ROBUST ESTIMATORS – INFLUENCE FUNCTION

***Risk and Asset Allocation* - Springer – `symmys.com`**

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www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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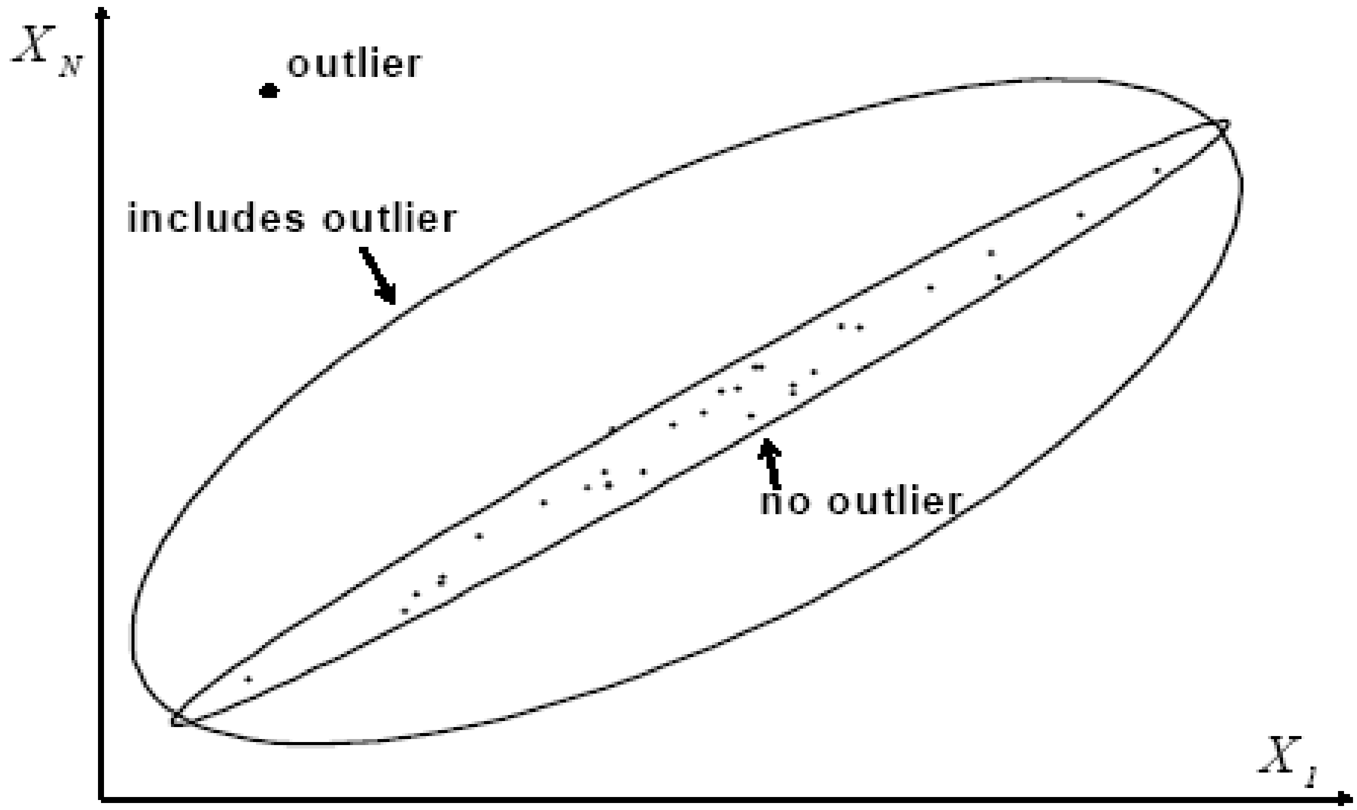

Fig. 4.18. Sample estimators: lack of robustness

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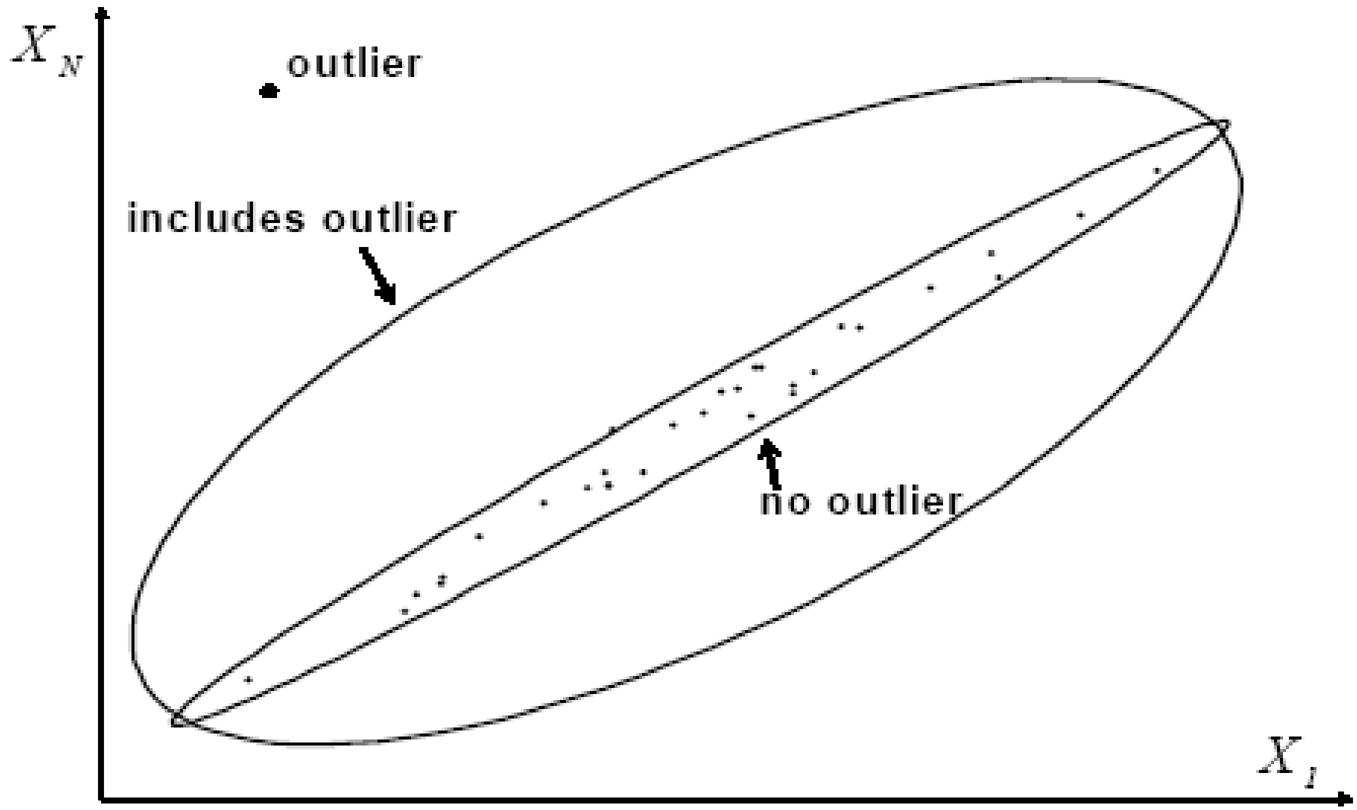
$$\boxed{\mathrm{SC}\left(\mathbf{x},\widehat{\mathbf{G}}\right)\equiv T\widehat{\mathbf{G}}\left(\mathbf{x}_1,\ldots,\mathbf{x}_T,\mathbf{x}\right)-T\widehat{\mathbf{G}}\left(\mathbf{x}_1,\ldots,\mathbf{x}_T\right)} \qquad (4.166)$$



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- $$\boxed{\mathrm{SC}\left(\mathbf{x}, \widehat{\mathbf{G}}\right) \equiv T\widehat{\mathbf{G}}\left(\mathbf{x}_1, \ldots, \mathbf{x}_T, \mathbf{x}\right) - T\widehat{\mathbf{G}}\left(\mathbf{x}_1, \ldots, \mathbf{x}_T\right)} \tag{4.166}$$

$$f_{i_T} \mapsto (1-\epsilon) f_{i_T} + \epsilon\delta^{(\mathbf{x})}. \tag{4.183}$$

$$f_{i_T} \equiv \frac{1}{T}\sum_{t=1}^{T} \delta^{(\mathbf{x}_t)} \qquad \epsilon \equiv 1/(T+1)$$

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- $$\boxed{\widehat{\mathbf{G}} \equiv \widetilde{\mathbf{G}}\left[f_{i_T}\right]} \quad (4.167)$$

- **non-parametric**

$$\boxed{\widehat{\mathbf{G}} \equiv \mathbf{G}\left[f_{i_T}\right]} \quad (4.169)$$

- **maximum likelihood**

$$\boldsymbol{\psi}(\mathbf{x}, \boldsymbol{\theta}) \equiv \frac{\partial}{\partial\boldsymbol{\theta}} \ln\left(f_{\boldsymbol{\theta}}(\mathbf{x})\right) \quad (4.176)$$

$$\widetilde{\boldsymbol{\theta}}[h] : \quad \int_{\mathbb{R}^N} \boldsymbol{\psi}\left(\mathbf{x}, \widetilde{\boldsymbol{\theta}}\right) h(\mathbf{x})\, d\mathbf{x} \equiv \mathbf{0}. \quad (4.175)$$

$$\boxed{\widehat{\boldsymbol{\theta}} \equiv \widetilde{\boldsymbol{\theta}}\left[f_{i_T}\right]} \quad (4.177)$$

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- $$\boxed{\mathrm{SC}\left(\mathbf{x},\widehat{\mathbf{G}}\right)\equiv T\widehat{\mathbf{G}}\left(\mathbf{x}_1,\ldots,\mathbf{x}_T,\mathbf{x}\right)-T\widehat{\mathbf{G}}\left(\mathbf{x}_1,\ldots,\mathbf{x}_T\right)} \qquad (4.166)$$

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$$f_{i_T}\equiv\frac{1}{T}\sum_{t=1}^{T}\delta^{(\mathbf{x}_t)} \qquad \epsilon\equiv 1/\left(T+1\right)$$

- $$\boxed{\widehat{\mathbf{G}}\equiv\widetilde{\mathbf{G}}\left[f_{i_T}\right]} \qquad (4.167)$$

$$\mathrm{SC}\left(\mathbf{x},\widehat{\mathbf{G}}\right)\equiv\frac{1-\epsilon}{\epsilon}\left\{\widetilde{\mathbf{G}}\left[\left(1-\epsilon\right)f_{i_T}+\epsilon\delta^{(\mathbf{x})}\right]-\widetilde{\mathbf{G}}\left[f_{i_T}\right]\right\} \qquad (4.184)$$

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- $$\boxed{\mathrm{SC}\left(\mathbf{x}, \widehat{\mathbf{G}}\right) \equiv T\widehat{\mathbf{G}}\left(\mathbf{x}_1, \ldots, \mathbf{x}_T, \mathbf{x}\right) - T\widehat{\mathbf{G}}\left(\mathbf{x}_1, \ldots, \mathbf{x}_T\right)} \quad (4.166)$$

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$$\boxed{\mathrm{IF}\left(\mathbf{x}, f_{\mathbf{X}}, \widehat{\mathbf{G}}\right) \equiv \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left(\widetilde{\mathbf{G}}\left[(1-\epsilon) f_{\mathbf{X}} + \epsilon \delta^{(\mathbf{x})}\right] - \widetilde{\mathbf{G}}\left[f_{\mathbf{X}}\right] \right)} \quad (4.185)$$

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$$\widehat{\mathrm{E}} \equiv \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}_t \quad (4.196)$$

$$\widehat{\mathrm{Cov}} \equiv \frac{1}{T}\sum_{t=1}^{T} \left(\mathbf{x}_t - \widehat{\mathrm{E}}\right)\left(\mathbf{x}_t - \widehat{\mathrm{E}}\right)' \quad (4.197)$$

$$\mathrm{IF}\left(\mathbf{x}, f_{\mathbf{X}}, \widehat{\mathrm{E}}\right) = \mathbf{x} - \mathrm{E}\left\{\mathbf{X}\right\} \quad (4.198)$$

$$\mathrm{IF}\left(\mathbf{x}, f_{\mathbf{X}}, \widehat{\mathrm{Cov}}\right) = \left(\mathbf{x} - \mathrm{E}\left\{\mathbf{X}\right\}\right)\left(\mathbf{x} - \mathrm{E}\left\{\mathbf{X}\right\}\right)' - \mathrm{Cov}\left\{\mathbf{X}\right\} \quad (4.199)$$

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$$\widehat{\boldsymbol{\mu}} = \sum_{t=1}^{T} \frac{w\left(\mathrm{Ma}^2\left(\mathrm{x}_t, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}\right)\right)}{\sum_{s=1}^{T} w\left(\mathrm{Ma}^2\left(\mathrm{x}_s, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}\right)\right)} \mathrm{x}_t \quad (4.203)$$

$$\widehat{\boldsymbol{\Sigma}} = \frac{1}{T}\sum_{t=1}^{T} \left(\mathrm{x}_t - \widehat{\boldsymbol{\mu}}\right)\left(\mathrm{x}_t - \widehat{\boldsymbol{\mu}}\right)' w\left(\mathrm{Ma}^2\left(\mathrm{x}_t, \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}\right)\right) \quad (4.204)$$

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$$\boxed{w(z) \equiv -2\frac{g'(z)}{g(z)}.} \quad (4.205)$$

$$f_{\boldsymbol{\theta}}(\mathrm{x}) \equiv \frac{1}{\sqrt{|\boldsymbol{\Sigma}|}} g\left(\mathrm{Ma}^2(\mathrm{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma})\right) \quad (4.201)$$

$$\mathrm{Ma}^2(\mathrm{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) \equiv (\mathrm{x} - \boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\mathrm{x} - \boldsymbol{\mu}) \quad (4.202)$$

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$$\boxed{\left\|\mathrm{IF}\left(\mathbf{x}, f_{\mathbf{X}}, \left(\widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}\right)\right)\right\| \propto \|\boldsymbol{\psi}\|} \quad (4.208)$$

$$\boldsymbol{\psi}(\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) \equiv \begin{pmatrix} w\left(\mathrm{Ma}^2(\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma})\right)(\mathbf{x} - \boldsymbol{\mu}) \\ w\left(\mathrm{Ma}^2_{\mathbf{x},\boldsymbol{\mu},\boldsymbol{\Sigma}}\right) \mathrm{vec}\left[(\mathbf{x} - \boldsymbol{\mu})(\mathbf{x} - \boldsymbol{\mu})'\right] - \mathrm{vec}\left[\boldsymbol{\Sigma}\right] \end{pmatrix} \quad (4.207)$$

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normal

$$w \equiv 1$$

Cauchy

$$w(z) = \frac{N+1}{1+z} \quad (4.209)$$

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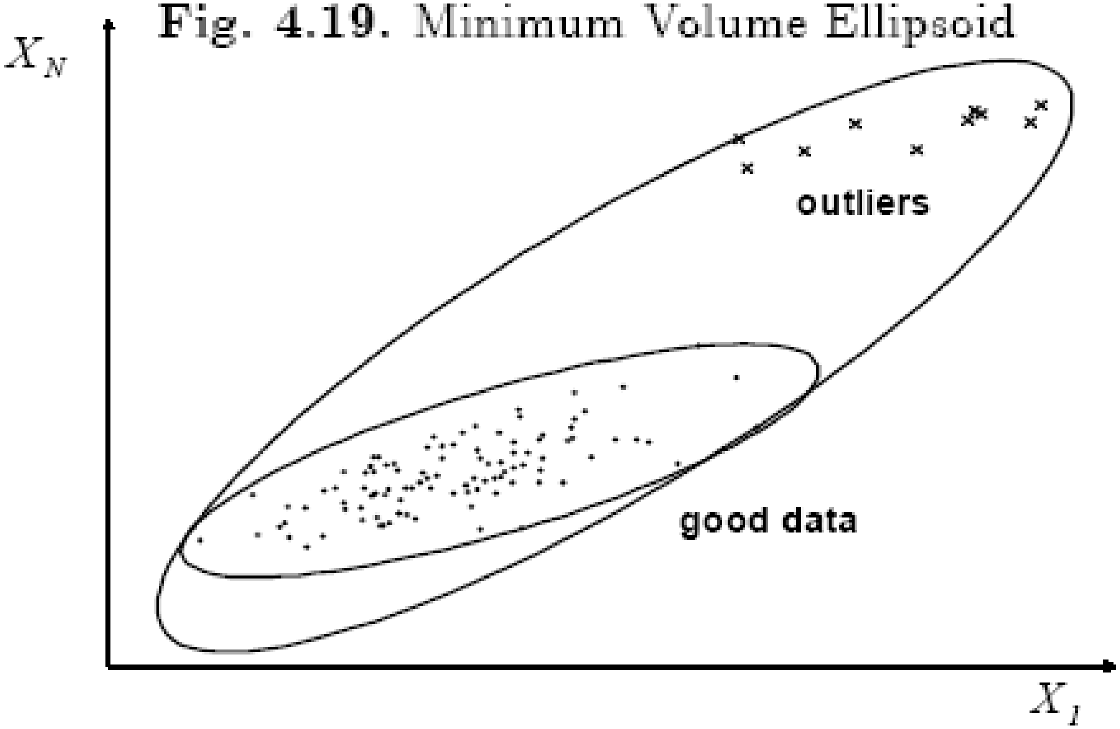
$$\mathcal{E}^{q}_{\boldsymbol{\mu},\boldsymbol{\Sigma}} \equiv \left\{ \mathrm{x} \in \mathbb{R}^{N} \text{ such that } (\mathrm{x}-\boldsymbol{\mu})' \, \boldsymbol{\Sigma}^{-1} (\mathrm{x}-\boldsymbol{\mu}) \leq q^{2} \right\} \quad (4.231)$$

Fig. 4.19. Minimum Volume Ellipsoid



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$$\mathrm{Ma}^{\boldsymbol{\mu},\boldsymbol{\Sigma}}_{t} \equiv \sqrt{(\mathrm{x}_t-\boldsymbol{\mu})' \boldsymbol{\Sigma}^{-1} (\mathrm{x}_t-\boldsymbol{\mu})}. \quad (4.234)$$

$$\mathrm{Vol}\left\{ \mathcal{E}^{q_{T_G}}_{\boldsymbol{\mu},\boldsymbol{\Sigma}} \right\} = \gamma_N \left(\mathrm{Ma}^{\boldsymbol{\mu},\boldsymbol{\Sigma}}_{T_G:T} \right)^N \sqrt{|\boldsymbol{\Sigma}|}. \quad (4.236)$$

$$q_{T_G} \equiv \mathrm{Ma}^{\boldsymbol{\mu},\boldsymbol{\Sigma}}_{T_G:T} \quad (4.235)$$

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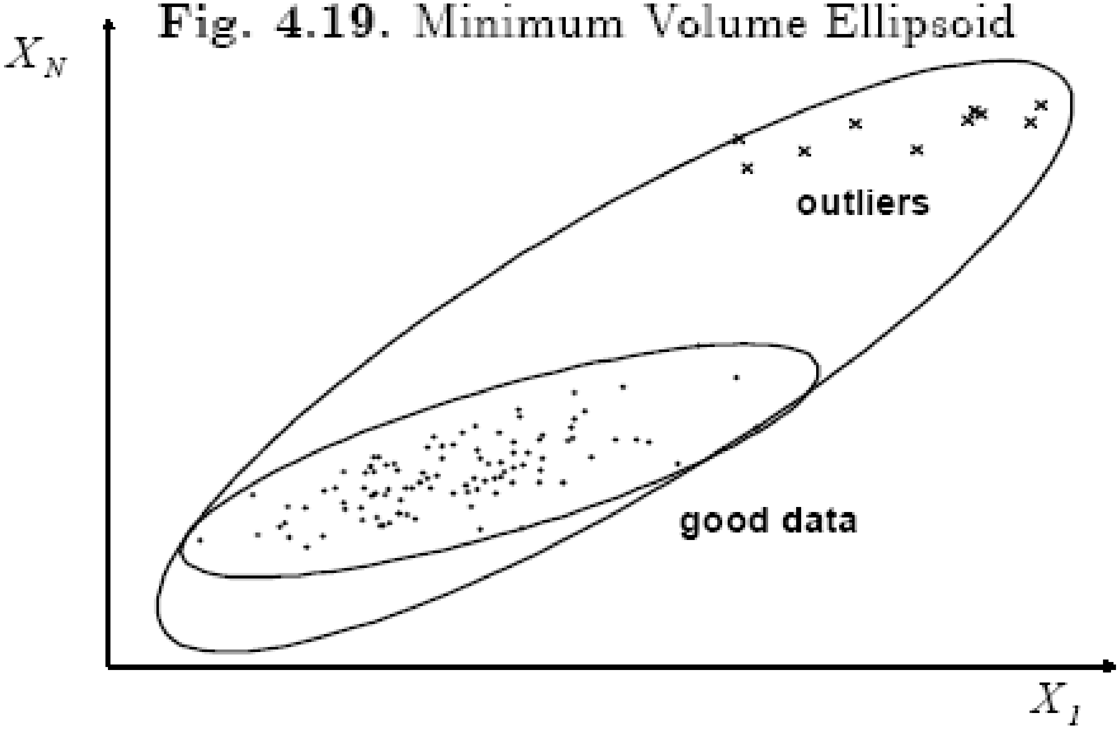

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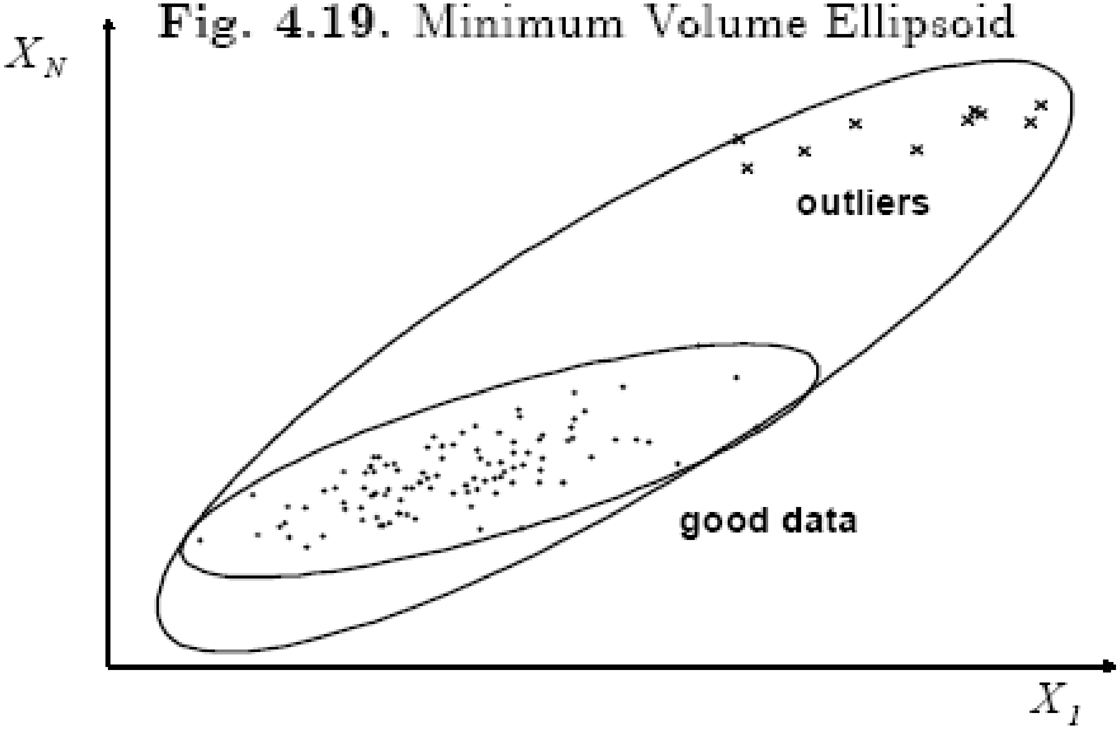

$$\boxed{\left(\widehat{\boldsymbol{\mu}}_{T_G}, \widehat{\boldsymbol{\Sigma}}_{T_G} \right) = \underset{\boldsymbol{\mu},\boldsymbol{\Sigma}\succeq \mathbf{0}, |\boldsymbol{\Sigma}|=1}{\operatorname{argmin}} \left\{ \mathrm{Ma}^{\boldsymbol{\mu},\boldsymbol{\Sigma}}_{T_G:T} \right\}} \quad (4.237)$$