INVESTOR'S OBJECTIVES EVALUATION: QUANTILE & VaR

Risk and Asset Allocation **- Springer – *symmys.com***

Attilio Meucci

www.symmys.com

Formulas and figures in this presentation refer to the book **Risk and Asset Allocation**, Springer.

The notation, say, (5.24) refers to Formula 24 in Chapter 5 of the book

The notation, say, (T4.12) refers to Formula 12 in the Technical Appendices for Chapter 4, which can be downloaded from www.symmys.com

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$$\mathbb{P}\left\{w_T - W_{T+\tau} < L_{\max}\right\} \geq c. \quad (5.155)$$

$$\mathrm{VaR}_c\left(\boldsymbol{\alpha}\right) \quad (5.158)$$

$$\Psi_{\alpha} \equiv W_{T+\tau}(\alpha) - w_T \qquad (5.156)$$

$$Q_{\Psi_{\alpha}}(1-c) \geq -L_{\max}. \qquad (5.157)$$

$$\Updownarrow$$

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$$\alpha \mapsto \mathrm{Q}_c(\alpha) \equiv Q_{\Psi_\alpha}(1-c) \quad (5.159)$$

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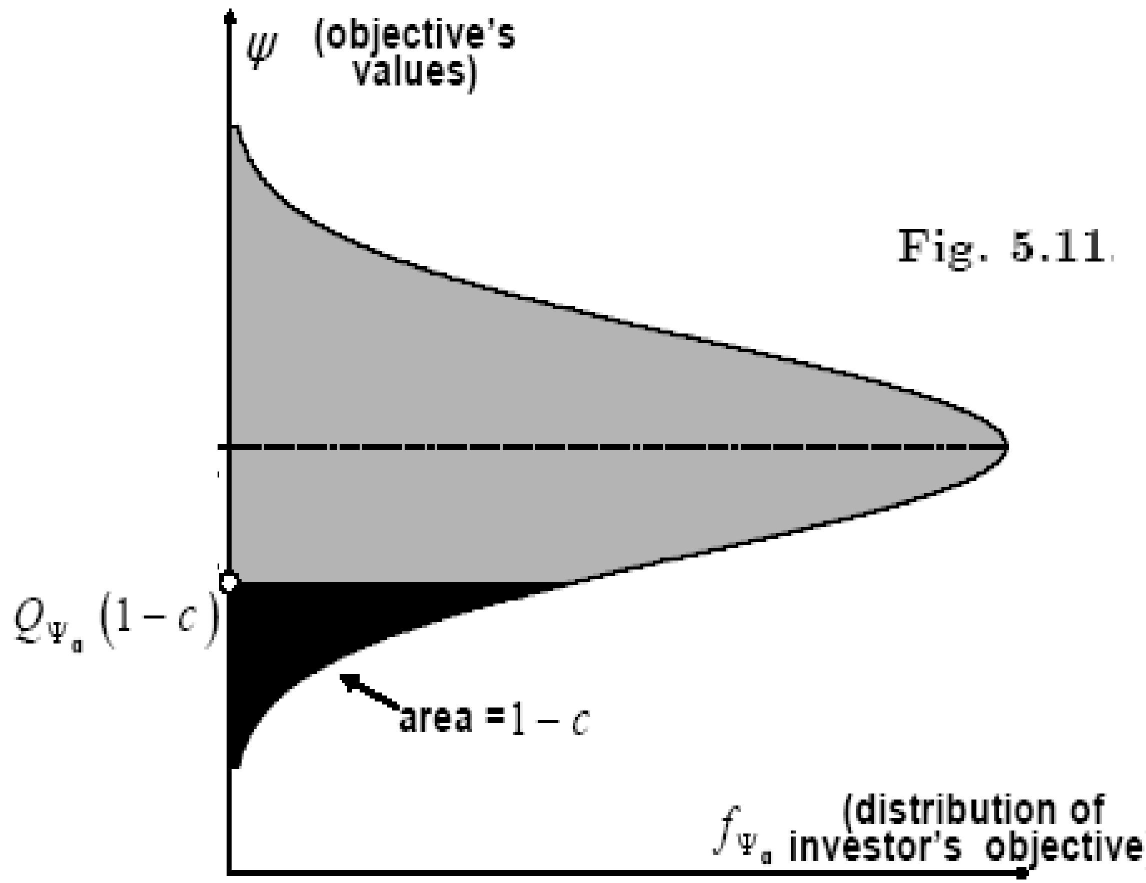

Fig. 5.11

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$$\boxed{\Psi_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}'\mathbf{M}} \quad (5.10)$$

$$\boldsymbol{\alpha} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \equiv Q_{\Psi_{\boldsymbol{\alpha}}}(1-c) \quad (5.159)$$

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \mu_{\boldsymbol{\alpha}} + \sigma_{\boldsymbol{\alpha}} \operatorname{erf}^{-1}(1-2c) \quad (5.175)$$

$$\Psi_{\boldsymbol{\alpha}} \sim \mathrm{N}\left(\mu_{\boldsymbol{\alpha}}, \sigma_{\boldsymbol{\alpha}}^2\right) \quad (5.173)$$

$$\left\{\begin{array}{l} \mu_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\boldsymbol{\mu} - \mathbf{P}_T) \\ \sigma_{\boldsymbol{\alpha}}^2 \equiv \boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \end{array}\right. \quad (5.174)$$

$$\Psi_{\boldsymbol{\alpha}} \equiv \boldsymbol{\alpha}'(\mathbf{P}_{T+\tau} - \mathbf{p}_T) \quad (5.9)$$

$$\mathbf{P}_{T+\tau} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \quad (5.172)$$

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- **Money-equivalence**

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \mu_{\boldsymbol{\alpha}} + \sigma_{\boldsymbol{\alpha}}\,\mathrm{erf}^{-1}(1-2c) \quad (5.175)$$

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- **Money-equivalence**

- **Estimability**

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto F_{\Psi_{\boldsymbol{\alpha}}} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.160)$$

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- **Consistence with stochastic dominance**

$$Q_{\Psi_{\boldsymbol{\alpha}}}(p) \geq Q_{\Psi_{\boldsymbol{\beta}}}(p) \ \text{ for all } p \in (0,1) \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha}) \geq \mathrm{Q}_c(\boldsymbol{\beta}) \quad (5.161)$$

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- **Money-equivalence**

- **Estimability**

$$\boldsymbol{\alpha} \mapsto \Psi_{\boldsymbol{\alpha}} \mapsto F_{\Psi_{\boldsymbol{\alpha}}} \mapsto \mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.160)$$

- **Sensibility**

$$\Psi_{\boldsymbol{\alpha}} \geq \Psi_{\boldsymbol{\beta}} \text{ in all scenarios } \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha}) \geq \mathrm{Q}_c(\boldsymbol{\beta}) \quad (5.162)$$

- **Consistence with stochastic dominance**

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- **Constancy**

$$\Psi_{\mathbf{b}} = \psi_{\mathbf{b}} \Rightarrow \mathrm{Q}_c(\mathbf{b}) = \psi_{\mathbf{b}} \quad (5.163)$$

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$$\mathrm{Q}_c(\lambda \boldsymbol{\alpha}) = \lambda \, \mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.164)$$

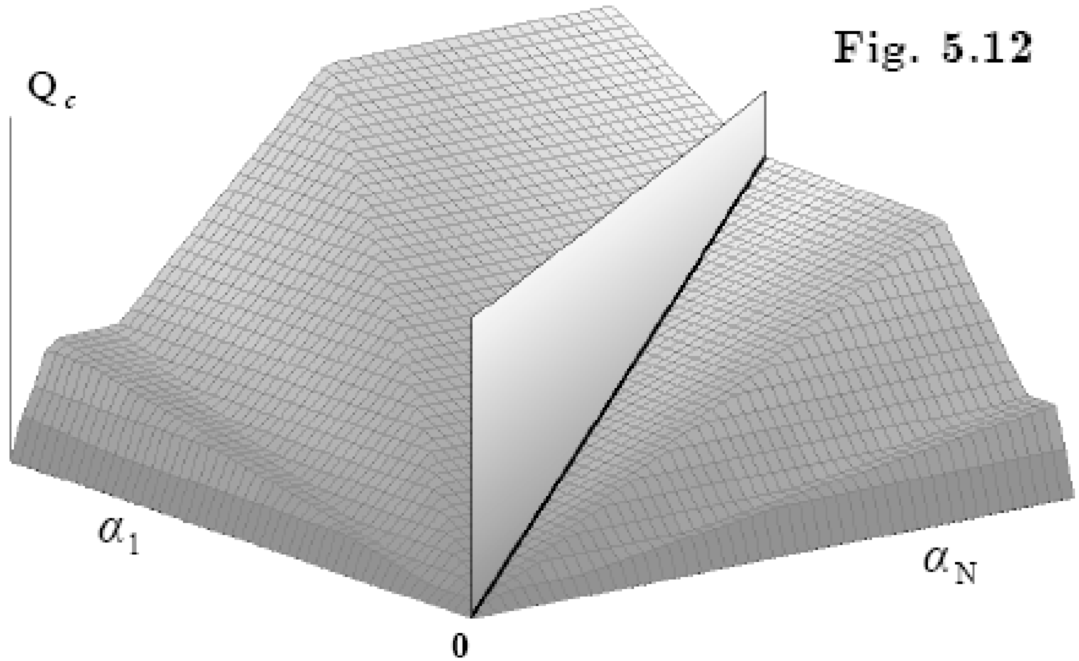

Fig. 5.12

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$$\mathrm{Q}_c(\lambda\boldsymbol{\alpha}) = \lambda \mathrm{Q}_c(\boldsymbol{\alpha}) \quad (5.164)$$

$$\Downarrow$$

Euler:

$$\mathrm{Q}_c(\boldsymbol{\alpha}) = \boldsymbol{\alpha}' \frac{\partial \mathrm{Q}_c(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}} \quad (5.188)$$

$$\frac{\partial \mathrm{Q}_c(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha}} = \boldsymbol{\mu} - \mathbf{p}_T + \frac{\boldsymbol{\Sigma}\boldsymbol{\alpha}}{\sqrt{\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha}}} \sqrt{2} \operatorname{erf}^{-1}(1-2c) \quad (5.189)$$

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$$\Psi_{\mathbf{b}} \equiv 1 \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha} + \lambda\mathbf{b}) = \mathrm{Q}_c(\boldsymbol{\alpha}) + \lambda \quad (5.165)$$

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- ~~Concavity~~

$$\frac{\partial^2 \mathrm{Q}_c(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}'\partial\boldsymbol{\alpha}} = -\left.\frac{\partial \ln f_{\Psi_{\boldsymbol{\alpha}}}(\psi)}{\partial\psi}\right|_{\psi=\mathrm{Q}_c(\boldsymbol{\alpha})} \mathrm{Cov}\{\mathrm{M}|\Psi_{\boldsymbol{\alpha}} = \mathrm{Q}_c(\boldsymbol{\alpha})\} - \left.\frac{\partial\,\mathrm{Cov}\{\mathrm{M}|\Psi_{\boldsymbol{\alpha}} = \psi\}}{\partial\psi}\right|_{\psi=\mathrm{Q}_c(\boldsymbol{\alpha})} \quad (5.191)$$

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- ~~super- additivity~~

- ~~Concavity~~

- ~~Risk aversion~~/~~propensity~~/~~neutrality~~

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- ~~super- additivity~~

- Co-monotonic additivity

$$(\boldsymbol{\alpha}, \boldsymbol{\delta}) \text{ co-monotonic } \Rightarrow \mathrm{Q}_c(\boldsymbol{\alpha} + \boldsymbol{\delta}) = \mathrm{Q}_c(\boldsymbol{\alpha}) + \mathrm{Q}_c(\boldsymbol{\delta}) \quad (5.167)$$

- ~~Concavity~~

- ~~Risk aversion/propensity/neutrality~~