Problem 1 (16 points)

Write down the equation for a traveling wave (in the x-z plane) on a string which satisfies the following: The wave propagates in the +z-direction with a speed of 100 m/sec, the amplitude (in the x-direction) is 0.005 m, and the frequency is 400 Hz.

$$x = x_0 \sin(kz - \omega t)$$

$$x_0 = 0.005$$

$$\text{speed} = \frac{\omega}{k} = 100$$

$$f = \frac{\omega}{2\pi} = 400$$

$$\rightarrow \quad \omega = 800\pi, \quad k = 8\pi$$

$$x = 0.005 \sin(8\pi z - 800\pi t)$$

Problem 2 (23 points)

A standing electromagnetic plane wave is given by $\vec{E} = 3\hat{z}\cos(\frac{\pi}{2}y)\sin(10^8\pi t)$ V/m. y is in meters, t in seconds, and all angles are in radians.

a. (10 points) What is the wavelength (in m) of the wave, and what is the index of refraction of the medium?

b. (13 points) What is the maximum value (in V/m) that the electric field will have at the location x = 3 m, y = 0.5 m, and z = 2 m?

a. $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\frac{\pi}{2}} = 4$

$\text{speed} = \frac{\omega}{k} = 2\times10^8$ m/s.

$n = \frac{c_{vacc}}{\text{speed}} = 1.5$

b. $3\times\frac{1}{\sqrt{2}}\times1 = \frac{3}{\sqrt{2}}$ V/m

Problem 3 (23 points)

A circuit consists of a resistor of 5 Ω, a capacitor of 1 μF, and an ideal self-inductor of 0.01 H. All three are **in series** with a power supply that generates an EMF of $10\sin(\omega t)$ Volt. The internal resistance of the power supply is negligibly small. **The system is at resonance**.

a. (10 points) What is the time averaged power (in Watt) generated by the power supply?

We decrease the frequency of the power supply such that the reactance $(\frac{1}{\omega C} - \omega L)$ becomes 5 Ω; the maximum EMF remains 10 V.

b. (13 points) What now will be the time averaged power generated by the power supply?

$\bar{P} = \overline{I^2 R} = \frac{\overline{V^2}}{R} = \frac{\overline{100\sin^2(\omega t)}}{5} = 10$ W

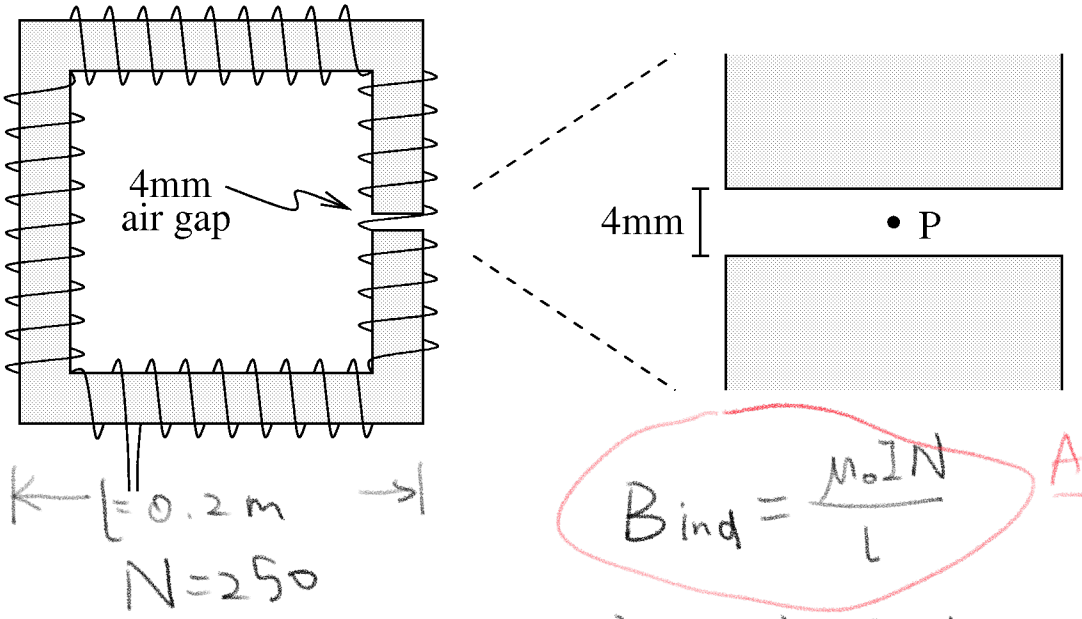
5 W

orig $P = IV$

$= \frac{V_0}{Z}\cos(\omega t - \phi) \cdot V_0 \cdot \cos\omega$

Problem 4 (22 points)

An iron bar which is 80 cm long is bent in a shape as shown. The ends of the bar do not touch; they are 4 mm apart. There is a coil with 1000 turns (see the figure). The current through the coil is 2 A. $\kappa_M = 200$ for the 80 cm long iron core. What is the approximate magnetic field strength (in Tesla) inside the air gap at P?



$B_{ind} = \frac{\mu_0 I N}{L}$ Ampere's Law

$B_{ins} = k_M B_{ind}$

$\oint \frac{1}{k_M} B\,dl = \mu_0 I_{pene}$ Maxwell

$B\left(\frac{0.8}{200} + 0.004\right) = \mu_0 \times 1000 \times 2$

Problem 5 (16 points)

Here follow 4 "True-False" questions. **For each correct answer you get 4 points, for each incorrect answer, you lose 4 points. You have the option of leaving one or more questions unanswered in which case you will neither gain nor lose points. Your total score for this problem will not be negative.** Please write only a "T" or an "F" in each box, or leave it blank.

a. Red laser light has a smaller wavelength in water than in air.

---

b. The time-averaged Poynting vector in a standing electromagnetic wave is zero.

---

c. The following traveling electromagnetic wave is linearly polarized:

$E_x = 0$; $E_y = E_0\, sin(kx + \omega t)$, $E_z = -2E_0\, sin(kx + \omega t)$

---

d. In the case where there is total reflection of light off the surface between two media, the angle of incidence must exceed a certain value which depends on the index of refraction of *both* media.