(a) Pseudo-code for Deriving the Yield-to-Maturity (YTM) Curve

Algorithm 1 Derive YTM Curve from Selected Bonds

Input: For each day, a set of bonds with attributes:

- Coupon Rate (c)
- Maturity Date and Years until Maturity (T)
- Clean Price

Output: Yield-to-Maturity (y) for each bond and a superimposed 5-year yield curve for each day.

for each day in the dataset **do**

  Preprocess bonds:

  1. Convert the *Maturity Date* to a date object.
  2. Compute `days_since_last_coupon` using the day convention:

$$\text{if month} < 7,\ \text{start date} = \text{Jan 1; else, start date} = \text{Jun 30.}$$

  3. Compute the *Dirty Price* as:

$$\text{Dirty Price} = \text{Clean Price} + \left(\frac{\texttt{days\_since\_last\_coupon}}{365}\right)(c \times 100).$$

  Sort the bonds by increasing "Years until Maturity".
  Initialize an empty map `ytm_rates`.
  for each bond in the sorted list **do**
    Let T denote the bond's time-to-maturity (in years).
    if $T < 0.5$ (i.e., a short-term bond with one coupon) **then**
      Compute yield directly using:

$$y = -\frac{\ln\left(\frac{\text{Dirty Price}}{100+c\times 50}\right)}{T}.$$

    else
      Define the coupon payment as:

$$N = 100 \times \frac{c}{2}.$$

      Let $n = \lfloor 2T \rfloor$ be the number of coupon payments.
      Define the pricing function:

$$f(y) = \sum_{i=1}^{n-1} N\, e^{-y\, t_i} + (100 + N)\, e^{-yT} - \text{Dirty Price},$$

      where t_i are the times of the coupon payments.
      Solve for y such that $f(y) = 0$ using a numerical method (e.g., `fsolve`) with an initial guess (e.g., $y_0 = 0.05$).
    end if
    Save the computed yield y associated with maturity T in `ytm_rates`.
  end for
  Interpolation: For maturities in the interval $[1, 5]$ years that do not have a directly computed yield, interpolate between the nearest available yields.
  Plot: Superimpose the 5-year yield curve (i.e., plot y vs. T for $T \in [1, 5]$) for the current day on a common graph.
end for

(b) Pseudo-code for Deriving the 1–5 Year Spot Curve

Algorithm 2 Derive Spot Curve from Selected Bonds

Input: For each day, a set of bonds with attributes:

- Coupon Rate (c)
- Maturity Date
- Clean Price

Output: Spot rates $r(T)$ for maturities $T \in [1, 5]$ years.

for each day in the dataset **do**

  For each bond:

  1. Compute `days_since_last_coupon` using the day convention:

$$\text{if month} < 7, \text{ start date} = \text{Jan 1 of current year; else, start date} = \text{Jun 30 of current year.}$$

  2. Compute the *dirty price*:

$$\text{Dirty Price} = \text{Clean Price} + \left(\frac{\texttt{days\_since\_last\_coupon}}{365}\right)(c \times 100).$$

  Sort bonds in increasing order of "Years until Maturity".

  Initialize an empty map `spot_rates`.

  for each bond in the sorted list **do**

    Let T be the bond's time-to-maturity (in years).

    if $T < 0.5$ (i.e., a bond with only one coupon payment) **then**

      Compute spot rate directly:

$$r(T) = -\frac{\ln\left(\frac{\text{Dirty Price}}{100 + c \times 50}\right)}{T}.$$

    else

      Let $n = \lfloor 2T \rfloor$ be the number of coupon payments.

      Set `residual_price` $\leftarrow$ Dirty Price.

      for each coupon payment time t_i, $i = 1, \ldots, n-1$, corresponding to maturities less than T **do**

        Retrieve or **interpolate** the spot rate $r(t_i)$ from `spot_rates`.

        Subtract the present value of the coupon:

$$\texttt{residual\_price} \mathrel{-}= \left(100 \times \frac{c}{2}\right) \times e^{-r(t_i)t_i}.$$

      end for

      Compute the spot rate at T:

$$r(T) = -\frac{\ln\left(\frac{\texttt{residual\_price}}{100 + 100 \times \frac{c}{2}}\right)}{T}.$$

    end if

    Save $r(T)$ in `spot_rates` with key T.

  end for

  Plot: Superimpose the spot curve (i.e., $r(T)$ vs. T for $T \in [1, 5]$) for this day onto a common graph.

end for

(c) Pseudo-code for Deriving the 1-Year Forward Curve

Algorithm 3 Derive 1-Year Forward Curve from Spot Rates

Input: For each day, the spot curve $r(T)$ for maturities $T \in [1, 5]$ years.

Output: 1-year forward rates $f(1, T)$ for maturities $T \in [2, 5]$ years (i.e., forward rates from time 1 to time T).

for each day in the dataset **do**

For each bond or maturity point T (with $T \geq 1$):

1. Retrieve or interpolate the spot rate $r(T)$.
2. Compute the *future price* of a zero-coupon bond maturing at T, evaluated at $t = 1$:

$$P(1, T) = e^{-r(T)(T-1)}.$$

3. Compute the natural logarithm: $L(T) = \ln(P(1, T))$.

For each maturity point T (from the available set, e.g., $T \in \{2, 3, 4, 5\}$):

1. **If** T **is an interior point:** Approximate the derivative using the central difference formula:

$$f(1, T) \approx -\frac{1}{2}\left(\frac{L(T + \Delta T) - L(T)}{\Delta T} + \frac{L(T) - L(T - \Delta T)}{\Delta T}\right).$$

2. **If** T **is a boundary point:** Use a forward (or backward) difference approximation:

$$f(1, T) \approx -\frac{L(T + \Delta T) - L(T)}{\Delta T} \quad \text{(for the lower boundary)}$$

or

$$f(1, T) \approx -\frac{L(T) - L(T - \Delta T)}{\Delta T} \quad \text{(for the upper boundary)}.$$

Save the computed forward rate $f(1, T)$ for each maturity T.

Plot: Superimpose the forward curve (i.e., $f(1, T)$ vs. T for $T \in [2, 5]$) for this day onto a common graph.

end for