用到的公式 [1]

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目录

1 二体问题

1.1 核心公式

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} \tag{1.1}$$

1.2 推导用到的向量公式

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}) \tag{1.2}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c} \tag{1.3}$$

1.3 能量 E 守恒

$$\dot{\vec{r}} \cdot \ddot{\vec{r}} + \dot{\vec{r}} \cdot \frac{\mu}{r^3}\vec{r} = 0$$

$$\rightarrow E = \frac{v^2}{2} + (c - \frac{\mu}{r}) = \text{const} \tag{1.4}$$

1.4 角动量 $\vec{h}$ 守恒

$$\vec{h} = \vec{r} \times \dot{\vec{r}} \tag{1.5}$$

1.5 近地点方向 $\vec{B} = \mu\vec{e}$

$$\dot{\vec{r}} \times \vec{h} = \frac{\mu}{r}\vec{r} + \vec{B} \tag{1.6}$$

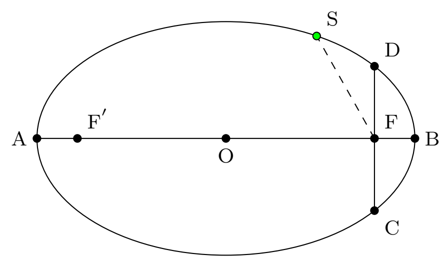
1.6 运动轨迹[1]

$$r = \frac{h^2/\mu}{1 + B/\mu \cos\theta} = \frac{p}{1 + e\cos\theta} \tag{1.7}$$

$$r = \begin{cases} a & e = 0, \text{圆} \\ \frac{a(1-e^2)}{1+e\cos\theta} & 0 < e < 1, \text{椭圆} \\ \frac{p}{1+\cos\theta} & e = 1, \text{抛物线} \\ \frac{a(e^2-1)}{1\pm e\cos\theta} & 1 < e, \text{双曲线} \end{cases} \tag{1.8}$$

以椭圆为例

[1]不包含指向质心的直线降落



F: prime focus, 代表中心天体 F′: secondary/vacant focus
AB: major axis, 长度为 $2a$ F′: focal distance, 焦距, 长度为 $2c$
CD: latus rectum, 长度为 $2p$
A: apoapsis(apogee/aphelion/aposelenium)
B: periapsis(perigee/perihelion/periselenium)

1.7 活力公式

取近地点代入能量积分中

$$\frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a} \tag{1.9}$$

1.8 anomaly 转换

θ: True anomaly(真近点角), E: Eccentric anomaly(偏近点角), M: Mean anomaly(平近点角).

$$a - r = ae\cos E \tag{1.10}$$

$$M = n(t - \tau) = E - e\sin E \tag{1.11}$$

1.9 Conversion between rv and classical orbit elements

rv **to** σ(注意 arccos)

$$
\begin{cases}
e=\frac{|\dot{\vec{r}}\times\vec{h}-\frac{\mu}{r}\vec{r}|}{\mu}\\
a=\frac{h^2}{\mu(1-e^2)}\\
i=\arccos\frac{\vec{h}\cdot(0,0,1)}{h}\\
\Omega=\arccos\frac{(0,0,1)\times\vec{h}\cdot(1,0,0)}{|(0,0,1)\times\vec{h}|}\\
\omega=\arccos\frac{\vec{B}\cdot((0,0,1)\times\vec{h})}{B|(0,0,1)\times\vec{h}|}\\
\theta=\arccos\frac{\vec{r}\cdot\vec{B}}{rB}
\end{cases}
\quad or \begin{cases}
a=-\frac{\mu r}{v^2r-2\mu}\\
e=\sqrt{1-\frac{h^2}{\mu a}}\\
i=\arccos\frac{\vec{h}\cdot(0,0,1)}{h}\\
\Omega=\arccos\frac{(0,0,1)\times\vec{h}\cdot(1,0,0)}{|(0,0,1)\times\vec{h}|}\\
\omega=\arccos\frac{\vec{B}\cdot((0,0,1)\times\vec{h})}{B|(0,0,1)\times\vec{h}|}\\
\theta=\arccos\frac{\vec{r}\cdot\vec{B}}{rB}
\end{cases}
\tag{1.12}
$$

σ **to** rv

$$
\begin{cases}
A=\begin{bmatrix}\cos(-\Omega) & \sin(-\Omega) & \\ -\sin(-\Omega) & \cos(-\Omega) & \\ & & 1\end{bmatrix}\begin{bmatrix}1 & & \\ & \cos(-i) & \sin(-i)\\ & -\sin(-i) & \cos(-i)\end{bmatrix}\begin{bmatrix}\cos(\omega+\theta) & \sin(\omega+\theta) & \\ -\sin(\omega+\theta) & \cos(\omega+\theta) & \\ & & 1\end{bmatrix}\\
\vec{r}=A\begin{bmatrix}\frac{p}{1+e\cos\theta}\\ 0\\ 0\end{bmatrix}\\
\vec{v}=A\begin{bmatrix}\frac{he\sin\theta}{p}\\ \frac{h(1+e\cos\theta)}{p}\\ 0\end{bmatrix}
\end{cases}
\tag{1.13}
$$

2 定轨

3 解析解

3.1 经典摄动法

根据第1章, 无摄运动的解的形式为 (其中 c_1,c_2,c_3,c_4,c_5,c_6 为常数)

$$\boldsymbol{r}=\boldsymbol{f}(c_1,c_2,c_3,c_4,c_5,c_6,t) \tag{3.14}$$

$$\dot{\boldsymbol{r}}=\boldsymbol{g}(c_1,c_2,c_3,c_4,c_5,c_6,t) \tag{3.15}$$

而受摄二体问题运动方程的一般形式:

$$\ddot{\boldsymbol{r}} = \boldsymbol{F}_0(\boldsymbol{r}) + \boldsymbol{F}_1(\boldsymbol{r}, \dot{\boldsymbol{r}}, t; \varepsilon) \tag{3.16}$$

$$\boldsymbol{F}_0(\boldsymbol{r}) = -\frac{\mu}{r^2}(\frac{\boldsymbol{r}}{r}) \tag{3.17}$$

利用求解常微分方程中的常数变易法, 保持(3.14)和(3.15)解的形式不变, 令 $c_1, c_2, c_3, c_4, c_5, c_6$ 是与时间相关的变量, 满足

$$\left.\begin{aligned} \sum_{i=1}^{6} \frac{\partial \boldsymbol{f}}{\partial c_i}\frac{dc_i}{dt} = 0 \\ \sum_{i=1}^{6} \frac{\partial \boldsymbol{g}}{\partial c_i}\frac{dc_i}{dt} = \boldsymbol{F}_1 \end{aligned}\right\} \tag{3.18}$$

3.2 平均根数法

构造根数 $\sigma = (a, e, i, \Omega, \omega, M_0)^T$ 解的形式

$$\begin{cases} \sigma(t) = \bar{\sigma}(t) + \sigma_l^{(1)}(t) + \sigma_l^{(2)}(t) + \cdots + \sigma_s^{(1)}(t) + \sigma_s^{(2)}(t) + \cdots \\ \bar{\sigma}(t) = \bar{\sigma}^{(0)}(t) + \sigma_1(t - t_0) + \sigma_2(t - t_0) + \cdots \\ \bar{\sigma}^{(0)}(t) = \bar{\sigma}_0 + \delta\bar{n}(t - t_0) \\ \bar{\sigma}_0 = \sigma_0 - [\sigma_l^{(1)}(t_0) + \sigma_l^{(2)}(t_0) + \cdots + \sigma_s^{(1)}(t_0) + \sigma_s^{(2)}(t_0) + \cdots] \end{cases} \tag{3.19}$$

其中 $\delta = (0, 0, 0, 0, 0, 1)^T$

参考文献

[1] R.R. Bate, D.D. Mueller, and J.E. White. *Fundamentals of Astrodynamics*. Dover Books on Aeronautical Engineering Series. Dover Publications, 1971.