Evolutionary Justifications for Non-Bayesian Beliefs

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Abstract

This paper suggests that the evolutionarily optimal belief of an agent's intrinsic reproductive ability is systematically different from the posterior belief obtained by the perfect Bayesian updating. In particular, the optimal belief depends on how risk-averse the agent is. Although the perfect Bayesian updating remains evolutionarily optimal for a risk-neutral agent, it is not for any other. Specifically, the belief is always positively biased for a risk-averse agent, and the more risk-averse an agent is, the more positively biased the optimally updated belief is. Such biased beliefs align with experimental findings and also offer an alternative explanation to the empirical puzzle that people across the population appear overconfident by consistently overestimating their personal hereditary traits.

Keywords: Non-Bayesian belief, risk-aversion, evolutionary economics, overconfidence, bias

JEL: C73, D83

1 Introduction

Daily observations, empirical evidence, and experimental findings suggest that under many circumstances human beings are not perfect Bayesian updaters, especially when they learn about their personal traits such as beauty and intelligence. One particular systematic deviation from the perfect Bayesian updating is **positive bias**: people tend to over-react to positive signals indicating their possibility of high ability and under-react to negative ones indicating otherwise. As a consequence, people are unrealistically overconfident about their personal characteristics.

The fact that the asymmetric updating results in population-wide overestimation of personal ability and persistently hyped beliefs even with overwhelming contrary indications is reflected in studies self-reporting many personal traits. 88% of American drivers believe that they drive more safely than the median driver does (Svenson, 1981) and 75% of Harvard undergraduate students think they have above median IQ among their peers, even with repeated informative signals indicating their true tested results (Möbius et al., 2012). Although many attribute the result to cognitive limitations like selective recall and selective information acquisition, while maintaining the perfect Bayes Rule evidence is too overwhelming and magnitude is too colossal to refuse entertaining the possibility of an updating system other than the perfect Bayesian and a resulting non-Bayesian posterior. This is especially true for beliefs about own traits - it is after all the same group of Harvard undergraduates who show perfect Bayesian updating behavior about other people's abilities in the same study. An increasing number of papers assume people have direct belief utilities because they care about self-esteem (Benabou and Tirole, 2002; Kőszegi, 2006; Eil and Rao, 2011), but this assumption almost directly implies positively biased beliefs.

This paper attempts to justify the seemingly imperfect human behavior - positively biased updates and beliefs - from an evolutionary standpoint, without assuming a belief utility or cognitive deficiencies. If such seemingly imperfect updating behavior is so prevalent, maybe it is key to our survival, or those who have adopted such updating behavior are the fittest and have adapted and survived to this date. The personal traits are esteemed because they affect the reproductive efficiencies, so the belief should be formed in a way that achieves maximal reproductive success. Although an agent cares about the survival of her offspring, she wants to maximize her *utility* about the survival of offspring. When the agent is not risk-neutral, utility maximization is not the same goal as the primary evolutionary goal of maximizing the expected survival rates of offspring. When a principal has an unaligned goal with an agent, the literature in contract theory and mechanism design considers possible compensation schemes by a principal to an agent to align the two goals so that the agent takes the effort level the principal desires. In this paper, the possible financial scheme is impossible, but the principal, in this case Nature, can choose to manipulate the agent's belief about her own ability. Nature achieves his goal of maximized growth while the agent still maximizes her utility. The agents who have the evolutionarily optimal beliefs would have the highest overall expected growth and survival rate. We will show concretely that risk-averse agents exhibit positively biased beliefs, as the way that evolution corrects risk deficiency.

Non-Bayesian updating has been widely studied by psychologists, evolutionary biologists, philosophers, and behavioral economists. This paper, in a broad sense, attempts to

explain phenomena observed from psychological and behavioral experiments, by abiding by philosophical rules and utilizing techniques developed by evolutionary economists. Although there are many experiments confirming the non-Bayesian behaviors of human in various settings, the theoretical literature explaining such behaviors is relatively scant and they do not provide or suggest an evolutionary link (Epstein, 2006; Epstein et al., 2008, 2010). There are only a few papers providing evolutionary justifications to risk aversion and Bayesian updating. Okasha (2012) shows that Bayesian updating is evolutionarily optimal when the agents are "rational" in a philosophical sense - essentially having von Neumann-Morgenstern utility. Levy (2010) shows that constant relative risk-averse (CRRA) utility function is evolutionarily optimal if the agent's objective is to have descendants forever.

The result that non-Bayesian updaters and believers dominate the competition sharply contrasts with some rational expectations results in the financial market. In the financial market, those investors who make inaccurate predictions about the market partly due to imperfect Bayesian updating are driven out in the competitive equilibrium (Sandroni, 2000). However, in the reproduction market, the ones who make reproductive decisions from perfectly updated beliefs are the ones who are driven out and will be extinct in the long run.

Section 2 defines and characterizes the evolutionarily optimal posterior. Section 3 shows that belief is consistently higher than true population proportion of high abilities. Section 4 concludes by pointing out limitations of the current model and possible future directions.

2   Evolutionarily Optimal Posterior

Let us introduce the setup and demonstrate the key results in a simple model with the imperfectly observable reproductive trait taking two possible values. In particular, we show that the evolutionarily optimal posterior is always higher than the perfect Bayesian posterior, for all signals.

An agent chooses a reproductive action based on her belief about her reproductive trait to maximize her expected utility about the survival of the offspring. The agent's action a and trait x determine the survival rate or the number of her offspring, which we call the reproduction function. The trait can be either high (H) or low (L) but the agent does not directly observe it. The reproduction function $F(a, x)$ is assumed to be continuously differentiable, increasing, and concave in a, with the boundary condition $F(0, x) = 0$. Furthermore, assume that $F(a, H) > F(a, L)$, i.e., exerting the same effort, an agent with high ability produces more than an agent with low ability. It is increasingly costly to exert effort in mating activities, so the cost function $c(\cdot)$ is assumed to be increasing and weakly convex, $c'(a) > 0, c''(a) \geq 0$. The agent derives utility $u(\cdot)$ from the net benefit $F(a, x) - c(a)$ she gets, with $u'(\cdot) > 0$.

Since the agent does not perfectly observe the trait x she has, she forms a posterior belief μ from a signal s coupled with a prior μ_0 inherited possibly from her parent. The signal can be H or L, and the signal generating process is publicly known,

$$
\begin{aligned}
\Pr(s = H | x = H) &= p_1, \\
\Pr(s = H | x = L) &= p_2.
\end{aligned}
$$

In particular, the perfect Bayesian posterior μ^B can be expressed as

$$\text{logit}\left(\mu^B\right) = \text{logit}\left(\mu_0\right) + \mathbf{1}_{s=H}\lambda_H + \mathbf{1}_{s=L}\lambda_L,$$

where μ^B, μ_0 are shorthands for $\mu^B\left(H\right), \mu_0\left(H\right)$ as a slight abuse of notation, and $\text{logit}\left(\mu\right) = \log\left(\frac{\mu}{1-\mu}\right)$. $\lambda_s = \log\left(\frac{\Pr(s|x=H)}{\Pr(s|x=L)}\right)$ represents the log-likelihood of being high type given signal s. In particular, $\lambda_H = \log\left(\frac{p_1}{p_2}\right)$ and $\lambda_L = \log\left(\frac{1-p_1}{1-p_2}\right)$.

Agent's Problem

Given the posterior belief μ, she chooses the effort level a to maximize expected utility,

$$u_A\left(a|\mu\right) = \mu u\left(F\left(a, H\right) - c\left(a\right)\right) + \left(1 - \mu\right) u\left(F\left(a, L\right) - c\left(a\right)\right)$$

So the FOC is

$$0 = \mu u'\left(F\left(a, H\right) - c\left(a\right)\right)\left(F_a\left(a, H\right) - c'\left(a\right)\right) + \left(1 - \mu\right) u'\left(F\left(a, L\right) - c\left(a\right)\right)\left(F_a\left(a, L\right) - c'\left(a\right)\right)$$

Or rearrange, a is chosen so that the following equation is satisfied,

$$\frac{\mu}{1 - \mu} = \left|\frac{F_a\left(a, L\right) - c'\left(a\right)}{F_a\left(a, H\right) - c'\left(a\right)}\right| \cdot \frac{u'\left(F\left(a, L\right) - c\left(a\right)\right)}{u'\left(F\left(a, H\right) - c\left(a\right)\right)} \tag{1}$$

Nature's Problem

Consider the agent's problem regarding a reproductive decision. Consider that x is an imperfectly observable reproductive trait (IQ, EQ, psychological fitness, for example), and a is the agent's effort spent in searching and mating with the convex cost of the effort that represents a reduction in one's own survival and fitness (frequenting dating sites such as bars and websites takes nontrivial effort and opportunity cost), with $F\left(a, x\right)$ guiding the number (and quality) of offspring an agent produces. The objective of Nature, a perfect Bayesian updater, is then to maximize the overall expected growth of the population,

$$u_N\left(a\right) = \mu^B\left(F\left(a, H\right) - c\left(a\right)\right) + \left(1 - \mu^B\right)\left(F\left(a, L\right) - c\left(a\right)\right)$$

Its FOC is

$$\frac{\mu^B}{1 - \mu^B} = \left|\frac{F_a\left(a^*, L\right) - c'\left(a^*\right)}{F_a\left(a^*, H\right) - c'\left(a^*\right)}\right| \tag{2}$$

where a^* is the evolutionarily optimal action Nature wants the agent to take, given prior μ_0 and signal s.

If Nature can manipulate the agent's belief to induce her to choose the evolutionarily optimal action, then (1) becomes

$$\frac{\mu^*}{1 - \mu^*} = \left|\frac{F_a\left(a^*, L\right) - c'\left(a^*\right)}{F_a\left(a^*, H\right) - c'\left(a^*\right)}\right| \cdot \frac{u'\left(F\left(a^*, L\right) - c\left(a^*\right)\right)}{u'\left(F\left(a^*, H\right) - c\left(a^*\right)\right)}.$$

Plugging in (2),

$$\text{logit}(\mu^*) = \text{logit}(\mu^B) + \log\left[\frac{u'(F(a^*, L) - c(a^*))}{u'(F(a^*, H) - c(a^*))}\right]. \tag{3}$$

Because $F(\cdot, L) < F(\cdot, H)$, when $u(\cdot)$ is concave, the second term on RHS of (3) is positive, which we refer to as the **risk-averse bias** $B(a^*)$. The evolutionarily optimal posterior belief of a risk neutral agent is the perfect Bayesian posterior, there is no bias; so it is a bias stemmed from risk aversion of the agent. When the utility function is CRRA or CARA, the bias perfectly correlates with the risk aversion factor, since a^* does not depend on the risk aversion factor but only on Bayesian posterior.

Example 1. If the utility is CRRA, $u(C) = C^{1-\rho}/(1-\rho)$, $\rho \geq 1$, $u'(C) = C^{-\rho}$,

$$B(a^*) = \rho \log\left|\frac{F(a^*, H) - c(a^*)}{F(a^*, L) - c(a^*)}\right|.$$

Example 2. If the utility is CARA, $u(C) = K - \exp(-\alpha C)$, $\alpha \geq 0$, $u'(C) = \alpha \exp(-\alpha C)$,

$$B(a^*) = \alpha[F(a^*, H) - F(a^*, L)].$$

The bias is positive for any risk-averse agent and negative for any risk-loving agent.

Proposition 1. *When the agent is risk-averse (risk-loving), the evolutionarily optimal posterior is a non-Bayesian posterior, positively (negatively) biased towards the high type compared to the perfect Bayesian posterior.*

Proof to Proposition 1. Because $F(\cdot, L) < F(\cdot, H)$ and $u(\cdot)$ is concave (convex),

$$u'(F(a^*, L) - c(a^*)) > (<)u'(F(a^*, H) - c(a^*)).$$

By (3),

$$B(a^*) = \log\left[\frac{u'(F(a^*, L) - c(a^*))}{u'(F(a^*, H) - c(a^*))}\right] > (<)\log 1 = 0.$$

$\square$

A few comments are in order. The evolutionarily optimal posterior is invariant to the order of arrival of a stream of i.i.d. signals. The perfect Bayesian posterior is invariant to the order of arrival of a stream of i.i.d. signals, given $\{s_t\}$ and prior μ_0, the posterior μ^B is

$$\text{logit}(\mu^B) = \text{logit}(\mu_0) + \#\{t : s_t = H\}\lambda_H + \#\{t : s_t = L\}\lambda_L,$$

Since a^* only depends on μ^B, it is invariant to the order of arrival, then by (3), μ^* is invariant to the order of arrival of $\{s_t\}$. Therefore, whether the agent makes the reproductive decision after appearance of one or more signals does not affect the reproductive outcome or the personal posterior belief.

As a result of the evolutionary correction, the long run survival of an agent does not depend on her risk aversion factor but only on her true trait. A non-Bayesian belief corrects the possible evolutionary sub-optimal reproductive decision a risk-averse agent can make, and the level of correction depends on the degree of risk aversion so that agents with the same Bayesian posterior make the same reproductive decision. Without sexual production or mutation, only the agents of high type will survive, and the evolutionarily optimal updating makes an agent realize that she is of high type faster, and of low type slower.

3 Population Posterior

Investigation of the population evolutionarily optimal posterior belief shows that regardless of the population composition, as long as they are risk-averse, more people believe that they are of high type than there really are. Since we are survivors and winners of millions of years of evolutionary struggles, this result possibly explains the aforementioned findings that people are overconfident about their intrinsic skills.

The result can be directly understood from the asymmetric belief. Every agent believes she is more likely to be a high type than a perfect Bayesian would believe. Regardless of the evidence (stream of signals) a person receives, her belief about herself being a high type is always higher than the Bayesian posterior belief.

Suppose the population is composed of proportion q realized high type and proportion $1 - q$ realized low type agents after the previous action. Suppose that after each time an agent takes an action, she observes the outcome of her action and infers her true type (since $F(a, x)$ is bijective, knowing what effort a she exerted and observing F is enough to uncover her true trait x). After each action, there is a probability ϵ that she mutates: her true type switches from one to another. Therefore, an agent's prior μ_0 after an action is $1 - \epsilon$ if she is high or ϵ if low.

We construct a population posterior and use it as a criterion to evaluate the percentage of people believing they are of high type. For any posterior $\mu = \{\mu_A\}_A$, the expected population posterior is defined as the total population belief that they are a high type.

$$q(\mu) = \int_A \mu_A dA$$

If every agent is a perfect Bayesian, given the signal generating process, their population posterior should be the same as the population prior, which is the same as the population composition, q, $q^B \equiv q(\mu^B) = q$. On the other hand, any agent risk-averse A has $\mu_A^* > \mu_A^B$, so in population, $q(\mu^*) > q(\mu^B)$, a relatively small portion of risk-seeking agents will not alter the population belief.

Proposition 2. *If most agents are risk-averse in the population, the evolutionarily optimal population posterior belief about high type is strictly greater than the population composition of high type.*

Even though every agent knows that in the population, there is only a proportion q of high type agents, the aggregate of individual beliefs is higher than it. This finding explains the perverse scenario mentioned in Introduction that objective aggregates of desirable personal characteristics are always lower than their subjective individual reports. The key to the result is the imperfect observability of personal characteristics and possibility of mutation.

4 Conclusion

The paper shows that in order to maximize the expected number of offspring, an agent with nonlinear preference has a belief different from the belief obtained by perfect Bayesian updating. In particular, for any risk-averse agent, she thinks more highly of herself than she

does if she is a perfect Bayesian. Therefore, the results suggest that evolution and survival play a role in the widespread existence of non-Bayesian belief, especially about a person's own trait that influences reproductive decisions.

Although the paper provides a possible evolutionary channel to persistent overconfidence across the population, it fails to characterize the exact way how this optimal overconfidence is sustained. In terms of terminology of the model, the paper is able to rationalize the misalignment of the optimal posterior and the perfect Bayesian posterior, but it fails to characterize the updating *rule*[1] that can consistently achieve and sustain the optimal posterior. The problem is especially conspicuous when the agent updates after each of many sequentially observed noisy signals. Characterizing or approximating such evolutionarily optimal updating rule would be interesting and useful.

It is also interesting to explore why and how risk aversion and non-Bayesian belief/updating rule could be evolutionarily optimal at the same time. If we treat the objective of having descendants forever to be the goal for each individual as in Levy (2010) and the objective of maximizing expected population growth to be the goal for the entire group, an evolutionarily optimal non-Bayesian updating is justified. While each individual agent needs to be risk-averse to have descendants forever, the ones who dominate the population are those following a non-Bayesian updating rule that, coupled with risk-averse utility, maximizes the expected number of descendants in each generation.

Finally, a model with multiple signals and/or attributes may be more realistic and possibly help to generate more insights including but not limited to conservative updating.

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