Value at Risk – MATLAB Application of Copulas on US and Indian Markets

A Rolling-over VaR Study of Portfolio of Indices

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Abstract

Diversification globally has become a style more than a strategy and the need of the hour boils down to the efficient investment identification, better diversification of the risk, and better risk – adjusted return. One of the ways of such diversification is the investment in the indices, which has a portfolio of varied securities, globally. Such an investment immediately calls for a better exposure measurement to analyse the various investment parameters such as the portfolio variance, the Value at Risk, the associated return, the End Tail Loss and so on. This paper measures one such parameter, the Value at Risk (VaR) using the bivariate Gaussian Copula distribution implemented in MATLAB for the Dow-Jones index and the National Stock Exchange index. From a business point of view, with huge investments being made in India by Foreign Institutional Investors (FIIs) and vice-versa, this study holds much relevance with the risk identification and its mitigation. Retail investors are the least ones to be informed about the markets and this might serve as a platform for them to study their investment risk as a measure of VaR. For an academician the correlation, its dependence on the on copulas and its study would always be beneficial for risk management.

Introduction

Entering the global market has become easy now. Now its efficient management in-terms of risk is mandatory when taking decisions for global investment. The foreign exchange rate risk is the foremost factor to be examined on. With lots of information available on the same, the need is to mitigate the investment risk inters if the Value at Risk and the End Tail Losses. This paper measures one such parameter, the Value at Risk (VaR) using the bivariate Gaussian Copula distribution implemented in MATLAB for the Dow-Jones index and the National Stock Exchange index. From a business point of view, with huge investments being made in India by Foreign Institutional Investors (FIIs) and vice-versa, this study holds much relevance with the risk identification and its mitigation. Retail investors are the least ones to be informed about the markets and this might serve as a platform for them to study their investment risk as a measure of VaR. For an academician the correlation, its dependence on the on copulas and its study would always be beneficial for risk management.

Copula functions represent a methodology which has recently become the most significant new tool to handle in a flexible way the co-movement between markets, risk factors and other relevant variables studied in finance.[*referecne]. This tool has been used in statistics for arriving at the multi-variant distribution function without making any approximations. This is slowly replacing the present normal assumption very commonly taken by the financial analysts. The black schools formula with its basic normal assumption is a under threat of being corrected.

These new developments have caused standard tools of financial mathematics to become suddenly obsolete. The reason has to be traced back to the overwhelming evidence of non-normality of the probability distribution of financial assets returns, which has become popular well beyond the academia and in the dealing rooms. May be for this reason, and these new environments, non-normality has been described using curious terms such as the “smile effect”, which traders now commonly use to define strategies, and the “fat-tails” problem, which is the major topic of debate among risk managers and regulators. The result is that now-a-days no one would dare to address any financial or statistical problem connected to financial markets without taking care of the issue of departures from normality.

There are three kinds of issues that are to be addressed in the contemporary world of financial management. The first is the non-normality of returns, which makes the standard Black and Scholes option pricing approach obsolete. The second is the incomplete market issue, which introduces a new dimension to the asset pricing problem – that of the choice of the right pricing kernel both in asset pricing and risk management. The third is credit risk, which has seen a huge development of products and techniques in asset pricing. The copula modeling has been used in credit risk management in large and the shift is toward its application in various other fields of modeling.

We are applying the copulas for calculating the Value at Risk for the portfolio with Dow Jones index investment and the National Stock Exchange. MATLAB code is written for the same and the VaR at 95 th percentile is calculated. The rolling over copula VaR is calculated from the year 2002 with a 5 year daily index study.

Brief Literature review

The Copula functions, the general theory of which was developed in the late 1950s by Sklar (1959), have recently been successfully applied to various domains of financial risk management. Joe (1997) and Nelsen (1999) have presented a good introduction to the copula theory. Although copulas have been only recently used in the financial area, there are available a number of applications in this area. Closer to home, they have been used for a number of years in actuarial science, and are now making major inroads into applied finance. In the actuarial field, they have been applied to such problems as bivariate survival analysis, loss modeling, and the analysis of indemnity claims.

The works of Bouy'e et al. (2000), Embrechts, McNeil and Straumann (2002) and Embrechts, Lindskog and McNeil (2003) have provided general examples of applications of copula in finance. For instance, Cherubini and Luciano (2001) estimated the VaR using the Archimedean copula family and the historical empirical distribution in the estimation of marginal distributions; Rockinger and Jondeau (2001) used the Plackett copula with GARCH processs with innovations modeled by the Student "t" asymmetrical generalized distribution of Hansen (1994) and proposed a new measure of conditional dependence; Georges (2001) used the normal copula to model options time of exercise and for derivative pricing; Meneguzzo and Vecchiato (2002) used copula for modeling the risk of credit derivatives; and Fortin and Kuzmics (2002) used convex linear combinations of copula for estimating the VaR of a portfolio of indices. Mbrechts, McNeil and Straumann (2002) and Embrechts, Ho-ingandJuri (2003) used copula to model extreme value and risk limits.

Background

Introduction to correlation

Correlation is a measure of the relation between two or more variables. The measurement scales used should be at least interval scales, but other correlation coefficients are available to handle other types of data. Correlation coefficients can range from -1.00 to +1.00. The value of -1.00 represents a perfect negative correlation while a value of +1.00 represents a perfect positive correlation. A value of 0.00 represents a lack of correlation.

$$Correl(X,Y) = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \sum (y-\bar{y})^2}}$$

Where x and y are instances of the data to be studied and $\bar{x}$ is the mean of those instances.

The significance level calculated for each correlation is a primary source of information about the reliability of the correlation. The significance of a correlation coefficient of a particular magnitude will change depending on the size of the sample from which it was computed. The test of significance is based on the assumption that the distribution of the residual values (i.e., the deviations from the regression line) for the dependent variable y follows the normal distribution, and that the variability of the residual values is the same for all values of the independent variable x. However, Monte Carlo studies suggest that meeting those assumptions closely is not absolutely crucial if your sample size is not very small and when the departure from normality is not very large. It is impossible to formulate precise recommendations based on those Monte- Carlo results, but many researchers follow a rule of thumb that if your sample size is 50 or more then serious biases are unlikely, and if your sample size is over 100 then you should not be concerned at all with the normality assumptions.

Introduction to VaR

Value at risk is a single, summary, statistical measure of possible portfolio losses. Specifically, value at risk is a measure of losses due to 'normal' market movements. Losses greater than the value at risk are suffered only with a specified small probability. Subject to the simplifying assumptions used in its calculation, value at risk aggregates all of the risks in a portfolio into a single number suitable for use in the board room, reporting to regulators, or disclosure in an annual report. Once one crosses the hurdle of using a statistical measure, the concept of value at risk is straight forward to understand. It is simply away to describe the magnitude of the likely losses on the portfolio.

The VaR figure has two important characteristics. The first is that it provides a common consistent measure of risk across different positions and risk factors. It enables us to measure the risk associated with a fixed – income position, say, in away that is comparable to and consistent with a measure of the risk associated with equity positions. VaR provides us with a common risk yardstick, and this yardstick makes it possible for institutions to manage their risks in new ways that were not possible before. The other characteristic of VaR is that it takes account of the correlations between different risk factors. If two risks off set each other, the VaR allows forth is offset and tells us that the overall risk is fairly low. If the same two risks don't offset eachother, theVaR takes this into account as well and gives us a higher risk estimate. Clearly, a risk measure that accounts for correlations is essential if we are to be able to handle portfolio risks in a statistically meaningful way.[* reference]

For example: if a portfolio of stocks has a one-day 5% VaR of Rs 1 lakh, there is a 5% probability that the portfolio will fall in value by more than Rs 1 lakh over a one day period, assuming markets are normal and there is no trading. Informally, a loss of Rs 1 lakh or more on this portfolio is expected on 1 day in 20.

Introduction to MATLAB

MATLAB, short for Matrix Laboratory, is a simple and flexible programming environment for a wide range of problems such as signal processing, optimization, linear programming, financial modeling and so on. The basic MATLAB software package can be extended by using add-on toolboxes. Examples of such toolboxes are: Signal Processing, Filter Design, Statistics and Symbolic Math, GRCH toolbox etc. MATLAB is an interpreted language. This implies that the source code is not compiled but interpreted on the fly. This is both an advantage and a disadvantage. MATLAB allows for easy numerical calculation and visualization of the results without the need for advanced and time consuming programming. The disadvantage is that it can be slow, especially when bad programming practices are applied.

Introduction to Copulas

The technique now most often used for pricing credit derivatives when there are many underlyings is that of the copula. The copula function is a way of simplifying the default dependence structure between many underlyings in a relatively transparent manner. The clever trick is to separate the distribution for default for each individual name from the dependence structure between those names. So you can rather easily analyze names one at a time, for calibration purposes, forexample, and then bring them all together in a multivariate distribution. Mathematically the copula way of representing the dependence (one marginal distribution per underlying, and a dependence structure) is no different from specifying a multivariate density function. But it can simplify the analysis. The copula approach in effect allow us to readily go from a single-default world to a multiple- default world almost seamlessly. And by choosing the nature of the dependence, the copula function, we can explore models with richer 'correlations' than we have seen so far in the multivariate Gaussian world. For example, having a higher degree of dependence during big market moves is quite straight forward.

Definition of Two dimensional Copula :

Consider a joint distribution function **H(x,y)** with **domain [-∞,∞]x[-∞,∞] & range[0,1] Є I** and With marginal function, **H(x,∞) = F(x) = u & H(∞,y) = G(y) = v** , then range of **F(x)** and **G(y) Є I**. Consider a function **C(u,v)** such that, **C(u,v) = C[F(x),G(y)] = H(x,y)** then **C(u,v)** is called a copula function if it satisfies the following properties.

1) **Dom C(u,v) = [0,1]x[0,1] Є I & Range C(u,v) = [0,1] Є I**
2) **C(u,v) is a Ground function i.e C(u,0) = 0 = C(0,v)**
3) **C(u,v) is an increasing function i.e C(u,1) = u & C(1,v) = v**
4) **For every u_1, u_2, v_1, v_2 in I such that $u_1 \leq u_2$ & $v_1 \leq v_2$ function C(u,v) satisfies the inequality,**

$C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \leq 0$

Then function C(u,v) defined as a copula function which relates the Marginal Distribution Function F(x) and G(y) of H(x,y) with function H(x,y) itself. There are many predefined copula function which are commonly used like Gaussian Copula, Archimedean copula, elliptical copula, normal copula and many others.

Gaussian Copula:

The Gaussian (or normal) copula is the copula of the multivariate normal distribution. In fact, the random vector **$X=(X_1,\ldots,X_n)$** is multivariate normal if :

1. The univariate margins $F_1, \ldots, F_n$ are Gaussians;

2. The dependence structure among the margins is described by a unique copula function C (the normal copula) such that

$$C_R^{Ga}(u_1,\ldots,u_n) = \Phi_R\left(\phi^{-1}(u_1),\ldots,\phi^{-1}(u_n)\right)$$

where Φ_R is the standard multivariate normal d.f. with linear correlation matrix R and ϕ^{-1} is the inverse of the standard univariate Gaussian d.f. Multivariate normal is commonly used in risk management applications to simulate the distribution of the n risk factors affecting the value of the trading book (market risk) or the distribution of the n systematic factors influencing the value of the credit worthiness index of a counterparty (credit risk).

If n=2 , we get the Gaussian Copula function

$$C_R^{Ga}(u,v) = \int_{-\infty}^{\phi^{-1}(u)} \int_{-\infty}^{\phi^{-1}(v)} \frac{1}{2\pi(1-R_{12}^2)^{1/2}} \exp\left\{ -\frac{s^2 - 2R_{12}st + t^2}{2(1-R_{12}^2)} \right\} dsdt$$

where R_{12} is simply the linear correlation coefficient between the two random variables.

Objectives

1. To find out the correlation between the Indian and the American market form the perspective of better diversification.
2. To fine out the VaR of the portfolio of security in the index of both Dow Jones (US) and national Stock Exchange (Indian).
3. To develop a MATLAB code for Copula VaR of the portfolio.
4. To find out the VaR for the same with the Dow Jones and the NSE indices as samples.
5. To compare the rolling over correlations and its impact on the Copula VaR.

Research Methodology

The steps adopted in the research is analysed and narrated as under:

1. The analysis is done for the correlation between the Dow Jones index (DJI) and the National Stock Exchange index (NSEI) for the period of 9 years from 2002 – 2010 February. The data for the index values are obtained from website.

2. Selection of the stock exchanges was done using convenience sampling. The crux of the paper is to bring in the concept of VaR for the portfolio of the indices and its dependence on the correlation.

3. Rolling over daily mean, daily variance and standard deviation are taken for the 5 year periods atarting from 2002-06 to 2006-10 with one year increment.

4. The covariance and the correlation for the indices for each period are studied and the results plotted.

5. The VaR for the individual indices are also found out using the parametric VaR method using a normal assumption. The logarithmic returns are taken for each day and the 95th percentile deviation is taken and the VaR is given as **VaR = [1.645*Target value*standard deviation (daily)]**. Such VaR is found out for all the rolling over years

and the individual VaR is noted down. The next step involves the identification of the portfolio of the index and the same procedure is followed. The diversified and the undiversified vaR values are also noted.

6. Now to compare it with the VaR found out using the copula we have found out the density function of the individual univariate distributions. To accomplish this we have implemented the Gaussian Copula function in the MATLAB code, that takes in the correlation and the univariate distribution as input and gives the CDF as the output. Assumption made is that the integral that runs from infinity can be assumed to be equally valid when taken from -5. Since area under the curve of a normal distribution with a range of -5 to 5 gives us almost 100% coverage this assumption holds good. After obtaining the copula CDF's they are arranged in the ascending order and the 5th percentile value gives the Copula VaR. This exercise is done for the all rolling over years and is compared. The code used for the same is been given as under.
7. MATLAB Code:

```
% Defining the function – Gaussian Function
function output = fun_1(x,y)
output = 1/2/(1-CORREL*CORREL)^0.5/pi*exp(-(x.^2-
2.*CORREL.*x.*y+y.^2)/2/(1- CORREL*CORREL));

% Integrating the above Gaussian function if copula
output_ans = zeros(#DATA,1);
for i = 1 : #DATA
     result1 = dblquad('fun_1',-5,u(i),-5,v(i));

% Storing the CDF in the array
output_ans(i,1)= result1;
end
```

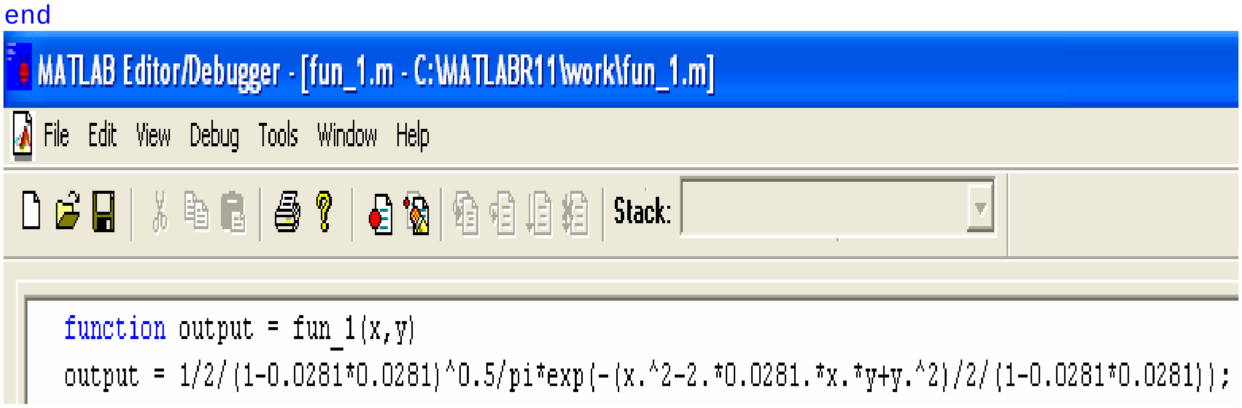

Figure 1: Screen shot of the code – Gaussian function

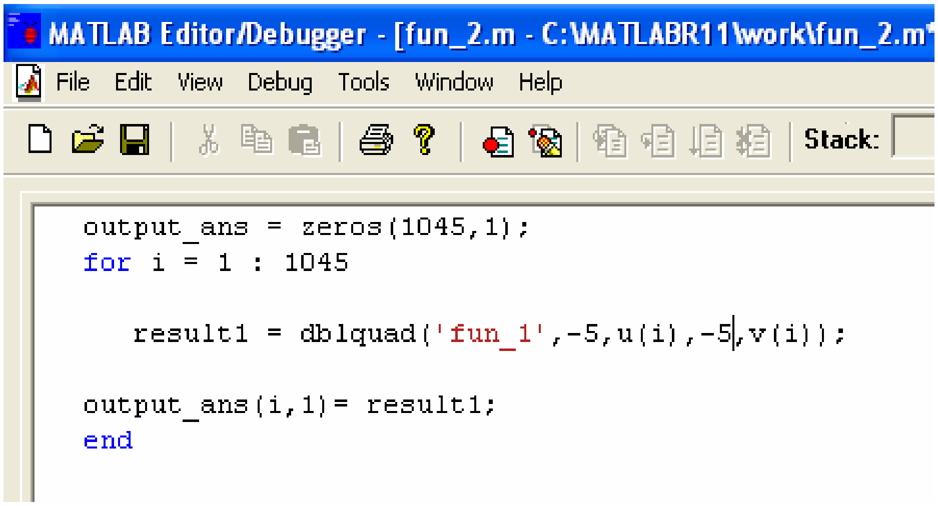

Figure 2: Screen shot of the code – Integration of the Function

8. The output is regressed with the univariate z values and the graph that is obtained by the linear assumption is plotted for a specific correlation.

9. After obtaining the Copula VaR the relationship between the correlation and the Copula VaR is studied and the advantages are noted down.

Data Analysis and Results

The Dow Jones index return and the NSE index return is plotted as under for a period of 9 years from 2002 till date.

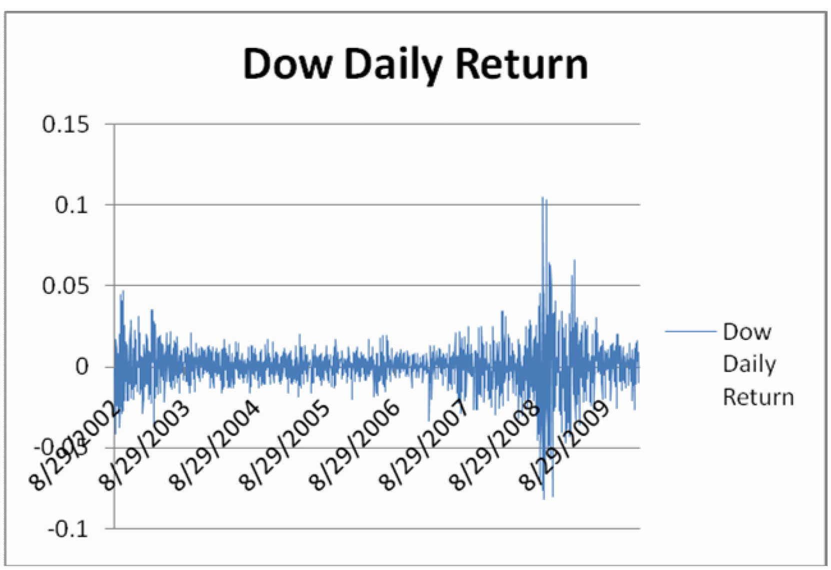

Figure 3: Dow Daily return

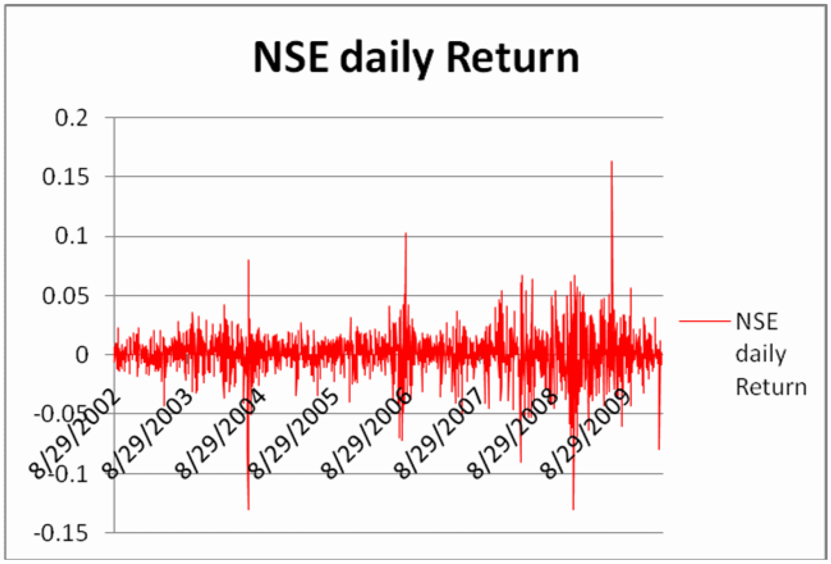

Figure 4: NSE Daily return

VaR @ 95% confidence interval						
	2002-10	**2002-06**	**2003-07**	**2004-08**	**2005-09**	**2006–10**
Investment (50-50 wt)	7663.045	8103.625	9598.61	5793.37	7747.375	7663.045
Correlation of Returns	0.022321	0.001489	0.008056	0.015268	0.028106	0.028106
Parametric DOW	220.6174	182.6124	173.5846	180.5853	238.7036	257.5505
Parametric NSE	145.8571	88.51587	144.8430	85.57718	163.4313	170.6857
Parametric Portfolio	135.5573	107.7025	113.2209	96.33621	144.2344	154.1776
Copula	146.9000	99.34000	101.3300	122.5467	155.4120	186.0500

Table 1: Consolidated VaR @ 95% significance Interval

Correlation of the daily returns

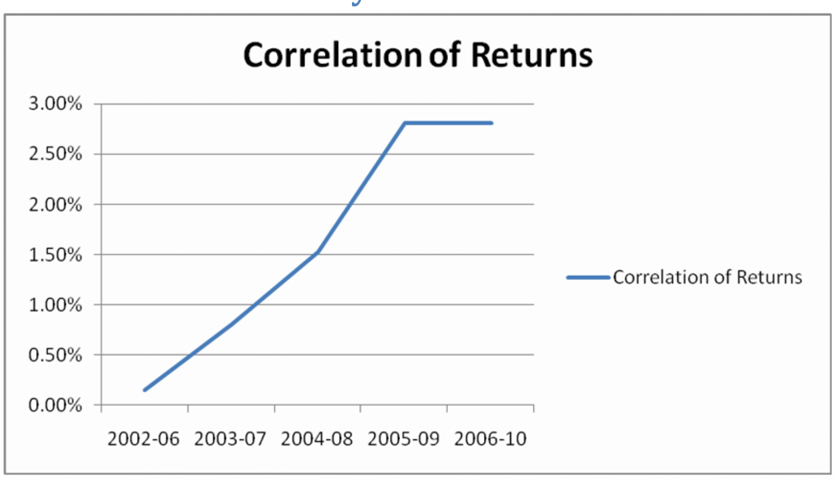

Figure 5: The correlation of the returns over the years

The correlation of between the Indian Market and the Dow has been increasing steadily and this has in impact on the copula VaR also. With the increase in the correlation the vaR has also increased because of the increase in the variance of the portfolio.

Diversified and Un-Diversified VaR

Using the normal returns assumption we have calculated the VaR at 95% confidence interval and it has been plotted as under.

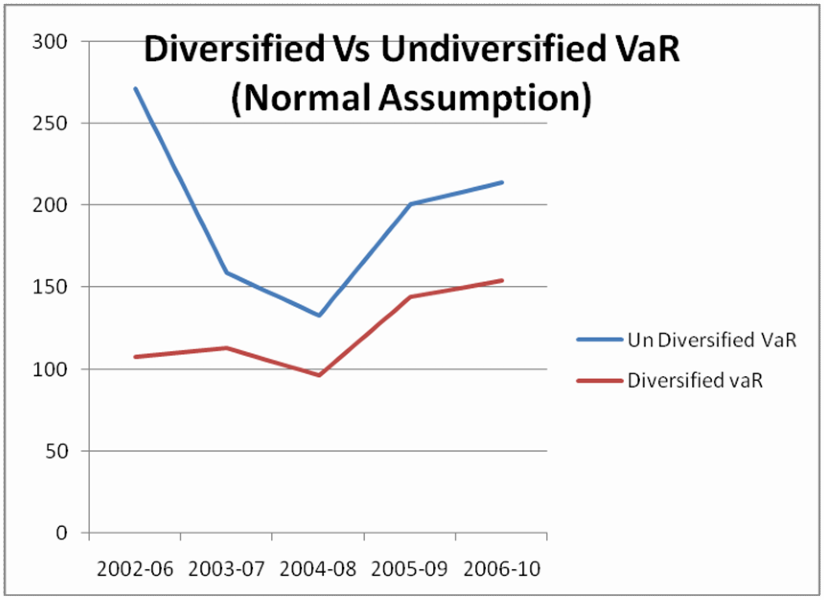

Figure 6: Diversified and Undiversified VaR

It can be seen that the benefit f diversification is always there and is shown by the lesser VaR for the same period.

Diversified Var and Copula VaR

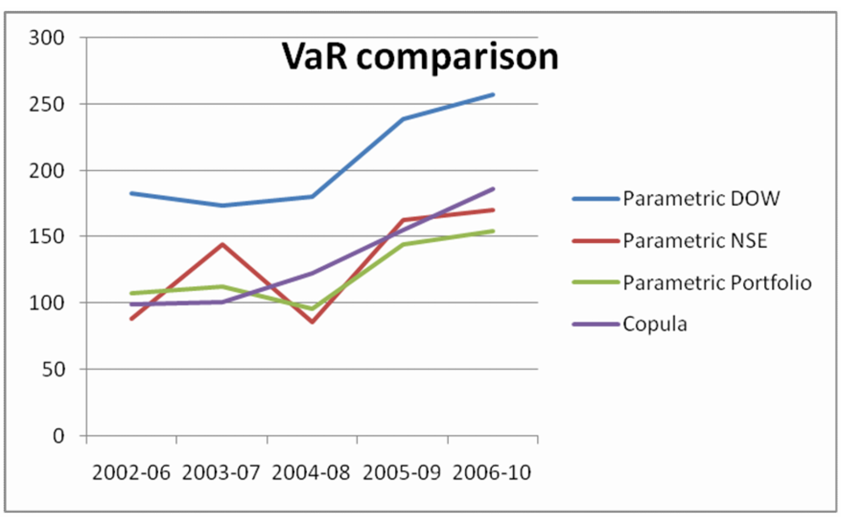

Figure 7: Diversified VaR and Copula vaR

A subtle conclusion arising from the figure is that the copula VaR has a greater dependency in the correlation of the indices. During the first few years the copula VaR was lower than the parametric VaR using the normal assumption.

Regressed with linear assumption

The relationship between the VaR copula and the univariate distributions are assumed to be linear and the graph is plotted in the 3-D space as shown below.

The regression coefficients and the R^2 values are also shown.

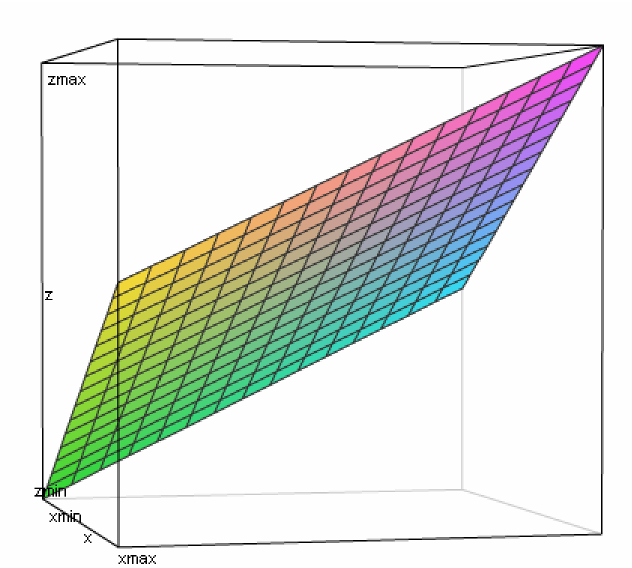

Figure 8: Linear regression graph

Summary Table:

Regression Statistics	
Multiple R	0.843222
R Square	0.711024
Adjusted R Square	0.710717
Standard Error	0.102612
Observations	1888

Table 2 : The regression statistics

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	48.83522	24.41761	2319.012	0
Residual	1885	19.84777	0.010529		
Total	1887	68.68299			

Table 3: The ANOVA comparison

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.314457	0.002362	133.1562	0	0.309825	0.319088	0.309825	0.319088
P/L in terms of z Dow	0.117522	0.002363	49.73946	0	0.112888	0.122156	0.112888	0.122156
P/L in terms of z NSE	0.107304	0.002363	45.41481	0	0.10267	0.111938	0.10267	0.111938

Table 4 : The regression coefficient

Therefore the copula VaR can be given as

Copula VaR = 0.3144 + 0.1175*DOW_Z + 0.1073*NSE_Z

Scope and Conclusion

Industry application:

This can be a very useful tool for risk mitigation. The VaR measured using copula is more reliable in terms of the basic assumption made than the normal assumption. This can further be developed as user friendly software which will directly give the VaR values of the selected portfolios. And few risk mitigation techniques also be incorporated in the same so as to make this kit complete.

Individual Application:

High net-worth individuals going global diversification can opt for this method so that this part of the risk can be incorporated and hedged.

Academicians:

A comprehensive study of the copula and its application can bring insight into the various nuances and various financial modeling can be done on the same.

Therefore, the present study highlights the following:

1. The correlation of the Indian market with that of the Dow Jones has been increasing. And this has increased the variance of the portfolio of the indices.

2. The diversified VaR (Normal vaR of portfolio) has always been less than the individual VaR and has been greater than the Copula VaR for initial few years (2002-06 and 2003-07).

3. At 95% significance level the Copula VaR is highly dependent on the correlation. This is evident from the fact that, with increase in the correlation the copula VaR value shave increased. This can also be proved with the increased variance of the portfolio due to increased correlation.

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