L1. POWERS OF TEN - UNITS - DIMENSIONS - MEASUREMENTS - UNCERTAINTIES - DIMENSIONAL ANALYSIS - SCALING ARGUMENTS

-Powers of Ten (film)

-Units

L (length): m

T (time): second

M (mass): kg

-All other quantities in physics can be derived from these fundamental quantities.

$$[\text{speed}] = \frac{[\text{L}]}{[\text{T}]} \qquad [\text{volume}] = [\text{L}]^3$$

$$[\text{density}] = \frac{[\text{M}]}{[\text{L}]^3} \qquad [\text{acceleration}] = \frac{[\text{L}]}{[\text{T}]^2}$$

-Any measurement that you make without the knowledge of its uncertainty is completely meaningless.

-Dimensional Analysis

$$t \propto h^\alpha m^\beta g^\gamma$$

$$[T]^1 = [L]^\alpha [M]^\beta \frac{[L]^\gamma}{[T]^{2\gamma}}$$

$$[M] : (\beta = 0) \qquad [L] : (\alpha + \gamma = 0) \qquad [T] : (1 = -2\gamma)$$

$$\rightarrow \qquad \gamma = -1/2 \qquad \alpha = +1/2$$

$$\rightarrow \qquad t = C\sqrt{\frac{h}{g}} \propto \sqrt{h}$$

L2. 1D KINEMATICS - SPEED - VELOCITY - ACCELERATION

-Velocity

$$\bar{v}_{t_1 t_2} = \frac{x_{t_2} - x_{t_1}}{t_2 - t_1}$$

-Average Speed vs. Average Velocity

-Instantaneous Velocity

$$v_t = \lim_{\Delta t \to 0} \frac{x_{t+\Delta t} - x_t}{\Delta t} = \frac{dx}{dt}$$

$$v = \frac{dx}{dt}$$ (the first derivative of the position w.r.t. time)

-Average Acceleration

$$\bar{a}_{t_1 t_2} = \frac{v_{t_2} - v_{t_1}}{t_2 - t_1} \ m/s^2 \qquad \bar{a} = \frac{\Delta v}{\Delta t}$$

-Instantaneous Acceleration

$$a_t = \lim_{\Delta t \to 0} \frac{v_{t+\Delta t}}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

(the first derivative of velocity versus time)

(also the second derivative of position versus time)

-1D Motion with Constant Acceleration

$$x = C_1 + C_2 t + C_3 t^2$$

$$v = C_2 + 2C_3 t$$

$$a = 2C_3$$

$$C_1 = x_0 \qquad C_2 = v_0 \qquad C_3 = \frac{1}{2} a_0$$

$$x = x_0 + v_0 t + \frac{1}{2} a_0 t^2$$

L3. VECTORS - DOT PRODUCTS - CROSS PRODUCTS - 3D KINEMATICS

-Decomposition of a Vector

$$\overrightarrow{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

-Dot Product (Scalar Product)

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{B} = |A|\ |B| \cos\theta \qquad (\theta: \text{ angle between } \vec{A},\ \vec{B})$$

-Cross Product (Vector Product)

$$\vec{A} \times \vec{B} = \vec{C}$$

$$\vec{C} = (A_y B_z - A_z B_y)\hat{x} + (A_z B_x - A_x B_z)\hat{y} + (A_x B_y - A_y B_x)\hat{z}$$

$$\vec{C} = C_x \hat{x} + C_y \hat{y} + C_z \hat{z}$$

$$|C| = |A|\ |B| \sin\theta \qquad (\theta: \text{ angle between } \vec{A},\ \vec{B})$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

-Decomposition of 3D Vectors: r, v, and a

$$\vec{r_t} = x_t \hat{x} + y_t \hat{y} + z_t \hat{z}$$

$$\vec{v_t} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \dot{x}\hat{x} + \dot{y}\hat{y} + \dot{z}\hat{z}$$

$$\vec{a_t} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \ddot{x}\hat{x} + \ddot{y}\hat{y} + \ddot{z}\hat{z}$$

L4. 3D KINEMATICS - FREE FALLING REFERENCE FRAMES

-Shape of the Projectile Trajectory

$$y = \tan\alpha \cdot x - \frac{1}{2} g \frac{x^2}{(v_0 \cos\alpha)^2}$$ (a parabola)

$$0 = v_y = v_0 \sin\alpha - gt \Rightarrow t_P = \frac{v_0 \sin\alpha}{g}$$ (time at highest point)

$$h = y(t_P) = \frac{(v_0 \sin\alpha)^2}{g} - \frac{1}{2}\frac{(v_0 \sin\alpha)^2}{g} = \frac{(v_0 \sin\alpha)^2}{2g}$$ (height at highest point)

$$t_S = 2t_P = \frac{2v_0 \sin\alpha}{g}$$ (time traveled)

$$OS = v_x t_S = \frac{v_0^2 \sin 2\alpha}{g}$$ (distance traveled)

-Demo: Monkey Hunter

L5. Circular Motion - Centrifuges Moving - Reference Frames - Perceived Gravity

-Circular Motion

T (s): period

$f = \frac{1}{T}$ (s^{-1} or hertz): frequency

$\omega = \frac{2\pi}{T}$: angular velocity (how many radians per second)

$$v = \frac{2\pi r}{T} = \omega r$$

$\left| a_c \right| = \frac{v^2}{r} = \omega^2 r$: centripetal acceleration

-Perceived Gravity

L6. NEWTON'S LAWS

-Newton's First Law and Inertial Reference Frames

-Galileo (the law of inertia): A body at rest remains at rest and a body in motion continues to move at constant velocity along a straight line unless acted upon by an external force.

-Newton (*Principia*): Every body perseveres in its state of rest or of uniform motion in a right line unless it is compelled to change that state by forces impressed upon it.

-When does the First Law work?

"inertial" frame of reference: a frame in which there are no accelerations of any kind.

-Can Newton's Law be proven?

The answer is no, because it's impossible to be sure that your reference frame is without any accelerations.

-Do we believe in this?

Yes, we do.

We believe in it since it is consistent <u>within the uncertainty of the measurements</u> with all experiments that have been done.

-Newton's Second Law

-Newton: A force action on a body gives it an acceleration which is in the direction of the force and has a magnitude given by ma.

$$\overrightarrow{F} = m\overrightarrow{a}$$

unit: $kg \cdot m/s^2$, or “newton”, N

-Newton’s Third Law

Newton: If one object exerts a force on another, the other exerts the same force in opposite direction on the one.

Action $= -$ Reaction

L7. WEIGHT - PERCEIVED GRAVITY - WEIGHTLESSNESS FREE FALL - ZERO GRAVITY IN ORBIT (MISNOMER)

L8. FRICTION

-Friction

$$N = mg$$

$F_{f,max} = \mu \cdot N$ (μ: the friction coefficient)

$\mu_s > \mu_k$ (static friction coefficient is larger than kinetic friction coefficient)

-Measurements of the Coefficient of Static Friction

$$mg\sin\alpha - F_f = 0$$

$$mg\sin\alpha - \mu_s mg\cos\alpha = 0$$

$$\mu_s = \tan\alpha$$

-Ways to Reduce Friction

L9. First Exam Review

-Demo: two fingers under a bar moving towards each other

alternate finger stops, why?