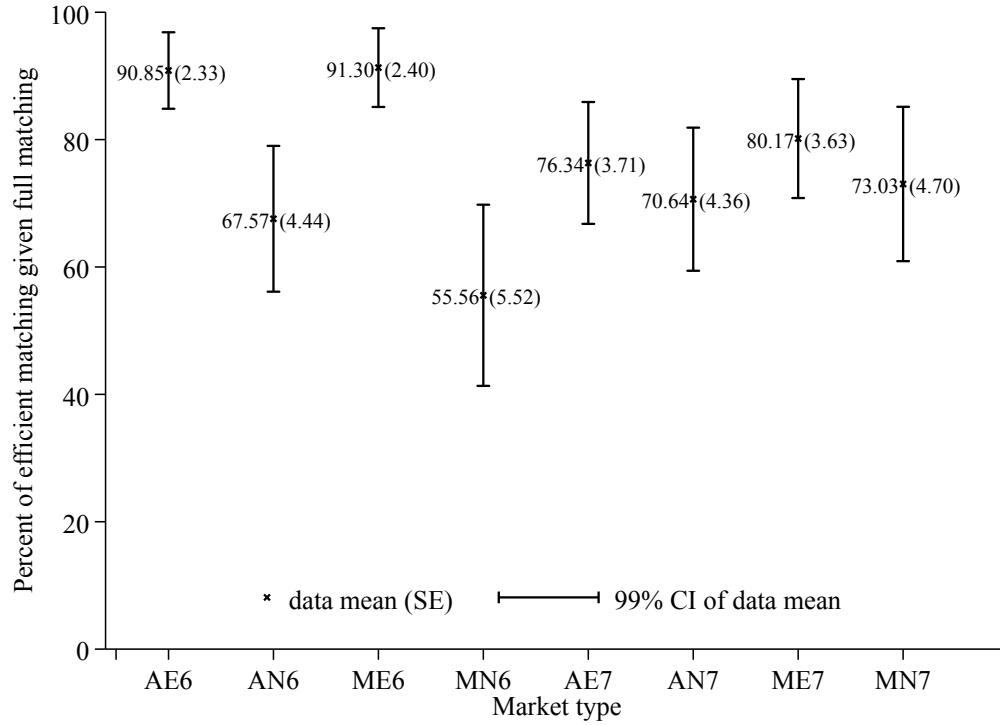
in AE7. Note that in balanced markets, when the maximum number of pairs is achieved, the minimum surplus is 80% and a random full matching generates 90% of the maximum surplus.

Figure 5: Percentage of efficient surplus achieved
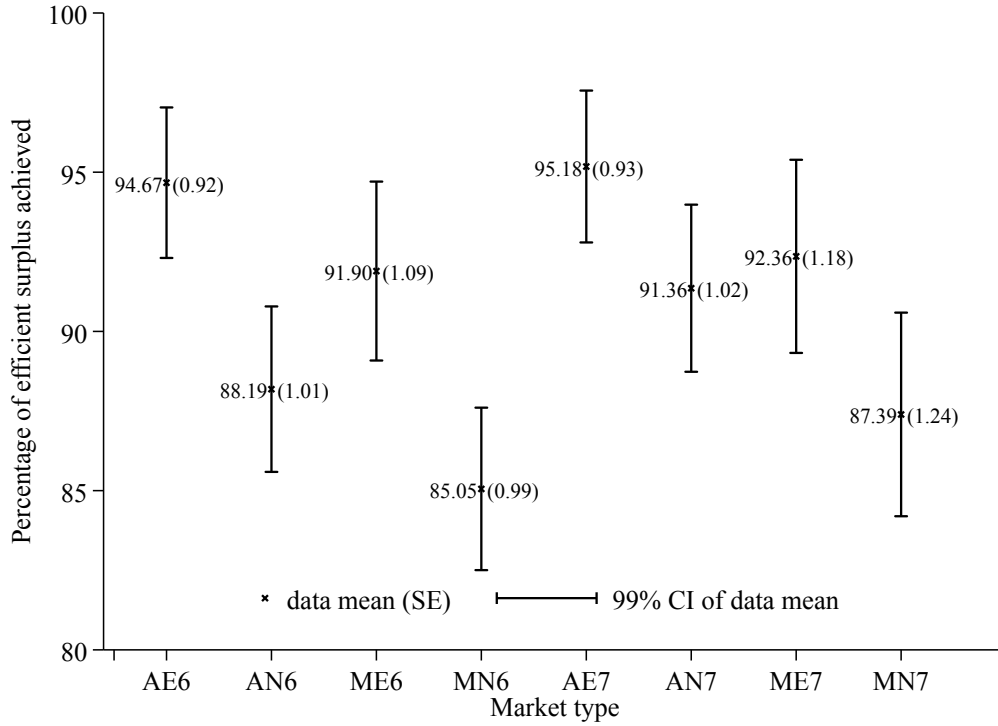
4.1.3 Stable outcome

An outcome is stable when not only matching is efficient, but also the combination of individual payoffs derived from pairwise surplus divisions is in the core. Hence, reaching a stable outcome—an efficient matching along with a stable division of surpluses—is more stringent than achieving an efficient matching. Due to the transferable nature of the payoffs, the matching in any stable outcome is efficient.

Hypothesis 1d. *A stable outcome is achieved, given that efficient matching is achieved.*

Table 3 shows the probability that an outcome is stable, given that the matching is efficient. In AE6 and ME6—the balanced ESIC (equal-splits in the core) markets—in the vast majority of cases, when subjects match efficiently, they also divide up the surplus in a way that cannot be improved on by any blocking pair. However, AN6 and MN6—the balanced markets without equal splits in the core—witness efficient matching less frequently (as shown in Figure 4b), and even when efficient matching is reached, blocking pairs are more likely to exist. Strictly speaking, only 14.3% of AN6 markets and 16.1% of MN6 markets do not have any blocking pair. These markets have blocking pairs that can modestly improve their payoffs: 72.6%-14.3%=58.3% of AN6 markets have blocking pairs that can improve by fewer than 5 units of payoff, and 71.4%-39.2%=32.1% of MN6 markets have blocking pairs that can improve by 5 to 10 units of payoff.

Table 3: Probability of a stable outcome given that matching is efficient

	AE6	AN6	ME6	MN6	AE7	AN7	ME7	MN7
	stable: no blocking pair							
	%	%	%	%	%	%	%	%
stable	92.1	14.3	73.1	16.1	0.0	0.0	0.0	0.0
not stable	7.9	85.7	26.9	83.9	100.0	100.0	100.0	100.0
	stable5: no blocking pair improves more than 5 units of payoff							
	AE6	AN6	ME6	MN6	AE7	AN7	ME7	MN7
	%	%	%	%	%	%	%	%
stable5	98.6	72.6	88.5	39.3	7.1	0.0	0.0	10.0
not stable5	1.4	27.4	11.5	60.7	92.9	100.0	100.0	90.0
	stable10: no blocking pair improves more than 10 units of payoff							
	AE6	AN6	ME6	MN6	AE7	AN7	ME7	MN7
	%	%	%	%	%	%	%	%
stable10	99.3	84.5	96.2	71.4	52.9	6.7	10.8	72.5
not stable10	0.7	15.5	3.8	28.6	47.1	93.3	89.2	27.5

In our unbalanced markets, stable outcomes always involve a matched subject and an unmatched subject who get zero payoff. Strictly speaking, a stable outcome is not reached in any of our experimental games with unbalanced markets, because no matched subject gets zero. Even with a looser definition of stability, a significant portion of unbalanced markets have blocking pairs that can improve by more than 10 units of payoff, but they do not form a match by the end of the game. This significant discrepancy between theory and experiment in stable payoffs suggests that players are behaving in a way that is systematically different from what the cooperative theory predicts.

4.2 Determinants of aggregate outcomes

Several papers report how strategic complexity affects plays in games. Bednar et al. (2012) demonstrate that the prevalent strategies in games that are less cognitively demanding are more likely to be used in games that are more cognitively demanding. He and Wu (2020) and Luhan et al. (2017) show that subjects may not use a certain efficient strategy due to its complexity, but instead settle on some simpler but inefficient strategy. Under nonassortative matching, the stable matching pattern is less obvious. Hence, mixed nonassortative—even when equal-splits is in the core—may be perceived as more complex to subjects and more cognitively demanding. Consequently, subjects may settle on inefficient matching patterns such as the ones on the diagonals or accept payoffs that are not supported in the core.

Equal-splits has been widely supported in the literature on bargaining experiments, especially when it is coupled with efficiency. Two arguments are commonly used to support equal splits

observed in the data: the focal point theory of Schelling (1960) and distributional social prefer-ences (Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000). When equal-splits is at odds with efficiency, there is mixed evidence on the trade-offs between the two; see Roth and Malouf (1979); Hoffman and Spitzer (1982); Roth and Murnighan (1982); Roth et al. (1989); Ochs and Roth (1989), Herreiner and Puppe (2010); Roth (1995); Camerer (2003); Anbarci and Feltovich (2013, 2018); Isoni et al. (2014); and Galeotti et al. (2018), among many others, on reporting and understand-ing equal splits in bargaining experiments. In our experiment, efficiency is aligned with stable matching. Hence, when equal-splits is in the core, it does not conflict with efficiency. However, when it is not in the core, subjects will face trade-offs between equality and efficiency, which may negatively affect the rate of matching, the rate of stable matching, and overall efficiency.

Hypothesis 1e (Determinants of outcomes in balanced markets). *For balanced markets, (i) the number of matched pairs, (ii) the number of efficiently matched pairs, and (iii) the percentage of efficient surplus achieved are the same (i) in assortative markets as in nonassortative markets, and (ii) in ESIC markets as in ESNIC markets.*

Figures 3b and 4a provide the following qualitative comparisons of balanced markets that contradict the hypothesis. First, the assortative markets (AE6 and AN6) have a higher number of matched pairs, a higher number of efficiently matched pairs, and a higher aggregate surplus than the nonassortative markets (ME6 and MN6). Second, the ESIC markets (AE6 and ME6) have a higher number of matched pairs, a higher number of efficiently matched pairs, and a higher surplus than ESNIC markets (AN6 and MN6). We confirm the statistical significance of these comparisons for balanced markets by running OLS regressions with the following specification:

$$y_i = \beta_1 \cdot \text{assortative}_i + \beta_2 \cdot \text{ESIC}_i + \beta_3 \cdot \text{assortative}_i \cdot \text{ESIC}_i + \beta_4 \cdot \text{round}_i + \beta_5 \cdot \text{order}_i + c + \varepsilon_i, \quad (1)$$

where i indicates the index of the game (out of 728 balanced markets), y_i is the dependent variable ((log) number of matched pairs in game i, (log) number of efficiently matched pairs in game i, or (log) surplus in game i), assortative$_i$ is an indicator of whether game i is assortative, ESIC$_i$ is an indicator of whether game i has equal splits in the core, round$_i$ is the round (out of 7) the same market has been played, and order$_i$ is the order (out of 4) the game is played in.

Table 4 reports the regression results. Compared with nonassortative markets, assortative markets have 7.75% more matched pairs, 14.8% more efficiently matched pairs, and 4.11% more surplus.[13] Compared with other markets, ESIC markets have 13.6% more matched pairs, 38.7% more efficiently matched pairs, and 7.78% more surplus.

---

[13]Note that 66 rounds had no efficiently matched pair. To include them in the regression, we define the dependent variable to be $\log(1+\text{number of efficiently matched pairs})$.

Table 4: Determinants of outcomes in balanced markets

	(1) log matches	(2) log (efficient matches+1)	(3) log surplus
assortative	0.0775***	0.148***	0.0411*
	(3.97)	(3.47)	(2.28)
ESIC	0.136***	0.387***	0.0778***
	(6.98)	(9.08)	(4.31)
assortative*ESIC	-0.0441	-0.135*	-0.00706
	(-1.60)	(-2.25)	(-0.28)
round	0.00708*	0.0160*	0.00847**
	(2.05)	(2.12)	(2.66)
order	0.0199**	0.0257	0.0218***
	(3.22)	(1.91)	(3.82)
constant	0.786***	0.682***	5.032***
	(31.27)	(12.40)	(216.09)
N	728	728	728

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Learning mildly improves matching outcome. Each additional round of play of the same game is associated with 0.71% more matched pairs, 1.6% more efficiently matched pairs, and 0.85% more surplus, and each 7 rounds of play of other games ahead of the current game are associated with 1.99% more matched pairs, 2.57% more efficiently matched pairs, and 2.18% more surplus. These results are statistically significant at at least the 95% level. In the appendix, we provide robustness checks with alternative specifications of the regressions regarding the dependent variables (no log) and rounds of plays. The results are quite consistent with those under our current specifications.

Overall, for balanced markets, equal-splits in the core is a crucial determinant of efficient matches and surpluses; in comparison, assortativity plays a less important role. And to a much less extent but at a statistically significant level, experience with the negotiation process slightly increases matching and efficiency.

Furthermore, we consider the determinants of outcomes when both balanced and unbalanced markets are included. Table 5 presents the results for the following regression model:

$$
y_i = \beta_1 \text{assortative}_i + \beta_2 \text{ESIC}_i + \beta_3 \text{balanced}_i + \beta_4 \text{assortative}_i \text{ESIC}_i + \beta_5 \text{assortative}_i \text{balanced}_i
$$
$$
+ \beta_6 \text{round}_i + \beta_7 \text{round}_i \text{balanced}_i + \beta_8 \text{order}_i + \beta_9 \text{order}_i \text{balanced}_i + c + \varepsilon_i, \tag{2}
$$

where balanced$_i$ indicates whether the market in game i is balanced. Table 5 presents the results. Controlling for other changes, assortativity increases the number of matches by 4.1%, the number

Table 5: Determinants of outcomes, all markets

	(1) log matches	(2) log (efficient matches+1)	(3) log surplus
assortative	0.0410**	0.0645*	0.0432**
	(2.75)	(1.96)	(3.01)
ESIC	0.136***	0.387***	0.0778***
	(7.36)	(9.48)	(4.37)
balanced	-0.141***	-0.259***	-0.0746*
	(-4.11)	(-3.42)	(-2.26)
assortative*ESIC	-0.0441	-0.135*	-0.00706
	(-1.69)	(-2.35)	(-0.28)
assortative*balanced	0.0365	0.0835	-0.00210
	(1.54)	(1.59)	(-0.09)
round	0.0138***	0.0296***	0.0130***
	(3.71)	(3.60)	(3.63)
round*balanced	-0.00675	-0.0136	-0.00455
	(-1.36)	(-1.24)	(-0.95)
order	0.00571	0.0217	0.00694
	(0.86)	(1.48)	(1.08)
order*balanced	0.0142	0.00405	0.0149
	(1.60)	(0.21)	(1.74)
constant	0.928***	0.941***	5.106***
	(37.54)	(17.25)	(214.74)
N	1,288	1,288	1,288

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

of efficiently matched pairs by 6.45%, and the surplus by 4.32%; equal-splits in the core increases the number of matched pairs by 13.6%, the number of efficiently matched pairs by 38.7%, and the surplus by 7.78%; market thickness increases the number of matches by 14.1%, the number of efficient matches by 25.9%, and the surplus by 7.56%. Playing an additional round of any game (i.e., the round effect) increases the matching by 1.38%, efficient matching by 2.96%, and surplus by 1.30%, but the order effect disappears when unbalanced markets are included.

In summary, including the unbalanced markets, equal-splits in the core continues to play a prominent role in determining matching and efficiency and assortativity plays a lesser role, both statistically and quantitatively. Market thickness helps increase matching and efficiency. Results with alternative dependent variables and alternative specifications (Tables B7 to B12) in Appendix B serve as robustness checks, and obtain similar conclusions.

Instead of the number of matched and efficiently matched pairs, we can examine whether full, efficient, and stable matching, respectively, is achieved in a probit model. We use the same

specifications as previously, specifications (1) and (2). Table 6 presents the results for balanced markets. Equal-splits in the core increases the probability of a full matching by 31.7 percentage points (pp), that of an efficient matching by 41.1 pp, and that of a stable matching by 47.6 pp; alternative measures of stability yield similar results. Assortativity increases the probabilities of full and efficient matching by 17.2 pp and 16.2 pp, respectively. However, assortativity does not statistically significantly increase the probability of a stable matching, but does increase the probability of no blocking pair that improves the current gain by at least 5 or 10 units of surplus (by 22.4 pp and 21.2 pp, respectively).

Table 6: Determinants of full, efficient, and stable matching in balanced markets

	(1) full	(2) efficient	(3) stable	(4) stable5	(5) stable10
assortative	0.172***	0.162***	0.0232	0.224***	0.212***
	(3.67)	(3.40)	(0.60)	(5.00)	(4.54)
ESIC	0.317***	0.411***	0.476***	0.516***	0.488***
	(6.77)	(8.63)	(12.22)	(11.53)	(10.46)
assortative*ESIC	-0.0893	-0.107	0.164**	-0.0972	-0.151*
	(-1.35)	(-1.59)	(2.97)	(-1.54)	(-2.29)
round	0.0151	0.0285***	0.0223**	0.0185*	0.0206*
	(1.83)	(3.39)	(3.24)	(2.35)	(2.50)
order	0.0450**	0.0519***	0.0438***	0.0606***	0.0538***
	(3.04)	(3.45)	(3.55)	(4.28)	(3.65)
constant	0.269***	0.0598	-0.153**	-0.109	-0.00129
	(4.46)	(0.97)	(-3.04)	(-1.90)	(-0.02)
Observations	728	728	728	728	728

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 7 presents the results for the analysis for all balanced and unbalanced markets. Equal-splits in the core (AE6 and ME6) has the same effects as mentioned for Table 6. Assortativity (AE6, AN6, AE7, and AN7) does not play a significant role in achieving efficient or stable matching. Furthermore, market thickness helps with matching: A balanced market is associated with 31.9 pp less full matching and efficient matching, and 15.3 pp less stable matching.

4.3   Individual payoffs: division of surplus

We consider the individual payoffs when efficient matching is reached and compare to existing solutions that refine the core. Table 8 presents comparisons between cooperative solutions and the experimental payoffs. Among single-valued solutions, we provide (1) the Shapley value, which

Table 7: Determinants of full, efficient, and stable matching, all markets

	(1) full	(2) efficient	(3) stable	(4) stable5	(5) stable10
assortative	0.107**	0.0536	-0.0286	-0.0250	-0.00714
	(3.00)	(1.36)	(-1.08)	(-0.82)	(-0.20)
ESIC	0.317***	0.411***	0.476***	0.516***	0.488***
	(7.15)	(8.40)	(14.47)	(13.67)	(11.03)
balanced	-0.319***	-0.319***	-0.156*	-0.117	-0.0531
	(-3.88)	(-3.51)	(-2.56)	(-1.66)	(-0.65)
assortative*equal	-0.0893	-0.107	0.164***	-0.0972	-0.151*
	(-1.43)	(-1.55)	(3.52)	(-1.82)	(-2.42)
assortative*balanced	0.0646	0.108	0.0518	0.249***	0.219***
	(1.14)	(1.72)	(1.23)	(5.13)	(3.85)
round	0.0326***	0.0424***	0.00179	0.00402	0.0192*
	(3.65)	(4.30)	(0.27)	(0.53)	(2.15)
round*balanced	-0.0175	-0.0139	0.0205*	0.0145	0.00141
	(-1.47)	(-1.06)	(2.33)	(1.44)	(0.12)
order	0.0129	0.0121	0.0214	0.0221	0.0257
	(0.81)	(0.69)	(1.81)	(1.63)	(1.61)
order*balanced	0.0321	0.0398	0.0223	0.0384*	0.0281
	(1.51)	(1.70)	(1.42)	(2.12)	(1.32)
constant	0.588***	0.379***	0.00357	0.00714	0.0518
	(9.93)	(5.79)	(0.08)	(0.14)	(0.88)
N	1,288	1,288	1,288	1,288	1,288

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

assigns each player a payoff relative to how "important" that player is to overall surplus attainable; (2) the nucleolus, the lexicographical center of core payoffs; (3) the fair division point, the midpoint between the row- and column-optimal payoffs; (4) the median stable matching, which gives each player their median payoff. Among these solutions, the nucleolus and median stable matching do not match the payoffs when the matching is efficient (except for AE6). The fair division point performs well in the balanced ESIC markets, but not in the unbalanced markets. Another good performer is the midpoint in the core (not in the table), which coincides with our noncooperative limit payoffs in many games (AE6, AN6, ME6, and all games in Agranov and Elliott (2021)), but differs from our prediction in MN6. This market type distinguishes our prediction from the midpoint in the core, and demonstrates its better performance.[14] The limit equilibrium values from the noncooperative game we present next match well with our experimental values

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[14]The midpoint in the core is the average of the minimum and maximum possible payoffs of each player in the core. We are not aware of a noncooperative foundation.

across all markets.

Table 8: Comparison of experimental payoffs with cooperative and noncooperative solutions

	value	experiment (SE)	noncoop-erative limit	Shapley value	nucleolus	fair division	median stable matching
			our model	Shapley (1953)	Schmeidler (1969)	Thompson (1980)	Schwarz and Yenmez (2011)
AE6	m1	15.28 (.22)	15	18.17	15	15	15
	m2	29.92 (.13)	30	31.33	30	30	30
	m3	54.85 (.31)	55	50.50	55	55	55
AN6	m1	45.68 (.998)	50	46.17	55	50	55
	m2	31.38 (.56)	30	31.33	30	30	35
	m3	22.62 (.95)	20	22.50	15	20	15
ME6	m1	30.65 (.35)	30	31.83	32.5	30	32.22
	m2	49.14 (.33)	50	45.67	47.5	50	47.78
	m3	20.21 (.17)	20	21.83	20	20	20
MN6	m1	27.49 (.77)	30	28.00	17.5	20	18.34
	m2	41.27 (.91)	40	31.67	20	25	20.56
	m3	30.38 (1.45)	30	27.67	22.5	20	22.78
AE7	m1	21.34 (.37)	15	22.31	30	30	30
	m2	32.48 (.33)	30	36.31	45	45	45
	m3	55.27 (.37)	55	55.98	70	70	70
AN7	m1	56.28 (.71)	50	53.98	70	70	52.5
	m2	32.12 (.51)	30	37.14	45	45	67.5
	m3	21 (.46)	20	26.31	30	30	40
ME7	m1	31.33 (.38)	30	37.14	52.5	50	70
	m2	50.41 (.44)	50	52.48	67.5	70	50
	m3	26.48 (.50)	20	27.48	40	40	30
MN7	m1	30.81 (.49)	30	31.25	35	35	35.81
	m2	41.38 (.81)	40	34.75	35	40	37.5
	m3	32.55 (.52)	30	35.39	40	40	40

Note. The table shows the average payoffs in the efficient matching in the experiment versus cooperative and noncooperative values.

5   Noncooperative theory

Existing cooperative solutions—either set-valued ones like the core or singleton-valued ones like the nucleolus—depart from the experimental results in systematic ways. To rationalize the individual payoffs in the experiment, consider the following continuous-time model that captures the essence of our experimental setup. At time zero, no one is matched. At each instant $t \geq 0$,

any agent can propose to anyone on the other side of the market. A person receiving a proposal must accept or reject the proposal within time length Δ. Neither a proposer nor a receiver of a proposal can make another proposal within the time length Δ. At each instant, when several offers are made simultaneously, proposals from one side of the market are randomly selected to be sent, and whenever tie-breaking is needed next, proposals from the other side of the market are sent.[15] When a proposal is accepted, the match becomes temporary, and the temporary match and the temporarily agreed upon division of surplus are publicly announced. People who are temporarily matched can still propose to anyone on the other side of the market, other than their matched partner. The game ends when there is no new proposal in the last $\Delta \cdot (1 + \varepsilon)$ units of time, where $\varepsilon \in (0, 1)$, and all matches become final. Suppose each individual has a discount rate of r. Define $\delta \equiv e^{-r\Delta}$. Taking $\Delta \to 0$ is equivalent to taking $\delta \to 1$.

We consider the Markov perfect equilibria of the game. At each instant, the state of the game is summarized by the temporary matching μ and the temporary payoffs $\{U_m\}_{m \in M}$ and $\{V_w\}_{w \in W}$. Because of the rule whereby agents cannot make another offer before Δ units of time, in equilibrium, effectively, actions occur only at times that are integer multiples of Δ. Barring technical details, given the specific tie-breaking rule, we can alternatively think of a discrete-time model in which agents have discount factors δ and, at the initial period agents on one side of the market are randomly chosen to propose. In subsequent periods the two sides alternate in making proposals, and the game ends when there is no proposal in a period.

5.1 Balanced markets

Suppose there is a unique efficient matching μ^* in a balanced matching market, as in the four balanced markets in our experiment. Consider the following (Markov perfect) equilibrium in which players propose to their partners in the efficient matching. At time zero, each man $m \in M$ proposes to woman $\mu^*(m) \in W$ with the surplus division U_m^p to m and $s_{m\mu^*(m)} - U_m^p$ to $\mu^*(m)$, and each woman $w \in W$ proposes to man $\mu^*(w) \in M$ with the surplus division $s_{\mu^*(w)w} - V_w^p$ to $\mu^*(w)$ and V_w^p to w. Each man $m \in M$ accepts the highest acceptable offer, where an offer above $\delta \cdot U_m^r$ is weakly acceptable and U_m^r is the optimal value when m rejects the current offer. Each woman $w \in W$ accepts the highest offer, where an offer above $\delta \cdot V_w^r$ is weakly acceptable and V_w^r is the optimal value when w rejects the current offer. At each instant after time zero, each person makes an offer that maximizes their payoff given the current temporary payoffs, and each person accepts the highest acceptable offer if it is above their current temporary payoff. On the

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equilibrium path, each man $m \in M$ proposes to woman $\mu^*(m) \in W$ and each woman $w \in W$ proposes to man $\mu^*(w) \in M$ with the division specified above, and each person accepts the offer at time zero and does not make another offer. The proposal each man $m \in M$ makes to woman $\mu^*(m) \in W$ at time zero yields him a payoff of

$$U_m^p = s_{m\mu^*(m)} - \max\left\{\delta \cdot V_{\mu^*(m)}^r, \max_{m' \in M\backslash m}\left\{s_{m'\mu^*(m)} - U_{m'}^p\right\}\right\}, \tag{3}$$

where

$$V_{\mu^*(m)}^r = s_{m\mu^*(m)} - \max\left\{\delta \cdot U_m^p, \max_{w' \in W\backslash\mu^*(m)}\left\{s_{mw'} - \left[s_{\mu^*(w')w'} - U_{\mu^*(w')}^p\right]\right\}\right\}. \tag{4}$$

Note that $U_{m'}^p$ is the payoff of m' when $\mu^*(m')$ accepts, and $s_{\mu(w')w'} - U_{\mu(w')}^p$ is the payoff of w' when w' accepts. The offer man $m \in M$ proposes to woman $\mu^*(m) \in W$ is $s_{m\mu^*(m)} - U_m^p$, which is the maximum of (i) $\delta \cdot V_{\mu^*(m)}^r$, the continuation value that woman $\mu^*(m) \in W$ can get if she rejects, and (ii) $\max_{m' \in M\backslash\{m\}}\left\{s_{m'\mu^*(m)} - U_{m'}^p\right\}$, the highest possible deviation payoff that another man $m' \in M\backslash\{m\}$ can offer to $\mu^*(m)$. The expected payoff that woman $\mu^*(m) \in W$ gets if she rejects, $V_{\mu^*(m)}^r$, results from her proposing to man $m \in M$, while ensuring that no other woman $w' \in W\backslash\{w\}$ is able to offer $s_{mw'} - [s_{\mu^*(w')w'} - U_{\mu^*(w')}^p]$ to $m \in M$ to poach him. Analogously, the proposal each woman $w \in W$ makes to man $\mu^*(w) \in M$ at time zero is

$$V_w^p = s_{\mu^*(w)w} - \max\left\{\delta \cdot U_{\mu^*(w)}^r, \max_{w' \in W\backslash w}\left\{s_{\mu^*(w)w} - V_{w'}^p\right\}\right\}, \tag{5}$$

where

$$U_{\mu^*(w)}^r = s_{\mu^*(w)w} - \max\left\{\delta \cdot V_w^p, \max_{m' \neq \mu(w)}\left\{s_{m'w} - \left[s_{m\mu^*(m)} - V_{\mu^*(m)}^p\right]\right\}\right\}. \tag{6}$$

Note that when $\delta = 1$, all core payoffs satisfy the system of $n_M + n_W$ equations for $\{U_m^p\}_{m \in M}$ and $\{V_w^p\}_{w \in W}$. When $\delta < 1$, we can show that there is a unique set of payoffs $\{U_m^p\}_{m \in M}$ and $\{V_w^p\}_{w \in W}$ that satisfy the system of equations. The proofs are provided in Appendix C.

Theorem 1. *For any $\delta \in (0, 1)$, there exists a unique solution to the system of equations* (3)–(6).

Theorem 1 establishes the existence of a unique solution to the system of equations with efficient matching, which is supported as a MPE. This result contrasts Proposition 2, which shows that the set of stable payoffs is not a singleton in the canonical cooperative model. Furthermore, Proposition 3 implies that we should expect equal splits as the unique equilibrium outcome in the limit if and only if equal splits is in the core.

Proposition 3. *Suppose $s_{mw} > 0$ for any $m \in M$ and $w \in W$. There exists a $\underline{\delta} \in (0, 1)$, such that*

Table 9: Payoffs in comparable experiments in the literature

Nalbantian and Schotter (1995)		m1/m2/m3			w1/w2/w3		
surplus matrix	type	theory	efficient	all	theory	efficient	all
(4, 3, 3; 3, 4, 3; 3, 3, 4)	AE6	2	2.21	2	2	1.79	2
Agranov and Elliott (2021)		m1/w2			m2/w1		
(20, 15; 0 20)	AE4	10	10.0 (0.03)	9.8 (0.12)	10	10.0 (0.03)	9.8 (0.09)
(20, 25; 0 20)	AN4	7.5	7.5 (0.18)	6.2 (0.35)	12.5	12.3 (0.08)	12.3 (0.16)
(20, 30; 0 20)	AN4	5	4.9 (0.18)	3.6 (0.05)	15	15.0 (0.14)	14.2 (0.05)
Agranov et al. (2021)		m1/w1		m2/w2		m3/w3	
surplus matrix	type	theory	all	theory	all	theory	all
(8, 16, 24; 16, 32, 48; 24, 48, 72)	AE6	4	4.11 (0.06)	16	16.07 (0.37)	36	35.86 (0.1)
(8, 32, 56; 32, 48, 64; 56, 64, 72)	AN6	16	16.07 (0.37)	24	23.81 (0.25)	40	38.87 (0.45)

Note. We consider the CIEA setting in Nalbantian and Schotter (1995), Experiment III in Agranov and Elliott (2021), and the complete-information setting in Agranov et al. (2021). Agranov et al. (2021) does not separate efficient matches from all matches, while the other two papers do.

for any $\delta \in (\underline{\delta}, 1)$, when equal-splits is in the core, the equilibrium values are

$$U_m^p = \frac{s_{m\mu^*(m)}}{1+\delta} \text{ for any } m \in M \text{ and } V_w^r = \frac{s_{\mu^*(w)w}}{1+\delta} \text{ for any } w \in W.$$
$$V_w^p = \frac{s_{\mu^*(w)w}}{1+\delta} \text{ for any } w \in W \text{ and } U_m^r = \frac{s_{m\mu^*(m)}}{1+\delta} \text{ for any } m \in M.$$

When equal-splits is not in the core, there exists a $\underline{\delta} \in [0, 1)$, such that for any $\delta \in [\underline{\delta}, 1)$, the equilibrium values above are not satisfied.

The expected equilibrium payoffs are $U_m \equiv U_m^p/2 + [s_{m\mu^*(m)} - V_{\mu^*(m)}^p]/2$ for each $m \in M$ and $V_w \equiv V_w^p/2 + [s_{\mu^*(w)w} - U_{\mu^*(w)}^p]/2$ for each $w \in W$. These values as $\delta \to 1$ coincide with the payoffs in the experiment. Notably, $U_m^p = U_m^r$ and $V_w^p = V_w^r$ as $\delta \to 1$ in the games with equal-splits in the core, but $U_m^p \neq U_m^r$ and $V_w^p \neq V_w^r$ in the two games with equal splits not in the core, even in the limit as $\delta \to 1$. This suggests that on one hand, in markets with equal splits in the core, outside options do not play a role in equilibrium and agents effectively engage in Nash/Rubinstein bargaining in pairs; in other words, market forces are minimal. On the other hand, in markets with equal splits not in the core, outside threats alter bargaining and influence equilibrium payoffs, and market forces play a significant role. These distinctions between markets with and without equal splits in the core are also observed in noncooperative games with permanently accepted offers (Elliott and Nava, 2019; Talamàs, 2020; Agranov et al., 2021; Agranov and Elliott, 2021). By calculating the equilibrium payoffs in the four balanced markets, we formalize the following hypothesis:[16]

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[16]We only demonstrate men's payoffs as women's payoffs are pinned down by men's in efficient matching.

Hypothesis 2a. *The average individual payoffs for men in the four balanced markets are $U_1 = 15$, $U_2 = 30$, and $U_3 = 55$ in AE6; $U_1 = 50$, $U_2 = 30$, and $U_3 = 20$ in AN6; $U_1 = 30$, $U_2 = 50$, and $U_3 = 20$ in ME6; and $U_1 = 30$, $U_2 = 40$, and $U_3 = 30$ in MN6.*

Figure 1a shows how well the data matches the theoretical predictions of our model in the balanced markets. The theoretically predicted payoffs in the efficient matching fall in the 99% confidence interval of the data mean. The theory not only matches well with the average payoff, but also with the more detailed realized behavior. The modal outcome matches the theoretical prediction, shown in Figure B1a, which presents the histograms of payoffs of individuals in the efficient matching, with bandwidth of 1.[17] In addition, our theory matches other experimental results in the literature with comparable experimental settings, as summarized in Table 9. By our categorization, all of the surplus matrices in previous experiments are assortative and symmetric, while some have equal splits in the core and some do not. In comparison, we continue to vary whether equal-splits is in the core, and examine markets with nonassortative and asymmetric surplus matrices.

5.2 Unbalanced markets

Consider an unbalanced market in which two individuals are identical in terms of the surplus they generate with anyone on the other side of the market; in the four unbalanced markets in our experiment, we have $w^*, w^{**} \in W$ such that $s_{mw^*} = s_{mw^{**}}$ for all $m \in M$. There are two efficient matching outcomes μ^* and μ^{**} such that between w^* and w^{**}, only w^* is matched and only w^{**} is matched, respectively.

There are various (Markov perfect) equilibrium outcomes in this unbalanced market, in the spirit of the folk theorem. To fix ideas, consider the simplest unbalanced matching market of one man m^* and two women w^* and w^{**}, with either pair being able to generate a surplus of $s^* > 0$. In the first type of equilibrium, man m^* proposes to either woman w^* or woman w^{**} a division of the surplus s^* into s^* for himself and 0 for her; woman w^* and woman w^{**} propose to man m^* the same division; man m^* accepts a payoff weakly above s^*; and each woman accepts any division of surplus. The equilibrium outcome is a core outcome in an unbalanced matching market, and is what we call a competitive outcome, since the two women are competing to benefit the man on the short side of the market. However, in this dynamic noncooperative setting, there are other equilibrium outcomes. Consider the following equilibrium strategies. When man m^* is unmatched, woman w^* proposes to man m^* the Rubinstein division of surplus s^* with $s^*/(1+\delta)$ for her and $\delta \cdot s^*/(1+\delta)$ for the man; man m^* proposes to woman w^* the Rubinstein division $s^*/(1+\delta)$

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[17]The same pattern holds if we consider all matched individuals—not just the matched individuals in the efficient matching—as shown in Figure B2.

for himself and $\delta \cdot s^*/(1+\delta)$ for woman w^*, and accepts any offer above $\delta \cdot s^*/(1+\delta)$ and above his current temporary payoff. When m^* is matched with w^{**}, woman w^* proposes to man m^* the competitive division of surplus s^* with s^* for man m^* and 0 for woman w^*, and man proposes to woman w^* the same competitive offer. Woman w^{**} does not propose or accept any offer. This is an optimal strategy for woman w^{**}, as she knows that any proposal to or any acceptance of proposal from man m^* would still lead to a zero payoff for her. We call this equilibrium outcome a noncompetitive outcome, since the agents on the long side of the market, the women, are not competing. Finally, using this "grim-trigger" type of strategy, any equilibrium outcome that yields a payoff U between $\delta \cdot s^*/(1+\delta)$ and s^* for man m^* is possible if woman w^{**} accepts any offer that yields a payoff weakly above $s^* - U$. This results in a partially competitive outcome in which men benefit from some competition but not maximally. This indeterminacy resonates with Rubinstein and Wolinsky (1990), a noncooperative setting with permanently accepted offers.

We can generalize these arguments to the unbalanced matching markets with more individuals in which there are two identical women w^* and w^{**}. Consider a generalization of the noncompetitive equilibrium described above, each man $m \in M$ and each woman $w \in W \backslash \{w^{**}\}$ behave as if they are in the equilibrium in the balanced market with μ^* being the equilibrium matching with woman $w^{**} \in W$ remaining unmatched, and woman $w^{**} \in W$ does not attempt to make or accept a proposal. For any woman $w \in W \backslash \{w^{**}\}$, whenever man $\mu^*(w) \in M$ is temporarily matched with woman w^{**}, woman w would choose to make a proposal that yields a payoff $s_{\mu^*(w)w^{**}}$ for man $\mu^*(w)$. Given this grim-trigger strategy of any woman $w \in W \backslash \{w^{**}\}$, woman w^{**} has no (strict) incentive to propose to any man, because she knows that eventually she receives a payoff of 0. Analogously, we can have matching μ^{**} to be sustained in equilibrium in a similar way. In this type of equilibrium, despite the imbalance of the market, the short side does not benefit from it.

The second class of equilibria generalizes the other extreme of competitive equilibrium in which women w^* and w^{**} compete for man $\mu^*(w^*) = \mu^{**}(w^{**}) \equiv m^*$. In this equilibrium, both woman w^* and woman w^{**} propose to man m^* a division of the surplus $s_{m^*w^*} = s_{m^*w^{**}} \equiv s^*$ with payoff s^* for man m^* and 0 for herself; meanwhile, man m^* proposes to either woman w^* or woman w^{**} the same division of surplus.

These offers yield a payoff of $U_{m^*} = s^*$ for man m^* and payoff 0 for w^* and w^{**}. There are two possibilities for the other pairs of agents. First, they may be unaffected by these competitions between w^* and w^{**}, since they continue to get the noncompetitive outcome in equilibrium, and they can maintain those noncompetitive outcomes by invoking grim-trigger strategies. Second, agents on the long side of the market may be influenced by the competition with the unmatched woman w^{**}. To maximally deter the unmatched woman, the matched women may actively choose to offer $s_{\mu^*(w)w^{**}}$ to man $\mu^*(m)$, so that he has no incentive to match with woman w^{**}, and woman

w^{**} has no way to poach man $\mu^*(m)$. The maximum deterrence is to offer $s_{\mu^*(w)w^{**}}$ to man, but any payoff between $s_{\mu^*(w)w} - V_w^p$ and $s_{\mu^*(w)w^{**}}$ for man $\mu^*(w)$ can be supported in equilibrium for any woman w, generating a range of equilibrium outcomes.

In the experiment, the core payoffs—the competitive outcome—are not the most plausible predictions for these unbalanced matching markets. As a consequence, any refinement of the core with the cooperative approach will not yield a satisfying prediction for the unbalanced markets. Rather, we observe a range of payoffs for men and women between the competitive outcome and the noncompetitive outcome, as shown by the histograms of realized individual payoffs. This multiplicity is also observed in other Boehm-Bawerk types of unbalanced markets. Leng (2020) meticulously follows the continuous-time setup of Perry and Reny (1994) that supposedly generates only core outcomes; a range of noncore outcomes analogous to our noncompetitive outcomes arises in markets with unequal numbers of participants on the two sides (to be precise, markets with one seller and two buyers).

Hypothesis 2b. *The lower bounds of the average individual payoffs for men in the four unbalanced markets are $U_1 = 15, U_2 = 30,$ and $U_3 = 55$ in AE7; $U_1 = 50, U_2 = 30,$ and $U_3 = 20$ in AN7; $U_1 = 30,$ $U_2 = 50,$ and $U_3 = 20$ in ME7; and $U_1 = 30, U_2 = 40,$ and $U_3 = 30$ in MN7.*[18]

Adding one player to a balanced market shrinks the core. The payoffs of the players on the short side of the market increase, and those of the players on the long side decrease; some matched players' payoffs are driven to zero in the cases we consider in our experiment. However, experimentally, players' average payoffs do not change that drastically, as shown in Figure 1b. No participants end up with the competitive core outcome of zero payoffs. The noncompetitive outcome is much more frequent. Figure B1b shows that the modal payoffs of matched players remain to be the noncompetitive payoffs; the same pattern holds if we consider all matched individuals—not just the matched individuals in efficient matching—as shown in Figure B2. Furthermore, notably, although they are on the long side of the market, women with the highest bargaining power slightly gain in the unbalanced market (Figure B3). This in general supports our prediction of a noncompetitive equilibrium in the unbalanced markets.

Overall, there is some competition, which is an equilibrium outcome in our noncooperative model. There is enough competition to reject the noncompetitive outcome as the sole outcome, but competition does not drive the relevant players' payoffs to zero or affect other players' payoffs drastically. The payoffs stay close to the noncompetitive outcome. In general, if the observed payoffs are not predicted by our noncompetitive limit payoffs, then the observed payoffs are between our noncompetitive limit payoffs and the lower (upper) bound of core payoffs for players on the short (long) side of the market.

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[18]Note that these numbers are indentical to those in Hypothesis 2a because they correspond to the equilibrium payoffs in the non-competitive equilibrium.

6 Other experimental results

6.1 Bargaining activities

The aggregate surplus gradually increases from time zero (Figure B4) through a series of proposals. In the balanced markets, the number of proposals is 12.4% fewer in assortative settings, and 30.5% fewer in the settings with equal splits in the core (Column (2) in Table 10). The number of proposals also decreases by round: An additional round decreases the number of proposals by 2.93%, and having played a game 7 rounds rounds of other market games ahead of the current market decreases the number of proposals by 9.96%, which averages to 1.42% per round (Column (2) of Table 10). The effect of assortativity disappears in the analysis regarding balanced and unbalanced markets (Columns (3)-(4) of Table 10), but the effect of equal-splits in the core persists. A balanced market increases the average number of proposals per player per round by 27.6% (Column (4) of Table 10).

Table 10: Determinants of number of proposals per player per round, balanced and all markets

	(1) proposal	(2) log proposal	(3) proposal	(4) log proposal
assortative	-0.251*	-0.124*	-0.0454	-0.0232
	(-2.29)	(-2.39)	(-0.53)	(-0.62)
ESIC	-0.483***	-0.305***	-0.483***	-0.305***
	(-4.42)	(-5.88)	(-4.58)	(-6.55)
assortative*ESIC	0.225	0.0964	0.225	0.0964
	(1.46)	(1.31)	(1.51)	(1.46)
round	-0.0388*	-0.0293**	-0.0229	-0.0136
	(-2.01)	(-3.19)	(-1.08)	(-1.45)
order	-0.159***	-0.0996***	0.0162	-0.00121
	(-4.60)	(-6.07)	(0.43)	(-0.07)
balanced			0.432*	0.276**
			(2.21)	(3.20)
assortative*balanced			-0.205	-0.101
			(-1.52)	(-1.69)
round*balanced			-0.0159	-0.0156
			(-0.56)	(-1.25)
order*balanced			-0.175***	-0.0984***
			(-3.47)	(-4.41)
constant	3.041***	1.209***	2.609***	0.933***
	(21.57)	(18.08)	(18.52)	(15.00)
N	728	728	1,288	1,288

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The average number of all proposals (accepted, rejected, ignored, and expired) per player per round is between 1.98 and 2.76 across different types of games (Figure 6). There does not appear to be a decline or increase in the number of proposals across periods.

Figure 6: Number of proposals

(a) Number of proposals per player per round

(b) Total number of proposals per round
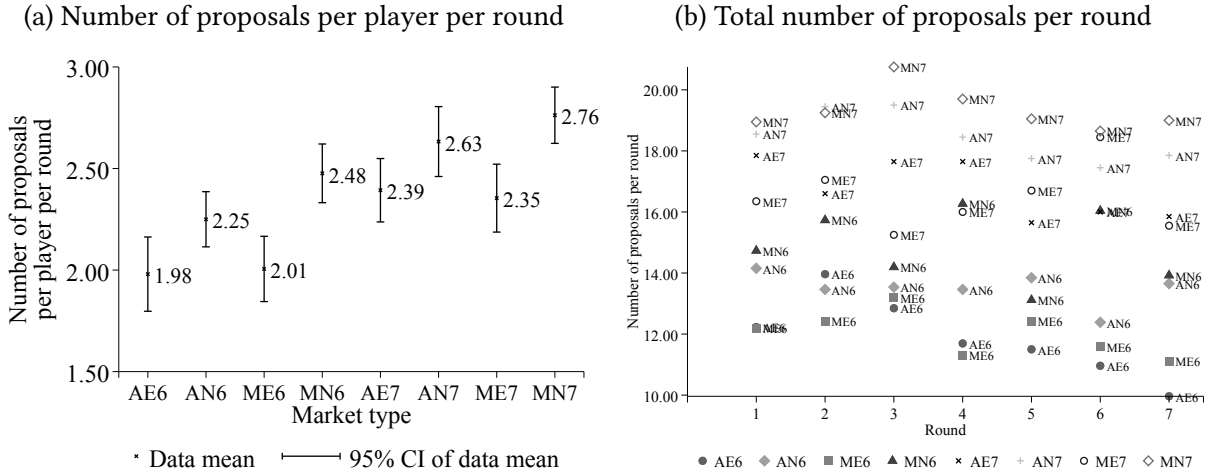
Figure 7 illustrates the proportion of equal-splits (±2) in proposed, temporarily accepted, and permanent offers. Equal-splits offers dominate the markets with equal splits in the core (AE6 and ME6). There are also considerably more equal-split offer in AE7 and ME7, although they are not ESIC.

Figure 7: Proportion of equal-splits offers

(a) All offers

(b) Temporarily accepted offers

(c) Permanent offers
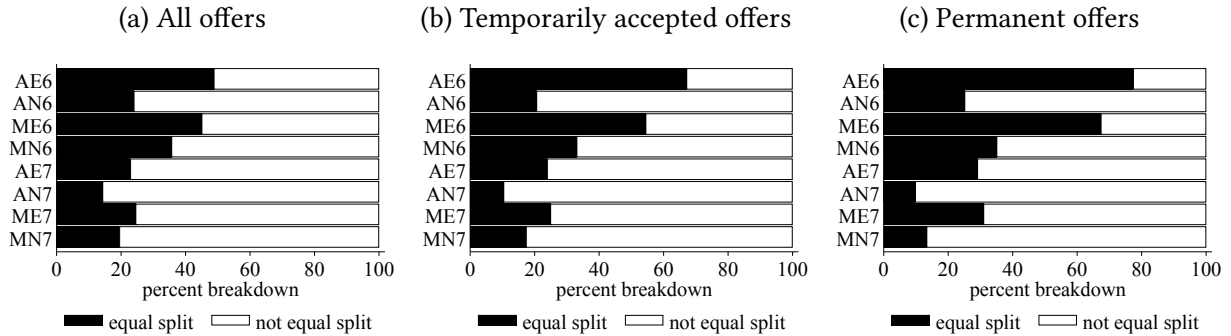
6.2 Reasons for being unmatched in balanced markets

Some subjects are unmatched in 33.6% of balanced markets (15.9% of AE6, 39.0% of AN6, 24.2% of ME6, and 56.5% of MN6). Overall, 5.31% of agents in AE6, 13.19% in AN6, 8.61% in ME6, and 19.05% in MN6 are unmatched. It is worthwhile to understand the reasons they end up

unmatched, because a significant amount of potential surpluses is left unrealized, and the loss due to being unmatched far exceeds the loss due to inefficient mismatches.

To this end, we categorize a few reasons why people are left unmatched. Namely, we define four categories. A person is **unlucky** if he/she was matched within the last 30 seconds of the game (i.e., after 150 seconds of the game) but was left unmatched by the end. A person is **unattractive** if he/she was unmatched for the last 30 seconds, was never proposed to, and proposed to others but was rejected by others. A person is **picky** if the person was unmatched for the last 30 seconds, did not propose to anyone in the last 30 seconds, and rejected any incoming proposals in the last 30 seconds of the game. A person is **trying** if the person has both been rejected and rejected in the last 30 seconds of the game.

Table 11 lists the reason for being unmatched. The leading factor for being unmatched is that a person is suddenly released from a match within 30 seconds of the end of the game. About half of the singles (48.3% in AE6, 47.9% in AN6, 63.0% in ME6, and 48.6% in MN6) are left unmatched for this reason. Among the rest of the singles, a little less than half are left unmatched because they are unattractive, i.e., in the last 30 seconds, their offers were not accepted and no one ever proposed to them. Among the last quarter of the singles, half were picky—i.e., they did not make any offer and rejected all incoming proposals in the last 30 seconds—half were actively participating without success.[19]

Table 11: Reasons for being unmatched

Single reason	AE6	AN6	ME6	MN6	Total
	%	%	%	%	%
unlucky	48.3	47.9	63.0	48.6	51.0
unattractive	22.4	20.8	14.1	26.4	22.1
picky	13.8	20.1	9.8	11.1	13.7
trying	15.5	11.1	13.0	13.9	13.1
Total	100.0	100.0	100.0	100.0	100.0

Table 12 shows how the environment leads to being unmatched. Overall, there is no strong evidence that the different unmatched types show up in different ways in different configurations. The "individual efficient surplus" is the theoretically predicted total surplus an individual can generate in the match. The "individual random surplus" is the expected total surplus an individual gets with his/her partner. For example, for ■ (blue square) m_1 in AE, the individual efficient surplus is 30, and the individual random surplus is $(30+40+50)/3 = 40$. The larger these factors, the higher the surplus an individual can provide. Therefore, as row 2 of Table 12 shows, a higher indi-

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[19]We also check whether some subjects tend to always be unlucky, picky, unattractive or trying, and this is not the case. The majority of subjects who have been unlucky, picky, unattractive, or trying experienced this only once or twice.

Table 12: Determinants of reasons for being unmatched

	(1) unmatched	(2) unlucky	(3) unattractive	(4) picky	(5) trying
individual efficient s	0.0286	-0.534	0.650*	0.174	-0.189
	(0.44)	(-1.65)	(2.26)	(0.77)	(-0.84)
individual random s	-0.106***	0.277***	-0.349***	-0.0614	0.0434
	(-6.34)	(3.41)	(-4.82)	(-1.08)	(0.77)
assortative	-0.0612***	-0.00623	-0.0291	0.0828*	-0.0237
	(-4.57)	(-0.11)	(-0.60)	(2.17)	(-0.63)
ESIC	-0.108***	0.129*	-0.129*	-0.0209	-0.00318
	(-8.03)	(2.06)	(-2.32)	(-0.48)	(-0.07)
assortative*ESIC	0.0301	-0.132	0.110	-0.0440	0.0477
	(1.59)	(-1.34)	(1.25)	(-0.64)	(0.70)
round	-0.00251	0.0184	-0.0184	-0.00221	-0.00295
	(-1.03)	(1.70)	(-1.91)	(-0.29)	(-0.39)
period	-0.00241***	0.00183	-0.00313	-0.00353	0.00287
	(-3.98)	(0.64)	(-1.22)	(-1.76)	(1.45)
Constant	0.547	1.534	-0.901	-0.315	0.730
	(1.89)	(1.03)	(-0.68)	(-0.30)	(0.71)
N	4,368	502	502	502	502

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

vidual random surplus is associated with a lower chance of being unmatched, and—conditional on being unmatched—a lower chance that an agent is left single for being unattractive.

6.3 Demographic characteristics

We investigate whether individual characteristics have any effects on the number of matches and payoffs in each configuration. Using regressions with individual fixed effects, Table 13a shows the effect of age, gender, grade, and major on the number of matched pairs a subject reaches in each of the four balanced markets. There is hardly any effect of these characteristics, except that in AN6, subjects from economics or business have a higher number of matches, and in MN6, male students have a higher number of matches. In Table 13b, we can see the effects of these characteristics on payoffs, and only male students in the MN6 game earn a higher payoff compared with female students. For the unbalanced markets, Tables 14a and 14b present the effect of these individual characteristics on the number of matched pairs and payoffs, respectively. The only significant finding is that male students earn less than female students in AN7. These results indicate a modest role of gender and major in the two-sided matching markets.

Table 13: Individual characteristics determinants of outcomes in balanced markets

(a) Being matched

	AE6 Log Matches	AN6 Log Matches	ME6 Log Matches	MN6 Log Matches
Age	0.013	0.003	0.009	-0.006
	(1.08)	(0.13)	(0.41)	(-0.39)
Male	-0.015	-0.050	0.014	0.060*
	(-0.65)	(-1.27)	(0.36)	(2.58)
Grade of study	-0.019	0.013	-0.012	0.012
	(-1.20)	(0.45)	(-0.36)	(0.59)
Econ/Business	0.004	0.062*	-0.035	-0.018
	(0.18)	(2.02)	(-0.82)	(-0.71)
Constant	1.626***	1.678***	1.535***	1.879***
	(8.22)	(5.08)	(4.05)	(7.15)
Observations	1,092	1,092	1,092	1,092

t statistics in parentheses

clustered at individual level

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

(b) Payoffs

	AE6 payoff	AN6 payoff	ME6 payoff	MN6 payoff
Round (1-7)	-0.433	-0.082	0.030	-0.021
	(-1.44)	(-0.32)	(0.11)	(-0.08)
Age	0.706	0.856	0.283	-0.564
	(1.03)	(1.24)	(0.41)	(-0.91)
Male	-0.331	-1.468	-0.763	2.325*
	(-0.28)	(-1.14)	(-0.59)	(2.17)
Grade of study	-0.517	0.042	0.018	0.938
	(-0.58)	(0.04)	(0.02)	(1.09)
Econ/Business	-0.315	1.643	-2.015	0.186
	(-0.28)	(1.43)	(-1.67)	(0.18)
Constant	18.99	12.04	22.21	37.85***
	(1.63)	(1.04)	(1.91)	(3.57)
Observations	1,092	1,092	1,092	1,092

t statistics in parentheses

clustered at individual level

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 14: Individual characteristics determinants of being matched in unbalanced markets

(a) Being matched

	AE7 Log Matches	AN7 Log Matches	ME7 Log Matches	MN7 Log Matches
Age	0.006	0.006	0.002	0.009
	(0.73)	(0.45)	(0.28)	(0.77)
Male	-0.034	-0.086	0.004	0.016
	(-1.05)	(-1.90)	(0.12)	(0.32)
Grade of study	-0.002	-0.003	0.004	-0.003
	(-0.83)	(-0.54)	(1.46)	(-0.51)
Econ/Business	0.029	-0.019	0.036	0.023
	(0.65)	(-0.38)	(0.75)	(0.42)
Constant	1.620***	1.620***	1.640***	1.423***
	(8.99)	(6.29)	(10.10)	(5.44)
Observations	980	980	980	980

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

(b) Payoffs

	AE7 payoff	AN7 payoff	ME7 payoff	MN7 payoff
Round (1-7)	0.352	0.281	0.023	0.556
	(1.18)	(0.87)	(0.08)	(1.80)
Age	0.072	0.264	-0.133	0.252
	(0.22)	(0.76)	(-0.45)	(0.68)
Male	-2.125	-2.962*	0.077	-0.465
	(-1.44)	(-2.15)	(0.07)	(-0.32)
Grade of study	-0.025	0.050	0.039	-0.034
	(-0.17)	(0.30)	(0.26)	(-0.23)
Econ/Business	-0.360	0.170	1.264	0.857
	(-0.20)	(0.12)	(0.93)	(0.50)
Constant	25.30***	20.03**	27.81***	17.10*
	(3.52)	(2.76)	(4.53)	(2.15)
Observations	980	980	980	980

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

7 Conclusion

We experimentally investigated an influential class of matching models that has received extensive theoretical and empirical scrutiny. We find that factors that are abstracted away in the basic apparatus play important roles in determining the rate of matching, stability, and efficiency. Specifically, (i) whether agents can sort on their productivity and (ii) whether agents can split their surpluses by half as a sustainable outcome both influence the outcome of the two-sided matching market. Most interestingly, a noncooperative extension of the cooperative model provides near-perfect predictions of the payoffs participants obtained in the experiment.

Our contributions are threefold. First, we experimentally investigate how different features of the setting—whether efficient matching is assortative and whether equal-splits is in the core—affect the rate of matching, the rate of efficient matching, and the percentage of efficient surplus achieved. Second, we provide a noncooperative theory that makes a unique prediction of individual payoffs in the balanced markets, which is experimentally supported by our results and results in the literature. Third, we investigate unbalanced markets and find that noncompetitive outcomes may arise both theoretically and experimentally.

Our experiment serves as an initial step in understanding decentralized matching and bargaining markets by considering 3-by-3 and 3-by-4 markets. Interesting next steps worth pursuing include investigating (i) the outcome when the market is larger (e.g., 6 by 6) in order to study the effects of market thickness on stable bargaining outcomes; (ii) the effects of more unbalanced ratio of the two sides (e.g., 3 by 6) and hence more competition on aggregate and individual outcomes of the market; (iii) the effects of different bargaining protocols on outcomes; and (iv) the effects of asymmetric information on outcomes.

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Online appendices

A   Experiment instructions

Experimental instructions are in Chinese. We present the English translation for balanced markets. The instructions for unbalanced markets are appropriately modified. Figure 8 presents a screenshot of the experiment.

Figure 8: A (translated) screenshot of the experiment

Instructions for balanced markets

Welcome page

Welcome to this experiment on decision-making. Please read the following instructions carefully.

This experiment will last about two hours. During the experiment, do not communicate with other participants in any way. If you have any question at any time, please raise your hand, and an experimenter will come and assist you privately.

At the beginning of the experiment, you will be randomly assigned to a group of six participants, and this is fixed throughout the experiment. Each participant sits behind behind a private computer, and all decisions are made on the computer screen. This is an anonymous experiment:

Experimenters and other participants cannot link your name to your desk number, and thus will not know your identity or that of other participants who make the specific decisions.

Payoffs

Throughout the experiment, your earnings are denoted in points. Your earnings depend on your own choices and the choices of other participants. At the end of the experiment, your earnings will be converted to RMB at the following rate: 12 points = 1 RMB. In addition, you will receive 20 RMB as show-up fee. This a show-up fee is added to your earnings during the experiment. Your total earnings will be paid to you privately at the end of the experiment.

There are three cold colors and three warm colors in experimental roles. Cold colors are Blue, Cyan, and Green. Warm colors are Pink, Red, and Yellow. In each of the matching games (there are 28 games in total), each of the six participants will be randomly assigned one of the six role colors. In these matching games, a cold color can only be matched with a warm color, and vice versa. Two cold colors and two warm colors cannot be matched. For example, a Cyan can match with a Pink (if they both want to).

When a cold color is matched with a warm color, they can share their total earnings. The total earnings of two colors are depicted in the table below. In this table, you can see that a Blue and a Yellow can share total earnings of 10 points. That is, their total earnings must equal 10.

	🔴(Pink)	🔴(Red)	🟡(Yellow)
🟦	50	20	10
🟦	20	30	60
🟩	30	50	20

Matching Stage

In order to reach a match, all of the six participants will go through a short matching stage that lasts for 3 minutes.

Proposing. Each participant can propose to any of the other three colors on the opposite side of the market. When proposing to someone, you can first click that color in the screen, and decide how you want to share the total earnings.

For example, if the Red (proposer) wants to propose to the Green (receiver), the Red has to decide how to allocate the total 60 points between them. Once the proposal is made, the Green will receive a notification of the proposal on his or her private information board. The notification contains all of the information about the proposal (who proposes and how many points each gets). Note that except for the Green (the receiver of the proposal), other people will not receive any information about this proposal.

Accepting/rejecting proposals. When a proposal is made from a proposer to a receiver, the receiver has 30 seconds to either accept or reject the proposal.

If the receiver rejects the proposal within 30 seconds or does not accept it within the 30 seconds, this proposal is no longer valid and will disappear on the receiver's private information board.

If the receiver accepts the proposal within 30 seconds, a temporary match between the receiver and the proposer is made. Once a temporary match is made, a matching posting will appear on the public information board with full information (who matched and how many points each gets).

Before the receiver decides to accept or reject a proposal (and before the 30 seconds are over), the proposer of this proposal is not able to make any proposals to any other colors (or to make a new proposal to the same receiver); however, the proposer of this proposal can accept a proposal from others. In this case, his or her previous proposal becomes invalid.

Moreover, it is possible that one participant receives multiple proposals from different proposers at the same time. In this case, the receiver can choose to accept at most one proposal (or reject all of them).

Temporary match. Once a temporary match is made, the two people in this match are still able to make proposals to others, and they can also receive proposals from other proposers.

In the former case, if one's new proposal is accepted, then the previous temporary match is ended, and a new temporary match is formed. In this case, the person who is previously matched with him or her will be notified, and the matching posting will be updated on the public board.

As long as the matching stage has not ended, one can always break his or her current temporary match by forming a new temporary match (by proposing and accepting, or by accepting another proposal). One cannot break a current temporary match without forming a new match. If one is passively broken up with by someone within the last 15 seconds of the 3-minute market, he or she will be granted 15 seconds to make new proposals to others.

Permanent match. When the matching stage ends at the 3-minute mark, all of the temporary matches at the end of the matching stage become permanent. All participants with a permanent match will receive the points allocated to him or her in the match (as made by the proposers), and all of the remaining participants are unmatched, and will receive zero points. Once everyone receives his or her points, the game is finished.

Repetition

In this experiment, you will play four different matching games. In each of the matching games the procedures are the same; the only difference is the game payoff. The game payoff matrix will be shown to you once a new game is being played. Each of the matching games will be repeated for 7 rounds. Therefore, there are 28 rounds in total for the entire experiment. Throughout the 28 rounds, you will stay in the same group of six participants. Before the start of the 28 rounds, you will also have the opportunity to play one practice round. The goal of the

practice round is to let you get familiar with the procedure; the points you receive in this round will not be included in your final earnings.

All of the six participants in a group can also see the matching results from past rounds. The matching results contain information about which colors are matched with each other and the number of points they earned in the match.

Earnings

At the end of the experiment, you will receive the sum of the 240 points (endowed in the beginning) and the points from each round. Your total earnings in the experiment are equal to the total points divided by 12.

B Robustness checks

This section contains robustness checks of the empirical results with alternative specifications.

B.1 Hypothesis 1e: Determinants of outcomes in balanced markets

To check the robustness of our results regarding Hypothesis 1e, we present the results from regressions with alternative dependent variables and alternative specifications: We consider (i) the outcomes of interest directly as dependent variables in addition to their logged values, and (ii) the following four specifications, in which specification (3) is the leading specification we presented in the main text.

$$(1) \quad y_i = \beta_1 \cdot \text{sort}_i + \beta_2 \cdot \text{ESIC}_i + c + \varepsilon_i,$$

$$(2) \quad y_i = \beta_1 \cdot \text{sort}_i + \beta_2 \cdot \text{ESIC}_i + \beta_3 \cdot \text{sort}_i \cdot \text{ESIC}_i + \beta_7 + c + \varepsilon_i,$$

$$(3) \quad y_i = \beta_1 \cdot \text{sort}_i + \beta_2 \cdot \text{ESIC}_i + \beta_3 \cdot \text{sort}_i \cdot \text{ESIC}_i + \beta_4 \cdot \text{round}_i + \beta_5 \cdot \text{order}_i + c + \varepsilon_i,$$

$$(4) \quad y_i = \beta_1 \cdot \text{sort}_i + \beta_2 \cdot \text{ESIC}_i + \beta_3 \cdot \text{sort}_i \cdot \text{ESIC}_i + \beta_4 \cdot \text{round}_i + \beta_6 \cdot \text{period}_i + c + \varepsilon_i,$$

where i is the index of a game (out of 728 balanced markets); y_i is the number of matched pairs in Table B1, and the log of it in Table B2; sort_i is the indicator of whether the market played in the game is assortative; ESIC_i is the indicator of whether the market has equal splits in the core; round_i is the round (out of 7) the same market has been played; order_i is the order (out of 4) the game is played in; and period_i is the overall period (out of 28) the game is played in.

Tables B1 and B2 present the results for determinants of the number of matched pairs. All else equal, assortativity increases the number of matched pairs by 0.129 to 0.183 (or by 5.39% to 7.75%), whereas equal splits in the core increases the number of matches by 0.277 to 0.328 (or by 11.4% to 13.6%), depending on whether learning over time is controlled for. The evidence

Table B1: Determinants of the number of matched pairs

	(1) matches	(2) matches	(3) matches	(4) matches
assortative	0.129***	0.176***	0.183***	0.183***
	(3.83)	(3.69)	(3.87)	(3.87)
ESIC	0.277***	0.324***	0.328***	0.328***
	(8.23)	(6.80)	(6.93)	(6.93)
assortative*ESIC		-0.0934	-0.101	-0.101
		(-1.39)	(-1.50)	(-1.50)
round			0.0165*	0.00972
			(1.97)	(1.13)
order			0.0473**	
			(3.16)	
period				0.00676**
				(3.16)
constant	2.457***	2.434***	2.246***	2.294***
	(84.12)	(72.20)	(36.80)	(43.39)
N	728	728	728	728

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table B2: Determinants of the logged number of matched pairs

	(1) log matches	(2) log matches	(3) log matches	(4) log matches
assortative	0.0539***	0.0745***	0.0775***	0.0775***
	(3.88)	(3.79)	(3.97)	(3.97)
ESIC	0.114***	0.135***	0.136***	0.136***
	(8.20)	(6.85)	(6.98)	(6.98)
assortative*ESIC		-0.0410	-0.0441	-0.0441
		(-1.48)	(-1.60)	(-1.60)
round			0.00708*	0.00424
			(2.05)	(1.19)
order			0.0199**	
			(3.22)	
period				0.00284**
				(3.22)
constant	0.876***	0.866***	0.786***	0.806***
	(72.74)	(62.31)	(31.27)	(37.02)
N	728	728	728	728

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

suggests that whether equal split is in the core plays a more important role than assortativity in determining the number of matches. Tables B1 and B2 also show evidence that learning mildly improves the expected number of matches over time. Having played the same game for one more round increases the number of matches by about 0.07%, and having played any game for one more round increases the number of matches by about 0.28%. Having played 7 more other games increases the number of matches by 1.99% (the order coefficient in Table B2).

Table B3: Determinants of the number of efficiently matched pairs

	(1) eff match	(2) eff match	(3) eff match	(4) eff match
assortative	0.220**	0.368***	0.380***	0.380***
	(3.02)	(3.58)	(3.72)	(3.72)
ESIC	0.841***	0.989***	0.995***	0.995***
	(11.54)	(9.62)	(9.74)	(9.74)
assortative*ESIC		-0.297*	-0.309*	-0.309*
		(-2.04)	(-2.14)	(-2.14)
round			0.0433*	0.0321
			(2.40)	(1.72)
order			0.0784*	
			(2.43)	
period				0.0112*
				(2.43)
constant	1.552***	1.478***	1.103***	1.181***
	(24.61)	(20.34)	(8.37)	(10.35)
N	728	728	728	728

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Tables B3 and B4 present the results for determinants of the number of efficiently matched pairs. Assortativity increases the number of efficiently matched pairs by 0.22 to 0.38 (or by 7.84% to 14.8%), and equal-splits in the core increases the number of efficiently matched pairs by 0.841 to 0.995 (or by 38.1% to 43.4%), depending on whether learning over time is controlled for.

Tables B5 and B6 present the results for determinants of the surplus. All else equal, assortativity increases surplus by 3.78% to 4.11%, and equal-splits in the core increases surplus by 7.61% to 7.78%, depending on whether learning is controlled for. There is some gain from learning. Having the same game one more round increases efficiency by about 0.85%, and having played any game one more round increases efficiency by about 0.31%. Having played 7 more other games increases efficiency by 2.18% (the order coefficient in specification (3) in Table B6). Since in terms of both the surplus and the number of matches the order effect disappears when we control for period number, we conclude that there is no order effect caused by spillovers of playing previous

Table B4: Determinants of the logged number of efficiently matched pairs

	(1) log (eff+1)	(2) log (eff+1)	(3) log (eff+1)	(4) log (eff+1)
assortative	0.0784**	0.144***	0.148***	0.148***
	(2.58)	(3.37)	(3.47)	(3.47)
ESIC	0.319***	0.385***	0.387***	0.387***
	(10.52)	(9.00)	(9.08)	(9.08)
assortative*ESIC		-0.131*	-0.135*	-0.135*
		(-2.17)	(-2.25)	(-2.25)
round			0.0160*	0.0123
			(2.12)	(1.58)
order			0.0257	
			(1.91)	
period				0.00368
				(1.91)
constant	0.845***	0.812***	0.682***	0.708***
	(32.16)	(26.84)	(12.40)	(14.86)
N	728	728	728	728

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table B5: Determinants of surplus

	(1) surplus	(2) surplus	(3) surplus	(4) surplus
assortative	5.907**	6.264*	6.781*	6.781*
	(2.94)	(2.21)	(2.42)	(2.42)
ESIC	13.32***	13.68***	13.94***	13.94***
	(6.64)	(4.82)	(4.98)	(4.98)
assortative*ESIC		-0.714	-1.232	-1.232
		(-0.18)	(-0.31)	(-0.31)
round			1.315**	0.835
			(2.66)	(1.63)
order			3.363***	
			(3.79)	
period				0.480***
				(3.79)
constant	170.3***	170.1***	156.2***	159.5***
	(98.03)	(84.75)	(43.20)	(50.96)
N	728	728	728	728

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table B6: Determinants of log surplus

	(1) log surplus	(2) log surplus	(3) log surplus	(4) log surplus
assortative	0.0359**	0.0378*	0.0411*	0.0411*
	(2.78)	(2.07)	(2.28)	(2.28)
ESIC	0.0743***	0.0761***	0.0778***	0.0778***
	(5.75)	(4.16)	(4.31)	(4.31)
assortative*ESIC		-0.00370	-0.00706	-0.00706
		(-0.14)	(-0.28)	(-0.28)
round			0.00847**	0.00536
			(2.66)	(1.63)
order			0.0218***	
			(3.82)	
period				0.00312***
				(3.82)
constant	5.123***	5.122***	5.032***	5.054***
	(457.79)	(396.12)	(216.09)	(250.59)
N	728	728	728	728

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

games. In all of our analyses, we pool all data together.

B.2   Determinants of outcomes, all markets

Tables B7 and B8 show the determinants of the number of matched pairs when both balanced and unbalanced markets are considered. Assortativity and ESIC continue to have significant influences on market outcome: Assortative markets have 0.104 to 0.118 (or 4.10% to 4.83%) more matched pairs, and ESIC markets have 0.277 to 0.327 (or 11.4% to 13.6%) more matched pairs. Also, an additional player in unbalanced markets increases matched pairs. In particular, 0.280 to 0.318 more pairs are matched in unbalanced markets on average, which increases the matching rate by 11.5% to 13.4%.

Tables B9 and B10 show that assortativity does not increase the number of efficiently matched pairs at a statistically significant level. In comparison, having equal splits in the core increases the number of efficiently matched pairs by 0.841 to 0.994 (or by 31.9% to 38.7%). Market thickness increases the number of efficiently matched pairs by 0.665 to 0.784 (or by 26.1% to 30.3%). Note that there are significantly more efficiently matched pairs in balanced assortative (AE6 and AN6) markets.

Tables B11 and B12 show that assortativity increases surplus by 3.91% to 4.32%; having equal

splits in the core increases surplus by 7.43% to 7.73%; and market thickness increases surplus by 5.39% to 5.66%, and all are statistically significant at the 99.9% significance level.

Since experience with the negotiation has a mild yet statistically significant influence on outcomes, besides controlling for learning effects, we also consider the determinants of the outcomes when the learning effect is minimal. We consider the determinants in the first games subjects play (i.e., first period of 28) and the first time a particular game is played (i.e., first round of 7 for each game). The results are presented in Tables B13 and B14. The small number of groups (26) causes the test to lose statistical power.

B.3 Individual payoffs

Figure B1 shows the histogram of payoffs for all efficiently matched individuals. Figure B2 shows the histograms of payoffs for all matched individuals—rather than individuals in efficient matching only, as in the main text—in the balanced and unbalanced markets. Figure B3 shows the average payoffs of me and women in balanced versus unbalanced markets, by time.

B.4 Other experimental results

Table B15 shows alternative specifications for regression on determinants of the number of proposals for balanced markets. The alternative specifications yield conclusions similar to our leading specification (3), presented in Column (2) of Table 10.

Figure B4 shows the percentage of surplus achieved by time for balanced and unbalanced markets. Figure B5 reports the word clouds of subjects' responses to the following questions regarding their behavior. In general, subjects are mostly behaving to maximize their payoffs, with very minimal concerns for fairness.

Table B7: Determinants of the number of matched pairs, all markets

	(1) matches	(2) matches	(3) matches	(4) matches
assortative	0.118***	0.104**	0.104**	0.104**
	(4.90)	(2.84)	(2.86)	(2.86)
ESIC	0.277***	0.324***	0.327***	0.327***
	(8.66)	(7.16)	(7.28)	(7.28)
balanced	-0.280***	-0.316***	-0.318***	-0.318***
	(-9.62)	(-7.68)	(-7.81)	(-7.81)
assortative*ESIC		-0.0934	-0.0984	-0.0984
		(-1.46)	(-1.55)	(-1.55)
assortative*balanced		0.0723	0.0773	0.0773
		(1.24)	(1.34)	(1.34)
round			0.0239***	0.0192**
			(4.00)	(3.12)
order			0.0326**	
			(3.06)	
period				0.00466**
				(3.06)
constant	2.743***	2.750***	2.573***	2.606***
	(125.41)	(106.52)	(58.47)	(67.72)
N	1,288	1,288	1,288	1,288

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table B8: Determinants of the logged number of matched pairs, all markets

	(1) log matches	(2) log matches	(3) log matches	(4) log matches
assortative	0.0483***	0.0410**	0.0410**	0.0410**
	(4.86)	(2.72)	(2.75)	(2.75)
ESIC	0.114***	0.135***	0.136***	0.136***
	(8.64)	(7.21)	(7.33)	(7.33)
balanced	-0.115***	-0.131***	-0.132***	-0.132***
	(-9.55)	(-7.74)	(-7.87)	(-7.87)
assortative*ESIC		-0.0410	-0.0431	-0.0431
		(-1.55)	(-1.65)	(-1.65)
assortative*balanced		0.0335	0.0356	0.0356
		(1.40)	(1.50)	(1.50)
round			0.0100***	0.00806**
			(4.07)	(3.18)
order			0.0137**	
			(3.11)	
period				0.00196**
				(3.11)
constant	0.994***	0.997***	0.923***	0.937***
	(110.16)	(93.68)	(50.88)	(59.06)
N	1,288	1,288	1,288	1,288

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table B9: Determinants of the number of efficiently matched pairs, all markets

	(1) eff matches	(2) eff matches	(3) eff matches	(4) eff matches
assortative	0.185***	0.139	0.139	0.139
	(3.45)	(1.71)	(1.73)	(1.73)
ESIC	0.841***	0.989***	0.994***	0.994***
	(11.79)	(9.82)	(9.97)	(9.97)
balanced	-0.665***	-0.779***	-0.784***	-0.784***
	(-10.26)	(-8.51)	(-8.66)	(-8.66)
assortative*ESIC		-0.297*	-0.307*	-0.307*
		(-2.08)	(-2.18)	(-2.18)
assortative*balanced		0.229	0.239	0.239
		(1.77)	(1.87)	(1.87)
round			0.0586***	0.0489***
			(4.42)	(3.57)
order			0.0682**	
			(2.87)	
period				0.00974**
				(2.87)
constant	2.234***	2.257***	1.852***	1.920***
	(45.88)	(39.30)	(18.94)	(22.46)
N	1,288	1,288	1,288	1,288

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table B10: Determinants of the logged number of efficiently matched pairs, all markets

	(1)	(2)	(3)	(4)
	log (eff+1)	log (eff+1)	log (eff+1)	log (eff+1)
assortative	0.0724***	0.0645	0.0645*	0.0645*
	(3.31)	(1.95)	(1.96)	(1.96)
ESIC	0.319***	0.385***	0.387***	0.387***
	(10.97)	(9.37)	(9.48)	(9.48)
balanced	-0.261***	-0.301***	-0.303***	-0.303***
	(-9.89)	(-8.07)	(-8.18)	(-8.18)
assortative*ESIC		-0.131*	-0.135*	-0.135*
		(-2.26)	(-2.34)	(-2.34)
assortative*balanced		0.0795	0.0832	0.0832
		(1.51)	(1.59)	(1.59)
round			0.0219***	0.0184**
			(4.04)	(3.30)
order			0.0240*	
			(2.47)	
period				0.00343*
				(2.47)
constant	1.109***	1.113***	0.966***	0.990***
	(55.82)	(47.51)	(24.15)	(28.31)
N	1,288	1,288	1,288	1,288

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table B11: Determinants of surplus, all markets

	(1) surplus	(2) surplus	(3) surplus	(4) surplus
assortative	6.289***	6.786**	6.786**	6.786**
	(4.22)	(3.00)	(3.04)	(3.04)
ESIC	13.32***	13.68***	13.86***	13.86***
	(6.73)	(4.88)	(5.00)	(5.00)
balanced	-9.901***	-9.640***	-9.819***	-9.819***
	(-5.50)	(-3.79)	(-3.90)	(-3.90)
assortative*ESIC		-0.714	-1.071	-1.071
		(-0.18)	(-0.27)	(-0.27)
assortative*balanced		-0.522	-0.165	-0.165
		(-0.14)	(-0.05)	(-0.05)
round			1.665***	1.334***
			(4.52)	(3.51)
order			2.322***	
			(3.52)	
period				0.332***
				(3.52)
constant	180.0***	179.8***	167.3***	169.6***
	(133.06)	(112.46)	(61.58)	(71.41)
N	1,288	1,288	1,288	1,288

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table B12: Determinants of log number of matched pairs, all markets

	(1) log matches	(2) log matches	(3) log matches	(4) log matches
assortative	0.0483***	0.0410**	0.0410**	0.0410**
	(4.86)	(2.72)	(2.75)	(2.75)
ESIC	0.114***	0.135***	0.136***	0.136***
	(8.64)	(7.21)	(7.33)	(7.33)
balanced	-0.115***	-0.131***	-0.132***	-0.132***
	(-9.55)	(-7.74)	(-7.87)	(-7.87)
assortative*ESIC		-0.0410	-0.0431	-0.0431
		(-1.55)	(-1.65)	(-1.65)
assortative*balanced		0.0335	0.0356	0.0356
		(1.40)	(1.50)	(1.50)
round			0.0100***	0.00806**
			(4.07)	(3.18)
order			0.0137**	
			(3.11)	
period				0.00196**
				(3.11)
constant	0.994***	0.997***	0.923***	0.937***
	(110.16)	(93.68)	(50.88)	(59.06)
N	1,288	1,288	1,288	1,288

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table B13: Determinants of outcomes in balanced markets, first games

	(1) log matches	(2) log (efficient matches+1)	(3) log surplus
assortative	-0.106	-0.231	-0.0904
	(-0.94)	(-1.21)	(-0.90)
ESIC	-0.135	0.183	-0.196
	(-1.16)	(0.92)	(-1.87)
assortative*ESIC	0.193	0.329	0.250
	(1.21)	(1.22)	(1.75)
constant	1.031***	0.924***	5.235***
	(12.48)	(6.59)	(70.83)
N	26	26	26

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table B14: Determinants of outcomes in balanced markets, first rounds

	(1) log matches	(2) log (efficient matches+1)	(3) log surplus
assortative	0.120*	0.354**	0.0756
	(2.06)	(3.37)	(1.35)
ESIC	0.151*	0.588***	0.0888
	(2.60)	(5.61)	(1.59)
assortative*ESIC	-0.0888	-0.354*	-0.0274
	(-1.08)	(-2.39)	(-0.35)
order	-0.00173	0.0192	0.0111
	(-0.09)	(0.58)	(0.63)
constant	0.827***	0.557***	5.044***
	(13.14)	(4.92)	(83.72)
N	104	104	104

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table B15: Determinants of the logged number of proposals

	(1) log proposals	(2) log proposals	(3) log proposals	(4) log proposals
assortative	-0.0682	-0.109*	-0.124*	-0.124*
	(-1.81)	(-2.04)	(-2.39)	(-2.39)
ESIC	-0.257***	-0.297***	-0.305***	-0.305***
	(-6.80)	(-5.57)	(-5.88)	(-5.88)
assortative*ESIC		0.0810	0.0964	0.0964
		(1.07)	(1.31)	(1.31)
round			-0.0293**	-0.0150
			(-3.19)	(-1.59)
order			-0.0996***	
			(-6.07)	
period				-0.0142***
				(-6.07)
constant	2.607***	2.627***	3.001***	2.901***
	(79.73)	(69.59)	(44.89)	(50.11)
N	728	728	728	728

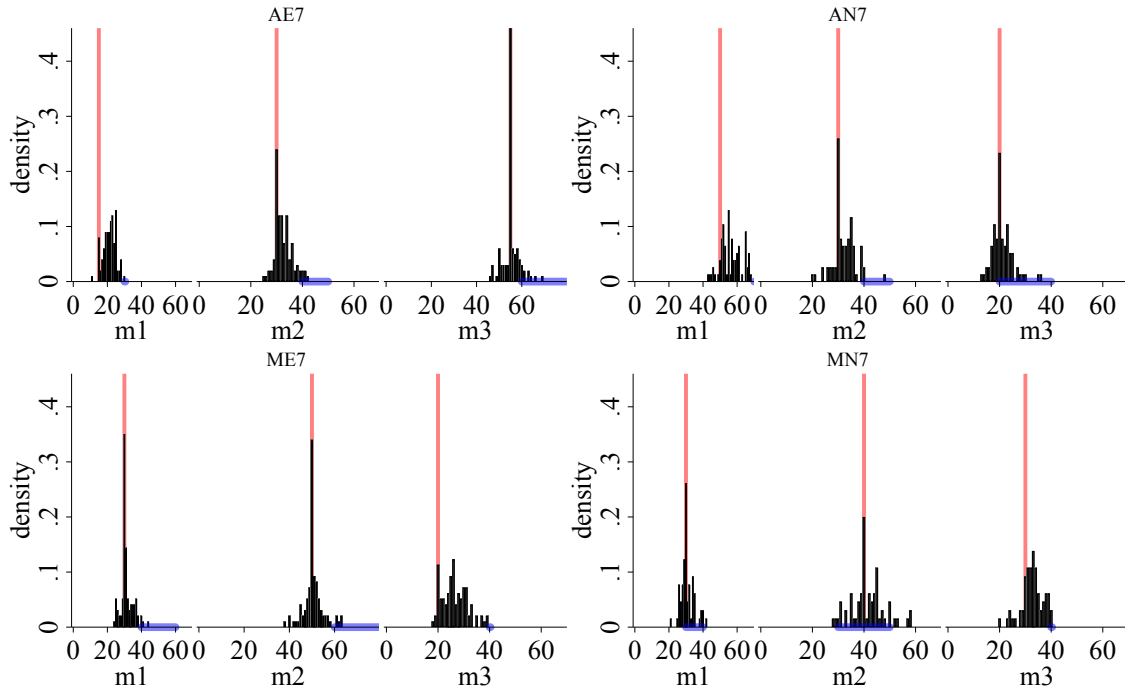*t* statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Figure B1: Histogram of payoffs in the efficient matching
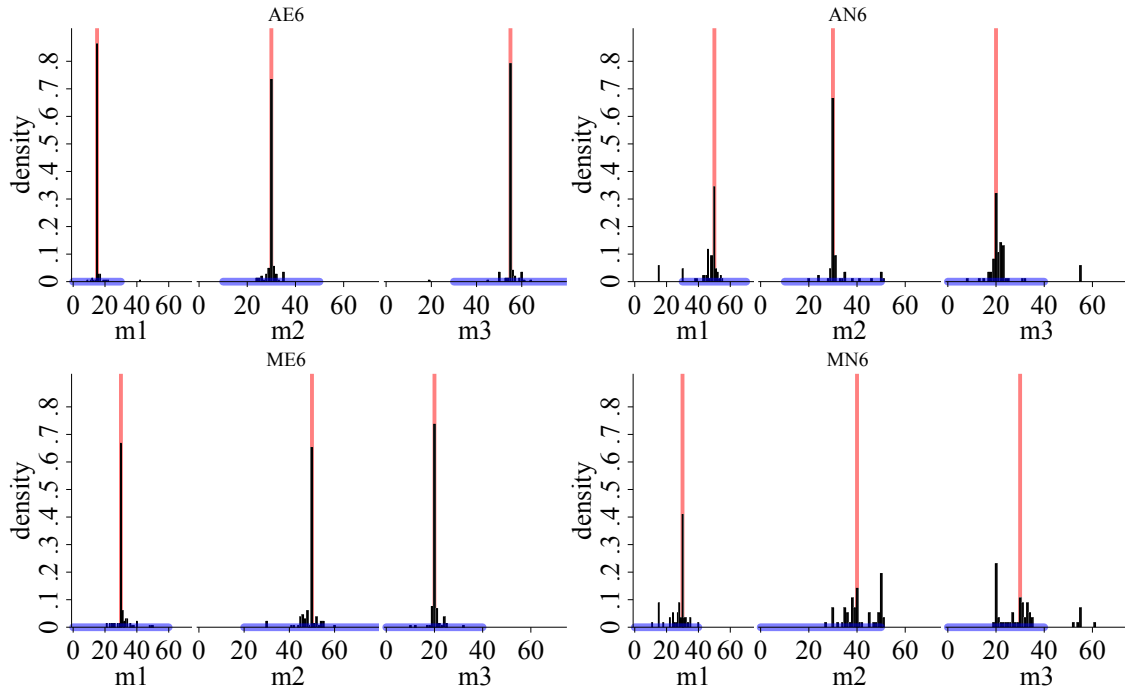
(a) Balanced market

(b) Unbalanced market

Note. Blue horizontal lines represent the range of core payoffs in the cooperative model. Red vertical lines represent the noncompetitive limit payoffs in the noncooperative model. The histogram is the black lines.

Figure B2: Histogram of payoffs for matched individuals
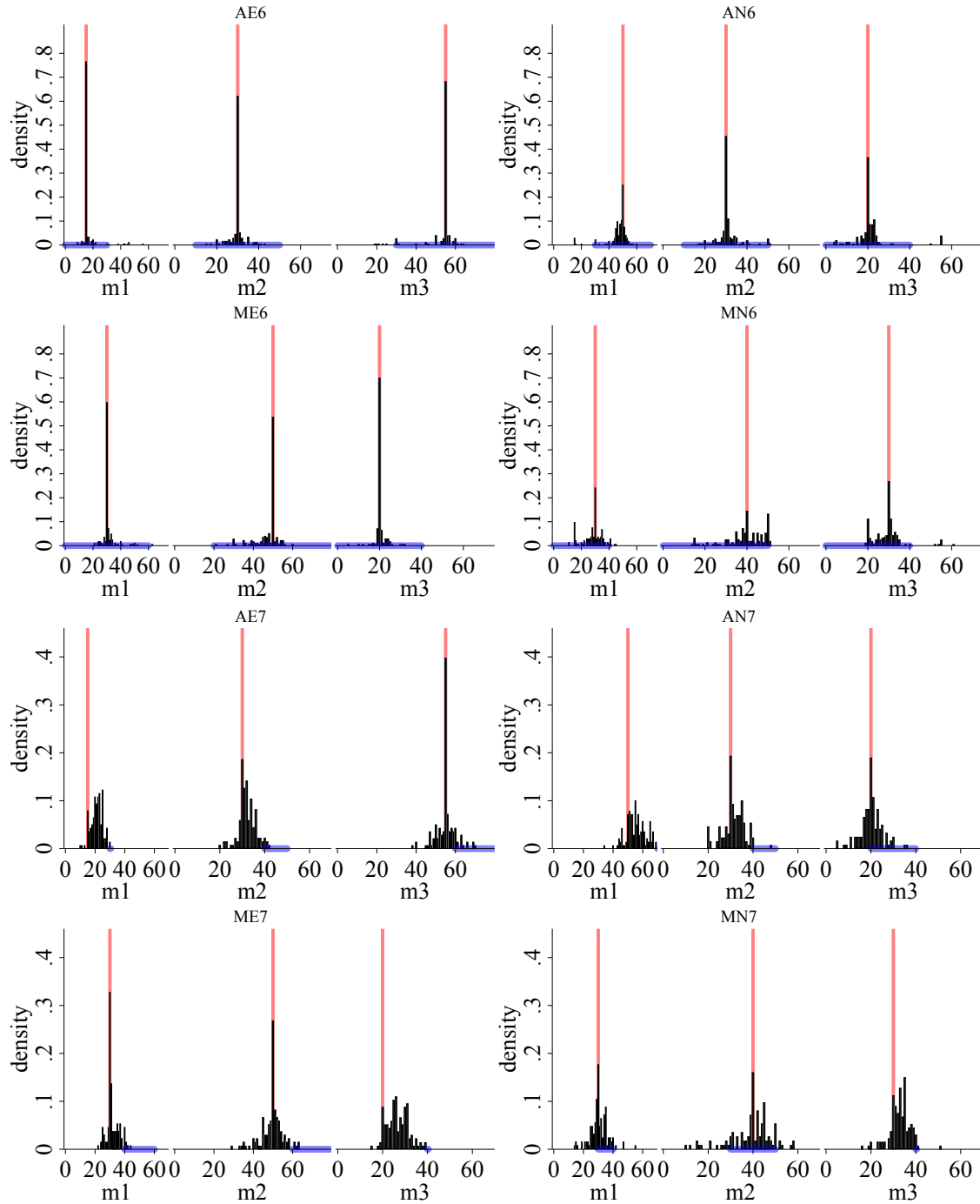

Note. Blue horizontal lines represent the range of core payoffs in the cooperative model. Red vertical lines represent the limit payoffs in the noncooperative model.

Figure B3: Men's and women's payoffs in balanced versus unbalanced markets
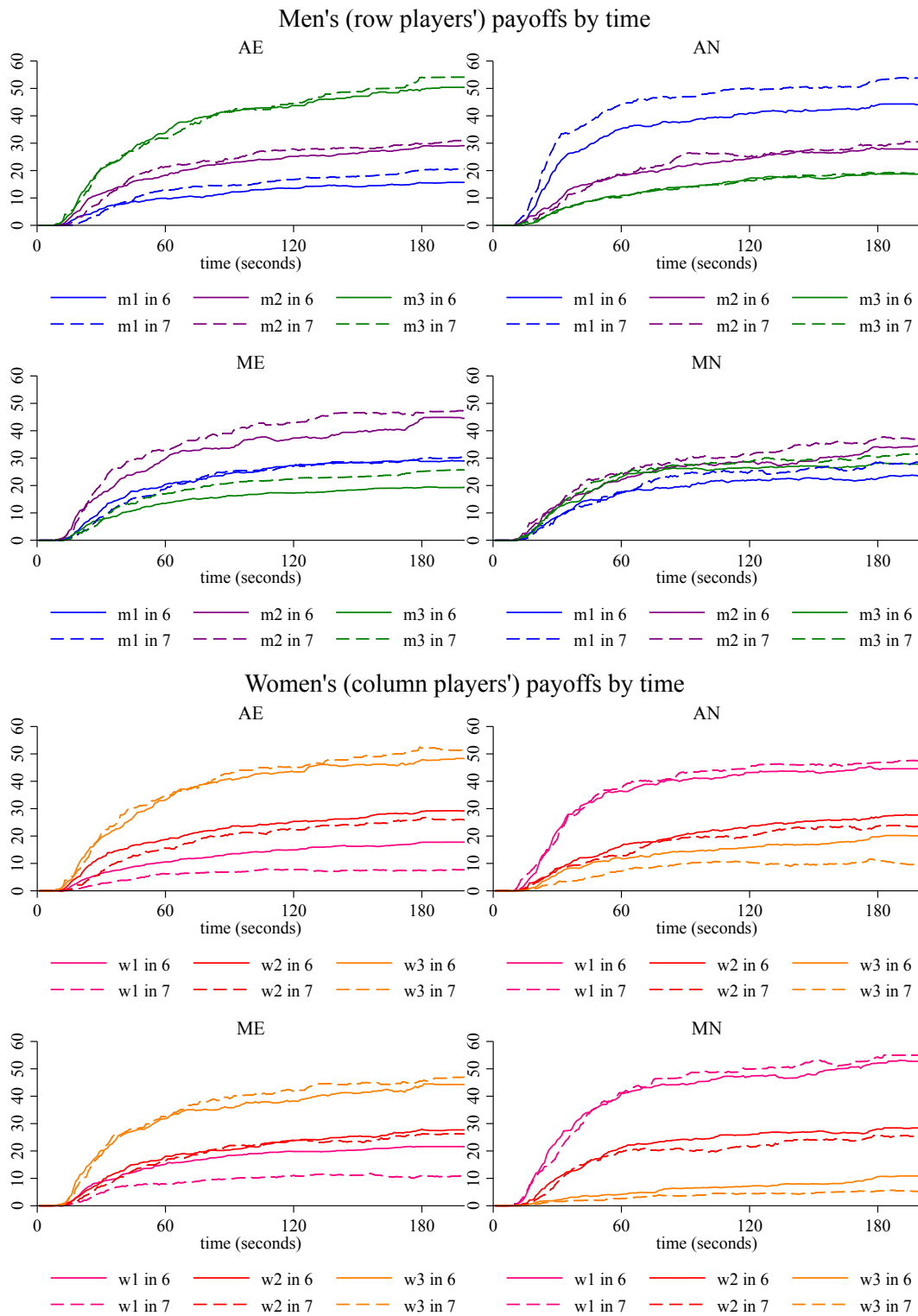
Figure B4: Percent of surplus achieved by time

(a) Balanced markets

(b) Unbalanced markets

Figure B5: Word cloud of responses to questions

(a) "How did you decide to make a proposal?"

(b) "How did you decide to accept or reject a proposal?"

(c) "Were you unmatched in some rounds? If so, why?"

Note. Subjects answers were in Chinese. We present the word clouds of their responses in Chinese and their Google-translated responses in English without correction.
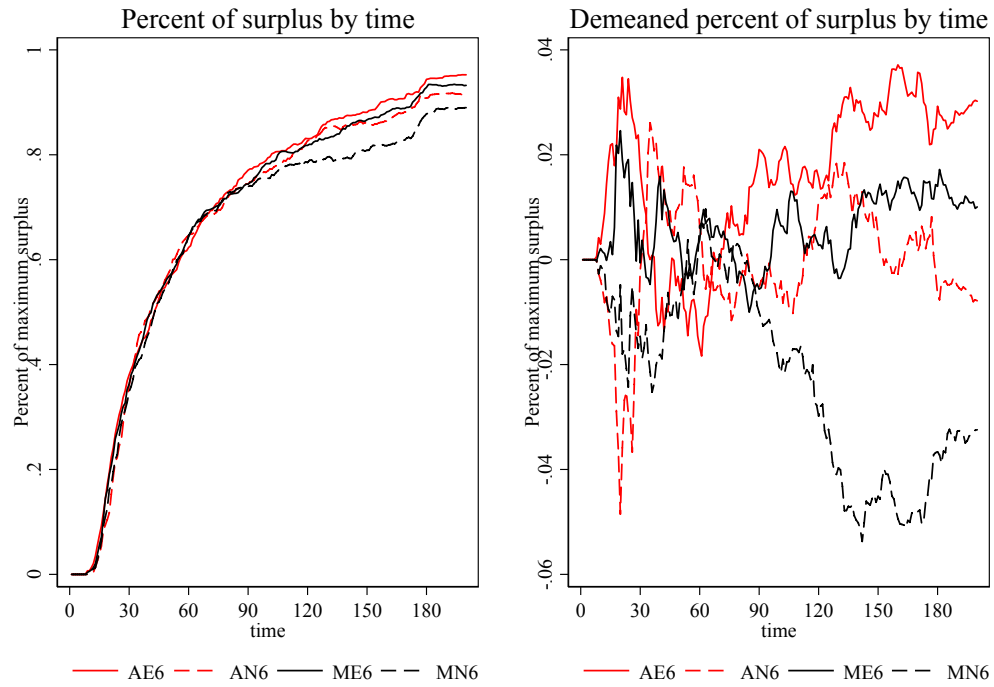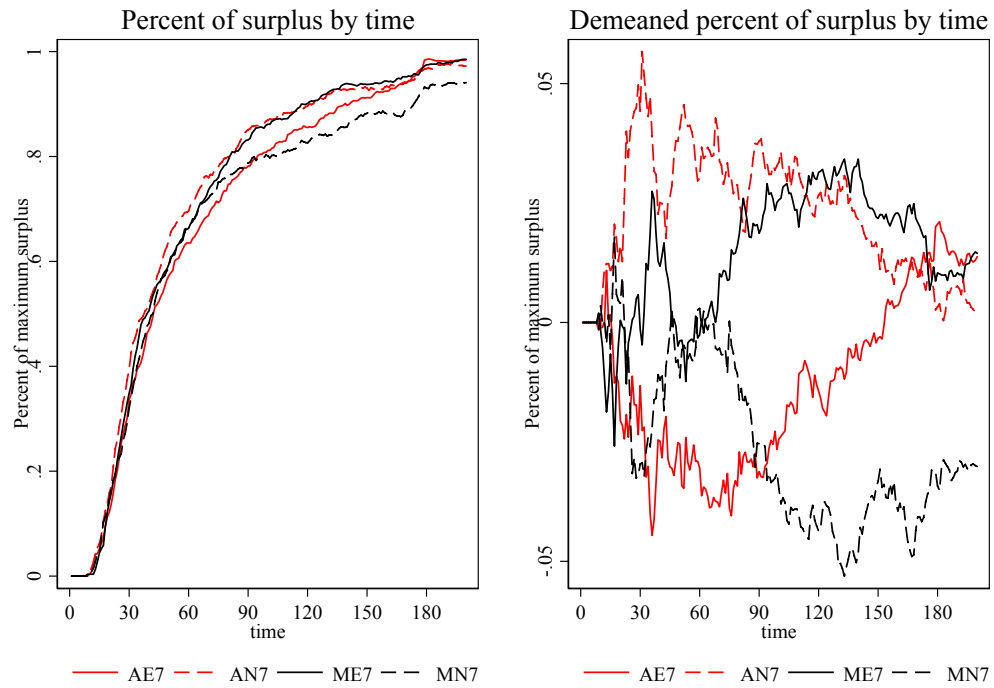
C Omitted proofs

For Theorem 1, it suffices to show the following Lemmas 1, 2, and 3.

Lemma 1. *(1) There is at most one solution to the system of equations given a matching μ and a discount factor $\delta < 1$. (2) If there exists a solution given μ and δ, then there exists a solution given μ and any $\delta' < \delta$.*

Proof of Lemma 1. Fix a matching μ. Consider the system of equations for the cases in which men are the proposers at time zero:

$$U_m^p = s_{m\mu(m)} - \max\left\{\delta \cdot V_{\mu(m)}^r, \max_{m' \in M \setminus m}\left\{s_{m'\mu(m)} - U_{m'}^p\right\}\right\},$$

where

$$V_{\mu(m)}^r = s_{m\mu(m)} - \max\left\{\delta \cdot U_m^p, \max_{w' \in W \setminus \mu(m)}\left\{s_{mw'} - \left[s_{\mu(w')w'} - U_{\mu(w')}^p\right]\right\}\right\};$$

For notational convenience, we follow the notations from max algebra to define $a \oplus b \equiv \max\{a, b\}$ and $\sum_{i \in \{1, \cdots, I\}}^{\oplus} a_i \equiv a_1 \oplus \cdots \oplus a_I$. Consider the following system of $n_M + n_W$ equations with $n_M + n_W$ unknowns $U_{m_1}^p, \cdots, U_{m_M}^p, V_{w_1}^r, \cdots, V_{w_W}^r$.

$$\begin{cases} U_{m_1}^p = s_{m_1\mu(m_1)} - \delta V_{\mu(m_1)}^r \oplus \sum_{m' \neq m_1}^{\oplus}\left[s_{m'\mu(m_1)} - U_{m'}^p\right], \\ \ddots \\ U_{m_{n_M}}^p = s_{m_{n_M}\mu(m_{n_M})} - \delta V_{\mu(m_{n_M})}^r \oplus \sum_{m' \neq m_{n_M}}^{\oplus}\left[s_{m'\mu(m_{n_M})} - U_{m'}^p\right], \\ V_{w_1}^r = s_{\mu(w_1)w_1} - \delta U_{\mu(w_1)}^p \oplus \sum_{w' \neq w_1}^{\oplus}\left[s_{\mu(w_1)w'} - \left[s_{\mu(w')w} - U_{\mu(w')}^p\right]\right], \\ \ddots \\ V_{w_{n_W}}^r = s_{\mu(w_{n_W})w_{n_W}} - \delta U_{\mu(w_{n_W})}^p \oplus \sum_{w' \neq w_{n_W}}^{\oplus}\left[s_{\mu(w_{n_W})w'} - \left[s_{\mu(w')w} - U_{\mu(w')}^p\right]\right]. \end{cases}$$

Consider and rearrange the equation for U_m^p, for any $m \in M$:

$$U_m^p + \delta V_{\mu(m)}^r \oplus \sum_{m' \neq m}^{\oplus}\left[s_{m'\mu(m)} - U_{m'}^p\right] = s_{m\mu(m)}.$$

Then, by using the slack variable methods, we can rewrite this nonlinear equation as a set of n_M linear equations and one nonlinear condition with n_M additional unknowns $x_{mm_1}, \cdots, x_{mm_{n_M}}$:

$$U_m^p + \delta V_{\mu(m)}^r + x_{mm} = s_{m\mu(m)},$$
$$U_m^p + \left[s_{m'\mu(m)} - U_{m'}^p\right] + x_{mm'} = s_{m\mu(m)} \quad \text{for any } m' \neq m,$$
$$x_{mm} \cdot \prod_{m' \neq m} x_{mm'} = 0.$$

We can rearrange the equation for V_w^r and apply the slack variable method to it for any $w \in W$ in a similar fashion, Then we can rewrite the entire problem as a linear programming problem with

$n_M^2 + n_W^2 + n_M + n_W$ variables

$$\min \quad \sum_{m' \in M} \sum_{m \in M} x_{mm'} + \sum_{w' \in W} \sum_{w \in W} x_{ww'},$$

subject to the following $n_M^2 + n_W^2$ main constraints:

$$U_m^p + \left[s_{m'\mu(m)} - U_{m'}^p \right] + x_{mm'} - s_{m\mu(m)} \geq 0, \quad \forall m' \in M \backslash m, \forall m \in M,$$
$$U_m^p + \delta V_{\mu(m)}^r + x_{mm} - s_{m\mu(m)} \geq 0, \quad \forall m \in M,$$
$$V_w^r + \left[s_{\mu(w)w'} - \left[s_{\mu(w')w} - U_{\mu(w')}^p \right] \right] + x_{ww'} - s_{\mu(w)w} \geq 0, \quad \forall w' \in W \backslash w, \forall w \in W,$$
$$V_w^r + \delta U_{\mu(m)}^p + x_{ww} - s_{\mu(w)w} \geq 0, \quad \forall w \in W;$$

and $n_M^2 + n_W^2 + n_M + n_W$ nonnegative constraints:

$$U_m^p \geq 0 \quad \forall m \in M, \quad V_w^r \geq 0 \quad \forall w \in W,$$
$$x_{mm'} \geq 0 \quad \forall m, m' \in M, \quad x_{ww'} \geq 0 \quad \forall w, w' \in W.$$

First, we argue that there is at most one solution to the minimization problem. Note that the constraints are noncolinear, because each of the main constraints contains a different $x_{mm'}$, x_{mm}, $x_{ww'}$ or x_{ww}. If the constraints are satisfied, then there exists a solution. If there exists a solution, there is a unique solution, because of the following argument. All the main constraints will be binding and not all $x_{mm'}$'s and $x_{ww'}$'s will be zero, so the optimal value—if it exists—is not zero. By Dantzig's sufficient uniqueness condition that for a linear program in canonical form the optimal value is positive, the solution is unique.

The proof for the system of equations when women are the proposers in period zero is identical. This establishes part (1) of the proposition.

Second, let C^δ be the constrained set for the minimization problem when the discount factor is δ. Then for $\delta' < \delta$, $C^{\delta'}$ is a closed subset of C^δ because the parts containing δ in the main constraints are nonnegative, which makes the constraints tightened as δ decreases. Since the objective function of the minimization problem is linear, we have that when there is a solution with δ, there will be a solution with $\delta' < \delta$.[20] This establishes part (2) of the proposition. $\square$

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[20]When the objective function is linear, then every indifference surface is a hyperplane with the normal vector being the gradient of the objective function. Now we use this gradient vector as an axis going through the origin. That is, moving in one direction on the axis is going in the same direction as the gradient, and the other going in the opposite direction. Then every point in the entire space lies on some indifference surface of the objective function and all points on the same indifference surface can be projected to a single point where this surface intersects the gradient axis. Hence, if a minimum occurs in the set C^δ, then it is necessarily the case that a lower bound is realized on projection of C^δ on the gradient axis (with lower bound being oriented according to the direction of lower objective values). Since $C^{\delta'}$ is a closed subset of C^δ, its projection on the gradient axis is a closed subset of the projection of C^δ on the gradient axis, which continues to have a lower bound. This immediately implies that a minimum continues to exist when restricted to $C^{\delta'}$. We thank Van Kolpin for the suggestion.

Lemma 1 shows that fixing a matching μ and a discount factor δ, if a solution exists, it is unique and for any discount factor smaller than δ, there exists a unique solution given μ. Proposition 1 leads to the main result on surplus division:

Lemma 2. *For any $\delta \in (0, 1)$, there exists a solution to the system of equations with μ^*.*

Since we already know that there exists a solution with efficient matching when $\delta = 1$, by Proposition 1 part (2), we must have a solution with efficient matching for any $\delta < 1$. This directly gives us Lemma 2.

Lemma 3. *Any inefficient matching μ cannot be supported by the system of equations.*

Proof of Lemma 3. Suppose μ is an inefficient matching: The total surplus s^μ from this inefficient matching is less than the total surplus s^{μ^*} from the unique efficient matching μ^*. Suppose there is a solution to the system of equations for μ. Then since for any man $m \in M$,

$$U_m^p = s_{m\mu(m)} - \max\left\{\delta V_{\mu(m)}^r, \max_{m' \in M\setminus m}\{s_{m'\mu(m)} - U_{m'}^p\}\right\},$$

we must have that and for any $m' \in M\setminus m$,

$$U_m^p \leq s_{m\mu(m)} - (s_{m'\mu(m)} - U_{m'}^p).$$

In particular, the inequality holds for the man $\mu^*(\mu(m))$ that woman $\mu(m)$ would have matched with in the efficient matching μ^*:

$$U_m^p \leq s_{m\mu(m)} - \left(s_{\mu^*(\mu(m))\mu(m)} - U_{\mu^*(\mu(m))}^p\right). \tag{Um}$$

By the same logic, we have the following for each woman in W:

$$V_w^p \leq s_{\mu(w)w} - \left(s_{\mu(w)\mu^*(\mu(w))} - V_{\mu^*(\mu(w))}^p\right). \tag{Vw}$$

Sum all (Um) and (Vw) for all $m \in M$ and $w \in W$, we get

$$\sum_{m \in M} U_m^p + \sum_{w \in W} U_w^p \leq \sum_{m \in M} s_{m\mu(m)} - \sum_{m \in M}\left[s_{\mu^*(\mu(m))\mu(m)} - U_{\mu^*(\mu(m))}^p\right]$$
$$+ \sum_{w \in W} s_{\mu(w)w} - \sum_{w \in W}\left[s_{\mu(w)\mu^*(\mu(w))} - V_{\mu^*(\mu(w))}^p\right],$$

which can be simplified as follows:

$$2s^{\mu^*} \leq 2s^\mu.$$

This is impossible. We conclude that μ cannot be supported by the system of equations. $\square$

Next, we consider what the unique solution to the system of equations looks like when equal split is or is not in the core. We present the following results:

Proof of Proposition 3. Since equal split is the core, for any $m' \in M$, we must have

$$s_{m'\mu^*(m)} - \frac{1}{2}s_{m'\mu^*(m')} \leq \frac{1}{2}s_{m\mu^*(m)}.$$

This implies that

$$
\begin{aligned}
s_{m'\mu^*(m)} - U^p_{m'} &= s_{m'\mu^*(m)} - \frac{1}{1+\delta}s_{m'\mu^*(m')} < s_{m'\mu^*(m)} - \frac{1}{2}s_{m'\mu^*(m')} \\
&\leq \frac{1}{2}s_{m\mu^*(m)} < \frac{1}{1+\delta}s_{m\mu^*(m)} = V^r_{\mu^*(m)}.
\end{aligned}
$$

Hence, there exists a uniform lower bound $\underline{\delta} \in (0,1)$ such that for any $\delta \in (\underline{\delta},1)$, $s_{m'\mu^*(m)} - U^p_{m'} < \delta V^r_{\mu^*(m)}$ for any $m' \in M \backslash m$ and any $m \in M$.[21] This implies that for any $\delta \in (\underline{\delta},1)$, for any $m \in M$,

$$
\begin{aligned}
U^p_m &= s_{m\mu^*(m)} - \delta \cdot \max\left\{V^r_{\mu^*(m)}, \max_{m' \in M \backslash m}\left\{s_{m'\mu^*(m)} - U^r_{m'}\right\}\right\} \\
&= s_{m\mu^*(m)} - \delta \cdot V^{\mu^*}_{\mu^*(m)},
\end{aligned}
$$

which is automatically satisfied given $U^p_m = V^r_{\mu^*(m)} = s_{m\mu^*(m)}/(1+\delta)$. Similarly, we obtain the same conclusion for the case when women are the proposers.

When equal-splits is not in the core, there exist $m, m' \in M$, such that $s_{m\mu^*(m)} + s_{m'\mu^*(m')} < 2s_{m\mu^*(m')}$ or $s_{m\mu^*(m)} + s_{m'\mu^*(m')} < 2s_{m'\mu^*(m)}$ or both. Without loss of generality, assume that $s_{m\mu^*(m)} + s_{m'\mu^*(m')} < 2s_{m\mu^*(m')}$. Assume that

$$U^p_m = \frac{s_{m\mu^*(m)}}{1+\delta}, \text{ for any } m \in M; \quad V^r_w = \frac{s_{\mu^*(w)w}}{1+\delta}, \text{ for any } w \in W.$$

Then we must have

$$
\begin{aligned}
\delta V^r_{\mu^*(m)} &\geq \max_{m'' \in M \backslash m}\left\{s_{m''\mu^*(m)} - U^p_{m''}\right\} \geq s_{m'\mu^*(m)} - U^p_{m'} \\
\Rightarrow \quad \frac{\delta s_{m\mu^*(m)} + s_{m'\mu^*(m')}}{1+\delta} &\geq s_{m\mu^*(m')}.
\end{aligned}
$$

Since $s_{m\mu^*(m)} + s_{m'\mu^*(m')} < 2s_{m\mu^*(m')}$, there exists a $\underline{\delta} \in [0,1)$, such that for any $\delta \in [\underline{\delta},1)$, the above inequality does not hold, implying that it cannot be a solution. Similarly, we obtain the same conclusion for the case when women are the proposers. $\square$

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[21]The existence of such a lower bound for each pair of m and m' requires $s_{m'\mu^*(m')}$ to be strictly positive. Hence, as long as we assume that $s_{mw} > 0$ for any $m \in M$ and $w \in W$, we ensures the existence of a uniform lower bound.