# 用到的公式 [1]

haodayizhia

2023 年 9 月 14 日

## 目录



# 1 二体问题

## 1.1 核心公式

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} \tag{1.1}$$

## 1.2 推导用到的向量公式

$$\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}(\vec{a}\cdot\vec{c})-\vec{c}(\vec{a}\cdot\vec{b}) \tag{1.2}$$

$$\vec{a}\cdot(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\cdot\vec{c} \tag{1.3}$$

## 1.3 能量 $E$ 守恒

$$\begin{aligned}\dot{\vec{r}}\cdot\ddot{\vec{r}}+\dot{\vec{r}}\cdot\frac{\mu}{r^3}\vec{r}=0\\ \rightarrow E=\frac{v^2}{2}+(c-\frac{\mu}{r})=\mathrm{const}\end{aligned} \tag{1.4}$$

## 1.4 角动量 $\vec{h}$ 守恒

$$\vec{h}=\vec{r}\times\dot{\vec{r}} \tag{1.5}$$

## 1.5 近地点方向 $\vec{B}=\mu\vec{e}$

$$\dot{\vec{r}}\times\vec{h}=\frac{\mu}{r}\vec{r}+\vec{B} \tag{1.6}$$

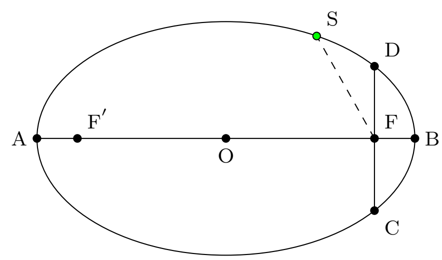
## 1.6 运动轨迹[1]

$$r=\frac{h^2/\mu}{1+B/\mu\cos\theta}=\frac{p}{1+e\cos\theta} \tag{1.7}$$

$$r=\begin{cases}a & e=0,\text{圆}\\ \frac{a(1-e^2)}{1+e\cos\theta} & 0<e<1,\text{椭圆}\\ \frac{p}{1+\cos\theta} & e=1,\text{抛物线}\\ \frac{a(e^2-1)}{1\pm e\cos\theta} & 1<e,\text{双曲线}\end{cases} \tag{1.8}$$

以椭圆为例

[1] 不包含指向质心的直线降落




F: prime focus, 代表中心天体 F′: secondary/vacant focus

AB: major axis, 长度为 $2a$ F′: focal distance, 焦距, 长度为 $2c$

CD: latus rectum, 长度为 $2p$

A: apoapsis(apogee/aphelion/aposelenium)

B: periapsis(perigee/perihelion/periselenium)

## 1.7 活力公式

取近地点代入能量积分中

$$\frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a} \tag{1.9}$$

## 1.8 anomaly 转换

$\theta$: True anomaly(真近点角), $E$: Eccentric anomaly(偏近点角), $M$: Mean anomaly(平近点角).

$$a - r = ae\cos E \tag{1.10}$$

$$M = n(t-\tau) = E - e\sin E \tag{1.11}$$

### 1.9 Conversion between rv and classical orbit elements

$rv$ **to** $\sigma$(注意 arccos)

$$\begin{cases} e = \frac{|\dot{\vec{r}}\times\vec{h}-\frac{\mu}{r}\vec{r}|}{\mu} \\ a = \frac{h^2}{\mu(1-e^2)} \\ i = \arccos\frac{\vec{h}\cdot(0,0,1)}{h} \\ \Omega = \arccos\frac{(0,0,1)\times\vec{h}\cdot(1,0,0)}{|(0,0,1)\times\vec{h}|} \\ \omega = \arccos\frac{\vec{B}\cdot((0,0,1)\times\vec{h})}{B|(0,0,1)\times\vec{h}|} \\ \theta = \arccos\frac{\vec{r}\cdot\vec{B}}{rB} \end{cases} \quad or \begin{cases} a = -\frac{\mu r}{v^2 r-2\mu} \\ e = \sqrt{1-\frac{h^2}{\mu a}} \\ i = \arccos\frac{\vec{h}\cdot(0,0,1)}{h} \\ \Omega = \arccos\frac{(0,0,1)\times\vec{h}\cdot(1,0,0)}{|(0,0,1)\times\vec{h}|} \\ \omega = \arccos\frac{\vec{B}\cdot((0,0,1)\times\vec{h})}{B|(0,0,1)\times\vec{h}|} \\ \theta = \arccos\frac{\vec{r}\cdot\vec{B}}{rB} \end{cases} \tag{1.12}$$

$\sigma$ **to** $rv$

$$\begin{cases} A = \begin{bmatrix} \cos(-\Omega) & \sin(-\Omega) & \\ -\sin(-\Omega) & \cos(-\Omega) & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & \\ & \cos(-i) & \sin(-i) \\ & -\sin(-i) & \cos(-i) \end{bmatrix} \begin{bmatrix} \cos(\omega+\theta) & \sin(\omega+\theta) & \\ -\sin(\omega+\theta) & \cos(\omega+\theta) & \\ & & 1 \end{bmatrix} \\ \vec{r} = A \begin{bmatrix} \frac{p}{1+e\cos\theta} \\ 0 \\ 0 \end{bmatrix} \\ \vec{v} = A \begin{bmatrix} \frac{he\sin\theta}{p} \\ \frac{h(1+e\cos\theta)}{p} \\ 0 \end{bmatrix} \end{cases} \tag{1.13}$$

# 2 定轨

## 参考文献

[1] R.R. Bate, D.D. Mueller, and J.E. White. *Fundamentals of Astrodynamics*. Dover Books on Aeronautical Engineering Series. Dover Publications, 1971.