

MISSING DATA – EM ALGORITHM

Risk and Asset Allocation - Springer – *symmys.com*

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www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.


The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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$$\mathbf{X} \equiv \begin{pmatrix} \mathbf{X}_A \\ \mathbf{X}_B \end{pmatrix} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (2.160)$$

$$\boldsymbol{\mu} \equiv \begin{pmatrix} \mu_A \\ \mu_B \end{pmatrix}, \quad \boldsymbol{\Sigma} \equiv \begin{pmatrix} \Sigma_{AA} & \Sigma_{AB} \\ \Sigma_{BA} & \Sigma_{BB} \end{pmatrix} \quad (2.161)$$


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$$\mathbf{X} \equiv \begin{pmatrix} \mathbf{X}_A \\ \mathbf{X}_B \end{pmatrix} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (2.160)$$

$$\boldsymbol{\mu} \equiv \begin{pmatrix} \boldsymbol{\mu}_A \\ \boldsymbol{\mu}_B \end{pmatrix}, \quad \boldsymbol{\Sigma} \equiv \begin{pmatrix} \boldsymbol{\Sigma}_{AA} & \boldsymbol{\Sigma}_{AB} \\ \boldsymbol{\Sigma}_{BA} & \boldsymbol{\Sigma}_{BB} \end{pmatrix} \quad (2.161)$$

$$\boldsymbol{\mu}_B | \mathbf{x}_A \equiv \boldsymbol{\mu}_B + \boldsymbol{\Sigma}_{BA} \boldsymbol{\Sigma}_{AA}^{-1} (\mathbf{x}_A - \boldsymbol{\mu}_A) \quad (2.165)$$

$$\mathbf{X}_B | \mathbf{x}_A \sim N(\boldsymbol{\mu}_B | \mathbf{x}_A, \boldsymbol{\Sigma}_B | \mathbf{x}_A) \quad (2.164)$$

$$\boldsymbol{\Sigma}_B | \mathbf{x}_A \equiv \boldsymbol{\Sigma}_{BB} - \boldsymbol{\Sigma}_{BA} \boldsymbol{\Sigma}_{AA}^{-1} \boldsymbol{\Sigma}_{AB} \quad (2.166)$$

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$$\begin{pmatrix} X_{t,\text{mis}(t)} \\ X_{t,\text{obs}(t)} \end{pmatrix} \sim N \left(\begin{pmatrix} \mu_{\text{mis}(t)} \\ \mu_{\text{obs}(t)} \end{pmatrix}, \begin{pmatrix} \Sigma_{\text{mis}(t),\text{mis}(t)} & \Sigma_{\text{mis}(t),\text{obs}(t)} \\ \Sigma_{\text{obs}(t),\text{mis}(t)} & \Sigma_{\text{obs}(t),\text{obs}(t)} \end{pmatrix} \right) \quad (4.257)$$

$$\mu_B|x_A \equiv \mu_B + \Sigma_{BA}\Sigma_{AA}^{-1}(x_A - \mu_A) \quad (2.165)$$

$$\Sigma_B|x_A \equiv \Sigma_{BB} - \Sigma_{BA}\Sigma_{AA}^{-1}\Sigma_{AB} \quad (2.166)$$

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$$\begin{pmatrix} X_{t,\text{mis}(t)} \\ X_{t,\text{obs}(t)} \end{pmatrix} \sim N \left(\begin{pmatrix} \mu_{\text{mis}(t)} \\ \mu_{\text{obs}(t)} \end{pmatrix}, \begin{pmatrix} \Sigma_{\text{mis}(t),\text{mis}(t)} & \Sigma_{\text{mis}(t),\text{obs}(t)} \\ \Sigma_{\text{obs}(t),\text{mis}(t)} & \Sigma_{\text{obs}(t),\text{obs}(t)} \end{pmatrix} \right) \quad (4.257)$$

$$x_{t,\text{obs}(t)}^{(u)} \equiv x_{t,\text{obs}(t)} \quad (4.261)$$

$$x_{t,\text{mis}(t)}^{(u)} \equiv \mu_{\text{mis}(t)}^{(u)} \quad (4.262)$$

$$\mu_B | x_A \equiv \mu_B + \Sigma_{BA} \Sigma_{AA}^{-1} (x_A - \mu_A) \quad (2.165)$$

$$+ \Sigma_{\text{mis}(t),\text{obs}(t)}^{(u)} \left(\Sigma_{\text{obs}(t),\text{obs}(t)}^{(u)} \right)^{-1} \left(x_{t,\text{obs}(t)} - \mu_{\text{obs}(t)}^{(u)} \right)$$

$$\Sigma_B | x_A \equiv \Sigma_{BB} - \Sigma_{BA} \Sigma_{AA}^{-1} \Sigma_{AB} \quad (2.166)$$

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$$\begin{pmatrix} \mathbf{X}_{t,\text{mis}(t)} \\ \mathbf{X}_{t,\text{obs}(t)} \end{pmatrix} \sim N \left(\begin{pmatrix} \boldsymbol{\mu}_{\text{mis}(t)} \\ \boldsymbol{\mu}_{\text{obs}(t)} \end{pmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_{\text{mis}(t),\text{mis}(t)} & \boldsymbol{\Sigma}_{\text{mis}(t),\text{obs}(t)} \\ \boldsymbol{\Sigma}_{\text{obs}(t),\text{mis}(t)} & \boldsymbol{\Sigma}_{\text{obs}(t),\text{obs}(t)} \end{pmatrix} \right) \quad (4.257)$$

$$\mathbf{x}_{t,\text{obs}(t)}^{(u)} \equiv \mathbf{x}_{t,\text{obs}(t)} \quad (4.261)$$

$$\mathbf{x}_{t,\text{mis}(t)}^{(u)} \equiv \boldsymbol{\mu}_{\text{mis}(t)}^{(u)} \quad (4.262)$$

$$\boldsymbol{\mu}_B | \mathbf{x}_A \equiv \boldsymbol{\mu}_B + \boldsymbol{\Sigma}_{BA} \boldsymbol{\Sigma}_{AA}^{-1} (\mathbf{x}_A - \boldsymbol{\mu}_A) \quad (2.165)$$

$$+ \boldsymbol{\Sigma}_{\text{mis}(t),\text{obs}(t)}^{(u)} \left(\boldsymbol{\Sigma}_{\text{obs}(t),\text{obs}(t)}^{(u)} \right)^{-1} \left(\mathbf{x}_{t,\text{obs}(t)} - \boldsymbol{\mu}_{\text{obs}(t)}^{(u)} \right)$$

$$\mathbf{C}_{t,\text{obs}(t),\text{mis}(t)}^{(u)} \equiv \mathbf{0}, \quad \mathbf{C}_{t,\text{obs}(t),\text{obs}(t)}^{(u)} \equiv \mathbf{0}, \quad (4.263)$$

$$\mathbf{C}_{t,\text{mis}(t),\text{mis}(t)}^{(u)} \equiv \boldsymbol{\Sigma}_{\text{mis}(t),\text{mis}(t)}^{(u)} \quad (4.264)$$

$$\boldsymbol{\Sigma}_B | \mathbf{x}_A \equiv \boldsymbol{\Sigma}_{BB} - \boldsymbol{\Sigma}_{BA} \boldsymbol{\Sigma}_{AA}^{-1} \boldsymbol{\Sigma}_{AB} \quad (2.166)$$

$$- \boldsymbol{\Sigma}_{\text{mis}(t),\text{obs}(t)}^{(u)} \left(\boldsymbol{\Sigma}_{\text{obs}(t),\text{obs}(t)}^{(u)} \right)^{-1} \boldsymbol{\Sigma}_{\text{obs}(t),\text{mis}(t)}^{(u)}.$$

estimate

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$$\begin{pmatrix} \mathbf{X}_{t,\text{mis}(t)} \\ \mathbf{X}_{t,\text{obs}(t)} \end{pmatrix} \sim N \left(\begin{pmatrix} \boldsymbol{\mu}_{\text{mis}(t)} \\ \boldsymbol{\mu}_{\text{obs}(t)} \end{pmatrix}, \begin{pmatrix} \boldsymbol{\Sigma}_{\text{mis}(t),\text{mis}(t)} & \boldsymbol{\Sigma}_{\text{mis}(t),\text{obs}(t)} \\ \boldsymbol{\Sigma}_{\text{obs}(t),\text{mis}(t)} & \boldsymbol{\Sigma}_{\text{obs}(t),\text{obs}(t)} \end{pmatrix} \right) \quad (4.257)$$

$$\mathbf{x}_{t,\text{obs}(t)}^{(u)} \equiv \mathbf{x}_{t,\text{obs}(t)} \quad (4.261)$$

$$\mathbf{x}_{t,\text{mis}(t)}^{(u)} \equiv \boldsymbol{\mu}_{\text{mis}(t)}^{(u)} \quad (4.262)$$

$$\boldsymbol{\mu}_B | \mathbf{x}_A \equiv \boldsymbol{\mu}_B + \boldsymbol{\Sigma}_{BA} \boldsymbol{\Sigma}_{AA}^{-1} (\mathbf{x}_A - \boldsymbol{\mu}_A) \quad (2.165)$$

$$+ \boldsymbol{\Sigma}_{\text{mis}(t),\text{obs}(t)}^{(u)} \left(\boldsymbol{\Sigma}_{\text{obs}(t),\text{obs}(t)}^{(u)} \right)^{-1} \left(\mathbf{x}_{t,\text{obs}(t)} - \boldsymbol{\mu}_{\text{obs}(t)}^{(u)} \right)$$

$$\mathbf{C}_{t,\text{obs}(t),\text{mis}(t)}^{(u)} \equiv \mathbf{0}, \quad \mathbf{C}_{t,\text{obs}(t),\text{obs}(t)}^{(u)} \equiv \mathbf{0}, \quad (4.263)$$

$$\mathbf{C}_{t,\text{mis}(t),\text{mis}(t)}^{(u)} \equiv \boldsymbol{\Sigma}_{\text{mis}(t),\text{mis}(t)}^{(u)} \quad (4.264)$$

$$\boldsymbol{\Sigma}_B | \mathbf{x}_A \equiv \boldsymbol{\Sigma}_{BB} - \boldsymbol{\Sigma}_{BA} \boldsymbol{\Sigma}_{AA}^{-1} \boldsymbol{\Sigma}_{AB} \quad (2.166)$$

$$- \boldsymbol{\Sigma}_{\text{mis}(t),\text{obs}(t)}^{(u)} \left(\boldsymbol{\Sigma}_{\text{obs}(t),\text{obs}(t)}^{(u)} \right)^{-1} \boldsymbol{\Sigma}_{\text{obs}(t),\text{mis}(t)}^{(u)}.$$

estimate

$$\boldsymbol{\mu}^{(u+1)} \equiv \frac{1}{T} \sum_t \mathbf{x}_t^{(u)} \quad (4.265)$$

$$\boldsymbol{\Sigma}^{(u+1)} \equiv \frac{1}{T} \sum_t \left[\mathbf{C}_t^{(u)} + \left(\mathbf{x}_t^{(u)} - \boldsymbol{\mu}^{(u)} \right) \left(\mathbf{x}_t^{(u)} - \boldsymbol{\mu}^{(u)} \right)' \right] \quad (4.266)$$

update