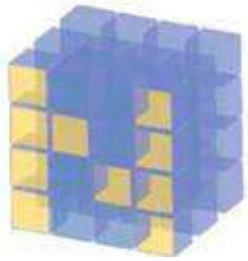
Image Transformation Based on Complex Mapping

PPT by Hanzhi Zhou

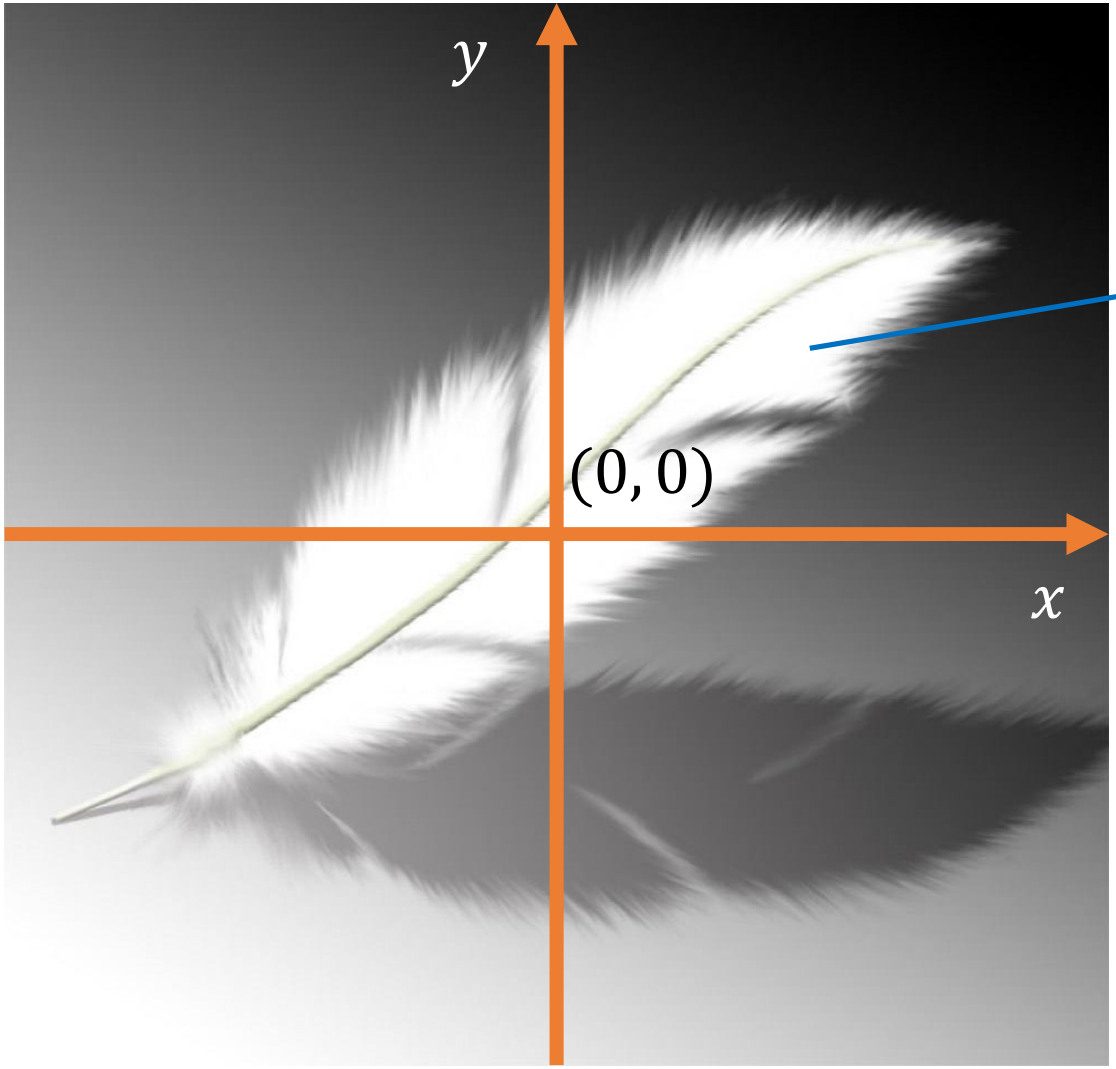


Numpy

The fundamental package needed for scientific computing with Python

matplotlib

Background Information



$$\mathbb{R}^2 \mapsto \mathbb{C}$$

$$(x, y) \mapsto x + yi$$

$$f(x + yi) = z \mapsto (Re(z), Im(z))$$

Complex Mapping f: $\mathbb{C} \mapsto \mathbb{C}$

$$f(z) = c \cdot z, where\ c \in \mathbb{C}$$

$$f(z) = z^a, where\ a \in \mathbb{R}$$

$$f(z) = \frac{a}{z}, where\ a \in \mathbb{R}$$

$$f(z) = \frac{a}{Re(z)} + i \cdot \frac{a}{Im(z)}, where\ a \in \mathbb{R}$$

$$f(z) = |z| * e^{i \cdot g(arg(z))}, where\ g: \mathbb{R} \mapsto \mathbb{R}$$

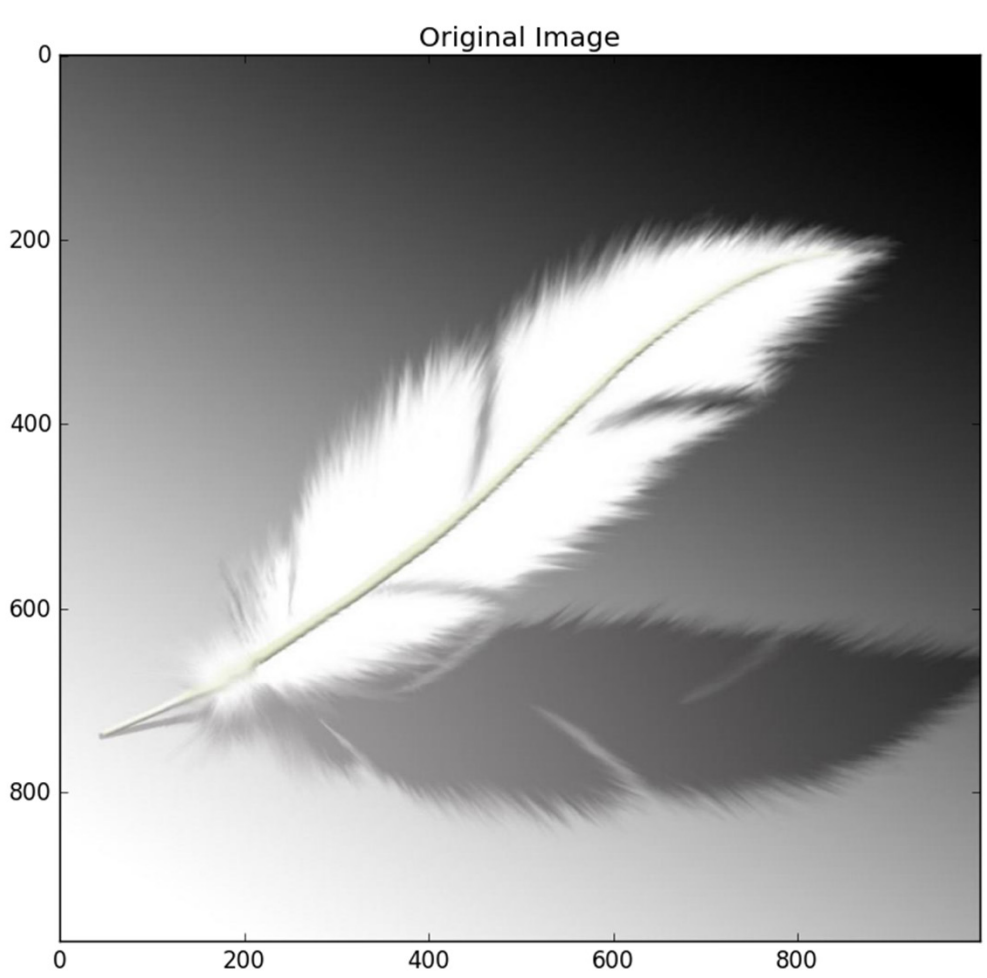
$$f(z) = c * z$$

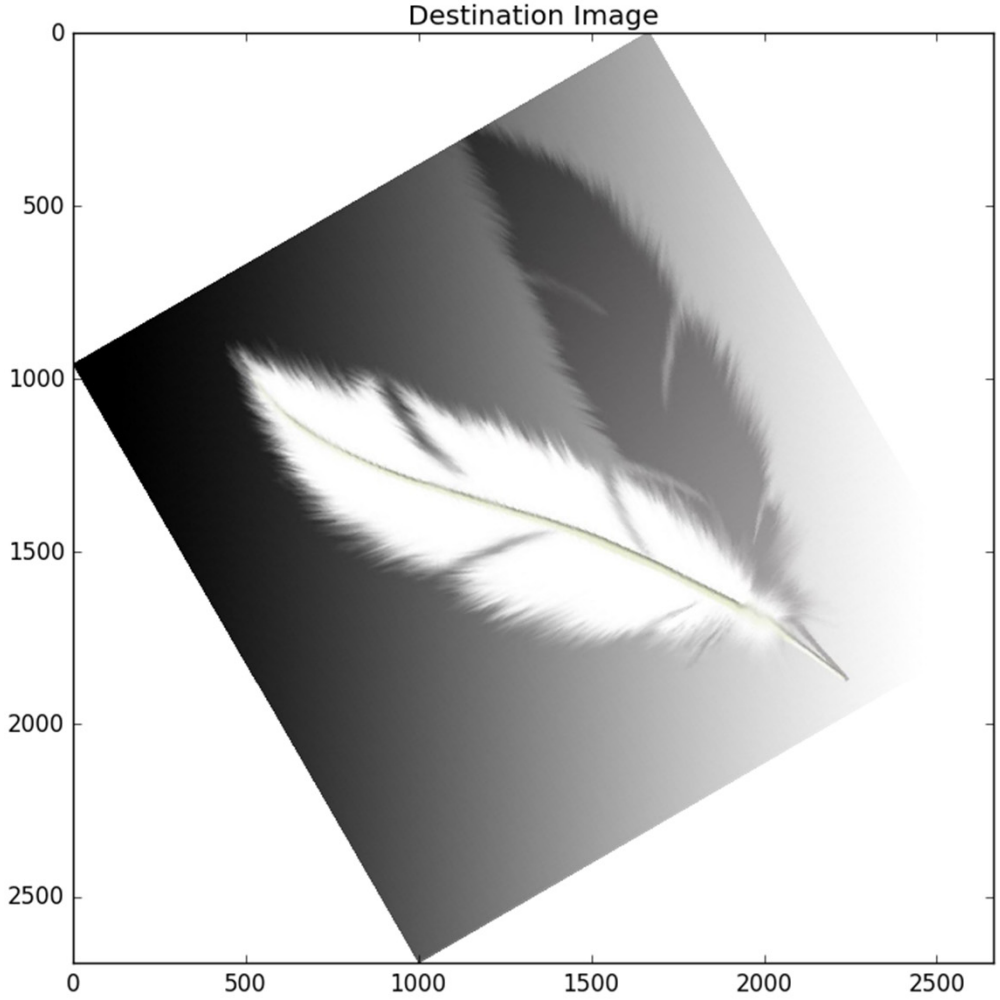
$$c * z = |c||z|e^{i*[\arg(c)+\arg(z)]}$$

Magnification by scale factor |c|

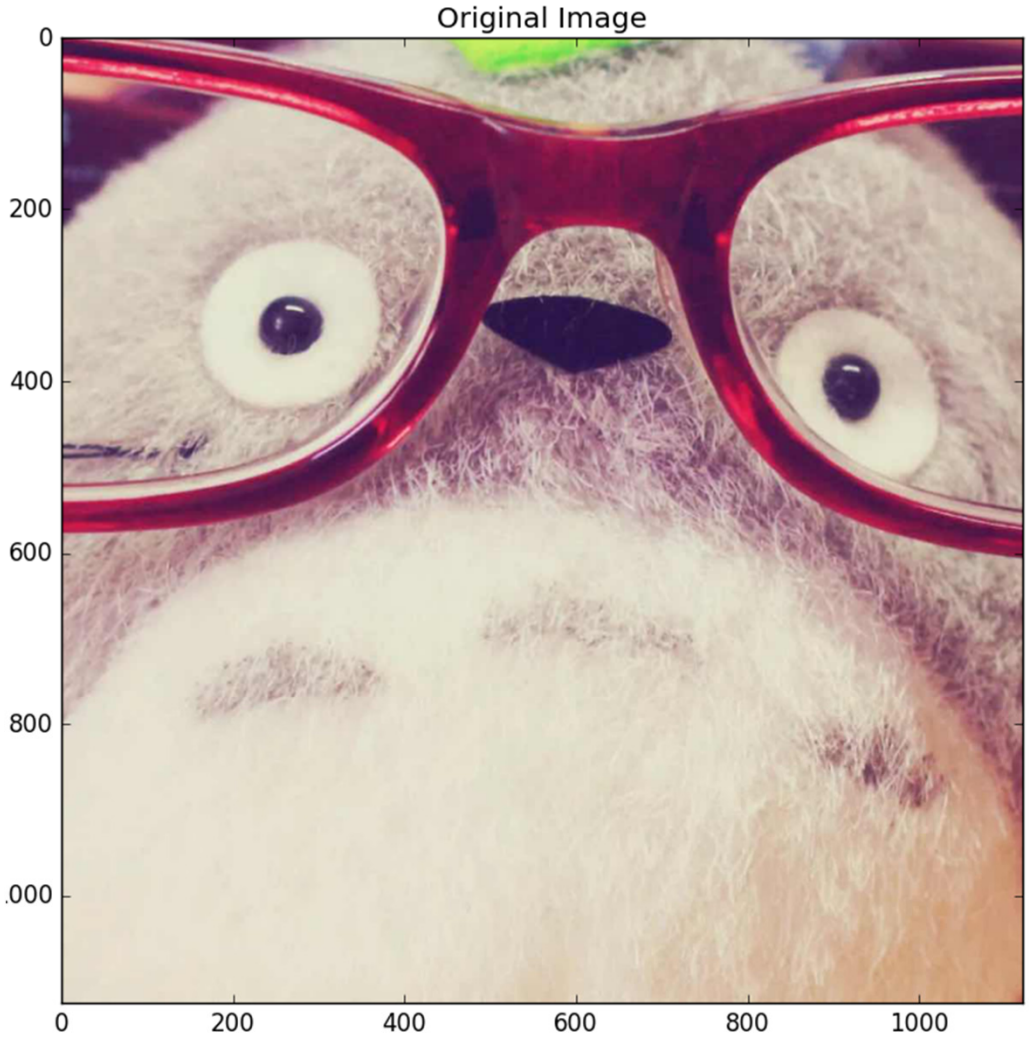
counterclockwise rotation by arg(c)

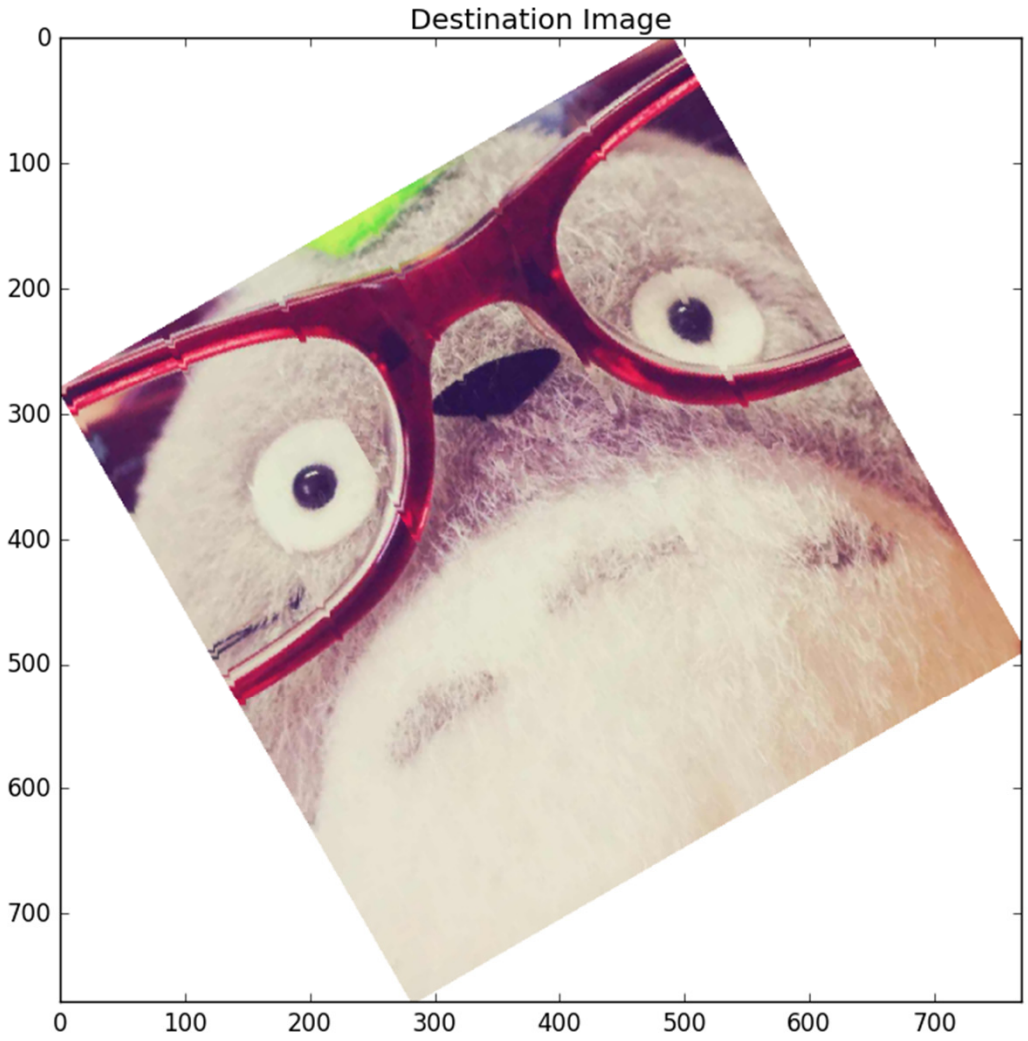
$$f(z) = c \cdot z, where\ z \in \mathbb{C}, c = 2e^{i\frac{2\pi}{3}}$$





$$f(z) = c \cdot z, where\ z \in \mathbb{C}, c = \frac{1}{2} e^{i\frac{\pi}{6}}$$





$$f(z) = z^a, where\ a \in \mathbb{R}$$

$$z^a = |z|^a \cdot e^{ia\theta}$$

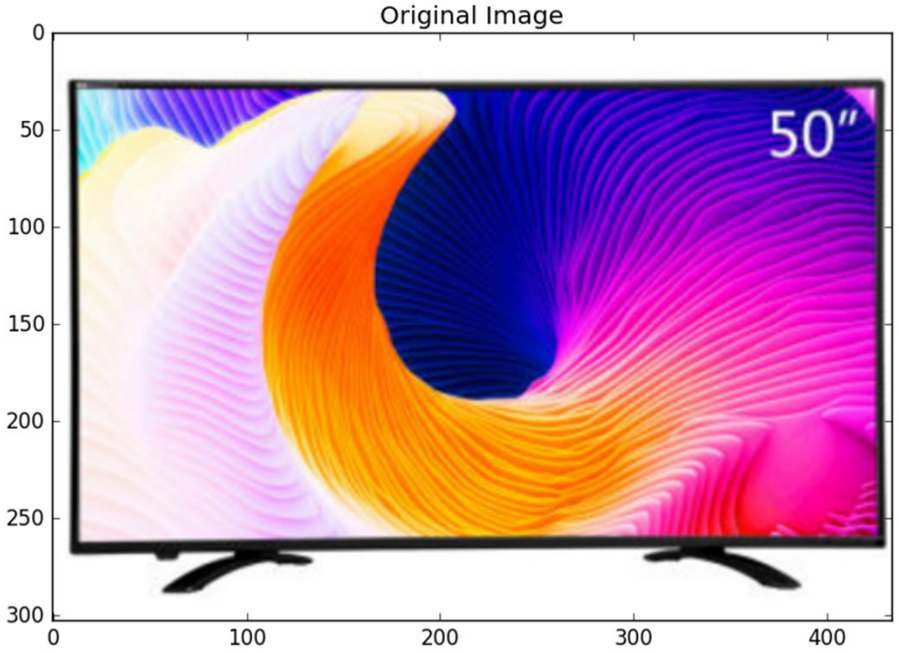
Particularly

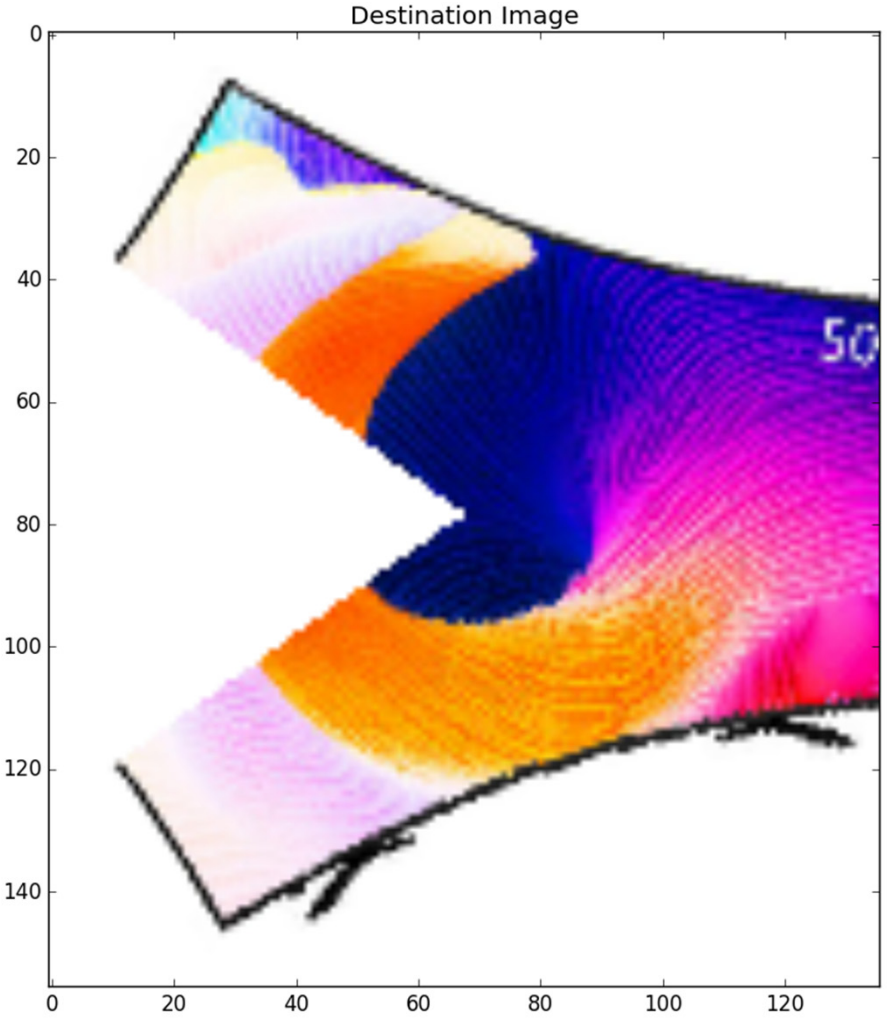
$$z^{-1} = \frac{1}{z} = \frac{1}{|z|} \cdot e^{i(-\theta)}$$

Magnification by scale factor $|z|^{a-1}$

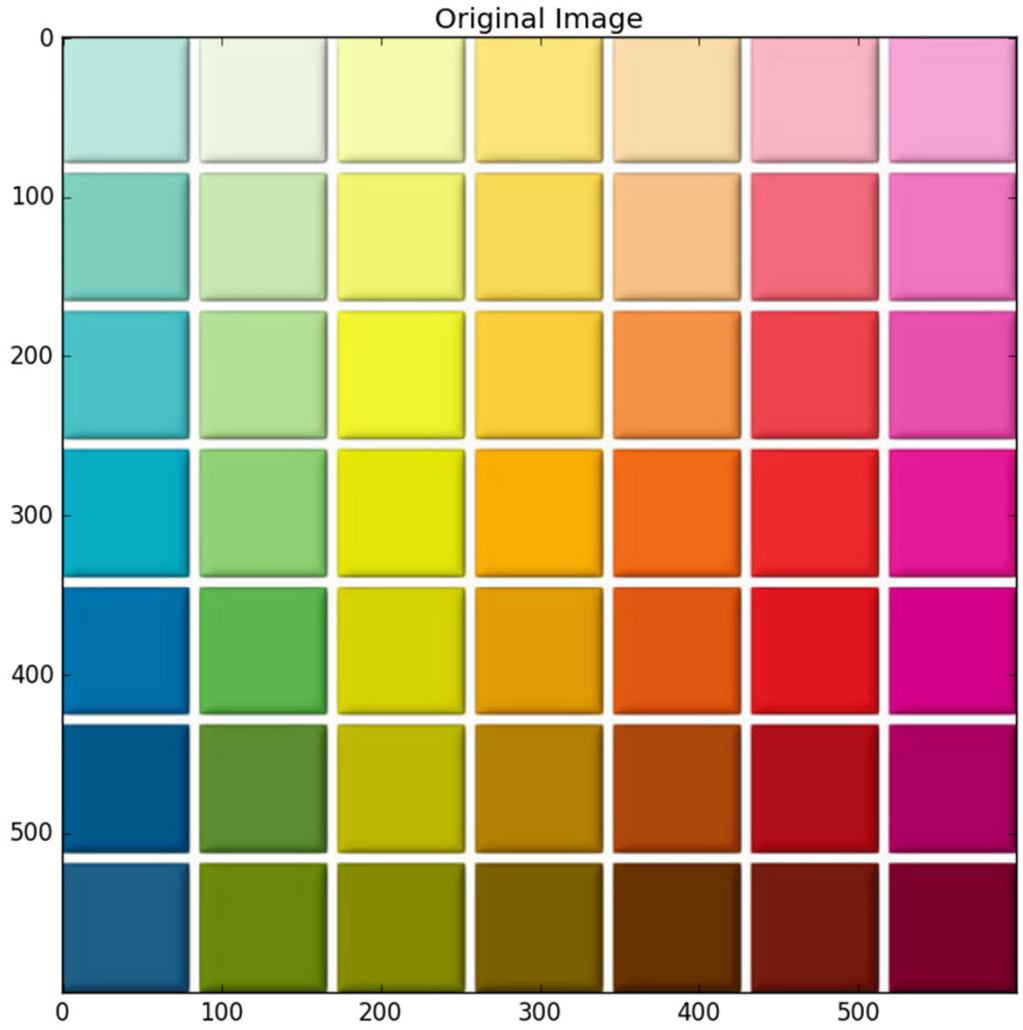
Counterclockwise rotation by $(a-1)\theta$

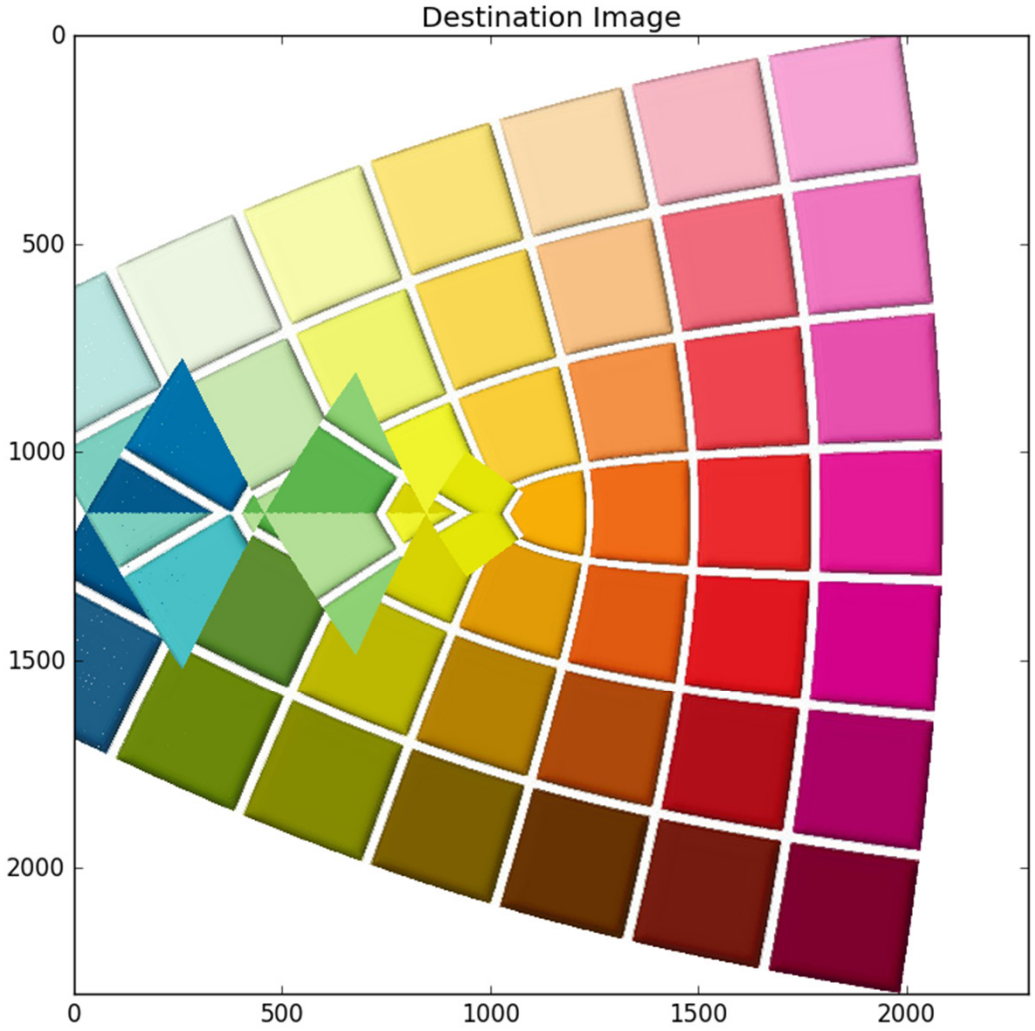
$$f(z) = z^a, where\ a = 0.8$$



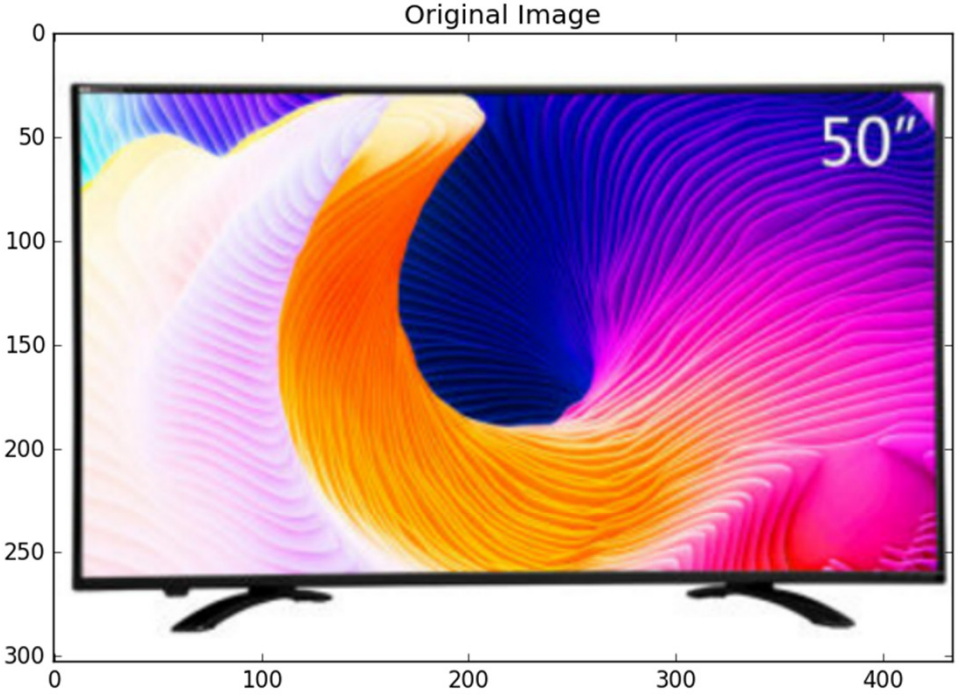


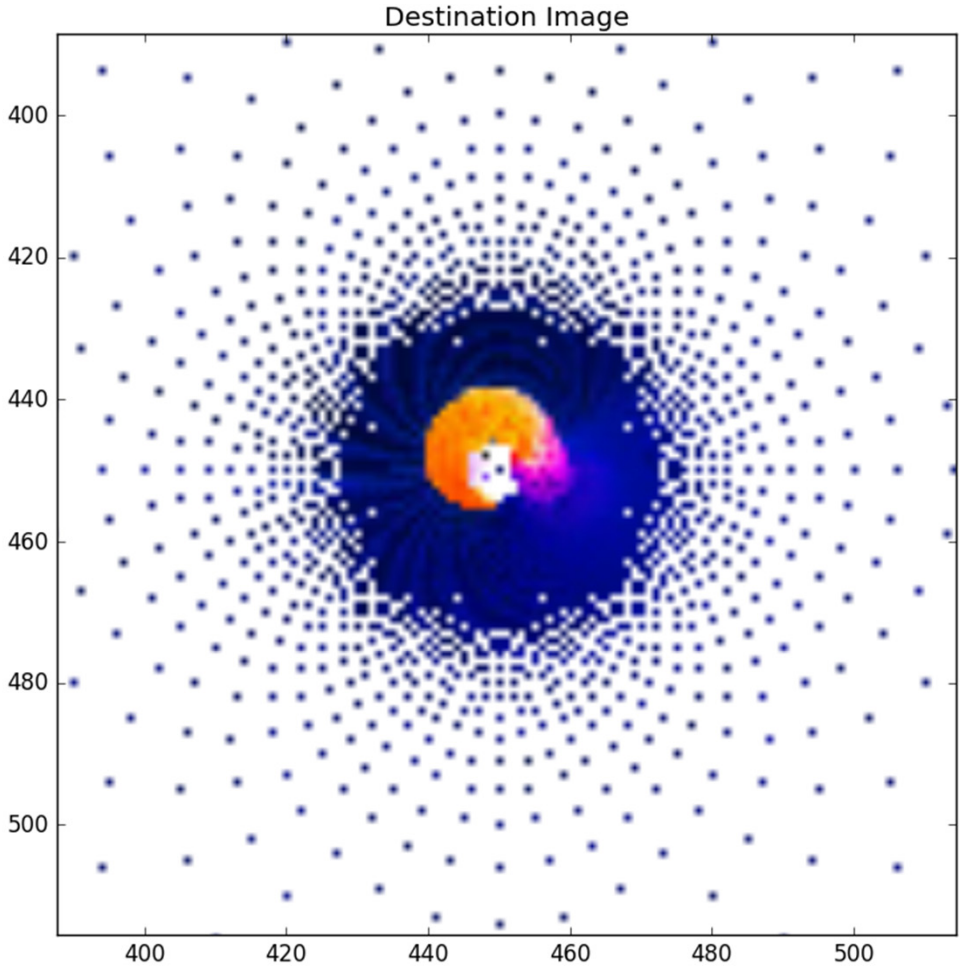
$$f(z) = z^a, where\ a = 1.2$$



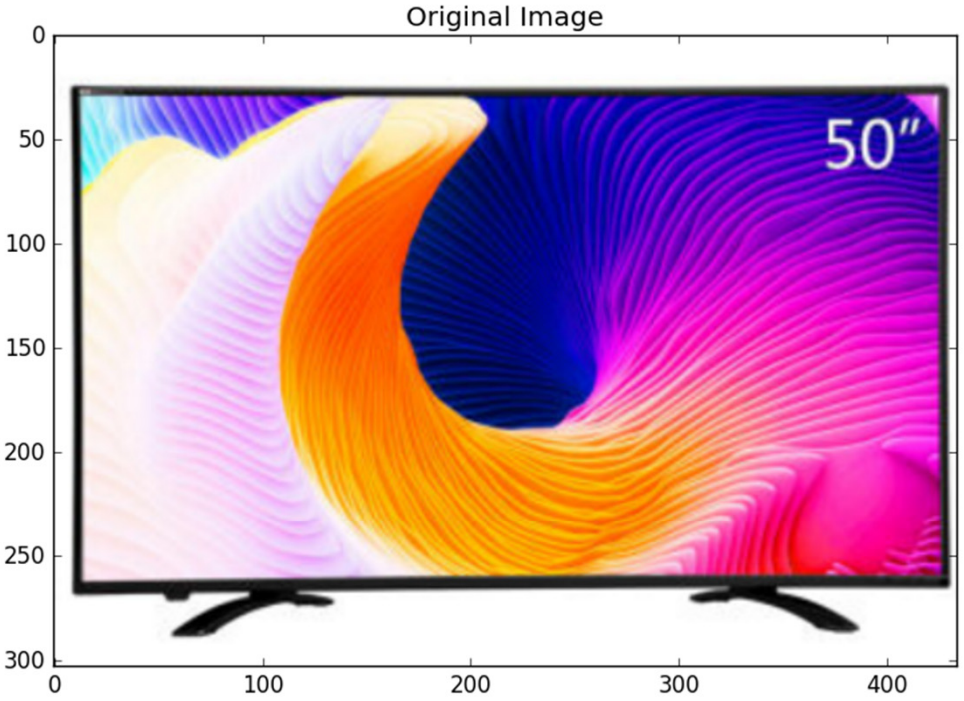


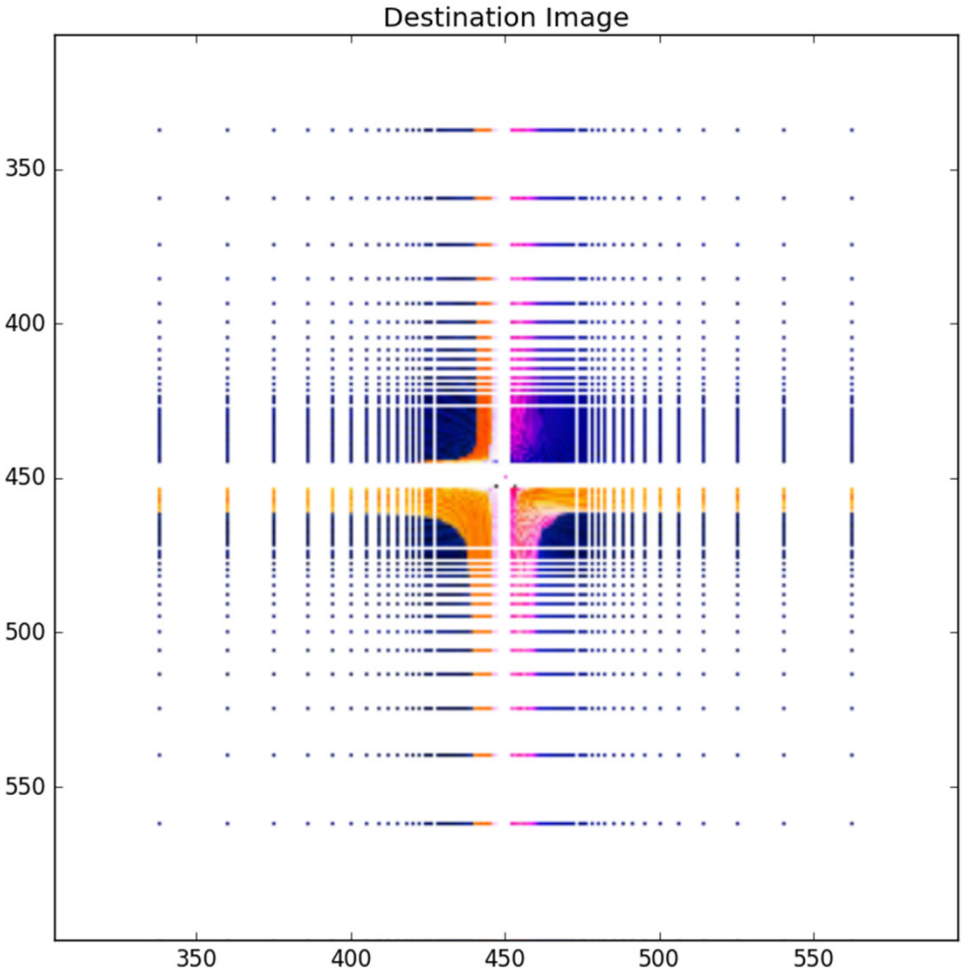
$$f(z) = \frac{a}{z}, where\ a = 450$$



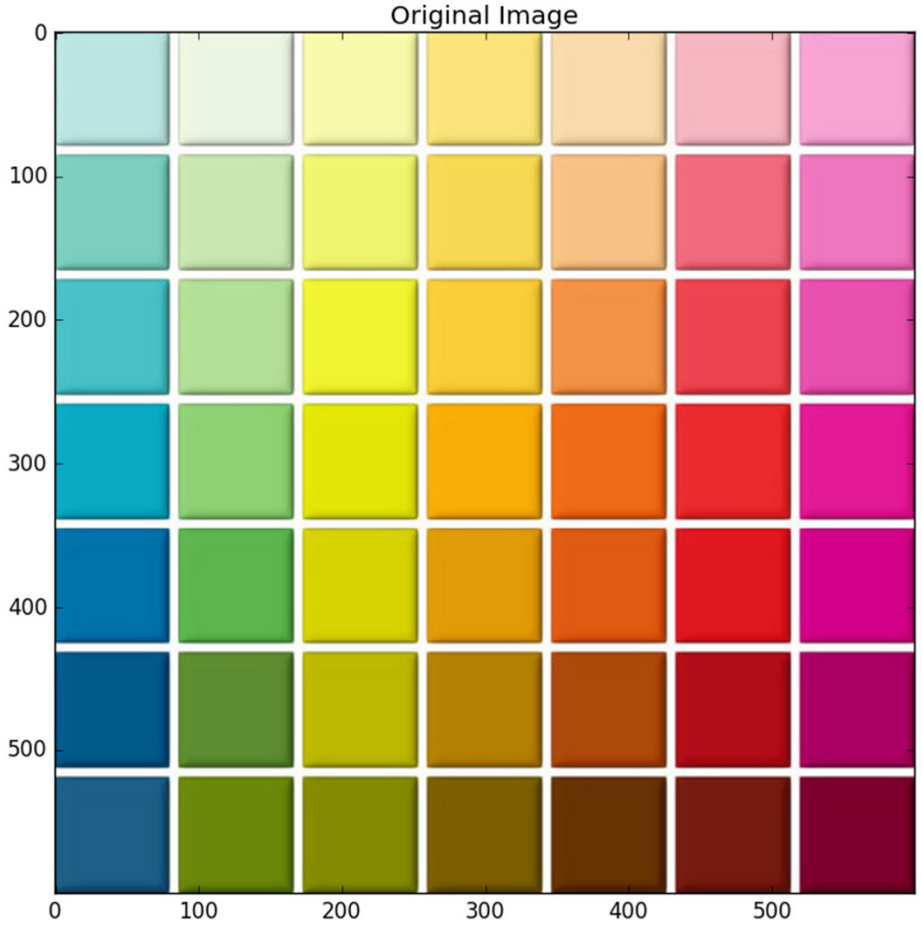


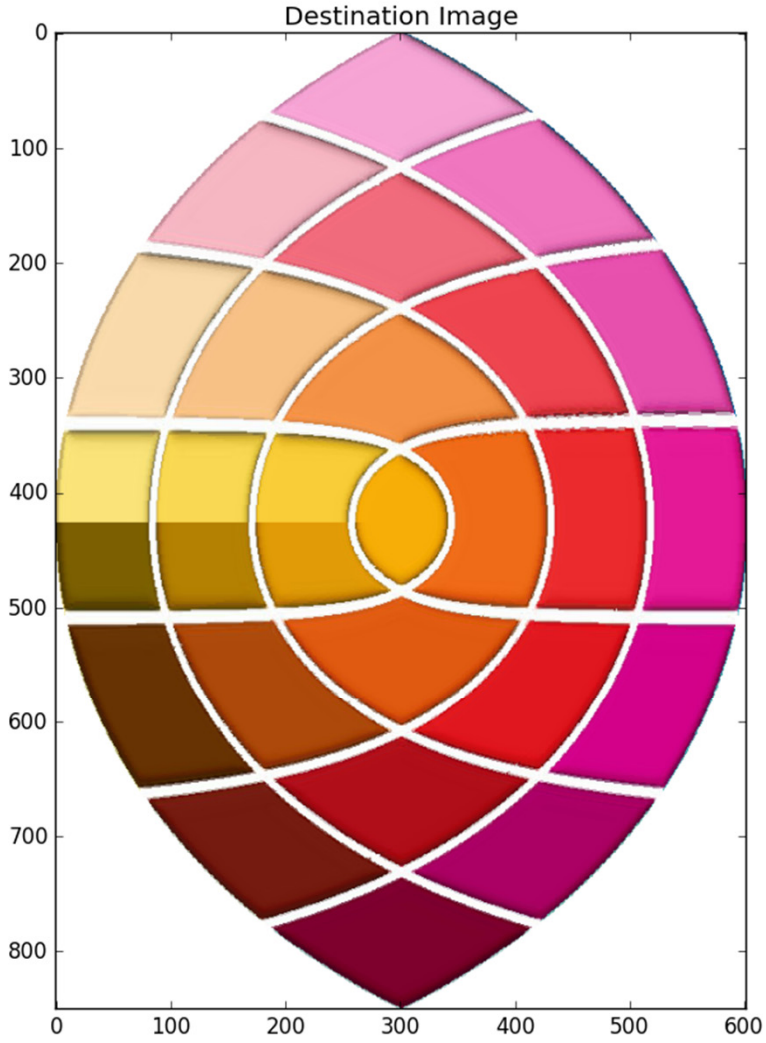
$$f(z) = \frac{a}{Re(z)} + i \cdot \frac{a}{Im(z)}, where\ a = 450$$



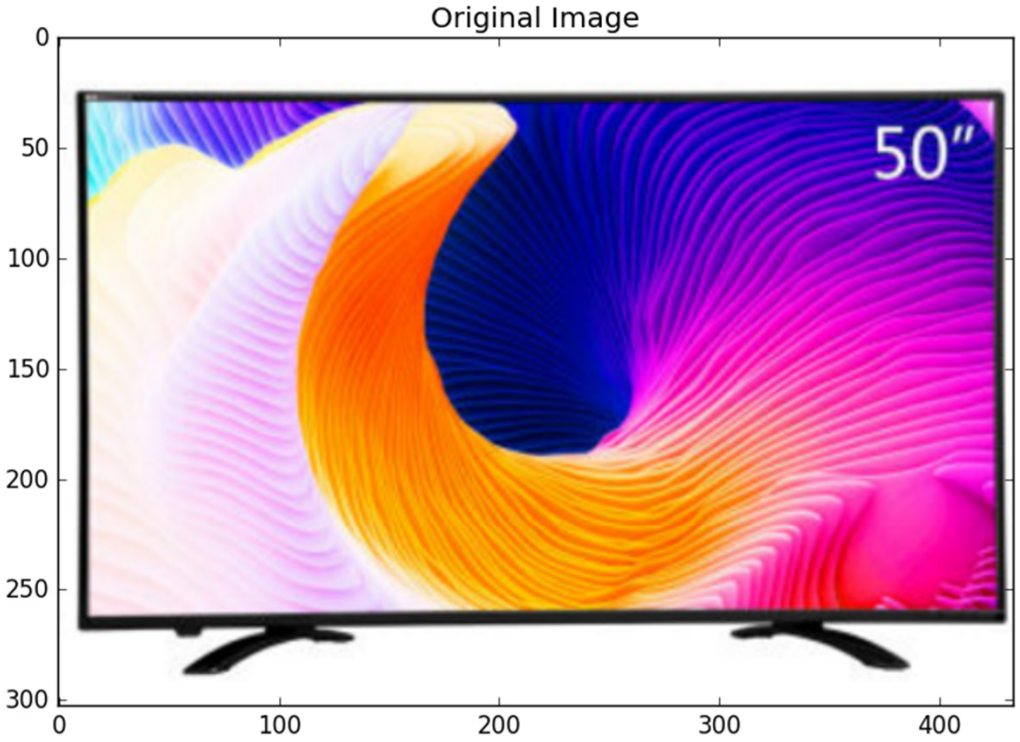


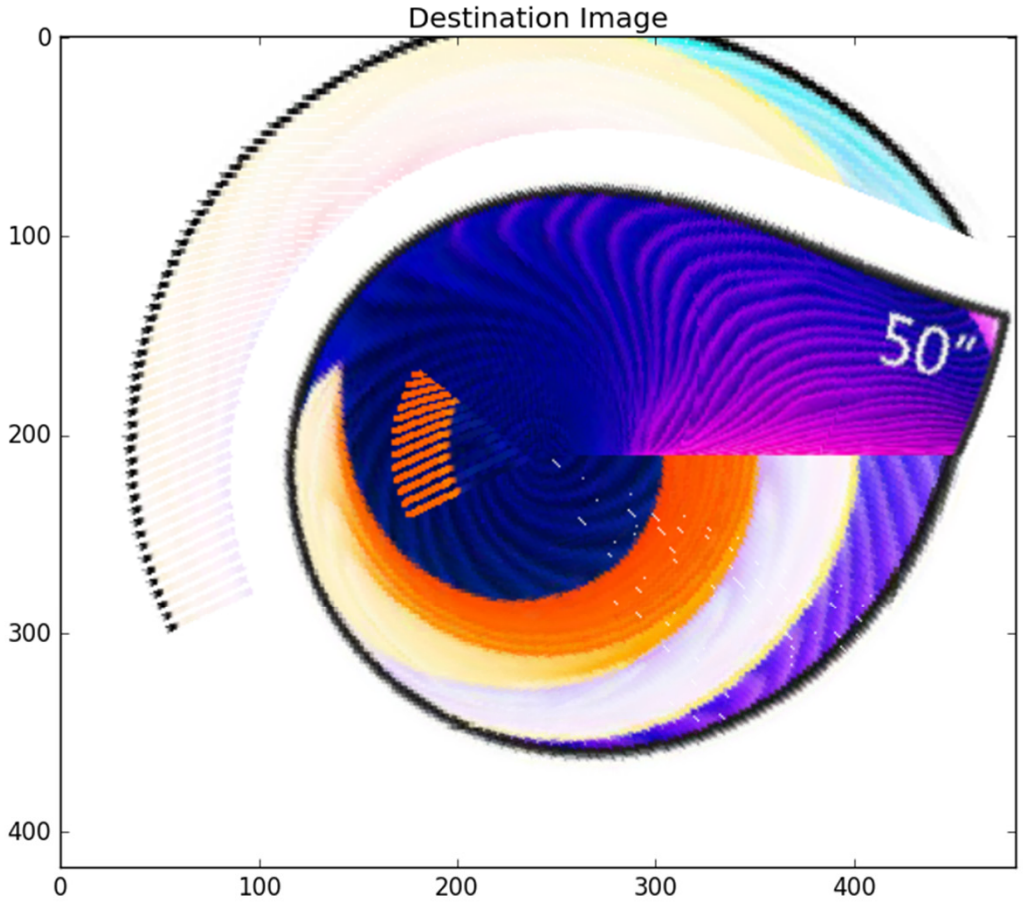
$$f(z) = |z| * e^{i \cdot g(arg(z))}, where\ g(x) = 2x$$





$$f(z) = |z| * e^{i \cdot g(arg(z))}, g(x) = x^2$$





Thanks for listening