Digital Villages*

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Abstract

A notable phenomenon in China is the rise of digital villages: less-developed rural areas where a considerable portion of the population is supported and subsidized by e-commerce platforms such as Alibaba to manufacture and sell products online. We argue that these platform actions are not purely philanthropic but are strategically designed to enhance profitability. The platform provides subsidies to new sellers early on to incentivize entry and reduce learning costs, later recouping these subsidies as sellers gain experience and increase their sales. Using a dynamic two-period model of two-sided markets, we analyze the intertemporal and cross-side pricing strategies of the monopoly platform. Our findings reveal that sellers' network externalities and their learning-by-doing reinforce each other, incentivizing the platform to subsidize them. Our study bridges two traditionally distinct fields: industrial organization and economic development.

Keywords: digital villages, two-sided markets, dynamic pricing, price discrimination, economic development

JEL Codes: D62, O12, L11, L81

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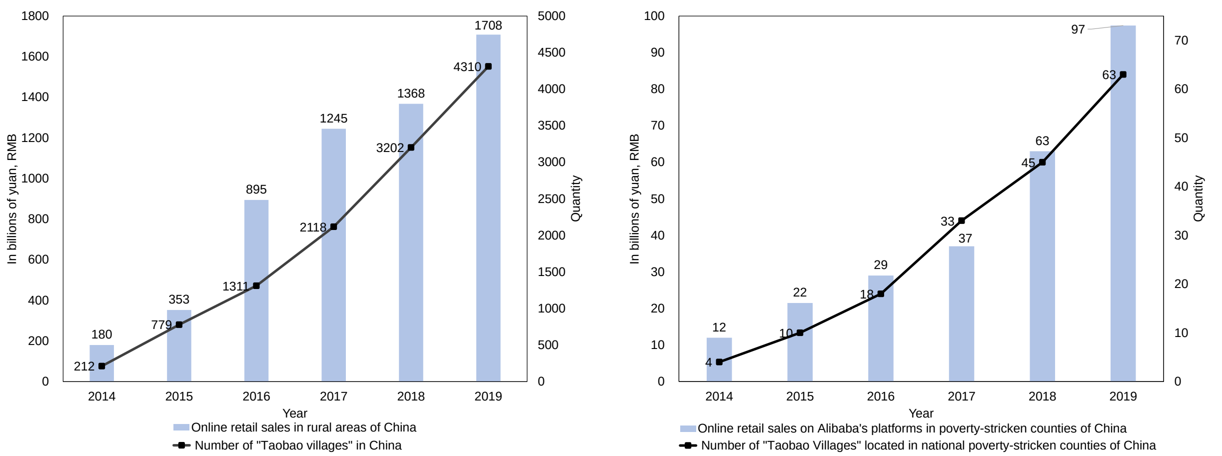
1 Introduction

Digital marketplaces have grown rapidly in recent years. E-commerce giants, such as Amazon, eBay, Mercado Libre, Alibaba, JD, and Temu, link buyers and sellers through their online trading services. A notable phenomenon is the rise of digital villages: rural villages with a high volume of transactions and a significant number or proportion of online stores (Luo and Niu, 2019). Defined by AliResearch (2015), a "Taobao village" on Alibaba's platforms (mainly including Taobao and Tmall) is an administrative village in which the number of active online shops exceeds 100 or the ratio of the active online shops to the local households exceeds 10%, and the annual turnover in e-commerce exceeds 10 million yuan (approximately US$1.52 million).[1] In the span of five years from 2014 to 2019, online sales in rural China increased almost ten-fold from $27.27 billion to $258.83 billion, and the number of Taobao villages increased more than twenty-fold from 212 to 4,310 (Figure 1(a)). When considering only less developed areas (LDAs)—832 poverty-stricken counties—in China, online retail sales on Alibaba's platforms increased eight-fold from $1.82 billion in 2014 to $14.76 billion in 2019, and during the same period, the number of Taobao villages increased more than fifteen-fold from 4 to 63 (Figure 1(b)).[2]



(a) All rural counties

(b) Poverty-stricken counties

Figure 1: Online retail sales and the number of Taobao Villages in rural China, 2014–2019
Notes: Data sources are provided in the Supplementary Appendix.

What role have e-commerce giants played in driving economic development in LDAs? Take Alibaba as an example. In October 2014, Alibaba launched the "1,000 Counties and 10,000 Villages" program (also known as the rural Taobao model). This program planned to invest 10 billion

[1]Dollar to yuan nominal exchange rate used in this paper is 6.6, the average from 2014 to 2019.

[2]In 2014, the State Council Leading Group Office of Poverty Alleviation and Development identified 832 counties in China as poverty-stricken because of their extremely low income per capita. We refer to them as LDAs.

yuan ($1.52 billion) over three to five years and aimed to build an e-commerce service system with 100,000 administrative villages in 1,000 counties in rural areas of China.[3] In December 2017, Alibaba initiated the Poverty Alleviation Fund, investing 10 billion yuan ($1.52 billion) over five years. This fund aimed at poverty reduction and alleviation in five target areas: e-commerce, ecology, education, health, and women.[4]

In 2018, Alibaba held a 58.2% share of China's e-commerce market, significantly surpassing other platforms.[5] Why would a platform with significant market power be incentivized to engage heavily in seemingly philanthropic public affairs? How does a profit-maximizing company like Alibaba benefit from investing in LDAs? The existing literature lacks a targeted theory to analyze these paramount questions related to two-sided markets and economic development. This paper aims to fill this void.

By incorporating the learning-by-doing of sellers in the theory of two-sided markets with heterogeneous sellers and buyers, we build a novel two-period model to explore both intertemporal and cross-side pricing strategies of a monopoly platform (or any platform with market power). Our model proceeds as follows. The profit-maximizing monopoly platform sets membership fees (which may be negative) for buyers and sellers in the two periods, and then after observing these prices, the buyers and sellers simultaneously make choices between this platform and an outside option in each period. All agents are rational and can anticipate the responses of other agents in each period. The utility function of buyers and sellers participating in online sales on this platform is the simplified version of Weyl (2010): Buyers and sellers are heterogeneous in membership values, while they obtain homogeneous interaction values from online trade, respectively, and all buyers and sellers obtain zero utility from offline trade.[6] More specifically, the membership values of buyers are positive (we call them membership benefits) in the two periods, and the membership values of sellers

[3]According to AliResearch (2015) and World Bank and Alibaba (2018), the following three outcomes are noteworthy: (1) By the end of 2018, there were 30,000 village-level service stations and a rural service team at the village level of nearly 60,000 people (including part-time and full-time workers). This program incubated 160 regional agricultural brands; (2) By the end of 2017, Taobao University had built 11 e-commerce training bases that conducted 133 e-commerce courses to train entrepreneurs; (3) By the end of 2017, Ant Financial, a subsidiary of Alibaba Group, had provided $1.70 billion in loans to entrepreneurs in poor counties and underdeveloped areas.

[4]World Bank and Alibaba (2018) highlight three notable outcomes of this program. First, in 2019, poverty-stricken counties recorded sales revenues of $14.76 billion on Alibaba's platforms. Second, in 2018, Alibaba trained over 260,000 people, both employed and self-employed in e-commerce and cloud computing, and opened nine e-commerce training bases in impoverished counties. Third, the program trained 18,200 women and helped 10,600 women gain employment in e-commerce.

[5]In 2018, the second and third biggest e-commerce giants, JD and Pinduoduo, owned 16.3% and 5.2% of the retail e-commerce sales shares in China, respectively. The data are from eMarketer, a research company focused on digital transformation (see https://www.emarketer.com).

[6]One could argue that the specification of unidimensional heterogeneity is similar to that of Armstrong (2006), but our framework allows for the outside option and does not involve the so-called "cost of distance" under the Hotelling-based framework. In this sense, our framework is closer to that of Weyl (2010).

are negative (we call them membership costs) in period 1. Due to learning-by-doing, sellers who participated in online sales in period 1 have lower membership costs in period 2, while the costs of new entrants remain unchanged. This specification for sellers' membership costs is novel, and the heterogeneity of sellers' membership costs can be interpreted as the difference in online business ability.

In the baseline model, the platform can charge different fees for newly entered sellers and old sellers in period 2, which is called third-degree price discrimination, while in period 1, only a unified fee for all sellers is charged.[7] Meanwhile, the platform can only charge the same fee for all buyers in each period. In equilibrium, the platform has an incentive to subsidize sellers in period 1 if the network externality each online seller makes is higher than each online buyer makes or if the growth in the business ability (i.e., learning-by-doing effect) of each online seller is high enough; the cross-side externality and learning-by-doing effect of each online seller can mutually reinforce each other in reducing the platform's charges (or increasing its subsidies) to sellers in period 1 when the externality of each seller is high enough, and that of each buyer is not too high. The theoretical results also indicate that both the prices for buyers in period 1 and old sellers in period 2 are higher than the price for sellers in period 1. Intuitively, although sellers must bear membership costs in the initial period, the relatively higher externality and the expected increase in business ability are taken into account when the platform implements cross-side and intertemporal pricing strategies. This subsidization mechanism for sellers explains the monopoly e-commerce platform's considerable investment of resources, such as training unskilled merchants, building village-level service stations, and providing loans in LDAs. Moreover, we show that the numbers of online buyers in the two periods are equal, and the numbers of online sellers in the two periods are equal, because the platform can internalize the cross-side network externalities through its dynamic pricing power.

In the extended model, we consider an environment where third-degree price discrimination toward sellers is prohibited in period 2. We find that, compared with the baseline model, the conditions for subsidizing sellers are relaxed in period 1. This is because the platform's pricing flexibility for sellers decreases in period 2, while the platform values the increments in online business abilities and the cross-side network externalities generated by the sellers. As in the baseline model, the buyer fee is higher than the seller fee in period 1, revealing the platform's cross-side price structure. Only when the increment in business ability obtained by each online seller is high enough will the platform charge sellers a lower fee in period 1 than in period 2. This demonstrates that the platform's intertemporal pricing strategy for sellers depends on the improvement of their online business abilities. We also find that if the growth in online sellers' business ability is significant enough, the cross-side price difference between buyers and sellers is larger in period 1 than in pe-

[7] In practice, price discrimination between entrants and existing members, especially on the sellers' side, is usually implemented by the platform; see related discussions in Cabral (2019).

riod 2, and the intertemporal price difference between periods 1 and 2 is greater for sellers than for buyers.

A comparative analysis of the equilibria yields three main findings. First, when price discrimination is prohibited, the platform's aggregate profit and the prices charged to sellers in both periods decrease, while the number of online sellers in both periods and online buyers in period 1 increase. This aligns with the traditional interpretation of price discrimination. Due to cross-side network externalities in a dynamic framework, the next two findings differ from those in the conventional price discrimination literature. Second, the price for buyers in period 2 increases when price discrimination is prohibited. Third, changes in the price for buyers in period 1 and the number of online buyers in period 2 are ambiguous and depend on the magnitude of cross-side network externalities.

Welfare analysis demonstrates that in the baseline model, the total surplus of sellers is equal across the two periods, and the total surplus of buyers is also equal across the two periods. However, when price regulation (i.e., prohibiting third-degree price discrimination between new and old sellers) is implemented, although the total surplus of sellers remains equal across the two periods, the total surplus of buyers declines. Hence, due to cross-side network externalities and the intertemporal enhancement of sellers' business abilities, the regulation on price discrimination will dynamically reduce the total surplus of buyers. We further analyze the impact of price regulation on social welfare through several numerical examples.Model calibration shows that price regulation reduces social welfare only when the parameters of cross-side network externalities for buyers and sellers and the increase in each seller's business ability are sufficiently small. For most feasible parameters, price regulation significantly increases social welfare.

The rest of this paper is organized as follows. The remainder of this section reviews the related literature. Section 2 presents the baseline model, and Section 3 analyzes its equilibrium. Section 4 analyzes the equilibrium when price discrimination is prohibited, and Section 5 investigates the welfare effects of the prohibition of price discrimination. Section 6 provides a further discussion of our model, and Section 7 concludes. All proofs are included in the Online Appendix.

Related literature

The most important contribution of our paper is that we bridge the gap between two fields in economics—industrial organization and economic development—by incorporating learning-by-doing among merchants in LDAs into a dynamic two-sided market framework with a monopoly platform. The theory of two-sided markets is proposed in seminal works by Rochet and Tirole

(2003, 2006), Caillaud and Jullien (2003), and Armstrong (2006).[8] More recent papers, such as Weyl (2010), White and Weyl (2010, 2016), Jullien and Pavan (2019), Karle et al. (2020), and Tan and Zhou (2021), further develop the literature on two-sided markets. These papers offer general frameworks for exploring conventional topics in industrial organization, such as pricing, competition, and platform entry in the presence of cross-group externalities. Existing theoretical and empirical studies cover a range of industries, such as payment systems (Bedre-Defolie and Calvano, 2013; Dolfen et al., 2019; Li et al., 2020; Rochet and Tirole, 2003; Wright, 2012), informational intermediaries via the internet (Caillaud and Jullien, 2003), video games (Hagiu, 2006; Landsman and Stremersch, 2011; Lee, 2013; Zhou, 2017; Zhu and Iansiti, 2012), media markets (Anderson and Coate, 2005; Anderson et al., 2018; Athey et al., 2013; Ferrando et al., 2004), newspapers (Argentesi and Filistrucchi, 2007; Chandra and Collard-Wexler, 2009; Fan, 2013; Seamans and Zhu, 2014), magazines (Kaiser and Wright, 2006), sport card conventions (Jin and Rysman, 2015), labor matching markets (Lee and Schwarz, 2017), and online real estate trade (Karle et al., 2020). However, incentives for a monopoly platform to promote economic development in LDAs have not been the focus of theoretical studies. In recent years, some empirical papers have examined the development of e-commerce in rural China, for example, Luo and Niu (2019) and Couture et al. (2018, 2021). In particular, the rapid development of digital villages has attracted much academic and policy attention (Ding et al., 2018; Luo and Niu, 2019; Qi et al., 2019). For example, Fan et al. (2018) demonstrate that e-commerce can reduce spatial consumption inequality by lowering fixed market-entry costs and distance-based trade barriers; Luo and Niu (2019) highlight the role of e-commerce in fostering entrepreneurship. Moreover, several studies have explored how learning-by-doing fosters economic development in theory (Arrow, 1962; Lucas, 1988). More broadly, Nunn (2020) reviews the literature on economic development from a historical perspective. However, the formation of sellers' learning-by-doing and the role of monopoly platforms in economic development remain underexplored. In summary, the development of e-commerce in LDAs and two-sided platforms have been treated separately in the literature. Our paper links them together.

Another contribution of our paper is that it enriches the literature on dynamic pricing in two-sided markets. The pricing strategy of a monopolistic two-sided platform has been explored in pioneering studies. For example, Rochet and Tirole (2003) and Armstrong (2006) examine the price structure in the benchmark model with a monopoly platform, and Weyl (2010) develops an analytical framework centered on monopoly pricing in two-sided markets. However, these models are static and cannot be used to analyze the dynamic pricing strategy of the monopoly platform driving e-commerce development in LDAs. Although some theoretical articles have studied dynamic games with network effects, they do not explicitly model the dynamic game among three

[8] Previously, network externalities in the information communication technology industry were studied in pioneer papers like Katz and Shapiro (1985, 1986) and Farrell and Saloner (1985, 1986).

rational parties—buyers, sellers, and the monopoly platform—in a two-sided market. For example, Doganoglu (2003), Cabral (2011), Radner et al. (2014), Biglaiser and Crémer (2020) and Halaburda et al. (2020) focus solely on one-sided network effects of consumers, rather than cross-side externalities in two-sided markets. Although Chen and Tse (2008) and Cabral (2019) consider dynamic pricing in two-sided markets, the agents in Cabral (2019) only take current payoffs into account, and the numbers of buyers and sellers in Chen and Tse (2008) only depend on the anticipated market segment in the subsequent period.[9] Using a two-period model of two-sided markets, Lam (2017) analyzes the impact of switching costs on price competition between two symmetric platforms.[10] In contrast, our model explores cross-sided and intertemporal pricing strategies implemented by a monopoly platform, characterizing the distinct ability distributions of buyers and sellers and deriving closed-form price solutions in a dynamic framework.[11]

Our paper also contributes to the literature on price regulation and price discrimination in two-sided markets.[12] Price discrimination between groups of agents on the two sides is well documented in early studies, for example, Caillaud and Jullien (2003), Armstrong (2006) and Weyl (2010). However, when the two sides consist of different types of agents (e.g., buyers and sellers), the price difference between the two sides does not qualify as price discrimination.[13] A few papers have recognized price discrimination on different types of agents on each side. For example, using a model of a two-sided monopoly platform, Jeon et al. (2022) document second-degree price discrimination on two types of agents on one side and analyze the impact of price regulation on social welfare.[14] Liu and Serfes (2013) explore perfect price discrimination within each group in two-sided markets using a static model. In contrast, our dynamic model allows for the characterization of third-degree price discrimination across different groups of sellers in period 2, with the grouping of sellers occurring endogenously at the end of period 1.

[9]Doganoglu (2003) and Radner et al. (2014) assume that consumers are myopic. Moreover, Chen and Tse (2008) and Cabral (2019) do not explicitly characterize the cross-side network externalities between buyers and sellers as in Rochet and Tirole (2003), Armstrong (2006), and Weyl (2010).

[10]There are only two types of agents—consumers and platforms—in Lam's paper, although the consumers are distributed on the different sides. To simplify the analysis, Lam also assumes that the interaction benefits each consumer obtains from any consumer on the other side are the same no matter which side this consumer belongs to.

[11]Our research topic and the model setup are quite different from those of Lam (2017). Lam's model focuses on the effects of switching costs on the first-period price competition and social welfare in two-sided markets.

[12]Some papers have examined price discrimination in two-sided markets (Evans, 2003; Gomes and Pavan, 2016; Jeon et al., 2022; Liu and Serfes, 2013; Rysman, 2009; Wang and Wright, 2017; Zhang and Liu, 2016). Weisman and Kulick (2010), referring to the Notice of Proposed Rulemaking issued by the Federal Communications Commission in the United States, provide a detailed discussion of price discrimination and two-sided markets in the regulation circumstance of net neutrality.

[13]The price difference between different sides is called "cross-subsidization" in some papers, for example, Gomes and Pavan (2016), Cabral (2019) and Tan and Zhou (2021).

[14]Choi et al. (2015), Böhme (2016), and Lin (2020) also analyze second-degree price discrimination in two-sided markets.

2 Baseline Model

In the digital economy, three groups of agents—a monopoly platform, a unit mass of heterogeneous buyers, and a unit mass of heterogeneous sellers—live for two periods. The platform has pricing power over both buyers and sellers in the two periods and aims to maximize its aggregate discounted profit. Given the fees charged by the platform, each buyer and seller chooses to join it or take the outside option (e.g., another platform or offline).

2.1 Sellers' Best Responses

The target of each seller is to maximize the aggregate discounted utility from selling products in the two periods. For each seller in each period, there are two choices: selling online and offline. Suppose that the utility of offline selling is constant with value $\underline{U}^s$. Meanwhile, in the two periods, seller i's utilities from selling on the e-commerce platform are expressed as

$$U^s_{i,1} = B^s_{i,1} + \alpha N^b_1 - P^s_1, \tag{1}$$

$$U^s_{i,2} = \underbrace{B^s_{i,1} + \mathbf{1}^i c}_{\equiv B^s_{i,2}} + \alpha N^b_2 - \underbrace{\left[\left(1 - \mathbf{1}^i\right) P^s_{21} + \mathbf{1}^i P^s_{22}\right]}_{\equiv P^s_{i,2}}, \tag{2}$$

$$\mathbf{1}^i = \begin{cases} 1, & \text{if seller } i \text{ sells online in period 1;} \\ 0, & \text{if seller } i \text{ does not sell online in period 1,} \end{cases} \tag{3}$$

where $\alpha \in (0, 1)$ denotes the interaction benefit from each buyer on the other side; $B^s_{i,t}$ $(t = 1, 2)$ are the benefits of participating in online sales, which are also interpreted as seller i's ability to run a business online; $c \in (0, 1)$ is a constant increment in the sellers' business ability from participating in e-commerce; N^b_t $(t = 1, 2)$ are the numbers of buyers on the digital platform.

The platform charges each seller a unified fee P^s_1 in period 1 and charges seller i fee $P^s_{i,2}$ in period 2 that depends on the seller's age; for concreteness, the fee the platform charges each seller is P^s_{21} if the seller was not affiliated with the platform in period 1 and P^s_{22} otherwise. As an indicator function, $\mathbf{1}^i$ equals one if the seller i sells online in period 1 and zero if not. Here, a latent assumption is that in period 2, the monopoly platform is allowed to implement third-degree price discrimination on sellers. Later, we will adjust this assumption and analyze the economic implications of prohibiting price discrimination.

Poor sellers face a relatively higher startup cost due to their lower degree of education and the lower internet penetration in rural areas compared to rich sellers.[15] Suppose that $B_{i,t}$ obeys a

[15]This comparison implies that when a poor seller and a rich seller face the same entrepreneurship program, it is more difficult for the poor one to accomplish it. Some might argue that the rich seller has a relatively higher opportunity cost compared with the poor seller, but we do not focus on this comprehensive comparison result in our model.

uniform distribution on interval $[-1, 0]$ in period 1. With the deepening of the digital economy in rural regions, poor merchants who are affiliated with the platform in period 1 obtain the learning-by-doing benefit that is available in period 2. In this scenario, the law of motion of business ability is

$$B_{i,2}^{s} = B_{i,1}^{s} + c, \tag{4}$$

As documented in Spence (1981), the hypothesis of learning curve (or learning-by-doing) implies that the unit cost of making a product of a firm decreases with the accumulation of experience. Stokey (1988) studies the role of learning by doing in driving economic growth. Minniti and Bygrave (2001) also argue that the knowledge of entrepreneurs from past experiences determines the sequence of their choices. In contrast, all old (heterogeneous) sellers obtain the same level of cost reduction c from period 1 to period 2 in our model. This can be seen as a simplified version of the learning-by-doing setup in the literature.

However, if the merchant i chooses not to sell products on the digital platform in period 1, the law of motion of the business ability becomes

$$B_{i,2}^{s} = B_{i,1}^{s}, \tag{5}$$

which means there is no enhancement in the business ability of the seller i.

Consider that a unit mass of completely heterogeneous sellers is uniformly distributed on the interval $[-1, 0]$ in period 1. If there exists a utility indifference point in period 1, the distribution of business ability will have a gap in period 2.[16] The reason for this gap is that the sellers who are affiliated with the platform in period 1 experience an enhancement in business ability, while the rest of the sellers do not.

In general, given the prices implemented by the monopoly platform, each seller's homing choices in the two periods are concluded as a two-period game as in Figure 2. $\widehat{U}_{i,2}^{s}$ denotes seller i's utility in period 2 when the seller did not choose to join the platform in period 1, and $\widetilde{U}_{i,2}^{s}$ denotes seller i's utility in period 2 under the condition that this seller had chosen the platform in period 1. Then, seller i's maximizing problem can be formed as

$$\max\left\{\underline{U}^{s} + \delta \max\left\{\widehat{U}_{i,2}^{s}, \underline{U}^{s}\right\}, U_{i,1}^{s} + \delta \max\left\{\widetilde{U}_{i,2}^{s}, \underline{U}^{s}\right\}\right\}, \tag{6}$$

where δ is the discount factor. For the sake of brevity, we assume that the utility of both buyers and sellers is intertemporally additive with a zero discount rate, that is, $\delta = 1$.

[16] At this critical point, the seller can obtain the same level of utility from selling online and offline.

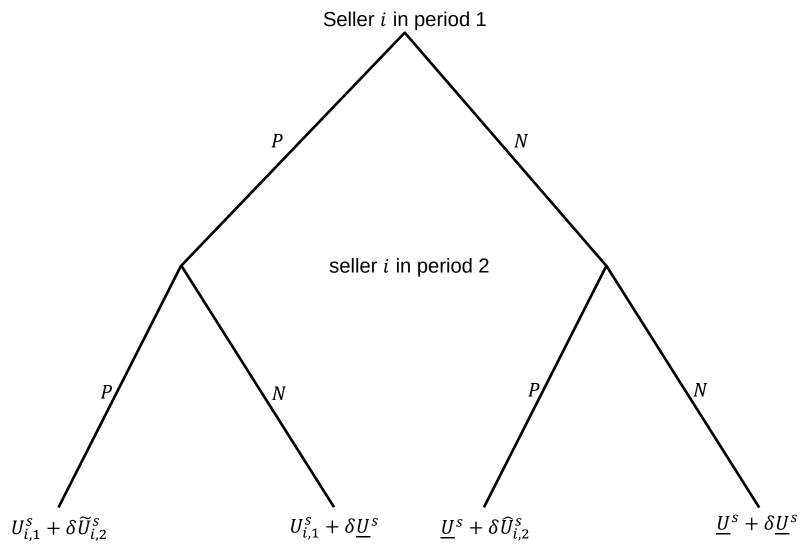


Figure 2: Two-period Homing-choice Game of Each Seller

Specifically, these two types of utilities $\widehat{U}_{i,2}^s$ and $\widetilde{U}_{i,2}^s$ in period 2 can be expressed as

$$\widehat{U}_{i,2}^s = B_{i,1}^s + \alpha N_2^b - P_{21}^s,$$
$$\widetilde{U}_{i,2}^s = B_{i,1}^s + c + \alpha N_2^b - P_{22}^s.$$

Next, we describe the homing choice of sellers when third-degree price discrimination toward sellers is allowed in period 2. Sellers are completely heterogeneous because of the difference in their ability to run an online business. The seller's homing choice in the two periods can be characterized as shown in Figure 3. Although the figure only presents a specific scenario of sellers' homing choice, the logic of their best responses can be seen. Here, the seller k is the critical one in period 1, and the

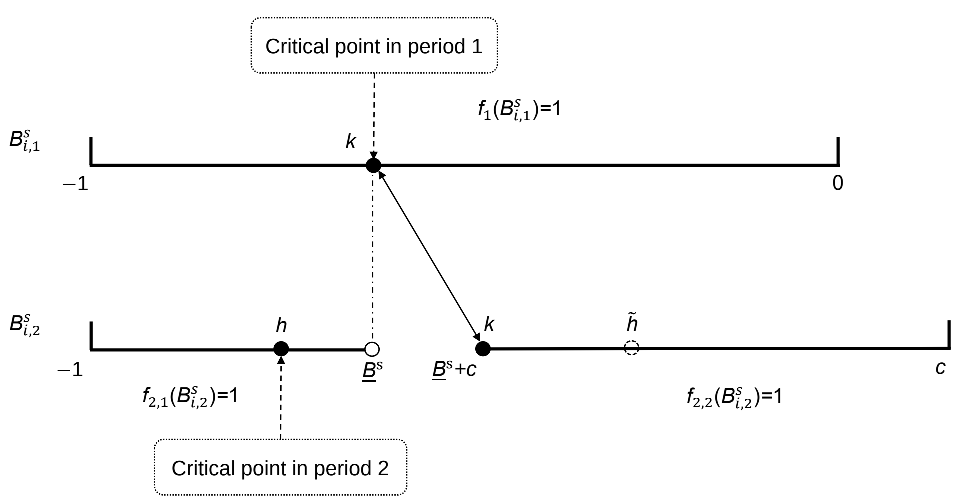


Figure 3: Distribution of Completely Heterogeneous Sellers in Two Periods

seller h (and the seller $\widetilde{h}$) is (are) at the critical point(s) in period 2. For the sake of analysis, let $\underline{B}^s$ denote the business ability of seller k in period 1. Moreover, $f_1(B^s_{i,1})$ is the probability density of $B^s_{i,1}$ in period 1, and $f_{2,1}(B^s_{i,2})$ and $f_{2,2}(B^s_{i,2})$ denote the probability densities of $B_{i,2}$ on the intervals $[-1, \underline{B}^s)$ and $[\underline{B}^s + c, c]$ in period 2, respectively. All these probability densities equal 1.

Theoretically, given the prices proposed by the monopoly platform, there are four scenarios for the critical seller(s) in the two periods as follows.

Scenario A (the sellers on the platform are the same in both periods). The seller h and the seller k are the same, which implies that three conditions, $U^s_{k,1} = \underline{U}^s$, $\widetilde{U}^s_{k,2} = \underline{U}^s$, and $U^s_{k,1} + \widetilde{U}^s_{k,2} = \underline{U}^s + \widehat{U}^s_{k,2}$, are satisfied. In period 1, given price P^s_1, the critical condition $U^s_{k,1} = \underline{U}^s$ is satisfied. The reason is that in the second (last) period, the platform will charge a high enough fee such that the critical seller's utility is indifferent between online and offline sales. From this, we can obtain the number of sellers choosing the platform in period 1 as

$$\begin{aligned} N^s_1(P^s_1, N^b_1) &= \Pr\left(B^s_{i,1} \geq \underline{B}^s\right) \\ &= \int_{\underline{U}^s + P^s_1 - \alpha N^b_1}^{0} f_1(B^s_{i,1}) dB^s_{i,1} \\ &= \alpha N^b_1 - P^s_1 - \underline{U}^s. \end{aligned} \tag{7}$$

From the last two conditions, we have that the number of sellers affiliated with the platform in period 2 is

$$N^s_2 = N^s_1 \tag{8}$$

only if the following two conditions are satisfied:

$$P^s_{22} = -N^s_1 + c + \alpha N^b_2 - \underline{U}^s, \tag{9}$$

$$P^s_{21} = P^s_{22} - c. \tag{10}$$

Scenario B (some sellers enter the platform in period 2). There is only one critical seller h in period 2, and this seller has weaker business ability compared with seller k, as shown in Figure 3; that is, the seller h's business ability is on the interval $[-1, \underline{B}^s)$. In this scenario, four conditions, $\widetilde{U}^s_{k,2} = \underline{U}^s$, $\widehat{U}^s_{h,2} = \underline{U}^s$, $U^s_{k,1} + \widetilde{U}^s_{k,2} = \underline{U}^s + \widehat{U}^s_{k,2}$ and $\widehat{U}^s_{h,2} < \widehat{U}^s_{k,2}$, should be satisfied. From the first condition, we can indirectly obtain the number of sellers on the platform in period 1:

$$\begin{aligned} N^s_1(P^s_{22}, N^b_2) &= \Pr\left(B^s_{i,1} \geq \underline{B}^s\right) \\ &= \int_{\underline{U}^s + P^s_{22} - \alpha N^b_2 - c}^{0} f_1(B^s_{i,1}) dB^s_{i,1} \\ &= \alpha N^b_2 + c - P^s_{22} - \underline{U}^s. \end{aligned} \tag{11}$$

From the second condition, we can derive the number of sellers on the platform in period 2:

$$\begin{aligned} N_2^s(P_{21}^s, N_2^b) &= \Pr\left(B_{i,2}^s \geq B_{h,2}^s\right) \\ &= \int_{\underline{U}^s + P_{21}^s - \alpha N_2^b}^{\underline{B}^s} f_{21}(B_{i,2}^s) dB_{i,2}^s + \int_{\underline{B}^s + c}^{c} f_{22}(B_{i,2}^s) dB_{i,2}^s \\ &= \alpha N_2^b - P_{21}^s - \underline{U}^s. \end{aligned} \tag{12}$$

Moreover, the first and third conditions imply that $U_{k,1}^s = \widehat{U}_{k,2}^s$; that is,

$$P_1^s = \alpha N_1^b + P_{21}^s - \alpha N_2^b. \tag{13}$$

Scenario C (some sellers leave the platform in period 2). There is only one critical seller $\widetilde{h}$ in period 2, and this seller has stronger business ability compared to seller k. The seller $\widetilde{h}$'s business ability is on the interval $(\underline{B}^s + c, c]$. In this scenario, four conditions, $U_{k,1}^s = \underline{U}^s$, $\widetilde{U}_{\widetilde{h},2}^s = \underline{U}^s$, $\widetilde{U}_{k,2}^s < \widetilde{U}_{\widetilde{h},2}^s$ and $\widehat{U}_{k,2}^s \leq \underline{U}^s$, are satisfied. Then, from the first condition, we have the same expression for the number of sellers on the platform in period 1 as in equation (7). Based on the second condition, we obtain the number of sellers on the platform in period 2 as

$$\begin{aligned} N_2^s(P_{22}^s, N_2^b) &= \Pr\left(B_{i,2}^s \geq B_{\widetilde{h},2}^s\right) \\ &= \int_{\underline{U}^s + P_{22}^s - \alpha N_2^b}^{c} f_{22}(B_{i,2}^s) dB_{i,2}^s \\ &= \alpha N_2^b + c - P_{22}^s - \underline{U}^s. \end{aligned} \tag{14}$$

When the last condition binds, we have

$$\begin{aligned} P_{21}^s &= \underline{B}^s + \alpha N_2^b - \underline{U}^s \\ &= -N_1^s + \alpha N_2^b - \underline{U}^s. \end{aligned} \tag{15}$$

Scenario D (some sellers leave the platform, but some sellers enter the platform in period 2). There are two critical sellers, h and $\widetilde{h}$, in period 2. Compared to the critical seller k, the seller h has weaker business ability and the seller $\widetilde{h}$ has stronger business ability. In this scenario, the following five conditions, $\widehat{U}_{h,2}^s = \underline{U}^s$, $\widetilde{U}_{\widetilde{h},2}^s = \underline{U}^s$, $\underline{U}^s + \widehat{U}_{k,2}^s = U_{k,1}^s + \underline{U}^s$, $\widehat{U}_{h,2}^s < \widehat{U}_{k,2}^s$, and $\widetilde{U}_{k,2}^s < \widetilde{U}_{\widetilde{h},2}^s$, are satisfied. From the first condition, we have the number of sellers who have higher business ability compared with seller h:

$$\widetilde{N}_2^s = \alpha N_2^b - P_{21}^s - \underline{U}^s. \tag{16}$$

Then, we can derive the number of sellers who are only affiliated with the platform in period 2 as

$$N_{21}^s = \widetilde{N}_2^s - N_1^s = \alpha N_2^b - P_{21}^s - \underline{U}^s - N_1^s. \tag{17}$$

From the second condition, we obtain the number of sellers who have higher business ability than seller $\widetilde{h}$

$$N_{22}^s = \alpha N_2^b + c - P_{22}^s - \underline{U}^s. \tag{18}$$

We then obtain the number of sellers affiliated with the platform in period 2 from equations (17) and (18); that is,

$$\begin{aligned} N_2^s &= N_{21}^s + N_{22}^s \\ &= 2\alpha N_2^b + c - P_{21}^s - P_{22}^s - 2\underline{U}^s - N_1^s. \end{aligned} \tag{19}$$

The third condition implies the following relationship between P_1^s and P_{21}^s:

$$P_{21}^s = \alpha N_2^b - \alpha N_1^b + P_1^s. \tag{20}$$

2.2 Buyers' Best Responses

There is a unit mass of completely heterogeneous buyers in the economy. Each buyer faces the choice between buying goods on the online platform or from offline stores. Meanwhile, buyers are distinguished by their ability to conduct business online. We assume that the utility of buying offline is a constant value $\underline{U}^b$. The buyer j's utility from buying online in period t ($t = 1, 2$) is formed as

$$U_{j,t}^b = B_j^b + \beta N_t^s - P_t^b, \tag{21}$$

where $\beta \in (0, 1)$ denotes buyer's interaction benefit from each seller. B_j^b is the level of business ability of the buyer j, which follows a uniform distribution on the interval $[0, 1]$. There is no enhancement of buyers' online business abilities as there is for sellers; thus, the distribution of buyers does not change over time. N_t^s indicates the number of sellers who choose to join the online platform. The monopoly platform charges the buyers the unified price P_t^b.

Consider that the probability density of B_j^b is $h(B_j^b) = 1$, corresponding to domain $[0, 1]$. Theoretically, there are three scenarios for buyers' choices when the aggregate utility of two periods is not less than $2\underline{U}^b$. First, the utility is greater than $\underline{U}^b$ in period 1 but less than $\underline{U}^b$ in period 2. This scenario cannot occur because buyers will anticipate and choose offline buying in period 2. Second, the buyer bears a utility that is less than $\underline{U}^b$ in period 1 and then obtains a higher utility in period 2. However, if the buyer deviates from this strategy by choosing offline buying in period 1, he will achieve a higher utility $\underline{U}^b$ in that period, and the utility he obtains from the platform in period 2 will not be affected.[17] Third, each buyer will choose a level of utility that is not less than $\underline{U}^b$ in each period. This scenario is the best response for each buyer.

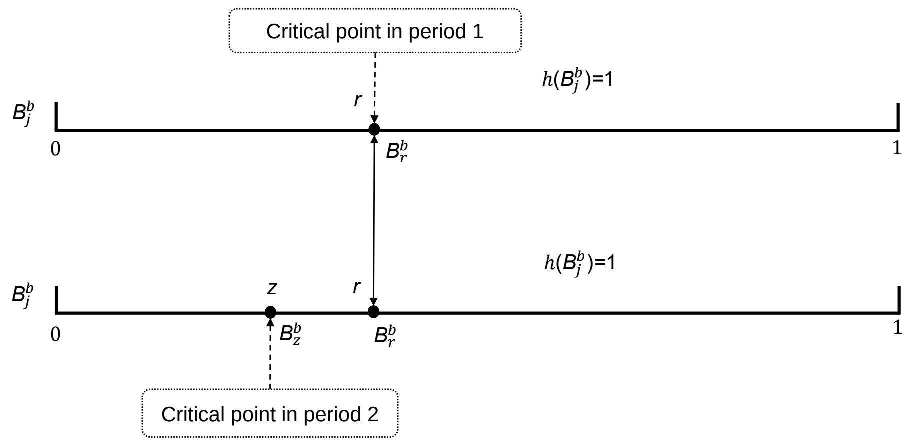


Figure 4: Distribution of Completely Heterogeneous Buyers in Two Periods

The homing choice of buyers is described in Figure 4. Buyer r is the critical buyer in period 1, and buyer z is that in period 2. Then, when $U^b_{j,t} = \underline{U}^b (t = 1, 2)$ are satisfied, we have

$$B^b_r = \underline{U}^b + P^b_1 - \beta N^s_1, \tag{22}$$

$$B^b_z = \underline{U}^b + P^b_2 - \beta N^s_2. \tag{23}$$

In each period, if a buyer has greater business ability than the critical buyer, he will affiliate with the online platform. Thus, we have the numbers of buyers affiliated with the online platform in the two periods as

$$\begin{aligned} N^b_1(P^b_1, N^s_1) &= \Pr(B^b_j \geq B^b_r) \\ &= \int_{\underline{U}^b + P^b_1 - \beta N^s_1}^{1} h(B^b_j) dB^b_j \\ &= 1 + \beta N^s_1 - \underline{U}^b - P^b_1, \end{aligned} \tag{24}$$

and

$$\begin{aligned} N^b_2(P^b_2, N^s_2) &= \Pr(B^b_j \geq B^b_z) \\ &= \int_{\underline{U}^b + P^b_2 - \beta N^s_2}^{1} h(B^b_j) dB^b_j \\ &= 1 + \beta N^s_2 - \underline{U}^b - P^b_2. \end{aligned} \tag{25}$$

Combining the best responses of all buyers and buyers, we can obtain homing-choice equilibria in the four scenarios discussed above. We can conclude homing-choice equilibria as follows.

[17] As a particle of the economy, this buyer's choice does not affect the platform's price or other buyers' choices. This also implies that buyers cannot form a collective action to negotiate with the monopoly platform.

Scenario A. The equilibrium of homing choices can be solved by combining buyers' choices (24) and (25) with sellers' choices (7) and (8). Furthermore, the equalities (9) and (10) need to hold in equilibrium.

Scenario B. From four equalities, (24), (25), (11) and (12), we derive the buyers' and sellers' homing choice equilibria. Note that the last condition, $\widehat{U}^s_{\hat{h},2} < \widehat{U}^s_{k,2}$, and the equality (13) should be satisfied in equilibrium.

Scenario C. Putting equations (24) and (25) together with equations (7) and (14), we have the homing choice equilibria for buyers and sellers. Note that in equilibrium, the two conditions, inequality $\widetilde{U}^s_{\tilde{k},2} < \widetilde{U}^s_{\tilde{h},2}$ and equality (15), should be satisfied.

Scenario D. Although the number of sellers on the platform in period 1 cannot be expressed directly, it is included in equation (19). Therefore, by combining equations (24) and (25) with equations (19) and (20), the equilibrium homing choices of buyers and sellers are obtained. Moreover, the last two conditions, $\widehat{U}^s_{\hat{h},2} < \widehat{U}^s_{k,2}$ and $\widetilde{U}^s_{\tilde{k},2} < \widetilde{U}^s_{\tilde{h},2}$, have not yet been considered and need to be guaranteed in equilibrium.

Note that, up to now, all prices for buyers and sellers in the two periods are given. We present the buyers' and sellers' homing choice equilibria in these four scenarios in Online Appendix A.1.

2.3 Monopoly Platform's Dynamic Pricing Strategy

According to the best-response choices of buyers and sellers on prices, the monopoly platform sets the prices for the two periods. The monopoly platform's aggregate profit is

$$\pi_a = \max_{\mathbf{P}_a} \left[P^s_1 N^s_1 + P^b_1 N^b_1 + \frac{1}{1+r} \left(P^s_{21} N^s_{21} + P^s_{22} N^s_{22} + P^b_2 N^b_2 \right) \right], \tag{26}$$

when third-degree price discrimination is allowed. Here, the list of platform prices is $\mathbf{P}_a = \left\{ P^s_1, P^b_1, P^s_{21}, P^s_{22}, P^b_2 \right\}$. r is the interest rate on the platform's profit from period 1 to period 2. To simplify this dynamic problem, but not jeopardize the core mechanics, we assume that $\underline{U}^s = 0$, $\underline{U}^b = 0$, and $r = 0$.

2.4 Nash Equilibrium

If the platform is allowed to practice third-degree price discrimination, there are two prices for sellers in period 2 (i.e., P^s_{21} and P^s_{22}). Four possible homing choices in equilibrium are described in Subsection 2.1. The discussion of these four scenarios describes the results of combining the best responses of buyers and sellers under all the possible prices to which the platform has access. The final results after a two-stage game indicate that *Scenario A* is the Nash equilibrium as described in the following lemma:

Lemma 1. *When third-degree price discrimination is allowed, the platform's optimal pricing strategy indicates that Scenario A, characterized in Subsection 2.1, is a unique Nash equilibrium.*

Proof. See Online Appendix B. □

The proof of Lemma 1 shows that *Scenarios B, C, and D* tend to cross the discontinuity point of the sellers' distribution, and that *Scenario A* is the only Nash equilibrium after the optimization of the monopoly platform. Two features in this scenario are worth noting. One is that the price for sellers who only want to enter the platform in period 2 (i.e., P^s_{21}) is a commitment or threat fee because no seller chooses this strategy in the final Nash equilibrium. Another feature is that in *Scenario A* , sellers sell online or offline in both periods, which implies that the monopoly platform is more powerful in charging sellers a higher fee in period 2 due to their enhanced business ability.

Next, we present and discuss the homing choices of buyers and sellers, the prices set by the platform, and the two-period profit the platform obtains in Nash equilibrium. The monopoly platform sets five prices in the two periods:

$$P^b_1 = P^b_2 = \frac{4 - 2\alpha(\alpha+\beta) - c(\alpha-\beta)}{2(2-\alpha-\beta)(2+\alpha+\beta)}, \tag{27}$$

$$P^s_1 = P^s_{21} = \frac{2\alpha - 2\beta + \alpha c(\alpha+\beta) - 2c}{2(2-\alpha-\beta)(2+\alpha+\beta)}, \tag{28}$$

$$P^s_{22} = c + \frac{2\alpha - 2\beta + \alpha c(\alpha+\beta) - 2c}{2(2-\alpha-\beta)(2+\alpha+\beta)}. \tag{29}$$

Thus, putting these equilibrium prices back into equations (64), (65), (66), and (67), we obtain the homing choice equilibria for the two periods as

$$N^s_1 = N^s_2 = \frac{\alpha+\beta+c}{(2-\alpha-\beta)(2+\alpha+\beta)}, \tag{30}$$

$$N^b_1 = N^b_2 = \frac{4 + c(\alpha+\beta)}{2(2-\alpha-\beta)(2+\alpha+\beta)}. \tag{31}$$

Thus, the two-period profit of the monopoly platform in equilibrium is

$$\pi = \frac{4 + c^2 + 2c(\alpha+\beta)}{2(2-\alpha-\beta)(2+\alpha+\beta)}. \tag{32}$$

3 Equilibrium Analysis

In this section, we analyze the equilibria of the baseline model. Before we analyze the equilibria, we need to provide a feasible set of parameters as shown in the following lemma.

Lemma 2. *To establish* $0 < N_1^s, N_1^b, N_2^s, N_2^b < 1$ *and* $\pi > 0$, *we have a feasible set of parameters as* $0 < \alpha, \beta < 1$, $0 < \alpha + \beta < \sqrt{2}$, *and* $0 < c < \min\left\{1, \frac{4-2(\alpha+\beta)^2}{\alpha+\beta}\right\}$.

Proof. See Online Appendix C. □

Intuitively, if the network externality parameters $\{\alpha, \beta\}$ and the enhancement of business ability c are too large, the outside options for buyers and sellers, such as offline selling and buying, will disappear in the economy. Throughout the following analysis in this subsection, Lemma 2 always holds.[18]

All prices in our model can be interpreted as *membership fees*, as discussed in Rochet and Tirole (2006). These fees measure the allocation of the gross surplus between buyers and sellers. For example, consider a one-shot trade on the monopoly platform. The highest willingness of a buyer to pay for an online transaction is W, and the seller's reserve price is R. The platform charges the fees P^b and P^s to buyers and sellers, respectively. This transaction occurs only when $P^b + P^s \leq W - R$. Then, prices P^b and P^s reflect the (latent) bargaining power of buyers and sellers, respectively. The higher the price or fee charged by the online platform, the weaker the agent's bargaining power. More importantly, the two prices are comparable, which will be useful for analyzing the price structure in equilibrium later.

Subsidization is a common pricing strategy not only in traditional one-sided markets but also in two-sided markets. In static multi-sided markets, cross-subsidization is attributed to cross-side externalities (Tan and Zhou, 2021). Subsidies (that is, negative prices) are also discussed in dynamic models (Cabral, 2011, 2019; Halaburda et al., 2020). Moreover, Cabral (2019) points out that under a dynamic framework, cross-side and intertemporal externalities are addressed when a platform determines the optimal pricing strategy.

In our model, in the beginning, all the sellers are poor with negative business ability distributed on the interval $[-1, 0]$, which reduces the utility they obtain from online transactions. If the platform charges a positive price for participating sellers in the initial period, the only source of positive utility for sellers is the cross-side network externality. However, buyers have positive ability, which is uniformly distributed on the interval $[0, 1]$. A common incentive the platform has is that to maintain transactions in two-sided markets, the platform may subsidize sellers to some extent to ensure that their utility level is at least higher than that of the offline option. This motivation can be interpreted as cross-subsidization, as described in Tan and Zhou (2021). Before analyzing the price structures, we explore the first-period price that the platform charges (poor) sellers.

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[18] In the literature relevant to two-sided markets, the parameters are usually limited to a certain range. For example, Armstrong (2006) provides an assumption on the relationship between the network externality parameters and the differentiation parameters, to avoid a corner solution. Choi (2010) also imposes some assumptions on parameters to obtain certain homing choices.

Theorem 1 (Initial Price for Online Sellers When Allowing Price Discrimination)**.** *Suppose that third-degree price discrimination is allowed in period 2. Given the feasible set for parameters in Lemma 2, in equilibrium:*

1. *If either (i) $\beta > \alpha$, or (ii) $\alpha > \beta$ and $c > \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)}$, the price for sellers in period 1 is negative (that is, $P_1^s < 0$). If $\alpha > \beta$ and $c < \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)}$, the opposite result applies (that is, $P_1^s > 0$).*
2. *P_1^s is decreasing in c and β.*
3. *If $\alpha < \frac{2\sqrt{2}}{3}$ and $\beta > \frac{2-\alpha^2-2\sqrt{1-\alpha^2}}{\alpha}$, we have $\frac{\partial^2 P_1^s}{\partial c \partial \beta} < 0$. If (i) $\alpha < \frac{2\sqrt{2}}{3}$ and $\beta < \frac{2-\alpha^2-2\sqrt{1-\alpha^2}}{\alpha}$, or (ii) $\alpha > \frac{2\sqrt{2}}{3}$, we have the opposite result (that is, $\frac{\partial^2 P_1^s}{\partial c \partial \beta} > 0$).*

Proof. See Online Appendix D. □

Item 1 in Theorem 1 shows that the platform has the incentive to subsidize sellers in the initial period when their network externality or growth in business ability is high enough. First, when $\beta > \alpha$, the price P_1^s is negative. Inequality $\beta > \alpha$ implies that sellers have a stronger market power than buyers because, on the platform, a buyer obtains more from each seller than a seller obtains from each buyer. Hence, the platform is more afraid of losing valuable sellers and chooses to subsidize them to attract greater participation in online transactions. The result that price P_1^s is decreasing in β in item 2 also provides evidence of the subsidy decision the platform may make.[19] Second, although the network externality each seller produces is lower than that each buyer produces, if the enhancement of sellers' business ability is high enough (that is, $c > \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)}$), the platform also subsidizes sellers on it in period 1. The reason is that the platform anticipates that in period 2, these old sellers will gain more utility from their business ability than before, and this utility increment is the basis for the platform to charge higher fees. Furthermore, as shown in item 2, an increase in the increment of business ability c will induce a lower price for sellers in period 1. This result is due to the latent intertemporal pricing strategy.[20] Item 3 in the theorem implies that if the externality of each seller (i.e., β) is high enough and the externality of each buyer (i.e., α) is not too high, β and c (i.e., learning-by-doing effect) are mutually reinforcing in reducing platform's charges (or increasing its subsidies) to sellers. The reason is that in this case, the platform has more substantial incentives to use the learning-by-doing benefit to attract sellers, thus providing more price concessions to sellers in period 1.

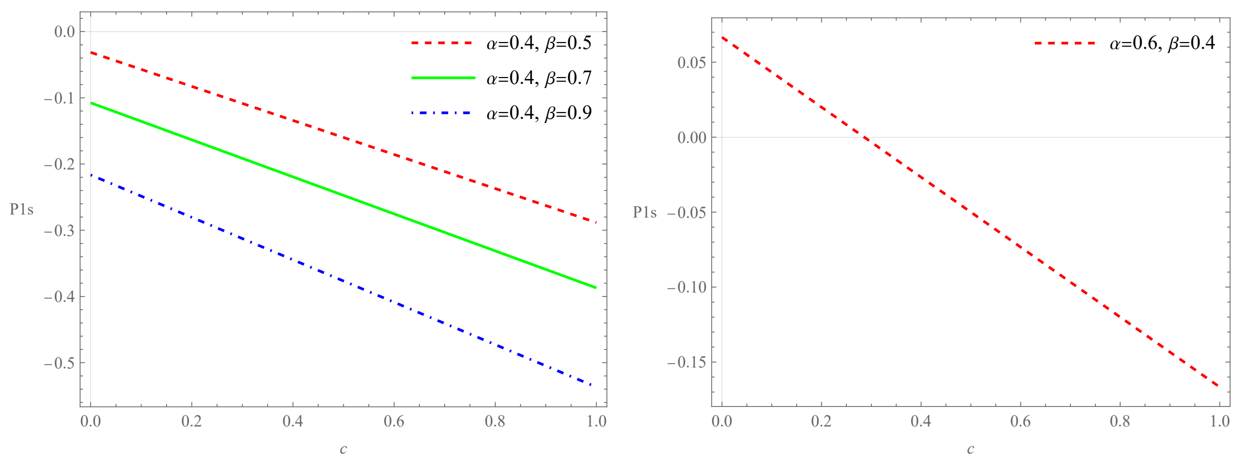
Although Cabral (2019) points out that subsidizing participants at an early stage may be optimal in dynamic pricing theory with learning-by-doing or network effects, the coordinated effect between the learning-by-doing benefit and the cross-sided network externalities has not attracted much attention. In the model of Cabral (2019), the cross-sided network externalities are not explic-

[19]A special feature in our model is that the sellers' price charged by the monopoly platform depends on their own externality parameter α, which is different from that in Armstrong (2006).

[20]Later, we will discuss more details of the intertemporal pricing attributions.

itly characterized, and agents are assumed to be myopic. However, in our model, all agents have rational expectations, and the cross-sided network externalities and the learning-by-doing benefit are simultaneously explicitly featured within a unified framework. Our model shows a closed-form solution of prices for buyers and sellers in the two periods and thus provides the concrete conditions under which the platform will subsidize early sellers.[21]

Figure 5 shows several numerical examples to illustrate the conditions of the subsidy to online sellers in period 1 and the impact of changes in c and β on P_1^s. Figure 5(a) presents the situation in which $\beta > \alpha$. The scenario in which $\alpha > \beta$ is presented in Figure 5(b). These numerical examples show that P_1^s is decreasing in β (see Figure 5(a)) and c (see Figures 5(a) and 5(b)).



(a) $\alpha = 0.4$, $\beta = 0.5, 0.7, 0.9$

(b) $\alpha = 0.6$, $\beta = 0.4$

Figure 5: Relationship between the Price for Sellers in Period 1 and the Increment in Online Business Ability c

Notes: Panel (a) plots the relationships between P_1^s and c in three numerical examples, (1) $\alpha = 0.4$ and $\beta = 0.5$; (2) $\alpha = 0.4$ and $\beta = 0.7$; and (3) $\alpha = 0.4$ and $\beta = 0.9$, respectively, with $0 < c < 1$. Panel (b) plots the relationship between P_1^s and c when $\alpha = 0.4$ and $\beta = 0.6$ with $0 < c < 1$.

Next, we analyze the features of cross-side price structures, intertemporal price structures, and the interaction of the two. We also discuss the effects of network externalities and the enhancement of business ability on these price structures.

Theorem 2 (Cross-Side Price Structure When Allowing Price Discrimination)**.** *Suppose that third-degree price discrimination is allowed in period 2. Given the feasible set for parameters in Lemma 2, in equilibrium:*

1. *In period 1, buyers are charged a higher fee than sellers; that is,* $P_1^b > P_1^s$.
2. *The cross-side price difference* $P_1^b - P_1^s$ *is increasing in* c *and* β *and decreasing in* α.

[21] Halaburda et al. (2020) present the dynamic competition between the two platforms with one-sided network effects and homogeneous consumers, while they focus on the quality and focal status of the two competing platforms.

3. *If (i) $\beta > \frac{1+\alpha}{2}$ or (ii) $\beta < \frac{1+\alpha}{2}$ and $c < \frac{2(1-\alpha)}{3-2\beta-\alpha}$, in period 2, the price for buyers is higher than that for sellers who are affiliated with the platform in the two periods (that is, $P_2^b > P_{22}^s$). However, if $\beta < \frac{1+\alpha}{2}$ and $c > \frac{2(1-\alpha)}{3-2\beta-\alpha}$, the opposite result applies.*
4. *The cross-side price difference $P_2^b - P_{22}^s$ is increasing in β and decreasing in c and α.*

Proof. See Online Appendix E. □

Theorem 2 highlights the cross-side price structures in the two periods. In period 1, the conclusion is that the platform always charges a higher fee to buyers than to sellers. This occurs for the same reason as explained earlier—buyers have an absolute advantage in obtaining utility from their own ability compared to sellers. The cross-side price difference depends on two externality parameters (α and β) and improvement in business ability c. The cross-side price difference measures the (latent) relative bargaining power between buyers and sellers when negotiating with the monopoly platform.[22] Consider that α and β denote the values of the network externalities of each buyer and seller, respectively, in two-sided markets. The phenomenon that $P_1^b - P_1^s$ is increasing in β and decreasing in α is obvious. Moreover, when the platform anticipates a higher enhancement of the business ability of sellers, the platform tends to increase the relative price between buyers and sellers in the initial period. As shown in Theorem 1, P_1^s decreases as c increases. However, the change in P_1^b is not clear with an increase in c because it depends on the relative size between α and β.[23] Even if P_1^b may decrease as c increases, the relative price $P_1^b - P_1^s$ will increase with an increase in c, as proven in Online Appendix E, because the increase in c changes the relative market power between buyers and sellers.

In period 2, the cross-side price structure changes. Here, two main features are noteworthy. One is that period 2 is the last period, and the game ends on this date. It also implies that there is no further change in behavior, which the platform needs to consider. Another feature is that sellers who sell online in period 1 have achieved enhanced business ability, which gives them an increase in utility. Then, as described in the theorem, when the buyers' network externality parameter β is high enough or the business ability increment c of sellers is low enough, the platform charges buyers a higher fee than sellers. It is straightforward that $P_2^b - P_{22}^s$ is increasing in β and decreasing in α from the explanation of cross-side price structure in period 1.[24] One difference is that the price gap between buyers and sellers in period 2 is decreasing in the business ability increment c. From equation (29), we can derive that $\frac{\partial P_{22}^s}{\partial c} = \frac{6-(\alpha+\beta)(\alpha+2\beta)}{2(4-(\alpha+\beta)^2)} > 0$. Intuitively, the greater is the utility that sellers obtain from improvement in their ability, the higher is the fee the platform charges them.

[22]In reality, pricing negotiations may not occur between the monopoly platform and buyers or sellers.

[23]From equality (27), we can derive that the derivative of P_1^b with respect to c is $\frac{\beta-\alpha}{8-2(\alpha+\beta)^2}$. Hence, if $\beta > \alpha$, P_1^b is increasing in c. If $\beta < \alpha$, the opposite result applies.

[24]These results are consistent with some conclusions in the literature. For example, Hagiu (2009) points out that the platform will extract more profits from agents on the more powerful side compared to those on the other side.

Since $P_1^b = P_2^b$, the analysis of the derivative of P_2^b with respect to c is the same as that for P_1^b. Even if P_2^b is increasing in c when $\beta > \alpha$, the price difference $P_2^b - P_{22}^s$ will decrease with an increase in c due to increased growth in P_{22}^s. This also means that when poor sellers become more capable of running the online business through learning-by-doing, the platform tends to charge them an asymmetrically higher fee than buyers.

Theorem 3 (Intertemporal Price Structure When Allowing Price Discrimination)**.** *Suppose that third-degree price discrimination is allowed in period 2. Given the feasible set for parameters in Lemma 2, in equilibrium:*

1. *The prices on buyers in the two periods are equal; that is,* $P_2^b = P_1^b$.
2. *The prices on sellers in the two periods satisfy that* $P_{21}^s = P_1^s$ *and* $P_{22}^s = P_1^s + c$.

Proof. See Online Appendix F. □

Theorem 3 states that (i) there is no difference in price for buyers and new sellers in the two periods, and (ii) the price for old sellers in period 2 is c greater than the price in period 1. The distribution of buyer ability and the externality parameters of the network remain unchanged during the two periods. Then, the platform's optimal pricing strategy for buyers is to set the same price in both periods. A further result of this pricing strategy for buyers is that the number of buyers who buy on the platform is the same in the two periods. However, in period 2 the price for sellers who choose to join the online platform in period 1 will be adjusted upward by the business ability increment c. The reason for this upward trend is that this promotion of business skills guarantees that old sellers can afford higher expenses than previously. Overall, the intertemporal price structure will only change as the business ability on the same side changes over time.[25]

The discussion of cross-side and intertemporal price structures has provided some insights into economic development in the digital economy. The rest of this section discusses the dynamic change in the cross-side price structure over time.

Proposition 1 (Intertemporal Cross-Side Price Structure When Allowing Price Discrimination)**.** *Suppose that third-degree price discrimination is allowed in period 2. Given the feasible set of parameters in Lemma 2, in equilibrium, the cross-side price difference between buyers and sellers in period 1 is greater than that in period 2 (that is,* $P_2^b - P_{22}^s < P_1^b - P_1^s$*); the intertemporal difference in cross-side price differences satisfies* $(P_2^b - P_{22}^s) - (P_1^b - P_1^s) = -c$.

Proof. See Online Appendix G. □

[25] If price discrimination on the part of sellers in period 2 is prohibited, the results here will change. The next section provides details on this scenario.

Proposition 1 indicates that the platform tends to set a higher cross-side price difference between buyers and sellers in period 1 and the change in this price difference completely depends on the increment in online sellers' skills. More specifically, the source of this intertemporal cross-side change is only the increase in online sellers' prices from period 1 to 2, as discussed above. A key reason is that the increment in online sellers' ability can be anticipated at the beginning because of complete information, but it is realized in the next and last period. This theorem also implies that, according to the learning-by-doing effect, the enhancement in business ability of one type of agent in two-sided markets will affect the cross-side and intertemporal price structures and the platform enterprise's profit. Although these agents only have relatively low initial skills in running a business, the platform includes them in online transactions through intertemporal and/or cross-side price adjustments. The intertemporal and cross-side pricing strategy implemented by the monopoly platform also serves its goal of maximizing aggregate two-period profit, and hence the platform has an incentive to do this.

4 Prohibiting Price Discrimination

In this section, we model the game when third-degree price discrimination toward sellers is prohibited in period 2. We also analyze the platform's change in the pricing strategy in equilibrium compared with allowing price discrimination. Given the prices for buyers in the two periods, the best responses of buyers are the same as those characterized in Subsection 2.2. A main difference occurs in the platform's pricing strategy toward sellers. It is obvious that both sellers' best responses and the platform's two-period profit change.

4.1 Adjusted Sellers' Best Responses and the Platform's Profit

When third-degree price discrimination is prohibited, the best responses of sellers will change because the flexibility of the platform's pricing strategy has decreased. Consider that in period 2, all online sellers face a unified price P_2^s proposed by the platform, which means that P_{21}^s and P_{22}^s above are replaced by P_2^s. An observation is that $\widetilde{U}_{i,2}^s > \widehat{U}_{i,2}^s$ because of the enhancement of seller i's business ability. More importantly, there may be a newly emerging phenomenon that although some poor sellers face a lower utility in period 1 compared with offline returns, their rational choice is to participate in online sales at the beginning because the enhancement in business ability will bring them enough benefit in the next period. [26]

There is only one critical seller in each period because of the unified price and continuous

[26] The lower utility in period 1 implies that these poor sellers have a relatively low standard of living and have to borrow money to survive. A potential assumption here is that the financial market is complete for all sellers.

ability distribution. In theory, given the two-period prices for buyers and sellers proposed by the platform, there are three scenarios for the homing choice of this critical seller (Figure 3 illustrates the critical seller's strategy).

Scenario A (the sellers on the platform are the same in both periods). The seller h and the seller k are the same, which implies that three conditions, $U^s_{k,1} = \underline{U}^s$, $\widetilde{U}^s_{h,2} = \underline{U}^s$, and $B^s_{k,1} = B^s_{h,1}$, are satisfied. From these three conditions, we have that the numbers of sellers affiliated with the platform in the two periods are

$$N^s_1(P^s_1, N^b_1) = \alpha N^b_1 - P^s_1 - \underline{U}^s, \tag{33}$$

and

$$N^s_2 = N^s_1 \tag{34}$$

with the following relationship between P^s_1 and P^s_2

$$P^s_2 = \alpha N^b_2 + c + P^s_1 - \alpha N^b_1. \tag{35}$$

Then, we obtain the homing choice equilibria for buyers and sellers from equations (24), (25), (33), and (34) (see Online Appendix A.2). Meanwhile, the relationship between P^s_1 and P^s_2, as shown in equation (35), must be guaranteed in equilibrium.[27]

Scenario B (some sellers enter the platform in period 2). The critical seller h in period 2 has weaker business ability than the seller k. Hence, $\widehat{U}^s_{h,2} = \underline{U}^s$, $U^s_{k,1} + \widetilde{U}^s_{k,2} = \underline{U}^s + \widehat{U}^s_{k,2}$ and $\widehat{U}^s_{h,2} < \widehat{U}^s_{k,2}$. Then, from the first two conditions, the numbers of sellers in the two periods can be expressed as

$$N^s_1(P^s_1, N^b_1) = \alpha N^b_1 + c - P^s_1 - \underline{U}^s, \tag{36}$$

and

$$N^s_2(P^s_2, N^b_2) = \alpha N^b_2 - P^s_2 - \underline{U}^s. \tag{37}$$

From equations (24), (25), (36) and (37), the homing choice equilibria can be obtained (see Online Appendix A.2). In addition, the third condition $\widehat{U}^s_{h,2} < \widehat{U}^s_{k,2}$ must hold in equilibrium.

Scenario C (**some sellers leave the platform in period 2**). The critical seller $\widetilde{h}$ in period 2 has a stronger business ability than the seller k. In this scenario, $U^s_{k,1} = \underline{U}^s$, $\widetilde{U}^s_{\widetilde{h},2} = \underline{U}^s$ and $\widetilde{U}^s_{k,2} < \widetilde{U}^s_{\widetilde{h},2}$.

[27]It could be argued that an alternative method to solve the homing choice equilibria in *Scenario A* would be to obtain N^s_2 from condition $\widetilde{U}^s_{h,2} = \underline{U}^s$ (that is, $N^s_2 = c + \alpha N^b_2 - P^s_2 - \underline{U}^s$) and then utilize the relation $N^s_1 = N^s_2$ to obtain N^s_1. Therefore, the relationship between the two prices for sellers becomes $P^s_1 = P^s_2 + \alpha N^b_1 - \alpha N^b_2 - c$. This method is wrong because the monopoly platform will apply a backward induction method to develop the pricing strategy. Hence, the game requires that P^s_2 is the result of the platform's best response in period 2 when P^s_1 is given, and then P^s_1 will be chosen by the platform to maximize the two-period profit.

The first two conditions imply that the numbers of sellers in the two periods are

$$N_1^s(P_1^s, N_1^b) = \alpha N_1^b - P_1^s - \underline{U}^s, \tag{38}$$

and

$$N_2^s(P_2^s, N_2^b) = \alpha N_2^b + c - P_2^s - \underline{U}^s. \tag{39}$$

By combining (24) and (25) with equations (38) and (39), we can solve the homing choice problem of all buyers and sellers (see Online Appendix A.2). Meanwhile, the third condition $\widetilde{U}_{k,2}^s < \widetilde{U}_{\tilde{h},2}^s$ should be satisfied in equilibrium.

When third-degree price discrimination is prohibited, the aggregate profit of the monopoly platform in the two periods is expressed as

$$\pi_b = \max_{\mathbf{P}_b} \left[P_1^s N_1^s + P_1^b N_1^b + \frac{1}{1+r} \left(P_2^s N_2^s + P_2^b N_2^b \right) \right], \tag{40}$$

where the list of the platform prices is $\mathbf{P}_b = \left\{ P_1^s, P_1^b, P_2^s, P_2^b \right\}$. All the other parameters are the same as previously described. Here, we also assume that $\underline{U}^s = 0$, $\underline{U}^b = 0$, and $r = 0$ for simplicity.

4.2 Nash Equilibrium

Under the economic environment that prohibits price discrimination, we use the "tilde" script overhead to distinguish the equilibrium results from the results of allowing price discrimination. After a two-period game, the final equilibrium results in different scenarios demonstrate the following lemma.

Lemma 3. *When third-degree price discrimination is prohibited, the platform's optimal pricing strategy indicates that Scenario A, featured in Subsection 4.1, is a unique Nash equilibrium.*

Proof. See Online Appendix H. □

In this Nash equilibrium, the prices for the two periods for buyers and sellers set by the monopoly platform are

$$\widetilde{P_1^b} = \frac{6 - 3\alpha(\alpha+\beta) - 2c(\alpha-\beta)}{2(6-(\alpha+\beta)(\alpha+2\beta))}, \tag{41}$$

$$\widetilde{P_1^s} = \frac{\alpha(\alpha(\alpha+\beta)+2) - 4\beta - 2c(2-\alpha(\alpha+\beta))}{2(6-(\alpha+\beta)(\alpha+2\beta))}, \tag{42}$$

$$\widetilde{P_2^b} = \frac{6 + 2\beta c - \alpha^2 - \alpha\beta}{2(6-(\alpha+\beta)(\alpha+2\beta))}, \tag{43}$$

$$\widetilde{P_2^s} = c - \frac{\alpha(\alpha(\alpha+\beta)-2) + 4\beta + c(4-2\alpha\beta)}{2(6-(\alpha+\beta)(\alpha+2\beta))}. \tag{44}$$

Putting these equilibrium prices back into equalities (80), (81), (82), and (83), we then obtain

$$\widetilde{N_1^s} = \widetilde{N_2^s} = \frac{2(\alpha+\beta+c)}{6-(\alpha+\beta)(\alpha+2\beta)}, \tag{45}$$

$$\widetilde{N_1^b} = \frac{6+(\alpha+\beta)(\alpha+2c)}{2(6-(\alpha+\beta)(\alpha+2\beta))}, \tag{46}$$

$$\widetilde{N_2^b} = \frac{6+2\beta c - \alpha^2 - \alpha\beta}{2(6-(\alpha+\beta)(\alpha+2\beta))}. \tag{47}$$

Thus, in the final Nash equilibrium, the optimal two-period profit of the platform is

$$\widetilde{\pi} = \frac{36 - (\alpha+\beta)(\alpha^2(\alpha+\beta)+4\alpha+16\beta) - 2(c^2+2(\alpha+\beta)c)(\alpha^2+2\alpha\beta+2\beta^2-4)}{2(6-(\alpha+\beta)(\alpha+2\beta))^2}. \tag{48}$$

4.3 Equilibrium Analysis

This subsection analyzes the equilibria when prohibiting price discrimination. Some results for the cross-side and intertemporal price structures that were discussed earlier also hold when price discrimination is prohibited. Therefore, we place greater emphasis on the difference in price strategy. To interpret the effect of price discrimination on the equilibria, we also compare the equilibrium results of these two cases.

Lemma 4. *To establish* $0 < \widetilde{N_1^s}, \widetilde{N_1^b}, \widetilde{N_2^s}, \widetilde{N_2^b} < 1$ *and* $\widetilde{\pi} > 0$ *in equilibrium, we have a feasible set of parameters as* $0 < \alpha < 1$, $0 < \beta < \min\left\{1, \frac{\sqrt{\alpha^2+96}-7\alpha}{8}\right\}$, *and* $0 < c < \min\left\{1, \frac{6-4\beta^2-7\alpha\beta-3\alpha^2}{2(\alpha+\beta)}\right\}$.

Proof. Putting equilibria (45), (46), (47), and (48) together with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$, we derive that $0 < \widetilde{N_1^s}, \widetilde{N_1^b}, \widetilde{N_2^s}, \widetilde{N_2^b} < 1$ and $\widetilde{\pi} > 0$ are satisfied if one of following four conditions holds:

(1) $0 < \alpha \le \frac{1}{6}\left(\sqrt{73}-7\right)$, $0 < \beta \le \frac{1}{8}\sqrt{\alpha^2-4\alpha+100} - \frac{1}{8}(2+7\alpha)$, and $0 < c < 1$;

(2) $0 < \alpha \le \frac{1}{6}\left(\sqrt{73}-7\right)$, $\frac{1}{8}\sqrt{\alpha^2-4\alpha+100} - \frac{1}{8}(2+7\alpha) < \beta < 1$, and $0 < c < \frac{6-4\beta^2-7\alpha\beta-3\alpha^2}{2(\alpha+\beta)}$;

(3) $\frac{1}{6}\left(\sqrt{73}-7\right) < \alpha < 1$, $0 < \beta \leq \frac{1}{8}\sqrt{\alpha^2-4\alpha+100}-\frac{1}{8}(2+7\alpha)$, and $0 < c < 1$;

(4) $\frac{1}{6}\left(\sqrt{73}-7\right) < \alpha < 1$, $\frac{1}{8}\sqrt{\alpha^2-4\alpha+100}-\frac{1}{8}(2+7\alpha) < \beta < \frac{\sqrt{\alpha^2+96}-7\alpha}{8}$, and $0 < c < \frac{6-4\beta^2-7\alpha\beta-3\alpha^2}{2(\alpha+\beta)}$.

These four conditions can be further expressed as $0 < \alpha < 1$, $0 < \beta < \min\left\{1, \frac{\sqrt{\alpha^2+96}-7\alpha}{8}\right\}$, and $0 < c < \min\left\{1, \frac{6-4\beta^2-7\alpha\beta-3\alpha^2}{2(\alpha+\beta)}\right\}$. □

Lemma 4 provides the feasible set of parameters. All the equilibrium analyses that follow in this subsection are within this feasible set.

First, we explore the equilibrium price for online sellers in period 1 because subsidization to (poor) sellers is the basis and beginning of economic development in rural areas.

Theorem 4 (Initial Price for Online Sellers When Prohibiting Price Discrimination)**.** *Suppose that third-degree price discrimination is prohibited in period 2. Given the feasible set of parameters in Lemma 4, in equilibrium, if (i) $\beta > \alpha - \frac{2\alpha\left(1-\alpha^2\right)}{4-\alpha^2}$ or (ii) $\beta < \alpha - \frac{2\alpha\left(1-\alpha^2\right)}{4-\alpha^2}$ and $c > \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)} - \frac{\alpha\left(1-\frac{\alpha(\alpha+\beta)}{2}\right)}{2-\alpha(\alpha+\beta)}$, the price for sellers in period 1 is negative (that is, $\widetilde{P_1^s} < 0$). If $\beta < \alpha - \frac{2\alpha\left(1-\alpha^2\right)}{4-\alpha^2}$ and $c < \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)} - \frac{\alpha\left(1-\frac{\alpha(\alpha+\beta)}{2}\right)}{2-\alpha(\alpha+\beta)}$, the opposite result applies (that is, $\widetilde{P_1^s} > 0$).*

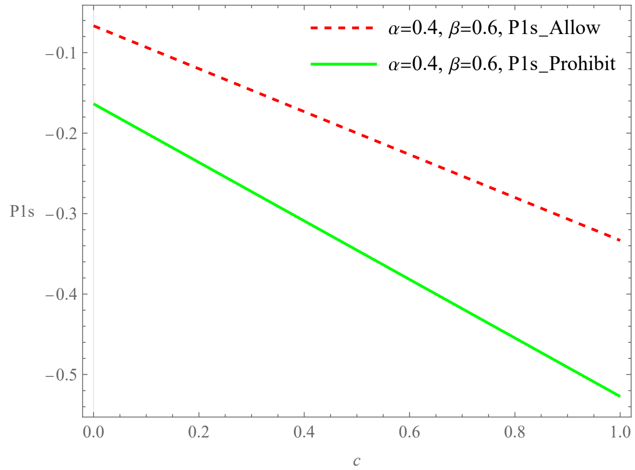
Proof. See Online Appendix I. □

Theorem 4 demonstrates the platform's price strategy toward sellers in period 1 in equilibrium. In contrast to allowing price discrimination as described in Theorem 1, the conditions of subsidization are relaxed when third-degree price discrimination is prohibited. More specifically, recalling $\alpha \in (0,1)$ and $\beta \in (0,1)$, both terms $\frac{2\alpha(1-\alpha^2)}{4-\alpha^2}$ and $\frac{\alpha\left(1-\frac{\alpha(\alpha+\beta)}{2}\right)}{2-\alpha(\alpha+\beta)}$ are positive; thus, the feasible zone (β, c) for a negative price for sellers in period 1 is expanded. The economic intuition behind this result is as follows: when price discrimination is prohibited, the flexibility of pricing toward sellers decreases because the threat fee cannot be charged if the sellers enter the platform in period 2, which also means that the platform's (latent) bargaining power decreases. Here, an implicit guess is that, at least in period 1, the number of online sellers may increase. We will discuss these comparable results later. Moreover, we use two numerical examples, (1) $\alpha = 0.4$ and $\beta = 0.6$ and (2) $\alpha = 0.6$ and $\beta = 0.4$, to verify our analysis intuitively.

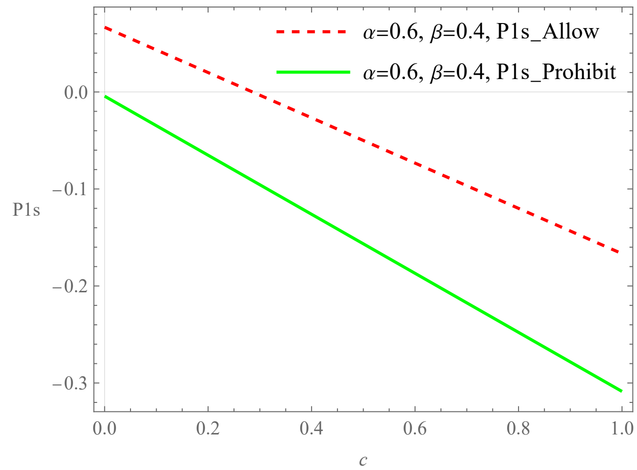
Next, we present the price structures in equilibrium. As shown in Theorem 5, most of the results for the cross-side price structure are the same as those in Theorem 2, and we can also interpret them as we did previously.

Theorem 5 (Cross-Side Price Structure When Prohibiting Price Discrimination)**.** *Suppose that third-degree price discrimination is prohibited in period 2. Given the feasible set for parameters in Lemma 4, in equilibrium:*

1. *In period 1, the price for buyers is greater than that on sellers; that is, $\widetilde{P_1^b} > \widetilde{P_1^s}$.*



(a) $\alpha = 0.4, \beta = 0.6$



(b) $\alpha = 0.6, \beta = 0.4$

Figure 6: Relationship between the Price for Sellers in Period 1 and the Increment in Online Business Ability c

Notes: Panel (a) plots the relationships between P_1^s, $\widetilde{P_1^s}$ and c, when $\alpha = 0.4$ and $\beta = 0.6$. Panel (b) plots the relationships between P_1^s, $\widetilde{P_1^s}$ and c, when $\alpha = 0.6$ and $\beta = 0.4$. The feasible set for c is $(0, 1)$.

2. *The price difference $\widetilde{P_1^b} - \widetilde{P_1^s}$ is increasing in c and β.*
3. *In period 2, if the two conditions $\beta < \frac{1}{8}(-\alpha^2 - 3\alpha - 6) + \frac{1}{8}\sqrt{\alpha^4 - 10\alpha^3 + 5\alpha^2 + 68\alpha + 68}$ and $c > \frac{-\alpha^3 - \alpha^2\beta + \alpha^2 + \alpha\beta + 2\alpha - 4\beta - 6}{2\alpha^2 + 4\alpha\beta + 4\beta^2 + 2\beta - 8}$ are satisfied, the price for sellers is higher than that on buyers, that is, $\widetilde{P_2^s} > \widetilde{P_2^b}$. Otherwise, $\widetilde{P_2^b} \geq \widetilde{P_2^s}$.*
4. *The price difference $\widetilde{P_2^b} - \widetilde{P_2^s}$ is decreasing in c and increasing in β.*

Proof. See Online Appendix J. □

Theorem 6 (Intertemporal Price Structure When Prohibiting Price Discrimination)**.** *Suppose that third-degree price discrimination is prohibited in period 2. Given the feasible set for parameters in Lemma 4, in equilibrium:*

1. *The price on buyers in period 2 is higher than that in period 1; that is, $\widetilde{P_2^b} > \widetilde{P_1^b}$.*
2. *The price difference $\widetilde{P_2^b} - \widetilde{P_1^b}$ is increasing in c, α, and β.*
3. *If $c > \frac{\alpha^2(\alpha+\beta)}{6-2\alpha^2-3\alpha\beta-2\beta^2}$, the price for sellers in period 2 is greater than that in period 1 (that is, $\widetilde{P_2^s} > \widetilde{P_1^s}$). If $c < \frac{\alpha^2(\alpha+\beta)}{6-2\alpha^2-3\alpha\beta-2\beta^2}$, the opposite result applies.*
4. *The price difference $\widetilde{P_2^s} - \widetilde{P_1^s}$ is increasing in c and decreasing in α and β.*

Proof. See Online Appendix K. □

We now present and explain the intertemporal price structure when price discrimination is prohibited, based on Theorem 6. Here, a direct observation is that the intertemporal price structures are very different from those in Theorem 3. This adjustment of its pricing strategy is the platform's best response to the change in price regulation. First, we discuss the intertemporal price structure on the buyer's side. The platform charges buyers a higher fee in period 2 than in period 1. The reason is that the monopoly platform has an incentive to attract more sellers in period 1 because the platform's bargaining power with sellers decreases in period 2. Hence, one of the ways to attract sellers is to charge buyers a relatively lower price in period 1 to attract more buyers (more sellers enter the platform in period 1 because of greater network externality). However, in the second and last period, there are no longer any intertemporal attributes (not only for buyers but also for sellers), and the price for buyers will be greater. An increase in c causes an increase in intertemporal price difference $\widetilde{P_2^b} - \widetilde{P_1^b}$. We can interpret this from two aspects. One is that although the enhancement in business ability only occurs on the seller's side, the rational platform can expect it and thus will subsidize buyers more or charge them less in period 1 if $\beta < \alpha$.[28] Another aspect is that a larger c implies that sellers directly obtain higher utility from participating in online selling in period 2, and sellers' relative dependence on the network externality of buyers decreases. Hence, the platform charges buyers a higher fee because it is no longer afraid that the sellers will leave.[29] Moreover, the intertemporal price difference $\widetilde{P_2^b} - \widetilde{P_1^b}$ is also increasing in α and β. Note that we can easily obtain $\frac{\partial \widetilde{P_2^b}}{\partial \alpha} > 0$ and $\frac{\partial \widetilde{P_2^b}}{\partial \beta} > 0$ from equation (43), which implies that an increase in the network externality each buyer or seller produces leads to buyers being charged a higher fee in period 2. The reason is that both α and β are the basis of the platform's prices, especially in the last period. The higher the values these two parameters are, the higher the fees (or more room) they are charged by the monopoly platform. However, the relationships between $\widetilde{P_1^b}$ and α, β are not clear.[30] Two possible scenarios may occur. One is that an increase in the externality parameter will decrease the price $\widetilde{P_1^b}$. Another scenario is that this relationship is positive, but the increasing speed is lower than that in period 2. Intuitively, the interaction between the intertemporal and cross-side considerations determines that the platform will not apply the pricing strategy as in the second and last period.

Second, we discuss the intertemporal price structure on the seller's side. Only when the increment in business ability is high enough will the platform charge online sellers a higher fee in period 2. One possible reason is that the platform must balance sellers and buyers to set an optimal pricing strategy. Consider a situation in which $c < \frac{\alpha^2(\alpha+\beta)}{6-2\alpha^2-3\alpha\beta-2\beta^2}$ is satisfied. The platform may tend to

[28]Recalling the equilibrium price $\widetilde{P_1^b}$ in equation (41), we can derive that $\frac{\partial \widetilde{P_1^b}}{\partial c} = \frac{\beta-\alpha}{6-(\alpha+\beta)(\alpha+2\beta)}$. Recalling $\alpha \in (0,1)$ and $\beta \in (0,1)$, we have $\frac{\partial \widetilde{P_1^b}}{c} < 0$ if $\beta < \alpha$. Of course, the relationships between $\widetilde{P_2^b} - \widetilde{P_1^b}$ and α and β also establish if $\beta > \alpha$, because $\widetilde{P_2^b}$ increases faster than $\widetilde{P_1^b}$ as α or β increases.

[29]From the equilibrium price $\widetilde{P_2^b}$ in equation (43), we have that $\frac{\partial \widetilde{P_2^b}}{\partial c} = \frac{\beta}{6-(\alpha+\beta)(\alpha+2\beta)} > 0$ when we consider $\alpha \in (0,1)$ and $\beta \in (0,1)$.

[30]We can derive that $\frac{\partial \widetilde{P_1^b}}{\partial \alpha} = \frac{3\beta(\alpha+\beta)^2-12\alpha-c\left(\alpha^2-2\alpha\beta-5\beta^2+6\right)}{(6-(\alpha+\beta)(\alpha+2\beta))^2}$ and $\frac{\partial \widetilde{P_1^b}}{\partial \beta} = \frac{-3\alpha(\alpha+\beta)^2+12\beta+2c\left(-2\alpha^2-2\alpha\beta+\beta^2+3\right)}{(6-(\alpha+\beta)(\alpha+2\beta))^2}$ from equation (41). The signs of these two derivatives are not clear and depend on the relationships between c, α, and β.

set a lower price for sellers in period 2 to attract or keep sellers but shift price pressure to buyers. The last result in this theorem provides some intuitions about the price structure. An increase in business ability implies that the base of pricing increases in period 2, and therefore, the platform charges sellers a higher fee. However, it is not straightforward that $\widetilde{P_2^s} - \widetilde{P_1^s}$ is decreasing in α and β.[31] A possible mechanism is as follows: the platform tends to subsidize buyers in period 1 to attract more sellers with the future enhancement of business ability (or the learning-by-doing effect), and then in period 2, the platform puts the pressure of charging on buyers by retaining sellers.

To interpret the platform's dynamic pricing with economic development, we consider the interaction between intertemporal and cross-side price structures. The result is characterized by the following proposition.

Proposition 2 (Intertemporal Cross-Side Price Structure When Prohibiting Price Discrimination)**.** *Suppose that third-degree price discrimination is prohibited in period 2. Given the feasible set for parameters in Lemma 4, in equilibrium:*

1. *If* $c > \frac{\alpha(\alpha+1)(\alpha+\beta)}{6-2\alpha^2-3\alpha\beta-\alpha-2\beta^2}$, *we have* $\widetilde{P_2^b} - \widetilde{P_2^s} < \widetilde{P_1^b} - \widetilde{P_1^s}$ *(or* $\widetilde{P_2^b} - \widetilde{P_1^b} < \widetilde{P_2^s} - \widetilde{P_1^s}$*).*
2. *If* $c < \frac{\alpha(\alpha+1)(\alpha+\beta)}{6-2\alpha^2-3\alpha\beta-\alpha-2\beta^2}$, *we have* $\widetilde{P_2^b} - \widetilde{P_2^s} > \widetilde{P_1^b} - \widetilde{P_1^s}$ *(or* $\widetilde{P_2^b} - \widetilde{P_1^b} > \widetilde{P_2^s} - \widetilde{P_1^s}$*).*

Proof. See Online Appendix L. □

Proposition 2 shows that when the improvement in business ability is high enough, the platform will impose greater price pressure on the cross-side price structure in period 1 (i.e., $\widetilde{P_1^b} - \widetilde{P_1^s}$) and the intertemporal price on seller's side (i.e., $\widetilde{P_2^s} - \widetilde{P_1^s}$). Intuitively, information on the growth of online sellers' business ability is complete and can be expected by all agents, but its value is generated only in period 2 (the increased utility of sellers who choose to join the platform at the beginning). Therefore, when the potential growth of these (poor) sellers' skill is sizable, the platform tends to charge them less or subsidize them more in period 1. When these sellers become more capable of running the online business, the platform increases its fee for sellers more than for buyers in period 2.

Proposition 3. *Suppose that third-degree price discrimination is prohibited in period 2. Given the feasible set of parameters in Lemma 4, in equilibrium, although the number of sellers who were affiliated with the platform in period 1 is equal to that in period 2 (that is,* $\widetilde{N_1^s} = \widetilde{N_2^s}$*), the number of online buyers in period 1 is greater than that in period 2 (that is,* $\widetilde{N_2^b} < \widetilde{N_1^b}$ *); moreover, this gap* $\left|\widetilde{N_2^b} - \widetilde{N_1^b}\right|$ *is increasing in* α*,* β*, and* c*.*

Proof. See Online Appendix M. □

[31] The derivatives of $\widetilde{P_1^s}$ and $\widetilde{P_2^s}$ with respect to α and β are complicated, and their signs are not clear.

Proposition 3 states that the number of online sellers does not change, but the number of online buyers decreases over time. In equilibrium, the numbers of buyers and sellers on the platform are very different from those in a scenario that allows third-degree price discrimination (see equations (30) and (31)). When price discrimination toward sellers is allowed in period 2, the numbers of buyers affiliated with the platform are equal in the two periods. However, when price discrimination is prohibited, the platform must transfer the pressure of charging to buyers because of the loss of pricing flexibility on the sellers' side; thus, the number of online buyers in period 2 is less than the number in period 1.

4.4 Comparative Analysis of Equilibria

In this section, we compare the equilibrium prices and numbers in two cases. Next, for simplicity, we offer the definition $\phi \equiv \alpha + \beta$, indicating the additive network externality of buyers and sellers in two-sided markets. From the equilibrium results, we obtain the following observation:

Remark 1. When third-degree price discrimination is allowed, the homing choice equilibria and the aggregate profit of the platform in the two periods do not depend on the allocation of ϕ between α and β, although the prices depend on this allocation. However, when price discrimination is prohibited, the allocation of ϕ between α and β is relevant for the homing choice equilibria and the platform's aggregate profit.

The reason is that when price discrimination is allowed, the platform can more flexibly use network externality parameters α and β, which are the basis of profit and utility. Through price discrimination, the platform has enough power to balance sellers and buyers, and the platform internalizes the allocation of ϕ. However, when price discrimination is prohibited, the platform cannot operate. Hence, the difference between α and β is relevant for the numbers of buyers and sellers and the platform's profit in equilibrium.

Price discrimination in the traditional economic literature means that a monopoly can operate with a higher price and lower production. However, in two-sided markets, the monopoly platform only plays the role of an intermediary or bridge in linking buyers and sellers. Thus, prohibiting price discrimination may cause some unconventional outcomes. The following proposition summarizes all the comparisons pertaining to profits, prices, and numbers.

Proposition 4. *Given the feasible sets for parameters in Lemmas 2 and 4, in equilibrium:*

1. *The two-period total profit the monopoly platform obtains under price discrimination is higher than that under no price discrimination. That is, $\pi > \widetilde{\pi}$ is always established.*
2. *In each period, the number of sellers who are affiliated with the platform when price discrimination toward sellers in period 2 is allowed is less than that when this price discrimination is prohibited. That is, $N_1^s < \widetilde{N_1^s}$ and $N_2^s < \widetilde{N_2^s}$ are always established.*

3. *In each period, the price for sellers who sell on the platform when allowing price discrimination is higher than that when prohibiting price discrimination. That is,* $P_1^s > \widetilde{P_1^s}$ *and* $P_{22}^s > \widetilde{P_2^s}$ *are always satisfied.*
4. *The intertemporal price difference for online sellers when price discrimination is allowed is greater than when price discrimination is prohibited; that is,* $\widetilde{P_2^s} - \widetilde{P_1^s} < P_{22}^s - P_1^s$*.*
5. *In period 1, the number of buyers who choose to join the platform when price discrimination is allowed is less than that when price discrimination is prohibited (that is,* $N_1^b < \widetilde{N_1^b}$*). In period 2, if* $\alpha < \frac{1}{2}\left(\sqrt{17}-3\right)$ *and* $\beta > \frac{\sqrt{8\alpha^2+1}-1-\alpha^2}{\alpha}$*, the number of buyers on the platform when price discrimination is allowed is less than the number when price discrimination is prohibited (that is,* $N_2^b < \widetilde{N_2^b}$*); if either (i)* $\alpha \geq \frac{1}{2}\left(\sqrt{17}-3\right)$ *or (ii)* $\alpha < \frac{1}{2}\left(\sqrt{17}-3\right)$ *and* $\beta < \frac{\sqrt{8\alpha^2+1}-1-\alpha^2}{\alpha}$ *are satisfied, the number of buyers on the platform when price discrimination is allowed is greater than when price discrimination is prohibited (that is,* $N_2^b > \widetilde{N_2^b}$*).*
6. *In period 1, if* $\alpha > \beta$*, the price is higher for buyers affiliated with the platform when price discrimination is allowed than when it is prohibited (that is,* $P_1^b > \widetilde{P_1^b}$*); if* $\alpha < \beta$*, the price is lower for buyers who are affiliated with the platform when price discrimination is allowed than when it is prohibited (that is,* $P_1^b < \widetilde{P_1^b}$*). In period 2, the price is lower for buyers who choose to join the platform when price discrimination is allowed than when it is prohibited (that is,* $P_2^b < \widetilde{P_2^b}$*).*
7. *The intertemporal price difference for online buyers is lower when price discrimination is allowed than when it is prohibited (that is,* $\widetilde{P_2^b} - \widetilde{P_1^b} > P_2^b - P_1^b$*).*

Proof. See Online Appendix N. □

In Proposition 4, the first result is intuitive: the prices set by the platform under the prohibition of price discrimination are also feasible when price discrimination is allowed. The next three results (items 2, 3, and 4) are comparative results pertaining to the sellers' side. The change in numbers and prices (terms 2 and 3, respectively) are consistent with the traditional interpretation of price discrimination because price regulation occurs on the sellers' side. The price regulation also reduces the intertemporal price discrimination toward sellers (term 4). Intuitively, although price discrimination only occurs in period 2, the monopoly platform expects the effect of price regulation and then changes its pricing strategy in both periods. The comparative results on the buyers' side are described in terms 5, 6, and 7. Buyers are indirectly affected by the prohibition of price discrimination on the sellers' side. Hence, the change on the buyers' side is novel and different from that on the sellers' side. In period 1, the number of online buyers increases due to the prohibition of price discrimination, while the price for buyers can increase after prohibiting price discrimination if the network externality each online buyer produces is lower than each online seller produces. In period 2, the platform always has an incentive to charge buyers a much higher fee when price discrimination is prohibited, while the number of online buyers under price regulation can be greater than that under price discrimination if the network externality of each buyer is low enough and/or the network externality of each seller is high enough. Moreover, the last item also explains that the price regulation on sellers leads to expanded intertemporal price discrimination on the buyers' side because the platform can transfer price pressures across sides and periods.

5 Social Welfare Analysis

This section analyzes the buyers' and sellers' surplus under different pricing policies and explores the effect of price regulation on social welfare.

5.1 Allowing Third-Degree Price Discrimination

As shown in equations (30) and (31), the critical buyers and sellers who are indifferent between online and offline are the same in the two periods on each side. Hence, only the agents who have higher business ability than the critical ones choose to join the online platform. Next, we calculate the *sellers' surplus* and *buyers' surplus*. The sum of these two types of surplus and the platform's profit is the *social welfare*.

Sellers' surplus. In equilibrium, the business abilities of critical seller k in two periods are $B^s_{k,1} = \underline{U}^s + P^s_1 - \alpha N^b_1 = -\frac{c+\alpha+\beta}{4-(\alpha+\beta)^2}$ and $B^s_{k,2} = B^s_{k,1} + c = c - \frac{c+\alpha+\beta}{4-(\alpha+\beta)^2}$, respectively. Then, the sellers' surplus in period 1 can be calculated as

$$W^s_1 = \int_{B^s_{k,1}}^{0} U^s_{i,1} \cdot f_1\left(B^s_{i,1}\right) dB^s_{i,1} = \frac{(c+\alpha+\beta)^2}{2\left(4-(\alpha+\beta)^2\right)^2}. \tag{49}$$

The utility expression in equation (1), probability density function $f_1(B^s_{i,1}) = 1$, and the expressions of $\{B^s_{k,1}, N^b_1, P^s_1\}$ in equilibrium have been applied to this deduction. Meanwhile, in the same way as for period 1, we obtain the sellers' surplus in period 2:

$$W^s_2 = \int_{B^s_{k,2}}^{c} U^s_{i,2} \cdot f_{2,2}\left(B^s_{i,2}\right) dB^s_{i,2} = \frac{(c+\alpha+\beta)^2}{2\left(4-(\alpha+\beta)^2\right)^2}, \tag{50}$$

where we have already taken into account the utility expression in equation (2), probability density $f_{2,2}(B^s_{i,2}) = 1$ and the expressions of $\{B^s_{k,2}, N^b_2, P^s_{22}\}$ in equilibrium. Therefore, we have the sellers' surplus in the two periods:

$$W^s = W^s_1 + W^s_2 = \frac{(c+\alpha+\beta)^2}{\left(4-(\alpha+\beta)^2\right)^2}. \tag{51}$$

Buyers' surplus. Because there is no enhancement in buyers' business ability, their distribution does not change from period 1 to period 2. Then, in equilibrium, we can obtain the critical buyers' business abilities in the two periods, that is, $B_r^b = \underline{U}^b + P_1^b - \beta N_1^s = \frac{4-(\alpha+\beta)(c+2(\alpha+\beta))}{2(4-(\alpha+\beta)^2)}$ and $B_z^b = \underline{U}^b + P_2^b - \beta N_2^s = \frac{4-(\alpha+\beta)(c+2(\alpha+\beta))}{2(4-(\alpha+\beta)^2)}$, respectively. Critical buyers, r and z are the same because $B_z^b = B_r^b$. The buyers' surplus in period 1 is

$$W_1^b = \int_{B_r^b}^{1} U_{j,1}^b \cdot h\left(B_j^b\right) dB_j^b = \frac{(c(\alpha+\beta)+4)^2}{8\left(4-(\alpha+\beta)^2\right)^2}. \tag{52}$$

We can calculate the buyers' surplus in period 2 in the same way as in period 1; that is,

$$W_2^b = \int_{B_z^b}^{1} U_{j,2}^b \cdot h\left(B_j^b\right) dB_j^b = \frac{(c(\alpha+\beta)+4)^2}{8\left(4-(\alpha+\beta)^2\right)^2}. \tag{53}$$

The utility expression in equation (21), probability density function $h(B_j^b) = 1$, and the expressions of $\{B_r^b, B_z^b, N_1^s, N_2^s, P_1^b, P_2^b\}$ in equilibrium are utilized in equations (52) and (53). Then, we obtain the buyers' surplus in the two periods:

$$W^b = W_1^b + W_2^b = \frac{(c(\alpha+\beta)+4)^2}{4\left(4-(\alpha+\beta)^2\right)^2}. \tag{54}$$

Social welfare. From the *sellers' surplus* in equation (51), the *buyers' surplus* in equation (54), and the platform's profit in equation (32), we can obtain the *social welfare* in the two periods, which is expressed as

$$W = W^s + W^b + \pi = \frac{c^2\left(12-(\alpha+\beta)^2\right) + 4c(\alpha+\beta)\left(8-(\alpha+\beta)^2\right) - 4(\alpha+\beta)^2 + 48}{4\left(4-(\alpha+\beta)^2\right)^2}. \tag{55}$$

There are two important implications for social welfare. One is that the emergence of the platform can improve social welfare (i.e., $W > 0$ with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$), even if the platform is monopolistic. The reason is that the network externality ϕ and the increment in business ability c, as the foundations of social welfare in the digital economy, are activated by the monopoly platform.[32] Another implication is that welfare on each side and *social welfare* only rely on ϕ and c, and the allocation of ϕ between α and β does not matter. A possible interpretation is that each side has a network externality to the other side, and the allocation of ϕ between α and β is perfectly internalized by the monopoly platform.

We summarize the comparisons of surplus across periods on each side and the comparative statics results reflecting the effects of ϕ and c on *social welfare* in the following proposition.

[32]Recall the definition $\phi \equiv \alpha + \beta$ we mentioned above.

Proposition 5. *Suppose that third-degree price discrimination is allowed in period 2. Given the feasible set for parameters in Lemma 2, in equilibrium:*

1. *The total surplus of all sellers affiliated with the platform is equal in the two periods; that is,* $W_1^s = W_2^s$.
2. *The total surplus of all buyers affiliated with the platform is equal in the two periods; that is,* $W_1^b = W_2^b$.
3. *Social welfare is increasing in c and ϕ. Furthermore, all the partial derivatives of social welfare with respect to c and ϕ are increasing in c and ϕ.*

Proof. See Online Appendix O. □

Parts 1 and 2 in Proposition 5 state that from period 1 to period 2, there is no dynamic change in the surplus levels of sellers and buyers. One explanation is that, anticipating the increment in business ability, all the participants take this attribute into account when they make intertemporal choices. Part 3 shows that as the learning-by-doing effect c and/or the sum of network externality ϕ increases, *social welfare* exhibits quadratic growth, and c and ϕ are mutually reinforcing in determining social welfare.

5.2 Prohibiting Third-Degree Price Discrimination

When price discrimination is prohibited, the *sellers' surplus*, *buyers' surplus*, and *social welfare* can be calculated in the same way as in Subsection 5.1. All the notations for critical buyers and sellers (k, r, and z) are the same as above, although they may denote different buyers and sellers.

Sellers' surplus. In equilibrium, the business abilities of critical seller k in the two periods are $B_{k,1}^s = \underline{U}^s + P_1^s - \alpha N_1^b = -\frac{2(c+\alpha+\beta)}{6-(\alpha+\beta)(\alpha+2\beta)}$ and $B_{k,2}^s = B_{k,1}^s + c = c - \frac{2(c+\alpha+\beta)}{6-(\alpha+\beta)(\alpha+2\beta)}$, respectively. Then, we can obtain the sellers' surplus in each period:

$$\widetilde{W}_1^s = \int_{B_{k,1}^s}^{0} U_{i,1}^s \cdot f_1\left(B_{i,1}^s\right) dB_{i,1}^s = \frac{2(c+\alpha+\beta)^2}{(6-(\alpha+\beta)(\alpha+2\beta))^2}, \tag{56}$$

$$\widetilde{W}_2^s = \int_{B_{k,2}^s}^{c} U_{i,2}^s \cdot f_{2,2}\left(B_{i,2}^s\right) dB_{i,2}^s = \frac{2(c+\alpha+\beta)^2}{(6-(\alpha+\beta)(\alpha+2\beta))^2}, \tag{57}$$

where the utility expressions (equations (1) and (2)), the probability density functions ($f_1(B_{i,1}^s) = 1$ and $f_{2,2}(B_{i,2}^s) = 1$), and the expressions of $\{B_{k,1}^s, B_{k,2}^s, \widetilde{N_1^b}, \widetilde{N_2^b}, \widetilde{P_1^s}, \widetilde{P_2^s}\}$ in equilibrium have been taken into account. Hence, the sellers' surplus in the two periods equals

$$\widetilde{W}^s = \widetilde{W}_1^s + \widetilde{W}_2^s = \frac{4(c+\alpha+\beta)^2}{(6-(\alpha+\beta)(\alpha+2\beta))^2}. \tag{58}$$

Buyers' surplus. The buyers' surplus in each period can be expressed as

$$\widetilde{W_1^b} = \int_{B_r^b}^{1} U_{j,1}^b \cdot h\left(B_j^b\right) dB_j^b = \frac{(6+(\alpha+\beta)(2c+\alpha))^2}{8(6-(\alpha+\beta)(\alpha+2\beta))^2}, \tag{59}$$

$$\widetilde{W_2^b} = \int_{B_z^b}^{1} U_{j,2}^b \cdot h\left(B_j^b\right) dB_j^b = \frac{(6+2\beta c-\alpha(\alpha+\beta))^2}{8(6-(\alpha+\beta)(\alpha+2\beta))^2}. \tag{60}$$

The utility expression of buyers in equation (21), probability density function $h(B_j^b) = 1$, and the expressions of $\{B_r^b, B_z^b, \widetilde{N_1^s}, \widetilde{N_2^s}, \widetilde{P_1^b}, \widetilde{P_2^b}\}$ in equilibrium are applied in equations (59) and (60). Then, we obtain the buyers' surplus in the two periods:

$$\widetilde{W^b} = \widetilde{W_1^b} + \widetilde{W_2^b} = \frac{(2\beta c+6-\alpha(\alpha+\beta))^2 + ((\alpha+\beta)(2c+\alpha)+6)^2}{8(6-(\alpha+\beta)(\alpha+2\beta))^2}. \tag{61}$$

Social welfare. The sum of the *sellers' surplus* in equation (58), *buyers' surplus* in equation (61), and the platform's profit in equation (48) is the *social welfare*:

$$\begin{aligned}\widetilde{W} &= \widetilde{W^s} + \widetilde{W^b} + \widetilde{\pi} \\ &= \frac{\begin{pmatrix} -(\alpha+\beta)(\alpha(\alpha(\alpha+\beta)-8)+16\beta) - 2c^2\left(\alpha^2+2\alpha\beta+2\left(\beta^2-8\right)\right) \\ -2c\left(11\left(\alpha^2-4\right)\beta+\alpha\left(3\alpha^2-38\right)+16\alpha\beta^2+8\beta^3\right)+108 \end{pmatrix}}{4(6-(\alpha+\beta)(\alpha+2\beta))^2}.\end{aligned} \tag{62}$$

Proposition 6. *Suppose that third-degree price discrimination is prohibited in period 2. Given the feasible set for parameters in Lemma 4, in equilibrium:*

1. *The total surplus of all the sellers affiliated with the platform is equal in the two periods; that is, $\widetilde{W_1^s} = \widetilde{W_2^s}$.*
2. *The total surplus of all buyers affiliated with the platform in period 1 is greater than in period 2; that is, $\widetilde{W_1^b} > \widetilde{W_2^b}$.*
3. *Social welfare is increasing in c, α, and β. The partial derivatives of social welfare with respect to c, α, and β are increasing in c, α, and β, respectively.*

Proof. See Online Appendix P. □

Proposition 6 demonstrates the dynamic change in surplus on each side and the effects of network externalities and learning-by-doing growth on social welfare when third-degree price discrimination is prohibited. Parts 1 and 2 show that the surplus levels of sellers in the two periods are equal as in Proposition 5, while the surplus of buyers in period 1 is greater than that in period 2. The reason is intuitive: In period 1, due to the learning-by-doing effect among sellers, the platform aims to attract more sellers through price concessions to buyers, but in the second and last period, the platform has no incentive to charge buyers a low fee. As in Proposition 5, part 3 here also indicates the importance of learning-by-doing growth c and network externalities α and β for social welfare.

5.3 Allowing versus Prohibiting Price Discrimination

Based on the results calculated above in the different cases, we obtain the difference in social welfare caused by price regulation. From expressions (55) and (62), we obtain the difference in social welfare, ΔW, as

$$\Delta W = \widetilde{W} - W = \left\{ \begin{array}{l} \dfrac{\left(\begin{array}{l} (\alpha+\beta)(\alpha(8-\alpha(\alpha+\beta)) - 16\beta) + 2c^2\left(-\alpha^2 - 2\alpha\beta + 2\left(8-\beta^2\right)\right) \\ + 2c\left(11\left(4-\alpha^2\right)\beta + \alpha\left(38-3\alpha^2\right) - 16\alpha\beta^2 - 8\beta^3\right) + 108 \end{array} \right)}{4(6-(\alpha+\beta)(\alpha+2\beta))^2} \\ - \dfrac{48 - 4(\alpha+\beta)^2 + c^2\left(12-(\alpha+\beta)^2\right) + 4c(\alpha+\beta)\left(8-(\alpha+\beta)^2\right)}{4\left(4-(\alpha+\beta)^2\right)^2} \end{array} \right\} \tag{63}$$

Consider that it is difficult to obtain the sign of ΔW in theory. Several numerical examples are used to analyze the sign and magnitude of the difference in social welfare caused by price regulation. First, we consider two numerical examples: (1) $\alpha = 0.6$, $\beta = 0.4$ and (2) $\alpha = 0.4$, $\beta = 0.6$. Based on the feasible sets provided by Lemmas 2 and 4, we can easily derive that the feasible set is $c \in (0, 1)$.

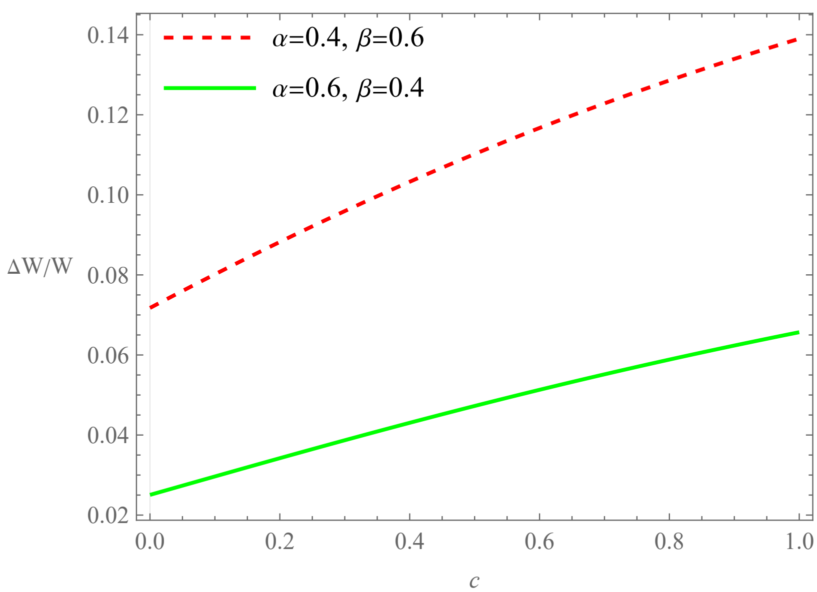


Figure 7: Relationship between Improvement in Social Welfare and the Learning-by-Doing Effect

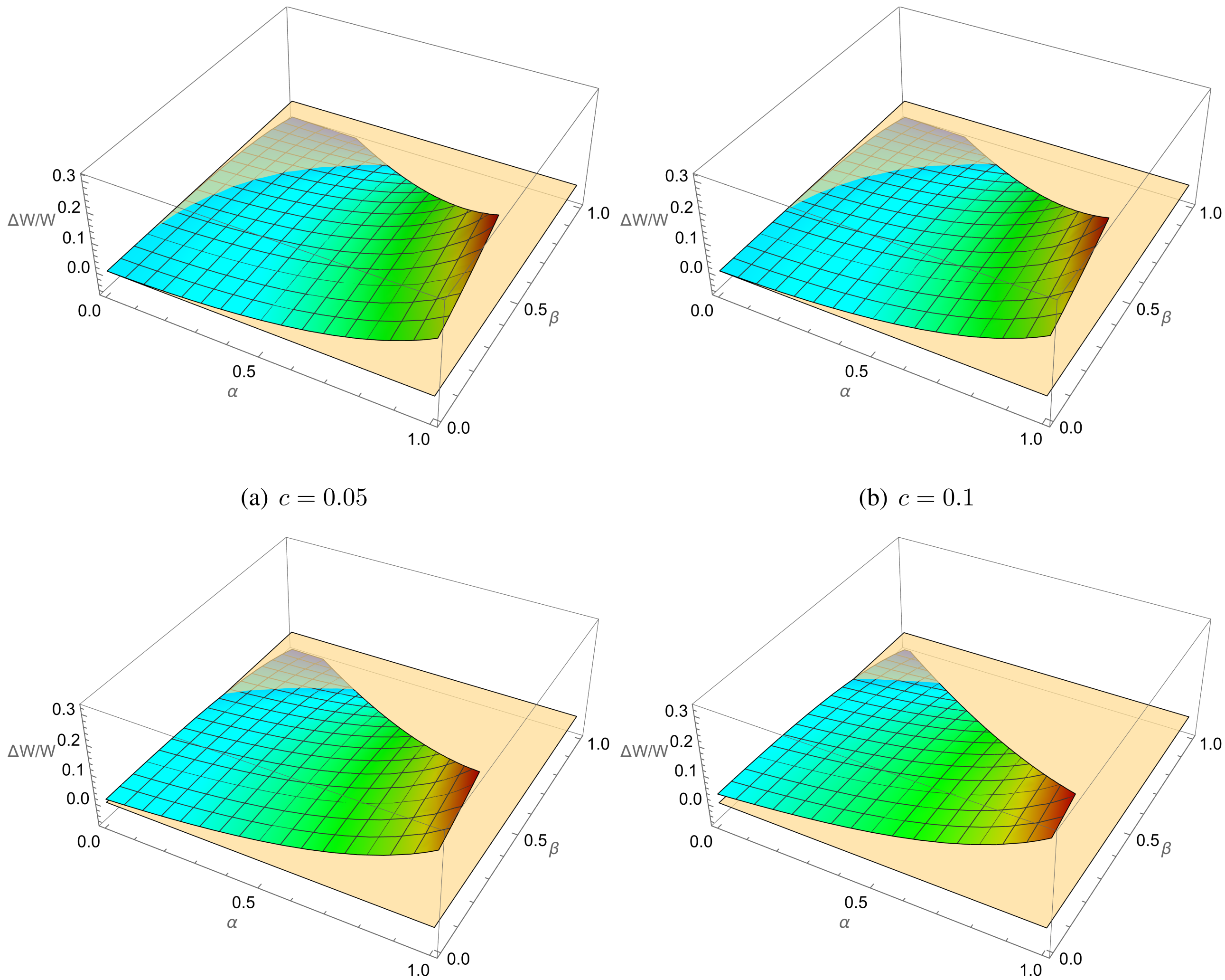
The numerical results for $\Delta W/W$ in these two examples are described in Figure 7. Here, three features are worth noting. First, the sign of ΔW is positive, which is intuitive as in the literature on antitrust issues. Second, the higher the learning-by-doing increment, the greater the percentage improvement in social welfare due to price regulation. Third, by means of price regulation, the degree of improvement in social welfare depends on the allocation of ϕ between α and β. In the figure, compared to the example with $\alpha = 0.6$ and $\beta = 0.4$, the example with $\alpha = 0.4$ and $\beta = 0.6$ always has a higher percentage of improvement in social welfare.

We discuss the effect of network externalities on the improvement in social welfare caused by price regulation. Based on the feasible sets provided by Lemmas 2 and 4, we consider the numerical examples with $c = 0.05, 0.1, 0.5$, and 0.9. If $c = 0.05$, the feasible zones of α and β are subject to (1) $0 < \beta \leq \frac{1}{80}\left(\sqrt{9681} - 71\right)$ and $0 < \alpha < 1$ or (2) $\frac{1}{80}\left(\sqrt{9681} - 71\right) < \beta < 1$ and $0 < \alpha < \frac{1}{60}\sqrt{100\beta^2 + 20\beta + 7201} - \frac{1}{60}(1 + 70\beta)$. If $c = 0.1$, the feasible zones of α and β are subject to (1) $0 < \beta \leq \frac{1}{10}\left(\sqrt{151} - 9\right)$ and $0 < \alpha < 1$ or (2) $\frac{1}{10}\left(\sqrt{151} - 9\right) < \beta < 1$ and $0 < \alpha < \frac{1}{30}\sqrt{25\beta^2 + 10\beta + 1801} - \frac{1}{30}(1 + 35\beta)$. If $c = 0.5$, the feasible zones of α and β are subject to (1) $0 < \beta \leq \frac{1}{2}\left(\sqrt{6} - 2\right)$ and $0 < \alpha < 1$ or (2) $\frac{1}{2}\left(\sqrt{6} - 2\right) < \beta < 1$ and $0 < \alpha < \frac{1}{6}\sqrt{\beta^2 + 2\beta + 73} - \frac{1}{6}(1 + 7\beta)$. If $c = 0.9$, the feasible zones of α and β are subject to (1) $0 < \beta \leq \frac{1}{10}\left(\sqrt{151} - 11\right)$ and $0 < \alpha < 1$ or (2) $\frac{1}{10}\left(\sqrt{151} - 11\right) < \beta < 1$ and $0 < \alpha < \frac{1}{30}\sqrt{25\beta^2 + 90\beta + 1881} - \frac{1}{30}(9 + 35\beta)$. The numerical results for $\Delta W/W$ in these examples are shown in Figure 8. There are three main features. First, the percentage of social welfare improvement caused by price regulation increases with increasing α or β. Second, only when α is low enough and β is high enough will price regulation potentially cause a decline in social welfare. This feature implies that price regulation can improve social welfare in most of the feasible zone. Third, an increase in c reduces the feasible zone in which the price regulation has a negative welfare effect (see the dynamic change in the feasible zone from Figure 8(a) to Figure 8(b), to Figure 8(c), and then to Figure 8(d)).

Next, we analyze the decomposition of the social welfare change caused by price regulation. Consider $W_1^s = W_2^s$ and $\widetilde{W_1^s} = \widetilde{W_2^s}$. We only need to compare the total surplus of online sellers in the case of price regulation, $\widetilde{W_s}$, to that of price discrimination, W_s.

Remark 2. Given the feasible sets for the parameters in Lemmas 2 and 4, in equilibrium, the total surplus of the sellers on the platform will increase if third-degree price discrimination is prohibited; that is, $\widetilde{W^s} > W^s$ (obviously, there are $\widetilde{W_1^s} > W_1^s$ and $\widetilde{W_2^s} > W_2^s$).

We can easily obtain this proposition taking into account $\alpha \in (0, 1)$, $c \in (0, 1)$, and $\beta \in (0, 1)$. The proposition indicates that prohibiting third-degree price discrimination can improve sellers' surplus in the two periods. The reason is that price regulation enhances the bargaining power of sellers. However, the theoretical analysis of the buyers' surplus change is quite complicated. We consider a numerical example with $c = 0.1$, and Figure 9 shows all numerical results. Figure 9(a) demonstrates the theoretical result $\frac{\widetilde{W^s} - W^s}{W^s} > 0$ Proposition 2 implies. From figures 9(b), 9(c),



(a) $c = 0.05$

(b) $c = 0.1$

(c) $c = 0.5$

(d) $c = 0.9$

Figure 8: Relationship between Social Welfare Improvement and Network Externalities Given Different Values of c

Notes: The light yellow region denotes the zero plan; that is $\Delta W/W = 0$ in this plan.

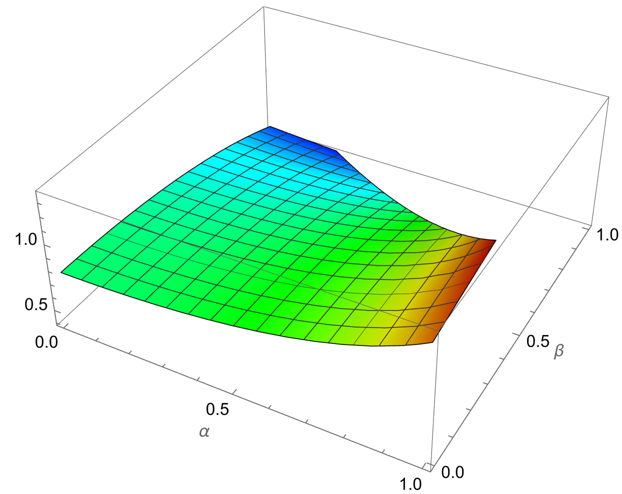
and 9(d), we can conclude that although in period 1, buyers benefit from price regulation, if the externality generated by each buyer is too weak compared to the externality generated by each seller, buyers are likely to suffer a great loss in period 2, resulting in the loss of the total surplus of buyers. Therefore, price regulation may lead to a decline in social welfare if the surplus of buyers in period 2 decreases too much due to this regulation.

To sum up, the numerical analysis shows that only when all of the cross-side network externalities produced by each type of agent and the increment in online business ability each old seller obtains are small enough will the price regulation (prohibiting price discrimination) cause a decline in social welfare. In most of the feasible zones of the parameters, the price regulation significantly increases social welfare. Hence, the impact of price discrimination between different types of agents on one side is different from that between different groups of agents on two sides, as discussed in Weyl (2010). Due to the dynamic framework with a monopoly platform and an outside option, the impact of price discrimination in our paper is also different from that under platform competition, as shown in Liu and Serfes (2013). In an economy without externalities, an increase in output is a necessary condition for third-degree price discrimination to improve social welfare (Varian, 1985), while Yoshida (2000) documents that an increase in output is a sufficient condition for third-degree price discrimination to cause welfare deterioration in an input market. In contrast, in two-sided markets, the output is not explicitly defined, making the welfare analysis of third-degree price discrimination quite different from traditional markets.

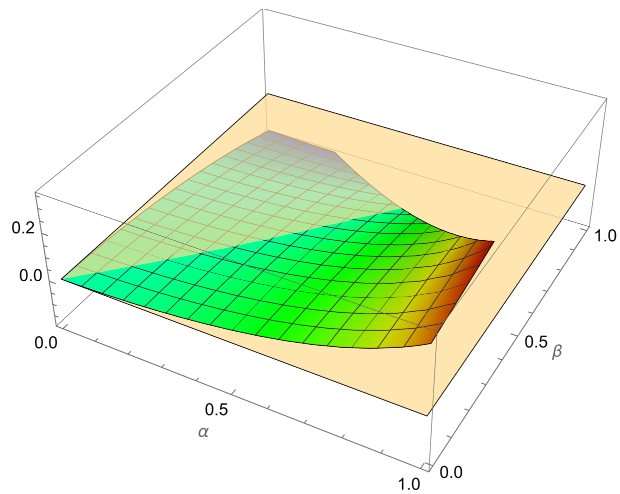
6 Further Discussion

As mentioned in Section 2, our model focuses on poor sellers in rural areas to analyze the economic phenomenon of emerging "digital villages." Compared to the literature on the learning-by-doing hypothesis that emphasizes productivity improvement or cost reduction due to increased production volume within a firm, our model focuses on the improvement of business ability of more microscopic individuals (sellers) on the digital platform. In this sense, these individuals have the characteristics of entrepreneurs. Thus, the learning-by-doing benefit of old sellers over time is very similar to human capital accumulation, permanently improving business ability. Although there are only two periods in our model, we can expect continued benefits for sellers from the learning-by-doing effect in the future. Including high-skill sellers from metropolitan areas in the model will complicate the analysis, while the core mechanics have not been changed (and thus the main results of this paper may still hold). Indeed, the logic in our model can be widely applied to any dynamic case with cross-side network externalities in imperfectly competitive markets.

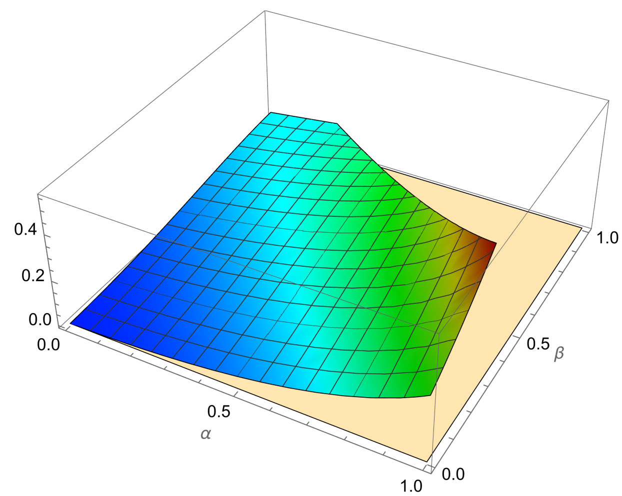
Educational investment is widely recognized as a key determinant of economic development. Acquiring knowledge and skills through various channels such as preschool programs, schools, and



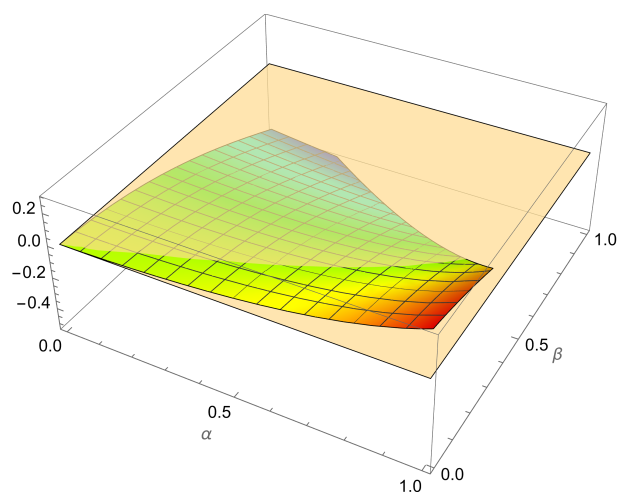
(a) $\frac{\widetilde{W}^s - W^s}{W^s}$



(b) $\frac{\widetilde{W}^b - W^b}{W^b}$



(c) $\frac{\widetilde{W}_1^b - W_1^b}{W_1^b}$



(d) $\frac{\widetilde{W}_2^b - W_2^b}{W_2^b}$

Figure 9: Decomposition of Social Welfare Change with Respect to Network Externalities (α, β)
Notes: Here, we consider a numerical example with $c = 0.1$. The light yellow region denotes the zero plan.

formal training programs has been shown to improve productivity (Behrman, 2010). In rural areas, education investments are predominantly funded by the government, while enterprise-based skills training rarely reaches impoverished populations. In industries that lack network effects, providing skills training to the poor becomes profitable only when the learning-by-doing effect is sufficiently strong. Hypothetically, if the financial market functioned perfectly, the poor would have incentives to invest in their own education and skills. However, in industries characterized by one-sided network effects, such as the telephone and electronic payment industries, the exchange of physical goods is minimal, limiting the positive network externalities that the rural poor can generate. Digital technology, which facilitates the establishment of two-sided markets, has disrupted this landscape. When cross-side network effects become sufficiently strong, digital platform enterprises are incentivized to provide skills training to the rural poor, particularly during the early stages of economic development. As a result, high-quality agricultural products from rural areas gain access to online markets, significantly broadening consumer choices and enhancing social welfare.

7 Conclusion

By incorporating the learning-by-doing of sellers into the theory of two-sided markets with heterogeneous sellers and buyers and a monopoly platform, we provide a novel dynamic model to analyze a new platform-based phenomenon of economic development—emerging "digital villages" in China. The most important contribution of our paper is that we fill the void in understanding the interaction between two fields, industrial organization and economic development. Our paper also provides a tractable framework to explore the dynamic pricing strategy of a monopoly platform in a situation where price discrimination occurs within one group (on the sellers' side), along with the enhancement of the business ability of online sellers.

The key result is that due to the anticipated growth of the business ability of online sellers and cross-side network externalities, the rational and profit-maximizing platform tends to charge sellers a lower fee in period 1 by manipulating cross-side and intertemporal price structure. In particular, if (i) the network externality that each online seller produces is higher than that which each online buyer produces or (ii) the growth in the business ability of online sellers is high enough, the platform will subsidize sellers in period 1. When the externality of each seller is high enough and the externality of each buyer is not too high, the cross-side externality and learning-by-doing effect of each online seller can mutually reinforce each other in reducing the platform's charges (or increasing its subsidies) to sellers in period 1. With the regulation on price discrimination between new and old sellers in period 2, (1) the prices for sellers in the two periods will decrease, and (2) the numbers of online sellers in the two periods and the number of online buyers in period 1 will increase; however, (3) the changes in both the price for buyers in period 1 and the number of online buyers in period 2 are ambiguous, which is very different from the literature. Moreover, we analyze the impact of prohibiting price discrimination on social welfare and find that in most of the feasible zones of the parameters, this regulation significantly increases social welfare.

Our dynamic model of two-sided markets can be used to analyze other aspects of economic development. For example, two-sided platforms tend to invest substantial funds or resources during the early stages of development to cultivate users' stickiness, habits, and cross-side networks. Moreover, our theory can be extended to address at least the following two fundamental theoretical problems: dynamic development on the buyers' side and on both sides of the market. Other important issues, such as endogenous investment decisions (made by platforms, buyers, or sellers) and income (or wealth) inequality caused by heterogeneous abilities in the digital economy, are left for future research.

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Online Appendix

A Homing Choice Equilibria

When all prices are given by the digital platform, homing choice equilibria are formed after all buyers' and sellers' best responses.

A.1 Allowing Third-Degree Price Discrimination

Scenario A: By combining buyer choices (24) and (25) with seller choices (7) and (8), we derive the homing choice equilibria:

$$N_1^b = \frac{P_1^b + \beta(P_1^s + \underline{U}^s) + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{64}$$

$$N_1^s = \frac{\alpha(P_1^b + \underline{U}^b - 1) + P_1^s + \underline{U}^s}{\alpha\beta - 1}, \tag{65}$$

$$N_2^b = \frac{\beta(\alpha(P_1^b - P_2^b) + P_1^s + \underline{U}^s) + P_2^b + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{66}$$

$$N_2^s = \frac{\alpha(P_1^b + \underline{U}^b - 1) + P_1^s + \underline{U}^s}{\alpha\beta - 1}. \tag{67}$$

In addition, we need to maintain the equalities (9) and (10) in equilibrium.

Scenario B: From equalities (24), (25), (11), and (12), we have the buyers' and sellers' homing choice equilibria:

$$N_1^b = \frac{\beta(\alpha\beta(c + P_{21}^s - P_{22}^s) - c + P_{22}^s + \alpha P_2^b + \underline{U}^s) - \alpha\beta P_1^b + P_1^b + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{68}$$

$$N_1^s = c + \frac{P_{21}^s + \alpha(P_2^b + \underline{U}^b - 1) + \underline{U}^s}{\alpha\beta - 1} + P_{21}^s - P_{22}^s, \tag{69}$$

$$N_2^b = \frac{\beta(P_{21}^s + \underline{U}^s) + P_2^b + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{70}$$

$$N_2^s = \frac{P_{21}^s + \alpha(P_2^b + \underline{U}^b - 1) + \underline{U}^s}{\alpha\beta - 1}. \tag{71}$$

Inequality $\widehat{U}_{h,2}^s < \widehat{U}_{k,2}^s$ and equality (13) should also be satisfied in equilibrium.

Scenario C: Putting equations (24) and (25) together with equations (7) and (14), we have the homing choice equilibria for buyers and sellers:

$$N_1^b = \frac{P_1^b + \beta(P_1^s + \underline{U}^s) + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{72}$$

$$N_1^s = \frac{\alpha(P_1^b + \underline{U}^b - 1) + P_1^s + \underline{U}^s}{\alpha\beta - 1}, \tag{73}$$

$$N_2^b = \frac{\beta(-c + P_{22}^s + \underline{U}^s) + P_2^b + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{74}$$

$$N_2^s = \frac{-c + P_{22}^s + \alpha(P_2^b + \underline{U}^b - 1) + \underline{U}^s}{\alpha\beta - 1}. \tag{75}$$

In equilibrium, inequality $\widetilde{U}_{k,2}^s < \widetilde{U}_{\tilde{h},2}^s$ and equality (15) should be satisfied.

Scenario D: By combining equations (24) and (25) with equations (19) and (20), the homing choice equilibria of buyers and sellers are

$$N_1^b = \frac{\alpha(-\beta c + P_1^b + P_2^b + 2\underline{U}^b - 2) + \alpha\beta(2P_1^s - P_{21}^s + P_{22}^s + 2\underline{U}^s) - P_1^s + P_{21}^s}{2\alpha(\alpha\beta - 1)}, \tag{76}$$

$$N_1^s = \frac{\alpha\beta(-c + 2\alpha(P_1^b + \underline{U}^b - 1) + 2P_1^s - P_{21}^s + P_{22}^s + 2\underline{U}^s) - \alpha P_1^b - P_1^s + P_{21}^s + \alpha P_2^b}{2\alpha\beta(\alpha\beta - 1)}, \tag{77}$$

$$N_2^b = \frac{\alpha(-\beta c + P_1^b + P_2^b + 2\underline{U}^b - 2) + P_1^s + \alpha\beta(P_{21}^s + P_{22}^s + 2\underline{U}^s) - P_{21}^s}{2\alpha(\alpha\beta - 1)}, \tag{78}$$

$$N_2^s = \frac{\alpha\beta(-c + P_{21}^s + P_{22}^s + 2\alpha(P_2^b + \underline{U}^b - 1) + 2\underline{U}^s) + \alpha P_1^b + P_1^s - P_{21}^s - \alpha P_2^b}{2\alpha\beta(\alpha\beta - 1)}. \tag{79}$$

Meanwhile, conditions $\widehat{U}_{h,2}^s < \widehat{U}_{k,2}^s$ and $\widetilde{U}_{k,2}^s < \widetilde{U}_{\tilde{h},2}^s$ must be guaranteed in equilibrium.

A.2 Prohibiting Third-Degree Price Discrimination

Scenario A: From equations (24), (25), (33), and (34), we can obtain the homing choice equilibria for buyers and sellers as

$$N_1^b = \frac{P_1^b + \beta(P_1^s + \underline{U}^s) + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{80}$$

$$N_1^s = \frac{\alpha(P_1^b + \underline{U}^b - 1) + P_1^s + \underline{U}^s}{\alpha\beta - 1}, \tag{81}$$

$$N_2^b = \frac{\beta(\alpha(P_1^b - P_2^b) + P_1^s + \underline{U}^s) + P_2^b + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{82}$$

$$N_2^s = \frac{\alpha(P_1^b + \underline{U}^b - 1) + P_1^s + \underline{U}^s}{\alpha\beta - 1}, \tag{83}$$

Meanwhile, the relationship between P_1^s and P_2^s, as shown in equation (35), holds in equilibrium.

Scenario B: From equations (24), (25), (36), and (37), the homing choice equilibria can be obtained.

$$N_1^b = \frac{\beta(-c + P_1^s + \underline{U}^s) + P_1^b + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{84}$$

$$N_1^s = \frac{-c + \alpha(P_1^b + \underline{U}^b - 1) + P_1^s + \underline{U}^s}{\alpha\beta - 1}, \tag{85}$$

$$N_2^b = \frac{P_2^b + \beta(P_2^s + \underline{U}^s) + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{86}$$

$$N_2^s = \frac{\alpha(P_2^b + \underline{U}^b - 1) + P_2^s + \underline{U}^s}{\alpha\beta - 1}. \tag{87}$$

In addition, the third condition $\widehat{U}_{h,2}^s < \widehat{U}_{k,2}^s$ holds in equilibrium.

Scenario C: By combining equations (24) and (25) with equations (38) and (39), we can solve the homing choice problem of all buyers and sellers as

$$N_1^b = \frac{P_1^b + \beta(P_1^s + \underline{U}^s) + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{88}$$

$$N_1^s = \frac{\alpha(P_1^b + \underline{U}^b - 1) + P_1^s + \underline{U}^s}{\alpha\beta - 1}, \tag{89}$$

$$N_2^b = \frac{\beta(-c + P_2^s + \underline{U}^s) + P_2^b + \underline{U}^b - 1}{\alpha\beta - 1}, \tag{90}$$

$$N_2^s = \frac{-c + \alpha(P_2^b + \underline{U}^b - 1) + P_2^s + \underline{U}^s}{\alpha\beta - 1}. \tag{91}$$

Meanwhile, condition $\widetilde{U}_{k,2}^s < \widetilde{U}_{\tilde{h},2}^s$ should be satisfied in equilibrium.

B Proof of Lemma 1

Proof. All possible scenarios are described above (i.e., *Scenarios A, B, C*, and *D* in subsection 2.1). First, we check whether *Scenario B* is the optimal choice of both buyers and sellers after the monopoly platform sets the optimal prices. It is noteworthy that $N_{22}^s = N_1^s$ and $N_{21}^s = N_2^s - N_{22}^s$ in this scenario. By making decisions on P_1^b, P_2^b, P_{21}^s, and P_{22}^s, the platform's problem is to maximize profit (26) subject to equalities (13), (68), (69), (70), and (71). The final equilibrium prices are

$$\begin{aligned}
P_1^b &= \frac{2-\alpha(\alpha+\beta)-c(\alpha-\beta)}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
P_2^b &= \frac{2-\alpha(\alpha+\beta)}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
P_{21}^s &= \frac{\alpha-\beta}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
P_{22}^s &= \frac{\alpha-\beta+c\,(2-(\alpha+\beta)^2)}{(2-\alpha-\beta)(2+\alpha+\beta)}.
\end{aligned}$$

Putting these equilibrium prices back into equations (68), (69), (70), and (71), we obtain that

$$\begin{aligned}
N_1^b &= \frac{c(\alpha+\beta)+2}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
N_1^s &= \frac{\alpha+\beta+2c}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
N_2^b &= \frac{2}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
N_2^s &= \frac{\alpha+\beta}{(2-\alpha-\beta)(2+\alpha+\beta)}.
\end{aligned}$$

Meanwhile, according to (13), we have

$$P_1^s = \frac{\alpha(c(\alpha+\beta)+1)-\beta}{(2-\alpha-\beta)(2+\alpha+\beta)}.$$

Consider that the condition $\widehat{U}_{h,2}^s < \widehat{U}_{k,2}^s$ requires that $N_2^s > N_1^s$. However, recalling $0<\alpha+\beta<2$, the gap between N_2^s and N_1^s in equilibrium is

$$N_2^s - N_1^s = -\frac{2c}{(2-\alpha-\beta)(2+\alpha+\beta)} < 0,$$

which contradicts the necessary condition $\widehat{U}_{h,2}^s < \widehat{U}_{k,2}^s$. Then, *Scenario B* does not exist.

Second, we consider *Scenario C*. Under this scenario, $N_{21}^s = 0$ and $N_{22}^s = N_2^s$. By choosing P_1^b, P_2^b, P_1^s and P_{22}^s, the monopoly platform maximizes profit (26) subject to equalities (15), (72), (73), (74), and (75). Then, we have final equilibrium prices:

$$P_1^b = \frac{2 - \alpha(\alpha + \beta)}{(2 - \alpha - \beta)(2 + \alpha + \beta)},$$
$$P_1^s = \frac{\alpha - \beta}{(2 - \alpha - \beta)(2 + \alpha + \beta)},$$
$$P_2^b = \frac{2 - \alpha(\alpha + \beta) - c(\alpha - \beta)}{(2 - \alpha - \beta)(2 + \alpha + \beta)},$$
$$P_{22}^s = \frac{\alpha - \beta + c(2 - \beta(\alpha + \beta))}{(2 - \alpha - \beta)(2 + \alpha + \beta)}.$$

Reconsidering (72), (73), (74), and (75) under equilibrium prices, we obtain

$$N_1^b = \frac{2}{(2 - \alpha - \beta)(2 + \alpha + \beta)},$$
$$N_1^s = \frac{\alpha + \beta}{(2 - \alpha - \beta)(2 + \alpha + \beta)},$$
$$N_2^b = \frac{2 + c(\alpha + \beta)}{(2 - \alpha - \beta)(2 + \alpha + \beta)},$$
$$N_2^s = \frac{\alpha + \beta + 2c}{(2 - \alpha - \beta)(2 + \alpha + \beta)}.$$

Putting the final equilibrium results back into equation (15), we have

$$P_{21}^s = \frac{\alpha(c(\alpha + \beta) + 1) - \beta}{(2 - \alpha - \beta)(2 + \alpha + \beta)}.$$

The necessary condition $\widetilde{U}_{k,2}^s < \widetilde{U}_{\tilde{h},2}^s$ implies that $N_2^s < N_1^s$ in this scenario. However, in equilibrium, we have

$$N_2^s - N_1^s = \frac{2c}{(2 - \alpha - \beta)(2 + \alpha + \beta)} > 0,$$

which does not satisfy the above necessary condition $\widetilde{U}_{k,2}^s < \widetilde{U}_{\tilde{h},2}^s$. Hence, *Scenario C* is not an equilibrium result.

Third, we need to verify whether *Scenario D* is an equilibrium result. We can obtain N_{21}^s and N_{22}^s from equations (17) and (18), respectively. The platform's problem is to maximize (26) subject to equalities (76), (77), (78), and (79), according to the platform's decisions on P_1^b, P_2^b, P_1^s, P_{21}^s, and P_{22}^s. The optimal prices are

$$\begin{aligned}
P_1^b &= \frac{4-2\alpha(\alpha+\beta)-c(\alpha-\beta)}{2(2-\alpha-\beta)(2+\alpha+\beta)},\\
P_1^s &= \frac{(\alpha-\beta)(c(\alpha+\beta)+4)}{4(2-\alpha-\beta)(2+\alpha+\beta)},\\
P_2^b &= \frac{4-2\alpha(\alpha+\beta)-c(\alpha-\beta)}{2(2-\alpha-\beta)(2+\alpha+\beta)},\\
P_{21}^s &= \frac{(\alpha-\beta)(c(\alpha+\beta)+4)}{4(2-\alpha-\beta)(2+\alpha+\beta)},\\
P_{22}^s &= \frac{4\alpha-4\beta+c(8-(\alpha+\beta)(\alpha+3\beta))}{4(2-\alpha-\beta)(2+\alpha+\beta)}.
\end{aligned}$$

Reconsidering the homing choice equilibria, we have

$$\begin{aligned}
N_1^b &= \frac{c(\alpha+\beta)+4}{2(2-\alpha-\beta)(2+\alpha+\beta)},\\
N_1^s &= \frac{\alpha+\beta+c}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
N_2^b &= \frac{c(\alpha+\beta)+4}{2(2-\alpha-\beta)(2+\alpha+\beta)},\\
N_2^s &= \frac{\alpha+\beta+c}{(2-\alpha-\beta)(2+\alpha+\beta)}.
\end{aligned}$$

Putting these equilibrium results back into expression (17), we have

$$N_{21}^s = -\frac{c}{4} < 0,$$

which violates the necessary condition $\widehat{U}_{h,2}^s < \widehat{U}_{k,2}^s$ because this condition implies $N_{21}^s > 0$. Thus, this scenario is not an equilibrium result.

In summary, *Scenarios B, C*, and *D* cannot form a final equilibrium. Thus, we conclude that only *Scenario A* is a final equilibrium result. □

C Proof of Lemma 2

Proof. Combining the final equilibria (30), (31), and (32) with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$, we have that $0 < N_1^s, N_1^b, N_2^s, N_2^b < 1$ and $\pi > 0$ are established if (i) $0 < \alpha, \beta < 1$, $0 < \alpha+\beta \le \frac{1}{4}\left(\sqrt{33}-1\right)$, and $0 < c < 1$, or (ii) $0 < \alpha, \beta < 1$, $\frac{1}{4}\left(\sqrt{33}-1\right) < \alpha+\beta < \sqrt{2}$, and $0 < c < \frac{4-2(\alpha+\beta)^2}{\alpha+\beta}$. These conditions can be expressed as $0 < \alpha, \beta < 1$, $0 < \alpha+\beta < \sqrt{2}$, and

$0 < c < \min\left\{1, \frac{4-2(\alpha+\beta)^2}{\alpha+\beta}\right\}$. $\square$

D Proof of Theorem 1

Proof. Recall $\phi = \alpha + \beta$. P_1^s in equality (28) can be rearranged as

$$P_1^s = \frac{2(\alpha-\beta) - c(2-\alpha(\alpha+\beta))}{2(4-(\alpha+\beta)^2)}.$$

Obviously, the denominator $2(4-(\alpha+\beta)^2)$ and the term $2-\alpha(\alpha+\beta)$ in the numerator are positive because $\alpha \in (0,1)$ and $\beta \in (0,1)$. Recalling $c \in (0,1)$, when $\alpha < \beta$, we have $P_1^s < 0$. When $\alpha > \beta$, there are two additional scenarios. One is that if $c > \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)}$, then $P_1^s < 0$. The other is that if $c < \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)}$, then there is $P_1^s > 0$.

Consider $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. The derivatives of P_1^s with respect to c and β satisfy

$$\frac{\partial P_1^s}{\partial c} = \frac{\alpha(\alpha+\beta)-2}{2\left(4-(\alpha+\beta)^2\right)} < 0,$$

$$\frac{\partial P_1^s}{\partial \beta} = \frac{1}{4}\left(-\frac{(1-\alpha)(c+2)}{(2-\alpha-\beta)^2} - \frac{(\alpha+1)(2-c)}{(2+\alpha+\beta)^2}\right) < 0.$$

Meanwhile, we have the cross derivative $\frac{\partial^2 P_1^s}{\partial c \partial \beta}$ as

$$\frac{\partial^2 P_1^s}{\partial c \partial \beta} = \frac{\alpha\beta^2 - (4-2\alpha^2)\beta + \alpha^3}{2(2-\alpha-\beta)^2(2+\alpha+\beta)^2}.$$

Consider that $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. We have $\frac{\partial^2 P_1^s}{\partial c \partial \beta} > 0$ if (i) $\alpha < \frac{2\sqrt{2}}{3}$ and $\beta < \frac{2-\alpha^2-2\sqrt{1-\alpha^2}}{\alpha}$, or (ii) $\alpha > \frac{2\sqrt{2}}{3}$. Under these assumptions, we also have $\frac{\partial^2 P_1^s}{\partial c \partial \beta} < 0$ if $\alpha < \frac{2\sqrt{2}}{3}$ and $\beta > \frac{2-\alpha^2-2\sqrt{1-\alpha^2}}{\alpha}$. $\square$

E Proof of Theorem 2

Proof. From equalities (27) and (28), we have

$$P_1^b - P_1^s = \frac{(2+c)(1-\alpha)}{2(2-\alpha-\beta)}.$$

Obviously, with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$, we have $P_1^b > P_1^s$.

Meanwhile, under these three assumptions, by taking derivatives of $P_1^b - P_1^s$ with respect to c, β, and α, we have the following results:

$$\frac{\partial(P_1^b - P_1^s)}{\partial c} = \frac{1-\alpha}{2(2-(\alpha+\beta))} > 0,$$
$$\frac{\partial(P_1^b - P_1^s)}{\partial \beta} = \frac{(1-\alpha)(c+2)}{2(2-(\alpha+\beta))^2} > 0,$$
$$\frac{\partial(P_1^b - P_1^s)}{\partial \alpha} = -\frac{(1-\beta)(c+2)}{2(2-(\alpha+\beta))^2} < 0.$$

From inequalities (27) and (29), $P_2^b - P_{22}^s$ can be expressed as

$$P_2^b - P_{22}^s = \frac{2(1-\alpha) - c(3-2\beta-\alpha)}{4-2(\alpha+\beta)}.$$

All of $2(1-\alpha)$, $(3-2\beta-\alpha)$ and $4-2(\alpha+\beta)$ are positive because $\alpha \in (0,1)$ and $\beta \in (0,1)$. Thus, considering $c \in (0,1)$, we find that, if $\beta > \frac{1+\alpha}{2}$, there is $P_2^b - P_{22}^s > 0$ for all $0 < c < 1$; if $\beta < \frac{1+\alpha}{2}$, $P_2^b - P_{22}^s > 0$ is satisfied only when $c < \frac{2(1-\alpha)}{3-2\beta-\alpha}$. However, if $\beta < \frac{1+\alpha}{2}$ and $c > \frac{2(1-\alpha)}{3-2\beta-\alpha}$, we have $P_2^b - P_{22}^s < 0$.

By taking derivatives of $P_2^b - P_{22}^s$ with respect to c, α, and β, we have the following results:

$$\frac{\partial(P_2^b - P_{22}^s)}{\partial c} = \frac{\alpha+2\beta-3}{2(2-(\alpha+\beta))} < 0,$$
$$\frac{\partial(P_2^b - P_{22}^s)}{\partial \alpha} = -\frac{(1-\beta)(c+2)}{2(2-(\alpha+\beta))^2} < 0,$$
$$\frac{\partial(P_2^b - P_{22}^s)}{\partial \beta} = \frac{(1-\alpha)(c+2)}{2(2-(\alpha+\beta))^2} > 0,$$

because $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. □

F Proof of Theorem 3

Proof. We obtain $P_1^b = P_2^b$ and $P_1^s = P_{21}^s$ directly from the equalities (27) and (28). Meanwhile, putting (28) together with (29), we have $P_{22}^s = P_1^s + c$. □

G Proof of Proposition 1

Proof. From equalities (27), (28), and (29), we have the following relation:

$$\frac{P_2^b - P_{22}^s}{P_1^b - P_1^s} = \frac{2(1-\alpha) - c(3-\alpha-2\beta)}{(1-\alpha)(c+2)} < 1,$$

with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. Recalling that $P_1^b > P_1^s$, we then obtain $P_2^b - P_{22}^s < P_1^b - P_1^s$. Recalling that $P_1^b = P_1^s$, $P_1^s = P_{21}^s$, and $P_{22}^s = P_1^s + c$, we can easily obtain $(P_2^b - P_{22}^s) - (P_1^b - P_1^s) = -c$.

□

H Proof of Lemma 3

Proof. All possible scenarios are discussed in Subsection 4.1 (*Scenarios A, B*, and *C*). First, we check whether *Scenario B* is the optimal choice after the monopoly platform sets all prices on buyers and sellers in the two periods. By choosing four prices $\{\widetilde{P_1^b}, \widetilde{P_1^s}, \widetilde{P_2^b}, \widetilde{P_2^s}\}$, the platform's problem is to maximize profit (40) subject to equalities (84), (85), (86), and (87). Then we obtain the optimal prices

$$\begin{aligned}
\widetilde{P_1^b} &= \frac{-\alpha(\alpha+\beta) - c(\alpha-\beta) + 2}{(-\alpha-\beta+2)(\alpha+\beta+2)},\\
\widetilde{P_1^s} &= \frac{\alpha-\beta+c(2-\beta(\alpha+\beta))}{(-\alpha-\beta+2)(\alpha+\beta+2)},\\
\widetilde{P_2^b} &= \frac{2-\alpha(\alpha+\beta)}{(-\alpha-\beta+2)(\alpha+\beta+2)},\\
\widetilde{P_2^s} &= \frac{\alpha-\beta}{(-\alpha-\beta+2)(\alpha+\beta+2)}.
\end{aligned}$$

By utilizing these equilibrium prices, the homing choice equilibria become the following:

$$\begin{aligned}
\widetilde{N_1^b} &= \frac{2+c(\alpha+\beta)}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
\widetilde{N_1^s} &= \frac{\alpha+\beta+2c}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
\widetilde{N_2^b} &= \frac{2}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
\widetilde{N_2^s} &= \frac{\alpha+\beta}{(2-\alpha-\beta)(2+\alpha+\beta)}.
\end{aligned}$$

An additional necessary condition is $\widehat{U}^s_{h,2} < \widehat{U}^s_{k,2}$, which implies that $\widetilde{N^s_2} > \widetilde{N^s_1}$. However, in equilibrium, the difference in the number of sellers in the two periods is

$$\widetilde{N^s_2} - \widetilde{N^s_1} = -\frac{2c}{(2-\alpha-\beta)(2+\alpha+\beta)} < 0,$$

which means the condition $\widehat{U}^s_{h,2} < \widehat{U}^s_{k,2}$ cannot be satisfied. Therefore, *Scenario B* is not a final equilibrium.

Next, we explore *Scenario C*. By taking into account four prices $\{\widetilde{P^b_1}, \widetilde{P^s_1}, \widetilde{P^b_2}, \widetilde{P^s_2}\}$, the platform maximizes profit (40) subject to equalities (88), (89), (90), and (91). The optimal prices the platform charges are

$$\begin{aligned}
\widetilde{P^b_1} &= \frac{2-\alpha(\alpha+\beta)}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
\widetilde{P^s_1} &= \frac{\alpha-\beta}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
\widetilde{P^b_2} &= \frac{2-\alpha(\alpha+\beta)-c(\alpha-\beta)}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
\widetilde{P^s_2} &= \frac{\alpha-\beta+c(2-\beta(\alpha+\beta))}{(2-\alpha-\beta)(2+\alpha+\beta)}.
\end{aligned}$$

On the basis of these prices, we obtain the homing choice results in final equilibrium:

$$\begin{aligned}
\widetilde{N^b_1} &= \frac{2}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
\widetilde{N^s_1} &= \frac{\alpha+\beta}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
\widetilde{N^b_2} &= \frac{2+c(\alpha+\beta)}{(2-\alpha-\beta)(2+\alpha+\beta)},\\
\widetilde{N^s_2} &= \frac{\alpha+\beta+2c}{(2-\alpha-\beta)(2+\alpha+\beta)}.
\end{aligned}$$

In equilibrium, the condition $\widetilde{U}^s_{k,2} < \widetilde{U}^s_{\tilde{h},2}$ must be satisfied. This condition indicates that $\widetilde{N^s_2} < \widetilde{N^s_1}$. However, in equilibrium, the gap between $\widetilde{N^s_2}$ and $\widetilde{N^s_1}$ is

$$\widetilde{N^s_2} - \widetilde{N^s_1} = \frac{2c}{(2-\alpha-\beta)(2+\alpha+\beta)} > 0,$$

which means the necessary condition $\widetilde{U}^s_{k,2} < \widetilde{U}^s_{\tilde{h},2}$ cannot be satisfied. Thus, *Scenario C* is not a final equilibrium result.

In summary, neither *Scenario B* nor *Scenario C* exists. Hence, we conclude that only *Scenario*

A forms a final equilibrium. $\square$

I Proof of Theorem 4

Proof. $\widetilde{P_1^s}$ in equality (42) can be rearranged as

$$\begin{aligned}\widetilde{P_1^s} &= \frac{\alpha(\alpha(\alpha+\beta)+2)-4\beta+2c(\alpha(\alpha+\beta)-2)}{2(6-(\alpha+\beta)(\alpha+2\beta))} \\ &= \frac{((\alpha^2+2)\alpha-(4-\alpha^2)\beta)+2c(\alpha(\alpha+\beta)-2)}{2(6-(\alpha+\beta)(\alpha+2\beta))}.\end{aligned}$$

Obviously, the denominator $2(6-(\alpha+\beta)(\alpha+2\beta))$ is positive, and the term $2c(\alpha(\alpha+\beta)-2)$ in the numerator is negative because $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. If $\beta > \frac{(\alpha^2+2)\alpha}{4-\alpha^2} = \alpha - \frac{2\alpha(1-\alpha^2)}{4-\alpha^2}$, the first term $(\alpha^2+2)\alpha-(4-\alpha^2)\beta$ in the numerator is also negative, and then we have $P_1^s < 0$. If $\beta < \alpha - \frac{2\alpha(1-\alpha^2)}{4-\alpha^2}$, only when $c > \frac{4\beta-\alpha(\alpha(\alpha+\beta)+2)}{2((\alpha(\alpha+\beta))-2)} = \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)} - \frac{\alpha\left(1-\frac{\alpha(\alpha+\beta)}{2}\right)}{2-\alpha(\alpha+\beta)}$ is also satisfied, we have $\widetilde{P_1^s} < 0$. Moreover, if $\beta < \alpha - \frac{2\alpha(1-\alpha^2)}{4-\alpha^2}$ and $c < \frac{2(\alpha-\beta)}{2-\alpha(\alpha+\beta)} - \frac{\alpha\left(1-\frac{\alpha(\alpha+\beta)}{2}\right)}{2-\alpha(\alpha+\beta)}$, we obtain $\widetilde{P_1^s} > 0$. $\square$

J Proof of Theorem 5

Proof. From equalities (41) and (42), we have

$$\widetilde{P_1^b} - \widetilde{P_1^s} = \frac{(1-\alpha)(\alpha(\alpha+\beta+4)+4\beta+2c(\alpha+\beta+2)+6)}{2(6-(\alpha+\beta)(\alpha+2\beta))} > 0,$$

because $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. By taking derivatives of $\widetilde{P_1^b} - \widetilde{P_1^s}$ with respect to c and β, we have

$$\frac{\partial(\widetilde{P_1^b} - \widetilde{P_1^s})}{\partial c} = \frac{(1-\alpha)(\alpha+\beta+2)}{6-(\alpha+\beta)(\alpha+2\beta)} > 0,$$

$$\frac{\partial(\widetilde{P_1^b} - \widetilde{P_1^s})}{\partial \beta} = \frac{(1-\alpha)\left((\alpha+4)(\alpha+\beta)^2+12(\alpha+\beta+1)+2c\left(\alpha^2+2(\alpha+2)\beta+3\alpha+\beta^2+3\right)\right)}{((\alpha+\beta)(\alpha+2\beta)-6)^2} > 0.$$

because $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$.

From equalities (43) and (44), we have

$$\widetilde{P_2^b} - \widetilde{P_2^s} = \frac{6-8c+(\alpha-1)\alpha^2-2\alpha+2\alpha^2c+((\alpha-1)\alpha+4\alpha c+2c+4)\beta+4c\beta^2}{2(6-(\alpha+\beta)(\alpha+2\beta))}.$$

By combining with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$, we have $\widetilde{P_2^b} < \widetilde{P_2^s}$ if two conditions $c > \frac{-\alpha^2\beta-\alpha^3+\alpha^2+\alpha\beta+2\alpha-4\beta-6}{2\alpha^2+4\alpha\beta+4\beta^2+2\beta-8}$ and $\beta < \frac{1}{8}\left(-\alpha^2-3\alpha-6\right)+\frac{1}{8}\sqrt{\alpha^4-10\alpha^3+5\alpha^2+68\alpha+68}$. Otherwise, we have $\widetilde{P_2^b} \geq \widetilde{P_2^s}$.

Obviously, by taking the derivative of $\widetilde{P_2^b} - \widetilde{P_2^s}$ with respect to c and β, we obtain

$$\frac{\partial(\widetilde{P_2^b} - \widetilde{P_2^s})}{\partial c} = \frac{(\alpha+\beta)^2+\beta^2+\beta-4}{6-(\alpha+\beta)(\alpha+2\beta)} < 0,$$

$$\frac{\partial(\widetilde{P_2^b} - \widetilde{P_2^s})}{\partial \beta} = \frac{\begin{Bmatrix} ((\alpha-1)\alpha+4)\beta^2+(12-2(2-\alpha)\alpha(\alpha+1)\beta+12\beta) \\ +\alpha(6-(2-\alpha)\alpha(\alpha+1))+c\left(\alpha^3-\alpha^2-2(\alpha-1)\beta^2+8\beta+6\right) \end{Bmatrix}}{((\alpha+\beta)(\alpha+2\beta)-6)^2} > 0,$$

because $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. $\square$

K Proof of Theorem 6

Proof. From equations (41) and (43), we have

$$\widetilde{P_2^b} - \widetilde{P_1^b} = \frac{\alpha(c+\alpha+\beta)}{6-(\alpha+\beta)(\alpha+2\beta)} > 0,$$

with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. Meanwhile, with these three assumptions, taking derivatives of $\widetilde{P_2^b} - \widetilde{P_1^b}$ with respect to c, α, and β, respectively, we obtain that

$$\frac{\partial(\widetilde{P_2^b} - \widetilde{P_1^b})}{\partial c} = \frac{\alpha}{6-(\alpha+\beta)(\alpha+2\beta)} > 0,$$
$$\frac{\partial(\widetilde{P_2^b} - \widetilde{P_1^b})}{\partial \alpha} = \frac{-2\beta(\alpha+\beta)^2+6(2\alpha+\beta)+c\left(\alpha^2-2\beta^2+6\right)}{(6-(\alpha+\beta)(\alpha+2\beta))^2} > 0,$$
$$\frac{\partial(\widetilde{P_2^b} - \widetilde{P_1^b})}{\partial \beta} = \frac{\alpha\left(2(\alpha+\beta)^2+c(3\alpha+4\beta)+6\right)}{(6-(\alpha+\beta)(\alpha+2\beta))^2} > 0.$$

From equations (42) and (44), we have the following expression:

$$\widetilde{P_2^s} - \widetilde{P_1^s} = \frac{c\left(6-2\alpha^2-3\alpha\beta-2\beta^2\right)-\alpha^2(\alpha+\beta)}{6-(\alpha+\beta)(\alpha+2\beta)}.$$

The denominator $6-(\alpha+\beta)(\alpha+2\beta)$ is always positive and the term $6-2\alpha^2-3\alpha\beta-2\beta^2$ ranges within $(0,1)$ when the feasible set in Lemma 4 holds. We then have that if $c > \frac{\alpha^2(\alpha+\beta)}{6-2\alpha^2-3\alpha\beta-2\beta^2}$, we have $\widetilde{P_2^s} > \widetilde{P_1^s}$; if $c < \frac{\alpha^2(\alpha+\beta)}{6-2\alpha^2-3\alpha\beta-2\beta^2}$, we have that $\widetilde{P_2^s} < \widetilde{P_1^s}$.

Furthermore, in the feasible set provided by Lemma 4, the derivatives of $\widetilde{P_2^s} - \widetilde{P_1^s}$ with respect to c, α, and β satisfy

$$\frac{\partial(\widetilde{P_2^s} - \widetilde{P_1^s})}{\partial c} = \frac{6 - 2\alpha^2 - 3\alpha\beta - 2\beta^2}{6 - \alpha^2 - 3\alpha\beta - 2\beta^2} > 0,$$

$$\frac{\partial(\widetilde{P_2^s} - \widetilde{P_1^s})}{\partial \alpha} = \frac{\alpha\left(6\alpha^2\beta + \alpha^3 + 9\alpha\beta^2 - 18\alpha + 4\beta^3 - 12\beta + 3\alpha\beta c + 4\beta^2 c - 12c\right)}{((\alpha+\beta)(\alpha+2\beta)-6)^2} < 0,$$

$$\frac{\partial(\widetilde{P_2^s} - \widetilde{P_1^s})}{\partial \beta} = -\frac{\alpha^2\left(2(\alpha+\beta)^2 + c(3\alpha+4\beta)+6\right)}{((\alpha+\beta)(\alpha+2\beta)-6)^2} < 0.$$

□

L Proof of Proposition 2

Proof. From equalities (41), (42), (43), and (44), we have

$$(\widetilde{P_2^b} - \widetilde{P_2^s}) - (\widetilde{P_1^b} - \widetilde{P_1^s}) = \frac{\alpha(\alpha+1)(\alpha+\beta) - c\left(6 - (2\alpha^2 + 3\alpha\beta + \alpha + 2\beta^2)\right)}{6 - (\alpha+\beta)(\alpha+2\beta)}.$$

The term $6 - (2\alpha^2 + 3\alpha\beta + \alpha + 2\beta^2) = 6 - (2(\alpha+\beta)^2 + \alpha(1-\beta))$ is positive in the feasible set provided by Lemma 4. Then, if $c > \frac{\alpha(\alpha+1)(\alpha+\beta)}{6-(2\alpha^2+3\alpha\beta+\alpha+2\beta^2)}$, we have $(\widetilde{P_2^b} - \widetilde{P_2^s}) - (\widetilde{P_1^b} - \widetilde{P_1^s}) < 0$. If $c < \frac{\alpha(\alpha+1)(\alpha+\beta)}{6-(2\alpha^2+3\alpha\beta+\alpha+2\beta^2)}$, we obtain $(\widetilde{P_2^b} - \widetilde{P_2^s}) - (\widetilde{P_1^b} - \widetilde{P_1^s}) > 0$. □

M Proof of Proposition 3

Proof. $\widetilde{N_1^s} = \widetilde{N_2^s}$ can be easily obtained from the equality (30). From equations (46) and (47), we have $\widetilde{N_2^b} - \widetilde{N_1^b} = -\frac{\alpha(\alpha+\beta+c)}{6-(\alpha+\beta)(\alpha+2\beta)}$, which is negative because $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. Taking derivatives of $\widetilde{N_2^b} - \widetilde{N_1^b}$ with respect to c, α, and β, we obtain that

$$\frac{\partial(\widetilde{N_2^b} - \widetilde{N_1^b})}{\partial c} = -\frac{\alpha}{6 - (\alpha+\beta)(\alpha+2\beta)} < 0,$$

$$\frac{\partial(\widetilde{N_2^b} - \widetilde{N_1^b})}{\partial \alpha} = \frac{2\beta(\alpha+\beta)^2 - 6(2\alpha+\beta) - c\left(\alpha^2 - 2\beta^2 + 6\right)}{(6 - (\alpha+\beta)(\alpha+2\beta))^2} < 0,$$

$$\frac{\partial(\widetilde{N_2^b} - \widetilde{N_1^b})}{\partial \beta} = -\frac{\alpha\left(2(\alpha+\beta)^2 + c(3\alpha+4\beta)+6\right)}{(6 - (\alpha+\beta)(\alpha+2\beta))^2} < 0,$$

in the feasible set provided by Lemma 4. Within the feasible set, we also have $\left|\widetilde{N_2^b} - \widetilde{N_1^b}\right| = -(\widetilde{N_2^b} - \widetilde{N_1^b})$. Thus, we conclude that the number gap $\left|\widetilde{N_2^b} - \widetilde{N_1^b}\right|$ is increasing in α, β, and c. $\square$

N Proof of Proposition 4

Proof. Based on equalities (30) and (45), we have

$$\begin{aligned}N_1^s - \widetilde{N_1^s} &= N_2^s - \widetilde{N_2^s} \\ &= \left(\frac{1}{4-(\alpha+\beta)^2} - \frac{2}{6-(\alpha+\beta)(\alpha+2\beta)}\right)(\alpha+\beta+c) < 0,\end{aligned}$$

within the feasible sets of parameters in Lemmas 2 and 4.

From equations (31) and (46), we obtain

$$N_1^b - \widetilde{N_1^b} = -\frac{(\alpha+\beta)(2-\alpha(\alpha+\beta))(\alpha+\beta+c)}{2(2-\alpha-\beta)(2+\alpha+\beta)(6-(\alpha+\beta)(\alpha+2\beta))} < 0,$$

within the feasible sets of parameters in Lemmas 2 and 4. From equations (31) and (47), the difference between N_2^b and $\widetilde{N_2^b}$ can be expressed as

$$N_2^b - \widetilde{N_2^b} = \frac{\left(6\alpha - \alpha^3 - 2\left(1+\alpha^2\right)\beta - \alpha\beta^2\right)(\alpha+\beta+c)}{2\left(4-\left(\alpha^2+2\alpha\beta+\beta^2\right)\right)\left(6-\left(\alpha^2+3\alpha\beta+2\beta^2\right)\right)}.$$

The denominator and the term $\alpha+\beta+c$ in the numerator are positive with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. We then consider the term $6\alpha - \alpha^3 - 2\left(1+\alpha^2\right)\beta - \alpha\beta^2$. In theory, the solutions to $6\alpha - \alpha^3 - 2\left(1+\alpha^2\right)\beta - \alpha\beta^2 = 0$ are

$$\begin{aligned}\beta_1 &= -\frac{\sqrt{8\alpha^2+1}+1+\alpha^2}{\alpha} < 0, \\ \beta_2 &= \frac{\sqrt{8\alpha^2+1}-1-\alpha^2}{\alpha} > 0,\end{aligned}$$

because $\alpha \in (0,1)$. The inequality $\beta_2 > 1$ means that $\alpha \geq \frac{1}{2}\left(\sqrt{17}-3\right)$ with $\alpha \in (0,1)$. In this scenario, $6\alpha - \alpha^3 - 2\left(1+\alpha^2\right)\beta - \alpha\beta^2 > 0$ for all $\beta \in (0,1)$. Consider $\alpha \in (0,1)$ and $\beta \in (0,1)$. If $\alpha < \frac{1}{2}\left(\sqrt{17}-3\right)$ and $\beta < \beta_2 = \frac{\sqrt{8\alpha^2+1}-1-\alpha^2}{\alpha}$, the relation $6\alpha - \alpha^3 - 2\left(1+\alpha^2\right)\beta - \alpha\beta^2 > 0$ is also satisfied. However, if $\alpha < \frac{1}{2}\left(\sqrt{17}-3\right)$ and $\beta > \beta_2 = \frac{\sqrt{8\alpha^2+1}-1-\alpha^2}{\alpha}$, we have that $6\alpha - \alpha^3 - 2\left(1+\alpha^2\right)\beta - \alpha\beta^2 < 0$.

From equations (28) and (42), the price difference $P_1^s - \widetilde{P_1^s}$ satisfies

$$P_1^s - \widetilde{P_1^s} = \frac{(2-\alpha(\alpha+\beta))^2(\alpha+\beta+c)}{2(2-\alpha-\beta)(2+\alpha+\beta)(6-(\alpha+\beta)(\alpha+2\beta))} > 0,$$

because $\alpha \in (0,1)$ and $\beta \in (0,1)$. Equations (29) and (44) mean that

$$P_{22}^s - \widetilde{P_2^s} = \frac{(4-4\alpha\beta+\alpha^2(2-\alpha-\beta)(2+\alpha+\beta))(\alpha+\beta+c)}{2(2-\alpha-\beta)(2+\alpha+\beta)(6-(\alpha+\beta)(\alpha+2\beta))}.$$

Because the denominator and the term $c+\alpha+\beta$ are positive with $\alpha \in (0,1)$ and $\beta \in (0,1)$, we only need to explore whether the term $4-4\alpha\beta+\alpha^2(2-\alpha-\beta)(2+\alpha+\beta)$ is positive. This term can be expressed as $-\alpha^2\beta^2 + (-2\alpha^3-4\alpha)\beta - \alpha^4 + 4\alpha^2 + 4$. Two solutions to $-\alpha^2\beta^2 + (-2\alpha^3-4\alpha)\beta - \alpha^4 + 4\alpha^2 + 4 = 0$ satisfy

$$\beta_1 = -\frac{2\sqrt{2}\sqrt{\alpha^4+\alpha^2}+2\alpha+\alpha^3}{\alpha^2} < 0,$$
$$\beta_2 = \frac{2\sqrt{2}\sqrt{\alpha^4+\alpha^2}-2\alpha-\alpha^3}{\alpha^2} > 1,$$

with $\alpha \in (0,1)$ and $\beta \in (0,1)$. Then, under these two assumptions, we have that $P_{22}^s > \widetilde{P_2^s}$.

From equations (27) and (41), we have

$$P_1^b - \widetilde{P_1^b} = \frac{(\alpha-\beta)(2-\alpha(\alpha+\beta))(\alpha+\beta+c)}{2(2-\alpha-\beta)(2+\alpha+\beta)(6-(\alpha+\beta)(\alpha+2\beta))}.$$

The denominator, $2-\alpha(\alpha+\beta)$ and $\alpha+\beta+c$ are positive with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. Then we have that, if $\alpha > \beta$,we obtain that $P_1^b > \widetilde{P_1^b}$; if $\alpha < \beta$, we obtain that $P_1^b < \widetilde{P_1^b}$. From equalities (27) and (43), we obtain that

$$P_2^b - \widetilde{P_2^b} = -\frac{(\alpha(6-(\alpha+\beta)(\alpha+3\beta))+2\beta)(\alpha+\beta+c)}{2(2-\alpha-\beta)(2+\alpha+\beta)(6-(\alpha+\beta)(\alpha+2\beta))} < 0,$$

in the feasible sets provided by Lemmas 2 and 4. That is, inequality $P_2^b < \widetilde{P_2^b}$ always holds.

From equalities (27), (41), and (43), we have that $\widetilde{P_2^b} - \widetilde{P_1^b} = \frac{\alpha(\alpha+\beta+c)}{6-(\alpha+\beta)(\alpha+2\beta)} > P_2^b - P_1^b = 0$.

From equalities (28), (29), (42), and (44), we have that $P_{22}^s - P_1^s = c > \widetilde{P_2^s} - \widetilde{P_1^s} = \frac{\alpha^2(\alpha+\beta+c)}{(\alpha+\beta)(\alpha+2\beta)-6} + c$ with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$.

From equations (32) and (48), the difference in profit satisfies

$$\pi - \widetilde{\pi} = \frac{(4-\alpha(4\beta-\alpha(-\alpha-\beta+2)(\alpha+\beta+2)))(\alpha+\beta+c)^2}{2(-\alpha-\beta+2)(\alpha+\beta+2)(6-(\alpha+\beta)(\alpha+2\beta))^2} > 0,$$

within the feasible sets of parameters in Lemmas 2 and 4, which means that $\pi > \widetilde{\pi}$ is always satisfied. $\square$

O Proof of Proposition 5

Proof. Based on the equalities (49) and (50), we derive $W_1^s = W_2^s$. Meanwhile, from the equalities (52) and (53), we have $W_1^b = W_2^b$. As shown in equation (55), social welfare depends only on c and ϕ. The allocation of ϕ between α and β does not matter. Then, the first- and second-order derivatives of W satisfy

$$\frac{\partial W}{\partial c} = \frac{2c\left(12-\phi^2\right)+4\phi\left(8-\phi^2\right)}{4\left(4-\phi^2\right)^2} > 0,$$

$$\frac{\partial W}{\partial \phi} = \frac{-\left(c^2+4\right)\phi^3+20\left(c^2+4\right)\phi-2c\phi^4+24c\phi^2+64c}{2\left(4-\phi^2\right)^3} > 0,$$

$$\frac{\partial^2 W}{\partial c^2} = \frac{12-\phi^2}{2\left(4-\phi^2\right)^2} > 0,$$

$$\frac{\partial^2 W}{\partial \phi^2} = \frac{-3\left(c^2+4\right)\phi^4+88\left(c^2+4\right)\phi^2+80\left(c^2+4\right)-4c\phi^5+64c\phi^3+576c\phi}{2\left(4-\phi^2\right)^4} > 0,$$

$$\frac{\partial^2 W}{\partial \phi \partial c} = \frac{c\left(20-\phi^2\right)\phi-\phi^4+12\phi^2+32}{\left(4-\phi^2\right)^3} > 0,$$

with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. $\square$

P Proof of Proposition 6

Proof. From equations (56) and (57), we can obtain that $\widetilde{W_1^s} = \widetilde{W_2^s}$. From equations (59) and (60), we have that

$$\widetilde{W_2^b} - \widetilde{W_1^b} = -\frac{\alpha(c+\alpha+\beta)(6+c(\alpha+2\beta))}{2(6-(\alpha+\beta)(\alpha+2\beta))^2} < 0$$

with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$.

Equation (62) can be expressed as

$$\widetilde{W} = \frac{\begin{pmatrix} (\alpha+\beta)(\alpha(8-\alpha(\alpha+\beta))-16\beta)+2c^2\left(-\alpha^2-2\alpha\beta+2\left(8-\beta^2\right)\right) \\ +2c\left(11\left(4-\alpha^2\right)\beta+\alpha\left(38-3\alpha^2\right)-16\alpha\beta^2-8\beta^3\right)+108 \end{pmatrix}}{4(6-(\alpha+\beta)(\alpha+2\beta))^2}.$$

The derivatives of $\widetilde{W}$ with respect to c, α, and β satisfy

$$\frac{\partial \widetilde{W}}{\partial c} = \frac{(\alpha+\beta)(76-6\beta^2-6(\alpha+\beta)^2)+4c(16-\beta^2-(\alpha+\beta)^2)+4\beta(3-(\alpha+\beta)^2)}{4(6-(\alpha+\beta)(\alpha+2\beta))^2} > 0,$$

$$\frac{\partial \widetilde{W}}{\partial \alpha} = \frac{\left\{\begin{array}{l} -2(\alpha+\beta)\left(2\alpha^2\left(\beta^2-1\right)+\alpha^3\beta+\alpha\beta\left(\beta^2-13\right)-20\beta^2\right) \\ -12(22\alpha+25\beta)+2c^2\left(3\alpha^2\beta+\alpha^3+\alpha\left(5\beta^2-26\right)+4\beta^3-42\beta\right) \\ +c\left(30\alpha^2\left(\beta^2-2\right)+13\alpha^3\beta+3\alpha^4+2\alpha\beta\left(18\beta^2-79\right)+4\left(4\beta^4-23\beta^2-57\right)\right) \end{array}\right\}}{2((\alpha+\beta)(\alpha+2\beta)-6)^3} > 0,$$

$$\frac{\partial \widetilde{W}}{\partial \beta} = \frac{\left\{\begin{array}{l} 2(\alpha+\beta)\left(\alpha^2\left(\beta^2-11\right)+2\alpha^3\beta+\alpha^4-4\alpha\beta+16\beta^2\right)-12(25\alpha+28\beta) \\ +2c^2\left(5\alpha^2\beta+2\alpha^3+6\alpha\left(\beta^2-7\right)+4\beta\left(\beta^2-13\right)\right) \\ +c\left(6\left(7\alpha^2-20\right)\beta^2+\left(25\alpha^2-244\right)\alpha\beta+7\alpha^4-118\alpha^2+40\alpha\beta^3+16\beta^4-264\right) \end{array}\right\}}{2((\alpha+\beta)(\alpha+2\beta)-6)^3} > 0,$$

with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. Meanwhile, all the second-order derivatives satisfy

$$\frac{\partial^2 \widetilde{W}}{\partial c^2} = \frac{16-\beta^2-(\alpha+\beta)^2}{(6-(\alpha+\beta)(\alpha+2\beta))^2} > 0,$$

$$\frac{\partial^2 \widetilde{W}}{\partial \alpha^2} =$$

$$\frac{\left\{\begin{array}{l} 2(\alpha+\beta)\times \\ \left(\alpha^3\left(2\beta^2-3\right)+\alpha^4\beta-15\alpha^2\beta+\alpha\left(-2\beta^4-57\beta^2+312\right)-\beta^3\left(\beta^2+54\right)+444\beta\right) \\ +c^2\left(-28\alpha^2\left(\beta^2-4\right)-12\alpha^3\beta-3\alpha^4+6\alpha\beta\left(62-7\beta^2\right)-26\beta^4+296\beta^2+156\right) \\ +c\left(\alpha^3\left(84-48\beta^2\right)+16\alpha^2\beta\left(23-6\beta^2\right)-15\alpha^4\beta-3\alpha^5\right) \\ +c\left(6\alpha\left(-16\beta^4+75\beta^2+174\right)+4\beta\left(-9\beta^4+37\beta^2+375\right)\right)+792 \end{array}\right\}}{((\alpha+\beta)(\alpha+2\beta)-6)^4} > 0,$$

$$\frac{\partial^2 \widetilde{W}}{\partial \beta^2} =$$

$$\frac{\left\{\begin{array}{l} -6(\alpha+\beta)\left(\alpha^3\left(3\beta^2-11\right)+\alpha^2\beta\left(\beta^2-24\right)+3\alpha^4\beta+\alpha^5-212\alpha+8\beta\left(2\beta^2-29\right)\right) \\ +c^2\left(-56\left(\alpha^2-8\right)\beta^2+6\left(124-7\alpha^2\right)\alpha\beta-13\alpha^4+296\alpha^2-48\alpha\beta^3-24\beta^4+312\right) \\ +c\left(\alpha^3\left(334-128\beta^2\right)+4\alpha^2\beta\left(267-34\beta^2\right)-75\alpha^4\beta-19\alpha^5\right) \\ +c\left(16\alpha\left(-6\beta^4+65\beta^2+120\right)-32\beta\left(\beta^4-9\beta^2-72\right)\right)+1008 \end{array}\right\}}{((\alpha+\beta)(\alpha+2\beta)-6)^4} > 0,$$

with $\alpha \in (0,1)$, $c \in (0,1)$, and $\beta \in (0,1)$. $\square$

Supplementary Appendix of the Paper "Platform Dynamics and Economic Development",

by *Danxia Xie*, *Buyuan Yang*, and *Hanzhe Zhang*

(not for publication)

1. Data on the number of "Taobao villages" in China and the number of "Taobao villages" located in national poverty-stricken counties of China are from *China Taobao Village Research Report (2020)* (Chinese title: 中国淘宝村研究报告 2020).

2. Data on online retail sales in rural areas of China come from *E-commerce in China 2019* (Chinese title: 中国电子商务报告 2019).

3. Data sources of online retail sales on Alibaba's platforms in poverty-stricken counties of China:

   (1) The datum for 2014 is from *2014 county and rural e-commerce data* (Chinese title: 2014 年县域暨农村电商数据).

   (2) The datum for 2015 is from *2015 China County E-commerce Report* (Chinese title: 2015 年中国县域电子商务报告).

   (3) The datum for 2016 is from *E-commerce Development: Experience from China.*

   (4) The datum for 2017 is from *New Country, New Consumption, New Business: Rural Business Research Report* (Chinese title: 新乡村 新消费 新商业——农村商业研究报告).

   (5) The datum for 2018 is from *Alibaba Poverty Alleviation Work Report 2018* (Chinese title: 阿里巴巴脱贫工作报告 2018).

   (6) The datum for 2019 is from *Alibaba's 2020 Fiscal Year Public Welfare "Financial Report"* (Chinese title: 阿里巴巴 2020 财年公益"财报").