NON-PARAMETRIC ESTIMATORS

Risk and Asset Allocation **- Springer –** *`symmys.com`*

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www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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$$\boxed{\mathbf{G}\left[f_{\mathbf{X}}\right] \equiv \text{"unknown truth"}} \qquad (4.6)$$

$$\boxed{\textbf{information } i_T \mapsto \textbf{number } \widehat{\mathbf{G}}} \qquad (4.9)$$

$$i_T \equiv \{\mathbf{x}_1, \ldots, \mathbf{x}_T\} \qquad (4.8)$$

$$\boxed{\mathbf{G}\left[f_{\mathbf{X}}\right] \equiv \text{"unknown truth"}} \quad (4.6)$$

$$\boxed{\textbf{information } i_T \mapsto \widehat{\mathbf{G}}\left[i_T\right] \equiv \mathbf{G}\left[f_{i_T}\right]} \quad (4.36)$$

$$f_{i_T}(\mathbf{x}) \equiv \frac{1}{T} \sum_{t=1}^{T} \delta^{(\mathbf{x}_t)}(\mathbf{x}) \quad (4.35)$$

$$\boxed{\lim_{T \to \infty} F_{i_T}(\mathbf{x}) = F_{\mathbf{X}}(\mathbf{x})} \quad (4.34)$$

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$$G\left[f_X\right] \equiv \int_{-\infty}^{+\infty} x f_X(x)\, dx. \quad (4.7)$$

$$\widehat{G}\left[i_T\right] \equiv \int_{\mathbb{R}^N} \mathbf{x} f_{i_T}(\mathbf{x})\, d\mathbf{x} \equiv \frac{1}{T}\sum_{t=1}^{T} x_t. \quad (4.41)$$

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$$\int_{-\infty}^{q_p[f_X]} f_X(x)\, dx \equiv p, \quad (4.38)$$

$$\widehat{q}_p[i_T] \equiv x_{[pT]:T} \quad (4.39)$$

NON-PARAMETRIC ESTIMATORS – ORDINARY LEAST SQUARES

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$$\mathbf{X} = \mathbf{B}\mathbf{F} + \mathbf{U}. \quad (4.50)$$

$$\boxed{\mathbf{B}_r \equiv \mathrm{E}\left\{\mathbf{X}\mathbf{F}'\right\} \mathrm{E}\left\{\mathbf{F}\mathbf{F}'\right\}^{-1}} \quad (3.121)$$

$$\boxed{\widehat{\mathbf{B}}\left[i_T\right] \equiv \left(\sum_t \mathbf{x}_t \mathbf{f}_t'\right)\left(\sum_t \mathbf{f}_t \mathbf{f}_t'\right)^{-1}} \quad (4.52)$$

$$i_T \equiv \left\{\mathbf{x}_1, \mathbf{f}_1, \ldots, \mathbf{x}_T, \mathbf{f}_T\right\} \quad (4.51)$$

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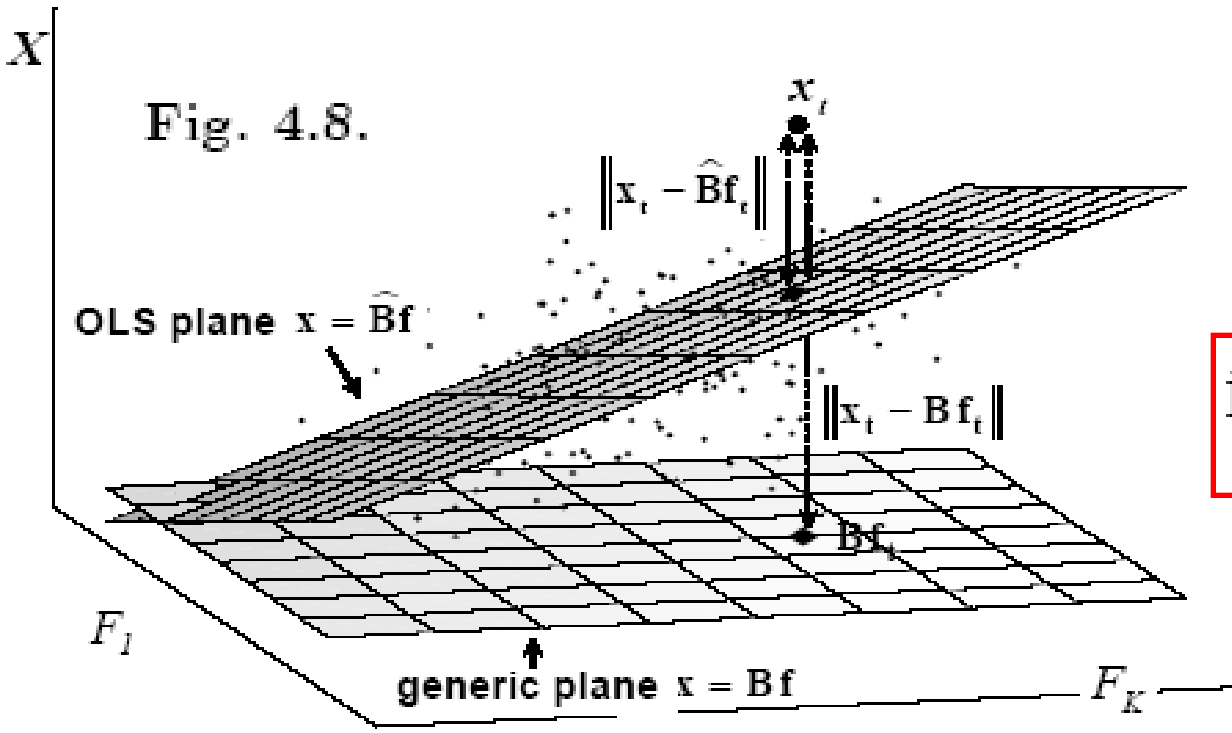

$$\boxed{\widehat{\mathbf{B}} = \underset{\mathbf{B}}{\operatorname{argmin}} \sum_t \left\| \mathbf{x}_t - \mathbf{B}\mathbf{f}_t \right\|^2} \quad (4.53)$$

NON-PARAMETRIC ESTIMATORS – SAMPLE MEAN/COVARIANCE

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$$\boxed{\mathbf{G}\left[f_{\mathbf{X}}\right] \equiv \text{"unknown truth"}} \quad (4.6)$$

$$\boxed{\text{information } i_T \mapsto \widehat{\mathbf{G}}\left[i_T\right] \equiv \mathbf{G}\left[f_{i_T}\right]} \quad (4.36)$$

$$\boxed{\mathrm{E}\left\{\mathbf{X}\right\}} \quad (2.54)$$

$$\boxed{\widehat{\mathrm{E}}\left[i_T\right] = \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_t} \quad (4.41)$$

$$\boxed{\mathrm{Cov}\left\{\mathbf{X}\right\}} \quad (2.67)$$

$$\boxed{\widehat{\mathrm{Cov}}\left[i_T\right] = \frac{1}{T}\sum_{t=1}^{T}\left(\mathbf{x}_t - \widehat{\mathrm{E}}\left[i_T\right]\right)\left(\mathbf{x}_t - \widehat{\mathrm{E}}\left[i_T\right]\right)'} \quad (4.42)$$

NON-PARAMETRIC ESTIMATORS – SAMPLE MEAN/COVARIANCE

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$$\boxed{\mathbf{G}\left[f_{\mathbf{X}}\right] \equiv \text{"unknown truth"}} \tag{4.6}$$

$$\boxed{\mathrm{E}\left\{\mathbf{X}\right\}} \tag{2.54}$$

$$\boxed{\mathrm{Cov}\left\{\mathbf{X}\right\}} \tag{2.67}$$

$$\boxed{\textbf{information } i_T \mapsto \widehat{\mathbf{G}}\left[i_T\right] \equiv \mathbf{G}\left[f_{i_T}\right]} \tag{4.36}$$

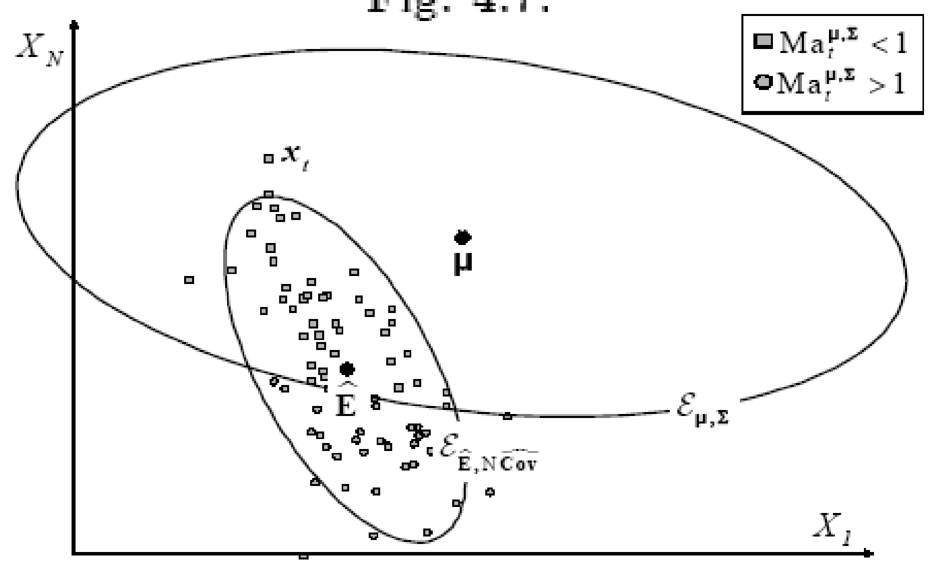
$$\boxed{\widehat{\mathrm{E}}\left[i_T\right] = \frac{1}{T}\sum_{t=1}^{T} \mathrm{x}_t} \tag{4.41}$$

$$\boxed{\widehat{\mathrm{Cov}}\left[i_T\right] = \frac{1}{T}\sum_{t=1}^{T}\left(\mathrm{x}_t - \widehat{\mathrm{E}}\left[i_T\right]\right)\left(\mathrm{x}_t - \widehat{\mathrm{E}}\left[i_T\right]\right)'} \tag{4.42}$$

$$\mathcal{E}_{\boldsymbol{\mu},\boldsymbol{\Sigma}} \equiv \left\{\mathrm{x} \in \mathbb{R}^N \text{ such that } (\mathrm{x}-\boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\mathrm{x}-\boldsymbol{\mu}) \leq 1\right\} \tag{4.45}$$

$$\mathrm{Ma}_t^{\boldsymbol{\mu},\boldsymbol{\Sigma}} \equiv \mathrm{Ma}\left(\mathrm{x}_t, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \equiv \sqrt{\left(\mathrm{x}_t-\boldsymbol{\mu}\right)' \boldsymbol{\Sigma}^{-1}\left(\mathrm{x}_t-\boldsymbol{\mu}\right)} \tag{4.46}$$

Fig. 4.7.



NON-PARAMETRIC ESTIMATORS – SAMPLE MEAN/COVARIANCE

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$$\boxed{\mathrm{E}\left\{\mathbf{X}\right\}} \quad (2.54)$$

$$\boxed{\widehat{\mathrm{E}}\left[i_T\right] = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t} \quad (4.41)$$

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$$\boxed{\widehat{\mathrm{Cov}}\left[i_T\right] = \frac{1}{T}\sum_{t=1}^{T}\left(\mathbf{x}_t - \widehat{\mathrm{E}}\left[i_T\right]\right)\left(\mathbf{x}_t - \widehat{\mathrm{E}}\left[i_T\right]\right)'} \quad (4.42)$$

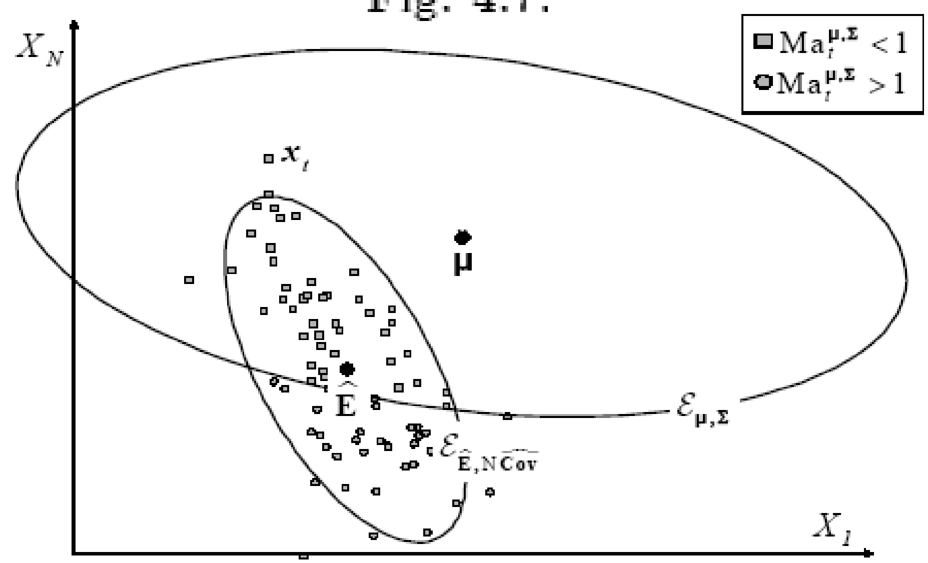
$$\mathcal{E}_{\boldsymbol{\mu},\boldsymbol{\Sigma}} \equiv \left\{\mathbf{x} \in \mathbb{R}^N \text{ such that } (\mathbf{x} - \boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu}) \leq 1\right\} \quad (4.45)$$

$$\mathrm{Ma}_t^{\boldsymbol{\mu},\boldsymbol{\Sigma}} \equiv \mathrm{Ma}\left(\mathbf{x}_t, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \equiv \sqrt{(\mathbf{x}_t - \boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\mathbf{x}_t - \boldsymbol{\mu})} \quad (4.46)$$

$$\overline{r^2}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \equiv \frac{1}{T}\sum_{t=1}^{T}\left(\mathrm{Ma}_t^{\boldsymbol{\mu},\boldsymbol{\Sigma}}\right)^2 \quad (4.47)$$

$$\boxed{\begin{gathered}\left(\widehat{\mathrm{E}}, N\widehat{\mathrm{Cov}}\right) = \underset{(\boldsymbol{\mu},\boldsymbol{\Sigma})\in\mathcal{C}}{\operatorname{argmin}}\left[\mathrm{Vol}\left\{\mathcal{E}_{\boldsymbol{\mu},\boldsymbol{\Sigma}}\right\}\right] \\ \overline{r^2}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \equiv 1\end{gathered}} \quad (4.48)$$

Fig. 4.7.



NON-PARAMETRIC ESTIMATORS – KERNELS

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$$f_{i_T}(\mathbf{x}) \equiv \frac{1}{T}\sum_{t=1}^{T} \delta^{(\mathbf{x}_t)}(\mathbf{x}) \quad (4.35)$$

$$\boxed{\textbf{information } i_T \mapsto \widehat{\mathbf{G}}\left[i_T\right] \equiv \mathbf{G}\left[f_{i_T;\epsilon}\right]} \quad (4.56)$$

$$f_{i_T} \mapsto f_{i_T;\epsilon} \equiv \frac{1}{T}\sum_{t=1}^{T} \frac{1}{(2\pi)^{\frac{N}{2}}\,\epsilon^N} e^{-\frac{1}{2\epsilon^2}(\mathbf{x}-\mathbf{x}_t)'(\mathbf{x}-\mathbf{x}_t)}. \quad (4.55)$$