Capstone Project

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November 2022

Abstract

In our project, we explore Faraday's Law and its applicability to guitar pickups. We constructed pickup coils and electric guitar string and performed Fast Fourier Transforms on the potential signals collected to test various theories. We test validity of a model of guitar string/pickup and experiment with Noise Reduction using a Humbucker coil and find that our initial model is insufficient to represent a true guitar string. We also find that Humbucker coils do indeed reduce noise in output.

1 Introduction

We chose to investigate electric guitars, as well as the implications Faraday's laws have on them. In this lab, we are going to explore the functionality of pick-up coils in electric guitars via Faraday's law. In both experiments, we ignore the solenoid's edge effects and idealize the uniform magnetic field.

1.1 Experiment A, Evaluating Pickup Models

We want to verify our pick-up model by experiment results in this experiment.

1.2 Experiment B, "Hum" Noise Cancellation

In this experiment, we want to check how noise is reduced with two opposite sides of pickup coils compared to one.

2 Theory

2.1 Faraday's Law

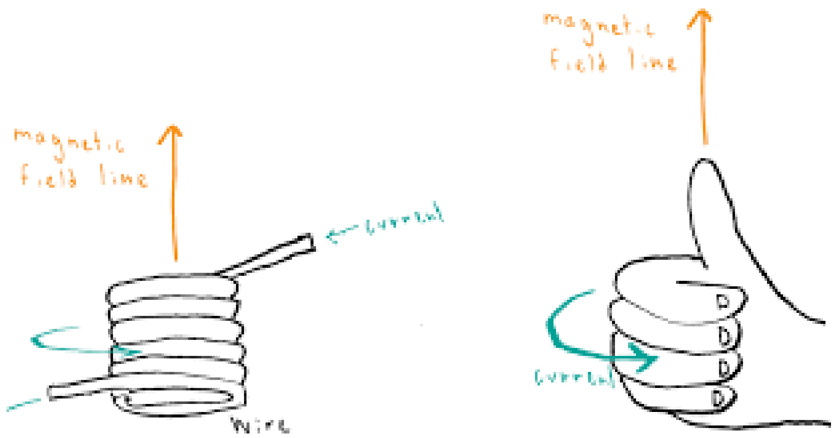
The function generator will make the ferromagnetic wire vibrate in the z direction, and therefore by Faraday's law,

$$\epsilon = -\frac{d\Phi}{dt}$$

where

$$\Phi = \iint_S \overrightarrow{\mathbf{B}} \cdot d\overrightarrow{\mathbf{A}}$$

there should be an induced EMF around the coil in the *theta* direction. Because by the right hand rule, the current in the *theta* direction have a magnetic field in the z direction. And that magnetic field from the current should counter the effect of the magnetic change of the coil by Lent's law. Here's the graph of how the current should be:



2.2 Damped Harmonic Oscillator

In real oscillators, friction, or damping, slows the motion of the system. Due to frictional force, the velocity decreases in proportion to the acting frictional force. While in a simple undriven harmonic oscillator the only force acting on the mass is the restoring force, in a damped harmonic oscillator there

is in addition a frictional force which is always in a direction to oppose the motion. In many vibrating systems the frictional force F_f can be modeled as being proportional to the velocity v of the object: $F_f = -cv$, where c is called the viscous damping coefficient.
The balance of forces (Newton's second law) for damped harmonic oscillators is then

$$\mathbf{F} = -kx - c\frac{dx}{dt} = m\frac{d^2x}{dt^2},$$

which can be rewritten into the form

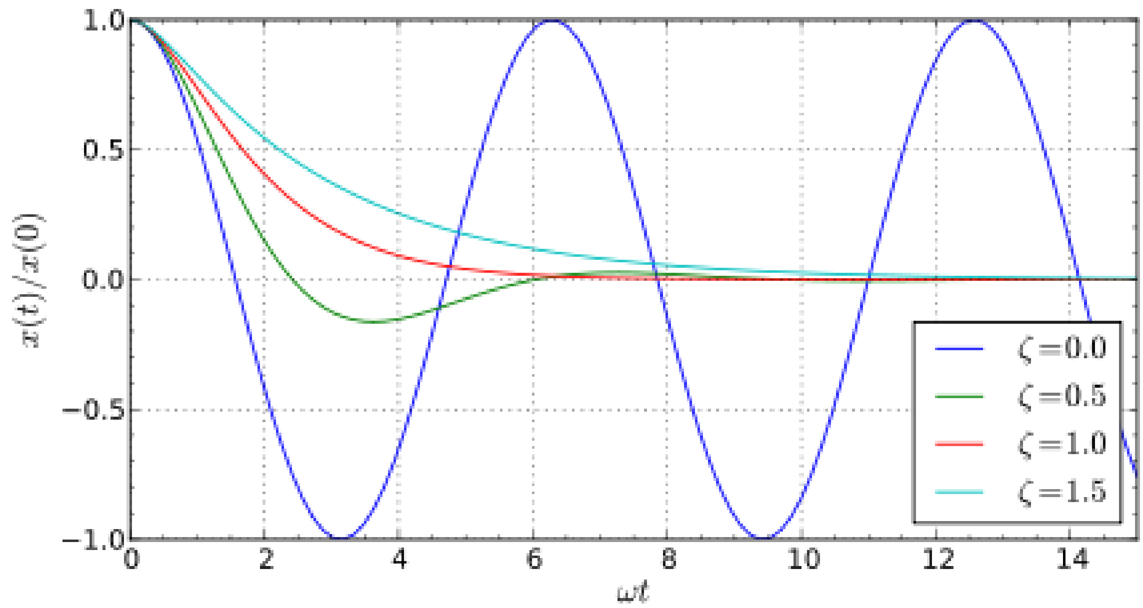
$$\frac{d^2x}{dt^2} + 2\zeta\omega_0\frac{dx}{dt} + {\omega_0}^2 x = 0,$$

where

- $\omega_0 = \sqrt{\frac{k}{m}}$ is called the "undamped angular frequency of the oscillator"
- $\zeta = \frac{c}{2\sqrt{mk}}$ is called the "damping ratio".

The value of the damping ratio ζ critically determines the behavior of the system. A damped harmonic oscillator can be:

- Overdamped ($\zeta > 1$): The system returns (exponentially decays) to steady state without oscillating. Larger values of the damping ratio ζ return to equilibrium more slowly.
- Critically damped ($\zeta = 1$): The system returns to steady state as quickly as possible without oscillating (although overshoot can occur if the initial velocity is nonzero). This is often desired for the damping of systems such as doors.
- Underdamped ($\zeta < 1$): The system oscillates (with a slightly different frequency than the undamped case) with the amplitude gradually decreasing to zero. The angular frequency of the underdamped harmonic oscillator is given by $\omega_1 = \omega_0\sqrt{1-\zeta^2}$, the exponential decay of the underdamped harmonic oscillator is given by $\lambda = \omega_0\zeta$.



Solving the differential equation for damped harmonic oscillation with velocity-dependent damping results in the model of the form:

$$x(t) = Ae^{-\beta t}\cos(\omega t + \phi)$$

2.2.1 Our Model

We consider the simple case where the wire is only oscillating in one direction, and approximate the magnetic field induced within the coil simplistically by a damped harmonic oscillation and velocity-dependent damping:

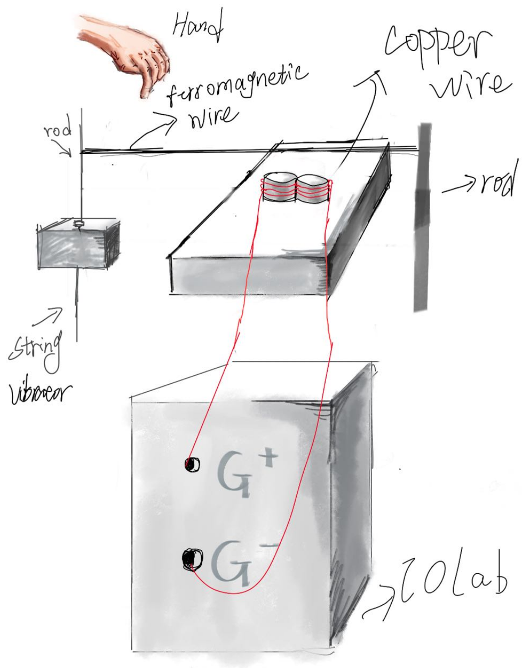
$$\vec{B}(t) = B_0 e^{-\beta t}\cos(\omega t + \phi)\hat{z}$$

Therefore, the model for our pick-up coil should be

$$\begin{aligned}\epsilon &= -\frac{d\Phi}{dt} = -A\frac{\partial B}{\partial t} = A(B_0\beta e^{-\beta t}\cos(\omega t + \varphi) - B_0\omega e^{-\beta t}\sin(\omega t + \varphi)\\ &= AB_0 e^{-\beta t}(\beta\cos(\omega t + \varphi) - \omega\sin(\omega t + \varphi))\end{aligned}$$

2.2.2 Humbucker Coils

Humbucker coils are able to reduce noise by measuring the signal via two pickup coils, such that the second one has the opposite magnetic field and the direction of the coiling is opposite. In this way, the magnitude of the wave itself across both coils should the same (it gets reversed twice once from the opposite field and another from the opposite coiling direction), but the magnitude of any extra signals gets reversed across the coil measurements (only due to the opposite coiling direction). In this way, adding the measurements from the two pickups cancels the noise and enhances the frequencies of the wire.



3 Methods

3.1 Guitar Pickup Construction

We use an iPhone application to test the frequency of the guitar we built. After setting that frequency on a function generator, we use the oscilloscope and IOlab to measure the induced emf caused by vibrating the guitar string(which is by Faraday's law, the change of magnetic flux induces an emf). Then we export the data from the oscilloscope and IOLab to verify if that fits our model

3.2 Tuning the String

For the first record of Pickup 1, the string was tuned to a frequency roughly around G$\#_3$ (207.65 Hz) using the iPhone Applications *Tuner Lite* and *iTablaPro* (iTablaPro is able to handle lower frequencies). Tuning was done by ear, so human error would have been involved in the process.

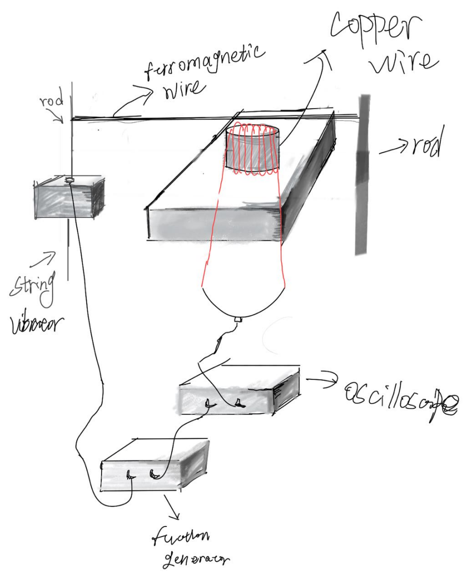
For the second and third records, the string was instead tuned to C$\#_3$ (130.81 Hz) as difficulties were encountered when attempting to tighten the string further (the points at which the string was tightened would snap under the higher tensions).

3.3 Measurements

Quantity	**Measurement**	**Device**
Magnet Diameter	1.280 ± 0.005 cm	Calipers
Cardboard Pickup Outer Length	5.740 ± 0.005 cm	Calipers
Cardboard Pickup Inner Length	3.330 ± 0.005 cm	Calipers
Cardboard Pickup Outer Width	1.300 ± 0.005 cm	Calipers
Coil Major Axis[1]	4.445 ± 0.005 cm	Calipers
Coil Minor Axis	2.080 ± 0.005 cm	Calipers
Height of Wire	9.51 ± 0.05 cm	Meter Stick
Height of Vibrator	6.10 ± 0.05 cm	Meter Stick

3.4 Overall Design For Experiment A

Here's the initial expected design and actual construction for Experiment A:



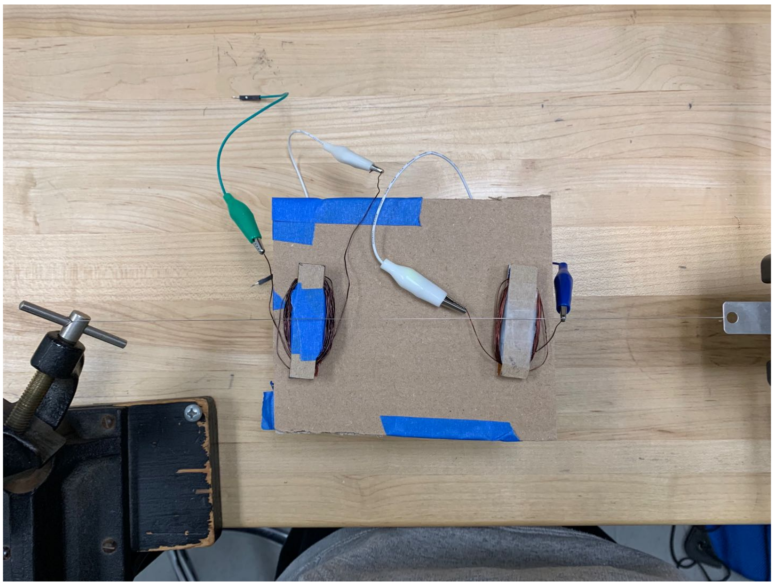
[1]The Major and Minor axes (length and width) of the coil were difficult to measure due to different parts of the coil being at different lengths, but we choose to approximate the coil as a thin ellipse or rectangle whose dimensions are measured using the maximum dimensions physically.

We first connect a ferromagnetic wire (which is made of Nickel) between the rod and string vibrator. We tried our best to make the wire as tighten as possible so that it's more like a guitar string and make sure the wire won't loose during the vibration. The string vibrator is connected to the function generator, which is then connect to one channel one of the Oscilloscope so that we can see the output voltage. The reason we put the magnet on the top of the box is that we want to make sure the magnet is close enough to the string, so that there will be a stronger magnetic field around the wire, and therefore we can get a stronger change of magnetic flux and induced EMF, which will be shown on the channel two of the Oscilloscope. After checking out what the frequency should be for the guitar string using the iPhone application, we set that frequency on the function generator and start the measurement for the induced EMF. However, our first design didn't work out. There weren't showing anything on the channel two of the Oscilloscope. And there were some wired things we observed that we don't know how to explain. So we disconnect the Oscilloscope and magnet, but moving hand ups and downs from a laptop, there were some signal in the channel two. Here's our failure graph:

The reason behind this failure might be due to the reason that there's a strong magnet in the laptop, so we moved the laptop away. And we coiled the magnet in the wrong direction. We coiled the wire in the Φ direction. However, as the change of magnetic flux is in the z direction, the induced EMF should be in the θ direction by the right hand rule. So we also changed our way of coiling the magnet and redesign the experiment (See appendix for more).

3.5 Overall Design For Experiment B

Because we want to explore how noise is reduced with two pick-up coils which are in the opposite direction, we repeat the setup in experiment A, but instead of putting one pickup coil below the guitar string, we put two. And we put those two coils in series within the circuit. Then by hand plucking the string, we can measure the new induced EMF and new audio sound which we can use to analyze. Here's the graph of our design:



However, there shows nothing in the IOLab when we plucking the guitar wire (Unfortunately, we didn't save the graph for this failed experiment design). The spike is so small that is not good for analyzing. The reason behind this might be the breadboard and wire. The breadboard maybe broken so that the circuit is actually an open circuit. Or that there are too much wire (because we need to make sure two coils are in series), and each one will dissipate some current energy, and since the induced EMF is not big, there won't be a significant signal in the IOLab. Therefore, instead of putting two coils in series, we decide to measure each of the coil individually during the plucking and try to add the overall effect (See appendix for more).

4 Analysis

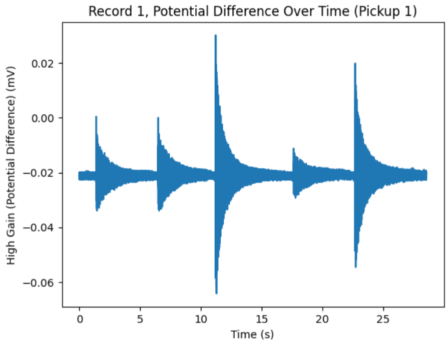
4.1 Evaluating Pickup Models

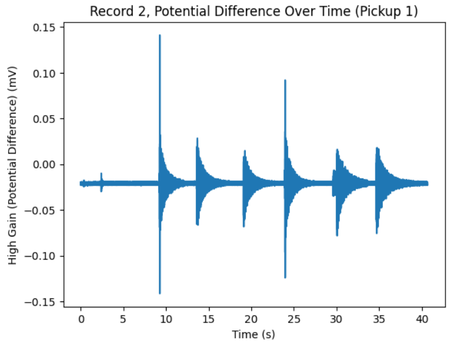
According to our theory,

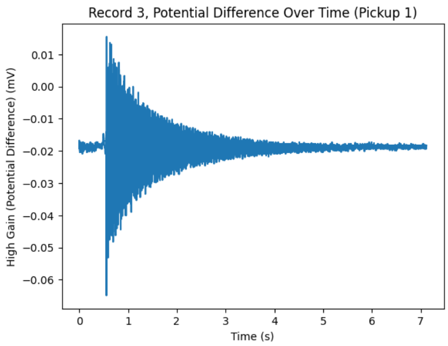
$$\epsilon = AB_0 e^{-\beta t}(\beta \cos(\omega t + \phi) - \omega \sin(\omega t + \phi))$$

so the potential measured (ϵ) has frequency ω which is the same as the oscillation of the string. Therefore, the frequency of the note which we tune the string to should be equal to the frequency of the oscillations in potential produced in the pickup.

Our initial experiments consisted of recording the potential difference across two leads coming from an individual pickup. Plotting this difference over time yielded the following plots:

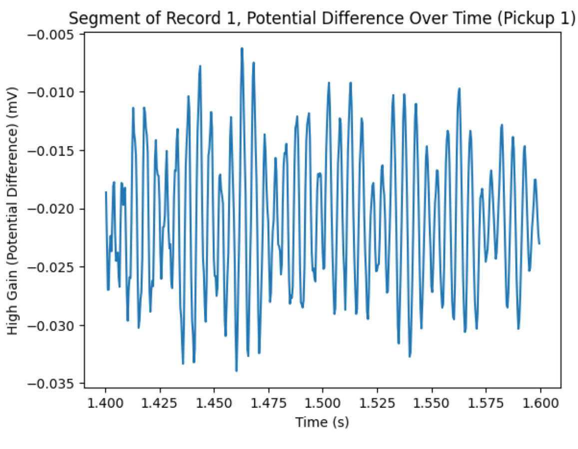




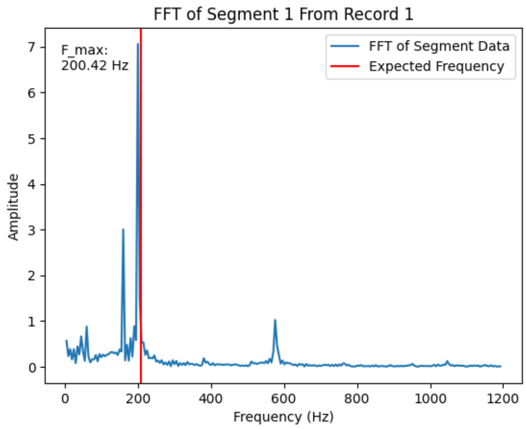


The plot appears to be a high frequency sinusoid which decreases in amplitude over time. This matches with our expectations, as the string behaves as a damped harmonic oscillator. Each spike in potential corresponds with a time when the string was plucked.

We first analyze a small interval of the data from Record 1 in which the string underwent only a small amount of damping. The interval from $t = 1.4$ to $t = 1.6$ was considered relatively arbitrarily (the first pluck was chosen) as a relatively small segment (with less dampening) after a pluck was performed:



As expected, a high frequency sinusoidal function appears to be surrounded by lower frequency ones, representing the interference between the different sounds at different frequencies. Performing a Fast Fourier Transform (FFT) on our this segment yields the following plot in the frequency domain:

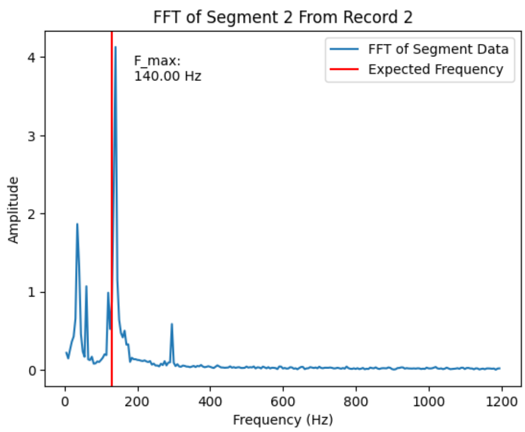


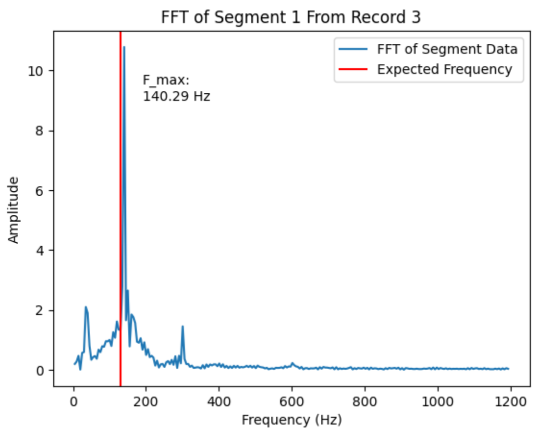
This shows that our experimental frequency detected via the potential signal was 200.42 Hz. We expected a frequency corresponding to G$\#_3$, or 207.65 Hz. To perform an agreement test we must estimate the error on each of these measurements. We take the uncertainty of the dominant frequency to be some unknown $\alpha_{f_{\text{exp}}} > 0$. To measure the uncertainty in the expected frequency, we assume that a musically inclined individual is able to detect the pitch of a note up to a half-step[2], so an estimate of our confidence of the frequency is given by the interval $(196.00, 220.00)$. In other words, our uncertainty is $\alpha_{f_{\text{acc}}} = 207.65 - 196.00 = 11.65$ (we take the smaller difference since this would imply agreement with the larger uncertainty). An agreement test shows:

$$|207.65 - 200.42| = 7.23 < 11.65 < \sqrt{11.65^2 + \alpha_{f_{\text{exp}}}^2}$$

so our experimental results do indeed agree with our theoretical results. This supports our model.[3]

The above was repeated with both other records:





Agreement tests with the C$\#_3$ frequency show:

$$|140.00 - 138.59| = 1.41 < 7.78 < \sqrt{7.78^2 + \alpha_{f_{\text{exp}}}^2}$$

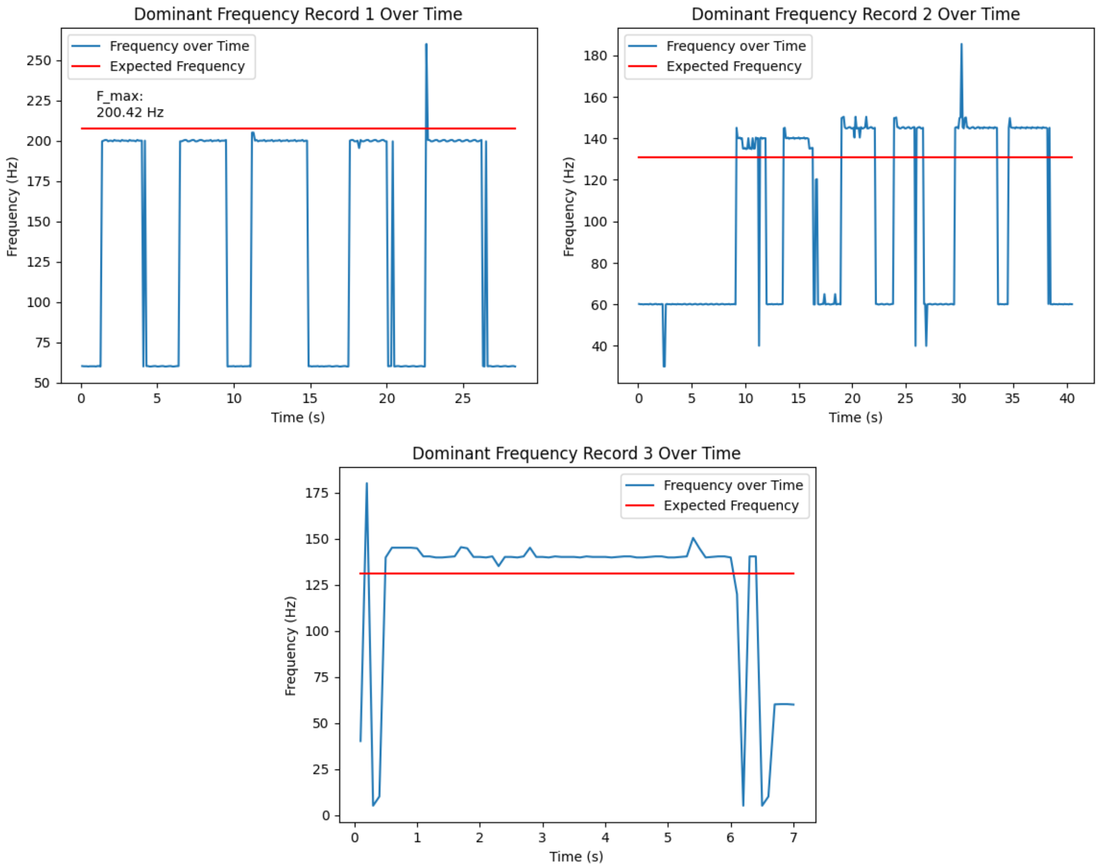
$$|140.29 - 138.59| = 1.70 < 7.78 < \sqrt{7.78^2 + \alpha_{f_{\text{exp}}}^2}$$

which both show agreement. The small error between the measured frequencies for Record 2 and Record 3 also display the precision of our measurements.

Let f_d be the dominant frequency (point of maximum amplitude in the transformed graph). We may also see how f_d changes as a function of time, $f_d(t)$. We do this by plotting the dominant frequency of the wave over a small interval around a point for each point in time:

[2] A half step is the smallest interval in Western music theory

[3] Note that this took the frequency in our model to be the dominant frequency and ignored other frequencies detected. To the extent that only one frequency is in play, our model seems accurate.

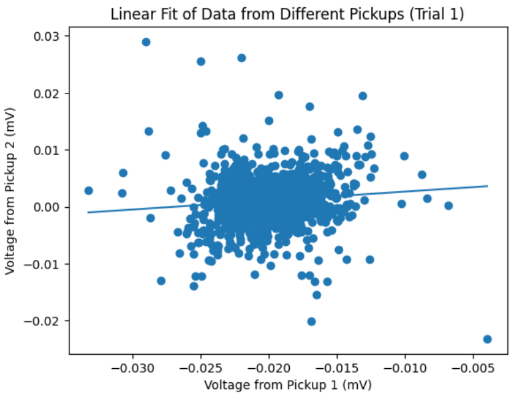


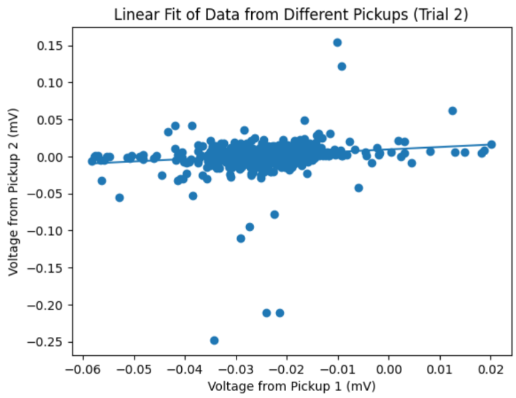
These plots show more clearly the frequency over time for each of the records. “Segment n” in the title of the segment FFT plots shown previously correspond to the nth pluck in the above plots. Notice that the frequencies actually appear to drop not to zero but to a lower point of 60 Hz. This is likely due to background frequencies which makes sense since 60 Hz is also the frequency that AC current is commonly transmitted at. It is also interesting to note that the spikes shown in the plot are likely due to times when we plucked too violently, and the increase in the dominant frequency over time in Record 2 is due to us manually tightening of the string during the process (which would have changed the frequency, and is thus why we used segment 2 for analysis above).

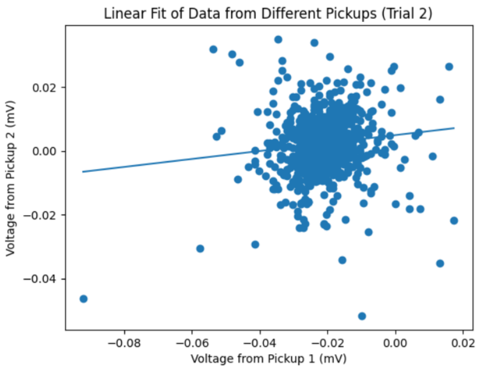
We were also able to convert our potential signals back into the original audio signals via the scipy.io.wavefile module. While we are unable to attach the audio files to this PDF, they are available at the following links:

- Record 1
- Record 2
- Record 3

4.2 “Hum” Noise Cancellation

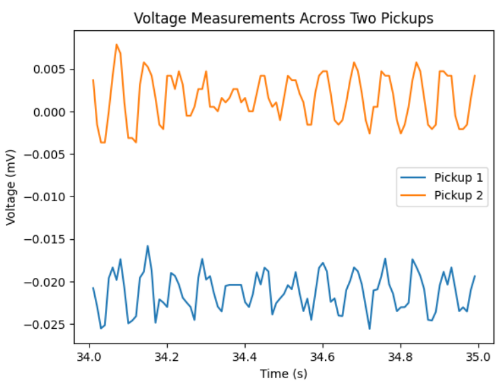


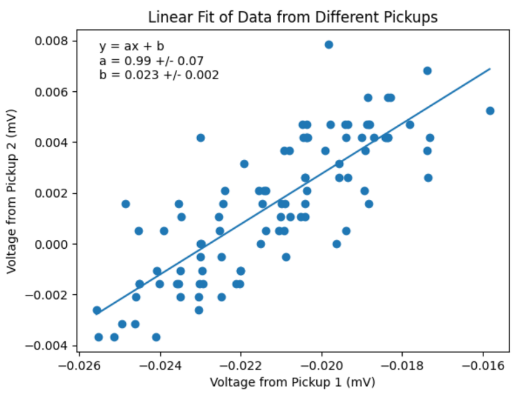




In order to cancel noise, we use two coils which were made identically. However, since initial experiments proved that the coils were not identical, we fit the data from the coil above using a linear model. We do this because we expect the data to be linearly correlated by Faraday’s law, being directly proportional to the area and number of coils. Since only Trial 2 appeared to give a linear shape, we proceed with the rest of the analysis on Trial 2.

We started by collecting a small segment (from $t = 34$ to $t = 35$ of the data from trial 2. This segment was chosen because it was a pluck of smaller amplitude, which may reduce errors in recorded signal (as we found in part 1). The data from both pickups is shown below, along with the linear plot of the data from both pickups.





Our linear fit between the two potential signals was:

$$y = ax + b$$

$$a = 0.99 \pm 0.07$$

$$b = 0.023 \pm 0.002$$

with a reduced-Ξ^2 value of 10.12. Since this is very large, we use a data-driven model to generate the above statistics. The above model shows the relationship between the two signals, which allows us to now "normalize" before combination to remove error. Specifically, if our signals were $V_1(t)$ and $V_2(t)$, we have

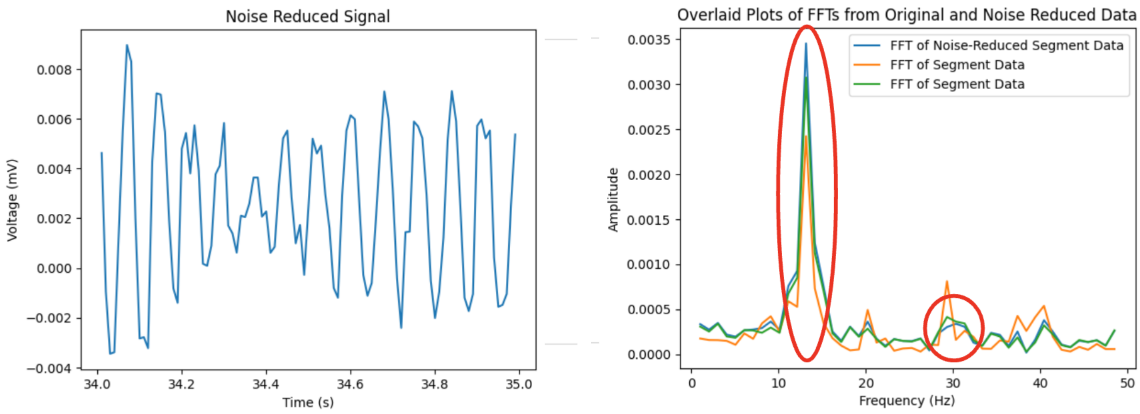
$$V_2(t) \approx aV_1(t) + b$$

Thus the new signal

$$V_r(t) = V_2(t) + aV_1(t) + b$$

combines the signals so that the frequencies cancel and add as would be expected by our theory of Humbuckers.

To finally determine whether our Humbucker setup resulted in reduced noise, we first created a new "reduced noise" signal by combining the signals in the ratio above. This resulted in the first plot below. We then plotted the Fast Fourier Transforms of all three signals, $V_1(t)$, $V_2(t)$, and $V_r(t)$:



By our theory, we expect to see addition of the signals coming from the guitar and reduction/cancellation of the signals coming from external sources. These features are circled on the graph. The first oval shows our dominant frequency for which the FFT of the Noise-Reduced signal is higher than either of the original signals. The second oval shows a portion where the Noise-Reduced signal was lower than either of the other signals, showing high frequency noise which was reduced. While design limitations such as the signals from both pickups being different may have prevented our setup from reducing noise as ideally as expected, we indeed see signs of reduction in the above plot.

5 Conclusion

Through the lab, we test the validity of model of a guitar string and also experiment with the noise reduction using humbucker coils. We find out that our initial model is insufficient to represent a true guitar string, and in order to fit the data, we use the FFT. We also find that the noise is indeed reduced with humbucker coils.

We redesign our experiment set up a lot during the whole lab, each time wasting lots of time. We should instead try to analyze our experiment set up first and first think about all the possibilities that we could have to design the experiment, in hat case, we can save lots of time and maybe get a better fit for the data. Thus, in future lab, we should derive the equation more based on the theory and come up with a better model.

There are many errors during the experiment in which we can try to reduce. There are human errors that sometimes we pluck the wire too hard which will impact the tuning of the wire or the distance between wire and magnet. There are systematic errors in wire, calipers, IOLab (for example, two IOLabs will give different time scale), and noise. There are experimental errors that strong magnet is in close proximity to wire and that could cause wire to oscillate differently than we expected. There are also some random errors such as the loosing of wire during the oscillation. Though we make the wire tighten manually during oscillation, we can't make it as tighten as possible, which will cause some variance in the data.

Thus, in order to reduce those errors, we could improve our experimental design in the future. For example, we can implement a better method for keeping the wire straight. We could also build a better coil and a better block (a block that makes the coil close to the wire but not so close). And indeed, we could also come up with a theory of how close should the magnet be with the wire. We can come up with a standard of the coil so that two coils are identical and the noise can be reduced more and show a better graph. We could also improve the measurement tools to reduce the systematic errors. We could also use two laptops to measure each IOLab so that the time scale would fit and will be easier and more accurate for us to analyze.

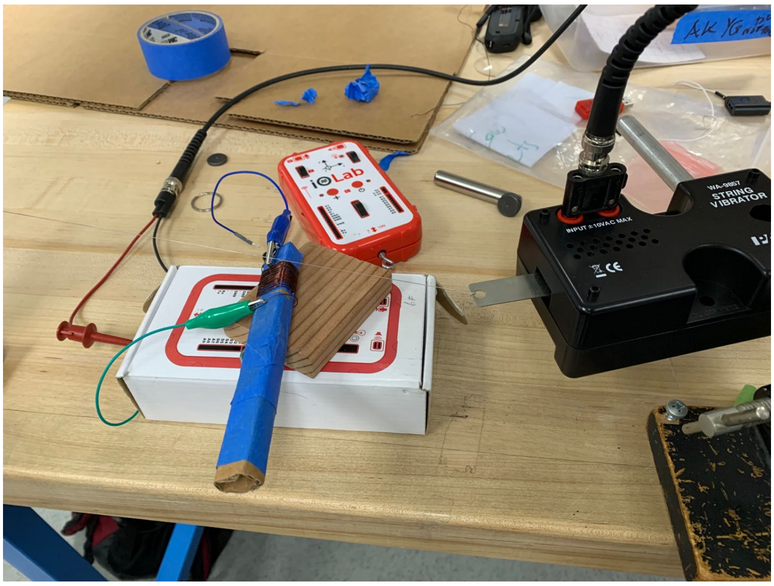
Also, in the future, if we make a good humbucker coils, we could also compare the sound quality of the reduced noises of humbucker with the digital noise reduction.

6 Appendix

6.1 Redesign of Experiment A

6.1.1 First Redesign: New Coil

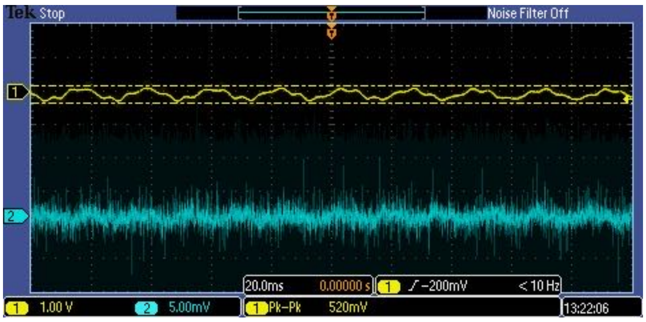
This time we remove the laptop, and make a triangular cube cardboard which we can easily make the magnet fixed so that the magnet won't touch the wire easily. And we coil the wire on the outside of the cardboard in θ direction. Here's how our new coil looks like:



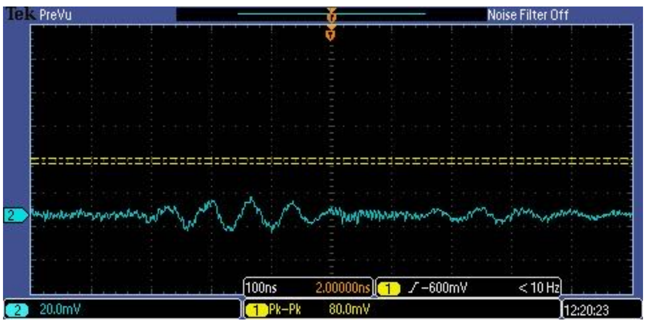
However, we still seeing nothing in the channel two of the Oscilloscope. And the really wired part is that when we disconnect the magnet with the Oscilloscope, and use our hand move ups and downs from the magnet, we did see some pattern in the channel two (which is the blue line). And that pattern, though we didn't analyze it and didn't know how that happened, look like the model

$$\varepsilon = AB_0 e^{-\beta t}(\beta cos(\omega t + \phi) - \omega sin(\omega t + \phi))$$

Here's the graph shown in Oscilloscope when we connecting the magnet and Oscilloscope:



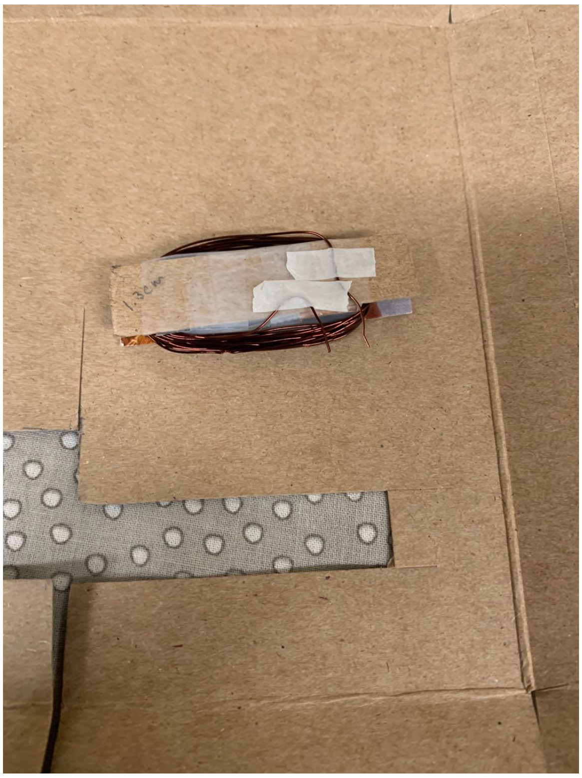
Here's the graph shown in Oscilloscope when we connecting the magnet and Oscilloscope:



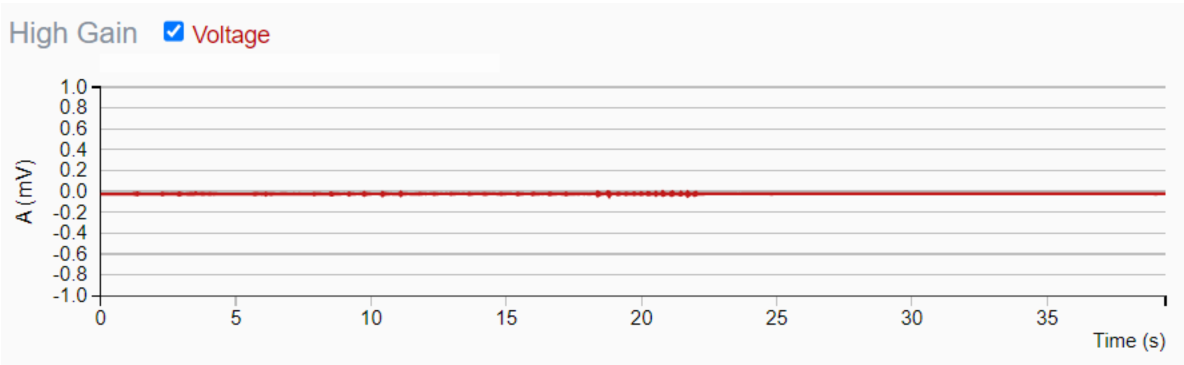
The reason behind not showing anything on channel two when connecting the magnet and Oscilloscope might due to the reason that the cardboard is thick so that it acts like a insulator, therefore there's no current through the wire. Also, because the Oscilloscope acted wired so far so we decided not to use it anymore to measure the EMF. Instead, we use the IO-Lab to measure the induced EMF.

6.1.2 Second Redesign: New Coil and IOLab

This time we put two magnet in series and use a wire to coil around these two magnet in the θ direction. And in case these two strong magnet stick together, we cut the cardboard to hold the top and bottom of the magnets so that two magnets can be fixed in position. And instead of using the Oscilloscope, we connect the magnet with the IOLab. Here's the graph of our new coil:



This time, though we saw some spikes on the IOLab, it's not very much apparent to observe or analyze. Here's how the IOLab looks like:



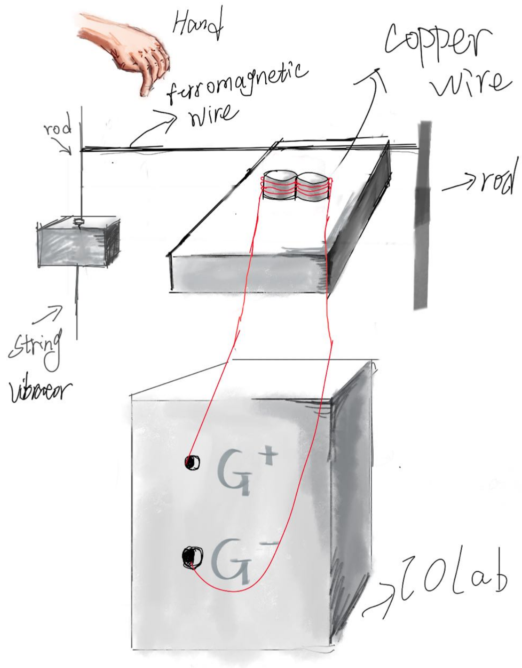
Hence, instead of using the function generator, which certainly can give us a accurate oscillation frequency, but has small oscillating amplitude. And that small amplitude maybe the reason why the spike is so small. If the vibrating amplitude is small, then the change of magnetic flux is small, because the magnetic flux is defined as

$$\Phi = \iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

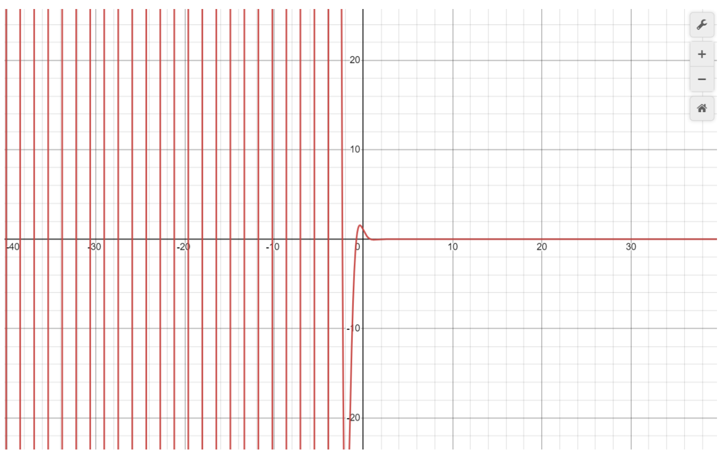
Therefore, because the amplitude is small, the area is very small, though the magnetic field is large, but is certainly not big enough compared to the area. So, instead of using the function generator, we decide use the hand plucking to make the area bigger.

6.1.3 Third Redesign: Hand Plucking

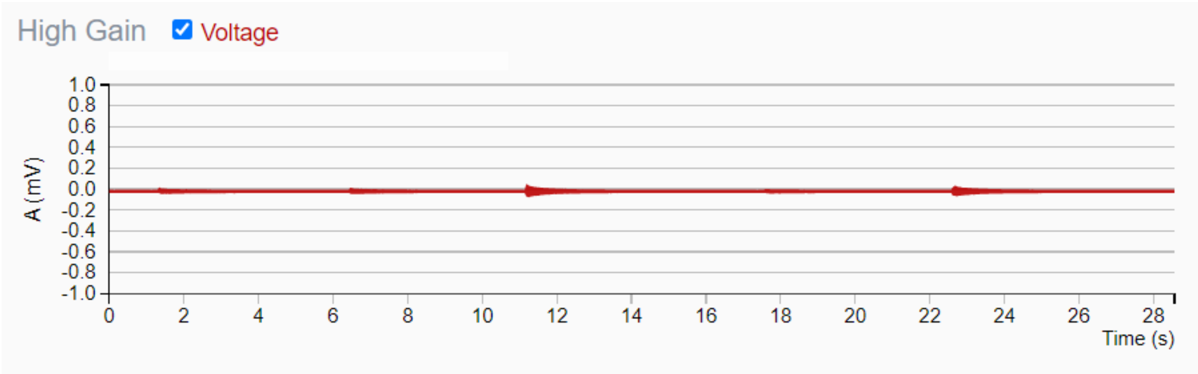
This time instead of using a function generator to oscillate the Nickel wire, we use hand to pluck the wire. In this way, we could get more are in which magnetic filed change, and get a more apparent graph in IOLab. So far, we removed the function generator and Oscilloscope. Here's how we expected graph should be like:



This time we finally succeed in getting the expected graph. The desired graph of a exponential function times a trigonometric function should look like (We use the online graphing website www.desmos.com):



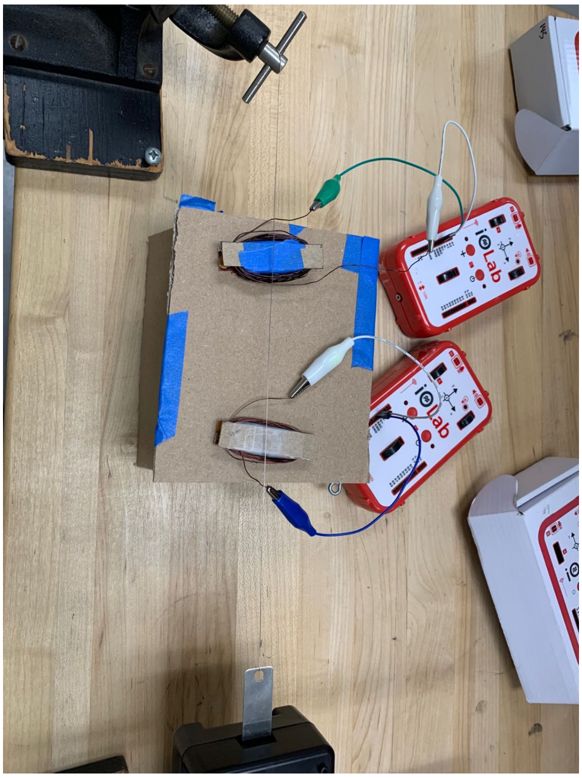
And here's the graph of our experiment result which we use to analyze:



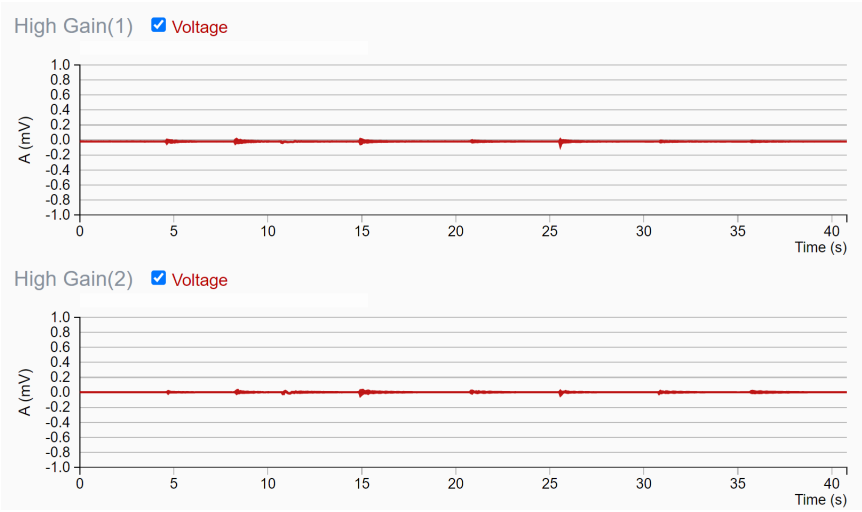
As can seen from the graph, the spikes are different. At about 11 second, the spike is significant larger than other spikes, that's because we pluck the string with different strength. That human error and the resulting analysis of induced EMF and audio can be seen in the 'Analysis' Part.

6.2 Redesign of Experiment B

Our new idea for Experiment B is to use two IOLabs to measure each of the pick-up coil individually. And in the end, we can analyze each of the graph and try to add them to see the overall effect. Here's the graph of our new design:



This time we successfully get the expected graph in each of the IOLabs. Here's the graph of our data:



But there are some issues with using two IOLabs individually. Because of the reaction time, there are some tiny time difference when turning on the IOLab, so that the time scale of each IOLab is different, and making it harder for us to analyze, which can cause some problems when fitting the data (See 'Analysis' part for more).

7 References

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