Preference Evolution in Different Matching Markets

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Abstract

We examine preference evolution under different matching market arrangements, when preferences are influenced by own choices and parental preferences. The dynamical system exhibits pitchfork bifurcation as the degree of sorting varies: Multiple stable equilibria arise under sufficiently random matching, but a unique equilibrium exists under sufficiently assortative matching. Market-differential evolutionary trajectories after transitory and permanent shocks allow us to shed light on the effects of marriage-market structure on social phenomena such as female labor force participation after World War II and the persistence of gender norms in developing countries.

Keywords: preference evolution, matching market, pitchfork bifurcation

JEL: C73, C78, Z13

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1 Introduction

Classic economic analyses often treat preferences as fixed and exogenous. Recent contributions investigate how preferences evolve across generations over time. One approach subjects preferences to natural selection and provides an evolutionary foundation for preferences, such as those on risk, time, and altruism.[1] Another approach assumes that preferences are shaped by family and society in a cultural transmission process.[2]

However, in these studies, preference is often assumed to only be affected by the preference or choice of one parent (or two parents treated as a unit). Intergenerational transmission of preferences is two-sided in reality, as both parents' preferences and choices influence their children genetically and culturally.[3] If both parents influence preferences, how parents pair in the matching market determines the effectiveness of parental influences. Therefore, the organization of the matching market must be taken into consideration to obtain a complete picture of the evolution of preferences in societies. To the best of our knowledge, the intergenerational transmission of preferences under two-sided matching is understudied.[4] The central goal of this paper is to investigate how different marriage market structures lead to different evolutionary processes and

[1]See Robson and Samuelson (2011); Alger and Weibull (2019); and Newton (2018) for surveys of the literature of preference evolution. The literature has studied preferences on risk (Robson, 1996b; Roberto and Szentes, 2017; Robson and Samuelson, 2019); time (Rogers, 1994; Robson and Samuelson, 2007; Robson and Szentes, 2008; Robson and Samuelson, 2009; Iantchev et al., 2012; Robson and Szentes, 2014); overconfidence (Zhang, 2013; Gannon and Zhang, 2020); social preferences, including altruism, reciprocity, and morality (Güth and Yaari, 1992; Güth, 1995; Sethi and Somanathan, 2001; Dekel et al., 2007; Alger and Weibull, 2010, 2013); and the interaction between institutions and evolution (Wu, 2017; Besley and Persson, 2018; Besley, 2020).

[2]Initiated by Bisin and Verdier (2000, 2001), a body of research seeks to explain a wide variety of cultural phenomena; see Bisin and Verdier (2011) for an extensive survey. Bisin and Verdier (2000) and Bisin et al. (2004) explain the persistence of ethnic differences and the coexistence of religious preferences in the United States, respectively. Fernández et al. (2004) attribute the increasing female labor force participation in the United States to the intergenerational transmission of gender norms after a temporary increase in female labor force participation triggered by World War II. Doepke and Zilibotti (2006) show that the rise of the middle class during the British Industrial Revolution was associated with the transmission of work ethic and patience. Tabellini (2008) demonstrates that historical institutional qualities may have a long-run impact on the current societal level of generalized trust through cultural transmission. Kuran and Sandholm (2008) account for psychological forces that drive the evolution of culture. Cheung and Wu (2018) provide a continuous-trait extension of the binary-trait Bisin-Verdier model. Bisin and Verdier (2017) model the co-evolution of culture and institutions.

[3]In the terminology of evolutionary economics, many previous models assume that reproduction is asexual, but in reality it is not.

[4]Several other papers consider preference formation in the presence of a two-sided marriage market, but they do not systematically investigate the importance of its structure. Robson (1996a) considers risk-taking in an assortative marriage market without frictions. Bisin and Verdier (2000) consider preference formation in a model with choices in random or assortative matching markets. Fernández et al. (2004) consider female labor force participation in a random marriage market. Mailath and Postlewaite (2006) demonstrate the social value of unproductive heritable traits in a stable matching model with intergenerational transmission. Bisin and Tura (2020) study cultural integration in a model of assortative marriage market and collective household decisions on fertility and cultural socialization. A recent paper (Cigno et al., 2020) is an exception that considers the effect of different two-sided matching technologies on the evolution of taste for filial attention and its implications on family rules.

distributions of preferences.[5]

In the model, matching technologies differ in the degree of assortativity, ranging from the least assortative—i.e., completely random matching—to the most assortative—i.e., perfectly positive assortative matching.[6]

Each person can be one of two preference types. We start with a simple model in which a man's type is inherited and a woman's type is by choice.[7] We will generalize the model so that both men and women inherit from both parents and make choices to determine their types. A woman's choice depends on whom she can marry—which is determined by the matching technology—and the cost associated with the choice. A woman's choice shapes her son's preference through intergenerational transmission.[8] For an example, a man's type represents his preference for either a working wife (type a) or a nonworking wife (type b), and a woman's type reflects whether she participates in the formal labor force (action a) or not (action b).[9]

The evolution of preferences differs by the matching technology. On the one hand, under random matching, as the fraction of type-a men increases, more women will be attracted to choose action a, as there is a higher chance of marrying a type-a man. Hence, the interaction between men and women takes a form similar to a coordination game; since men inherit their types from their mothers, there is intertemporal complementarity in women's actions. We find that generically, there exist two stable equilibria: one with type a being predominant and another one with type b being predominant.[10] On the other hand, under assortative matching, the marital prospect of a type-a woman is better when there are fewer type-a women. Therefore, the interaction between men and women takes a form similar to an anti-coordination game; there is intratemporal competition between women, in addition to intertemporal complementarity. We find that there always exists a unique stable equilibrium. In general, equilibria resemble those under the per-

[5]A recent empirical literature documents the historical determinants of preferences including agricultural technologies, geography, language, and family structure. See Giuliano (2020) and Nunn (2020) for recent surveys. However, the organization of the marriage market has not been considered yet.

[6]Such a comparative approach to understanding the impact of assortativity has been applied to study cooperation (Bergstrom, 2003; Bilancini et al., 2018); asexual preference evolution (Alger and Weibull, 2013); and income inequality (Kremer, 1997; Fernández and Rogerson, 2001).

[7]Obviously, the simple model can be applied to the mirror case in which men's types are by choice and women's types are by inheritance.

[8]See Fernández (2013) and Fernández et al. (2004) for evidence supporting the notion that men's preferences for working women are significantly affected by whether their mothers work. In the general analysis, we allow for a more general transmission mechanism.

[9]For another example, in the mirroring model in which women inherit their preferences and men choose actions, a woman's type is her preference for a blue or green beard, and a man's action is to dye his beard blue or green.

[10]There is an additional equilibrium with a more balanced distribution of types, but it is never stable. Many models that study cultural evolution feature multiple equilibria. See Hazan and Maoz (2002) and Fernández (2013) for models with multiple possible evolutionary paths of female labor force participation; Bénabou and Tirole (2006); Mailath and Postlewaite (2006); and Guiso et al. (2009) for models with multiple social norms; and Tabellini (2008), Bidner and Francois (2010, 2013), Belloc and Bowles (2013), Bisin and Verdier (2017), Besley and Persson (2018) and Besley (2020) for models with multiple institution-culture pairs.

fectly random setting when a sufficiently high proportion of couples are matched randomly and otherwise resemble those in the perfectly assortative setting.

The results demonstrate that the number and properties of equilibria crucially depend on the underlying two-sided matching technology. The matching technology influences not only who matches with whom but also, more importantly, individual choices that shape future generations' preferences and choices. The number and properties of equilibria in turn determine how shocks may impact the evolution of preferences under different matching technologies. Notably, a temporary shock to preferences and behavior can bring permanent paradigm shift only if the marriage market is sufficiently random and the shock itself is sufficiently large; otherwise the dynamic reverts back to the original equilibrium.

Differences in evolutionary trajectories after shocks enable us to shed light on a wide range of phenomena. First, our model may speak to how World War II contributed to the growth in female labor force participation through the channel of preference evolution in the United States, because it served as a tremendous transitory shock that boosted female labor force participation during the war. If couples sort predominantly randomly on the dimension of gender role attitudes, our model predicts that such a shock is able to overcome frictions in the marriage market and move social attitudes about working women, as well as the female labor force participation rate, to what they are today. Second, we can partly attribute the persistence of traditional gender norms in developing countries to the higher assortativeness on preferences. Our model predicts that under assortative matching, the dynamic always moves toward the unique equilibrium regardless of the transitory shock, which explains why a social norm persists as well as why neither a government campaign to change the preferences of a generation nor a temporary social or political event may result in a permanent change. Finally, our model is consistent with the findings that the historical male-biased sex ratio has a persistent effect on men having a more traditional gender attitude toward women. The male-biased sex ratio altered the bargaining position within a household, which permanently changed men's marital preferences. Our model demonstrates that a permanent shock to marital preferences will result in a permanent shift in the distribution of preferences in the long run in the direction consistent with the evidence.

The rest of the paper is organized as follows. Section 2 presents the model that illustrates the main insights. Section 3 investigates equilibria under different matching technologies—random matching, assortative matching, and any matching that mixes random and assortative matching. Section 4 investigates the evolution of preferences after transitory and permanent shocks. Section 5 discusses model implications. Section 6 presents the general model. Section 7 concludes.

2 The Model

We use the simplest possible model in this section to illustrate the main insights. Each person can be one of two types. In the simple model, each man's type is inherited from his mother, and a woman's type is determined by her choice. A motivating example would be that men's types represent their preferences for either a working wife or a nonworking wife, and women's types reflect whether they participate in the formal labor force or not. In Section 6, we generalize the model to one in which both men and women have types and choices and their types are determined jointly by inheritance from both parents and their own choices.

2.1 Basic Setup

There is a unit mass of men and a unit mass of women every period. All men and women pair up and each pair reproduces two children, one son and one daughter; equivalently, each child is a male or a female with equal probabilities. Each person is either type a or type b. Let p_t denote the mass of type-a men in period t. Assume that men's types are determined through intergenerational transmission, which is specified in Section 2.2. Before she enters the marriage market, each woman chooses to become type a or b by choosing action a or b, respectively. Whom they can marry is determined by the matching technology in the marriage market, which is specified in Section 3.

The cost difference in actions a and b is heterogeneous. We normalize the cost of action b to 0 and denote the cost of action a by c. Assume the cost is distributed according to a differentiable and strictly increasing distribution F with associated density f. Assume the density f is *single-peaked*: There exists a $\widehat{c}$ such that $f(c) \leq f(c') \leq f(\widehat{c})$ for any c and c' such that $c < c' < \widehat{c}$ or $c > c' > \widehat{c}$. For example, bell-shaped distributions, triangular distributions, and uniform distributions satisfy the condition.

Let $u_{t_w t_m}$ denote a type-t_w woman's utility from marrying a type-t_m man.[11]. We do not impose additional assumptions on the utility function other than *homophily*: $u_{aa} > u_{ab}$ and $u_{bb} > u_{ba}$.

The cost of choosing an action and the utility obtained through marriage, which depends on the matching technology, jointly determine a woman's optimal action choice. Let q_t denote the mass of women choosing action a in period t.

[11]In terms of the motivating example we provided at the beginning of this section, u_{ab} represents a working woman's utility of marrying a man who prefers a non-working wife. Note that before the lifted marriage bar in 1964, most women worked until marriage, and decided to stop working after being married. However, they did decide whether to invest in education before marriage. Hence, if we treat a as the decision to invest in education, then u_{ab} represents the utility of a woman who invested in education but decided to stop working after marrying a men who do not prefer a working wife.

2.2 Intergenerational Transmission

Let $\alpha_m(t_m, t_w)$ denote the probability that a son is type a given his father's type t_m and his mother's type t_w. One can impose different assumptions on $\alpha_m(t_m, t_w)$. We give two examples used in the literature.

Example 1 (Random-parent-to-son transmission). *$\alpha_m(a, a) = 1$, $\alpha_m(a, b) = \alpha_m(b, a) = \frac{1}{2}$, and $\alpha_m(b, b) = 0$. When both parents are of the same type, a son would adopt that type for sure. Otherwise, a son would randomly become either type a or type b. In other words, a homogamous marriage has a superior transmission technology compared with a heterogamous marriage, which is assumed in the model of Bisin and Verdier (2000) and is empirically supported by Dohmen et al. (2012) on the transmission of risk preferences and trust attitudes. This is a special case of the vertical transmission mechanism in Cavalli-Sforza and Feldman (1981), and is also considered in Mailath and Postlewaite (2006).*

Example 2 (Mother-to-son transmission). *$\alpha_m(t_m, t_w) = 1_{t_w=a}$. Each son's preference is solely influenced by his mother's type. A mother's influence on her son is documented in Fernández et al. (2004) and Fernández (2013).*

These two examples are special cases of a more general specification. Assume that a son adopts his father's type with probability h and his mother's type with probability $1 - h$, for $h \in [0, 1]$. When $h = \frac{1}{2}$, we have the first example. When $h = 0$, we have the second example. The value of h does not change the main results of the model. Therefore, for illustrative purposes, we focus on the simplest case: $h = 0$. In this case, the evolutionary dynamic of preferences is simply

$$p_{t+1} = q_t.$$

That is, the mass of type-a men in a period is the mass of action a women in the previous period.[12]

[12] The intergenerational transmission model we consider differs from that of Bisin and Verdier (2000, 2001) in two crucial ways. First, we only model the vertical transmission from parents to children without considering the oblique transmission in which children adopt preferences from peers or role models. We argue that adding the oblique transmission would not significantly affect the main insights of the model. To see why, suppose that a son instead adopts his mother's type with probability $\phi \in (0, 1)$, and randomly adopts the type of a role model in the society otherwise. In this case, the dynamic is given by

$$p_{t+1} = \phi q_t + (1 - \phi)\frac{p_t + q_t}{2} = \frac{1 + \phi}{2} q_t + \frac{1 - \phi}{2} p_t.$$

Compared with the dynamic generated in the case without oblique transmission, the new dynamic would result in the same stationary equilibria, which are defined in Section 2.3, and would behave similarly except for the speed of convergence to equilibria. Second, we do not explicitly model the decision process of parents to transmit their types to their children, which is a crucial factor for the phenomenon of cultural heterogeneity of Bisin and Verdier (2000, 2001). The main insights of our paper are instead driven by the incentives in the marriage market determined by its two-sided matching technology. Adding the parents-to-children decision process would not change either the

2.3 Equilibrium

The intergenerational transmission process gives rise to a dynamic that describes the evolution of preferences. Subsequently, we are interested in the stationary equilibria of the dynamic under different matching technologies. In a stationary equilibrium, each woman chooses her type to maximize her expected payoff, and the distribution of types is the same across periods. Any stationary equilibrium can be simply characterized by a cutoff cost c^*: Any woman with a cost below c^* chooses action a, and any woman with a cost above c^* chooses action b.

We say an equilibrium c^* is *stable under positive perturbations*, or *positive-stable*, if there exists an $\epsilon > 0$ such that women's optimal cutoff converges to c^* when there is initially mass $F(c^*)+\epsilon$ of type-a men. Similarly, we say an equilibrium c^* is *stable under negative perturbations*, or *negative-stable*, if there exists an $\epsilon > 0$ such that women's optimal cutoff converges to c^* when there is initially fraction $F(c^*)-\epsilon$ of type-a men. We say an equilibrium is *stable* if it is both positive-stable and negative-stable, is *unstable* if it is neither positive-stable nor negative-stable, and is *partially stable* if it is neither stable nor unstable (positive-stable but not negative-stable, or negative-stable but not positive-stable).

3 Equilibria under Different Matching Markets

In this section, we characterize equilibria of the simple model under different matching technologies. We first consider a marriage market with completely random matching, which can be thought of as an environment with high frictions such that people are unable to sort according to types. Second, we consider assortative matching in which women are free to match with men they like, though they may need to compete with one another when there is a shortage of likable men. Given the assumption of homophily, homogamous marriages will be the most frequent in such an environment. Finally, we investigate intermediate cases by varying the level of friction.

3.1 Random Matching

Suppose men and women are randomly matched. That is, in period t, given mass p_t of type-a men, any woman marries a type-a man with probability p_t and a type-b man with probability $1 - p_t$. Under the random matching technology, compared with action b, action a for a woman yields a gain $u_{aa} - u_{ba}$ when she marries a type-a man, which happens with probability p_t, and a loss $u_{bb} - u_{ab}$ when she marries a type-b man, which happens with probability $1 - p_t$. Hence, a woman chooses action a if and only if the (net) cost of the action is lower than the expected benefit, or equivalently, the cost is lower than a cutoff cost that depends on the distribution of

number of equilibria or their properties. Therefore, we abstract away from the parents-to-children decision process to elucidate the marriage-market effect.

men's preferences:

$$c \leq p_t(u_{aa} - u_{ba}) - (1 - p_t)(u_{bb} - u_{ab}) \equiv c_R(p_t).$$

The cutoff cost function $c_R(p_t)$ has a positive slope $\Delta \equiv (u_{aa} - u_{ab}) + (u_{bb} - u_{ba})$, which is the sum of the gains from homogamous marriages. A positive slope of the function means that more women choose action a when more men are type a. A steeper slope, which results from higher gains from homogamous marriages, leads women to be more responsive to changes in the distribution of men's preference types. Since the distribution of men's preference types is determined by the choices made by women from the previous generation, the slope Δ serves as a measure of the intertemporal complementarity between the choices of women.

When the cutoff cost in period t is c_t, because the mass of type-a men is determined by the mass of women choosing action a in the previous period—$p_{t+1} = F(c_t)$—the cutoff cost in period $t+1$ is $c_R(F(c_t))$. The change in men's and women's type distributions is $F(c_R(F(c_t))) - F(c_t)$, and the change in the cutoff cost is

$$c_R(F(c_t)) - c_t \equiv \psi_R(c_t).$$

When $\psi_R(c_t)$ is positive (negative), the cutoff increases (decreases), so more (fewer) women choose action a in the current period than in the previous period. Stationary equilibrium c^* satisfies $\psi_R(c^*) = 0$.

The slope of ψ_R is $\psi_R'(c) = f(c)\Delta - 1$. If $f(\widehat{c})\Delta > 1$, then the slope of $\psi_R(c)$ is negative unless c is sufficiently close to $\widehat{c}$. Namely, there are two solutions to $\psi_R'(c) = 0$, denoted by $\underline{c}$ and $\overline{c} > \underline{c}$. When $c < \underline{c}$ or $c > \overline{c}$, $\psi_R(c)$ is decreasing, and when $c \in (\underline{c}, \overline{c})$, $\psi_R(c)$ is increasing. The function $\psi_R(c)$ when $f(\widehat{c})\Delta > 1$ is depicted in Figure 1a. When $\psi_R(c)$ is decreasing, the dynamic is converging, and an equilibrium is stable if there is any. When $\psi_R(c)$ is increasing, the dynamic is diverging, and an equilibrium is unstable if there is any.[13] To summarize, we have the following characterization of equilibria under random matching.

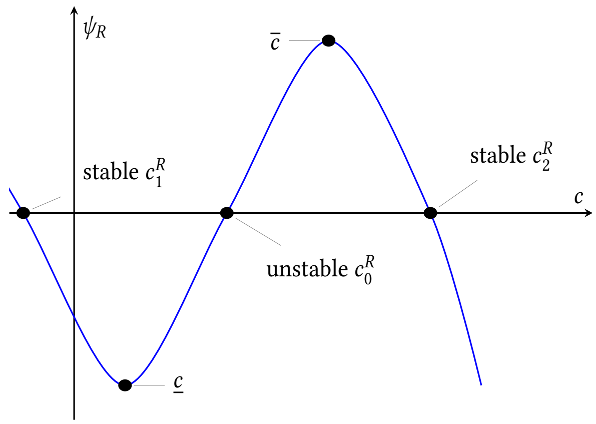
Proposition 1 (Equilibria under Random Matching). *Suppose agents are randomly matched. If $f(\widehat{c})\Delta > 1$ and $\psi_R(\underline{c}) < 0 < \psi_R(\overline{c})$, where $\underline{c}$ and $\overline{c}$ are the two solutions to $\psi_R'(c) = 0$, there are two stable equilibria $c_1^R < \underline{c}$ and $c_2^R > \overline{c}$ and one unstable equilibrium $c_0^R \in (\underline{c}, \overline{c})$.*

The conditions $f(\widehat{c})\Delta > 1$ and $\psi_R(\underline{c}) < 0 < \psi_R(\overline{c})$ are necessary and sufficient for the existence of two stable equilibria. The case with two stable equilibria described in Proposition 1 is the case we will focus on, and it is depicted in Figure 1a. We characterize, in the proof of Proposition 1,

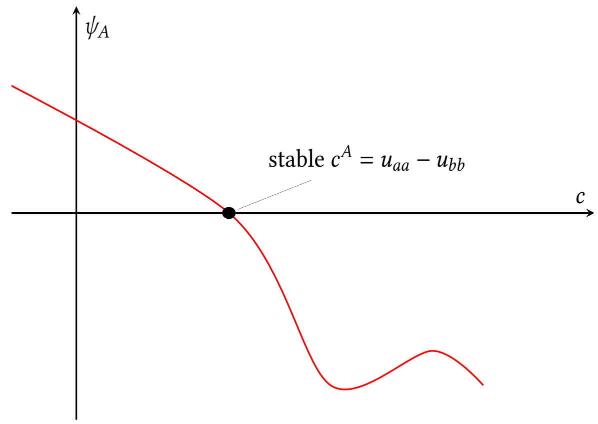
[13] The dynamic is converging and any equilibrium is stable when $\psi_R(c)$ is decreasing, because if $\psi_R(c^*) = 0$, $\psi_R(c)$ for any $c < c^*$ is positive, so $c_R(F(c)) > c$, and $\psi_R(c)$ for any $c > c^*$ is negative, so $c_R(F(c)) < c$. The dynamic is diverging and any equilibrium is unstable when $\psi_R(c)$ is increasing, because if $\psi_R(c^*) = 0$, $\psi_R(c)$ for any $c < c^*$ is negative, so $c_R(F(c)) < c < c^*$, and $\psi_R(c)$ for any $c > c^*$ is positve, so $c_R(F(c)) > c > c^*$.

Figure 1: Equilibria under Random Matching versus Assortative Matching.

(a) Random Matching: Two stable equilibria c_1^R and c_2^R and one unstable equilibrium c_0^R.



(b) Assortative Matching: One stable equilibrium c^A and no other equilibrium.



the cases in which these conditions do not hold. There is one stable equilibrium, and potentially, another partially stable equilibrium.

Another way to present the case we consider is provided in Figure 2a. The graph depicts the relation between p_t and p_{t+1}. The existence of two stable equilibria relies on $F(C_R(p_t))$ being sufficiently "S-shaped." A reduction in the variance of F and/or an increase in Δ helps to make $F(C_R(p_t))$ more "S-shaped." In other words, it is more likely to have two stable equilibria when the environment is less volatile and/or the intertemporal complementarity is stronger.

The dynamic incentive structure of our model under random matching is similar to that of an evolutionary model of coordination games. Women are trying to "coordinate" on the action that matches the prevalent type of men, which is inherited from the actions of the previous generation of women, leading to two distinct social conventions: one with type-a men predominant and more women choosing action a, and another with type-b men predominant and more women choosing action b.

3.2 Assortative Matching

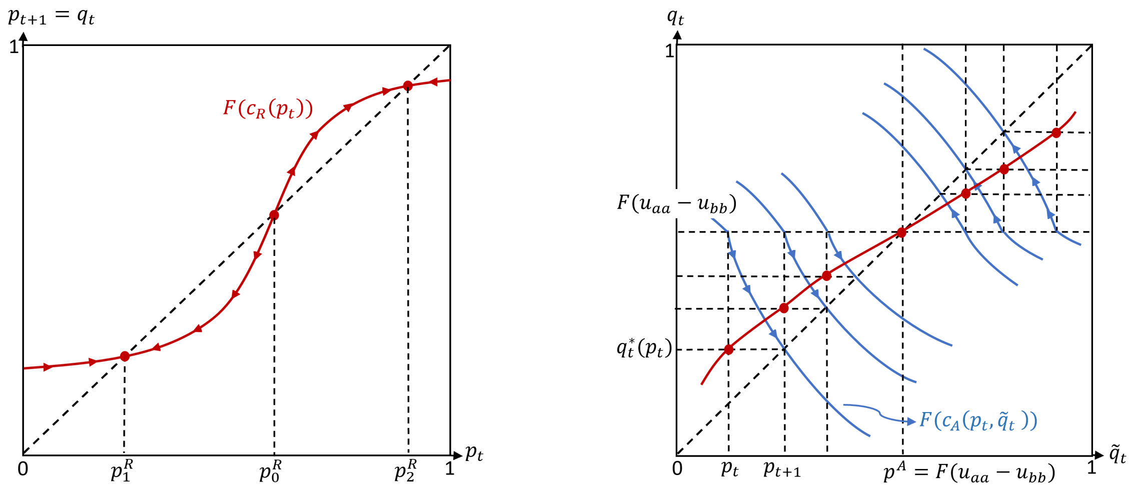
Suppose men and women are positive assortatively matched. When the distributions of types are identical across sexes, the matching would exhibit perfect assortativity, as type-a men and women marry, and type-b men and women marry. When there is an imbalance of types, there are cross-type marriages. For example, if there are more type-a women than type-a men, then there are some cross-type marriages between type-a women and type-b men.

The payoff difference between the two actions depends on the relative distributions of preferences of men and women. Let q_t represent the mass of women choosing action a in period t. Suppose $q_t > p_t$. A type-a woman marries a type-a man with probability p_t/q_t and marries

Figure 2: An Alternative Representation of the Equilibria.

(a) Random Matching: Two stable equilibria p_1^R and p_2^R and one unstable equilibrium p_0^R.

(b) Assortative Matching: One stable equilibrium p^A and no other equilibrium.



a type-b man with probability $1 - p_t/q_t$, so that a woman's expected payoff from action a is $\frac{p_t}{q_t}u_{aa} + (1 - \frac{p_t}{q_t})u_{ab} - c$. A type-$b$ woman marries a type-b man for sure, so that her payoff from action b is u_{bb}. We can follow a similar logic to derive a woman's expected payoff when $q_t = p_t$ and when $q_t < p_t$. In summary, a woman chooses action a if and only if $c \le c_A(p_t, q_t)$, where

$$c_A(p_t, q_t) = \begin{cases} \frac{p_t}{q_t}u_{aa} + \left(1 - \frac{p_t}{q_t}\right)u_{ab} - u_{bb} & q_t > p_t \\ u_{aa} - u_{bb} & q_t = p_t \\ u_{aa} - \left(\frac{p_t - q_t}{1 - q_t}u_{ba} + \frac{1 - p_t}{1 - q_t}u_{bb}\right) & q_t < p_t \end{cases}.$$

Note that the function $c_A(p_t, q_t)$ is continuous and strictly increasing in p_t, and is continuous and strictly decreasing in q_t. That is, when there are more type-a men, more women would choose action a, but when there are more type-a women, fewer women would choose a. Hence, two effects on the choices of women are present under assortative matching: intertemporal complementarity and intratemporal competition.

Under random matching, women's optimal decisions are purely driven by the distribution of men's preferences and do not depend on other contemporaneous women's actions. In contrast, under assortative matching, *women are playing a game with one another because their decisions take into account what other women choose.* Given p_t, the mass of type-a men in the market, the mass of women choosing action a in period t is given by $F(\widetilde{c})$, where $\widetilde{c}$ is the unique value that satisfies the following equation.[14]

$$c_A(p_t, F(\widetilde{c})) - \widetilde{c} = 0.$$

In a stationary equilibrium of the simple model, the distributions of preference types must be identical for the two sexes. Otherwise, the mass of type-a men will change in the next period.[15] Also, the equilibrium cutoff cost c^A must coincide with the unique cutoff cost simultaneously determined by all women's choices. Therefore, it satisfies

$$c_A(F(c^A), F(c^A)) - c^A = 0.$$

Since there is no imbalance in types across sexes in the stationary equilibrium, a type-a woman gets u_{aa} and a type-b woman gets u_{bb}. Hence, the equilibrium cutoff cost c^A is the difference between the two homophily payoffs, $u_{aa} - u_{bb}$, and the equilibrium mass of type-a men is $F(c^A)$.

To determine the stability of the unique equilibrium, we need to check that the dynamic is converging. Namely, define the change in the cutoff costs,

$$\psi_A(c) \equiv \widetilde{c}(c) - c,$$

where $\widetilde{c}(c)$ is the current period's cutoff cost when the previous period's cutoff cost is c. We need to check that $\psi_A(\cdot)$ is decreasing at the equilibrium.[16] In summary,

Proposition 2 (Equilibria under Assortative Matching). *Suppose agents are positively assortatively matched. There exists a unique equilibrium $c^A = u_{aa} - u_{bb}$, and it is stable and stationary.*

The case described in Proposition 2 is depicted in Figure 1b. The intuition for Proposition 2 is best described by Figure 2b. Let $\widetilde{q}_t$ denote the belief of a woman in period t about the proportion of women in period t choosing action a. Suppose the proportion of type-a men is smaller than the equilibrium one: $p_t < p^A = F(u_{aa} - u_{bb})$. If a woman believes that $\widetilde{q}_t$ equals p_t, then she should expect that the fraction of women choosing action a equals $F(u_{aa} - u_{bb}) = p^A$, which is greater than $\widetilde{q}_t = p_t$. Hence, the woman's belief is not correct. Since an equilibrium of the game played by all women requires equilibrium knowledge, the woman should adjust her belief up until her belief is consistent with the actual fraction of women choosing action a, $\widetilde{q}_t = q_t^*(p_t)$, which results in an increase in the fraction of type-a men in the next period. The hypothetical

[14] The uniqueness of the solution follows from continuity and strict monotonicity of c_A in q_t.

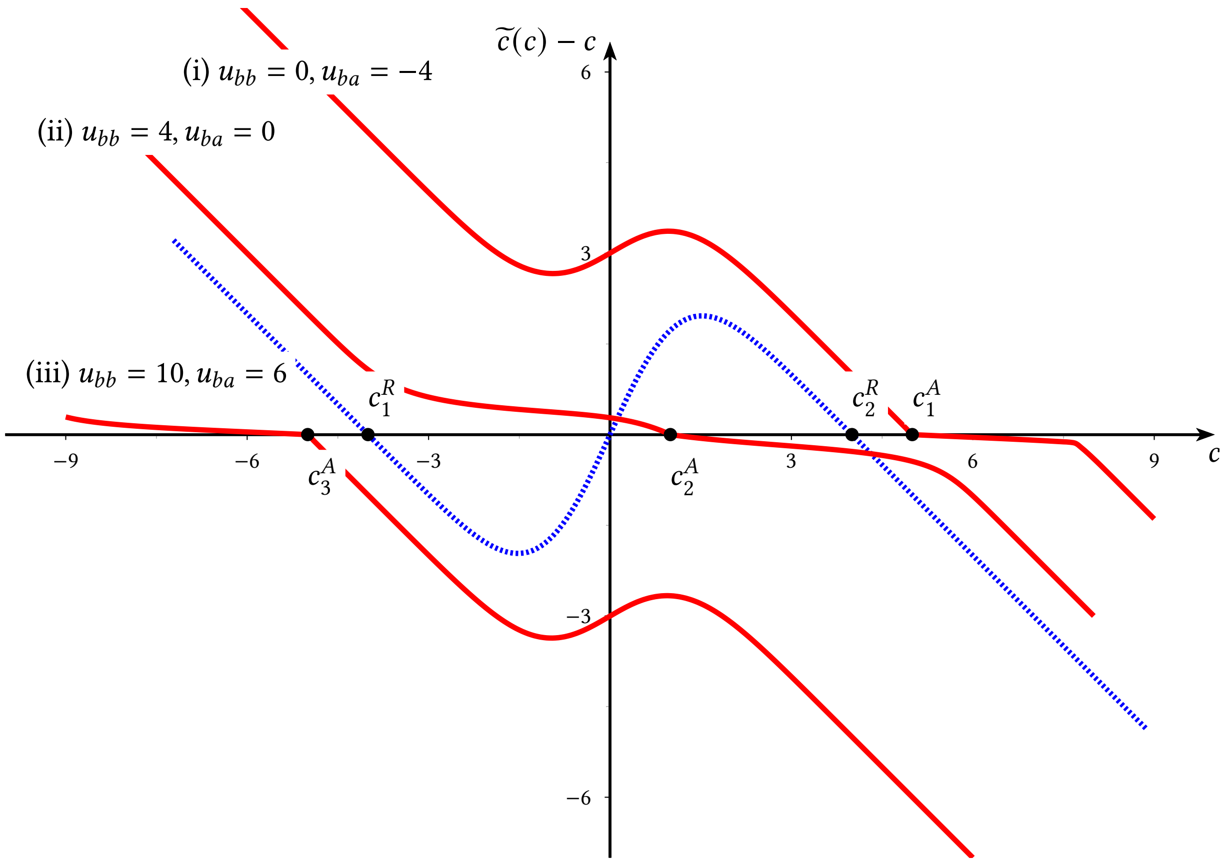
[15] We show in the proof of Proposition 2 the nonexistence of a nonstationary equilibrium.

[16] The function $\psi_A(\cdot)$ may not be decreasing at all c, as illustrated by Figure 1b, but for the purpose of proving a unique equilibrium, it suffices to show that it satisfies a single-crossing property: $\psi_A(c^A) = 0$, $\psi_A(c) > 0$ for any $c < c^A$, and $\psi_A(c) < 0$ for any $c > c^A$.

belief adjustment process described above is depicted by the blue curves in the graph. The red curve depicts the dynamic relation between p_t and p_{t+1} (if we replace the x-axis label with p_t and the y-axis label with p_{t+1}), which leads to the unique equilibrium p^A.

Interestingly, even though more women tend to "coordinate" on an action when more men are of the corresponding type under assortative matching because of the intertemporal complementarity, the evolutionary trajectory in fact resembles that of an anti-coordination game because of the intratemporal competition. More specifically, the better prospect of marrying a type-a man induces more women to compete for type-a men when the fraction of type-a men, p_t, is smaller than p^A. Similarly, the opposite is true when $p_t > p^A$.

Figure 3: Random-Matching versus Assortative-Matching Stable Equilibria.



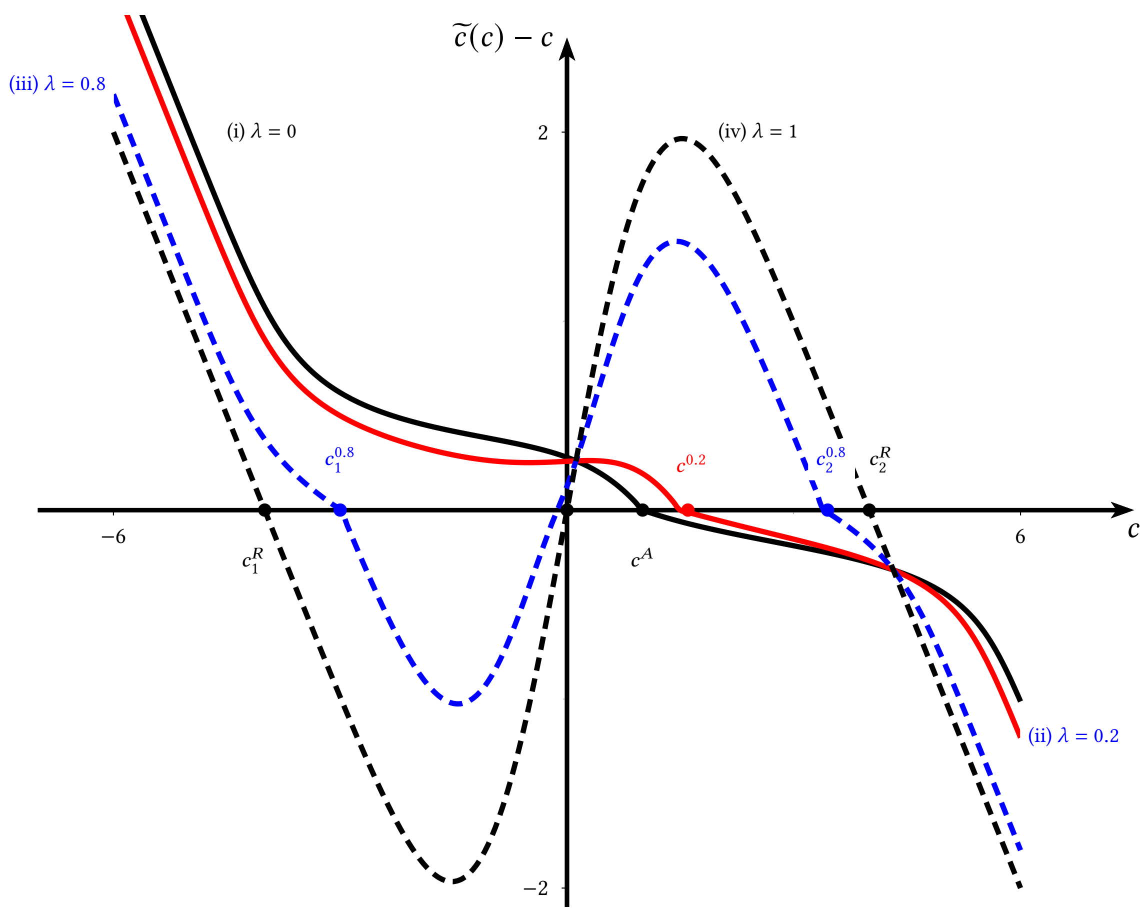
Note: $c \sim N(0,1)$, $u_{aa} = 5$, and $u_{ab} = 1$. We keep $u_{bb} - u_{ba} = 4$ so that the two equilibria under random matching are characterized by the intersections the blue dashed line and the x-axis: $c_1^R = -4$ and $c_2^R = 4$. The one assortative-matching equilibrium can be (i) $c_1^A = 5$ when $u_{bb} = 0$ and $u_{ba} = -4$, bigger than, (ii) $c_2^A = 1$ when $u_{bb} = 4$ and $u_{ba} = 0$, between, or (iii) $c_3^A = -5$ when $u_{bb} = 10$ and $u_{ba} = 6$, smaller than the two random-matching equilibria.

It is worth noting that the equilibrium distribution of types is *not necessarily* more balanced under assortative matching than under random matching, ceteris paribus. Figure 3 shows the possible relationships between the two stable equilibria under random matching and the unique stable equilibrium under assortative matching. The equilibrium mass of type-a women under assortative matching can be (i) bigger than, (ii) between, or (iii) smaller than the two possible equilibrium masses of type-a women under random matching.

3.3 Mixed Matching

Finally, we combine the two extremes—the random matching market and the assortative matching market—and consider the intermediate cases in which both markets operate. Suppose that each person marries in the random matching pool with probability λ and in the assortative matching pool with probability $1 - \lambda$. Therefore, λ captures the degree of randomness—or, in other words, the level of friction—in the matching market.

Figure 4: Stable Equilibria under Mixed Matching.



Note: Fix $c \sim N(0, 1)$, $u_{aa} = 5$, $u_{bb} = 4$, $u_{ab} = 1$, and $u_{ba} = 0$. (i) $\lambda = 0$ (perfectly assortative matching): A unique stable equilibrium c^A; (ii) $\lambda = 0.2$ (predominantly assortative matching): A unique stable equilibrium $c^{0.2}$; (iii) $\lambda = 0.8$ (predominantly random matching): Two stable equilibria $c_1^{0.8}$ and $c_2^{0.8}$; (iv) $\lambda = 1$ (perfectly random matching): Two stable equilibria c_1^R and c_2^R.

When random matching is prevalent, there may exist two stable equilibria, but when assortative matching is prevalent, there is only one stable equilibrium. Figure 4 demonstrates four cases, in which λ takes the value of 0, 0.2, 0.8, and 1, respectively. When $\lambda = 0.2$, there is one stable equilibrium, which resembles the equilibrium under assortative matching. When $\lambda = 0.8$, there are two stable equilibria, which resembles those under random matching. Moreover, equilibria in the intermediate mixed-matching environment are between the stable equilibria in the extreme cases of random and assortative matching environments. For example, let c_1^R and c_2^R denote the two random-matching stable equilibria and c^A the unique assortative-matching stable equilibrium. The two stable equilibria when $\lambda = 0.8$, $c_1^{0.8}$ and $c_2^{0.8}$, are between c_1^R and c^A and between c^A and c_2^R, respectively.

Furthermore, we can show that there is one stable equilibrium if the degree of friction is lower than some critical degree λ^*, and there are two stable equilibria otherwise. We call a marriage market with $\lambda \leq \lambda^*$ **predominantly assortative** and a marriage market with $\lambda > \lambda^*$ **predominantly random**. The existence of a unique critical degree of friction λ^* that separates the number of stable equilibria depends on the fact that the dynamic describing the change in the cutoff cost, ψ_λ, is a linear combination of ψ_R and ψ_A. Figure 5 shows bifurcation diagrams, i.e., the set of equilibria as λ shifts from 0 to 1. In the language of bifurcation theory, we have a *pitchfork bifurcation*: The system transitions from having one fixed point to having three fixed points as frictions increase.[17]

Proposition 3 (Equilibria under Mixed Matching). *There exists a critical degree of friction λ^* such that there is one stable equilibrium when $\lambda \leq \lambda^*$, and there are two stable equilibria when $\lambda > \lambda^*$.*

We can measure λ from observational data. Suppose we observe the distributions of preference types for men and women (summarized by $\widehat{p}$ and $\widehat{q}$) and the matches between different preference type; in particular, there is a mass $\widehat{\mu}_{aa}$ of type-a matches. We know that $\widehat{\mu}_{aa} = (1-\lambda)\min\{\widehat{p},\widehat{q}\} + \lambda\widehat{p}\widehat{q}$, which yields[18]

$$\widehat{\lambda} = \frac{\min\{\widehat{p},\widehat{q}\} - \widehat{\mu}_{aa}}{\min\{\widehat{p},\widehat{q}\} - \widehat{p}\widehat{q}}.$$

In our discussion of model implications, observed assortativeness, $1-\widehat{\lambda} = \frac{\widehat{\mu}_{aa}-\widehat{p}\widehat{q}}{\min\{\widehat{p},\widehat{q}\}-\widehat{p}\widehat{q}}$, will provide an indication of whether the dynamical system has one or multiple stable equilibria.

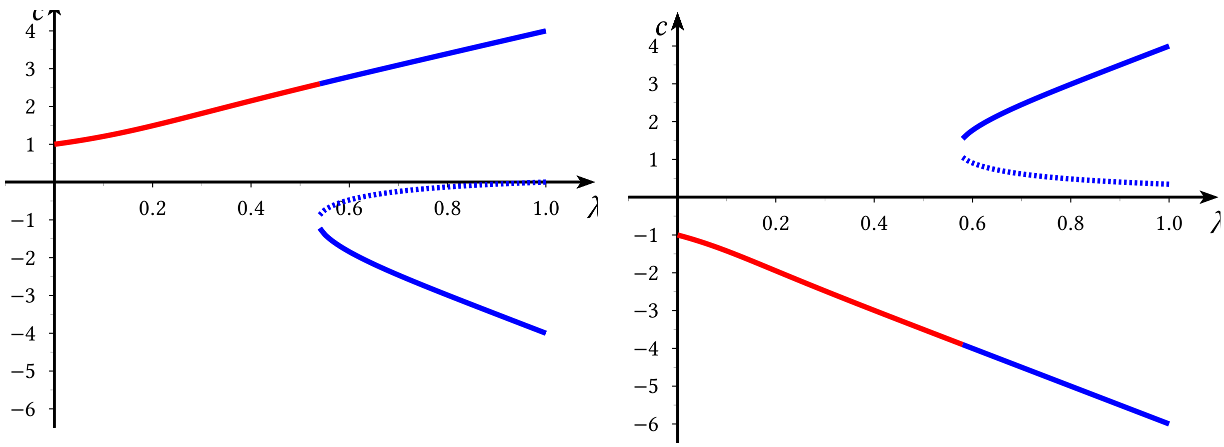
[17] More precisely, the dynamical system has a supercritical imperfect pitchfork bifurcation.

[18] We can derive the same characterization from any other mass of matches (i.e., $\widehat{\mu}_{ab}$, $\widehat{\mu}_{ba}$, or $\widehat{\mu}_{bb}$), so λ is exactly identified.

Figure 5: Pitchfork Bifurcation.

Note: $c \sim N(0,1)$, $u_{aa} = 5$, $u_{ab} = 1$, and $u_{ba} = 0$. The solid lines represent stable equilibria, and the dotted lines represent unstable equilibria. The red lines represent the low level of friction in which there is a unique equilibrium.

(a) $u_{bb} = 4$. **(b)** $u_{bb} = 6$.



3.4 Welfare Comparison

Only women's marriage utilities and costs are defined in the simple model, so we use the average payoff of women as the criterion for welfare analysis. To obtain sharp predictions, we assume that the range of c is $[u_{ab} - u_{bb}, u_{aa} - u_{ba}]$. In this case, the two stable equilibria under random matching are $c_1^R = u_{ab} - u_{bb}$ and $c_2^R = u_{aa} - u_{ba}$, and the average payoffs of women for these two equilibria are $W(c_1^R) = u_{bb}$, and $W(c_2^R) = u_{aa} - \int_{u_{ab}-u_{bb}}^{u_{aa}-u_{ba}} c dF(c)$. The unique stable equilibria under assortative matching $c^A = u_{aa} - u_{bb}$ corresponds to an average payoff of women that equals to

$$W(c^A) \quad = \quad F(c^A)u_{aa} + (1 - F(c^A))u_{bb} - \int_{u_{ab}-u_{bb}}^{c^A} c dF(c).$$

We have $W(c^A) > W(c_1^R)$ and $W(c^A) > W(c_2^R)$. Hence, assortative matching gives a higher average payoff to women than random matching. Nevertheless, the different equilibria may have implications outside the model's specifications. For example, if action a corresponds to female labor force participation, then high female labor participation may be beneficial to the entire society.

4 Evolution of Preferences

In this section, we investigate how the effects of transitory and permanent changes in preferences and matching technology on the equilibrium distribution of preferences differ by marriage institution.

4.1 Transitory Changes

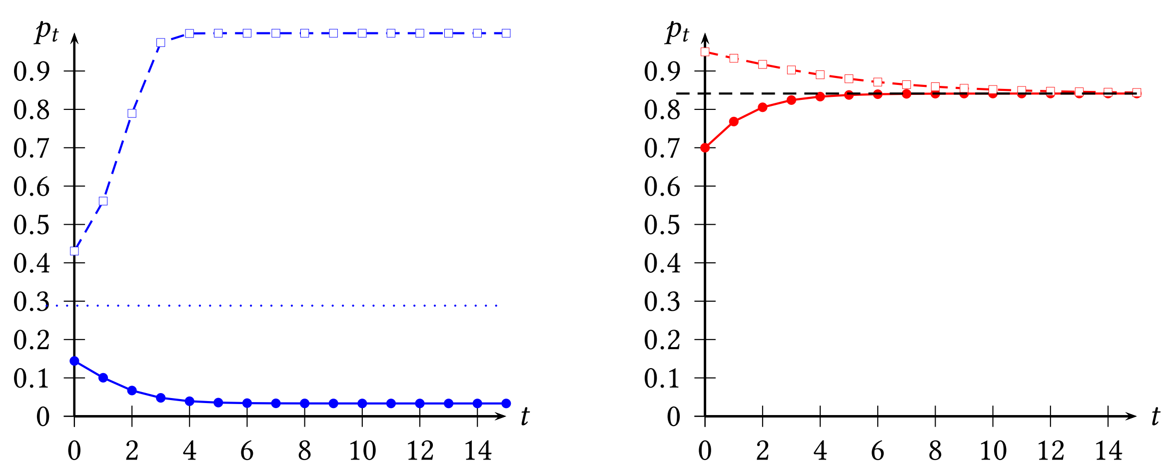
When random matching is sufficiently prevalent, there are two stable equilibria, c_1^* and $c_2^* > c_1^*$, and an unstable equilibrium, $c_0^* \in (c_1^*, c_2^*)$. A sufficiently large transitory shock can move the system from one stable equilibrium to the other. For example, suppose the system is initially at the equilibrium c_1^* with fewer type-a men. There is a shock that results in an increase in the mass of women—or equivalently, the mass of type-a men—choosing action a from $F(c_1^*)$ to $p_0 = F(c_0) > F(c_1^*)$. If the shock is so large—i.e., $c_0 > c_0^*$—that all women with costs lower than c_0^* choose action a, then this transitory change enables the population to escape from the basin of attraction of the c_1^* equilibrium to that of the c_2^* equilibrium, resulting in a change in the long-run outcome. Otherwise, if the shock is not large enough—i.e., $c_0 < c_0^*$—then the population initially has a higher mass of type-a agents due to the temporary shock, but later reverts to the equilibrium distribution.[19]

Figure 6a demonstrates the evolution of preferences after a small temporary deviation from the lower stable equilibrium as well as a large temporary deviation from the lower stable equilibrium. The economy moves toward the higher stable equilibrium after the large temporary deviation, but reverts back to the original equilibrium after the small temporary deviation.

Figure 6: Evolution of Preferences After a Transitory Change.

(a) Predominantly random matching. A large transitory shock can have a long-run impact (dashed). A small shock cannot (solid).

(b) Predominantly assortative matching. A large transitory shock can't have a long-run impact (dashed). Neither can a small shock (solid).



Note: Fix $c \sim N(0,1)$, $u_{aa} = 3$, $u_{bb} = 2$, $u_{ab} = 0$, and $u_{ba} = 0$.

When assortative matching is sufficiently prevalent, there is only one stable equilibrium. Hence, any transitory shock in the distribution of types does not lead to a persistent change

[19] If c_0 happens to be exactly c_0^*, then the shock shifts the system to the unstable equilibrium.

in the equilibrium. Figure 6b demonstrates the evolution of preferences after a temporary deviation from the stable equilibrium. The distribution initially moves away from the equilibrium due to the shock, but the equilibrium immediately reverts to the unique stable equilibrium after either a positive temporary change or a negative temporary change. To summarize, we have the following proposition.

Proposition 4 (Evolution After a Transitory Change in Preferences or Costs). *Consider a temporary change from a stable equilibrium cutoff cost to* c_0*.*

1. *Suppose there are two stable equilibria* c_1^* *and* c_2^* *and one unstable equilibrium* $c_0^* \in (c_1^*, c_2^*)$*. If (i) the original equilibrium is* c_1^* *and* $c_0 > c_0^*$ *or (ii) the original equilibrium is* c_2^* *and* $c_0 < c_0^*$*, the system moves to the other stable equilibrium. If* $c_0 = c_0^*$*, the system moves to the unstable equilibrium. Otherwise, the equilibrium is unchanged.*

2. *Suppose there is one stable equilibrium* c^**. The system initially changes to* c_0 *but reverts back to* c^* *afterward.*

4.2 Permanent Changes

The equilibrium changes in intuitive ways after a permanent shock to either preferences or women's cost of choosing action a, regardless of the structure of the marriage market.

Proposition 5 (Evolution After a Permanent Change in Preferences or Costs). *Type* a *becomes strictly more prevalent in equilibrium when (i)* u_{aa} *increases; (ii)* u_{ab} *increases and* $\lambda \neq 0$*; (iii)* u_{ba} *decreases and* $\lambda \neq 0$*; (iv)* u_{bb} *decreases; or (v)* F *decreases first-order stochastically.*

Consider a predominantly assortative environment so that there is always a unique stable equilibrium. If the marriage market becomes less assortative, there might be more or fewer people choosing action a and becoming type a in equilibrium, as illustrated by the red lines in Figures 5a and 5b, respectively.

Proposition 6 (Evolution After a Permanent Change in Matching Technology). *Suppose* $\lambda < \lambda^*$ *so that there is a unique stable equilibrium. When* λ *increases, equilibrium* c^λ *decreases, i.e., there is a lower mass of type-a men and women when marriages become less assortative, if and only if* $(1 - F(c^A))(u_{aa} - u_{ab}) > F(c^A)(u_{bb} - u_{ba})$*.*

The variation in the number of stable equilibria discussed in Section 3.3 suggests that a significant change to the matching technology (from predominantly random to predominantly assortative or vice versa) can potentially serve as an effective policy instrument. For example, the matching is initially random and the population is situated at the stable equilibrium, with type-a people dominating. Suppose such an equilibrium is undesirable from a societal perspective.

Policy makers can seek to reduce frictions such that the matching technology becomes more assortative, and consequently the population can possibly move to a more balanced state with both types coexisting. Or conversely the government can also reduce assortativeness to move to a progressive norm. For example, India has incentivized with cross-caste marriages (Hortacsu et al., 2019).

5 Model Implications

We present three applications to suggest that different marriage institutions can lead to different changes and persistent patterns of societal preferences.

5.1 Female Labor Force Participation in Developed Countries

Studies have documented the profound impact of gender role attitudes on female labor force participation.[20] Most notably, Fernández et al. (2004) show that men whose mothers worked were more likely to find wives who worked, by using regional variation in the influence of World War II as a shock to female labor force participation in the United States.[21] They suggest an intergenerational transmission mechanism: Compared with men who have nonworking mothers, those with working mothers are more likely to marry working wives, suggesting a stronger preference for working wives.

Our model suggests that a tremendous transitory event like World War II could result in a permanent increase in female labor force participation through intergenerational transmission of gender role attitudes, but only if men and women were sufficiently randomly sorted on the dimension of attitudes toward women working. A transitory positive shock in mothers' work does not always increase labor force participation for women of future generations: When the marriage market is predominantly assortative, a transitory shock does not lead to a permanent change, because there is a unique stable equilibrium. When the marriage market is predominantly random, a transitory shock must be large enough to overcome frictions in the marriage market and shift from the equilibrium with fewer working women to one with more working women.

We provide suggestive evidence that the matching between husband's mother's work behavior and wife's work behavior in the United States is quite random (Figure 7).[22] The General Social

[20]Fortin (2005) shows the impact of cultural beliefs about women's appropriate role on women's labor market outcomes in OECD countries. Fernández and Fogli (2005) show that labor force participation rates in their parents' countries of origin predict those rates of second-generation American women. Fernández (2007) shows that attitudes toward working women in parents' countries of origin can explain second-generation American women's work behavior.

[21]Goldin (1991), Acemoglu and Autor (2004), and Goldin and Olivetti (2013) also study the effect of World War II on female labor supply, which persisted for decades after the war.

[22]Since we assume that a son's preference is determined by his mother, the matching between husband's mother's work behavior and wife's work behavior reflects the matching between husband's preference and wife's work behavior.

Surveys (Smith et al., 2019) ask respondent's mother's work history when he is growing up (summarized as MOMWORK from MAWRKGRW, MAWORK14, and MAWRK16 in different years of survey), in addition to respondent's wife's work behavior (WIFEWORK). Fernández et al. (2004) show a positive correlation between the work behavior of these two individuals. However, the assortativeness, as measured by $1-\widehat{\lambda}$, is quite low: The average is 0.168 across the different years.[23] In comparison, the assortativeness on college education in the censuses (Ruggles et al., 2020) hovers around 0.5 to 0.7 across different years and metropolitan statuses. A more direct measure of men's preference for women's labor force participation is available from men's response to the question "Should women work?" (FEWORK) in most surveys from 1972 to 1998. The average assortativeness between husband's approval and wife's actual work behavior is 0.239, still quite low.

Compared with Fernández et al. (2004), our model incorporates a richer set of matching technologies.[24] Furthermore, how the dynamic operates given different matching technologies depends solely on the incentives created by the matching technology, free of any particular functional forms used in the model. Our model also clearly demonstrates, in Section 4.1, that WWII as a transitory shock plays a key role in changing the female labor force participation and the result crucially depends on the matching technology being random. In their model, however, the dynamic is either on an upward path where transitory shocks play no role, or on a downward path, escaping from which instead requires permanent shocks to economic fundamentals. Finally, our model suggests that reducing frictions in the marriage market, such that the entire society is transformed into a more assortative environment, can potentially help a society to escape from the equilibrium with low female labor force participation when large transitory shocks are absent.[25]

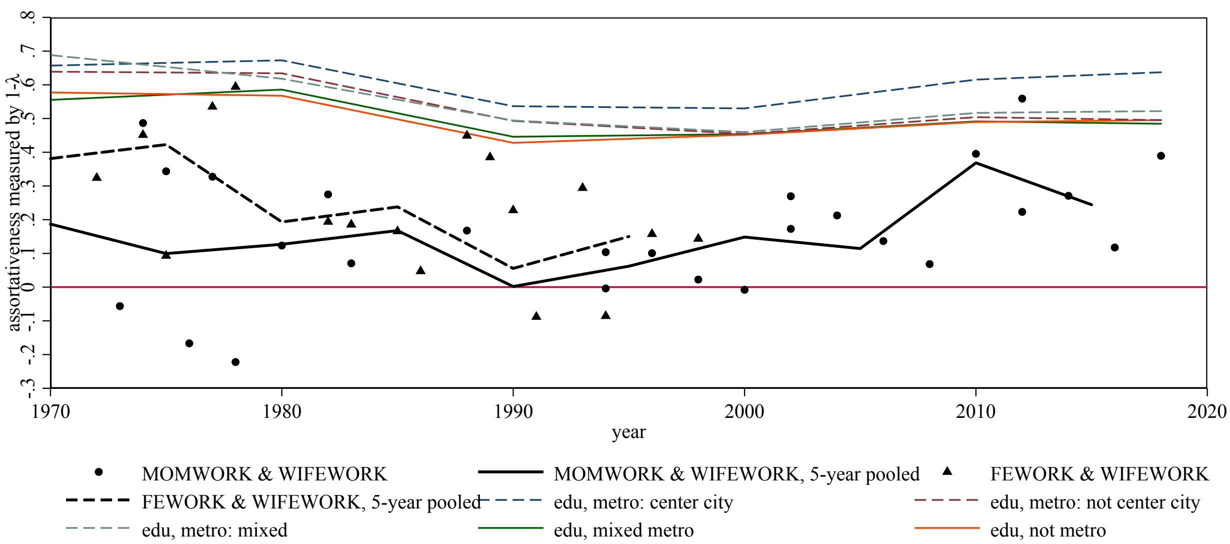
[23]Recall that the method to measure assortativity is provided at the end of Section 3.3.

[24]Their model resembles our random matching model. Men have two types: preferring a working wife and preferring a nonworking wife. A man's type is directly determined by whether his mother works. Before marriage, each woman chooses an education level that determines her wage distribution, which in turn affects her decision to work if she gets married. The marriage market consists of one round of random matching and the marriage decision is made after a pair is matched. Each woman can decide to get married or to stay single. They find that a woman's effort level is always increasing in the proportion of men who like a working wife. However, this does not necessarily result in an increase in the proportion of men who like a working wife across generations, because women can stay single. In general, depending on the functional forms, the model generates two possible dynamic paths: (i) an upward path leading to a steady state with men who like a working wife being the majority in the population, and (ii) a downward path leading to a steady state with no man preferring a working wife. Therefore, there are two possibilities for the population to evolve to the state in which most men prefer a working wife (which is accompanied by a high female labor force participation rate). First, the evolutionary dynamic is already situated on the upward path, such that the composition of the population is moving to the desired steady state. Second, the evolutionary dynamic is on the downward path and factors such as war, the expansion of service sectors, labor-saving household technology, and decreasing importance of marriage bar may shift the curve to the upward path.

[25]Pande (2018) suggests that raising a low rate of female labor force participation will "require behavioral interventions that address social norms."

Figure 7: Assortativeness between Men's Mother's and Wife's Work Behavior.

Note: Following Fernández et al. (2004), we investigate white married men who are between 30 and 50 years old. Mother's work history MOMWORK collects several variables in the General Social Surveys (Smith et al., 2019): (i) MAWK16 (mother's employment when respondent growing up) in 1973-1978, 1980, 1982, and 1983; (ii) MAWORK14 (Did mom work before respondent was 14?) in 1988, 1994, 2002, and 2012; and (iii) MAWRKGRW (mother's employment when respondent was 16) every other year from 1994 to 2018. FEWORK is respondent's response from the question "Should women work?" available for most years from 1972 to 1998. WIFEWORK is defined to be one if wife is working full time or is temporarily leaving work and zero otherwise (i.e., SPWRKSTA is 1 or 3); alternative measures—such as counting part-time work as working—would further reduce the assortativeness. Each dot represents the measure of assortativeness by year, and each line represents the measure by pooled 5-year data. More refined regional estimates are less reliable, because each year contains on average 150 eligible responses. The matching assortativeness on college education is imputed from 1970–2000 decennial censuses, and 2010 and 2018 5-year American Community Surveys (Ruggles et al., 2020); the metropolitan status is categorized by the variable METRO.



5.2 Gender Norms in Developing Countries

While developed countries have experienced a tremendous transformation toward more equal gender norms and increasing female labor force participation and educational attainment, traditional gender role attitudes such as preferences for female chastity and practices such as child marriage, purdah, and female genital circumcision persist in Africa, the Middle East, and South Asia. Why did traditional gender norms persist in these regions while transformation toward gender equality is observed in many parts of the rest of world? We believe that marriage market assortativity affects the transmission of preferences—as we have demonstrated in our model—is a plausible explanation.

Consider our model in which men's type b represents a preference for a modest and domestic wife or a preference for female chastity, and type a is the opposite. For women, action a is the decision to participate in the labor force or to receive formal education, and action b is the opposite. As we have argued, if there is a relatively high degree of assortativity in the marriage market along the dimension of gender norms, there is a unique stable equilibrium. If the cost of choosing action a for women is sufficiently high, which is true in the regions we consider, then the unique equilibrium should feature strong traditional gender norms and a low female labor participation rate.[26] Moreover, the equilibrium is resilient to transitory events, which means that there is still a long way ahead for globalization and interventions by governments or international agencies to change the status quo.

Again, we provide suggestive evidence from India Human Development Survey 2011-2012 (Desai et al., 2015) that the assortativeness can be much higher in developing countries than in developed countries. The assortativeness between a woman's actual work behavior (GR46) and whether she is allowed to work if job is suitable (GR49) is 0.54, and the assortativeness between a whether a woman is willing to work (GR48) and whether she is allowed to work if job is suitable (GR49) is 0.95, both much higher than the assortativeness of 0.24 between American women's work and men's preference. The assortativeness between a woman and her mother-in-law on school attendance is 0.87, and that on literacy is 0.84.

One feature that distinguishes these regions from the rest of the world is the assortativeness of preferences and behavior between husband's and wife's families partially due to the prevalence of arranged marriages (Goode, 1970; Cherlin, 2012), and arranged marriages are deeply connected with the above described traditional gender norms. For example, 95 percent of all marriages are still arranged in South Asia (Rubio, 2014), and there is universal demand for female chastity.[27] Traditional gender role attitudes and practices in regions where marriages are mostly arranged

[26] In rural and less developed areas, the high cost of choosing action a can be attributed to the lack of governmental support for the elderly and the absence of a market for household services. These factors raise the opportunity cost of working or receiving higher education for women and raise the value parents place on a submissive and home-oriented daughter-in-law.

[27] Even a slight possibility of losing her virginity will reduce a bride's desirability (Desai and Andrist, 2010). As a result, parents who benefit from delivering a virgin bride will try their best to prevent their daughter from contacting the opposite sex or searching for potential partners (Edlund and Lagerlöf, 2004). An effective way for parents to preserve a daughter's virginity is to marry her at a young age. Wahhaj (2018) quotes the following paragraph from Rozario (1992) on the case of Bangladesh to support his argument that in societies with predominantly arranged marriages, child marriage results from the fact that age signals a woman's poor quality of women:

> Many ... parents prefer to have their daughters marry as young as possible. About 15-16 years old is seen as ideal, while 18 years is considered too old, particularly if a girl begins to visit friends and neighbours outside the household and thereby cast doubt on her purity. (Rozario, 1992)

Men's preferences for female purity also result in the practice of purdah, which is adopted in certain Muslim and Hindu societies to segregate women from men, and it seems that the practice is transmitted across generations:

> [Women who practice purdah] look forward to being able to arrange their children's marriages and exert an element of power in that important decision. They certainly expect their sons to marry girls who have been carefully shielded by purdah from temptation. (White, 1977)

severely limit women's mobility and reduce their chances of education and work (Jayachandran, 2015). There is a negative correlation between arranged marriage and female participation in the formal labor market and a negative correlation between arranged marriage and women's educational attainment (Rubio, 2014).

Arranged marriages result in more homogamous marriages in certain preferences than freewill marriages do, for the following reasons. First, arranged marriages have fewer information and search frictions than freewill marriages. Arranged marriages are usually based on known qualities of families and children. Through their social networks, parents usually have wide access to potential candidates and they may be better at evaluating the candidates' characteristics. Under freewill marriages, in contrast, people must search for partners on their own with imperfect information about certain characteristics of their potential partners, and long courtships are often required. In addition, arranged marriages are usually organized locally, and naturally the relatively small size of the marriage market leads to a higher degree of assortativity, while freewill marriages occur in larger marriage markets. Studies have shown a positive correlation between freewill marriage and urbanization.[28] In an urban area, due to the sheer size of the market, the marriage market is inevitably more random.[29] Second, freewill marriages often involve match-specific qualities that are idiosyncratic to the couple and not predictable according to observable traits. The match-specific quality can be interpreted as affection or attraction between a couple, and it adds randomness to the matching process (Fernández et al., 2005; Huang et al., 2017). Match-specific qualities, however, are usually not a factor in arranged marriages, since they are not important in the considerations for parents even if the parents are altruistic. In certain countries, the practice of blind marriage serves as a way to prevent love from standing in the way of achieving the goals of parents in arranged marriages.

As a result, freewill marriages should exhibit more randomness in the dimensions that families care more about in arranged marriages, such as female chastity, education, and labor force participation. Appendix C provides evidence that Indian couples in arranged marriages have more closely aligned preferences for family values such as women's work and desired number of children, drawing from the two waves of India Human Development Survey in 2005 and 2011-2012.

[28] Rubio (2014) finds that the transition from arranged marriage to freewill marriage is correlated with increases in urbanization across countries. Cherlin (2012) describes the rise of a "hybrid form" of arranged marriage with the daughter's consent in the urban middle class in India. Huang et al. (2017) document that in the early 1990s, 48 percent of rural couples and 14.5 percent of urban couples were married by parent-involved matchmaking in China.

[29] It is worth mentioning that the distinction between marriage markets in urban and rural areas allows us to explain why the same temporary shock to behavior may move the social norm in cities more than in villages.

5.3 Cultural Norms in the Long Run

A recent literature has documented the historical roots of contemporary gender role attitudes (Alesina et al., 2013; Hensen et al., 2015; Teso, 2018; Xue, 2020). The idea is that the short-run outcome of a certain historical incident may imprint onto people's preferences and beliefs, which are transmitted through generations until today, even though the circumstances that caused the incident have long since changed.

Grosjean and Khattar (2019) show that the male-biased sex ratio caused by the British policy of sending convicts to Australia has a persistent effect of men having more traditional gender attitudes toward women even now, although the gender balance was quickly restored after the importation of convicts stopped. They argue that the male-biased sex ratio changed the bargaining position between men and women, leading to women enjoying more leisure in the short run. This in turn became part of the preferences, and persisted through cultural transmission. Moreover, they argue that homogamous marriages reinforce the persistence. They find that in areas with a higher percentage of homogamous marriages, a male-biased sex ratio leads to a more traditional gender view, while it is not the case in areas with a lower percentage of homogamous marriages. Our model can account for these empirical regularities.

Consider the simple model with type a referring to a man's preference for a working wife and type b referring to the opposite. Action a represents a woman's participation in the work force and action b represents the opposite. Let the male-biased sex ratio be a shock that fundamentally changes people's utility in marriage. In particular, it leads to an increase in u_{bb}, the utility of a woman who chooses to stay home marrying a man who prefers a more traditional breadwinner-housewife family. According to Proposition 5(iv), an increase in u_{bb} will increase the prevalence of type b regardless of the matching technology. Hence, regardless of the underlying matching technology, an increase in u_{bb} has a persistent effect of lowering the proportion of type-a men in the population, which matches the main observation of Grosjean and Khattar (2019).

In addition, our model predicts that a sufficiently large cultural shock that promotes equal gender norms can shift men in regions with low prevalence of homogamous marriages (i.e., predominantly random matching) to have more progressive gender role attitudes, but not men in regions with high prevalence of homogamous marriages (i.e., predominantly assortative matching). This also explains observed variations in gender role attitudes across regions with different marriage markets.

6 The General Model

We generalize the simple model by allowing both men and women to have types and actions and having each agent's final preference determined by both parents' preferences and the choices they make.

Consider a unit mass of men and a unit mass of women every period. There are two types available to all agents: a and b. Each agent's life has two periods: childhood and adulthood. During childhood, an agent adopts an initial type from their parents through intergenerational transmission. During adulthood, an agent chooses either action a or b. The initial type of an agent determines the cost of choosing different actions for them when they enter adulthood. For example, suppose type a represents a preference for diligence, while type b represents a preference for leisure. Action a represents an occupation that requires diligence, and action b is the opposite. Then an agent who has a preference for diligence in their childhood is likely to have a lower cost for choosing an occupation that requires diligence when they enter the adulthood than one who has a taste for leisure in their childhood.

The action chosen in adulthood determines the final type for an agent. For example, consider an agent who has a taste for leisure in their childhood. Even though they are less likely to choose an occupation that requires hard work, as long as they choose it, they will develop a preference for diligence. Observe that although the choice made in adulthood determines the final type of an agent, intergenerational transmission indirectly influences the choice made by the agent through determining their initial type.

Let p_t^0 and q_t^0 denote the mass of men and women whose initial type is a in period t. Let α_t^m and α_t^w denote the mass of men and women whose initial type is a who choose action a in their adulthood in period t. Let β_t^m (β_t^w) denote the mass of men (women) whose initial type is b who choose action a in adulthood in period t. Let p_t and q_t denote the mass of men and women whose final type is a in period t, respectively. We have $p_t = p_t^0\alpha_t^m + (1-p_t^0)\beta_t^m$, and $q_t = q_t^0\alpha_t^w + (1-q_t^0)\beta_t^w$.

After choosing their actions and forming their final types in adulthood, all men and women enter the marriage market to find a partner. Assume that all men and women pair up, and each pair produces two children, one son and one daughter.

We normalize the cost of action b to 0 and denote the cost of action a by c_ρ^g for an individual whose gender is $g \in \{m, f\}$ and initial type is $\rho \in \{a, b\}$. Assume the cost is distributed according to a differentiable and strictly increasing distribution F_ρ^g with associated single-peaked density f_ρ^g, for $g \in \{m, f\}$ and $\rho \in \{a, b\}$.

Let $u_{t_i t_j}^i$ denote a type-t_i agent's utility from marrying a type-t_j agent of the opposite gender, for $i \neq j$ and $i, j \in \{m, f\}$. Assume homophily in types: $u_{aa}^m > u_{ab}^m$ and $u_{bb}^m > u_{ba}^m$; $u_{aa}^w > u_{ab}^w$ and $u_{bb}^w > u_{ba}^w$.

The intergenerational transmission process is characterized as follows. Suppose that a son has a probability h^m of inheriting his father's type and a probability $1 - h^m$ of inheriting his mother's type. A daughter has a probability $h^w \in [0, 1]$ of inheriting her father's type and a probability $1 - h^w$ of inheriting her mother's type. This intergenerational transmission process gives rise to a dynamic system that characterizes the evolution of preferences:

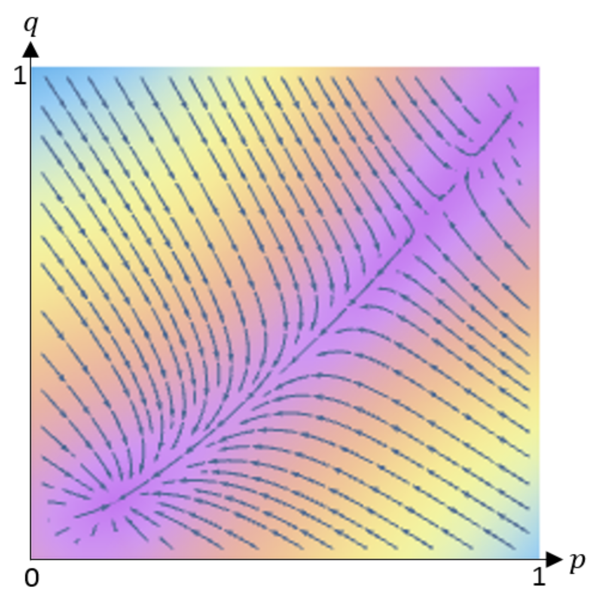
$$p_t = (h^m p_{t-1} + (1-h^m)q_{t-1})\alpha_t^m + (1 - h^m p_{t-1} - (1-h^m)q_{t-1})\beta_t^m;$$
$$q_t = (h^w p_{t-1} + (1-h^w)q_{t-1})\alpha_t^w + (1 - h^w p_{t-1} - (1-h^w)q_{t-1})\beta_t^w.$$

Appendix B characterizes the equilibria of the general model. Generically, as in the simple model, there are multiple stable equilibria under random matching and a unique stable equilibrium under assortative matching. Figure 8 provides numerical demonstration.

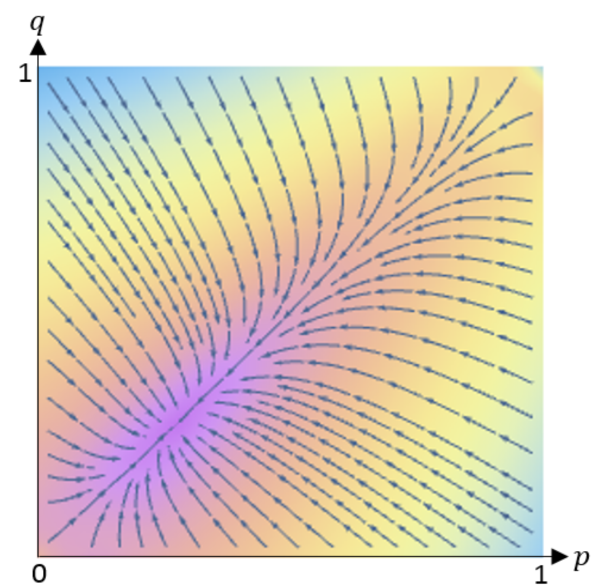
Figure 8: Equilibria in the General Model.

Note: The horizontal axis represents the mass of type-a men, and the vertical axis the mass of type-a women. The graph illustrates equilibria and directions of evolution to the equilibria when $h^m = 0.6$, $h^w = 0.4$, $u_{aa}^m = 4$, $u_{ab}^m = 3$, $u_{ba}^m = 1$, $u_{bb}^m = 4$, $u_{aa}^w = 4$, $u_{ab}^w = 2$, $u_{ba}^w = 2$, $u_{bb}^w = 4$, $F_a^m = F_a^w \sim N[0,1]$ and $F_b^m = F_b^w \sim N[5,5]$.

(a) Random Matching: Two stable equilibria ($p_1^* = 0.15$, $q_1^* = 0.10$) and ($p_2^* = 0.96$, $q_2^* = 0.92$) and one unstable equilibrium ($p_0^* = 0.78$, $q_0^* = 0.68$).



(b) Assortative Matching: One stable equilibrium ($p^* = 0.24, q^* = 0.24$) and no unstable equilibrium.



Random Matching. Under random matching, a man chooses action a if and only if

$$c \leq q_t(u_{aa}^m - u_{ba}^m) + (1-q_t)(u_{ab}^m - u_{bb}^m) \equiv k_m^R(q_t),$$

where $k_m^R(q_t)$ denotes the cutoff cost for men. We have $\alpha_t^m = F_a^m(k_m^R(q_t))$ and $\beta_t^m = F_b^m(k_m^R(q_t))$. Similarly, a woman chooses action a if and only if

$$c \leq p_t(u_{aa}^w - u_{ba}^w) + (1-p_t)(u_{ab}^w - u_{bb}^w) \equiv k_w^R(p_t),$$

where $k_w^R(p_t)$ denotes the cutoff cost for women. We have $\alpha_t^w = F_a^w(k_w^R(p_t))$ and $\beta_t^w = F_b^w(k_w^R(p_t))$.

Assortative Matching. Under assortative matching, a man chooses action a if and only if

$$c \leq k_m^A(p_t, q_t) = \begin{cases} \frac{q_t}{p_t} u_{aa}^m + \left(1 - \frac{q_t}{p_t}\right) u_{ab}^m - u_{bb}^m & p_t \geq q_t, \\ u_{aa}^m - \left(\frac{q_t - p_t}{1 - p_t} u_{ba}^m + \frac{1 - q_t}{1 - p_t} u_{bb}^m\right) & p_t < q_t, \end{cases}$$

where $k_m^A(p_t, q_t)$ denotes the cutoff cost for men. We have $\alpha_t^m = F_a^m(k_m^A(p_t, q_t))$ and $\beta_t^m = F_b^m(k_m^A(p_t, q_t))$. Similarly, a woman chooses action a if and only if

$$c \leq k_w^A(p_t, q_t) = \begin{cases} \frac{p_t}{q_t} u_{aa}^w + \left(1 - \frac{p_t}{q_t}\right) u_{ab}^w - u_{bb}^w & q_t \geq p_t, \\ u_{aa}^w - \left(\frac{p_t - q_t}{1 - q_t} u_{ba}^w + \frac{1 - p_t}{1 - q_t} u_{bb}^w\right) & q_t < p_t, \end{cases}$$

where $k_w^A(p_t, q_t)$ denotes the cutoff cost for women. We have $\alpha_t^w = F_a^w(k_w^A(p_t, q_t))$ and $\beta_t^w = F_b^w(k_w^A(p_t, q_t))$.

7 Conclusion

This paper examines the intergenerational transmission of preferences under different organizations of the marriage market. We find that different organizations of the marriage market influence the evolution of preferences. Namely, there are multiple stable equilibria when the degree of frictions in matching is large, and there is one stable equilibrium when the degree of frictions is small.

Market-differential effects of transitory and permanent shocks on preference evolution help us explain a set of phenomena. First, to be able to explain how the equilibrium permanently shifts due to a transitory shock to individual choices or preferences (for example, more women work today in the United States due to the transitory increase in World War II), we must be working under a sufficiently frictional matching market. Second, the prevalence of arranged marriages may help explain the persistence of backward gender norms. Finally, a small initial difference may lead to a big difference in preferences in the long run, which explains the long-term impact of sex ratio on gender role attitudes in Australia. The purpose of this work is to point out the possible importance of the marriage market structure in influencing the evolution of preferences. Future work should quantify the claimed importance of this marriage-market effect in relation to other well-established effects.

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A Omitted Proofs

Proof of Proposition 1 (Equilibria under Random Matching).
Stationary equilibrium c^* satisfies

$$\psi_R(c^*) = c_R(F(c^*)) - c^* = 0.$$

The slope of ψ_R is

$$\psi_R'(c) = c_R'(F(c))f(c) - 1 = \Delta f(c) - 1.$$

Since we assume $f(\widehat{c})\Delta > 1$, and f is single-peaked, there exist two solutions, $\underline{c}$ and $\overline{c}$, to the equation $\psi_R'(c) = \Delta f(c) - 1 = 0$. As a result, $\psi_R(c)$ is strictly decreasing for any $c < \underline{c}$ and for any $c > \overline{c}$. For $c \to -\infty$ or $c \to \infty$, $f(c) \to 0$, so $\psi_R'(c) \to -1$. Furthermore, we assume $\psi_R(\underline{c}) < 0 < \psi_R(\overline{c})$. Therefore, there is a $c_1^R < \underline{c}$ and a $c_2^R > \overline{c}$ such that $\psi_R(c_1^R) = \psi_R(c_2^R) = 0$. Because $\psi_R(c)$ is strictly decreasing around c_1^R and c_2^R, $c_R(F(c)) > c$ for any c smaller than but sufficiently close to c_i^R and $c_R(F(c)) < c$ for any c larger than but sufficiently close to c_i^R, $i = 1, 2$. Hence, the dynamic around the equilibrium costs c_1^R and c_2^R is converging, so these two equilibria are stable.

Furthermore, $\psi_R(c)$ is strictly increasing for any $c \in (\underline{c}, \overline{c})$. And by the assumption that $\psi_R(\underline{c}) < 0 < \psi_R(\overline{c})$ and continuity of $\psi_R(c)$, there exists a $c_0^R \in (\underline{c}, \overline{c})$ such that $\psi(c_0^R) = 0$. Since $\psi(c)$ is strictly increasing around c_0^R and $\psi(c_0^R) = 0$, $c_R(F(c)) < c$ for any c smaller than but sufficiently close to c_0^R, and $c_R(F(c)) > c$ for any c larger than but sufficiently close to c_0^R. The dynamic around equilibrium c_0^R is diverging, so the equilibrium c_0^R is unstable.

Figure A.1 illustrates the four possible scenarios when $f(\widehat{c})\Delta > 1$ but the assumption $\psi_R(\underline{c}) < 0 < \psi_R(\overline{c})$ does not hold. There is always one stable equilibrium. If $f(\widehat{c})\Delta \leq 1$, then $\psi_R(c)$ is always decreasing and there is one and only one equilibrium, and the equilibrium is stable.

There may exist nonstationary equilibria; for example, a nonstationary equilibrium in which the cutoff alternates between c_1 and c_2 such that $c_2 = c_R(F(c_1))$ and $c_1 = c_R(F(c_2))$. However, these nonstationary equilibria are unstable. $\square$

Proof of Proposition 2 (Equilibria under Assortative Matching).
Let $\widetilde{c}(c)$ denote the cutoff cost in a period when c is the cutoff cost in the previous period. By definition, $\widetilde{c}(c)$ solves

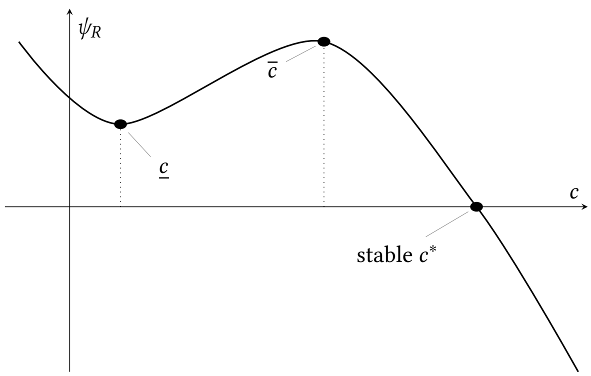
$$c_A(F(c), F(\widetilde{c})) - \widetilde{c} = 0.$$

Define $\psi_A(c)$ as

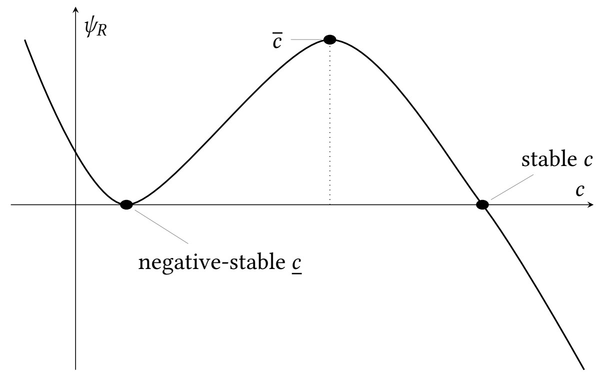
$$\psi_A(c) \equiv \widetilde{c}(c) - c.$$

Figure A.1: Equilibria in Nongeneric Cases under Random Matching.

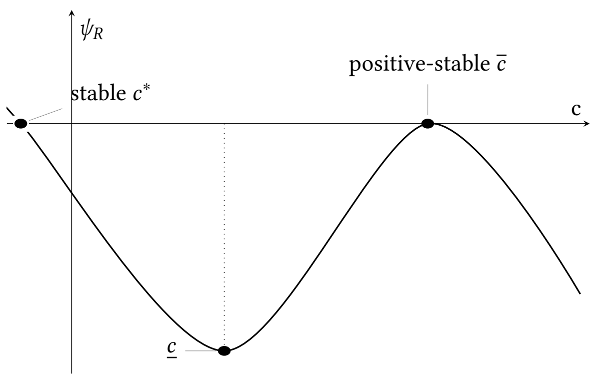
(a) $0 < \psi_R(\underline{c}) < \psi_R(\overline{c})$: A stable equilibrium $c^* > \overline{c}$ and no other equilibrium.



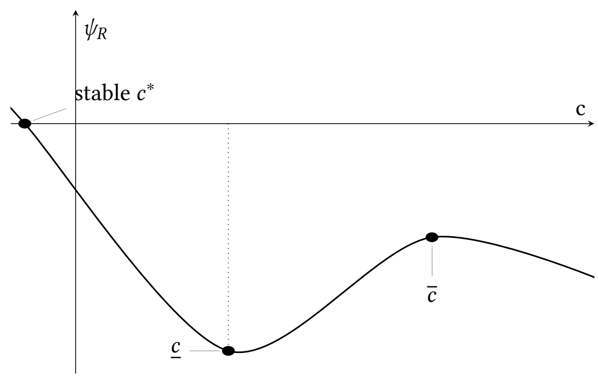
(b) $0 = \psi_R(\underline{c}) < \psi_R(\overline{c})$: A stable equilibrium $c^* > \overline{c}$ and a negative-stable positive-unstable equilibrium $\underline{c}$.



(c) $\psi_R(\underline{c}) < \psi_R(\overline{c}) = 0$: A stable equilibrium c^* and a positive-stable negative-unstable equilibrium $\overline{c}$.



(d) $\psi_R(\underline{c}) < \psi_R(\overline{c}) < 0$: A stable equilibrium c^* and no other equilibrium.



The slope of $\psi_A(c)$ is

$$\psi_A'(c) = \widetilde{c}'(c) - 1,$$

where $\widetilde{c}'(c)$ satisfies

$$c_{A1} f(c) + c_{A2} f(\widetilde{c}(c))\widetilde{c}'(c) - \widetilde{c}'(c) = 0,$$

which simplifies to

$$\widetilde{c}'(c) = \frac{c_{A1} f(c)}{1 - c_{A2} f(\widetilde{c}(c))},$$

where

$$c_{A1} = \begin{cases} \frac{1}{F(\widetilde{c}(c))}(u_{aa} - u_{ab}) & \widetilde{c}(c) > c \\ \frac{1}{1-F(\widetilde{c}(c))}(u_{bb} - u_{ba}) & \widetilde{c}(c) < c \end{cases}, \text{ and } c_{A2} = \begin{cases} -\frac{1}{F(\widetilde{c}(c))}\frac{F(c)}{F(\widetilde{c}(c))}(u_{aa} - u_{ab}) & \widetilde{c}(c) > c \\ -\frac{1}{1-F(\widetilde{c}(c))}\frac{1-F(c)}{1-F(\widetilde{c}(c))}(u_{bb} - u_{ba}) & \widetilde{c}(c) < c \end{cases}.$$

The slope of $\psi_A(c)$ is

$$\psi_A'(c) = \frac{c_{A1} f(c) + c_{A2} f(\widetilde{c}(c)) - 1}{1 - c_{A2} f(\widetilde{c}(c))}.$$

More specifically,

$$\psi_A'(c)[1-c_{A2}f(\widetilde{c}(c))] = \begin{cases} \frac{F(c)}{F(\widetilde{c}(c))}(u_{aa}-u_{ab})\left[\frac{f(c)}{F(c)}-\frac{f(\widetilde{c}(c))}{F(\widetilde{c}(c))}\right]-1 & \widetilde{c}(c) > c \\ \frac{1-F(c)}{1-F(\widetilde{c}(c))}(u_{bb}-u_{ba})\left[\frac{f(c)}{1-F(c)}-\frac{f(\widetilde{c}(c))}{1-F(\widetilde{c}(c))}\right]-1 & \widetilde{c}(c) < c \end{cases}.$$

To have a stationary equilibrium c^A, we must have $\widetilde{c}(c^A) = c^A$. Therefore, in equilibrium, $\widetilde{c}(c^A)$ must satisfy

$$c_A(F(c), F(c)) - c = 0.$$

This equation simplifies to

$$u_{aa} - u_{bb} - c = 0.$$

Therefore, there is a unique cost $c^A = u_{aa} - u_{bb}$ that satisfies the equation. Because $1 - c_{A2}f(\widetilde{c}) > 0$, $\lim_{c \uparrow c^A} \psi_A'(c) = -1/(1 - c_{A2}f(\widetilde{c})) < 0$ and $\lim_{c \downarrow c^A} \psi_A'(c) = -1/(1 - c_{A2}f(\widetilde{c})) < 0$, the unique equilibrium is stable.

It remains to show that there does not exist a nonstationary equilibrium. Suppose there exists a nonstationary equilibrium with alternating cutoff costs c_1 and c_2. Then $c_2 = \widetilde{c}(c_1)$ and $c_1 = \widetilde{c}(c_2)$. Without loss of generality, suppose $c_2 > c_1$. Then, because $\widetilde{c}$ is strictly increasing, $\widetilde{c}(c_2) > \widetilde{c}(c_1)$, which means $c_1 > c_2$, a contradiction with the premise. Following the same logic, there cannot exist a sequence $\{c_1, c_2, \cdots, c_T\}$ such that $c_{t+1} = \widetilde{c}(c_t)$ for any $t = 1, \cdots, T-1$, and $c_1 = \widetilde{c}(c_T)$. □

Proof of Proposition 3 (Equilibria under Mixed Matching).
Next period's cutoff $\widetilde{c}(c)$ given current period's cutoff c satisfies $\widetilde{\psi}_\lambda(c) = 0$, where

$$\widetilde{\psi}_\lambda(c) = \lambda c_R(F(c)) + (1-\lambda)c_A(F(c), F(\widetilde{c}(c))) - \widetilde{c}(c).$$

A stationary equilibrium c^* satisfies $\psi_\lambda(c^*) = 0$, where

$$\psi_\lambda(c) = \lambda c_R(F(c)) + (1-\lambda)c_A(F(c), F(c)) - c,$$

which is simplified to

$$\psi_\lambda(c) = \lambda c_R(F(c)) + (1-\lambda)(u_{aa} - u_{bb}) - c,$$

because $c_A(p, q) = u_{aa} - u_{bb}$ when $p = q$. The slope of $\psi_\lambda(c)$ is

$$\psi_\lambda'(c) = \lambda f(c)\Delta - 1.$$

If $\lambda \leq 1/(\Delta f(\widehat{c}))$, then the slope $\psi_\lambda'(c)$ is negative for almost any c, and there is a unique stable equilibrium.

If $\lambda > 1/(\Delta f(\widehat{c}))$, then there is a range of c such that the slope $\psi'(c)$ is positive in the range and negative otherwise. Let $\underline{c}_\lambda$ and $\overline{c}_\lambda$ denote the smallest and the largest c such that the slope $\psi'_\lambda(c)$ is nonnegative when the degree of marriage frictions is λ. In other words, $\underline{c}_\lambda$ and $\overline{c}_\lambda$ are the two solutions of $\lambda\Delta f(c) = 1$. If $\psi_\lambda(\underline{c}_\lambda) < 0 < \psi_\lambda(\overline{c}_\lambda)$, then there are two stable equilibria. Otherwise, there is one stable equilibrium. When there is one stable equilibrium, either $\psi_\lambda(\underline{c}_\lambda) \geq 0$ or $\psi_\lambda(\overline{c}_\lambda) \leq 0$.

To show that there is a unique λ^* such that there are two stable equilibria for $\lambda > \lambda^*$ and there is one stable equilibrium for $\lambda \leq \lambda^*$, it suffices to show that if there is a unique stable equilibrium under λ then there is a unique stable equilibrium under λ' for any $\lambda' < \lambda$.

Let $\lambda > \lambda' > 1/(\Delta f(\widehat{c}))$. Otherwise, the unique stable equilibrium is satisfied because $\psi'_\lambda(c) < 0$ for any c. Since $\lambda\Delta f(c) > \lambda'\Delta f(c)$ for any c, we must have

$$\underline{c}_\lambda < \underline{c}_{\lambda'} < \widehat{c} < \overline{c}_\lambda < \overline{c}_{\lambda'},$$

which by extension,

$$\underline{c} = \underline{c}_1 < \underline{c}_\lambda < \underline{c}_{\lambda'} < \widehat{c} < \overline{c}_\lambda < \overline{c}_{\lambda'} < \overline{c} = \overline{c}_1.$$

In words, the range of c in which $\psi_\lambda(c)$ is increasing is shrinking as λ decreases to $1/(\Delta f(\widehat{c}))$.

Suppose there is one stable equilibrium c_λ under λ. We discuss two possible cases: (1) $c^A > c_0^R$ and (2) $c^A < c_0^R$. First, suppose $c^A > c_0^R$. There must exist a stable equilibrium c_λ larger than c^A, because $\psi_\lambda(c)$ is continuous, and $\psi_\lambda(c^A) > 0$ and $\lim_{c\to\infty} \psi_\lambda(c) < 0$ together imply that $\psi_\lambda(c_\lambda) = 0$ for some $c_\lambda > c^A$. As a result, there is no other stable equilibrium. Then $\psi_\lambda(\underline{c}_\lambda) > 0$. We can show that $\psi_{\lambda'}(\underline{c}_{\lambda'}) > 0$ for any $\lambda' < \lambda$. Suppose $\underline{c}_\lambda < c_0^R$. Because $\psi'_\lambda(c) > 0$ for any c between $\underline{c}_\lambda$ and $\underline{c}'_\lambda$, $\psi_\lambda(\underline{c}_\lambda) > \psi_\lambda(c_\lambda)$. Because $\psi_A(c_{\lambda'}) > \psi_R(c_{\lambda'})$, $\psi_{\lambda'}(\underline{c}_\lambda) > \psi_\lambda(\underline{c}_{\lambda'})$. The case with $c^A < c_0^R$ is the mirror image of the case with $c^A > c_0^R$. There must exist a stable equilibrium c_λ smaller than c_0^R. There is no other equilibrium, and $\psi_\lambda(\overline{c}_\lambda) > 0$. We can then show that $\psi_{\lambda'}(\overline{c}_{\lambda'}) < 0$. □

Proof of Proposition 4 (Evolution After a Transitory Change in Preferences or Costs). For the first part of the proposition, suppose c_1^* and c_2^* are the two stable equilibria and c_0^* is the unstable equilibrium in between. Let $\widetilde{\psi}_\lambda(c) = \widetilde{c}(c) - c$ be the difference between the current period cutoff c and the next period cutoff $\widetilde{c}(c)$. We must have $\widetilde{\psi}(c) > 0$ for all $c \in (c_0^*, c_2^*)$ and $\widetilde{\psi}(c) < 0$ for all $c \in (c_2^*, \infty)$, though $\widetilde{\psi}(c)$ may not be monotonic in those ranges. Otherwise, there may exist other stable equilibria: If $\widetilde{\psi}(c') = 0$ for some c', then $\psi(c') = 0$, and c' is an equilibrium, contradicting the claim that only c_0^*, c_1^*, and c_2^* are equilibria. Similarly, we must also have $\widetilde{\psi}(c) < 0$ for all $c \in (c_1^*, c_0^*)$ and $\widetilde{\psi}(c) > 0$ for all $c \in (-\infty, c_1^*)$. Therefore, only a shock $c_0 > c_0^*$ when the original equilibrium is c_1^* or a shock $c_0 < c_0^*$ when the original equilibrium is c_2^* results in a dynamic that converges to a different equilibrium in the long run.

For the second part of the proposition, suppose c^* is the unique stable equilibrium. Since $\widetilde{\psi}_\lambda(c) < 0$ as $c \to \infty$ and $\widetilde{\psi}_\lambda(c) > 0$ as $c \to -\infty$, we must have $\widetilde{\psi}_\lambda(c) \geq 0$ for all $c < c^*$ and $\widetilde{\psi}_\lambda(c) \leq 0$ for all $c > c^*$. Again, note that the proof does not need $\psi_\lambda(c)$ to be monotonic in c. $\square$

Proof of Proposition 5 (Evolution after a permanent change in preferences or costs). Stable equilibrium c^* satisfies $\psi_\lambda(c^*) = 0$, where $\psi_\lambda(c^*)$ can be expanded and simplified to

$$\psi_\lambda(c^*) = \lambda F(c^*)(u_{aa} + u_{bb} - u_{ab} - u_{ba}) - \lambda(u_{bb} - u_{ab}) + (1-\lambda)(u_{aa} - u_{bb}) - c^*.$$

Since $\psi'_\lambda(c^*) = \lambda f(c^*)(u_{aa} + u_{bb} - u_{ab} - u_{ba}) - 1 = \lambda f(c^*)\Delta - 1 \leq f(c^*)\Delta - 1 < 0$ at any stable equilibrium c^*, c^* would increase as a variable v increases if

$$\frac{\partial \psi_\lambda(c^*)}{\partial v} > 0.$$

Similarly, c^* would decrease as a variable v increases if $\partial\psi_\lambda(c^*)/\partial v < 0$, and c^* would not change as a variable v increases if the derivative is zero. Hence, locally, it is sufficient to derive the sign of $\partial\psi_\lambda(c^*)/\partial v$ for any v. The derivative of $\psi_\lambda(c^*)$ with respect to each of the five variables of interest is as follows.

(i). $\dfrac{\partial \psi_\lambda(c^*)}{\partial u_{aa}} = 1 - \lambda(1 - F(c^*)) > 0.$

(ii). $\dfrac{\partial \psi_\lambda(c^*)}{\partial u_{ab}} = \lambda[1 - F(c^*)] > 0$ if $\lambda \neq 0$.

(iii). $\dfrac{\partial \psi_\lambda(c^*)}{\partial u_{ba}} = -\lambda F(c^*) < 0$ if $\lambda \neq 0$.

(iv). $\dfrac{\partial \psi_\lambda(c^*)}{\partial u_{bb}} = \lambda F(c^*) - 1 < 0.$

(v). $\dfrac{\partial \psi_\lambda(c^*)}{\partial F(c^*)} = \lambda\Delta > 0$ if $\lambda \neq 0$. If $\lambda = 0$, the decrease in $F(c)$ itself still results in a strict decrease in the prevalence of type a.

$\square$

Proof of Proposition 6 (Evolution after a permanent change in matching technology). The equilibrium cutoff is simply characterized by

$$\lambda c_R(F(c)) + (1-\lambda)c_A(F(c), F(c)) - c = 0.$$

Explicitly, the LHS is

$$\lambda F(c)(u_{aa} - u_{ab} + u_{bb} - u_{ba}) + \lambda(u_{ab} - u_{bb}) + (1-\lambda)(u_{aa} - u_{bb}) - c \equiv \psi_\lambda(c).$$

It has a slope of $\lambda f(c)\Delta - 1$. If $\lambda > 1/(f(\widehat{c})\Delta)$ and $\psi(\underline{c}) < 0 < \psi(\overline{c})$, where $\underline{c}$ and $\overline{c} > \underline{c}$ are the two solutions of $f(c)\Delta = 1/\lambda$, then there are two stable equilibria characterized by $c_1^* < \underline{c}$ and $c_2^* > \overline{c}$. Consider the equation characterizing the equilibrium cutoff,

$$\lambda F(c^*)(u_{aa} - u_{ab} + u_{bb} - u_{ba}) + \lambda(u_{ab} - u_{bb}) + (1 - \lambda)(u_{aa} - u_{bb}) - c^* = 0.$$

Applying the implicit function theorem and taking the derivative of the equation, we get

$$F(c^*)(u_{bb} - u_{ba}) + (F(c^*) - 1)(u_{aa} - u_{ab}) - c'(\lambda) + \lambda f(c^*)\Delta c'(\lambda) = 0.$$

Rearranging, we have

$$c'(\lambda) = \frac{F(c^*)(u_{bb} - u_{ba}) - (1 - F(c^*))(u_{aa} - u_{ab})}{1 - \lambda f(c^*)\Delta}.$$

Since $\lambda f(c^*)\Delta - 1$ is the slope of the LHS of the equation, it is negative, and the denominator is positive. Therefore, $c'(\lambda)$ has the same sign as $F(c^*)(u_{bb} - u_{ba}) - (1 - F(c^*))(u_{aa} - u_{ab})$. □

B Equilibria in the General Model

B.1 Equilibria under Random Matching

Let (p, q) denote an equilibrium. It satisfies

$$(h^m p + (1 - h^m)q)F_a^m(k_m^R(q)) + (1 - h^m p - (1 - h^m)q)F_b^m(k_m^R(q)) - p = 0, \tag{R1}$$

$$(h^w p + (1 - h^w)q)F_a^w(k_w^R(p)) + (1 - h^w p - (1 - h^w)q)F_b^w(k_w^R(p)) - q = 0, \tag{R2}$$

where

$$k_m^R(q) = q(u_{aa}^m - u_{ba}^m) + (1 - q)(u_{ab}^m - u_{bb}^m),$$

$$k_w^R(p) = p(u_{aa}^w - u_{ba}^w) + (1 - p)(u_{ab}^w - u_{bb}^w).$$

Since for any p, there is a q that satisfies (R2), we have that

$$(h^w p + (1 - h^w)q(p))F_a^w(k_w^R(p)) + (1 - h^w p - (1 - h^w)q(p))F_b^w(k_w^R(p)) - q(p) = 0.$$

By the implicit function theorem,

$$\begin{aligned} (h^w + (1 - h^w)q')F_a^w + (-h^w - (1 - h^w)q')F_b^w - q' & \\ +(h^w p + (1 - h^w)q)f_a^w\Delta^w + (1 - h^w p - (1 - h^w)q)f_b^w\Delta^w &= 0, \end{aligned}$$

where

$$\Delta^w = u^w_{aa} - u^w_{ab} + u^w_{bb} - u^w_{ba} > 0.$$

Simplify and rearrange the above expression:

$$q' = \frac{h^w(F^w_a - F^w_b) + f^w \Delta^w}{1 - (1 - h^w)(F^w_a - F^w_b)} > 0,$$

where

$$f^w = (h^w p + (1 - h^w)q) f^w_a + (1 - h^w p - (1 - h^w)q) f^w_b \in (\min\{f^w_a, f^w_b\}, \max\{f^w_a, f^w_b\}).$$

The slope of the LHS of equation (R1) given $q(p)$ is

$$(h^m + (1 - h^m)q')(F^m_a - F^m_b) + f^m \Delta_m q' - 1,$$

where

$$f^m = (h^m p + (1 - h^m)q) f^m_a + (1 - h^m p - (1 - h^m)q) f^m_b \in (\min\{f^m_a, f^m_b\}, \max\{f^m_a, f^m_b\}),$$

and

$$\Delta^m = u^m_{aa} - u^m_{ab} + u^m_{bb} - u^m_{ba} > 0.$$

Plugging in q', we can show that the LHS of (R1) has the same sign as the following expression:

$$\left[\frac{(1 - h^m)(F^m_a - F^m_b) + f^m \Delta^m}{1 - h^m(F^m_a - F^m_b)}\right] \cdot \left[\frac{h^w(F^w_a - F^w_b) + f^w \Delta^w}{1 - (1 - h^w)(F^w_a - F^w_b)}\right] - 1 \equiv K(p).$$

Suppose $K(p) = 0$ has two solutions, denoted by $\underline{p}$ and $\overline{p}$. We must have $K(p) < 0$ for $p \in (0, \underline{p}) \cup (\overline{p}, 1)$ and $K(p) > 0$ for $p \in (\underline{p}, \overline{p})$. Furthermore, if the LHS of (R1) is negative when $p = \underline{p}$ and is positive when $p = \overline{p}$, then there must exist two stable equilibria lying in $(0, \underline{p})$ and $(\overline{p}, 1)$, respectively (because the LHS of (R1) is nonnegative when $p = 0$ and is nonpositive when $p = 1$), and one unstable equilibrium lying in $(\underline{p}, \overline{p})$.

B.2 Equilibria under Assortative Matching

Let (p, q) denote an equilibrium. It satisfies

$$(h^m p + (1 - h^m)q) F^m_a(k^A_m(p, q)) + (1 - h^m p - (1 - h^m)q) F^m_b(k^A_m(p, q)) - p = 0, \tag{A1}$$

$$(h^w p + (1 - h^w)q) F^w_a(k^A_w(p, q)) + (1 - h^w p - (1 - h^w)q) F^w_b(k^A_w(p, q)) - q = 0, \tag{A2}$$

where

$$k_m^A(p,q) = \begin{cases} \frac{q}{p}u_{aa}^m + (1-\frac{q}{p})u_{ab}^m - u_{bb}^m & p \geq q \\ u_{aa}^m - \left(\frac{q-p}{1-p}u_{ba}^m + \frac{1-q}{1-p}u_{bb}^m\right) & p < q \end{cases},$$

and

$$k_w^A(q,p) = \begin{cases} \frac{p}{q}u_{aa}^w + (1-\frac{p}{q})u_{ab}^w - u_{bb}^w & p < q \\ u_{aa}^w - (\frac{p-q}{1-q}u_{ba}^w + \frac{1-p}{1-q}u_{bb}^w) & p \geq q \end{cases}.$$

Since for any p, there is a q that satisfies (A2), we have that

$$(h^w p + (1-h^w)q(p))F_a^w(k_w^A(p,q(p))) + (1-h^w p - (1-h^w)q(p))F_b^w(k_w^A(p,q)) - q(p) = 0.$$

By the implicit function theorem,

$$\begin{aligned} (h^w + (1-h^w)q')F_a^w + (-h^w - (1-h^w)q')F_b^w - q' & \\ +(h^w p + (1-h^w)q)f_a^w \cdot (k_{wp}^A + k_{wq}^A q') + (1-h^w p - (1-h^w)q)f_b^w \cdot (k_{wp}^A + k_{wq}^A q') &= 0, \end{aligned}$$

where $k_{wp}^A > 0$ and $k_{wq}^A < 0$ represent

$$k_{wp}^A = \begin{cases} \frac{1}{q}\left(u_{aa}^w - u_{ab}^w\right) & p < q \\ \frac{1}{1-q}\left(u_{bb}^w - u_{ba}^w\right) & p \geq q \end{cases}, \quad k_{wq}^A = \begin{cases} -\frac{p}{q}\frac{1}{q}\left(u_{aa}^w - u_{ab}^w\right) & p < q \\ -\frac{1-p}{1-q}\frac{1}{1-q}\left(u_{bb}^w - u_{ba}^w\right) & p \geq q \end{cases}.$$

Rearranging the above expression, we get

$$q' = \frac{h^w(F_a^w - F_b^w) + f^w k_{wp}^A}{1 - (1-h^w)(F_a^w - F_b^w) - f^w k_{wq}^A} > 0,$$

where

$$f^w = (h^w p + (1-h^w)q)f_a^w + (1-h^w p - (1-h^w)q)f_b^w \in (\min\{f_a^w, f_b^w\}, \max\{f_a^w, f_b^w\}).$$

The denominator minus the numerator of q' is

$$1 - (F_a^w - F_b^w) - f^w(k_{wq}^A + k_{wp}^A) = 1 - (F_a^w - F_b^w) - f^w \times \begin{cases} (1-\frac{p}{q})\frac{1}{q}(u_{aa}^w - u_{ab}^w) & p < q \\ (1-\frac{1-p}{1-q})\frac{1}{1-q}(u_{bb}^w - u_{ba}^w) & p \geq q \end{cases}. \quad \text{(A3)}$$

As long as (A3) is nonnegative, q' is weakly smaller than 1. The slope of the LHS of equation (A1), given $q(p)$, is

$$(h^m + (1-h^m)q')(F_a^m - F_b^m) + f^m \cdot (k_{mp}^A + k_{mq}^A q') - 1,$$

where

$$f^m = (h^m p + (1-h^m)q) f_a^m + (1 - h^m p - (1-h^m)q) f_b^m \in (\min\{f_a^m, f_b^m\}, \max\{f_a^m, f_b^m\}),$$

and $k_{mp} < 0$ and $k_{mq} > 0$ represent

$$k_{mp}^A = \begin{cases} -\frac{q}{p}\frac{1}{p}(u_{aa}^m - u_{ab}^m) & p \geq q \\ -\frac{1-q}{1-p}\frac{1}{1-p}(u_{bb}^m - u_{ba}^m) & p < q \end{cases}, \quad k_{mq}^A = \begin{cases} \frac{1}{p}(u_{aa}^m - u_{ab}^m) & p \geq q \\ \frac{1}{1-p}(u_{bb}^m - u_{ba}^m) & p < q \end{cases}.$$

Plugging in q', we can show that the slope of the LHS of (A1) has the same sign as the following expression:

$$\left[\frac{(1-h^m)(F_a^m - F_b^m) + f^m k_{mq}^A}{1 - h^m(F_a^m - F_b^m) - f^m k_{mp}^A}\right] \cdot \left[\frac{h^w(F_a^w - F_b^w) + f^w k_{wp}^A}{1 - (1-h^w)(F_a^w - F_b^w) - f^w k_{wq}^A}\right] - 1.$$

The numerator minus the denominator of the first term is simplified as

$$1 - (F_a^m - F_b^m) - f^m(k_{mq}^A + k_{mp}^A) = 1 - (F_a^m - F_b^m) - f^m \times \begin{cases} (1 - \frac{q}{p})\frac{1}{p}(u_{aa}^m - u_{ab}^m) & p \geq q \\ (1 - \frac{1-q}{1-p})\frac{1}{1-p}(u_{bb}^m - u_{ba}^m) & p < q \end{cases}. \quad \text{(A4)}$$

As long as (A4) is nonnegative, the first term is weakly smaller than 1. Coupled with q' smaller than 1, the LHS of (A1) must be decreasing and we have a unique equilibrium. Furthermore, since the LHS of (A1) is nonnegative when $p = 0$ and is nonpositive when $p = 1$, even if the LHS of (A1) is upward-sloping for small p (or for big p), there is still a unique equilibrium.

Let us look at the special case in which men and women are completely symmetric. In this case, (A1) and (A2) imply that $p = q$, which in turn implies that (A3) and (A4) are

$$1 - (F_a^w - F_b^w) > 0,$$
$$1 - (F_a^m - F_b^m) > 0,$$

respectively. Hence, the slope of the LHS of (A1) is always negative, and there must exist a unique stable equilibrium in this case.

C Evidence from Arranged Marriages in India

We use India Human Development Survey-II (IHDS-II), 2011-2012, to verify our assumption that arranged marriages are more assortative in marital preferences and characteristics, as well as our predictions that (the more assortative) arranged marriages are associated with more backward (male-dominated) norms in marriage and work, and in fertility preferences and actualization. Arranged marriages are defined as those marriages in which parents/relatives alone choose the husband (MH4A=3) and the woman does not have a say in the choice (MH4B=0). Non-arranged marriages are those marriages in which (i) a woman chooses on her own (MH4A=1); (ii) the woman and parents/relatives jointly choose together (MH4A=2); or (iii) parents/relatives choose alone (MH4A=3), but a woman has a say in the choice (MH4B=1).[30]

Table C.1 shows summary statistics of arranged marriages: 5 percent of women choose their husband alone, 21.9 percent of women choose jointly with their parents, 30.6 percent of women have a say in their parents' choice, and 42.5 percent of women do not have a say in their parents' choices.

Table C.1: Marriage Type

Item	Number	Percent
Woman chooses	1,968.0	5.0
Woman and parents/relatives jointly choose	8,605.0	21.9
Parents/relatives choose, woman has a say	11,991.0	30.6
Parents/relatives choose, woman has no say	16,672.0	42.5
Total	39,236.0	100.0

Table C.2: Preference Homophily in Arranged Marriages

	workpref	morekidspref	whennextkidpref	nmorekidspref
	b/t	b/t	b/t	b/t
arranged=1	0.011***	0.105***	0.003	0.001
	(4.07)	(13.43)	(0.65)	(0.08)
Constant	0.945***	0.533***	0.969***	0.956***
	(512.02)	(105.92)	(367.87)	(196.44)
Observations	26,677	16,179	6,505	2,769

Subsequently, we show how arranged marriages are associated with more homophily in preferences, more homophily in social and economic status, more backward norms in work and marriages, preferences for more children and sons, and having more children (but not ending up with more sons). First, arranged marriages are associated with more assortative matching in preferences. Table C.2 shows that arranged marriages are associated with 1.1 percent more chance of having the same preference for whether women want to work and 10.5 percent more chance of having the same preference for having more children as well as more homophily in preferences for when to have the next child and how many more children to have.

[30] Jacob (2016) defines arranged marriages in the same way. In contrast to our paper, which focuses on the associations of arranged marriages with marital preferences and with the alignment of husband's and wife's preferences,

Table C.3: Social and Economic Status Homophily in Arranged Marriages

	samecaste	sameeconstatus	samecollege	sameEnglish
	b/t	b/t	b/t	b/t
arranged=1	0.014***	0.014***	0.034***	0.031***
	(6.58)	(3.75)	(8.41)	(7.15)
Constant	0.944***	0.159***	0.785***	0.781***
	(613.35)	(65.13)	(287.29)	(267.60)
Observations	39,077	39,143	39,236	34,401

Second, arranged marriages are associated with more assortative matching in social and economic status. Table C.3 shows that arranged marriages are associated with 1.4 percent more chance of marrying within the same caste, 1.4 percent more chance of marrying someone of the same or better economic status, 3.4 percent more chance of marrying someone of the same educational level, and 3.1 percent more chance of speaking English.

Table C.4: Backward Norms in Arranged Marriages

	purdah	illiterate	husbdecideswork	husbdecideskidmarr
	b/t	b/t	b/t	b/t
arranged=1	0.297***	0.262***	0.011*	0.035***
	(62.84)	(53.76)	(2.15)	(7.52)
Constant	0.452***	0.279***	0.422***	0.672***
	(136.48)	(93.42)	(128.42)	(214.87)
Observations	39236	39233	39236	39236

Finally, arranged marriages are associated with more male-dominated norms in marital preferences and behavior. Table C.4 shows that arranged marriages are associated with 29.7 percent more chance of practicing purdah, 26.2 percent more chance of being illiterate, 1.1 percent more chance that the husband decides whether the wife can work, and 3.5 percent more chance that the husband decides whom children marry. Figure C.2 confirms the positive correlation between percent of male-dominated norms and percent of arranged marriage in different Indian states. Arranged marriages are associated with more children and a higher percentage of sons desired. Table C.5 shows that compared to women in non-arranged marriages, women in arranged marriages want 0.345 more children, 0.263 more sons, 0.083 more daughters, and 2.3 percent more sons. Arranged marriages are associated with more actual children but not more actual sons. Table C.6 shows that women in arranged marriages have 1.207 more children, 0.598 more sons, and 0.598 more daughters, but virtually the same percent of sons as those in non-arranged marriages despite their preference for a higher composition of sons.

Jacob investigates the effects of arranged marriages on marital life and child development.

Figure C.2: Correlation between Percent of Backward Norms and Percent of Arranged Marriages in Different Indian States.

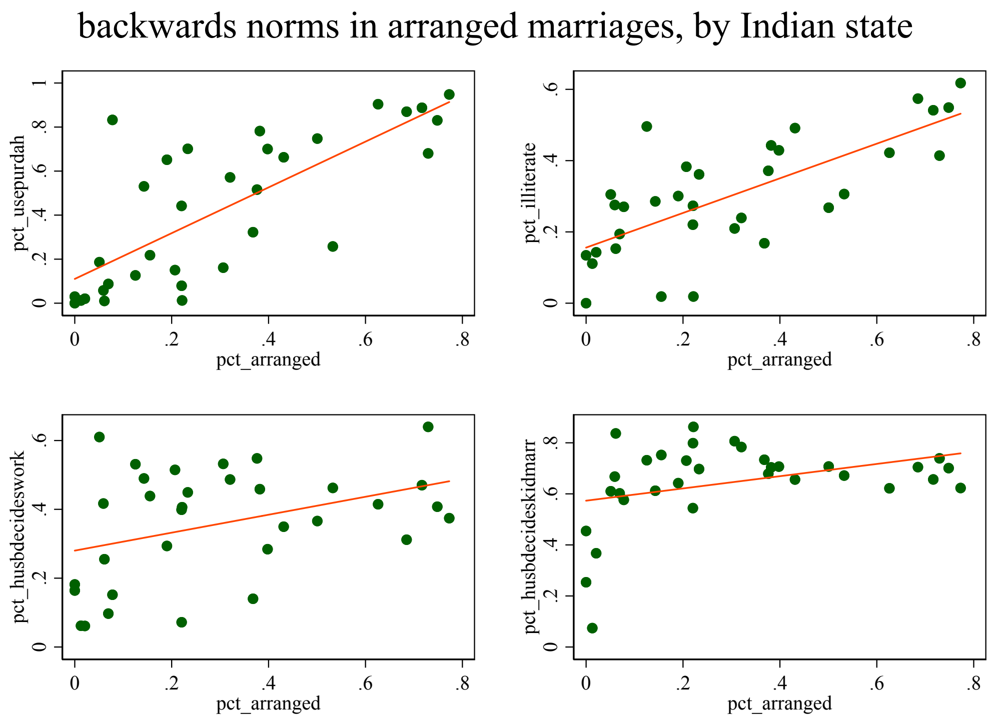


Table C.5: Children Desired in Arranged Marriages

	nkidswanted b/t	nsonswanted b/t	ndaughterswanted b/t	psonswanted b/t
arranged=1	0.345***	0.263***	0.083***	0.023***
	(35.07)	(40.65)	(16.32)	(14.35)
Constant	2.261***	1.238***	1.097***	0.541***
	(376.60)	(330.05)	(338.58)	(524.53)
Observations	37,430	34,567	34,246	34,505

Table C.6: Children Realized in Arranged Marriages

	nkids b/t	nsons b/t	ndaughters b/t	psons b/t
arranged=1	1.207***	0.609***	0.598***	-0.003
	(47.23)	(38.24)	(33.49)	(-0.66)
Constant	2.422***	1.259***	1.163***	0.540***
	(169.35)	(136.33)	(115.37)	(175.92)
Observations	24,452	24,452	24,452	22,695