Investing in Mutual Funds with Regime Switching

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Abstract

This paper proposes a Bayesian framework that allows an investor to optimally choose a portfolio of mutual funds in the presence of regime switching in stock market returns. I find that the existence of 'bull' and 'bear' regimes in market returns significantly impacts investor fund choices and that ignoring the regimes imposes large utility costs. For example, an investor with perfect prior confidence in the Capital Asset Pricing Model but who rules out the possibility of managerial skill, would experience a utility loss of 90% or 267 basis points per month in certainty equivalent terms, when failing to account for the regimes. Alternatively, consider an investor whose prior beliefs attach a 5% probability to the event that asset returns will deviate from the CAPM's predictions by ± 4% per year. The cost of ignoring regime switches for such an investor ranges between 81 and 100 basis points per month depending on her prior beliefs in managerial skill.

Investing in Mutual Funds with Regime Switching

There is now compelling evidence that economic systems occasionally transition from one state or regime to another. For example, the macroeconomy periodically switches between booms and recessions. Similarly, stock markets periodically transition between 'bull' and 'bear' market states with each state being characterized by distinctive dynamics. The presence of distinct regimes in economic time series can potentially have a significant impact on investor decisions.[1] One such decision concerns the selection of a portfolio of mutual funds by an investor. The importance of the fund selection decision may be gauged by the size of the assets invested in mutual funds, by the considerable resources devoted by investors to this task, and by their appetite for fund performance statistics and rankings that are widely disseminated by mass media outlets. A natural question that arises in this context is: "Are the potential regime shifts in the economy important for the fund selection decision, and if so, how should investors account for them in their decision making process?"

This paper develops a framework for choosing a portfolio of mutual funds in the presence of regime switching in stock market returns. Specifically, I adopt a Bayesian methodology that allows for regime uncertainty to be incorporated in the investment decision of the investor. I apply the proposed framework to study the optimal choices made by fund investors. The key findings, discussed below, are that the existence of regimes in market returns exerts a strong influence on investor fund choices. Furthermore, ignoring the existence of regimes imposes significant utility costs on investors.

I consider the problem of a mean-variance optimizing investor who chooses a portfolio of no-load stock mutual funds with the highest ex ante Sharpe ratio. The universe of funds available to the investor includes 513 no-load mutual funds that exist as of December 2004 in the CRSP Survivor-Bias Free US Mutual Fund Database. The investor believes that the stock market returns are characterized by two

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[1] For example, Ang and Bekaert (2002, 2004) study the impact of bull and bear market regimes on international asset allocation strategies and find that ignoring regime switching is costly when the investment set includes a conditionally risk free asset. In a multi-asset context, Sa-Aadu, Shilling, and Tiwari (2005) show that the optimal investor portfolios are tilted towards tangible assets such as real estate and precious metals during the bad economic states.

regimes, labeled the ‘bull’ and the ‘bear’ regimes. I consider a two-state Markov regime switching model in order to capture the dynamics of stock market returns. In the present context an appealing feature of a Markov regime switching model is that it can offer important diagnostic information through time. Such a model is particularly suited to the task of analyzing the performance of managed fund portfolios since it can provide a measure of fund performance that takes into account a fund manager’s dynamic factor exposure strategy.

Note that in addition to the uncertainty regarding the economic states, a fund investor also faces two other sources of uncertainty in making her fund selection decision. To see this, recall that the usual procedure for evaluating fund performance requires the investor to rely on a factor model. The investor estimates the slope and intercept parameters in a regression of excess fund returns on the excess returns of certain benchmark assets specified by an asset pricing model. The estimated intercept in such a regression, i.e., the fund alpha, is customarily viewed as a measure of skill or value added by the fund manager. The first uncertainty concerns the investor’s prior belief regarding the degree of pricing error afflicting the asset pricing model used by her in evaluating fund performance. The second uncertainty relates to the investor’s prior beliefs regarding the degree of skill possessed by the fund managers. For example, an investor, before examining the data, could potentially have complete confidence in a model such as the Capital Asset Pricing Model (CAPM). Alternatively, she could be completely skeptical about the validity of the model. At the same time, the investor may possess a range of prior beliefs regarding the skill of mutual fund managers. In each case, the prior beliefs together with the sample evidence shape the investment choices made by the investor. As described in the next section, I examine the investment decisions of investors under a range of beliefs regarding both the validity of the benchmark asset pricing models utilized for performance evaluation, as well as managerial skill.

I explore a Bayesian framework that allows investors to make inference about mutual fund performance in the presence of regime switching in market returns. The investor’s inference problem includes the estimation of the regime switching model, and the identification of the states. The identification of the states allows the investor to obtain estimates of the state-dependent parameters of the fund specific regression model used to evaluate fund performance. The investor combines her prior beliefs with the sample evidence to obtain estimates of the predictive return distribution of fund returns. The moments of the predictive return distribution are then utilized by the investor in choosing the optimal portfolio of funds. Note that the incorporation of regime switching in the decision problem of the investor makes the task of obtaining the predictive return distribution, non-trivial. A key feature of the proposed framework of this paper is the use of the Gibbs sampling procedure to estimate the relevant parameters of interest. The use of the Gibbs sampling procedure makes it possible to estimate a high-dimensional system involving over 500 funds. Importantly, the framework allows for decision making in the context of a large number of assets without the need to specify or optimize the complete likelihood function – a task that would be extremely difficult, if not altogether infeasible, in the context of a regime switching model with several hundred assets and unobserved states.

I find that across a range of prior beliefs regarding the pricing error of the CAPM and the 4-factor Carhart (1997) model, as well as fund manager skill, accounting for regime switching in market returns exerts a strong influence on the optimal fund choices of the investor. In order to gauge the economic significance of regime switching for the fund selection decision I consider the ex ante utility of a mean-variance optimizing investor who recognizes the existence of regimes in market returns. I calculate the certainty equivalent loss experienced by this investor if she were to hold a portfolio that is optimal from the perspective of an investor who fails to account for regime switches in market returns. I find that the economic costs of ignoring regime switching are substantial. For example, an investor who has perfect prior confidence in the CAPM and whose prior beliefs rule out the possibility of managerial skill would experience a loss of 267 basis points per month in certainty equivalent terms. This represents a 90% loss in certainty equivalent terms relative to the investor's optimal portfolio choice. The corresponding utility loss from ignoring regime switching for an investor who has perfect prior confidence in the 4-factor Carhart model is 372 basis points per month, representing a 63% reduction relative to her optimal portfolio.

The costs of ignoring regime switching are somewhat lower for investors with a lesser degree of prior confidence in the model, but they continue to be significant. For instance, consider an investor who regards the CAPM with a degree of skepticism and whose prior beliefs attach a 5% probability to the event that asset returns will deviate from the CAPM's predictions by ± 4% per year. For such an investor the cost of ignoring regime switching still varies between 81 and 100 basis points per month depending on the strength of her prior beliefs in managerial skill.

This paper makes two contributions to the literature on investors' mutual fund selection decision. First, it proposes a formal Bayesian framework to allow investors to incorporate regime switching uncertainty in their decision process. The proposed framework makes it feasible to address regime switching uncertainty even in the context of a portfolio allocation decision involving several hundred mutual funds. Second, the paper provides an assessment of the economic value of accounting for regime switching in market returns when selecting a portfolio of mutual funds. This paper is related to a number of recent studies that analyze the mutual fund choice decision within a Bayesian framework. It is closest in spirit to a series of important papers by Pástor and Stambaugh (2002a, 2002b) who develop a Bayesian framework that allows investors to combine prior beliefs about manager skill and model mispricing with the sample evidence in choosing a portfolio of funds. The present paper extends the Bayesian econometric framework developed by Pástor and Stambaugh to allow for the incorporation of regime switching uncertainty in the investor's fund selection decision. The results of this study suggest that this is potentially quite important from the standpoint of the investor's utility.

In related work Baks, Metrick, and Wachter (2001) investigate the set of prior beliefs about managerial skill that would imply zero investment in active mutual funds for a mean-variance investor. They find that even under extremely skeptical prior beliefs, there is an economically significant allocation to active funds. Jones and Shanken (2005) study how inference about an individual fund's performance is affected by learning about the cross-sectional dispersion in the performance of a large number of other funds. Avramov and Wermers (2005) analyze the mutual fund investment decision in the presence of predictable returns. Busse and Irvine (2005) find that Bayesian estimates of fund alphas based on the

Pástor and Stambaugh (2002a, 2002b) framework are able to predict future fund performance better than the standard frequentist measures of fund alphas. In contrast to this paper, none of the above studies allows for the possibility of regime switching in asset returns. Consequently, the present study addresses an unexplored issue in this literature, namely, the potential impact of regimes in market returns on the fund selection decision of investors.

The rest of the paper proceeds as follows. Section I outlines the decision framework and the Bayesian methodology employed. Section II describes the data and the empirical results. Concluding remarks are offered in Section III.

I. Methodology and Decision Framework

I model the mutual fund selection problem of a Bayesian investor who recognizes the possibility that stock market returns are subject to two distinct regimes, labeled as 'bull' and 'bear' regimes for the sake of convenience. The investor's objective is to choose the portfolio of funds with the highest ex ante Sharpe ratio. In evaluating the candidate mutual funds, the investor makes use of her subjective prior beliefs regarding the skill possessed by, or equivalently, the value added by fund managers. The investor also has prior beliefs about the degree of pricing error inherent in an asset pricing model that is employed to evaluate fund performance. These prior beliefs when combined with sample evidence allow the investor to make an inference about the predictive return distributions for the set of candidate mutual funds available for investment at a point in time. The estimated moments of the predictive return distributions are then used as inputs in the optimization problem of the investor.

The possibility of regime switching in market portfolio returns makes the above problem non-trivial. One complication is that the state variable governing the evolution of regimes is unobserved. Furthermore, even the simplest model of regime switching, gives rise to an extremely high-dimensional system as the number of candidate mutual funds available for investment is quite large. Below I describe the methodology used to address these issues.

A. Specification of the Regime-Switching Model

I adopt a parsimonious two-state Markov regime switching model to capture the dynamics of the stock market returns.[2] The model captures the notion that the market portfolio returns are subject to 'bull' and 'bear' regimes. Specifically, I model the market portfolio return as a stochastic process that is subject to changes in its mean and variance due to shifts in the underlying state or regime represented by an unobserved variable S_t, $S_t \in S = (1, 2)$, that is described by a 2-state Markov chain. More formally, the stochastic processes governing the market returns can be expressed as

$$r_{m,t} | (S_t = 1) \sim N(\mu_1, \sigma_1^2)$$
$$r_{m,t} | (S_t = 2) \sim N(\mu_2, \sigma_2^2)$$

The unobserved variable S_t evolves according to a two-state, first-order Markov-switching process with transition probability matrix given by

$$\begin{pmatrix} P & (1-P) \\ (1-Q) & Q \end{pmatrix}$$

To estimate the parameters of the above model, I adopt the Bayesian estimation approach of Kim and Nelson (1999). Under this approach, both the Markov-switching variable S_t, (t = 1, 2, 3, …T), and the model's unknown parameters, μ_1, μ_2, σ_1^2, σ_2^2, P, and Q, are treated as random variables. Under the assumption that, conditional on the vector $\widetilde{S}_T = [S_1 \quad S_2 \quad S_3 \quad .... \quad S_T]$, the transition probabilities P and Q are independent of the other parameters of the model and the observed data, Bayesian estimation of the model can be carried out using the Gibbs sampling procedure. The procedure is discussed further later in this section and details are provided in the Appendix. The next sub-section describes the inference problem of the investor who assesses the performance of funds in light of her prior beliefs regarding benchmark model accuracy and managerial skill.

[2] Unless otherwise noted, I use the term 'return(s)' to denote the rate of return in excess of the risk free return.

B. Making inference about fund performance in the presence of model pricing uncertainty

The conventional measure of the skill of a fund manager is the fund alpha defined with respect to a benchmark model. In the context of the regime-switching framework, consider the following regression of a fund's returns on a set of *k* benchmark asset returns:

$$r_{A,t} = \alpha_A^{S_t} + B_A^{S_t} r_{B,t} + \varepsilon_{A,t} \tag{1}$$

where $r_{A,t}$ is the fund's excess return in month *t*, and $r_{B,t}$ is the *k x 1* vector of excess returns on the benchmark assets relevant to the pricing model. A particular asset pricing model specifies the set of benchmark assets that should be used to evaluate fund performance. Of course, it is well known that if the asset pricing model is mis-specified, the above alpha may be non-zero even in the absence of true skill on the part of the fund manager. Hence, the relevant question for an investor is how best to disentangle model pricing error from true skill?

In order to distinguish between the pricing error in a model and managerial skill, consider the following multivariate regression involving excess returns on *m* non-benchmark assets:

$$r_{N,t} = \alpha_N^{S_t} + B_N^{S_t} r_{B,t} + \varepsilon_{N,t} \tag{2}$$

Here $r_{N,t}$ denotes the $m \, x \, 1$ vector of excess returns on *m* non-benchmark assets while $r_{B,t}$ denotes the excess returns on the *k* benchmark assets returns relevant to an asset pricing model. Let the variance-covariance matrix of $\varepsilon_{N,t}$ be denoted by Σ^{S_t}. Clearly, a non-zero estimate of $\alpha_N^{S_t}$ provides evidence against perfect pricing ability of the candidate asset pricing model. If the investor admits the possibility of less than perfect pricing ability of the model, then a better measure of skill may obtained by the intercept in the following regression of individual fund excess returns on the *p (= m + k)* passive asset returns:

$$r_{A,t} = \delta_A^{S_t} + c'^{S_t}_{AN} r_{N,t} + c'^{S_t}_{AB} r_{B,t} + u_{A,t} \tag{3}$$

where the variance of $u_{A,t}$ is denoted by σ_u^2. Conditional on the realized state, the error terms are assumed to be independently and identically normally distributed across time and uncorrelated across funds. Note that the skill measure $\delta_A^{S_t}$ is defined with respect to a broader set of passive assets compared to the conventional measure, $\alpha_A^{S_t}$ in Equation (1). Clearly, the improvement in inference made possible by such a measure is partly a function of the choice of the additional non-benchmark assets used to define $\delta_A^{S_t}$. A given set of non-benchmark assets selected by an investor may not necessarily lead to the correct inference about manager skill. Nevertheless, errors in $\delta_A^{S_t}$ as a skill measure imply the inadequacy of $\alpha_A^{S_t}$, while the converse is not necessarily true. Substituting the right hand side of Equation (2) in (3) yields:

$$r_{A,t} = \left(\delta_A^{S_t} + c'^{S_t}_{AN}\alpha_N\right) + \left(c'^{S_t}_{AN}B_N^{S_t} + c'^{S_t}_{AB}\right)r_{B,t} + \left(c'^{S_t}_{AN}\varepsilon_{N,t} + u_{A,t}\right) \tag{4}$$

The first term within parenthesis on the right hand side of Equation (4) may be interpreted as the fund's alpha, conditional on the state or regime, when the investor accounts for uncertainty about model pricing ability and managerial skill.

C. Specification of Prior Beliefs

Investors' prior beliefs about model pricing and skill are specified as follows. First consider the parameters of Equation (2). The prior distribution for the covariance matrix Σ of the error terms $\varepsilon_{N,t}$ is specified as inverted Wishart:

$$\left(\Sigma^{S_t}\right)^{-1} \sim W\left(H^{-1}, \nu\right) \tag{5}$$

The prior precision matrix H is specified as $H = s^2(\nu - m - 1)I_m$, so that $E\left(\Sigma^{S_t}\right) = s^2 I_m$. I use an empirical Bayes approach to set the value of s^2 equal to the average of the diagonal elements of the sample OLS estimate of Σ using data for the period 1962 to 2004. I set the value of ν, the prior degrees of freedom, equal to $m+3$ in order to ensure that the prior contains little information. The priors for the slope coefficients B_N in Equation (2) are assumed to be diffuse. Conditional on Σ^{S_t}, the prior for $\alpha_N^{S_t}$ is specified as:

$$\alpha_N^{S_t} \left| \Sigma^{S_t} \right. \sim N\left(0, \sigma^2_{\alpha N}\left(\frac{1}{s^2}\Sigma^{S_t}\right)\right) \tag{6}$$

The above specification links the conditional prior covariance matrix for α_N to Σ and is similar to that employed by Pástor and Stambaugh (1999, 2002a, 2002b) and Pástor (2000).[3] A variety of prior beliefs regarding the pricing ability of an asset pricing model can be allowed for by choosing different values for $\sigma_{\alpha N}$, the standard deviation of the marginal prior distribution for the elements of α_N.[4] For example, the beliefs of an investor who has perfect confidence in the pricing ability of the model, can be represented by $\sigma_{\alpha N} = 0$. Note that this is equivalent to setting α_N equal to zero indicating that the benchmark assets have perfect ability to price the non-benchmark assets. At the other end of the spectrum, diffuse prior beliefs can be represented by $\sigma_{\alpha N} = \infty$. Prior beliefs representing less than perfect, but moderate degrees of confidence may be represented by setting $\sigma_{\alpha N}$ equal to non-zero, finite positive values.

Next consider the priors for elements of Equation (3). The prior distribution of σ_u^2, the variance of the individual fund error term $u_{A,t}$ is specified as inverted gamma:

$$\sigma_u^2 \sim \frac{\nu_0 s_0^2}{\chi_{\nu_0}^2}$$

Conditional on σ_u^2, the prior for managerial skill for a given fund, δ_A, is specified to be identical across regimes, as a Normal distribution,

$$\delta_A^{S_t} \left| \sigma_u^2 \sim N\left(\delta_0 , \left(\frac{\sigma_u^2}{E\left(\sigma_u^2\right)} \right) \sigma_\delta^2 \right) \right. \tag{7}$$

[3] The prior specification in Equation (6) is motivated by the recognition that one can achieve portfolios of passive assets with large Sharpe ratios if the elements of $\alpha_N^{S_t}$ are large when the elements of Σ^{S_t} are small (MacKinlay (1995)). Making the conditional prior covariance matrix of α_N proportional to Σ^{S_t}, as in (6), results in a lower prior probability of such an event relative to the case when the elements of α_N are distributed independently of Σ^{S_t}.

[4] This measure of pricing uncertainty was proposed by Pástor and Stambaugh (1999).

Note that under the above specification, the prior variance of δ_A is directly proportional to fund residual variance, σ_u^2. Intuitively, if the benchmark assets do a poor job of explaining the variance of the fund's returns (i.e., σ_u^2 is high), the manager is more likely to be able to deliver a large value of δ_A. To examine different prior beliefs regarding skill, σ_δ, the marginal prior standard deviation of δ_A is set to different values. For example, extreme prior skepticism about managerial skill is captured by specifying $\sigma_\delta = 0$. At the other extreme, the beliefs of an investor who admits the possibility of essentially unbounded managerial skill, may be characterized by $\sigma_\delta = \infty$. Finite, positive values of σ_δ can be used to characterize modest prior beliefs in managerial skill.

The prior mean level of skill, δ_0, is specified to be the same across regimes and set equal to (the negative of) the costs incurred by the fund. Specifically, when the prior belief of the investor rules out the possibility of managerial skill ($\sigma_\delta = 0$), following Pástor and Stambaugh (2002b), I specify

$$\delta_0 = -\frac{1}{12}\left(Expense + 0.01xTurnover\right)$$

where *Expense* denotes the average annual expense ratio for the fund and *Turnover* represents the average annual turnover of the fund. Intuitively, in the absence of skill, the fund's skill measure should simply reflect its operational costs. Multiplying the fund portfolio turnover by 0.01 is equivalent to assuming a round trip transaction cost of 1 percent for the fund. This is roughly equal to the 95 basis point estimate provided by Carhart (1997) based on a cross-sectional regression of the estimated fund alphas on fund characteristics such as turnover. For investor beliefs that admit the possibility of managerial skill, I specify $\delta_0 = -\frac{1}{12}\left(Expense\right)$. Such a specification implicitly assumes that when the fund manager is believed to be skilled, portfolio turnover is not necessarily a deadweight cost that negatively impacts fund performance. In other words, given the possibility of managerial skill, high portfolio turnover is likely to be accompanied by high performance.

Next consider the priors for the slope coefficients in Equation (3). Let the vector c_A be defined as $c_A^{S_t} = \left(c'^{S_t}_{AN} \quad c'^{S_t}_{AB} \right)$. The conditional prior distribution of $c_A^{S_t}$ is specified as

$$c_A^{S_t} \Big| \sigma_u^2 \sim N\left(c_0, \left(\frac{\sigma_u^2}{E\left(\sigma_u^2\right)} \right) \Phi_c \right) \tag{8}$$

I use an empirical Bayes procedure to choose values for the mean vector c_0 and the covariance matrix of slope coefficients Φ_c. Specifically, their values are set equal to the sample cross-sectional moments of $\hat{c}_A$, the OLS estimate of c_A, for all funds having the same investment objective as the subject fund. Similarly, the estimated cross-sectional mean and variance of the fund-specific residuals, $\hat{\sigma}_u^2$, are utilized in the above specification of the prior for c_A. Note that the prior beliefs with respect to the fund specific coefficients c_A are assumed to be similar across the two regimes.

The priors for the conditional expected benchmark returns vector, $E_B^{S_t}$, and the covariance matrix of benchmark returns, $V_{BB}^{S_t}$, are assumed to be diffuse:

$$p\left(E_B^{S_t}\right) \propto 1$$
$$p\left(V_{BB}^{S_t}\right) \propto \left|V_{BB}\right|^{-(k+1)/2}$$

Finally, the prior distributions for the transition probabilities, P and Q, are assumed to be independent beta distributions with hyperparameters $u_{i,j}, i, j = 1, 2$:

$$P \sim beta\left(u_{11}, u_{12}\right)$$
$$Q \sim beta\left(u_{22}, u_{21}\right)$$

Let R denote the returns on the benchmark and non-benchmark assets, as well as the mutual funds, through month T and let θ denote the parameters of the model. With the above complete specification of priors, the investor forms her posterior beliefs in light of the sample information:

$$p(\theta|R) \propto p(\theta) \times p(R|\theta)$$

In choosing the fund portfolio with the highest Sharpe ratio, the investor makes use of the predictive return distributions for the candidate funds. The next subsection formally describes the investment problem of the investor as well as the choice of benchmark model/assets and the non-benchmark assets utilized in making inference about individual fund performance.

D. The Investor's Decision Problem

The investor chooses a portfolio of no-load stock mutual funds with the highest ex ante Sharpe ratio as of December 2004. The investor uses the moments of the predictive distribution of fund returns in computing the Sharpe ratios and in solving the optimization problem. The predictive return distribution may be expressed as,

$$p(r_{T+1}|R) = \int_{\theta} p(r_{T+1}|R,\theta)\, p(\theta|R)\, d\theta \tag{9}$$

In the context of the regime switching model considered here and given the large number of individual funds analyzed, the predictive density is not readily obtained. Note that when the investment universe consists of several hundred mutual funds, there are potentially several thousand parameters to be estimated. Accordingly, parameter estimation via the usual method of optimizing the associated likelihood function (see, for example, Hamilton (1989)) becomes extremely difficult.

In order to address this problem I adopt the Gibbs sampling procedure. The use of the Gibbs sampler for the Bayesian analysis of Markov-switching models was popularized by Albert and Chib (1993). The Gibbs sampling procedure is a Markov chain Monte Carlo method that allows the approximation of joint and marginal distributions by sampling repeatedly from the known conditional distributions. The technique is particularly suited to the kind of problem considered here in which the joint density may not be known. However, if the set of conditional densities are known, one can sequentially sample from the conditional density of each parameter (or blocks of parameters), beginning with an arbitrary starting value for the some initial parameters. The unobserved state variable S_T is also treated as an unknown parameter and is generated from its distribution conditional on the other parameters of the model. After a suitable number of burn-in iterations, the Gibbs-sampler is expected to have converged and the subsequent draws of the parameters can be used to conduct inference.

In the context of this paper I employ the Gibbs sampling procedure to obtain estimates of the parameters of interest including the moments of the predictive return distribution of the funds. For each case representing an investor with certain set of prior beliefs, 1000 Gibbs draws are made after discarding an initial burn-in set of 1000 draws.[5] The (state dependent) moments of the predictive return distribution of the funds estimated from these draws are then used by the investor in her optimization problem. The relevant moments are detailed in the appendix. When the investor takes the possibility of regime switching into account she uses the two sets of moments (i.e., one set per state) along with the inferred stationary probabilities of each state, to construct the (unconditional) vector of expected excess returns and covariance matrix for the 513 funds. These moments form the basis of fund allocations under regime switching.

To assess the economic costs of ignoring regime switching, I also compute the optimal fund allocations from the perspective of an investor who ignores regime switching in market returns. These allocations are based on the moments of the predictive return distribution estimated without accounting for regime switching in the market returns. A comparison of the certainty equivalent rates of return (CERs) for the optimal portfolios formed under regime switching versus when regime switching is ignored, provides an measure of the utility costs of ignoring regime switching. I compute the certainty equivalent rate of return (CER) for a given portfolio chosen by the investor, assuming a mean-variance objective,

$$CER = E_p - \frac{1}{2}\lambda\sigma_p^2$$

where in this context E_p and σ_p^2 denote the expected return and variance of the investor's overall portfolio that includes an investment in the one month U.S. T-bills in addition to the optimal fund portfolio. The risk aversion coefficient λ, is set equal to 2.25, which is the level of risk aversion at which an investor would allocate 100% to the CRSP value weighted market index portfolio over the period 1962

[5] I use a number of formal diagnostic procedures to ensure that the Gibbs sampler has achieved convergence. These include the use of diagnostics proposed by Raftery and Lewis (1995) and Geweke (1992).

to 2004 if the investment universe was restricted solely to this portfolio and the risk free T-bills. The overall portfolio of the investor precludes any short positions in the optimal mutual fund portfolio but allows for long positions (by borrowing at the T-bill rate) subject to a 50% margin requirement in accordance with *Regulation T* of the Federal Reserve Board.

As evident from the earlier discussion, another issue facing the investor concerns the choice of the benchmark model and the non-benchmark assets. I examine investment decisions for beliefs centered on two of the widely used models in the performance evaluation arena. The first model is the Capital Asset Pricing Model. The benchmark return in this case is the market portfolio return. The other model utilized is the 4-factor Carhart model that includes a momentum factor (UMD) in addition to the three Fama-French factors namely, the excess market portfolio return (RMRF), and factor mimicking portfolios for size (SMB), and book-to-market (HML) effects in stock returns. In each case I employ ten industry portfolios as non-benchmark assets. The ten portfolios represent the Durables, Energy, Health, Manufacturing, Non-durables, Retail, Technology, Telecom, and Utility sectors as well as a Miscellaneous category.[6]

II. Empirical Analysis

A. Sample Description

I obtain a sample of domestic no-load stock mutual funds from the CRSP Survivor-Bias Free US Mutual Fund Database. Funds are selected from one of three categories, namely, "Aggressive Growth", "Growth and Income", and "Growth" based on classification codes provided by Wiesenberger ("OBJ"), ICDI ("ICDI_OBJ"), and Strategic Insight ("SI_OBJ"). To be eligible for inclusion, a fund is required to be in existence as of December 2004 and to have at least six years of returns history. Sector funds and specialized funds are specifically excluded. I also exclude multiple share classes of the same fund. This selection procedure yields a sample of 513 unique no-load funds which is described in Table I. As can be seen from the table, funds in the "Growth" category are the most numerous although the "Growth and

[6] Data on all benchmark and non-benchmark portfolios are obtained from the website maintained by Ken French. I thank him for making these data available.

Income" funds account for the bulk of the assets at $362.76 billion. The "Aggressive Growth" funds have the highest average expense ratio at 1.09 percent while the "Growth and Income" category has the lowest expense ratio at 0.57 percent. The latter category of funds also has the lowest annual turnover rate at 41.5 percent.

B. Estimates of the Regime Switching Model

Table II provides estimates of the two-state regime switching model using data on the market portfolio monthly (excess) returns for the period 1962 to 2004. The table presents the posterior means and the standard deviations (in parenthesis) for the parameters of interest. As may be seen from the table, there is evidence of two distinct regimes or states in the data. The first state is characterized by low market excess return of -1.3% per month compared to 2.3% per month in the second state. The returns in the first state are also nearly twice as volatile compared to the second state. Hence, State 1 may be viewed as the 'bear' state while State 2 may be viewed as the 'bull' state.[7] Both states also appear to be persistent with transition probabilities in excess of 0.50. Furthermore, note that the 'bull' state is more persistent than the 'bear' state. Figure I plots the time series of the posterior mean of the probability of the market being in the 'bear' regime. As can be seen from the figure the probability of being in the 'bear' regime peaks during some well known episodes in the stock market including the market crash of October 1987, as well as the market declines during April 1970, October 1974, March 1980, and August, 1998, among others. Interestingly, the 'bear' market probability is seen to be at an all time low during late 1995 – a period highlighted by the initial public offering of equity by Netscape which marked the start of the technology driven boom in the market over the next several years. Next, I examine the impact of these regimes on the fund choices of investors.

[7] I use the labels 'bull' and 'bear' simply for the sake of convenience in distinguishing the two states. Clearly, the two states identified here do not correspond to say, a technical analyst's definition of what a 'bull' and 'bear' market state might be.

C. Optimal Mutual Fund Choices When Ignoring Regime Switching

Tables III and IV report the composition of portfolios with the highest ex ante Sharpe ratios when the investor ignores the possibility of regime switching in the data and centers her beliefs on the CAPM or the 4-factor Carhart model. Results are presented for the two sets of prior investor beliefs with respect to model pricing error uncertainty. These cases are characterized by distinct values for the prior beliefs about the annualized standard deviation of the model pricing error. In the first case ($\sigma_{\alpha N} = 0$) , the investor has perfect confidence in the ability of each model to price non-benchmark assets. In the second case ($\sigma_{\alpha N} = 2\%$), the investor has a moderate degree of confidence in the model. In economic terms, a belief that $\sigma_{\alpha N} = 2\%$, implies that the investor *a priori* attaches a 5% probability to the event that the expected return on non-benchmark asset will deviate from its CAPM (or Carhart model) prediction by ± 4% per year.

For each set of beliefs concerning model pricing error, the investor entertains three priors with regard to the uncertainty surrounding the skill possessed by fund managers. In the first case ($\sigma_{\delta} = 0$), the investor completely rules out the possibility of managerial skill. At the other extreme, the investor believes that there is no limit on the magnitude of the skill possessed by fund managers. The intermediate case of $\sigma_{\delta} = 2\%$, represents modest prior confidence in the skill of fund managers. In economic terms this case represents an investor belief that there is a 2.5% probability of the fund manager delivering a positive abnormal performance of 400 basis points per year.

Note from Panel A of Table III that in the cases where the prior beliefs of the investor rule out the possibility of managerial skill (i.e., the cases in which $\sigma_{\delta} = 0$), her portfolio is generally weighted towards index funds such as SPDRs or DIAMONDS, or towards funds that may mimic the index funds. Not surprisingly, in these cases the correlation of the chosen fund portfolio with the value-weighted market portfolio is quite high at 96 percent, as seen in Panel B of the table. With less than complete confidence in the CAPM's pricing ability ($\sigma_{\delta} = 0$) or when prior beliefs admit the possibility of managerial skill ($\sigma_\delta > 0$), the optimal fund portfolios consist exclusively of actively managed funds. As expected, in these cases, the chosen portfolios' correlations with the market portfolio are markedly lower.

The qualitative patterns noted above also hold true for the optimal fund portfolios formed when investor beliefs are centered on the 4-factor Carhart model that includes the returns on a factor mimicking portfolio for the momentum factor in addition to the three Fama-French factors. When the investor's prior beliefs rule out the possibility of managerial skill, the optimal fund portfolio is chosen to mimic the portfolio representing the optimal combination of the four benchmark factors. As the degree of prior confidence in the pricing model is lowered, and when the prior beliefs allow for the possibility of managerial skill, the optimal fund portfolios are more heavily invested in active funds. Collectively, the results in Tables III and IV highlight the importance of prior beliefs regarding model pricing error and fund manager skill in determining the optimal fund choices of the investor.

D. Optimal Mutual Fund Choices With Regime Switching

I next examine the composition of optimal fund portfolios when investors explicitly account for the possibility of regime switching. Recall that an investor who accounts for regime switching in market returns uses as her optimization inputs, the weighted average of the two sets of state-dependent moments of the fund return distributions. The weights represent the stationary probabilities for the two states as inferred from the estimated transition probability matrix. The implied stationary probabilities for states 1 and 2 are 0.34 and 0.66, respectively. Tables V and VI present results for the cases when investor beliefs are centered on the CAPM, and the 4-factor Carhart model. For each case representing a combination of prior beliefs in the model under consideration and managerial skill, the tables report the top five fund holdings in the optimal portfolio. It is apparent from Table V (a similar conclusion emerges from Table VI) that accounting for potential regime switches significantly impacts the optimal fund choices. In particular, it may be inferred that allocations are now spread out over a larger number of funds as the top five holdings collectively account for less than 20 percent of the portfolio. Furthermore, the allocations to index funds appear to be diminished even in cases where the possibility of managerial skill is ruled out

($\sigma_\delta = 0$). Intuitively, under regime switching, suitable combinations of active funds exist that may dominate pure index fund portfolio combinations. To see this, note that an investor who accounts for regime switching in market returns, is aware of the fact that the risk premium on the market portfolio (RMRF) is in fact negative in the 'bear' market state. Hence, her optimal fund allocation would reflect a desire to hedge against this outcome in the 'bear' state. Accordingly, her exposure to index funds that stay fully invested in the market would be lower relative to the optimal fund portfolio of an investor who ignores the existence of distinct regimes in market returns. Of course, the relevant question to ask is "Does recognition of regime switching matter from the perspective of investor welfare?" I address this issue in the next subsection.

E. Is it Costly to Ignore Regime Switching When Selecting Mutual Funds?

Table VII presents the differences in certainty equivalent rates of return (CER) for fund portfolios that are optimally chosen under a given set of beliefs and when accounting for regime switches relative to portfolios that are optimal for the same beliefs but when regime switching is ignored. From the perspective of an investor who believes in regime switching, the latter set of portfolios is likely to be sub-optimal. The relevant question is whether the differences are meaningful in the eyes of the investor. The CER differences reported in Table VII help answer this question. The certainty equivalents are computed using the predictive moments perceived by the investor who accounts for the possibility of regime switches. The investor is assumed to optimize her utility defined over the mean and variance of the fund portfolio. She is also assumed to have a coefficient of relative risk aversion equal to 2.25.[8] In calculating the CER figures, short positions in fund portfolios are ruled out as most funds disallow short sales. Investors are allowed to take long positions in the chosen optimal fund portfolio by borrowing at the risk free rate subject to a 50% margin requirement that is consistent with the Federal Reserve Board's *Regulation T*. Intuitively, the CER differentials provide an economic measure of the importance of regime

[8] As noted previously, this value characterizes the risk aversion of an investor who would have allocated 100% to the market portfolio during the period 1962-2004, if the investment universe consisted solely of the market portfolio and one month U.S. T-bills.

switching for the investor's mutual fund selection decision. Another way to interpret these differences is to think of them as the utility loss experienced by an investor who believes in regime switching but is forced to hold the sub-optimal portfolio based on ignoring the possibility of regime switches.

Panel A of Table VII reports the CER differences when the investor is less than completely skeptical (i.e., $\sigma_{\alpha N} < \infty$) about the pricing ability of the two models considered here. It is clear that the costs of ignoring regime switching are substantial in economic terms. An investor who has complete faith in the pricing ability of the CAPM and who rules out the possibility of fund manager skill, experiences a 90% reduction in certainty equivalent terms (267 basis points per month) if forced to ignore the possibility of regime switches. It is worth emphasizing that an investor with complete confidence in the pricing ability of the CAPM but who recognizes the existence of two distinct regimes, will take into account the fact that the market portfolio's expected return in the 'bear' state is in fact, quite poor. Her optimal fund portfolio will reflect this possibility and will be tailored to provide a hedge against such a market downturn. On the other hand, an investor with complete confidence in the CAPM, but who ignores the existence of regimes will choose to always hold a portfolio of funds that has a high correlation with the market portfolio.

The CER differences decline as the possibility of managerial skill is admitted or when the confidence in the CAPM is moderated. Note however, that even when the prior beliefs of the investor rule out any limits on the possibility of managerial skill ($\sigma_{\delta} = \infty$) and when confidence in the CAPM is less than perfect ($\sigma_{\alpha N} = 2\%$), ignoring regime switching still results in a reduction in CER of 81 basis points per month which represents a 56% loss relative to the optimal fund portfolio.

Similar conclusions emerge when considering prior beliefs centered on the 4-factor Carhart model. For instance, when the investor has perfect confidence in the model's pricing ability and rules out the possibility of fund manager skill, ignoring regime switching leads to a loss in CER of 372 basis points per month or a 63% reduction relative to the optimal fund portfolio. As confidence in the model is moderated or as the investor becomes less skeptical about the possibility of fund manager skill, the CER differences decline. Nevertheless, even in the case where the investor admits the possibility of unbounded managerial skill levels and has a moderate confidence in the model's pricing ability, the utility costs of ignoring regime switching are substantial at 60 basis points per month.

Panel B of Table VII presents results for the case when the investor is completely skeptical about the pricing ability of the two models, i.e., when $\sigma_{\alpha N} = \infty$, even though her beliefs are anchored on one of the models. Even in this case we find that the utility costs of ignoring regime switching continue to be substantial. For example, when the investor anchors her beliefs on the CAPM but is extremely skeptical of managerial skill ($\sigma_{\delta} = 0$), her perceived utility loss from ignoring regime switching is 73 basis points in certainty equivalent terms, representing a 67% reduction relative to her optimal fund portfolio choice. The corresponding utility loss for beliefs anchored on the 4-factor Carhart model under extreme skepticism about managerial skill, is a decrease in CER of 121 basis points per month, i.e., a reduction of 85% relative to the optimal portfolio choice. Admitting the possibility of managerial skill mitigates these differences although they continue to be large in economic terms.

In summary, the results of this subsection suggest that the economic costs of ignoring regime switching in the fund investment decision are substantial. This holds true across a range of beliefs regarding uncertainty about model pricing error and fund manager skill. I also find that the costs are most pronounced when the investor has high confidence in the relevant asset pricing model.

III. Conclusion

This paper makes two contributions to the literature on investors' mutual fund selection decision within a Bayesian framework. First, it proposes a framework that allows an investor to incorporate regime switching uncertainty in their decision process. The proposed framework makes it feasible to address regime switching uncertainty in the context of a portfolio allocation decision involving several hundred mutual funds. Second, the paper provides an assessment of the economic value of accounting for regime switching in market returns when selecting a portfolio of mutual funds.

Specifically, I consider the problem of a mean-variance optimizing investor who chooses a portfolio of no-load stock mutual funds with the highest ex ante Sharpe ratio. The universe of funds available to the investor consists of 513 no-load stock mutual funds with at least six years of return history and which exist as of December 2004 in the CRSP Survivor-Bias Free US Mutual Fund Database. The investor believes that the stock market returns are characterized by two regimes, labeled the 'bull' and the 'bear' regimes. I consider a two-state Markov regime switching model in order to capture the dynamics of stock market returns. The proposed framework allows the investor to incorporate prior beliefs regarding pricing error in the asset pricing model used for performance evaluation as well as beliefs about managerial skill. Hence, the framework proposed here extends the analysis of Pastor and Stambaugh (2002a, 2002b) by allowing for regime uncertainty to be considered in addition to investor uncertainty regarding model pricing error and fund manager skill.

I find that for a range of prior beliefs regarding the pricing error of the CAPM and the 4-factor Carhart model, and fund manager skill, recognizing regime switching in market returns exerts a powerful influence on the fund choices of the investor. In order to gauge the economic significance of regime switching for the fund selection decision I calculate the certainty equivalent loss experienced by the investor if she were to ignore the regime switches in market returns. I find that the economic costs of ignoring regime switching are substantial. For example, an investor who has perfect prior confidence in the CAPM and whose prior beliefs rule out the possibility of managerial skill would experience a loss of 90% (267 basis points per month) in certainty equivalent terms. The corresponding utility loss from ignoring regime switching for an investor who has perfect prior confidence in the 4-factor Carhart model is 372 basis points per month, representing a 63% reduction relative to her optimal portfolio.

The costs of ignoring regime switching for investors with a lesser degree of prior confidence in the model continue to be substantial. Consider for instance, an investor who regards the CAPM with a degree of skepticism and whose prior beliefs attach a 5% probability to the event that asset returns will deviate from the CAPM's predictions by ± 4% per year. For such an investor the cost of ignoring regime switching still varies between 81 and 100 basis points per month depending on her prior beliefs in managerial skill.

The central message of this paper is that it is important for investors to recognize the potential regime switches in benchmark returns when evaluating and investing in mutual funds. The analysis presented here can be extended in a number of directions. For example, an obvious extension would be to examine the performance of fund selection strategies that account for regime switches in benchmark returns when making decisions in real-time. Similarly, the analysis can be readily adapted to alternative investor objectives when choosing the mutual fund portfolio. Examples of such alternative objectives may include the optimization of the information ratio, or the maximization of a suitable utility function defined over terminal wealth in a multi-period setting. I look forward to exploring these issues in future extensions of the paper.

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Appendix

General description of the model in the two-state framework

I begin by generating the sequence of states $S_T\left(t=1,2,3,\ldots\ldots,T\right)$ conditional on the set of other parameters in the model including the transition probabilities *P* and *Q*. The conditioning features of the model allow the states to be simulated via the Gibbs sampling procedure. Specifically, the multi-move Gibbs sampling algorithm (see, for example, Kim and Nelson (1999)) is employed for this purpose. Briefly, the generation of the states involves the following procedure. Beginning with arbitrary initial values for the parameters of the model, the following steps may be iterated until convergence is obtained:

Step 1. Generate each S_t from the joint posterior conditional density $g\left(S_t \middle| R_{mT},\theta\right)$ where θ is the vector of the model parameters, and $R_{mT}=\left(R_{m1},R_{m2},\ldots..,R_{mT}\right)$ represent the historical excess returns on the market portfolio. Alternatively, one may generate the entire vector S_T as one block from $g\left(S_t \middle| R_{mT},\theta\right)$.

Step 2. Generate the transition probabilities, *P* and *Q* from the conditional density, $g\left(P,Q \middle| S_t\right)$.

Step 3. Generate θ from $g\left(\theta \middle| R_{mT},S_t\right)$.

Suppose that there are *L* different draws of the complete vector of states. For each draw of the states, I partition the data into two sets, according to the associated states. Using data for a particular state and conditional on the state, one can obtain the relevant posterior moments for the benchmark and non-benchmark asset as well as the conditional moments for the fund returns. The following derivation of the moments is based on Pástor and Stambaugh (2002a, 2002b) who derive the relevant moments in the case without regime switching. In order to avoid clutter, I intentionally suppress the superscripts relating to the states, S_t.

First consider the distribution of the excess returns on the passive assets, i.e., the *m* non-benchmark and the *k* benchmark assets. Define $Y=(r_{N,1,}\cdots,r_{N,T})'$, $X=(r_{B,1,}\cdots,r_{B,T})'$, *and* $Z=(\iota_T\ \ X)$, where ι_T denotes a *T*-vector of ones. Also define the $(k+1)\times m$ matrix $G=(\alpha_N\ \ B_N)'$, and let $g=vec(G)$. For the *T* observations *t=1,...,T*, the regression model in Equation (2) can be written as

$$Y=ZG+U,\quad vec(U)\sim N(0,\Sigma\otimes I_T), \tag{A.1}$$

where $U=(\varepsilon_{N,1,}\cdots,\varepsilon_{N,T})'$. and the superscripts, S_t, are suppressed. Let E_B and V_{BB} denote the mean and covariance matrix of the normal distribution for $r_{B,t}$, let θ_p denote the parameters of the joint distribution of the passive asset returns $(G,\sum,E_B,and\ V_{BB})$, and define the $T\times P$ sample matrix of passive returns, $R_P=(X\ \ Y)$. Next consider the statistics:

$$\hat{G} = (Z'Z)^{-1} Z'Y,\ \hat{g} = vec(\hat{G}),\ \hat{\Sigma} = (Y - Z\hat{G})'(Y - Z\hat{G})/T,\ \hat{E}_B = X'\iota_T/T, \text{ and}$$
$$\hat{V}_{BB} = (X - \iota_T \hat{E}_B')'(X - \iota_T \hat{E}_B')/T$$

Given the assumed priors mentioned previously, the posterior distributions of the covariance matrix of residuals from (A.1) and the slope coefficients, are given by

$$\Sigma^{-1}\big|R_p \sim W\left(T + \nu - k,\ H + T\hat{\Sigma} + \hat{G}'AG\right)$$

$$g\big|\Sigma, R_p \sim N\left(\tilde{g}, \Sigma \otimes F^{-1}\right)$$

The posterior moments of the slope coefficients and the covariance matrix are given by

$$\tilde{g} = E\left(g\big|R_p\right) = \left(I_m \otimes F^{-1}Z'Z\right)\hat{g}$$
$$\tilde{\Sigma} = E\left(\Sigma\big|R_p\right) = \frac{1}{T + \nu - m - k - 1}\left(H + T\hat{\Sigma} + \hat{G}'A\hat{G}\right)$$

where $A = Z'(I_T - ZF^{-1}Z')Z$, $F = (D + Z'Z)$ and D is $(k+1)x(k+1)$ matrix whose (1,1) element is $\frac{s^2}{\sigma^2_{\alpha N}}$, and all other elements are zero.

The relevant posterior moments of the benchmark returns are:

$$\tilde{E}_B = (E_2|R) = \hat{E}_2,\ \tilde{V}_{BB} = E(V_{BB}|R) = \frac{T}{T-k-2}\hat{V}_{BB}$$

where the tildes are used to denote the posterior moments. Further, define $r_{P,T+1} = (r'_{N,T+1} \quad r'_{B,T+1})'$ as the vector of all passive asset returns. The predictive mean vector of the passive asset returns is then given by

$$E^*_P = E(r_{P,T+1} \mid R_P) = \begin{pmatrix} \tilde{\alpha}_N + \tilde{B}_N \tilde{E}_N \\ \tilde{E}_B \end{pmatrix}, \tag{A.2}$$

where $\tilde{\alpha}_N$ and $\tilde{B}_N$ are obtained using $\tilde{g} = vec((\tilde{\alpha}_N \quad \tilde{B}_N)')$. Partition the predictive covariance matrix of the passive assets as

$$V^*_P = Var(r_{P,T+1} \mid R_P) = \begin{bmatrix} V^*_{NN} & V^*_{NB} \\ V^*_{BN} & V^*_{BB} \end{bmatrix}$$

Note that $V^*_P = \begin{bmatrix} B_N V_{BB} B'_N + \Sigma & B_N V_{BB} \\ V_{BB} B'_N & V_{BB} \end{bmatrix}$.

Suppressing the superscripts for the regimes or states, the regression model in Equation (3), can be written as

$$\begin{aligned} r_{A,T+1} &= \delta_A + c'_A r_{P,T+1} + u_{T+1} \\ &= [1 \ \ r'_{P,T+1}]\phi_A + u_{T+1}, \end{aligned} \tag{A.3}$$

where $\phi_A = (\delta_A \ c'_A)'$. Let *R* denote the returns on the mutual funds and the passive assets through period *T*, and let θ_A denote the set of fund-specific parameters ϕ_A and σ^2_u. Under the assumed priors, the posterior distribution is given by

$$\phi_A \mid R_P, r_A, \sigma_u \sim N(\tilde{\phi}_A, \sigma^2_u(\Lambda_0 + Z'_A Z_A)^{-1}), \tag{A.4}$$

$$\sigma_u^2 \mid R_{P,} r_A \sim \frac{h_A}{\chi_T^2 + v_0} \tag{A.5}$$

where

$$\tilde{\phi}_A = \left(\Lambda_0 + Z_A' Z_A\right)^{-1} \left(\Lambda_0 \phi_0 + Z_A' r_A\right) \tag{A.6}$$

$$h_A = v_0 s_0^2 + r_A' r_A + \phi_0' \Lambda_0 \phi_0 - \tilde{\phi}_A' (\Lambda_0 + Z_A' Z_A) \tilde{\phi}_A. \tag{A.7}$$

and $\Lambda_0 = \dfrac{v_0 s_0^2}{v_0 - 2} \begin{bmatrix} \sigma_\delta^2 & 0 \\ 0 & \Phi_c \end{bmatrix}^{-1}$

The fund's expected return, E_A, is calculated as

$$E_A = \delta_A + c_A' E_P = \phi_A' \begin{bmatrix} 1 \\ E_P \end{bmatrix}, \tag{A.8}$$

where

$$E_P = \begin{bmatrix} \alpha_N + B_N E_B \\ E_B \end{bmatrix} \tag{A.9}$$

is the vector of expected returns on all passive assets. The fund's standard deviation of returns, δ_A, is calculated from the same equation as

$$\sigma_A^2 = c_A' V_P c_A + \sigma_u^2, \tag{A.10}$$

where

$$V_P = \begin{bmatrix} B_N V_{BB} B_N' + \Sigma & B_N V_{BB} \\ V_{BB} B_N' & V_{BB} \end{bmatrix} \tag{A.11}$$

Under the assumed (beta) conjugate prior distributions for the transition probabilities *P* and *Q*, their conditional posterior distributions are independent beta distributions. These are noted below in the description of the Gibbs sampling algorithm.

Details of the Gibbs sampling procedure

In order to obtain the regime-dependent moments of the funds' return distributions, I draw samples from the joint posterior distribution of the relevant parameters as described below.

1. Draw a T-dimensional vector of states $S_T \left(t = 1, 2, 3, \ldots\ldots, T\right)$ based on the procedure described earlier in this appendix.

2. Draw from the posterior distribution of the transition probabilities P and Q

$$g\left(P \middle| S_T\right) = beta\left(u_{11} + n_{11}, u_{12} + n_{12}\right)$$
$$g\left(Q \middle| S_T\right) = beta\left(u_{22} + n_{22}, u_{21} + n_{21}\right)$$

where $u_{i,j}, i, j = 1, 2$ are the known hyperparameters of the of the prior (beta) distribution for P and Q, and n_{ij} refers to the transitions from state i to j in a particular draw of the vector S_T.

3. Draw the covariance matrix of non-benchmark asset returns:

$$\left(\Sigma^{S_t}\right)^{-1} | R \sim W\left(T + \nu - k,\ \left(H + T\hat{\Sigma} + \hat{G}' A \hat{G}\right)^{-1}\right)$$

where the parameters of the Wishart distribution are state dependent but the superscript S_t is suppressed for these parameters.

4. Draw from the conditional distribution of the parameter vector g:

$$g^{S_t} | \Sigma^{S_t} \sim N\left(\tilde{g}^{S_t},\ \Sigma^{S_t} \otimes \left(F^{S_t}\right)^{-1}\right)$$

5. Draw from the covariance matrix of benchmark asset returns:

$$\left(V_{BB}^{S_t}\right)^{-1} | R \sim W\left(T - 1,\ \left(T\hat{V}_{BB}^{S_t}\right)^{-1}\right)$$

6. Draw from the expected return distribution of the benchmark returns:

$$E_B^{S_t} \left| V_{BB}^{S_t}, R \sim N\left(\hat{E}_B,\ \left(\frac{1}{T}\hat{V}_{BB}^{S_t}\right)\right)\right.$$

The above sampling scheme yields draws of the posterior moments for the passive (benchmark and non-benchmark) assets:

$$E_p^{S_t} = \begin{pmatrix} \tilde{\alpha}_N^{S_t} + B_N^{S_t} \tilde{E}_N^{S_t} \\ \tilde{E}_B^{S_t} \end{pmatrix}, \text{ and}$$

$$V_P^{S_t} = \begin{bmatrix} V_{NN}^{S_t} & V_{NB}^{S_t} \\ V_{BN}^{S_t} & V_{BB}^{S_t} \end{bmatrix}$$

Conditional on each draw of $E_p^{S_t}$ and $V_p^{S_t}$, one can draw from the posterior distribution of the moments of the individual fund's returns.

7. Draw from the distribution of the fund-specific residual variance for each of the 513 funds:

$$\sigma_u^2 \mid R_{P,} r_A \sim \frac{h_A}{\chi_T^2 + v_0}$$

8. Draw from the conditional distribution of the state-dependent fund specific regression parameters, $\phi_A^{S_t}$ for each of the 513 funds:

$$\phi_A^{S_t} \left| R_p, r_A, \sigma_u \right. = N\left(\tilde{\phi}_A^{S_t}, \sigma_u^2\left(\Lambda_0 + Z_A'^{S_t} Z_A^{S_t}\right)^{-1}\right)$$

9. Draw from the state-dependent conditional posterior return distribution of each of the 513 funds:

$$r_{A,T+1}^{S_t} = N\left(\delta_A^{S_t} + c_A'^{S_t} r_{p,T+1}, \sigma_u^2\right)$$

Thus, with each iteration of the Gibbs sampler, I obtain a set of returns for each of the 513 funds for each of the two states, 1 and 2.

10. Repeat Steps 1 through 9.

The above Gibbs sampling procedure is started with arbitrary initial values. The first 1000 draws are discarded to minimize the impact of the initial values. The draws from the subsequent 1000 iterations are retained. The mean and covariance matrix of the 1000 returns observations for the 513 funds for each state are used as inputs in the investor's optimization problem of choosing the fund portfolio with the highest ex ante Sharpe ratio.

Table I
Descriptive Statistics for Sample of No-Load Mutual Funds

This table presents descriptive statistics for the sample of 513 stock mutual funds obtained from the CRSP Survivor-Bias Free US Mutual Fund Database. The sample consists of no-load funds belonging to three categories and having at least six years of available returns history through December 2004. The fund categories are “Aggressive Growth” (AG), “Growth and Income” (GI), and “Growth” (GR). Funds are classified into one of these categories based on the classification codes provided by Weisenberger and Strategic Insight. Aggregate TNA (total net assets) refers to the aggregate market value of fund assets as of December 2004 and is shown in billions of dollars. Expense ratio is the percentage of investment that shareholders pay for the fund’s operating expenses. Turnover represents the minimum of aggregate purchases or sales of securities during the year, divided by the average TNA. In order to calculate the averages for expense ratio, turnover and stock holdings, I first calculate the yearly TNA-weighted average across all funds within a category during the period 1962 to 2004, and then report a time-series average of the cross-sectional averages.

Item	Fund Category		
	AG	GI	GR
Number of funds	44	184	285
Aggregate TNA	15.90	362.76	183.18
Average expense ratio (%)	1.09	0.57	0.82
Average turnover (%)	79.0	41.5	83.2
Average stock holding (%)	90.38	90.20	90.13
Largest fund as of Dec 2004	Fidelity OTC fund	Vanguard 500 Index/Inv	Fidelity Contra fund
Largest fund TNA	8.14	84.17	44.48

Table II
Estimates of the Regime Switching Model

This table presents parameter estimates from a two-state Markov regime switching model for the monthly returns on the CRSP value-weighted market index in excess of the one-month US T-bill returns for the period 1962-2004. The model is estimated using a Gibbs sampling procedure described in the text. The table shows the posterior means and standard deviations (in parenthesis) of the respective parameters.

Panel A: Estimates of Transition Probabilities	
	Probability Estimate
P	0.63 (0.12)
Q	0.81 (0.09)

Panel B: Estimates of Mean and Standard Deviation of Excess Market Return		
	Mean	Std. Dev
State 1	-0.013 (0.006)	0.061 (0.001)
State 2	0.023 (0.006)	0.033 (0.001)

Table III
Optimal Fund Portfolios for Beliefs Centered on the CAPM: Ignoring Regime Switching

Panel A of the table shows the portfolios of no-load funds with the highest ex ante Sharpe ratios when the investor ignores regime switching. Investor prior beliefs are characterized by varying levels of confidence in the CAPM and varying levels of confidence in the skill of fund managers. Portfolio weights in some columns may not sum to 100 percents as funds having less than 2 percent allocation under each scenario are not shown. Panel B shows the correlation of each fund portfolio with the value-weighted market portfolio of stocks computed with respect to the predictive distribution used to derive the fund portfolio. The universe of candidate funds available for investment consists of 513 no-load stock mutual funds with at least six years of return history through December 2004.

Pricing error uncertainty, $\sigma_{\alpha N}$ (percent per year)	0	0	0	2	2	2
Skill Uncertainty, σ_{δ} (percent per year)	0	2	∞	0	2	∞
Panel A: Portfolio Allocation (%)						
ABN AMRO Veredus Aggressive Growth Fund/N		4	6			9
Ameristock Mutual Fund	5			3		
Cambiar Opportunity Fund		9	9		7	1
CGM Focus Fund		4	4			5
Clipper Focus Fund/PBHG		2	2			
Clipper Fund		1	2			
Columbia Mid-Cap Growth Fund/Z		2	2			
Delphi Value Fund/Retail		1	3			
DFA Invest Grp US Large Cap Value Port	47			4		
Dow Industrials DIAMONDS	9			4		
Excelsior Value and Restructuring		2	0			
Fidelity Advisor Dynamic Cap Apprec/Instl		3	2			
Fidelity New Millennium		6	4			
GMO Tr US Sector Fund/III		2				
Hartford Capital Appreciation Fund/Y		8	6		2	
Hartford HLS Capital Appreciation/IB		3	2		13	12
Henlopen Fund		2	1			
ING Corporate Leaders Trust Fund Series B/A				6		
LWAS/DFA US High Book to Market Port	39			9		
MFS New Discovery Fund/I			2			2
MFS Strategic Value Fund/I		3	5		4	
Nuveen NWQ Multi-Cap Value Fund/R		3	4			
Prudent Bear Fund		1	1		5	6
Santa Barbara Group:Bender Growth Fund/Y		3	2			
SEI Index Funds S&P 500 Index Port/E				6		
Sequoia Fund		5	4			
SPDRs				35		
SSgA Aggressive Equity Fund		5	9			20
Strong Enterprise Fund/Inv		7	6		15	9
Undiscovered Managers Behavioral Growth/Inv		2	1			
Undiscovered Managers Behavioral Value/Instl		14	11		52	36
Value Line Special Situations Fund		3	3			
Vanguard Institutional Index/Instl Plus				33		
Yacktman Focused Fund		2	3			
Panel B: Characteristics of Fund Portfolios						
Correlation with Market Portfolio (percent)	96	94	86	96	83	83

Table IV
Optimal Fund Portfolios for Beliefs Centered on the Carhart Model: Ignoring Regime Switching

This table shows the portfolios of no-load funds with the highest ex ante Sharpe ratios when the investor ignores regime switching. Investor prior beliefs are characterized by varying levels of confidence in the 4-factor Carhart model and varying levels of confidence in the skill of fund managers. Portfolio weights in each column may not sum to 100 percents as funds having less than 2 percent allocation under each scenario are not shown. The universe of candidate funds available for investment consists of 513 no-load stock mutual funds with at least six years of return history through December 2004.

Pricing error uncertainty, $\sigma_{\alpha N}$ (percent per year)	0	0	0	2	2	2
Skill uncertainty, σ_{δ} (percent per year)	0	2	∞	0	2	∞
Panel A: Portfolio Allocation (%)						
Cambiar Opportunity Fund		16	19		21	20
CGM Focus Fund	9			18		
DFA Invest Grp US Large Cap Value Port	8					
Fidelity Advisor Dynamic Cap Apprec/Instl		11	11			
Fidelity Independence		11	9			
Fidelity New Millennium		3	2			
GMO Tr US Sector Fund/III	27			81		
Hartford Capital Appreciation Fund/Y		11	12			
Hartford HLS Capital Appreciation/IB		1	1		11	11
IPS New Frontier Fund		4	4			
LWAS/DFA US High Book to Market Port	9					
Mutual Qualified Fund/Z	45	23	16			
PBHG Large Cap Growth Concentrated		2	2			
Prudent Bear Fund					7	6
SSgA Aggressive Equity Fund			4			13
Strong Enterprise Fund/Inv		12	13		20	20
Undiscovered Managers Behavioral Value/Instl		6	6		41	30

Table V
Top 5 Holdings of Optimal Fund Portfolios for Beliefs Centered on the CAPM With Regime Switching

This table shows the top 5 holdings of portfolios of no-load funds with the highest ex ante Sharpe ratios when the investor accounts for possibility that market returns follow a two-state Markov regime switching model. Investor prior beliefs are characterized by varying levels of confidence in the CAPM and varying levels of confidence in the skill of fund managers. The universe of candidate funds available for investment consists of 513 no-load stock mutual funds with at least six years of return history through December 2004.

Pricing error uncertainty, $\sigma_{\alpha N}$ (percent per year)	0	0	0	2	2	2
Skill uncertainty, σ_{δ} (percent per year)	0	2	∞	0	2	∞
Panel A: Portfolio Allocation (%)						
ABN AMRO Montag & Caldwell Growth Fund/I					3	
American Century Stgc Alloc Agg/Inv	3					
Aquinas Growth Fund				4		
Boston Partners Large Cap Value/Invest						4
Columbia Disciplined Value Fund/Z						3
Credit Suisse Select Equity/Cmn		4				
Dodge & Cox Stock Fund			3			
Dreyfus BASIC S&P 500 Stock Index Fund				3		
Dreyfus Premier Intrinsic Value Fund/R			4			
Fidelity Large Cap Stock						4
GE Funds:US Equity Fund/Y					3	
GMO Tr Growth Fund/III	3					
Goldman Sachs Growth & Income/Inst					2	
Harris Insight Fds Index Fund/Instl						3
Hartford HLS Capital Appreciation/IB	4					
ING Corporate Leaders Trust Fund Series B/A					4	
Janus Aspen Srs:Growth & Income/Ist		3				
LKCM Equity		3				
MainStay Funds All Cap Value/I					2	
Morgan Stanley Instl:Focus Equity/A			3			
Morgan Stanley Instl:Value Equity/B		3				
MTB Grp of Funds:Multi Cap Growth/Instl I		2				
Pacific Capital Value/Y				4		
SEI Index Funds S&P 500 Index Port/E	3					
Smith Barney S&P 500 Index/A						3
Strategic Partners Equity Fund/Z				3		
Strong Advisor US Value Fund/Z			3			
Transamerica Premier Index Fund/Inv				3		
UBS PACE Fds:Large Co Value Equity/P	3					
USAA Mutual Fund:First Start Growth Fund			3			

Table VI
Top 5 Holdings of Optimal Fund Portfolios for Beliefs Centered on the Carhart Model With Regime Switching

This table shows the top 5 holdings of portfolios of no-load funds with the highest ex ante Sharpe ratios when the investor accounts for possibility that market returns follow a two-state Markov regime switching model. Investor prior beliefs are characterized by varying levels of confidence in the 4-factor Carhart model and varying levels of confidence in the skill of fund managers. The universe of candidate funds available for investment consists of 513 no-load stock mutual funds with at least six years of return history through December 2004.

Pricing error uncertainty, $\sigma_{\alpha N}$ (percent per year)	0	0	0	2	2	2
Skill uncertainty, σ_{δ} (percent per year)	0	2	∞	0	2	∞
Panel A: Portfolio Allocation (%)						
ABN AMRO Montag & Caldwell Growth Fund/I	2					
ABN AMRO Value Fund/N			2			
American Century Equity Growth/Inv					5	
American Century Select/Inv					2	
Bridges Investment		2				
Chesapeake Core Growth Fund						3
Dreyfus Premier Value Fund/R			2			
Evergreen Large Cap Equity/I		2				
Fidelity Dividend Growth				3		
Fidelity Freedom 2020						3
Fidelity Independence		2				
Goldman Sachs CORE Large Cap Growth/Inst						3
Hartford HLS Capital Appreciation/IA				3		
Henlopen Fund	3					
IPS Millennium Fund	2					
Nations Strategic Growth/Prim A	2					
Oppenheimer Equity Fund/Y	2					
ProFunds:Bear/Iv						4
Reynolds Funds Opportunity Fund			2			
Schwab MarketTrack All Equity Portfolio				3		
Schwab US MarketMasters Fd/Inv		2				
Selected American Shares/S		2				
State Street Research:Investment Trust/S			2			
State Street Research:Large Cap Value Fd/S					3	
Stratus Growth Portfolio/Instl					3	
T Rowe Price Spectrum Fds Growth Fund				4		
TCW Galileo Large Cap Value Fund/I						3
WP Stewart & Co Growth Fund					3	

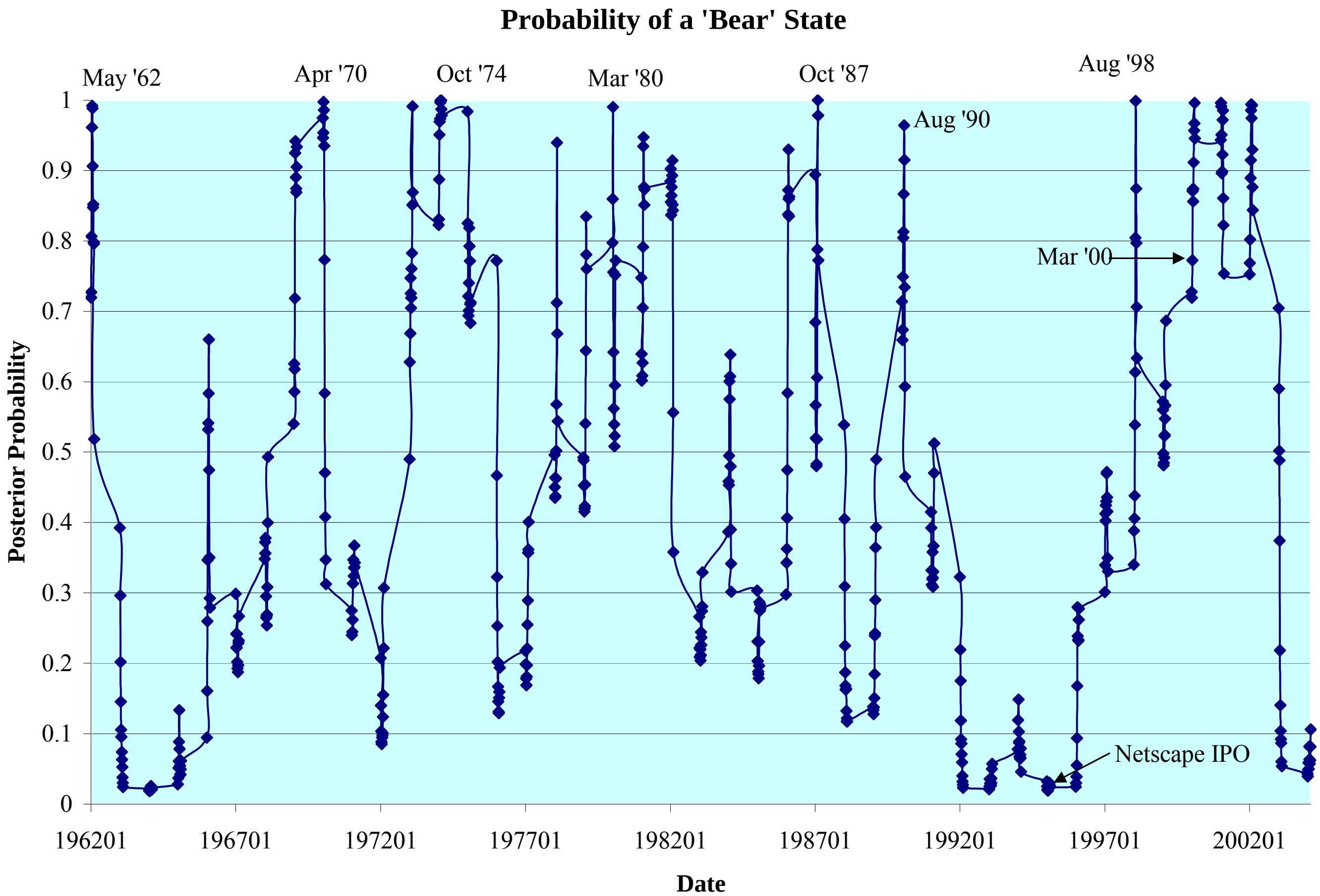
Table VII
Certainty Equivalent Differences for Optimal Fund Portfolios

This table compares the certainty equivalent rates of return (CER) for portfolios that have the highest ex ante Sharpe ratio under regime switching relative to the corresponding optimal portfolios for the case when regime switching is ignored. Both portfolios are formed under a given set of investor prior beliefs regarding model pricing error uncertainty and skill uncertainty. The table reports the CER differences for each pair of portfolios. In each case the CER difference represents the utility loss suffered by a mean-variance optimizing investor who is aware of the regime switches in market returns, but is forced to invest in the fund portfolio that is optimal when regime switching is ignored. The relative risk aversion of the investor is assumed to be 2.25. The universe of candidate funds available for investment consists of 513 no-load stock mutual funds with at least six years of return history through December 2004.

Pricing error uncertainty, $\sigma_{\alpha N}$ (percent per year)	0	0	0	2	2	2
Skill uncertainty, σ_{δ} (percent per year)	0	2	∞	0	2	∞
Panel A: CER differences when beliefs are centered on an asset pricing model						
Model	CER Differences					
CAPM						
CER Diff (bps per month)	267	171	154	87	100	81
CER Diff (as percent of optimal CER)	90	57	49	70	71	56
Carhart 4 factor Model						
CER Diff (bps per month)	372	278	247	90	118	60
CER Diff (as percent of optimal CER)	63	46	40	69	80	42

Panel B: CER differences under complete skepticism about asset pricing models			
Skill uncertainty, σ_{δ} (percent per year)	0	2	∞
Model	CER Differences		
CAPM			
CER Diff (bps per month)	73	58	74
CER Diff (as percent of optimal CER)	67	44	57
Carhart 4-factor Model			
CER Diff (bps per month)	121	77	97
CER Diff (as percent of optimal CER)	85	60	60

Figure I


Probability of a 'Bear' State
May '62
Apr '70
Oct '74
Mar '80
Oct '87
Aug '90
Aug '98
Mar '00
Netscape IPO
Posterior Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
196201
196701
197201
197701
198201
198701
199201
199701
200201
Date