

An Evolutionary Justification for Overconfidence

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Overconfidence

- ▶ An *overconfident* individual overestimates his or her innate ability.
- ▶ There is overwhelming empirical evidence of overconfidence. It is one of the most well-established biases: Oskamp (1965), Kidd (1970), Svenson (1981), Cooper et al. (1988), Bondt et al. (1989), Russo and Schoemaker (1992), Babcock and Loewenstein (1997), Guthrie et al. (2001).
- ▶ Perhaps overconfidence is an evolutionarily desirable trait...

Main Results

- ▶ This paper provides an evolutionary justification for overconfidence.
- ▶ We show in a standard two-player resource-fighting game, when players have heterogeneous and possibly biased confidence levels about their chance of winning the game, overconfident players dominate in equilibrium.
 - ▶ The result holds regardless of whether a player knows opponent's confidence level.
 - ▶ The dominance of overconfident players always holds, regardless of the equilibrium players play or how we choose the equilibrium.
- ▶ We do not need to assume any bias or constraint in Nature as in majority of previous work.

Literature: Survival of Biased Beliefs

- ▶ Optimal biased beliefs in strategic settings (indirect evolutionary approach): Güth and Yaari (1992), Heifetz et al. (2007a), Heifetz et al. (2007b), Dekel et al. (2007), Johnson and Fowler (2011).
- ▶ Optimal biased beliefs in incomplete asset markets (market selection hypothesis): Blume and Easley (1992), Sandroni (2000), Mailath and Sandroni (2003), Blume and Easley (2009b), Blume and Easley (2009a), Blume and Easley (2010), Beker and Chattopadhyay (2010), Coury and Sciubba (2012), Condie and Phillips (2016).
- ▶ Optimal biased beliefs of a single decision maker with other bias: Zhang (2013), Herold and Netzer (2015), Benabou and Tirole (2002), Compte and Postlewaite (2004), Zabojnik (2004), Benoit and Dubra (2011), Harris and Hahn (2011).

A Motivating Example

- i and j play the following game. Their **payoffs** are

$i \setminus j$	Fight	Not Fight
Fight	$1_i \cdot 6 + 1_j \cdot 3 - 2, 1_j \cdot 6 + 1_i \cdot 3 - 2$	6, 3
Not Fight	3, 6	3, 3

where 1_i denotes that i wins the game.

- In reality, i and j have the same chance of winning,
 $\Pr(1_i) = \Pr(1_j) = 0.5$.

Two Unbiased Players

- A type $\theta_i = 0.5$ player versus another type $\theta_j = 0.5$ player, their **perceived utilities** are the same as their payoffs.

$i \setminus j$	Fight	Not Fight
Fight	$\frac{1}{2}6 + \frac{1}{2}3 - 2 = 2.5, \frac{1}{2}6 + \frac{1}{2}3 - 2 = 2.5$	6, 3
Not Fight	3, 6	3, 3

- Three equilibria
1. (Fight, Not Fight): (6, 3).
 2. (Not Fight, Fight): (3, 6).
 3. ($\frac{6}{7} \circ \text{Fight} + \frac{1}{7} \circ \text{Not Fight}, \frac{6}{7} \circ \text{Fight} + \frac{1}{7} \circ \text{Not Fight}$): (3, 3).
- The two players on average get $x \in [3, 4.5]$.

An Overconfident Player

- An overconfident type $\theta_i = 0.8$ player **perceives** his utility to be

$i \setminus j$	Fight	Not Fight
Fight	$(0.8)(6) + (0.2)(3) - 2 = 3.4$	6
Not Fight	3	3

- Therefore, it is a **strictly dominant** strategy to play Fight.

Overconfident Player versus Unbiased Player

- ▶ An overconfident type $\theta_i = 0.8$ player and an unbiased type $\theta_j = 0.5$ player play the following game as they each observe the other player's confidence type,

$i \setminus j$	Fight	Not Fight
Fight	3.4, 2.5	6, 3
Not Fight	3, 3	3, 3

- ▶ The unique Nash equilibrium is $(\theta_i = 0.8 \text{ Fight}, \theta_j = 0.5 \text{ Not Fight})$.
- ▶ Equilibrium utilities and payoffs are 6 for the overconfident and 3 for the unbiased.

Two Overconfident Players

- ▶ Two overconfident types $\theta_i = 0.8$ and $\theta_j = 0.8$ players play their perceived game,

$i \setminus j$	Fight	Not Fight
Fight	3.4, 3.4	6, 3
Not Fight	3, 3	3, 3

- ▶ Both perceive a dominant strategy of Fight. The unique Nash equilibrium is (Fight, Fight).
- ▶ Both get an expected utility of 3.4 but an actual expected payoff of 2.5: the winner gets $4 = 6 - 2$ and the loser gets $1 = 3 - 2$.

Population Game

- ▶ In summary,
 - ▶ Two unbiased players: on average each gets $x \in [3, 4.5]$.
 - ▶ Overconfident player versus unbiased player: overconfident player gets 6 and unbiased player gets 3.
 - ▶ Two overconfident players: both Fight and on average each gets 2.5.
- ▶ Population game
 - ▶ A continuum of players.
 - ▶ Players randomly pairwise match to play.
 - ▶ How many offspring (fitness) they have depend on the payoff.
 - ▶ In equilibrium, proportion p^* of players is overconfident.
 - ▶ Overconfident and unbiased players' payoffs equate.

Equilibrium Distribution of Confidence

- ▶ Proportion p^* of players is overconfident.
- ▶ Type-0.8 players' average payoff is

$$\pi_{0.8}^* = p^*(2.5) + (1 - p^*)(6)$$

- ▶ Type-0.5 players' average payoff is

$$\pi_{0.5}^* = p^*(3) + (1 - p^*)x$$

where $x \in [3, 4.5]$.

- ▶ $\pi_{0.8}^* = \pi_{0.5}^*$ implies

$$\frac{1 - p^*}{p^*} = \frac{0.5}{6 - x}.$$

Since $3 \leq x \leq 4.5$, $3/4 \leq p^* \leq 6/7$.

- ▶ 75% to 85.7% of players are overconfident!

General Model

$i \setminus j$	Fight	Not Fight
Fight	$1_i \cdot R + 1_j \cdot r - c, 1_j R + 1_i r - c$	R, r
Not Fight	r, R	r, r

where

- ▶ $r < R$: It is worth to play Fight.
- ▶ $\frac{1}{2}(R - r) < c$: The unbiased player does not have a dominant strategy to Fight.
- ▶ $c < R - r$: It is a strictly dominant strategy for a type $\theta = 1$ player to Fight.

Population Game

- ▶ A continuum of players of confidence types $\theta \in [0, 1]$ is in the population.
- ▶ They randomly pairwise match to fight for the resource, observing each other's confidence type.
- ▶ A type θ_i player and a type θ_j player play the perceived game

$i \setminus j$	Fight	Not Fight
Fight	$\theta_i R + \theta_i r - c, \theta_j R + \theta_j r - c$	R, r
Not Fight	r, R	r, r

- ▶ Their payoffs are determined by

$i \setminus j$	Fight	Not Fight
Fight	$\frac{1}{2}R + \frac{1}{2}r - c, \frac{1}{2}R + \frac{1}{2}r - c$	R, r
Not Fight	r, R	r, r

Equilibrium when Confidence is Always Observed

Definition

An equilibrium consists of an equilibrium distribution of types represented by a CDF F^* and PDF f^* on $[0, 1]$ and an equilibrium strategy $\sigma_\theta^* : [0, 1] \rightarrow [0, 1]$ such that

1. For each $\theta_i, \theta_j \in [0, 1]$, $(\sigma_{\theta_i}^*(\theta_j), \sigma_{\theta_j}^*(\theta_i))$ is a Nash equilibrium in the perceived game played between type θ_i and θ_j players.
2. π_θ^* equalizes across all θ , where

$$\begin{aligned}\pi_{\theta_i}^* = & \int_0^1 [\sigma_{\theta_i}^*(\theta_j)\sigma_{\theta_j}^*(\theta_i)(\frac{1}{2}R + \frac{1}{2}r - c) \\ & + \sigma_{\theta_i}^*(\theta_j)(1 - \sigma_{\theta_j}^*(\theta_i))R + (1 - \sigma_{\theta_i}^*(\theta_j))r]f^*(\theta_j)d\theta_j.\end{aligned}$$

Critical Type θ^*

- ▶ Type $\theta^* = c/(R - r)$ is a critical confidence type.
- ▶ For any type $\theta \geq \theta^*$ player, Fight is a dominant strategy.
- ▶ For any type $\theta < \theta^*$ player, Fight is not a dominant strategy.
- ▶ Since $c > (R - r)/2$ by assumption, $\theta^* > 1/2$.
- ▶ Type $\theta \geq \theta^*$ players are certainly overconfident (sup-critical) and type $\theta < \theta^*$ players are possibly overconfident (sub-critical).

Pairwise Games

- ▶ $\theta_i < \theta^*$ versus $\theta_j < \theta^*$: three equilibria (Fight, Not Fight), (Not Fight, Fight), (mixed) $\Rightarrow (R, r), (r, R), (r, r)$. The average payoff is $x \in [r, (R + r)/2]$.
- ▶ $\theta_i \geq \theta^*$ versus $\theta_j < \theta^*$: unique equilibrium (Fight, Not Fight) $\Rightarrow (R, r)$.
- ▶ $\theta_i \geq \theta^*$ versus $\theta_j \geq \theta^*$: unique equilibrium (Fight, Fight) $\Rightarrow ((R + r)/2 - c, (R + r)/2 - c)$.

Equilibrium Distribution

Proposition

In equilibrium, strictly more than half of agents have confidence level above $\theta^ = c/(R - r) > 1/2$.*

- ▶ Suppose proportion p^* are $\theta \geq \theta^*$ and proportion $1 - p^*$ are $\theta < \theta^*$.
- ▶ Average fitness of a sup-critical player:

$$\pi_{\theta \geq \theta^*} = p^*[(R + r)/2 - c] + (1 - p^*)R.$$

- ▶ Average fitness of a sub-critical player:

$$\pi_{\theta < \theta^*} = p^*r + (1 - p^*)x, \quad x \in [r, (R + r)/2].$$

- ▶ $\pi_{\theta \geq \theta^*} = \pi_{\theta < \theta^*}$ yields $p^* = \frac{R-x}{\frac{R+r}{2} + c - x}$.

- ▶ Since $c < R - r$,

$$p^* > \frac{R - x}{R + \frac{1}{2}(R - r) - x} = 1 - \frac{\frac{1}{2}(R - r)}{R + \frac{1}{2}(R - r) - x}.$$

- ▶ Since $x \leq (R + r)/2$,

$$p^* > 1 - \frac{\frac{1}{2}(R - r)}{R + \frac{1}{2}(R - r) - \frac{1}{2}(R + r)} = \frac{1}{2}.$$

Confidence is Never Observed

Definition

An equilibrium consists of an equilibrium distribution of types represented by a CDF F^* and PDF f^* on $[0, 1]$ and an equilibrium strategy $\sigma_\theta^* : \emptyset \rightarrow [0, 1]$ such that

1. For each $\theta_i \in [0, 1]$,

$$\sigma_{\theta_i}^* \in \arg \max_{\sigma \in [0, 1]} \int_0^1 \{ \sigma [\sigma_{\theta_j}^* (\theta_i R + (1 - \theta_i) r - c) + (1 - \sigma_{\theta_j}^* (\theta_i)) R] + (1 - \sigma) r \} f^*(\theta_j) d\theta_j.$$

2. π_θ equalizes across all $\theta \in [0, 1]$, where

$$\pi_{\theta_i} = \int_0^1 [\sigma_{\theta_i}^* \sigma_{\theta_j}^* (\frac{1}{2} R + \frac{1}{2} r - c) + \sigma_{\theta_i}^* (1 - \sigma_{\theta_j}^*) R + (1 - \sigma_{\theta_i}^* (\theta_j)) r] f^*(\theta_j) d\theta_j.$$

Critical Type θ^*

- ▶ There is a critical type θ^* who is indifferent between Fight and Not Fight.
- ▶ Any type $\theta > \theta^*$ player strictly prefers Fight and any type $\theta < \theta^*$ player strictly prefers Not Fight.
- ▶ Let p^* denote the proportion of type $\theta > \theta^*$ players, Δ^* proportion of type θ^* players, and $1 - \Delta^* - p^*$ proportion of type $\theta < \theta^*$ players.

Equilibrium Distribution of Confidence

Proposition

At least 2/3 of the population are overconfident or unbiased, and at most 1/3 are underconfident.

- ▶ When a type θ player chooses Fight, his expected utility is

$$u_{\theta} = (p^* + \Delta^* \sigma^*)[\theta R + (1 - \theta)r - c] + (1 - p^* - \Delta^* \sigma^*)R.$$

- ▶ It must hold for the critical type θ^* , $u_{\theta^*} = r$.

$$(p^* + \Delta^* \sigma^*)[\theta^* R + (1 - \theta^*)r - c] + (1 - p^* - \Delta^* \sigma^*)R = r.$$

- ▶ Second, the fitnesses of all confidence types must equalize,

$$(p^* + \Delta^* \sigma^*)\left[\frac{1}{2}R + \left(1 - \frac{1}{2}\right)r - c\right] + (1 - p^* - \Delta^* \sigma^*)R = r.$$

- ▶ From the two equations above, $\theta^* = 1/2$.

- ▶ And,

$$p^* + \Delta^* \sigma^* = \frac{R - r}{\frac{1}{2}(R - r) + c}.$$

Since $c < R - r$, we have $p^* + \Delta^* \sigma^* > 2/3$.

Conclusion

- ▶ This paper provides an evolutionary justification for overconfidence.
- ▶ In a class of resource-fighting games, under different settings of observability of players' beliefs, overconfident players overpopulate.
- ▶ Remark 1: We only consider players of the same ability. We can look at stationary joint distributions of abilities and beliefs.
- ▶ Remark 2: We can investigate partially observable beliefs.
- ▶ Remark 3: We are agnostic about equilibrium selection and additional restrictions on evolutionary stability. Although there is the possibility of multiple equilibria and a range of stable distribution of confidence levels, the central result that most of the players are overconfident in equilibrium holds with very little restrictions.

THANK YOU!

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