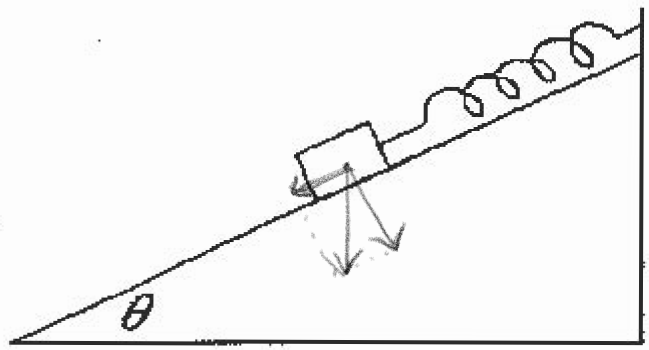
$F = C_1 rv + C_2 r^2 v^2$ $\quad$ $F = mMG/r^2$ $\quad$ $F = dp/dt$

$U = -mMG/r$ $\quad$ $U = mgh$ $\quad$ $U = kx^2/2$

Problem 1 (42 points).

A block of mass m rests on an incline which makes an angle θ with the horizontal plane (see figure). There is friction between the block and the surface. The static friction coefficient μ_s is larger than the kinetic friction coefficient, μ_k. The block is attached to a "massless" spring of spring constant k. In the absence of any forces on the spring, its (relaxed) length would be ℓ.



a. (6) We pull on the block and extend the spring till its length is $\ell + x$. What is the maximum extension, x_{max}, of the spring for which the block will remain stationary when released?

$\sin\theta\, mg + \cos\theta\, mg\, \mu_s = k\, x_{max}$ ✓

b. (6) In this position, show a free body diagram for the block. Indicate all forces that act on the block and give their magnitudes.

$\cos\theta\, mg$ $\quad$ $\cos\theta\, mg\mu_s$ $\quad$ $k x_{max}$ $\quad$ mg ✓

In the following three questions, use the symbol x_{max}.

(c.2) $(x_{max} - x)k = (\mu_s - \mu_k)\cos\theta\, mg$

c. (10) In this position the block is then gently touched at time $t = 0$. It starts moving. For what value of x will the block reach its maximum speed?

(c.1) $a = kx - \sin\theta\, mg - \cos\theta\, mg\, \mu_k \quad x = \dfrac{\sin\theta\, mg + \cos\theta\, mg\mu_k}{k}$ ✓

d. (10) As the block moves, the spring will get shorter. At some point in time, t_1, the extension is x. How much work was done by (i) gravity, (ii) the spring force, and (iii) by friction between $t = 0$ and t_1.

(i) $\sin\theta\,(x_{max} - x)mg$ $\quad$ (ii) $\dfrac{(x_{max}+x)}{2} k\,(x_{max} - x)$ $\quad$ (iii) $(x_{max} - x)\cos\theta\, mg\mu_k$ ✓

e. (10) As the block moves up-hill, the spring gets shorter. What is a necessary requirement for the spring to become at least as short as its relaxed length ℓ?

$(-\text{i} + \text{ii} - \text{iii})(0) \geq 0$ ✓

Problem 2 (32 points).

a. (6) I throw an object of mass m up from the ground at an angle of 45° with the vertical. There is a substantial airdrag on the object. It reaches its highest point after 2 sec. Will it take longer or shorter than 2 sec to fall back to the ground or will it take the same amount of time? Explain your answer clearly.
[g = 10 m/sec^2] $U + F_{ru} h = KE_u \quad U - F_{rd} = KE_d$

$\Rightarrow \quad KE_u > KE_d \Rightarrow V_u > V_d \Rightarrow t_u < t_d$ ✓

b. (6) A pendulum is hanging from the ceiling of an elevator. Its period (at small angles) is T sec when the elevator is at rest. We now accelerate the elevator downwards with 5 m/sec^2. What is the period now? Be quantitative.
[g = 10 m/sec^2] . $T \propto \frac{1}{\sqrt{g}} \quad T_n = \sqrt{2}\, T$ ✓

c. (6) We release at zero speed an oil drop of radius r in air at 1 atmosphere. The density of the oil is ρ. How small should the oil drop be so that the drag force is dominated by the viscous term which is proportional with the speed? C_1 and C_2 are the coefficients (for 1 atmosphere air) for the viscous and the pressure term, respectively.

A particle moves in one dimension as a function of time: $x = -0.3 \sin(2t + \pi/4)$.
x is in meters, t in sec.

d. (6) What is the frequency (in Hz) of this simple harmonic oscillation?

e. (8) What are the times (in sec) at which the speed of the particle is maximum?

$C_1 r V_c = C_2 r^2 V_c^2 \quad a = \frac{V\rho g - C_1 r V}{V\rho} = g - \frac{C_1 r V}{\frac{4}{3}\pi r^3 \rho} = g - \frac{3 C_1 V}{4\pi r^2 \rho}$

$V_c = \frac{C_1}{C_2 r}$

at V_c $\quad a << 0 \quad g << \frac{3 C_1^2}{4\pi C_2 r^3 \rho} \quad r << \sqrt[3]{\frac{3 C_1^2}{4\pi C_2 \rho g}}$ ✓

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d. $\omega = 2 \quad T = \frac{2\pi}{\omega} = \pi \quad f = \frac{1}{T} = \frac{1}{\pi}$ Hz ✓

e. $\dot{x} = -0.6 \cos(2t + \pi/4)$

$2t + \pi/4 = n\pi \quad t = \left(\frac{n}{2} - \frac{1}{8}\right)\pi$ ✓

Problem 3 (26 points)

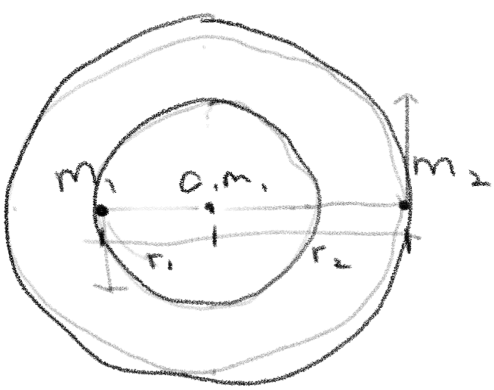
A binary star system consists of two stars of mass m_1 and m_2 orbiting about each other. The orbits of the stars are circles of radii r_1 and r_2 centered on the center of mass of the system.

a. (6) Make a drawing (sketch) of the two orbits. Indicate the positions of the center of mass, and of the stars m_1 and m_2. Mark r_1 and r_2 and indicate the direction of motion for each star.

b. (5) What is the magnitude of the gravitational force that m_1 exerts on m_2?

c. (5) What is the magnitude of the acceleration of m_1 and of m_2?

d. (10) Derive the orbital period of this binary system. Express your answer in terms of r_1, r_2, m_1, m_2, and G.



$$m_1 a_1 = m_2 a_2 \qquad m_1 v_1 = m_2 v_2$$

$$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \frac{v_2^2}{v_1^2} \cdot \frac{r_1}{r_2}$$

$$\frac{m_1}{m_2} = \frac{v_2}{v_1} = \sqrt{\frac{m_1 r_2}{m_2 r_1}}$$

$$\downarrow$$

$$\sqrt{\frac{r_1}{m_2}} = \sqrt{\frac{r_2}{m_1}}$$

$$F = \frac{m_1 m_2 G}{(r_1+r_2)^2}$$

$$a_1 = \frac{m_2 G}{(r_1+r_2)^2} = \omega^2 r_1$$

$$\omega = \sqrt{\frac{m_2 G}{(r_1+r_2)^2 r_1}}$$

$$T = \frac{2\pi}{\omega} = 2\pi(r_1+r_2)\sqrt{\frac{r_1}{m_2 G}}$$

$$= 2\pi(r_1+r_2)\sqrt{\frac{r_2}{m_1 G}}$$

$$= 2\pi(r_1+r_2)\sqrt{\frac{r_1+r_2}{(m_1+m_2)G}}$$