18.01 Single Variable Calculus
Fall 2006

18.01 Exam 2

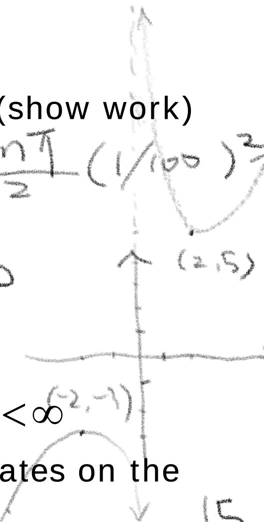
Tuesday, Oct. 17, 2006

Problem 1. (15 pts.) Estimate the following to two decimal places (show work)

a. (8 pts.) $\sin(\pi + 1/100)$ $\approx \sin\pi + \cos\pi \cdot 1/100 - \frac{\sin\pi}{2}(1/100)^2$

$= -0.01$

b. (7 pts.) $\sqrt{101}$ $\approx \sqrt{100} + \frac{1}{2\sqrt{100}}\cdot(101-100) = 10.05$



Problem 2. (20 pts.) Sketch the graph of $y = \frac{4}{x} + x + 1$ on $-\infty < x < \infty$
and label all critical points and infection points with their coordinates on the graph along with the letter "C" or "I"

$x^2 + x + 4 = 0 \to$ None

$y'' = \frac{8}{x^3}$ $x>0$ $y''>0$, $x<0$ $y''<0$

$y' = -\frac{4}{x^2} + 1 = 0 \to x = \pm 2$ $y = \begin{cases} 5 \\ -3 \end{cases}$

$x \neq 0$ $x \to \pm\infty$ $y \to \pm\infty$ $x \to \pm 0$ $y \to \pm\infty$

Problem 3. (20 pts.) An architect plans to build a triangular enclosure with a fence on two sides and a wall on the third side. Each of the fence segments has fixed Length L. What is the length x of the third side if the region enclosed has the largest possible area? Show work and include an argument to show that your answer really gives the maximum area.

$A = u\sqrt{L^2 - u^2}$ $(u = \frac{x}{2})$

$h = \sqrt{L^2 - \frac{x^2}{4}}$

$\frac{d}{du}A = \sqrt{L^2-u^2} + u \cdot \frac{-2u}{2\sqrt{L^2-u^2}} = \frac{L^2 - 2u^2}{\sqrt{L^2-u^2}} = 0$ $u = \frac{1}{\sqrt{2}}L$ $x = \sqrt{2}L$

Problem4. (15 pts) A rocket has launched straight up, and its altitude is h = $10t^2$ feet after t seconds. You are on the ground 1000 feet from the launch site. The line of sight from you to the rocket makes an angle θ with the horizontal. By how many Radians per second is θ changing ten seconds after the launch?

when $u < \frac{1}{\sqrt{2}}L$ $\frac{dA}{du} > 0$
" $>$ " " $<$ "

$\tan\theta = \frac{10t^2}{1000}$

Write down on which intervals the function is: $\frac{d\theta}{dt}\frac{1}{\cos^2\theta} = \frac{20}{1000}t$

Increasing: $0 \sim +\infty$

$\theta' = \frac{20000t}{100000+t^2}$ $\frac{d\theta}{dt} = \frac{20\cos^2\theta}{1000}t$

Decreasing: None.

$\theta'' = \frac{20000}{100000+t^2} - \frac{40000t^2}{(10000+t^2)^2}$

$\cos\theta = \frac{1000}{\sqrt{1000^2 + (10t^2)^2}}$

Concave down: $100 \sim +\infty$

$= \frac{200000000 - 20000t^2}{(10000+t^2)^2}$

$\frac{d\theta}{dt} = \frac{20000t}{1000^2 + (10t^2)^2}$

$= \frac{2000}{1001}$ $(t=10)$

Problem 5. a. (10 pts) Evaluate the following indefinite integrals

+C

i. $\int \cos(3x)dx$ $d\sin(3x) = 3\cos(3x)\,dx \rightarrow \frac{1}{3}\sin(3x)$

ii. $\int xe^{(x^2)}dx$ $de^{(x^2)} = 2xe^{x^2}dx \longrightarrow \frac{1}{2}e^{(x^2)}$ +C

b. (10 pts) Find $y(x)$ such that $y^1 = \frac{1}{y^3}$ and $y(0)=1$

$\frac{dy}{dx} = \frac{1}{y^3}$ $y^3dy = dx$ $\frac{1}{4}y^4 = x + c$ $c = \frac{1}{4}$

$y = \sqrt[4]{4x+1}$

Problem 6. (10 pts.) Suppose that $f'(x)=e^{(x^2)}$, and $f(0)=10$
One can conclude from the mean value theorem that

$$A < f(1) < B$$

for which numbers A and B? $B = f(0) + f'(1)(b-0)$

$f(1) = f(0) + f'(c)(1-0)$ $B = 10 + e \cdot b$.

for some c, $0 \leq c \leq 1$

B

$f'(0) = 1$

$10 + 1 < f(1) < 10 + e$ ...

$f'(1) = e$