

# Measuring the Specific Heat and the Adiabatic Index of the Air

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May 5, 2023

## Abstract

In this experiment, we used Pasco ideal gas apparatus to measure the specific heat of constant volume  $c_V$ , the specific heat of constant pressure  $c_P$ , and the adiabatic index  $\gamma$  of the air. By letting the air inside the apparatus with an initial temperature of 0 °C to exchange heat with water of higher temperature, we obtained  $c_V = 61.0 \pm 5.7 \text{ J/(g}\cdot\text{K)}$  and  $c_P = 87.9 \pm 5.8 \text{ J/(g}\cdot\text{K)}$  based on the temperature change of the water and the air. While both  $c_V$  and  $c_P$  don't agree with their theoretical values of  $c_V = 0.718 \text{ J/(g}\cdot\text{K)}$  and  $c_P = 1.005 \text{ J/(g}\cdot\text{K)}$ , their ratio is given by  $c_P/c_V = 1.4 \pm 0.2$ , which matches with the theoretical value of  $\gamma \equiv c_P/c_V = 1.4$ . With the apparatus covered with Styrofoam, a good thermal insulator, we tracked the relationship between the pressure and the volume of the air inside the apparatus, with which we obtained  $\gamma = 1.19 \pm 0.02$ . This value also disagrees with the theoretical value. By analyzing the possible sources of error, we conclude that for specific heat measurement, improvements can be made by determining the specific heat of the metal canister of the apparatus and using a liquid other than water with lower specific heat; for adiabatic index measurement, improvements can be made by choosing a better thermal insulating material and using a piston with better airtightness, greater load-bearing capacity, and more precise meter scale in place of the piston on the Pasco apparatus.

# 1 Objectives

In this experiment, we want to measure the following properties of the air in the lab: the specific heat of constant volume  $c_V$ , the specific heat of constant pressure  $c_P$ , and the adiabatic index  $\gamma$ . We will then compare our measured values to the theoretical values of which are given by  $c_V = 0.718 \text{ J/(g}\cdot\text{K)}$ ,  $c_P = 1.005 \text{ J/(g}\cdot\text{K)}$ , and  $\gamma = 1.4$  at room temperature [1][4].

## 2 Theory & Background

### 2.1 Specific Heat of Constant Volume and Constant Pressure

During the process of heat transfer, the specific heat  $c$  of the object satisfies the following relation:

$$Q = cm\Delta T = cm|T_f - T_i| \quad (1)$$

where  $m$  is the mass of the object, and  $T_i$ ,  $T_f$  are the initial and final temperatures of the object before and after the process.

If we allow the object to exchange heat with water, whose specific heat  $c_w = 4.184 \text{ J/(g}\cdot\text{K)}$  and mass  $m_w$  are known to us, then in theory they will have different initial temperatures but share the same final temperature, and the heat gained/lost by the object should be the same as the heat lost/gained by the water, i.e.

$$Q = cm|T_f - T_{i,a}| = c_w m_w |T_f - T_{i,w}| \quad (2)$$

In this experiment, we want the air to exchange heat with the water. However, since we need something to contain the air, the heat exchange caused by the container should also be accounted. Since the container must have the same initial and final temperature as the air it contains, Eq.(2) should be modified to become:

$$Q = (c_a m_a + c_c m_c) |T_f - T_{i,a}| = c_w m_w |T_f - T_{i,w}| \quad (3)$$

where  $c_a$ ,  $c_c$  are the specific heat of the air and the container, and  $m_a$ ,  $m_c$  are the mass of the air and the container.

Rearranging this equation, we can get the specific heat of the air  $c_a$ :

$$c_a = \frac{c_w m_w |T_f - T_{i,w}| - c_c m_c |T_f - T_{i,a}|}{m_a |T_f - T_{i,a}|} = \frac{1}{m_a} \left( \frac{|T_f - T_{i,w}|}{|T_f - T_{i,a}|} c_w m_w - c_c m_c \right) \quad (4)$$

Since it is difficult to measure the mass of the air directly, we can use ideal gas law to calculate the mass of the air based on its initial pressure  $P_i$ , initial volume  $V_i$ , and initial temperature  $T_{i,a}$  using the theoretical molar mass of the air  $M_a = 28.97 \text{ g/mol}$  [2, p.25], as shown below:

$$\begin{aligned} P_i V_i &= n_a R T_{i,a} = \frac{m_a}{M_a} R T_{i,a} \\ \Rightarrow m_a &= \frac{P_i V_i M_a}{R T_{i,a}} \end{aligned} \quad (5)$$

We can then substitute  $c_a$  in Eq.(4) with  $c_V$  and  $c_P$  as we conduct the experiment isochorically and isobarically, and thus the specific heat of constant volume  $c_V$  and constant pressure  $c_P$  of the air can be obtained. Theoretically, we should expect  $c_V = 0.718 \text{ J/(g}\cdot\text{K)}$  and  $c_P = 1.005 \text{ J/(g}\cdot\text{K)}$  [4].

## 2.2 Adiabatic Index

Theoretically the adiabatic index  $\gamma$  of the air is given by:

$$\gamma = \frac{c_P}{c_V} \quad (6)$$

The air mostly consists of diatomic molecules. At room temperature, which is about 298 K, most of the air molecules will have 5 degrees of freedom, 3 of which are translational and 2 of which are rotational [1]. By equipartition theorem,  $c_V$  is given by:

$$c_V = \frac{\text{number of degrees of freedom}}{2} k_B = \frac{5}{2} k_B \quad (7)$$

where  $k_B$  is the Boltzmann constant.

Then, based on the relation between  $c_P$  and  $c_V$ , we have:

$$c_P = c_V + k_B = \frac{5}{2} k_B + k_B = \frac{7}{2} k_B \quad (8)$$

By Eq.(6), the adiabatic index  $\gamma$  of the air should thus be  $\frac{7}{5} = 1.4$ .

During an adiabatic process, we have the equation

$$PV^\gamma = C \quad (9)$$

where  $C$  is a constant. After rearrangement and taking the log of both sides, we have:

$$\begin{aligned} V^\gamma &= C \cdot \frac{1}{P} \\ \Rightarrow \quad \gamma \ln V &= \ln \left( C \cdot \frac{1}{P} \right) \\ \Rightarrow \quad \ln \frac{1}{P} &= \gamma \ln V - b \end{aligned} \quad (10)$$

where  $b \equiv \ln C$ .

Therefore, by plotting  $\ln \frac{1}{P}$  vs.  $\ln V$  and fitting a linear model to it, we can obtain an experimental value for the adiabatic index of the air by taking the slope of the linear fit.

## 3 Procedure

In this experiment, we used the Pasco ideal gas apparatus shown in Figure 1. There was a millimeter scale on the side of the chamber with a movable piston so that we could read off the position of the piston  $h_p$  to calculate the volume of the gas inside the chamber. The

material made up of the air canister was believed to be aluminum, which has a specific heat of  $c_{Al} = 0.91 \text{ J}/(\text{g}\cdot\text{K})$  [3].

We first conducted several measurements: the diameter  $d_c$  and the height  $h_c$  of the canister, the diameter  $d_t$  and length  $l_t$  of the tube, and the diameter of the piston  $d_p$ . With these, we can later calculate the total volume  $V$  of the gas inside the apparatus to be:

$$V = \pi \left( h_c \left( \frac{d_c}{2} \right)^2 + l_t \left( \frac{d_t}{2} \right)^2 + h_p \left( \frac{d_p}{2} \right)^2 \right) \quad (11)$$

In addition, we also measured the mass of the metal canister  $m_c$ , so that we could use it to calculate the heat contributed by the canister.

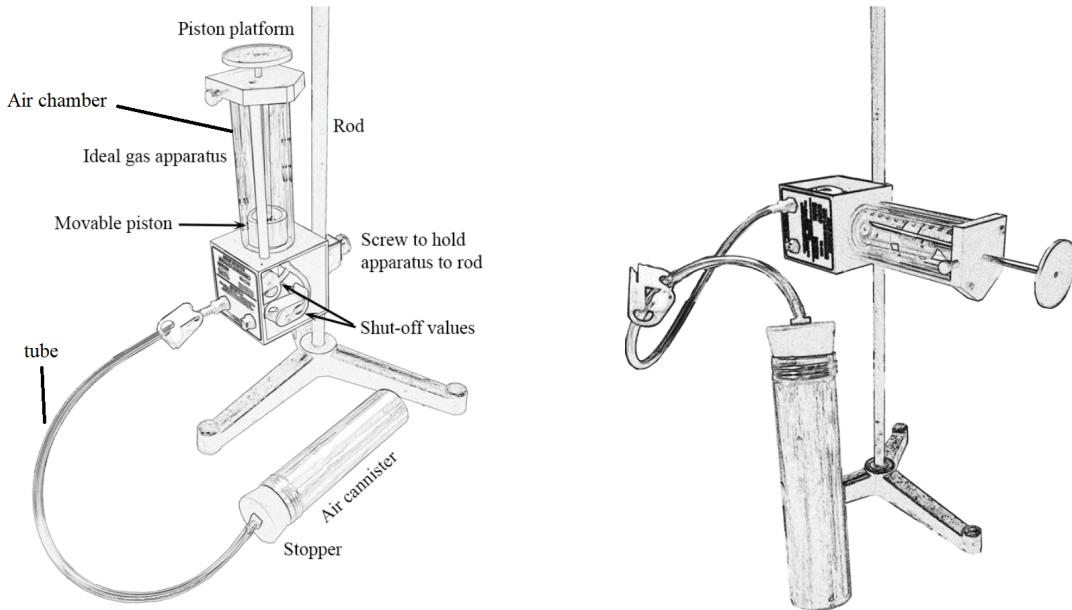


Figure 1: The Pasco ideal gas apparatus with each part labeled. The left is the apparatus in the upright position, and the right is in the horizontal position.

### 3.1 Specific Heat Measurement

In this part, we want to measure the specific heat of constant volume  $c_V$  and constant pressure  $c_P$  of the air.

Since the ideal gas apparatus can have some leakage at the piston, in order to minimize this leakage, the apparatus was held sideways, as shown in the right diagram of Figure 1.

We first took a Styrofoam cup and added water and sufficient ice into it to make an ice bath, the temperature of which was assumed to be  $0 \text{ }^\circ\text{C}$  as long as the ice was not fully melted. We then put some water of higher temperature into a Styrofoam cup and measured the total mass of the water and the cup ( $m_w + m_{cup}$ ). After that, we immersed the canister into the ice bath, with the valve on the tube open and the piston allowed to move freely (see Figure 2). While waiting for the canister to reach the temperature of the

ice bath, we put a thermal probe, which was connected to the Logger Pro software on a computer through a LabQuest, into the higher-temperature water.



Figure 2: Immersing the canister in the ice bath

The reason we used an ice bath instead of a heat bath was because it took a long time for the heater to heat up the water in it to a relatively stable temperature, which was our first major challenge during the experiment. Even after the temperature was relatively stable, there could still be some small fluctuations, which could add to the uncertainty in our measurement. An ice bath, however, could always ensure a 0 °C temperature as long as the ice was not fully melted.

We then adjusted the position of the piston so that its bottom was right at 40 mm on the millimeter scale. After that, we closed the valve on the tube to stop air from coming in and out of the apparatus, which ensured a constant mass of the air inside the apparatus throughout the trial. The initial pressure of the air  $P_i$  should be the same as the atmospheric pressure, which was measured by a Phone app called “phyphox.” For  $c_V$  measurement, we fixed the piston so that the volume of the air would not change. On the other hand, for  $c_P$  measurement, we still allowed the piston to move freely throughout the trial.

We clicked ”start collection” on the Logger Pro software to start recording the water temperature change detected by the thermal probe. After the initial temperature of the water was well-recorded for a while, we quickly removed the canister from the ice bath and immersed it fully into the higher-temperature water. We also capped the higher-temperature water cup with another Styrofoam cup to minimize heat transfer between the water and its surroundings (see Figure 3). After the temperature of the water became relatively stable, we looked at the flat sections before and after the immersion of the canister on the temperature vs. time graph made by the Logger Pro software, through which we can determine the initial and final temperature of the water  $T_{i,w}$  and  $T_f$ , respectively.

We then repeated the experiment to obtain 5 trials of data in total for the measurement of both  $c_V$  and  $c_P$ .

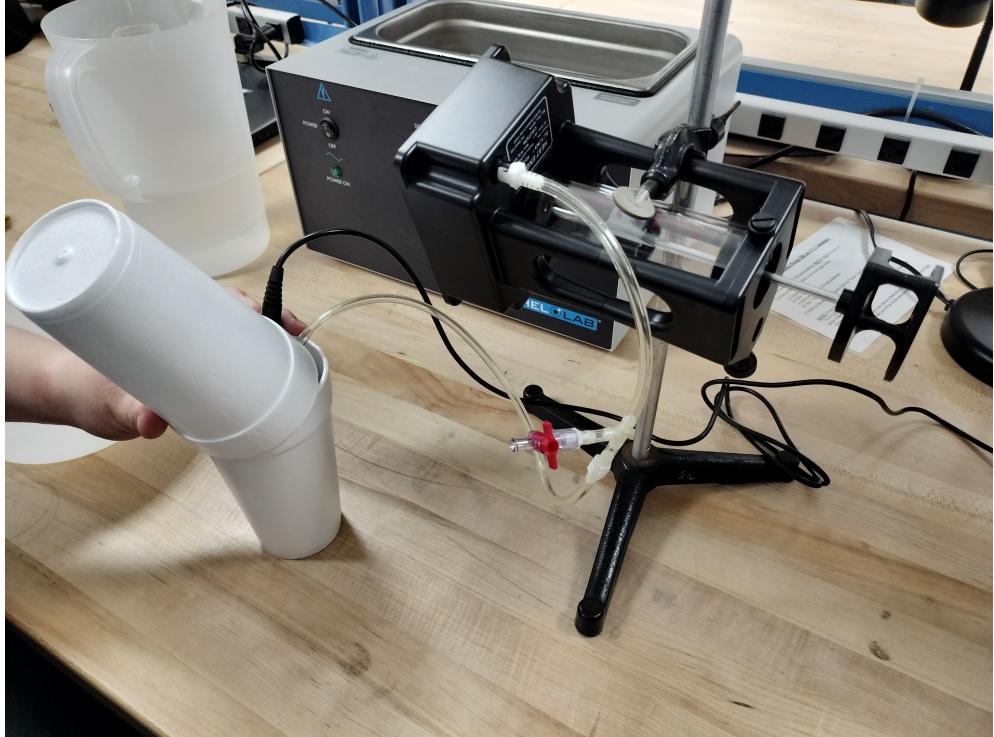


Figure 3: A picture of ongoing heat transfer between the canister, the air, and the water

### 3.2 Adiabatic Index Measurement

In this part, we want to measure the adiabatic index of the air.

To minimize heat transfer between the air in the canister and the environment, we cut Styrofoam from the Styrofoam cups and taped it around the canister as a cover (see Figure 4). Since Styrofoam has a low thermal conductivity, as measured in Lab 6, we could assume the air inside the canister would not exchange heat with its surroundings after the canister was covered if we conduct the experiment quickly enough. Also, since the tube and the wall of the air chamber of the apparatus are made of rubber and plastics respectively, both of which have low thermal conductivity, we could assume there was no heat transfer through the tube and the wall of the chamber when our experiment was conducted quickly enough, e.g. within 5 seconds.



Figure 4: The ideal gas apparatus with the metal canister covered with Styrofoam.

We would then add weight onto the piston to see the relation between the pressure and the volume of the air in the apparatus. However, due to the defect of the apparatus, there was air leakage at the piston. The longer we conducted a trial, the more leakage there would be; the more weight we added, the more quickly the air would leak through the piston. Therefore, based on our experience in Lab 7, in order to minimize leakage while enabling us to add weight on the piston, we put the chamber upside down, as shown in Figure 4. We also decided to reset our setup after each trial of the experiment to minimize the effect of the leakage. In other words, we wanted to ensure at the beginning of each trial, our initial temperature, volume, and pressure of the air inside the apparatus were the same.

For each trial, we first put the canister in the ice bath to make sure the initial temperature was at 0 °C (i.e. the Styrofoam cover was not put on yet). We opened the valve and put the piston at the same initial height, 40 mm. In this way, our initial volume and pressure were the same in each trial. Then, we closed the valve and quickly moved the canister from the ice bath into the Styrofoam cover. Immediately after this, We added weight to the piston and read off the final position of the piston on the millimeter scale. We then removed the canister from the Styrofoam cover, put it back into the ice bath, and repeated the procedure above for more trials.

## 4 Data & Analysis

### 4.1 General Measurements

The following table shows the measured values for the quantities that we will use for both parts of the experiment.

Quantity	Value
Height of the canister $h_c$	$11.000 \pm 0.005$ cm
Diameter of the canister $d_c$	$4.630 \pm 0.005$ cm
Length of the tube $l_t$	$74.3 \pm 0.5$ cm
Diameter of the tube $d_t$	$0.35 \pm 0.01$ cm
Diameter of the piston $d_p$	$3.25 \pm 0.01$ cm
Mass of the Styrofoam cup $m_{cup}$	$5 \pm 1$ g
Mass of the canister $m_c$	$30 \pm 1$ g

Table 1: Measured values for the quantities used in both parts of the experiment.

The uncertainty in the mass measurement came from the precision error of the scale we used. While we used a caliper to measure  $h_c$ ,  $d_c$ ,  $d_t$ , and  $d_p$ , only the uncertainty in  $h_c$  and  $d_c$  comes from the precision error of the caliper we used. Since  $d_t$  was very small, we assumed there should be a greater uncertainty in our measurement, which is why we have 0.01 cm instead of 0.005 cm.  $d_p$  also has 0.01 cm because the plastic layer outside the air chamber is a little elastic, so we assumed a greater uncertainty than the precision error of the caliper. Finally, the uncertainty in  $l_t$  is the greatest since we could hardly pull the tube straight and do the measurement. Since some part of the tube was inevitably bent, we assume a greater uncertainty.

### 4.2 Specific Heat Measurement

In this part, the atmospheric pressure was measured to be  $100.90 \pm 0.01$  kPa, the uncertainty of which came from the last digit of the barometer on the Phone app “phyphox.”

The following two tables show the data we collected for the constant volume experiment and the constant pressure experiment. The calculated values for  $c_V$  and  $c_P$  are also listed. Remember that the piston was always kept at 40 mm at the beginning, and  $c_V$  and  $c_P$  are calculated with Eq.(4), in which the mass and the volume of the air are given by Eq.(5) and Eq.(11).

Trial	$m_w + m_{cup}$ (g)	$T_{i,w}$ (°C)	$T_f$ (°C)	Calculated $c_V$ (J/(g·K))
1	212	19.59	18.62	62.4
2	213	18.84	17.94	57.4
3	225	19.89	19.02	52.0
4	188	18.30	17.16	82.2
5	189	17.71	16.80	50.7

Table 2: Data collected for  $c_V$  measurement.

Trial	$m_w + m_{cup}$ (g)	$T_{i,w}$ (°C)	$T_f$ (°C)	Calculated $c_P$ (J/(g·K))
1	217	42.69	40.40	80.2
2	231	17.96	16.94	103.2
3	232	17.43	16.60	70.6
4	232	16.63	15.72	96.4
5	212	20.54	19.36	88.9

Table 3: Data collected for  $c_P$  measurement.

The uncertainty for all the mass measurement above is 1 g, which came from the precision error of the scale we used. The uncertainty in the temperature measurement is 0.01 °C, the last digit of the thermometer reading.

Since the difference between trials is relatively large, we choose to calculate the uncertainty in the final result with the use of standard deviation and standard error. The following table shows the related statistical parameters calculated from the 5 trials of  $c_V$  and  $c_P$  values, with units omitted in the first four columns. The final results with uncertainties are also reported below.

Quantity	Mean	Standard deviation	Standard error	Result with uncertainty
$c_V$	61.0	12.8	5.7	$61.0 \pm 5.7$ J/(g·K)
$c_P$	87.9	12.9	5.8	$87.9 \pm 5.8$ J/(g·K)

Table 4: Related statistical parameters for  $c_V$  and  $c_P$  data, with the final result reported with uncertainty.

Theoretically,  $c_V = 0.718$  J/(g· K) and  $c_P = 1.005$  J/(g· K). Our experimental value clearly do not agree with the theoretical values. However, as we calculate their ratio, we get  $c_P/c_V = 1.4 \pm 0.2$ , which agrees with the theoretical adiabatic index of the air.

### 4.3 Adiabatic Index Measurement

In this part, the atmospheric pressure was measured to be  $101.57 \pm 0.01$  kPa, the uncertainty of which also came from the last digit of the barometer on the Phone app “phyphox.”

The following table shows the weight added onto the piston and the corresponding final position of the piston, through which we can later calculate the pressure  $P$  and volume  $V$  of the gas inside the apparatus:

Weight added (g) (uncertainty: 1 g)	Final piston position (mm) (uncertainty: 0.5 mm)
0	40.0
50	41.0
100	42.5
200	45.0
250	46.5
300	48.0
350	49.5
400	51.0

Table 5: Weight added to the piston and the corresponding final position of the piston.

The uncertainty in the piston position comes from the precision error of the millimeter scale on the piston. For the weight, the uncertainty comes from the precision error of the scale we used to measure the weight.

Since the apparatus was placed upside down, the pressure  $P$  of the air inside the apparatus will be the atmospheric pressure  $P_{atm}$  minus the pressure  $P_m$  created by the weight added, which is given by  $P_m = F_m/A = mg/A$ , where  $A$  is the area of the piston. Given the position of the piston and the quantities in Table 1, the volume  $V$  of the air can be calculated with Eq.(11).

We then plot  $\ln \frac{1}{P}$  vs.  $\ln V$ , and fit a linear model to the data points using weighted least-squares linear regression, as shown below:

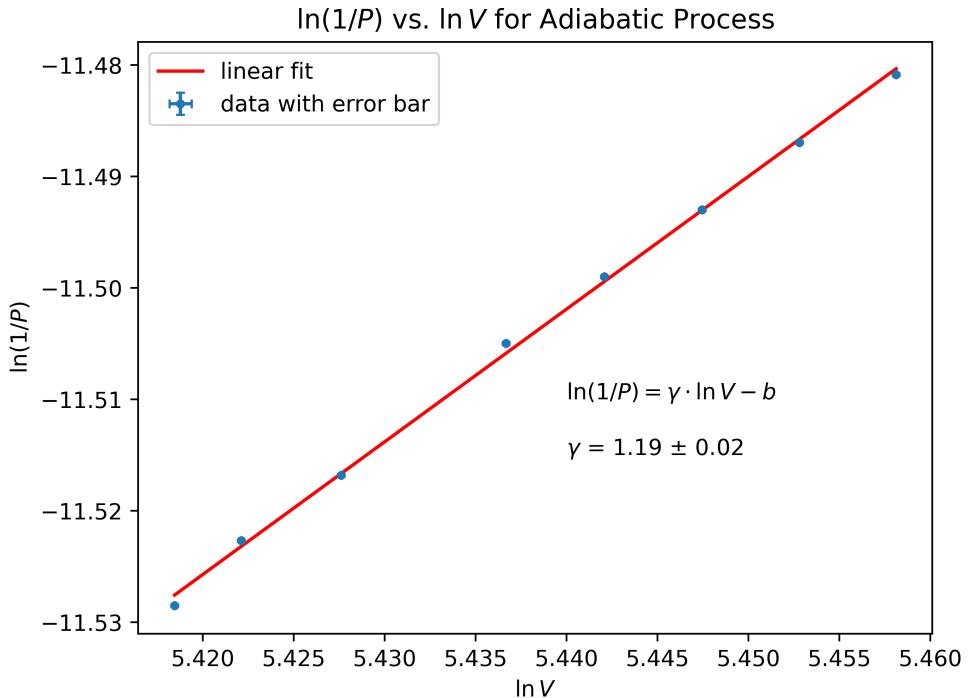


Figure 5:  $\ln \frac{1}{P}$  vs.  $\ln V$  with a fitted linear model generated by weighted least-squares linear regression. The error bars on the data points are very small.

The adiabatic index is then given by the slope of the linear fit, which, as shown in the

plot, is  $\gamma = 1.19 \pm 0.02$ . This value does not agree with the theoretical value of  $\gamma = 1.4$ .

## 4.4 Discussion

### Specific Heat Measurement

In this part of the experiment, the major contributor to the error in our measurement may be the specific heat of the canister. It was believed that the canister was made of aluminum. However, on the website of Pasco, there is no information about the material made up of the canister in this apparatus. Therefore, it is reasonable to doubt the specific heat of the canister we used for our calculations. We then derive that, if we want to obtain the theoretical values of  $c_V$  and  $c_P$  through our original calculations, the specific heat of the canister should roughly be around  $1.8 \text{ J/(g}\cdot\text{K)}$  instead of  $0.91 \text{ J/(g}\cdot\text{K)}$ .

It is also possible that as we transferred the metal canister from the ice bath into the higher-temperature water, the canister might carry some water drops from the ice bath on its surface, which would also take some heat from the higher-temperature water. So in reality we might have  $Q_{water} - Q_{canister} = Q_{air} + Q_{drop}$ , but in our calculation we just considered  $Q_{air} + Q_{drop}$  to be  $Q_{air}$ , which could potentially be much greater than the actual  $Q_{air}$  and thus lead to a much greater specific heat value calculated.

In addition, as we immersed the canister into the higher-temperature water and started tracking the temperature change, it is also possible that the final temperature of the water wasn't uniform during the short amount of time when we were measuring. Therefore, depending on where we put the thermal probe in the water, we might get a different temperature reading for the water. Considering that the measurement of temperature in this experiment requires precision, this could potentially contribute to a great fluctuation in our measurement among different trials.

Since the mass of the air in the apparatus was very small comparing to the mass of the canister and the water, it is inevitable that we will have a great error in our measurement. For future experiment, besides determining the specific heat of the canister, we can also make improvement by having more air to exchange heat with liquid that has a smaller specific heat (e.g. oil). With this, we can expect to see a more drastic change in the temperature of the liquid, and thus less precision will be required when measuring the temperature change. In the end, we can expect to obtain a more accurate specific heat measurement.

### Adiabatic Index Measurement

One possible reason that our experimental adiabatic index does not agree with the theoretical value may be that there was heat transfer between the Styrofoam cover and the canister. Remember that before the canister was put into the Styrofoam cover in each trial, we first put it in the ice bath to ensure a  $0^\circ\text{C}$  initial temperature. However, during this time, the Styrofoam cover could get to a temperature different from  $0^\circ\text{C}$ , not to mention the first trial when the Styrofoam was initially at room temperature. With the heat potentially from the Styrofoam cover, the process might not be fully adiabatic, so our measurement could go off.

Another major culprit was the leakage at the piston. As we added more weight to the

piston, the leakage became faster, and thus it required our reading of the piston position to be faster. This means that for greater weight added, our reading might not only have lower precision, but also poorer accuracy. The insufficient precision of the millimeter scale on the piston also added to this problem. Since the piston could not bear very large weight, we could only exert pressure that produced limited changes in the piston position. Because the change in piston position was not much greater than the precision error of the millimeter scale, our measurement of the change in the volume of the air could have had a much greater uncertainty than we originally expected.

One solution for future experiment is to use a larger piston with better airtightness, greater load-bearing capacity, and more precise meter scale on the side. With this, we can then add more weight onto the piston to create a greater volume change so that the precision error of the meter scale will not have much effect on the measurement. With such a piston, we can also conduct the experiment without the need to reset the apparatus every time, so that the heat transfer between the cover and the piston mentioned above will no longer be a problem. With that said, we should try to use a better thermal insulating material other than Styrofoam.

## 5 Conclusion

With the Pasco ideal gas apparatus, we were able to measure the specific heat of constant volume  $c_V$ , specific heat of constant pressure  $c_P$ , and the adiabatic index  $\gamma$  of the air. By calculating the heat exchange between water and the air in the apparatus, we obtained  $c_V = 61.0 \pm 5.7 \text{ J/(g}\cdot\text{K)}$  and  $c_P = 87.9 \pm 5.8 \text{ J/(g}\cdot\text{K)}$ , with their ratio to be  $c_P/c_V = 1.4 \pm 0.2$ . While the ratio matches with the theoretical value, both  $c_V$  and  $c_P$  are larger than the theoretical values by roughly two orders of magnitude. The possible causes of such disagreement may be the inaccurate specific heat of the canister, the heat contributed by the cold water drops on the canister, and the non-uniform temperature of the water. By exerting pressure on the piston of the apparatus and measuring the change in volume, we arrived at an adiabatic index of  $\gamma = 1.19 \pm 0.02$ , which also does not agree with the theoretical value. The possible cause of this disagreement may be the heat exchange of the canister with the Styrofoam cover, the leakage at the piston, and the insufficient precision of the millimeter scale on the piston. For future measurement of the specific heat, we can improve by finding out the specific heat of the canister and using a liquid with smaller specific heat to exchange heat with the air and the canister. For future measurement of the adiabatic index, we can improve by choosing a better thermal insulating material and using a piston with better airtightness, greater load-bearing capacity, and more precise meter scale.

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