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Modeling and Estimation Techniques for Portfolio Management

AGENDA

PORTFOLIO MODELING - mean variance and representations

MARKET MODELING - location dispersion ellipsoid and PCA

ESTIMATION - non parametric, ML, robust, shrinkage, Bayesian

REFERENCES

AGENDA

PORTFOLIO MODELING - mean variance and representations

MARKET MODELING - location dispersion ellipsoid and PCA

ESTIMATION - non parametric, ML, robust, shrinkage, Bayesian

REFERENCES

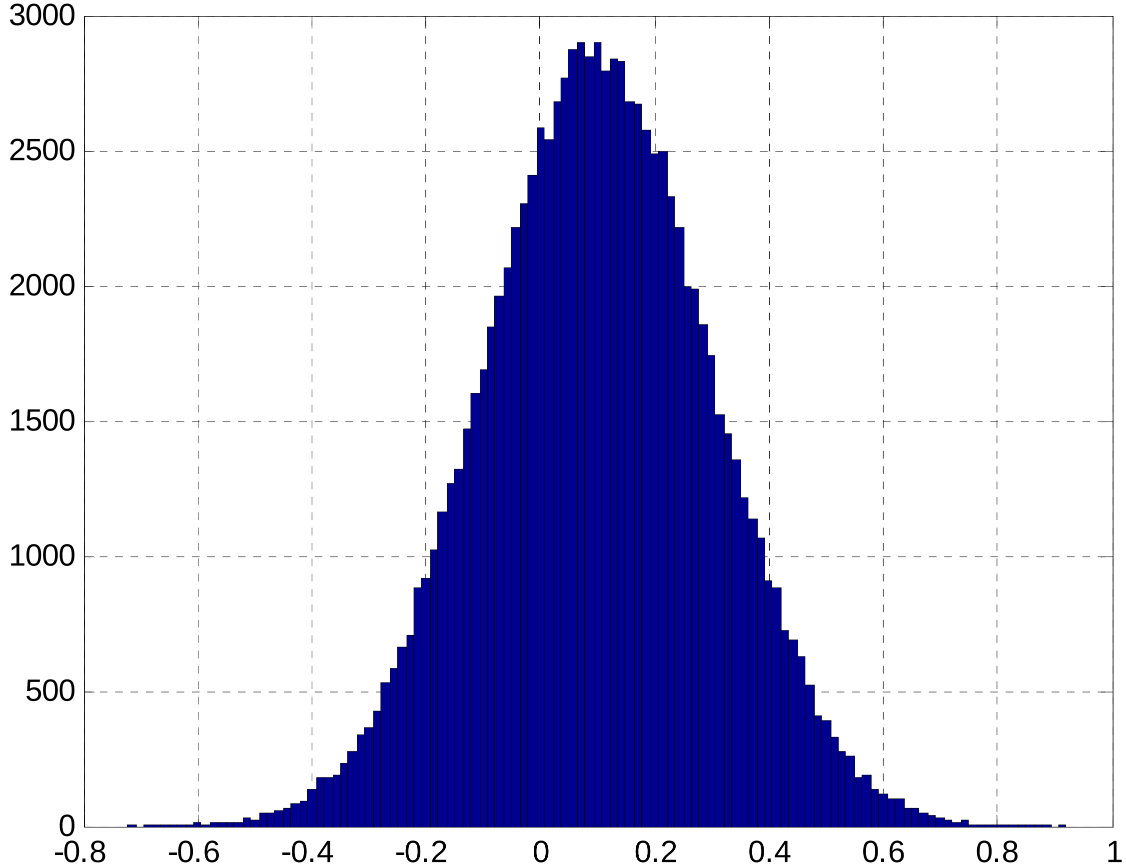
PORTFOLIO MODELING - REPRESENTATION AS HISTOGRAM

$R^{\tau}_{t+\tau} \equiv P_{t+\tau} / P_t - 1$: linear return (random variable) $\Leftrightarrow$ $\underset{J \times 1}{\mathcal{R}}$: vector of simulations

outcome frequency

$\approx$

true pdf f_R



return outcome in each scenario

PORTFOLIO MODELING - MEAN VARIANCE

$R_{t+\tau}^{\tau} \equiv P_{t+\tau} / P_t - 1$: linear return (random variable)

$m \equiv \mathrm{E}\left\{R_{t+\tau}^{\tau}\right\}$: expected return = **GOOD**

$S \equiv \mathrm{Var}\left\{R_{t+\tau}^{\tau}\right\}$: variance of return = **BAD**

$\Updownarrow$

$s \equiv \mathrm{Sd}\left\{R_{t+\tau}^{\tau}\right\}$: standard deviation of return

example

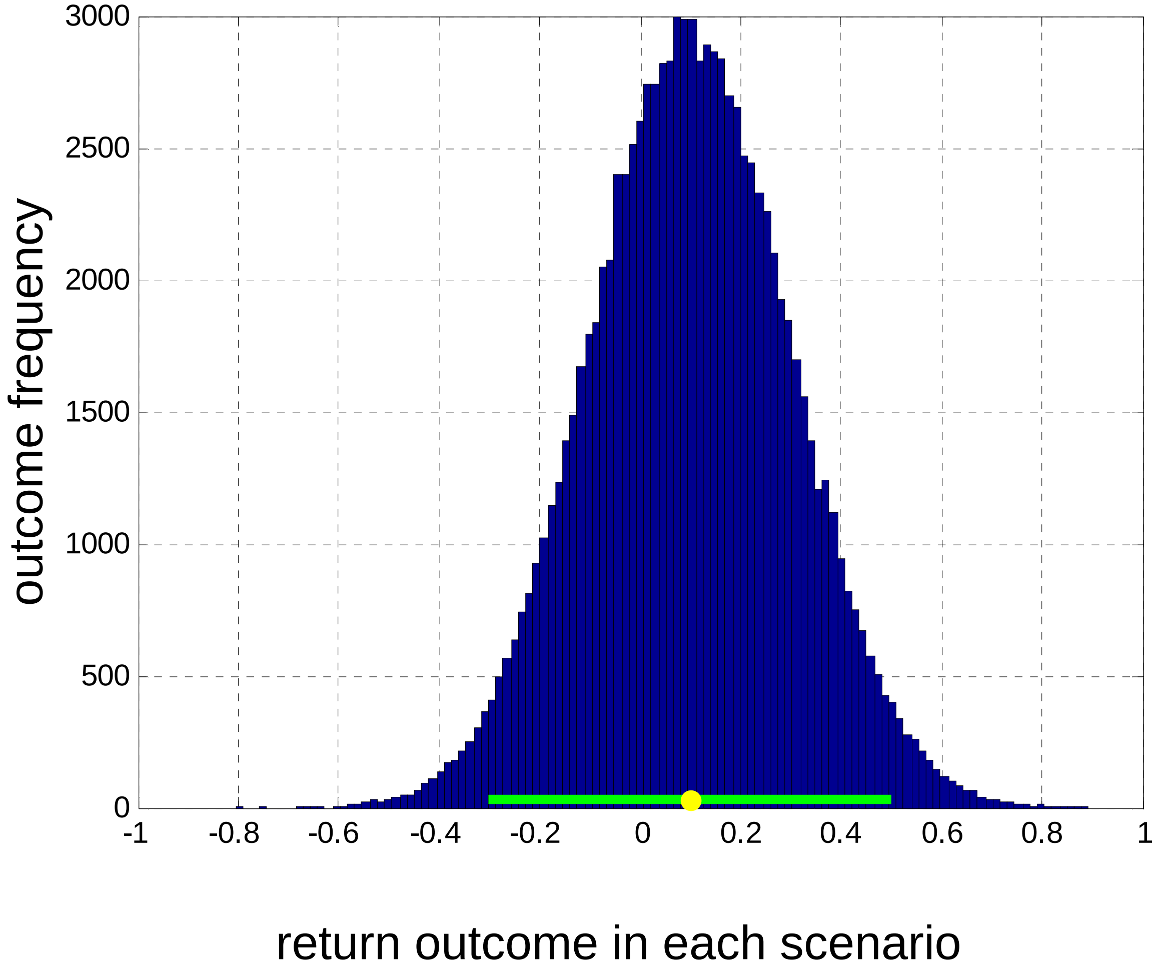
$m \equiv 10\%$

$S \equiv (20\%)^2$

$s \equiv 20\%$

PORTFOLIO MODELING - REPRESENTATION OF MEAN VARIANCE INPUTS

$R_{t+\tau}^{\tau} \equiv P_{t+\tau} / P_t - 1$: linear return (random variable)



example

$m \equiv 10\%$

$S \equiv (20\%)^2$

$s \equiv 20\%$

location dispersion bar

<u>expected value</u>
= center of bar

<u>standard deviation</u>
= length of bar

AGENDA

PORTFOLIO MODELING - mean variance and representations

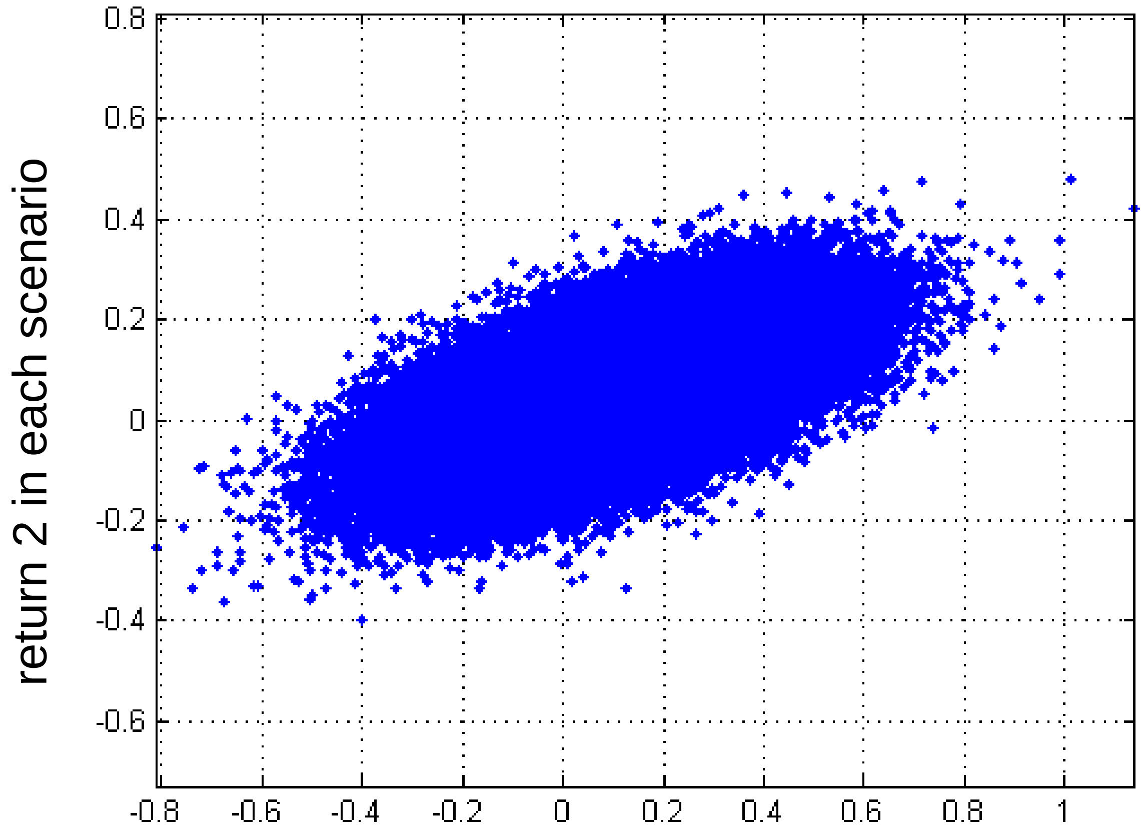
MARKET MODELING - location dispersion ellipsoid and PCA

ESTIMATION - non parametric, ML, robust, shrinkage, Bayesian

REFERENCES

MARKET MODELING - REPRESENTATION AS SIMULATIONS

$\boldsymbol{R}_{t+\tau}^{\tau} \equiv \boldsymbol{P}_{t+\tau} ./ \boldsymbol{P}_{t} - \mathbf{1}$: linear returns (random variables) $\Leftrightarrow \underset{J \times N}{\mathcal{R}}$: panel of simulations



return 1 in each scenario

MARKET MODELING - MEAN VARIANCE

$\boldsymbol{R}_{t+\tau}^{\tau} \equiv \boldsymbol{P}_{t+\tau} ./ \boldsymbol{P}_t - \mathbf{1}$: linear returns (random variables)

example

$\boldsymbol{w}$: relative portfolio weights

$$\boldsymbol{w} \equiv \begin{pmatrix} 40\% \\ 60\% \end{pmatrix}$$

$m_{\boldsymbol{w}} = ?$: portfolio expected return = **GOOD**

$$m_{\boldsymbol{w}} \equiv ?$$

$S_{\boldsymbol{w}} = ?$: portfolio variance = **BAD**

$$S_{\boldsymbol{w}} \equiv ?$$

MARKET MODELING - SUMMARY STATISTICS

$\boldsymbol{R}^{\tau}_{t+\tau} \equiv \boldsymbol{P}_{t+\tau} ./ \boldsymbol{P}_t - \mathbf{1}$: linear returns (random variables)

example

$\boldsymbol{m} \equiv \mathrm{E}\left\{\boldsymbol{R}^{\tau}_{t+\tau}\right\}$: expected returns

$$\boldsymbol{m} \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

$\boldsymbol{S} \equiv \mathrm{Cov}\left\{\boldsymbol{R}^{\tau}_{t+\tau}\right\}$: covariance of returns

$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times 0.6 \\ 20\% \times 10\% \times 0.6 & (10\%)^2 \end{pmatrix}$$

$\boldsymbol{w}$: relative portfolio weights

$$\boldsymbol{w} \equiv \begin{pmatrix} 40\% \\ 60\% \end{pmatrix}$$

$m_{\boldsymbol{w}} \equiv \boldsymbol{w}'\boldsymbol{m}$: portfolio expected return **= GOOD**

$$m_{\boldsymbol{w}} \equiv 40\% \times 10\% + 60\% \times 5\%$$

$S_{\boldsymbol{w}} \equiv \boldsymbol{w}'\boldsymbol{S}\boldsymbol{w}$: portfolio variance **= BAD**

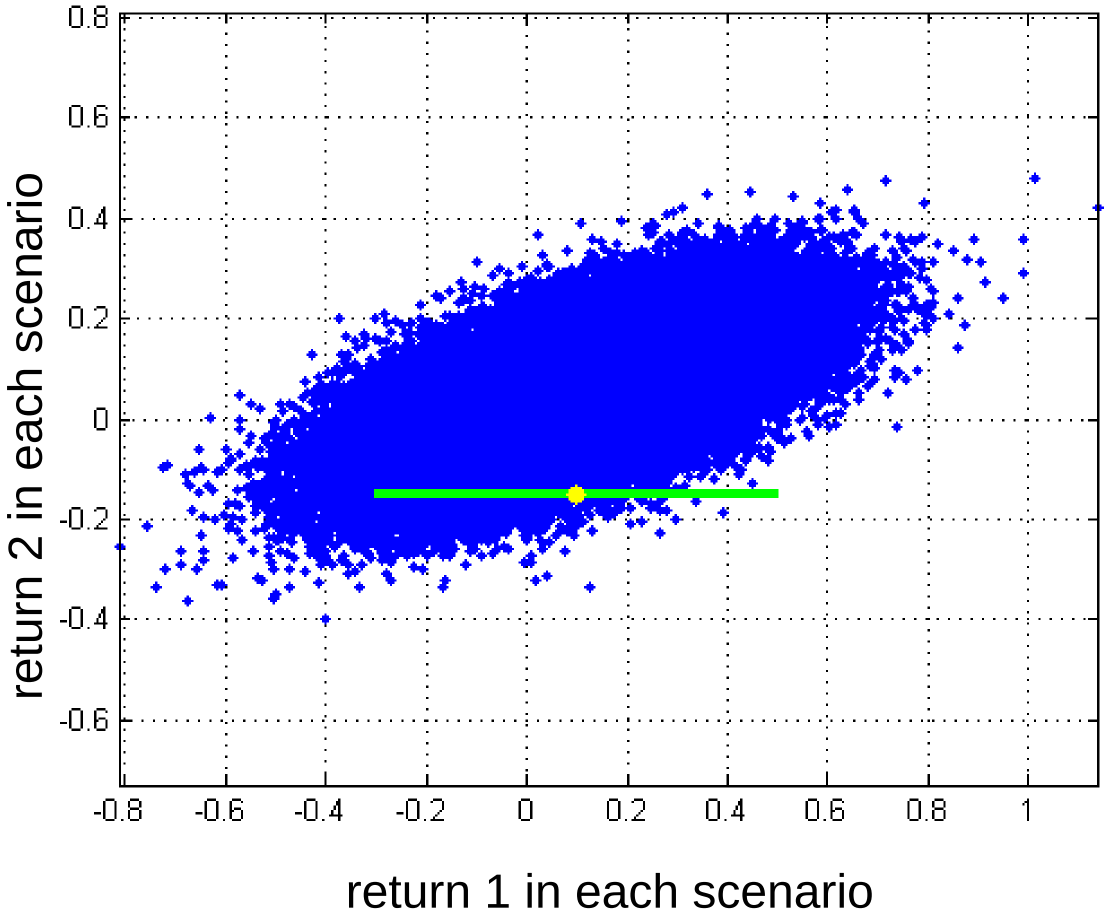
$$S_{\boldsymbol{w}} \equiv (40\% \times 20\%)^2 + (60\% \times 10\%)^2 + 2 \times 40\% \times 60\% \times 20\% \times 10\% \times 0.6$$

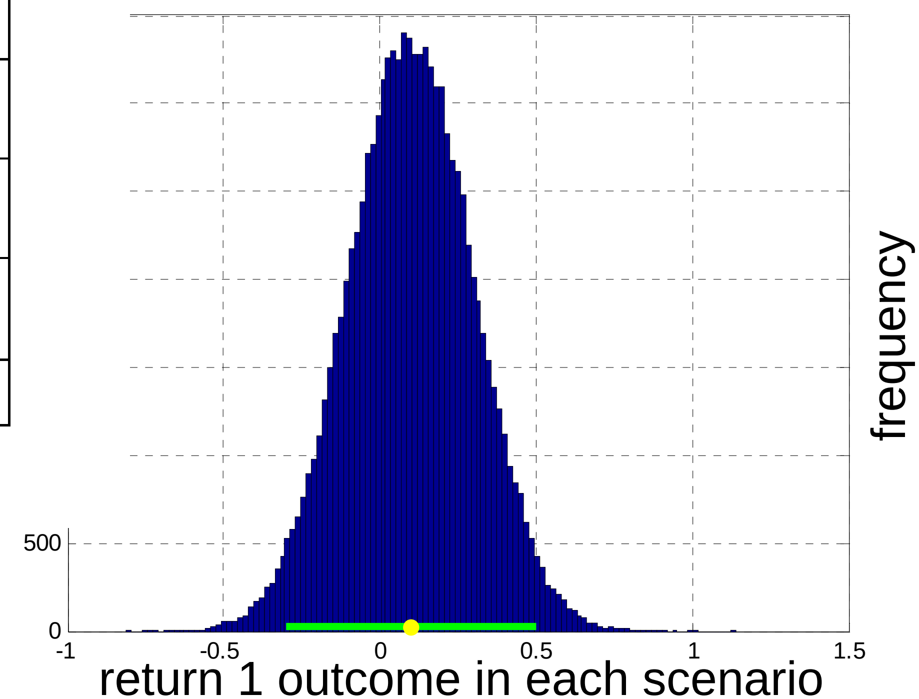
MARKET MODELING - LOCATION DISPERSION BOX

$\boldsymbol{R}_{t+\tau}^{\tau} \equiv \boldsymbol{P}_{t+\tau} ./ \boldsymbol{P}_{t} - \mathbf{1}$: linear returns (random variables)

$$\boldsymbol{m} \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times 0.6 \\ 20\% \times 10\% \times 0.6 & (10\%)^2 \end{pmatrix}$$





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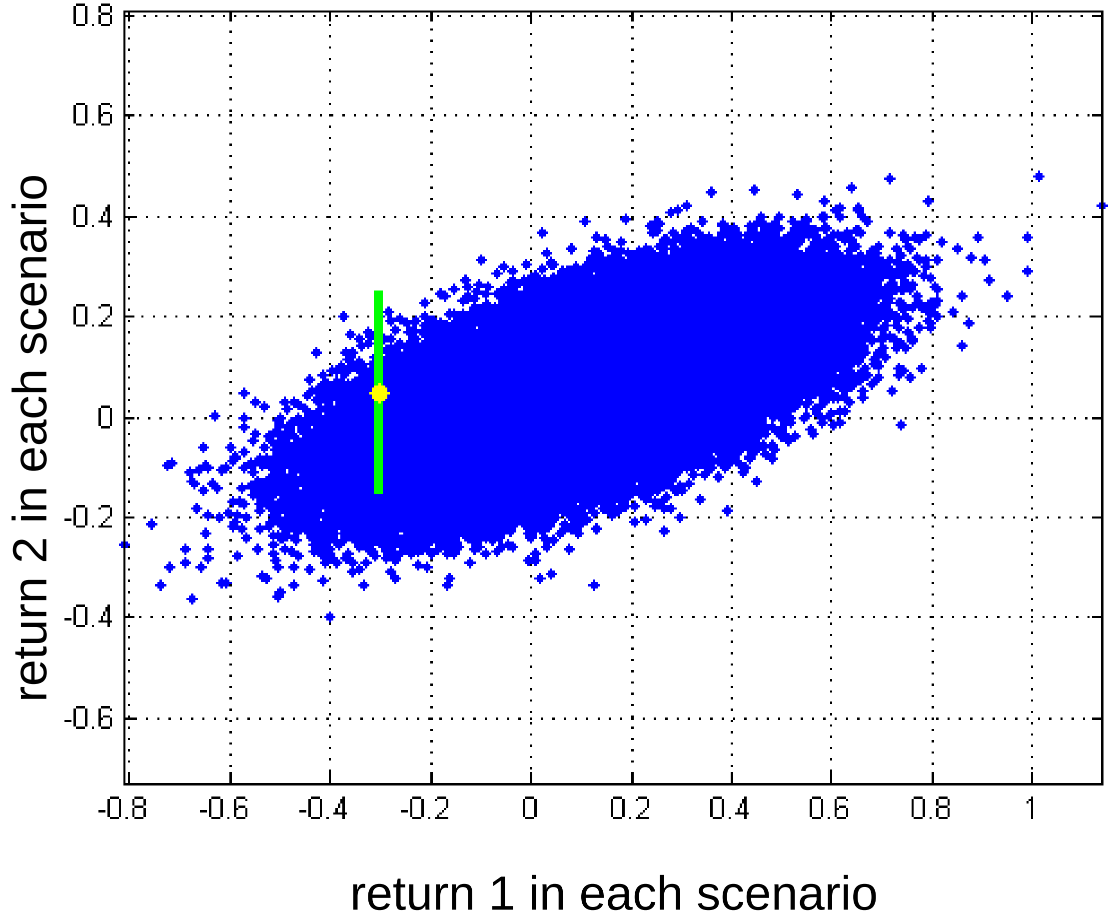

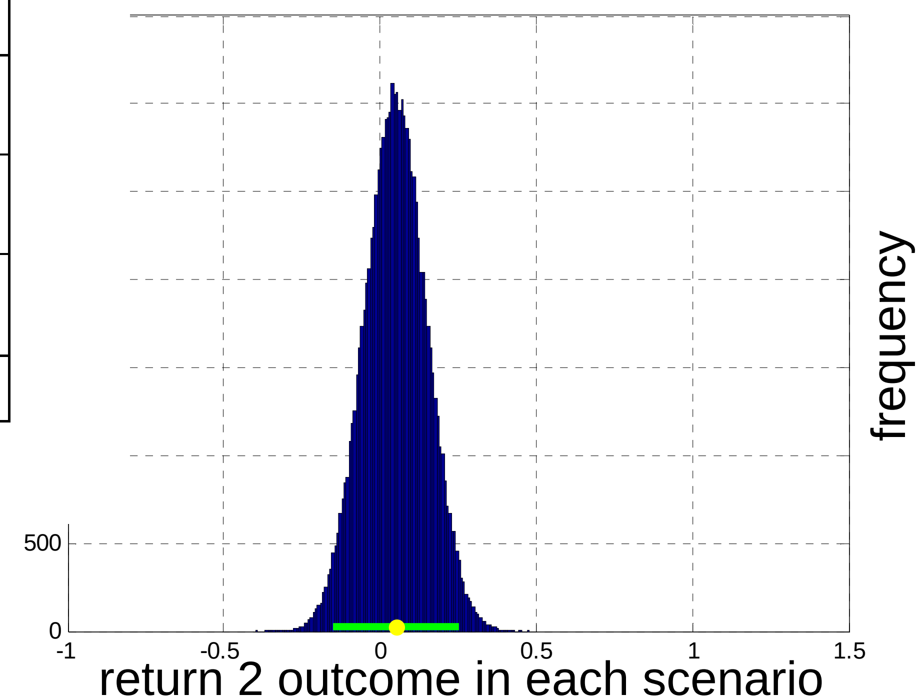

MARKET MODELING - LOCATION DISPERSION BOX

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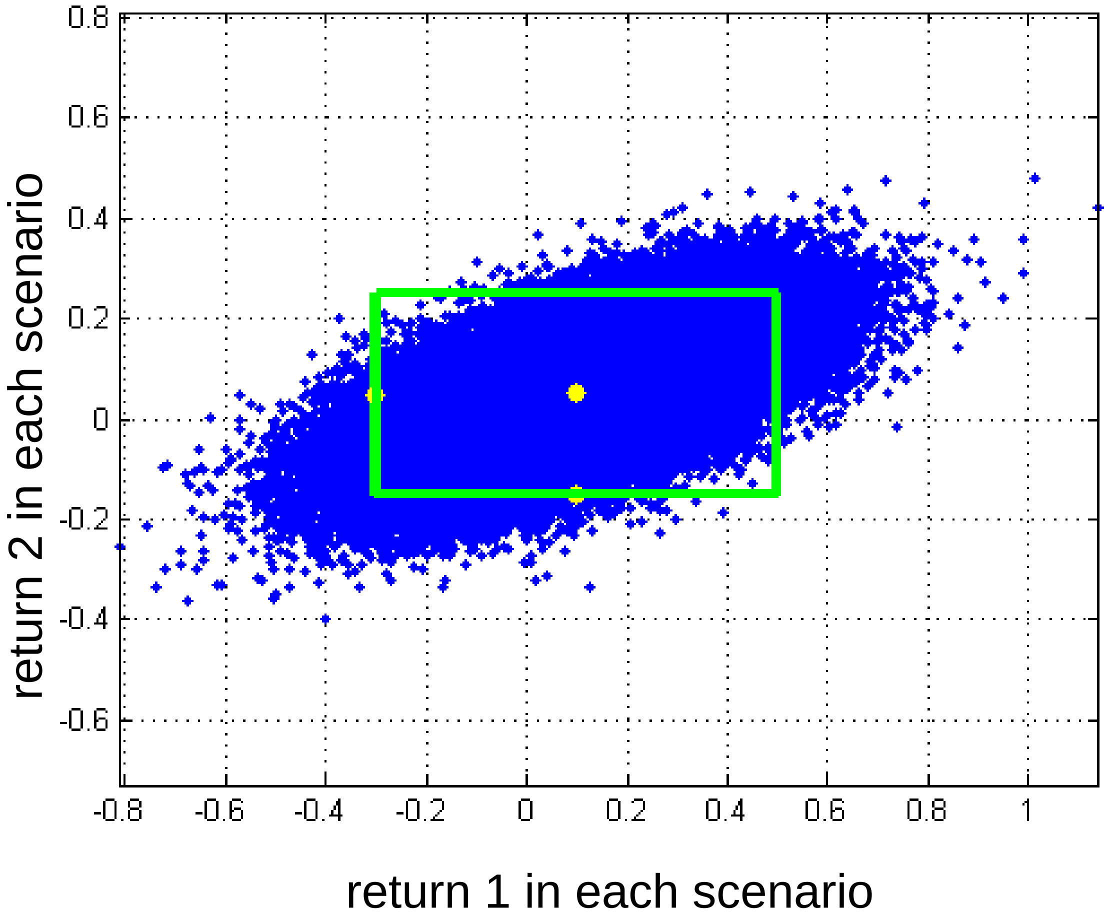


$$\boldsymbol{m} \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times 0.6 \\ 20\% \times 10\% \times 0.6 & (10\%)^2 \end{pmatrix}$$

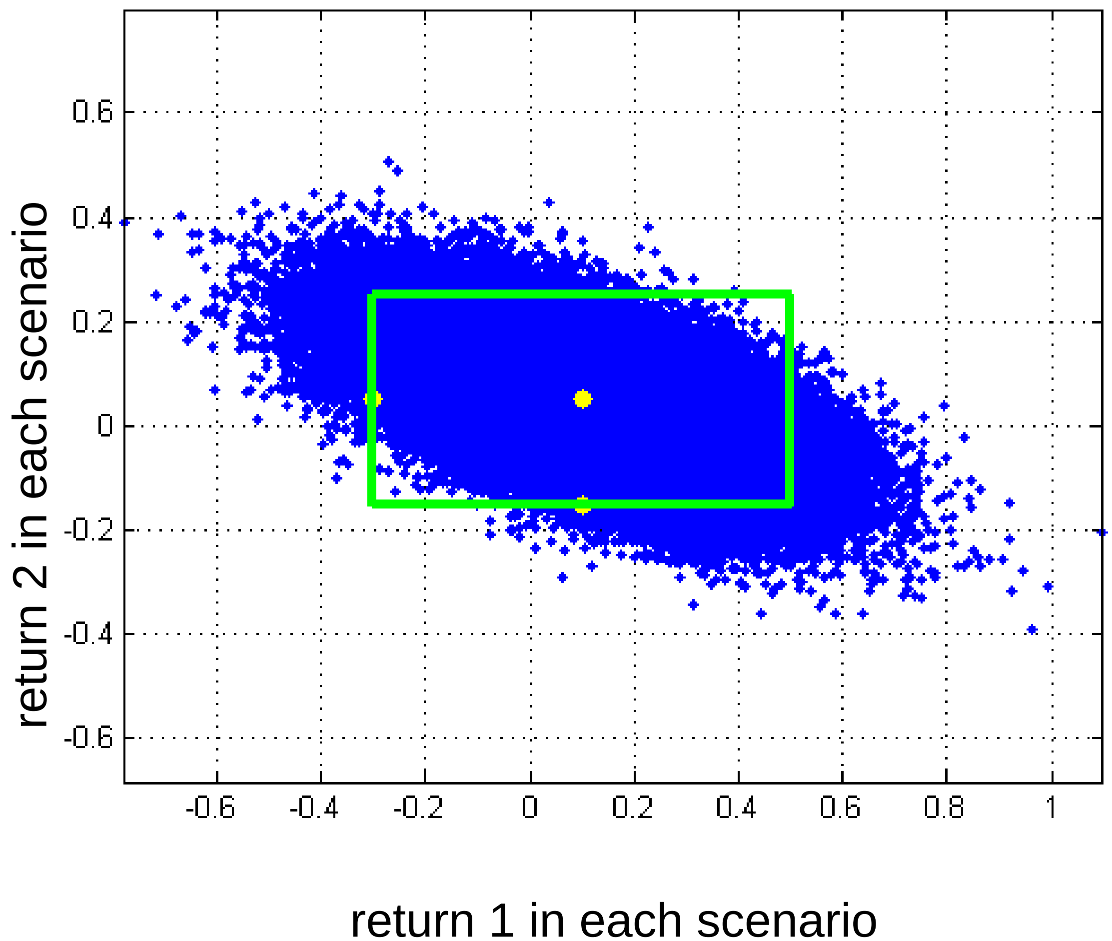
location dispersion box represents

expected value of single securities
= center of box

standard deviation of single securities
= sides of box

MARKET MODELING - LOCATION DISPERSION BOX

$\boldsymbol{R}_{t+\tau}^{\tau} \equiv \boldsymbol{P}_{t+\tau} ./ \boldsymbol{P}_t - \boldsymbol{1}$: linear returns (random variables)



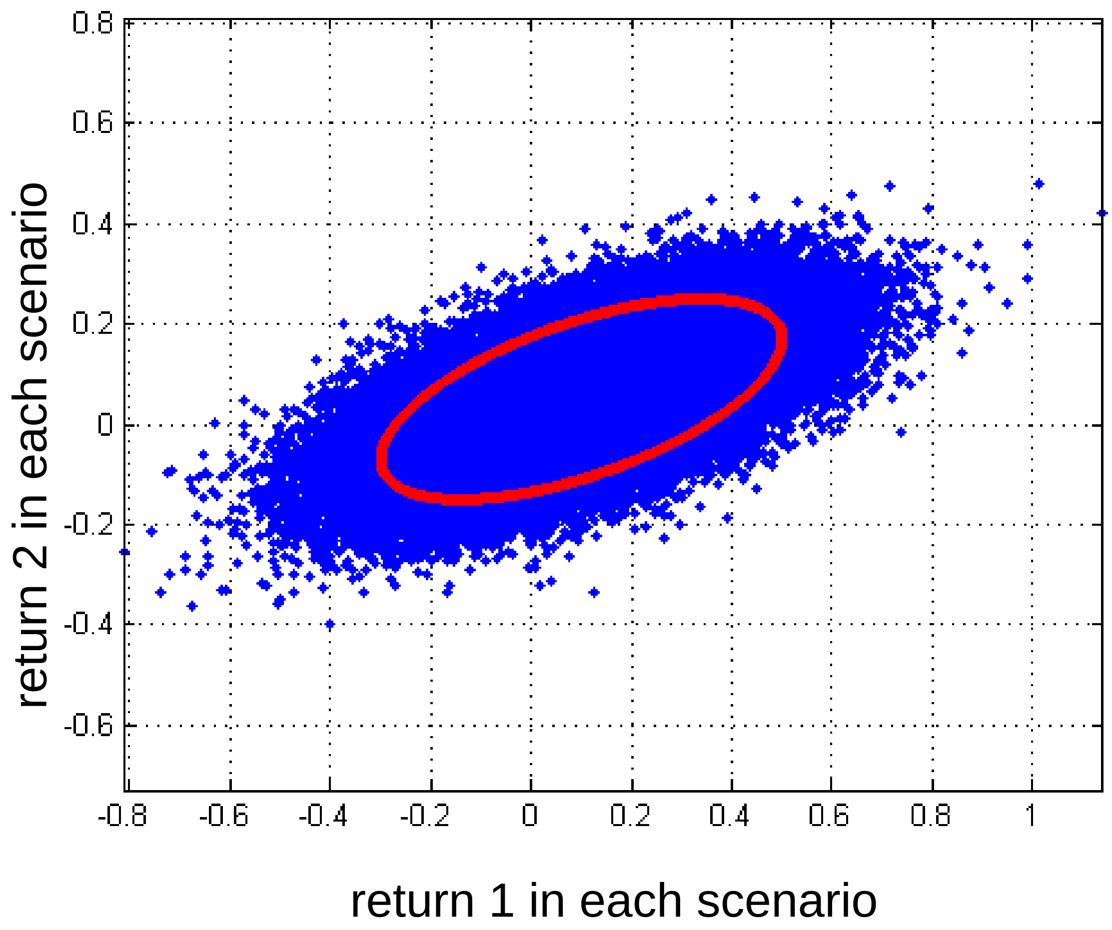
$$\boldsymbol{m} \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times -0.6 \\ 20\% \times 10\% \times -0.6 & (10\%)^2 \end{pmatrix}$$

location dispersion box does <u>not</u> represent <u>correlations</u>

MARKET MODELING - LOCATION DISPERSION ELLIPSOID

$$\boldsymbol{R}_{t+\tau}^{\tau} \equiv \boldsymbol{P}_{t+\tau} ./\, \boldsymbol{P}_t - \mathbf{1}$$: linear returns (random variables)



$$\boldsymbol{m} \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

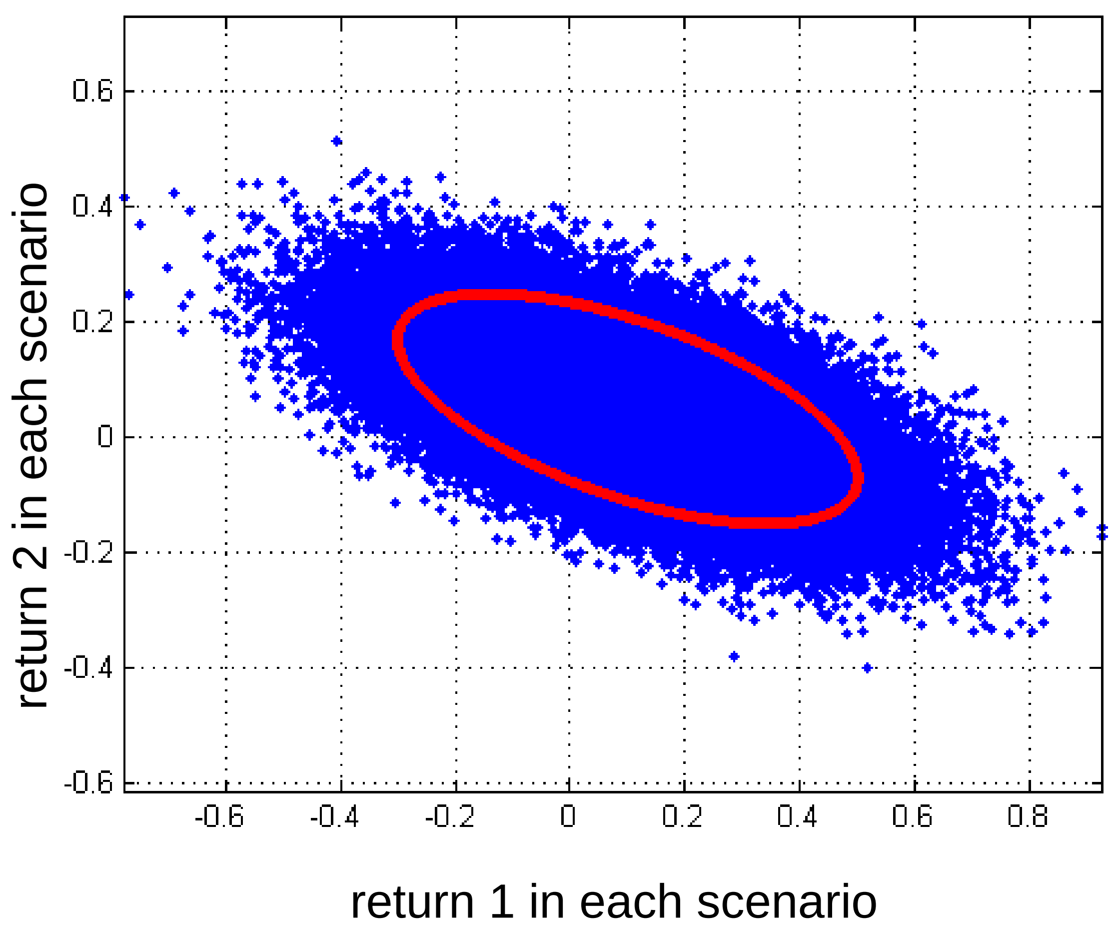
$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times 0.6 \\ 20\% \times 10\% \times 0.6 & (10\%)^2 \end{pmatrix}$$

location dispersion ellipsoid

$$(\boldsymbol{r} - \boldsymbol{m})' \boldsymbol{S}^{-1} (\boldsymbol{r} - \boldsymbol{m}) \equiv \text{constant}$$

MARKET MODELING - LOCATION DISPERSION ELLIPSOID

$R^{\tau}_{t+\tau} \equiv P_{t+\tau} ./ P_t - 1$: linear returns (random variables)



$$m \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

$$S \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times -0.6 \\ 20\% \times 10\% \times -0.6 & (10\%)^2 \end{pmatrix}$$

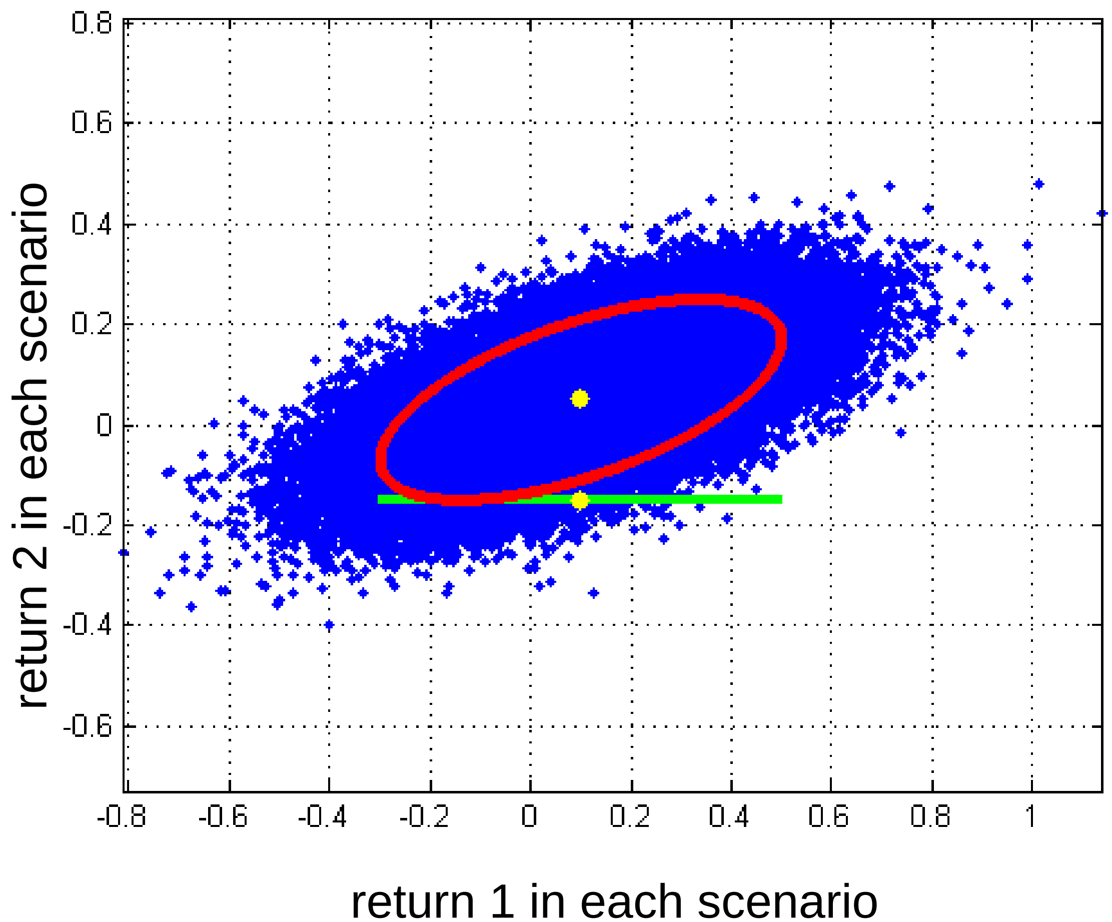
location dispersion ellipsoid

$$(r - m)' S^{-1} (r - m) \equiv \text{constant}$$

represents correlation

MARKET MODELING - LOCATION DISPERSION ELLIPSOID

$\boldsymbol{R}^{\tau}_{t+\tau} \equiv \boldsymbol{P}_{t+\tau}./\boldsymbol{P}_t - \boldsymbol{1}$: linear returns (random variables)



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$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times 0.6 \\ 20\% \times 10\% \times 0.6 & (10\%)^2 \end{pmatrix}$$

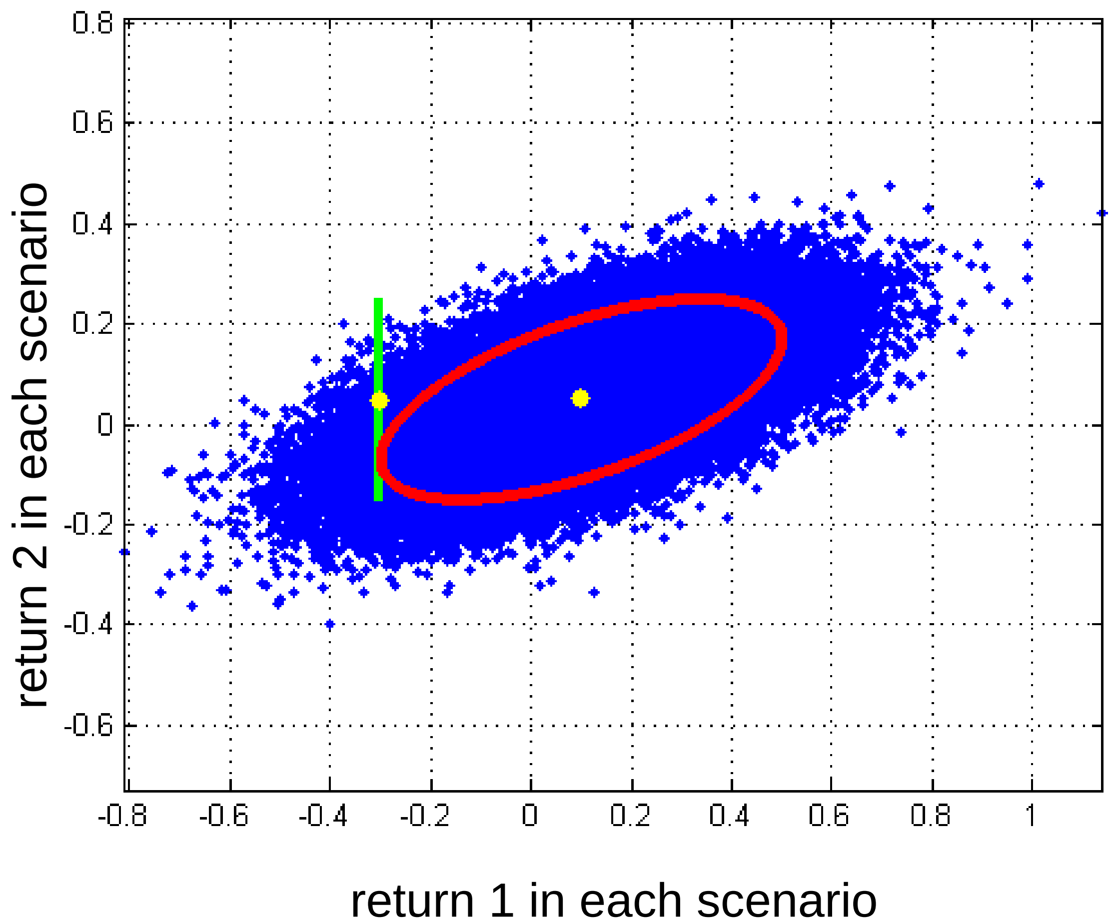
location dispersion ellipsoid

$$(\boldsymbol{r} - \boldsymbol{m})' \boldsymbol{S}^{-1} (\boldsymbol{r} - \boldsymbol{m}) \equiv \text{constant}$$

includes location-dispersion
bar information…

MARKET MODELING - LOCATION DISPERSION ELLIPSOID

$$\boldsymbol{R}^{\tau}_{t+\tau} \equiv \boldsymbol{P}_{t+\tau} ./ \boldsymbol{P}_{t} - \mathbf{1}$$: linear returns (random variables)



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$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times 0.6 \\ 20\% \times 10\% \times 0.6 & (10\%)^2 \end{pmatrix}$$

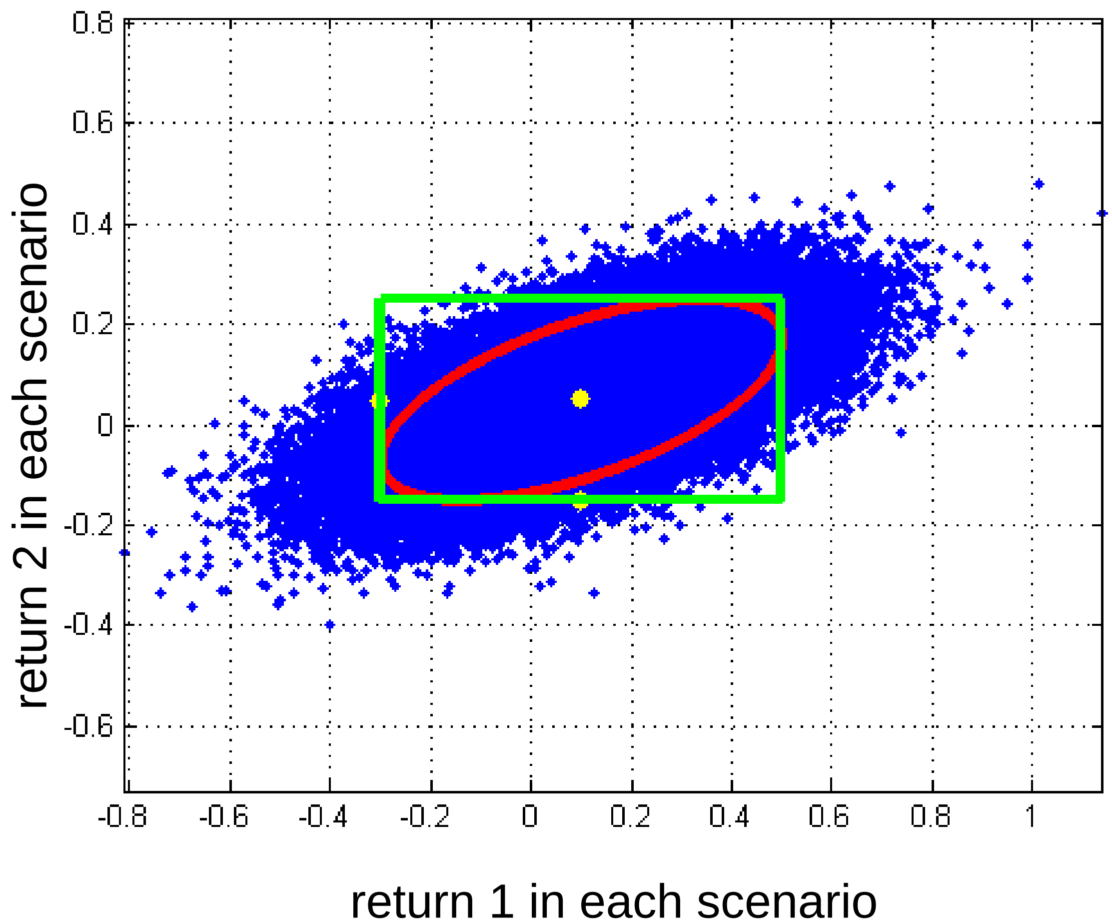
location dispersion ellipsoid

$$(\boldsymbol{r} - \boldsymbol{m})' \boldsymbol{S}^{-1} (\boldsymbol{r} - \boldsymbol{m}) \equiv \text{constant}$$

…includes location-dispersion bar information…

MARKET MODELING - LOCATION DISPERSION ELLIPSOID

$\boldsymbol{R}_{t+\tau}^{\tau} \equiv \boldsymbol{P}_{t+\tau} ./ \boldsymbol{P}_{t} - \mathbf{1}$: linear returns (random variables)



$$\boldsymbol{m} \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times 0.6 \\ 20\% \times 10\% \times 0.6 & (10\%)^2 \end{pmatrix}$$

location dispersion ellipsoid

$$(\boldsymbol{r} - \boldsymbol{m})' \boldsymbol{S}^{-1} (\boldsymbol{r} - \boldsymbol{m}) \equiv \text{constant}$$

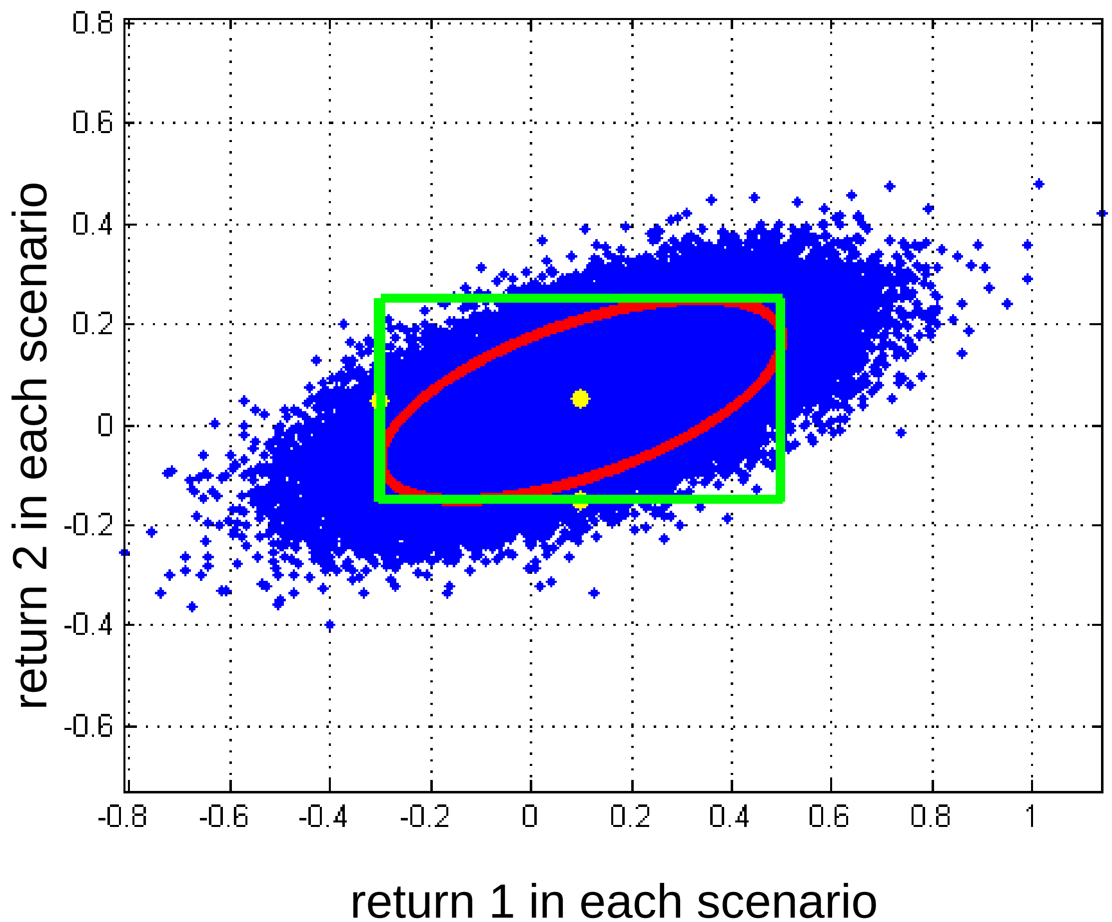
includes box information:

location-dispersion box is the only box that enshrouds the ellipsoid

MARKET MODELING - LOCATION DISPERSION ELLIPSOID

$\boldsymbol{R}^{\tau}_{t+\tau} \equiv \boldsymbol{P}_{t+\tau} ./ \boldsymbol{P}_t - \mathbf{1}$: linear returns (random variables)

example



$$\boldsymbol{m} \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times 0.6 \\ 20\% \times 10\% \times 0.6 & (10\%)^2 \end{pmatrix}$$

location dispersion ellipsoid

$$(\boldsymbol{r} - \boldsymbol{m})' \boldsymbol{S}^{-1} (\boldsymbol{r} - \boldsymbol{m}) \equiv \text{constant}$$

represents exp. value and covariance

expected value = center

covariance = shape and orientation

MARKET MODELING - LOCATION DISPERSION ELLIPSOID NON-NORMAL

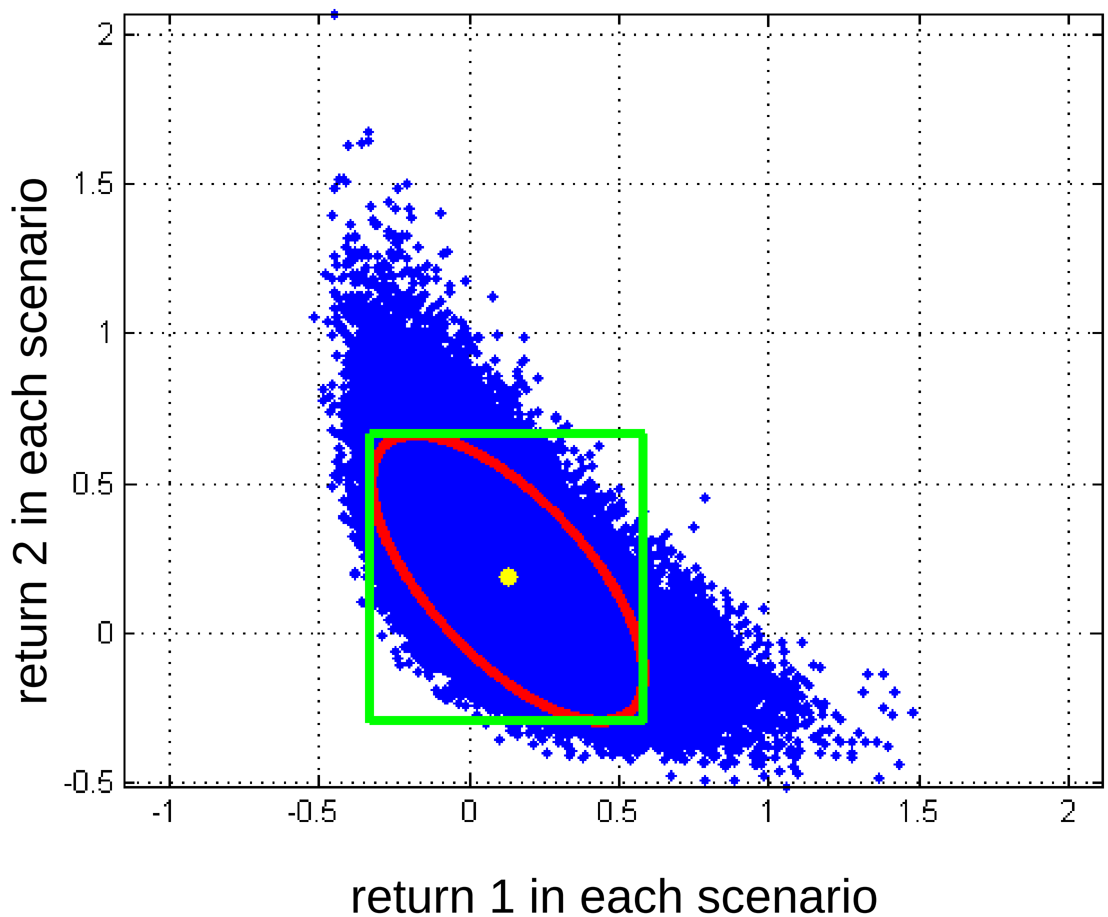
$\boldsymbol{R}_{t+\tau}^{\tau} \equiv \boldsymbol{P}_{t+\tau} . / \boldsymbol{P}_{t} - \mathbf{1}$: linear returns

example (Black-Scholes)

$$\ln\left(\boldsymbol{P}_{t+\tau} . / \boldsymbol{P}_{t}\right) \sim \mathrm{N}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right)$$

$$m_n = e^{\mu_n + \frac{1}{2}\Sigma_{nn}} - 1$$

$$S_{nm} = e^{\mu_n + \frac{1}{2}\Sigma_{nn} + \mu_m + \frac{1}{2}\Sigma_{mm}} \left(e^{\Sigma_{nm}} - 1\right)$$



location dispersion ellipsoid

$$\left(\boldsymbol{r} - \boldsymbol{m}\right)' \boldsymbol{S}^{-1} \left(\boldsymbol{r} - \boldsymbol{m}\right) \equiv \text{constant}$$

represents exp. value and covariance

<u>expected value</u> = center

<u>covariance</u> = shape and orientation

MARKET MODELING - PRINCIPAL COMPONENT ANALYSIS

example

$$\boldsymbol{R}^{\tau}_{t+\tau} \equiv \boldsymbol{P}_{t+\tau} ./ \boldsymbol{P}_t - \mathbf{1}$$: linear returns (random variables)

$$\boldsymbol{m} \equiv \mathrm{E}\left\{ \boldsymbol{R}^{\tau}_{t+\tau} \right\}$$: expected returns

$$\boldsymbol{m} \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

$$\boldsymbol{S} \equiv \mathrm{Cov}\left\{ \boldsymbol{R}^{\tau}_{t+\tau} \right\}$$: covariance of returns

$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times 0.6 \\ 20\% \times 10\% \times 0.6 & (10\%)^2 \end{pmatrix}$$

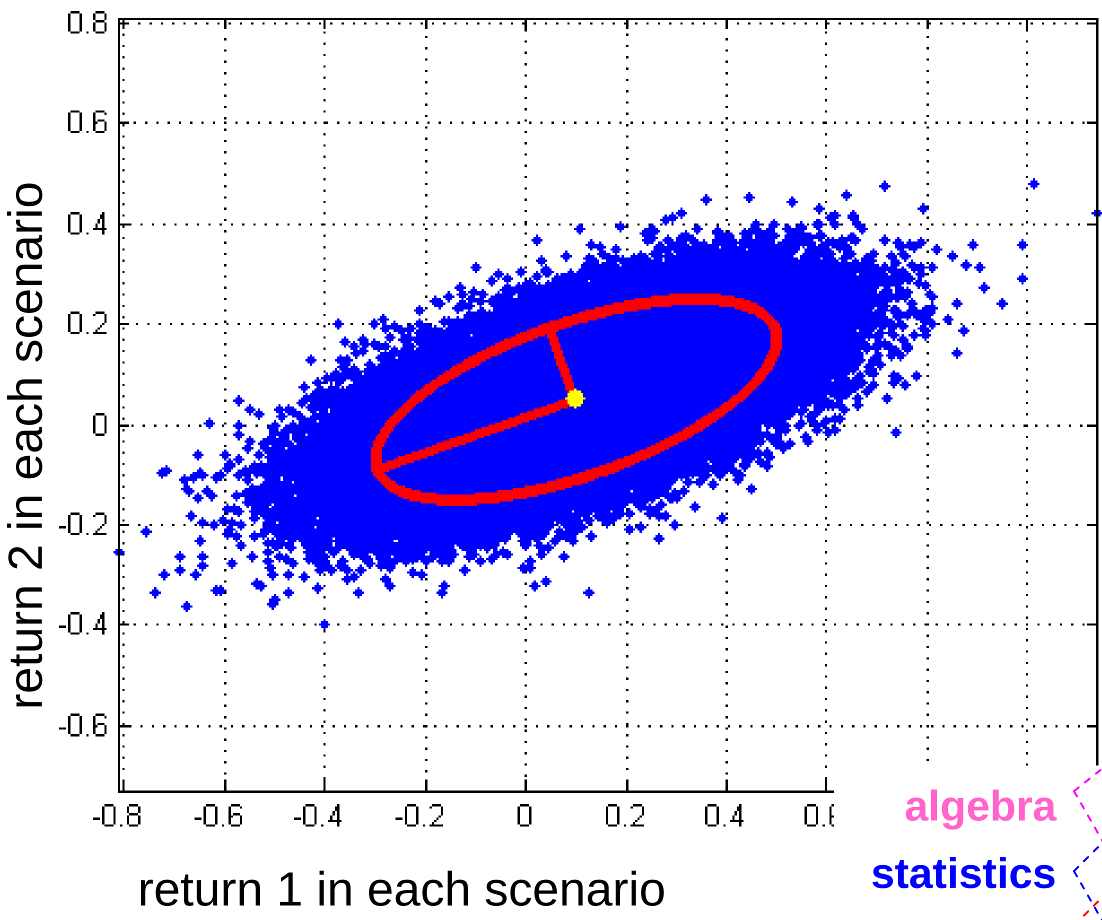
$$\boldsymbol{S} \equiv \boldsymbol{E} \boldsymbol{\Lambda} \boldsymbol{E}'$$: principal component decomposition

orthogonal eigenvectors (columns)	positive eigenvalues (diagonal)	algebra
⇕	⇕	
uncorrelated directions	variance in uncorrelated directions	statistics

MARKET MODELING - LOCATION DISPERSION ELLIPSOID AND PCA

$\boldsymbol{R}_{t+\tau}^{\tau} \equiv \boldsymbol{P}_{t+\tau} ./ \boldsymbol{P}_{t} - \mathbf{1}$: linear returns (random variables)

example



$$\boldsymbol{m} \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times 0.6 \\ 20\% \times 10\% \times 0.6 & (10\%)^2 \end{pmatrix}$$

location dispersion ellipsoid

$$(\boldsymbol{r} - \boldsymbol{m})' \boldsymbol{S}^{-1} (\boldsymbol{r} - \boldsymbol{m}) \equiv \text{constant}$$

represents PCA:

algebra

statistics

geometry

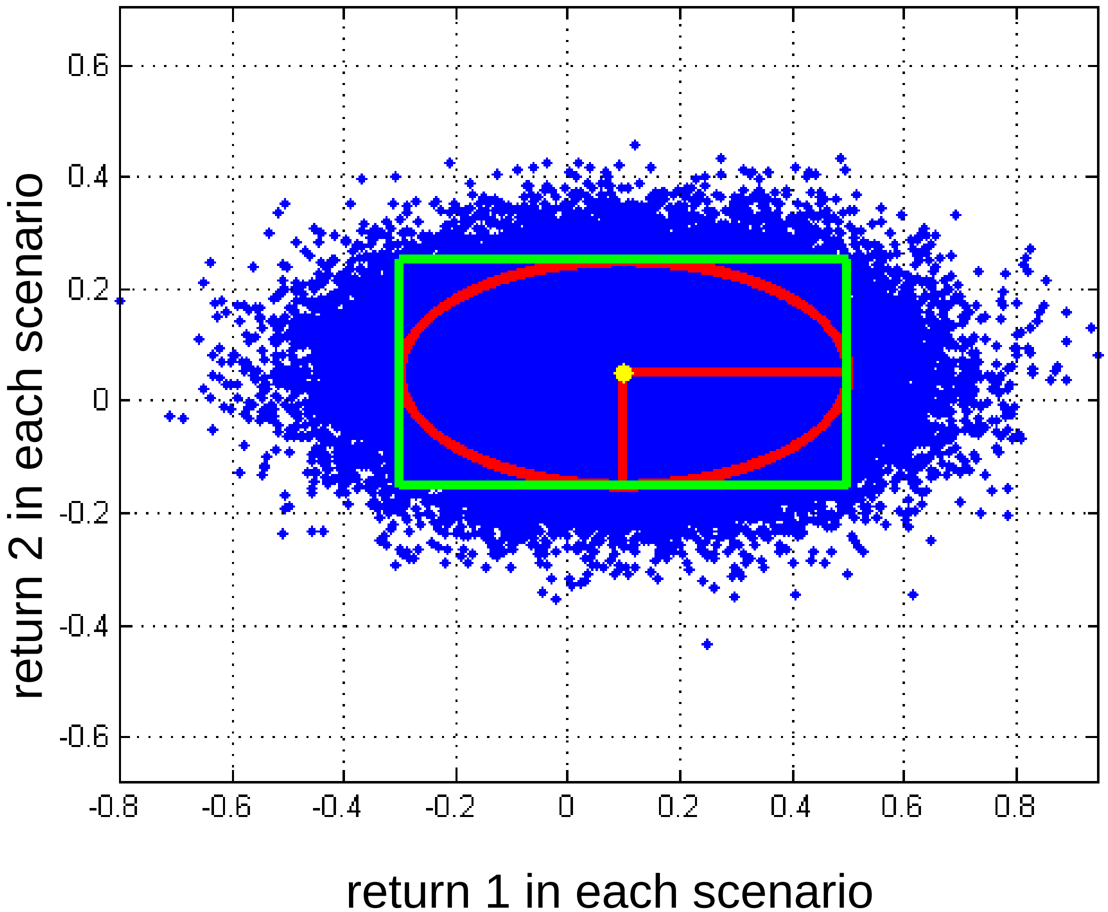
orthogonal eigenvectors =
uncorrelated directions =
<u>directions</u> of principal axes

square root of eigenvalues =
volatility in uncorrelated directions =
<u>length</u> of principal axes

MARKET MODELING - NO CORRELATION

$\boldsymbol{R}^{\tau}_{t+\tau} \equiv \boldsymbol{P}_{t+\tau} ./ \boldsymbol{P}_{t} - \mathbf{1}$: linear returns (random variables)

example



$$\boldsymbol{m} \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

$$\boldsymbol{S} \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times \mathbf{0} \\ 20\% \times 10\% \times \mathbf{0} & (10\%)^2 \end{pmatrix}$$

location dispersion ellipsoid

$$(\boldsymbol{r} - \boldsymbol{m})' \boldsymbol{S}^{-1} (\boldsymbol{r} - \boldsymbol{m}) \equiv \text{constant}$$

represents PCA:

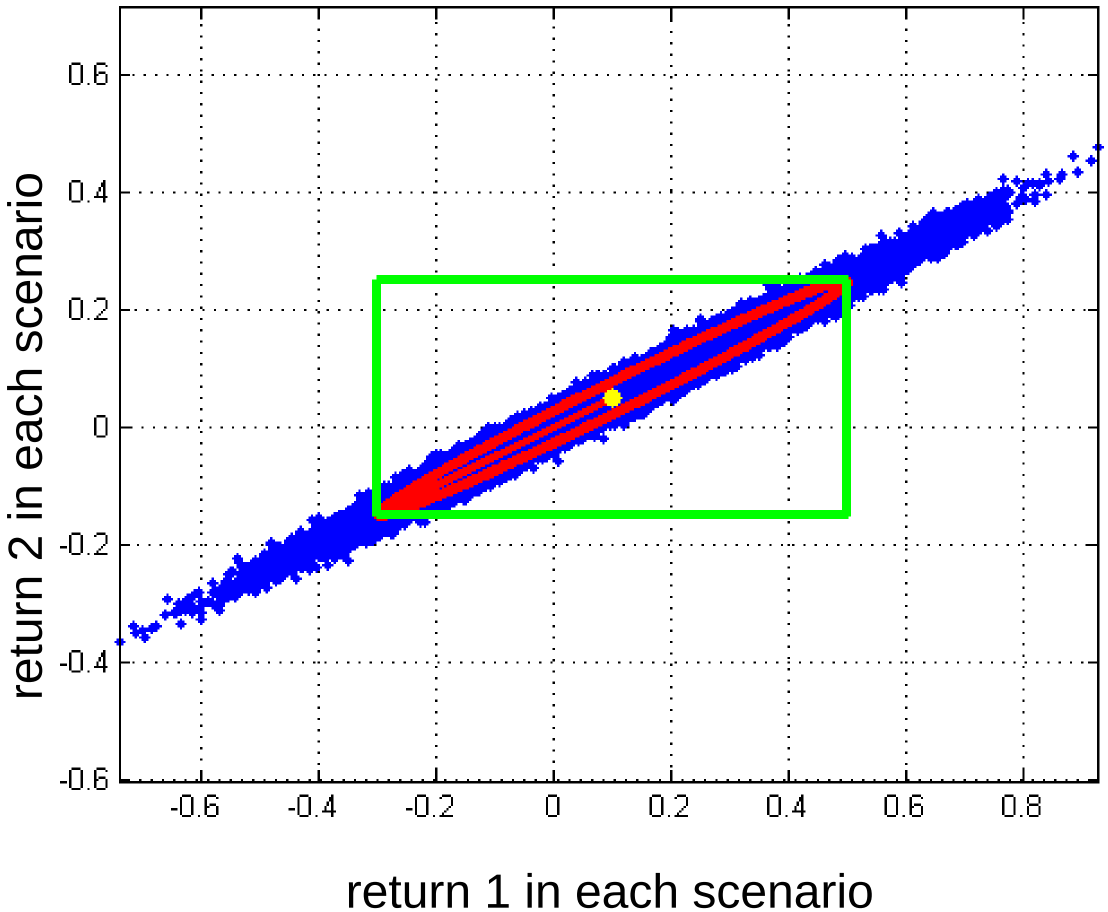
orthogonal eigenvectors =
uncorrelated directions =
<u>directions</u> of principal axes

square root of eigenvalues =
volatility in uncorrelated directions =
<u>length</u> of principal axes

MARKET MODELING - ILL-CONDITIONED MARKETS

$R^{\tau}_{t+\tau} \equiv P_{t+\tau} ./ P_t - 1$: linear returns (random variables)

example



$$m \equiv \begin{pmatrix} 10\% \\ 5\% \end{pmatrix}$$

$$S \equiv \begin{pmatrix} (20\%)^2 & 20\% \times 10\% \times \mathbf{0.99} \\ 20\% \times 10\% \times \mathbf{0.99} & (10\%)^2 \end{pmatrix}$$

location dispersion ellipsoid

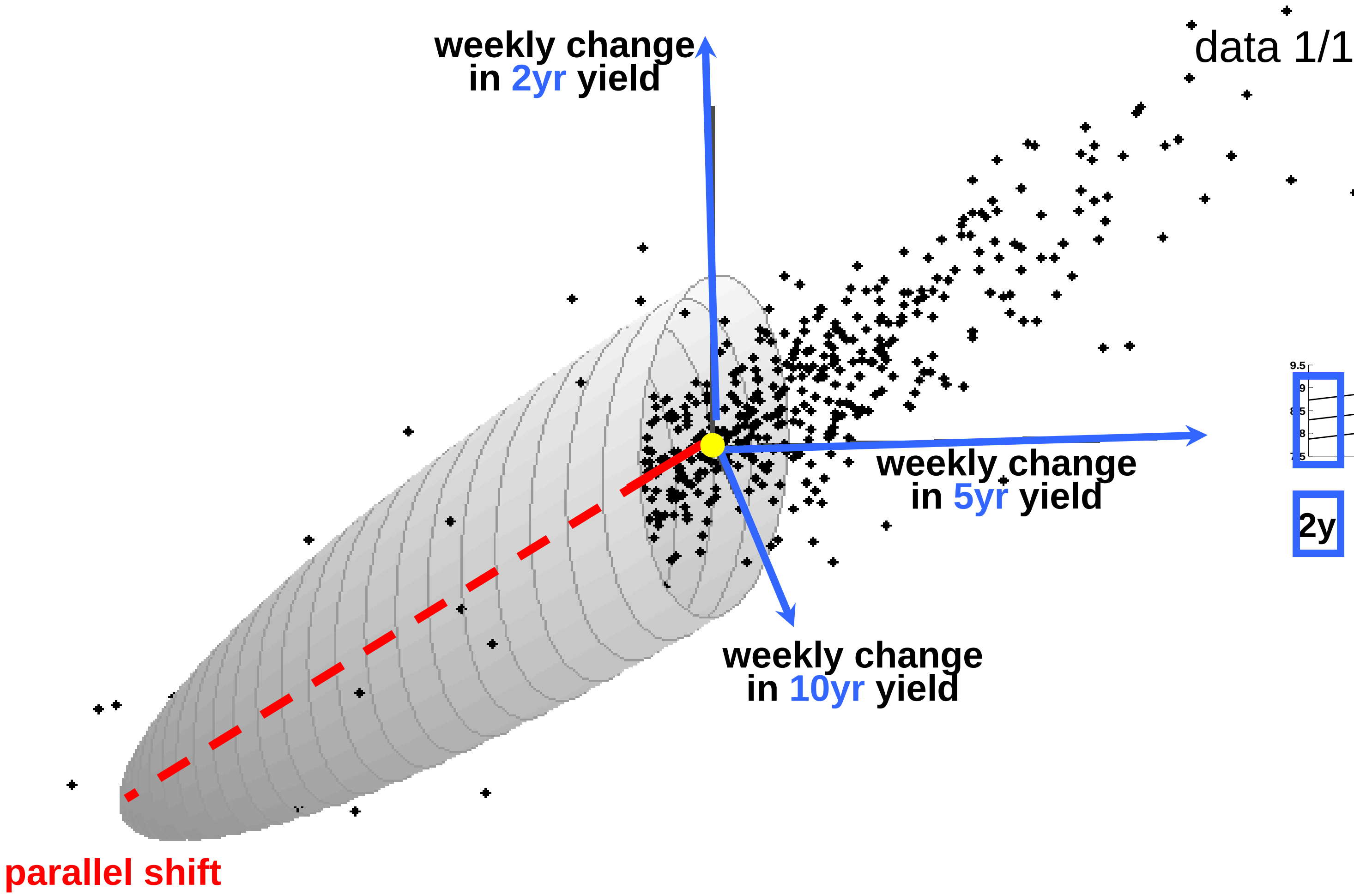
$$(r - m)' S^{-1} (r - m) \equiv \text{constant}$$

represents PCA:

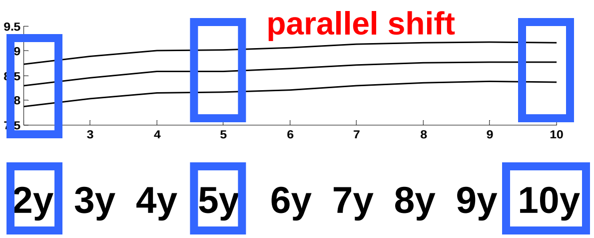
orthogonal eigenvectors =
uncorrelated directions =
<u>directions</u> of principal axes

square root of eigenvalues =
volatility in uncorrelated directions =
<u>length</u> of principal axes

MARKET MODELING - SWAP PCA

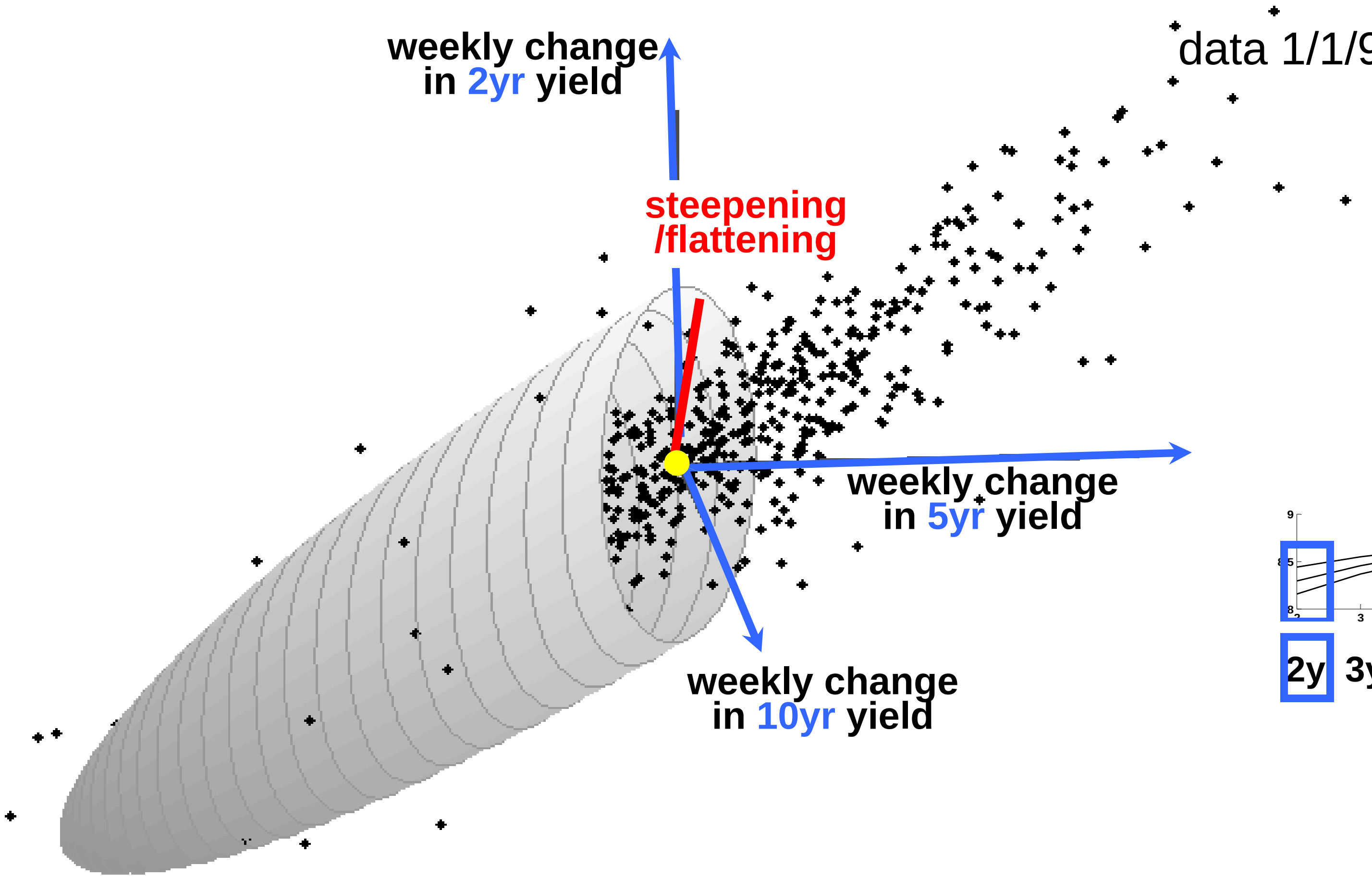


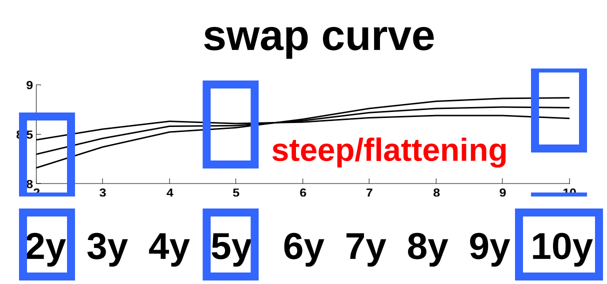
swap curve



parallel shift dominates the movements of the swap curve

MARKET MODELING - SWAP PCA

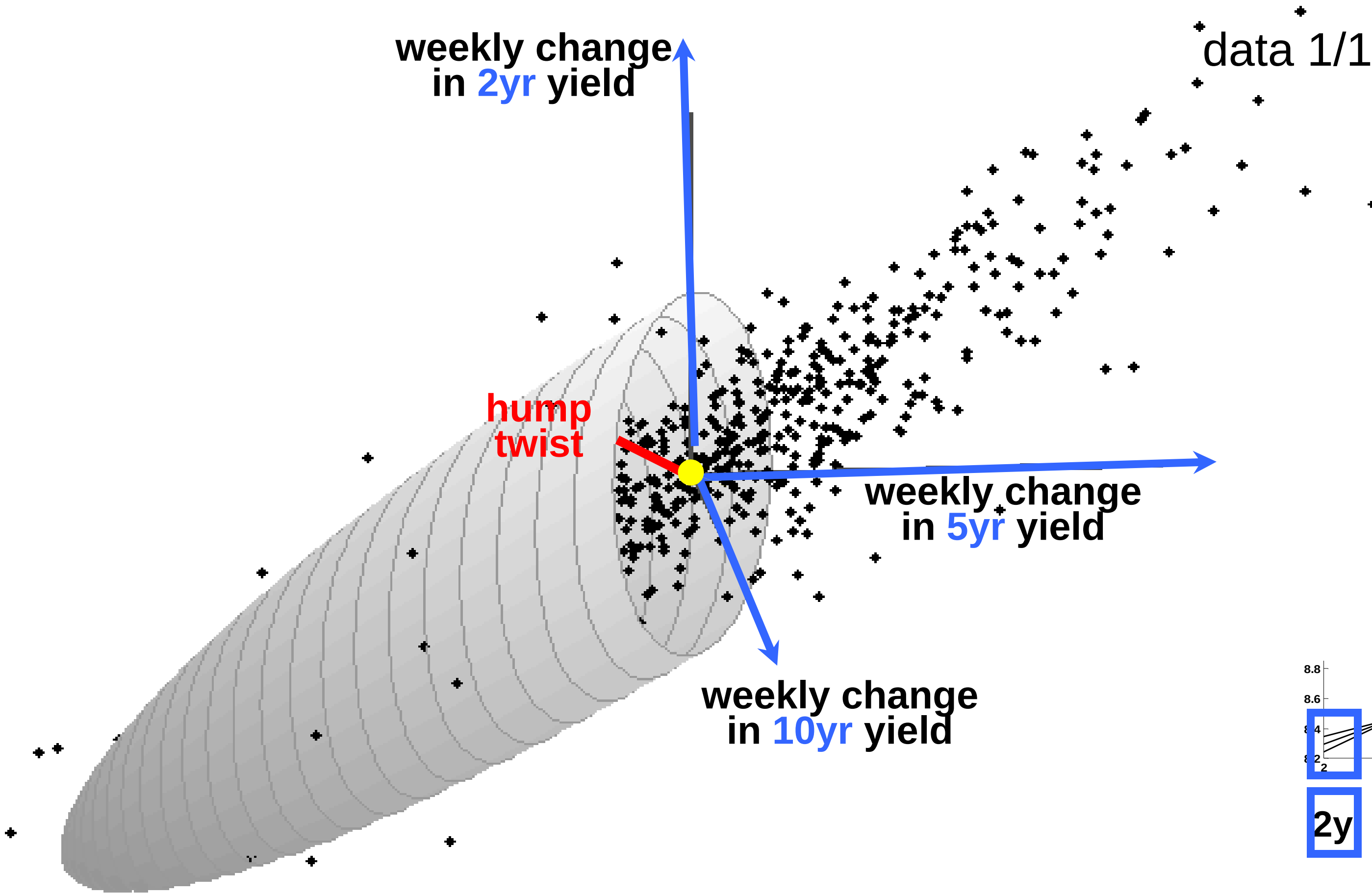




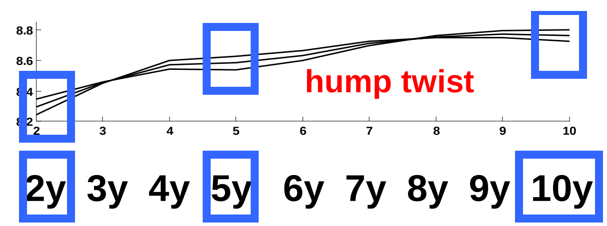
parallel shift dominates the movements of the swap curve

MARKET MODELING - SWAP PCA

data 1/1/90 – 12/31/02

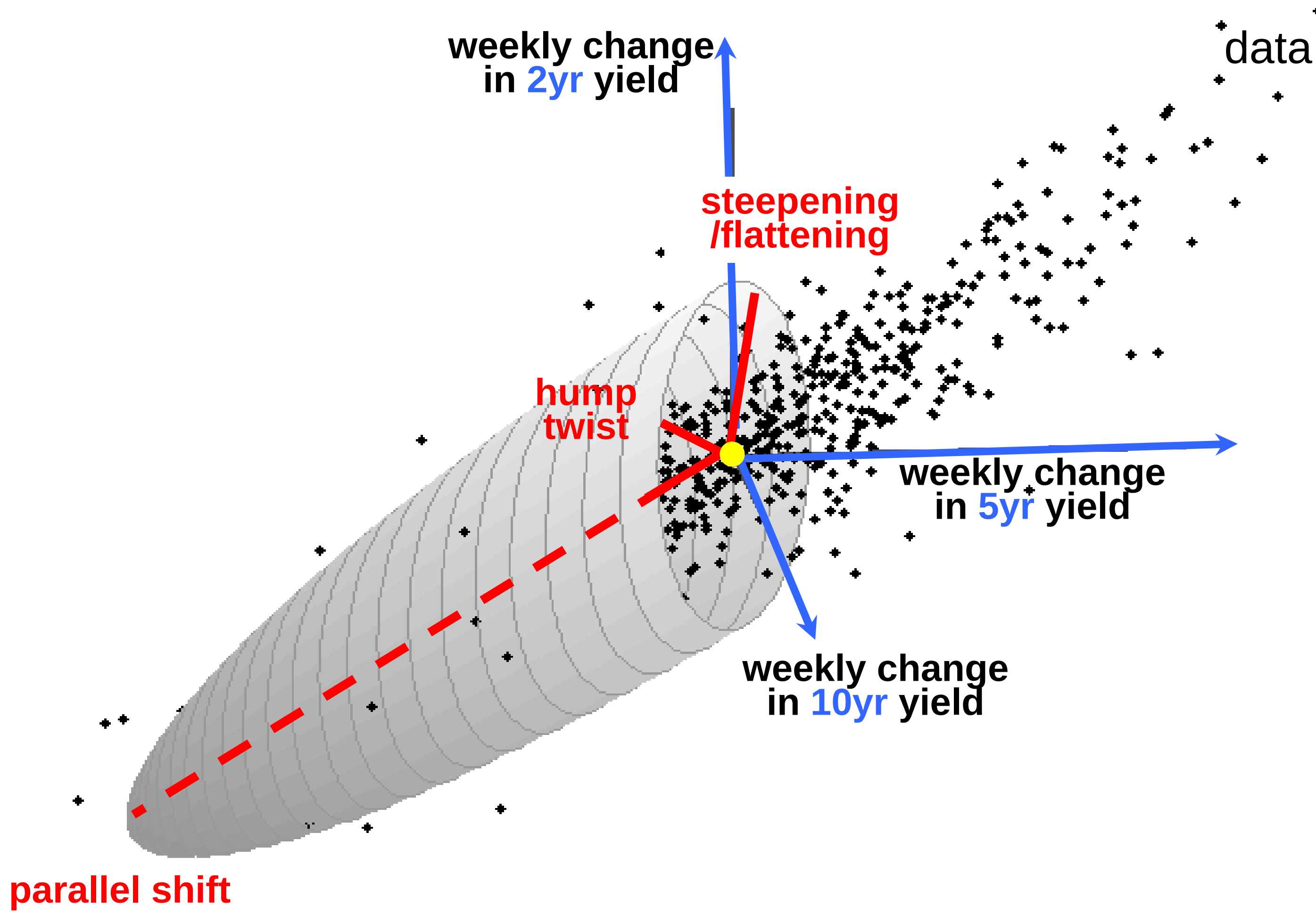


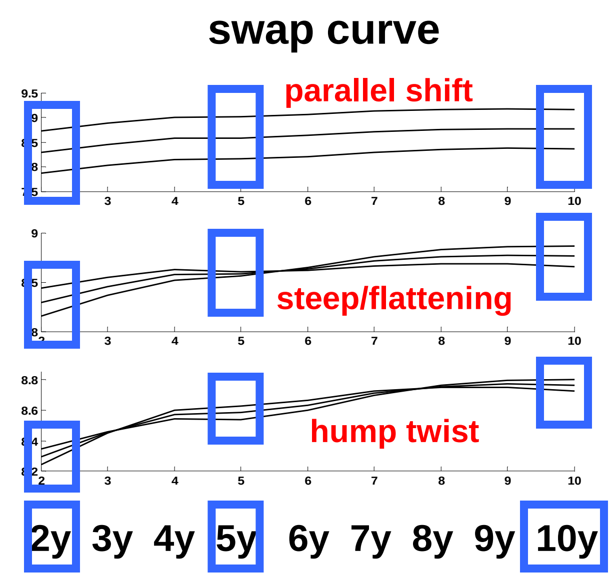
swap curve



parallel shift dominates the movements of the swap curve

MARKET MODELING - SWAP PCA





parallel shift dominates the movements of the swap curve

AGENDA

PORTFOLIO MODELING - mean variance and representations

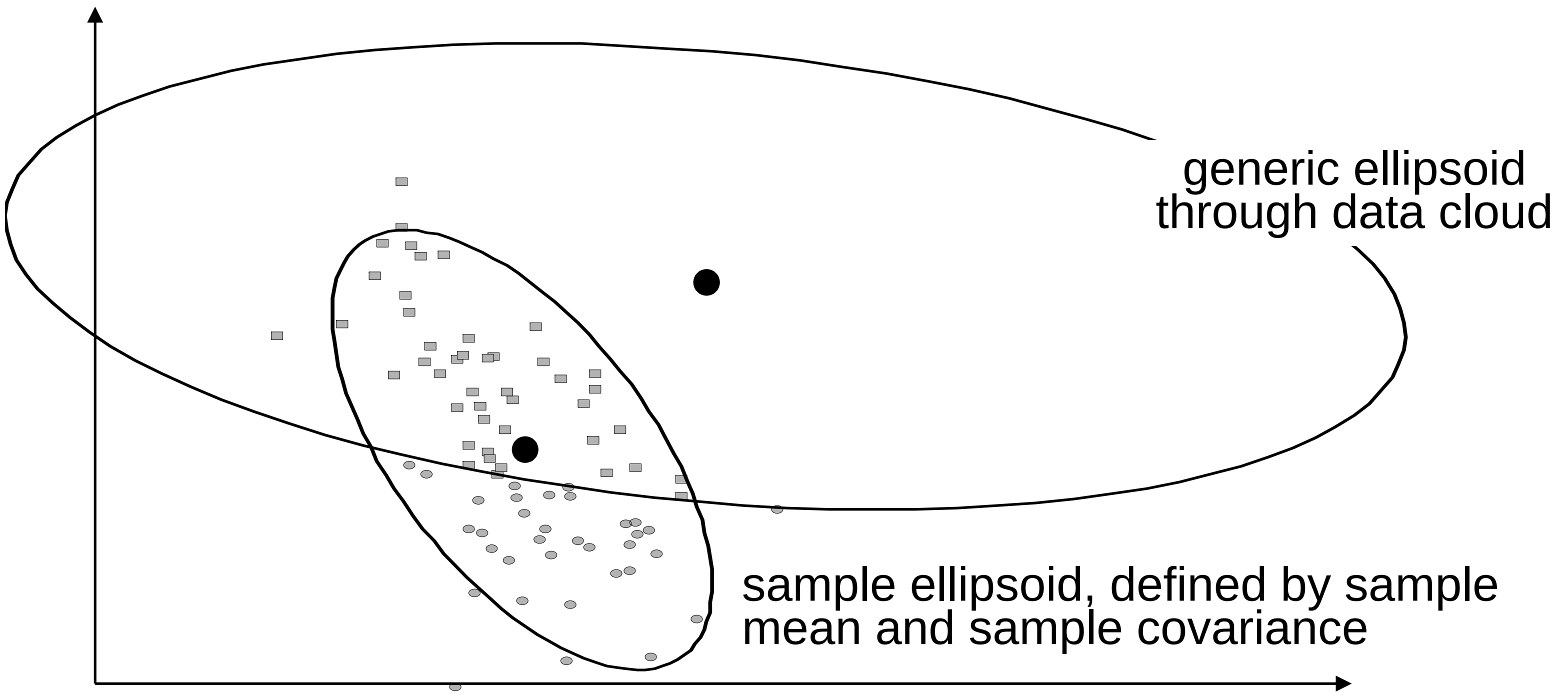
MARKET MODELING - location dispersion ellipsoid and PCA

ESTIMATION - non parametric, ML, robust, shrinkage, Bayesian

REFERENCES

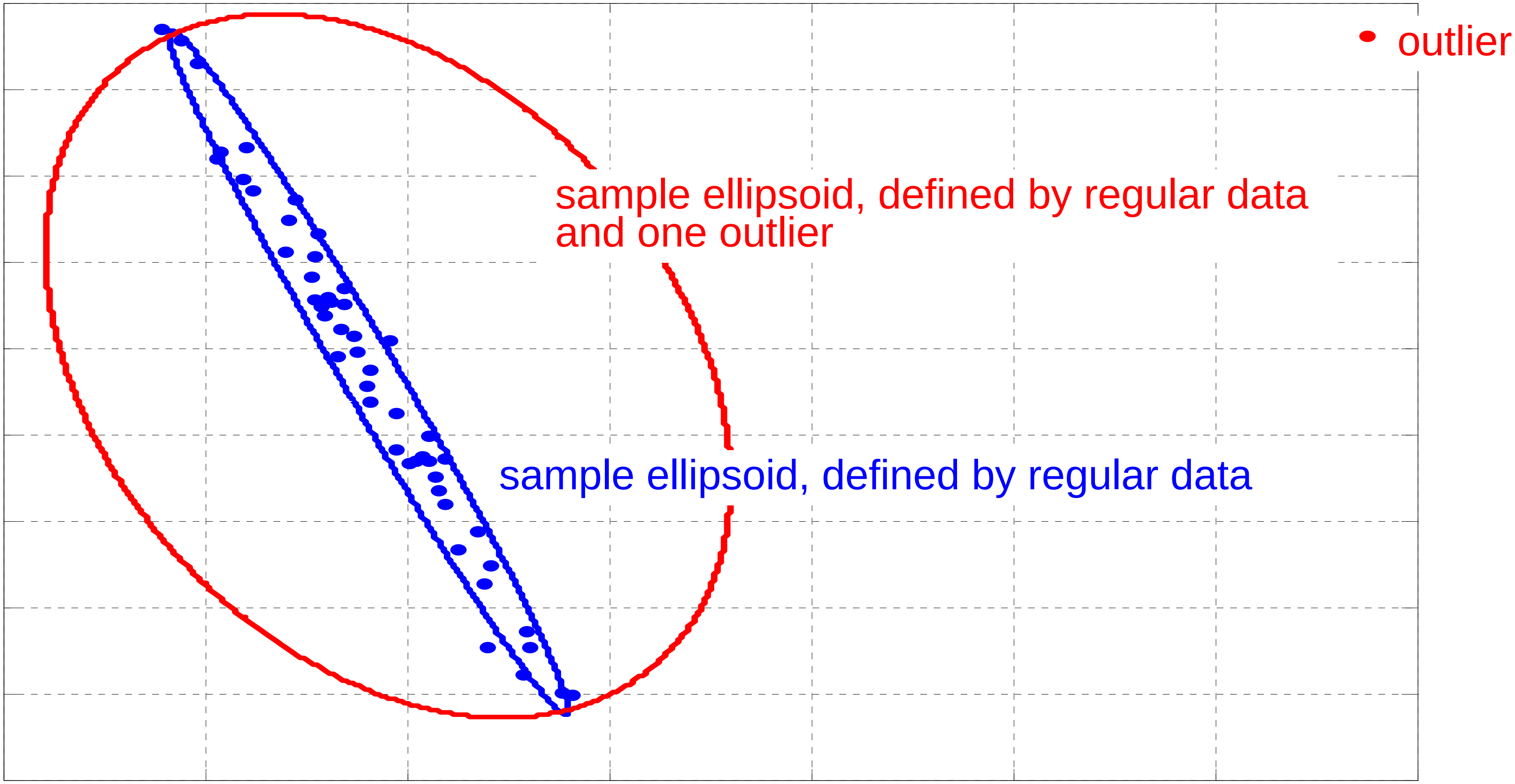
ESTIMATION - NON PARAMETRIC

sample mean: $\widehat{\boldsymbol{m}} \equiv \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{r}_t$ sample covariance: $\widehat{\boldsymbol{S}} \equiv \frac{1}{T}\sum_{t=1}^{T}\left(\boldsymbol{r}_t - \widehat{\boldsymbol{m}}\right)\left(\boldsymbol{r}_t - \widehat{\boldsymbol{m}}\right)'$



the sample ellipsoid is the smallest ellipsoid through the data cloud (i.e. average Mahalanobis distance =1)

ESTIMATION - NON PARAMETRIC



sample ellipsoid tries “too hard” to fit the data

$\Longrightarrow$ sample mean and covariance are **not robust**

ESTIMATION - ROBUSTNESS MEASURES: INFLUENCE FUNCTION

generic estimator: $\widehat{\boldsymbol{G}} \equiv \widehat{\boldsymbol{G}}\left(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_T\right)$ **information = time series**

sensitivity curve: $\mathrm{SC}\left(\boldsymbol{r}^*, \widehat{\boldsymbol{G}}\right) \equiv T\left[\widehat{\boldsymbol{G}}\left(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_T, \boldsymbol{r}^*\right) - \widehat{\boldsymbol{G}}\left(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_T\right)\right]$

normalization **artificial outlier**

$\Downarrow$

influence function: $\mathrm{IF}\left(\boldsymbol{r}^*, f_{\boldsymbol{r}}, \widehat{\boldsymbol{G}}\right) \equiv \lim_{T \to \infty} \mathrm{SC}\left(\boldsymbol{r}^*, \widehat{\boldsymbol{G}}\right)$

example

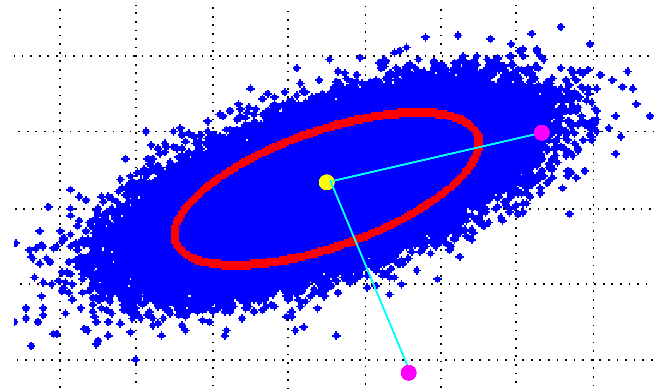
$$\widehat{\boldsymbol{m}} \equiv \frac{1}{T} \sum_{t=1}^{T} \boldsymbol{r}_t$$

$$\mathrm{IF}\left(\boldsymbol{r}^*, f_{\boldsymbol{r}}, \widehat{\boldsymbol{m}}\right) = \boldsymbol{r}^* - \mathrm{E}\left\{\boldsymbol{R}\right\}$$

not bounded

the influence function measures the marginal robustness to one outlier in the limit of infinite information

ESTIMATION - MAXIMUM LIKELIHOOD (HEAVY TAILS)



MLE mean: $\widehat{\boldsymbol{m}} \equiv \frac{1}{\sum_{s=1}^{T} w_s} \sum_{t=1}^{T} w_t \boldsymbol{r}_t$

MLE scatter: $\widehat{\boldsymbol{S}} \equiv \frac{1}{T} \sum_{t=1}^{T} w_t \left(\boldsymbol{r}_t - \widehat{\boldsymbol{m}}\right)\left(\boldsymbol{r}_t - \widehat{\boldsymbol{m}}\right)'$

MLE (Cauchy) weights: $w_t \equiv \frac{\text{constant}}{1 + \boxed{\left(\boldsymbol{r}_t - \widehat{\boldsymbol{m}}\right) \widehat{\boldsymbol{S}}^{-1} \left(\boldsymbol{r}_t - \widehat{\boldsymbol{m}}\right)'}}$

(square) <u>Mahalanobis distance</u>:

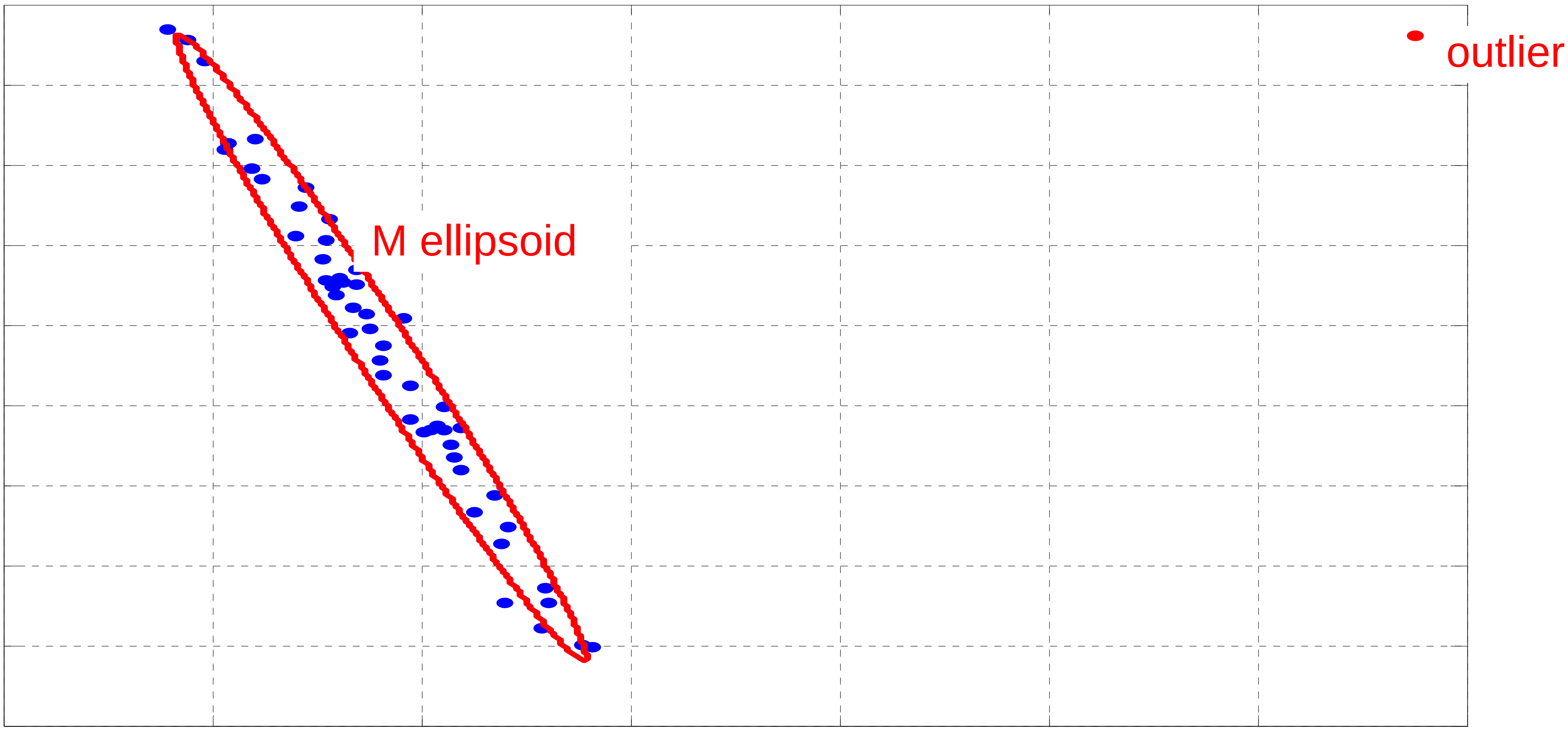
volatility- and correlation-weighted distance of observation from mean

MLE ellipsoid under heavy tailed assumption demotes outliers

$\Longrightarrow$ influence function is bounded

$\Longrightarrow$ respective mean and scatter matrix are **<u>robust</u>**

ESTIMATION - M ESTIMATORS



M ellipsoid demotes outliers without assumption on the distribution

$\Longrightarrow$ influence function is bounded

$\Longrightarrow$ respective mean and scatter matrix are **robust**

ESTIMATION - ROBUSTNESS MEASURES: BREAKDOWN POINT

examples

generic estimator: $\widehat{\boldsymbol{G}} \equiv \widehat{\boldsymbol{G}}\left(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_T\right)$ **information = time series**

mean $\widehat{\boldsymbol{m}} \equiv \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{r}_t$

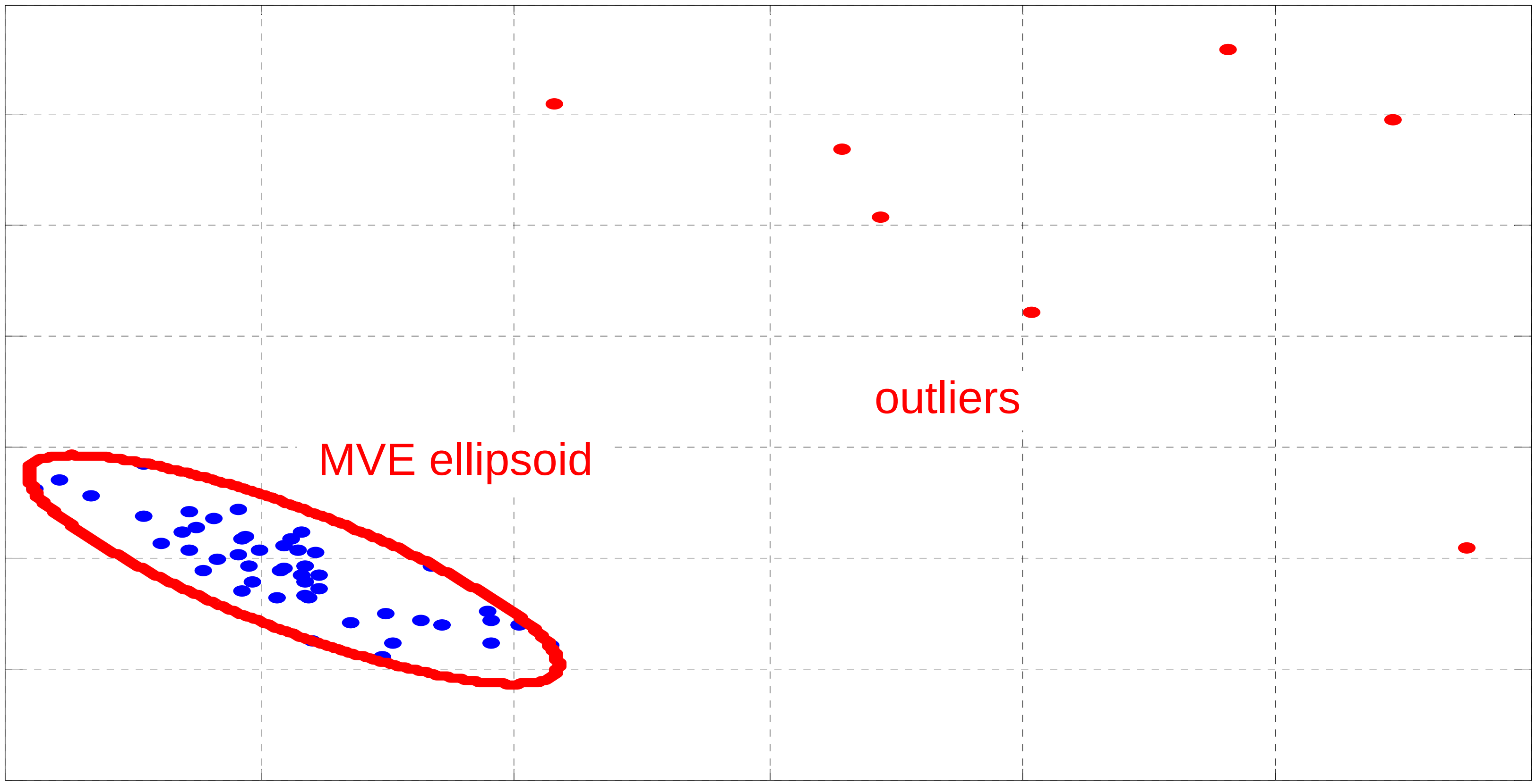
median $\widehat{\boldsymbol{\mu}} \equiv \boldsymbol{r}_{\frac{T}{2}:T}$

breakdown point: $\mathrm{BP}\left(\widehat{\boldsymbol{G}}\right) \equiv$ maximum percentage of data that can be changed without distorting the estimation

$\mathrm{BP}\left(\widehat{\boldsymbol{m}}\right) = 0$ $\mathrm{BP}\left(\widehat{\boldsymbol{\mu}}\right) = \frac{1}{2}$

the breakdown point measures the global robustness to outliers

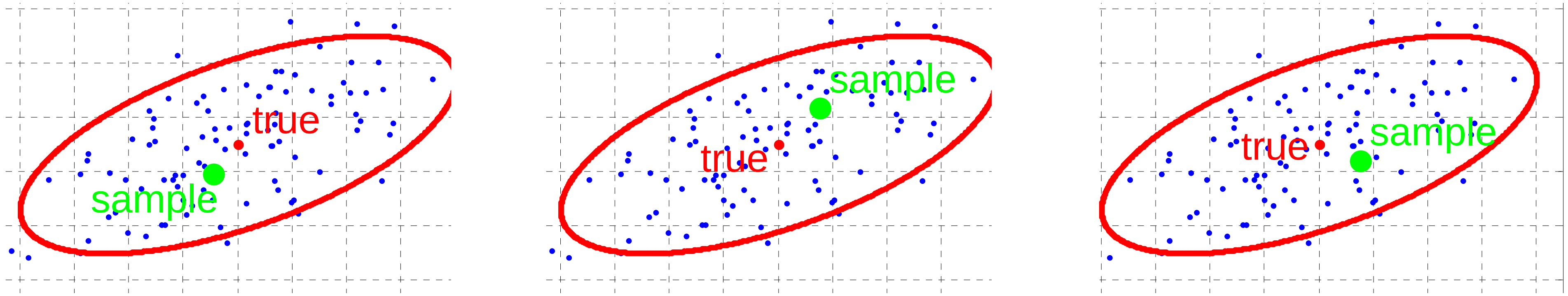
ESTIMATION - HIGH BREAKDOWN ESTIMATORS



like the median, the Minimum Volume Ellipsoid neglects outliers

$\Rightarrow$ MVE-mean and MVE-scatter are **robust**

ESTIMATION – SHRINKAGE OF LOCATION



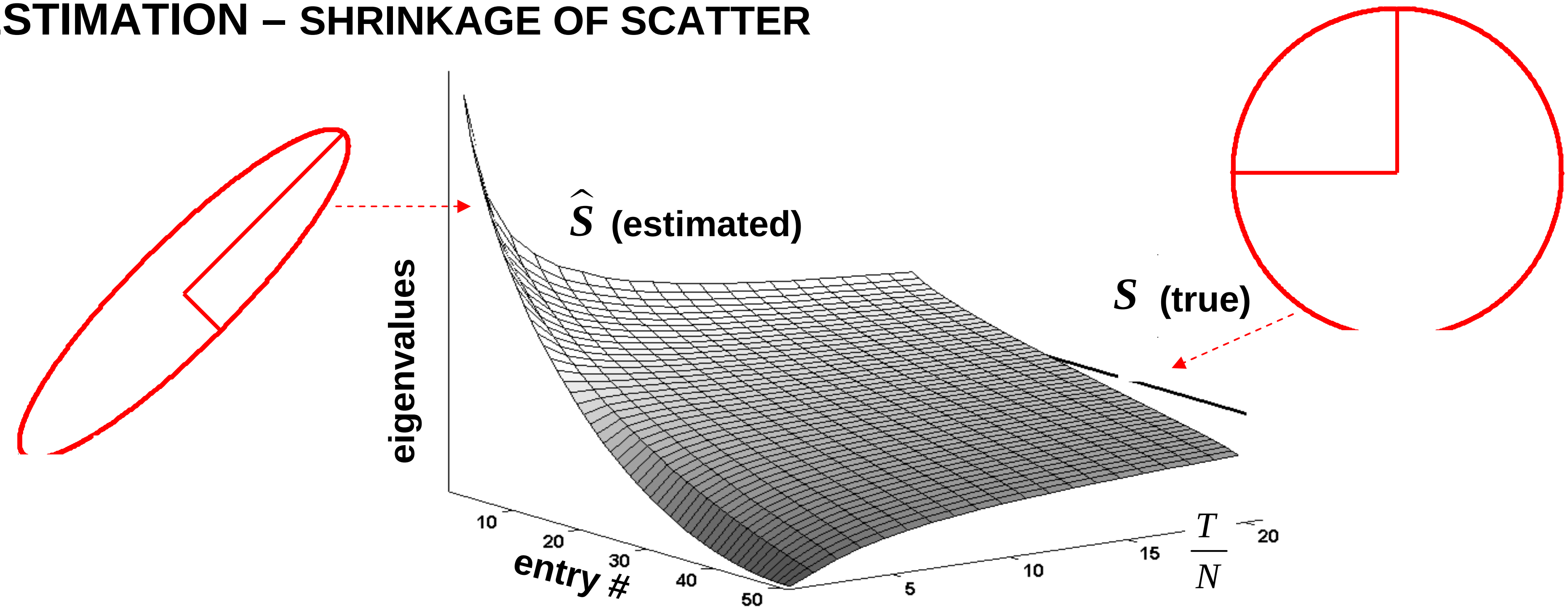
the sample mean $\widehat{\boldsymbol{m}} \equiv \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{r}_t$ is <u>not admissible</u> because it is very <u>inefficient</u>:

outcome of the estimation process is scattered around the true value

Stein shrinkage: $\widehat{\boldsymbol{m}}_{SR} \equiv (1-s)\widehat{\boldsymbol{m}} + s\boldsymbol{m}_0$

unbiased **efficient**

ESTIMATION – SHRINKAGE OF SCATTER



the sample covariance $\widehat{\boldsymbol{S}} \equiv \frac{1}{T}\sum_{t=1}^{T}\left(\boldsymbol{r}_t - \widehat{\boldsymbol{m}}\right)\left(\boldsymbol{r}_t - \widehat{\boldsymbol{m}}\right)'$ is <u>not admissible</u> because

the eigenvalues are scattered away from their true value

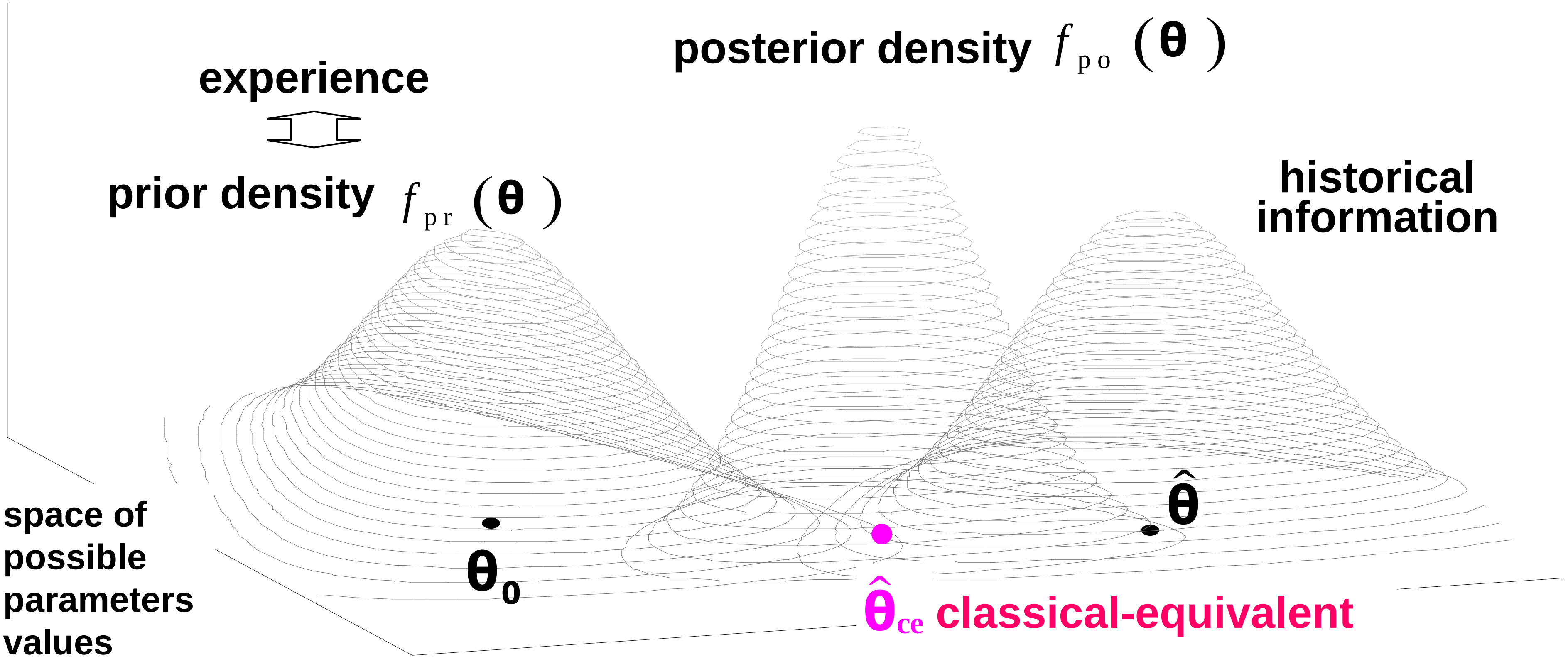
Ledoit-Wolf shrinkage: $\widehat{\boldsymbol{S}}_{SR} \equiv (1-s)\,\widehat{\boldsymbol{S}} + s\,\frac{\mathrm{tr}\left(\widehat{\boldsymbol{S}}\right)}{N}\boldsymbol{I}$ **spherical**

ESTIMATION - Bayesian shrinkage to prior

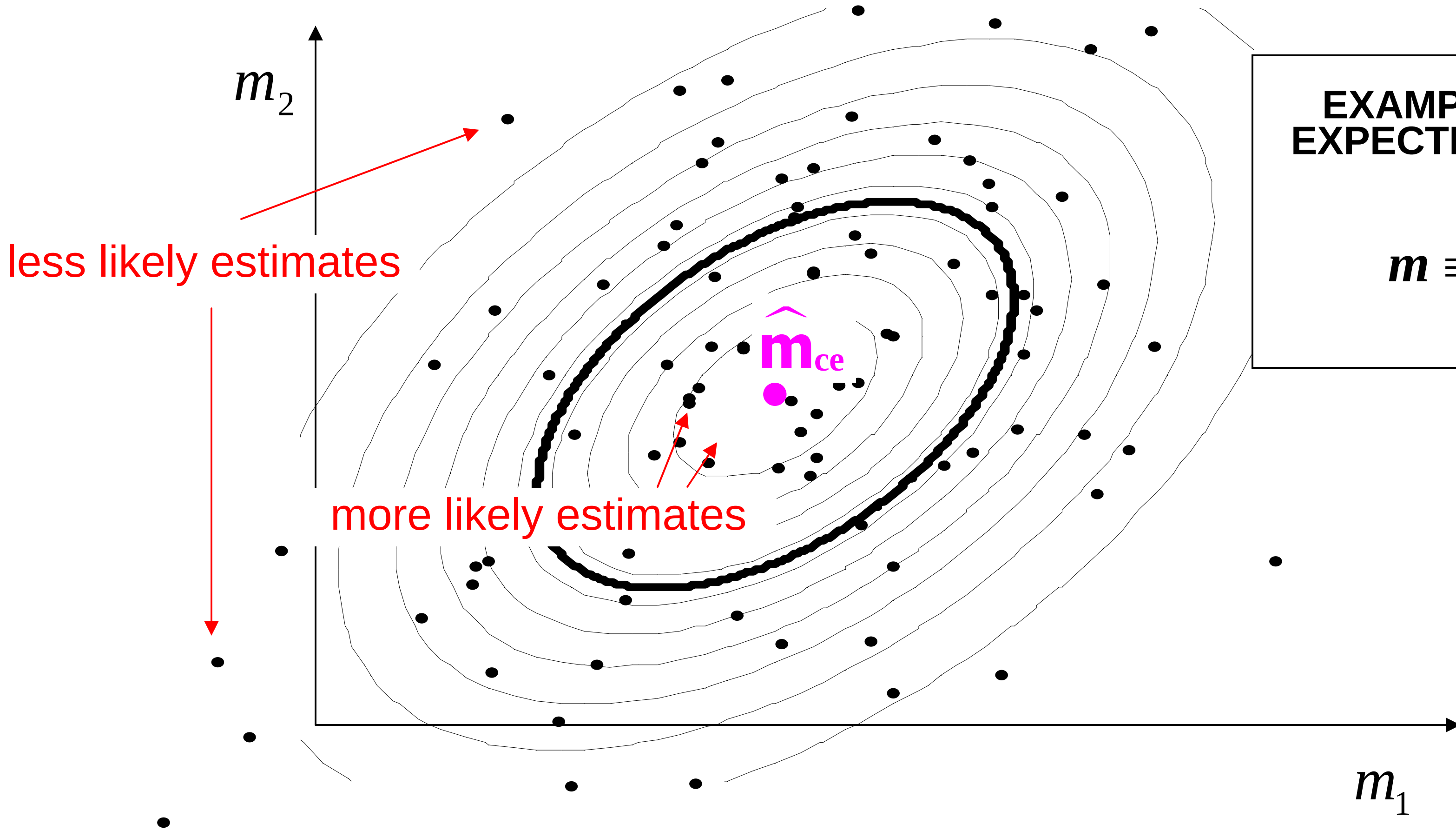
The Bayesian approach to estimation of the generic market parameters $\boldsymbol{\theta} \equiv (\boldsymbol{m}, \boldsymbol{S})$ differs from the classical approach in two respects:

• it blends historical information from time series analysis with experience

• the outcome of the estimation process is a (posterior) distribution, instead of a number



ESTIMATION - Bayesian shrinkage to prior: expectations

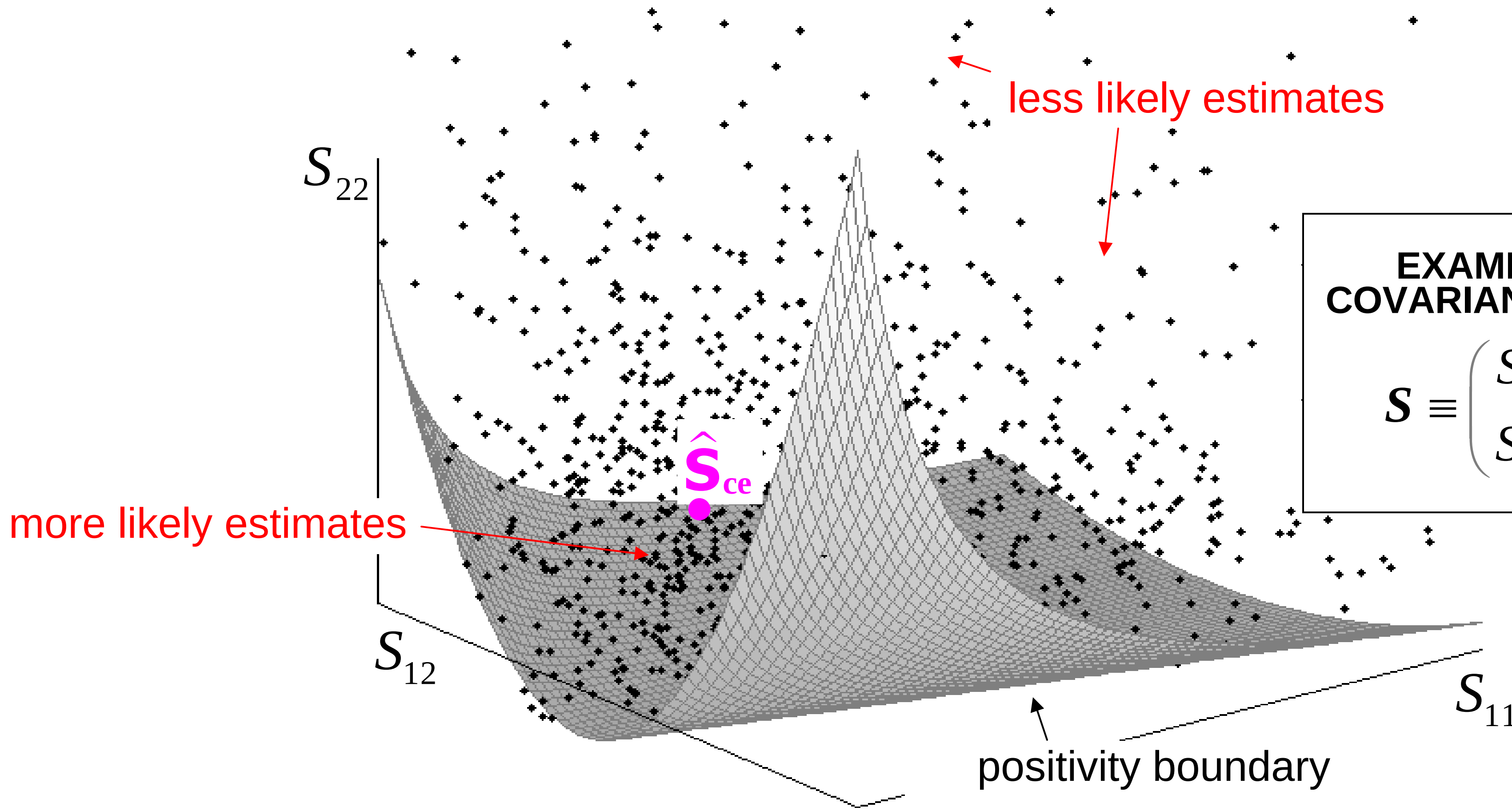


EXAMPLE: 2-dim EXPECTED VALUES

$$\boldsymbol{m} \equiv \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$$

in the Bayesian approach the expected values of the returns are a random variable

ESTIMATION - Bayesian shrinkage to prior: covariances



EXAMPLE: 2x2 COVARIANCE MATRIX

$$\boldsymbol{S} \equiv \begin{pmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{pmatrix}$$

in the Bayesian approach the covariance matrix of the returns is a random variable

ESTIMATION - Bayesian shrinkage to prior: NIW example

We make the following assumptions:

• The market is composed of equity-like securities, for which the returns are independent and identically distributed across time

• The estimation interval coincides with the investment horizon

• The linear returns are normally distributed:

$$\boldsymbol{R}_{t+\tau}^{\tau} \mid \boldsymbol{m}, \boldsymbol{S} \sim \mathrm{N}\left(\boldsymbol{m}, \boldsymbol{S}\right)$$

We model the investor's prior as a normal-inverse-Wishart distribution:

$$\boldsymbol{m} \mid \boldsymbol{S} \sim \mathrm{N}\left(\boldsymbol{m}_0, \frac{\boldsymbol{S}}{T_0}\right), \quad \boldsymbol{S}^{-1} \sim \mathrm{W}\left(\nu_0, \frac{\boldsymbol{S}_0^{-1}}{\nu_0}\right)$$

where

$\left(\boldsymbol{m}_0, \boldsymbol{S}_0\right)$: investor's experience on $\left(\boldsymbol{m}, \boldsymbol{S}\right)$

$\left(T_0, \nu_0\right)$: investor's confidence on $\left(\boldsymbol{m}_0, \boldsymbol{S}_0\right)$

ESTIMATION - Bayesian shrinkage to prior: NIW example

Under the above assumptions, the posterior distribution is normal-inverse-Wishart

$$\boldsymbol{m} \mid \boldsymbol{S} \sim \mathrm{N}\left(\boldsymbol{m}_1, \frac{\boldsymbol{S}}{T_1}\right), \qquad \boldsymbol{S}^{-1} \sim \mathrm{W}\left(\nu_1, \frac{\boldsymbol{S}_1^{-1}}{\nu_1}\right)$$

where

$$\widehat{\boldsymbol{m}} \equiv \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{r}_t \qquad \widehat{\boldsymbol{S}} \equiv \frac{1}{T}\sum_{t=1}^{T}\left(\boldsymbol{r}_t - \widehat{\boldsymbol{m}}\right)\left(\boldsymbol{r}_t - \widehat{\boldsymbol{m}}\right)'$$

$$T_1 \equiv T_0 + T \qquad \nu_1 \equiv \nu_0 + T$$

$$\widehat{\boldsymbol{m}}_1 \equiv \frac{1}{T_1}\left[T_0 \boldsymbol{m}_0 + T\widehat{\boldsymbol{m}}\right] \qquad \widehat{\boldsymbol{S}}_1 \equiv \frac{1}{\nu_1}\left[\nu_0 \boldsymbol{S}_0 + T\widehat{\boldsymbol{S}} + \frac{\left(\boldsymbol{m}_0 - \widehat{\boldsymbol{m}}\right)\left(\boldsymbol{m}_0 - \widehat{\boldsymbol{m}}\right)'}{\frac{1}{T_0} + \frac{1}{T}}\right]$$

AGENDA

PORTFOLIO MODELING - mean variance and representations

MARKET MODELING - location dispersion ellipsoid and PCA

ESTIMATION - non parametric, ML, robust, shrinkage, Bayesian

REFERENCES

REFERENCES

- This presentation

  symmys.com > Teaching > Talks > ***Modeling and Estimation Techniques for Portfolio Management***

- Implementation code (MATLAB)

  symmys.com > Book > Downloads > MATLAB

- Comprehensive discussion of
  - Modeling
  - Estimation
  - Location-dispersion ellipsoid
  - Satisfaction maximization
  - Quantitative portfolio-management
  - Risk-management
  - Estimation risk
  - Black-Litterman allocation
  - Bayesian techniques
  - Robust techniques
  - More …

  symmys.com > Book > A. Meucci, ***Risk and Asset Allocation*** - Springer

Springer Finance

Attilio Meucci

Risk and Asset Allocation

Springer