Risk Balanced Portfolio Construction

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Abstract

Given the question whether market weighted benchmark portfolios are efficient, various new concepts have been suggested and attracted investors' attention. This paper examines the difference between portfolio construction strategies using the low volatility anomaly, diversification or the idea to equalize risk contributions for portfolio selection decisions. Minimum variance, most diversified and equally weighted risk contribution portfolios have in common, that their construction does not rely on expected average returns but rather focus on risk information instead. However, while the minimum variance approach equalizes risk contribution on a marginal basis, the most diversified and equally weighted risk contributions portfolios favor diversification in their selection process. In this article, we give theoretical insight and provide a comparison of these portfolio construction approaches to market capitalization and equal weighted portfolios and show that minimum variance investing is still worth the while. Regression analysis is performed to attribute excess returns to size, value, momentum and volatility.

Keywords: Minimum Variance Portfolio, Maximum Diversification, Equal Risk Contribution, Portfolio Construction

JEL Codes: G10; G11; G14.

1 Introduction

Given the question whether market capitalization weighted benchmark portfolios are efficient, various new concepts have been suggested and attracted investors' attention. This paper examines the difference between three risk balanced portfolio construction strategies. Based on the low volatility anomaly introduced by Haugen and Baker (1991), minimizing portfolio variance, maximizing diversification and the idea to equalize total risk contributions have been proposed as alternative portfolio construction approaches.

Assuming that the rational investor seeks to maximize the expected return for a given level of volatility or equivalently seeks to minimize portfolio's ex ante risk for any given expected return, Markowitz (1952, 1959) triggered the development of modern portfolio theory with the introduction of his mean variance framework. The concept of portfolio efficiency quantifies the link that exists between risk and return of a portfolio and the complete set of optimal (or efficient) portfolios forms the mean-variance frontier. Built upon the mean variance framework, the Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965) and Mossin (1966) states that under some certain conditions and taking different levels of risk tolerance into account, the portfolio that provides the highest reward per unit of risk, better known as maximum Sharpe ratio (MSR) portfolio should be held by all investors.

Although mean-variance analysis and the CAPM are two pillars of modern finance, these models have been scrutinized since their introduction. Especially the simplified assumptions, namely the aim of rational and risk averse investors to maximize economic utilities without influencing prices, having homogeneous investment views based on all information to be available at the same time to all investors, trading without any costs and holding a well diversified portfolio, have been strongly criticized. While Roll (1977) passed criticism on the observability of the tangential portfolio, Merton (1980) found that already small changes in return estimates can lead to completely different optimal weights in a portfolio construction process. In a nutshell, due to the CAPM general assumptions and the question about availability of risk and return estimates, the mean variance framework is known to have difficulties in its practical implementation.

Nevertheless, given theoretical support by first the CAPM and second the fact that many investors assumed resulting differences to be neglected, the adoption of the market capitalization weighted equity portfolio as most obvious proxy for the MSR portfolio, hence the portfolio to track for passive equity investors and as a reasonable benchmark for active equity investors has been encouraged. Following Martellini (2008), this standard practice of constructing stock market indices has recently been subject to renewed scrutiny.

Extending the work from Merton (1987), Malkiel and Xu (2006) found that if investors do not hold the market portfolio, unsystematic risk is positively related to stock returns. According to Martellini (2008), taken together with the fact that asset pricing theory implies a positive premium for systematic risk, these findings suggest a positive relationship between total volatility and expected returns. However, Haugen and Baker (1991) were the first to provide empirical evidence for the inefficiency of market capitalization weighted indices and simultaneously focusing on risk in their alternative portfolio construction process. Repeatedly investing into a stock portfolio constructed to expose investors to minimum risk as measured by variance provided a higher Sharpe ratio than the Wilshire 5000 index in the period 1972-1989 and therewith inspired further risk based investing approaches.

Based on the discovery of the so called "low volatility anomaly" and encouraged by the fact that expected returns are difficult to obtain with a reasonable estimation error, Chan et al. (1999), Jagannathan and Ma (2003) and Clarke et al. (2006) extended testing to the efficiency of global minimum variance portfolios. While all portfolios on the efficient frontier are designed to minimize risk for a given return, the minimum variance portfolio minimizes risk without any expected return estimation (see Clarke et al., 2006).

This paper adopts the question of how to construct an optimal portfolio and contributes to existing literature by comparing minimum variance investing to two related concepts. The second concept aims to maximize diversification given as the ratio of average volatility of the stocks in the portfolio to portfolio volatility and has been proposed by Choueifaty and Coignard (2008). The equal risk contribution approach by Maillard et al. (2010) finally yields a portfolio that is similar to a minimum variance portfolio subject to a diversification constraint. We examine the difference between these three strategies using different objective functions for portfolio selection decisions. While the minimum variance approach is designed to minimize risk and equalizes risk contribution on a marginal basis, the most diversified and equal risk contributions portfolios favor diversification in their selection process.

Besides risk based investing, alternative concepts like naive diversification have been introduced, used in practice and proven to be efficient out of sample by DeMiguel et al. (2009).

This is the first paper to provide a comparison of minimum variance, maximum diversification and equal risk contribution approaches to market capitalization and equal weighted portfolios. With regard to a non-generally admitted definition of diversification (see Meucci, 2009) we scrutinize the construction approaches using diversification as main criterion. Hence, the results by Choueifaty and Coignard (2008) for the Eurozone can only be replicated when using a large cap index as underlying stock universe. Given a broad European market capitalization index as universe we show that minimum variance investing is still worth the while. Although the minimum variance portfolio is very similar to the most diversified portfolio it achieves a slightly better Sharpe ratio in the period August 1991 - December 2010. Maillard et al. (2010) compared minimum variance investing to equally weighting and equal risk contribution portfolios in an asset allocation and sector allocation context, respectively. They concluded that the equal risk contribution approach might be considered a good trade off between the other two methods in terms of absolute level of risk, risk budgeting and diversification. Focusing on equity portfolio construction we perform the comparison for a broad European equity universe and find that minimum variance as well as most diversified portfolios considerably outperform equal risk contribution portfolios, followed by equally weighted portfolios. The market capitalization weighted portfolio show fewest efficiency in terms of risk adjusted performance.

The remainder of this paper is organized as follows. First, we define minimum variance, maximum diversification and equal risk contribution portfolios and examine their theoretical properties. Second, we compare empirical results to market capitalization weighted and equally weighted portfolios. We further perform regression analysis to attribute excess returns to size, value, momentum and volatility in the sense Fama and French (1993, 1996), Carhart (1997) and Clarke et al. (2010). The last section concludes.

2 Risk Balanced Portfolio Construction

This section encompasses a detailed overview of already introduced risk balanced portfolio construction techniques. While very similar in not requiring any excess return forecasts and focusing on risk as stock selection criteria only, we elaborate major differences for minimum variance, maximum diversification and equal risk contribution portfolios. For each approach we proceed as follows. First, we introduce the optimization problem to be solved. Second, we provide mathematical details. Finally we draw consequences and economically interpret different objectives and their resultant influence on optimal portfolio weights.

2.1 Minimum variance investing

We consider an investable universe of N risky assets with σ_i^2 the variance for asset i, σ_{ij} the covariance between asset i and j and $\boldsymbol{\Omega} = (\sigma_{ij})_{i,j=1}^N$ the covariance matrix. Furthermore let $\boldsymbol{C}$ be the correlation matrix and $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_N)'$ the vector of asset volatilities. A portfolio is denoted by its weight vector $\boldsymbol{w} = (w_1, \ldots, w_N)'$ and we can therewith define total portfolio risk as:

$$\sigma(\boldsymbol{w}) = \sqrt{\boldsymbol{w}'\boldsymbol{\Omega}\boldsymbol{w}}. \tag{1}$$

According to Clarke et al. (2006), the minimum variance portfolio at the left most tip of the mean variance frontier has the unique property that security weights are independent of expected returns on the individual securities. To construct a portfolio, which under the budget constraint minimizes variance, the following optimization problem has to be solved:

$$\begin{aligned} \min \quad & \tfrac{1}{2}\boldsymbol{w}'\boldsymbol{\Omega}\boldsymbol{w}, \\ \text{s.t.} \quad & \boldsymbol{w}' \cdot \mathbf{1} = 1 \end{aligned} \tag{2}$$

with $\mathbf{1} = (1, \ldots, 1)'$ the summation vector. Quadratic optimization with equality constraints can be solved in closed form. Therefore, the optimization problem has to be reduced to an unconstrained optimization problem by introducing Lagrange multipliers (see Chincarini and Kim, 2006). Hence, for the optimization problem (2) the optimal portfolio given as:

$$\boldsymbol{w}^* = \frac{\boldsymbol{\Omega}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}} \tag{3}$$

providing a portfolio variance of:

$$\sigma^2(\boldsymbol{w}^*) = t(\boldsymbol{w}^*)\boldsymbol{\Omega}\boldsymbol{w}^* = \frac{1}{\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1}}. \tag{4}$$

In this study we further consider inequality constraints in order to account for investability of assets. Depending on a stock's market capitalization, we allow for a reasonable and implementable amount of portfolio weight. Following Chincarini and Kim (2006), this leads to a quadratic optimization problem which cannot be solved with a closed-form solution and requires numerical solutions instead. Note, that this implication holds for all risk balanced portfolio construction approaches we analyze in this study. Finally, the following minimum variance portfolio optimization problem is considered:

$$\begin{aligned} \min \quad & \tfrac{1}{2}\boldsymbol{w}'\boldsymbol{\Omega}\boldsymbol{w}, \\ \text{s.t.} \quad & \boldsymbol{w}' \cdot \mathbf{1} = 1, \\ & 0 \leq w_{i,large} \leq 4\%, \\ & 0 \leq w_{i,mid} \leq 2\%, \\ & 0 \leq w_{i,small} \leq 1\%. \end{aligned} \tag{5}$$

Regardless the set of linear constraints Roll and Ross (1977) provide a better understanding and further details of the investment weights in the minimum variance portfolio (MVP). The ith asset's weight is proportional to $\sigma_i^2 + \sum_{i \neq j} \sigma_{i,j}$, or the sum of the ith row of the inverse covariance matrix. In terms of proportionality we obtain:

$$\boldsymbol{w}^* \propto \boldsymbol{\Omega}^{-1} \cdot \mathbf{1}, \tag{6}$$

or more detailed:

$$\boldsymbol{w}^* \propto \overbrace{\mathrm{diag}(\boldsymbol{\sigma})^{-1} \cdot \boldsymbol{C}^{-1} \cdot \mathrm{diag}(\boldsymbol{\sigma})^{-1}}^{\Omega^{-1}} \cdot \mathbf{1}. \tag{7}$$

Finally, we can interpret the portfolio construction approach as simultaneously minimizing volatilities and correlations on portfolio level with the objective of equalized risk contributions on marginal basis which leads us to:

$$\partial_{w_i}\sigma(\boldsymbol{w}) = \partial_{w_j}\sigma(\boldsymbol{w}), \tag{8}$$

with:

$$\partial_{w_i}\sigma(\boldsymbol{w}) = \frac{\partial\sigma(\boldsymbol{w})}{\partial w_i} = \frac{w_i\sigma_i^2 + \sum\limits_{j \neq i} w_j\sigma_{ij}}{\sigma(\boldsymbol{w})} \tag{9}$$

the marginal risk contribution of asset i. Following Maillard et al. (2010) a small increase in any asset weight w_i will lead to the same increase in the total risk of the portfolio on ex ante basis. Minimum variance portfolio optimization is well known in practice and modern portfolio theory. Besides, maximum diversification and equal risk contribution are two further portfolio construction approaches with different objectives and correspondingly different portfolio structure.

2.2 Maximum Diversification

Given all stocks in the universe with the same volatility, the minimum variance portfolio is equal to the most diversified portfolio (MDP), which is designed to maximize the diversification ratio defined by Choueifaty and Coignard (2008). In order to construct a portfolio which is minimally exposed to exogenous shocks, hence with equal correlations across all stocks in the resultant portfolio, they propose to focus on diversification as stock selection criterion and introduced the diversification ratio of any portfolio $\boldsymbol{w}$ as the ratio of the weighted average of volatilities to portfolio volatility. The optimization problem to solve turns out to be:

$$\begin{aligned} \max \quad & D(\boldsymbol{w}) = \tfrac{\boldsymbol{w}' \cdot \boldsymbol{\sigma}}{\sqrt{\boldsymbol{w}' \boldsymbol{\Omega} \boldsymbol{w}}}, \\ \text{s.t.} \quad & \boldsymbol{w}' \cdot \mathbf{1} = 1, \\ & 0 \leq w_{i,large} \leq 4\%, \\ & 0 \leq w_{i,mid} \leq 2\%, \\ & 0 \leq w_{i,small} \leq 1\%. \end{aligned} \tag{10}$$

As for the minimum variance optimization problem, the objective function to maximize diversification contains a quadratic risk term. Given the same inequality constraints as for the minimum variance solution suggests that this problem can also only be solved by using numerical approximations. We can easily see that the solution in our setting, hence long only MDPs with $w_i \geq 0$, $i = 1, ..., N$, will provide a diversification ratio $D(\boldsymbol{w}) \geq 1$. Furthermore, given expected excess returns of assets proportional to their volatilities, maximizing the diversification ratio is formally equivalent to maximizing the portfolio's Sharpe ratio (see Martellini, 2008). Since the maximum diversification approach leads to a portfolio in which the correlation across all stocks is the same without any dominant asset pushing performance more than any other constituent and due to the fact that all assets in the MDP have the same positive correlation to it, we obtain:

$$\rho_{i,MDP} = \rho_{j,MDP}, \quad i, j = 1, ..., N. \tag{11}$$

Following Choueifaty and Coignard (2008) the proportionality of the optimal weight vector is given as:

$$\boldsymbol{w}^* \propto \text{diag}(\boldsymbol{\sigma})^{-1} \cdot \boldsymbol{C}^{-1} \cdot \mathbf{1}. \tag{12}$$

Hence, the methodology is very similar to the minimum variance construction process except for the fact, that the inverse correlation matrix is multiplied by asset volatilities only once. While the minimum variance strategy accounts for both, variance and correlations between single assets, the Choueifaty and Coignard (2008) suggest to focus on diversification considered as minimal correlations across all assets, regardless the number of constituents in the portfolio. The MDP is consequently constructed by minimizing pairwise correlations and finally rescaling resultant weights by corresponding asset volatilities. Due to the characteristic of a diversified portfolio in terms of equal correlations across all assets, no single asset will dominate the MDP's risk adjusted performance.

2.3 Equal Risk Contribution

Besides minimum variance and maximum diversification strategies we analyze a risk balanced portfolio construction approach which focuses on diversification. Nevertheless, instead of focussing on equal pairwise correlations as in the MDP, equally weighted risk contribution (ERC) portfolios are based on the idea to find a risk balanced portfolio of all stocks in the universe and to equalize risk contributions from different components of the portfolio (see Maillard et al., 2010). The ERC approach has already been proposed by Qian (2005) and can be understood as located between the naive diversification portfolio and the MVP.

Due to the fact that we use a broad European universe for our empirical analysis, we can employ the same constraints as for minimum variance and maximum diversification so that the following optimization problem has to be solved to get an optimal portfolio in terms of equal contribution to total portfolio risk across all universe constituents:

$$\begin{aligned} \min \quad & f(\boldsymbol{w}) = \textstyle\sum_{i=1}^{N} \sum_{j=1}^{N} \left[w_i(\boldsymbol{\Omega w})_i - w_j(\boldsymbol{\Omega w})_j \right]^2 \\ \text{s.t.} \quad & \boldsymbol{w}' \cdot \mathbf{1} = 1, \\ & 0 \leq w_{i,large} \leq 4\%, \\ & 0 \leq w_{i,mid} \leq 2\%, \\ & 0 \leq w_{i,small} \leq 1\%. \end{aligned} \tag{13}$$

Maillard et al. (2010) defined $\boldsymbol{\Omega w}$ as the ith row of the vector issued from the product of $\boldsymbol{\Omega}$ and $\boldsymbol{w}$ and due to the endogenous solution they finally proposed solve the optimization problem by using a sequential quadratic programming (SQP) algorithm. In order to provide more details of the resultant portfolio and based on the definition of marginal risk contribution, we introduce total risk contribution of asset i as $\sigma_i(\boldsymbol{w}) = w_i \times \partial_{w_i}\sigma(\boldsymbol{w})$. Accordingly, portfolio risk is given as:

$$\sigma(\boldsymbol{w}) = \sum_{i=1}^{n} \sigma_i(\boldsymbol{w}). \tag{14}$$

Finally, the optimal weights to provide equal risk contributions to total portfolio risk across all stocks in the portfolio can be characterized with:

$$w_i \times \partial_{w_i}\sigma(\boldsymbol{w}) = w_j \times \partial_{w_j}\sigma(\boldsymbol{w}) = \sigma(\boldsymbol{w})/n, \tag{15}$$

for all $i, j = 1, ..., N$. Having $w_i = w_j$ for equally weighted portfolios in mind, the relationship to naive diversification as well as to minimum variance strategies can immediately be established. A similar statement with respect to proportionality of optimal weights as in the descriptions above requires the introduction of a components' sensitivity to the ERC portfolio. Following Maillard et al. (2010), we start from the definition of covariance between the returns of the ith asset and the returns of the ERC portfolio, $\sigma_{i\boldsymbol{w}^*} = cov(r_i, \sum_j w_j^* r_j) = \sum_j w_j^* \sigma_{ij}$ and deduce $\sigma_i(\boldsymbol{w}^*) = w_i \sigma_{i\boldsymbol{w}^*}/\sigma(\boldsymbol{w}^*)$. Consequentially, $\beta_i = \sigma_{iw^*}/\sigma^2(\boldsymbol{w}^*)$ and $\sigma_i(\boldsymbol{w}^*) = w_i^* \beta_i \sigma(\boldsymbol{w}^*)$. Based on the definition of the ERC portfolio, we obtain:

$$w_i^* \propto \frac{\beta_i^{-1}}{\sum_{j=1}^{N} \beta_j^{-1}} = \frac{\beta_i^{-1}}{N}. \tag{16}$$

To summarize, the optimal weight of component i in the ERC portfolio is inversely proportional to its beta or sensitivity to the ERC portfolio. Therewith the ERC approach ensures to contain all stocks in the given universe and furthermore satisfies to provide equal risk contributions across all constituents. Although the maximum diversification and equal risk contribution approach have the idea of a well diversified portfolio in common, optimal portfolio will be completely different except the case when correlation across all assets is constant (see Maillard et al., 2010).

In the next section we review the data our empirical tests of the three introduced risk balanced portfolio construction approaches are based on. Furthermore, a comparison of their risk adjusted performance to naive diversification and the index portfolio will be provided. Finally, we control the exposure to Carhart (1997) factors as well as the Clarke et al. (2010) VMS factor for respective portfolio returns.

3 Empirical Results

We construct minimum variance, maximum diversification, equal risk contribution and equal weighted portfolios using the MSCI Europe Index as universe from August 1991 through December 2010. On average, the MSCI Europe Index contains 569 stocks which ensures a broad and implementable European universe for our analysis. Finally, Datastream Total Return Indices denoted in Euro as well as Datastream Germany EU-Mark 3 Months Middle Rate are used for performance tracking purposes.

A wide range of methodologies exist to estimate covariance matrices. Besides Bayesian shrinkage, or the Connor and Korajczyk (1988) asymptotic principal components procedure, a lot of techniques have evolved and been suggested by literature. As a matter of practicability, we use a commercial risk model provided by Northfield Information Services, Inc. [1]. The "Single Country & Regional Equity Risk Model" for Europe is applied for covariance matrix estimation.

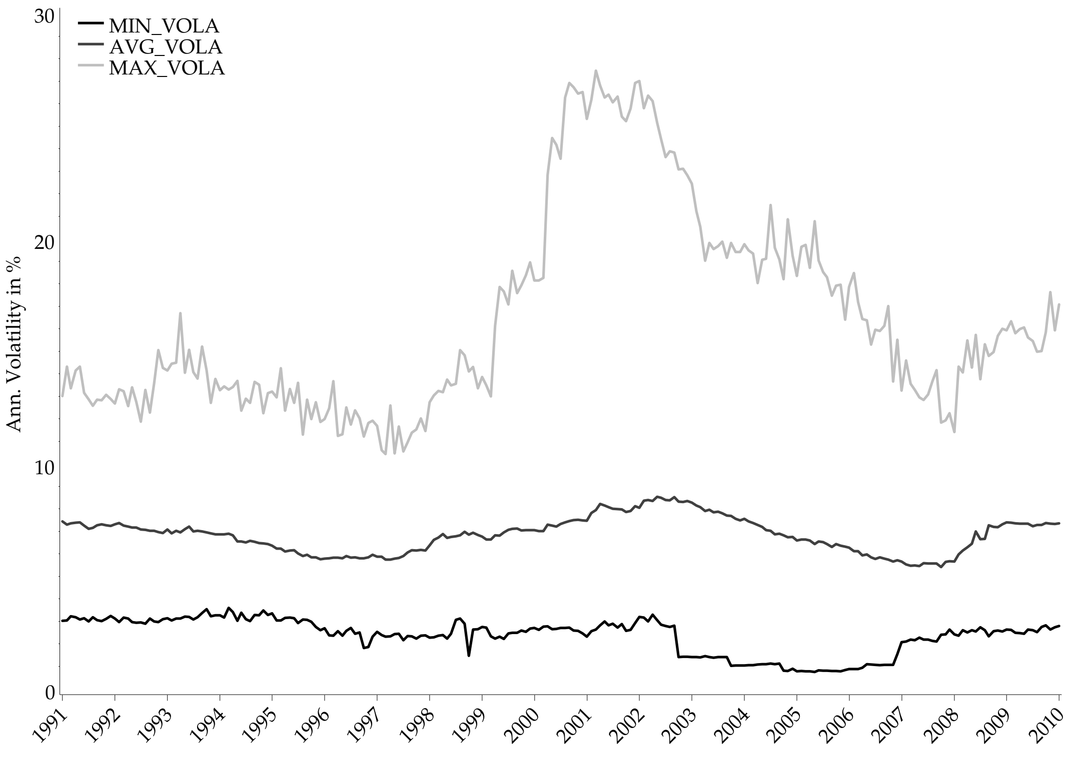

Figure 1: Northfield Volatility Estimates

[1]Northfield Information Services, Inc., 77 North Washington St. , 9th floor, Boston, Massachusetts 02114 USA

The macroeconomic risk model uses a history of 60 months of data to estimate the covariance matrix. Figure 1 shows the minimum, maximum and average estimated annualized volatility across MSCI Europe Index constituents. The peak of the maximum volatility chart can be easily explained with increased overall volatility during the "TMT-bubble". The Northfield risk model is widely used in practice and is appreciated for its suitability in equity portfolio management.

Regarding the implementation of risk balanced portfolio construction approaches, we use numerical routines in SAS Procedure IML at the end of each month beginning in August 1991 to get optimal portfolio weights. In order to realize more suitable results, we impose additional constraints to the optimization process depending on a stock's size, i.e., maximum weight for small caps is 1%, maximum weight for mid caps is 2% and maximum weight for large caps is 4%. We do not implement any sector or country constraints but apply the budget constraint and require security weights summing up to 1, i.e., long only portfolios. The analysis does not take any transaction costs into account.

The procedure for our empirical tests is as follows: first, we select the MSCI Europe Index from August 1991 through December 2010 and delete stocks without any risk data provided by Northfield. Note, that on average only 3.5% of index constituents are excluded due to missing risk estimates, so that the fundamental characteristics of the MSCI Europe Index as universe are not affected. Second, we calculate the covariance matrix and stock volatilities for the underlying universe. Performing portfolio optimization for minimum variance, maximum diversification and equal risk contribution strategies yields optimal security weights for each approach. Performance tracking of risk balanced, equal weighted and index portfolios is provided by using subsequent total or excess returns, respectively. Carhart (1997) factor portfolio returns have also been calculated in order to control exposure of the resultant portfolios to size, value and momentum.

3.1 Portfolio Results

Risk balanced portfolio construction techniques provide superior risk adjusted performance than the index portfolio or naive diversification approaches. While the MSCI Europe Index yields an annualized return of 3.88% and an annualized volatility of 15.89%, the equal weighted portfolio delivers an annualized return of 5.03%, but facing the highest level of volatility at 17.91%. These figures compare to 5.29% and 15.54% for the ERC portfolio, to be seconded only by MDP (5.55% and 12.42%) and MVP (5.63% and 11.76%). Especially during market crisis for instance after the burst of the "TMT bubble" in March 2000 or after the failure of Lehman Brothers Holdings Inc. during the subprime crisis, naive diversification and the MSCI Europe Index suffered very high losses. Table 1 presents an overview of the results further broken up into two subperiods, both including bull as well as bear markets.

Full Period: Aug 1991 - Dec 2010	MVP	MDP	ERC	EW	MSCI Europe
Annualized Return	9.82%	9.73%	9.48%	9.20%	7.66%
Annualized Excess Return	5.63%	5.55%	5.29%	5.03%	3.75%
Annualized Volatility	11.76%	12.42%	15.54%	17.91%	15.89%
Sharpe Ratio	0.48	0.45	0.34	0.28	0.24
Subperiod: Aug 1991 - Dec 2000					
Annualized Return	15.32%	15.48%	14.50%	15.14%	16.09%
Annualized Excess Return	9.68%	9.83%	8.89%	9.94%	10.80%
Annualized Volatility	12.24%	13.18%	14.47%	14.60%	14.31%
Sharpe Ratio	0.79	0.75	0.61	0.68	0.75
Subperiod: Jan 2001 - Dec 2010					
Annualized Return	4.92%	4.62%	4.99%	3.94%	0.36%
Annualized Excess Return	1.99%	1.70%	2.05%	0.64%	-2.42%
Annualized Volatility	11.19%	11.56%	16.45%	20.49%	17.06%
Sharpe Ratio	0.18	0.15	0.12	0.03	neg.

Table 1: Performance Stats - Risk Balanced Portfolio Construction

While the ERC portfolio benefits from its well diversified portfolio regarding total risk contribution, both the MDP and MVP show superior Sharpe ratios, confirming prior results by Choueifaty and Coignard (2008) and Clarke et al. (2006). However, while Choueifaty and Coignard (2008) presented empirical results indicating the MDP to be the most efficient portfolio construction approach compared to minimum variance, market capitalization and equal weighted portfolios, we can only replicate their results by using a European large cap universe. Given a broad European equity benchmark such as the MSCI Europe Index, we find the minimum variance approach to yield slightly superior risk adjusted performance than the MDP. Regarding upward trend markets from August 1991 through December 2000, minimum variance and maximum diversification approaches realize similar excess returns than the MSCI Europe Index, but with substantially lower realized volatility. However, the ERC as well as the naive diversification portfolio do not provide an appropriate risk return trade off compared to the MSCI Europe Index. Figure 2 shows the total performance of the different strategies.

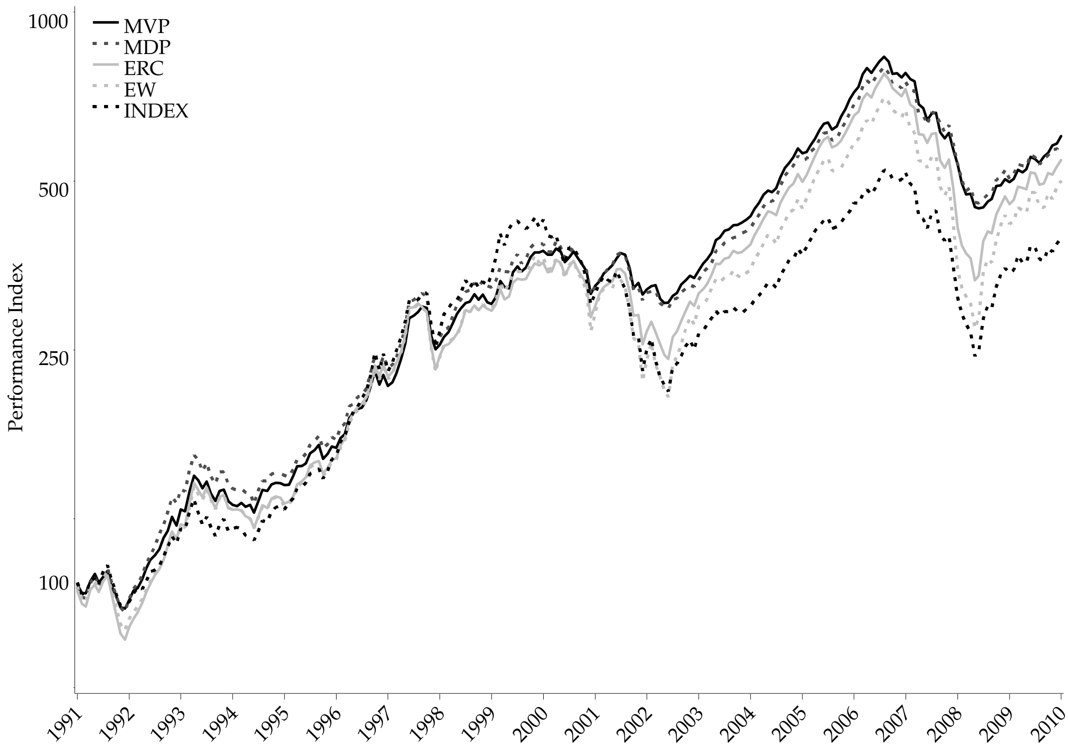

Figure 2: Performance - Risk Balanced Portfolio Construction

In downward markets, as expected the MVP, MDP and ERC portfolio benefit from their risk balanced portfolios. Due to an annualized excess return of -2.42% the Sharpe ratio of the MSCI Europe Index is negative, whereas the MVP with 1.99% excess return and 11.19% volatility, followed by the MDP with 1.70% excess return and 11.56% volatility provide considerably superior risk adjusted performance. In order to further analyze the characteristics of the introduced risk balanced portfolio construction approaches, we calculated the 60 months trailing standard deviation of portfolio returns which from September 1996 through December 2010 is presented in Figure 3.

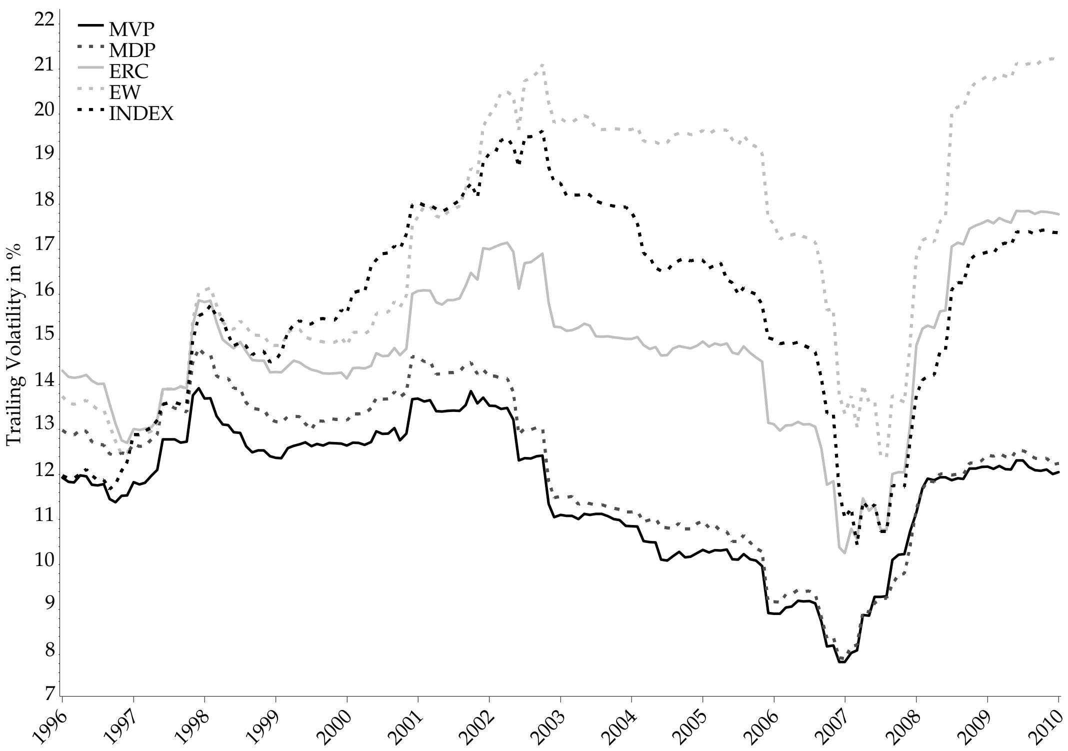

Figure 3: Trailing Portfolio Volatility

Confirming the intuition that risk based portfolio construction leads to portfolios with a risk reducing characteristic, the MVP as well as the MDP show considerably lower volatility magnitude than the ERC, equal weighted and the MSCI Europe Index portfolio. Further, in volatility intense market crisis like the subprime crisis ex post portfolio volatility for the MVP and MDP increases by about 50% from 8% to 12%, whereas the MSCI Europe Index volatility rises from 10% to over 17%, followed by the equal weighted and ERC portfolio with increases up to 75% to a level of approximately 21%, respectively. Again, while the MVP yields equal risk contributions on a marginal basis, the maximum diversification approach focuses on equal correlations of all assets having a positive portfolio weight in the MDP. According to Meucci (2009), portfolio managers comprehend a portfolio to be well diversified if it is not heavily exposed to individual shocks. However, there exists no broadly accepted, general definition of diversification. In fact, to maximize diversification does not mean to have a large number of assets with small weights in the optimal portfolio. Far from it, using the same constraints for all methodologies the maximum diversification approach leads to a similar concentrated portfolio as the MVP. Figure 4 plots the number of securities in the MVP, MDP and ERC portfolio over time.

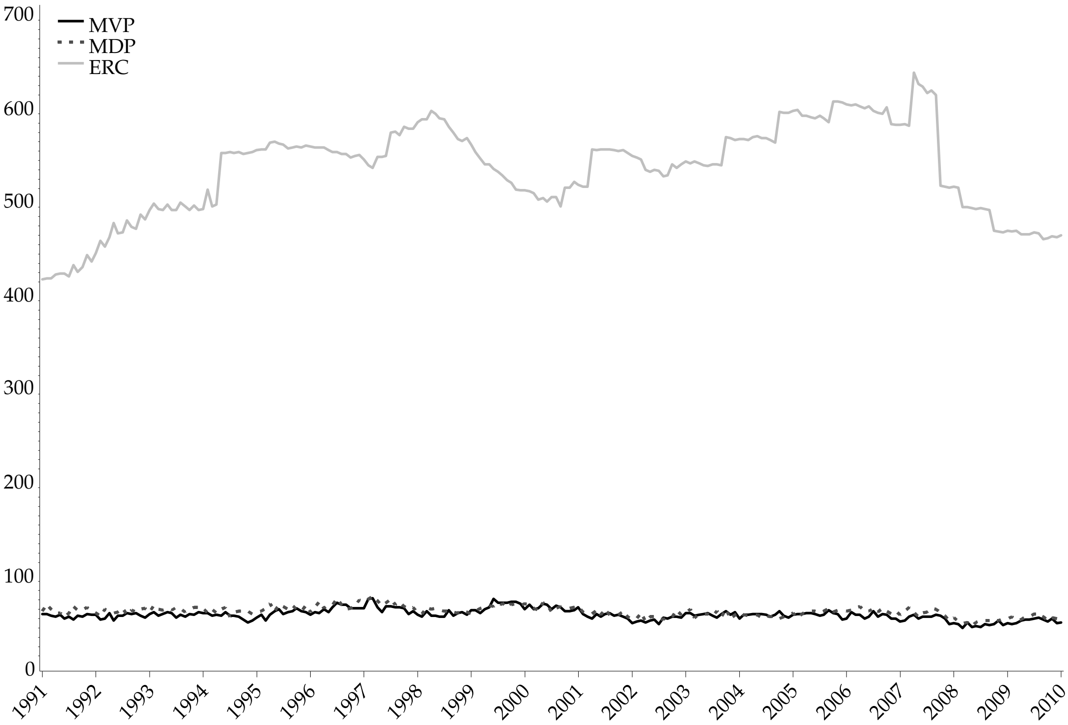

Figure 4: Number of stocks

The number of assets in the ERC portfolio varies from 413 to 634, depending on the number of assets in the MSCI Europe Index and the availability of risk data. The MDP and MVP consist of 40 to 74 assets from August 1991 through December 2010. Implementing a greater stock universe as well as additional constraints into the portfolio construction process definitely increases possibilities to further diversify the portfolio or minimize total risk, respectively.

3.2 Regression Analysis

In order to attribute performance of risk balanced portfolio construction approaches to common risk factors we first calculate factor portfolio returns according to the Fama-French methodology.

Multivariate regressions of excess portfolio returns on MSCI Europe Index as market proxy (in excess of risk-free rate) and further risk factors are performed based on 231 monthly observations from September 1991 through December 2010. Besides Fama and French's (1993) three-factor model including the market, SMB (small-minus-big market capitalization) and HML (high-minus-low book-to-market ratio) and the extended version by Carhart (1997) including UMD (up-minus-down past return), we follow Clarke et al. (2010) and introduce the VMS (volatile-minus-stable) factor to analyze the overall risk exposure of each strategy. Figure 5 shows cumulative returns of each risk factor.

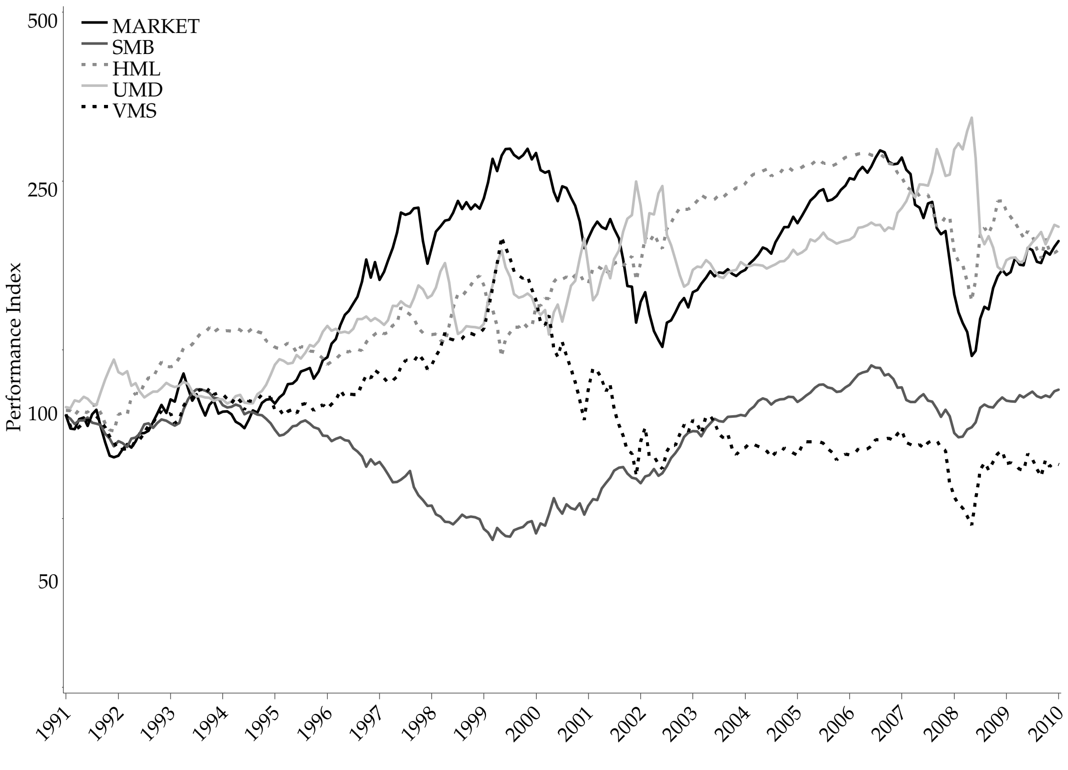

Figure 5: Carhart factor returns

Again, focusing on the burst of the "TMT bubble" 2000-2001 or the peak of the liquidity crisis in 2008 we find value, as measured with HML begins to outperform. Associated with the high value but low quality rallye driven by the banking sector in March 2009, all factors except momentum perform well. The SMB chart documents large caps outperforming small caps until 2000, followed by the opposing trend until mid 2007. Great drops in the VMS chart in 2000 or in fall 2008 describe investor's willingness to pay for stability. According to Clarke et al. (2010), the negative cumulative VMS return suggests that portfolios with lower exposure to idiosyncratic risk have typically not been penalized and that approaches to construct portfolios with lower exposure to risk has become popular. We adopt this development and conduct multivariate regressions of MVP, MDP and ERC portfolio excess returns on first, Fama and French (1993) factors, second Carhart (1997) factors and third Carhart (1997) factors supplemented by adding the VMS factor. Regression coefficients and corresponding t-statistics can be found in the following table.

Full Period: Sep 1991 - Dec 2010	Intercept	MKT	SMB	HML	UMD	VMS
Minimum Variance	0.26%	0.63	0.25	-0.07		
t-stat	(2.13)	(21.76)	(4.26)	(-1.81)		
Maximum Diversification	0.24%	0.67	0.30	-0.06		
t-stat	(1.91)	(22.58)	(5.08)	(-1.49)		
Equal Risk Contribution	0.11%	0.86	0.52	0.11		
t-stat	(1.80)	(56.72)	(17.06)	(5.23)		
Minimum Variance	0.11%	0.66	0.26	0.09	0.18	
t-stat	(0.94)	(23.60)	(4.77)	(1.80)	(5.38)	
Maximum Diversification	0.09%	0.69	0.32	0.10	0.18	
t-stat	(0.74)	(24.40)	(5.63)	(2.02)	(5.30)	
Equal Risk Contribution	0.05%	0.87	0.53	0.18	0.08	
t-stat	(0.76)	(59.20)	(17.99)	(7.11)	(4.57)	
Minimum Variance	0.10%	0.79	0.33	0.01	0.12	-0.24
t-stat	(0.94)	(23.71)	(6.34)	(0.20)	(3.73)	(-6.33)
Maximum Diversification	0.08%	0.79	0.37	0.04	0.14	-0.17
t-stat	(0.71)	(22.30)	(6.61)	(0.86)	(3.99)	(-4.36)
Equal Risk Contribution	0.04%	0.95	0.57	0.13	0.04	-0.15
t-stat	(0.76)	(56.73)	(21.58)	(5.62)	(2.60)	(-8.04)

Table 2: Regression Analysis, full period 1991 - 2010

The first panel of Table 2 shows the Fama-French regression results with the MVP, MDP or ERC portfolio excess return as dependent variable and the market, size and value as explanatory variables. The intercept can be interpreted as alpha in excess of implemented risk factors over time. Subsequently, the second panel shows the results for Carhart regressions whereas results for the VMS factor added to the Carhart factors as explanatory variables can be found in the last panel.

Estimated intercepts of 0.26% for the MVP and 0.24% for the MDP having a significant t-statistic in the Fama-French framework indicate a positive risk adjusted return over time. These strategies further provide a quite defensive market beta of approximately two thirds, an enhanced small cap exposure and finally a small negative exposure to value. In contrast, the ERC portfolio which typically invests in all index constituents has a full market participation, significantly higher small cap exposure and finally a positive exposure to value.

Including the momentum factor into the analysis leads to reduced but still positive intercepts and small changes in the strategies' exposure to value, whereas market beta remains equal. Specifically, alpha and statistically significant positive momentum exposure leads to the intuition, that trying to catch up the low volatility anomaly in minimum variance, maximum diversification or equal risk contribution strategies is tantamount to providing exposure to momentum.

Extending the Carhart regression by adding the volatility risk factor further yields interesting results. While we see a slight increase in market and value exposure for all strategies, value exposure remains neutral. Positive but statistically significant momentum exposure and the same magnitude of intercepts across the strategies almost replicate the results of the four factor model. However, comparing the VMS exposure of minimum variance, maximum diversification and equal risk contribution strategies shows significant negative factor loadings for each strategy, equivalent to strong positive exposure to less volatile stocks. With an estimated coefficient of -0.15 the ERC portfolio shows the smallest exposure to stable stocks, followed by the MDP with a VMS pattern of -0.17. Remarkably, the minimum variance strategy further enhances the investors' exposure to low volatility. The estimated volatility coefficient at -0.24 is 40% higher than for the other strategies indicating that minimum variance exhibits superior reduction of total risk.

4 Conclusion

We present a detailed analysis of three risk balanced portfolio construction approaches, namely minimum variance, maximum diversification and equal risk contribution. After mathematical definitions we extend prior research from Clarke et al. (2006), Choueifaty and Coignard (2008) and Maillard et al. (2010) and perform empirical tests using a broad European stock universe. Furthermore, we compare the results of minimum variance, maximum diversification and equal risk contribution approaches as alternative index concepts in the sense of Martellini (2008) to naive diversification as well as the MSCI Europe Index as market proxy.

Motivated by the already introduced volatility anomaly by Haugen and Baker (1991) and in line with later work by Grinold (1992), we find that the MVP, MDP and ERC portfolio provide higher risk adjusted performance in terms of Sharpe ratios than the market capitalization weighted MSCI Europe Index.

This article aims to compare portfolio construction approaches that do not require any excess return forecasts and only include risk forecasts to yield an optimal portfolio. Recent literature provides different results especially when comparing either maximum diversification or equal risk contribution to the minimum variance strategy. Note, that elaborated outperformance of maximum diversification in contrast to minimum variance by Choueifaty and Coignard (2008) can only be replicated when implementing a large cap universe into the optimization process, whereas Maillard et al. (2010) compares equal risk contribution to minimum variance in an asset allocation framework. This is the first study to analyze these three strategies in a portfolio construction framework based on a broad and implementable stock universe.

Given the MSCI Europe Index as universe, we find that minimum variance is still investing worth wile. Both annualized return and annualized volatility are superior for the observed period from August 1991 through December 2010, followed by the maximum diversification and equal risk contribution approach. The MVP risk adjusted performance is also superior for analyzed subperiods including two opposed market phases.

Finally, all risk balanced portfolio construction methodologies have in common that focusing on assets with low volatility to reduce risk of the resultant portfolio is tantamount to build up momentum exposure, whereas the minimum variance approach exhibits superior reduction of total risk, measured by the VMS factor by Clarke et al. (2010).

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Endnote

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