Overcoming Borrowing Stigma: The Design of Lending-of-Last-Resort Policies

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Abstract

How should the government effectively provide liquidity to banks during periods of financial distress? During the 2008-2010 crisis, banks avoided borrowing from the Fed's long-standing discount window (DW), but actively participated in its special monetary program, the Term Auction Facility (TAF), although both programs had the same borrowing requirements. Using a dynamic adverse selection model with endogenous borrowing stigma costs, we explain how the introduction of TAF increased banks' borrowing and willingness to pay. Using DW borrowing and TAF bidding data, we confirm our theoretical prediction that weaker banks borrowed relatively more from the DW.

Keywords: lending of last resort, discount window stigma, Term Auction Facility, adverse selection

JEL: G01, E52, D44, E58

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"For [...] the competitive format of the auctions, TAF has not suffered the stigma of conventional discount window lending and has proved effective for injecting liquidity into the financial system... Another possible reason that TAF has not suffered from stigma is that auctions are not settled for several days, which signals to the market that auction participants do not face an immediate shortage of funds."

— Bernanke (2010) to the US House of Representatives

1 Introduction

Financial crises are typically accompanied by liquidity shortages in the entire banking sector. How should the central bank lend to depository institutions to increase liquidity during such episodes? The answer is not obvious. The discount window (DW) has been the primary lending facility used by the Federal Reserve, but it was severely underutilized when the interbank market froze at the beginning of the financial crisis in late 2007. A main reason for the underutilization is believed to be the stigma associated with DW borrowing: Tapping the discount window conveys a negative signal about borrowers' financial condition to their counterparties, competitors, regulators, and the public.[1]

In response to the credit crunch and banks' reluctance to borrow from DW, the Fed created a temporary program, the Term Auction Facility (TAF), in December 2007. TAF held an auction every other week and provided a preannounced amount of loans with identical loan maturity, collateral margins, and eligibility criteria to those of the DW.

Surprisingly, TAF provided much more liquidity than DW: Figure 1a shows that the outstanding balance in TAF far exceeded that in DW during 2007–2010.[2] Even more surprisingly, banks sometimes paid a higher interest rate to obtain liquidity through the auction: Figure 1b shows that the *stop-out rate*—the rate that clears the auction—was higher than the concurrent *discount rate*—the rate readily available in DW—in 21 of the 60 auctions, especially from March to September 2008, the peak of the financial crisis.[3]

[1]Banks have regularly paid more for loans from the interbank market than for loans they could readily get from DW (Peristiani, 1998; Furfine, 2001, 2003, 2005). Although the Fed does not publicly disclose which institutions have received loans from DW, the Board of Governors publishes weekly the total amount of DW lending by each of the 12 Federal Reserve Districts. Therefore, a surge in total DW borrowing could send the market scrambling to identify the loan recipients. Because of the interconnectedness of the interbank lending market, it is not impossible for other banks to infer which institutions went to the discount window. Market participants and social media could also infer from other activities.

[2]The outstanding balance in DW made up at most 33.4 percent of the total outstanding balance between 2007 and 2010. See Figure 2a for DW's balance as a percent of the total balance week by week between 2007 and 2010.

[3]The stop-out rate ranged from 1.5 percentage points above (on September 25, 2008) to 0.83 percentage points below (on December 4, 2008) the concurrent discount rate. The stop-out rate was above the concurrent discount rate

This episode suggests the importance of the design of emergency lending programs to effectively cope with liquidity shortages. More specifically, it raises a series of questions about lending-of-last-resort policies. Why could TAF overcome the stigma and generate more borrowing than DW? Shouldn't the same stigma also prevent banks from participating in TAF? How did banks decide to borrow from DW and/or TAF? Was there any systematic difference between the banks that borrowed from the two facilities? How could the program be further improved? The answers to these questions are unclear, even to the policy makers involved (Armantier and Sporn, 2013; Bernanke, 2015).

This paper provides a comprehensive analysis of lending of last resort in the presence of borrowing stigma. We introduce a model in which banks have private information about their financial condition. Weaker banks have more urgent liquidity needs and enjoy higher borrowing benefits. Two lending facilities are available. An auction is held once to allocate a set amount of liquidity, and DW is always available—before, during, and after the auction. Importantly, TAF delays its release of funds. Borrowing from each facility incurs a stigma cost, which is endogenously determined by the financial condition of participating banks.

In equilibrium, banks self-select into different programs. The weakest banks borrow immediately from the DW because they are desperate for liquidity and cannot afford to wait. Stronger banks, in contrast, are lured to participate in the auction because the potential of borrowing cheap renders the auction more attractive than DW. Their liquidity needs are not as imperative, and they value the lower expected price in the auction more than weaker banks do. Of the banks that participate in TAF, some may bid higher than the discount rate because they would like to avoid the discount window stigma brought by association with the weakest banks. As a result, the clearing price in the auction may exceed the discount rate. Of the banks that have lost in TAF, relatively weaker ones might still borrow from DW.

Our model demonstrates that TAF, used in accordance with DW, could increase liquidity provision through three channels. First, by setting a low reserve price in the auction, TAF attracted moderately weak banks (that would have borrowed from DW without TAF) to participate and take their chances on borrowing cheap. Second, participating banks can submit bids to internalize any stigma cost associated with TAF, so TAF also attracted moderately strong banks (that would not have borrowed at all without TAF) to participate. Finally, due to the selection by stronger banks into the auction, the auction stigma is endogenously lower than the discount window stigma, which further encourages stronger banks to participate in TAF. Hence, the combination of TAF and DW expands the set of banks who try to, and may obtain, liquidity, thus increasing

for almost all auctions between March 2008 (when Bear Stearns filed for bankruptcy) and September 2008 (when Lehman Brothers filed for bankruptcy). See Figure 2b for the difference between the stop-out rate and the concurrent discount rate auction by auction.

the overall supply of short-term credit to the economy.

We use granular data on DW and TAF borrowing during the crisis to verify one of the model's main predictions: Weaker banks borrowed more from the DW. We show that compared with TAF banks, DW banks have higher leverage, a lower tier-1 capital to risk-weighted asset ratio, and a lower fraction of private-label mortgage-backed securities—all signs of financial weakness—on their balance sheets. Moreover, exploring the credit guarantee programs implemented in G7 countries—especially Canada, France, and Germany—in October 2008, we show that following these policies, banks in those countries increased their borrowing from TAF and reduced their borrowing from DW, compared to their peers in the US. Finally, we show that prior to the borrowing dates, DW banks had persistently higher credit default swap spreads—higher risks of default—than TAF banks.

The paper contributes to the theoretical and empirical literature on the lending-of-last-resort policies. Theoretically, our paper improves the understanding of appropriate interventions during a financial crisis. More specifically, it contributes to the literature that studies government intervention in markets plagued by adverse selection by (i) incorporating dynamics and (ii) distinguishing stigma costs associated with different programs (Philippon and Skreta, 2012; Tirole, 2012; Ennis and Weinberg, 2013; La'O, 2014; Lowery, 2014; Fuchs and Skrzypacz, 2015; Gauthier et al., 2015; Li et al., 2016; Ennis, 2017; Che et al., 2018; Li et al., 2018). In these studies, either there is no explicit stigma cost associated with a government-sponsored facility, or stigma costs are implicitly assumed to be uniform across all programs. In contrast, our paper endogenizes and differentiates the stigma costs associated with different programs in a dynamic setting.

Empirically, to the best of our knowledge, our paper is the first to combine micro-level data on DW borrowing and TAF bidding and link them to banks' fundamentals. Prior papers largely focus on measuring either DW stigma or the subsequent economic effects of TAF borrowing. To prove the existence of DW stigma, Peristiani (1998) and Furfine (2001, 2003, 2005) offer evidence that banks prefer the federal funds market to DW, and Armantier et al. (2015) show that more than half of TAF participants submitted bids above the discount rate during the 2007-2008 financial crisis. The TAF was shown to be effective in reducing liquidity concerns (Wu, 2011), lowering LIBOR McAndrews et al. (2017), and conferred a benefit on the real economy (Moore, 2017). Cassola et al. (2013) study the financial crisis based on bidding data of the European central bank from January 4 to December 11, 2007—closely predating December 17, 2007, when the first TAF auction was conducted—and find that banks respond not only to the costs of obtaining liquidity elsewhere but also strategically to other banks' bids.

The rest of the paper is organized as follows. Section 2 describes lending-of-last-resort facilities during the financial crisis. Section 3 sets up the model. Section 4 characterizes the equilibrium of the model and discusses liquidity provision under different settings. Section 5 presents empirical evidence consistent with the predictions of the model. Section 6 concludes. The appendix contains omitted proofs, figures, and tables.

2 Background

Stress in the interbank lending market began to loom in the summer of 2007. In June, two of Bear Stearns' mortgage-heavy hedge funds reported large losses. On July 31, they declared bankruptcy. On August 9, BNP Paribas, France's largest bank, barred investors from withdrawing money from investments backed by US subprime mortgages, citing evaporated liquidity as the main reason. Subsequently, many other banks and financial institutions experienced liquidity dry-ups in wholesale funding (in the form of asset-based commercial paper or repurchase agreements).

With the growing scarcity of short-term funding, banks were supposed to borrow from the lender of last resort (LOLR). In the US, the role of LOLR has largely been fulfilled by the discount window, which allows eligible institutions—mostly commercial banks—to borrow money from the Federal Reserve on a short-term basis to meet temporary shortages of liquidity caused by internal and external disruptions.[4] Discount window loans were extended to sound institutions with good collateral. Since its founding in 1913, the Fed has never lost a penny on a discount window loan. However, banks were reluctant to use the discount window, due to the widely held perception that a stigma was associated with borrowing from the Fed. As advised by Bagehot (1873), a penalty—1 percentage point above the target federal funds rate—was charged on discount window loans, with the goal of encouraging banks to look first to private markets for funding. However, this penalty generated a side effect for banks: Banks would look weak if it became known that they had borrowed from the Fed.

Individual banks' discount window borrowing was kept confidential.[5] However, banks were nervous that investors, in particular money market participants, could guess when they had come to the window by observing banks' behavior and carefully analyzing the Fed's balance sheet figures.[6] For example, the Fed had to disclose weekly the level of discount window borrowing at both the aggregate and district level.[7]

[4]The discount window was once an actual teller window staffed by a lending officer.

[5]The Dodd-Frank Act required the disclosure of details of discount window loans after July 2010 on a 2-year lag from the date on which the loan is made.

[6]According to Bernanke (2015), Ron Logue, CEO of State Street, approached the Boston Fed and asked whether the weekly district-by-district reporting of loan totals could be eliminated. The request was turned down for legal reasons and concerns about market-wide confidence.

[7]The stigma associated with borrowing from the government was also significant in the UK. In August 2007, Barclays twice tapped the emergency lending facility offered by the Bank of England. The news came out on Thursday, August 30, when the Bank of England said it had supplied almost 1.6 billion pounds as a lender of last resort, without naming the borrower(s). Journalists and the market scrambled to find out. Barclays declined to confirm that

The Fed subsequently made a few changes to discount window policies. In particular, on August 16, 2007, it halved the interest rate penalty on discount window loans. The maturity of loans was also extended from overnight to up to 30 days with an implicit promise of further renewal. Moreover, the Fed tried to persuade some leading banks to borrow at the window, thereby suggesting that borrowing did not equal weakness. On August 17, Timothy Geithner and Donald Kohn hosted a conference call with the Clearing House Association, claiming that the Fed would consider borrowing at the discount window "a sign of strength." Following the call, on August 22, Citi announced that it was borrowing $500 million for 30 days. JPMorgan Chase, Bank of America, and Wachovia subsequently announced that they had borrowed the same amount, increasing the total amount borrowed at the discount window by $2 billion. However, the four big banks—with the borrowing stigma in mind—made it clear in their announcements that they did not need the money. Thirty days later, the discount window borrowing fell back to $207 million.[8] On December 11, 2007, the Fed lowered its discount rate to 4.75%, but the attempt was unsuccessful in injecting liquidity to the financial system.

To further relieve stress in the short-term lending market, the Fed implemented the Term Auction Facility in December 2007. The first auction, held on December 17, released $20 billion in the form of 28-day loans. The participation requirement was the same as for DW.[9] The Fed received over $61 billion in bids and released the full $20 billion to 93 institutions. In February 2008, Dick Fuld, CEO of Lehman Brothers, urged the Fed to include Wall Street investment banks in auctions, which would require invoking Section 13(3) to allow the Fed to have authority to lend to non-bank institutions, but the Fed refused. From March to September 2008, the stop-out rate in TAF consistently exceeded the concurrent discount rate. The final auction was held on

it had used the central bank's standing borrowing facility, but later, it cited a technical breakdown in the clearing system as the reason for the large pile of cash. In its statement, Barclays said, "The Bank of England sterling standby facility is there to facilitate market operations in such circumstances. Had there not been a technical breakdown, this situation would not have occurred." Its share fell 2.5 pounds immediately after the statement, which cast doubt on its 45 billion pound bid to take over the Dutch bank ABN Amro.

Shin (2009) described the bank run on Northern Rock, UK's fifth-largest mortgage lender. In the UK, there was no government deposit insurance, and banks relied on an industry-funded program that only partially protected depositors. On September 13, 2007, the BBC broke the news that Northern Rock had sought the Bank of England's support. The next morning, the Bank of England announced that it would provide emergency liquidity support. It was only *after* the announcement—that is, after the central bank had announced its intervention to support the bank—that retail depositors started queuing outside the branch offices.

[8] Records released later show that JPMorgan and Wachovia returned most of the money the next day, whereas Bank of America and Citi—already showing signs of problems—kept the money for a month.

[9] The rule of the auction was as follows. On Monday, banks phoned their local Fed regional banks to submit their bids specifying their interest rate (and loan amount) and to post collaterals. On Tuesday, the Fed secretly informed the winners and publicly announced the stop-out rate (as well as the number of banks receiving loans), determined by the highest losing bid (or the minimum reserve price if the auction was undersubscribed). On Thursday, the Fed released the loans to the banks. Throughout the whole auction process, banks were free to borrow from DW. The following Monday, each regional Fed published total lending from last week; banks may be inferred from these summaries or other channels.

March 8, 2010, as the auctions had been consistently undersubscribed since 2009.

As shown in Figure 1, TAF was clearly more successful than DW in providing liquidity, and banks were also willing to pay a higher interest rate in TAF than the concurrent discount rate in DW. As Bernanke (2015) acknowledged, before implementing TAF, policy makers were also concerned that the stigma that had kept banks away from the discount window would also be attached to the auctions. The program was implemented as "give it a try and see what happens," but turned out to be quite successful. Why TAF was more successful is still unclear.

3 The Model

There are n banks. Each bank is endowed with one unit of an illiquid asset. Before the asset pays off, each bank faces a liquidity shock. The probability that a bank may be affected by the liquidity shock is privately known by the bank. Each bank can borrow from two facilities: the discount window (DW), which provides liquidity before any liquidity shock hits, and the Term Auction Facility (TAF), which provides liquidity after an early liquidity shock may have hit the bank. Borrowing banks may incur a penalty if detected borrowing. The penalty depends on the facility one borrows from and the average financial condition of the other banks that borrow from the same facility. Figure 3 sketches the sequence of events, which we will describe in detail next.

3.1 Preferences, Technology, and Shocks

All banks are risk neutral and do not discount future cash flows. Each bank has one unit of long-term, illiquid assets that will mature at the end of the game. The asset generates cash flows R upon maturity, but nothing if liquidated early. Each bank may be hit with a liquidity shock à la Holmström and Tirole (1988). Let $1-\theta_i \in [0,1]$ be the probability that the liquidity shock affects bank i, where θ_i follows the independently and identically distributed cumulative distribution function (cdf) F with associated probability density function (pdf) f on the support $[0,1]$. Assume that F is log-concave. This assumption is not restrictive, as many standard distributions satisfy it; it is imposed to guarantee equilibrium uniqueness.[10] Throughout the paper, we assume that θ_i is private information and only known by the bank itself. We drop subscript i whenever no confusion arises. Type θ is also referred to as a bank's financial strength.[11] We sometimes refer

[10]Distributions with a log-concave pdf, which implies a log-concave cdf, include normal; exponential; uniform over any convex set; logistic; extreme value; Laplace; chi; Dirichlet if all parameters are no less than 1; gamma if the shape parameter is no less than 1; beta if both shape parameters are no less than 1; Weibull if the shape parameter is no less than 1; and chi-square if the number of degrees of freedom is no less than 2. Distributions with a log-concave cdf but non-log-concave pdf include log-normal; Pareto; Weibull if the shape parameter is smaller than 1; and gamma if the shape parameter is smaller than 1. Student's t, Cauchy, and F distributions are not log-concave for all parameters (Bagnoli and Bergstrom, 2005).

[11]In reality, one can proxy a bank's strength θ by either its reserve of liquid assets or the level of its demandable liabilities that can evaporate in a flash.

to a type-θ bank as bank θ.

Before the liquidity shock hits, each bank has opportunities to borrow. Receiving a loan with interest rate r will help the bank defray the liquidity shock and bring a net benefit of $(1-\theta)R$ at the cost of interest rate r. Bank θ's expected payoff from borrowing a rate-r loan is $(1-\theta)R-r$ if it receives the loan immediately, and $\delta(1-\theta)R-r$ if an early liquidity shock hits with probability $1-\delta$ before it receives the loan.[12] As will become clear later on, the specific functional form of the borrowing benefit does not matter for any of our results. What matters is that the benefit is lower if the bank is stronger or if the interest rate is higher.

We describe the two lending facilities in the next subsection.

3.2 Lending Facilities

Any bank is able to borrow from either the discount window or the Term Auction Facility.[13]

3.2.1 Discount Window

The discount window is a facility that offers loans at a fixed interest rate r_D, which is commonly referred to as the discount rate and is exogenously set by the Federal Reserve. Since a bank can always borrow from the discount window with certainty, the net borrowing benefit is $(1-\theta)R-r_D$.

3.2.2 Term Auction Facility

The Term Auction Facility allocates preannounced m units of liquidity through an auction. In the auction, banks that decide to participate simultaneously submit their sealed bids, which are required to be higher than the preannounced minimum bid r_A. After receiving all of the bids, the auctioneer ranks them from highest to lowest. The auction takes a uniform-price format: All winners pay the same interest rate, which is referred to as the stop-out rate s, and losers do not pay anything. If there are fewer bids than the units of liquidity provided, each bidder receives a loan and pays r_A. If there are more bidders than the total liquidity, each of the m highest bidders receives one unit of liquidity by paying the highest *losing* bid. Formally, suppose there are l bidders in total. If $l \leq m$, each bidding bank receives a loan by paying $s = r_A$. If $l > m$, each of the m highest bidding banks receives one unit of liquidity by paying the $m+1^{\text{st}}$ highest bid. The remaining $l-m$ banks do not pay anything and, of course, do not receive any liquidity.

[12] According to Bernanke (2015), one main reason to implement the Term Auction Facility was that it would take time to conduct an auction and determine the winning bids, so that borrowers would receive funds with a delay, and thus signal that they were not desperate for cash.

[13] The interbank market essentially froze during the 2007-2008 financial crisis. We will describe Appendix A.1.10 an extension in which the interbank market is wellfunctioning and demonstrate that no results change.

We have modeled the TAF auction as an extended second-price auction: All winning parties pay the highest losing bid. In reality, TAF is closer to an extended first-price auction: All winning banks pay the lowest winning bid. The two auctions generate the same revenue for the auction and the same expected payoffs for the bidders, by the revenue equivalence theorem (Myerson, 1981), and consequently make the same borrowing decisions. We present the analysis with the extended second-price auction because it is notationally simpler, as it is a weakly dominant strategy for each bank to simply bid the maximum interest rate it is willing to pay (Vickrey, 1961).[14]

In reality, winners receive their TAF funds 3 days after the auction. We assume there is a probability, $1-\delta$, that an early liquidity shock hits each bank before it receives the funds. Hence, the expected net borrowing benefit of a winner who pays stop-out rate s is $\delta(1-\theta)R-s$. Losers, upon learning the result of the auction, may borrow from the discount window if needed.

3.3 Borrowing Stigma Costs

Banks are assumed to incur a facility-dependent stigma cost. We have argued that a key reason that banks were reluctant to borrow from the lender of last resort is stigma cost. Detected borrowing may signal financial weakness to counterparties, investors, and regulators. Although θ is private information, the public can still draw inferences based on whether the bank has borrowed or which facility the bank has used if it has borrowed. We assume that upon detection, the public can perfectly tell whether the borrowing has been achieved through the discount window or the auction.

We capture the notion of stigma cost in a parsimonious way. We assume that after all of the borrowing is complete, banks that have successfully borrowed may be detected independently. Denote the probability of a bank's being detected borrowing from a particular facility to be p. This penalty can be understood as the combined cost in the bank's deteriorated reputation, a reduced chance to find counterparties, or the cost of a heightened chance of runs and increasing withdrawals by creditors. Let G_D and G_A be the type distributions of the banks that have borrowed from DW and from TAF, respectively. Let the stigma cost depend linearly on the expected financial condition of the bank. That is, for any detected borrowing decision $\omega \in \{D, A\}$,

$$k_\omega \equiv k(G_\omega) = K - \kappa \int_0^1 \theta dG_\omega(\theta).$$

Assume $\kappa < \delta R$ so that the detection penalty is increasing at a slower rate than the real benefit of borrowing for worse banks. For expositional convenience, without loss of generality, let's

[14]In contrast, in the first-price auction, banks shade their bids, which depend on the liquidity supply and other participating banks.

normalize the stigma cost of a bank believed to have an unconditional average condition to be 0, $k_{\emptyset} \equiv 0$, that is, $K \equiv \kappa \int_0^1 \theta dF(\theta)$.

3.4 Definition of Equilibrium

In summary, the setting is summarized by the return R, type distribution F of banks, discount rate r_D in DW, number m of units of liquidity auctioned, minimum bid r_A in TAF, and the penalty function $k : G \mapsto \mathbb{R}_+$ attached to different belief distributions of bank's type.

Without loss of generality, we restrict each bank's strategy to be type-symmetric. Each bank θ's strategy can be succinctly described by $\sigma(\theta) = (\sigma_{D1}(\theta), \sigma_A(\theta), \beta(\theta), \sigma_{D2}(\theta))$, where $\sigma_\omega(\theta)$ is the probability of borrowing from $\omega \in \{D1, A, D2\}$, and $\beta(\theta)$ is its bid if it participates in the auction. Given strategies σ, beliefs about the financial situation can be inferred by Bayes' rule; in this case, we say that aggregate strategies σ generate posterior belief system $G = (G_A, G_D)$.

Definition 1. *Borrowing and bidding strategies σ^* and belief system G^* form an equilibrium if (i) each type-θ bank's strategy $\sigma^*(\theta)$ maximizes its expected payoff given belief system G^*, and (ii) the belief system G^* is consistent with banks' aggregate strategies σ^*.*

Clearly, the best (i.e., type-1) bank has no intention of borrowing at all, because it would pay a price, incur a stigma cost, and receive no benefit from borrowing. We assume that the borrowing benefit of the worst (i.e., type-0) bank is sufficiently high that it has a strict incentive to borrow even given the most pessimistic belief about banks that borrow: $R - r_D - k\left(\underline{G}\right) > 0$, where $\underline{G}(\theta) = 1$ for all $\theta > 0$.

4 Equilibrium and Liquidity Provision

We present the solutions of two benchmark designs—only DW and only TAF—before presenting the solution of the actual design (DW and TAF with a delayed release of funds). Solving the two benchmark designs helps to obtain and demonstrate the solution of the general design. Then we discuss three alternative designs (DW and TAF with immediate release of funds, two DWs with different releases of funds, and two DWs with different interest rates), and discuss why they do not improve liquidity provision. Finally, we discuss alternative detection technologies, and show why the detection technology we assume is the most realistic.

4.1 Only DW

We start by examining the equilibrium when the government only sets up the discount window. The optimal borrowing decision can be characterized by one threshold: Weaker banks borrow from the discount window, and stronger banks do not borrow at all.

Note (again) that the best bank never borrows, because it knows that a liquidity shock could never affect it and therefore it never needs the liquidity; instead, borrowing incurs an interest cost as well as a stigma cost. The larger is the probability a liquidity shock affects the bank, the more incentive the bank has to borrow. If the assumption $r_D < R - k(\underline{G})$ holds, the worst bank has a strict incentive to borrow from the discount window.

Furthermore, there is a unique equilibrium, which is guaranteed by the assumption of a log-concave cdf F.

Theorem 1 (Equilibrium with only DW). *Suppose only DW is available, i.e., $m = 0$. There exists a unique equilibrium characterizable by a threshold $\theta^{DW} > 0$: Banks $\theta \in [0, \theta^{DW}]$ borrow from DW, and banks $\theta \in (\theta^{DW}, 1]$ do not borrow. The equilibrium discount window stigma is*

$$k^{DW}(\theta^{DW}) = K - \kappa \int_0^{\theta^{DW}} \theta dF(\theta)/F(\theta^{DW}),$$

where the threshold θ^{DW} satisfies

$$(1 - \theta^{DW})R - r_D - pk^{DW}(\theta^{DW}) = 0. \tag{DW}$$

The discount window provides liquidity to all banks worse than θ^{DW}, but banks better than θ^{DW} do not borrow because the real economic benefits of borrowing to save the unrealized assets are dwarfed by the interest cost and the stigma cost. The change in the returns, interest rate, and stigma costs will affect liquidity provision as follows.

Proposition 1 (Liquidity Provision with only DW). *The expected total liquidity to be provided with only DW, L^{DW}, is $nF(\theta^{DW})$. It increases as (i) the return R increases, (ii) the discount rate r_D decreases, (iii) the probability of detection p decreases, and (iv) the stigma severity κ decreases.*

How total liquidity depends on the change in the distribution of banks' types is interesting, though: It may decrease when banks face higher liquidity risks overall.

Proposition 2 (Market Condition and Liquidity Provision with only DW). *Total liquidity with only DW, L^{DW}, (i) decreases when the distribution of banks $\theta \leq \theta^{DW}$ shifts in a first-order stochastically dominated (FOSD) way, and (ii) changes ambiguously when the type distribution F shifts in a FOSD way.*

When banks worse than θ^{DW} face even higher liquidity risks than before, banks that borrow from DW are perceived to be of even lower quality than before. As a result, the stigma cost rises, and bank θ^{DW}, which was indifferent between borrowing from DW and not, is no longer interested in borrowing. In other words, *the worsened conditions of infra-marginal borrowing banks adversely affect the borrowing decision of the marginal borrowing bank.* As a result of the stigma cost, the discount window may not be effectively providing liquidity when the worst banks become worse. In general, when all banks face higher liquidity risks, more banks would not necessarily borrow because the heightened stigma cost may dominate the worsened liquidity risks. The fact that banks were initially reluctant to borrow from DW before the introduction of TAF suggests that the worst banks in the economy were facing higher liquidity risks.

4.2 Only TAF

Next, we examine the equilibrium when the government only sets up the auction. The equilibrium can also be characterized by one threshold: Weaker banks bid their willingness to pay in the auction, and stronger banks do not participate in the auction and do not borrow at all.

Theorem 2 (Equilibrium with only TAF). *Suppose only TAF is available, i.e., $m > 0$ and $r_D \geq R - k(\underline{G})$. Furthermore, suppose $\delta R - pk(0) > r_A$. There exists a unique equilibrium characterized by a threshold θ^{TAF}: (i) banks $\theta \in \left[0, \theta^{TAF}\right]$ bid $\beta^{TAF}(\theta) = \delta(1-\theta)R - pk_A$ in TAF, and (ii) banks $\theta \in \left(\theta^{TAF}, 1\right]$ do not bid. Equilibrium auction stigma is*

$$k^{TAF}(\theta^{TAF}) = K - \kappa \int_0^{\theta^{TAF}} \int_0^{\theta_s} \frac{\theta dF(\theta)}{F(\theta_s)} h(\theta_s) d\theta_s - \kappa \int_{\theta^{TAF}}^1 \int_0^{\theta^{TAF}} \frac{\theta dF(\theta)}{F(\theta^{TAF})} h(\theta_s) d\theta_s,$$

where $h(\theta_s) = \binom{n}{m} F^{m-1}(\theta_s) f(\theta_s)(1-F(\theta_s))^{n-m}$ is the pdf of the m^{th} weakest bank, and the threshold θ^{TAF} satisfies

$$\delta R(1-\theta^{TAF}) - r_A - pk_A^{TAF}(\theta^{TAF}) = 0. \tag{TAF}$$

TAF alone is not necessarily more effective than DW in providing liquidity. If the facilities are used alone, it is unclear which one will provide more liquidity. The combination of DW and TAF is needed to increase liquidity provision compared with the DW-only design.

4.3 DW and TAF

We now solve for the equilibrium when both the discount window and Term Auction Facility with delayed release of funds are available. We will first describe a bank's bidding strategy in TAF, followed by its incentives in choosing between DW and TAF. We show that of the banks willing to borrow from DW, stronger ones have more incentives to bid in TAF than to borrow immediately from DW, which is the key force behind the separation of banks into the two facilities in equilibrium.

Lemma 1. *Only banks $\theta \le \theta_D$ would borrow from the discount window if they have lost in the auction, where $\theta_D = 1-(r_D+pk_D)/R$.*

Lemma 2. *Banks $\theta \in (\theta_1, \theta_A]$ participate in the auction, where*

$$\theta_1 = 1 - \frac{r_D - r_A + pk_D - pk_A}{(1-\delta)R}, \quad \theta_A = 1 - \frac{r_A + pk_A}{\delta R}.$$

and bid

$$\beta(\theta) = \begin{cases} r_D + pk_D - pk_A - (1-\delta)R(1-\theta) & \text{if } \theta < \theta_D \\ \delta R(1-\theta) - pk_A & \text{if } \theta \ge \theta_D \end{cases}.$$

Note that bids are increasing in θ when $\theta < \theta_D$ and decreasing in θ when $\theta \ge \theta_D$. Therefore, bank θ_D has the highest willingness to pay, and banks further away from θ_D have lower willingness to pay. The bids are increasing at the rate of $(1-\delta)R$ for banks worse than θ_D, and are decreasing at the rate of δR for banks better than θ_D. Winners in the auction are going to be the banks that are the closest to θ_D. In other words, banks $\theta < \theta_D$ that would be willing to borrow in the discount window are attracted to participate in the auction, and banks $\theta > \theta_D$ that would not have borrowed from the discount window are also attracted to participate in the auction. For any bank, as long as its willingness to pay is above r_A, it will participate in the auction by submitting a bid higher than r_A. Figure 4 shows the bids in TAF (if there is no minimum bid) and the optimal facility choice of different banks.

The bids may be higher than the concurrent discount rate, because of the difference in the stigma cost between the two borrowing facilities. Banks close to θ_D are willing to bid more than r_D, up to $pk_D - pk_A$ more to be exact, to avoid the stigma cost.

Lemma 3 (Equilibrium with Both DW and TAF: High Chance of Early Liquidity Shock). *Suppose DW and TAF are both available, and there is a sufficiently high chance of an early liquidity shock: $m > 0$, $r_D < R - k(\underline{G})$, and $\delta \le \left[r_A + k(\theta^{DW})\right] / \left[r_D + pk^{DW}(\theta^{DW})\right]$. In the unique equilibrium, banks $\theta \in [0, \theta^{DW}]$ borrow from DW, and banks $\theta \in (\theta^{DW}, 1]$ do not borrow.*

Therefore, delaying the release of the funds from TAF for too long will render the program ineffective.

Theorem 3 (Equilibrium with Both DW and TAF: Low Chance of Early Liquidity Shock). *Suppose DW and TAF are both available, and there is a sufficiently low chance of an early liquidity shock: $m > 0$, $r_D < R - k(\underline{G})$, and $\delta > \left[r_A + k(\theta^{DW})\right] / \left[r_D + pk^{DW}(\theta^{DW})\right]$. Suppose $\delta R \ge p\kappa$ and $(1-\delta)R \ge p\kappa$. In the unique equilibrium, there exist three thresholds θ_1, θ_D, and θ_A such that (i) banks $\theta \in [0, \theta_1]$ borrow from the discount window before the auction; (ii) banks $\theta \in (\theta_1, \theta_D]$ bid in the auction and borrow from the discount window if they lose in the auction; (iii) banks $\theta \in (\theta_D, \theta_A]$*

bid in the auction and do not borrow if they lose in the auction; and (iv) banks $\theta \in (\theta_A, 1]$ neither borrow from the discount window nor participate in the auction.

Theorem 3 immediately implies:

Corollary 1. *In equilibrium, discount window stigma k_D^* is larger than auction stigma k_A^*.*

Three forces separate banks borrowing in DW and those in TAF. First, the possibility of early liquidation as a result of the delayed release of funds in TAF forces the worst banks to borrow from DW, and deters them from participating in TAF. Second, the exclusion of the worst banks from the auction increases the discount window stigma and decreases the auction stigma, thus further attracting more banks to borrow from TAF for TAF's lower stigma cost. Finally, the competitive nature of the auction attracts banks that would not have borrowed with only DW by offering them a chance to borrow cheaper than the discount rate. TAF serves as a substitute for DW for banks that are close to and worse than θ^{DW}. They substitute into borrowing in the auction from borrowing in the discount window. TAF serves as a complement for DW in terms of total lending. Banks that are close to and better than θ^{DW} substitute into borrowing in the auction from not borrowing.

For total liquidity, consider the expected marginal borrower. The expected marginal borrower is better than θ^{DW}, because they borrow from the auction, and the distribution of the types of banks participating in the auction in the DW and TAF setting first-order stochastically dominates the distribution of the types of banks borrowing from DW.

Proposition 3 (Liquidity Provision with Both DW and TAF). *The combination of TAF and DW provides more total liquidity in expectation than does DW alone: $L^* > L^{DW}$. The liquidity provided by DW decreases when TAF is introduced.*

4.4 Alternative Designs

In addition to the always available DW, instead of creating a TAF with delayed release of funds, could the Fed have improved liquidity provision with the addition of (i) a TAF with immediate release of funds, (ii) a DW with delayed release of funds, or (iii) a DW with a different interest rate? We explore these possibilities next.

4.4.1 DW and Immediate TAF

Suppose TAF immediately releases funds to winners, and DW is always available. This is essentially a special case of the DW-and-TAF design above, with probability $1 - \delta = 0$ of encountering a liquidity shock between winning the auction and receiving the loan. DW no longer possesses an immediacy advantage, so all of the weakest banks bid in the auction first. All of the banks that would borrow from DW after losing in the auction—banks $\theta \le \theta'_D$—bid the same rate $r_D + pk_D - pk_A$, and all of the banks that would not borrow from DW after losing in the auction—banks $\theta > \theta'_D$—bid lower rates. In summary, as Figure 5 illustrates, banks $\theta \in [0, \theta'_A]$ participate in the auction. Winners receive loans from TAF, and losers with sufficiently weak financial conditions—banks $\theta \le \theta'_D$—borrow from DW afterward.

Proposition 4 (Equilibrium with DW and Immediate TAF). *Suppose TAF releases funds immediately and DW is always available. In the unique equilibrium, there exist two thresholds θ_D and θ_A such that banks $\theta \in [0, \theta_D]$ bid in TAF and borrow from DW if they lose in TAF, and banks $\theta \in (\theta_D, \theta_A)$ bid in TAF and do not borrow if they lose in TAF.*

Compared with the original design, this design provides less liquidity for three reasons. First, the weakest banks—banks $\theta \le \theta_1$—no longer immediately borrow from DW but participate in the auction, so they take away liquidity from stronger banks that would not have borrowed from DW if they lose in the auction, i.e., banks $\theta \in [\theta_D, \theta_A]$. Second, stronger banks that would not have borrowed from DW are less incentivized to participate in the auction, as their willingness to pay in the auction is lower. Third, there is less separation between DW and TAF banks, so the additional rate that banks are willing to pay in TAF is lower. There is a countervailing force whereby banks borrowing from DW are stronger than banks borrowing from DW in the original case—thus expanding the number of banks willing to borrow from DW—but this force cannot overcome the other three forces to provide more liquidity.

4.4.2 DWs with Immediate and Delayed Release of Funds

If the delay in releasing funds is important, why doesn't the Fed simply set up a separate discount window D' that releases funds later? The main problem with this separate discount window is that banks are separated into the two facilities only for certain combinations of discount rate r_D and discount factor δ. Let's explore this possibility and see how this design does not inject liquidity as desired. Suppose DW D' charges the interest rate $r_{D'}$.

Proposition 5. *Suppose DW D releases funds immediately and DW D' releases funds with a delay. Suppose $\delta R \ge p\kappa$ and $(1-\delta) \ge p\kappa$. In the unique equilibrium, there exist two thresholds θ_1 and θ_2 such that banks $\theta \in [0, \theta_1]$ borrow from DW D, and banks $\theta \in [0, \theta_2]$ borrow from DW D' if they do not borrow from DW D.*

To guarantee the separation of banks into two facilities, there must be an intermediate chance of an early liquidity shock:

$$\frac{r_D + pk_D^*}{R}\left[1 - \frac{r_{D'} + pk_{D'}^*}{r_D + pk_D^*}\right] < 1 - \delta < 1 - \frac{r_{D'} + pk_{D'}^*}{r_D + pk_D^*}.$$

Otherwise, all banks borrow early (when the chance of an early liquidity shock is high) or borrow late (when the chance of an early liquidity shock is low). The possible inability to separate banks into two facilities may render the design less useful, as the main purpose of such a design is to separate banks to inject liquidity to stronger banks with a delay. The DW-and-TAF design circumvents this potential problem by setting a relatively low minimum required bid to attract banks to participate in the auction and to allow individual bids, so that those willing to pay the most emerge as winners and separate themselves from other banks.

4.4.3 Cheap and Expensive DWs

Setting up two DWs with different interest rates does not provide more liquidity, because setting up a cheaper DW and a more expensive DW always provides less liquidity than setting up only the cheaper of the two DWs.

Proposition 6 (Equilibrium with a More Expensive DW). *Suppose DW D charges interest rate r_D and DW D' charges interest rate $r_{D'} > r_D$. In equilibrium, banks are indifferent between the two DWs. The design offers less liquidity than setting up only the cheaper DW D.*

5 Empirical Analysis

Our theory predicts that the banks that borrowed more from DW over time were fundamentally weaker than the banks that borrowed more from TAF. In this section, we examine this hypothesis using data from various sources, including banks' regulatory reporting, subsequent failure, and credit default swap (CDS) spread. Throughout this section, all analysis is conducted at the bank holding company (BHC) level. Although under Section 23A of the Federal Reserve Act, it is illegal for a member bank to channel funds borrowed from LOLR to other affiliates within the same BHC, temporary exemptions of Section 23A were granted in late 2007 (Bernanke, 2015). Therefore, by conducting our analysis at the BHC level, we implicitly assume an efficient internal capital market within a BHC, which is consistent with the evidence in Cetorelli and Goldberg (2012) and Ben-David et al. (2017).

5.1 Descriptive Statistics of DW and TAF Borrowing

Let us start by describing the BHCs' borrowing behaviors from DW and TAF. The main dataset we use is obtained through Bloomberg and includes 407 institutions that borrowed from the Fed between August 1, 2007 and April 30, 2010. These data were released by the Fed on March 31, 2011, under a court order, after Bloomberg filed a lawsuit against the Fed.[15] The data contain information on each institution's daily outstanding balance of its borrowing from DW, TAF, and five other related programs. We will merge this dataset with the banks' regulatory database, equity returns, and CDS spreads to study how financial conditions affected the BHCs' borrowing decisions.

Since the Bloomberg dataset was collected by scraping over 29,000 pages of PDF files released from the Fed, data processing could be compromised. To evaluate the data's quality, we calculate the aggregate weekly outstanding balance in DW and TAF programs from the Bloomberg dataset and compare these numbers with the official ones released by Board of Governors of the Federal Reserve System (2019). Figure 6 shows the comparison. Clearly, the Bloomberg data managed to capture the vast majority of borrowing in both DW and TAF.

Table 1 provides the summary statistics of the BHCs' borrowing behavior during the crisis. Approximately 73 percent of borrowing institutions (313 out of 407) are banks, together with diversified financial services (mostly asset management firms), insurance companies, savings and loans, and other financial service firms. Foreign banks that borrowed through their American subsidiaries were also included in the Bloomberg database. Banks' choices of borrowing facilities were heterogeneous: 260 borrowing institutions tapped both facilities, 18 used only TAF, and 86 used only DW. Borrowing frequencies in both programs exhibit large skewness. While the median bank tapped the discount window twice, the Alaska USA Federal Credit Union used it 242 times. Similarly, for the 60 TAF auctions, while the median bank borrowed only three times, Mitsubishi UFJ Financial Group borrowed 28 times. On average, TAF lent more liquidity ($3,174 million) than DW ($1,529 million) to an average bank, consistent with the evidence in Figure 1a that TAF was more successful in providing liquidity. However, the Dexia Group—the BHC that borrowed the most from DW—borrowed approximately $190 billion over the 3-year period, far exceeding $100 billion from the largest borrower in TAF (Bank of America Corporation). This evidence suggests that DW banks were in need of larger amount of liquidity than TAF banks.

[15]For details, see Torres (2011). In May 2008, Bloomberg News reporter Mark Pittman filed a FOIA request with the Fed, requesting data about details of discount window lending and collateral. Unsurprisingly, it was stonewalled by the Fed. In November 2008, Bloomberg LP's Bloomberg News filed a lawsuit challenging the Fed, with the Fox News Network later filing a similar lawsuit. Other news organizations also showed support by filing legal briefs. In March 2011, the US Supreme Court ruled that the Fed must release information on the discount window loans in response to the lawsuits. Later that month, the Fed released the data, in the form of 894 PDF files with more than 29,000 pages on two CD-ROMS. Bloomberg News later published an exhaustive analysis that included the detailed data.

5.2 Evidence from Banks' Fundamentals

5.2.1 Domestic Banks

We link the Bloomberg data to FR Y-9C reports, the Consolidated Financial Statements for Holding Companies. The Y-9C reports collect financial-statement data from BHCs on a quarterly basis, which are then published in the Federal Reserve Bulletin. All domestic BHCs are required to submit these reports within 40 or 45 calendar days following the end of a quarter. While this merge allows us to use proxies for banks' financial condition, it unfortunately eliminates all foreign banks from the borrowing sample, which took out about 60% of total TAF loans (Benmelech, 2012). Of the 289 American banks that borrowed from either DW or TAF, we managed to merge Y-9C reports to 135 of them. These banks account for 42.2% of all American banks' loans from DW, and 81.8% from TAF.[16]

To explore how the BHCs' financial condition affects their borrowing from DW and TAF, we estimate the following specification:

$$\frac{DW_{it}}{DW_{it}+TAF_{it}} = \alpha + \beta_1 \cdot x_{it} + \beta_2 \cdot x_{i,t-1} + \Gamma \cdot [\text{Size}_{it}, ROA_{it}] + Q_t + \varepsilon_{it}. \tag{1}$$

The left-hand side of Equation (1) is BHC i's DW borrowing in quarter t divided by its total DW and TAF borrowing in the same period. It measures use of the DW relative to the TAF. On the right-hand side, x_{it} is one of the proxies for BHC i's financial condition in quarter t, including its tier-1 capital to risk-weighted asset ratio (T1RWA), book leverage, and liquid asset to asset ratio.[17] In all regressions, Q_t is the quarter fixed effect to take into account variations in aggregate economic conditions, and banks' size and ROAs are included as additional controls. Note that we do not include BHC fixed effects in these regressions, since, as shown in Table 1, a majority of the BHCs only borrow from DW or TAF once throughout the sample period. BHC fixed effects will effectively absorb all of the explanatory power. Moreover, we use the *contemporaneous* measurement of banks' financial condition while controlling for the lagged measurements.[18] Since these risk measurements were not available until at least 30 days after the quarter ended, we interpret

[16] There are several reasons behind the missing matches. First, many borrowers were credit unions or savings and loans holding companies that did not file Y-9C reports. For example, US Central Federal Credit Union took out $39,101 million in loans from the two facilities. Another example is Washington Mutual Inc. Even though it had an RSSD 2550581, it was an S&L holding company instead of a bank holding company. Therefore, it was regulated by the Office of Thrift Supervision and did not file a Y-9C report. Second, there are certain thresholds for reporting Y-9C. For example, banks with assets below $1 billion did not have to report. Finally, there were several mergers and acquisitions during the crisis period. For example, Wachovia borrowed $34,460 million from DW from 2007 Q3 to 2008 Q4, with the majority ($29,000 million) borrowed in 2008 Q4. However, Wachovia was acquired by Wells Fargo in 2008 Q4, and thus did not file a Y-9C report that quarter.

[17] In a Y-9C report, the tier-1 capital to risk-weighted assets is defined as bhck8274/bhcka223, and book leverage is defined as 1-(bhck3210/bhck2170).

[18] Our results are unchanged if we only control for either $x_{i,t}$ or $x_{i,t-1}$.

the contemporaneous risk measurements as the part of banks' fundamentals that are not entirely observed by the public.

Table 2 reports the results. Columns differ in the measurement used for bank fundamentals. Column (1) shows that once a bank's tier-1 capital to risk-weighted asset ratio goes up by 1 percent, the same bank borrows 4.9 percent less from the DW, with the total amount borrowed from the DW and TAF unchanged. The result is economically and statistically significant, suggesting that better capitalized banks tried to avoid borrowing from the DW. Column (2) confirms a similar result when we use book leverage. Column (3) examines the link between a bank's liquid assets and its borrowing decisions, where the definition of liquid assets follows Flannery et al. (2017) and Chen et al. (2019).[19] Clearly, banks with more liquid assets tend to borrow less from the DW, controlling for the past level of liquid-asset holdings. Finally, in Columns (4) and (5), we run a horserace across all risk measures, including the level of core deposits (Ellul and Yerramilli, 2013), liquid assets, private-label MBS, unused loan commitments, and short-term wholesale funding. Notably, banks' capital ratio and book leverage can significantly predict its borrowing from the DW relative to the TAF.[20]

Did DW and TAF loans capture potentially unobservable risks in banks' fundamentals? In particular, did these loans predict changes in banks' fundamentals? To answer this question, we estimate the following specification:

$$x_{i,t+1} = \alpha + \beta_1 \cdot x_{it} + \beta_2 \cdot \frac{DW_{it}}{DW_{it} + TAF_{it}} + \Gamma \cdot [\text{Size}_{it}, ROA_{it}] + Q_t + \varepsilon_{it}, \tag{2}$$

where x_{it} is still one of the previous proxies for BHC i's financial condition in quarter t.

Table 3 reports the results. Across all three columns, the results show that the relative borrowing from DW could have additional predictive power regarding a bank's tier-1 capital ratio, book leverage, and liquid asset holdings in the next quarter.

5.2.2 Foreign Banks

Specification (1) suffers from potential endogeneity issues. To address concern about omitted variables, we further employ a difference in difference (DID) approach and explore the international aspects of borrowing banks. Notably, following the bankruptcy of Lehman Brothers and the increasing pressure in the financial market, several countries undertook interventions to combat the potential crisis. The announcement dates of country-specific policies were staggered, however, as these policies could be largely driven by political bargaining and renegotiation. The

[19] This definition includes cash, federal funds sold, reverse repos, and securities excluding MBS/ABS.

[20] Since tier-1 capital ratio and book leverage are highly correlated (correlation≈ -0.7), we don't control for both in the same regression.

staggered structure offers us an ideal setup to study the difference in these countries' banks' decisions to borrow from the lender of last resort in the US. In October 2008, leaders from the G7 countries met and established a plan of action that aimed to stabilize financial markets, restore the flow of credit, and support global economic growth. Following the meeting, all of the G7 countries except Japan immediately announced to launch credit guarantee programs that effectively reduced the liquidity risk faced by domestic financial institutions (Yale Program on Financial Stability, 2019). In this subsection, we compare the decisions to borrow from the two emergent lending facilities by banks in G7 countries versus the US and study whether they switched more from DW borrowing to TAF borrowing following the announcement of these credit guarantee programs. In particular, we estimate the following equations on a biweekly basis using data from 2008 Q3:

$$\log(1+DW_{iw}) = \alpha + T_i + \lambda_w + \delta_{DW}(T_i \times \lambda_w) + \varepsilon_{iw}, \tag{3}$$

$$\log(1+TAF_{iw}) = \alpha + T_i + \lambda_w + \delta_{TAF}(T_i \times \lambda_w) + \varepsilon_{iw}. \tag{4}$$

In the specification, $\log(1+DW_{iw})$ and $\log(1+TAF_{iw})$ are bank i's total amount of borrowing within two weeks w. T_i is a dummy variable indicating whether bank i falls into the treated country (Canada/France/Germany/Italy/UK), and λ_w is a time dummy variable that equals 1 following the credit guarantee program and 0 otherwise.[21] The coefficients we are interested in are δ_{DW} and δ_{TAF}. Our theory predicts that $\delta_{TAF} > \delta_{DW}$, since these credit-guarantee programs effectively reduce the riskiness of banks. Note that we restrict the sample to 2008 Q3, and thus do not control for any additional variables that measure banks' health. Table 4 presents the results across all countries.

5.3 Evidence from Bank Failure

Next, we study whether banks that borrowed more from DW were also more likely to fail subsequently. To do so, we manually collect data on whether a bank failed, was acquired, or got nationalized by the government by December 31, 2011. Our results are robust to the choice of this particular ending date. In the borrowing sample, 36 financial institutions failed by December 31, 2011. Of these, 11 failed in 2008, eight in 2009, seven in 2010, and 10 in 2011. We study whether banks that borrowed more from DW were more likely to fail.

Table 5 reports the results. Column (1) shows that compared with a bank that only borrowed from TAF, a bank that solely borrowed from DW was more likely to fail within the same quarter by an additional probability of 0.7%. Column (2) shows that the additional probability of a bank

[21] We eliminate Japan, since the Japanese policy was not announced until 2010 Q1.

failing eventually is 12.5%. Both results are economically and statistically significant, implying that DW banks were riskier than TAF banks.

5.4 Evidence from CDS Spreads

In this subsection, we take advantage of the high frequency of the Bloomberg data and match borrowing banks with their CDS spreads in the Markit database. Since only very large banks have CDS contracts outstanding, we could match 70 of them, which accounts for 24.8% of DW borrowing and 79.4% of TAF borrowing.

Figure 7 plots the level of 5-year CDS spreads around borrowing dates, after removing fixed effects of BHC, month, and CDS rating. Two observations are prominent. First, prior to the borrowing event, DW banks have persistently higher CDS spreads than TAF banks. The difference (about 0.05) is significant relative to the standard deviation (less than 0.002), implying that prior to the borrowing, DW banks have a higher probability of default as acknowledged by the CDS price. Second, following both borrowing events, BHCs' CDS spreads drop within the next 5 days, even though it seems that TAF banks drop slightly more than DW banks. Two reasons can potentially explain the difference in drop. First, TAF banks in general take out larger loans, and therefore their funding constraint is more relaxed. Second, if borrowing from DW and TAF has an identical probability of being detected, TAF borrowing suffers a lower level of stigma cost.

Formally, we estimate the following specification(s):

$$y_{it} = \alpha + \beta \cdot \text{CDS}_{i,t-1} + \gamma \cdot \text{CDS rating}_{i,t-1} + Q_m + \gamma_i + \varepsilon_{it}, \tag{5}$$

where y_{it} is a dummy variable that takes one if a bank borrows from DW or TAF. Table 6 reports the results. In Column (1), y_{it} equals 1 if BHC i borrows from DW on date t and 0 if BHC i borrows from TAF on date t. The coefficient shows that if the BHC's 5-year CDS spreads on date $t-1$ increases by 100 basis points, its probability to borrow from DW, as opposed to TAF, increases by 0.1%. In Columns (2) and (3), $y_{it} = 0$ if BHC i does not borrow from either DW or TAF. In Column (2), $y_{it} = 1$ if it borrows from DW, whereas in Column (3), $y_{it} = 1$ if it borrows from TAF. Clearly, the results show that lagged CDS can predict DW borrowing but not TAF borrowing.

6 Conclusion

In this paper, we investigate how the Term Auction Facility mitigates the stigma associated with borrowing from the discount window when it was used in accordance with the discount window. We constructed an auction model with endogenous participation and showed that optimal auction bidding strategies that internalized any stigma associated with the auction increased participation and consequently mitigated the borrowing stigma.

The theoretical predictions we derived from the model are consistent with empirical observations. First, banks with strong financial health are reluctant to borrow from the discount window due to their reluctance to associate themselves with banks worse than them (Akerlof, 1970). Second, when both DW and TAF are available, weaker banks borrow from DW, and stronger ones participate in TAF. Of those that lose in the auction, weaker ones borrow from DW. Third, we show that the introduction of TAF may or may not expand the set of banks that obtain liquidity; it is the combination of TAF and DW that mitigates borrowing stigma and increases liquidity provision. Lastly, the stop-out rate of TAF may be higher or lower than the discount rate.

Our theoretical and empirical analyses together provide a better understanding of the role of a special monetary program, the Term Auction Facility, played during the financial crisis, and suggest how to better design lending-of-last-resort programs in the future. After comparing with several alternative designs, we conclude that the Fed's design of DW and delayed-funds-release TAF achieved its intended goal of lowering the borrowing stigma by separating the banks into distinct groups, encouraging participation by stronger banks, and providing more liquidity to the economy. The improvement over the current design is a quantitative matter of setting the more appropriate discount rate, minimum bid, and number of days to delay the release of funds. We leave this important quantitative exercise to future researches.

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A Appendix

A.1 Omitted Proofs

A.1.1 Proof of Theorem 1

Bank θ prefers borrowing from DW over not borrowing if and only if

$$u_D(\theta) = (1-\theta)R - r_D - pk_D - (1-p)k_\emptyset \geq 0.$$

Since we normalize $k_\emptyset$ to be 0, we can simplify the condition to

$$(1-\theta)R - r_D - pk_D \geq 0.$$

Clearly, the gain from borrowing from DW is strictly decreasing in θ. Therefore, for any given k_D, bank θ borrows from the discount window if and only if

$$\theta \leq 1 - \frac{r_D + pk_D}{R}.$$

Therefore, there exists a threshold—let's denote it by θ^{DW}—such that bank θ^{DW} is indifferent between borrowing from DW and not borrowing; banks worse than θ^{DW} borrow from DW; and banks better than θ^{DW} do not borrow. In equilibrium, k_D depends on θ^{DW}:

$$k_D = K - \kappa \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F\left(\theta^{DW}\right)}.$$

Plugging equilibrium k_D into the equilibrium condition above, we see that θ^{DW} is determined by

$$\left(1-\theta^{DW}\right)R - r_D - p\left[K - \kappa \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F\left(\theta^{DW}\right)}\right] = 0,$$

which is rearranged as

$$R - r_D - \theta^{DW}R + p\kappa \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F\left(\theta^{DW}\right)} = 0. \tag{DW}$$

The terms involving θ^{DW} can be rearranged as

$$-\theta^{DW}(R - p\kappa) - p\kappa\left[\theta^{DW} - \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F\left(\theta^{DW}\right)}\right].$$

The first term, $-\theta^{DW}(R - p\kappa)$, is decreasing in θ^{DW}, because $R > 1 > p\kappa$. For the second term, $-p\kappa\left[\theta^{DW} - \int_0^{\theta^{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})}\right]$, the expression in the square brackets is *mean advantage over inferiors*, as Bagnoli and Bergstrom (2005) name it. Because the distribution is assumed to be log-concave, by Bagnoli and Bergstrom (2005, Theorem 5), the term in the square brackets is weakly increasing in θ^{DW}, so the second term is weakly decreasing in θ^{DW}.In summary, the left-hand side of Equation (DW) is strictly decreasing in θ^{DW}.

To show the existence of a unique solution to Equation (DW), it remains to show that its left-hand side is positive for $\theta^{DW} = 0$ and negative for $\theta^{DW} = 1$. When $\theta^{DW} = 0$, the left-hand side is

$$R - r_D - p\kappa \int_0^1 \theta dF(\theta) = R - r_D - pK > 0,$$

where the equality follows from the normalization of $K = \kappa \int_0^1 \theta dF(\theta)$, and the inequality comes from the assumption that $R > r_D + pK$. When $\theta^{DW} = 1$, the left-hand side is

$$-r_D + p\kappa \int_0^1 \theta dF(\theta) = -r_D + pK < 0,$$

where the inequality follows from $r_D > 1 > pK$. Hence, there is a unique equilibrium. □

A.1.2 Proof of Proposition 1

The left-hand side of Equation (DW) strictly shifts up when (i) R increases, (ii) r_D decreases, (iii) p increases, or (iv) κ increases. Since the left-hand side of Equation (DW) is strictly decreasing in θ^{DW}, the equilibrium θ^{DW} increases as a result of any of the changes (i)-(iv). □

A.1.3 Proof of Proposition 2

The left-hand side of Equation (DW) strictly shifts down when F for $\theta < \theta^{DW}$ shifts in a first-order stochastically dominated way, because the only term affected by the change, $\int_0^{\theta_{DW}} \frac{\theta dF(\theta)}{F(\theta^{DW})}$, strictly decreases. Hence, the new threshold $\widetilde{\theta}^{DW}$ is strictly smaller than θ^{DW}. Total liquidity expected to be provided, $\widetilde{L}^{DW} = nF(\widetilde{\theta}^{DW})$, is also smaller than $L^{DW} = nF(\theta^{DW})$. □

A.1.4 Proof of Theorem 2

Bank θ bids (gross) interest rate $\beta(\theta)$ such that its payoff from winning in the auction with this rate is the same as the payoff from not borrowing,

$$\delta(1-\theta)R - \beta(\theta) - pk_A = 0.$$

In other words, the bid is the bank's maximum willingness to pay (WTP) for the loan:

$$\beta(\theta) = \delta(1-\theta)R - (pk_A).$$

Note that the bid is strictly decreasing in θ. Therefore, worse banks are willing to bid higher interest rates. Consequently, given any stigma cost k_A, there exists a threshold bank θ^{TAF} such that banks worse than θ^{TAF} are willing to bid more than the minimum bid r_A, and all banks better than θ^{TAF} are not willing to bid more than r_A. Bank θ^{TAF} bids exactly the prespecified minimum bid r_A:

$$\beta(\theta^{TAF}) = r_A \Rightarrow \theta^{TAF} = 1 - \frac{pk_A + r_A}{\delta R}.$$

Now, consider the equilibrium stigma cost:

$$k_A(\theta^{TAF}) = K - \kappa \int_0^{\theta^{TAF}} \int_0^{\theta_s} \frac{\theta dF(\theta)}{F(\theta_s)} dH(\theta_s) - \kappa \int_{\theta^{TAF}}^1 \int_0^{\theta^{TAF}} \frac{\theta dF(\theta)}{F(\theta^{TAF})} dH(\theta_s),$$

where $H(\theta_s)$ is the distribution of the m^{th} weakest bank of all; that is, $H(\theta_s) = \int_0^{\theta_s} h(\theta)d\theta$, where

$$h(\theta) = \binom{n}{m} F^{m-1}(\theta) f(\theta)(1-F(\theta))^{n-m}.$$

Rearranging the expression for θ^{TAF}, we have

$$[\delta R - r_A] - [\delta R \theta^{TAF} + pk_A(\theta^{TAF})] = 0. \tag{TAF}$$

The terms in the first pair of square brackets do not depend on θ^{TAF}. The terms in the second pair of square brackets can be expanded and rearranged as

$$(\delta R - p\kappa)\theta^{TAF} + pK + p\kappa \int_0^{\theta^{TAF}} \int_{\theta_s}^{\theta^{TAF}} \frac{\theta dF(\theta)}{F(\theta_s)} dH(\theta_s) + p\kappa \left[\theta^{TAF} - \int_0^{\theta^{TAF}} \frac{\theta dF(\theta)}{F(\theta^{TAF})} dH(\theta_s) \right].$$

The square bracket in the integral is increasing in θ^{TAF}, and the second term is also increasing in θ because each term in the integral (Bagnoli and Bergstrom, 2005, mean advantage over inferiors) is positive, as long as $\delta R > p\kappa$. The term in the third pair of square brackets in Equation (TAF) is decreasing in θ^{TAF}. Therefore, the left-hand side of Equation (TAF) is strictly decreasing in θ^{TAF}.

To show the existence of a unique solution to Equation TAF, it remains to show that its left-hand side is positive for $\theta^{TAF} = 0$ and negative for $\theta^{TAF} = 1$. When $\theta^{TAF} = 0$, its left-hand side is $\delta R - r_A - pk(0) > 0$, and when $\theta^{TAF} = 1$, its left-hand side equals $-r_A < 0$. Hence, there is a unique equilibrium. □

A.1.5 Proof of Lemma 1

Bank θ would borrow in DW if and only if $(1-\theta)R - r_D - pk_D \geq 0$, which simplifies to $\theta \geq \theta_D \equiv 1 - (r_D + pk_D)/R$. □

A.1.6 Proof of Lemma 2

Banks that could still get a positive payoff from borrowing in the discount window if they lose in the auction are willing to pay up to $\beta^D(\theta)$:

$$R(1-\theta) - r_D - pk_D = \delta R(1-\theta) - c - \beta^D(\theta) - pk_A.$$

Rearrange:

$$\beta^D(\theta) = r_D + pk_D - pk_A - (1-\delta)R(1-\theta) - c.$$

Note that the bid is increasing in θ, for $\theta < \theta_D$.

On the other hand, for banks that could not get a positive payoff from borrowing in the discount window, they are willing to pay up to $\beta^N(\theta)$:

$$0 = \delta R(1-\theta) - c - \beta^N(\theta) - pk_A.$$

Rearrange:

$$\beta^N(\theta) = \delta R(1-\theta) - c - pk_A.$$

Note that the bid is decreasing in θ, for $\theta > \theta_D$.

Altogether, the maximum WTP in the auction is

$$\beta(\theta) = \begin{cases} \beta^D(\theta) = r_D + pk_D - pk_A - (1-\delta)R(1-\theta) - c & \text{if } \theta < \theta_D, \\ \beta^N(\theta) = \delta R(1-\theta) - c - pk_A & \text{if } \theta \geq \theta_D. \end{cases}$$

Bank θ participates in the auction if its maximum WTP in the auction is greater than the minimum required bid r_A—that is, if the bank's type is between θ_1 and θ_A, where $\beta^D(\theta_1) = r_A$ and $\beta^N(\theta_A) = r_A$. Solving for those conditions and simplifying, we get

$$\theta_1 = 1 - \frac{r_D - r_A + pk_D - pk_A - c}{(1-\delta)R}, \quad \text{and} \quad \theta_A = 1 - \frac{r_A + c + pk_A}{\delta R}.$$

□

A.1.7 Proof of Lemma 3

By Lemma 1, banks borrow from the discount window if and only if

$$\theta \leq \theta_D = 1 - \frac{r_D + pk_D}{R}.$$

Of these banks, some are willing to wait for the auction if and only if

$$\theta > \theta_1 = 1 - \frac{r_D - r_A + pk_D - pk_A}{(1-\delta)R}.$$

Banks that borrow from the discount window would not participate in the auction if and only if $\theta_1 \geq \theta_D$, which is

$$1 - \frac{r_D - r_A + pk_D - pk_A}{(1-\delta)R} \geq 1 - \frac{r_D + pk_D}{R}.$$

The inequality can be simplified to

$$r_D + pk_D \geq \frac{r_D - r_A + pk_D - pk_A}{1-\delta},$$

which further simplifies to

$$r_D + pk_D - \delta(r_D + pk_D) \geq r_D + pk_D - r_A - pk_A,$$

which can be further simplified to $\delta \leq (r_A + pk_A)/(r_D + pk_D)$. Hence, in equilibrium, if $\delta \leq r_A/(r_D + pk_D^*)$, banks that would borrow from the discount window if they lost in the auction would not participate in the auction in the first place.

Knowing the condition derived above, we can directly verify that banks $\theta \in [0, \theta^{DW}]$ borrowing from the discount window immediately is part of an equilibrium. When banks $\theta \in [0, \theta^{DW}]$ borrow from the discount window, the equilibrium discount window stigma is $k_D^* = k_D^{DW}(\theta^{DW})$, and since we have the assumption $\delta \leq r_A/\left[r_D + pk_D^{DW}(\theta^{DW})\right]$, by the condition derived above, we have that no discount window bank would be willing to participate in the auction. Furthermore, since bank θ^{DW}, which should have the highest WTP in the auction, is not willing to participate in the auction, no bank will participate in the auction. $\square$

A.1.8 Proof of Theorem 3

An equilibrium is determined by three thresholds, θ_1, θ_D, and θ_A, where

$$\theta_D = 1 - \frac{r_D + pk_D}{R},$$

$$\theta_1 = 1 - \frac{r_D + pk_D - r_A - pk_A}{(1-\delta)R},$$

$$\theta_A = 1 - \frac{r_A + pk_A}{\delta R}.$$

Rearranging the three equations, we have

$$(1-\theta_D)R - r_D - pk_D = 0, \tag{DW2}$$

$$(1-\theta_1)(1-\delta)R - r_D - pk_D = r_A + pk_A, \tag{DW1}$$

$$(1-\theta_A)\delta R = r_A + pk_A. \tag{A}$$

The stigma costs are

$$k_D(\theta_D, \theta_1, \theta_A) = K - \kappa \frac{\int_0^{\theta_1} \theta dF(\theta) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)}{F(\theta_1) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)},$$

and

$$k_A(\theta_1, \theta_A) = K - \kappa \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_{s2}(s)} \frac{\theta dF(\theta)}{F(\theta_{s2}(s)) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A),$$

where $[\theta_{s1}(s), \theta_{s2}(s)]$ is the interval of types of banks winning the auction when s is the stop-out rate, and $H(s|\theta_1, \theta_2)$ is the distribution of the stop-out rate.

Plugging $k_A(\theta_1, \theta_A)$ into Equation (A), we have

$$\delta R - r_A - pK - (\delta R - p\kappa)\theta_A - p\kappa \left[\theta_A - \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_{s2}(s)} \frac{\theta dF(\theta)}{F(\theta_{s2}(s)) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A) \right] = 0.$$

The expression in the square brackets is mean advantage over inferiors for an order statistics distribution. Then by Chen et al. (2009), the order statistics distribution is log-concave. Hence, by Bagnoli and Bergstrom (2005, Theorem 5), the expression in the square brackets is increasing in θ_A. If $\delta R > p\kappa$, then the left-hand side of the equation above is strictly decreasing in θ_A. For each fixed θ_1, there is a unique θ_A that satisfies the equation. Let $\widetilde{\theta}_A(\theta_1)$ represent this function, and note that $\widetilde{\theta}_A(\theta_1)$ is strictly increasing in θ_1.

Plugging k_D into Equation (DW2) and rearranging, we have

$$R - r_D - pK - \theta_D R + p\kappa \frac{1}{\Delta} \int_0^{\theta_1} \theta dF(\theta) - p\kappa \frac{1}{\Delta} \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \widetilde{\theta}_A(\theta_1)) = 0,$$

where $\Delta = F(\theta_1) + \int_{r_A}^{\infty} \int_{\theta_{s1}(s)}^{\theta_D} \frac{dF(\theta)}{F(\theta_D) - F(\theta_{s1}(s))} dH(s|\theta_1, \theta_A)$ represents the denominator in the fractional part of the expression of k_D. The terms that include θ_D can be rearranged as

$$-\theta_D(R-p\kappa)-p\kappa\left[\theta_D-\frac{\int_0^{\theta_1}\theta dF(\theta)+\int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{\theta dF(\theta)}{F(\theta_D)-F(\theta_{s1}(s))}dH(s|\theta_1,\widetilde{\theta}_A(\theta_1))}{F(\theta_1)+\int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{dF(\theta)}{F(\theta_D)-F(\theta_{s1}(s))}dH(s|\theta_1,\widetilde{\theta}_A(\theta_1))}\right].$$

Again, the expression in the square brackets is mean advantage over inferiors for a truncated order statistics distribution, which continues to be log-concave, so it is increasing in θ_D. Therefore, for each θ_1, there is a unique θ_D that satisfies Equation (DW2). Let $\widetilde{\theta}_D(\theta_1)$ represent this function.

Plugging $\widetilde{\theta}_D(\theta_1)$, $\widetilde{\theta}_A(\theta_1)$, k_D, and k_A into Equation (DW1), we have

$$-r_D-r_A+(1-\delta)R-\theta_1(1-\delta)R-pk_D(\theta_1,\widetilde{\theta}_D(\theta_1),\widetilde{\theta}_A(\theta_1))-pk_A(\theta_1,\widetilde{\theta}_A(\theta_1))=0.$$

Using the same trick as before, we extract and rearrange all the terms that include θ_1:

$$\begin{aligned}&-\theta_1[(1-\delta)R-p\kappa]-pk_A(\theta_1,\widetilde{\theta}_A(\theta_1))\\&-p\kappa\left[\theta_1-\frac{\int_0^{\theta_1}\theta dF(\theta)+\int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{\theta dF(\theta)}{F(\theta_D)-F(\theta_{s1}(s))}dH(s|\theta_1,\theta_A)}{F(\theta_1)+\int_{r_A}^{\infty}\int_{\theta_{s1}(s)}^{\theta_D}\frac{dF(\theta)}{F(\theta_D)-F(\theta_{s1}(s))}dH(s|\theta_1,\theta_A)}\right].\end{aligned}$$

The expression is strictly decreasing for the same reason as in the previous argument, as long as $(1-\delta)R > p\kappa$. Therefore, there is a unique θ_1. □

A.1.9 Proof of Proposition 3

From the previous proof we see that the equilibrium condition for the banks that borrow from DW in the DW-and-TAF setting is

$$(1-\theta_D^*)R-r_D-pk_D^*=0.$$

Compare this condition to the equilibrium condition for banks that borrow from DW in the DW-only setting:

$$(1-\theta^{DW})R-r_D-pk^{DW}=0.$$

As long as $k^{DW} < k_D^*$, fewer banks are willing to borrow from DW in the DW-and-TAF setting. This condition indeed holds, because the strongest banks of the banks worse than θ_D win in the auction.

For total liquidity, consider the expected marginal borrower. The expected marginal borrower is better than θ^{DW}, because they borrow from the auction, and in the DW-and-TAF setting, the type distribution of banks winning in TAF first-order stochastically dominates that of banks borrowing form DW. ◻

A.1.10 Interbank Market

Suppose banks can borrow from the interbank market at a rate of $r > r_D$. The borrowing benefit is then $(1-\theta)R - r - p_I k_I$, where now I denotes borrowing from the interbank market. A bank borrows from the interbank market if and only if

$$(1-\theta)R - r > (1-\theta)R - r_D - pk_D.$$

The condition is simplified to $r < r_D + pk_D$. Hence, banks are willing to pay a higher interest rate in the interbank market to avoid the discount window, consistent with the empirical evidence. But for sufficiently large r, interbank market borrowing is not optimal even if it is available. ◻

A.1.11 Proof of Proposition 4

Banks $\theta \le \theta_D$ prefer borrowing from DW to not borrowing, where $\theta_D = 1 - (r_D + pk_D)/R$, as characterized in the proof of Proposition 1. Banks $\theta \le \theta_D$ bid $\beta(\theta) = r_D + pk_D - pk_A$, which follows from $(1-\theta)R - r_D - pk_D = (1-\theta)R - \beta(\theta) - pk_A$. If they participate in the auction, banks $\theta > \theta_D$ would bid $\beta(\theta) = (1-\theta)R - pk_A$, which follows from $(1-\theta)R - \beta(\theta) - pk_A = 0$. Only banks θ such that $\beta(\theta) \ge r_A$ participate in the auction. That is, only banks $\theta \le \theta_A$ participate in the auction, where $\theta_A = 1 - (r_A + pk_A)/R$ is derived from $(1-\theta_A)R - pk_A = r_A$.

Fix cutoffs θ_D and θ_A. The stigma cost of borrowing from DW is

$$k_D(\theta_D) = K - p\kappa \int_0^{\theta_D} \theta \frac{dF(\theta)}{F(\theta_D)}.$$

The stigma cost $k_A(\theta_D, \theta_A)$ of borrowing from TAF is lower, as some banks stronger than θ_D may obtain liquidity from TAF:

$$K - p\kappa \left[\int_0^{\theta_D} \int_0^{\theta_D} \frac{\theta dF(\theta)}{F(\theta_D)} dH(\theta_s) + \int_{\theta_D}^{\theta_A} \int_0^{\theta'} \frac{\theta dF(\theta)}{F(\theta')} dH(\theta_s) + \int_{\theta_A}^{1} \int_0^{\theta_A} \frac{\theta dF(\theta)}{F(\theta_A)} dH(\theta_s) \right],$$

where $H(\theta_s)$ is the distribution of the m^{th} weakest bank, that is, $H(\theta_s) = \displaystyle\int_0^{\theta_s} h(\theta) d\theta$, where

$$h(\theta_s) = \binom{n}{m} F^{m-1}(\theta_s) f(\theta_s) [1 - F(\theta_s)]^{n-m}.$$

In equilibrium, θ_D^* is uniquely pinned down by $R(1-\theta) - r_D - pk_D(\theta_D) = 0$, and θ_A^* is uniquely pinned down by $R(1-\theta_A) - r_A - pk_A(\theta_A, \theta_D^*) = 0$. The uniqueness follows from the monotonicity of the left-hand side of the two equations, which is argued in previous proofs. $\square$

A.1.12 Proof of Proposition 5

Bank θ, by borrowing in DW D, gets $u_D(\theta) = (1-\theta)R - r_D - pk_D$, and by borrowing in DW D' gets $u_{D'}(\theta) = \delta(1-\theta)R - r_{D'} - pk_{D'}$. Therefore, bank θ prefers borrowing from D to borrowing from D' if and only if

$$u_D(\theta) = (1-\theta)R - r_D - pk_D \geq u_{D'}(\theta) = \delta(1-\theta)R - r_{D'} - pk_{D'},$$

which is rearranged as

$$(1-\delta)(1-\theta)R - (r_D - r_{D'}) - (pk_D - pk_{D'}) \geq 0.$$

Hence, banks $\theta \leq \theta_1$ borrow from DW D, where

$$\theta_1 = 1 - \frac{(r_D - r_{D'}) + (pk_D - pk_{D'})}{(1-\delta)R}.$$

Furthermore, bank θ prefers borrowing from DW D' to not borrowing if and only if

$$u_{D'}(\theta) = \delta(1-\theta)R - r_{D'} - pk_{D'} \geq 0,$$

which is rearranged as

$$\theta \leq \theta_2 \equiv 1 - \frac{r_{D'} + pk_{D'}}{\delta R}.$$

To have banks borrowing from DW D', we must have $\theta_2 > \theta_1$, that is,

$$1 - \frac{r_{D'} + pk_{D'}}{\delta R} > 1 - \frac{(r_D - r_{D'}) + (pk_D - pk_{D'})}{(1-\delta)R},$$

$$\frac{r_{D'} + pk_{D'}}{\delta} < \frac{(r_D - r_{D'}) + (pk_D - pk_{D'})}{(1-\delta)},$$

which is rearranged as

$$\delta(r_D + pk_D) > r_{D'} + pk_{D'}.$$

Since banks $\theta \in [0, \theta_1]$ borrow from DW D, and banks $\theta \in (\theta_1, \theta_2]$ borrow from DW D', the stigma costs are

$$k_D(\theta_1) = K - \kappa \int_0^{\theta_1} \frac{\theta dF(\theta)}{F(\theta_1)} \quad \text{and} \quad k_{D'}(\theta_1, \theta_2) = K - \kappa \int_{\theta_1}^{\theta_2} \frac{\theta dF(\theta)}{F(\theta_2) - F(\theta_1)}.$$

Equilibrium θ_1 and θ_2 satisfy

$$(1-\delta)(1-\theta_1)R-(r_D-r_{D'})-pk_D(\theta_1)+pk_{D'}(\theta_1,\theta_2)=0 \quad \text{and} \tag{D1}$$

$$\delta(1-\theta_2)R-r_{D'}-pk_{D'}(\theta_1,\theta_2)=0. \tag{D2}$$

Plug $k_D(\theta_1)$ into and rearrange the left-hand side of Equation (D1):

$$(1-\delta)R-(r_D-r_{D'})-pK-[(1-\delta)R-p\kappa]\theta_1-p\kappa\left[\theta_1-\int_0^{\theta_1}\frac{\theta dF(\theta)}{F(\theta_1)}\right]+pk_{D'}(\theta_1,\theta_2).$$

The expression is strictly decreasing in θ_1 as long as $(1-\delta R)>p\kappa$. In addition, the expression is strictly decreasing in θ_2. Therefore, given any θ_2, there is a unique $\theta_1(\theta_2)$ that satisfies Equation (D1), and $\theta_1(\theta_2)$ is strictly decreasing in θ_2. Plug $k_{D'}(\theta_1,\theta_2)$ into and rearrange Equation (D2):

$$\delta R-r_{D'}-pK-(\delta R-p\kappa)\theta_2-p\kappa\left[\theta_2-\int_{\theta_1(\theta_2)}^{\theta_2}\frac{\theta dF(\theta)}{F(\theta_2)-F(\theta_1(\theta_2))}\right]=0. \tag{D2'}$$

Consider the derivative of $\theta_2-\int_{\theta_1(\theta_2)}^{\theta_2}\frac{\theta dF(\theta)}{F(\theta_2)-F(\theta_1(\theta_2))}$ with respect to θ_2. Fixing $\theta_1(\theta_2)$, the derivative is positive, because the expression is a mean advantage over inferiors for the truncated cdf $F(\theta)$ between $\theta_1(\theta_2)$ and θ_2. The derivative with respect to $\theta_1(\theta_2)$ is decreasing, but $\theta_1'(\theta_2)<0$. Hence, the derivative overall is increasing. Therefore, the left-hand side of Equation (D2) is strictly decreasing in θ_2, as long as $\delta R>p\kappa$, and there is a unique θ_2 that satisfies Equation (D2'). □

A.1.13 Proof of Proposition 6

Bank θ gets $(1-\theta)R-r_D-pk_D$ from D, and gets $(1-\theta)R-r_{D'}-pk_{D'}$ from D'. All banks are indifferent between the two facilities if $r_D+pk_D^*=r_{D'}+pk_{D'}^*$. Therefore, the average bank borrowing from D is worse than the average bank borrowing from D', and consequently the average bank of all borrowing banks is better than the average bank borrowing from D. The marginal bank θ^* satisfies $(1-\theta^*)R-r_D-pk_D^*=0$. However, if the average bank of all banks $\theta\in[0,\theta^*]$ is better than the average bank borrowing from D, $(1-\theta^*)R-r_D-p\left[K-\kappa\int_0^{\theta^*}\frac{\theta dF(\theta)}{F(\theta^*)}\right]>0$. Some banks $\theta>\theta^*$ would have borrowed if only D with interest rate $r_D<r_{D'}$ were offered. □

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Figure 1: Borrowing Amounts and Rates in DW and TAF from 2008 to 2010

(a) **Borrowing Amount** (b) **Borrowing Rates**

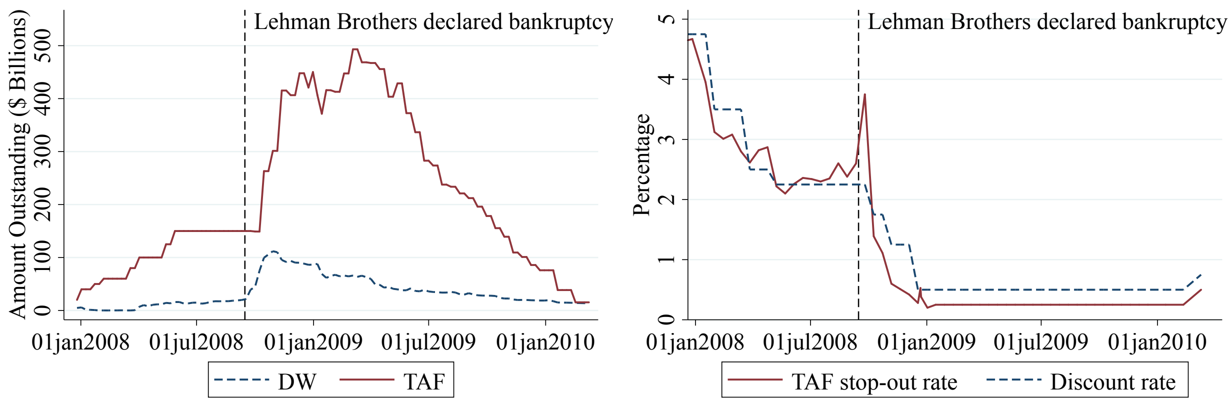

Figure 2: Differences in Borrowing Amounts and Rates in DW and TAF from 2008 to 2010

(a) **DW Balance as a Fraction of Total Balance** (b) **TAF Rate minus DW Rate**

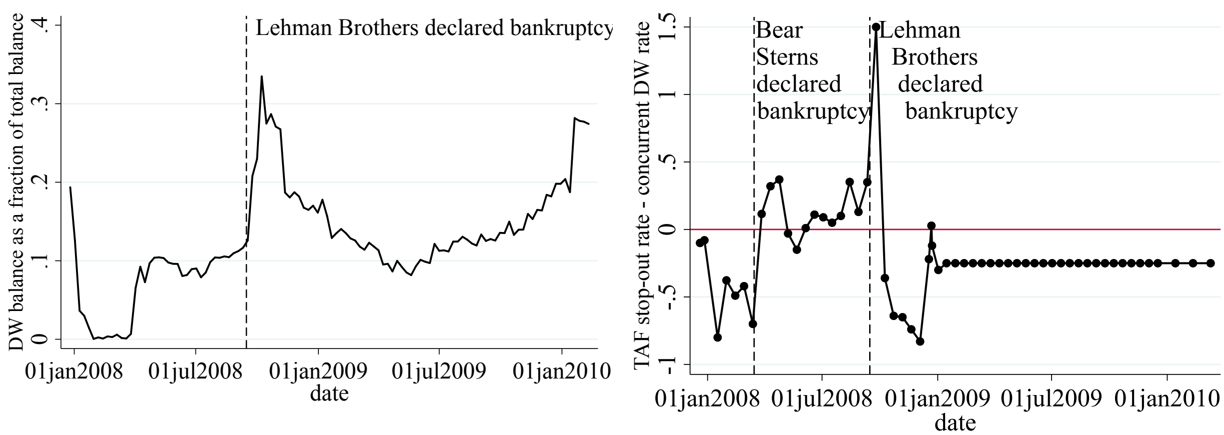

Figure 3: Timeline of the Model

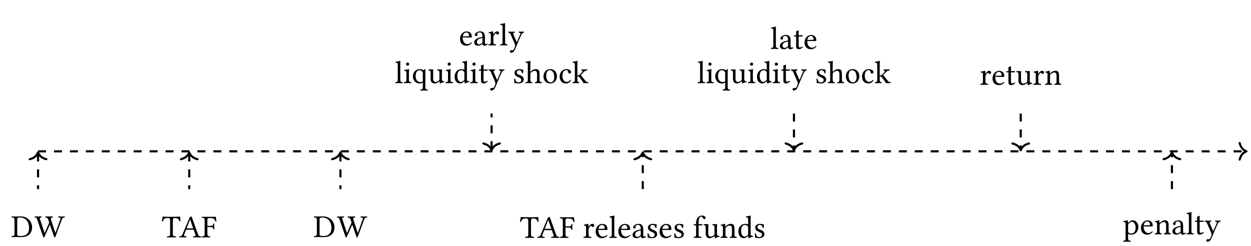

Figure 4: Facility Choice and TAF Bids in the DW-and-TAF Design

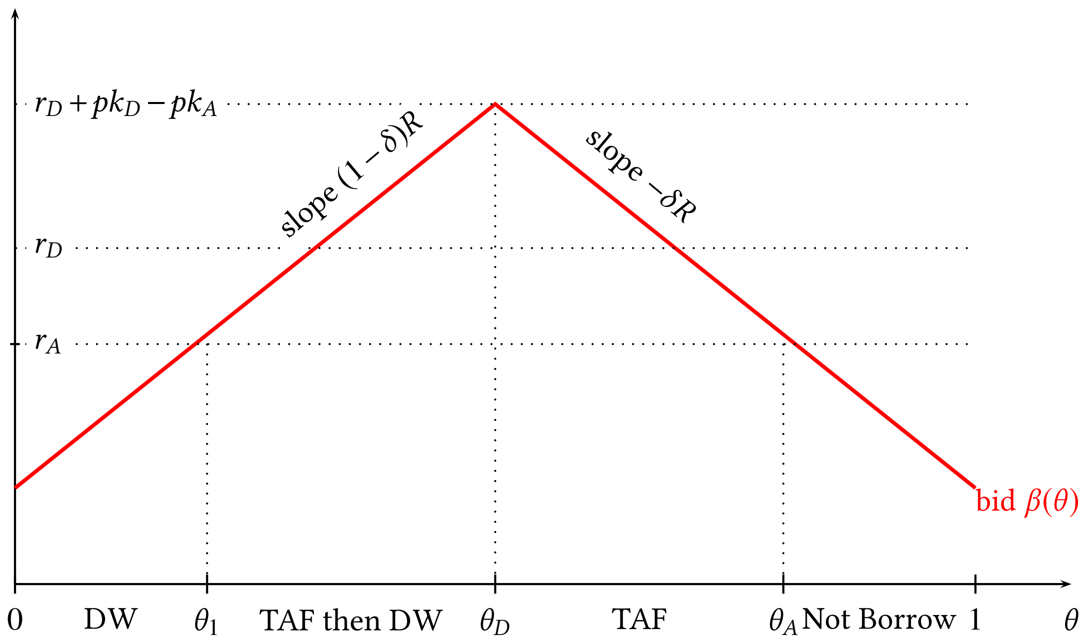

Figure 5: Facility Choice and TAF Bids in the DW-and-Immediate-TAF Design

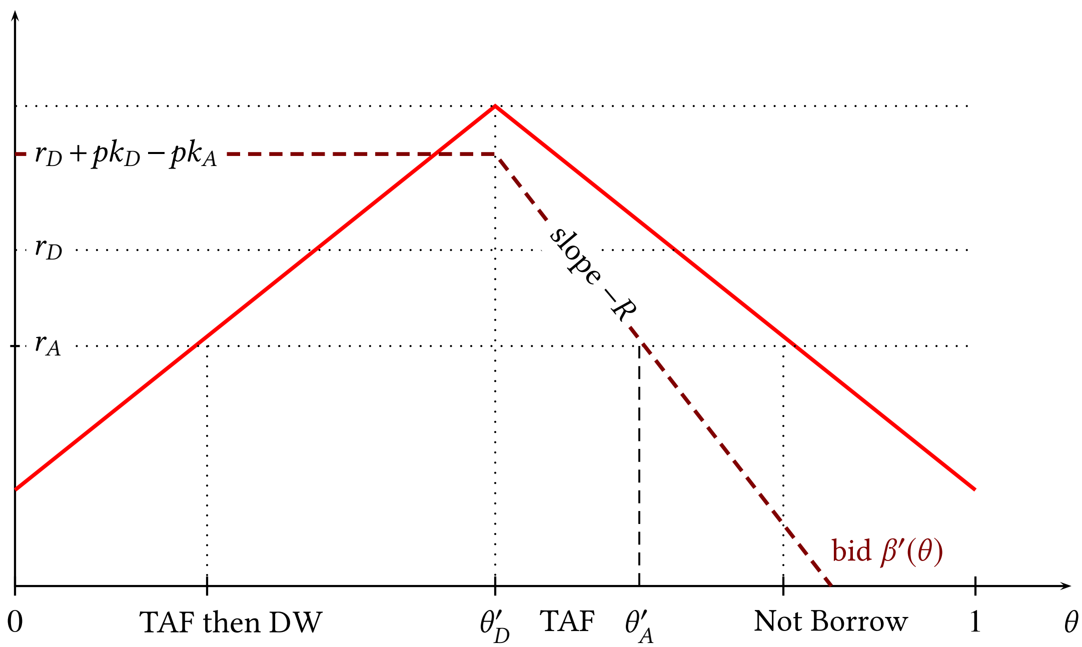

Figure 6: Comparison of Bloomberg Data and Fed Data

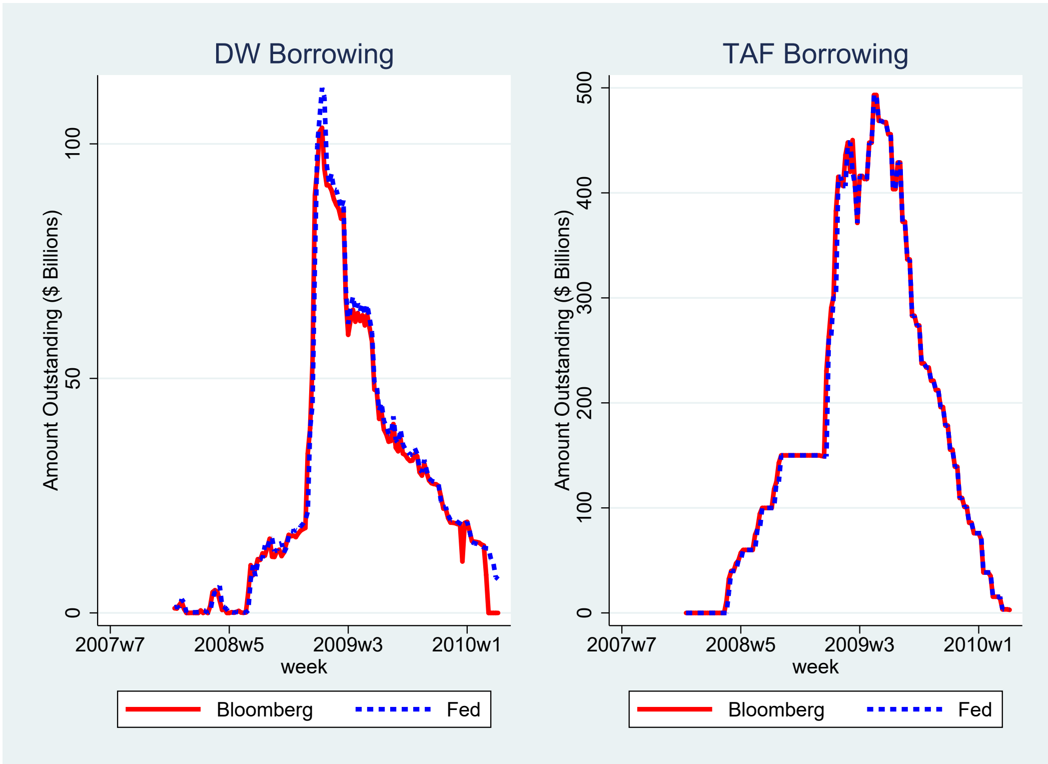

Figure 7: CDS Spreads around Borrowing Events

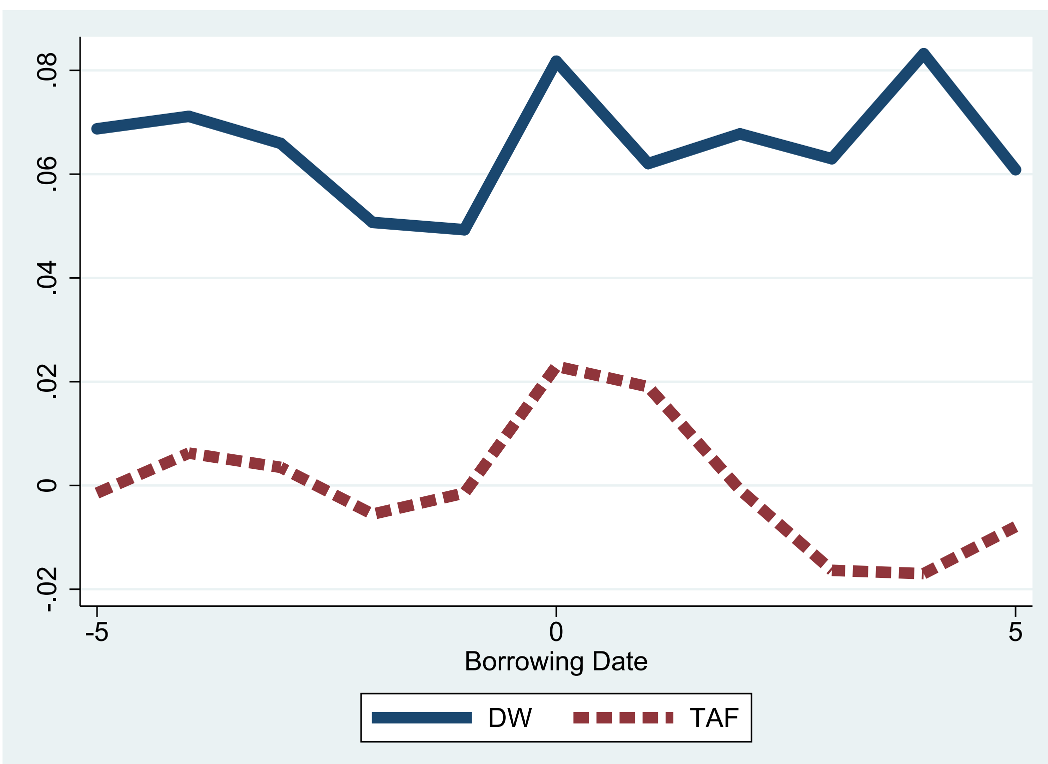

Table 1: Summary Statistics of Bloomberg Data

	N	Mean	Max	Min	SD	10^{th}	50^{th}	90^{th}
No. of Borrowers	407							
Banks	313							
Diversified Financial Services	24							
Insurance Companies	12							
Savings and Loans	30							
Market Cap on Aug 1, 2007 (MM)		28525	399089	11	49876.8	107	7331	81813
Foreign Banks	92							
DW-only banks	18							
TAF-only banks	86							
borrow both	260							
Total DW events		12	242	0	28.7	0	2	35
Total TAF events		5	28	0	5.1	0	3	13
Total DW amount (MM)		1529	190155	0	10393.8	0	20	1809
Total TAF amount (MM)		3174	100167	0	10727.5	0	58	7250
Number of days in debt to Fed		323	814	28	196.8	85	306	606

Table 2: LOLR Borrowing and Bank Fundamentals

	T1RWA	Lev	Liquid Asset/Asset	All	All
cond	-4.899***	3.316	-1.448*		
	(1.758)	(2.030)	(0.776)		
Lcond	3.501*	-1.294	1.779**		
	(1.850)	(2.068)	(0.796)		
Tier 1 Capital/Risk-Weighted Assets				-3.733***	
				(1.233)	
Book Leverage					2.185**
					(1.109)
Core Deposits/Assets				0.045	0.070
				(0.237)	(0.239)
Liquid Assets (CGHV) / Total Assets				0.199	-0.320
				(0.365)	(0.335)
MBS/Assets				1.502*	1.785**
				(0.783)	(0.779)
Unused commitments / total assets				0.206	0.247
				(0.363)	(0.366)
Short-Term Wholesale Funding/Assets				0.078	-0.003
				(0.327)	(0.330)
ROA	2.138	2.518	-2.529	8.093	7.455
	(4.265)	(4.322)	(4.239)	(5.417)	(5.524)
log(Size)	-0.056***	-0.054***	-0.060***	-0.074***	-0.062***
	(0.009)	(0.009)	(0.009)	(0.014)	(0.014)
Observations	571	571	571	381	381
Adjusted R^2	0.124	0.121	0.119	0.174	0.163

Table 3: LOLR Borrowing and Future Bank Fundamentals

	T1RWA	Lev	Liquid Asset/Asset
DW/(DW+TAF)	-0.001	0.002**	-0.003
	(0.001)	(0.001)	(0.003)
cond	0.927***	0.951***	0.952***
	(0.021)	(0.018)	(0.015)
ROA	0.236**	-0.258***	-0.650**
	(0.108)	(0.094)	(0.266)
log(Size)	0.000*	-0.000	-0.000
	(0.000)	(0.000)	(0.001)
Observations	574	574	574
Adjusted R^2	0.813	0.850	0.887

Table 4: DID and LOLR Borrowing

	All DW	All TAF	CAN DW	CAN TAF	DEU DW	DEU TAF	FRA DW	FRA TAF
DID	-0.423	1.281	-2.059**	5.224**	0.595	1.713	-3.195	-1.127
	(1.161)	(1.577)	(1.043)	(2.222)	(1.813)	(2.334)	(2.998)	(3.115)
Post	0.040	1.779***	0.051	2.048***	0.040	1.779***	0.040	1.779***
	(0.255)	(0.290)	(0.230)	(0.274)	(0.255)	(0.290)	(0.255)	(0.290)
Treat	1.354	10.890***	-0.323	12.524***	2.401	8.893***	2.417	14.780***
	(0.968)	(1.359)	(1.034)	(2.208)	(1.501)	(2.046)	(2.817)	(2.667)
Constant	2.333***	1.996***	2.332***	2.050***	2.333***	1.996***	2.333***	1.996***
	(0.220)	(0.231)	(0.181)	(0.191)	(0.220)	(0.231)	(0.220)	(0.231)
Observations	2728	2728	2560	2560	2632	2632	2560	2560
Adjusted R^2	0.001	0.159	0.001	0.089	0.009	0.082	-0.000	0.063

Table 5: LOLR Borrowing and Bank Failure

	Fail this quarter	Fail during crisis
DW/(DW+TAF)	0.007*	0.125**
	(0.004)	(0.050)
Constant	0.003	0.050***
	(0.002)	(0.019)
Observations	1586	364
Adjusted R^2	0.001	0.020

Table 6: CDS Spreads and Borrowing Events

	(1) DW/TAF	(2) DW/None	(3) TAF/None
Lagged 5y CDS spread	0.129**	0.004***	0.001
	(0.058)	(0.001)	(0.002)
Constant	1.257***	0.030***	0.004
	(0.279)	(0.007)	(0.010)
N	707	33440	33617
R^2	0.466	0.043	0.016