L19. ROTATING RIGID BODIES - MOMENT OF INERTIA - PARALLEL AXIS AND PERPENDICULAR AXIS THEOREM - ROTATIONAL KINETIC ENERGY - FLY WHEELS - NEUTRON STARS - PULSARS

-Rotating Rigid Bodies

$$v = \omega R = \dot{\theta} R$$

$$a_{tan} = \dot{\omega} R = \ddot{\theta} R = \alpha R$$ (α: angular acceleration rad/s^2)

$$x \to \theta \qquad v \to \omega \qquad a \to \alpha \qquad +$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad \to$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

-Moment of Inertia (m: mass, measurement of inertia) 惯性矩/转动惯量

$$K_{disk} = \frac{\omega^2}{2} \sum m_i r_i^2$$

$\sum m_i r_i^2 = I$: moment of inertia

$$K_{disk} = \frac{1}{2} I_c \omega^2 \qquad \to \qquad m \to I$$

$$I_{disk_c} = \frac{1}{2} m R^2$$

Sphere: $I = \frac{2}{5} m R^2$ Rod: $I = \frac{1}{12} m l^2$

-Parallel Axis and Perpendicular Axis Theorem

$$I_{l'} = I_l + m d^2$$

$$I_z = I_x + I_y$$

L20. ANGULAR MOMENTUM - TORQUES - CONSERVATION OF ANGULAR MOMENTUM - SPINNING NEUTRON STARS - STELLAR COLLAPSE

-Angular Momentum (Moment of Momentum) 角动量/动量矩

$$\overrightarrow{L}_Q = \vec{r}_Q \times \overrightarrow{p} = (\vec{r}_Q \times \overrightarrow{v})m$$

$$|L_Q| = mvr_Q \sin\theta$$

$$r_Q \sin\theta = r_{\perp_Q}$$

-Torques (Moment of Force) 力矩

$$\frac{d}{dt}(\overrightarrow{L}_Q) = \frac{d}{dt}(\vec{r}_Q \times \overrightarrow{p}) \qquad \rightarrow$$

$$\frac{d\overrightarrow{L}_Q}{dt} = \frac{d\vec{r}_Q}{dt} \times \overrightarrow{p} + \vec{r}_Q \times \frac{d\overrightarrow{p}}{dt} = \overrightarrow{v} \times \overrightarrow{p} + \vec{r}_Q \times \overrightarrow{F} \qquad \rightarrow$$

$$\frac{d\overrightarrow{L}_Q}{dt} = \vec{r}_Q \times \overrightarrow{F} = \vec{\tau}_Q \qquad \text{(Torque)}$$

-the Spin Angular Momentum (a stationary axis through the center of mass)

$$L_{c_i} = m_i r_i v_i = m_i r_i^2 \omega$$

$$L_{disk_c} = \omega \sum_i m_i r_i^2 = I_c \omega \qquad \text{(regardless of reference point C)}$$

-Conservation of Angular Momentum

$$\frac{d\overrightarrow{L}_Q}{dt} = \vec{\tau}_{Q\ external}$$

L21. TORQUES - OSCILLATING BODIES - HOOPS

$|\tau_Q| = I_Q \alpha_Q \qquad (F = ma)$

$|L_Q| = I_Q \omega_Q \qquad (p = mv)$

e.g.1 Pendulum of a Ruler

$$|\tau_{P_t}| = Mgb \sin\theta_t = -I_P \alpha_t$$

$$Mgb\theta + I_P \ddot{\theta} = 0$$ (small angle approximation, $\sin\theta = \theta$)

$$\ddot{\theta} + \left(\frac{Mgb}{I_P}\right)\theta = 0$$

$$\omega = \sqrt{\frac{Mgb}{I_P}} \qquad T = 2\pi\sqrt{\frac{I_P}{Mgb}} = 2\pi\sqrt{\frac{\frac{1}{12}l^2 + b^2}{gb}}$$

e.g.2 Pendulum of a Hula Hoop

L22. KEPLER'S LAWS - ELLIPTICAL ORBITS - SATELLITES - CHANGE OF ORBITS - HAM SANDWICH

-Kepler's Laws

1. orbits are ellipses, sun is at the focus
2. equal areas — equal times
3. $T^2 \propto (mean\ distance)^3$

-Circular Orbits

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

$$v = \frac{2\pi R}{T} = \sqrt{\frac{MG}{R}}$$

$$E_{total} = K + U = \frac{1}{2}mv^2 - \frac{mMG}{R} = -\frac{mMG}{2R}$$

$$v_{escape} = \sqrt{2}v = \sqrt{\frac{2MG}{R}}$$

-Elliptical Orbits (a: semi major axis)

$$T^2 = \frac{4\pi^2 a^3}{MG}$$

$$E_{total} = K + U = \frac{1}{2}mv^2 - \frac{mMG}{r} = -\frac{mMG}{2a}$$

$$v_{escape} = \sqrt{\frac{2MG}{r}}$$

L23. DOPPLER EFFECT - BINARY STARS - NEUTRON STARS AND BLACK HOLES

L24. ROLLING MOTION - GYROSCOPES - VERY NON-INTUITIVE

-Rolling Motion (pure roll)

$$v_Q = v_{circumference} = \omega R \quad \rightarrow$$

$$a = \dot{\omega}R = \alpha R \quad \rightarrow$$

$$\tau_Q = RF_f = I_Q\alpha = I_Q\frac{a}{R} \quad \rightarrow$$

$$Ma = Mg\sin\beta - F_f = Mg\sin\beta - I_Q\frac{a}{R^2} \quad \rightarrow$$

$$a = \frac{MR^2 g\sin\beta}{MR^2 + I_Q} \quad + \quad I_Q = \frac{1}{2}MR^2 \quad \rightarrow$$

$$a = \frac{2}{3}g\sin\beta$$

-Gyroscopes

-Precession

-Angular Momentum (moment of momentum) will change to the Direction of the Torque (moment of force)

$$\omega_{pr} = \frac{\tau}{L_s}$$

L25. STATIC EQUILIBRIUM - STABILITY - ROPE WALKER

L26. ELASTICITY - YOUNG'S MODULUS

$$\frac{F}{A} = Y\frac{\Delta l}{l}$$ (Young's Modulus)

L27. FLUID MECHANICS - PASCAL'S PRINCIPLE - HYDROSTATICS - ATMOSPHERIC PRESSURE - OVER PRESSURE IN LUNGS AND TIRES

$$P = \frac{F}{A}$$ (unit: $\frac{N}{m^2}$, or Pa)

$$\lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = P$$ (Pascal's Principle)

$$\lim_{\Delta y \to 0} \frac{P_{y+\Delta y} - P_y}{\Delta y} = -\rho_y g = \frac{dP}{dy}$$ (Hydrostatic Pressure)

$$\int_{P_1}^{P_2} dP = -\rho g \int_{y_1}^{y_2} dy \qquad \rightarrow \qquad P_1 - P_2 = \rho g (y_2 - y_2)$$

-Atmospheric Pressure

Barometric Pressure

L28. HYDROSTATICS - ARCHIMEDES' PRINCIPLE - FLUID DYNAMICS - WHAT MAKES YOUR BOAT FLOAT? - BERNOULLI'S EQUATION

$$F_b = A\rho_{fluid}gh$$ (Buoyant Force)

-Bernoulli's Equation

$$\frac{1}{2}\rho v^2 + \rho g y + P_y = C$$ (Conservation of Energy per unit volume)