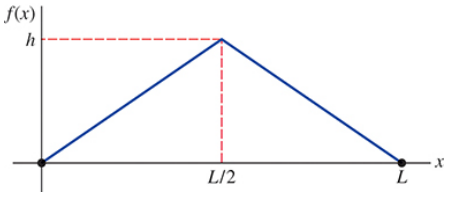
Engineering Mathematics (2)

Assignment#3 (Due: 2024/05/07)

1. (a) A string is secured to the x-axis at x=0 and at x=L and its initial displacement $u(x,0)=f(x)$, $0<x<L$, is shown in the figure below. Find the vertical displacement $u(x, t)$ if the string is released from the rest. Further, show that the solution of the problem can be written as $u(x,t)=\frac{1}{2}\left[f(x+at)+f(x-at)\right]$.



(b) If $L=\pi$, $h=1$ and the speed of wave on a string $a=1$, use Python, Matlab, Excel…to plot the vertical displacement $u(x, t)$ at time t=0, 0.2, 0.7, 1.0, 1.6, and 1.9.

2. (a) Solve Laplace equation for a rectangular plate subject to the given boundary conditions:

$$u(0,y)=0,\quad u(1,y)=0$$
$$u(x,0)=100,\quad u(x,1)=200$$

(b) If possible, refer to Figure 13.5.2 shown in textbook (page 737), use Python to graph the surface defined by $u(x, t)$ (3D-plot) and isotherms (contour-plot).

3. Solve the boundary-value problem

$$k\frac{\partial^2 u}{\partial x^2}-hu=\frac{\partial u}{\partial t},\quad 0<x<1,\quad t>0$$
$$u(0,t)=0,\quad u(\pi,t)=u_0,\quad t>0$$
$$u(x,0)=0,\quad 0<x<\pi$$

The PDE is a form of the heat equation when heat is lost by convection from the lateral surface of a thin rod into a medium at temperature zero.

4. Solve the boundary-value problem

$$k\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t},\quad 0<x<1,\quad t>0$$
$$u(0,t)=0,\quad \left.\frac{\partial u}{\partial x}\right|_{x=1}=-h\left[u(1,t)-u_0\right],\quad h>0,\ t>0$$
$$u(x,0)=f(x),\quad 0<x<1$$

5. The temperature in a semi-infinite solid is modeled by the boundary problem

$$k\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t},\quad x>0,\quad t>0$$
$$u(0,t)=u_0,\quad \lim_{x\to\infty}u(x,t)=0,\quad t>0$$
$$u(x,0)=0,\quad x>0$$

where u_0 is nonzero constant. Solve for $u(x,t)$ by Laplace transform. (Hint: Laplace transforms of the error and complementary error functions given in Table 15.1.1)