INVESTOR'S OBJECTIVES EVALUATION: STOCHASTIC DOMINANCE

Risk and Asset Allocation* - Springer – *symmys.com

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www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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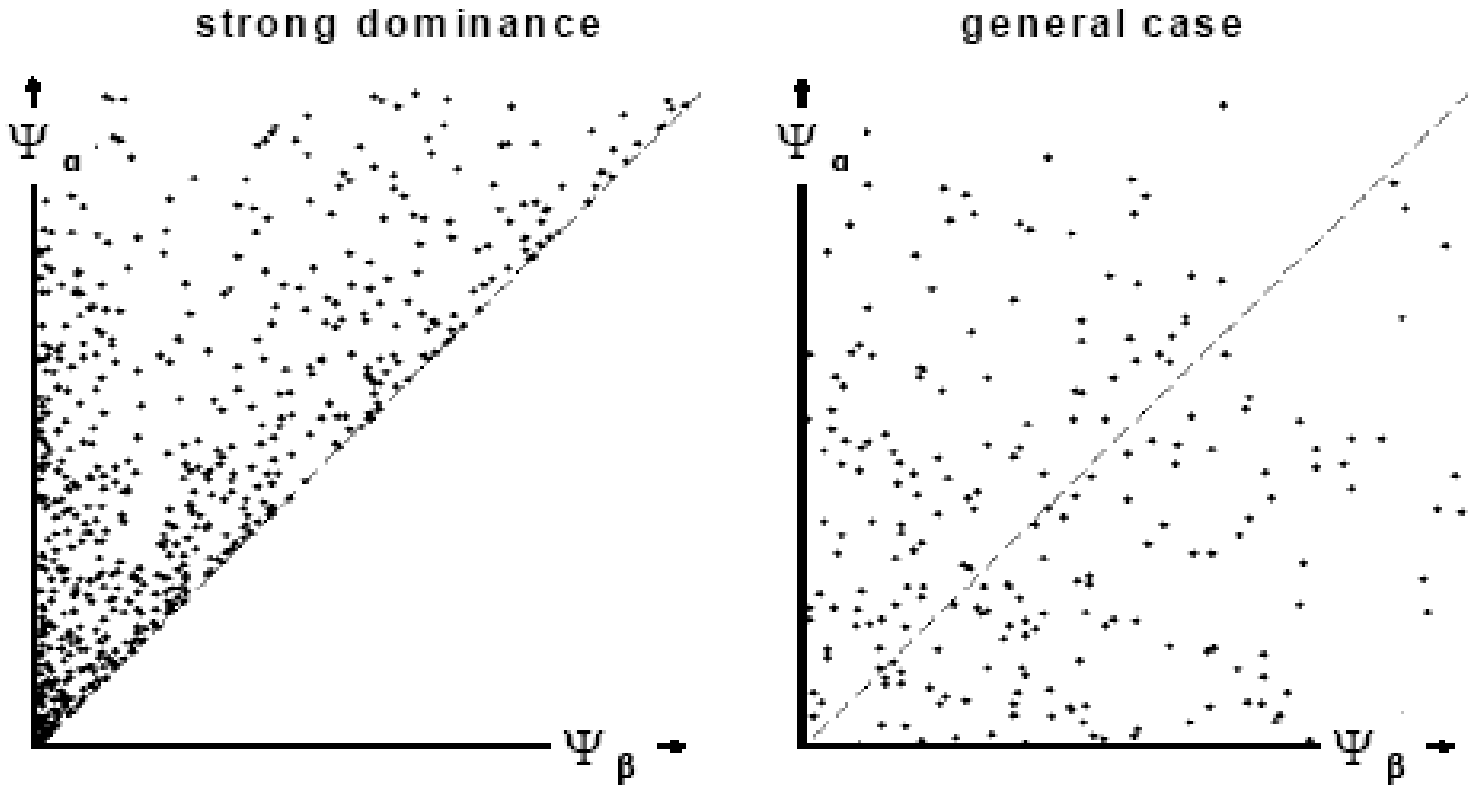
$$\Psi_\alpha = \alpha' \mathbf{M} \quad (5.10)$$

strong dom.: $\Psi_\alpha \geq \Psi_\beta$ in all scenarios. (5.31)

Fig. 5.1



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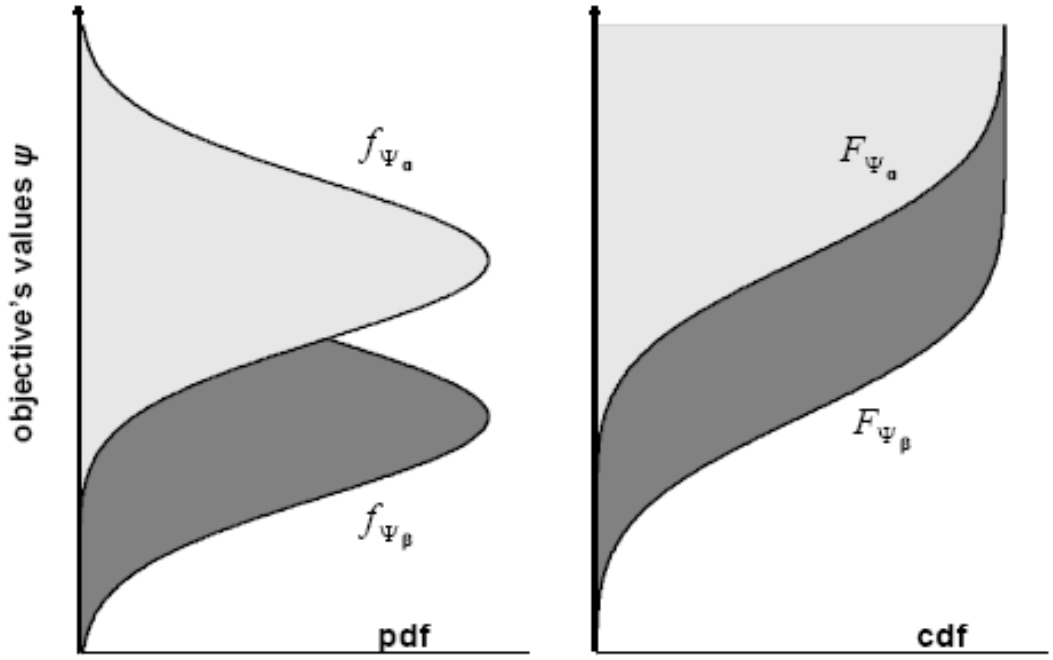
$$\Psi_{\alpha} = \alpha' \mathbf{M} \quad (5.10)$$

strong dom.: $\Psi_{\alpha} \geq \Psi_{\beta}$ in all scenarios. (5.31)

weak dom: $Q_{\Psi_{\alpha}}(p) \geq Q_{\Psi_{\beta}}(p)$ for all $p \in (0,1)$ (5.36)

Fig. 5.2. Weak dominance



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$$\Psi_{\alpha} = \alpha' \mathbf{M} \quad (5.10)$$

strong dom.: $\Psi_{\alpha} \geq \Psi_{\beta}$ in all scenarios. (5.31)

weak dom: $Q_{\Psi_{\alpha}}(p) \geq Q_{\Psi_{\beta}}(p)$ for all $p \in (0,1)$ (5.36)

SSD: $\mathrm{E}\left\{-(\Psi_{\alpha} - \psi)^{-}\right\} \geq \mathrm{E}\left\{-(\Psi_{\beta} - \psi)^{-}\right\}$ for all $\psi \in (-\infty, +\infty)$ (5.43)

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$$\Psi_\alpha = \alpha' \mathbf{M} \quad (5.10)$$

strong dom.: $\Psi_\alpha \geq \Psi_\beta$ in all scenarios. (5.31)

weak dom: $Q_{\Psi_\alpha}(p) \geq Q_{\Psi_\beta}(p)$ for all $p \in (0,1)$ (5.36)

second-order stochastic dominance

$$\mathcal{I}^2 [f_{\Psi_\alpha}](\psi) \leq \mathcal{I}^2 [f_{\Psi_\beta}](\psi) \quad (5.44)$$

$$\mathcal{I}^2 [f_\Psi](\psi) \equiv \mathcal{I}[F_\Psi](\psi) \equiv \int_{-\infty}^{\psi} F_\Psi(s)\, ds \quad (5.45)$$

SSD: $\mathrm{E}\left\{-(\Psi_\alpha - \psi)^-\right\} \geq \mathrm{E}\left\{-(\Psi_\beta - \psi)^-\right\}$ for all $\psi \in (-\infty, +\infty)$ (5.43)

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$$\Psi_\alpha = \alpha' \mathbf{M} \quad (5.10)$$

strong dom.: $\Psi_\alpha \geq \Psi_\beta$ in all scenarios. (5.31)

first-order dominance

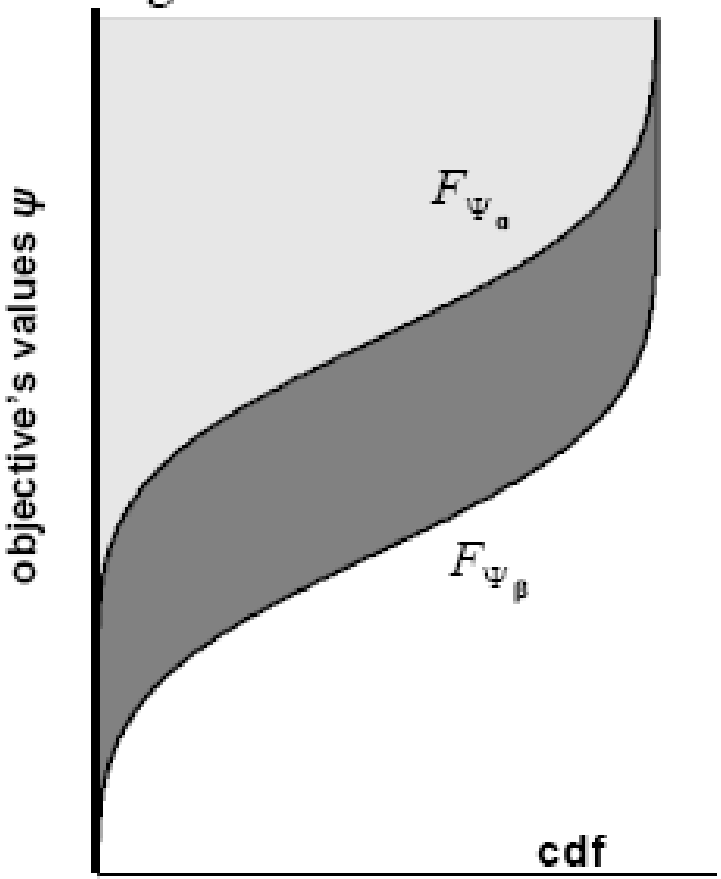
$$F_{\Psi_\alpha}(\psi) \leq F_{\Psi_\beta}(\psi) \text{ for all } \psi \in (-\infty, +\infty) \quad (5.36)$$

weak dom: $Q_{\Psi_\alpha}(p) \geq Q_{\Psi_\beta}(p)$ for all $p \in (0,1)$ (5.36)

second-order stochastic dominance

$$\mathcal{I}^{(2)}[f_{\Psi_\alpha}](\psi) \leq \mathcal{I}^{(2)}[f_{\Psi_\beta}](\psi) \quad (5.44)$$

Fig. 5.2. Weak dominance



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$$\Psi_{\alpha} = \alpha' \mathbf{M} \quad (5.10)$$

strong dom.: $\Psi_{\alpha} \geq \Psi_{\beta}$ in all scenarios. (5.31)

first-order dominance (weak)

$$F_{\Psi_{\alpha}}(\psi) \leq F_{\Psi_{\beta}}(\psi) \text{ for all } \psi \in (-\infty, +\infty) \quad (5.36)$$

second-order stochastic dominance

$$\mathcal{I}^2 [f_{\Psi_{\alpha}}](\psi) \leq \mathcal{I}^2 [f_{\Psi_{\beta}}](\psi) \quad (5.44)$$

order-q dominance

q-dom.: $\mathcal{I}^q [f_{\Psi_{\alpha}}](\psi) \leq \mathcal{I}^q [f_{\Psi_{\beta}}](\psi)$ (5.46)

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$$\Psi_{\alpha} = \alpha' \mathbf{M} \quad (5.10)$$

strong dom.: $\Psi_{\alpha} \geq \Psi_{\beta}$ in all scenarios. (5.31)

$\Downarrow$

first-order dominance (weak)

$$F_{\Psi_{\alpha}}(\psi) \leq F_{\Psi_{\beta}}(\psi) \text{ for all } \psi \in (-\infty, +\infty) \quad (5.36)$$

$\Downarrow$

second-order stochastic dominance

$$\mathcal{I}^2 [f_{\Psi_{\alpha}}](\psi) \leq \mathcal{I}^2 [f_{\Psi_{\beta}}](\psi) \quad (5.44)$$

$\Downarrow$

order-q dominance

q-dom.: $\mathcal{I}^q [f_{\Psi_{\alpha}}](\psi) \leq \mathcal{I}^q [f_{\Psi_{\beta}}](\psi)$ (5.46)

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$$\boxed{\Psi_{\alpha} = \alpha' \mathbf{M}} \quad (5.10)$$

order zero dominance (strong)

$$\Psi_{\alpha} \geq \Psi_{\beta} \text{ in all scenarios.} \quad (5.31)$$

$$\Downarrow$$

first-order dominance (weak)

$$F_{\Psi_{\alpha}}(\psi) \leq F_{\Psi_{\beta}}(\psi) \text{ for all } \psi \in (-\infty, +\infty) \quad (5.36)$$

$$\Downarrow$$

second-order stochastic dominance

$$\mathcal{I}^2 [f_{\Psi_{\alpha}}](\psi) \leq \mathcal{I}^2 [f_{\Psi_{\beta}}](\psi) \quad (5.44)$$

$$\Downarrow$$

order-q dominance

$$q\text{-dom.: } \mathcal{I}^q [f_{\Psi_{\alpha}}](\psi) \leq \mathcal{I}^q [f_{\Psi_{\beta}}](\psi) \quad (5.46)$$

$$0\text{-dom.} \Rightarrow 1\text{-dom.} \Rightarrow \cdots \Rightarrow q\text{-dom.} \quad (5.47)$$