Pre-Matching Gambles

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  - Financial portfolios $\rightarrow$ entrepreneurs-investors market

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   - A carefully designed tax scheme yields a unique efficient equilibrium with reduced income inequality.

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3. Applications to efficiency, inequality, and tax.

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3. Moderately risk-averse agents prefer to take unfair gambles.

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- $\sigma_m(\widehat{x}_m)$ and $\sigma_w(\widehat{x}_w)$ represent gambling choices.

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  - The map from (v_m, v_w) to (σ_m, σ_w) is continuous.

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Claim

In general, a person can prefer a second-order stochastically dominated investment gamble with lower expected matching characteristics. (This result helps to rationalize observed seemingly irrational/risk-loving career choice, for example, entrepreneurship).

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- x_m and x_w share the entire surplus,

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- x_m marries woman $x_w(x_m)$ that gives him highest payoff,

$$\mathbf{x}_w(x_m) \in \text{argmax}_{x_w \in \text{supp}(\mu_w)}[s(x_m, x_w) - v_w(x_w)].$$

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$$\underbrace{\mathbb{E}\left[s\left(x_m, \widehat{x}_w\right) \cancel{-v_w\left(\widehat{x}_w\right)}\right] - \left[s\left(\widehat{x}_m, \widehat{x}_w\right) \cancel{-v_w\left(\widehat{x}_w\right)}\right]}_{\text{surplus contribution effect}}$$
$$+$$
$$\underbrace{\mathbb{E}\left\{\left[s\left(x_m, \mathbf{x}_w(x_m)\right) - v_w\left(\mathbf{x}_w(x_m)\right)\right] - \left[s\left(x_m, \widehat{x}_w\right) - v_w\left(\widehat{x}_w\right)\right]\right\}}_{\text{competitive rematching effect}\geq 0}$$

Competitive Rematching Effect under ITU

$$
\begin{gathered}
\mathbb{E}\left[v_m\left(x_m\right)\right]-v_m\left(\widehat{x}_m\right) \\
= \\
\mathbb{E}\phi\left(x_m, \mathbf{x}_w\left(x_m\right), v_w\left(\mathbf{x}_w\left(x_m\right)\right)\right)-\phi\left(\widehat{x}_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right) \\
-\mathbb{E}\phi\left(x_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right)+\mathbb{E}\phi\left(x_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right) \\
= \\
\underbrace{\mathbb{E}\phi\left(x_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right)-\phi\left(\widehat{x}_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right)}_{\text{surplus contribution effect}} \\
+ \\
\underbrace{\mathbb{E}\left\{\phi\left(x_m, \mathbf{x}_w\left(x_m\right), v_w\left(\mathbf{x}_w\left(x_m\right)\right)\right)-\phi\left(x_m, \widehat{x}_w, v_w\left(\widehat{x}_w\right)\right)\right\}}_{\text{competitive rematching effect}\geq 0}
\end{gathered}
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  2. Fixed supply of status goods (one-sidedness).

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- Another implication is that efficiency leads to inevitable inequality.

An Example with Two Equilibria

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- Mass 1 of characteristics 2 men.

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- Surplus $s(x_m, x_w) = x_m x_w$.

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   - $SW^* = (0.5)(3)(3) + (0.5)(1)(1) = 5$.

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2. The gambling equilibrium creates inequality.
3. The government has no revenue.

Remedy 1: Tax on Matching Payoffs

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3. Government generates positive tax revenue
   - $\tau = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot (-0.5) = \frac{1}{2}$.

Remedy 2: Tax on Matching Types (Incomes)

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3. Government generates positive tax revenue
   - $\tau = \frac{1}{2} \cdot (0.5) + \frac{1}{2} \cdot (0.5) = 1$.

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- Two-sided gambling could be socially efficient but cause inequality; could be equal but socially inefficient. (Carefully designed) taxation could eliminate inefficiency, mitigate inequality, and generate positive revenue.

THANK YOU!

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