Attilio Meucci

Managing Diversification

COMMON MEASURES OF DIVERSIFICATION

DIVERSIFICATION DISTRIBUTION

MEAN-DIVERSIFICATION FRONTIER

CONDITIONAL ANALYSIS

REFERENCES

Managing diversification: common measures of diversification

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$

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weight-based definitions

$$\mathcal{D}_{Her} \equiv 1 - \mathbf{w}'\mathbf{w}.$$

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$$\mathcal{D}_{Her} \equiv 1 - \mathbf{w}'\mathbf{w}.$$

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Managing diversification: common measures of diversification

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$

weight-based definitions

$$\mathcal{D}_{Her} \equiv 1 - \mathbf{w}'\mathbf{w}.$$

$$\mathcal{D}_{BP} \equiv - \sum_{n=1}^{N} w_n \ln (w_n)$$

$$\mathcal{D}_{HK}^{(\gamma)} \equiv - \left(\sum_{n=1}^{N} w_n^{\gamma} \right)^{\frac{1}{\gamma - 1}}, \quad \gamma > 0.$$

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Correlations? Volatilities?

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$$\mathcal{D}_{IP} \equiv 1 - \mathbf{w}'\mathbf{C}\mathbf{w}$$

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Correlations?

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Short positions?

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factor-based definition

$$R_n \equiv \sum_{k=1}^{K} \beta_{n,k} F_k + \epsilon_n \qquad R_{\epsilon} \equiv \mathbf{w}'\boldsymbol{\epsilon}$$

$$\mathcal{D}_{IS} \equiv 1 - \frac{\operatorname{Var}\{R_{\epsilon}\}}{\operatorname{Var}\{R_{\mathbf{w}}\}}$$

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Inside idiosyncratic?

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$$\boxed{\mathrm{Var}\{R_{\mathbf{w}}\} \equiv \sum_{n=1}^{N} \mathrm{Var}\{w_n R_n\}}$$

if correlations = 0

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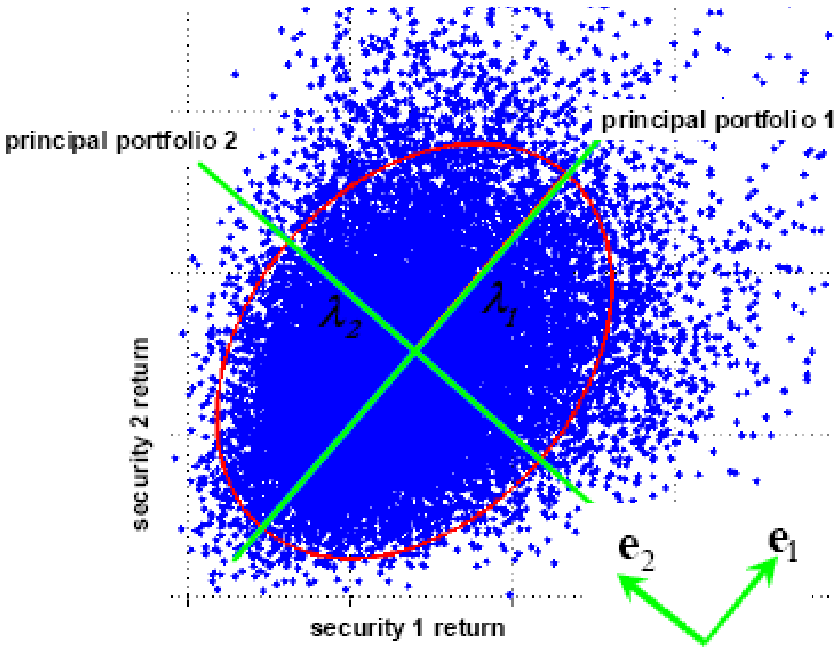
$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}.$$

$$\mathrm{Var}\left\{R_{\mathbf{w}}\right\} \neq \sum_{n=1}^{N} \mathrm{Var}\left\{w_n R_n\right\}$$

if correlations ≠ 0

Managing diversification – diversification distribution



$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}$$

$$\Sigma \equiv \mathrm{Cov}\{\mathbf{R}\}$$

$$\downarrow$$

$$\mathbf{E}'\Sigma\mathbf{E} \equiv \mathbf{\Lambda}$$ **PCA**

$$\mathbf{E} \equiv (\mathbf{e}_1, \ldots, \mathbf{e}_N)$$ eigenvectors

$$\mathbf{\Lambda} \equiv \mathrm{diag}\left(\lambda_1^2, \ldots, \lambda_N^2\right)$$ eigenvalues

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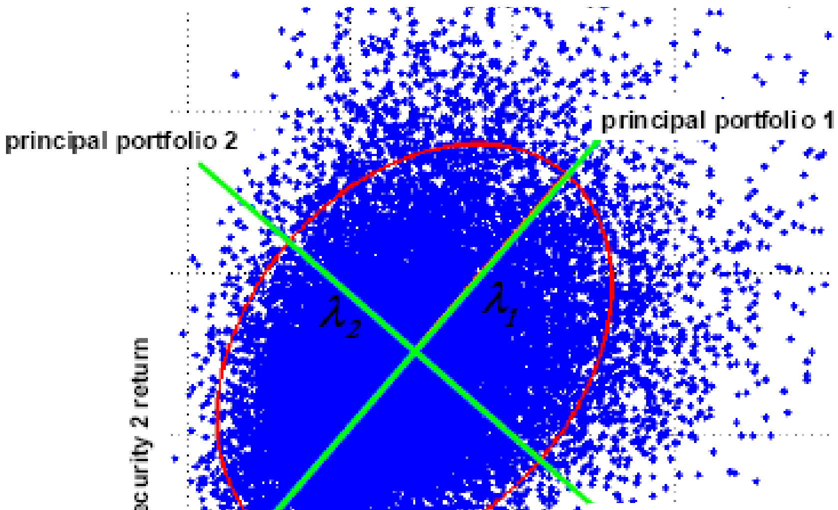

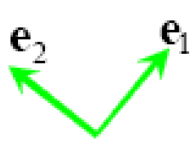

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PCA

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eigenvectors

$\Updownarrow$

$$\mathbf{e}_n \equiv \underset{\mathbf{e}'\mathbf{e}\equiv 1}{\operatorname{argmax}} \{\mathbf{e}'\mathbf{\Sigma}\mathbf{e}\}$$

uncorrelated, maximum variance portfolios

$$\mathbf{e}'\mathbf{\Sigma}\mathbf{e}_j \equiv 0 \text{ for all existing } \mathbf{e}_j$$

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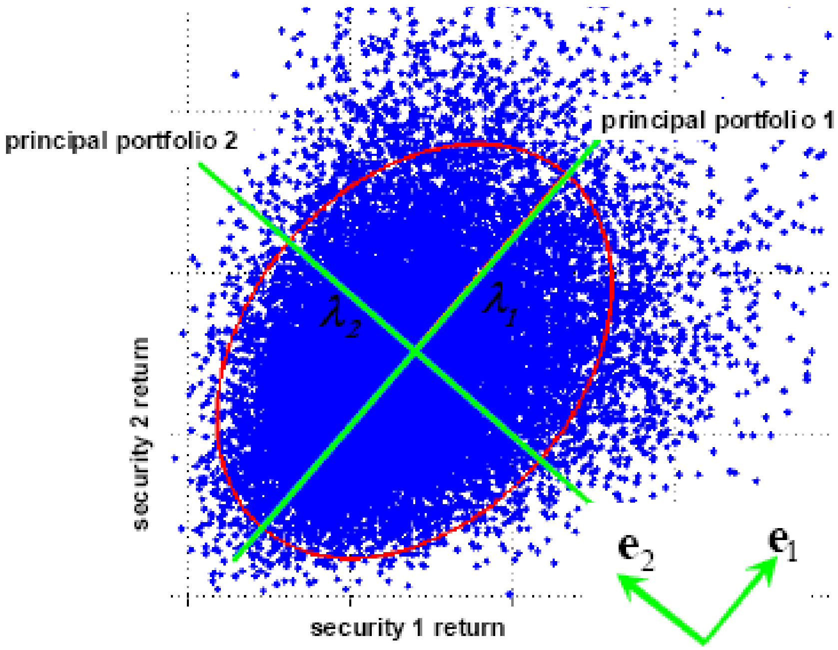

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$$\lambda_n^2 \equiv \mathrm{Var}\{\mathbf{e}_n'\mathbf{R}\}$$

variances of uncorrelated, maximum variance portfolios

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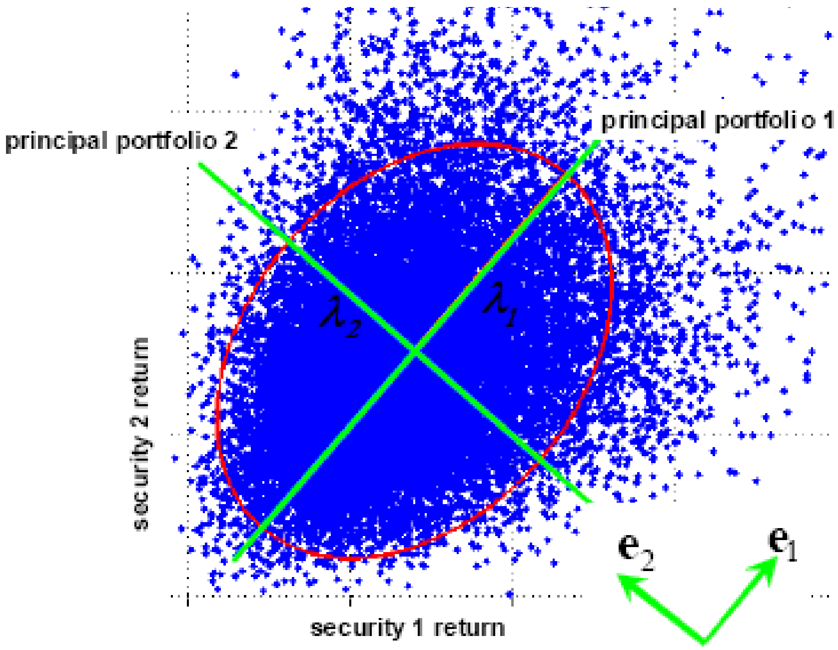

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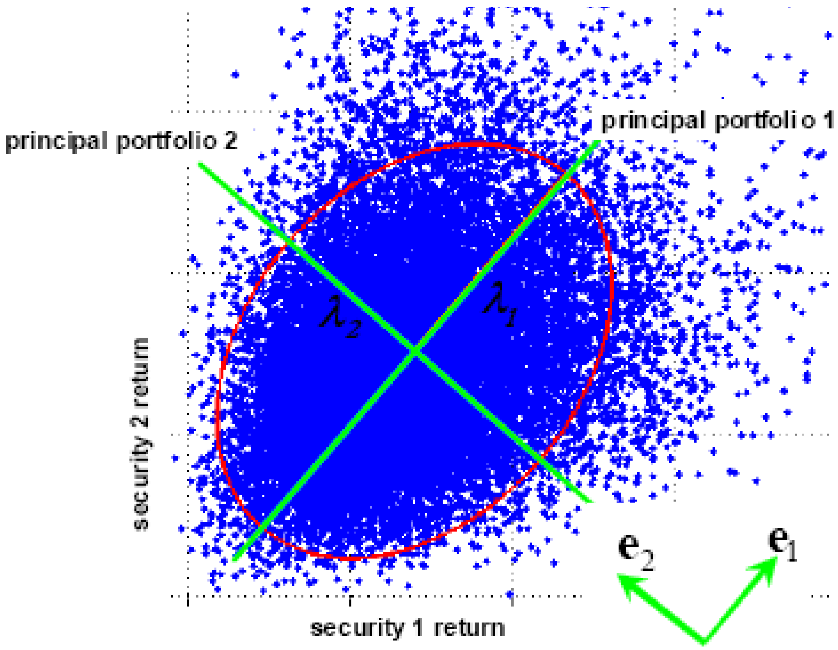

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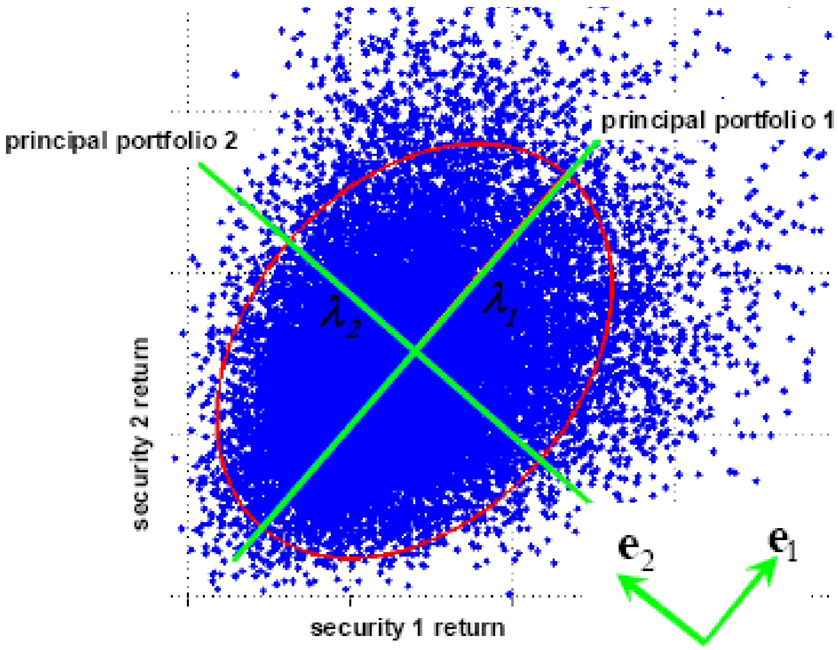

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principal variances

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contribution to original portfolio variance from n-th principal portfolio:

$$\boxed{\mathrm{Var}\{R_{\mathbf{w}}\} \equiv \sum_{n=1}^{N} v_n}$$

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$\Updownarrow$

$s_n \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Sd}\{R_{\mathbf{w}}\}}$ volatility concentration curve

contribution to original portfolio volatility from n-th principal portfolio: "hot spots"

Managing diversification – diversification distribution

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contribution to original portfolio r-square from n-th principal portfolio

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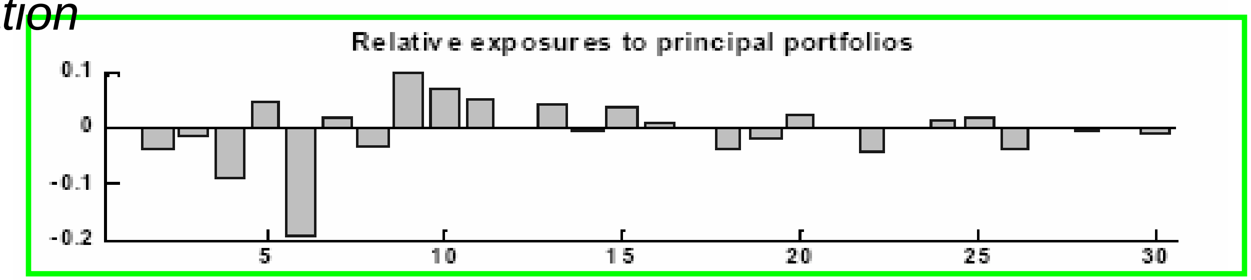
Managing diversification



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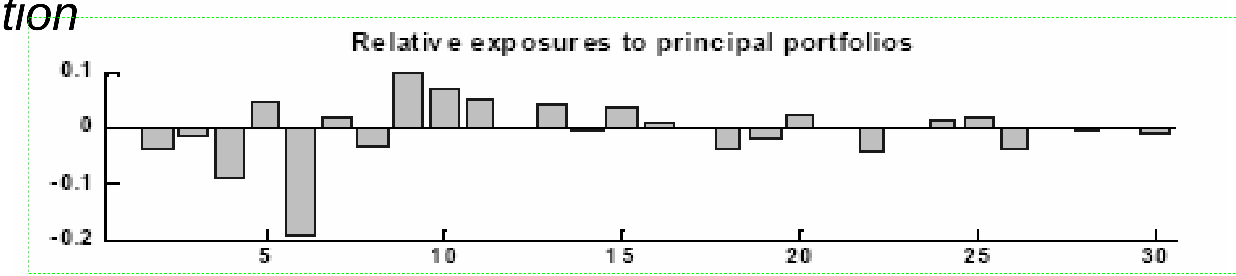

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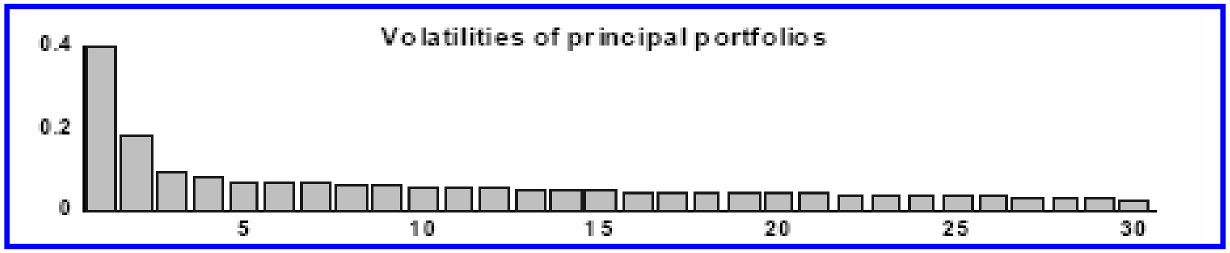

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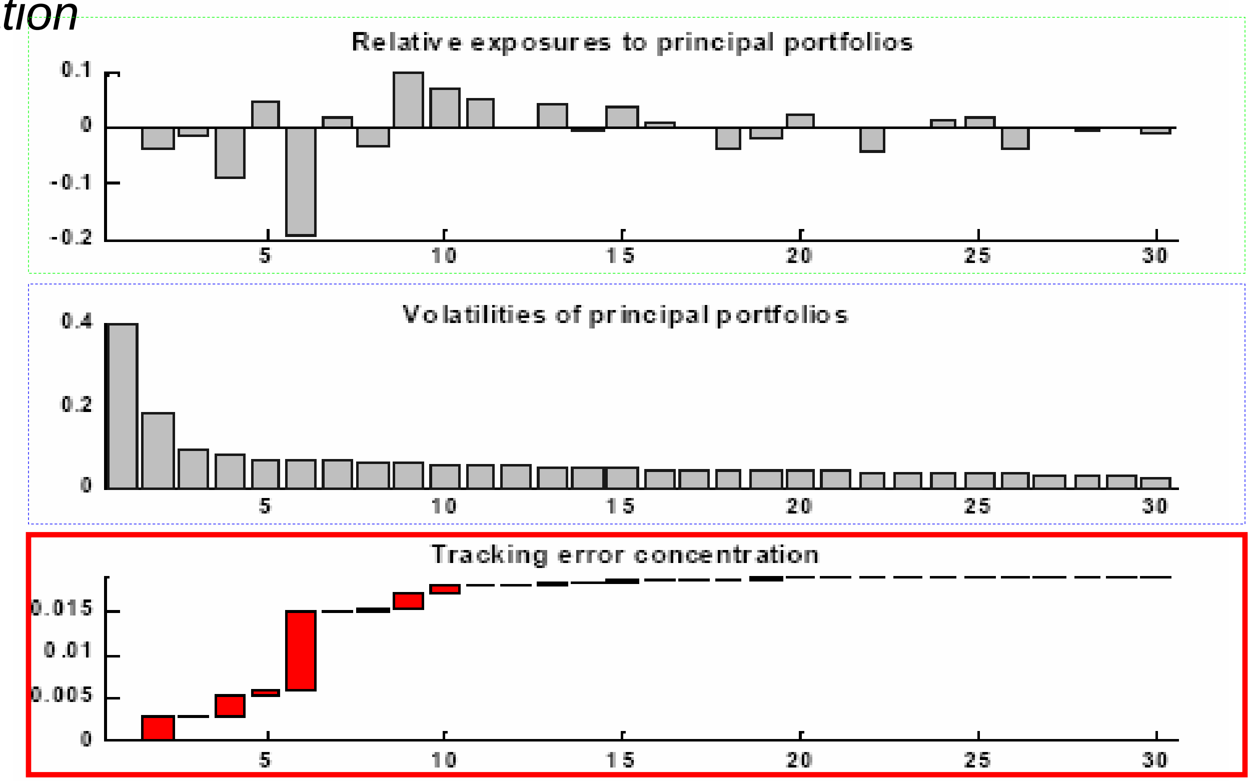

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$\boxed{p_n} \equiv \dfrac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathbf{w}}\}}$ diversification distribution: “probability mass”

Managing diversification – mean-diversification frontier

entropy: $$-\sum_{n=1}^{N} \boxed{p_n} \ln \boxed{p_n}$$

$\Updownarrow$

diversification

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Managing diversification – mean-diversification frontier

effective number of bets

$$\mathcal{N}_{Ent} \equiv \exp\left(-\sum_{n=1}^{N} p_n \ln p_n\right)$$

$\Updownarrow$

diversification

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Managing diversification – mean-diversification frontier

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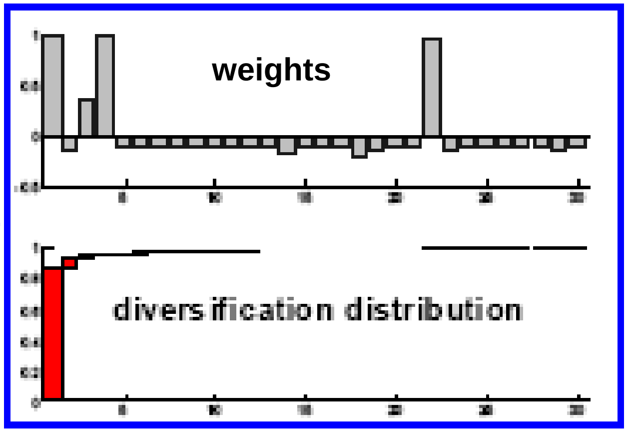
full concentration $\boxed{\mathcal{N}_{Ent} \approx 1}$



$$p_n \equiv \frac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathrm{w}}\}}$$ diversification distribution: “probability mass”

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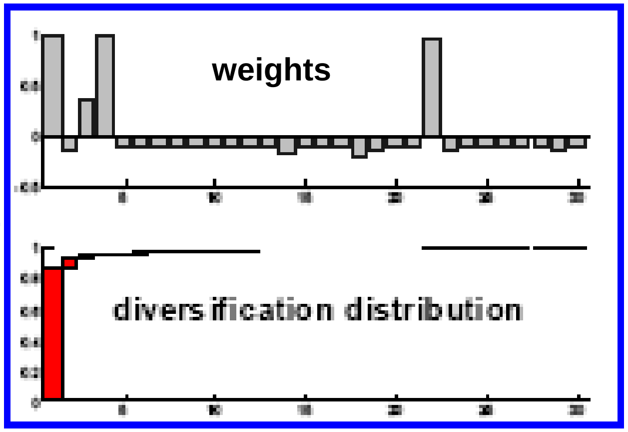
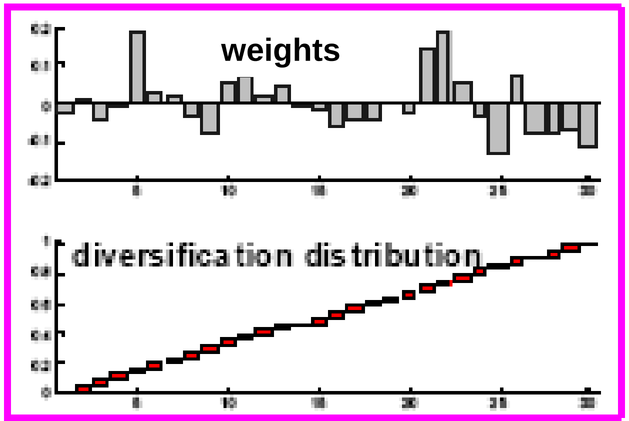
full concentration $\boxed{\mathcal{N}_{Ent} \approx 1}$

full diversification $\boxed{\mathcal{N}_{Ent} \approx N}$





$$p_n \equiv \frac{\widetilde{w}_n^2 \lambda_n^2}{\mathrm{Var}\{R_{\mathrm{w}}\}}$$ diversification distribution: “probability mass”

Managing diversification – mean-diversification frontier

effective number of bets

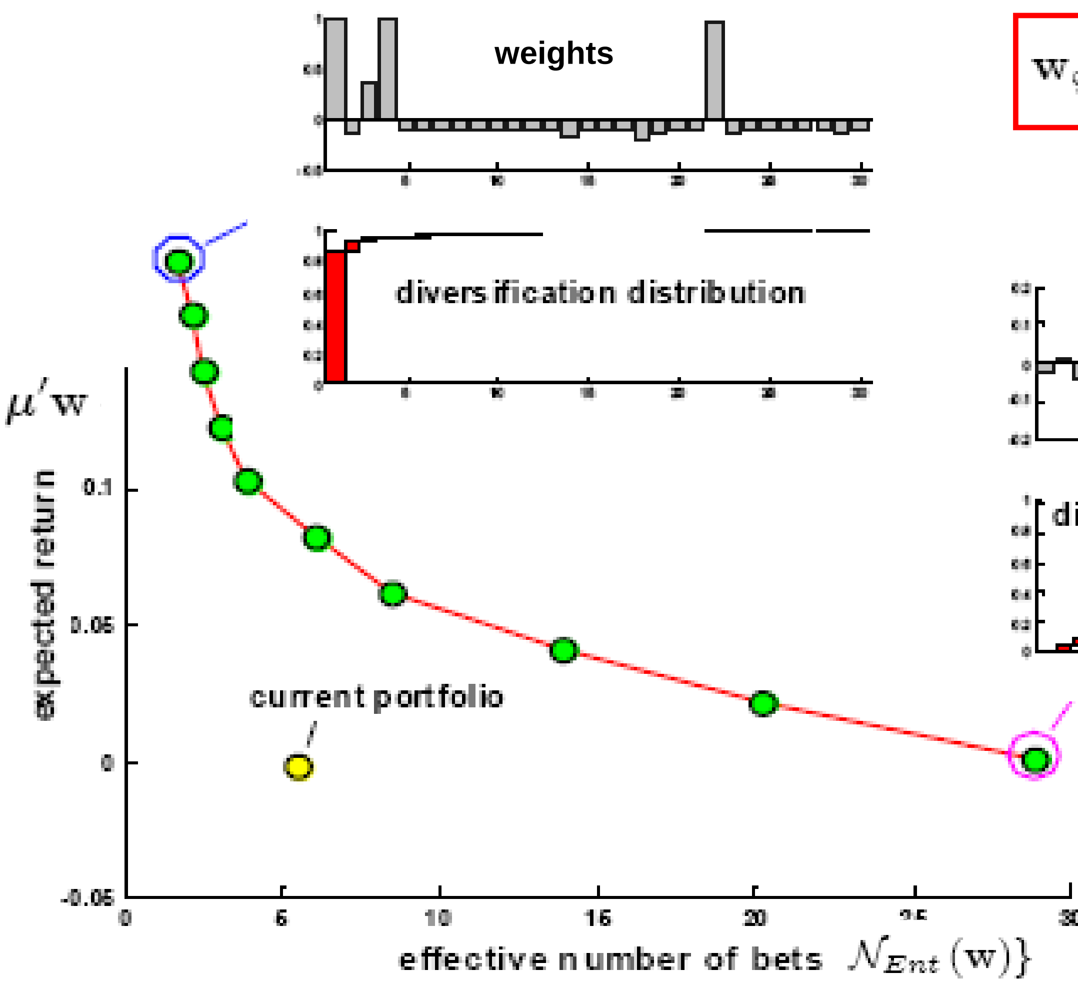
$$\mathcal{N}_{Ent} \equiv \exp\left(-\sum_{n=K+1}^{N} p_n \ln p_n\right)$$

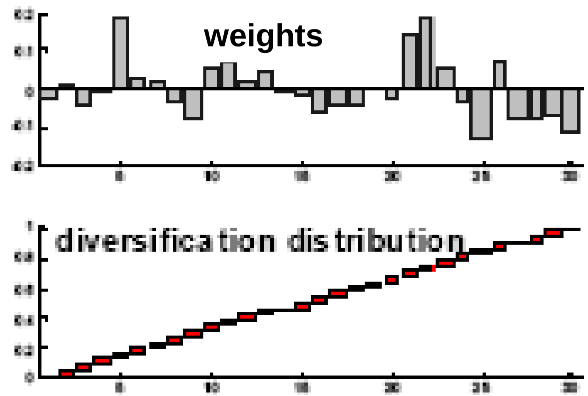
full concentration $\mathcal{N}_{Ent} \approx 1$

full diversification $\mathcal{N}_{Ent} \approx N$.

mean-diversification frontier

$$\boxed{\mathbf{w}_{\varphi} \equiv \underset{\mathbf{w} \in \mathcal{C}}{\operatorname{argmax}} \left\{ \varphi \mu' \mathbf{w} + (1-\varphi) \mathcal{N}_{Ent}(\mathbf{w}) \right\}}$$





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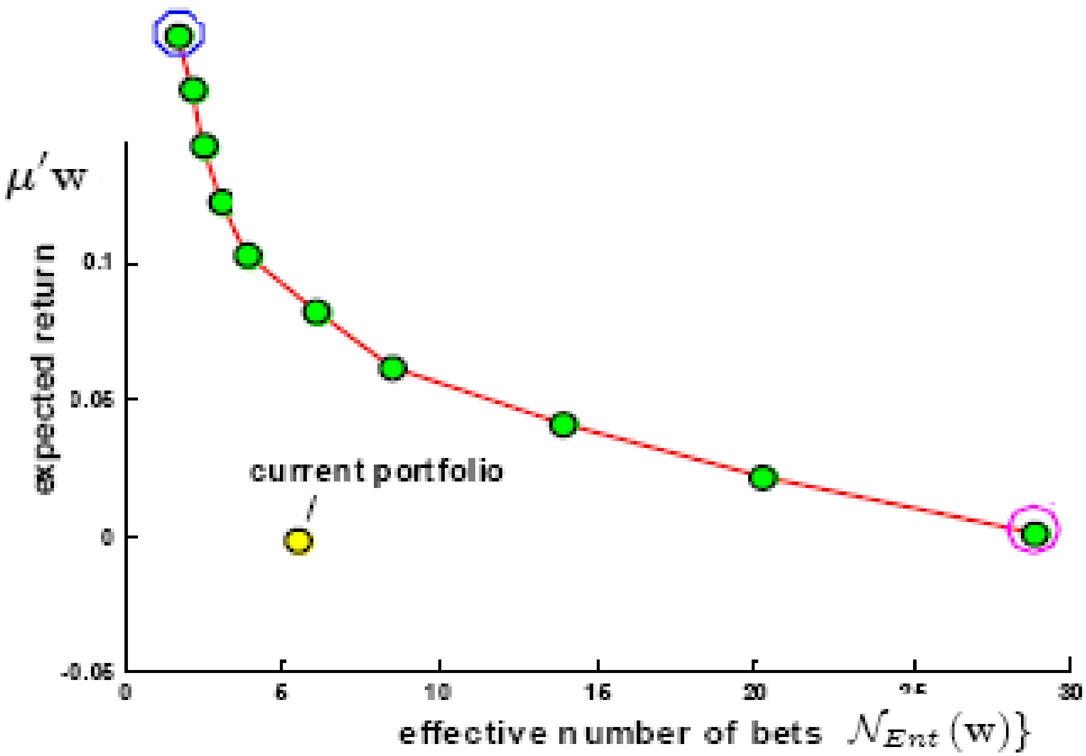

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transaction costs

$$\boxed{\mu'\mathbf{w} \mapsto \mu'\mathbf{w} - \mathcal{T}(\mathbf{w}, \mathbf{w}_{cur})}$$

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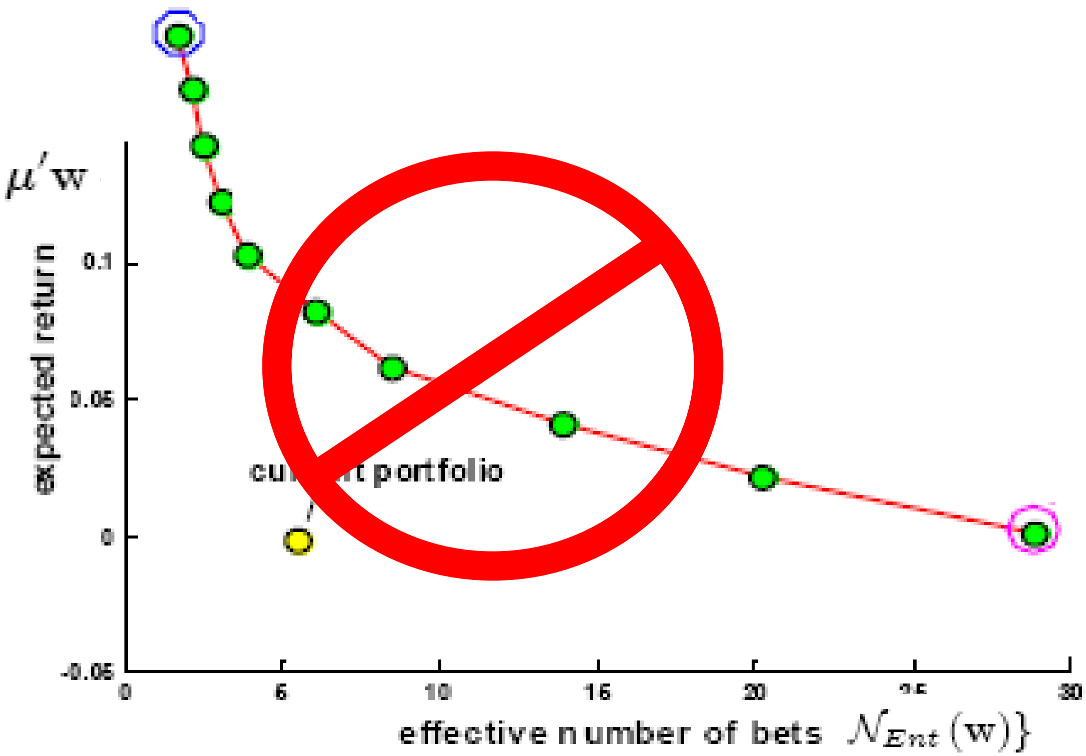

best few trades

Managing diversification – mean-diversification frontier

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$$\mathbf{S}_{\{1,3\}} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$\boxed{\Delta\mathbf{w} \equiv \mathbf{S}_{I_M}\mathbf{x},}$$

only trade few securities at a time

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selection heuristic

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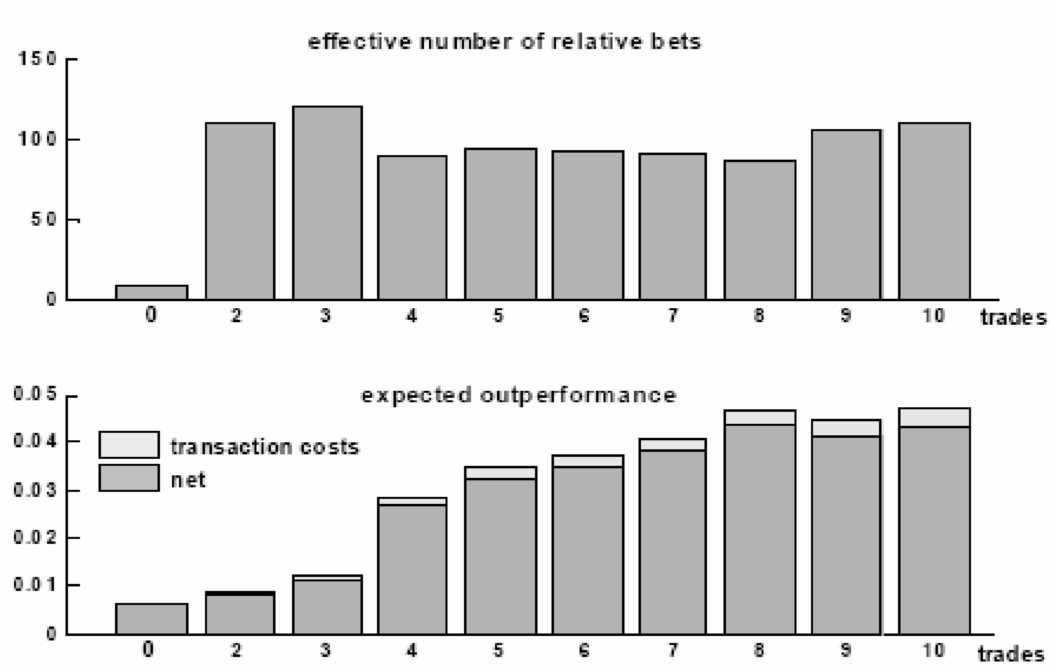

COMMON MEASURES OF DIVERSIFICATION

DIVERSIFICATION DISTRIBUTION

MEAN-DIVERSIFICATION FRONTIER

CONDITIONAL ANALYSIS

REFERENCES

Managing diversification – conditional analysis

$$R_{\mathbf{w}} \equiv \mathbf{w}'\mathbf{R}$$

$$\Sigma \equiv \mathrm{Cov}\{\mathbf{R}\}$$

$$\downarrow$$

$$\mathbf{E}'\boldsymbol{\Sigma}\mathbf{E} \equiv \boldsymbol{\Lambda}$$

PCA

$$\mathbf{E} \equiv (\mathbf{e}_1, \ldots, \mathbf{e}_N)$$

principal portfolios

$$\mathbf{e}_n \equiv \underset{\mathbf{e}'\mathbf{e}\equiv 1}{\operatorname{argmax}} \{\mathbf{e}'\boldsymbol{\Sigma}\mathbf{e}\}$$

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principal variances

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constraints

$$\mathbf{A}\Delta\mathbf{w} \equiv \mathbf{0}$$

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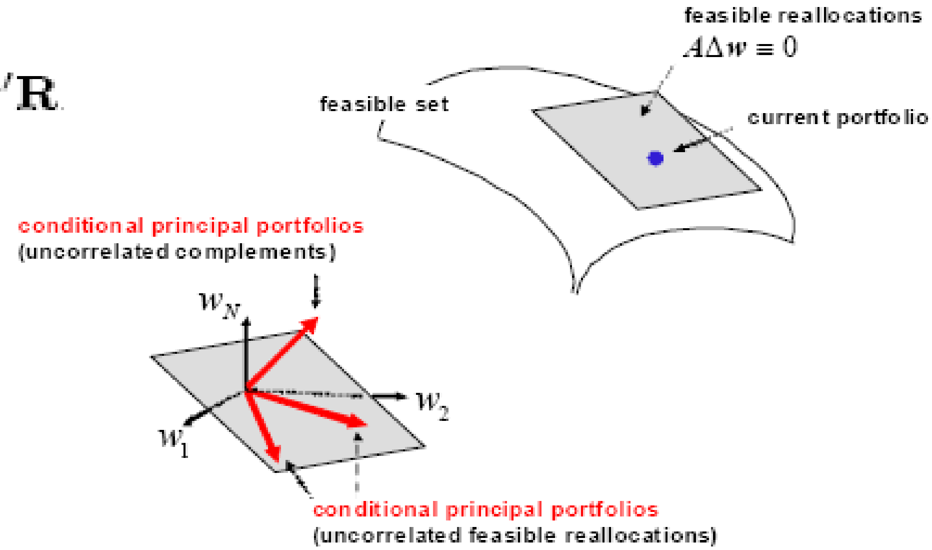

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REFERENCES

Managing diversification – references

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  Attilio Meucci, "Managing Diversification"
  ***The Risk Magazine* - May 2009**
  extended version available at
  www.symmys.com > Research > Working Papers

- **MATLAB examples:**

  www.symmys.com > Teaching > MATLAB

- **This presentation:**

  www.symmys.com > Teaching > Talks