CORNISH-FISHER EXPANSION

Risk and Asset Allocation **- Springer – *symmys.com***

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Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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$$Q_X(p) = \mathrm{E}\{X\} + \mathrm{Sd}\{X\}\left[z(p) + \frac{1}{6}\left(z^2(p) - 1\right)\mathrm{Sk}\{X\}\right] + \cdots \tag{5.179}$$

quantile of <u>generic</u> distribution

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examples

$$k=3: \quad \mathrm{Sk}\{X_l, X_m, X_n\} \equiv [\mathrm{Sk}\{\mathbf{X}\}]_{lmn} \equiv \frac{\mathrm{CM}^{\mathbf{X}}_{lmn}}{\mathrm{Sd}\{X_l\}\,\mathrm{Sd}\{X_m\}\,\mathrm{Sd}\{X_n\}}. \qquad (2.95)$$

$$k=4: \quad \mathrm{Ku}\{X_l, X_m, X_n, X_p\} \equiv [\mathrm{Ku}\{\mathbf{X}\}]_{lmnp} \equiv \frac{\mathrm{CM}^{\mathbf{X}}_{lmnp}}{\mathrm{Sd}\{X_l\}\,\mathrm{Sd}\{X_m\}\,\mathrm{Sd}\{X_n\}\,\mathrm{Sd}\{X_p\}} \qquad (2.96)$$