-1D Motion with Constant Acceleration

$$x = x_0 + v_0 t + \frac{1}{2} a_0 t^2$$

-Circular Motion

T (s): period

$f = \dfrac{1}{T}$ (s^{-1} or hertz): frequency

$\omega = \dfrac{2\pi}{T}$: angular velocity (how many radians per second)

$v = \dfrac{2\pi r}{T} = \omega r$

$\left| a_c \right| = \dfrac{v^2}{r} = \omega^2 r$: centripetal acceleration

-EXAM I

-Newton's Law

$\overrightarrow{F} = m\overrightarrow{a}$ (unit: $kg \cdot m/s^2$, or "newton", N)

$Action = -Reaction$

-Friction

$N = mg$

$F_{f,max} = \mu \cdot N$ (μ: the friction coefficient)

$\mu_s > \mu_k$ (static friction coefficient is larger than kinetic friction coefficient)

-Spring: Hooke's Law (ideal springs)

$F = -kx$ (k: spring constant, unit: N/m)

-SHO

-Dynamic Equations of a Displaced Spring

$$ma = -kx \;\rightarrow\; m\ddot{x} + kx = 0 \;\rightarrow\; \ddot{x} + \frac{k}{m}x = 0$$

$$x = A\cos(\omega t + \phi)$$ (A: the amplitude, ω: angular frequency)

$$\ddot{x} = -A\omega^2\cos(\omega t + \phi) = -\omega^2 x$$

$$\therefore \omega^2 = \frac{k}{m} \quad \rightarrow \quad \omega = \sqrt{\frac{k}{m}}$$

$$T = \frac{2\pi}{\omega} \quad \rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}}$$ (independent of amplitude and phase angle)

-Dynamic Equations of a Pendulum

-small-angle approximations

$$\theta << 1$$

-Consequence

1. $\cos\theta \approx 1$

2. excursion in y-direction is way smaller than in x-direction so that $\ddot{y} \approx 0$

-Work

$$W_{AB} = \int_A^B F\mathrm{d}x$$ (work, unit: Nm, or J, “joule”)

-Kinetic Energy

$$\frac{1}{2}mv^2 = KE = K$$

-Work-Energy Theorem

$$W_{AB} = \mathrm{KE}_B - \mathrm{KE}_A$$

-Gravity

-Gravity is a Conservative Force

$$\mathrm{U}_B - \mathrm{U}_A = mgh$$

-Newton's Law of Universal Gravitation

$$F = \frac{mMG}{r^2}$$

$$G = 6.67 \times 10^{-11} Nm^2/kg^2$$

-Gravitational Potential Energy

$$W_{WL} = \int_{\infty}^{R} \frac{mMG}{r^2} dr = -\frac{mMG}{r}\Big|_{\infty}^{R} = -\frac{mMG}{R}$$

-Resistive Forces

$$\vec{F}_{res} = -(k_1 v + k_2 v^2)\hat{v}$$

$$(k_1\ unit : km/m/s, k_2\ unit : km/m^3)$$

First term: Viscous term, Second term: Pressure term

-Terminal Velocity

$$mg = C_1 r v + C_2 r^2 v^2$$

-Two Regimes and the Critical Velocity

-Potential Energy

-Potential Energy of the Spring

$$U = \frac{1}{2}kx^2 \qquad \frac{dU}{dx} = -F_x$$

-of the Gravitational Force

$$F = -\frac{dU}{dy} = -mg$$

-Power

$$P = \frac{dW}{dt} = \frac{\vec{F}\cdot d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$$ (unit: joules per second, or W(att))

-Momentum

$$\vec{p} = m\vec{v}$$ (p: momentum, unit: $kg \cdot m/s$)

-Conservation of Momentum

$$\frac{d\vec{p}_{total}}{dt} = \vec{F}_{net,external}$$

-Center of Mass of a System

$$\vec{F}_{net,external} = M_{total}\vec{a}_{cm}$$

-EXAM II

-Impulse

$$\vec{I} = \int_0^{\Delta t} \vec{F} dt$$ (I: impulse, unit, same as momentum $kg \cdot m/s$)

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt} \quad \rightarrow$$

$$= \int_{p_i}^{p_f} d\vec{p} = \vec{p}_f - \vec{p}_i$$

-Rotating Rigid Bodies

$$v = \omega R = \dot{\theta} R$$

$$a_{tan} = \dot{\omega} R = \ddot{\theta} R = \alpha R$$ (α: angular acceleration rad/s^2)

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

-Moment of Inertia (m: mass, measurement of inertia) 惯性矩/转动惯量

$$K_{disk} = \frac{\omega^2}{2}\sum m_i r_i^2$$

$\sum m_i r_i^2 = I$: moment of inertia

$$K_{disk} = \frac{1}{2} I_c \omega^2 \quad \rightarrow \quad m \rightarrow I$$

$$I_{disk_c} = \frac{1}{2} m R^2$$

Sphere: $I = \frac{2}{5} m R^2$ Rod: $I = \frac{1}{12} m l^2$

-Parallel Axis and Perpendicular Axis Theorem

$$I_{l'} = I_l + m d^2$$

$$I_z = I_x + I_y$$

-Angular Momentum (Moment of Momentum) 角动量/动量矩

$$\overrightarrow{L}_Q = \vec{r}_Q \times \overrightarrow{p} = (\vec{r}_Q \times \overrightarrow{v})m$$

$$|L_Q| = mvr_Q \sin\theta$$

$$r_Q \sin\theta = r_{\perp_Q}$$

-the Spin Angular Momentum (a stationary axis through the center of mass)

$$L_{c_i} = m_i r_i v_i = m_i r_i^2 \omega$$

$$L_{disk_c} = \omega \sum_i m_i r_i^2 = I_c \omega$$ (regardless of reference point C)

-Conservation of Angular Momentum

$$\frac{d\overrightarrow{L}_Q}{dt} = \vec{\tau}_{Q\ external}$$

-Torques (Moment of Force) 力矩

$$\frac{d}{dt}(\overrightarrow{L}_Q) = \frac{d}{dt}(\vec{r}_Q \times \overrightarrow{p}) \qquad \rightarrow$$

$$\frac{d\overrightarrow{L}_Q}{dt} = \frac{d\vec{r}_Q}{dt} \times \overrightarrow{p} + \vec{r}_Q \times \frac{d\overrightarrow{p}}{dt} = \overrightarrow{v} \times \overrightarrow{p} + \vec{r}_Q \times \overrightarrow{F} \qquad \rightarrow$$

$$\frac{d\overrightarrow{L}_Q}{dt} = \vec{r}_Q \times \overrightarrow{F} = \vec{\tau}_Q$$ (Torque)

$$|\tau_Q| = I_Q \alpha_Q \qquad (F = ma)$$

$$|L_Q| = I_Q \omega_Q \qquad (p = mv)$$