MAXIMUM LIKELIHOOD ESTIMATORS

Risk and Asset Allocation* - Springer – *symmys.com

Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

MAXIMUM LIKELIHOOD ESTIMATORS

$$\boxed{\mathbf{G}\left[f_{\mathbf{X}}\right] \equiv \text{"unknown truth"}} \quad (4.6)$$

$$\boxed{\textbf{information } i_T \mapsto \textbf{number } \widehat{\mathbf{G}}} \quad (4.9)$$

$$i_T \equiv \{\mathbf{x}_1, \ldots, \mathbf{x}_T\} \quad (4.8)$$

MAXIMUM LIKELIHOOD ESTIMATORS

Risk and Asset Allocation **- Springer –** `symmys.com`

$$\boxed{\mathbf{G}\left[f_{\mathbf{X}}\right] \equiv \text{"unknown truth"}} \quad (4.6)$$

$$\boxed{\textbf{information } i_T \mapsto \textbf{number } \widehat{\mathbf{G}}} \quad (4.9)$$

$$f_{\boldsymbol{\theta}}\left(i_T\right) \equiv f_{\boldsymbol{\theta}}\left(\mathbf{x}_1\right) \cdots f_{\boldsymbol{\theta}}\left(\mathbf{x}_T\right). \quad (4.65)$$

$$\widehat{\boldsymbol{\theta}}\left[i_T\right] \equiv \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\operatorname{argmax}} f_{\boldsymbol{\theta}}\left(i_T\right) \quad (4.66)$$

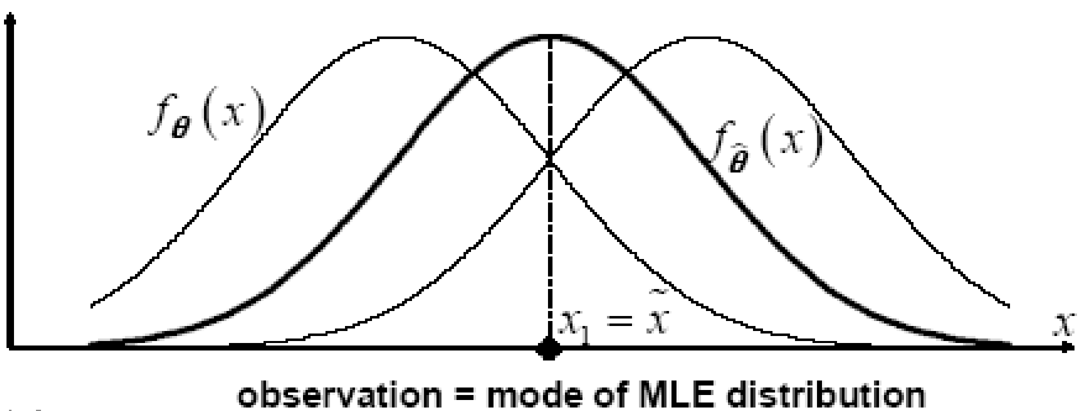

Fig. 4.10

MAXIMUM LIKELIHOOD ESTIMATORS

***Risk and Asset Allocation* - Springer – `symmys.com`**

$$\boxed{\mathbf{G}[f_{\mathbf{X}}] \equiv \text{"unknown truth"}} \quad (4.6)$$

$$\boxed{\text{information } i_T \mapsto \text{number } \widehat{\mathbf{G}}} \quad (4.9)$$

$$f_{\boldsymbol{\theta}}(i_T) \equiv f_{\boldsymbol{\theta}}(\mathbf{x}_1) \cdots f_{\boldsymbol{\theta}}(\mathbf{x}_T). \quad (4.65)$$

$$\widehat{\boldsymbol{\theta}}[i_T] \equiv \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\operatorname{argmax}} f_{\boldsymbol{\theta}}(i_T) \quad (4.66)$$

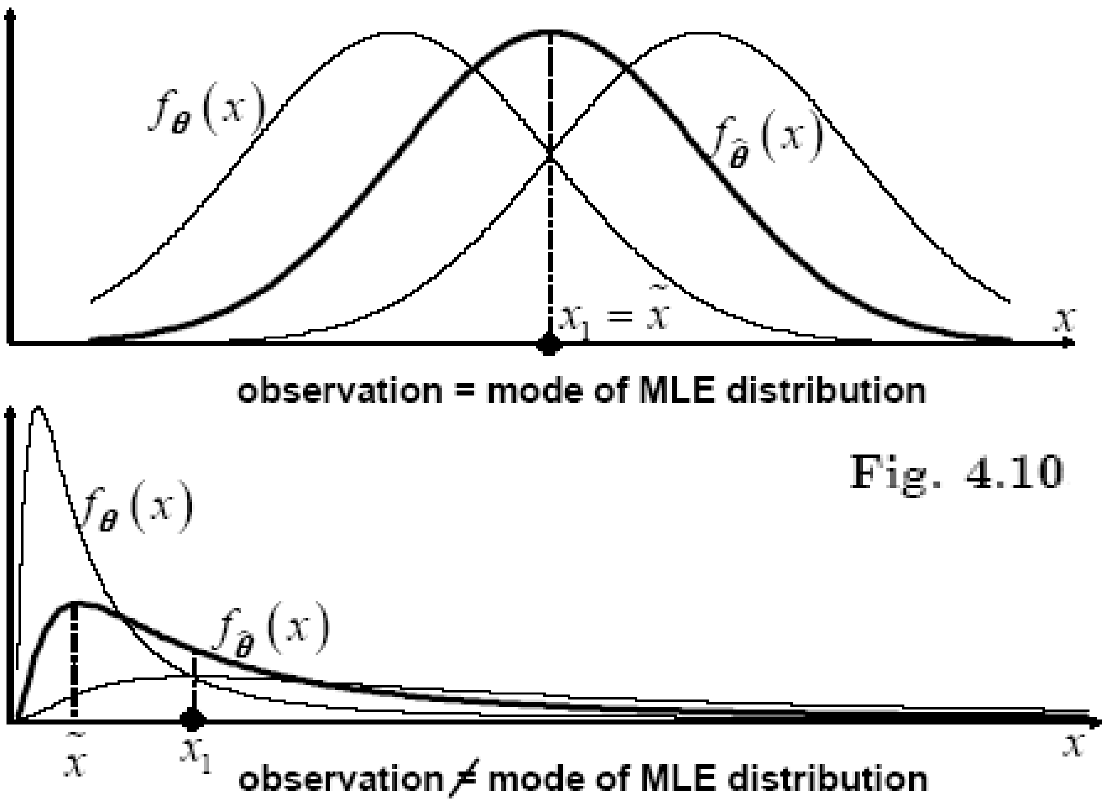

Fig. 4.10

MAXIMUM LIKELIHOOD ESTIMATORS

Risk and Asset Allocation **- Springer –** `symmys.com`

$$\boxed{\mathbf{G}\left[f_{\mathbf{X}}\right] \equiv \text{"unknown truth"}} \qquad (4.6)$$

$$\boxed{\textbf{information } i_T \mapsto \textbf{number } \widehat{\mathbf{G}}} \qquad (4.9)$$

$$f_{\boldsymbol{\theta}}\left(i_T\right) \equiv f_{\boldsymbol{\theta}}\left(\mathbf{x}_1\right) \cdots f_{\boldsymbol{\theta}}\left(\mathbf{x}_T\right). \quad (4.65)$$

$$\widehat{\boldsymbol{\theta}}\left[i_T\right] \equiv \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\operatorname{argmax}}\, f_{\boldsymbol{\theta}}\left(i_T\right) \quad (4.66)$$

- $\widehat{g\left(\boldsymbol{\theta}\right)} = g\left(\widehat{\boldsymbol{\theta}}\right)$ (4.70)

- $\widehat{\boldsymbol{\theta}}\left[i_T\right] \sim \mathrm{N}\left(\boldsymbol{\theta}, \frac{\boldsymbol{\Gamma}}{T}\right)$ (4.71)

$$\boldsymbol{\Gamma} \equiv \operatorname{Cov}\left\{\frac{\partial \ln\left(f_{\boldsymbol{\theta}}\left(\mathbf{X}\right)\right)}{\partial \boldsymbol{\theta}}\right\} \quad (4.72)$$

MAXIMUM LIKELIHOOD ESTIMATORS – ELLIPTICAL

Risk and Asset Allocation **- Springer –** `symmys.com`

$$\mathbf{X} \sim \mathrm{El}(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g) \quad (4.73)$$

$$f_{\boldsymbol{\theta}}(\mathrm{x}) \equiv \frac{1}{\sqrt{|\boldsymbol{\Sigma}|}} g\left(\mathrm{Ma}^2(\mathrm{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma})\right) \quad (4.74)$$

$$\mathrm{Ma}(\mathrm{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) \equiv \sqrt{(\mathrm{x} - \boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\mathrm{x} - \boldsymbol{\mu})} \quad (4.75)$$

MAXIMUM LIKELIHOOD ESTIMATORS – ELLIPTICAL

***Risk and Asset Allocation* - Springer – `symmys.com`**

$$\mathbf{X} \sim \mathrm{El}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g\right) \qquad (4.73)$$

$$f_{\boldsymbol{\theta}}\left(\mathrm{x}\right) \equiv \frac{1}{\sqrt{\left|\boldsymbol{\Sigma}\right|}} g\left(\mathrm{Ma}^2\left(\mathrm{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right)\right) \qquad (4.74)$$

$$\mathrm{Ma}\left(\mathrm{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \equiv \sqrt{\left(\mathrm{x} - \boldsymbol{\mu}\right)' \boldsymbol{\Sigma}^{-1} \left(\mathrm{x} - \boldsymbol{\mu}\right)} \qquad (4.75)$$

$$\widehat{\boldsymbol{\mu}} = \sum_{t=1}^{T} \frac{w_t}{\sum_{s=1}^{T} w_s} \mathrm{x}_t \qquad (4.81)$$

$$\widehat{\boldsymbol{\Sigma}} = \frac{1}{T} \sum_{t=1}^{T} w_t \left(\mathrm{x}_t - \widehat{\boldsymbol{\mu}}\right) \left(\mathrm{x}_t - \widehat{\boldsymbol{\mu}}\right)' . \qquad (4.82)$$

$$w_t \equiv w\left(\mathrm{Ma}^2\left(\mathrm{x}_t, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}\right)\right) \qquad (4.80)$$

$$w\left(z\right) \equiv -2 \frac{g'\left(z\right)}{g\left(z\right)} \qquad (4.79)$$

MAXIMUM LIKELIHOOD ESTIMATORS – ELLIPTICAL

Risk and Asset Allocation - Springer – `symmys.com`

$$\mathbf{X} \sim \mathrm{El}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g\right) \quad (4.73)$$

$$f_{\boldsymbol{\theta}}\left(\mathrm{x}\right) \equiv \frac{1}{\sqrt{|\boldsymbol{\Sigma}|}} g\left(\mathrm{Ma}^2\left(\mathrm{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right)\right) \quad (4.74)$$

$$\mathrm{Ma}\left(\mathrm{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \equiv \sqrt{\left(\mathrm{x}-\boldsymbol{\mu}\right)' \boldsymbol{\Sigma}^{-1}\left(\mathrm{x}-\boldsymbol{\mu}\right)} \quad (4.75)$$

$$\widehat{\boldsymbol{\mu}} = \sum_{t=1}^{T} \frac{w_t}{\sum_{s=1}^{T} w_s} \mathrm{x}_t \quad (4.81)$$

$$\widehat{\boldsymbol{\Sigma}} = \frac{1}{T} \sum_{t=1}^{T} w_t \left(\mathrm{x}_t - \widehat{\boldsymbol{\mu}}\right)\left(\mathrm{x}_t - \widehat{\boldsymbol{\mu}}\right)' . \quad (4.82)$$

$$w_t \equiv w\left(\mathrm{Ma}^2\left(\mathrm{x}_t, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}\right)\right) \quad (4.80)$$

$$w\left(z\right) \equiv -2\frac{g'\left(z\right)}{g\left(z\right)} \quad (4.79)$$

$$g^{\mathrm{N}}\left(z\right) \equiv \frac{e^{-\frac{z}{2}}}{\left(2\pi\right)^{\frac{N}{2}}}, \quad (4.96)$$

$$w\left(z\right) \equiv 1 \quad (4.97)$$

MAXIMUM LIKELIHOOD ESTIMATORS – ELLIPTICAL

***Risk and Asset Allocation* - Springer – `symmys.com`**

$$\mathbf{X} \sim \mathrm{El}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}, g\right) \quad (4.73)$$

$$f_{\boldsymbol{\theta}}\left(\mathbf{x}\right) \equiv \frac{1}{\sqrt{\left|\boldsymbol{\Sigma}\right|}} g\left(\mathrm{Ma}^2\left(\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right)\right) \quad (4.74)$$

$$\mathrm{Ma}\left(\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \equiv \sqrt{\left(\mathbf{x} - \boldsymbol{\mu}\right)' \boldsymbol{\Sigma}^{-1} \left(\mathbf{x} - \boldsymbol{\mu}\right)} \quad (4.75)$$

$$\widehat{\boldsymbol{\mu}} = \sum_{t=1}^{T} \frac{w_t}{\sum_{s=1}^{T} w_s} \mathbf{x}_t \quad (4.81)$$

$$\widehat{\boldsymbol{\Sigma}} = \frac{1}{T} \sum_{t=1}^{T} w_t \left(\mathbf{x}_t - \widehat{\boldsymbol{\mu}}\right)\left(\mathbf{x}_t - \widehat{\boldsymbol{\mu}}\right)' . \quad (4.82)$$

$$w_t \equiv w\left(\mathrm{Ma}^2\left(\mathbf{x}_t, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}\right)\right) \quad (4.80)$$

$$w\left(z\right) \equiv -2\frac{g'\left(z\right)}{g\left(z\right)} \quad (4.79)$$

$$g^{\mathrm{N}}\left(z\right) \equiv \frac{e^{-\frac{z}{2}}}{\left(2\pi\right)^{\frac{N}{2}}}, \quad (4.96)$$

$$w\left(z\right) \equiv 1 \quad (4.97)$$

$$g^{\mathrm{Ca}}\left(z\right) = \frac{\Gamma\left(\frac{1+N}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\left(\pi\right)^{\frac{N}{2}}}\left(1+z\right)^{-\frac{1+N}{2}} \quad (4.83)$$

$$w_t = \frac{N+1}{1 + \mathrm{Ma}^2\left(\mathbf{x}_t, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}\right)} \quad (4.84)$$

MAXIMUM LIKELIHOOD ESTIMATORS – NORMAL (LOCATION)

Risk and Asset Allocation - Springer – `symmys.com`

normal assumption

$$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \qquad (4.95)$$

estimator definition

$$\widehat{\boldsymbol{\mu}}[i_T] = \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_t. \qquad (4.98)$$

MAXIMUM LIKELIHOOD ESTIMATORS – NORMAL (LOCATION)

***Risk and Asset Allocation* - Springer – `symmys.com`**

normal assumption $$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \qquad (4.95)$$

estimator definition $$\widehat{\boldsymbol{\mu}}[i_T] = \frac{1}{T} \sum_{t=1}^{T} \mathbf{x}_t. \qquad (4.98)$$

estimator distribution (replicability) $$\widehat{\boldsymbol{\mu}}[I_T] \sim \mathrm{N}\left(\boldsymbol{\mu}, \frac{\boldsymbol{\Sigma}}{T}\right) \qquad (4.102)$$

MAXIMUM LIKELIHOOD ESTIMATORS – NORMAL (LOCATION)

Risk and Asset Allocation **- Springer –** `symmys.com`

normal assumption $$\mathbf{X} \sim \mathrm{N}\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right) \quad (4.95)$$

estimator definition $$\widehat{\boldsymbol{\mu}}\left[i_T\right] = \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_t. \quad (4.98)$$

estimator distribution (replicability) $$\widehat{\boldsymbol{\mu}}\left[I_T\right] \sim \mathrm{N}\left(\boldsymbol{\mu}, \frac{\boldsymbol{\Sigma}}{T}\right) \quad (4.102)$$

Generalized t-tests…

Generalized p-values…

MAXIMUM LIKELIHOOD ESTIMATORS – NORMAL (LOCATION)

Risk and Asset Allocation - Springer – `symmys.com`

normal assumption $$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (4.95)$$

estimator definition $$\widehat{\boldsymbol{\mu}}[i_T] = \frac{1}{T} \sum_{t=1}^{T} \mathbf{x}_t. \quad (4.98)$$

estimator distribution (replicability) $$\widehat{\boldsymbol{\mu}}[I_T] \sim \mathrm{N}\left(\boldsymbol{\mu}, \frac{\boldsymbol{\Sigma}}{T}\right) \quad (4.102)$$

estimator evaluation (global) $$\mathrm{Loss}(\widehat{\boldsymbol{\mu}}, \boldsymbol{\mu}) \equiv [\widehat{\boldsymbol{\mu}}[I_T] - \boldsymbol{\mu}]' [\widehat{\boldsymbol{\mu}}[I_T] - \boldsymbol{\mu}] \quad (4.108)$$

estimator evaluation (summary)
$$\left\{ \begin{aligned} &\mathrm{Err}^2(\widehat{\boldsymbol{\mu}}, \boldsymbol{\mu}) = \frac{1}{T} \mathrm{tr}(\boldsymbol{\Sigma}). \quad (4.109) \\ &\mathrm{Inef}^2(\widehat{\boldsymbol{\mu}}) = \frac{1}{T} \mathrm{tr}(\boldsymbol{\Sigma}) \quad (4.110) \\ &\mathrm{Bias}^2(\widehat{\boldsymbol{\mu}}, \boldsymbol{\mu}) = 0. \quad (4.111) \end{aligned} \right.$$

MAXIMUM LIKELIHOOD ESTIMATORS – NORMAL (LOCATION)

***Risk and Asset Allocation* - Springer – `symmys.com`**

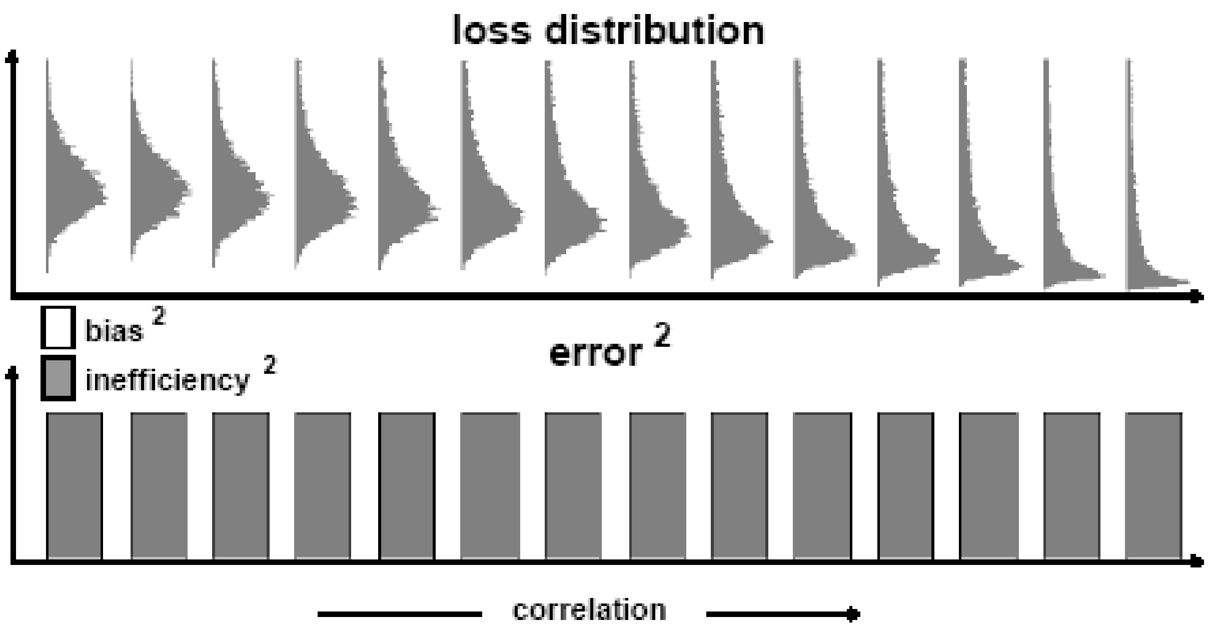

Fig. 4.11.

estimator evaluation (global)

$$\text{Loss}\left(\widehat{\boldsymbol{\mu}},\boldsymbol{\mu}\right)\equiv\left[\widehat{\boldsymbol{\mu}}\left[I_T\right]-\boldsymbol{\mu}\right]'\left[\widehat{\boldsymbol{\mu}}\left[I_T\right]-\boldsymbol{\mu}\right] \quad (4.108)$$

estimator evaluation (summary)

$$\text{Err}^2\left(\widehat{\boldsymbol{\mu}},\boldsymbol{\mu}\right)=\frac{1}{T}\operatorname{tr}\left(\boldsymbol{\Sigma}\right). \quad (4.109)$$

$$\text{Inef}^2\left(\widehat{\boldsymbol{\mu}}\right)=\frac{1}{T}\operatorname{tr}\left(\boldsymbol{\Sigma}\right) \quad (4.110)$$

$$\text{Bias}^2\left(\widehat{\boldsymbol{\mu}},\boldsymbol{\mu}\right)=0. \quad (4.111)$$

MAXIMUM LIKELIHOOD ESTIMATORS – NORMAL (SCATTER)

Risk and Asset Allocation - Springer – `symmys.com`

normal assumption $\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ (4.95)

estimator definition $\widehat{\boldsymbol{\Sigma}}[i_T] = \frac{1}{T} \sum_{t=1}^{T} (\mathbf{x}_t - \widehat{\boldsymbol{\mu}})(\mathbf{x}_t - \widehat{\boldsymbol{\mu}})'$ (4.99)

MAXIMUM LIKELIHOOD ESTIMATORS – NORMAL (SCATTER)

***Risk and Asset Allocation* - Springer – `symmys.com`**

normal assumption $$\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (4.95)$$

estimator definition $$\widehat{\boldsymbol{\Sigma}}[i_T] = \frac{1}{T}\sum_{t=1}^{T}(\mathbf{x}_t - \widehat{\boldsymbol{\mu}})(\mathbf{x}_t - \widehat{\boldsymbol{\mu}})' \quad (4.99)$$

estimator distribution (replicability) $$T\widehat{\boldsymbol{\Sigma}}[I_T] \sim \mathrm{W}(T-1, \boldsymbol{\Sigma}) \quad (4.103)$$

MAXIMUM LIKELIHOOD ESTIMATORS – NORMAL (SCATTER)

***Risk and Asset Allocation* - Springer – `symmys.com`**

normal assumption $\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ (4.95)

estimator definition $\widehat{\boldsymbol{\Sigma}}[i_T] = \frac{1}{T} \sum_{t=1}^{T} (\mathbf{x}_t - \widehat{\boldsymbol{\mu}})(\mathbf{x}_t - \widehat{\boldsymbol{\mu}})'$ (4.99)

estimator distribution (replicability) $T\widehat{\boldsymbol{\Sigma}}[I_T] \sim \mathrm{W}(T-1, \boldsymbol{\Sigma})$ (4.103)

Generalized t-tests…

Generalized p-values…

MAXIMUM LIKELIHOOD ESTIMATORS – NORMAL (SCATTER)

Risk and Asset Allocation **- Springer –** `symmys.com`

normal assumption $\mathbf{X} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ (4.95)

estimator definition $\widehat{\boldsymbol{\Sigma}}[i_T] = \frac{1}{T}\sum_{t=1}^{T}(\mathbf{x}_t - \widehat{\boldsymbol{\mu}})(\mathbf{x}_t - \widehat{\boldsymbol{\mu}})'$ (4.99)

estimator distribution (replicability) $T\widehat{\boldsymbol{\Sigma}}[I_T] \sim \mathrm{W}(T-1, \boldsymbol{\Sigma})$ (4.103)

estimator evaluation (global) $\mathrm{Loss}\left(\widehat{\boldsymbol{\Sigma}}, \boldsymbol{\Sigma}\right) \equiv \mathrm{tr}\left[\left(\widehat{\boldsymbol{\Sigma}}[I_T] - \boldsymbol{\Sigma}\right)^2\right]$ (4.118)

estimator evaluation (summary)

$$\mathrm{Err}^2\left(\widehat{\boldsymbol{\Sigma}}, \boldsymbol{\Sigma}\right) = \frac{1}{T}\left[\mathrm{tr}\left(\boldsymbol{\Sigma}^2\right) + \left(1 - \frac{1}{T}\right)[\mathrm{tr}(\boldsymbol{\Sigma})]^2\right] \quad (4.119)$$

$$\mathrm{Inef}^2\left(\widehat{\boldsymbol{\Sigma}}\right) = \frac{1}{T}\left(1 - \frac{1}{T}\right)\left[\mathrm{tr}\left(\boldsymbol{\Sigma}^2\right) + [\mathrm{tr}(\boldsymbol{\Sigma})]^2\right] \quad (4.120)$$

$$\mathrm{Bias}^2\left(\widehat{\boldsymbol{\Sigma}}, \boldsymbol{\Sigma}\right) = \frac{1}{T^2}\mathrm{tr}\left(\boldsymbol{\Sigma}^2\right) \quad (4.121)$$

MAXIMUM LIKELIHOOD ESTIMATORS – NORMAL (SCATTER)

***Risk and Asset Allocation* - Springer – `symmys.com`**

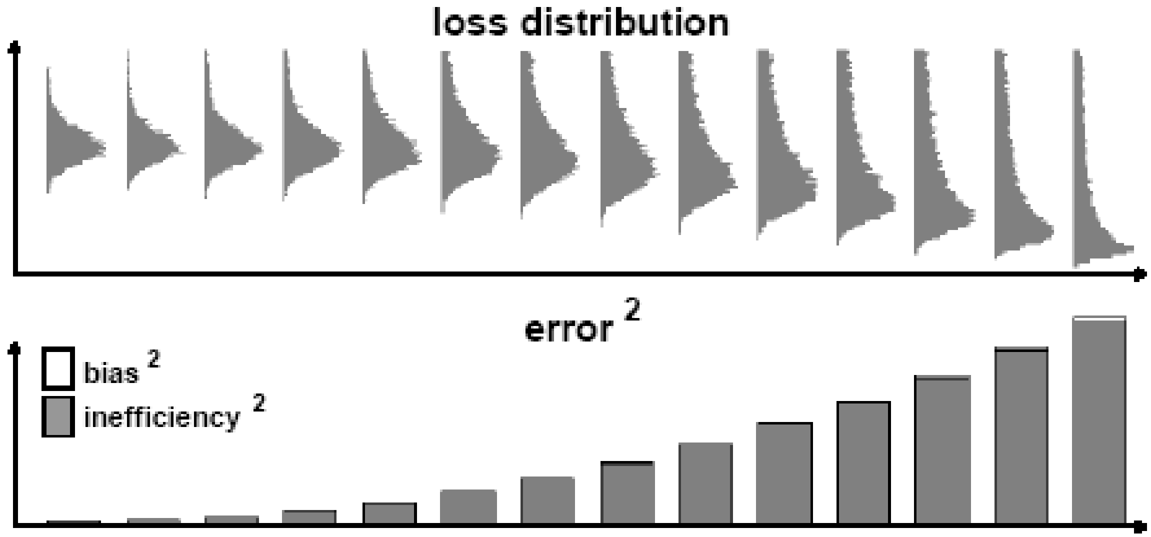

Fig. 4.12

estimator evaluation (global)

$$\operatorname{Loss}\left(\widehat{\Sigma}, \Sigma\right) \equiv \operatorname{tr}\left[\left(\widehat{\Sigma}\left[I_T\right] - \Sigma\right)^2\right] \qquad (4.118)$$

estimator evaluation (summary)

$$\operatorname{Err}^2\left(\widehat{\Sigma}, \Sigma\right) = \frac{1}{T}\left[\operatorname{tr}\left(\Sigma^2\right) + \left(1 - \frac{1}{T}\right)\left[\operatorname{tr}\left(\Sigma\right)\right]^2\right] \qquad (4.119)$$

$$\operatorname{Inef}^2\left(\widehat{\Sigma}\right) = \frac{1}{T}\left(1 - \frac{1}{T}\right)\left[\operatorname{tr}\left(\Sigma^2\right) + \left[\operatorname{tr}\left(\Sigma\right)\right]^2\right] \qquad (4.120)$$

$$\operatorname{Bias}^2\left(\widehat{\Sigma}, \Sigma\right) = \frac{1}{T^2}\operatorname{tr}\left(\Sigma^2\right) \qquad (4.121)$$

MAXIMUM LIKELIHOOD ESTIMATORS – NORMAL (LOADINGS)

Risk and Asset Allocation **- Springer –** `symmys.com`

normal assumption

$$\mathbf{X}|\mathbf{f} = \mathbf{B}\mathbf{f} + \mathbf{U}|\mathbf{f} \quad (4.88)$$

$$\mathbf{U}_t|\mathbf{f}_t \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma}) \quad (4.123)$$

estimator definition

$$\widehat{\mathbf{B}}[i_T] = \widehat{\boldsymbol{\Sigma}}_{XF}[i_T]\,\widehat{\boldsymbol{\Sigma}}_F^{-1}[i_T] \quad (4.126)$$

$$\widehat{\boldsymbol{\Sigma}}_{XF}[i_T] \equiv \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_t\mathbf{f}_t', \quad \widehat{\boldsymbol{\Sigma}}_F[i_T] \equiv \frac{1}{T}\sum_{t=1}^{T}\mathbf{f}_t\mathbf{f}_t'. \quad (4.127)$$

$$\widehat{\boldsymbol{\Sigma}}[i_T] = \frac{1}{T}\sum_{t=1}^{T}\left(\mathbf{x}_t - \widehat{\mathbf{B}}[i_T]\,\mathbf{f}_t\right)\left(\mathbf{x}_t - \widehat{\mathbf{B}}[i_T]\,\mathbf{f}_t\right)' \quad (4.128)$$

estimator distribution
(replicability)

$$\widehat{\mathbf{B}}[I_T|\mathbf{f}_1,\ldots,\mathbf{f}_T] \sim \mathrm{N}\left(\mathbf{B}, \frac{\boldsymbol{\Sigma}}{T}, \widehat{\boldsymbol{\Sigma}}_F^{-1}\right) \quad (4.129)$$

$$T\widehat{\boldsymbol{\Sigma}}[I_T|\mathbf{f}_1,\ldots,\mathbf{f}_T] \sim \mathrm{W}(T-K, \boldsymbol{\Sigma}) \quad (4.130)$$

Generalized t-tests, generalized p-values, estimator evaluation