Educational Rat Race, Positional Externalities, and Intergenerational Mobility

Ernest Liu
Princeton University

Shaoda Wang
Chicago Harris

Hanzhe Zhang
Michigan State University

Washington University in St. Louis
Friday, February 27, 2026

Educational rat race

- Competition for high-quality jobs or opportunities can induce parents and students to engage in excessive education.
  - analogous to a *rat race* in which everyone runs faster just to stay in place.
- In many societies, a fixed number of top positions (elite college slots, prestigious jobs) drives families to invest heavily in education to improve their children's chances of winning these opportunities.
- This paper develops a tractable overlapping-generations model to investigate this phenomenon
  - as well as income inequality and intergenerational mobility.

Positional externalities

- When skills and jobs are positively matched, the equilibrium features over-investment in education relative to the social optimum.
- This over-investment is driven by a *positional externality*:
  - individual families do not account for the fact that improving their child's rank in the skill distribution comes partly at the expense of others.
- Empirical evidence: We show that when income declines more steeply with college rank in a country, the share of private spending on education tends to increase in that country.

Income inequality and intergenerational mobility

- Given the optimal policy of parents in each generation, we derive a linear law of motion for (log) skill across generations.
- The process for log human capital is mean-reverting: children's log skill is an affine function of their parents' log skill.
- Under mild conditions, the economy converges to a stationary distribution of skills, which is lognormal.
- We also derive the intergenerational elasticity of income (IGE) in the stationary state.
    - The IGE is increasing in the strength of human capital transmission (education technology and any direct skill inheritance) but is mitigated by the randomness in job allocations.

Literature: Rat race and positional externalities

- The paper is related to premarital/pre-matching investments, e.g., Bhaskar and Hopkins (2016, JPE), Zhang (2021, JPE), Bhaskar, Li and Yi (2023, JPE).
    - recent surveys of investment and matching literature: Hopkins (2022) and Nöldeke and Samuelson (2024).
- Positional externalities have appeared in the context of status competition, e.g., Hopkins and Kornienko (2004, AER), Becker, Murphy and Werning (2005, JPE), Ray and Robson (2012, Ecma)
    - empirical analysis: Kim, Tertilt and Yum (2024, AER) quantify educational overinvestment due to status competition in South Korea

Literature: Intergenerational mobility

- We provide a general-equilibrium model of altruistic parents deciding on educational investments for children who compete in the labor market to derive a stationary law of motion for skill formation that involves aspects of nature and nurture, as well as intergenerational mobility.
  - Becker and Tomes (1979, JPE), Becker and Tomes (1986, JOLE) and Becker, Kominers, Murphy and Spenkuch (2018, JPE)
    - Relatedly, Becker and Barro (1988, QJE) and Barro and Becker (1989, Ecma) study fertility and intergenerational mobility in overlapping-generations models.
  - a stylized version of the skill formation of Cunha and Heckman (2007, AERPP), Cunha, Heckman and Schennach (2010, Ecma), and Heckman and Mosso (2014, ARE)

Roadmap

- Key economic insights of positional externalities in the educational rat race in the two-period model

- Empirical evidence

- Inequality and intergenerational mobility in the tractable overlapping-generations model

Parent's education investment choice

- Parent i in generation t has income $y_{i,t}$.
- Parent i chooses to spend $e_{i,t}$ to invest in a child's education.
- Education translates into a child's skill:

$$\ln s_{i,t+1} = \eta_e \ln e_{i,t} + {\varepsilon^s}_{i,t+1},$$

  where ${\varepsilon^s}_{i,t} \sim N\left(0, \sigma^2_{\varepsilon^s}\right)$ and $0 < \eta_e < 1$.

  - Later on we will generalize to a fully dynamic problem in which the parent's skill could also influence the child's (and subsequently the future generations') skill:

$$\ln s_{i,t+1} = \eta_e \ln e_{i,t} + \eta_s \ln s_{i,t} + \varepsilon^s_{i,t+1},$$

Child's labor market matching and income

- Skills are matched noisily with job qualities via the Gaussian copula:

$$\begin{pmatrix} \ln s_{i,t+1} \\ \ln q_{i,t+1} \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_{s,t+1} \\ \mu_q \end{pmatrix}, \begin{pmatrix} \sigma^2_{s,t+1} & \rho\, \sigma_{s,t+1}\sigma_q \\ \rho\, \sigma_{s,t+1}\sigma_q & \sigma^2_q \end{pmatrix} \right),$$

  where $0 \leq \rho \leq 1$.
  - The pool of opportunities is fixed, $\ln q_{i,t} \sim N\left(\mu_q, \sigma_q^2\right)$, but for any $\rho > 0$, higher skills are more likely to draw higher opportunities.
    - $\rho = 1$: perfectly positive-assortative matching.
    - $\rho = 0$: uniformly random matching.
- A child's income on the job is

$$\ln y_{i,t+1} = (1-\alpha) \ln s_{i,t+1} + \alpha \ln q_{i,t+1} + k.$$

Microfoundation when $\rho = 1$

- Cobb-Douglas matching output $O(i,j) = As_i^{1-\alpha} q_j^{\alpha}$.
- Perfectly transferable utility.
- Stable outcome is a pair of payoff function (x, y) such that
  - $y_i + x_j = O(i,j)$ for matched pairs (i,j), and
  - $y_i + x_j \geq O(i,j)$ for all feasible (i,j).
- Under lognormal distributions of skills and job qualities,

$$\ln s \sim N(\mu_s, \sigma_s) \text{ and } \ln q \sim N(\mu_q, \sigma_q),$$

  - Matching is perfectly positive assortative: $\Phi_s(s) = \Phi_q(q)$
  - Workers' payoffs are $\ln y_i = (1-\alpha)\ln s_i + \alpha \ln q_i +$ constant.

Microfoundation when $\rho < 1$

- Cobb-Douglas matching output $O(i,j) = As_i^{1-\alpha}q_j^{\alpha}$.
- Matching is positive assortative based on a latent index

$$\rho \cdot \frac{\ln s_i - \mu_s}{\sigma_s} + \sqrt{1-\rho^2} \cdot \varepsilon_i, \quad \varepsilon_i \sim N(0,1).$$

- Division of surplus is through Nash bargaining

$$y_i = \theta A s_i^{1-\alpha} q_j^{\alpha},$$

so

$$\ln y_i = \ln(\theta A) + (1-\alpha)\ln s_i + \alpha \ln q_i.$$

Parent's utility function

- We first analyze a simple problem in which the child consumes the entire income, generating utility $\ln y_{i,t+1}$.

- Each parent decides how much to spend on education, solving

$$u\left(y_{i,t}\right) = \max_{e_{i,t}} \left\{\ln\left(y_{i,t} - e_{i,t}\right) + \delta \mathbb{E}\left[\ln y_{i,t+1} | e_{i,t}\right]\right\}.$$

  - In the dynamic model, the child also decides on education investment for the grandchild, so altruistic parents have a dynastic utility function

$$u_t\left(y_{i,t}, s_{i,t}\right) = \max_{e_{i,t}} \left\{\ln\left(y_{i,t} - e_{i,t}\right) + \delta \mathbb{E}\left[u_{t+1}\left(y_{i,t+1}, s_{i,t+1}\right) | e_{i,t}, s_{i,t}\right]\right\}.$$

Expected job quality

- Define the percentile ranks of $s_{i,t+1}$ and $q_{i,t+1}$:

$$\Phi_s(s_{i,t+1}) \equiv \frac{\ln s_{i,t+1} - \mu_{s,t+1}}{\sigma_{s,t+1}} \text{ and } \Phi_q(q_{i,t+1}) \equiv \frac{\ln q_{i,t+1} - \mu_q}{\sigma_q}.$$

- The Gaussian copula job matching process implies

$$\Phi_q(q_{i,t+1}) = \rho \cdot \Phi_s(s_{i,t+1}) + \sqrt{1-\rho^2} \cdot \varepsilon^q_{i,t+1}.$$

- Given $\varepsilon^q_{i,t+1} \sim N(0,1)$,

$$\mathbb{E}\left[\frac{\ln q_{i,t+1} - \mu_q}{\sigma_q} \middle| s_{i,t+1} \right] = \rho \frac{\ln s_{i,t+1} - \mu_{s,t+1}}{\sigma_{s,t+1}}.$$

- Rearranged,

$$\mathbb{E}\left[\ln q_{i,t+1} | s_{i,t+1}\right] = \mu_q + \rho \frac{\sigma_q}{\sigma_{s,t+1}} \left(\ln s_{i,t+1} - \mu_{s,t+1}\right).$$

Expected income given skill

- Recall income for a job

$$\ln y_{i,t+1} = (1-\alpha)\ln s_{i,t+1} + \alpha \ln q_{i,t+1} + k.$$

- Hence, we can write income generation given skill as

$$\begin{aligned}
& \mathbb{E}[\ln y_{i,t+1} | s_{i,t+1}] \\
= \quad & (1-\alpha)\ln s_{i,t+1} + \alpha \left[\mu_q + \rho \frac{\sigma_q}{\sigma_{s,t+1}} (\ln s_{i,t+1} - \mu_{s,t+1})\right] + k \\
= \quad & \left(1 - \alpha + \alpha\rho \frac{\sigma_q}{\sigma_{s,t+1}}\right) \ln s_{i,t+1} + \text{constants}
\end{aligned}$$

Expected income given education

- Recall skill given education

$$\ln s_{i,t+1} = \eta_e \ln e_{i,t} + \varepsilon^s_{i,t+1}.$$

- Child's expected income given education $e_{i,t}$ is

$$\mathbb{E}\left[\ln y_{i,t+1} | e_{i,t}\right] = \underbrace{\left(1 - \alpha + \alpha\rho \frac{\sigma_q}{\sigma_{s,t+1}}\right)}_{\equiv S_{t+1}} \eta_e \ln e_{i,t} + \text{constants}$$

Parent's education investment decision

- Parent's investment decision solves

$$\max_{e_{i,t}} \left\{ \ln\left(y_{i,t} - e_{i,t}\right) + \delta \mathbb{E}\left[\ln y_{i,t+1} | e_{i,t}\right] \right\}.$$

- Plugging in the expected income, we translate the problem to

$$\max_{e_{i,t}} \left\{ \ln\left(y_{i,t} - e_{i,t}\right) + \delta S_{t+1} \eta_e \ln e_{i,t} + \text{constants} \right\}.$$

- First-order condition:

$$\frac{1}{y_{i,t} - e_{i,t}} = \delta S_{t+1} \eta_e \frac{1}{e_{i,t}}.$$

- We get a constant rate of income spent on education:

$$R_t \equiv \frac{e_{i,t}}{y_{i,t}} = \frac{\delta S_{t+1} \eta_e}{1 + \delta S_{t+1} \eta_e}.$$

Investment incentives

- Let's better understand

$$S_{t+1} = 1 - \alpha + \alpha\rho\frac{\sigma_q}{\sigma_{s,t+1}}; S = 1 - \alpha + \alpha\rho\frac{\sigma_q}{\sigma_s}$$

in

$$R_t \equiv \frac{e_{i,t}}{y_{i,t}} = \frac{\delta S_{t+1}\eta_e}{1 + \delta S_{t+1}\eta_e} = \frac{\delta\left(1 - \alpha + \alpha\rho\frac{\sigma_q}{\sigma_{s,t+1}}\right)\eta_e}{1 + \delta\left(1 - \alpha + \alpha\rho\frac{\sigma_q}{\sigma_{s,t+1}}\right)\eta_e}.$$

- Note that we can rewrite the maximization problem as

$$\max_{e_{i,t}}\left\{\underbrace{\ln\left(y_{i,t} - e_{i,t}\right)}_{\text{consumption}} + \underbrace{\delta\left(1 - \alpha\right)\eta_e \ln e_{i,t}}_{\text{child income due to skill}} + \underbrace{\delta\,\overbrace{\alpha\mathbb{E}\left[\ln q_{i,t+1}|e_{i,t}\right]}^{\alpha\rho\frac{\sigma_q}{\sigma_{s,t+1}}\eta_e \ln e_{i,t}}}_{\text{child income due to job quality}}\right\}.$$

Positional incentives

- The sensitivity of expected log job quality with respect to education

$$\frac{\mathrm{d}\mathbb{E}\left[\ln q_{i,t+1}|e_{i,t}\right]}{\mathrm{d}\ln e_{i,t}} = \alpha\rho\frac{\sigma_q}{\sigma_{s,t+1}},$$

can be decomposed into

$$\underbrace{\frac{\mathrm{d}\ln s_{i,t+1}}{\mathrm{d}\ln e_{i,t}}}_{\eta_e}\underbrace{\frac{\mathrm{d}\Phi_s(\ln s_{i,t+1})}{\mathrm{d}\ln s_{i,t+1}}}_{\frac{1}{\sigma_{s,t+1}}}\underbrace{\frac{\mathrm{d}\Phi_q\left(\mathbb{E}\left[\ln q_{i,t+1}|s_{i,t+1}\right]\right)}{\mathrm{d}\Phi_s(\ln s_{i,t+1})}}_{\rho}\underbrace{\frac{\mathrm{d}\mathbb{E}\left[\ln q_{i,t+1}|s_{i,t+1}\right]}{\mathrm{d}\Phi_q\left(\mathbb{E}\left[\ln q_{i,t+1}|s_{i,t+1}\right]\right)}}_{\sigma_q}$$

Investment rate R_t

Higher investment rate

$$R_t \equiv \frac{e_{i,t}}{y_{i,t}} = \frac{\delta S_{t+1}\eta_e}{1+\delta S_{t+1}\eta_e} = \frac{\delta\left(1-\alpha+\alpha\rho\frac{\sigma_q}{\sigma_{s,t+1}}\right)\eta_e}{1+\delta\left(1-\alpha+\alpha\rho\frac{\sigma_q}{\sigma_{s,t+1}}\right)\eta_e}$$

when

- the agent is more patient/altruistic (i.e., higher δ);
- education translates into more skills (i.e., higher η_e);
- correlation between skill and opportunity is higher (i.e., higher ρ)
- more investment translates into bigger gains in the skill quantiles (i.e., lower $\sigma_{s,t+1}$), and/or
- a higher skill quantile translates into bigger gains in expected job quality (i.e., higher σ_q).
    - parents invest in education to improve skill because higher skill (i) raises output and (ii) raises the expected opportunity (given a fixed job pool).

Equilibrium over-investment

- Socially optimal education investment is

$$R_t^{\text{opt}} \equiv \frac{e_{i,t}^{\text{opt}}}{y_{i,t}} = \frac{\delta\left(1-\alpha\right)\eta_e}{1+\delta\left(1-\alpha\right)\eta_e}.$$

- In contrast, privately optimal education investment is

$$R_t \equiv \frac{e_{i,t}}{y_{i,t}} = \frac{\delta\left(1-\alpha+\alpha\rho\frac{\sigma_q}{\sigma_{s,t+1}}\right)\eta_e}{1+\delta\left(1-\alpha+\alpha\rho\frac{\sigma_q}{\sigma_{s,t+1}}\right)\eta_e}.$$

- For any $\rho > 0$, the wedge is positive, implying an over-investment by parents, due to positive-assortative matching, a positional externality.

- To formally see this, note that a utilitarian planner solves

$$\max_{e_{i,t}} \left\{ \int_i \left[\ln\left(y_{i,t} - e_{i,t}\right) + \delta \mathbb{E}\left[\ln y_{i,t+1} | e_{i,t}\right]\right] \mathrm{d}i \right\}$$

subject to

$$\ln y_{i,t+1} = (1-\alpha)\ln s_{i,t+1} + \alpha \ln q_{i,t+1},$$

$$\ln s_{i,t+1} = \eta_e \ln e_{i,t} + \varepsilon^s_{i,t+1},$$

and

$$\ln q_{i,t+1} = \mu_q + \rho \frac{\sigma_q}{\sigma_{s,t+1}} \left(\ln s_{i,t+1} - \int_j \ln s_{j,t+1}\, \mathrm{d}j \right) + \sigma_q \sqrt{1-\rho^2}\varepsilon^q_{i,t+1},$$

where $\varepsilon^q_{i,t+1} \sim N(0,1)$. We can combine the three constraints to write

$$\mathbb{E}\left[\ln y_{i,t+1} | e_{i,t}\right] = \alpha\mu_q + (1-\alpha)\,\eta_e \ln e_{i,t} + \alpha\rho \frac{\sigma_q}{\sigma_{s,t+1}} (\ln s_{i,t+1} - \int_j \ln s_{j,t+1}\, \mathrm{d}j)$$

- Hence, the social planner's problem is

$$\max_{e_{i,t}} \quad \int_i \left[\ln\left(y_{i,t} - e_{i,t}\right) + \delta\left(1-\alpha\right)\eta_e \ln e_{i,t}\right] \, \mathrm{d}i + \alpha\rho\frac{\sigma_q}{\sigma_{s,t+1}} \underbrace{\left(\int_i \ln s_{i,t+1} \, \mathrm{d}i - \int_j \ln s_{j,t+1} \, \mathrm{d}j\right)}_{=0},$$

which implies a simplified problem of

$$\max_{e_{i,t}} \int_i \left[\ln\left(y_{i,t} - e_{i,t}\right) + \delta\left(1-\alpha\right)\eta_e \ln e_{i,t}\right] \, \mathrm{d}i$$

and a solution of

$$R_t^{\text{opt}} = \frac{e_{i,t}^{\text{opt}}}{y_{i,t}} = \frac{\delta\left(1-\alpha\right)\eta_e}{1+\delta\left(1-\alpha\right)\eta_e}.$$

Proposition 1 (Over-Investment Rat Race)

The decentralized equilibrium features over-investment in education relative to the social optimum as long as job assignment is positively assortative ($\rho > 0$). In particular, the equilibrium education rate R exceeds the efficient rate R^{opt}, with the wedge

$$R - R^{opt} = \frac{\delta\,\eta_e\,\alpha\,\rho\,\dfrac{\sigma_q}{\sigma_{s,t+1}}\,(1-\alpha)}{\Big[1+\delta\,(1-\alpha)\,\eta_e\Big]\;\Big[1+\delta\,\Big(1-\alpha+\alpha\,\rho\,\frac{\sigma_q}{\sigma_{s,t+1}}\Big)\,\eta_e\Big]} \;>\; 0 \quad \textit{for } \rho > 0.$$

Equivalently, parents invest too much in children's education, driven by the private incentive to boost their child's relative standing (the "rat race"), which the social planner would avoid.

Key testable implication

- The education investment rate

$$R \equiv \frac{e_i}{y_i} = \frac{\delta S \eta_e}{1 + \delta S \eta_e}$$

is increasing in skill-income elasticity

$$S = \frac{\mathrm{d}\mathbb{E}[\ln y_i | s_i]}{\mathrm{d} \ln s_i} = \frac{\mathrm{d}\mathbb{E}[\ln y_i | s_i]}{\mathrm{d}\Phi_s(\ln s_i)} \underbrace{\frac{\mathrm{d}\Phi_s(\ln s_i)}{\mathrm{d} \ln s_i}}_{>0}$$

Empirical education investment rate R: Measure 1

- **Household education expenditure rate**

$$\widehat{R} = \frac{\text{Total Private Household Expenditure on Education}}{\text{Gross Domestic Product (GDP)}} \times 100.$$

  - in harmonized national household survey data provided by the UNESCO Institute for Statistics (UIS)
  - It reflects the macroeconomic weight of education financing borne by households.
  - Private household education expenditure includes tuition payments, fees, textbooks, and other direct education-related expenditures reported in national household income and expenditure surveys.

Empirical education investment rate R: Measure 2

- **Household secondary education expenditure rate**

$$\widehat{R} = \frac{\left(\frac{\text{Total Household Expenditure on Secondary Education}}{\text{Total Secondary Enrollment}}\right)}{\text{GDP per capita}} \times 100.$$

  - constructed by UNESCO UIS using household survey data aligned with macroeconomic indicators from the World Bank's World Development Indicators (WDI) measures per-student household expenditure in secondary education (ISCED levels 2 and 3).
  - allows cross-country comparisons of the relative financial effort required for secondary schooling.
  - can be interpreted as the proportion of the average annual income required to finance one student's secondary education, serving as a proxy for the intensity of private investment in human capital.

Revelio Lab LinkedIn Data

- 12+ million college graduates from 2000 to 2015 worldwide with complete information on education, work, and country
  - 624+ million LinkedIn profiles in total
  - 129+ million reported bachelor degree information
- Revelio Lab estimated a salary based on companies, titles, countries, etc.

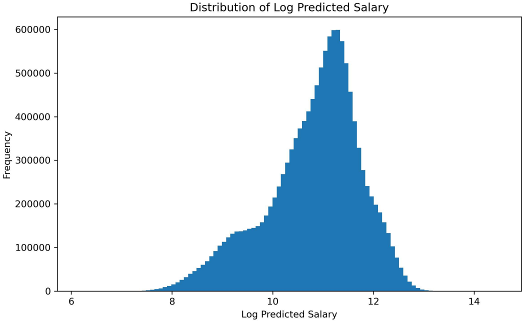
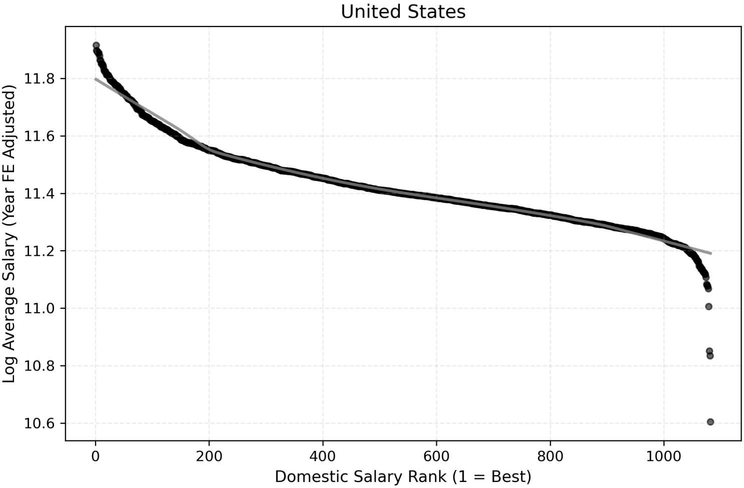


Table: Top 10 Universities by Earnings

Rank	US	China	Korea
1	Univ of Pennsylvania	Tsinghua University	KAIST
2	Duke University	Peking University	Seoul National University
3	Princeton University	University of Science and Technology of China	Yonsei University
4	Stanford University	Central University of Finance and Economics	Korea University
5	Yeshiva University	Beijing University of Posts and Telecommunications	Hongik University
6	Caltech	Zhejiang University	Ajou University
7	Yale University	Beihang University	Sogang University
8	MIT	Xiamen University	Chung-Ang University
9	George Washington Univ	Shanghai Jiao Tong University	Hanyang University
10	Bentley University	Nanjing University	Ewha Womans University

Average log salary by college rank



University rank gradient

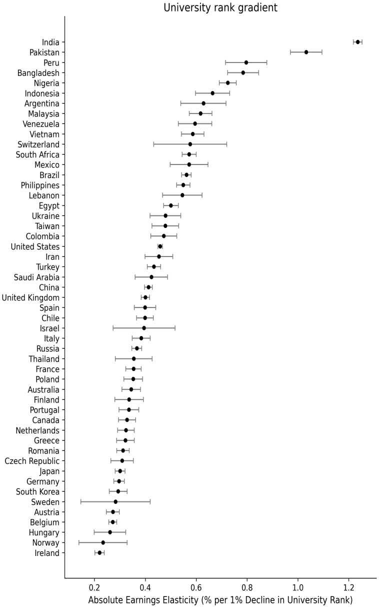
- Run regression:

$$\overline{\ln y}_{uc} = c + \beta_c \ln R_{uc} + \epsilon_{uc}$$

  - $\overline{\ln y}_{uc}$: average log salary of university u in country c
  - $\ln R_{uc}$: log rank of the university
  - $\widehat{\beta}_c$: percentile increase in income by percentile increase in rank

- University rank gradient:

$$|\widehat{\beta}_c| = \frac{\mathrm{d}\mathbb{E}[\ln y_i | s_i]}{\mathrm{d}\Phi_s(\ln s_i)}$$

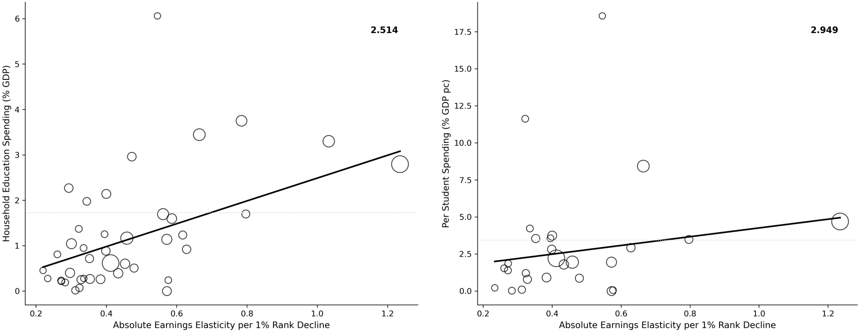

University rank gradient
India
Pakistan
Peru
Bangladesh
Nigeria
Indonesia
Argentina
Malaysia
Venezuela
Vietnam
Switzerland
South Africa
Mexico
Brazil
Philippines
Lebanon
Egypt
Ukraine
Taiwan
Colombia
United States
Iran
Turkey
Saudi Arabia
China
United Kingdom
Spain
Chile
Israel
Italy
Russia
Thailand
France
Poland
Australia
Finland
Portugal
Canada
Netherlands
Greece
Romania
Czech Republic
Japan
Germany
South Korea
Sweden
Austria
Belgium
Hungary
Norway
Ireland
0.2
0.4
0.6
0.8
1.0
1.2
Absolute Earnings Elasticity (% per 1% Decline in University Rank)

Positive relationships between university rank gradient and education investment rate

	(1) HH edu spend (% GDP)	(2) Per-student spend (% GDP pc)
β_c	2.514***	2.949***
	(0.324)	(0.881)
Constant	-0.025	1.310*
	(0.253)	(0.718)
Observations	41	27
R^2	0.607	0.310

Notes. Country-level weighted least squares (WLS) regressions. The regressor is the country-specific ranking gradient β_c, defined as the absolute earnings elasticity (in percentage points) per 1% decline in university rank. Weights are proportional to population (normalized by mean). Standard errors in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.10$.

Positive relationships between university rank gradient and education investment rate



Robustness checks

The positive relationships are robust to alternative specifications:

- Alternative years after graduation: 1-year, 5-year, 10-year
- Alternative subsamples
  - Postgraduate degrees: including or excluding PhDs
  - education and work in the same country
- Alternative measures of β_c
  - Explanatory power R^2 of a university fixed effects regression.
  - Sorting power ROC-AUC of university: how well university identity predicts the likelihood of entering high-income jobs

Overlapping generations model

- Time is discrete, indexed by generation $t = 0, 1, 2, \ldots$.
- Each generation t, there is a unit mass of families
  - indexed by $i \in [0, 1]$.
- Each family consists of one parent and one child.
- Each agent lives as a parent for one period, during which they work, earn income, consume, and decide how much to invest in their child's education.
- The child then becomes the parent of generation $t + 1$.

Parent's education investment choice

- Parent i has income $y_{i,t}$.
- Parent i chooses to spend $e_{i,t}$ to invest in a child's education.
- ~~Education translates into skill:~~

$$\cancel{\ln s_{i,t+1} = \eta_e \ln e_{i,t} + \varepsilon^s_{i,t+1},}$$

- Education and parent's skill translate into child's skill:

$$\ln s_{i,t+1} = \eta_e \ln e_{i,t} + \eta_s \ln s_{i,t} + \varepsilon^s_{i,t+1},$$

where $\varepsilon^s_{i,t} \sim N\left(0, \sigma^2_{\varepsilon^s}\right)$ and $0 < \eta_e < 1$.

Child's labor market matching

- Skills are matched noisily with job qualities via the Gaussian copula:

$$\begin{pmatrix} \ln s_{i,t+1} \\ \ln q_{i,t+1} \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_{s,t+1} \\ \mu_q \end{pmatrix}, \begin{pmatrix} \sigma_{s,t+1}^2 & \rho\, \sigma_{s,t+1} \sigma_q \\ \rho\, \sigma_{s,t+1} \sigma_q & \sigma_q^2 \end{pmatrix} \right),$$

  where $0 \leq \rho \leq 1$.

  - The pool of opportunities is fixed, $\ln q_{i,t} \sim N\left(\mu_q, \sigma_q^2\right)$, but for any $\rho > 0$, higher skills are more likely to draw higher opportunities.

- Once a child is matched with a job, income is generated:

$$\ln y_{i,t+1} = (1 - \alpha) \cdot \ln s_{i,t+1} + \alpha \cdot \ln q_{i,t+1}.$$

Parent's utility function

- ~~We first analyze a simple problem in which the child consumes the entire income, generating utility~~ $\cancel{\ln y_{i,t+1}}$~~.~~

- ~~Each parent decides on how much to spend on education, solving~~

$$\cancel{u\left(y_{i,t}\right) = \max_{e_{i,t}} \left\{\ln\left(y_{i,t} - e_{i,t}\right) + \delta \mathbb{E}\left[\ln y_{i,t+1} | e_{i,t}\right]\right\}}.$$

- Altruistic parents have a dynastic utility function

$$u_t\left(y_{i,t}, s_{i,t}\right) = \max_{e_{i,t}} \left\{\ln\left(y_{i,t} - e_{i,t}\right) + \delta \mathbb{E}\left[u_{t+1}\left(y_{i,t+1}, s_{i,t+1}\right) | e_{i,t}, s_{i,t}\right]\right\}.$$

Equilibrium utility

- In the two-period model, it is solved that

$$u(y_{i,t}) = \ln((1 - R_t)y_{i,t}) + S_{t+1}\eta_e \ln(R_t y_{i,t}) + \text{constants}.$$

- In the overlapping-generations model, guess and verify:

$$u_t(y_{i,t}, s_{i,t}) = A_t + B_t \ln(y_{i,t}) + C_t \ln(s_{i,t}).$$

Solving for equilibrium investment

- Plug $u_{t+1}(y_{i,t+1}, s_{i,t+1})$ in $u_t(y_{i,t}, s_{i,t})$:

$$\ln(y_{i,t} - e_{i,t}) + \delta\mathbb{E}\left[A_{t+1} + B_{t+1}\ln\left(y_{i,t+1}\right) + C_{t+1}\ln\left(s_{i,t+1}\right)|e_{i,t}, s_{i,t}\right]$$

- Note

$$\mathbb{E}[\ln s_{i,t+1}|e_{i,t}, s_{i,t}] = \eta_e \ln e_{i,t} + \eta_s \ln s_{i,t};$$

$$\mathbb{E}[\ln y_{i,t+1}|e_{i,t}, s_{i,t}] = S_{t+1}\mathbb{E}[\ln s_{i,t+1}|e_{i,t}, s_{i,t}] + \text{constants},$$

- The utility expression becomes

$$\ln(y_{i,t} - e_{i,t}) + \delta[B_{t+1}S_{t+1} + C_{t+1}][\eta_e \ln e_{i,t} + \eta_s \ln s_{i,t}] + \text{constants}$$

Equilibrium investment

- Equilibrium constant education investment rate:

$$R_t \equiv \frac{e_{i,t}}{y_{i,t}} = \frac{\delta \left[B_{t+1}S_{t+1} + C_{t+1}\right]\eta_e}{1 + \delta \left[B_{t+1}S_{t+1} + C_{t+1}\right]\eta_e}.$$

- We solve for nested expressions of

$$\begin{aligned} A_t &= \ln\left(1 - R_t\right) + \delta \left\{ A_{t+1} + \alpha \left[\mu_q - \rho \frac{\sigma_q}{\sigma_{s,t+1}} \mu_{s,t+1} \right] \right\} \\ &\quad + \delta \eta_e \left[B_{t+1}S_{t+1} + C_{t+1}\right] \ln R_t \\ B_t &= 1 + \delta \eta_e \left[B_{t+1}S_{t+1} + C_{t+1}\right] \\ C_t &= \delta \eta_s \left[B_{t+1}S_{t+1} + C_{t+1}\right] \end{aligned}$$

Law of motion for skill and income

- Income dynamics

$$\ln y_{i,t} = S_t \ln s_{i,t} + \alpha \left[\mu_q - \rho \frac{\sigma_q}{\sigma_{s,t}} \mu_{s,t} + \sigma_q \sqrt{1-\rho^2} \varepsilon^q_{i,t} \right],$$

where $\varepsilon^q_{i,t} \sim N(0,1)$.

- Skill dynamics

$$\begin{aligned} \ln s_{i,t+1} =& \eta_e \ln R_t + (\eta_e S_t + \eta_s) \ln s_{i,t} \\ & + \alpha \eta_e \left[\mu_q - \rho \frac{\sigma_q}{\sigma_{s,t}} \mu_{s,t} + \sigma_q \sqrt{1-\rho^2} \varepsilon^q_{i,t} \right] + \varepsilon^s_{i,t+1}. \end{aligned}$$

Stationary skill and income distributions

- Time-invariant skill variance σ_s^2:

$$\sigma_s^2 = (\eta_e S + \eta_s)^2 \sigma_s^2 + \alpha^2 \eta_e^2 \left(1 - \rho^2\right) + \sigma_\epsilon^2.$$

  - The dynamic path of skill inequality is

$$\sigma_{s,t+1}^2 = (\eta_e S_t + \eta_s)^2 \sigma_{s,t}^2 + \alpha^2 \eta_e^2 \left(1 - \rho^2\right) \sigma_q^2 + \sigma_{\varepsilon^s}^2.$$

- Time-invariant income variance:

$$\sigma_y^2 = S^2 \sigma_s^2 + \alpha^2 \left(1 - \rho^2\right) \sigma_q^2.$$

  - The dynamic path of income inequality is

$$\sigma_{y,t}^2 = S_t^2 \sigma_{s,t}^2 + \alpha^2 \left(1 - \rho^2\right) \sigma_q^2.$$

Income variance

$$\sigma_y^2 = \left[\eta_e\left((1-\alpha) + \alpha\,\rho\,\frac{\sigma_q}{\frac{X\,Y + \sqrt{X^2Y^2 + (1-X^2)\,(Y^2+Z)}}{1-X^2}}\right) + \eta_s\right]^2$$
$$\times\left(\frac{X\,Y + \sqrt{X^2Y^2 + (1-X^2)\,(Y^2+Z)}}{1-X^2}\right)^2 \;+\; \alpha^2\,(1-\rho^2)\,\sigma_q^2,$$

where

$$X = \eta_e^2\,(1-\alpha) + \eta_e\,\eta_s + \eta_s,$$
$$Y = \alpha\,\eta_e^2\,\rho\,\sigma_q,$$
$$Z = \alpha^2\,\eta_e^2\,(1-\rho^2)\,\sigma_q^2 + \eta_\varepsilon^2\,\sigma_\varepsilon^2.$$

Comparative statics of income variance

- $\alpha \to \uparrow$
- $\sigma_q \to \uparrow$
- $\eta_e \to \uparrow$
- $\eta_s \to \uparrow$
- $\eta_\varepsilon,\, \sigma_\varepsilon \to \uparrow$
- $\rho \to$ ambiguous
  - Direct effect: Higher ρ decreases $(1-\rho^2) \to$ tends to reduce the "match noise" component $\to$ tends to lower σ_y^2.
  - Indirect effect: Higher ρ increases S (through $\alpha\,\rho\,\frac{\sigma_q}{\sigma_s}$) $\to$ increases persistence $\to$ tends to raise σ_y^2.

Intergenerational mobility

- Intergenerational elasticity of income (IGE)

$$\iota = \frac{\mathrm{d}\ln y_{i,t+1}}{\mathrm{d}\ln y_{i,t}}.$$

- For any t,

$$\iota_t = \frac{S_{t+1}}{S_t} \cdot (\eta_e S_t + \eta_s).$$

- In the stationary equilibrium,

$$\iota = \eta_e S + \eta_s.$$

Conclusion

- Educational over-investment due to positional externalities
  - Empirical evidence consistent with key comparative statics
- A tractable overlapping-generations model with closed-form expressions of skill and income dynamics
  - Implications for intergenerational mobility

THANK YOU!

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