Measuring Assortativeness in Marriage: Axiomatic and Structural Investigations*

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Abstract

Assortative matching refers to the tendency of individuals with similar characteristics to form partnerships. Measuring the extent to which assortative matching differs between two economies is challenging when the marginal distributions of the characteristic along which sorting takes place (e.g., education) changes for either sex or both sexes. We show how the use of different measures can generate different conclusions. We axiomatize different measures (e.g., odds ratio, normalized trace, and likelihood ratio) and provide a structural interpretation of the odds ratio.

Keywords: assortative matching, sorting, homogamy

JEL: C78, J01

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1 Introduction

The study of sorting in the marriage market has recently attracted renewed attention. The degree of homogamy in marriage—defined as people's tendency to "marry their own"—has important consequences for family inequalities and intergenerational transmission of human capital. It is therefore surprising that the various studies on this topic have not reached a consensus on the evolution of homogamy over the recent years, and even on how to best measure homogamy (e.g., Fernández and Rogerson, 2001; Mare and Schwartz, 2005; Greenwood et al., 2014; Siow, 2015; Chiappori, Salanié, and Weiss, 2017; Eika, Mogstad, and Zafar, 2019; Ciscato and Weber, 2020; Gihleb and Lang, 2020; Hou et al., 2022).

The goal of this paper is to understand why different approaches to the same problem, using similar data, can reach opposite conclusions and, more generally, to clarify the theoretical issues underlying the choice of a particular measure of assortativeness. Our analysis considers two populations, women and men, sorting in marriage according to a characteristic, say education. Whenever the marginal distributions change—e.g., women's average education increases—matching patterns will change. The main problem faced by any measure of assortativeness is to disentangle the mechanical effects of such variations in the marginal distributions from deeper changes in the matching structure itself, for instance originating from changes in the gain generated by assortativeness along that characteristic. The latter represents what one would call "changes in assortativeness." Existing studies proposed various indices and stochastic orders to measure assortativeness and changes therein. They measure assortativeness in different ways, and may therefore generate diverging conclusions.

Rather than starting with specific measures and justifying them with selected nice properties they satisfy, we take an *axiomatic approach*: We start with a set of properties measures should satisfy to discover the appropriate measure(s) of assortative matching.

Technically, we start with three sets of basic axioms: (i) axioms of *invariance* to specify when two matchings are equally assortative, (ii) axioms of *monotonicity* to specify when one matching is more assortative than another, and (iii) axioms of *homogamy* to specify what matchings are the most assortative.[1] Namely, invariance axioms are *scale invariance* (invariance to scaling the market), *side invariance* (invariance to swapping sides), and *type invariance* (invariance to swapping types). Two sets of notions of monotonicity we consider are (i) *marginal monotonicity* to rank matchings with the same marginal distributions and (ii) *diagonal* and *off-diagonal monotonicity* (assortativeness increases when more like types match and decreases when more unlike types match). Finally, we introduce two definitions of the most assortative matching: *full homogamy* (every pair in the market is homogamous) and *maximum homogamy* (the maximum number of homogamous pairs is achieved in the market, although not every pair is necessarily homogamous).

Many commonly used measures fail to satisfy one or more of basic axioms. The *odds ratio*—the ratio of college and noncollege individuals' odds to match with the same type and with a different type—and *normalized trace*—the proportion of like types—satisfy all basic axioms. We state more crucial axioms that will distinctly characterize these measures, which we refer to as *characterization axioms*.

In two-type markets, the odds ratio is the unique measure, up to monotonic transformation, that satisfies the basic axioms plus *marginal independence* (Edwards, 1963) (Theorem 1). All measures that are monotonic transformations of the odds ratio (e.g., Yule's Q, Yule's Y, and log odds ratio) provide the same ordering of markets in terms of assortativeness, the result clarifies that the odds ratio carries ordinal meaning only. We also provide a structural interpretation of the odds ratio: The estimated supermodular core (the net benefit of assortative matching over nonassortative matching) is double the odds ratio in a Choo and Siow (2006) transferable-utilities matching framework. In this sense, the odds ratio provides a way to compare two matchings even if their marginal distributions differ.

We also define *decomposability* axioms: The assortativeness measure for any market is a weighted average or weighted sum of the measures for submarkets decomposed from the

[1]In this paper, we use axioms and properties interchangeably.

market. The *normalized trace*—the proportion of pairs of like types—is the unique measure (up to linear transformation) that satisfies the basic axioms plus *population decomposability*, i.e, the weight to average is population size (Theorem 2). For comparison, we also axiomatize *aggregate likelihood ratio* (e.g., used in Eika, Mogstad, and Zafar (2019))—the excess likelihood of like types pairing up relative to the hypothetical likelihood under random matching. It is the unique measure (up to multiplication) that satisfies selected basic axioms plus *random decomposability*, i.e., the weight to sum is the hypothetical proportion of pairs of like types under random matching (Theorem 3).

In general, in markets with more than two types (e.g., there are 14 detailed education categories in the US Census), *robustness to categorization* is a desired property: Assortativeness comparison should be robust to how education levels are categorized into two or several groups. We provide an impossibility result: No total preorder satisfies the minimal set of axioms of monotonicity and robustness to categorization (Theorem 4). As a result, we must resort to preorders, which provide partial rankings of matchings.

The rest of the paper is organized as follows. Section 2 sets up the model and discusses special matching patterns and commonly used measures. Section 3 lays out the basic axioms. Section 4 discusses (i) the axiomatic and structural justifications for the odds ratio and (ii) characterization of normalized trace and aggregate likelihood ratio. Section 5 provides empirical results on marital sorting in the US. Section 6 discusses results beyond two types, and Section 7 concludes.

2 Model, matchings, and measures

2.1 Model and objective

Each man and woman possesses a trait (e.g., college education or any other phenotype such as a psychological, biological, socioeconomic trait). We start with the setting that a trait is one of two types, e.g., θ_1 and θ_2. For expositional ease and alignment with our primary empirical application, we refer the two traits as college-educated and noncollege-educated. Consider the matching between men and women described by matrix/table $M = (a, b, c, d)$:

$$M = \begin{array}{ccc} m\backslash w & \theta_1 \text{ (college)} & \theta_2 \text{ (noncollege)} \\ \theta_1 \text{ (college)} & a & b \\ \theta_2 \text{ (noncollege)} & c & d \end{array}.$$

A cell describes the mass of pairs between a specific combination of types of men and women. To recover the mass of (matched) individuals of a specific gender and type, we sum a column or a row. For example, there is mass $a+b$ of college men. The marginal distribution for men is then described by $(a+b, c+d)$ and that for women is $(a+c, b+d)$. Assume a full support of types on both sides of the market: $(a+b)(a+c)(b+d)(c+d) > 0$. We refer to M as the *matching*, *matrix*, *market*, or *table*, and let $|M| \equiv a+b+c+d$ denote its population size.

Objective. Most ideally, we would like to find an *index*, i.e., a function $I : \mathbb{R}^4_+ \to \mathbb{R} \cup \{-\infty, +\infty\}$, or equivalently (if we focus on the ordinal information), a *total preorder* $\succeq$ on the set of matchings, $\mathbb{R}^4_+$. At least we would like to find a *preorder*, which provides a partial ranking of matchings. Note that while total preorders and preorders are ordinal, indices may have cardinal meanings.

2.2 Special matching patterns

When a person is matched with a partner of the same type with probability one, i.e., $b = c = 0$, we call such a matching *fully positive-assortative.* However, fully positive-assortative matching is feasible only when men and women have the same marginal distribution. More generally, we call a matching *maximally positive-assortative* if there is maximum mass of pairs of like types: $bc = 0$ (that is, $b = 0$ or $c = 0$ or both). Say, all individuals on the short side of the educated category (men in our case) marry an educated partner, but not every educated individual on the long side (women) does so, i.e., $c = 0$; intuitively, the only reason that we observe "mixed" couples is the lack of educated men. Every market has a unique maximally positive-assortative matching.

In two-by-two markets, we can also analogously define *fully negative-assortative* matching as one in which all individuals are matched with a partner of a different type with probability one, i.e., $a = d = 0$; and *maximally negative-assortative* (or equivalently, *minimally positive-assortative*) matching as one in which there is a maximal mass of pairs of unlike types (or equivalently, in two-by-two markets, minimal mass of pairs of like types), i.e., $ad = 0$ (that is, $a = 0$ or $d = 0$ or both).

Note that fully (positive or negative) assortative matching is maximally assortative, but maximally assortative matching is not necessarily fully assortative. Most—not all—of the preorders in use have maximally positive-assortative matching as the maximum element.

One way of defining positive and negative assortative matching is to compare to the random matching benchmark. We say that matching M is a *positive assortative matching (PAM)* if the mass of couples with equal education (the "diagonal" of matrix M) is strictly larger than what would obtain under random matching. Under random matching the mass of couples in which both spouses are college-educated is $(a+b)(a+c)/|M|$. Then we have PAM if and only if

$$a(a+b+c+d) > (a+b)(a+c), \tag{1}$$

or equivalently,

$$ad > bc. \tag{2}$$

This inequality also implies that more noncollege individuals marry each other than would be implied by random matching.[2] In other words, PAM arises when extra forces generate more matches between equally educated people than would happen for random reasons. *Negative assortative matching (NAM)* in two-type markets can be analogously defined: $ad < bc$.

[2]Mathematically, $d > (d+b)(d+c)/|M| \Leftrightarrow d(a+b+c+d) > (d+b)(d+c) \Leftrightarrow d(a+b) > b(d+c) \Leftrightarrow ad > bc$. The inequality also implies that $a + d >$.

2.3 Existing measures and their differences

We now review some most commonly used measures in the literature. Many are measuring the extent to which the left-hand side is over the right-hand side in equations (1) and (2).

Odds ratios. The *odds ratio (OR)* is probably the most widely used index:[3]

$$I_O(M) = \begin{cases} \dfrac{ad}{bc} & \text{if } b \neq 0 \text{ and } c \neq 0, \\ +\infty & \text{if } bc = 0. \end{cases}$$

The index ranges from 0 to $+\infty$. A few monotonic transformations of the odds ratio are used in the literature. They all yield the same preorder of assortativeness. *Log odds ratio* (*LOR*) is $I_o(M) \equiv \ln I_O(M)$. *Yule's Q* or *coefficient of association* (Yule, 1900) is $I_Q(M) \equiv \frac{I_O(M)-1}{I_O(M)+1}$. This is +1 when PAM, −1 when NAM, and 0 when random matching (uncorrelated). *Yule's Y* or *coefficient of colligation* (Yule, 1912) is $I_Y(M) \equiv \frac{\sqrt{I_O(M)}-1}{\sqrt{I_O(M)}+1}$.

Likelihood ratios. *Type-specific likelihood ratio*, used for instance by Eika, Mogstad, and Zafar (2019), measures marital sorting between men and women of the same type by the ratio of the actual probability of matching relative to what would occur at random:[4]

$$I_{L1}(M) \equiv \frac{\text{observed } \#\theta_1\theta_1}{\text{random baseline}} = \frac{a}{|M|} \Big/ \frac{a+b}{|M|}\frac{a+c}{|M|} = \frac{a(a+b+c+d)}{(a+b)(a+c)}.$$

The likelihood ratio based on θ_1 compares the realized mass of high-type pairs with the benchmark of the hypothetical mass of high-type pairs under random matching. The likeli-

[3]The odds ratio is popular in the demographic literature, as it can be directly derived from the log-linear approach; see, for instance Mare (2001), Mare and Schwartz (2005), and Bouchet-Valat (2014). In economics, it was used by Siow (2015) ("local odds ratio"), Chiappori et al. (2020), Chiappori, Salanié, and Weiss (2017), and Ciscato and Weber (2020), among many others.

[4]Ciscato, Galichon, and Goussé (2020) call this the homogamy rate. It is used to study sorting on career ambition (Almar et al., 2023) and on childhood family income percentile (Binder et al., 2023).

hood ratio based on θ_2 is analogously defined:

$$I_{L2}(M) \equiv \frac{\text{observed } \#\theta_2\theta_2}{\text{random baseline}} = \frac{d}{|M|} \Big/ \frac{d+b}{|M|}\frac{d+c}{|M|} = \frac{d(a+b+c+d)}{(d+b)(d+c)}.$$

One issue with the type-dependent likelihood ratio is that it requires the choice of a type as a benchmark; in other words, it fails *type invariance*, which we will define later on. Empirically, the likelihood ratio based on different types yields opposite conclusions on the direction of change in assortative matching (Zhang, 2024).

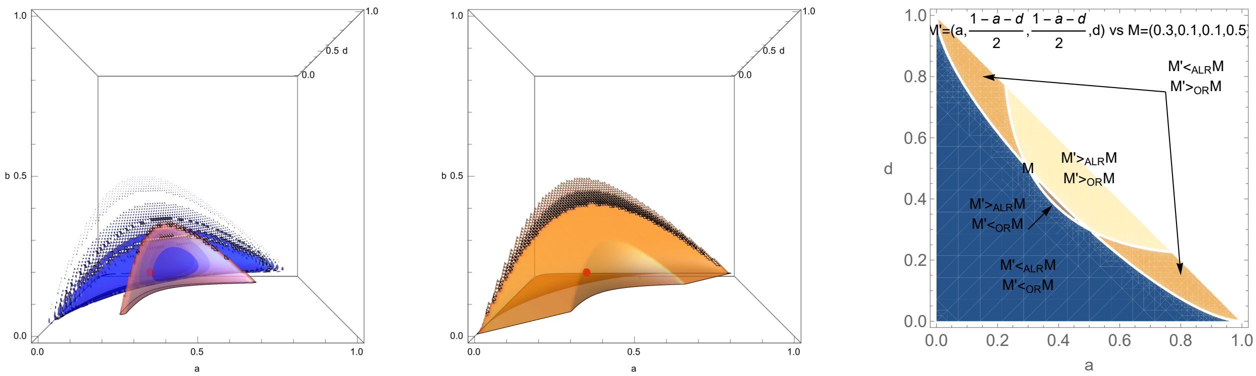
The *aggregate likelihood ratio* is a weighted average of the type-specific likelihood ratios, in which the weight on each type-specific likelihood ratio is the expected mass of pairs of like types under random matching.

$$\begin{aligned} I_L(M) &\equiv \frac{(a+b)(a+c)}{(a+b)(a+c)+(d+b)(d+c)} I_{L1}(M) + \frac{(d+b)(d+c)}{(a+b)(a+c)+(d+b)(d+c)} I_{L2}(M) \\ &= \frac{\text{observed } \#\theta_1\theta_1 + \#\theta_2\theta_2}{\text{random baseline}} = \frac{a+d}{|M|} \Big/ \left(\frac{a+b}{|M|}\frac{a+c}{|M|} + \frac{d+b}{|M|}\frac{d+c}{|M|} \right). \end{aligned}$$

Simplified, this is the ratio between the observed mass of couples of all like types and the counterfactual mass of pairs if individuals matched randomly.

Differences between the odds ratio and likelihood ratio. Eika, Mogstad, and Zafar (2019) use the likelihood ratios to find that educational assortativeness in the United States has increased recently, and Chiappori, Costa Dias, and Meghir (2024) use the odds ratio to find the opposite result. When will the two measures lead to different conclusions? Figure 1 provides an illustration. Fix a reference market, $M = (a, b, c, d) = (0.3, 0.1, 0.1, 0.5)$. Figure 1a shows what we call *iso-assortative surfaces* in the (a, b, d) three-dimensional space: The red iso-assortative surface collects the markets $(a, b, 1-a-b-d, d)$ that are equally assortative to M according to the aggregate likelihood ratio, and the blue iso-assortative surface collects the markets that are equally assortative to M according to the odds ratio. Because the iso-assortative surfaces are different, the sets of markets that are judged to be more assortative

Figure 1: Iso-assortative curves and conflicting conclusions



(a) Iso-assortative surface

(b) Conflicting region

(c) Iso-assortative curves and conflicting region

than M by the two measures will differ. The orange region in Figure 1b consists of the markets that are more assortative than M under the aggregate likelihood ratio but are less assortative than M under the odds ratio. Figure 1c illustrates *iso-assortative curves* and conflicting regions between the aggregate likelihood ratio and the odds ratio in a plane, by restricting the attention to markets in which $b = c$. Mathematically speaking, Figure 1c illustrates the sliced plane of $b = (1 - a - d)/2$ in the (a, b, d) space of Figures 1a and 1b. The iso-assortative surfaces or curves can be drawn for any measure, and can be used to compare any pairs of measures.

Normalized trace. A simple index that turns out to have nice properties is what we call the *normalized trace*.[5] It equals 1 if the matching is maximal PAM, equals 0 if the matching is minimal PAM, and equals the proportion of like types in the market otherwise:

$$I_{tr}(M) = \begin{cases} 1 & \text{if } bc = 0, \\ \frac{a+d}{a+b+c+d} \in (0,1) & \text{if } abcd \neq 0, \\ 0 & \text{if } ad = 0. \end{cases}$$

[5] Using the normalized trace, Cheremukhin, Restrepo-Echavarria, and Tutino (2024) study marital sorting in the US and Li (2024) and Li and Derdenger (2024) study matching between students and colleges.

Minimum distance. In the *minimum distance* approach of Fernández and Rogerson (2001) and Abbott et al. (2019), one constructs the convex combination of two extreme cases—random matching and maximum PAM—that minimizes the distance with the matching under consideration, and defines the weight of the maximum PAM component as the index. In two-type markets, it coincides with the *perfect-random normalization* of Liu and Lu (2006) and Shen (2020) in two-type markets, and is equal to

$$I_{MD}(M) = \frac{ad - bc}{(a + \min\{b, c\})(d + \min\{b, c\})}.$$

Correlation. Another natural index is the correlation between wife's and husband's educations, each considered as a Bernoulli random variable taking the value θ_1 with probability $\frac{a+b}{|M|}$ (resp., $\frac{a+c}{|M|}$) and θ_2 with probability $\frac{c+d}{|M|}$ (resp., $\frac{b+d}{|M|}$). This has been used in various contributions (e.g., Greenwood, Guner, and Knowles, 2003; Greenwood et al., 2014), either explicitly or through a linear regression framework.

$$I_{Corr}(M) = \frac{ad - bc}{\sqrt{(a+b)(c+d)(a+c)(b+d)}}.$$

One can readily check that, in this 2×2 case, the correlation index also coincides with Spearman's rank correlation, which exploits the natural ranking of education levels (C > HS). Equivalently, one can consider the χ^2 index, which is $\chi^2(a, b, c, d) = [I_{Corr}(a, b, c, d)]^2$.

3 Axioms of invariance, monotonicity, and homogamy

We introduce three sets of basic axioms/properties a measure should satisfy. They describe (i) when two matchings are equally assortative, (ii) when one matching is more assortative than another, and (iii) what matchings are the most assortative. For generality, whenever possible, we define axioms that a preorder—rather than an index—satisfies. We say that total preorder $\succeq$ is *induced by* index I if for all matchings M and M', $I(M) > I(M') \Leftrightarrow M \succ M'$

and $I(M) = I(M') \Leftrightarrow M \sim M'$. Two indices I and I' are *preorder-equivalent* if the total preorders $\succeq_I$ and $\succeq_{I'}$ they induce are equivalent. An index is said to satisfy an axiom if the total preorder induced by the index satisfies it.

3.1 Invariance axioms (INV)

We start with axioms that define two equally assortative markets. First, when a matching market is doubled or scaled up or down by a poisitive constant without changing the relative composition, the assortativeness should not be evaluated as changed.

> **[ScInv] Scale Invariance.** *For all M and $\lambda > 0$, $M \sim \lambda \cdot M$.*

Note that given ScInv, we can transform M into a contingency table by dividing each cell by $\lambda = |M|$. Next, the assortativeness should not be affected if we change the sides of the market. Effectively, we switch the masses of high-low (b) and low-high (c) pairs.

> **[SiInv] Side Invariance.** *The market is equally assortative when sides are switched:*
>
> $$\begin{array}{c|c} a & b \\ \hline c & d \end{array} \sim \begin{array}{c|c} a & c \\ \hline b & d \end{array}.$$

Neither should switching the types affect the assortativeness of the market. Swap the labels of the two types is essentially swapping high-high (a) and low-low (d) pairs and swapping high-low (b) and low-high (c) pairs.

> **[TInv] Type Invariance.** *The market is equally assortative when types are switched:*
>
> $$\begin{array}{c|c} a & b \\ \hline c & d \end{array} \sim \begin{array}{c|c} d & c \\ \hline b & a \end{array}.$$

These are basic axioms an appropriate measure should satisfy, so we refer to all three together as *invariance axioms (INV)*. While type-specific likelihood ratio fail type invariance, all aforementioned measures satisfy all three axioms of invariance (see summary in Table 1).

3.2 Monotonicity axioms

Next, we define notions of monotonicity that specify when one market is more assortative than another. We define two notions of monotonicity, one based on comparisons of markets of the same marginal distributions and the other based on comparisons of markets of different marginals. First, consider an alternative notion of monotonicity that compares markets with the same marginal distributions of men and women. Markets that share the same marginal distributions as (a, b, c, d) are $(a + \epsilon, b - \epsilon, c - \epsilon, d + \epsilon)$—essentially, a one-parameter family.

[MMon] Marginal Monotonicity. *Consider two markets M and M' with the same marginal distributions (i.e., $a + c = a' + c'$, $a + b = a' + b'$, $d + b = d' + b'$, and $d + c = d' + c'$). $M \succ M'$ if and only if $a > a'$ (equivalently, $b < b'$, $c < c'$, or $d > d'$).*

All aforementioned measures satisfy MMon.

Next, we compare markets that differ in marginal distributions, but differ in the minimal way: by one cell. For markets that maximally assortative, when pairs of like types (i.e., the terms on the diagonal of the matrix) increase, the matching becomes more assortative.

[DMon] Diagonal Monotonicity. *For all $M \gg 0$ and $\epsilon > 0$,*

$$\begin{array}{c|c} a+\epsilon & b \\ \hline c & d \end{array} \succ \begin{array}{c|c} a & b \\ \hline c & d \end{array} \text{ and } \begin{array}{c|c} a & b \\ \hline c & d+\epsilon \end{array} \succ \begin{array}{c|c} a & b \\ \hline c & d \end{array}.$$

We analogously define off-diagonal monotonicity: When pairs of unlike types (i.e., the terms off the diagonal of the matrix) increase, the matching becomes less assortative.

[ODMon] Off-Diagonal Monotonicity. *For all $M \gg 0$ and $\epsilon > 0$,*

$$\begin{array}{c|c} a & b \\ \hline c & d \end{array} \succ \begin{array}{c|c} a & b+\epsilon \\ \hline c & d \end{array} \text{ and } \begin{array}{c|c} a & b \\ \hline c & d \end{array} \succ \begin{array}{c|c} a & b \\ \hline c+\epsilon & d \end{array}.$$

Claim 1. *DMon and ODMon imply MMon.*

Proof of Claim 1. Suppose $M = (a, b, c, d)$ and $M' = (a', b', c', d')$ have the same marginal distributions, and suppose $a > a'$. Market M' can be represented as

$$M' = \begin{array}{c|c} a-(a-a') & b+(a-a') \\ \hline c+(a-a') & d-(a-a') \end{array} \prec \begin{array}{c|c} a & b+(a-a') \\ \hline c+(a-a') & d \end{array} \prec \begin{array}{c|c} a & b \\ \hline c & d \end{array} = M,$$

where the first $\prec$ follows from DMon and the second $\prec$ follows from ODMon. □

Conversely, however, MMon does not necessarily imply DMon or ODMon, because MMon solely specifies relations for markets with the same marginal distributions and by itself has no implications for markets with different marginal distributions.

Let us examine whether the aforementioned measures satisfy these monotonicity axioms. The type-specific likelihood ratio satisfies ODMon but fails DMon: $I_{L1}(1, 1, 1, 6) = 2.25 > I_{L1}(2, 1, 1, 6) \approx 2.22 > I_{L1}(6, 1, 1, 6) \approx 1.74$. To investigate further, we can take the partial derivative of I_{L1} with respect to a. DMon tends to fail when one of the cells is relatively small. Aggregate likelihood ratio satisfies ODMon but fails DMon:

$$\frac{\partial I_L}{\partial a} = \frac{(a+d)(d-a)(b+c)}{[(a+b)(a+c)+(d+b)(d+c)]^2} < 0 \text{ when } a > d.$$

Other aforementioned measures satisfy DMon and ODMon.

3.3 Homogamy axioms

A natural requirement is that any maximally positive-assortative matching should be a maximal element for preorder $\succeq$, which we follow Robbins (2009) and call it the *maximum homogamy (HMax)* property.

> **[HMax] Maximum Homogamy.** *For any M, $(a, b, 0, d) \succeq M$.*

A less restrictive condition suggests that fully positive-assortative matching should be perceived as the most positive-assortative; we call this the *full homogamy (HFull)* property (or weak HMax in Chiappori, Costa Dias, and Meghir (2024)).

> **[HFull] Full Homogamy.** *For any M, $(a, 0, 0, d) \succeq M$.*

Note that HMax is a stricter condition than HFull, because it requires not only markets of the form $(a, 0, 0, d)$ be maximal elements, but also markets of the form $(a, 0, c, d)$ or $(a, b, 0, d)$.

One can readily check that the odds ratio satisfies HFull and HMax, since it becomes infinite when $b = 0$ or $c = 0$. By construction, we have that $I_{MD}(M) \leq 1$ for all M and $I_{MD}(a, b, 0, d) = 1$. Therefore minimum distance satisfies HMax. The correlation index obviously satisfies HFull (since the correlation is then equal to 1). However, it does not satisfy HMax. For instance, $I_{Corr}(45, 5, 5, 45) = .8 > .41 = I_{Corr}(40, 40, 0, 20)$.

Likelihood ratios fail HMax and HFull: Compare markets $M = (5, 5, 5, 85)$ and $M' = (50, 0, 0, 50)$ corresponding, for instance, to two different cohorts in the same economy. The distribution of education is independent of gender in both M and M', but the number of educated people has increased from 10% to 50% between M and M'. Cohort M exhibits PAM in the usual sense (more people on the diagonal than would obtain under random matching); yet, 50% of educated people marry an uneducated spouse. Cohort M' displays perfect sorting, with all educated individuals marrying together. The college-based likelihood ratio yields $I_{L1}(M) = 5$ and $I_{L1}(M') = 2$: Assortativeness has *decreased* from M to M'.

3.4 Summary

Table 1 summarizes whether the measures satisfy the basic equivalence and monotonicity axioms. Type-specific likelihood ratio does not satisfy type symmetry, because relabeling the types would change the measure. Likelihood ratios do not satisfy DMon, HMax, and HFull. Correlation index fails HMax.

Table 1: Do the measures satisfy the invariance, monotonicity, and homogamy axioms?

	Invariance axioms			Monotonicity axioms			Homogamy axioms	
	ScInv	SiInv	TInv	MMon	DMon	ODMon	HMax	HFull
Odds ratio	✓	✓	✓	✓	✓	✓	✓	✓
Type-specific likelihood ratio	✓	✓	X	✓	X	✓	X	X
Aggregate likelihood ratio	✓	✓	✓	✓	X	✓	X	X
Normalized trace	✓	✓	✓	✓	✓	✓	✓	✓
Minimum distance	✓	✓	✓	✓	✓	✓	✓	✓
Correlation	✓	✓	✓	✓	✓	✓	X	✓

The odds ratio, normalized trace, and minimum distance satisfy all the basic axioms. That begs the question, what would be axioms that can uniquely characterize these measures? We answer this question next.

4 Characterization results

Table 1 shows that, except for the type-specific likelihood ratio, all of the measures listed satisfy the invariance axioms and at least one notion of monotonicity: marginal monotonicity. Hence, the basic axioms are not sufficient to distinguish the measures and provide a definitive answer regarding what measure to use.

We need additional axioms to distinguish and characterize the different measures. We will introduce what we call *characterization axioms*, such that a measure will be the unique one that satisfies both the basic axioms and an additional axiom, which essentially highlights the special property of the measure.

4.1 Odds ratio

4.1.1 Characterization axiom

We now provide the characterization result regarding marginal independence (Edwards, 1963) and the odds ratio (CCM). An preorder satisfies marginal independence if multiplying any row or any column of any market does not change the assortativeness order.

[MInd] Marginal Independence (Edwards, 1963). *For all $M \gg 0$ and $\lambda > 0$,*

$$\begin{array}{c|c} a & b \\ \hline c & d \end{array} \sim \begin{array}{c|c} \lambda a & \lambda b \\ \hline c & d \end{array} \sim \begin{array}{c|c} a & b \\ \hline \lambda c & \lambda d \end{array} \sim \begin{array}{c|c} \lambda a & b \\ \hline \lambda c & d \end{array} \sim \begin{array}{c|c} a & \lambda b \\ \hline c & \lambda d \end{array}.$$

Note that MInd is an ordinal property. Nonetheless, MInd is a strong condition. MInd implies ScInv, TInv, and SiInv. MInd and MMon together imply DMon and ODMon.

Claim 2. *MInd and MMon imply DMon and ODMon.*

In fact, more strongly, MMon, DMon, and ODMon are equivalent under MInd:

Claim 3. *Suppose a measure satisfies MInd. It satisfies DMon if and only if it satisfies ODMon if and only if it satisfies MMon.*

Note that an index and its monotonic transformation are preorder-equivalent.

Theorem 1. *An index satisfies MMon, MInd, and HMax if and only if it is a monotonic transformation of the odds ratio.*

Note that the sole role of HMax is to specify the maximum PAM to be the maximal element of the preorder.

It is interesting to refer to an older statistical literature that discusses the properties of measures of association in the case of paired attributes (i.e., in our case, husband's and wife's education). The property posed by Edwards (1963) states that the association should not be "*influenced by the relative sizes of the marginal totals*" (p. 110). That is, the measure should not change if one starts from a Table M and doubles the number of couples where the man is educated (while keeping unchanged the ratio of educated versus uneducated wives of educated men). Formally, for any (a, b, c, d) and any positive λ, it should hold that:

$$I(\lambda a, \lambda b, c, d) = I(\lambda a, b, \lambda c, d) = I(a, \lambda b, c, \lambda d) = I(a, b, \lambda c, \lambda d) = I(a, b, c, d).$$

Edwards (1963) justifies this property by posing that the measure must only be a function of *the proportion of educated women whose husband is educated and the proportion of uneducated women whose husband is educated* (and conversely), so that any population change that keeps these proportions constant should not affect the index. This imposes, in particular, that global changes in the marginal distributions, for instance a global increase in the number of educated women, should not systematically impact the index; only changes in the odds of marrying different types of spouses should matter. The condition was later generalized by Altham (1970) to the $n \times n$ case.

Among the indices just reviewed, only one—the odds ratio—satisfies Edwards's marginal invariance. It is interesting to consider how the other indices violate this requirement. Consider market $M_\lambda = (\lambda a, \lambda b, c, d)$ with $ad > bc$ (PAM) and $\lambda \geq 1$. Suppose λ increases. The minimum distance index *increases* since $\partial I_{MD}/\partial \lambda > 0$. The correlation and Spearman correlation may increase or decrease depending on parameters. The type-specific likelihood ratio *decreases* since $\partial I_L/\partial \lambda < 0$.

4.1.2 Structural interpretation

It is important to note that among the various indices, the odds ratio has a known structural interpretation. Specifically, assume that the observed matching behavior constitutes the stable equilibrium of a frictionless matching model under transferable utility. Assume, furthermore, that the surplus generated by a match between woman x belonging to category θ_i and man y belonging to category θ_j takes the separable form

$$s(x, y) = Z^{\theta_i \theta_j} + \alpha_x^{\theta_j} + \beta_y^{\theta_i}, \tag{3}$$

where Z is a deterministic component depending only on individual educations and the α's and β's are random shocks reflecting unobserved heterogeneity among individuals.[6] It is now well known (Graham, 2011; Chiappori, 2017) that, keeping constant the distribution of the shocks, assortativeness is related to the supermodularity of the matrix $Z^{\theta_i \theta_j}$, i.e., in the two-type case, to the sign of the *supermodular core* $Z^{\theta_1\theta_1} + Z^{\theta_2\theta_2} - Z^{\theta_1\theta_2} - Z^{\theta_2\theta_1}$. More importantly, if, following the seminal contribution by Choo and Siow (2006), one assumes that the random shocks follow type-1 extreme value distributions (the so-called Separable Extreme Value or SEV model), then the supermodular core equals twice the odds ratio I_O.

This structural interpretation is especially useful for disentangling possible changes in the value of different matches from the mechanical effect of variations in the marginal distributions of education among individuals: "structural changes," here, can only affect either the matrix Z or the distributions of random shocks. It is also useful for constructing counterfactual simulations, since the same structure can be applied to different distributions of education by genders, using standard techniques to solve for the stable equilibrium of the corresponding matching game.[7]

Two remarks can be made at this point. First, as clearly pointed out by Choo and Siow (2006) in their original contribution as well as by the subsequent literature (Galichon and Salanié, 2022; Chiappori and Salanié, 2016), this structural model can be identified (in the econometric sense) from matching patterns, but only under strong parametric restrictions. For instance, the initial Choo and Siow (2006) framework, which generates the odds ratio as an estimator of the supermodularity of the surplus, is *exactly* identified under the assump-

[6]Transferable-utility models satisfying condition (3) are referred to as separable models in the literature.
[7]See Chiappori et al. (2020) and Chiappori, Costa Dias, and Meghir (2020) for applications.

tion that the random shocks α and β follow a type-1 extreme value distribution. Indeed, the consensus among specialists is that, to be identified from the sole observation of matching patterns, any structural model would require strong parametric assumption regarding the distribution of random variables. It follows that, in general, the ranking (in terms of assortativeness, i.e., supermodularity of the deterministic surplus) may vary with the specific assumptions made on the distribution of the stochastic factors.

Several routes can be followed to overcome this limitation. One may, following Chiappori, Salanié, and Weiss (2017), consider repeated cross sections and impose restrictions on how the structural components change over time. Alternatively, Chiappori, Dias, and Meghir (2018) argue that direct observation of post-marital behavior provides additional information on the surplus (since the latter, under TU, is simply the sum of individual utilities and can be estimated from the observation of the demand functions); this information can be used in particular to relax parametric assumptions made on the stochastic components of the model. In a recent contribution, Gualdani and Sinha (2022) also show how partial (i.e., set) identification may obtain using general assumptions on the distribution of stochastic shocks (for instance, independence of taste shocks from covariates and quantile or symmetry restrictions).

Second and more importantly, while robust examples can be given where an assortativeness ranking is reversed when the assumptions regarding stochastic distributions are changed, one can nevertheless define conditions under which the ranking will be the same for any separable TU model, *irrespective of the stochastic distribution*, provided the latter satisfy some basic properties (such as independence). Specifically, Chiappori, Costa Dias, and Meghir (2020) show that if two PAM M and M' are such that:

$$\frac{a}{a+b} \geq \frac{a'}{a'+b'}, \quad \frac{a}{a+c} \geq \frac{a'}{a'+c'}, \quad \frac{d}{b+d} \geq \frac{d'}{b'+d'} \quad \text{and} \quad \frac{d}{c+d} \geq \frac{d'}{c'+d'}, \tag{4}$$

then irrespective of the stochastic distributions of α and β (provided they are independent from each other and from the observed characteristics), the deterministic surplus corresponding to M will be more supermodular than M'; this is the *Generalized Separable (GS)* criterion in Chiappori, Costa Dias, and Meghir (2020). As a result, for any stochastic distributions of α's and β's, while the numerical value of the supermodular cores will depend on the choice of distributions, the structural model *will always rank M above M' in terms of assortativeness.*

In other words, one can define a preorder $\succeq$ that is totally robust to changes in distributional assumptions. The price to pay for this generalization, however, is that the preorder is no longer complete: if some inequalities in (4) are satisfied while others are violated, the matchings simply cannot be compared.

4.2 Other characterization results

The next set of characterization axioms will endow the measures with cardinal interpretations. Namely, the assortativeness measure of a market will be a weighted sum or average of the measures of submarkets decomposed from the original market. Depending on the weights used, we will have different decomposability axioms that correspond to different measures.

4.2.1 Population decomposability and normalized trace

Consider the following axiom in which the weight to average is the population size of submarkets.

[PDec] Population Decomposability. For all markets $M \gg 0$ and $M' \gg 0$,

$$I(M + M') = \frac{|M|}{|M|+|M'|} I(M) + \frac{|M'|}{|M|+|M'|} I(M').$$

While, as explained above, MMon and the invariance axioms cannot imply DMon and ODMon. However, MMon and PDec and the invariance axioms (ScInv and TInv, to be exact) imply DMon and ODMon.

Claim 4. *ScInv, TInv, MMon, and PDec imply DMon and ODMon.*

We show in the theorem below that RPDec and PDec are characterization axioms for normalized trace. That is, normalized trace is the unique index, up to linear transformation, that satisfies PDec and the basic invariance and monotonicity axioms. Because Claims 1 and 4 combine to imply the equivalence of the two notions of monotonicity—DMon+ODMon and MMon—given PDec, the characterization result holds for both notions of monotonicity.

Theorem 2. *An index satisfies INV, MMon (or DMon and ODMon), HMax, and PDec if and only if it is a linear transformation of normalized trace.*

The index is uniquely determined if the range is specified: When the range is $[0, 1]$, the unique index is normalized trace.

4.2.2 Random decomposability and aggregate likelihood ratio

The aggregate likelihood ratio is characterized by a decomposability axiom in which the weight to sum is the expected mass of randomly matched pairs of like types.

[RDec] Random Decomposability. For all markets $M \gg 0$ and $M' \gg 0$,

$$I(M+M') = \frac{r(M)}{r(M+M')} I(M) + \frac{r(M')}{r(M+M')} I(M'),$$

where $r(M)$ indicates the expected mass of pairs of like types under random matching in market M:

$$r(M) \equiv \frac{a+b}{|M|}\frac{a+c}{|M|}|M| + \frac{d+b}{|M|}\frac{d+c}{|M|}|M| = \frac{(a+b)(a+c)+(d+b)(d+c)}{a+b+c+d}.$$

Theorem 3. *An index satisfies INV, MMon, and RDec if and only if it is proportional to the aggregate likelihood ratio.*

We make two comments. First, note that ALR does not satisfy DMon, ODMon, or HMax, so we do not have a characterization result that links those axioms and ALR. Second, note that the class of measures that satisfy the axioms in Theorem 3 must be a constant multiple of ALR. The class has to be a constant multiple of ALR because the weights $r(M)/r(M+M')$ and $r(M')/r(M+M')$ do not necessarily add up to be 1. Nonetheless, this characterization result gives ALR a cardinal interpretation.

To address the second comment that the weights do not add up to one, consider an axiom that involves weighted averages that add up to 1 and compares markets with proportional marginal distributions.

[MRDec] Marginal Random Decomposability. For all $M \gg 0$ and $M' \gg 0$ such that $M/|M|$ and $M'/|M'|$ have the same marginal distributions,

$$I(M+M') = \frac{r(M)}{r(M)+r(M')} I(M) + \frac{r(M')}{r(M)+r(M')} I(M').$$

ALR satisfies this axiom, but it is not the unique measure that does so. For example, the normalized trace—and as a result, any linear combination of ALR and NT—also satisfies this axiom.

5 Example: Educational homogamy in the US

As an illustration, consider the evolution of homogamy by education in the US over the recent decades. Our goal is to compare the answer given by various indices to the same basic question, namely: Did educational homogamy increase between different cohorts? Gihleb and Lang (2020) consider this issue from a general perspective, using a variety of indices (correlation, rank correlation, Goodman and Kruskal's γ, Kendall's τ adjusted and not adjusted for ties). A difference from our approach is that they consider changes affecting the entire distribution of education and many classes, whereas we focus on the top of the education distribution and consider two broad education "classes," thus fitting our 2×2 framework. Taking into account Gihleb and Lang's (2020) findings that different groupings of educational categories can change the results, we use three different splits: college and above versus some college, college and above versus less than college, and some college and above versus HS and below. We consider various cohorts, from individuals born in the 1930s to those born in the 1970s. In each case, we compute the difference between the values obtained for different cohorts for the indices described above (odds ratio, χ^2, minimum distance, and likelihood ratio).

Table 2: Marital assortativeness at the top of the distribution of education – comparing cohorts born between 1930-39 and 1970-75

	College vs Some College				College vs less than College				Some College vs HS and below			
	Odds ratio	χ^2	Min dist	Ll ratio	Odds ratio	χ^2	Min dist	Ll ratio	Odds ratio	χ^2	Min dist	Ll ratio
1940 vs 30	0.189	0.028	0.019	0.023	-0.259	0.032	-0.021	-1.125	-0.132	0.018	-0.017	-0.610
adj p-val	0.000	0.000	0.355	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.005	0.000
1950 vs 30	0.366	0.069	0.020	0.118	-0.521	0.020	-0.106	-1.469	-0.430	-0.026	-0.119	-0.904
adj p-val	0.000	0.000	0.137	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1960 vs 30	0.534	0.098	0.010	0.126	-0.486	0.044	-0.148	-1.619	-0.380	-0.021	-0.116	-0.968
adj p-val	0.000	0.000	0.477	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 30	0.580	0.097	0.022	0.071	-0.459	0.060	-0.080	-1.889	-0.137	0.006	-0.048	-1.030
adj p-val	0.000	0.000	0.123	0.000	0.000	0.000	0.000	0.000	0.000	0.272	0.000	0.000
1950 vs 40	0.178	0.041	0.002	0.094	-0.262	-0.013	-0.085	-0.344	-0.298	-0.044	-0.102	-0.294
adj p-val	0.000	0.000	0.857	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1960 vs 40	0.345	0.071	-0.009	0.102	-0.227	0.011	-0.127	-0.495	-0.248	-0.039	-0.099	-0.358
adj p-val	0.000	0.000	0.337	0.000	0.000	0.009	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 40	0.391	0.070	0.003	0.048	-0.201	0.028	-0.059	-0.765	-0.005	-0.012	-0.031	-0.419
adj p-val	0.000	0.000	0.778	0.000	0.000	0.000	0.000	0.000	0.845	0.014	0.000	0.000
1960 vs 50	0.167	0.029	-0.010	0.008	0.035	0.024	-0.042	-0.150	0.050	0.005	0.004	-0.064
adj p-val	0.000	0.000	0.352	0.108	0.100	0.000	0.000	0.000	0.029	0.182	0.697	0.000
1970 vs 50	0.214	0.029	0.001	-0.046	0.061	0.040	0.026	-0.421	0.293	0.032	0.071	-0.125
adj p-val	0.000	0.000	0.878	0.000	0.040	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 60	0.047	-0.001	0.012	-0.054	0.026	0.016	0.068	-0.270	0.243	0.027	0.068	-0.062
adj p-val	0.403	0.889	0.148	0.000	0.565	0.001	0.000	0.000	0.000	0.000	0.000	0.000

Notes: Columns identify each of the 2×2 sorting matrices. In each panel, row 1 shows estimates of the difference in the respective index between the latest and earliest cohorts; row 2 shows p-values for 2-sided significance testing adjusted for multiple hypothesis using the stepdown method for the three outcomes on the row (Romano and Wolf, 2005; Romano, Shaikh, and Wolf, 2008; Romano and Wolf, 2016). *Data source*: March extract of the US Current Population Survey, subsample of married individuals observed when aged 35-44 and born in 1930-59 to 1970-75.

Table 2 summarizes the results. For the oldest cohorts, born in the 1950s or earlier, estimates show some mixed patterns. The indices indicate unanimously that homogamy in marriage *increased* at the top of the education distribution over this period (first four columns in Table) but that, at the same time, it *decreased* when broader education groups are considered (middle and final sets of four columns in Table 2). These results are consistent with the Gihleb and Lang (2020) finding that the level of aggregation matters for what can be said about the direction of change in education homogamy in marriage.

From a theoretical perspective, of particular interest is the difference between the generation born in the 1950s and that born in the 1970s (second to last row of the the table). It is now widely agreed that the return to investments in human capital increased substantially during the 80s and later. Educational and marital choices by the older generation were mostly made before that period; on the contrary, individuals born in the 70s, when choosing both their education level and their spouse, were fully aware of the new context. Some authors have argued that, as a result, homogamy should increase between these two cohorts (for instance Chiappori, 2017; Chiappori, Salanié, and Weiss, 2017). This is indeed the conclusion obtained when considering three of the four indices—namely, odds ratio, χ^2 and minimum distance—for all levels of aggregation in education classes considered here. On the contrary, the likelihood ratio concludes that homogamy has decreased over the period (and actually over all periods under consideration for the 'College versus less than College' and 'Some College versus HS and below' splits)). More broadly, all indices but the likelihood ratio consistently show that the direction of change in homogamy by education started to change in the 1950s, towards increasing assortativeness. Given the previous analysis, a possible interpretation is that this difference is related to the global increase in the proportion of college graduates over the period, particularly among women; as discussed above, such an increase tends to mechanically reduce the likelihood ratio.

Finally, it is important to note that the Generalized Separable conditions (4) are never simultaneously satisfied in our data (although in one case the violations are not significant), as can be seen from Table 3. This implies that one can always find specific distributions for the random terms such that the ranking would be reversed, a conclusion similar to that of Gualdani and Sinha (2022).

6 Multiple types

There are 14 different education categories in the census, for example. It is worthwhile to consider how to compare assortativeness in markets of more than two types (if we must compare). Consider market $M = (\mu_{ij})_{i,j\in\{1,\cdots,n\}}$ for $n \geq 3$.

6.1 Extensions from two-type markets

We have some straightforward extensions from two-type markets. The aggregate likelihood ratio and normalized trace can be naturally extended to multi-type markets, but the odds ratio does not have a natural extension. *Normalized trace for multi-type markets* is

$$I_{tr}(M) = \begin{cases} 1 & \text{if } \mu_{ij}\mu_{ji} = 0 \ \forall i \text{ and } j \neq i \\ 0 & \text{if } \mathrm{tr}(M) \equiv \sum_{i=1}^{n} \mu_{ii} = 0 \\ \mathrm{tr}(M)/|M| & \text{otherwise.} \end{cases}$$

Aggregate likelihood ratio for multi-type markets is

$$I_L(M) \equiv \frac{tr(M)/|M|}{\sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{n}\mu_{ij}}{|M|}\right)\left(\frac{\sum_{j=1}^{n}\mu_{ji}}{|M|}\right)} = \frac{|M|(\sum_{i=1}^{n}\mu_{ii})}{\sum_{i=1}^{n}(\sum_{j=1}^{n}\mu_{ij})(\sum_{j=1}^{n}\mu_{ji})}.$$

Theorems 2 and 3 can be naturally extended so that normalized trace and the aggregate likelihood ratio are the unique indices that satisfy the sets of axioms (appropriately modified) in those theorems. Hence, normalized trace and the aggregate likelihood ratio can still be used to measure the assortativeness of multi-type markets. In addition, we can extend the measures to measure assortativeness in (i) markets with unmarried and (ii) one-sided homosexual markets. See Zhang (2024) for more detailed theoretical discussions and empirical analyses.

6.2 Robustness to categorization

Consider three types θ_i, θ_j, and θ_k. When two types θ_i and θ_j merge so that the three categories are partitioned to $\{\{i,j\}\{k\}\}$, the market given this categorization becomes

$$M|_{\{\{i,j\}\{k\}\}} = \begin{pmatrix} \mu_{ii} + \mu_{ij} + \mu_{ji} + \mu_{jj} & \mu_{ik} + \mu_{jk} \\ \mu_{ki} + \mu_{kj} & \mu_{kk} \end{pmatrix}.$$

We say that M is more assortative than M' if and only if $M|_{\{\{\{i,j\}\{k\}\}}$ is more assortative than $M|'_{\{\{i,j\}\{k\}\}}$ for all i and all $j \neq i$. For example, With normalized trace, for M and M' such that $|M| = |M'|$ and $\mathrm{tr}(M) = \mathrm{tr}(M')$, M is more assortative than M' if and only if $\mu_{12} + \mu_{21} \geq \mu'_{12} + \mu'_{21}$, $\mu_{13} + \mu_{31} \geq \mu'_{13} + \mu'_{31}$, and $\mu_{23} + \mu_{32} \geq \mu'_{23} + \mu'_{32}$.

Formally,

[RCat] Robustness to Categorization. Let $\mathcal{C}$ denote the collection of categorizations of types to be considered. Let $M|_C$ denote the market under categorization $C \in \mathcal{C}$. $M \succeq M'$ if and only if $M|_C \succeq M'|_C$ for any categorization $C \in \mathcal{C}$, and $M \succ M'$ if and only if $M|_C \succ M'|_C$ for any categorization $C \in \mathcal{C}$.

Theorem 4. *No total preorder satisfies MMon and RCat.*

A counterexample suffices for the claim. Although the counterexample is one of three-type markets, it suffices for the claim for markets with more than three types.

Proof of Theorem 4. Consider matchings

$$M = \begin{array}{c|c|c} 1/9 & 1/9 & 1/9 \\ \hline 1/9 & 1/9 & 1/9 \\ \hline 1/9 & 1/9 & 1/9 \end{array} \text{ and } M' = \begin{array}{c|c|c} 1/9-\epsilon & 1/9+\epsilon & 1/9 \\ \hline 1/9+\epsilon & 1/9 & 1/9-\epsilon \\ \hline 1/9 & 1/9-\epsilon & 1/9+\epsilon \end{array} = M + \begin{array}{c|c|c} -\epsilon & +\epsilon & 0 \\ \hline +\epsilon & 0 & -\epsilon \\ \hline 0 & -\epsilon & +\epsilon \end{array}.$$

When we group θ_1 and θ_2,

$$M|_{\{\{1,2\}\{3\}\}} = \begin{array}{c|c} 4/9 & 2/9 \\ \hline 2/9 & 1/9 \end{array} \text{ and } M'|_{\{\{1,2\}\{3\}\}} = \begin{array}{c|c} 4/9+\epsilon & 2/9-\epsilon \\ \hline 2/9-\epsilon & 1/9+\epsilon \end{array} = M|_{\{\{1,2\}\{3\}\}} + \begin{array}{c|c} +\epsilon & -\epsilon \\ \hline -\epsilon & +\epsilon \end{array}.$$

When we group θ_2 and θ_3,

$$M|_{\{\{1\}\{2,3\}\}} = \begin{array}{c|c} 1/9 & 2/9 \\ \hline 2/9 & 4/9 \end{array} \text{ and } M'|_{\{\{1\}\{2,3\}\}} = \begin{array}{c|c} 1/9-\epsilon & 2/9+\epsilon \\ \hline 2/9+\epsilon & 4/9-\epsilon \end{array} = M|_{\{\{1\}\{2,3\}\}} + \begin{array}{c|c} -\epsilon & +\epsilon \\ \hline +\epsilon & -\epsilon \end{array}.$$

By MMon,

$$M|_{\{\{1,2\}\{3\}\}} \prec M'|_{\{\{1,2\}\{3\}\}} \text{ and } M|_{\{\{1\}\{2,3\}\}} \succ M'|_{\{\{1\}\{2,3\}\}}.$$

Hence, there does not exist a total preorder that satisfies MMon and RCat. □

The impossibility result suggests that we must resort to partial orders to satisfy RCat. Existing partial orders such as supermodular stochastic order (Meyer and Strulovici, 2012, 2015) and positive quadrant dependence order (Anderson and Smith, 2024) are natural candidates. However, they only compare markets with the same marginals. Hence, further investigations to compare markets with differential marginal distributions are needed.

7 Concluding Remarks

It is relatively simple to estimate whether there is positive assortative matching in a stochastic marriage market along the dimensions of a characteristic such as education. However, measuring the extent to which such assortative matching differs between two economies or between two points in time for the same economy is challenging when the marginal distributions of the characteristics also change.

In this paper, we show that different measures (indices or preorders induces by these indices) may generate different conclusions regarding the evolution of educational homogamy over time. We first take an axiomatic approach to better understand the underlying properties of commonly used measures. Namely, we axiomatize the odds ratio, highlighting the marginal independence property. We discuss structural interpretation of the odds ratio. We also discuss the difficulties to compare assortativeness for markets with more than two types.

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A Omitted proofs and details

A.1 Omitted proofs for characterization results

Proof of Claim 2. Suppose $M = (a, b, c, d)$ and $M' = (a', b', c', d')$ have the same marginal distributions and $M \succ M'$. By MInd,

$$\begin{aligned}(a, b, c, d) &\sim (a, b, c\frac{c'}{c}, d\frac{c'}{c}) \sim (a, b\frac{c}{c'}\frac{d'}{d}, c', d\frac{c'}{c}\frac{c}{c'}\frac{d'}{d}) \sim (a\frac{b'}{b}\frac{c'}{c}\frac{d}{d'}, b\frac{b'}{b}\frac{c'}{c}\frac{d}{d'}\frac{c}{c'}\frac{d'}{d}, c', d') \\ &\sim (a'\frac{a}{a'}\frac{b'}{b}\frac{c'}{c}\frac{d}{d'}, b', c', d').\end{aligned}$$

Let $\delta \equiv \frac{a}{a'}\frac{b'}{b}\frac{c'}{c}\frac{d}{d'}$. By MMon, $a > a'$, $b < b'$, $c > c'$, and $d > d'$. Hence, $\delta > 1$. $(a\delta, b', c', d') \sim (a', b'/\delta, c', d') \sim (a', b', c'/\delta, d') \sim (a', b', c', d'\delta) \succ (a', b', c', d')$ for all a', b', c', d' implies DMon and ODMon. $\square$

Proof of Claim 3. Suppose preorder $\succeq$ satisfies MMon. Then we have for any δ,

$$(a, b, c, d) \succ (a - \delta, b + \delta, c + \delta, d + \delta).$$

By (repeatedly applying) MInd,

$$\begin{aligned}(a - \delta, b + \delta, c + \delta, d + \delta) &\sim (a, b + \delta, (c + \delta)\frac{a}{a - \delta}, d - \delta) \\ &\sim (a, b, (c + \delta)\frac{a}{a - \delta}, (d - \delta)\frac{b}{b + \delta}) \\ &\sim (a, b, c, d\frac{a - \delta}{a}\frac{b}{b + \delta}\frac{c}{c + \delta}\frac{d - \delta}{d}) \equiv (a, b, c, d\lambda(\delta)),\end{aligned}$$

where $\lambda(\delta)$ is strictly smaller than 1 for any $\delta > 0$ and is continuously decreasing in δ. With similar transformations, we have

$$\begin{aligned}
& (a-\delta, b+\delta, c+\delta, d+\delta) \\
\sim \quad & (a, b, c, d\lambda(\delta)) \sim (a\lambda(\delta), b, c, d) \sim (a, b/\lambda(\delta), c, d) \sim (a, b, c/\lambda(\delta), d).
\end{aligned}$$

Hence, for any $\delta > 0$, DMon holds:

$$(a, b, c, d) \succ (a, b, c, d\lambda(\delta)) \sim (\lambda(\delta)a, b, c, d),$$

and ODMon holds:

$$(a, b, c, d) \succ (a, b/\lambda(\delta), c, d) \sim (a, b, c/\lambda(\delta), d).$$

Reversely,

$$(a, b, c, d) \succ (a, b, c, d\lambda(\delta)) \sim (\lambda(\delta)a, b, c, d)$$

for any δ implies

$$(a, b, c, d) \succ (a, b/\lambda(\delta), c, d) \sim (a, b, c/\lambda(\delta), d)$$

for any δ, and

$$(a, b, c, d) \succ I(a-\delta, b+\delta, c+\delta, d-\delta)$$

for any δ, where MInd is repeatedly used again. □

Proof of Theorem 1. It is straightforward to check that the odds ratio and its monotonic transformation satisfy DMon, ODMon, and MInd.

It remains to show that there does not exist an index I that satisfies DMon, ODMon, and MInd, but is not order-equivalent to the odds ratio. Suppose by way of contradiction that such an index exists. Then, we must have for some $M = (a, b, c, d)$ and $M' = (a', b', c', d')$. One of the following four cases occurs: (i) $I(M) > I(M')$ and $Q(M) < Q(M')$, (ii) $I(M) <$

$I(M')$ and $Q(M) > Q(M')$, (iii) $I(M) = I(M')$ and $Q(M) \neq Q(M')$, and (iv) $I(M) \neq I(M')$ and $Q(M) = Q(M')$.

First, suppose that case (i) $I(M) > I(M')$ and $Q(M) < Q(M')$ occurs. Because $I(M) > I(M')$, by the implication of DM, $ad \neq 0$, and by the implication of ODM, $b'c' \neq 0$, and because $Q(M) < Q(M')$, similarly, $bc \neq 0$ and $a'd' \neq 0$. For each of the following steps, we invoke a part of MInd:

$$\begin{aligned}
I(a,b,c,d) &= I(\frac{b'}{b}a, \frac{b'}{b}b, c, d) \\
&= I(\frac{b'}{b}a\frac{a'}{a}\frac{b}{b'}, b', c\frac{a'}{a}\frac{b}{b'}, d) \\
&= I(a', b', c\frac{a'}{a}\frac{b}{b'} \cdot \frac{c'}{c}\frac{a}{a'}\frac{b'}{b}, d \cdot \frac{c'}{c}\frac{a}{a'}\frac{b'}{b}\frac{d'}{d'}) \\
&= I(a', b', c', d' \cdot \frac{ad}{bc} / \frac{a'd'}{b'c'}).
\end{aligned}$$

By premise,

$$I(a', b', c', d' \cdot \frac{ad}{bc} / \frac{a'd'}{b'c'}) > I(a', b', c', d').$$

By DM, this implies

$$\frac{ad}{bc} > \frac{a'd'}{b'c'}.$$

However, this implies $Q(M) > Q(M')$, which contradicts the premise that $Q(M) < Q(M')$.

For each of the four possibilities, if no cell of matrices M and M' is zero, using the same logic as above, a contradiction can be derived.

Suppose there is a cell that is zero. Suppose $I(M) = I(M')$. If $bc = 0$, then $I(M) = I(a, 0, 0, d) = I(M')$, then $b'c' = 0$. In this case, by DM, $Q(M) = Q(M')$. If $ad = 0$ instead, then $I(M) = I(0, b, c, 0) = I(M')$ implies $a'd' = 0$. In this case, by ODM, $Q(M) = Q(M')$. Hence, whenever there is a cell with zero in one of the matrices, $I(M) = I(M')$ and $Q(M) = Q(M')$, which prevents all four cases from happening. $\square$

Proof of Claim 4. Consider $M = (a, b, c, d) \gg 0$ and $M' = (a', b, c, d) \gg 0$, where $a' > a$.

We want to show that $I(M') > I(M)$. By TInv,

$$I(a, b, c, d) = I(d, c, b, a).$$

By PDec,

$$\frac{1}{2}I(a, b, c, d) + \frac{1}{2}I(d, c, b, a) = I\left(\frac{1}{2}(a + d), \frac{1}{2}(b + c), \frac{1}{2}(b + c), \frac{1}{2}(a + d)\right).$$

By ScInv,

$$I(a, b, c, d) = I\left(\frac{a + d}{a + b + c + d}, \frac{b + c}{a + b + c + d}, \frac{b + c}{a + b + c + d}, \frac{a + d}{a + b + c + d}\right).$$

By the same sequence of arguments by TInv, PDec, and ScInv,

$$I(a', b, c, d) = I\left(\frac{a' + d}{a' + b + c + d}, \frac{b + c}{a' + b + c + d}, \frac{b + c}{a' + b + c + d}, \frac{a' + d}{a' + b + c + d}\right).$$

Note that the two matrices on the right-hand side of the two equations above have the same marginals (each row or column sums to 1). By MMon, $a' > a$ implies

$$I(M) = I(a', b, c, d) > I(M') = I(a, b, c, d).$$

Hence, DMon is proved. ODMon can be shown analogously. □

Proof of Theorem 2. I first show that any index I that satisfies ScInv, TInv, SiInv, DMon, ODMon, and PDec is order-equivalent to—i.e., a monotonic transformation of—NT. Consider $M = (a, b, c, d) \gg 0$ and $M' = (a', b', c', d') \gg 0$. Here, we prove a lemma.

Lemma 1. *Any index that satisfies ScInv, TInv, SiInv, DMon, ODMon, and PDec is an increasing function of $a + d$ and a decreasing function of $b + c$.*

Proof of Lemma 1. Consider $M = (a, x, x, d)$, and consider $M_1 = (a/2, x - \lambda/2, \lambda/2, d/2)$

and $M_2 = (a/2, \lambda/2, x - \lambda/2, d/2)$ for some $\lambda \in (0, 2x)$. Note that $M_1 + M_2 = M$ and M_1 and M_2 have the same total mass. Hence, by PDec,

$$I(M) = \frac{1}{2} I(M_1) + \frac{1}{2}(M_2).$$

By SiInv, $I(M_1) = I(M_2)$. Hence, $I(M) = I(M_1) = I(M_2)$. By ScInv,

$$I(M_1) = I(2M_1) = I(a, 2x - \lambda, \lambda, d).$$

Hence, for all $\lambda \in (0, 2x)$,

$$I(a, 2x - \lambda, \lambda, d) = I(a, x, x, d).$$

Hence, we have shown that $I(a, b, c, d) = I(a, b', c', d)$ whenever $b + c = b' + c'$. By the same logic, and by SiInv and TInv, $I(a, b, c, d) = I(a', b, c, d')$ whenever $a + d = a' + d'$. I, by DMon, is strictly increasing in $a + d$ whenever $bc \neq 0$, and, by ODMon, is strictly decreasing in $b + c$ whenever $ad \neq 0$. □

If (i) $a + d > a' + d'$ and $b + c \leq b' + c'$ or (ii) $a + d \leq a' + d'$ and $b + c > b' + c'$, then, by Lemma 1, (i) $I(M) > I(M')$ or (ii) $I(M) < I(M')$, respectively. Suppose $a + d > a' + d'$ and $b + c > b' + c'$. Define

$$M'' = (a'', b'', c'', d'') = M' \cdot (b + c)/(b' + c').$$

By definition of M'', $b'' + c'' = b + c$, and $a'' + d'' = (a' + d') \cdot (b + c)/(b' + c')$. By ScInv of I, $I(M'') = I(M')$. The comparison of $a + d$ and $a'' + d''$ pins down the ordinal assortativeness relation between M and M'. That is,

$$\frac{a + d}{a'' + d''} = \frac{a + d}{b + c} \Bigg/ \frac{a' + d'}{b' + c'} > 1 \Leftrightarrow I(M) > I(M').$$

When $a + d < a' + d'$ and $b + c < b' + c'$, we can similarly pin down the ordinal assortativeness relation between M and M'. Note that for any $M = (a, b, c, d) \gg 0$,

$$I_{tr}(M) = \frac{(a+d)}{(a+b)+(c+d)} = \frac{a+d}{b+c} \Big/ \left(\frac{a+d}{b+c} + 1\right).$$

Hence, $I(M) > I(M')$ if and only if $I_{tr}(M) > I_{tr}(M')$.

Any nonlinear transformation of NT would violate PDec or RPDec, so any index that satisfies the stated axioms must be not only a monotonic transformation but also a linear transformation of NT. □

Proof of Theorem 3. It is straightforward to check that ALR satisfies the axioms, so it remains to show the other direction. We first show that any index I that satisfies ScInv, TInv, SiInv, MMon, and RDec is proportional to ALR. Consider $M = (a, b, c, d)$. By TInv,

$$I(M) = I\begin{pmatrix} a & b \\ c & d \end{pmatrix} = I\begin{pmatrix} d & c \\ b & a \end{pmatrix}. \tag{5}$$

Recall

$$r(M) \equiv \frac{a+b}{|M|} + \frac{a+c}{|M|} + \frac{d+b}{|M|}\frac{d+c}{|M|}|M| = \frac{(a+b)(a+c)+(d+b)(d+c)}{a+b+c+d}.$$

By RDec,

$$I\begin{pmatrix} a+d & b+c \\ b+c & a+d \end{pmatrix} \cdot r\begin{pmatrix} a+d & b+c \\ b+c & a+d \end{pmatrix} = I\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot r\begin{pmatrix} a & b \\ c & d \end{pmatrix} + I\begin{pmatrix} d & c \\ b & a \end{pmatrix} \cdot r\begin{pmatrix} d & c \\ b & a \end{pmatrix},$$

which, by (5), is simplified to

$$I\begin{pmatrix} a+d & b+c \\ b+c & a+d \end{pmatrix} \frac{|M|^2+|M|^2}{2|M|} = 2I(M)r(M) \Rightarrow I(M) = \frac{1}{2}\frac{|M|}{r(M)} I\begin{pmatrix} a+d & b+c \\ b+c & a+d \end{pmatrix}. \tag{6}$$

Because for any $\epsilon < \min\{a+d, b+c\}$,

$$\begin{pmatrix} a+d & b+c \\ b+c & a+d \end{pmatrix} = \begin{pmatrix} a+d-\epsilon & \epsilon \\ \epsilon & a+d-\epsilon \end{pmatrix} + \begin{pmatrix} \epsilon & b+c-\epsilon \\ b+c-\epsilon & \epsilon \end{pmatrix},$$

by RDec, for any $\epsilon < \min\{a+d, b+c\}$,

$$|M| I \begin{pmatrix} a+d & b+c \\ b+c & a+d \end{pmatrix} = (a+d) I \begin{pmatrix} a+d-\epsilon & \epsilon \\ \epsilon & a+d-\epsilon \end{pmatrix} + (b+c) I \begin{pmatrix} \epsilon & b+c-\epsilon \\ b+c-\epsilon & \epsilon \end{pmatrix}.$$

Plugging in the expression of $I(M)$ in (6), we get, for any $\epsilon < \min\{a+d, b+c\}$,

$$I(M) = \frac{1}{2r(M)} \left[(a+d) \cdot I \begin{pmatrix} a+d-\epsilon & \epsilon \\ \epsilon & a+d-\epsilon \end{pmatrix} + (b+c) \cdot I \begin{pmatrix} \epsilon & b+c-\epsilon \\ b+c-\epsilon & \epsilon \end{pmatrix} \right].$$

Take $\epsilon \to 0$ and by ScInv and $I(0,1,1,0) = 0$, we have

$$I(M) = \frac{1}{2} \frac{a+d}{r(M)} I \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Hence, any index that satisfies the above axioms is proportional to $(a+d)/r(M)$, the aggregate likelihood ratio. $\square$

B Omitted table

Table 3 shows that Generalized Separable conditions (4) are never simultaneously satisfied in our data.

Table 3: General test of assortativeness at the top of the distribution of education, comparing birth cohorts 1930-1959 to 1970-1975

	College vs Some College				College vs less than College				Some College vs HS and below			
	$\frac{a}{a+b}$	$\frac{a}{a+c}$	$\frac{d}{d+b}$	$\frac{d}{d+c}$	$\frac{a}{a+b}$	$\frac{a}{a+c}$	$\frac{d}{d+b}$	$\frac{d}{d+c}$	$\frac{a}{a+b}$	$\frac{a}{a+c}$	$\frac{d}{d+b}$	$\frac{d}{d+c}$
1940 vs 30	0.056	-0.007	0.058	-0.011	0.111	0.017	-0.045	-0.036	0.121	0.047	-0.070	-0.066
adj p-val	0.000	0.198	0.000	0.194	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000
1950 vs 30	0.081	-0.062	0.193	0.003	0.176	-0.033	-0.048	-0.080	0.201	0.041	-0.125	-0.177
adj p-val	0.000	0.000	0.000	0.764	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1960 vs 30	0.144	-0.073	0.250	-0.030	0.269	-0.057	-0.027	-0.123	0.278	0.020	-0.089	-0.258
adj p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 30	0.207	-0.075	0.262	-0.106	0.357	-0.050	-0.028	-0.185	0.337	0.046	-0.071	-0.305
adj p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1950 vs 40	0.025	-0.055	0.135	0.014	0.065	-0.050	-0.002	-0.044	0.081	-0.006	-0.055	-0.111
adj p-val	0.000	0.000	0.000	0.039	0.000	0.000	0.200	0.000	0.000	0.046	0.000	0.000
1960 vs 40	0.089	-0.066	0.191	-0.019	0.158	-0.075	0.018	-0.088	0.157	-0.027	-0.018	-0.192
adj p-val	0.000	0.000	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 40	0.151	-0.068	0.204	-0.095	0.246	-0.068	0.018	-0.149	0.216	-0.001	-0.000	-0.239
adj p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.979	0.945	0.000
1960 vs 50	0.064	-0.011	0.056	-0.033	0.094	-0.024	0.021	-0.043	0.076	-0.021	0.037	-0.081
adj p-val	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1970 vs 50	0.126	-0.013	0.069	-0.109	0.181	-0.018	0.020	-0.105	0.135	0.005	0.055	-0.128
adj p-val	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.071	0.000	0.000
1970 vs 60	0.063	-0.002	0.013	-0.076	0.088	0.007	-0.001	-0.062	0.059	0.026	0.018	-0.047
adj p-val	0.000	0.890	0.097	0.000	0.000	0.190	0.804	0.000	0.000	0.000	0.000	0.000

Notes: Columns identify each of the 2×2 sorting matrices. In each panel, row 1 shows estimates of the difference in the respective index between the latest and earliest cohorts; row 2 shows p-values for 2-sided significance testing adjusted for multiple hypothesis using the stepdown method for the three outcomes on the row(Romano and Wolf, 2005; Romano, Shaikh, and Wolf, 2008; Romano and Wolf, 2016). Data source: March extract of the US Current Population Survey, subsample of married individuals observed when aged 35-44 and born in 1930-1959 to 1970-1975.