Information Quality and Consumer Search for Experience Goods: Evidence from Online Shopping for Cameras

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Abstract

This paper develops and estimates a consumer search model for experience goods - products that are challenging to evaluate without direct use - available across multiple retailers. Beyond price uncertainty, consumers also face uncertainty about product suitability (match values) and form expectations about the quality of the match-value information available at different retailers, available for instance through consumer reviews. Consumers direct search across retailers based on their expectations about prices, retailer preferences, and expectations about match information quality. Consumers gain more precise match signals at retailers with higher quality match information, which makes finding a well-matched product more efficient. Analyzing clickstream data on camera searches, I document search behaviors that cannot be explained by models lacking expectations about information quality. Structural estimation indicates that larger retailers, such as Amazon and Walmart, provide higher-quality information, which I quantify enhances consumer welfare by 8.35%. Additionally, I show that retailers with superior information quality have an increased capacity to steer consumers and extract rents.

JEL Codes: D83, L15, L81

Keywords: Consumer Search, Learning, Information Quality, Online Retail, Retailer Differentiation

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1 Introduction

The rise of online commerce has simplified shopping for products online, particularly experience goods - products that are challenging to evaluate without direct use. Unlike simple search products, experience goods require consumers to obtain non-price information to learn which products match their needs. One way for them to do so is by studying consumer reviews. These reviews play an increasingly crucial role in allowing consumers to make informed decisions, yet research has traditionally focused on other factors such as prices, to study how consumers make search decisions. There is a growing literature that studies what makes the content of reviews helpful, but understanding their impact on actual consumer choices remains a challenge.

Since the work of Stigler (1961), in a typical search model consumers weigh the gains and costs of searching to determine which products to search for and when to stop searching. When products are available at multiple retailers, consumers typically direct search across retailers based on expectations about prices or their own preferences for retailers. This paper argues that for experience goods, since learning their match values for products is also important, consumers may direct search based also on expectations about where they may learn the most about match values with any given product (match information). For instance, they may have expectations on the quality of product reviews at different retailers, and prefer shopping at a retailer at which they expect to find more helpful reviews. This paper shows that incorporating match information expectations into standard search models is important to understand and rationalize observed search behavior, which I describe in detail below. For instance, they may help to explain why consumers are sometimes observed to continually search at one retailer, even for products that are available at cheaper retailers.

To understand this behavior, I develop and estimate a consumer search model in which consumers learn over time about prices and match values based on the information they obtain. Consumers search across differentiated products that are available at multiple retailers, until their uncertainty about the highest utility product is sufficiently resolved. The consumer's sequential search process is characterized as a dynamic programming problem, where every period the consumer decides which product to search and at which retailer, and when to stop searching. Consumers start with Normal priors, search to observe log-prices and receive noisy match signals, and then use Bayes rule' to update their beliefs. The model combines elements from Erdem et al. (2008) and Chick and Frazier (2012) to capture learning about prices and match values, while allowing them to be correlated. However, my model differs from the literature by capturing expectations of match information across retailers, specifically through the variance of match signals consumers receive at each retailer.

A precise (low-variance) signal induces consumers to update their beliefs more drastically, reducing the gain from a further search for the considered product. Thus, a more informative retailer allows consumers to find well-matched products more efficiently, which may drive search to this retailer.

The model also accounts for the previously mentioned channels that may rationalize retailer choices. Consumers may prefer to visit retailers that are perceived to offer low prices. In the model, consumers have rational expectations about the mean of the price distribution across retailers. Additionally, consumers may prefer certain retailers due to non-price factors like quick shipping, return policies, and/or the platform's ease-of-use. Retailers are thus vertically differentiated, which is captured in the model through fixed effects in consumption utility.

I apply the model to clickstream data tracking consumer search and purchase choices when shopping for cameras online, originally collected by Bronnenberg et al. (2016). Several stylized facts from the data suggest that match information expectations in addition to price expectations and retailer preferences are necessary to fully explain consumers' choices over retailers. I first establish that consumers tend to favor initiating their search for cameras at larger retailers like Amazon and Walmart, and tend to be less likely to subsequently visit other retailers for previously-searched products; consumers are "sticky" to large retailers. Interestingly, consumers are found to be most sticky to Walmart despite charging prices on average 2% higher than Amazon, suggesting that factors beyond price, such as information quality, are necessary to fully explain retailer choices. To further investigate the role of non-price information in explaining these search patterns, I estimate a reduced-form regression, which shows that after discovering a high-match product, consumers shopping at small retailers are more likely to then find this product elsewhere, even while controlling for prices at other retailers and retailer fixed effects. This behavior suggests that they did not find enough information about products they were interested in and left to find better match information.

The descriptive evidence suggests that non-price factors influence retailer choices, but cannot fully distinguish between retailer quality and the match information learning channels. So to understand this search behavior, I then structurally estimate the model to pin down information quality across retailers. In the data, for each consumer I observe a sequence of visits to product pages on platforms including three large retailers, Amazon, Best Buy, and Walmart, and the camera that they purchase, if any. I exploit variation in the evolution of search paths across consumers who search across retailers to identify retailer-specific signal variances. Specifically, the degree to which a consumer searches across products at the same retailer, or across multiple retailers for fewer products identifies signal variances.

The latter behavior indicates that consumers' initial searches for high expected utility products did not sufficiently resolve uncertainty about match values, requiring the consumer to conduct further search to be reasonably certain of the best-matched product. I find that the larger retailers, that the data patterns identify as favored retailers, are associated with lower signal variance. I quantify the benefit of high information quality by showing that if signal variance at the larger retailers increased to the level found at smaller retailers, mean consumption utility would fall by 8.35%, requiring consumers to conduct 20% more searches to achieve the same level of utility. To investigate the importance of the information channel in explaining retailer choices, I compare the fit of the estimated model to a version that assumes identical information quality across retailers. Removing retailer-specific information quality significantly worsens the model's fit. I also verify that simulating data based on the estimated model generates moments that more closely match those observed in the data.

I then evaluate the ability of a retailer with high information quality to extract rents. In particular, this paper studies search diversion or the act of steering consumers to a "focal" product in an attempt to induce search and purchase. For example, Amazon often prioritizes its own private-label brands, such as Amazon Basics, in users' search results making them more likely to attract clicks. Indeed, Ursu (2018) shows that products at the top of search results are more likely to be clicked on. There is some concern by regulatory bodies that a large retailer may be unfairly extracting rents by favoring their own products. Regulatory bodies have even enacted regulations aimed at curbing this practice, with the European Union's 2022 Digital Markets Act prohibiting online platforms from "self-preferencing behavior", including prioritizing their own products.[1]

The empirical literature on steering typically studies whether lowering search costs, for instance by prioritizing focal products in search results, increases search and purchase incidence. What is less studied in the literature are factors that convert steered searches into purchases. I study the interaction of steering and information quality, to learn whether offering high quality information enhances a retailer's ability to steer search *and* purchase. In principle, higher information quality can make a consumer place higher trust on the products they are steered towards, which may lead to more conversions of searches to purchases. I show that while search can be steered, corresponding purchases depend on the information quality at the retailer. If the focal products are generally a good match for most consumers, higher information quality increases the probability of purchasing the steered product and vice versa. Thus, whether steering hurts consumers depends on the information quality a retailer provides. Through a set of counterfactual exercises that simulate consumer choices, I

[1] For instance, Google was fined €2.4 billion for self-preferencing in its Shopping search results, and is currently under investigation for self-preferencing in its general search results.

show that (a) retailers can steer search to focal products by altering the information provision structure, and (b) whether the consumer is also steered to purchase the focal product depends on the combination of match value of the focal product and the quality of information provided by the retailer.

The rest of the paper is organized as follows. Section 2 discusses relevant literature. Section 3 outlines the model setting and shows that the consumer problem is well-founded and can be solved. Section 4 discusses the data that I fit the model to and provides descriptive evidence of differentiation in retailers' information provision and how this affects consumers' decisions. Section 5 discusses estimation and identification, Section 6 provides the estimation results, and Section 7 discusses relevant counterfactuals. Finally, Section 8 concludes.

2 Related Literature

The literature on search with differentiated retailers typically assumes that retailers are differentiated in quality, which for online retailers reflect factors like delivery times, price-matching incentives, etc (Moorthy and Zhang (2006), De Los Santos et al. (2017), Sullivan (2024)). This is often modelled as affecting the consumer's utility from consuming any product at a retailer. The literature also considers that price distributions may be differentiated across brands (Erdem et al. (2008)). I contribute to the understanding of retailer differentiation in search by considering differentiation in match information, or the quality of information at a retailer. This is inherently distinct from vertical quality differences because match information affects the evolution of a consumer's beliefs as they search. As I argue in the Data section, match information differences help rationalize observed retailer choices over the consumer's search path. Related to the understanding of match information at online retailers is the literature studying the impact of consumer reviews in search. Ma (2018) develops a sequential search model in which consumers can learn about product quality by choosing to read different types of reviews and thus endogenizing the source from which a consumer will receive a signal. However, they only consider search for differentiated products at one retailer. Relative to their work, my model considers products that are available at multiple retailers, and thus the consumer needs to consider where they are likely to get good match information prior to searching. Future work with more comprehensive data that includes review information could combine the two approaches to gain a deeper understanding of match information.

Theoretical work in search point to a pattern in the effect of both product and retailer differentiation on consumer behavior. Bakos (1998) show in a seminal paper that product differentiation reverses the classic prediction that lower search frictions lead to more competition and lower prices. Instead, they find that product differentiation can cause search to be driven by finding a good match and that this effect is larger when search frictions are lower.[2] Retailer differentiation is found to have a similar effect; Chen and Sudhir (2004) find that Bertrand competition only occurs without retailer differentiation, and price dispersion can be higher with differentiated retailers.

The empirical search literature aims to quantify search frictions faced by consumers in their search and purchase decisions by estimating both preferences and search costs (Hong and Shum (2006), Moraga-Gonzalez and Wildenbeest (2008), Kim et al. (2010), Koulayev (2014), Honka et al. (2019)). Papers in this literature generally follow the simultaneous search framework of Stigler (1961) or the sequential seach framework of Weitzman (1979). In both cases, consumers face uncertainty and search to reveal the utility of products by considering the gains of searching (i.e. finding a product with a high utility) and a cost of searching, typically a fixed cost c. In the latter case, consumers know the distribution of rewards and sequentially search to reveal the reward of individual products. Many models point to the optimal search rules derived by Weitzman (1979) which characterizes a reservation policy: consumers search in order of their product reservation utilities, stop when no product has a reservation utility greater than the realized rewards of searched alternatives, and buy the product with the greatest realized reward. This paper differs since search does not remove all uncertainty about a product's utility, and consumers instead update beliefs through Bayesian learning. This is essential for a setting in which consumers may learn about a product at multiple retailers throughout their search sequence.[3] This is similar to the literature on Bayesian learning in dynamic repeated choice models as in Erdem et al. (2008). Where my model differs is that learning occurs through repeated search choices prior to a purchase. In addition, my model identifies information quality differences providing new insights into how consumers direct their search across retailers.

The empirical search with learning literature is generally divided into two strands. The first strand models search across products where consumers learn about the distribution of utility (or prices) across all products over time (Koulayev (2013), De Los Santos et al. (2017)) following the framework of Rothschild (1974). Searching a product reveals information about its utility, and thus information about the entire utility distribution which they use with Bayes' rule to update beliefs. In contrast to my model, consumers learn about an individual product's price and match value over time, so that the consumer decides when it is optimal to learn about one product over another. These two types of consumer search problems

[2] Anderson and Renault (1999) have a similar finding in a discrete-choice model when product differentiation is high.

[3] This can also rationalize the choice to visit the same source multiple times.

are fundamentally different, and thus lead to different optimal behavior. The second strand, which my model follows, models uncertainty and learning about match values (Ke and Villas-Boas (2019), Ursu et al. (2020)). Search reveals information about an individual product's utility, which the consumer uses with Bayes' rule to update beliefs. As mentioned, my model combines elements of Erdem et al. (2008) and Chick and Frazier (2012), and adapts their framework to allow for information quality differences across retailers.

Some of the empirical search literature is interested in studying sellers' ability to steer search to a subset of their product offerings. The glaring concern is that sellers may steer search to serve their own benefit rather than to optimize consumer-product matching. Examples of firms conducting such behavior include supermarkets placing more popular products at the back of their stores (Petroski (2004)) and sponsored search results given by numerous online platforms such as Amazon and Google (McDevitt (2014)). In addition, Mattioli (2019) reports that Amazon's genuine search results are designed to boost its own products. Policy-makers in the EU have proposed and even implemented regulations aimed at curbing such seller behavior (Nocke and Rey (2024)). Whether search resolves all uncertainty of a product's utility or only partially resolves uncertainty (i.e. noisy search or learning), the literature finds that sellers can steer search by manipulating search frictions for certain products. Noisy learning amplifies this effect by limiting the gains from learning about match values. I contribute to the study of such seller behavior and the literature on platform design and optimal information provision (Dinerstein et al. (2018), De los Santos and Koulayev (2017), Bimpikis et al. (2024)) by analyzing the role of a retailer's information quality on its ability to steer.

While I do not directly observe the information consumers observe upon visiting a retailer, it is likely the case the product information contained in consumer reviews, for example, varies across retailers. The marketing literature has documented that consumer reviews have a significant impact on consumer search and purchase choices (Park et al. (2007)). Robert Grant and Kyriazis (2007) survey the literature for factors that affect consumer's perceived value of online product information, and find that consumers value the quality of product information available online. Guan and Lam (2019) conduct an eye-tracking experiment that simulates a forum where consumers submit product reviews. They find that the product ratings consumers leave strongly influence consumer decisions.

3 Model

3.1 Setting

A consumer seeks to purchase an alternative $j \in \{1, ..., J\}$ from one of the retailers $r \in \{1, \ldots, R\}$ or choose the outside option denoted $j = 0$. The consumption utility of the outside option is fixed and denoted by m, but the consumption utilities of the J alternatives consist of both fixed and random components. Before choosing an alternative to purchase a consumer can choose to sequentially search one or more of the J alternatives to infer their expected utilities. The consumer's search decision is over $a = (j, r)$ pairs. That is, the consumer must decide which camera to search for and at which specific retailer.[4] The part of the online search process where a consumer sees a ranking of products in their search results is captured by the different costs consumers would pay to search $a = (j, r)$, denoted $c_{ijr} > 0$. Products high in search rankings can be rationalized as having relatively lower search costs than products farther down in search rankings, so that search costs capture the awareness effect of search rankings as in Ursu (2018). The consumer's goal is to maximize their expected utility from the alternative chosen when search ceases net of all search costs incurred during their search process. Thus the consumer's problem is an optimal stopping problem. A consumer i's utility for purchasing j at retailer r is given by

$$U_{ijr} = X_j\beta - \alpha p_{jr} + \varepsilon_{ij} + \mu_{ij} \tag{1}$$

I assume consumers know the values of product characteristics X_j and the idiosyncratic shock ε_{ij} which has distribution $N(0, 1)$.[5] The term μ_{ij}, referred to as match value, models utility from product quality that is unobserved to the consumer and researcher. Utilities for product j differ across retailers due to the different prices each retailer r may offer. Consumers are initially uncertain about both the prices each retailer offers and the product's match value. I model the consumer learning the values of p_{jr} and μ_{ij} using a Bayesian framework. In addition, I allow for prices and quality to be correlated, so that the price observed for j is itself a signal of μ_{ij} as in Erdem et al. (2008). Consumers assume that match values are distributed according to $N(\mu_j, \sigma_\mu^2)$. Consumers know the variance term σ_μ^2 but do not know the mean of the match distribution, μ_j. Consumers also believe that the price p_{jr} is related to match values according to the relation:

$$p_{jr} = P_0 + \phi\mu_j + \eta_r + \omega_{jr}, \tag{2}$$

[4]This matches the data since each search corresponds to a particular product page on a platform.

[5]Assuming consumer knowledge of these two components is common in the empirical search literature. For instance, see Ursu et al. (2023).

where $\omega_{jr} \sim N(0, \sigma_\omega^2)$. The parameter ϕ controls the degree to which prices and match values are correlated. A positive value for ϕ would mean prices are positively correlated to match values and vice-versa. The variables η_r are deviations from the equilibrium price-match value relation for each retailer r. These terms capture the idea that certain retailers may have lower prices than others across all products in expectation. Consumers do not know prices perfectly because they are uncertain about μ_j and the time-invariant shock ω_{jr}.[6]

3.2 Beliefs and Learning

Consumers start with the following priors, at $t = 0$, for price and quality:

$$\mu_{ij} \sim N(\mu_0, \sigma_{\mu_0}^2), \quad p_{jr} \sim N(P_0 + \phi\mu_0 + \eta_r, \phi^2\sigma_{\mu_0}^2 + \sigma_\omega^2) \tag{3}$$

The terms $\mu_0, \sigma_{\mu_0}^2, P_0$, and σ_ω^2 are parameters to be estimated. The terms $\mu_0, \sigma_{\mu_0}^2$ capture consumers' prior information about the distribution of match values across products. A higher $\sigma_{\mu_0}^2$ would mean consumers start off more uncertain about the match values of products, making them more likely to search. Search reveals information about a product j's match value μ_j and the price offered at one retailer r. Consumers observe the price p_{jr} and receive an unbiased signal s_{ijrt} of the match value for j, at a cost c_{ijrt}. Thus consumers have perfect recall of prices. Since prices and match values are correlated, the price observed is itself a signal of the product's match value. Thus, the initial search at j, r reveals two pieces of information with which consumers infer match values. A revisit to j, r reveals only an additional match value signal.

Retailers are differentiated in match information provision. The signals that consumers receive differ across retailers in their precision; signals are drawn according to $N(\mu_j, \sigma_{jr}^2)$. Note that the signals are independent of prices, so that prices influence consumer learning over match values through only the price-match relation in (2).[7]

After observing the price and match value signal, consumers then update their beliefs about the price available at retailer r to the observed price so that uncertainty over the price at r is resolved after one visit to a retailer. The consumer also updates their belief about

[6] The price shock can be allowed to be time-varying, at the expense of model complexity. In my setting, 60% of consumers do not observe different prices for the same j, r combination at different points in their search path. Of the consumers with multiple price quotes, 90% have multiple quotes for only one j, r combination. Appendix B presents an extension to the model that can rationalize new prices upon revists.

[7] Consumer reviews may be influenced by the price of the product since they may be more critical of expensive products. Thus a more realistic model may allow for correlation in the match value signals with prices. It is unclear how this correlation could be separately identified from the price-quality relation parameter ϕ in (2).

the match value for j following Bayes' Rule, which gives the following posterior for μ:

$$\mu_{ij} \sim N(\mu_{ijt}, \sigma^2_{\mu_{ijt}}), \tag{4}$$

$$\mu_{ijt} = \mu_{ij,t-1}\frac{\sigma^2_{\mu_{ijt}}}{\sigma^2_{\mu_{ijt-1}}} + \underbrace{\frac{p_{jr} - P_0 - \eta_r}{\phi}}_{\mu_j \text{ implied by } p_{jr}} \frac{\sigma^2_{\mu_{ijt}}}{(\phi^2\sigma^2_{\mu_{ijt-1}} + \sigma^2_\omega)/\phi^2} + s_{ijrt}\frac{\sigma^2_{\mu_{ijt}}}{\sigma^2_{jr}}, \tag{5}$$

$$\sigma^2_{\mu_{ijt}} = [1/\sigma^2_{\mu_{ijt-1}} + \phi^2/(\phi^2\sigma^2_{\mu_{ijt-1}} + \sigma^2_\omega) + 1/\sigma^2_{jr}]^{-1} \tag{6}$$

The posterior variance, $\sigma^2_{\mu_{ijt}}$, depends on the variance associated with the price draw, and the variance associated with the match signal. As the latter two terms decrease, the posterior variance decreases. Thus, the variances associated with match information can be interpreted as the (inverse of) informativeness or quality of match information. Higher quality match information induces larger decreases in percieved variance of match values. Generally, as consumers receive information about match values, their beliefs become more precise.

The posterior mean is a weighted sum of three terms that represent the three sources of match information at t. The first term is the prior mean, $\mu_{ij,t-1}$, the second term is the value of μ_j that is implied by the observed price p_{jr} through the price-match relation in Eq. 2, and the last term is the match value signal, s_{ijrt}. The denominator in the weights of each term is the variance of the corresponding match information. If this variance is large so that match information is lower quality, then the consumer updates their beliefs less drastically. The numerator is the posterior variance $\sigma^2_{\mu_{ijt}}$. If this variance is small, so that the consumer's belief is precise, then the consumer places a smaller weight on new match information so that their belief changes less drastically. In fact, as the consumer's belief becomes more and more precise, the weights on new match information converge to zero, while the weights on the prior converge to one.

Lastly, the consumer's belief about the price for j at other retailers $r' \neq r$ updates through the price-match relation to

$$N(P_0 + \phi\mu_{ijt} + \eta_{r'}, \phi^2\sigma^2_{\mu_{ijt}} + \sigma^2_\omega). \tag{7}$$

If the consumer's posterior belief is greater than the prior, so that they believe j has a higher than previously expected match value, their belief about prices at other retailers will update to be higher than previously expected as well.

3.3 Consumer Problem

The relevant state variables can be summarized by:

$$\Theta_t = (\Theta_{0t}, \Theta_{1t}, ..., \Theta_{Jt}), \quad \text{where} \;\; \Theta_{jt} = (\mu_{jt}, \sigma^2_{\mu_{jt}}, P^M_{jrt}, \sigma^2_{P_{jrt}}|_{r\in\{1,..,R\}}).$$

The consumer's problem is to choose a policy π which determines at each time t the alternative j and a retailer r to be chosen for search or an alternative to purchase. More precisely, π defines a mapping $a(t, \Theta_t)$ (denoted $a(t)$ for brevity) at each t to one of $J(1+R)+1$ possible decisions.[8] Let Π be the set of such policies, and E_π indicates the expectation given the chosen policy π. In this setting, the expected value of the future stream of rewards given a policy $\pi \in \Pi$ is

$$V^\pi(\Theta) = E_\pi\left[\sum_{t=0}^{T-1}\left(-c_{a(t)}\right) + U_{J(T),T+1}\right] \tag{8}$$

Note that since search costs are strictly positive, the expected value for a policy that samples forever is $-\infty$. Thus for any optimal policy it is clear there must be some finite period T at which sampling ceases. The alternative chosen for purchase at T is denoted $J(T)$. The consumer's problem is to choose a policy to maximize the value function in (8).

$$V^*(\Theta_0) = \sup_{\pi\in\Pi} V^\pi(\Theta_0) \tag{9}$$

The value function can be characterized as a dynamic programming problem.

Proposition 1. $V^\pi(\Theta_0) = E\left[\max_{j=0,1,...,J} U_j|\Theta_0\right] - E_\pi\left[\sum_{t=0}^{T-1} c_{a(t)} + L_{J(T)}|\Theta_0\right]$ *where* $L_j = (\max_{k=0,1,...,J} U_k) - U_j$ *is the loss associated with selecting alternative* j*, for* $j = 0, 1, ..., J$*.*

Proof. The proof follows directly from the Appendix of Chick and Frazier (2012). □

The term $E\left[\max_{j=0,1,...,J} U_j|\Theta_0\right]$ is the expected reward of choosing the best alternative for purchase without any incurred cost, and does not depend on the policy π. Then the policy π maximizes $V^\pi(\Theta_0)$ when minimizing the sum of the total sampling cost and the opportunity cost of choosing an alternative that is not best $E_\pi\left[\sum_{t=0}^{T-1} c_{j(t)} + L_{J(T)}|\Theta_0\right]$.

Since both $c_{a(t)}$ and $L_{J(T)}$ are nonnegative, this problem satisfies the (P) assumption of Chapter 9 of Bertsekas and Shreve (1996). Proposition 9.8 of Bertsekas and Shreve (1996) then shows that the value function satisfies Bellman's recursion,

$$V^*(\Theta_t) = \max\left(\max_{a=\{1,...,J\}\times\{1,...,R\}} E\left[-c_a + V^\pi(\Theta_{t+1})|\Theta_t, a(t) = a\right], \max_{j=0,1,...,J} E\left[U_j|\Theta_t\right]\right) \tag{10}$$

[8] The general model presented here assumes each product j is available at all R retailers. This is without loss of generality.

Thus the consumer's problem can be equivalently stated as follows: at each time t they must either choose the best alternative to search in which case they incur cost $c_{a(t)}$, observe a price and match value signal, update their beliefs summarized by the state variables from Θ_t to Θ_{t+1}, and receive continuation value $V^{\pi}(\Theta_{t+1})$, or choose to stop searching and choose the best alternative seen so far to purchase and receive its expected utility $E\left[U_j|\Theta_t\right]$.

Proposition 2. *Any policy π whose decisions attain the maximum in Bellman's recursion in (10) is optimal, i.e., $V^{\pi}(\Theta_0) = V^{*}(\Theta_0)$.*

Proof. The proof follows directly from Proposition 9.12 of Bertsekas and Shreve (1996). □

We can therefore solve the consumer's problem using standard dynamic programming techniques. The observation above, which states that an optimal policy is one where the consumer does not continue searching forever can be restated in the context of the consumer's dynamic programming problem. The sequential process of searching, receiving information about prices and match values, and updating beliefs continues until the consumer decides to stop searching. In doing so, in every period, the consumer will weigh the benefit of searching against the cost of searching for each possible alternative. The consumer's benefit from searching (j, r) depends on the probability that they find p_{jr} is cheaper than expected or μ_j is higher than expected. Since their beliefs become more precise over time, the probability that either of these two gains from searching occur and are sizeable decreases over time. Since search costs are strictly positive, at some point, search costs will strictly outweigh the benefits to searching.

I return to the consumer problem in Section 5, where I describe how I use approximate value iteration (Bertsekas and Tsitsiklis (1996)) to solve for the consumer's value function. First, I describe the data and present stylized facts from the data that frame how consumers behave and the variation in the data that identifies the model; in particular the variation that identifies retailer signal variance.

4 Data

I apply the model to a clickstream dataset that records the sequences of consumers' searches for cameras online. The dataset was originally collected by Bronnenberg et al. (2016), henceforth referred to as BKM, who graciously made the dataset publicly available. The primary source for these data is the comScore web browsing and purchasing panel from 2010. The browsing panel includes a history of each webpage url the consumer visited; a consumer identifier and an indicator for whether any transactions are associated with the visit. The transaction panel provides further information on transactions, including prices and a matching consumer identifier. BKM also match the browsing data with product characteristics data, including the prices observed during search and paid for purchases.[9]

For each consumer, the dataset provides the sequence of URLs that consumers clicked to visit camera product pages on e-Commerce platforms. The platforms observed in the data include Amazon, Best Buy, Walmart, and various smaller retailers which I group as "Other" for the empirical analysis. The former three retailers account for more than 90% of observed purchases and 85% of observed searches. Amazon accounts for the majority of observed transactions at 38%, followed by Walmart at 36% and Best Buy at 14%. I interpret both the clicks to product pages that do or do not correspond to a purchase as a search choice. The last click to a product page for j at retailer r that corresponds to a purchase is interpreted as the consumer's final search, after which they stop their sampling process and purchase j at retailer r.[10] If the consumer has no transactions, the sequence of all their searches are used to construct one search path, and the search path is interpreted to end with the purchase of the outside option. If the consumer has any searches after their first transaction, I treat those as being part of an independent search sequence. The model considered in the paper cannot rationalize search sequences beyond the consumer's first, however a potential extension to the model can be to set the prior beliefs that consumers start with to their posterior beliefs from previous search sequences.[11]

The data sample consists of 8276 observations, where an observation is a click onto a product page. There are 1253 search sequences, 916 of which result in a purchase. I define a product as a unique combination of a brand (e.g. "Canon"), a model (e.g. "T2I"), and the megapixels characteristic.[12] Using this definition, there are 466 products that are ever-searched, and 200 that are ever-purchased.[13] I provide summary statistics on their characteristics in Table 1. Table 2 lists the ten most commonly purchased products.

4.1 Relation between Retailers and Search Behavior

In general, cameras are considered highly technical products, with several factors beyond its salient characteristics determining the quality of a camera. In the language of the marketing literature, they can be classified as experience rather than search products. There are numerous information sources available online that explain the functionality and inner

[9] For a full discussion of the construction of the dataset, see Bronnenberg et al. (2016).

[10] This ensures that the consumer purchases a product that they have previously searched atleast once.

[11] 27.3% of seach sequences are from consumers that have a previous sequence.

[12] Including one of the four continuous characteristics in the definition of a product is necessary since some brand-model combinations are associated with different values for the continous characteristics. I interpret this as a data limitation; perhaps a more granular model name exists but is not observed.

[13] Since I only have data on searches, products that are never searched are not observed.

Table 1: Product Characteristics

	Searched		Purchased	
	Mean	SD	Mean	SD
Product Characteristics				
Log Pixel	2.34	0.37	2.41	0.32
Log Zoom	1.52	0.66	1.56	0.68
Log Display	0.99	0.21	1.02	0.26
Observations	466		200	

workings of cameras, so for consumers that care about finding a good match, considerable search effort must be expended. These consumers are also more likely to care about the quality of information that retailers are able to provide. Indeed, Bae and Lee (2011) find that consumers respond more to product reviews for experience goods. Thus the set of consumer reviews retailers have solicited are important pieces of information that consumers consider when deciding where to shop. As discussed previously, clicks to product pages at different retailers are mapped to the model as the consumer learning about match values with differing levels of precision and cost. In the following I provide descriptive evidence to support the idea that search and learning behavior depends on which retailer the consumer visits. First, I provide summary statistics to highlight differences in induced search behavior across retailers. I separate sequences by the retailer at which each consumer conducts their first search, and provide statistics on the length of each search sequence, and the share of searches and purchases at the same retailer, given in Table 3.

Table 3 shows that search is generally quite limited. Consumers starting search at Walmart conduct the fewest searches with an average of 5.88, up to an average of 9.00 at Amazon, the greatest among large retailers. Among these total searches, consumers consider an average of 3.22 products at Walmart and 4.40 at Amazon. Interestingly, despite consumers conducting more search after an initial search at Amazon, Amazon maintains a high retention rate, with 83% of further searches conducted remaining at Amazon. Similarly, the next largest retailer, Walmart maintains an 88% search inertia, with similarly large values of purchase retention. Thus, consumers are observed to be "sticky" to these large retailers, especially relative to the smaller retailers which only have a search inertia of 52%.

To explore the source of this "stickiness" to large retailers, I first consider the role of prices, as we may suspect that consumers prefer to shop at cheaper retailers. Table 5 presents summary statistics on transactions for which the purchase price was above the

Table 2: Most Commonly Purchased Products

Product	Num. Purchases	Mean Price (USD)
Kodak C183	65	68.88
Nikon S3000	43	117.89
Nikon L22	31	90.84
Kodak C143	30	69.93
Canon SD1300	29	132.94
Sony W330	25	146.37
Sony W350	22	164.42
Fuji S1800	21	158.28
Kodak Z981	21	213.53
Canon T2I	18	855.18
Other Products	611	247.75
Total	916	245.45

minimum available price at any retailer. Consumers could have saved an average of about 4%, or $1.97 if they conducted further search to find a lower price on their preferred products. Table 4 presents summary statistics on the ratio of prices in observed transactions relative to the prices offered by Amazon, broken down by retailer. Here, we see that the retailer with the highest search inertia and purchase retention, Walmart, actually charges prices 2% higher than Amazon on average. While I do not ignore that consumers may have price expectations across retailers, in combination with the magnitude of dispersion, these patterns suggest that non-price factors may also be important in driving retailer choices.

To investigate further, I consider how consumers' choices change over their search paths. Table 6 presents a summary of consumers' product/retailer choices following a search at one of the four retailers. Column 4 shows that consumers are more likely to leave a small retailer to find a previously searched product at another retailer, relative to Amazon and Walmart which are largely favored for search. In fact, this is least likely at Walmart which is most expensive among large retailers.

Table 7 shows how often each retailer is chosen when first searching for a product. Column 1 shows once again that Amazon and Walmart are favored. Column 2 shows that they are still favored when consumers actively switch from the retailer associated with their previous search, suggesting that the pattern is not due to the ease of staying at a retailer for further search. This suggests that consumers that are at the smaller retailers actively choose to search across retailers rather than stay within a retailer. That the pattern holds in Column

Table 3: Descriptive Statistics by Initial Retailer

		Retailer			
		Amazon	Best Buy	Walmart	Other
Num. Searches	Mean	9.00	7.72	5.88	13.62
	SD	18.14	8.37	6.99	12.23
Num. Products	Mean	4.40	4.08	3.32	6.21
	SD	5.79	3.86	3.52	5.23
Search Inertia[1]	Mean	0.83	0.71	0.88	0.52
	SD	0.26	0.33	0.23	0.28
Purchase Retention[2]	Mean	0.77	0.59	0.82	0.37
	SD	0.42	0.49	0.38	0.49

[1] Search inertia is defined as the proportion of searches in each sequence that are at the same retailer as the initially searched retailer.
[2] Purchase retention is an indicator variable that takes a value of one when the consumer purchases at the same retailer as their initial search.

Table 4: Relative Prices

Retailer	N	Mean	SD	10%	25%	50%	75%	90%
Amazon	316	1.00			-			
Best Buy	95	1.01	0.05	0.97	0.99	1.01	1.04	1.06
Walmart	165	1.02	0.03	0.98	1.00	1.01	1.02	1.06
Other	47	1.04	0.05	0.98	1.00	1.04	1.06	1.07

To construct relative prices, I take the ratio of each transaction price with the closest (within a 7-day window) observed transaction price at Amazon.

3 when the consumer eventually searches across multiple retailers suggests consumer tastes for retailers is not the key driver of these first search patterns, rather the consumer favors a certain retailer to obtain information at when first looking for a product.

4.2 Empirical Evidence of Retailer Differentiation

This subsection presents reduced-form results to suggest that learning about match value is an important driver of search, and that retailers differ in their ability to provide reliable match information. A direct implication of the model is that consumers are more likely to direct search to retailers that provide more precise signals of match values. Subsequently, since Bayes' rule implies that more precise signals cause consumers to update their beliefs more drastically, conditional on receiving a precise signal of a high match value for j, the model also implies that they should be less likely to visit another retailer for j, and thus

Table 5: Transactions above min. price

	All Retailers	Largest Three
Share of Transactions (%)	39.14	38.10
Average payment above min. price ($)	1.97	1.90
Average payment above min. price (%)	4.30	4.18

Table 6: Subsequent Search Patterns by Retailer

	Retailer			
	Amazon	Best Buy	Walmart	Other
New Product, Same Retailer	60.58	58.90	62.02	47.53
New Product, New Retailer	12.55	17.91	10.20	22.50
Same Product, Same Retailer	21.40	17.25	23.60	18.37
Same Product, New Retailer	5.47	5.93	4.18	11.60

Each row gives $P(j_{t+1} \stackrel{?}{=} j_t, r_{t+1} \stackrel{?}{=} r_t)$ given that r_t is one of the four retailers.

more likely to purchase j at that retailer.

To test this hypothesis, I modify an idea from Hodgson and Lewis (2023) to conduct a pseudo-experiment. I construct a reduced-form measure of each product's underlying match value, denoted $\hat{\mu}_j$. This measures how much more likely a product is to be purchased than other products that are close in characteristic-space. Recall from the consumption utility in 1 that match values are the unobserved part of utility. Two products j and k with identical characteristics but with $\mu_j > \mu_k$ would mean that j will have a larger purchase probability than k. Thus, $\hat{\mu}_j$ is interpreted as a measure of match value, since products that are more likely to be purchased than similar products are also more likely to have high match values. The details on the construction of this measure are provided in the Appendix. Using this measure, I test whether some retailers induce more purchase and less search conditional on the consumer searching for a product that is purchased more often, relative to other similar products. To test this idea, I estimate a search-level regression of measures of purchase and search incidence on the estimated match value measures $\hat{\theta}_j$. The regression results are given in Table 8.

Induced searches differ significantly across retailers. First, as the model predicts, the effect of finding a product j with a high match value at retailer r is to increase the likelihood of purchasing j at r. The effect is also to decrease the likelihood of searching for j again at another retailer. Matching the search and purchase retention statistics in Table 3, the effect is most intense when searching at Amazon or Walmart. Other retailers are the least likely

Table 7: Consumer Search Retailer Choice

	First Searches	Switch for First Search[1]	Switch for First, At-least Two Retailers[2]
Amazon	39.06	32.20	30.45
Best Buy	15.74	18.80	18.64
Walmart	33.01	23.00	23.62
Other	12.19	26.00	27.30

[1] This is the probability that each retailer is chosen when first searching for a product, given that the consumer previously searched at a different retailer.
[2] As in Column 2, but with an additional condition that the consumer eventually searches for that product at more than one retailer.

to retain further search or purchase. These regressions control for product characteristics including log of price, as well as the minimum available price at the time of search. Thus, these results are suggestive of differences in information provision across retailers. Retailers that are best able to retain consumers for purchases and limit further search may be those that provide reliable product information so as to aid consumers finding a good match.

5 Estimation

To take the model to data, I make some simplifying assumptions on the parameters. First, to ensure that each variance terms is positive, I parameterize these terms using an exponential. The variance of all signals received at retailer r is given by:

$$\sigma_{jr}^2 = \exp(\delta_r) \ \ \forall j. \tag{11}$$

I use a similar parameterization for search costs:

$$c_{ijrt} = \exp(\gamma_r) + \xi_{ijrt}, \tag{12}$$

where ξ_{ijst} is a logit shock to search costs.[14] This shock captures unobserved variance in search costs due to factors like search result rankings and marketing efforts that may have brought the consumer to the product page. It also leads to a convenient expression for the

[14] While the logit shock assumption no longer ensures that search costs are strictly positive, which is required for the proofs of Propositions (1) and (2), if the cost shock distribution has small enough variance, then it is unlikely that a single product has consistently positive search costs so that it is optimal for the consumer to continue searching forever. In this case, it is still likely that Bellman recursion leads to an optimal solution.

Table 8: Effect of Match Values on Purchase and Search Incidence

	Purchase at Retailer	Revisits at other Retailers
$\hat{\theta}_t$	0.110	0.049
	(0.035)	(0.008)
$\hat{\theta} * 1(\text{Amazon})$	0.144	−0.062
	(0.030)	(0.009)
$\hat{\theta} * 1(\text{Best Buy})$	0.108	−0.042
	(0.049)	(0.013)
$\hat{\theta} * 1(\text{Walmart})$	0.120	−0.056
	(0.047)	(0.012)
Retailer Fixed Effects	Yes	
Product Characteristics	Yes	
Sequence Controls	Yes	
N	5044	

The outcome variable in column one is 1 when the product being searched is eventually purchased at the same retailer, and in column 2 is the proportion of remaining searches that are at other retailers. Sequence controls include the length of the search sequence and each search's position within the sequence.

consumer's value function, which simplifies computation as in Hodgson and Lewis (2023). Thus the parameters ρ to be estimated include the taste parameters (β, α), the prior parameters $(\mu_0, \sigma_\mu^2, \sigma_\omega^2, P_0, \phi)$, the learning parameters (μ, δ_r, η_r), and the search cost parameters (γ_r).

5.1 Likelihood Function

The expectation of the value function (10) with respect to the distribution of cost shocks is

$$V^*(\Theta_t, \rho) = \log\left(\exp(\max_{j=0,1,\dots,J} E\left[U_j|\Theta_t\right]) + \sum_{a=\{1,\dots,J\}\times\{1,\dots,R\}} \exp(E\left[-c_a + V^\pi(\Theta_{t+1})|\Theta_t, a(t)=a\right])\right), \tag{13}$$

and the probability of a consumer searching $a = (j, r)$ is

$$P(a(t)|\Theta_t, \rho) = \frac{\exp(E\left[-c_a + V^\pi(\Theta_{t+1})|\Theta_t, a(t)=a\right])}{\exp(\max_{j=0,1,\dots,J} E\left[U_j|\Theta_t\right]) + \sum_a \exp(E\left[-c_a + V^\pi(\Theta_{t+1})|\Theta_t, a(t)=a\right])}. \tag{14}$$

Note that the dependence of Equations (13) and (14) on the parameters ρ is made explicit. This requirement will be made clearer in the discussion of value function iteration in Section 5.2. Let T_i denote the last period in consumer i's search sequence; the period they purchased a product. Also, let $\hat{u}_i = \max_{j=0,1,\ldots,J} E\left[U_{ij}|\Theta_{T_i}\right]$. Then the likelihood of the consumer's search path is

$$L_i(\{a(t)\}_{t=0}^{T_i-1}, J(T_i)|\{\Theta_t\}_{t=0}^{T_i}, \rho) = \left(\Pi_{t=0}^{T_i-1} P_i(a(t)|\Theta_t)\right) P_i(0|\Theta_{T_i}) \left(1 - \frac{|u_{iJ(T)} - \hat{u}_i|}{R_{u_{ij}}}\right). \quad (15)$$

The model does not assign purchase probabilities to the chosen product, rather it is assumed that the product that is purchased has the highest expected utility for consumer i. Thus, to discipline the parameter estimates so that purchase choices are rationalized, I insert the last term into the likelihood. This term takes on a value of 1 when the chosen product $\hat{j}$ is the highest expected utility product, and gets closer to 0 as $u_{iJ(T)} - \hat{u}_i$ increases. The likelihood unconditional on realizations of utilities is

$$L_i(\{a(t)\}_{t=0}^{T_i-1}, J(T_i)|\rho) = \int L_i(\{a(t)\}_{t=0}^{T_i-1}, J(T_i)|\{\Theta_t\}_{t=0}^{T_i}, \rho) dG(u_i). \quad (16)$$

The integral is approximated numerically by averaging the expression inside the integral over draws of u_i.[15] Thus this is a simulated maximum likelihood approach, as is standard in the empirical search literature. The parameters ρ are chosen by maximizing the objective function:

$$L(\rho) = \Pi_{i=1}^{N} L_i(\{a(t)\}_{t=0}^{T_i-1}, J(T_i)|\rho). \quad (17)$$

5.2 Solving the Value Function

Note that the probability of searching a in equation 14 depends on the consumer's continuation value. The consumer's state variables consist of the mean and variance of the consumer's match value distribution and price distribution at each retailer for each product. Thus, the dimension of the state space is $2J(1+R) \approx 4500$. Since the state variables are also continuous, solving the value function at each possible state point is intractable even if the state space is discretized. The state space is first represented more compactly by the following augmented state variable:

$$\tilde{\Theta}_\tau = \left((s_{a(t)})_{t<\tau}, (p_{a(t)})_{t<\tau}, (a(t))_{t<\tau}, \rho\right).$$

[15] That is, draws of prices p_{jrt} and match value signals s_{jrt} at each t.

Given a consumer's priors, the history of match value signals and prices observed can be used with Bayes' rule in (4)-(6) to arrive back at the initial representation of the state. Thus, the augmented state variable encapsulates all relevant information for the consumer's problem. Note also that I include the parameter vector in the augmented state variable. The dimension of $\tilde{\Theta}_\tau$ is $3\tau + 23$ which is significantly smaller than $2J(1 + R)$ even for the longest search sequence (192 searches). Next, I adopt the approximate value iteration technique described by Bertsekas and Tsitsiklis (1996), which is similar to the approximation technique developed by Keane and Wolpin (1994) in the dynamic discrete-choice literature. The general steps to the procedure are as follows:

1. Initialize the value function to $\hat{V}_0(\Theta) = 0$.

2. Draw W values of Θ from a proposal distribution. Call the set of sampled states S_W.

3. Iterate the Bellman equation once at each state $\Theta_t \in S_W$. On the $k + 1$th iteration,

$$V_{k+1}(\Theta_t) = \log\left(\exp(\max_{j=0,1,\ldots,J} E\left[U_j|\Theta_t\right]) + \sum_a \exp(E\left[-c_a + \hat{V}_k(\Theta_{t+1})|\Theta_t\right])\right)$$

4. Estimate a neural network regression of $V_{k+1}(\Theta_t)$ on the sampled states Θ_t. Let $\hat{V}_{k+1}(\Theta)$ the predicted values of the neural network at any state Θ.

5. Repeat steps 2-4 until convergence. The final predicted value $\hat{V}(\Theta)$ is the final approximation of the value function.

The basic idea is to perform value iteration at a set of sampled states rather than all possible state points, and then approximate the value function at states which were not sampled through interpolation. That is, by using the iterated value function and sampled states $(V_{k+1}(\Theta), \Theta)|_{\Theta \in S_W}$ as data in a prediction architecture, we obtain a general value function $\hat{V}_{k+1}(\Theta)$ that can be evaluated for any state point. I follow Bertsekas and Tsitsiklis (1996) and use a neural network regression. In addition, the dimensionality of the state space is further reduced by using indices that capture the most important features of the state space. More precise details for each of these steps are provided in Appendix A, where I also show that the algorithm converges well.

5.3 Identification

First, I discuss the normalizations needed to take the model to data. As is standard in discrete choice models, the utility of the outside good is set to $m = 0$, so that the utility for other goods is measured relative to the outside good. For the price parameters, one of the η_r require a location normalization. Notice that we can increase p_0 by λ and decrease each η_r by λ while leaving behavior implied by the model unchanged. Thus, I apply a location normalization by setting $\eta_{\text{Other}} = 0$, so that η_r measures the mean log prices offered by each of the three biggest retailers relative to other retailers.

The identification arguments for the taste parameters and search cost parameters are identical to the empirical search literature.[16] Informally, in addition to variation in purchases of products with different characteristics identifying the taste parameters, variation in searches and search order provide additional identification power. That is, the degree to which products with certain characteristics appear early in consumers' search sequences helps to pin down the taste parameters. Search costs are identified from variation in the lengths of consumers' search sequences, since they do not affect the consumer's decision to purchase. Separate identification of retailer specific search costs comes from variation in consumers who search at each retailer; lower search cost retailers are more likely to be searched. Identification of the prior and learning parameters is unique since my model combines learning over prices and match values, however the logic follows similarly to the learning literature.[17] Informally, these parameters are identified through the dynamics of the model as in Keane and Wolpin (1994) and Erdem et al. (2008). As consumers search more, their purchase decisions are based more on the learning parameters rather than the prior parameters. So, variation in search sequence lengths across consumers identifies the prior and learning parameters. I provide a more detailed informal identification discussion below.

First, I discuss identification of the prior parameters $(\mu_0, \sigma_\mu^2, p_0)$. Consumers who search relatively little, as per Bayes' Rule, base more of their decisions on their prior beliefs which depend on $(\mu_0, \sigma_\mu^2, p_0, \eta_r)$. Thus the mean and variance of purchase probabilities for these kinds of consumers help identify the prior parameters. Unlike much of the learning literature, the assumption that signal variance and the consumer's initial prior variance are different means that a scale normalization is not required on either of these two variance terms. As in Erdem et al. (2008), the observed price distributions across retailers help pin down p_0, η_r.

In contrast, consumers who purchase after a longer search spell, base more of their decisions on the signals they observed, which depend on (μ, δ_r). Specifically, as consumers search match value beliefs μ_t converge to the true mean match value μ, and so purchase probabilities for consumers who search more help identify μ. The match value signal precision parameters δ_r affect consumers future search choices for the products that the consumer has

[16] For instance, see Ursu et al. (2023) for details.

[17] For instance, see Ursu et al. (2020) who estimate a model based on the Chick and Frazier (2012) framework.

already searched. The model implies that the consumer is less likely to continue searching j if they received a precise signal of its match value. Thus, the extent to which consumers switch search choices over time after searching at each of the four retailers pins down the δ_r. Thus, the mean and variance of purchase probabilities for these consumers identify the learning parameters.

Lastly, I consider the price variance σ_ω^2, and the price-quality relation parameter ϕ. The price variance parameter σ_ω^2 is identified from the variance in the observed price distribution. Finally, the price-quality parameter ϕ is pinned down by the price-quality relationship, given that the other parameters are identified.

6 Results

6.1 Parameter Estimates

Table 9: Estimation Results

Parameter	Estimate	SE	Parameter	Estimate	SE
Tastes (α, β)			Signal Variance (δ_r)		
Log Price	-1.682	(0.103)	Amazon	0.378	(0.059)
Log Pixel	1.718	(1.455)	Best Buy	0.984	(0.229)
Log Zoom	0.726	(0.051)	Walmart	0.588	(0.153)
Log Display	-0.682	(0.989)	Other	1.572	(0.274)
Amazon	0.202	(0.052)	Search Cost (γ_r)		
Best Buy	-0.050	(0.093)	Amazon	1.874	(0.072)
Walmart	0.667	(0.108)	Best Buy	2.393	(0.748)
Other	-0.605	(0.071)	Walmart	2.247	(0.680)
			Other	2.552	(0.639)
Match Learning			Price Beliefs		
Prior mean μ_0	1.796	(0.107)	P_0	4.934	(0.120)
Prior sd σ_μ	1.170	(0.083)	ϕ	0.572	(0.045)
Mean Match Value μ	5.225	(1.020)	Price Deviations (η_r)		
			Amazon	-0.078	(0.775)
			Best Buy	0.080	(1.536)
			Walmart	-0.042	(1.080)
Log-likelihood			-24832.01		
Search Sequences			1253		
Total Searches			8276		

Table (9) provides the estimated parameters from the estimation procedure outlined in Section 5. For estimation, the monte carlo approximation of the integral in (16) involves drawing 30 values of s_{jrt} and p_{jrt} at each t.

The estimates of the taste parameters suggest, as expected, that cameras with greater functionality, as measured by the pixel and zoom characteristics increase utility, while higher prices decrease utility. I find that search costs are considerable and significantly different across retailers, at \$1.21 per search when a consumer visits a product page on Amazon and up to \$2.39 per search when a consumer visits a product page on Walmart. Indeed, the majority of searches observed in the data are at Amazon and prices are generally lowest as well. Comparing learning across retailers, I find considerable differences in the signal parameters δ_r. Consumers learn most precisely at Amazon, and least precisely at Walmart. These variance estimates are quite large relative to the estimated mean match value at 9.61. This rationalizes the observation that consumers conduct several searches before making a purchase decision. Thus, different information sources have a significant impact on consumers' learning and search decisions.

In contrast to the results of Ursu et al. (2020), I find positive estimates of the prior mean and match value, which make sense since most search sequences in these data result in a purchase. The difference in their values also suggest that consumers start with a pessimistic view of their match values but improve as they search and learn about products. This is consistent with the data observation that most consumers purchase a product they have searched before, and possibly multiple times.

The price learning coefficients suggest that consumers direct search based more on the ability to learn match values. The price deviations are largely insignificant, matching the data patterns in Table 4 that prices among different retailers are similar on average. The price variation σ_P is relatively large at 1.04, suggesting that consumers still expect to receive price deviations when searching. Together, these estimates match Table 5 which suggests consumers can still expect to find small discounts for preferred products with additional searches.

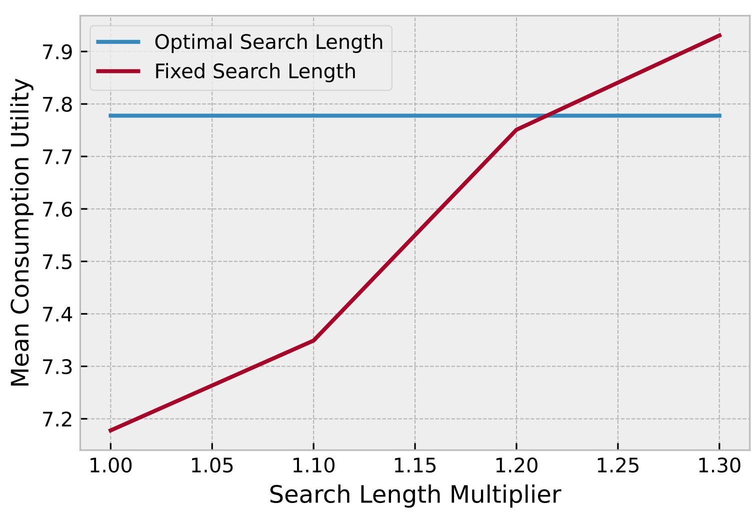
6.2 The Value of Information Quality

The parameter results from the previous section show that retailers differ in the quality of information provided to consumers. Consumers learn more efficiently at Amazon and Walmart, and more generally all three large retailers are estimated to provide higher quality information than other retailers. Thus, consumers who wish to find high quality match information may prefer searching and receiving match signals at the large retailers. In this section I quantify the benefit to consumers of the better information quality available at the larger retailers, which is measured in the model by the match value signal variances $\exp(\delta_r)$. To do this, I compare the estimated signal structure across retailers $(\hat{\delta}_r)$ to a counterfactual signal structure.

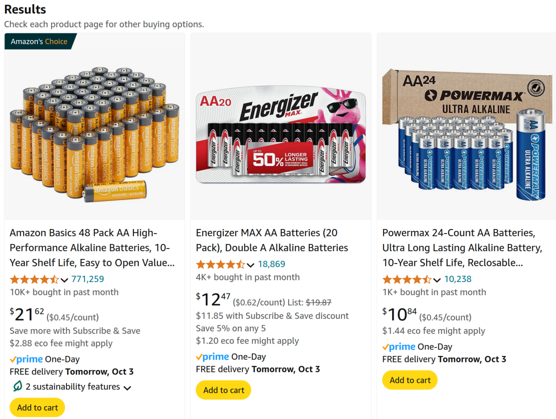
The first counterfactual is a uniform signal structure, where the signal variance at the larger retailers is set to the estimated signal variance at "other" retailers. That is, I set $\delta_r = \hat{\delta}_{Other}$ for all r. In this counterfactual signals are identical across retailers and, at the large retailers, more uninformative than in the estimated signal structure. The effect of this change is that if the consumer searches as much as in the estimated structure, their beliefs about utility are more noisy. Thus the consumer is more likely to purchase a poorly matched product. To expect to be matched with a product as well as they would be in the estimated structure, the consumer will have to search more.

So, I compare search-path outcomes for consumers in each of the two signal structures. Specifically, I compare how many more searches the consumer would need to conduct, in the counterfactual structure, to obtain the same consumption utility in expectation as in the estimated structure. For each consumer, I simulate a search path under the estimated signal structure where they behave optimally according to the model. The length of this search path is denoted l_i, the optimal search length. Then, holding utility draws fixed, I simulate a number of search paths where the search lengths are fixed to ml_i for $m \geq 1$.[18] This helps to isolate the effect of a change in the structure on the consumption utility obtained by consumers. Note that for this counterfactual I do not report total utility (consumption utility minus total search costs), since search lengths are restricted and not the optimal lengths under the counterfactual structure. Figure 1 plots the consumption utilities obtained in both signal structures.

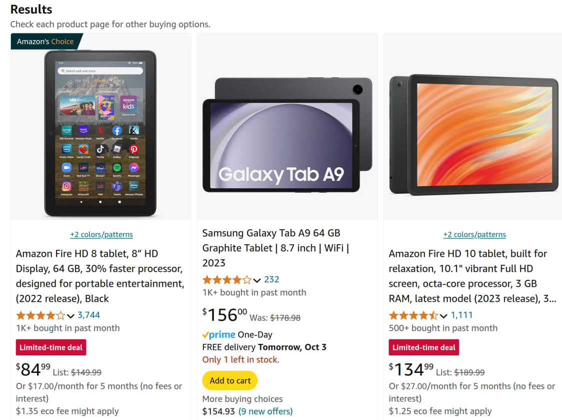
Figure 1: Signal Structure Counterfactuals



[18]More precisely, I simulate search paths where lengths are fixed to $l_i + k$ for $k \in \mathcal{Z}^+$. To compare search paths with different lengths, I convert $l_i + k$ to ml_i, and group search paths by m.



(a) Search Query: AA Batteries



(b) Search Query: Tablet

Figure 2: Examples of Steering on Amazon

The blue line plots the mean consumption utility for consumers' simulated search paths in the estimated signal structure. The red line plots the mean consumption utility for each consumer's set of counterfactual simulations for different values of m. When search lengths are identical, consumers are worse off under the uniform signal structure. As consumers increase their search efforts by searching more, their expected consumption utility gradually increases. The point where the two lines cross indicates that consumers need to conduct around 21% more searches to obtain the same level of consumption utility under the uniform structure. If the consumer searched the same nuber of times as in the estimated structure, they can expect to lose about 8.35% in consumption utility.

7 Counterfactuals

7.1 Steering

This section studies the interaction between information quality and a retailer's ability to steer consumer search. Figure 2 shows examples of Amazon steering customers to search for their own brands of products. When searching within a product category in which Amazon has its own brand, a common steering strategy used is for Amazon to raise awareness of its brand by placing it at the top of users' search results, thereby increasing the probability that a consumer clicks on it and visits its product page. While it is clear that this does induce a consumer to "search" for the product by visiting its page, since it is made visible by the high search ranking (Ursu (2018)), it is less clear how likely it is that this will increase the chance of purchasing the focal product. I hypothesize that the product information and consumer reviews the consumer sees upon visiting the product page can moderate whether consumers follow through by purchasing the focal product. Consumer reviews can signal to consumers whether the product is a good match, which likely increases purchase probability if the focal product is generally a good match and decreases purchase probability if its generally a poor match. Thus, the harm to consumers may be limited if the retailer also offers high quality match information through reliable consumer reviews. At the same time, if the product is a good match, then high quality information may increase the probability of purchase so that the retailer can extract rents by steering.

To test this, I simulate search paths where Amazon attempts to steer towards a focal product F by pushing the product to the top of users' search results. The change in search rankings is mapped to the model as a relative reduction in the focal product's search cost $c_{F,AMZN}$ by $f\%$ and an increase in non-focal search costs $c_{j,AMZN}$ by $f\%$ for $c \neq F$. In each simulation, the focal product F is randomly drawn from the set of products $\mathcal{J}$. I simulate search paths when the focal product is assigned either a high match value $\mu_F = \hat{\mu} + 2\hat{\sigma}_\mu$ or a low match value $\mu_F = \hat{\mu} - 2\hat{\sigma}_\mu$. Each simulation also varies the signal structure where $\delta_{AMZN} = m\hat{\delta}_{AMZN}$ for $m \in \{0.5, 1.0, 1.5\}$. I compare the search path outcomes for each combination of match value and signal structure to ask whether the quality of information consumers can access affects the probability of purchasing a focal product, when the product is generally a good or poor match for consumers.

Table 10: Steering on Amazon

	$m = 0.5$	$m = 1.0$	$m = 1.5$
High Match Value			
Consumption Utility	11.551	10.808	8.638
Purchase Probability	0.381	0.346	0.252
Low Match Value			
Consumption Utility	11.204	9.874	8.032
Purchase Probability	0.026	0.045	0.039

Each counterfactual estimate is obtained by simulating 1000 search paths. Search costs lowered for F and increased for $j \neq F$ by 25%.

Table 10 presents results for the simulations for $f = 25$. To compare outcomes, I report the mean consumption utility obtained across consumers and the proportion of purchases for the focal product.

As the literature finds, search can be successfully steered to the focal product by lowering search costs. Whether the consumer follows this with a purchase depends on the information quality and match value of the product. As expected, when the product is generally a good match, purchase probability is high at 34.6% when $m = 1$, and much lower at 4.5% when the product is a poor match. The large difference between these two probabilities is because the focal product is assigned a match value 2 standard deviations below or above other products. This means it is very likely to be the highest or lowest matches among all products. Note that the purchase probability is still relatively high for such a poorly matched product due to steering, since the search cost for the focal product is much lower than for other products. The effect of changing information quality depends on whether the product is a good match. When the product is a poor match, purchase probability decreases when information quality increases and vice versa. Thus, match information has a moderating effect on steering if the focal product is a poor match, but actually increases purchase probability when the focal product is a good match for consumers. In either case, consumers are less harmed when information quality is high since consumers learn quickly whether the focal product is a good match or whether they should continue their search effort.

These results highlight the importance of considering match information as contributing to retailer market power. Sticking to the Amazon example, their in-house brands like "Amazon Basics" and "Amazon Fire" often receive positive reviews and high ratings on their platform (Schley (2016)), and are regarded as cheaper options than other brands on the platform. Thus, it seems likely that the effect of Amazon's steering is to improve the visibility of their own brands and push consumers to purchase their products.

8 Conclusion

In this paper, I study consumers' decisions to search and learn about products before making a purchase decision. I develop a sequential search model in which consumers face uncertainty about their match values for products. In order to resolve this uncertainty, they can search to receive a noisy signal about their match value and update their beliefs in a Bayesian fashion. I adapt the model of Chick and Frazier (2012) by allowing for multiple sources of information, and thus multiple sources at which the consumer can sample each product. To simultaneously model uncertainty over prices, I embed the framework of Erdem et al. (2008). To study the effects of retailer competition, I endogenize the choice of retailer at which the consumer will search to find product information. Each retailer differs in its cost of search and the benefit to search in terms of the precision of its match value signal. I formulate the consumer problem as a dynamic programming problem and then apply the model to a dataset on consumers browsing for cameras online. The model is estimated using approximate value iteration as in Keane and Wolpin (1994) and Bertsekas and Tsitsiklis (1996). Estimation of the model reveals that different retailers differ in their estimated match value signal precision. That is, consumers learn differentially across retailers which helps rationalize data patterns where consumers exhibit preferences for search at a particular retailer.

Using the model, I estimate a set of counterfactuals to evaluate a retailer's ability to divert search by altering information provision. I find that consumers can be diverted to search and purchase a chosen product by improving information provision for the chosen product relative to others. Complementing the findings of the literature, I find that in the presence of competing retailers offering the same products, the ability to divert search online is weaker.

An important area for further work is in understanding how a consumer gains further information about the product by visiting different sources of information. I do not observe the exact information consumers read to learn about their match values. With these data, one can conduct interesting analysis on which type of information is most useful for learning.

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Appendix

A Approximate Value Iteration

This section gives further details about the approximate value iteration technique I employ to solve for the value function in 13. The implementation of the algorithm follows closely from Hodgson and Lewis (2023). Note that the augmented state variable includes the parameter vector ρ, and thus the final approximation of the value function obtained can be evaluated for any guess of ρ. Thus, this algorithm only needs to be computed once before estimating the model through maximum likelihood.

A.1 Sampling Augmented States

The augmented state variable, $\tilde{\Theta} = \big((s_{a(t)})_{t<\tau}, (p_{a(t)})_{t<\tau}, (a(t))_{t<\tau}, \rho\big)$, includes the history of match value signals, observed prices, and the set of product-retailer search choices the consumer made up to time τ. Recall that this encapsulated all relevant information for the consumer's problem. Thus, we will solve for $V(\tilde{\Theta})$. Since value function iteration at all state points is not possible, it is necessary to sample a subset S_W of all state points, and then interpolate the value function at states which were not sampled. The way in which states are sampled is not critical so long as the set S_W has good coverage of the state space. I follow the literature and simulate consumer choices through the model, and record the corresponding states.

First, I draw values of the parameters ρ. Given an initial guess, $\tilde{\rho}$, the sampled parameters $\tilde{\rho}_w$ are drawn from the $N(\tilde{\rho}, \Sigma_\rho)$ distribution, where Σ_ρ is a diagonal matrix. I draw 500 parameter vectors.

Next, to simulate the rest of the augmented state variables, given a parameter vector $\tilde{\rho}_w$, I simulate search sequences of length 100. Given an initial guess of the value function $\hat{V}(\tilde{\Theta})$, we can use the search probabilities from equation (14) to simulate consumer search choices. I follow Hodgson and Lewis (2023) to shut down the outside option, and increase the probability of each search choice as:

$$\tilde{P}_w(a|\tilde{\Theta}, \tilde{\rho}_w) = \Delta_0 P_w(a|\tilde{\Theta}, \tilde{\rho}_w) + \Delta_1 \frac{1}{JR},$$

where $\Delta_0 + \Delta_1 = 1$. This modification to search probabilities generates simulated search sequences that provide greater coverage of the state space than the optimal search sequences following search probabilities $P_w(a|\tilde{\Theta}, \tilde{\rho}_w)$ from equation (14).

The states at each time-period in the simulated search sequence are recorded as $((s_{a(t)})_{t<\tau}, (p_{a(t)})_{t<\tau},$

$(a(t))_{t<\tau}$. S_W is then these recorded states for each simulated parameter vector $\tilde{\rho}_w$. Thus, S_W consists of 50,000 simulated state points.

A.2 Approximation of the Value Function

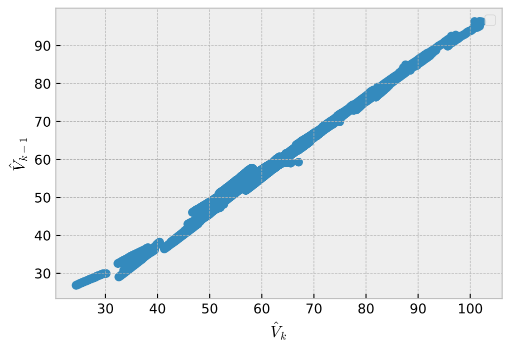
The next step is to compute the corresponding value function $V(\tilde{S}_{w\tau})$ for each state point $\tilde{S}_{w\tau} \in S_W$. Given an initial guess of the value function $\hat{V}_0 = 0$, this is obtained by iterating on the Bellman equation in (13). Now, we have 50,000 pairs of the tuple $(V(\tilde{S}_{w\tau}), \tilde{S}_{w\tau})$. To obtain the value function at all possible state points, we interpolate by using these pairs as data in a neural network regression. This is necessary since the next iteration of the Bellman equation at a new set of sampled states requires knowing the Value function at all possible state points that the consumer may reach after an additional search.

The procedure is as follows. First, note that the augmented state variable grows in dimension with each additional search. To fix the dimensionality of the augmented state space at a reasonable level, I generate a set of indices that sufficiently capture the most important features of the state space. First, since the state space includes the history of match value signals obtained, I replace $(s_{a(t)})_{t<\tau}$ with two measures; the average and maximum of match value signals observed. Similarly, I replace the history of prices observed $(p_{a(t)})_{t<\tau}$ with the average and maximum of prices observed. I also include the average and maximum of the expected utility across all products. Lastly, I replace the history of search choices $(a(t))_{t<\tau}$ with the number of searched alternatives, number of unsearched alternatives, and total searches at each retailer. I also append to these features the parameter vector $\tilde{\rho}_w$. This results in 41 features that represent the augmented state space.

I regress $V_k(\tilde{\Theta}_\tau)$ on these 41 features using a neural network with a single layer with 20 neurons and a rectified linear unit, ReLU, activation function. The neural network prediction at any state point $\hat{V}_k(\tilde{\Theta})$ is then used in the next iteration of the algorithm.

The algorithm iterates the sampling of states S_W, iteration of the Bellman equation (13), and the neural network procedure until convergence. Convergence is defined as the quantity $\sum_{S_W}(\hat{V}_k(\tilde{\Theta}) - \hat{V}_{k-1}(\tilde{\Theta}))$ falling below a critical value. Figure 3 plots $\hat{V}_k(\tilde{\Theta})$ against $\hat{V}_{k-1}(\tilde{\Theta})$ for the final iteration of the algorithm at the final set of sampled states S_W. I find that this algorithm works quite well; the convergence in Figure 3 was achieved after only 9 iterations.

Figure 3: Convergence of Value Function



B Model Extensions

Stochastic Prices

In the BKM data, for approximately 60% of all search paths, there is no $a = (j, r)$ for which the consumer sees different prices upon a revisit(s) in their search path. Of the consumers with multiple observed prices, roughly 90% see multiple prices for only one a. Thus for simplicity, in the main analysis I restrict the model so that all uncertainty about prices is removed after one search. That is, consumers behave as if p_{jr} is fixed over time.

The model can be relaxed so that prices are stochastic, by letting the price shock ω_{jr} be time-varying and independent across time, ω_{jrt}, as in Erdem et al. (2008). This can be accomodated with a suitable change in the Bayesian updating rules.

In this case, upon searching $a = (j, r)$, the consumer sees a price p_{jrt}, and updates their belief about the price distribution for the next time they visit a to

$$N(p^M_{jrt}, \sigma^2_{p_{jrt}}), \tag{18}$$

$$p^M_{jrt} = p^M_{jrt-1}\frac{\sigma^{-2}_{p_{jrt-1}}}{\sigma^{-2}_{p_{jrt}}} + p_{jrt}\frac{\sigma^{-2}_{\omega}}{\sigma^{-2}_{p_{jrt}}}, \tag{19}$$

$$\sigma^2_{p_{jrt}} = \left(1/\sigma^2_{jrt-1} + 1/\sigma^2_{\omega}\right)^{-1}. \tag{20}$$

Similar to the Bayesian updating rules for match values, as the consumer observes prices, their belief about the price distribution for a becomes more precise. In addition, their belief updates to a weighted average of the prior and the price that is observed. The rules in (4)-(6) and in (7) still apply to this case.