INVESTOR'S OBJECTIVES EVALUATION: SATISFACTION

Risk and Asset Allocation **- Springer –** *`symmys.com`*

Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \quad (5.48)$$

$$\boldsymbol{\alpha} \mapsto \mathcal{S}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.49)$$

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Sharpe ratio

$$\mathrm{SR}(\boldsymbol{\alpha}) \equiv \frac{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\}}{\mathrm{Sd}\{\Psi_{\boldsymbol{\alpha}}\}} \quad (5.51)$$

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Sharpe omega $$S\Omega_K(\alpha) \equiv \frac{\mathrm{E}\{\Psi_\alpha\} - K}{\widetilde{P}_K\{\Psi_\alpha\}} \quad \Longleftrightarrow \quad \textbf{omega} \quad \Omega_K(\alpha) \equiv S\Omega_K(\alpha) - 1$$

$$\widetilde{P}_K\{\Psi\} \equiv \mathrm{E}\{\max(K - \Psi, 0)\}$$

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Sortino ratio

$$So_K(\alpha) \equiv \frac{\mathrm{E}\{\Psi_\alpha\} - K}{\sqrt{\widetilde{P}^2_K\{\Psi_\alpha\}}}$$

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Kappa

$$\kappa^n_\lambda(\boldsymbol{\alpha}) \equiv \frac{\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \lambda}{\left(\widetilde{P}^n_\lambda\{\Psi_{\boldsymbol{\alpha}}\}\right)^{\frac{1}{n}}}$$

$$\widetilde{P}^n_K\{\Psi\} \equiv \mathrm{E}\{\max(K - \Psi, 0)^n\}$$

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- Money-equivalence

Sharpe ratio

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$$\cancel{\kappa_\lambda^n}(\boldsymbol{\alpha}) \equiv \frac{\mathrm{E}\{\Psi_\alpha\} - \lambda}{\left(\widetilde{P}_\lambda^n\{\Psi_\alpha\}\right)^{\frac{1}{n}}}$$

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- Money-equivalence

- Estimability

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto \left(f_{\Psi_{\boldsymbol{\alpha}}}, F_{\Psi_{\boldsymbol{\alpha}}}, \phi_{\Psi_{\boldsymbol{\alpha}}}\right) \mapsto \mathcal{S}(\boldsymbol{\alpha}) \quad (5.52)$$

$$f_{\psi} \mapsto \mathrm{E}\{\Psi\} \equiv \int_{\mathbb{R}} \psi f_{\psi}(\psi)\, d\psi. \quad (5.53)$$

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto f_{\Psi_{\boldsymbol{\alpha}}} \mapsto \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.54)$$

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- **Sensibility**

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios } \Rightarrow \mathcal{S}(\boldsymbol{\alpha}) \geq \mathcal{S}(\boldsymbol{\beta}) \quad (5.55)$$

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- **Consistence with stochastic dominance**

$$Q_{\Psi_{\boldsymbol{\alpha}}}(p) \geq Q_{\Psi_{\boldsymbol{\beta}}}(p) \text{ for all } p \in (0,1) \Rightarrow \mathcal{S}(\boldsymbol{\alpha}) \geq \mathcal{S}(\boldsymbol{\beta}) \quad (5.57)$$

$$\begin{aligned} \mathrm{E}\{\Psi\} &\equiv \int_{-\infty}^{+\infty} \psi f_{\psi}(\psi)\, d\psi \\ &= \int_0^1 Q_{\Psi}(u)\, du. \end{aligned} \quad (5.58)$$

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- Positive homogeneity

$$\Psi_{\lambda\boldsymbol{\alpha}} = \lambda \Psi_{\boldsymbol{\alpha}}, \quad \text{for all } \lambda \geq 0. \quad (5.64)$$

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$$\mathcal{S}(\lambda\boldsymbol{\alpha}) = \lambda\mathcal{S}(\boldsymbol{\alpha}), \quad \text{for all } \lambda \geq 0. \quad (5.65)$$

$$\mathrm{E}\{\Psi_{\lambda\boldsymbol{\alpha}}\} = \mathrm{E}\{\lambda\Psi_{\boldsymbol{\alpha}}\} = \lambda\,\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} \quad (5.66)$$

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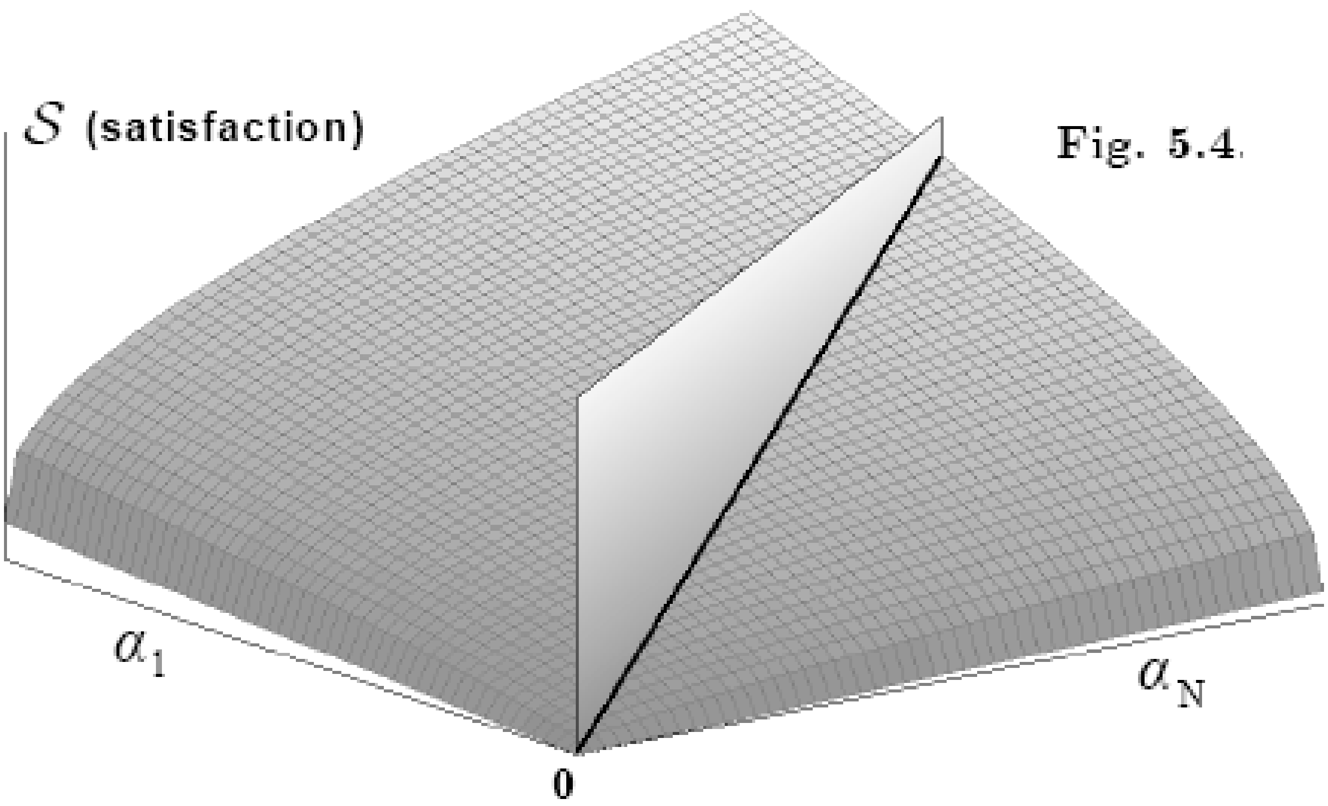

Fig. 5.4.

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$$\Downarrow \qquad\qquad \Downarrow$$

Euler:

$$\mathcal{S}(\boldsymbol{\alpha}) = \sum_{n=1}^{N} \alpha_n \frac{\partial\mathcal{S}(\boldsymbol{\alpha})}{\partial\alpha_n}. \quad (5.67)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}' \mathbf{P}_{T+\tau}, \quad (5.68)$$

$$\mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} = \sum_{n=1}^{N} \alpha_n \,\mathrm{E}\left\{P_{T+\tau}^{(n)}\right\} \quad (5.69)$$

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- **Positive homogeneity**

$$\mathcal{S}(\lambda\boldsymbol{\alpha}) = \lambda\mathcal{S}(\boldsymbol{\alpha}), \quad \text{for all } \lambda \geq 0. \quad (5.65)$$

- **Translation invariance**

$$\Psi_{\boldsymbol{\alpha}+\boldsymbol{\beta}} = \Psi_{\boldsymbol{\alpha}} + \Psi_{\boldsymbol{\beta}}. \quad (5.70)$$

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$$\mathcal{S}(\boldsymbol{\alpha} + \mathbf{b}) = \mathcal{S}(\boldsymbol{\alpha}) + \psi_{\mathbf{b}} \quad (5.71)$$

$$\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \mathrm{E}\{\Psi_{\boldsymbol{\alpha}+\lambda\mathbf{b}}\} = \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} + \lambda. \quad (5.73)$$

$$\Psi_{\boldsymbol{\alpha}+\boldsymbol{\beta}} = \Psi_{\boldsymbol{\alpha}} + \Psi_{\boldsymbol{\beta}}. \quad (5.70)$$

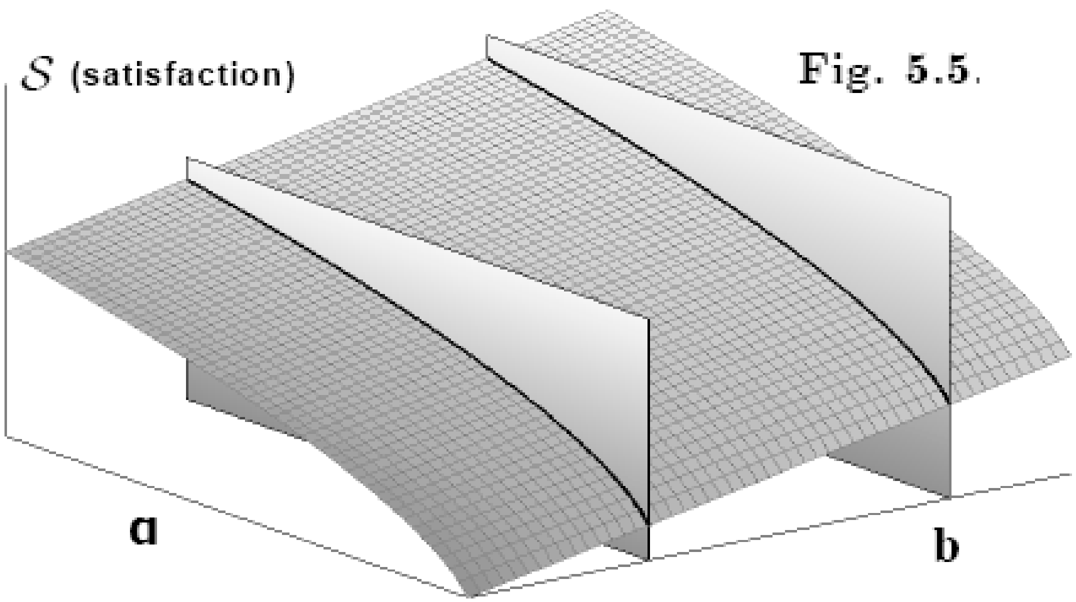

Fig. 5.5.

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- **super- additivity**

$$\mathcal{S}(\boldsymbol{\alpha} + \boldsymbol{\beta}) \geq \mathcal{S}(\boldsymbol{\alpha}) + \mathcal{S}(\boldsymbol{\beta}) \quad (5.75)$$

$$\mathrm{E}\{\Psi_{\boldsymbol{\alpha}+\boldsymbol{\beta}}\} = \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} + \mathrm{E}\{\Psi_{\boldsymbol{\beta}}\} \quad (5.77)$$

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- **Co-monotonic additivity**

$$(\boldsymbol{\alpha}, \boldsymbol{\delta}) \text{ co-monotonic } \Rightarrow \mathcal{S}(\boldsymbol{\alpha} + \boldsymbol{\delta}) = \mathcal{S}(\boldsymbol{\alpha}) + \mathcal{S}(\boldsymbol{\delta}) \quad (5.80)$$

$$\mathrm{E}\{\Psi_{\boldsymbol{\alpha}+\boldsymbol{\beta}}\} = \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} + \mathrm{E}\{\Psi_{\boldsymbol{\beta}}\} \quad (5.77)$$

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- **Concavity**

$$\mathcal{S}(\lambda\boldsymbol{\alpha} + (1-\lambda)\boldsymbol{\beta}) \geq \lambda\mathcal{S}(\boldsymbol{\alpha}) + (1-\lambda)\mathcal{S}(\boldsymbol{\beta}) \quad (5.81)$$

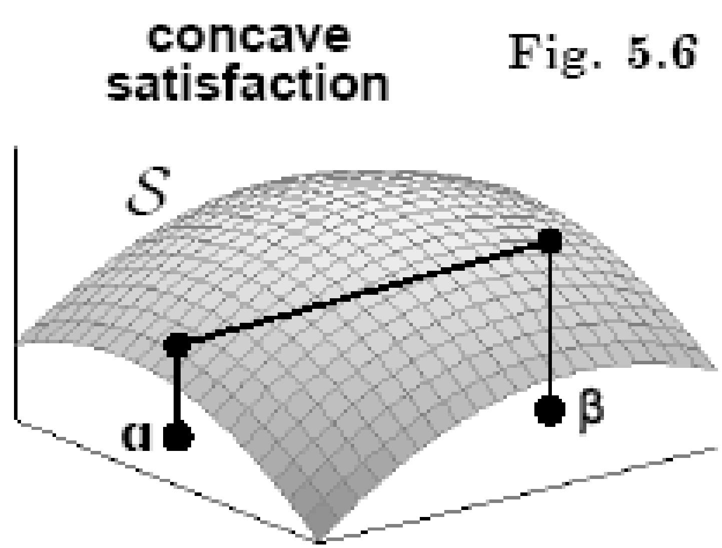

Fig. 5.6

$$\mathrm{E}\{\Psi_{\boldsymbol{\alpha}+\boldsymbol{\beta}}\} = \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} + \mathrm{E}\{\Psi_{\boldsymbol{\beta}}\} \quad (5.77)$$

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- **Risk aversion/propensity/neutrality**

$$\mathrm{RP}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \mathcal{S}(\boldsymbol{\alpha}) \quad (5.85)$$

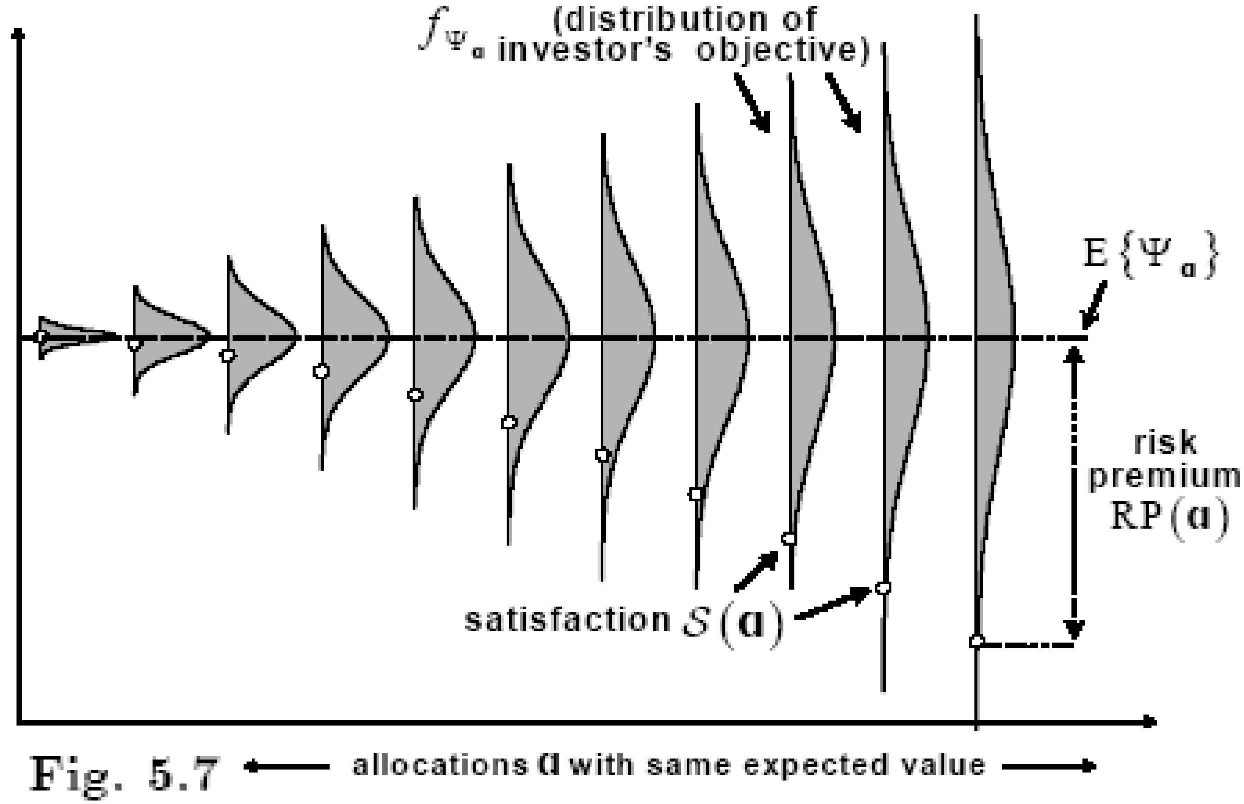

Fig. 5.7

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risk aversion: $\mathrm{RP}(\boldsymbol{\alpha}) \geq 0$ (5.86)

risk propensity: $\mathrm{RP}(\boldsymbol{\alpha}) \leq 0$ (5.87)

risk neutrality: $\mathrm{RP}(\boldsymbol{\alpha}) \equiv 0.$ (5.88)

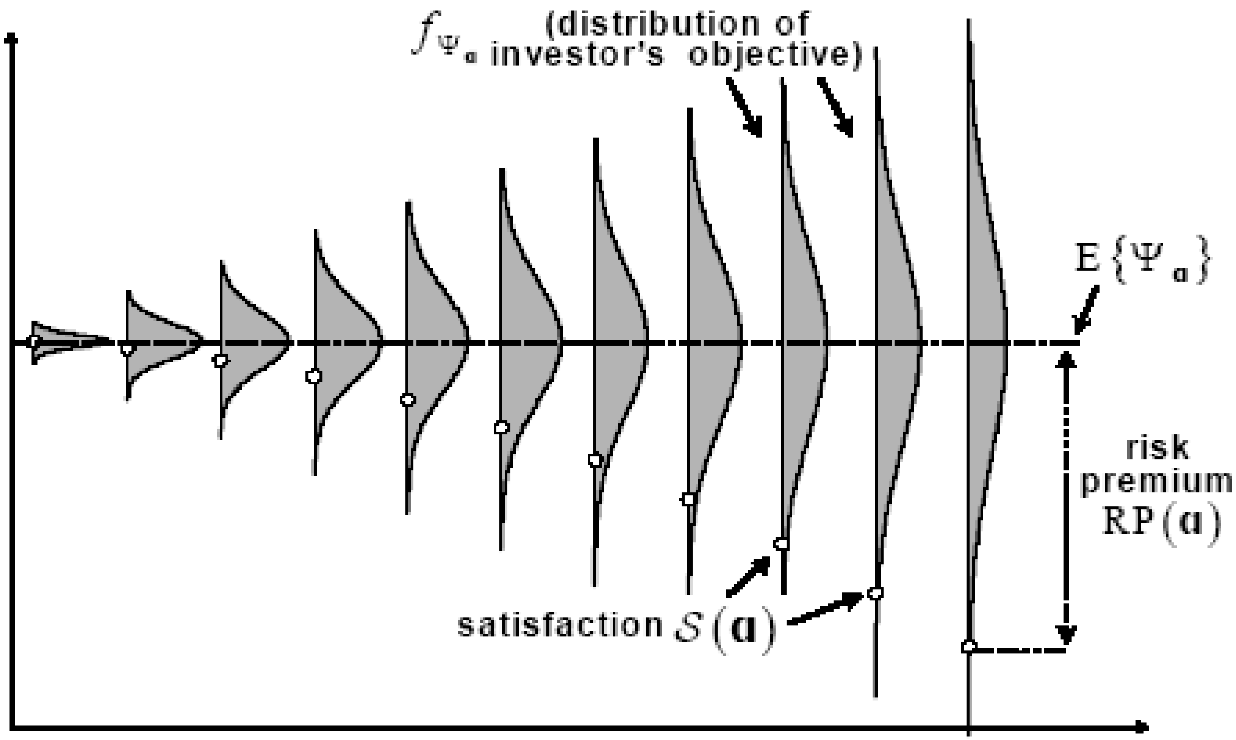

Fig. 5.7 ← allocations α with same expected value →

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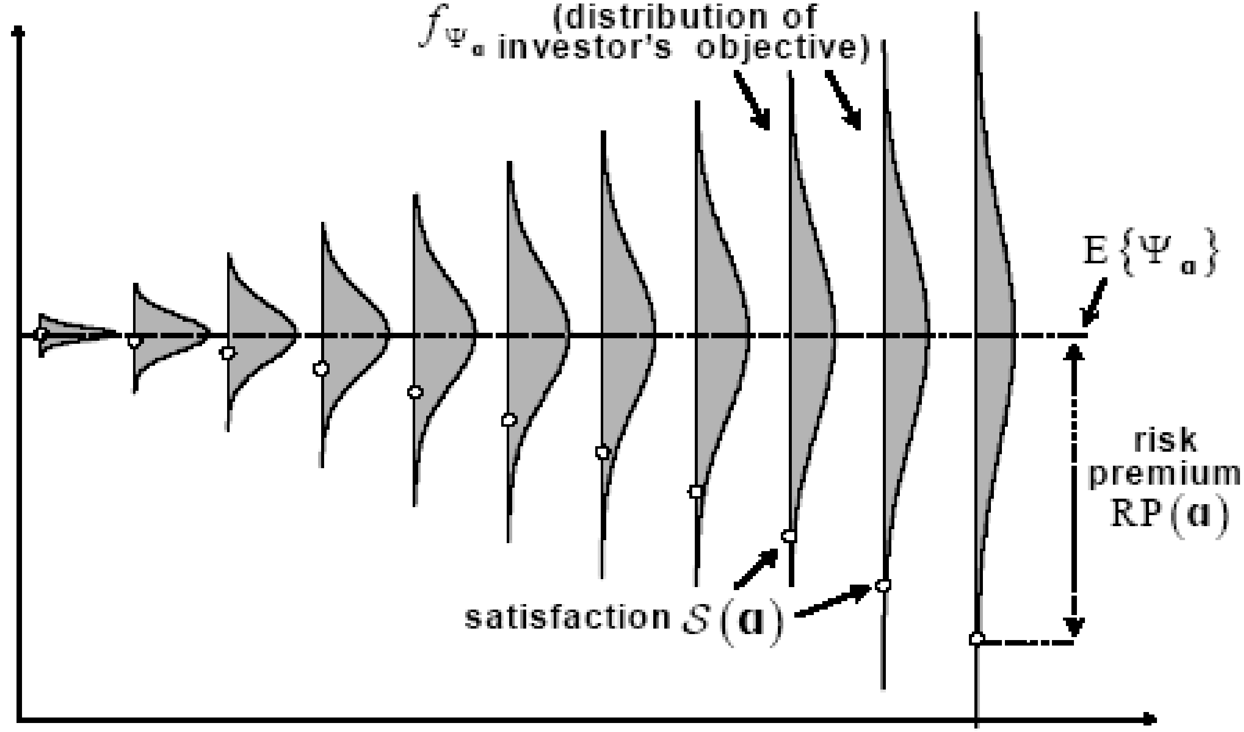

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$$\mathcal{S}(\boldsymbol{\alpha} + \mathbf{b}) = \mathcal{S}(\boldsymbol{\alpha}) + \psi_{\mathbf{b}} \quad (5.71)$$

- **super- additivity**

$$\mathcal{S}(\boldsymbol{\alpha} + \boldsymbol{\beta}) \geq \mathcal{S}(\boldsymbol{\alpha}) + \mathcal{S}(\boldsymbol{\beta}) \quad (5.75)$$

- **Co-monotonic additivity**

$$(\boldsymbol{\alpha}, \boldsymbol{\delta}) \text{ co-monotonic} \Rightarrow \mathcal{S}(\boldsymbol{\alpha} + \boldsymbol{\delta}) = \mathcal{S}(\boldsymbol{\alpha}) + \mathcal{S}(\boldsymbol{\delta}) \quad (5.80)$$

- **Concavity**

$$\mathcal{S}(\lambda\boldsymbol{\alpha} + (1-\lambda)\boldsymbol{\beta}) \geq \lambda\mathcal{S}(\boldsymbol{\alpha}) + (1-\lambda)\mathcal{S}(\boldsymbol{\beta}) \quad (5.81)$$

- **Risk aversion/propensity/neutrality**

$$\mathrm{RP}(\boldsymbol{\alpha}) \equiv \mathrm{E}\{\Psi_{\boldsymbol{\alpha}}\} - \mathcal{S}(\boldsymbol{\alpha}) \quad (5.85)$$