18.01 Single Variable Calculus
Fall 2006

$u = \frac{1}{x+1}$ $du = \frac{-1}{(x+1)^2}dx$ $v = \frac{1}{x}$

$uv' = \frac{1}{x+1} \cdot \frac{-1}{x^2}$

$= \int \frac{u}{u-1} du =$

$u = \frac{1}{x+1}$, $du = \frac{-1}{(x+1)^2}dx$, $x = \frac{1-u}{u}$

18.01 Exam 4

$$\overset{-1}{\frac{A}{(x+1)^2}} + \overset{-1}{\frac{B}{(x+1)}} + \overset{1}{\frac{C}{x}} = \frac{1}{x(x+1)^2}$$

Problem 1. (15 points) Evaluate $\int \frac{dx}{x(x+1)^2} = \frac{1}{x+1} + \ln|x+1| + \ln|x| + C$

$\cancel{u' = \ln x \quad u = \frac{1}{x}}$ $v = x^2$ $uv' = \frac{1}{x} \cdot 2x$

Problem 2. (15 points) Evaluate $\int (\ln x)x^2 dx = \int u'v = uv - \int uv' = x - 2$

Problem 3. (20 points) Use a trigonometric substitution to evaluate $\int_0^1 \frac{dx}{(4+x^2)^{3/2}}$
(Be careful evaluating the limits) $x = 2\tan u$ $dx = 2\sec^2 u\, du$

$= \frac{2\sec^2 u\, du}{(4\sec^2 u)^{3/2}} = \frac{du}{4\sec u} = \frac{\cos u\, du}{4}$ $\int_0^{\cot\frac{1}{2}} \frac{\cos u\, du}{4} = \frac{\sin u}{4}\Big|_0^{\cot\frac{1}{2}} = \frac{1}{4\sqrt{5}} - \frac{0}{4}$ ✓

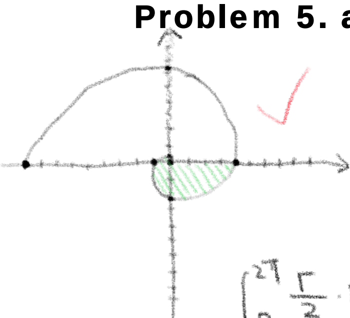
Problem 4. a. (10 points) Find an integral formula for the arc length of the curve $y = 2\sqrt{x+1}$ for $0 \le x \le 1$. Do not evaluate. $dy = 2 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x+1}} = \frac{1}{\sqrt{x+1}}dx$

$ds = \sqrt{(dy)^2 + (dx)^2} = \sqrt{\frac{x+2}{x+1}}dx$ $\int_0^1 ds = \int_0^1 \sqrt{\frac{x+2}{x+1}}dx$ ✓

b. (10 points) Find an integral formula for the surface area of the curve in part (a) rotated around the x-axis. Simplify the integrand and evaluate the integral.

$\int_0^1 2\pi y\, ds = \int_0^1 4\pi\sqrt{x+1} \cdot \sqrt{\frac{x+2}{x+1}}dx = 4\pi\int_0^1 \sqrt{x+2}\, dx = 4\pi \frac{2}{3}(x+2)^{\frac{3}{2}}\Big|_0^1 = 8\pi\left(3\sqrt{3} - \frac{2\sqrt{2}}{3}\right)$ ✓

Problem 5. a. (7 points) Sketch the spiral $r = \theta^2$, $0 \le \theta \le 3\pi$. Say how many times the curve meets the x-axis counting $\theta = 0$ as the first time, and mark those points with X-s. (Your sketch need not be accurate to scale.) 4 ✓



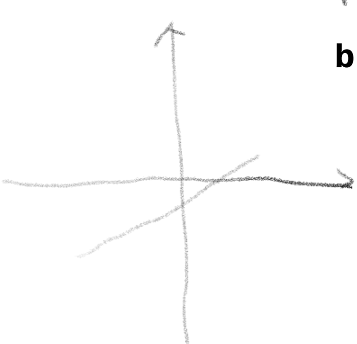
b. (8 points) On your picture, shade in the region $0 \le r \le \theta^2$, $0 \le \theta \le 2\pi$, and find its area.

$\int_0^{2\pi} \frac{r}{2} \cdot r\, d\theta = \int_0^{2\pi} \frac{\theta^4}{2} d\theta = \frac{\theta^5}{10}\Big|_0^{2\pi} = \frac{2^4\pi^5}{5}$ ✓

$= \int \frac{\sin\theta + \cos\theta\, d\theta}{1 - \sin 2\theta}$

Problem 6. a. (10 points) Find the equation in polar coordinates for the line $y = x - 1$ in the form r= $\int(\theta)$ ✓

$r\sin\theta = r\cos\theta - 1$ $r = \frac{1}{\cos\theta - \sin\theta} = \int \frac{\sin\theta + \cos\theta\, d\theta}{(\cos\theta - \sin\theta)^2}$



b. (5 points) Find the range of θ for the portion of line y=x-1 in the range $0 \le x \le \infty$. (It helps to draw a picture.)

$-\frac{\pi}{2} \sim \frac{\pi}{4}$ $0 \le r\cos\theta \le \infty$ ✓

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