Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

$$\boxed{\mathbf{G}\left[f_{\mathbf{X}}\right] \equiv \text{"unknown truth"}} \quad (4.6)$$

$$\boxed{\textbf{information } i_T \mapsto \textbf{number } \widehat{\mathbf{G}}} \quad (4.9)$$

$$i_T \equiv \{\mathbf{x}_1, \ldots, \mathbf{x}_T\} \quad (4.8)$$

$$(4.13) \quad \boxed{\widehat{\mathbf{G}}\left[i_T\right] \approx \mathbf{G}\left[f_{\mathbf{X}}\right]} \quad \textbf{?}$$

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$$G\left[f_X\right] \equiv \int_{-\infty}^{+\infty} x f_X(x)\, dx. \tag{4.7}$$

$$\widehat{G}\left[i_T\right] \equiv \frac{1}{T}\sum_{t=1}^{T} x_t. \tag{4.10}$$

$$\widehat{G}\left[i_T\right] \equiv x_1 x_T \tag{4.11}$$

$$\widehat{G}\left[i_T\right] \equiv 3. \tag{4.12}$$

$$\boxed{\widehat{\mathbf{G}}\left[i_T\right] \approx \mathbf{G}\left[f_{\mathbf{X}}\right]} \tag{4.13}$$

?

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$$i_T \equiv \{\mathbf{x}_1, \ldots, \mathbf{x}_T\} \mapsto I_T \equiv \{\mathbf{X}_1, \ldots, \mathbf{X}_T\} \quad (4.14)$$

$$\widehat{\mathbf{G}}\left[i_T\right] \mapsto \widehat{\mathbf{G}}\left[I_T\right] \quad (4.15)$$

$$\boxed{\operatorname{Loss}\left(\widehat{\mathbf{G}}, \mathbf{G}\right) \equiv \left\| \widehat{\mathbf{G}}\left[I_T\right] - \mathbf{G}\left[f_{\mathbf{X}}\right] \right\|^2} \quad (4.19)$$

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$$G\left[f_X\right] = \mu. \quad (4.18)$$

$$X_t \sim \mathrm{N}\left(\mu, \sigma^2\right) \quad (4.16)$$

$$\widehat{G}\left[I_T\right] \equiv \frac{1}{T}\sum_{t=1}^{T} X_t \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{T}\right) \quad (4.17)$$

$$\boxed{\mathrm{Loss}\left(\widehat{\mathbf{G}}, \mathbf{G}\right) \equiv \left\|\widehat{\mathbf{G}}\left[I_T\right] - \mathbf{G}\left[f_{\mathbf{X}}\right]\right\|^2} \quad (4.19)$$

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$$\operatorname{Loss}\left(\widehat{G}, G\right) \sim \operatorname{Ga}\left(1, \frac{\sigma^2}{T}\right) \quad (4.22)$$

$$\boxed{\operatorname{Loss}\left(\widehat{\mathbf{G}}, \mathbf{G}\right) \equiv \left\|\widehat{\mathbf{G}}\left[I_T\right] - \mathbf{G}\left[f_{\mathbf{X}}\right]\right\|^2} \quad (4.19)$$

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$$\mathrm{Err}\left(\widehat{\mathbf{G}},\mathbf{G}\right)\equiv\sqrt{\mathrm{E}\left\{\left\|\widehat{\mathbf{G}}\left(I_T\right)-\mathbf{G}\left[f_{\mathbf{X}}\right]\right\|^2\right\}}. \qquad (4.23)$$

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$$\mathrm{Err}\left(\widehat{G}, G\right) = \frac{\sigma}{\sqrt{T}}. \quad (4.24)$$

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$$\mathrm{Inef}^2\left[\widehat{\mathbf{G}}\right] \equiv \mathrm{E}\left\{\left\|\widehat{\mathbf{G}}\left[I_T\right] - \mathrm{E}\left\{\widehat{\mathbf{G}}\left[I_T\right]\right\}\right\|^2\right\} \quad (4.26)$$

$$\mathrm{Bias}^2\left[\widehat{\mathbf{G}}, \mathbf{G}\right] \equiv \left\|\mathrm{E}\left\{\widehat{\mathbf{G}}\left[I_T\right]\right\} - \mathbf{G}\left[f_{\mathbf{X}}\right]\right\|^2 \quad (4.25)$$

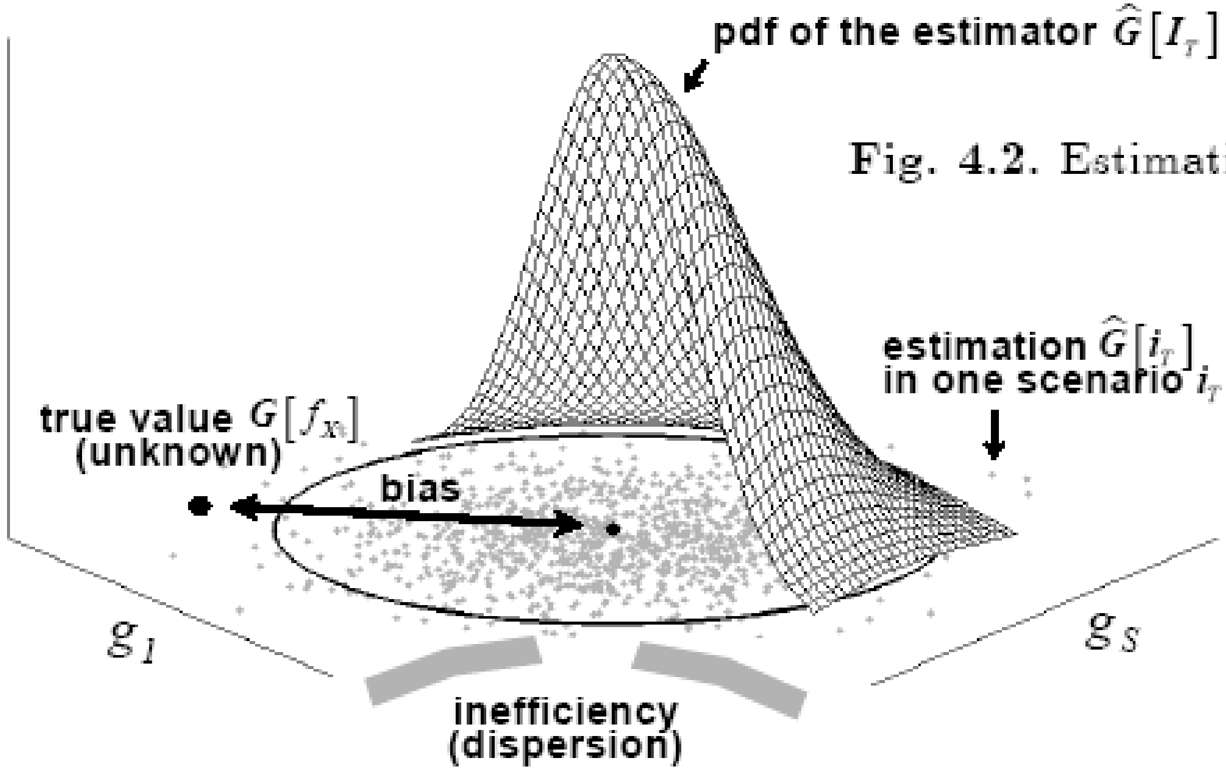

Fig. 4.2. Estimation: replicability, bias and inefficiency

$$\mathrm{Inef}^2\left[\widehat{\mathbf{G}}\right] \equiv \mathrm{E}\left\{\left\|\widehat{\mathbf{G}}\left[I_T\right] - \mathrm{E}\left\{\widehat{\mathbf{G}}\left[I_T\right]\right\}\right\|^2\right\} \quad (4.26)$$

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$$\mathrm{Inef}\left[\widehat{G}\right] = \frac{\sigma}{\sqrt{T}}. \quad (4.29)$$

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$$\mathrm{Bias}\left[\widehat{G}, G\right] = 0. \quad (4.28)$$

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$$\boxed{\mathrm{Err}^2\left[\widehat{\mathbf{G}},\mathbf{G}\right]=\mathrm{Bias}^2\left[\widehat{\mathbf{G}},\mathbf{G}\right]+\mathrm{Inef}^2\left[\widehat{\mathbf{G}}\right]} \quad (4.27)$$

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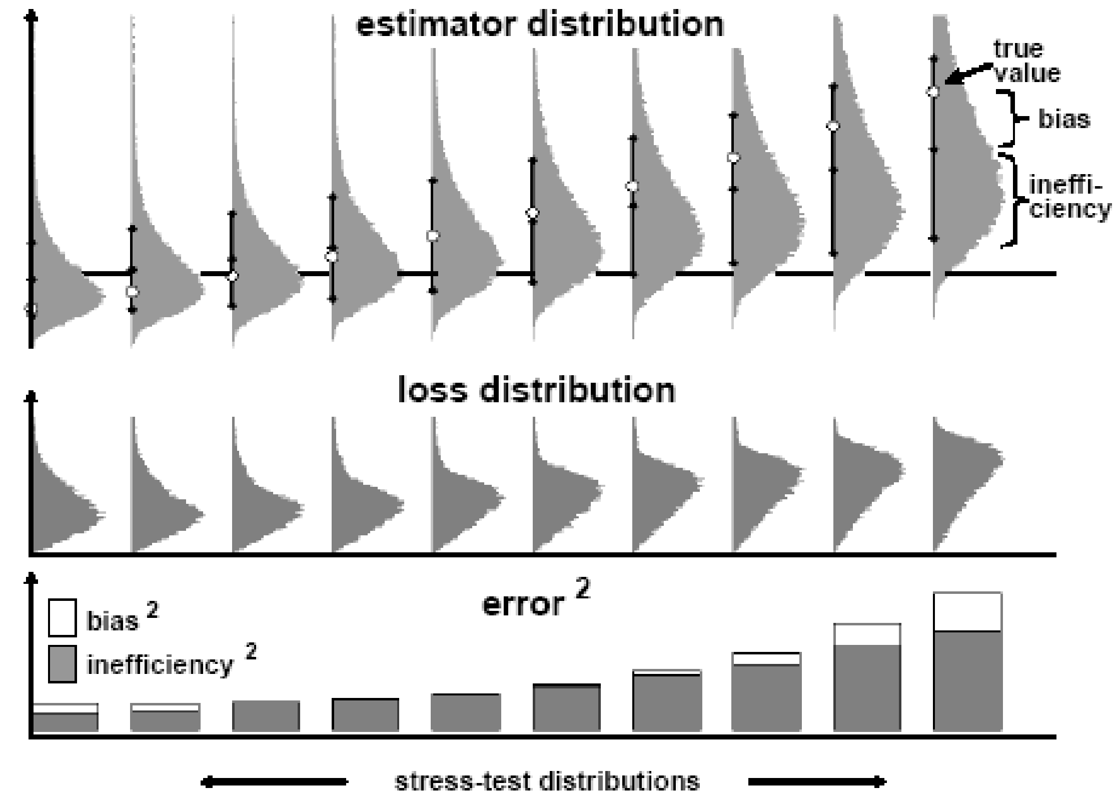

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