A Marriage-Market Perspective on Risk-Taking and Career Choices

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Abstract

The paper investigates how the marriage market affects risk-taking and career choices. First, it provides a new marriage-market justification for risk-taking. Second, because women for their shorter reproductive length cannot afford to wait to reap the benefits of a risky career, women are predicted to (a) be less likely to choose a risky career, (b) have lower within-gender income inequality, (c) marry earlier on average, and (d) are less likely to choose a risky career when unmarried. Evidence is consistent with these predictions. Key parameters in the model are shown to be easily recovered from aggregate statistics.

Keywords: marriage market, risk-taking, career choices, differential fecundity, gender differences

JEL: C78, D31, J1

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1 Introduction

The choices of a career and a marriage partner are two of the most important lifelong decisions. Many papers have studied these two decisions separately (since Becker (1964) and Becker (1973, 1974), respectively), but few have studied these two decisions jointly, especially in a general-equilibrium setting. The main purpose of this paper is *to study risk-taking and career choices in a general-equilibrium marriage-market framework*. The model we present is simple to state (in Sections 2 and 4) and to analyze (in Sections 3 and 4), and explains many observed patterns that have not been able to be explained jointly in a unified framework. The model makes the following five predictions, stated as five propositions in Section 5.

1. Risk-taking and career choices due to marriage-market incentives: men and women may be more inclined to choose a *risky career*, a career with an uncertain income path, than a *safe career*, a career with a certain income path, even if the risky career yields a *lower* expected income, because of the competitive nature of the marriage market.

2. Gender difference in pre-marital career choices: unmarried men are more likely than unmarried women to choose risky careers, because it is more costly for unmarried women to wait for the outcome of a risky career due to their shorter reproductive length.

3. Gender difference in income inequality: within-gender income inequality among men is larger than within-gender income inequality among women, because men are more likely than women to choose risky careers, and are voluntarily exposed to more income uncertainties.

4. Gender difference in marriage timing: men tend to choose risky careers and marry late whereas women tend to choose safe careers and marry early.

5. Gender difference in post-marital career choices: because of the possibility to improve marital prospects, unmarried men are more incentivized to choose risky careers than married men, and in contrast, because of the constraint in reproductive length that interferes with their career choice, unmarried women are more incentivized to choose safe careers than married women.

These predictions are supported by empirical and experimental evidence in the literature as well as by our own empirical analyses (in Section 6). Furthermore, we can estimate the magnitude of the effects of the gender difference in reproductive length on income differences from observed aggregate statistics about career choices and income distributions, and conduct counterfactual analyses (in Section 7).

In summary, the paper makes three contributions. First, the paper incorporates risk-taking and career choices into a general-equilibrium marriage-market framework. Although there have been previous papers based on general-equilibrium marriage-market frameworks to study how gender differences affect individuals' human capital investments and social roles (Bergstrom and Bagnoli, 1993; Siow, 1998; Iyigun and Walsh, 2007; Chiappori et al., 2009; Low, 2016; Zhang, 2017a,b), these papers do not consider how people voluntarily choose the level of income uncertainty they are exposed to. This paper shows surprising and important subtleties regarding the effects of the marriage market on risk-taking and career choices. The current framework with its endogenous determination of career choices, marriage timing, income distributions, marriage matching, and division of marriage surplus, enables us to derive a set of results that cannot be explained by empirically oriented partial-equilibrium frameworks that focus on one or two particular channels.[1] Despite of the model's complexity, we manage to keep the model tractable and solvable under closed form.

Second, the paper provides a new explanation of risk-taking based on the marriage market (Proposition 1). Many papers have provided reasons why people may take risks: overconfidence and social status concerns (Smith, 1776; Becker et al., 2005), preferences for lotteries (Friedman and Savage, 1948; Friedman, 1953), subsistence concerns (Rubin and Paul, 1979), polygamous marriages (Robson, 1992, 1996), and demand for amenities in big cities (Rosen, 1997). However, none of these papers shows that a monogamous marriage market induces people to take risks. In fact, the channel highlighted in this paper is not restrictive to marriage markets; any competitively organized two-sided one-to-one matching market encourages risk-taking.[2]

Third, this paper provides a unified understanding of gender differences in career choices, income inequalities, and marriage timing, only assuming a gender difference in reproductive fitness (Propositions 2 to 5). We do not assume other gender differences in competitiveness, risk preference, or overconfidence as in the previous literature.[3] However, it is worth noting that we are not denying (and cannot deny given the overwhelming evidence) that there are inherent gender differences in these aforementioned aspects. This paper merely suggests an additional gender difference that could result in observed socioeconomic gender differences. This approach helps us shed lights

[1] Previous partial-equilibrium papers focus on providing evidence that marital incentives affect college and career decisions but few have focused on the effects of the marital incentives on career choices (Goldin and Katz, 2002; Bailey, 2006; Bertrand et al., 2010; Lafortune, 2013; Bronson, 2013; Adda et al., 2017).

[2] As a complement to the current paper that focuses on the implications of the pre-marital career investments, Zhang (2015b) elaborates on the theoretical aspects that the competitive transferable-utilities matching market induces extreme gambles regardless of the shape of the surplus function, and investigates multiplicities and inefficiencies of equilibrium investments in two-sided matching markets.

[3] Previously, people have looked at gender differences in competitiveness: Niederle and Vesterlund (2007, QJE), Kleinjans (2008), Buser et al. (2014), Gill and Prowse (2014), and Wozniak et al. (2014); gender differences in risk preferences and expectations: Altonji and Blank (1999), Zafar (2013), Koellinger et al. (2013), Barbulescu and Bidwell (2012).

on a range of experimental and empirical findings in a unified way, discussed in detail in Section 6. The importance of this channel relative to other gender differences is partially addressed by the counterfactual analyses in Section 7.

The rest of the paper is organized as follows. Section 2 sets up the benchmark model. Section 3 shows how the marriage market encourages risk-taking behavior. Section 4 constructs a parametric extension of the benchmark model and characterizes its unique equilibrium in closed forms. Section 5 shows predictions of the model about gender differences. Section 6 presents evidence consistent with the predictions of the model. Section 7 recovers unobservable parameters of the model from simple aggregate statistics (income distributions and sex ratios in occupations) and conducts counterfactual analyses. Section 8 concludes.

2 Benchmark Model

We start with the following benchmark model to understand clearly how people have a marriage-market-driven incentive to choose a risky career.

Time is discrete and infinite: $t = 1, 2, \ldots$. At the beginning of each period, unit masses of men and women are born with heterogenous *income-earning abilities* x_m and x_w, distributed according to continuous and strictly increasing mass distributions F_m and F_w with supports $X_m \equiv [\underline{x}_m, \overline{x}_m] \subset \mathbb{R}$ and $X_w \equiv [\underline{x}_w, \overline{x}_w] \subset \mathbb{R}$.[4] Men and women make career and marriage decisions over the next two periods, described below and illustrated in Figure 1.

2.1 Career Choices

Each agent chooses a career at the beginning of the first period of his or her life. One can choose either a *safe career* or a *risky career*. The safe career compensates a person's human capital with certainty: an ability x_m man who chooses a safe career receives an income $y_m = x_m$ and an ability x_w woman who chooses a safe career receives an income $y_w = x_w$. The risky career noisily compensates a person's human capital: an ability x_m man who chooses a risky career receives an income $y_m = x_m + \varepsilon_m$ where ε_m is distributed according to a cumulative distribution function $\Phi_m(\cdot|x_m)$ and an ability x_w woman who chooses a risky career receives an income $y_w = x_w + \varepsilon_w$ where ε_w is distributed according to a cumulative distribution function $\Phi_w(\cdot|x_w)$.

In the benchmark model, suppose that a person who chooses the safe career enters the marriage market immediately in the current period and a person who chooses the risky career waits until the

[4] All the results continue to hold without the assumption of a balanced sex ratio. We will discuss how the results are affected when the sex ratio is imbalanced.

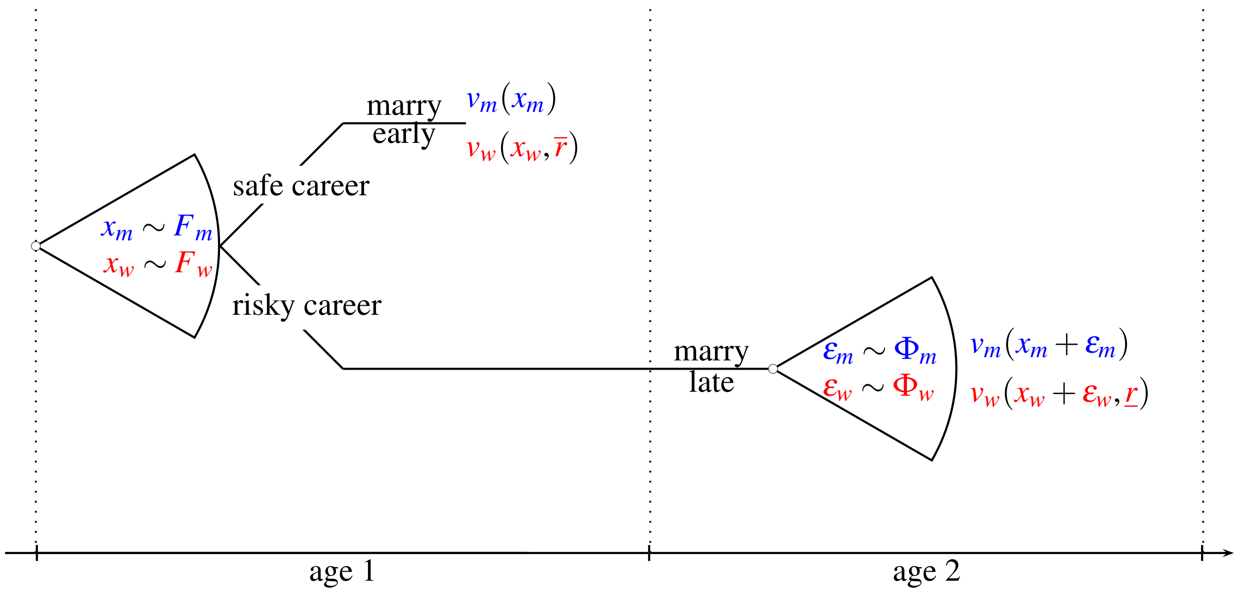

Figure 1: An individual's career and marriage decisions in the benchmark model.

income is realized in the next period to enter the marriage market.[5] Suppose each man or woman of the same income-earning ability chooses the same strategy: let $p_m(x_m)$ represent the probability that an ability x_m man chooses a risky career and $p_w(x_w)$ the probability that an ability x_w woman chooses a risky career.

2.2 The Marriage Market

The only gender difference in the model is that *women face a reproductive decline when they enter the marriage market late but men do not face a reproductive decline when they enter the marriage market late*. Namely, whereas men remain reproductively fit throughout the two periods, women who enter the marriage market early are reproductively fit ($r = \overline{r}$) but women who enter the marriage market late are reproductively less fit ($r = \underline{r} < \overline{r}$).[6]

Career choices $p_m(\cdot)$ and $p_w(\cdot)$ lead to income distributions G_m and G_w for men and women, respectively. For any y_m, men with an income below y_m include those men who have an ability below y_m and choose the safe career and those men who choose the risky career and realize an

[5]Later we will extend and allow separate career choice and marriage timing in the extended model, but we will see in Proposition 5 that it is not desirable for a person who chooses the safe career to marry late or for a person who chooses the risky career to marry early, so it is without loss of generality to assume that a risky career investor waits to marry and a safe career investor marries immediately.

[6]We choose to model the reproductive fitness as an additional dimension of women's characteristics for generality, but it is qualitatively equivalent to simply assume that a woman incurs a cost when entering the marriage market late. In the parametrized version of the model we present later, women's reproductive dimension is conveniently collapsed and each woman is represented by a single index encompassing income and reproductive fitness.

income below y_m:

$$G_m(y_m|p_m(\cdot)) \equiv \int_{\underline{x}_m}^{y_m} [1-p_m(x_m)]dF_m(x_m) + \int_{\underline{x}_m}^{\bar{x}_m} \Phi_m(y_m - x_m|x_m)p_m(x_m)dF_m(x_m), \quad (1)$$

Since women who choose a safe career and enter the marriage market in the first period are fit, the mass of fit women with an income below y_w is

$$G_w(y_w,\bar{r}|p_w(\cdot)) \equiv \int_{\underline{x}_w}^{y_w} [1-p_w(x_w)]dF_w(x_w), \quad (2)$$

and the mass of less fit women with an income below y_w is

$$G_w(y_w,\underline{r}|p_w(\cdot)) \equiv \int_{\underline{x}_w}^{\bar{x}_w} \Phi_w(y_w - x_w|x_w)p_w(x_w)dF_w(x_w). \quad (3)$$

The lifetime marriage surplus an income y_m man and an income y_w woman with reproductive fitness r produce is $s(y_m,y_w,r)$.[7] Normalize the surplus any unmarried produces to zero. Assume that the surplus is twice differentiable in incomes, and is strictly increasing in each income dimension and in reproductive fitness.

Men and women match and negotiate the division of their marriage surplus to reach a stable outcome in which no pair of a man and a woman could strictly improve their payoffs in the outcome. Formally:

Definition 1. *A* stable outcome *of the marriage market characterized by income distributions* (G_m, G_w) *consists of a* matching G *and* marriage payoff functions $(v_m(\cdot), v_w(\cdot,\cdot))$ *such that*

1. *Stable matching* $G(y_m,y_w,r)$ *describes the mass of couples with incomes lower than* y_m *and* y_w *such that the marginals of* G *are* G_m *and* G_w.

2. *Stable marriage payoffs* $v_m(y_m) \geq 0$ *and* $v_w(y_w,r) \geq 0$ *satisfy the following two stability conditions:*

   (a) *Every couple divides the marriage surplus: for any* (y_m,y_w,r) *in the support of* G, $v_m(y_m)+v_w(y_w,r) = s(y_m,y_w,r)$.

   (b) *No division of surplus could make any unmatched pair of man and woman strictly better off: for any* (y_m,y_w,r), $v_m(y_m)+v_w(y_w,r) \geq s(y_m,y_w,r)$.

By Theorem 2 of Gretsky et al. (1992), a stable outcome exists.

[7] The marriage surplus can be thought of as the result of a household production problem in which the husband and the wife allocate their time and resources to the production of private goods and public goods given their incomes and reproductive fitnesses. See Appendix for a household public good provision problem that justifies the use of specific surplus functions as well as transferable utilities.

2.3 Payoffs

Each person is risk neutral and does not discount. A person's payoff is simply his or her marriage payoff: an income y_m man's utility is $v_m(y_m)$, and an income y_w woman's utility is $v_w(y_w,\bar{r})$ if she marries in the first period of her life and is $v_w(y_w,\underline{r})$ if she marries in the second period of her life. An ability x_m man's expected payoff from strategy p_m when men's marriage payoff is $v_m(\cdot)$ is

$$u_m(p_m,x_m|v_m(\cdot)) \equiv p_m\mathbb{E}\left[v_m(x_m+\varepsilon_m)|x_m\right]+(1-p_m)v_m(x_m), \tag{4}$$

and an ability x_w woman's expected payoff from strategy p_w when women's marriage payoff is $v_w(\cdot,\cdot)$ is

$$u_w(p_w,x_w|v_w(\cdot,\cdot)) \equiv p_w\mathbb{E}\left[v_w(x_w+\varepsilon_w,\underline{r})|x_w\right]+(1-p_w)v_w(x_w,\bar{r}). \tag{5}$$

2.4 Equilibrium

In summary, the model's primitives are: ability distributions $F_m(\cdot)$ and $F_w(\cdot)$, income distributions from a risky career for each individual, $\Phi_m(\cdot|\cdot)$ and $\Phi_w(\cdot|\cdot)$, and the marriage surplus function $s(\cdot,\cdot,\cdot)$. Hence, $(F_m,F_w,\Phi_m,\Phi_w,s)$ summarizes the model. An equilibrium of the model is defined as follows. In the equilibrium, each agent chooses the career that maximizes his or her expected marriage payoff, and the marriage payoffs are the stable marriage payoffs in the marriage market induced by agents' career choices. Formally:

Definition 2. *$(p_m^*,p_w^*,G_m^*,G_w^*,G^*,v_m^*,v_w^*)$ is an* equilibrium *of $(F_m,F_w,\Phi_m,\Phi_w,s)$ if*

1. *$p_m^*(x_m)$ maximizes an x_m man's expected payoff when men's marriage payoff is $v_m^*(\cdot)$:*

$$p_m^*(x_m) \in \arg\max_{p_m\in[0,1]} u_m(p_m,x_m|v_m^*(\cdot)) \quad \forall x_m\in X_m,$$

   and $p_w^*(x_w)$ maximizes an x_w woman's expected payoff when women's marriage payoff is $v_w^*(\cdot)$:

$$p_w^*(x_w) \in \arg\max_{p_w\in[0,1]} u_w(p_w,x_w|v_w^*(\cdot)) \quad \forall x_w\in X_w.$$

2. *(G^*,v_m^*,v_w^*) is a stable outcome of the marriage market (G_m^*,G_w^*), where $G_m^*(\cdot)$ is induced by men's career choices $p_m^*(\cdot)$:*

$$G_m^*(y_m)=G_m(y_m|p_m^*(\cdot)) \quad \forall y_m,$$

and $G_w^*(\cdot,\bar{r})$ *and* $G_w^*(\cdot,\underline{r})$ *are induced by women's career choices* $p_w^*(\cdot)$*:*

$$G_w^*(y_w,\bar{r}) = G_w(y_w,\bar{r}|p_w^*(\cdot)) \quad \forall y_w.$$

$$G_w^*(y_w,\underline{r}) = G_w(y_w,\underline{r}|p_w^*(\cdot)) \quad \forall y_w.$$

Theorem 1. *An equilibrium exists.*

We can apply Glicksberg's fixed-point theorem to prove equilibrium existence (Theorem 1 of Zhang (2015a)). However, there is no systematic way to guarantee the uniqueness of the equilibrium without additional assumptions (see Section 5 of Zhang (2015b) for an example of multiple equilibria).

Partly because of the issue with potential multiple equilibria, in Section 4, we use a parametrized model with a unique equilibrium to derive additional implications as well as comparative statics. The parametrized model also extends the benchmark model by allowing risk aversion and endogenous determination of costs of choosing careers. Before we turn to the parametric model, we use the benchmark model to clearly demonstrate how the marriage market encourages risk-taking, and how this marriage-market incentive for risk-taking is independent of parametric assumptions.

3 Risk-Taking due to Marriage-Market Incentives

In this section, we show an inherent force in the competitive marriage market that encourages risk-taking. When the forces against risk-taking (concretely, concavity of the surplus function in the benchmark model and risk aversion in the parametric extension) are not strong enough, the market force that encourages risk-taking dominates and manifests in people's seemingly irrational choice of a risky career with a low expected income and a high income variance.

To understand this inherent market force that drives risk-taking, we must understand the competitive organization of the marriage market. The key property of the competitive marriage market is that *each person marries the partner that maximizes his or her marriage payoff*. To understand this key property, consider the stable division of the marriage surplus in the marriage market.

First, when an income y_m man marries a woman with characteristics $z_w(y_m)$, he and the woman divide up their marriage surplus: $v_m(y_m)+v_w(z_w(y_m)) = s(y_m,z_w(y_m))$. In other words, an income y_m man gets a payoff that is the total surplus he and a $z_w(y_m)$ woman generate net the $z_w(y_m)$ woman's payoff:

$$v_m(y_m) = s(y_m,z_w(y_m)) - v_w(z_w(y_m)). \tag{6}$$

Second, given women's stable marriage payoff schedule $v_w(\cdot)$, no woman can form a pair with the man and improve both of their payoffs. In other words, the total hypothetical surplus the man and any other woman generate must be lower than the sum of the payoffs they are getting in their current stable outcome: $v_m(y_m) + v_w(z_w) \geq s(y_m, z_w)$ for any $z_w \neq z_w(y_m)$. From a man's private perspective, his current payoff is better than the hypothetical payoff he could get by marrying any other type of woman:

$$v_m(y_m) \geq s(y_m, z_w) - v_w(z_w) \quad \forall z_w \neq z_w(y_m). \tag{7}$$

The two stability conditions (Equations 6 and 7) together yield

$$v_m(y_m) = s(y_m, z_w(y_m)) - v_w(z_w(y_m)) \geq s(y_m, z_w) - v_w(z_w) \quad \forall z_w \neq z_w(y_m), \tag{8}$$

that is, each man's marriage payoff is the most he can get; a man is married to the wife that maximizes his marriage payoff. There is no restriction for a man to be able to marry any woman, as long as he is willing to give to a woman her stable marriage payoff (or a payoff slightly above it). However, the deduction above shows that he has no strict incentive to marry anyone other than his partner in the stable outcome, because that partner provides him the highest marriage payoff given women's stable marriage payoff schedule.

How does this property encourage risk-taking in terms of career choices? Consider the following simple example to clearly see this effect in isolation. Suppose the marriage surplus is linear in the man's income and an ability $\tilde{y}_m$ man chooses between a safe career and a risky career whose income realization is simply a mean-preserving spread of the income obtained from the safe career. If the man always marries the same z_w woman regardless of his income realization y_m, his payoff is described by $v_m(y_m|z_w) = s(y_m, z_w) - v_w(z_w)$, a straight red line in Figure 2. Since the income from the risky career is assumed to be a mean-preserving spread of the income $\tilde{y}_m$ from the safe career, if a man were to always marry to a z_w woman, he will be indifferent between the risky career and safe career. However, the man marries the partner that gives him the highest payoff in the competitive marriage market, and that payoff-maximizing partner may not always be z_w. In fact, as Figure 2 illustrates, when the man realizes an income higher than $\tilde{y}_m$, he can achieve a better payoff by marrying z'_w or z''_w. Therefore, if there are three types of women in the marriage market, z_w, z'_w and z''_w, men's marriage payoff schedule is the upper envelope of three lines, which is weakly convex. Furthermore, if there is a continuum of types of women, men's marriage payoff function $v_m(y_m)$, is strictly convex, as illustrated by Figure 3. When the marriage payoff is strictly convex, the man has a strict incentive to choose the risky career as long as the risky career yields the same expected income as the safe career.

Therefore, when the marriage surplus is linear, men always choose a risky career if the risky career's income is a mean-preserving spread of the safe career's, due to the incentives provided by the competitive marriage market.

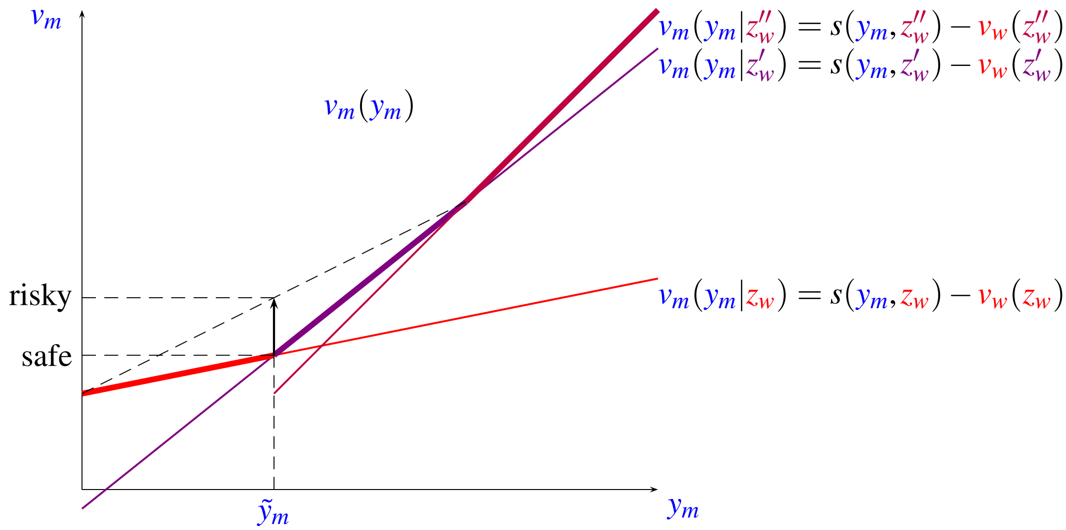

Figure 2: Linear surplus leads to weakly convex payoff.

Lemma 1. *If the marriage surplus is linear in the man's income and the risky career has the same expected income as the safe career, it is a strictly dominant strategy for each man to choose the risky career.*

When the surplus is linear and there is heterogeneity in types on the other side of the market, we show that the marriage payoff is strictly convex everywhere. When the surplus is concave, the marriage payoff is not strictly convex everywhere, but in certain ranges, the marriage payoff function may still be convex and people still have a strict incentive to choose the risky career, even if the risky career has a lower expected income than the safe career (see Figure 4).

Lemma 2. *An unmarried person might choose a career with a lower expected income and higher income uncertainty even if the marriage surplus function is strictly concave in income.*

This lemma helps to rationalize some seemingly irrational occupational choices. A person can be justified to choose a risky badly-paid career that has lower expected income and higher income variance, as long as he or she has marital concerns. As a result, anyone choosing a profession whose superstars are "overcompensated" may be well-justified. Some professions that exhibit this wage structure may include actors, singers, and lawyers. Previously, papers have explained such seemingly irrational choice by status concerns or overconfident; this paper provides an additional explanation based on the marriage market.

It is worth noting that the result does not rely on any supermodualrity or submodularity assumption of the surplus function. The surplus could have been strictly submodular (e.g. $s(y_m, y_w, r) = r(y_m + y_w - y_m y_w))$ and the result would still hold. The linearity of surplus in men's income is sufficient but not necessary. As long as the surplus function is weakly convex in men's income, the

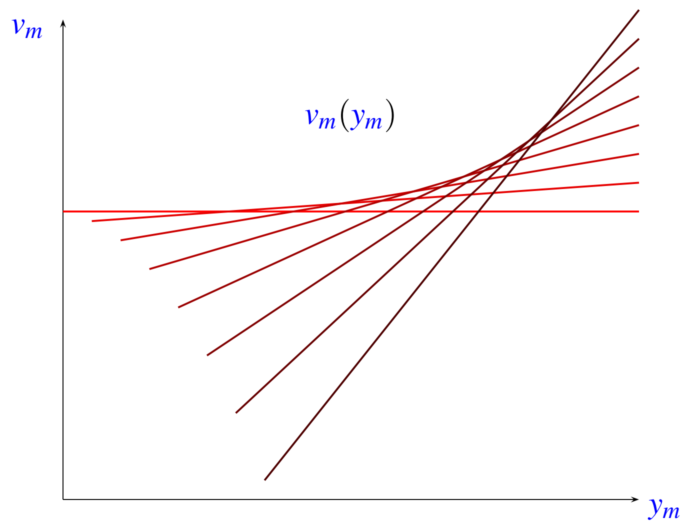

Figure 3: Linear surplus and heterogeneity lead to strictly convex payoff.

result continues to hold. The specific functional form of the marriage surplus does contribute to the risk-taking incentives but the key ingredient that drives the strict dominance of the risky career is the competitive nature of the marriage market, that is based on the stability conditions.

Finally, in contrast, since women face a reproductive decline associated with choosing the risky career, even though women have the same marriage-market incentive that encourages risk-taking, their reproductive decline acts as an additional cost that deters them from choosing the risky career. Exactly how much women are deterred from the risky career is captured by the parametric model, which we turn to next.

4 Parametric Extension

In this section, we extend and parametrize the benchmark model. We allow people to exhibit non-risk-neutral preferences and we also let the income realization from choosing a risky career to depend on the percentage of other people in the economy choosing the same career. Furthermore, in the next section, we separate career choice and marriage timing (i.e., someone who chooses a risky career can also marry early). To keep the model tractable and to derive closed-form solutions, we impose functional form assumptions on the income-earning ability distributions, on the income distributions of the risky career, as well as on the marriage surplus.

Time is still discrete and infinite: $t = 1, 2, \ldots$. At the beginning of each period, unit mass of men and unit mass of women, endowed with heterogeneous (log-income) abilities $x_m \sim N(\mu_{x_m}, \sigma^2_{x_m})$ and $x_w \sim N(\mu_{x_w}, \sigma^2_{x_w})$, choose between a safe career and a risky career. Anyone who chooses the safe career realizes his or her income and enters the marriage market in the current period, and anyone

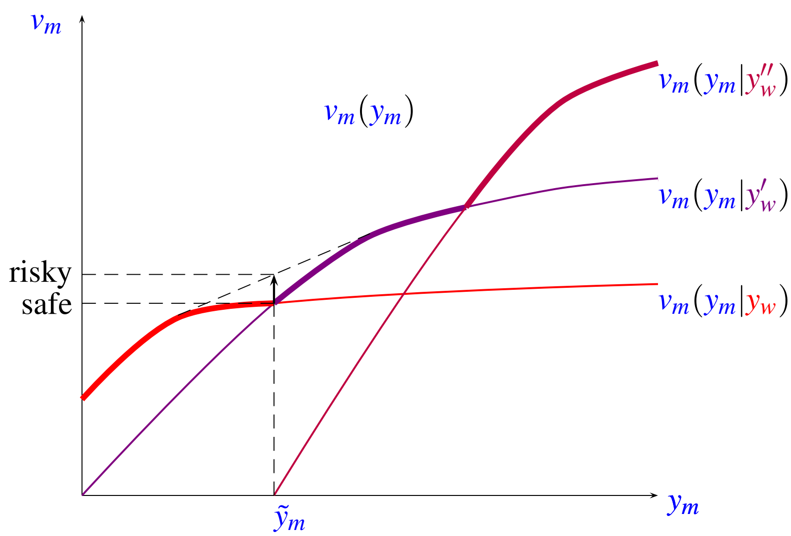

Figure 4: Concave surplus leads to partially convex payoff.

who chooses the risky career realizes his or her income and enters the marriage market in the next period.

4.1 The Marriage Market

In the marriage market, men are distinguished by incomes only but women are distinguished by incomes and reproductive fitnesses. Those women who enter the marriage market in the first period are more fit than those who enter in the second period. Namely, a log-income y_m man and a log-income y_w and fitness r woman produce a marriage surplus

$$s(y_m, y_w, r) = \exp(\alpha_m y_m + \alpha_w (y_w - 1_{r=\underline{r}} k)) \equiv= \exp(\alpha_m z_m + \alpha_w z_w).$$

The marriage surplus is simply in the Cobb-Douglas form in marriage types. Note that the marriage surplus is strictly increasing and strictly supermodular in marriage types z_m and $z_w = y_w - 1_{r=\underline{r}} k$ as well as in log-incomes y_m and y_w.

In the marriage market, men and women frictionlessly match and bargain over the division of their marriage surplus until a stable outcome is reached. A stable outcome of the marriage market is described by stable matching distributions $z_m(\cdot)$ and $z_w(\cdot)$ that are feasible as well as stable marriage payoff functions $v_m(\cdot)$ and $v_w(\cdot)$ such that (1) everyone gets a non-negative payoff: $v_m(z_m) \geq 0$ and $v_w(z_w) \geq 0$ for all z_m and z_w, (2) every marriage couple divides the surplus: $v_m(z_m) + v_w(z_w(z_m)) = s(z_m, z_w(z_m))$ and $v_m(z_m(z_w)) + v_w(z_w) = s(z_m(z_w), z_w)$ for all z_m and z_w, and (3) no pair of man and woman who are not married to each other have an incentive to marry each other: $v_m(z_m) + v_w(z_w) \geq s(z_m, z_w)$ for all z_m and z_w.

Stable matching is positive assortative, because the surplus is strictly supermodular. Given the type distributions $N(\mu_{z_m}, \sigma^2_{z_m})$ and $N(\mu_{z_w}, \sigma^2_{z_w})$, if z_m and z_w are matched, $(z_m - \mu_{z_m})/\sigma_{z_m} = (z_w - \mu_{z_w})/\sigma_{z_w}$. From the stability conditions, we can also derive stable marriage payoff functions, summarized as follows.

Lemma 3. *Suppose marriage types are normally distributed $N(\mu_{z_m}, \sigma^2_{z_m})$ and $N(\mu_{z_w}, \sigma^2_{z_w})$. Stable matching functions are*

$$z_m(z_w) = \frac{\sigma_{z_m}}{\sigma_{z_w}}(z_w - \mu_{z_w}) + \mu_{z_m}$$

$$z_w(z_m) = \frac{\sigma_{z_w}}{\sigma_{z_m}}(z_m - \mu_{z_m}) + \mu_{z_w}$$

Stable marriage payoff functions are

$$v_m(z_m) = \frac{\alpha_m}{\alpha_m + \alpha_w \frac{\sigma_{z_w}}{\sigma_{z_m}}} \exp(\alpha_m z_m + \alpha_w z_w(z_m))$$

$$v_w(z_w) = \frac{\alpha_w}{\alpha_w + \alpha_m \frac{\sigma_{z_m}}{\sigma_{z_w}}} \exp(\alpha_w z_w + \alpha_m z_m(z_w))$$

Since there is an equal mass of men and women in the marriage market, everyone marries immediately upon entering the marriage market, so from now on, it is equivalent to say “to enter the marriage market" and “to marry."

4.2 Career Choices

A safe career yields an ability x_m man a log-income $y_m = x_m$ and yields an ability x_w woman a log-income $y_w = x_w$. A risky career yields an ability x_m man a log-income $y_m = x_m - c + \varepsilon_m$ where c is a market-determined cost of taking the risky career, and $\varepsilon_m \sim N(t_m, s^2_m)$. Similarly, a risky career yields an ability x_w woman a log-income $y_w = x_w - c + \varepsilon_w$, where $\varepsilon_w \sim N(t_w, s^2_w)$. The cost c depends on the mass of people choosing the risky career. The more people choose it, the higher the cost, $c'(p_m + p_w) > 0$.

Let $p_m(x_m)$ and $p_w(x_w)$ be an ability x_m man's and an ability x_w woman's probability of choosing the risky career, respectively. Proportion $p_m = \int_{-\infty}^{\infty} p_m(x_m) d\Phi(\frac{x_m - \mu_{x_m}}{\sigma_{x_m}})$ of men and proportion $p_w = \int_{-\infty}^{\infty} p_w(x_w) d\Phi(\frac{x_w - \mu_{x_w}}{\sigma_{x_w}})$ of women choose the risky career, where Φ is the standard normal CDF. Since abilities are assumed to be normally distributed, men's income distribution is $LN(\mu_{y_m}, \sigma^2_{y_m})$ where $\mu_{y_m} = \mu_{x_m} + p_m(t_m - c)$ and $\sigma^2_{y_m} = \sigma^2_{x_m} + p_m \frac{s^2_m}{2}$, and women's income distribution is $LN(\mu_{y_w}, \sigma^2_{y_w})$ where $\mu_{y_w} = \mu_{x_w} + p_w(t_w - c)$ and $\sigma^2_{y_w} = \sigma^2_{x_w} + p_w \frac{s^2_w}{2}$.

Agents derive utilities from stable marriage payoffs according to the following functions:

$$u_m(v_m) = \frac{{v_m}^{1-\rho_m}}{1-\rho_m} \quad \text{and} \quad u_w(v_w) = \frac{{v_w}^{1-\rho_w}}{1-\rho_w}.$$

When $\rho_m = \rho_w = 0$, men and women are risk-neutral, and when $\rho_m, \rho_w > 0$, men and women are risk-averse.

We can now derive the optimal career choices.

Lemma 4. *Suppose marriage types are normally distributed $N(\mu_{z_m}, \sigma^2_{z_m})$ and $N(\mu_{z_w}, \sigma^2_{z_w})$. Let*

$$\delta_m \equiv t_m - c + (1-\rho_m)\left(\alpha_m + \alpha_w \frac{\sigma_{z_w}}{\sigma_{z_m}}\right)\frac{s_m^2}{2}$$

$$\delta_w \equiv t_w - c - k + (1-\rho_w)\left(\alpha_w + \alpha_m \frac{\sigma_{z_m}}{\sigma_{z_w}}\right)\frac{s_w^2}{2}$$

Every man chooses the risky career if $\delta_m > 0$, chooses the safe career if $\delta_m < 0$, and is indifferent between the risky career and the safe career if $\delta_m = 0$. Every woman chooses the risky career if $\delta_w > 0$, chooses the safe career if $\delta_w < 0$, and is indifferent between the risky career and the safe career if $\delta_w = 0$.

Remark 1. *δ_m and δ_w are key to understanding the model, so we elaborate on these two expressions. First, note that δ_m and δ_w do not depend on ability x_m and x_w: the incentive for choosing a risky career is the same for every man and for every woman. The incentive to choose risky career independent of ability is a result of the particular functional forms. Second, decompose δ_m into the following three terms:*

$$\delta_m = \left(t_m - c + \frac{s_m^2}{2}\right) + ((1-\rho_m)\alpha_m - 1)\frac{s_m^2}{2} + (1-\rho_m)\alpha_w \frac{\sigma_{z_w}}{\sigma_{z_m}}\frac{s_m^2}{2}.$$

The first term, $t_m - c + s_m^2/2$, is the difference in expected incomes between the risky career and the safe career. If $t_m - c = -s_m^2/2$, then the two careers yield the same expected income. The second term, $((1-\rho_m)\alpha_m - 1)s_m^2/2$, is the expected gain in own payoff from taking the risky career, without changing a marriage partner. If the marriage surplus is linear in the man's income and the man is risk-neutral, then this term is zero; if the marriage surplus is concave in the man's income and the man is risk-averse, then the term is negative. The third and final term, $(1-\rho_m)\alpha_w \frac{\sigma_{z_w}}{\sigma_{z_m}} \frac{s_m^2}{2}$, is the expected gain due to a changed partner in the marriage market. This term is positive regardless of the shape of the surplus function and the marriage-type distributions, as long as the man is not too risk-averse ($1-\rho_m > 0$) and there is some heterogeneity on the other side of the market

($\sigma_{z_w} > 0$). Similar decomposition can be done for

$$\delta_w = -k + \left(t_w - c + \frac{s_w^2}{2}\right) + ((1-\rho_w)\alpha_m - 1)\frac{s_w^2}{2} + (1-\rho_w)\alpha_m \frac{\sigma_{z_m}}{\sigma_{z_w}} \frac{s_w^2}{2}.$$

It has an additional term $-k$, which is the loss in payoff associated with declined reproductive fitness. All else equal, women have a lower expected gain from the risky career because of the reproductive fitness loss.

Certainty Equivalent. Suppose the risky career's log-income variance is s_m^2. The risky career only needs to have an expected log-income of

$$t_m - c + \frac{s_m^2}{2} = [1-(1-\rho_m)\alpha_m]\frac{s_m^2}{2} - (1-\rho_m)\alpha_w \frac{\sigma_{z_w}}{\sigma_{z_m}} \frac{s_m^2}{2}.$$

Analogously, for women, suppose the risky career's log-income variance is s_w^2. A woman is indifferent between the safe career and the risky career with an expected log-income of

$$t_w - c + \frac{s_w^2}{2} = k + [1-(1-\rho_w)\alpha_w]\frac{s_w^2}{2} - (1-\rho_w)\alpha_m \frac{\sigma_{z_m}}{\sigma_{z_w}} \frac{s_w^2}{2}.$$

Suppose all parameters are gender-symmetric, the certainty-equivalent log-incomes are

$$E_m(s^2) = t - c + \frac{s^2}{2} = [1-(1-\rho)\alpha]\frac{s^2}{2} - (1-\rho)\alpha \frac{\sigma_{z_w}}{\sigma_{z_m}} \frac{s^2}{2}.$$

$$E_w(s^2) = t - c + \frac{s^2}{2} = k + [1-(1-\rho)\alpha]\frac{s^2}{2} - (1-\rho)\alpha \frac{\sigma_{z_m}}{\sigma_{z_w}} \frac{s^2}{2}.$$

Figure 5 illustrates the (endogenously determined) careers indifferent to the safe career. For lower variance and intermediate average log-income, men prefer this job over the safe job but women prefer the safe job over this job. The preference is reversed for high-variance jobs. For a job with sufficiently high variance and intermediate average log-income, men prefer the safe job over this job but women prefer this job over the safe job.

4.3 Equilibrium Existence and Uniqueness

Definition 3. $(p_m^*(\cdot), p_w^*(\cdot), v_m^*(\cdot), v_w^*(\cdot))$ *is an* equilibrium *if*

1. $p_m^*(x_m)$ *maximizes each ability* x_m *man's expected utility and* $p_w^*(x_w)$ *maximizes each ability* x_w *woman's expected utility given equilibrium marriage payoff functions* $v_m^*(\cdot)$ *and* $v_w^*(\cdot)$ *as*

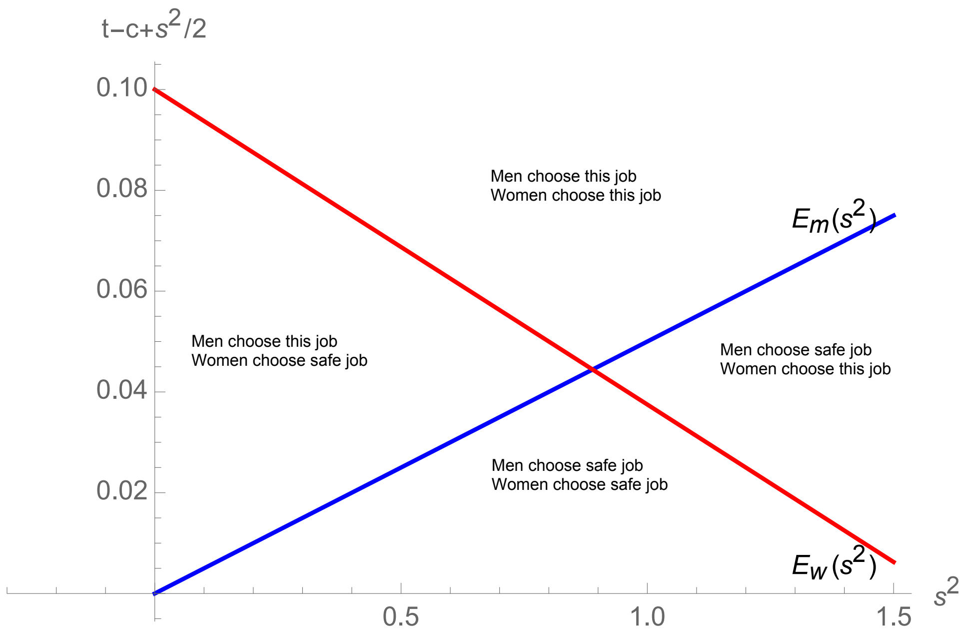

Figure 5: Certainty-Equivalent Average Log-Incomes.

well as market-determined cost $c^* = c(p_m^* + p_w^*)$ *where*

$$p_m^* = \int_{-\infty}^{\infty} p_m^*(x_m) d\Phi((x_m - \mu_{xm})/\sigma_{xm})$$

and

$$p_w^* = \int_{-\infty}^{\infty} p_w^*(x_w) d\Phi((x_w - \mu_{xw})/\sigma_{xw}).$$

2. *Equilibrium marriage payoff functions* $v_m^*(\cdot)$ *and* $v_w^*(\cdot)$ *are the stable marriage payoff functions of the marriage market with type distributions* $N(\mu_{z_m}, \sigma_{z_m}^2)$ *and* $N(\mu_{z_w}, \sigma_{z_w}^2)$ *where*

$$\mu_{z_m} = \mu_{y_m} + p_m(t_m - c), \quad \sigma_{z_m}^2 = \sigma_{y_m}^2 = \sigma_{x_m}^2 + \frac{p_m s_m^2}{2},$$

$$\mu_{z_w} = \mu_{y_w} + p_w(t_w - c - k), \quad \sigma_{z_w}^2 = \sigma_{y_w}^2 = \sigma_{x_w}^2 + \frac{p_w s_w^2}{2}.$$

Theorem 2. *An equilibrium always exists, and exists uniquely if* $\rho_m < 1$ *and* $\rho_w < 1$.

From now on, let's assume $\rho_m < 1$ and $\rho_w < 1$.

5 Predictions

First, since the model is now parametrized, it makes concrete exactly when a person prefers a risky career to a safe career, providing sharper predictions than Lemmas 1 and 2.

Proposition 1. *A risk-averse person could prefer a risky career with a lower income average and a higher income variance:*

1. *Between a safe career and a risky career with same expected income, a risk-neutral man would strictly prefer the risky career if the marriage surplus is linear in men's income.*

2. *Between a safe career and a risky career that returns lower expected income, a risk-averse man would strictly prefer the risky career if* $(1-\rho_m)(\alpha_m+\alpha_w\frac{\sigma_{y_w}}{\sigma_{y_m}})>1$*, and a risk-averse woman would strictly prefer the risky career if* $(1-\rho_w)(\alpha_m+\alpha_w\frac{\sigma_{y_w}}{\sigma_{y_m}})>1+2k/s_w^2$*.*

First, note that the first point above does not depend on various primitives of the model, notably distributions of initial abilities. Second, the second point above shows that, if the setting is gender-symmetric except for the reproductive cost k, then women are less likely to choose the risky career. The gender difference in the two expressions is $2k/s_w^2$. That is, higher reproductive cost and *lower* income variance of the risky career make women less inclined to choose a risky career. The result that women are less likely than men to choose a risky career continues to hold in the general equilibrium:

Proposition 2. *Suppose career opportunities are gender-symmetric (*$s_m=s_w\equiv s, t_m=t_w\equiv t$*) and variances in abilities are gender-symmetric (*$\sigma_{x_m}^2=\sigma_{x_w}^2\equiv\sigma_x^2$*). Men are more likely than women to choose the risky career.*

Because women are less likely to choose a risky career, consequently, women's income inequality is less than men's. Moreover, the disparity in the income inequality increases if the reproductive cost increases, if the risk aversion increases, and if the the risky career becomes more uncertain.

Proposition 3. *Suppose career opportunities are gender-symmetric (*$s_m=s_w\equiv s, t_m=t_w\equiv t$*).*

1. *Variance in log-earnings is greater for men than for women (*$\sigma_{y_m}^{*2}>\sigma_{y_w}^{*2}$*).*

2. *The ratio of the log-income variances,* $\sigma_{y_m}^{*2}/\sigma_{y_w}^{*2}$ *increases in career cost* k*, increases in risk aversion* R*, decreases in the risky career's log earnings variance* s^2*, increases in* α_w *if the ratio is smaller than* 2 *and decreases in* α_w *if the ratio is bigger than* 2*.*

Note that, for the result above, we do not need to assume that the ability distributions are gender-symmetric. For *any* underlying ability distributions, we will have the result that men's income inequality is larger than women's, as long as there is a reproductive difference.

In the basic model, career and marital choices are connected: one who chooses the risky career marries late, and one who chooses the safe career marries early. We separate career and marital choices in this section. There are four possible choices as a result: (1) choosing the safe career and marrying early, (2) choosing the safe career and marrying late, (3) choosing the risky career and marrying early with unresolved uncertainty in incomes, and (4) choosing the risky career and marrying late with resolved uncertainty in incomes.

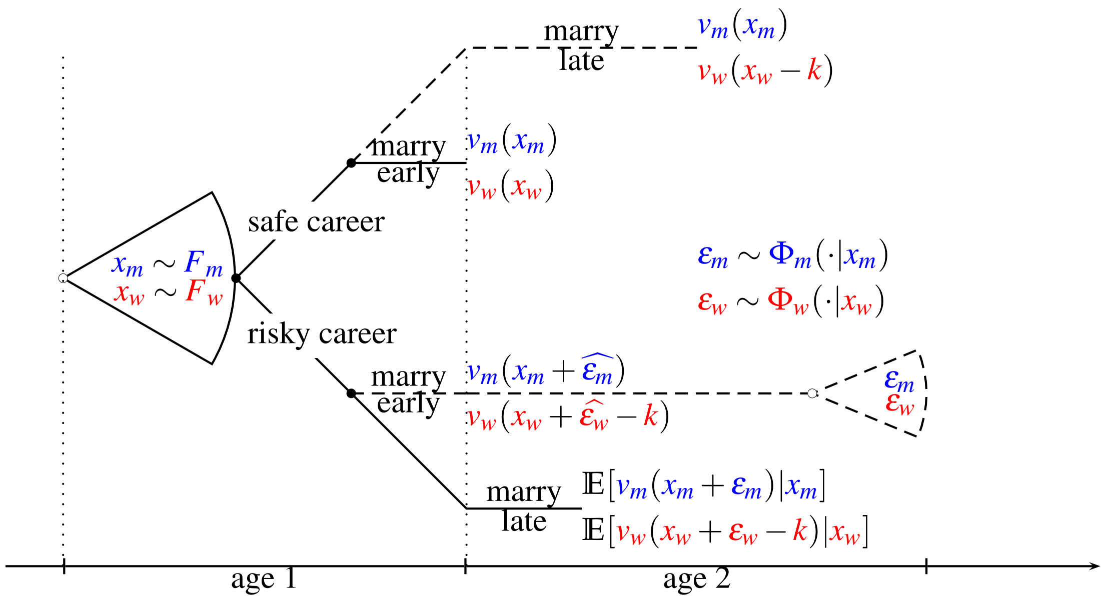

Figure 6: An individual's career and marriage decision tree in the parametric extension.

Suppose men and women can choose when to marry. Namely, an ability x_m man who chooses the safe career can choose to marry in the second period as a marriage type $z_m = x_m$, and an ability x_w woman who chooses the safe career can choose to marry in the second period as a marriage type $z_w = x_w - k$. An ability x_m man can choose the risky career and marry in the first period as a man who may realize a marriage type $z_m = x_m - c + \varepsilon_m$ where $\varepsilon_m \sim N(t_m, s_m^2/2)$. If he marries a marriage type z_w woman and realizes an income $x_m - c + \varepsilon_m$, they generate a marriage surplus $s(x_m - c + \varepsilon_m, z_w)$. Similarly, an ability x_w woman can choose the risky career and marry in the first period as a woman who may realize a marriage type $z_w = x_w - c - k + \varepsilon_w$. We show that even if marriage timing is separated from the career choice, anyone who chooses the safe career tends to marry in the first period and anyone who chooses the risky career tends to marry in the second period, as we have assumed throughout the paper.

Lemma 5. *Career choice and marriage age are related:*

1. *Any person who chooses the safe career marries in the first period.*

2. *Any person who chooses the risky career marries in the second period.*

To understand the result, realize that a person who has an incentive to choose a risky career only has the incentive to do so if he or she can marry to a different partner depending on the realization of his or her income from the risky career. For an ability x_m man, the four choices respectively yield: (1) $v_m(x_m)$, (2) $v_m(x_m)$, (3) $v_m(x_m + \widehat{\varepsilon_m}|x_m)$, (4) $\mathbb{E}[v_m(x_m + \varepsilon_m)|x_m]$, where $v_m(x_m + \widehat{\varepsilon_m}|x_m)$ represents the marriage payoff of an ability x_m man who chooses the risky career. First of all, there is no advantage in choosing the safe career and marrying late over choosing the safe career and marrying early - since delaying is always associated with some costs and/or discounting, when a man chooses the safe career, he might as well choose to marry early.

Proposition 4. *Suppose career opportunities are gender-symmetric ($s_m = s_w$ and $t_m = t_w$) and variances in abilities are gender-symmetric ($\sigma^2_{x_m} = \sigma^2_{x_w}$). Women marry earlier than men on average.*

Finally, let's compare the payoffs from not resolving and resolving the risky career's income uncertainty. If a man marries in the first period after choosing the risky career, the expected surplus he gets from marrying an income y_w woman is $\mathbb{E}[s(x_m + \varepsilon_m, y_w)|x_m]$. Hence, a man who chooses the risky career and marries early is treated as if he chooses the safe career and marries early. As a result, a male risk-taker is better off waiting to marry in the second period. This result again highlights that the marital benefits from the risky career come from the possibility to switch partners. Remember that the wife a man marries is the wife that maximizes his personal marriage payoff, so choosing the risky career while unmarried is better than choosing the risky career while married. In contrast, an unmarried woman faces the additional reproductive cost, so she is more inclined to choose a safe career. However, after she is married (and has kids), she no longer worries about the reproductive decline. If she can choose a different career, she may opt to choose a riskier career.

Proposition 5. *Marital status affects a person's career choice:*

1. *A married man is less likely than an unmarried man to choose the risky career.*

2. *A married woman is more likely than an unmarried woman to choose the risky career.*

6 Evidence

6.1 Risky Career Choice due to Marriage-Market Incentives

Proposition 1 predicts that the competitive nature of the marriage market encourages both men and women to engage in risk-taking behavior. The more competitive the marriage market becomes, the more likely men and women choose risky careers.

A canonical example of a risky career is entrepreneurship. Evidence suggests that the decision to become an entrepreneur is indeed influenced by the condition of the marriage market. Luo (2017) uses World War II casualty rates as exogenous variation in a natural experiment to establish causal relationship between the opportunity costs of marriage and female entrepreneurship. He finds that women in the counties with heavier male casualties and thus less favorable marriage-market conditions for women were more active in starting new businesses after the war ended, and this trend has continued to this day.

6.2 Gender Difference in Pre-Marital Career Choices

Proposition 2 predicts that women tend to choose a safe job and men tend to choose a risky job.

Entrepreneurship is a canonical example of a risky career because it has a high failure rate, takes a long time to become successful, and has huge monetary and non-monetary benefits after success. Figure 7 shows the percent of female entrepreneurs in the US since the 1940s. The percent has steadily increased but it is well below 50%. It is only around 22% nowadays. Similar patterns are found in other developed and developing countries. Koellinger et al. (2013) found that the the lower rate of female entrepreneurship is primarily due to women's lower propensity to start businesses rather than to differences in survival rates across genders.

Another extreme risk workers face is fatality. There is ample evidence that men choose occupations with much higher casualty rates and are compensated for the higher risks. According to the Bureau of Labor Statistics, 93.1% of fatal occupational injuries in the United States in 2013 were males. DeLeire and Levy (2001) investigate more closely the effects of marriage on risky occupational choices. Their results suggest that women choose safer jobs than men. Overall, men and women's different preferences for risk can explain about one-quarter of the fact that men and women choose different occupations.

A laboratory experiment conducted by Jung et al. (2016) closely captures the risky career choice in our model and supports the prediction about the gender difference. Subjects choose between a risky job and a secure job that involve the same task (typing). The risky job will not be available in any given period with a known probability but also pays more in order to compensate for the uncertainty. Women were more likely than men to select the secure job (accounting for

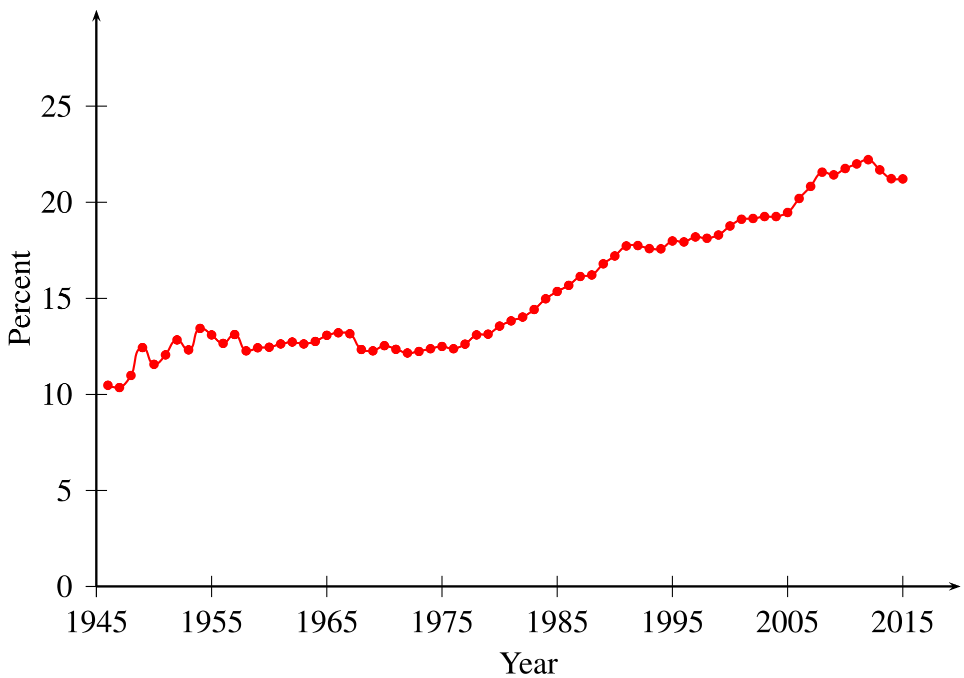

Figure 7: Percent of Female Entrepreneurs in the United States, 1945-2015 (Luo, 2017).

40% to 77% of the gender wage gap in the experiments).

6.3 Gender Difference in Income Inequalities

Proposition 3 predicts that the income inequality is greater for men than for women.

This is indeed the case in the data. Both the variances of (log-)incomes and (log-)wages are higher for (all) men than for (all) women. The same patterns hold if we restrict to college-educated men and women.

Other observations are also consistent with the predicted gender difference in income distributions. The fact that men are more likely than women to choose risky careers is also consistent with the observation that the top of the profession consists of men instead of women.[8] In addition, women in the STEM field face a "marriage-market" penalty.[9]

6.4 Gender Difference in Marriage Timing

Proposition 4 predicts that men marry later than women on average. Figure 8 shows the average marriage age of birth cohorts from 1900 to 1970 in the United States, and that men have always

[8] https://fourthwave.quora.com/Women-Running-Money?srid=C3RB.

[9] https://www.marketwatch.com/story/a-surprising-reason-why-college-women-arent-going-into-stem-2016-09-02.

married later than women on average. The same pattern holds around the world: men have married later on average than women in every country in the world through the recent decades (Bergstrom and Bagnoli, 1993; United Nations, 1990, 2000).

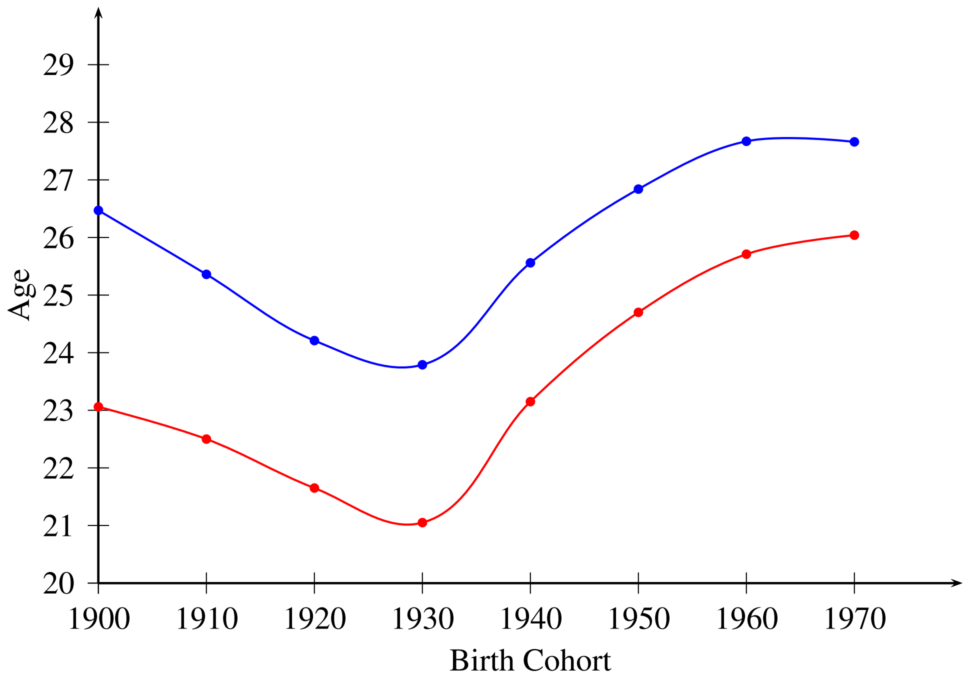

Figure 8: Average Marriage Age in the United States, Birth Cohorts 1900s-1970s.

7 Backed-Out Parameters

Suppose we observe the percentages of men and women choosing the risky career, p_m^* and p_w^*, as well as the income distributions $LN(\mu_{y_m}^*, \sigma_{y_m}^{*2})$ and $LN(\mu_{y_w}^*, \sigma_{y_w}^{*2})$. They are all equilibrium components of the model. From these equilibrium components, we can derive parameters in the model unobserved to econometricians, namely, (1) the risky career's income distributions, (2) the underlying ability distributions, and (3) the career cost.

1. First, the risky career's distribution of returns $\varepsilon \sim N(t-c, s^2)$.

   (a) The variance in log earnings for the risky career, s^2. The variance of log earnings for the risky career, s^2 can be recovered from

$$\sigma_{y_m}^{*2} = \sigma_x^2 + p_m^* s^2$$

$$\sigma_{y_w}^{*2} = \sigma_x^2 + p_w^* s^2$$

The two equations together yield

$$\widehat{s}^2 = \frac{\sigma_{y_m}^{*2} - \sigma_{y_w}^{*2}}{p_m^* - p_w^*}.$$

Note we need to assume the variance of abilities is gender-symmetric.

(b) The mean of log earnings for the risky career. From the equilibrium condition $\delta_m{}^* = 0$,

$$t - c^* + (1-R)(\alpha_m + \alpha_w \frac{\sigma_{y_m}^*}{\sigma_{y_w}^*})\frac{s^2}{2} = 0,$$

we get

$$\widehat{\nu} = t - c^* = -(1-R)\left(\alpha_m + \alpha_w \frac{1}{r^*}\right)\frac{\widehat{s}^2}{2} = -(1-R)\left(\alpha_m + \alpha_w \frac{1}{r^*}\right)\frac{1}{2}\frac{\sigma_{y_m}^{*2} - \sigma_{y_w}^{*2}}{p_m^* - p_w^*}.$$

Therefore, together, the return of the risky career is normally distributed $N(\widehat{\nu}, \widehat{s}^2)$.

2. Second, the ability distributions $N(\mu_x, \sigma_x^2)$. From

$$\sigma_{y_m}^{*2} = \sigma_x^2 + p_m^* s^2,$$

we get

$$\widehat{\sigma}_x^2 = \sigma_{y_m}^{*2} - p_m^* \widehat{s}^2 = \sigma_{y_m}^{*2} - p_m^* \frac{\sigma_{y_m}^{*2} - \sigma_{y_w}^{*2}}{p_m^* - p_w^*}$$

In addition, from

$$\mu_{y_m}^* = \mu_x + p_m^*(t - c^*),$$

we get

$$\widehat{\mu}_x = \mu_{y_m}^* - p_m^* \widehat{\nu}.$$

Therefore, the ability distributions are

$$N(\widehat{\mu}_x, \widehat{\sigma}_x^2) = (\mu_{y_m}^* - p_m^* \widehat{\nu}, \sigma_{y_m}^{*2} - p_m^* \frac{\sigma_{y_m}^{*2} - \sigma_{y_w}^{*2}}{p_m^* - p_w^*})$$

3. Perhaps most importantly, we can estimate women's reproductive cost k. The two equilibrium conditions $\delta_m{}^* = 0$ and $\delta_w{}^* = 0$ play important roles. Assume $\rho_m = \rho_w = R$.

$$t - c^* + (1-R)(\alpha_m + \alpha_w \frac{\sigma_{y_m}^*}{\sigma_{y_w}^*})\frac{s^2}{2} = 0$$

$$t - c^* + (1-R)(\alpha_w + \alpha_m \frac{\sigma^*_{y_w}}{\sigma^*_{y_m}})\frac{s^2}{2} = k$$

The two combining together:

$$(r^* - 1)(1-R)\left(\alpha_m + \alpha_w \frac{1}{r^*}\right)\frac{s^2}{2} = k$$

Finally,

$$\begin{aligned}\widehat{k} =& (r^* - 1)(1-R)\left(\alpha_m + \alpha_w \frac{1}{r^*}\right)\frac{\widehat{s}^2}{2} \\ =& (r^* - 1)(1-R)\left(\alpha_m + \alpha_w \frac{1}{r^*}\right)\frac{1}{2}\frac{\sigma^{*2}_{y_m} - \sigma^{*2}_{y_w}}{p^*_m - p^*_w}\end{aligned}$$

Suppose agents are risk-neutral and $\alpha_m = \alpha_w = 1/2$. We observe the variance of log-incomes are $\sigma^{*2}_{y_m} = 0.797$ and $\sigma^{*2}_{y_w} = 0.693$ (Table 1, Kopczuk et al. (2010)), the median incomes are 39403 and 26507, so $\mu^*_{y_m} = \log(39403)$ and $\mu_{y_w^*} = \log(26507)$, and $p^*_m = 60\%$ of men and $p^*_w = 20\%$ of women choose a risky career. Men's income distribution is observed $LN(10.5816, 0.797)$. Women's income distribution is observed $LN(10.1852, 0.693)$.

We yield "reasonable" estimates. The return of the risky career over the safe career is estimated to be $N(-0.125611, 0.26)$. The expected income from the risky career is estimated 100.44 percent of the expected income from the safe career. The equivalent income loss purely due to the reproductive loss is estimated 0.453768 percent. Ability distributions are estimated $N(10.657, 0.641)$. Figure 9 depicts backed-out ability distributions (black) and log-income distributions (blue for men and red for women).

8 Conclusion

In this paper, we embed the discussion of career choices and gender differences in career choices in an equilibrium marriage-market framework. In the model, a set of variables is simultaneously and endogenously determined: career choices, marriage timing, income distributions, matching between couples, and the division of marriage payoffs.

The model makes predictions consistent with evidence. We found that the competitive organization of the marriage market inherently induces men and women alike to choose risky careers. We found women's shorter reproductive length that imposes a gender-asymmetric career cost as a potential unifying factor to explain a set of gender differences in career choices, income inequality, and marriage age. Evidence supported our main predictions of the model. Finally, given

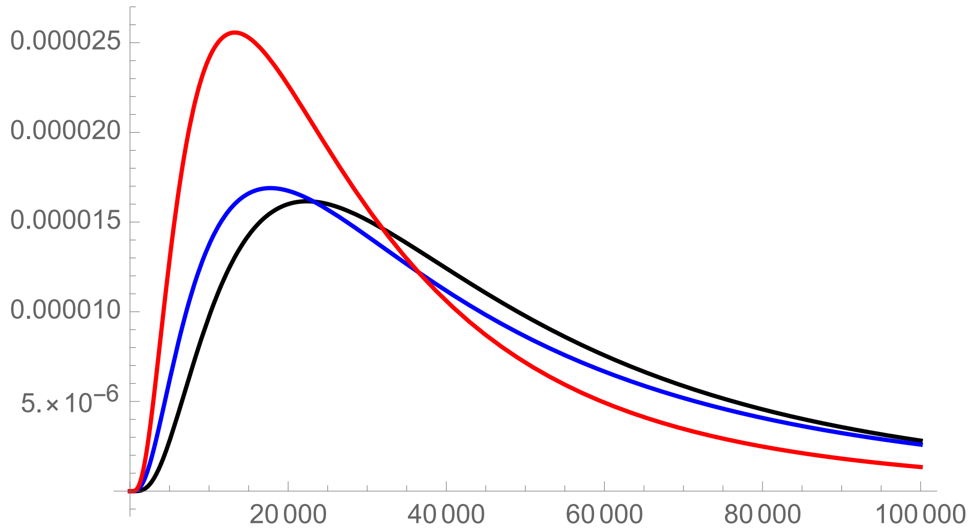

Figure 9: Income distributions and recovered underlying ability distributions.

the tractability of the parametric version of the model, we conducted counterfactual analyses to quantify consequences of eliminating gender-asymmetric career cost.

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Appendix

Public good provision justifies the surplus function. Consider a man with realized income y_m and a woman with realized income y_w. A single person's utility depends on the consumption of a public good and a private good: $u_m(Q, q_m) = q_m Q$ and $u_w(Q, q_w) = q_w Q$. They divide their incomes between the two goods to maximize their respective utilities. The utilities when they live alone are their reservation utilities $z_m(y_m) = \max_Q (y_m - Q)Q = (y_m/2)^2 = {y_m}^2/4$ and $z_w(y_w) = \max_Q (y_w - Q)Q = {y_w}^2/4$, respectively. The maximal total utility when the two marry (or live together) subject to the constraint $Q + q_m + q_w = y_m + y_w$ is

$$z(y_m, y_w) = \max_{Q, q_m, q_w} q_m Q + q_w Q = \max_Q (y_m + y_w - Q)Q = (y_m + y_w)^2/4.$$

The sum of private goods is determinate to be $q_m + q_w = (y_m + y_w)/2$, but the allocation of the private goods q_m and q_w is indeterminate. The surplus from the marriage of a couple (y_m, y_w) is hence

$$s(y_m, y_w) = z(y_m, y_w) - z(y_m) - z(y_w) = y_m y_w / 2.$$

The marital surplus is perfectly transferable between the two parties: to achieve a marital gain of v_m in combination with the reservation utility, the man consumes a private good of $q_m = [v_m + z(y_m)]/Q = 2[v_m + {y_m}^2/4]/(y_m + y_w)$. Similarly, the woman consumes a private good of $q_w = [v_w + z(y_w)]/Q = 2[v_m + {y_w}^2/4]/(y_m + y_w)$ to achieve a marital gain of v_w.

Proof of Theorem 1. Consider the following composite map

$$\Gamma : \mathscr{V} \rightrightarrows P_m \times P_w \to \mathscr{G}_m \times \mathscr{G}_w \rightrightarrows \mathscr{V},$$

where $\mathscr{V}$ is the set of stable marriage payoff functions $v_m : X_m \to \mathbb{R}_+$ and $v_w : X_w \to \mathbb{R}_+$, $P_m \times P_w$ is the set of career choice strategies $p_m : X_m \to [0,1]$ and $p_w : X_w \to [0,1]$, and $\mathscr{G}_m \times \mathscr{G}_w$ is the set of income distributions. By Glicksberg's fixed-point theorem, an equilibrium exists if $\mathscr{V}$, the set of stable marriage payoff functions, is non-empty, convex, and compact, and Γ is non-empty-valued, upper-hemicontinuous, convex-valued, and compact-valued. $\square$

Proof of Lemma 3. Fix a z_m. From the stability conditions,

$$v_m(z_m) = s(z_m, z_w(z_m)) - v_w(z_w(z_m))$$

and for all z_w,

$$v_m(z_m) \geq s(z_m, z_w) - v_w(z_w)$$

Therefore,

$$v_m(z_m) = \max_{z_w} s(z_m, z_w) - v_w(z_w)$$

and

$$z_w(z_m) \in \arg\max_{z_w} s(z_m, z_w) - v_w(z_w)$$

By the envelope theorem,

$$v_m'(z_m) = \frac{\partial s(z_m, z_w(z_m))}{\partial z_m} + \left(\frac{\partial s(z_m, z_w(z_m))}{\partial z_w} - {v_w}'(z_w(z_m)) \right) \frac{\partial z_w}{\partial z_m}$$

By the fact that

$$z_w(z_m) \in \arg\max_{z_w} s(z_m, z_w) - v_w(z_w)$$

We know

$$\frac{\partial s(z_m, z_w(z_m))}{\partial z_w} - {v_w}'(z_w(z_m)) = 0.$$

Therefore,

$$v_m'(z_m) = \frac{\partial s(z_m, z_w(z_m))}{\partial z_m} = \frac{\partial(\exp(\alpha_m z_m + \alpha_w z_w(z_m)))}{\partial z_m} = \alpha_m \exp(\alpha_m z_m + \alpha_w z_w(z_m)).$$

Then

$$\begin{aligned} v_m(z_m) = & \int_{-\infty}^{z_m} \alpha_m \exp\left(\alpha_m \tilde{z}_m + \alpha_w \left(\frac{\sigma_{z_w}}{\sigma_{z_m}} (\tilde{z}_m - \mu_{z_m}) + \mu_{z_w} \right) \right) d\tilde{z}_m \\ = & \frac{\alpha_m}{\alpha_m + \alpha_w \frac{\sigma_{z_w}}{\sigma_{z_m}}} \exp(\alpha_m z_m + \alpha_w z_w(z_m)). \end{aligned}$$

Similarly,

$$v_w(z_w) = \frac{\alpha_w}{\alpha_w + \alpha_m \frac{\sigma_{z_m}}{\sigma_{z_w}}} \exp(\alpha_w z_w + \alpha_m z_m(z_w)).$$

□

Proof of Lemma 4. An ability x_m man's expected utility gain from the risky career over the safe career is

$$\begin{aligned} \Delta_m(x_m) = & \mathbb{E}_{\varepsilon_m} [u_m(v_m(x_m - c + \varepsilon_m))] - u_m(v_m(x_m)) \\ = & \mathbb{E}_{\varepsilon_m} [u_m(v_m(x_m - c + \varepsilon_m)) - u_m(v_m(x_m))] \end{aligned}$$

where

$$u_m(v_m(x_m - c + \varepsilon_m)) - u_m(v_m(x_m))$$

$$
\begin{aligned}
&= \frac{1}{1-\rho_m}\left(\frac{\alpha_m}{\alpha_m+\alpha_w\frac{\sigma_{z_w}}{\sigma_{z_m}}}\right)^{1-\rho_m} \times \mathbb{E}_{\varepsilon_m}[\exp((1-\rho_m)(\alpha_m(x_m-c+\varepsilon_m)+\\
&\quad \alpha_w z_w(x_m-c+\varepsilon_m)))-\exp((1-\rho_m)(\alpha_m x_m+\alpha_w z_w(x_m)))]\\
&= \left(\frac{\alpha_m}{\alpha_m+\alpha_w\frac{\sigma_{z_w}}{\sigma_{z_m}}}\right)^{1-\rho_m} \frac{\exp[(1-\rho_m)(\alpha_m x_m+\alpha_w z_w(z_m))]}{1-\rho_m}\times\\
&\quad \mathbb{E}_{\varepsilon_m}\left[\exp\left((1-\rho_m)(\alpha_m+\alpha_w\frac{\sigma_{z_w}}{\sigma_{z_m}})(\varepsilon_m-c)\right)-1\right].
\end{aligned}
$$

Since $\mathbb{E}_{\varepsilon_m}[\exp(\alpha\varepsilon_m)]=\exp(\alpha t_m+\alpha^2 s_m^2/2)$, the expected value of the term in the square brackets becomes

$$
\exp\left((1-\rho_m)\left(\alpha_m+\alpha_w\frac{\sigma_{z_w}}{\sigma_{z_m}}\right)(t_m-c)+\left((1-\rho_m)\left(\alpha_m+\alpha_w\frac{\sigma_{z_w}}{\sigma_{z_m}}\right)\right)^2\frac{s_m^2}{2}\right)-1.
$$

Therefore,

$$
\begin{aligned}
\Delta_m(x_m) &= \left(\frac{\alpha_m}{\alpha_m+\alpha_w\frac{\sigma_{z_w}}{\sigma_{z_m}}}\right)^{1-\rho_m} \frac{\exp[(1-\rho_m)(\alpha_m x_m+\alpha_w z_w(x_m))]}{1-\rho_m}\times\\
&\left(\exp\left((1-\rho_m)\left(\alpha_m+\alpha_w\frac{\sigma_{z_w}}{\sigma_{z_m}}\right)\left(t_m-c+(1-\rho_m)\left(\alpha_m+\alpha_w\frac{\sigma_{z_w}}{\sigma_{z_m}}\right)\frac{s_m^2}{2}\right)\right)-1\right),
\end{aligned}
$$

and $\Delta_m(x_m)$ has the same sign as

$$
\delta_m \equiv t_m-c+(1-\rho_m)\left(\alpha_m+\alpha_w\frac{\sigma_{z_w}}{\sigma_{z_m}}\right)\frac{s_m^2}{2}.
$$

Similarly, an ability x_w woman 's expected utility gain from the risky career over the safe career, $\Delta_w(x_w)\equiv\mathbb{E}_{\varepsilon_w}[u_w(v_w(x_w-c-k+\varepsilon_w))]-u_w(v_w(x_w))$, is

$$
\begin{aligned}
&\left(\frac{\alpha_w}{\alpha_w+\alpha_m\frac{\sigma_{z_m}}{\sigma_{z_w}}}\right)^{1-\rho_w} \frac{\exp[(1-\rho_w)(\alpha_w x_w+\alpha_m z_m(x_w))]}{1-\rho_w}\times\\
&\left(\exp\left((1-\rho_w)\left(\alpha_w+\alpha_m\frac{\sigma_{z_m}}{\sigma_{z_w}}\right)\left(t_w-c-k+(1-\rho_w)\left(\alpha_w+\alpha_m\frac{\sigma_{z_m}}{\sigma_{z_w}}\right)\frac{s_w^2}{2}\right)\right)-1\right)
\end{aligned}
$$

and $\Delta_w(x_w)$ has the same sign as

$$
\delta_w \equiv t_w-c-k+(1-\rho_w)\left(\alpha_w+\alpha_m\frac{\sigma_{z_m}}{\sigma_{z_w}}\right)\frac{s_w^2}{2}.
$$

$\square$

Without Marriage Market. An ability x_m man's expected utility gain from the risky career over the safe career, *without the marriage market*, is

$$\Delta_m(x_m) = \mathbb{E}_{\varepsilon_m}[u_m(s(x_m - c + \varepsilon_m, z_w(x_m)) - v_w(z_w(x_m)) - u_m(v_m(x_m))]$$

where

$$\begin{aligned}
& u_m(s(x_m - c + \varepsilon_m, z_w(x_m)) - v_w(z_w(x_m)) - u_m(v_m(x_m)) \\
= \; & \frac{[s(x_m - c + \varepsilon_m, z_w(x_m)) - v_w(z_w(x_m))_+]^{1-\rho_m}}{1-\rho_m} - \frac{[v_m(x_m)]^{1-\rho_m}}{1-\rho_m} \\
= \; & \frac{\left[\exp[\alpha_m(x_m - c + \varepsilon_m) + \alpha_w z_w(x_m)] - \frac{\alpha_w}{\alpha_w + \alpha_m \frac{\sigma_{zm}}{\sigma_{zw}}} \exp[\alpha_m x_m + \alpha_w z_w(x_m))]\right]^{1-\rho_m}}{1-\rho_m} \\
& - \frac{\left[\frac{\alpha_m}{\alpha_m + \alpha_w \frac{\sigma_{zw}}{\sigma_{zm}}} \exp(\alpha_m x_m + \alpha_w z_w(x_m))\right]^{1-\rho_m}}{1-\rho_m} \\
= \; & \frac{\left[\left[\exp[\alpha_m(\varepsilon_m - c)] - \frac{\alpha_w}{\alpha_w + \alpha_m \frac{\sigma_{zm}}{\sigma_{zw}}}\right] \exp[\alpha_m x_m + \alpha_w z_w(x_m))]\right]^{1-\rho_m}}{1-\rho_m} \\
& - \frac{\left[\frac{\alpha_m}{\alpha_m + \alpha_w \frac{\sigma_{zw}}{\sigma_{zm}}} \exp(\alpha_m x_m + \alpha_w z_w(x_m)\right]^{1-\rho_m}}{1-\rho_m} \\
= \; & \frac{\exp[(1-\rho_m)(\alpha_m x_m + \alpha_w z_w(x_m))]}{1-\rho_m} \left[(\exp[\alpha_m \varepsilon_m - \alpha_m c] - K)_+^{1-\rho_m} - (1-K)^{1-\rho_m}\right]
\end{aligned}$$

where $K = \frac{\alpha_w \sigma_{zw}}{\alpha_w \sigma_{zw} + \alpha_m \sigma_{zm}}$. It remains to calculate

$$\mathbb{E}_{\varepsilon_m}\left[(\exp[\alpha_m(\varepsilon_m - c)] - K)^{1-\rho_m} - (1-K)^{1-\rho_m}\right]$$

Since $\varepsilon_m \sim \mathcal{N}(t_m, s_m^2)$, $\alpha_m \varepsilon_m - \alpha_m c \sim \mathcal{N}(\alpha_m(t_m - c), {\alpha_m}^2 s_m^2)$. $\square$

With Marriage Market. An ability x_m man's expected utility gain from the risky career over the safe career, *with the marriage market*, is

$$\begin{aligned}
& \mathbb{E}_{\varepsilon_m}[u_m(v_m(x_m - c + \varepsilon_m)) - u_m(v_m(x_m))] \\
= \; & \mathbb{E}_{\varepsilon_m}[s(x_m - c + \varepsilon_m, z_w(x_m - c + \varepsilon_m)) - v_w(z_w(x_m - c + \varepsilon_m)) - u_m(v_m(x_m))].
\end{aligned}$$

$\square$

Proof of Theorem 2. Define

$$\delta_m(p_m, p_w) \equiv t_m - c(p_m, p_w) + (1-\rho_m)\left(\alpha_m + \alpha_w\sqrt{\frac{\sigma_{x_w}^2 + p_w s_w^2}{\sigma_{x_m}^2 + p_m s_m^2}}\right)\frac{s_m^2}{2}$$

$$\delta_w(p_m, p_w) \equiv t_w - c(p_m, p_w) - k + (1-\rho_w)\left(\alpha_w + \alpha_m\sqrt{\frac{\sigma_{x_m}^2 + p_m s_m^2}{\sigma_{x_w}^2 + p_w s_w^2}}\right)\frac{s_w^2}{2}$$

Economically, $\delta_m(p_m, p_w)$ and $\delta_w(p_m, p_w)$ have the same sign as Δ_m and Δ_w where the proportion p_m of men and proportion p_w of women choose the risky career. Define correspondences

$$\rho_m(p_m, p_w) = \begin{cases} 0 & \text{if } \delta_m(p_m, p_w) < 0 \\ [0,1] & \text{if } \delta_m(p_m, p_w) = 0 \\ 1 & \text{if } \delta_m(p_m, p_w) > 0 \end{cases}$$

$$\rho_w(p_m, p_w) = \begin{cases} 0 & \text{if } \delta_w(p_m, p_w) < 0 \\ [0,1] & \text{if } \delta_w(p_m, p_w) = 0 \\ 1 & \text{if } \delta_w(p_m, p_w) > 0 \end{cases}$$

$\rho_m(p_m, p_w)$ and $\rho_w(p_m, p_w)$ represent the optimal probabilities of men and women who would choose the risky career if proportion ρ_m of men and proportion ρ_w of women choose the risky career. An equilibrium exists if the mapping

$$\rho(p_m, p_w) = (\rho_m(p_m, p_w), \rho_w(p_m, p_w))$$

has a fixed point. Since $\rho : [0,1]^2 \to [0,1]^2$ is upper hemicontinuous, convex-valued, and nonempty, by Kakutani's fixed point theorem, a fixed point exists.

Furthermore, we can show the uniqueness of equilibrium if $1-\rho_m > 0$ and $1-\rho_m > 0$. Note that because $1-\rho_m > 0$, $\delta_m(p_m, p_w)$ decreases in p_m. Fix p_w. Define $p_m(p_w)$ as follows,

$$p_m(p_w) = \begin{cases} 1 & \text{if } \delta_m(0, p_w) > \delta_m(1, p_w) > 0 \\ \text{solution of } \delta_m(p_m, p_w) = 0 & \text{if } \delta_m(0, p_w) > 0 > \delta_m(1, p_w) \\ 0 & \text{if } 0 > \delta_m(0, p_w) > \delta_m(1, p_w) \end{cases}$$

$p_m(p_w)$ is continuous and differentiable. We will show $\delta_w(p_m(p_w), p_w)$ strictly decreases in p_w.

$$\delta_w(p_m(p_w), p_w) = t_w - c(p_m(p_w), p_w) - k + \frac{s_w^2}{2}(1-\rho_w)\left(\alpha_w + \alpha_m\sqrt{\frac{\sigma_{x_m}^2 + p_m(p_w)s_m^2}{\sigma_{x_w}^2 + p_w s_w^2}}\right)$$

First note that,

$$\begin{aligned}\frac{d\sqrt{\frac{\sigma_{x_m}^2+p_m(p_w)s_m^2}{\sigma_{x_w}^2+p_w s_w^2}}}{dp_w} =& \sqrt{\frac{\sigma_{x_m}^2 + p_m(p_w)s_m^2}{\sigma_{x_w}^2 + p_w s_w^2}}\left(\frac{s_m^2/2}{\sigma_{x_m}^2 + p_m(p_w)s_m^2}p_m{}'(p_w) - \frac{s_w^2/2}{\sigma_{x_w}^2 + p_w s_w^2}\right)\\ \equiv& r_m(A_m p_m{}'(p_w) - A_w)\end{aligned}$$

Denote $\frac{s_w^2}{2}(1-\rho_w)$ by B_w. Hence,

$$\delta_w(p_m(p_w), p_w) = t_w - c(p_m(p_w), p_w) - k + B_w(\alpha_w + \alpha_m r_m)$$

andr

$$\begin{aligned}\frac{d\delta_w(p_m(p_w), p_w)}{dp_w} =& -c'(p)(p_m{}'(p_w) + 1) + B_w\alpha_m r_m(A_m p_m{}'(p_w) - A_w)\\ =& (B_w\alpha_m r_m A_m - c'(p))p_m{}'(p_w) - (A_w B_w \alpha_m r_m + c'(p))\end{aligned}$$

If $p_m{}'(p_w) = 0$, then we are done. If $p_m{}'(p_w) \neq 0$, then by the implicit function theorem,

$$\delta_m(p_m(p_w), p_w) = t_m - c(p_m(p_w), p_w) + B_m(\alpha_m \alpha_w r_w) = 0$$

where $B_w = (1-\rho_m)\frac{s_m^2}{2}$ and $r_w = 1/r_m$. Since

$$\frac{dr_w}{dp_w} = r_w(A_w - A_m p_m{}'(p_w))$$

We have

$$0 = \frac{d\delta_m(p_m(p_w), p_w)}{dp_w} = -c'(p)(p_m{}'(p_w) + 1) + B_m\alpha_w r_w(A_w - A_m p_m{}'(p_w))$$

Rearrange to get

$$p_m{}'(p_w) = \frac{A_w B_m \alpha_w r_w - c'(p)}{A_m B_m \alpha_w r_w + c'(p)}$$

Plugging into $\frac{d\delta_w}{dp_w}$, we get

$$\begin{aligned}
&\frac{d\delta_w(p_m(p_w),p_w)}{dp_w}\\
=&(A_mB_w\alpha_m r_m - c'(p))\frac{A_wB_m\alpha_w r_w - c'(p)}{A_mB_m\alpha_w r_w + c'(p)} - (A_wB_w\alpha_m r_m + c'(p))\\
=&\frac{(A_mB_w\alpha_m r_m - c'(p))(A_wB_m\alpha_w r_w - c'(p)) - (A_mB_m\alpha_w r_w + c'(p))(A_wB_w\alpha_m r_m + c'(p))}{A_mB_m\alpha_w r_w + c'(p)}\\
=&-\frac{(A_mB_w\alpha_m r_m + A_wB_m\alpha_w r_w + A_mB_m\alpha_w r_w + A_wB_w\alpha_m r_m)c'(p)}{A_mB_m\alpha_w r_w + c'(p)} < 0
\end{aligned}$$

Since $d\delta_w/dp_w < 0$, define p_w^* as follows

$$p_w^* = \begin{cases} 1 & \text{if } \delta_w(p_m(0),0) > \delta_w(p_m(1),1) > 0 \\ \text{solution of } \delta_w(p_m(p_w),p_w) = 0 & \text{if } \delta_w(p_m(0),0) > 0 > \delta_w(p_m(1),1) \\ 0 & \text{if } 0 > \delta_w(p_m(0),0) > \delta_w(p_m(1),1) \end{cases}$$

Then $(p_m(p_w^*), p_w^*)$ characterizes the unique equilibrium. □

Proof of Proposition 1. A man would strictly prefer the risky career if

$$\delta_m = t_m - c + (1-\rho_m)(\alpha_m + \alpha_w\frac{\sigma_{y_w}}{\sigma_{y_m}})\frac{s_m^2}{2} > 0$$

If $t_m - c = -\frac{s_m^2}{2}$ (the safe career and the risky career have the same expected income), $\rho_m = 0$, and $\alpha_m = 1$ (the marriage surplus is linear in income) then $\delta_m = \alpha_w \frac{\sigma_{y_w}}{\sigma_{y_m}}\frac{s_m^2}{2} > 0$. If $t_m - c = -\frac{s_m^2}{2} - e_m$, then $\delta_m = -e_m + ((1-\rho_m)(\alpha_m + \alpha_w\frac{\sigma_{y_w}}{\sigma_{y_m}}) - 1)\frac{s_m^2}{2}$. As long as $e_m < ((1-\rho_m)(\alpha_m + \alpha_w\frac{\sigma_{y_w}}{\sigma_{y_m}}) - 1)\frac{s_m^2}{2}$, a risk-averse man would strictly prefer the risky career that yields a lower expected income. Similarly, if $t_w - c = -\frac{s_w^2}{2} - e_w$, then $\delta_w = -k - e_w + ((1-\rho_w)(\alpha_w + \alpha_m\frac{\sigma_{y_m}}{\sigma_{y_w}}))\frac{s_w^2}{2}$. As long as $e_w < ((1-\rho_w)(\alpha_w + \alpha_m\frac{\sigma_{y_m}}{\sigma_{y_w}}))\frac{s_w^2}{2}$, a risk-averse woman would strictly prefer the risky career with a lower expected income. □

Proof of Proposition 3. The two parts are proved as follows.

1. Since $0 < p_m^* < 1$ and $0 < p_w^* < 1$, the two equilibrium premiums

$$\delta_m{}^* = t - c^* + (1-R)\left(\alpha_m + \alpha_w\frac{1}{r^*}\right)\frac{s^2}{2} = 0$$

$$
\begin{aligned}
\delta_w{}^* =& t - c^* - k + (1-R)(\alpha_w + \alpha_m r^*)\frac{s^2}{2} \\
=& t - c^* - k + r^*(1-R)\left(\alpha_m + \alpha_w \frac{1}{r^*}\right)\frac{s^2}{2} = 0
\end{aligned}
$$

where $r^* = \sigma^*_{y_m} / \sigma^*_{y_w}$. Subtract the two equations to get

$$
(r^* - 1)(1-R)\left(\alpha_m + \alpha_w \frac{1}{r^*}\right)\frac{s^2}{2} = k
$$

Since $1-R$ is assumed to be positive, in order for the left-hand side of the equation to be positive, $r^* > 1$, so $\sigma^*_{y_m} > \sigma^*_{y_w}$.

2. Rearranging the equilibrium condition

$$
(r-1)(1-R)(\alpha_m + \alpha_w \frac{1}{r})\frac{s^2}{2} = k,
$$

we get

$$
\begin{aligned}
(r-1)(\alpha_m r + \alpha_w) &= \frac{2k}{(1-R)s^2} \\
\alpha_m r^2 - \alpha_m r + \alpha_w r - \alpha_w &= \frac{2k}{(1-R)s^2} r \\
r^2 - \left(1 - \frac{\alpha_w}{\alpha_m} + \frac{1}{\alpha_m}\frac{2k}{(1-R)s^2}\right) - \frac{\alpha_w}{\alpha_m} &= 0
\end{aligned}
$$

Therefore,

$$
r^* = \frac{1}{2}\left[\left(1 - \frac{\alpha_w}{\alpha_m} + \frac{1}{\alpha_m}\frac{2k}{(1-R)s^2}\right) + \sqrt{\left(1 - \frac{\alpha_w}{\alpha_m} + \frac{1}{\alpha_m}\frac{2k}{(1-R)s^2}\right)^2 + 4\frac{\alpha_w}{\alpha_m}}\right].
$$

Obviously, r^* increases in k, decreases in $(1-R)$, and decreases in s^2. It is more complicated to derive $\partial r^* / \partial \alpha_w$.

$$
\begin{aligned}
\frac{\partial r^*}{\partial \alpha_w} =& \frac{1}{2}\left[-\frac{1}{\alpha_m} + \frac{1}{2}\frac{\left(1 - \frac{\alpha_w}{\alpha_m} + \frac{1}{\alpha_m}\frac{2k}{(1-R)s^2}\right) + \frac{4}{\alpha_m}}{\sqrt{\left(1 - \frac{\alpha_w}{\alpha_m} + \frac{1}{\alpha_m}\frac{2k}{(1-R)s^2}\right)^2 + 4\frac{\alpha_w}{\alpha_m}}}\right] \\
=& \frac{1}{2}\frac{1}{\alpha_m}\frac{2 - r^*}{\sqrt{\left(1 - \frac{\alpha_w}{\alpha_m} + \frac{1}{\alpha_m}\frac{2k}{(1-R)s^2}\right)^2 + 4\frac{\alpha_w}{\alpha_m}}}.
\end{aligned}
$$

$\frac{\partial r^*}{\partial \alpha_w} > 0$ if $r^* < 2$ and $\frac{\partial r^*}{\partial \alpha_w} < 0$ if $r^* > 2$.

$\square$

Proof of Proposition 2. $\sigma_{y_m}^{*2} = \sigma_x^2 + p_m^* s^2 > \sigma_x^2 + p_w^* s^2 = \sigma_{y_w}^{*2}$ implies

$$p_m^* = \frac{\sigma_{y_m}^2 - \sigma_x^2}{s^2} > \frac{\sigma_{y_w}^2 - \sigma_x^2}{s^2} = p_w^*.$$

$\square$

Proof of Lemma 5. The marriage timing is related to the career choice as follows.

1. A man who chooses the safe career gets a marriage payoff of $v_m(x_m)$ regardless of his marriage timing decision, so he is indifferent between marrying in the first period and marrying in the second period.

2. A woman who chooses the safe career gets a marriage payoff of $v_w(x_w)$ if she marries in the first period and gets $v_w(x_w - k) < v_w(x_w)$ if she marries in the second period, so she chooses to marry in the first period if she chooses the safe career.

3. An ability x_m man who chooses the risky career gets a marriage payoff of

$$s(x_m - c + \varepsilon_m, \tilde{z}_w) - v_w(\tilde{z}_w)$$

if $\tilde{z}_w$ is his wife's married type, and he realizes income $x_m - c + \varepsilon_m$. However, if he waits until the second period to get married, then he gets

$$v_m(x_m - c + \varepsilon_m) = s(x_m - c + \varepsilon_m, z_w(x_m - c + \varepsilon_m)) - v_w(z_w(x_m - c + \varepsilon_m))$$

By the stability condition, for any $\tilde{z}_w$

$$v_m(x_m - c + \varepsilon_m) \geq s(x_m - c + \varepsilon_m, \tilde{z}_w) - v_w(\tilde{z}_w)$$

Therefore, for any realization of ε_m, the man is weakly better off to wait until the second period to marry. Since for different ε_m, the man marries a different partner in the second period, it is almost always strictly better off to wait until the second period to marry. The expected marriage payoff is strictly higher when a man who chooses the risky career marries in the second period.

4. An ability x_w woman who chooses the risky career gets a marriage payoff of

$$s(\tilde{z}_m, x_w - c - k + \varepsilon_w) - v_m(\tilde{z}_m)$$

if $\tilde{z}_m$ is her husband's marriage type and she realizes income $x_w - c + \varepsilon_w$. However, if she waits until the second period to get married when she realizes income $x_w - c + \varepsilon_w$, then she gets

$$v_w(x_w - c + \varepsilon_w) = s(z_m(x_w - c + \varepsilon_w), x_w - c + \varepsilon_w) - v_m(z_m(x_w - c + \varepsilon_w)).$$

By the stability, for any $\tilde{z}_w$,

$$v_w(x_w - c + \varepsilon_w) \geq s(\tilde{z}_m, x_w - c + \varepsilon_w) - v_m(\tilde{z}_m).$$

Therefore, for any realization of ε_w, the woman is weakly better off to wait until the second period to marry. Since for different ε_w, the woman marries a different husband in the second period, it is almost always strictly better off to wait until the second period to marry. Hence, the expected marriage payoff is strictly higher when a woman who chooses the risky career marries in the second period.

□

Proof of Proposition 5. The effects of marital status on the risky career choice differ by gender in the following ways.

1. If a type z_m man who is married to a type $\tilde{z}_w$ woman chooses the risky career and he realizes an income $z_m - c + \varepsilon_m$, he gets a marriage payoff of $s(z_m - c + \varepsilon_m, \tilde{z}_w) - v_w(\tilde{z}_w)$. In contrast, if the same type z_m man who is unmarried chooses the risky career and he realizes an income $z_m - c + \varepsilon_m$, he gets a marriage payoff of $s(z_m - c + \varepsilon_m, z_w(z_m - c + \varepsilon_m)) - v_w(z_w(z_m - c + \varepsilon_m)) \geq s(z_m - c + \varepsilon_m, \tilde{z}_w) - v_w(\tilde{z}_w)$, and the inequality holds strictly as long as $-c + \varepsilon_m \neq 0$.

2. If a type z_w woman who is married to a type $\tilde{z}_m$ man chooses the risky career and realizes an income $z_w - c + \varepsilon_w$, she gets a marriage payoff of

$$s(\tilde{z}_m, z_w - c + \varepsilon_w) - v_m(\tilde{z}_m).$$

In contrast, if the same type z_w woman who is unmarried chooses the risky career and realizes an income $z_w - c + \varepsilon_w$, she gets marriage payoff of

$$s(z_m(z_w - c + \varepsilon_w - k), z_w - c + \varepsilon_w - k) - v_m(z_m(z_w - c + \varepsilon_w - k)).$$

The difference between the marriage payoffs can be written as

$$
\begin{aligned}
&[s(z_m(z_w - c + \varepsilon_w - k), z_w - c + \varepsilon_w - k) - v_m(z_m(z_w - c + \varepsilon_w - k))] \\
&- [s(\tilde{z}_m, z_w - c + \varepsilon_w - k) - v_m(\tilde{z}_m)] \\
&+ [s(\tilde{z}_m, z_w - c + \varepsilon_w - k) - v_m(\tilde{z}_m)] - [s(\tilde{z}_m, z_w - c + \varepsilon_w) - v_m(\tilde{z}_m)]
\end{aligned}
$$

The first two terms together are positive, but the third term is negative. The first two terms show that, by deciding whom to marry after the income is realized, an unmarried woman has a higher incentive to choose the risky career. The last term shows that, the absence of a reproductive decline gives a married woman a higher incentive to choose the risky career.

□