Project: Markov Random Field Segmentation of Images 550.431 and 580.466 Statistical Methods in Imaging - Spring 2014

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Your project will consist in developing, programming and experimenting segmentation algorithms based on Markov Random Fields. You can work alone or in pairs. In the later case, both students will receive the same grade.

You will experiment with simulated data that you will generate yourself and also with real images. The real images will come from the 3D image block which was used for Hmw 1 and available at `http://www.cis.jhu.edu/~bruno/StatForImagingSpring2013/`. It is an MRI image of the human brain. Some slices for training and testing have been prepared for you. Figure 1 is the slice with the first component set to 60 of the 3D MRI image block

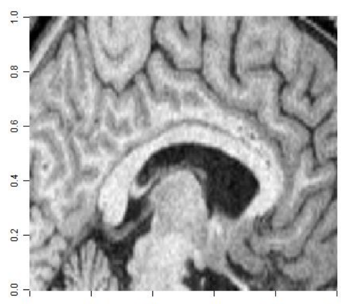

Figure 1: Slice with the first component set to the value 60 of the 3D MRI image block *Rdataset.m*

Rdataset.m.

We define now the parametric model which will be used. The graph on which the Markov Random Field is defined is $G = (S, E)$ where the set of vertices is the set of pixels of a 2-

dimensional image. The set of edges is determined by the 4 closest neighbors of each pixels. Note that pixels at the boundary of the image have only 2 or 3 neighbors depending if they are also corner pixels or not. The hidden segmentation is

$$y = (y_s); s \in S; y_s \in 1, \ldots, K \tag{1}$$

where K is the number of possible labels for the segmentation. You will focus on $K = 3$ labels first, although you can extend this to more labels if time permits. The prior distribution for y is the following Gibbs distribution over the graph $G = (S, E)$:

$$p_0(y) = \frac{1}{Z^0} \exp\left(\sum_{s \in S} \phi_s^0(y_s) + \sum_{<s,t> \in E} \psi_{st}^0(y_s, y_t)\right) \tag{2}$$

with

$$\phi_s^0(y_s) = \sum_{k=1}^{K} \lambda_k \delta(y_s, k) \tag{3}$$

Here $\delta(y_s, k) = 1$ if $y_s = k$ and $\delta(y_s, k) = 0$ if $y_s \neq k$. Also,

$$\psi_{st}^0(y_s, y_t) = \sum_{k_1=1}^{K} \sum_{k_2 \neq k_1} \gamma_{k_1,k_2} \delta(y_s = k_1 \text{ and } y_t = k_2) \tag{4}$$

Note that the prior distribution p_0 is parameterized by $K + K(K-1)/2$ parameters $(\lambda_1, \ldots, \lambda_K, \gamma)$.

We notate x the observed image.

$$x = (x_s), s \in S, x_s \in \mathbb{R} \tag{5}$$

We model the observed image given the segmentation y as follows:

$$p(x|y) = \prod_{s \in S} g(x_s, \mu_{y_s}, \sigma_{y_s}^2) \tag{6}$$

where $g(x_s, \mu_{y_s}, \sigma_{y_s}^2)$ is the density of the Normal distribution evaluated at x_s with mean μ_{y_s} and variance $\sigma_{y_s}^2$. Note that this distribution is parameterized by $2K$ parameters: $(\mu_1, \ldots, \mu_K, \sigma_1, \ldots, \sigma_K)$.

Having observed an image x, a segmentation of x is provided by y^* which is the function of x defined at each pixel s by

$$y_s^*(x) = \arg \max_{k \in \{1,\ldots,K\}} p(Y_s = k|x) \tag{7}$$

Note that $p(Y_s = k|x)$ is the marginal at pixel s of the conditional distribution $p(y|x)$.

As you know, for realistic image sizes, as the one in Figure 1, there is no exact algorithm available to compute $y_s^*(x)$. Instead, approximations must be made. The main goals of this project is to present, implement, and experiment with 2 algorithms for estimating $y_s^*(x)$ which have been discussed in class: the Gibbs sampler and the belief propagation algorithm. If you have time, you can present an alternative algorithm. You can choose the iterated conditional mode which is defined in J. Besag, “On the Statistical Analysis of Dirty Pictures”, J. Roy. Stat. Soc. B, vol. 48, pp. 259-302, 1986 or a graph cut algorithm. Alternatively, another improvement could consist in estimating the parameters of the model using stochastic gradient or using a more elaborate model.

Tasks:

1. **Computation of the posterior of x given y.** Compute
$$p_(y|x) \tag{8}$$

2. **Computation for the Gibbs sampler.** Compute
$$p(Y_s = k|x, y_{v(s)}) \tag{9}$$
where $v(s)$ are the neighbors of s.

3. **Use the Gibbs sampler to simulate images for** y. Choose $K = 3$. Choose images of size 270×160 as this is the size of the real images (Figure 1) on which you will run your algorithms. Choose the parameters by trial and error such that the generated images roughly resemble the real images.

4. **Simulate images of x given y.** Use the images of y simulated in the previous question in order to generate images of x given y. You will provide 5 images.

5. **Use the Gibbs sampler to estimate** $y^*(x)$. Use the Gibbs sampler to estimate $y^*(x)$ from each of the 5 images generated at the last question. In order to do this, run the Gibbs sampler for a long time and for eaxh pixel s, record the number of times where the values is k. Then, choose at each s for the value k which was the most frequent. Compare visually and quantitatively with the true segmentation y used to generate the images.

6. **Use the Belief Propagation algorithm to estimate** $y^*(x)$. Note that the Belief Propagation algorithm provides an estimate of $p(Y_s = k|x)$. Choose the value of k for which this quantity is maximum.

7. **Use the Gibbs sampler then the belief propagation algorithm to provide a segmentation of the real image.** Provide a visual analysis of the results as well as a quantitative analysis using the test images.

8. **Provide an improvement (for bonus points)**. One possibility is to use the stochastic gradient in order to estimate the parameters of the model with the help of the training images.

9. **Generate a 15 minutes power-point (or equivalent) presentation of your work.** Note that you can add extra slides which explain your work but which will not be presented orally.

10. **Do the oral presentation of your work Friday May** 8^{th}**, 9-12**. Split the talking time as equally as possible among the participants.

11. You will upload in Backboard your slides as well as your code.