An Investment-and-Marriage Model with Differential Fecundity: On the College Gender Gap

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Abstract

I build an investment-and-marriage model to provide a new explanation of the reversed college gender gap, i.e., more women than men are going to college. The explanation is based on differential fecundity and an equilibrium marriage-market effect. The model also sheds light on gender-specific relationships between age at marriage and midlife personal income for American men and women, and the evolving relationship between age at marriage and spousal income for American women.

Keywords: college gender gap, earnings gender gap, marriage age, nonassortative matching

JEL: C78, D1, J1

1   Introduction

More women than men have attended college in the United States and many other countries in recent decades, while women have continued to earn less than men on average (Goldin et al., 2006; Becker et al., 2010b,a). Many reasons—e.g., gender differences in noncognitive skills, risk aversion, or labor-market opportunities—have been proposed to explain separately the reversed college gender gap and the persistent earnings gender gap. I propose a new unifying explanation based on differential fecundity, i.e., women have a shorter fertility span than men. This gender difference results in a more significant fertility-income trade-off for women who make post-college career investments, which in turn leads to the earnings gender gap. However, the gender difference that deters women from making post-college career investments can encourage *more* women than men to make college investments through a subtle equilibrium marriage-market effect.

To illustrate this equilibrium effect and rationalize other empirical regularities, I build an investment-and-marriage model with differential fecundity. Men and women of a new generation enter the economy in each of the infinitely many periods in the model. Individuals differ in their probability—which I refer to as "ability"—of achieving a high lifetime income after an investment. Individuals with sufficiently low abilities forgo college and immediately marry. Those with sufficiently high abilities make a college investment and realize an uncertain income return. After the college investment, those who realize a high income marry, and those who realize a low income can take another income draw by making what I call a post-college career investment—e.g., obtaining additional education, receiving additional training, or searching for a new job. A college or career investment delays marriage and childbearing. A career investment delays women's marriage to a less fertile period of life and adversely affects their marital outcome, so it is especially costly for women.

Because a career investment is costlier than a college investment for women but not for men, only higher-ability women who realize a low income after college will make a career investment, whereas all men who realize a low income after college will make a career

investment.

Most surprisingly, in the unique equilibrium of the model, more women than men go to college and fewer women than men earn a high income. The reason that more women than men go to college is subtle, and operates through an equilibrium marriage-market channel. As pointed out above, because of differential fecundity, college-educated women are less likely than college-educated men to make career investments. Consequently, high-income women are scarcer than high-income men in both the labor market and the marriage market. As a result, high-income women are more "valuable" than high-income men in the marriage market, which provides an endogenously higher marriage-market incentive for women to go to college.

In essence, fewer women than men earn high incomes, because their shorter fertility span deters them from making post-college career investments, but more women than men go to college, because endogenously higher demand for fertile high-income women in the marriage market resulting from women's lower likelihood of career investments encourages more of them to make college investments.

The gender difference in career investment decisions also helps rationalize the observed gender difference in the relationships between marriage age and midlife personal income for Americans born in the twentieth century. The relationship has been hump-shaped for men (those who marry in their mid-twenties have a higher average midlife income than those who marry earlier or later) and positive for women (the later a woman marries, the more she earns on average). In the model, the lowest-ability men and women marry in the first period and earn low incomes without going to college, but the decision to marry in the second or third period differs by gender. Whereas only men who realize a high income after college marry in the second period, lower-ability women who realize a low income after college and all women who realize a high income after college marry in the second period. All men who realize a low income after college make a career investment and marry in the third period, earning a lower average income than those who marry in the second period; only higher-

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ability women who realize a low income after college make a career investment and marry in the third period, earning a higher average income than those who marry in the second period.

The prediction of the model also matches the observed hump-shaped relationship between age at marriage and spousal income first documented by Low (2020) for American women born in the twentieth century: Women who married in their mid-twenties had a higher average spousal income than those who married earlier or later. In the model, even though women who marry in the third period have a higher average income than those who marry in the second period, their marital outcome is adversely affected by their declined fertility. Furthermore, for cohorts born before the 1950s, women who married in their thirties had a lower average midlife spousal income than those who married around age 20, but for cohorts born after the 1950s, those who married in their thirties had a higher average midlife spousal income than those who married around age 20. This change can be explained by the declining importance of fertility relative to income in the marriage market: When fertility is less important in the marriage market, less fertile high-income women marry higher-income husbands than fertile low-income women.

In summary, I provide a parsimonious model to account for a collection of gender differences in college investment, earnings, and marriage. First and foremost, I provide a new and unified explanation of the college and earnings gender gaps using differential fecundity and the equilibrium marriage market, which contributes to the line of research that studies these two gender gaps, especially the effects of the marriage market on them (Iyigun and Walsh, 2007; Chiappori et al., 2009; Ge, 2011; Lafortune, 2013; Bruze, 2015; Greenwood et al., 2016; Chiappori et al., 2017). Second, I provide a detailed account and unified explanation of the relationships between age at marriage and midlife income, based on labor-market shocks and differential fecundity, which complements prior explanations based on search frictions (Becker, 1973, 1974) and information frictions (Bergstrom and Bagnoli, 1993) in the marriage market. Third, I provide a theory consistent with a previous fertility-based explanation of

the relationship between age at marriage and spousal income for women (Low, 2020), and demonstrate that differential fecundity is able to explain even more gender differences in economic and marital outcomes than the literature suggests (Siow, 1998; Greenwood et al., 2003; Coles and Francesconi, 2011, 2018; Díaz-Giménez and Giolito, 2013; Zhang and Zou, 2020; Gershoni and Low, ming).

Section 2 documents the stylized facts. Section 3 presents the model, characterizes its unique equilibrium, and explains the stylized facts. Section 4 concludes. Appendices collect omitted proofs and details.

2   Documenting the Stylized Facts

Figure 1 summarizes the three sets of stylized facts to be explained by the model.

(a) College and Earnings Gender Gaps. The college-educated share of Americans aged 35–39 was higher for men before year 2000 but higher for women after year 2000 (the left panel of Figure 1a), while the average labor income was consistently higher for men than for women (the right panel of Figure 1a). The coexistence of these two opposite gender gaps is not uniquely American: In 2010, women attended college at higher rates than men in 67 countries across all inhabited continents, but earned less than men on average in each of these countries. See Becker et al. (2010a,b) for the worldwide college gender gap since 1960, Goldin et al. (2006) for the American college gender gap in the twentieth century, and Goldin (1990, 2006, 2014) for the American earnings gender gap in the twentieth century.

(b) Relationships between Age at Marriage and Midlife Income for Men and Women. Men who married in their mid-twenties had a higher average midlife income than men who married earlier or later (Figure 1b). To match the three periods in my model, I refer to men who first married between ages 16 and 22, between ages 23 and 29, and between ages 30 and 39, as early, middle, and late grooms, respectively. Middle grooms born in almost every year between 1900 and 1979 earned a statistically and economically significantly higher

Figure 1: Stylized Facts

Note: I use the decennial censuses from 1960 to 2000 and 5-year American Community Surveys (ACS) in 2010 and 2015 in the Integrated Public Use Microdata Series (IPUMS) USA (Ruggles et al., 2020). Age at (first) marriage is either reported directly (AGEMARR) in the censuses of 1960, 1970, and 1980, or imputed from the year entering the current marriage (YRMARR) in the ACS since 2008 for those who have married once and stayed married. The measure of income is the reported pre-tax wage and salary income in the previous calendar year, inflation-adjusted to 1999 USD (i.e., INCWAGE×CPI99); similar relationships are obtained if total income (INCTOT) is used instead. Midlife income is measured by income between ages 41 and 50 whenever possible. Since spousal income was not reported in the 1950 census, I use the income between ages 51 and 60 in the 1960 census for the 1900s birth cohort. Since age at marriage was not present in IPUMS USA between 1980 and 2008, age at marriage and income between ages 41 and 50 are not simultaneously available for the 1940s or 1950s birth cohorts; I use the income between ages 61 and 70 for the 1940s birth cohort and the income between ages 51 and 60 for the 1950s birth cohort.
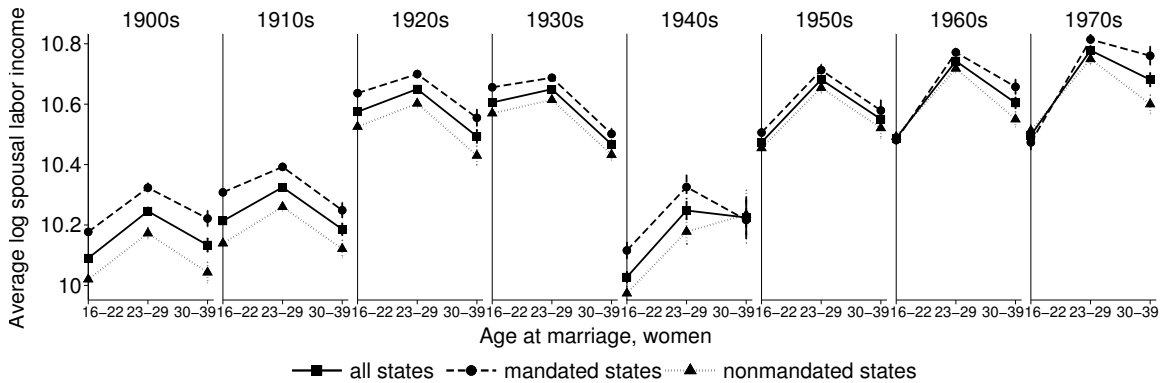
(a) Reversed College Gender Gap and Persistent Earnings Gender Gap
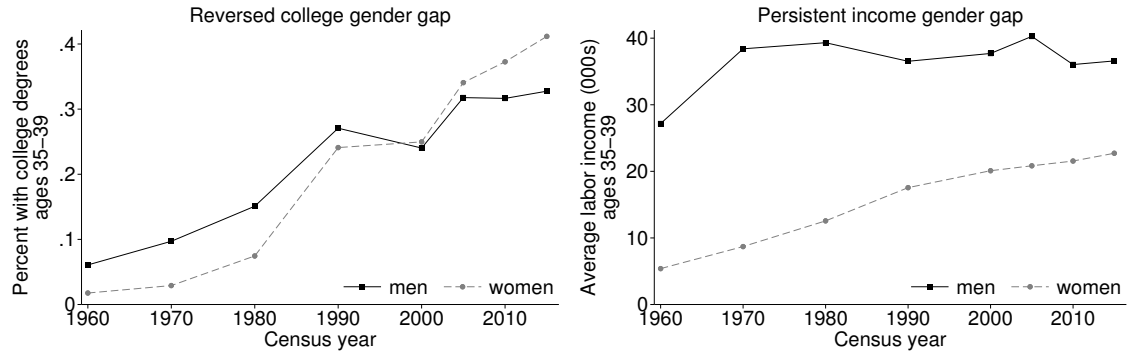
(b) Relationships between Marriage Age and Labor Income for Men and Women
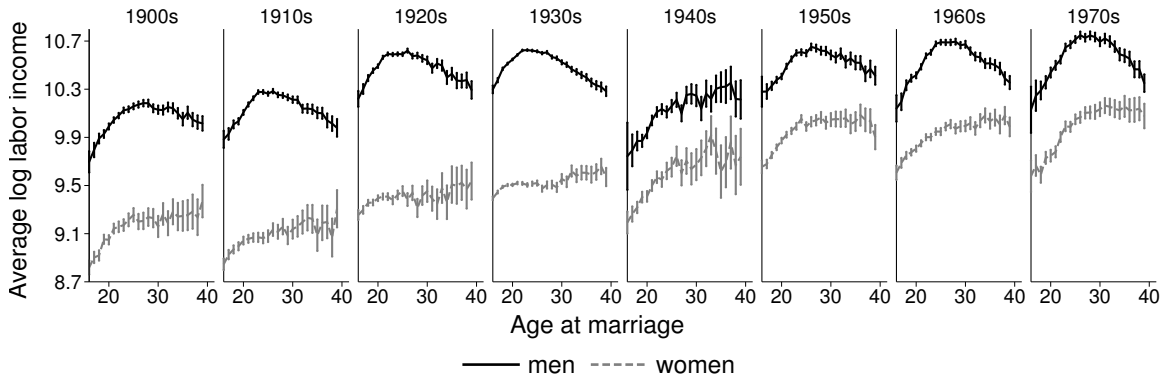
(c) Relationship between Marriage Age and Spousal Labor income for Women

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average midlife income than early and late grooms born in the same year. Compared with middle grooms, early grooms earned 11.8% less on average, and late grooms earned 12.9% less on average. In contrast, the later a woman married, the more she earned on average (Figure 1b). Early brides had on average 13.6% less midlife income than middle brides born in the same year, and middle brides had on average 4.2% less midlife income than late brides born in the same year. The observed gender difference in the relationships between age at marriage and personal income suggests the existence of a fundamental gender asymmetry that results in gender-differential marriage timing and human capital investment decisions.

(c) Relationship between Age at Marriage and Spousal Income for Women. Husbands of women who married in their mid-twenties earned a higher average midlife income than husbands of those who married earlier or later (Figure 1c). More precisely, husbands of early brides and late brides earned, respectively, 14.7% and 14.0% less midlife income on average than husbands of middle brides. Furthermore, the relationship has changed over time. Early brides in the pre-1950 birth cohorts had higher-income husbands than late brides (the first five panels of Figure 1c), but the pattern was reversed for post-1950 birth cohorts (the last three panels of Figure 1c). This change was more pronounced in the 13 states that passed laws between 1985 and 1995 requiring that infertility treatments be covered or offered by insurance (dashed lines with circles versus dotted lines with triangles in the last three panels of Figure 1c).[1] Because infertility treatment was (and still is) quite expensive, the laws reduced the costs for women and effectively extended the biological clocks of women in these states. We should expect that the marital outcome of early brides would have dropped more and the marital outcome of late brides would have improved more in these 13 states after the laws were passed. Indeed, the average spousal income of late brides statistically significantly surpassed that of early brides born after 1960 in the mandated states but not in the nonmandated states. This observation suggests that gender-differential fertility span

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[1]The 13 mandated states are Maryland (1985); Arkansas, Hawaii, Massachusetts, Montana, Texas (1987); California, Connecticut, Rhode Island (1989); New York (1990); Illinois, Ohio (1991); and West Virginia (1995). See Table 1 of Buckles (2007).

can help explain the observed relationship between age at marriage and spousal income for women.

3 Explaining the Stylized Facts

3.1 Model

Consider an economy over an infinite number of discrete periods. At the beginning of each period, a unit mass of men and a unit mass of women enter the economy. Each agent is endowed with a heterogeneous probability—"*ability*"—$\theta \in [0,1]$ to realize a high lifetime income after an investment. Let F_m and F_w denote the continuous and strictly increasing cumulative distribution of abilities for men and women, respectively. Men and women make investment and marriage decisions over the next three periods of their lives to maximize lifetime utility. Think of the three periods as ages 16–22, 23–29, and 30–39. Each agent pays investment costs and receives a reservation payoff z from working and an additional endogenously determined marriage payoff v if married. Each agent is risk neutral and does not discount.

Figure 2: An Agent's Investment Decisions.

Note: Each agent enters the marriage market after the return from investments is realized.
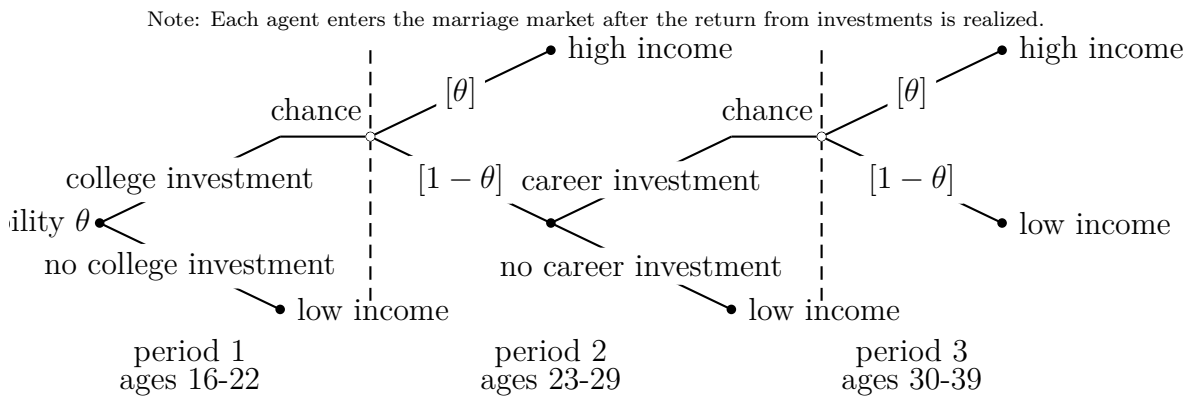
3.1.1 Investments

Figure 2 illustrates an agent's investment decisions. In period 1, each agent decides whether to go to college. Any agent who decides not to go to college earns a low lifetime income and enters the marriage market immediately. Any agent who decides to go to college pays a cost, c_m for a man and c_w for a woman, and is assumed to delay marriage.

In period 2, an ability-θ agent who went to college gets on the path to a high lifetime income with probability θ. Any agent who does not get on the path to a high lifetime income decides whether to make a post-college career investment, which is assumed to cost the same as a college investment. Examples of a career investment include obtaining additional education beyond college, obtaining additional training, and finding a new job. Any agent who does not make a career investment earns a low lifetime income and enters the marriage market in the second period. Any agent who makes a career investment gets another chance to improve their lifetime income but delays marriage.

In period 3, an ability-θ agent who made a career investment enters the marriage market with either a high lifetime income with probability θ or a low lifetime income otherwise.

3.1.2 Differential Fecundity

For simplicity, an agent can be either fertile—with a high probability of conceiving—or less fertile—with a low probability of conceiving. A man is fertile for all three periods, but a woman is fertile for only the first two periods and less fertile in the third period. In the marriage market, men are distinguished by income only, but women are distinguished by income and fertility. Let $\tau_m \in T_m = \{H, L\}$ and $\tau_w \in T_w = \{H, L, h, l\}$ denote the marital types for men and women, respectively. Letters H and h denote high-income types, and L and l denote low-income types; uppercase letters denote fertile types, and lowercase letters denote less fertile types.

Realized income and fertility—not ability—determine each agent's payoff. Each type-τ agent has a reservation utility of $z(\tau)$ without being married. A type-τ_m man and a type-

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τ_w woman would generate a total utility of $z(\tau_m, \tau_w)$ from marriage. Hence, the surplus due to marriage is $s_{\tau_m \tau_w} \equiv s(\tau_m, \tau_w) = z(\tau_m, \tau_w) - z(\tau_m) - z(\tau_w)$. Assume the marriage surplus is nonnegative, strictly increasing in income and fertility, strictly supermodular in incomes, and strictly supermodular in husband's income and wife's fertility level. Formally, let $\delta_{\tau_w} \equiv s_{H\tau_w} - s_{L\tau_w}$ denote the surplus difference between when a type-τ_w woman marries a high-income man and when she marries a low-income man. Strict supermodularity in incomes means $\delta_H > \delta_L$ and $\delta_h > \delta_l$, and strict supermodularity in husband's income and wife's fertility level means $\delta_H > \delta_h$ and $\delta_L > \delta_l$. The two assumptions together imply that δ_H is the largest and δ_l the smallest, and δ_h can be larger, smaller, or equal to δ_L. These supermodularity assumptions will help us pin down the stable matching patterns.

3.1.3   The Marriage Market

Overlapping generations of men and women meet and bargain over the division of their marriage surplus until they reach a stable outcome in which no one can improve his or her payoff. Formally, a marriage market is described by vectors of masses of marriage characteristics $G_m = (G_{mH}, G_{mL})$ and $G_w = (G_{wH}, G_{wh}, G_{wL}, G_{wl})$. A *stable outcome* of the marriage market (G_m, G_w) consists of *stable matching* $G = (G_{HH}, G_{Hh}, G_{HL}, G_{Hl}, G_{LH}, G_{Lh}, G_{LL}, G_{Ll})$ and *stable marriage payoffs* $v_m = (v_{mH}, v_{mL})$ and $v_w = (v_{wH}, v_{wh}, v_{wL}, v_{wl})$. Stable matching G satisfies *feasibility*: $\sum_{\tau_w} G_{\tau_m \tau_w} \leq G_{m\tau_m}$ for any $\tau_m \in T_m$ and $\sum_{\tau_m} G_{\tau_m \tau_w} \leq G_{w\tau_w}$ for any $\tau_w \in T_w$. Stable marriage payoffs v_m and v_w satisfy (i) *individual rationality*: $v_{m\tau_m} \geq 0$ for any $\tau_m \in T_m$ and $v_{w\tau_w} \geq 0$ for any $\tau_w \in T_w$ (every person receives at least as much as they would have if they had remained single); (ii) *pairwise efficiency*: $v_{m\tau_m} + v_{w\tau_w} = s_{\tau_m \tau_w}$ if $G_{\tau_m \tau_w} > 0$ (every married couple divides the entire marriage surplus); and (iii) *Pareto efficiency*: $v_{m\tau_m} + v_{w\tau_w} \geq s_{\tau_m \tau_w}$ for all $\tau_m \in T_m$ and $\tau_w \in T_w$ (no man-woman pair not married to each other can simultaneously improve their marriage payoffs by marrying each other). A stable outcome exists for any marriage market by Theorem 2 of Gretsky et al. (1992).

3.2   Unique Equilibrium

Define $\sigma_{m1}(\theta)$ and $\sigma_{m2}(\theta)$ as the probability of an ability-θ man investing in the first and second period, respectively, and define $\sigma_{w1}(\theta)$ and $\sigma_{w2}(\theta)$ for an ability-θ woman similarly. Strategies are summarized by functions $\sigma_m = (\sigma_{m1}, \sigma_{m2})$ and $\sigma_w = (\sigma_{w1}, \sigma_{w2})$. Strategies σ_m and σ_w are said to *induce* the marriage market (G_m, G_w) if the distributions of men's and women's marriage characteristics in each period are G_m and G_w, respectively, when men and women of every generation choose strategies σ_m and σ_w, respectively.

Definition 1. *A quadruple $(\sigma_m^*, \sigma_w^*, v_m^*, v_w^*)$ is an <u>equilibrium</u> if (i) $\sigma_m^*(\theta)$ and $\sigma_w^*(\theta)$, respectively, maximize each ability-θ man's and each ability-θ woman's expected utility when the marriage payoffs are v_m^* and v_w^*, and (ii) v_m^* and v_w^* are stable marriage payoffs of the marriage market (G_m^*, G_w^*) induced by σ_m^* and σ_w^*.*

3.2.1   Equilibrium Investments

Backward induction solves men's equilibrium investments. If an ability-θ man who receives a low-income offer after college decides to make a career investment, he incurs a cost c_m and expects a lifetime income gain $\theta(z_{mH} - z_{mL})$ and a lifetime marriage gain $\theta(v_{mH} - v_{mL})$. An ability-θ man makes a career investment if the expected gain outweighs the investment cost, that is, if his ability is above $\theta_m \equiv c_m/z_{mH} - z_{mL} + v_{mH} - v_{mL}$. (Ability-$\theta_m$ men are indifferent between investing and not investing.) A man goes through the same cost-benefit analysis to decide on optimal college investment. Therefore, in any equilibrium, any man with an ability above θ_m makes a college investment, and makes a career investment if he receives a low-income offer after college, while any man with an ablity below θ_m makes no investment.

Backward induction can also solve women's equilibrium investments. If an ability-θ woman who receives a low-income offer after college makes a career investment, her expected income gain is $\theta(z_{wH} - z_{wL})$ and her expected marriage gain is $\theta(v_{wh} - v_{wl}) - (v_{wL} - v_{wl})$, where the term $v_{wL} - v_{wl}$ represents her loss in marriage payoff due to fertility de-

cline. Therefore, she makes a career investment if and only if her ability θ is above

$\theta_{w2} \equiv c_w + v_{wL} - v_{wl}/z_{wH} - z_{wL} + v_{wh} - v_{wl}$. In contrast, a woman who makes a college investment does not expect an immediate fertility decline. An ability-θ woman makes a college investment if and only if her ability is above $\theta_{w1} \equiv c_w/z_{wH} - z_{wL} + v_{wH} - v_{wL}$. Note that $\theta_{w1} < \theta_{w2}$: Some women who make a college investment would not make a career investment. In summary, any woman with an ability above θ_{w2} makes a college investment and, in case her college investment fails, makes a career investment; any woman with an ability between θ_{w1} and θ_{w2} makes a college investment only; and any woman with an ability below θ_{w1} makes no investment.

The optimal investments induce distributions of marriage characteristics. Type-H men are those who have an ability above θ_m and receive a high-income offer after a college or career investment, so $G_{mH} = \int_{\theta_m}^{1}[\theta + (1-\theta)\theta]dF_m(\theta)$. Type-$L$ men consist of (i) all men with an ability below θ_m and (ii) men with an ability above θ_m who fail to receive a high income after college and career investments. Because there is a unit mass of men in each period's marriage market, the mass of low-income men is simply $G_{mL} = 1 - G_{mH}$.[2] Type-H women are those with an ability above θ_{w1} who succeed right after college: $G_{wH} = \int_{\theta_{w1}}^{1} \theta dF_w(\theta)$. Type-$h$ women are those with an ability above θ_{w2} who succeed only after their career investment: $G_{wh} = \int_{\theta_{w2}}^{1}(1-\theta)\theta dF_w(\theta)$. Type-$L$ women consist of (i) all women with an ability below θ_{w1} and (ii) women with an ability between θ_{w1} and θ_{w2} who fail after college and do not make a career investment: $G_{wL} = F_w(\theta_{w1}) + \int_{\theta_{w1}}^{\theta_{w2}}(1-\theta)dF_w(\theta)$. Finally, type-$l$ women are those with an ability above θ_{w2} who fail to receive a high income after making both college and career investments: $G_{wl} = 1 - G_{wH} - G_{wL} - G_{wh}$.

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[2] There is a unit mass of men and women in each period's marriage market because of the overlapping-generations nature of the model. The period-t marriage market consists of (i) those who are born in period $t-2$ and enter the marriage market in the third period of their lives, (ii) those who are born in period $t-1$ and enter the marriage market in the second period of their lives, and (iii) those who are in period t and enter the marriage market in the first period of their lives. Because the optimal investments are stationary, there is a unit mass of men and women in each period's marriage market.

3.2.2 Equilibrium Matching and Marriage Payoffs

Equilibrium stable matching is characterized as follows. First, because the marriage surplus is assumed to be strictly supermodular in incomes, given two equally fertile women, a higher-income woman almost surely marries a higher-income man; the modifier "almost surely" is necessary, because the marriage market consists of a continuum of men and women, as standard in the literature (Chiappori and Oreffice, 2008; Chiappori et al., 2012). Second, because the surplus is assumed to be strictly supermodular in husband's income and wife's fertility, given two women with the same income, a more fertile woman almost surely marries a higher-income man. The two results together imply that (i) type-H women almost surely marry higher-income husbands than women of any other type, and (ii) type-l women almost surely marry lower-income husbands than women of any other type. Whether a type-h woman or a type-L woman marries a higher-income husband depends on an additional condition. A type-h woman almost surely marries a man with a higher income than a type-L woman does if and only if $\delta_h > \delta_L$, and a type-L woman almost surely marries a higher-income husband than a type-H woman does if and only if $\delta_h < \delta_L$. In summary, stable matching is positive-assortative in men's income and women's type, provided that women's types are ranked according to (i) $H \succ h \succ L \succ l$ when $\delta_h > \delta_L$; (ii) $H \succ L \succ h \succ l$ when $\delta_h < \delta_L$; or (iii) $H \succ L \sim h \succ l$ when $\delta_h = \delta_L$.

Equilibrium marriage payoffs can be determined only up to a constant. Because there is an equal mass of men and women in the marriage market, there is a positive mass of marriages between the bottom-ranked type-L men and type-l women, so $v_{mL} + v_{wl} = s_{Ll}$, but neither v_{mL} nor v_{wl} is determinate. To determine stable marriage payoffs, it suffices to determine the marriage payoff differences between two adjacently ranked marriage types, because the sums of these differences, coupled with v_{mL} and v_{ml}, can represent any marriage payoff; for example, $v_{mH} = v_{mL} + (v_{mH} - v_{mL})$, and $v_{wH} = (v_{wH} - v_{wh}) + (v_{wh} - v_{wL}) + (v_{wL} - v_{wl}) + v_{wl}$. Appendix A shows the steps in using stability conditions to determine marriage payoff differences.

3.2.3 Equilibrium Existence and Uniqueness

Theorem 1. *An equilibrium exists. Equilibrium investments are uniquely determined, and equilibrium marriage payoffs are uniquely determined up to a constant.*

A key consequence of the derivation of the marriage payoff differences is that the marriage payoff difference between any two types can be represented by men's *marriage premium* $\pi_m = v_{mH} - v_{mL}$. Coupled with the fact that the three optimal investment thresholds, θ_m, θ_{w1}, and θ_{w2}, are uniquely determined by marriage payoff differences, any equilibrium can be simply represented by one real number, π_m. This reduction in dimensionality in the representation of an equilibrium is crucial for constructing the proof of equilibrium existence and uniqueness. The proof, presented in Appendix B, follows three steps. First, construct a correspondence that represents the demand for high-income men in the marriage market and a function that represents the supply. Second, argue that each intersection of the constructed demand and supply curves corresponds to an equilibrium. Third, show that the constructed demand and supply curves always intersect, proving equilibrium existence; and that the demand curve is downward-sloping and the supply curve is upward-sloping so that there is only one intersection between the two curves, proving equilibrium uniqueness.

3.3 Explanations

Proposition 1. *Suppose women are less fertile in the third period, while other primitives are gender-symmetric: investment costs $c_m = c_w$, labor-market opportunities $F_m = F_w$, income premiums $z_{mH} - z_{mL} = z_{wH} - z_{wL}$, and marriage surpluses $s_{HL} = s_{LH}$. Strictly more women than men go to college in equilibrium. Strictly fewer women than men earn a high lifetime income in equilibrium if $G_{mH}(\delta_l) > G_{wH}(\delta_l) + G_{wh}(\delta_l)$.*

Prior studies have explained the college gender gap using gender differences in the psychic and monetary costs of investments, labor-market opportunities, college income premiums, and marital roles. Proposition 1 states that even in a model that does not include any of these

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gender differences, it could be the case that more women than men attend college. Adding any of these gender differences to the model would only reinforce the female-dominated college gender gap. Furthermore, this college gender gap can be sustained even when the gender differences that deter women's college investments are included. Therefore, the model highlights a new fundamental force, rooted in differential fecundity and propagated through the marriage market, that contributes to the global college gender gap. At the same time, the earnings gender gap is maintained; this result is unattainable from previous models that explain the college gender gap without including additional gender differences.

Proof of Proposition 1. I first prove the college gender gap. Suppose by way of contradiction that weakly fewer women than men go to college in equilibrium: $1 - F_w(\theta^*_{w1}) \leq 1 - F_m(\theta^*_m)$. First, since $F_m = F_w$ by assumption, $F_w(\theta^*_{w1}) \geq F_m(\theta^*_m)$ implies $\theta^*_{w1} = c_w/(z_{wH} - z_{wL} + v^*_{wH} - v^*_{wL}) \geq \theta^*_m = c_m/(z_{mH} - z_{mL} + v^*_{mH} - v^*_{mL})$. Since $z_{wH} - z_{wL} = z_{mH} - z_{mL}$ by assumption, $v^*_{wH} - v^*_{wL} \leq v^*_{mH} - v^*_{mL}$.

Second, $\theta^*_{w2} > \theta^*_{w1}$, so strictly fewer women than men make a career investment in equilibrium. Since weakly fewer women go to college by our premise and strictly fewer women make a career investment, strictly fewer women than men earn a high income, i.e., $G^*_{wH} + G^*_{wh} < G^*_{mH}$. As a result, there is a positive mass of type-L women marrying high-income men. By pairwise efficiency, $v^*_{wL} = s_{HL} - v^*_{mH}$. Since there is always a positive mass of (H, H) couples, by pairwise efficiency, $v^*_{wH} = s_{HH} - v^*_{mH}$. The two pairwise efficiency conditions together imply $v^*_{wH} - v^*_{wL} = s_{HH} - s_{HL}$. By $s_{HL} = s_{LH}$, $v^*_{wH} - v^*_{wL} = s_{HH} - s_{HL} = s_{HH} - s_{LH} = \delta_H$. Because a positive mass of type-H men marries type-L women in equilibrium, $v^*_{mH} = s_{HL} - v^*_{wL}$. Furthermore, by Pareto efficiency, $v^*_{mL} \geq s_{LL} - v^*_{wL}$. The two conditions together imply $v^*_{mH} - v^*_{mL} \leq s_{HL} - s_{LL}$. Since the surplus is strictly super-modular in incomes, $v^*_{wH} - v^*_{wL} = \delta_H > \delta_L = v^*_{mH} - v_{mL}$. The two conclusions, $v^*_{wH} - v^*_{wL} \leq v^*_{mH} - v^*_{mL}$ and $v^*_{wH} - v^*_{wL} > v^*_{mH} - v^*_{mL}$, contradict each other. Therefore, there must be strictly more women than men going to college.

I now prove the earnings gender gap. Consider the assumption $G_{mH}(\delta_l) > G_{wH}(\delta_l) +$

$G_{wh}(\delta_l)$. It states that when men's stable marriage premium π_m is δ_l the lowest value possible, mass $G_{mH}(\delta_l)$ of high-income men is strictly greater than the mass $G_{wH}(\delta_l) + G_{wh}(\delta_l)$ of high-income women. That is, even when men have the smallest possible marriage premium $\pi_m = \delta_l = s_{Hl} - s_{Hl}$ and women have the largest possible marriage premium $\pi_w = s_{HH} - s_{HL}$, fewer women will end up with a high income than men. Therefore, the earnings gender gap always holds.   □

While I present a formal proof of the proposition by contradiction, I provide an economic explanation here. Define the *marriage premium* as the difference between the marriage payoffs of a fertile high-income earner and a fertile low-income earner, $\pi_i \equiv v_{iH} - v_{iL}$, $i = m, w$. The ability cutoffs for college investment are simply determined by the investment cost divided by the income premium and the marriage premium. Consider the gender-symmetric setting: $c_m = c_w$, $F_m = F_w$, and $z_{mH} - z_{mL} = z_{wH} - z_{wL}$.

If the marriage premiums π_m and π_w were exogenously fixed to be the same, the same number of men and women would go to college, and fewer women than men would make a career investment because of differential fecundity. Consequently, fewer women than men earn a high income. However, the marriage premiums are endogenously determined in the model. Precisely because fewer women than men earn a high income, women who earn a high income are scarcer and more "valuable" in the marriage market than men who achieve the same feat. Women's endogenously higher marriage premium prompts more women than men to make a college investment. When labor-market opportunities and investment costs are gender-symmetric, more women than men go to college if and only if the endogenous marriage premium is higher for women than for men.

Two drivers of the result are differential fecundity and the equilibrium marriage market. In a gender-symmetric equilibrium model without differential fecundity, the same number of men and women would go to college, make career investments, and earn a high income. In a model that incorporates differential fecundity but omits the equilibrium marriage market (i.e., marriage premiums are exogenously the same for the two genders), the same number

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of men and women would go to college, but fewer women than men would make a career investment and earn a high income; such a model would be able to capture the earnings gender gap, but not the college gender gap.

More precisely, the combination of differential fecundity and endogenous division of a supermodular marriage surplus is necessary to account for the observed gender gaps. Differential fecundity directly reduces women's career investments but does not directly increase their college investments. College and career investments are not direct substitutes to improve income, but endogenous marriage surplus division renders these investments *strategic substitutes*. Specifically, the decline in fertility directly discourages higher-ability college-investing women (women with an ability close to θ_{w2}^*) from making career investments, and indirectly encourages lower-ability women (women with an ability close to θ_{w1}^*) to make college investments through endogenous marriage surplus division. However, if the surplus is not strictly supermodular in incomes, regardless of whether there is differential fecundity, the same number of men and women would go to college.

Proposition 2. *The equilibrium relationship between age at marriage and income for men is hump-shaped. The equilibrium relationship between age at marriage and income for women is positive if $\int_{\theta_{w1}^*}^1 \theta d\theta / [\int_{\theta_{w1}^*}^{\theta_{w2}^*}(1-\theta)d\theta + \int_{\theta_{w1}^*}^1 \theta d\theta] \leq \int_{\theta_{w2}^*}^1 (1-\theta)\theta d\theta / \int_{\theta_{w2}^*}^1 (1-\theta)d\theta$, and hump-shaped otherwise.*

In the model, men who have an ability below θ_m^* marry in the first period; they earn low lifetime income without making any investment. Those who have an ability above θ_m^* and realize a high income after college marry in the second period. The remaining men who have an ability above θ_m^* fail to realize a high income after college, and consequently make a career investment and marry in the third period; some of them receive a high income and the rest receive a low income, so the average income is lower for late grooms than for middle grooms.

For the upward-sloping portion of the relationship, early grooms earn less than middle grooms on average, because early grooms invest less than middle grooms. Bergstrom and

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Bagnoli (1993) also predict a positive relationship between age at marriage and income for men, but there is a difference between their explanation and mine. In their model, high-income men wait to marry because they cannot credibly signal their earning ability when they are young. In contrast, there is no private information in my model. Even if a man can choose to marry during college, he weakly prefers to wait until after he resolves his post-college income uncertainty. The reason for delaying marriage in this model is rooted in the inherent nature of the marriage market. A man who has uncertainty about his future lifetime income may not be able to marry the woman he could marry when he has a high lifetime income for sure, so he chooses to delay marriage.

The downward-sloping portion of the relationship cannot be explained by Bergstrom and Bagnoli (1993), but can be explained by my model, as follows. Middle grooms earn more than late grooms on average, because middle grooms are college-educated men who realize a high income soon after college, and late grooms are college-educated men who fail to do so and end up with a lower income on average. Becker (1974) and Keeley (1979) also predict a negative relationship, but their explanation is different. Whereas higher-ability men in their models marry earlier because they encounter less marriage-market friction, higher-ability men in my model do so because they are less likely to encounter an adverse labor-market shock. Lower-income men involuntarily delay marriage due to marriage-market frictions in their models, but voluntarily delay marriage due to labor-market shocks in my model.

In equilibrium, early brides earn a low income because they are low-ability women (those with an ability below θ^*_{w1}) who do not go to college. Middle brides consist of all intermediate-ability women (those with an ability between θ^*_{w1} and θ^*_{w2}) and higher-ability women (those with an ability above θ^*_{w2}) who earn a high income right after college. Late brides are higher-ability women who do not receive a high income after college. The model predicts that early brides earn less than middle brides and late brides, but does not make a definitive prediction about whether middle brides or late brides earn more.

In the model, early brides earn less than middle brides, because early brides are those who

do not go to college. Middle brides, in contrast, are those who go to college; many end up with a high income. The impact of human capital investment on women's marriage timing is not considered in Becker (1974) and Keeley (1979) (who predict a positive relationship between age at marriage and income due to marriage-market frictions), as well as in Bergstrom and Bagnoli (1993) (who predict no relationship between age at marriage and income for women). My model naturally incorporates this effect.

In the model, middle brides tend to earn less than late brides, because middle brides mostly consist of intermediate-ability women who fail to receive a high income after college but nonetheless choose to marry; late brides are high-ability women who do not receive a high income right out of college but receive a high income with a large probability after career investments. In short, labor-market shocks and the fertility-income trade-off result in positive selection in delayed marriage. Becker (1974) predicts a positive relationship between age at marriage and income driven by marriage-market frictions.

Proposition 3. *The relationship between age at marriage and spousal income for women is hump-shaped in equilibrium.*

Early brides are fertile low-income earners, and middle brides are both fertile low-income earners and fertile high-income earners. Because fertile high-income women's husbands almost always have a higher income than fertile low-income women's, middle brides are predicted to have a higher average spousal income than early brides. Late brides consist of both high-income and low-income earners, but they are less fertile than middle brides. Since (i) for any two women with the same income, the more fertile one marries a higher-income husband in equilibrium, and (ii) late brides do not earn significantly more than middle brides on average, the average spousal income is predicted to be lower for late brides than for middle brides.

The key to explaining whether early brides or late brides marry higher-income husbands is nonassortative matching in incomes (Low, 2020). According to the model, early brides are fertile low-income earners, and late brides are less fertile women with a higher average

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income. If fertility is more important than income in the marriage market (i.e., $\delta_L > \delta_h$), type-L women marry higher-income husbands than type-h women, and consequently early brides' average spousal income would be lower than late brides'. Otherwise (i.e., $\delta_h > \delta_L$), it is possible that less fertile high-income women's husbands have a higher income than fertile low-income women's, and the average spousal income is higher for less fertile women than for fertile low-income women.

The fact that late brides have higher-income husbands than early brides in states with infertility treatment mandates suggests that the evolution of the difference in average spousal income between early brides and late brides is at least partially driven by the change in the relative importance of income and fertility in the marriage market. These changes in the marriage market can be thought of as a decrease in the demand for and/or an increase in the supply of "reproductive capital" (Low, 2020) in the marriage market.

On the one hand, the demand for reproductive capital has decreased. The desired and actual family size has decreased; the average desired number of children declined from 3.6 to 2.6 from 1960 to 2010 (Livingston and Cohn, 2010). Many families have shifted from a demand for quantity of children to a demand for quality, as Becker and Lewis (1973) predicted. Women's fertility has become less of a concern in marriage decisions than women's income and education. Second, an increase in income gain from college and career investments also contributes to a decrease in the relative importance of fertility; the benefit of the career investment in the labor market outweighs the cost of delayed marriage in the marriage market.

On the other hand, an increase in the supply of reproductive capital has been achieved by advances in medical technology such as in vitro fertilization, egg freezing, and more cost-effective maternal health services, all of which have resulted in a higher probability of remaining fertile and conceiving. Older women can have children with less financial burden, more physical ease, and fewer adverse health effects than in the past. Gershoni and Low (2020, ming) present causal evidence that policies that have made assistive reproduc-

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tive technology less expensive and more accessible directly improved the labor and marital outcomes of Israeli women who married late.

4   Conclusion

I build an equilibrium investment-and-marriage model with one gender asymmetry, differential fecundity, to account for a set of phenomena that have not previously been explained under a unified framework. Most notably, I (i) provide a new explanation of the college gender gap and the earnings gender gap, based on differential fecundity and an equilibrium marriage market, (ii) explain the relationships between age at marriage and personal income for both men and women, and (iii) encapsulate the previous explanation of the relationship between age at marriage and spousal income for women. I hope that this simple model serves as a stepping stone for future theoretical and empirical research.

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Appendix A.   Stable Marriage Payoff Differences

Figure A1 illustrates the two cases of the determination of stable marriage payoff difference between any two adjacently ranked types. In case (a), type-τ^* women marry both high-income men and low-income men with positive probabilities, so $v_{mH} + v_{w\tau^*} = s_{H\tau^*}$ and $v_{mL} + v_{w\tau^*} = s_{L\tau^*}$, which together imply $\pi_m = v_{mH} - v_{mL} = \delta_{\tau^*}$; the marriage payoff difference between any two adjacently ranked female types can be determined similarly, with the dashed arrows denoting the line of reasoning. In case (b), type $\tau \succeq \tau^*$ women almost surely marry high-income men, and type $\tau \prec \tau^*$ women almost surely marry low-income men. Since there is no type of women that marries both types of men with positive probabilities, π_m is indeterminate, and it can take any value between δ_τ and δ_{τ^*}, where τ is the female type ranked just below τ^*. This indeterminacy in π_m will dissipate in equilibrium, however, when marriage payoffs and investments are jointly determined. For women, almost all $\tau' \succeq \tau^*$ women marry high-income men and almost all $\tau \prec \tau^*$ women marry low-income men, so $v_{w\tau'} - v_{w\tau} = s_{H\tau'} - s_{L\tau} - \pi_m$.
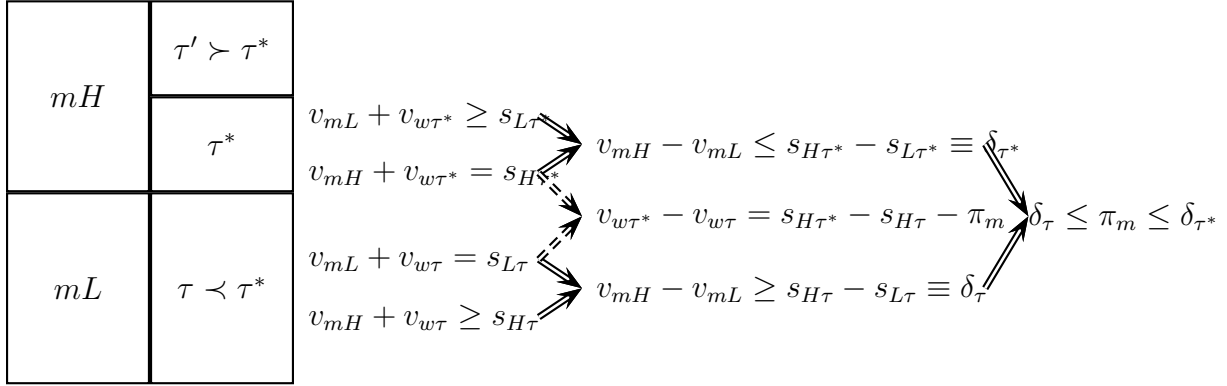
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Figure A1: Determination of Stable Marriage Payoff Difference

(a) **The mass of high-income men is strictly between the mass of women strictly higher ranked than τ^* and the mass of women weakly higher ranked than type τ^* for some $\tau^* \in T$.**
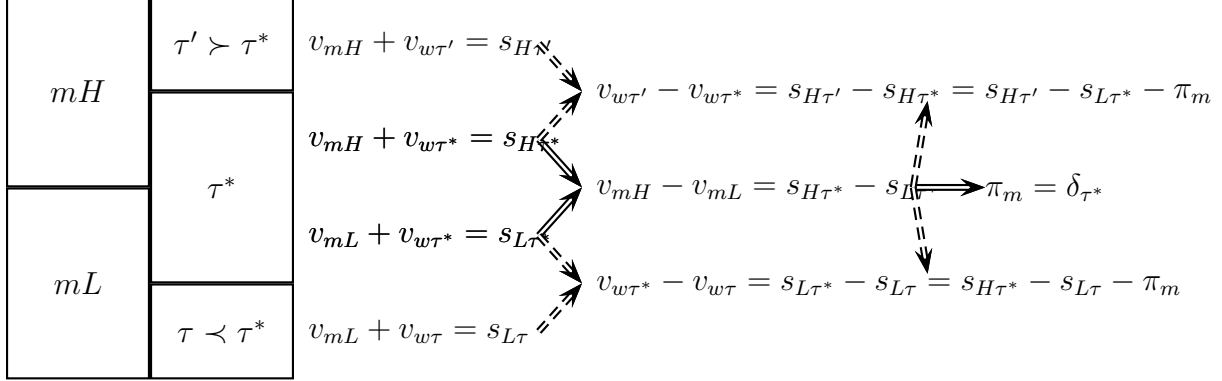
(b) **The mass of high-income men equals the mass of women weakly higher ranked than τ^* for some type $\tau^* \in T$.**

Appendix B.   Proof of Theorem 1

Let $\theta_m(\pi_m)$, $\theta_{w1}(\pi_m)$, and $\theta_{w2}(\pi_m)$ denote the ability cutoffs characterizing optimal human capital investments when men's stable marriage premium is π_m (and women's stable marriage-payoff differences are pinned down by π_m). Let $G_m(\pi_m)$ and $G_w(\pi_m)$ denote the induced distributions of men's and women's marriage characteristics, respectively, when the investment strategies are the ones characterized by the ability cutoffs $\theta_m(\pi_m)$, $\theta_{w1}(\pi_m)$, and $\theta_{w2}(\pi_m)$. Let $\Pi_m(G_m, G_w)$ denote the set of men's stable marriage premiums (and associated stable marriage payoffs of women) in the marriage market (G_m, G_w). Construct the

correspondence

$$D_{mH}(\pi_m) := \{G_{mH} \in [0,1] : \pi_m \in \Pi_m((G_{mH}, 1 - G_{mH}), G_w(\pi_m))\}.$$

For any $\pi_m \in [\delta_l, \delta_H]$, each element in the set $D_{mH}(\pi_m)$ is a mass G_{mH} of high-income men such that π_m is men's stable marriage premium in the marriage market $((G_{mH}, 1 - G_{mH}), G_w(\pi_m))$. Explicitly, (i) $D_{mH}(\pi_m) = [G_{w, \succ \tau_w^*}(\pi_m), G_{w, \succeq \tau_w^*}(\pi_m)]$ if $\pi_m = \delta_{\tau_w^*}$ for a certain type $\tau_w^* \in T_w$; and (ii) $D_{mH}(\pi_m) = G_{w, \succeq \tau_w^*}(\pi_m)$ if $\pi_m \in (\delta_{\tau_w'}, \delta_{\tau_w^*})$ for a pair of adjacently ranked types τ_w^* and $\tau_w' \prec \tau_w^*$.

I prove the claim that there exists an equilibrium in which men's stable marriage premium is π_m^* if and only if $G_{mH}(\pi_m^*) \in D_{mH}(\pi_m^*)$. First, the only if part. Suppose men's equilibrium marriage premium is π_m^*. The induced mass of high-income men is $G_{mH}(\pi_m^*)$, and the induced distribution of women's marriage characteristics is $G_w(\pi_m^*)$. Since $\pi_m^* \in \Pi_m((G_{mH}(\pi_m^*), 1 - G_{mH}(\pi_m^*)), G_w(\pi_m^*))$, by definition of $D_{mH}(\pi_m^*)$, we have $G_{mH}(\pi_m^*) \in D_{mH}(\pi_m^*)$. Reversely, the if only part. If $G_{mH}(\pi_m^*) \in D_{mH}(\pi_m^*)$, then by definition of $D_{mH}(\pi_m^*)$, $\pi_m^* \in \Pi_m((G_{mH}(\pi_m^*), 1 - G_{mH}(\pi_m^*)), G_w(\pi_m^*))$, so π_m^* is men's equilibrium marriage premium.

It follows from the claim above that an equilibrium exists if and only if the graph of function $G_{mH}(\cdot)$ and the graph of correspondence $D_{mH}(\cdot)$ intersect at least once. Equilibrium marriage-payoff differences and equilibrium investments are uniquely determined if and only if the graph of function $G_{mH}(\cdot)$ and the graph of correspondence $D_{mH}(\cdot)$ intersect once and only once. The existence of an equilibrium is guaranteed because $G_{mH}(\cdot)$ has a range $[0,1]$ and is continuous, and $D_{mH}(\cdot)$ has a range $[0,1]$ and is upperhemicontinuous.

It remains to prove equilibrium uniqueness. $G_{mH}(\pi_m) = \int_{\theta_m(\pi_m)}^1 \theta(2-\theta) dF_m(\theta)$ is strictly increasing in π_m because $\theta_m(\pi_m) = c_m/(z_{mH} - z_{mL} + \pi_m)$ is strictly decreasing in π_m. It suffices to show $D_{mH}(\pi_m)$ is weakly decreasing in the following sense: for any π_m and $\pi_m' > \pi_m$, $\max D_{mH}(\pi_m') \leq \min D_{mH}(\pi_m)$. For the remainder of the proof, we mechanically

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show that $D_{mH}(\pi_m)$ is decreasing. Depending on $\delta_h > \delta_L$, $\delta_h < \delta_L$, or $\delta_h = \delta_L$, $D_{mH}(\pi_m)$ is characterized differently. I discuss the three cases separately.

Case 1. Suppose $\delta_L > \delta_h$. Explicitly,

$$
D_{mH}(\pi_m) = \begin{cases}
[G_{w,\succeq h}(\pi_m), 1] & \text{if } \pi_m = \delta_l \\[4pt]
G_{w,\succeq h}(\pi_m) & \text{if } \pi_m \in (\delta_l, \delta_h) \\[4pt]
[G_{w,\succeq L}(\pi_m), G_{w,\succeq h}(\pi_m)] & \text{if } \pi_m = \delta_h \\[4pt]
G_{w,\succeq L}(\pi_m) & \text{if } \pi_m \in (\delta_h, \delta_L) \\[4pt]
[G_{wH}(\pi_m), G_{w,\succeq L}(\pi_m)] & \text{if } \pi_m = \delta_L \\[4pt]
G_{wH}(\pi_m) & \text{if } \pi_m \in (\delta_L, \delta_H) \\[4pt]
[0, G_{wH}(\pi_m)] & \text{if } \pi_m = \delta_H
\end{cases}
$$

It remains to show that (i) $G_{w,\succeq h}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_h)$, (ii) $G_{w,\succeq L}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$, and (iii) $G_{wH}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_L, \delta_H)$.

(i) To show $G_{w,\succeq h}(\pi_m) = 1 - \int_{\theta_{w2}(\pi_m)}^1 (1-\theta)^2 dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_h)$, it suffices to show $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_h)$. Men's stable marriage premium can be $\pi_m \in (\delta_l, \delta_h)$ only when $G_{mH} = G_{w,\succeq h}$. When $G_{mH} = G_{w,\succeq h}$, given men's stable marriage premium π_m, women's stable marriage-payoff differences are $v_{wL} - v_{wl} = s_{HL} - s_{Ll} - \pi_m$, $v_{wH} - v_{wL} = s_{HH} - s_{HL}$, and $v_{wh} - v_{wl} = s_{Hh} - s_{Ll} - \pi_m$, so

$$
\begin{aligned}
\theta_{w2}(\pi_m) &= \frac{c_w + (v_{wL} - v_{wl})}{z_{wH} - z_{wL} + (v_{wh} - v_{wl})} = \frac{c_w + (s_{HL} - s_{Ll} - \pi_m)}{z_{wH} - z_{wL} + (s_{Hh} - s_{Ll} - \pi_m)} \\[6pt]
&= \frac{c_w + (s_{HL} - s_{Ll}) - \pi_m}{z_{wH} - z_{wL} + (s_{Hh} + s_{Ll}) - \pi_m}.
\end{aligned}
$$

Since $\theta_{w2}(\pi_m) < 1$, $\theta'_{w2}(\pi_m) < 0$ when $\pi_m \in (\delta_l, \delta_h)$.

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(ii) To show $G_{w,\succeq L}(\pi_m) = 1 - \int_{\theta_{w2}(\pi_m)}^{1}(1-\theta)dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$, it suffices to show $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$. Men's stable marriage premium can be $\pi_m \in (\delta_h, \delta_L)$ only when $G_{mH} = G_{w,\succeq L}$. When $G_{mH} = G_{w,\succeq L}$, given men's stable marriage premium π_m, women's stable marriage-payoff differences are $v_{wL} - v_{wl} = s_{HL} - s_{Ll} - \pi_m$, $v_{wH} - v_{wL} = s_{HH} - s_{HL}$, and $v_{wh} - v_{wl} = s_{Lh} - s_{Ll}$, so

$$\theta_{w2}(\pi_m) = \frac{c_w + (s_{HL} - s_{Ll} - \pi_m)}{z_{wH} - z_{wL} + (s_{Lh} - s_{Ll})}.$$

Therefore, $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$.

(iii) To show $G_{wH}(\pi_m) = \int_{\theta_{w1}(\pi_m)}^{1}\theta dF_w(\theta) + \int_{\theta_{w2}(\pi_m)}^{1}(1-\theta)\theta dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_L, \delta_H)$, it suffices to show $\theta_{w1}(\pi_m)$ and $\theta_{w2}(\pi_m)$ are strictly increasing when $\pi_m \in (\delta_L, \delta_H)$. Men's stable marriage premium is $\pi_m \in (\delta_L, \delta_H)$ only when $G_{mH} = G_{wH}(\pi_m)$. When $G_{mH} = G_{wH}$, given men's stable marriage premium π_m, women's stable marriage-payoff differences are $v_{wL} - v_{wl} = s_{LL} - s_{Ll}$, $v_{wH} - v_{wL} = s_{HL} - s_{Ll} - \pi_m$, and $v_{wh} - v_{wl} = s_{Lh} - s_{Ll}$, so

$$\theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + s_{HH} - s_{LL} - \pi_m},$$

and

$$\theta_{w2}(\pi_m) = \frac{c_w + (s_{LL} - s_{Ll})}{z_{wH} - z_{wL} + (s_{Lh} - s_{Ll})}.$$

Therefore, both $\theta_{w1}(\pi_m)$ and $\theta_{w2}(\pi_m)$ are increasing when $\pi_m \in (\delta_L, \delta_H)$.

Case 2. Suppose $\delta_h \geq \delta_L$. Explicitly,

$$
D_{mH}(\pi_m) = \begin{cases}
[G_{w,\succeq L}(\pi_m), 1] & \text{if } \pi_m = \delta_l \\[1ex]
G_{w,\succeq L}(\pi_m) & \text{if } \pi_m \in (\delta_l, \delta_L) \\[1ex]
[G_{w,\succeq h}(\pi_m), G_{w,\succeq L}(\pi_m)] & \text{if } \pi_m = \delta_L \\[1ex]
G_{w,\succeq h}(\pi_m) & \text{if } \pi_m \in (\delta_L, \delta_h) \\[1ex]
[G_{wH}(\pi_m), G_{w,\succeq h}(\pi_m)] & \text{if } \pi_m = \delta_h \\[1ex]
G_{wH}(\pi_m) & \text{if } \pi_m \in (\delta_h, \delta_H) \\[1ex]
[0, G_{wH}(\pi_m)] & \text{if } \pi_m = \delta_H
\end{cases} .
$$

It suffices to show that (i) $G_{w,\succeq L}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_L)$, (ii) $G_{w,\succeq h}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_L, \delta_h)$, and (iii) $G_{wH}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_H)$.

(i) To show $G_{w,\succeq L}(\pi_m) = 1 - \int_{\theta_{w2}(\pi_m)}^{1}(1 - \theta)^2 dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_L)$, it suffices to show $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_L)$. Men's stable marriage premium can be $\pi_m \in (\delta_l, \delta_L)$ only when $G_{mH} = G_{w,\succeq L}$. When $G_{mH} = G_{w,\succeq L}$, given men's stable marriage premium π_m, women's stable marriage-payoff differences are $v_{wL} - v_{wl} = s_{HL} - s_{Ll} - \pi_m$, $v_{wH} - v_{wL} = s_{HH} - s_{HL}$, and $v_{wh} - v_{wl} = s_{Hh} - s_{Hl} - \pi_m$, so

$$
\theta_{w2}(\pi_m) = \frac{c_w + (s_{HL} - s_{Ll} - \pi_m)}{z_{wH} - z_{wL} + (s_{Hh} - s_{Hl} - \pi_m)}.
$$

Since $\theta_{w2}(\pi_m) < 1$, $\theta'_{w2}(\pi_m) < 0$ when $\pi_m \in (\delta_l, \delta_L)$.

(ii) To show $G_{w,\succeq h}(\pi_m)$, it suffices to show both $\theta_{w1}(\pi_m)$ and $\theta_{w2}(\pi_m)$ are strictly increasing when $\pi_m \in (\delta_h, \delta_L)$. Men's stable marriage payoff can be $\pi_m \in (\delta_h, \delta_L)$ only when $G_{mH} = G_{w,\succeq h}$. When $G_{mH} = G_{w,\succeq h}$, given men's stable marriage premium π_m, women's

stable marriage-payoff differences are $v_{wH} - v_{wL} = s_{HH} - s_{LL} - \pi_m$, $v_{wL} - v_{wl} = s_{LL} - s_{Ll}$, and $v_{wh} - v_{wl} = s_{Hh} - s_{Ll} - \pi_m$, so

$$\theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + (s_{HH} - s_{LL} - \pi_m)}$$

and

$$\theta_{w2}(\pi_m) = \frac{c_w + (s_{LL} - s_{Ll})}{z_{wH} - z_{wL} + (s_{Hh} - s_{Ll} - \pi_m)}.$$

Therefore, both $\theta_{w1}(\pi_m)$ and $\theta_{w2}(\pi_m)$ are strictly increasing when $\pi_m \in (\delta_L, \delta_h)$.

(iii) To show $G_{wH}(\pi_m) = \int_{\theta_{w1}(\pi_m)}^{1} \theta \, dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_H)$, it suffices to show $\theta_{w1}(\pi_m)$ is strictly increasing when $\pi_m \in (\delta_h, \delta_L)$. Men's stable marriage premium can be $\pi_m \in (\delta_h, \delta_L)$ only when $G_{mH} = G_{wH}$. When $G_{mH} = G_{wH}$, given men's stable marriage premium π_m, women's stable marriage-payoff difference $v_{wH} - v_{wL} = s_{HH} - s_{LL} - \pi_m$, so

$$\theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + s_{HH} - s_{LL} - \pi_m}.$$

Therefore, $\theta_{w1}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_h, \delta_L)$.

Case 3. Suppose $\delta_h = \delta_L$. Types are ranked as $H \succ L \sim h \succ l$. Let $\tau_2 := L \sim h$. Explicitly,

$$D_{mH}(\pi_m) = \begin{cases} [G_{w, \succeq \tau_2}(\pi_m), 1] & \text{if } \pi_m = \delta_l \\ G_{w, \succeq \tau_2}(\pi_m) & \text{if } \pi_m \in (\delta_l, \delta_{\tau_2}) \\ [G_{wH}(\pi_m), G_{w, \succeq \tau_2}(\pi_m)] & \text{if } \pi_m = \delta_{\tau_2} \\ G_{wH}(\pi_m) & \text{if } \pi_m \in (\delta_{\tau_2}, \delta_H) \\ [0, G_{wH}(\pi_m)] & \text{if } \pi_m = \delta_H \end{cases}.$$

It remains to show that (i) $G_{w,\succeq\tau_2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$, and (ii) $G_{wH}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$.

(i) To show $G_{w,\succeq\tau_2}(\pi_m) = 1 - \int_{\theta_{w2}(\pi_m)}^{1}(1-\theta)^2 dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$, it suffices to show $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$. Men's stable marriage premium can be $\pi_m \in (\delta_l, \delta_L)$ only when $G_{mH} = G_{w,\succeq\tau_2}$. When $G_{mH} = G_{w,\succeq\tau_2}$, given men's stable marriage premium π_m, women's stable marriage-payoff differences are $v_{wL} - v_{wl} = s_{HL} - s_{Ll} - \pi_m$, $v_{wH} - v_{wL} = s_{HH} - s_{HL}$, and $v_{wh} - v_{wl} = s_{Hh} - s_{Hl} - \pi_m$, so

$$\theta_{w2}(\pi_m) = \frac{c_w + s_{HL} - s_{Ll} - \pi_m}{z_{wH} - z_{wL} + s_{Hh} - s_{Hl} - \pi_m}.$$

Since $\theta_{w2}(\pi_m) < 1$, $\theta_{w2}(\pi_m)$ is strictly decreasing when $\pi_m \in (\delta_l, \delta_{\tau_2})$.

(ii) To show $G_{wH}(\pi_m) = \int_{\theta_{w1}(\pi_m)}^{1}\theta dF_w(\theta)$ is strictly decreasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$, it suffices to show $\theta_{w1}(\pi_m)$ is strictly increasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$. Men's stable marriage premium can be π_m only when $G_{mH} = G_{wH}$. When $G_{mH} = G_{wH}$, given men's stable marriage premium π_m, women's stable marriage-payoff difference $v_{wH} - v_{wL} = s_{HH} - s_{LL} - \pi_m$, so

$$\theta_{w1}(\pi_m) = \frac{c_w}{z_{wH} - z_{wL} + s_{HH} - s_{LL} - \pi_m}.$$

Therefore, $\theta_{w1}(\pi_m)$ is strictly increasing when $\pi_m \in (\delta_{\tau_2}, \delta_H)$.   *QED*