Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
B		2423449

Searching for Submersibles: a Localization-Search Model based on Ocean Dynamics

Summary

Nowadays, the exploration of marine resources gradually receives more and more attention. With the loss of the Titan submersible, the safety of submersibles has attracted widespread attention. In order to solve the problem of searching after the loss of a submersible, this paper built a **localization-search model**.

For task 1,to construct an accurate positioning model, we simplify the submersible into a **capsule** shape, conduct a force analysis according to **ocean dynamics**, combine with Newton's second law, establish a **positioning prediction model**, and take the Ionian Sea as an example, numerical simulation on the model ,and learn that when $t = 207.4s$, the depth of the submersible reaches the center of buoyancy point, $z = 175.3m$, from the model and sensitivity analysis, the key information to be accepted are the position r, current velocity u, and seawater density ρ.

For task 2, we collect and organize the function, price, availability, preparation speed and other parameters of each type of equipment by searching for information, by using **Analytic Hierarchy Process(AHP)**, we constructed a judgment matrix, and passed the consistency test, and got the score of each equipment, which provides a reference for the selection of equipment.

For task 3, we substitute the ocean current data into the localization prediction model established in Problem 1, so as to determine the main trajectory with the highest probability of trajectory, and use the synchronization point with the SAR equipment on this trajectory as the initial point to carry out the **moving rectangle search**. Taking the Ionian Sea as an example, it can be calculated that when $t^* = 201min$, the **synchronization point** is $r^* = (x^*, y^*) = (24032.5, 3458.7)$, and the search is carried out in a sector with a tensor angle of $\pi/6$, and the final probability of the search and rescue is stabilized at 94%, with the rescue time predicted to be 225min.

For task 4, in order to better extend the **"localization-search" model**, the Caribbean current data are substituted into the model of Problems 1 and 3, and the trajectories shown in Figure 15 are obtained, so that we can verify that the model is practical for different sea areas. For the case of a search vessel searching for a multi-finger submersible, we utilize **the shortest path algorithm** to determine the search order. Then the **multi-target Locked pursuit model** is established to search and rescue one by one.

Finally, the sensitivity of the model is fully tested.

key words: Ocean Dynamics; Locating-Search Model; Moving Rectangle Search

MEMO

FROM: Team 2423449 , MCM B
To: Government of Greece
Date: February 5, 2024
Dear Officials:

I am writing to provide you, on behalf of Maritime Cruises Mini-Submarines (MCMS), with a Memorandum of Assurance on securing safety for underwater adventures in the Ionian Sea and detailing our commitment. Our company has successfully developed and deployed manned submersibles capable of exploring the depths of the ocean for underwater adventures and shipwreck exploration. In addition, we have developed appropriate positioning models, equipment deployment and optimal rescue routes to ensure the safety of our visitors during their excursions.

1. Positioning model: We have fully considered the effects of gravity, mechanical propulsion, pressure gradient, Coriolis force, viscous force, turbulent stress and other external factors on the submersible when exploring under the sea, and created a three-dimensional capsule motion model by using differential equations, which can be used to realize the accurate positioning of the submersible through the transmission of information between the submersible and the main ship. Then we numerically simulate the time-dependent trajectory of the submersible through the relevant ocean data in the Ionian Sea, which lays the foundation for the next optimal rescue strategy.

In addition, the submersible is required to send critical information to the host ship at regular intervals in order to minimize uncertainty in case of unexpected events. In order to determine these key messages, we analyze the sensitivity of the parameters of the capsule model created by the differential equations, and finally identify the key factors to be transmitted as pressure, seawater density, current speed, and power of the submersible. In order to ensure that the submersible can transmit the above four parameters to the main ship in time, we equipped the submersible with real-time power diagnostic, communication system, global positioning system (GPS), underwater sonar, anemometer and other related instruments.

2. Preparation work on the main ship: We purchased appropriate rescue equipment according to the needs of safety and security work, which used the hierarchical analysis method to evaluate each type of equipment, taking into account the highest priority of search and rescue is time, so we chose the three main indicators of cost, practicability, and readiness, and constructed a judgment matrix (which passes the consistency test) to select the equipment, and ultimately, our company chose the Saab Seaeye Leopard ROV as our additional search equipment.

3. Equipment Deployment and Rescue Mode Determination: For solving the single target search and rescue, first we divided the search and rescue sea area into several grids, and then we divided the rescue into two steps, the first step is to roughly determine the sea area where the lost submersible is located, and the actual trajectory of the submersible is visualized accordingly, and we assumed that the search and rescue mission started 10 minutes after the loss of the link, and calculated the time of arriving at the current location of the lost submersible in the shortest possible time by using the corresponding pursuit formula. The time to reach the current location of the lost submersible in the shortest time was calculated using the corresponding pursuit formula, which was about 201.3 minutes. Then in the second step, we carried out a more detailed search and rescue in this small range of sea area, and chose the "Moving Rectangle Search" as our strategy, which was simulated by the value of 200 times, and the shortest time was about 23.3 minutes under the success rate of 94%, which meant that we could basically search and rescue in 224 minutes, which meant that we could basically search and rescue in 23 minutes. That is to say, we can basically realize the rescue of the trapped submersible in about 224 minutes.

For multi-target search and rescue, we use the shortest path algorithm to find out the shortest search and rescue sequence, i.e., the priority of search and rescue targets, and then use the single-target tracking search method to search according to the sequence, which not only optimizes the rescue time, but also takes into account the endurance of the search and rescue vessel.

After optimizing the search and rescue strategy, our company also applies the model and the search and rescue scheme to the Caribbean sea area for numerical simulation, and the results are also more satisfactory, which proves that our model has strong robustness and generalization ability, and is sufficient to cope with the emergency rescue tasks in reality.

Please consider the above suggestions and feel free to contact us for further discussion and fulfillment of the requirements of the relevant regulatory agencies.

Sincerely yours
MCM B Team 2423449

Contents

1 Introduction

1.1 Background

The ocean is the second largest space after land in the four strategic spaces of ”land, sea, air and sky” for human development, and it is a strategic development base for biological resources, energy, water resources and metal resources. As a great power tool for human exploration of the ocean and maintenance of maritime rights and interests, deep-sea submersibles will play an important and irreplaceable role. Submarine can carry a variety of electronic devices, mechanical equipment and personnel quickly and accurately reach a variety of deep-sea environments, for efficient exploration, scientific investigation and development of operational equipment, through the submarine's in-depth exploration, we were able to peep into the mysteries of the undersea world.On June 18, 2023, the **submersible Titan** was lost, and it took until June 22, 2023 for the ROV to locate the submersible's area, and the safety issues raised by the loss of the submersible have been widely publicized.

1.2 Restatement of the Problem

The Greek company MCMS builds submersibles that can carry people into the deep sea and lead tourists on expeditions to the seabed of the Ionian Sea in search of shipwrecks. In order to cope with various emergencies and to guarantee the safety of the tourists, a model is now needed that can predict the position of the submersible over a period of time when it is lost.

Therefore, our task is

1. **Locate:** (1) Build a model that predicts the position of the submersible over time. (2) Determine what information the submersible needs to send to the primary vessel based on the needs of the model so as to reduce uncertainty in the event of an accident. (3) Determine what equipment the submersible needs to be equipped with based on the need to collect the appropriate information.

2. **Prepare:** (1) Make a reasonable recommendation as to what search equipment needs to be carried on board the main ship (2) Make a reasonable plan, taking into account the costs associated with the availability, maintenance, readiness, and use of this equipment

3. **Search:** (1) Develop a model to determine the optimal initial deployment point and search pattern that will minimize the time to locate the lost submersible. (2) Determine the probability of finding the submersible based on time and cumulative search results

4. **Extrapolate:** (1) Extend the model to other tourist destinations, such as the Caribbean. (2) How would the model change if multiple submersibles were lost in the same sea area

1.3 Our Works

In this paper, we have done the following.

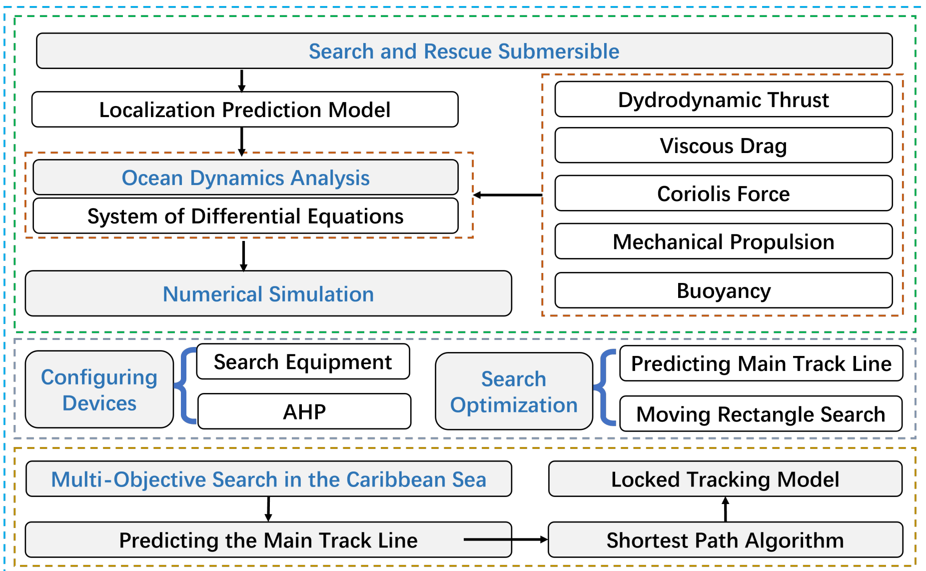


Figure 1: Analysis of Problematic Ideas

- Dynamic analysis of the submersible, development of a positioning prediction model, and visualization and stability analysis by numerical simulation
- Gathered information on functions, price, availability, maintenance requirements, speed of preparation, etc. of various types of search equipment, and evaluated them through the hierarchical analysis method to obtain suitable equipment, and recommended some of the rescue equipment.
- Using the established positioning prediction model to conduct multiple numerical simulations to obtain the probability distribution data to determine the predicted main track line, which in turn determines the initial deployment point, thereby minimizing the time to search for the lost submarine.
- When the search equipment arrives at the initial deployment point, it carries out a moving rectangular search along the tangent direction of the predicted main track line, and we have fitted the search and rescue probability as a function of time and cumulative search results by doing multiple numerical simulations and estimating the probability in terms of frequency.

- By collecting oceanographic parameters from other areas (e.g., the Caribbean Sea), the positional prediction model can still determine the predicted main track line and then apply the same search.
- In the case of multiple submarines, we consider the range of the search vessel, determine the search order by the shortest path algorithm, and conduct the search by the locked tracking method and the rectangular search method.

2 Assumptions and Notations

2.1 Notations

In this work, we use the symbols in Table 1 in the model construction. Other nonfrequent-used symbols will be introduced once they are used.

Table 1: Noations

Symbol	Definitions
t	Time
v	submarine speed
v_x, v_y, v_z	the components of the diver's velocity v in the X, Y, Z directions
u	current speed
F_T	current Thrust
F_V	viscous resistance
F_M	mechanical Propulsion
F_b	buoyancy force
A	judgment matrix t
$r*$	initial Deployment Points
m	Submersible quality
ρ	seawater density
P	mechanical output powe
x, y, z	horizontal east-west, north-south, and vertical displacements of the submersible
r	total displacement
v_x, v_y, v_z	speed of the submersible
u	currents and velocities
F	driving force on the submersible
a_x, a_y, a_z	acceleration of the submersible

2.2 Assumptions

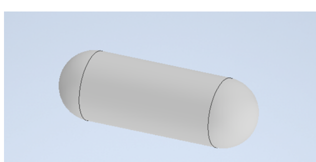
Due to the lack of necessary data, we make the following assumptions to help us perform modeling.

- **Assumption 1:**Assuming that the submarine loses contact due to signaling problems, and that there are no other serious accidents, the submarine will still be able to move with the currents after the power is turned off.

- **Assumption 2:** Assuming that after the loss of connection, the submersible stops power output and no longer floats and sinks by draining and absorbing water, i.e., no active motion.
- **Assumption 3:** It is assumed that the materials used in the construction of the submersible are so stiff that the change in volume is negligible, which facilitates the dynamics analysis.

3 Locate: Submarine Locating Model

When the submersible encounters a special fault into the drifting state, will be affected by the ocean currents, resulting in a change in the position of the submersible. For the drifting state of the submersible, in order to more convenient will be the ocean in the water situation and the force of the submersible to establish a more concise functional relationship, may wish to assume that the submersible is as shown in Figure (insert picture) two hemispheres sandwiched between the geometry of the cylinder, which the height of the cylinder for the sphere of two times the diameter of the sphere, i.e., $4R$, shaped like a capsule, as shown in Figure 2(a), in order to MIR2, as shown in Figure 2(b), can be known to be a reasonable assumption since it is similar to normal submersible geometry.



(a) Capsule model

(b) MIR 2

Figure 2: model simplification

3.1 Force Situation

The force analysis of the submersible, according to the ocean dynamics [1]can be seen, the submersible under the surface of the sea is mainly subjected to gravity, mechanical propulsion, pressure gradient force, Coriolis force [2], viscous force, turbulence stress, celestial gravitational tidal force, a total of seven forces. Among them, the turbulent stress, celestial tidal force and Coriolis force have very little effect on the state of the submersible compared to other forces, and the pressure gradient force can be approximated as the vector sum of buoyancy and a thrust force in the same direction as the current when the current velocity is not too large and the computational accuracy is not too high.

Therefore, in order to simplify the model, we neglect turbulent stresses, celestial gravitational tidal forces, and Corio's forces, and calculate the vector sum of buoyancy and thrust forces instead of pressure gradient forces.

3.1.1 Hydrodynamic Thrust

In the water flow through the elbow, will produce pressure on the elbow. Set AB and CD two sections at the average flow rate of v_1 and v_2, F_1 and F_2 are before and after the body of water for the AB and CD two sections at the total pressure; m for the $ABCD$ section of the body of water mass weight. Q is the flow rate of the water body through the unit section in unit time, and the density of water is ρ . The $ABCD$ water body part

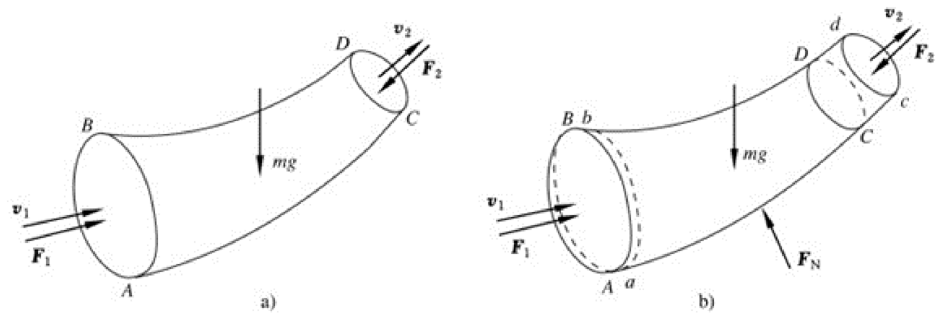


Figure 3: Fluid Bend Force Analysis

is selected as the mass system, as shown in the figure 3, where F_N is the binding force of the pipe on the water body. Let after dt time, the water body is displaced from the original $ABCD$ position to the new position $abcd$.

According to the momentum theorem we have

$$\rho Q dt(v_2 - v_1) = (mg + F_1 + F_2) + \rho Q(v_2 - v_1)$$

Thus, in order to change the direction of the fluid, the force acting on the fluid by the pipe wall should be

$$F_N = \rho Q(v_2 - v_1) \tag{1}$$

In the case of the capsule model, it is assumed that the initial velocity of the water flow is along the X-axis, and after collision with the sidewall of the capsule model, it flows out along the tangential direction of the sidewall, and the specific direction is related to the contact position From the analysis of the figure 4, it can be seen that in the X direction, the velocity changes from v to $v \sin\theta$, which is brought into the equation (1) ,

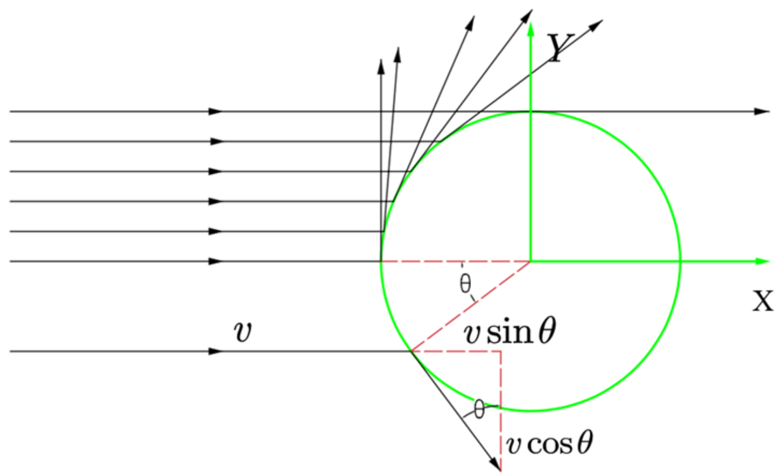


Figure 4: Current collision with submersible

and integrates to get

$$F_T = \rho Q \int_0^{\frac{\pi}{2}} v(1-\sin\theta)d\theta = \rho Q v(\frac{\pi}{2}-1) \tag{2}$$

In the Y direction, the thrusts generated by the upper and lower halves cancel each other out exactly, so that no thrust is generated in the Y direction. It follows that the current generates a thrust of $F = \rho Q v(\frac{\pi}{2}-1)$ in the direction parallel to it and no thrust in the direction perpendicular to it. From this, we obtain the components of the current thrust in the three directions as

$$\begin{cases} F_{Tx} = S_x \rho Q \Delta v_x(\frac{\pi}{2}-1), \Delta v_x = u_x - v_x, S_x = \pi R^2 \\ F_{Ty} = S_y \rho Q \Delta v_y(\frac{\pi}{2}-1), \Delta v_y = u_y - v_y, S_y = (8+\pi) R^2 \\ F_{Tz} = S_z \rho Q \Delta v_z(\frac{\pi}{2}-1), \Delta v_z = u_z - v_z, S_z = (8+\pi) R^2 \end{cases} \tag{3}$$

3.1.2 Viscous Resistance

In calculating the viscous drag, we assume the capsule model to be a hemisphere in the forward direction, and based on relevant empirical reports and research results[3], we can reasonably assume that the capsule model can be approximated to be a sphere in the forward direction and calculate the viscous drag using **Stokes' law**[4], which is given by

$$F_V = 6\pi\eta(v-u)R \tag{4}$$

where R is the radius of the sphere, is the velocity it is relative to the liquid, and η is the viscosity coefficient of the liquid.

Similarly, on the side of the capsule model, there will be viscous drag due to the component of velocity on the side, and it may be useful to set the

$$F_{Vy} = \alpha(v_y - u_y) \tag{5}$$

$$F_{Vz} = \beta(v_z - u_z) \tag{6}$$

α, β are the coefficients of viscous drag with relative velocity in Y, Z directions, respectively, and are related to fluid density and model shape.

3.1.3 Mechanical Propulsion

F_M is the mechanical propulsion, controlled by human, related to the mechanical structure, and is not related to the position of the waters, so $F_M = F_M(t)$, where we assume that the power output of the machinery is unchanged as P, according to $P = Fv$, get

$$F_M = \frac{P}{v_x} \tag{7}$$

3.1.4 Buoyancy

The density of seawater is affected by pressure, temperature and salinity, above 4°C, temperature decreases and density increases, pressure increases and density increases, the lower the altitude, the higher the pressure, so density increases with depth. The salinity of seawater in high latitudes increases with depth, while the salinity of seawater in middle and low latitudes decreases with depth, and the higher the salinity, the higher the density. After reviewing the literature[5]can be known, we can determine that the density and temperature into a primary function of the relationship between temperature and depth into a primary function of the relationship; density and pressure into a primary function of the relationship between pressure and depth into a primary function of the relationship between salinity and depth into a primary function of the relationship between salinity and depth, and so there are

$$\rho = \alpha(T - T_0) + \beta p_p + \gamma S + \rho_0$$

where,α, β, γ are the coefficients of variation of density with temperature, pressure, and salinity. Since temperature, pressure, and salinity are all primary functions of depth, the equation can be simplified as

$$\rho = \delta z + \rho_0 \tag{8}$$

Based on the basic dimensional parameters of the capsule model, we obtain

$$F_b = \rho g V = 16.76 \rho g R^3 = 16.76(\delta z + \rho_0) g R^3 \tag{9}$$

3.2 Dynamic Analysis

Using the last known position before the loss as the coordinate origin, choose the Cartesian Cartesian coordinate system shown in the figure.

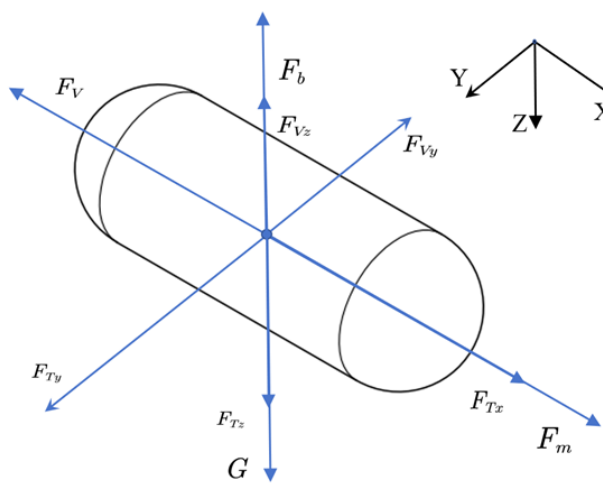


Figure 5: Force analysis of submersibles

Combining equations(3)(4)(7)(9) by force analysis we have

$$\begin{cases} \sum F_x = F_m + F_{Tx} - F_V \\ \qquad = \dfrac{P}{v_x} + \pi R^2 \rho(z) Q(u_x - v_x)(\dfrac{\pi}{2} - 1) - 6\pi\eta(v_x - u_x)R \\ \sum F_y = F_{Ty} - F_{Vy} \\ \qquad = (8 + \pi R^2)\rho(z) Q(u_y - v_y)(\dfrac{\pi}{2} - 1) - \alpha(v_y - u_y) \\ \sum F_z = G + F_{Tz} - F_{Vz} - F_b \\ \qquad = G + (8 + \pi R^2)\rho(z) Q(u_z - v_z)(\dfrac{\pi}{2} - 1) - \beta(v_z - u_z) - 16.76\rho(z) g R^3 \end{cases} \tag{10}$$

According to Newton's second law, there are

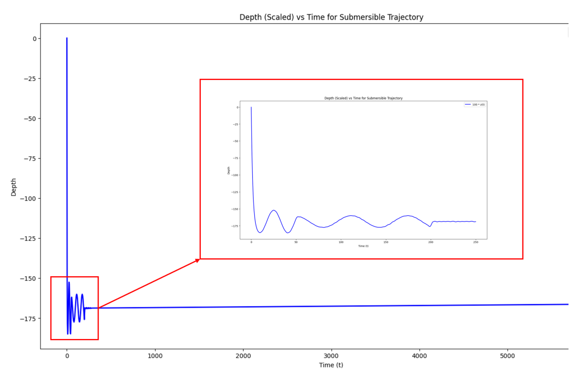
$$m\frac{d^2r}{dt^2} = m\frac{dv}{dt} = ma = \sum F$$

where$r(t) = (x(t), y(t), z(t))$ Taking the starting position as the origin of the coordinates, the position of the submersible is given by

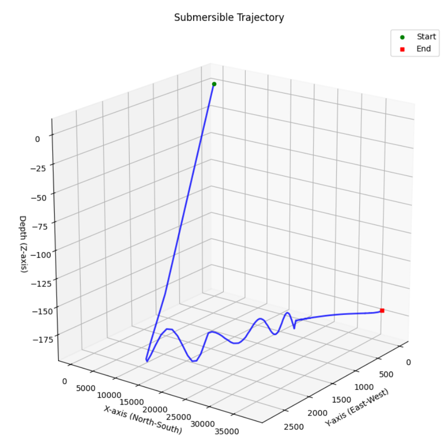
$$r(t) = \iint_0^t a(t)dtdt = \frac{1}{m}\iint_0^t F(t)dtdt \tag{11}$$

3.3 Trajectory Simulation

Simulation: In order to simulate the position of the submarine,we take the parameters of MIR2 submersible as an example. the weight is 17t, the width of the front is 2.6m, the radius can be approximated as 1.3m, and the sailing speed is 2 knots, and we set the Ionian Sea (**specific length and width**) as the simulated sea area, by collecting data on ocean currents and then let the initial state be$x(0) = y(0) = z(0) = 0$. We substitute each parameter of MIR2 into the model, the following simulation results can be obtained



(a) Variation of depth z with time t



(b) Submarine trajectory

Figure 6: numerical simulation

As shown in Figure 6(a),when $t = t_0 = 207.4s$,as shown in Figure 6(b)The depth of the submersible reached a steady, $z = 175.3m$.

4 Prepare: Equipment Evaluation Model on AHP

The assembled equipment can be divided into two categories: search equipment and rescue equipment, of which the underwater search equipment includes side-scan sonar, multibeam sonar, autonomous underwater unmanned aerial vehicles (AUVs), remotely operated underwater robots (ROVs), and so on.The current mainstream underwater search equipment is as follows search equipment.

1. **Portable Sonar:** This device can detect underwater objects by emitting sound waves, which helps to locate submersibles.

Table 2: equipment summary

	Cost	Availability	Maintenance	Preparation
side scan sonar	low	high	low	fast
multi-beam sonar	high	medium	low	fast
ROV	high	high	high	medium
AUV	High	medium	high	medium
UAV	Medium	medium	high	fast

2. **Deep-sea Camera:** Using a camera that can withstand high water pressure, it is possible to capture images in the deep sea to help locate submersibles.
3. **Unmanned Aerial Vehicle (UAV):** Drones can conduct aerial searches of vast areas of the sea and quickly discover the location of submersibles.
4. **Underwater Robot (Remotely Operated Vehicle, ROV):** ROV can be operated by controllers to conduct underwater search and survey and to obtain information about the submersible.

According to the relevant information, we know the price, availability, maintenance requirements, speed of preparation and many other parameters of various types of equipment, and organize the table(2) Cost is divided by order of magnitude of price, and Preparation represents the speed of preparation.

4.1 Construction of Judging Matrices

Select some mainstream models from various types of equipment, and search and organize to get the advantages and disadvantages of each device, because the sample size is small, so the use of hierarchical analysis of each model of equipment to evaluate the study, taking into account the highest priority of the search and rescue is the time, so we chose the three main indicators of the cost, utility, readiness, and according to these representative models, we have arrived at the figure through a comparison of Figure 7 shows the judgment matrix.

*The number of each cell is explained in the following Figure 8:

4.2 Consistency Test

Normalize the judgment matrix A

$$\overline{A_{ij}} = \frac{A_{ij}}{\sum_{k=1}^{n} A_{kj}}, i, j = 1, 2...n \tag{12}$$

A(Judging Matrix)	K	ET	KE	GOS	REMUS	TG	SSLROV	BROV	IP4G	TBGR
Klein3000	1	3	3	2	3	1/2	1/2	1/2	1/3	1/2
EdgeTech 4200-FS	1/3	1	2	3	2	3	1/2	1/3	1/2	1/2
Kongsberg EM2040P	1/3	1/2	1	4	3	3	1/3	1/3	1/2	1/2
Gavia Offshore Surveyor AUV	1/2	1/3	1/4	1	1/3	1/2	1/3	1/3	1/3	1/2
REMUS 100 (Kongsberg Hydroid)	1/3	1/2	1/3	3	1	2	1/2	1/3	1/2	1/2
Teledyne Gavia AUV	2	1/3	1/3	2	1/2	1	1/3	1/3	1/2	1/2
Saab Seaeye Leopard ROV	2	2	3	3	2	3	1	1/2	1/3	1/3
BlueROV2	2	3	3	3	3	3	2	1	1/2	2
Iridium Pilot 4G	3	2	2	3	2	2	3	2	1	2
Trimble BX992 GNSS Receiver	2	2	2	2	2	2	3	1/2	1/2	1

Figure 7: judgment matrix

Intensity of Value	Interpretation
1	Requirements i and j are of equal value.
3	Requirement i has a slightly higher value than j.
5	Requirement i has a strongly higher value than j.
7	Requirement i has a very strongly higher value than j.
9	Requirement i has an absolutely higher value than j.
2,4,6,8	These are intermediate scales between two adjacent judgments.
Reciprocals	If Requirement i has a lower value than j

Figure 8: Importance note

Obtain the normalized matrix$\overline{A}$,where $n = 10$ thus obtaining the eigenvectors.

$$W = \left[0.0985, 0.0824, 0.0783, 0.0351, 0.0571, 0.0579, 0.1139, 0.1653, 0.1823, 0.1292\right]^T \tag{13}$$

Calculating the maximum characteristic root (approximation) yields

$$\lambda_{max} = \frac{1}{n}\sum_{i=1}^{n}\frac{(\overline{A}W)_i}{a_i} = 9.0771 \tag{14}$$

Calculation of consistency indicators

$$CI = \frac{\lambda_{max} - n}{n - 1} = 0.1490$$
$$CR = \frac{CI}{RI} = 0.0575$$

Since $CR < 0.10$, consulting the RI table in the figure below Figure 9 shows that the consistency test of the judgment matrix passes,the resulting weights are therefore considered plausible

Order	1	2	3	4	5	6	7	8	9
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Figure 9: Order of RI

4.3 Equipment Recommendations

According to equation (13), analyzing the weight W of each type of submersible, we can see that Saab Seaeye Leopard ROV has the highest weight, and has high availability, quicker preparation time, very suitable for search and rescue, and suitable cost, and is a suitable search equipment for the main ship to always have on board.

5 Search: Optimization of Search

In the simulation of model 1, we get the parameters of the numerical simulation, after 207.4s, the lost diver will be stabilized at 175.3m below the horizontal plane, when t continues to increase, the depth of the lost diver will remain unchanged, at this time, we can consider the search of the two-dimensional plane of the X, Y, the radius of the search of the sonobuoy for 100m, i.e., when the search of the UAV and the diver are 100m apart, it will be considered that the search target is found. The search radius of sonar is 100m, i.e. when the search UAV is 100m away from the diver, it is considered that the search target is found.

Assuming the following scenario: the main ship and the submersible are 1000m apart, search and rescue will be carried out immediately after learning of the loss of connection, i.e., the initial deployment point of the UAV will be determined and the search UAV will be launched into the water to go to the initial deployment point, meanwhile, the rescue ship and the main ship will follow behind, and search and rescue will be carried out in accordance with a certain search pattern after arriving at the initial deployment point.

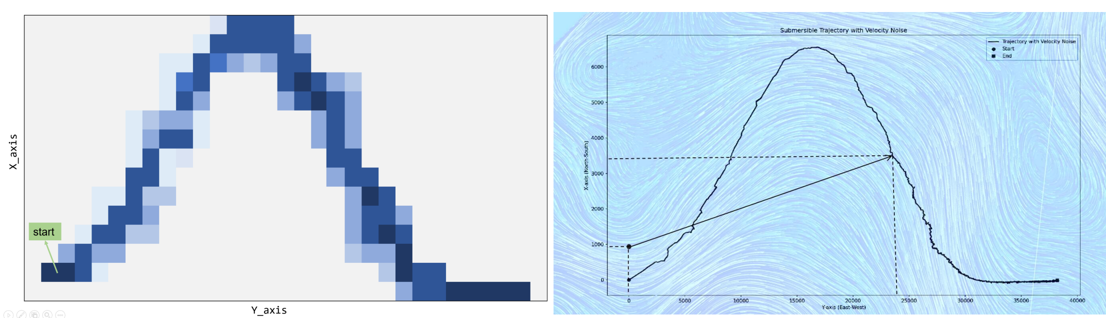
5.1 Setting of the Initial Deployment Point

Assuming that the propeller of the lost submersible fails, and the initial speed provided by the propeller after the loss of failure is weakened to 0 by the influence of current resistance, i.e., it enters into a drifting state, and according to the equation of the localization model (10), so that the propeller power $P = 0$, then the model becomes

$$\begin{cases} m\dfrac{d^2x}{dt^2} = \pi R^2\rho(z)Q(u_x - \dfrac{dx}{dt})(\dfrac{\pi}{2} - 1) - 6\pi\eta(\dfrac{dx}{dt} - u_x(t))R \\ m\dfrac{d^2y}{dt^2} = (8 + \pi R^2)\rho(z)Q(u_y - \dfrac{dy}{dt})(\dfrac{\pi}{2} - 1) - \alpha(v_y - u_y) \\ m\dfrac{d^2z}{dt^2} = G + (8 + \pi R^2)\rho(z)Q(u_z - \dfrac{dz}{dt})(\dfrac{\pi}{2} - 1) - \beta(v_z - u_z) - 16.76\rho(z)gR^3 \end{cases} \tag{15}$$

The square sea area is divided into $10m$ by $10m$ smaller. by dividing the square sea area into a straight line grid at 1m intervals.

Through each simulation, the predicted trajectory of the lost submersible from $t = 0$ to $t = 2h$ is obtained, and after 200 numerical simulations (if random perturbations are considered), the frequency of the submersible pathway for each small square is obtained, and with such a large number of simulations, the frequency is treated as a probability, and a probability distribution of trajectories is obtained as shown in Figure10(a) model, with the predicted trajectory with the highest probability as the main trajectory. Without



(a) Trajectory probability distribution

(b) Main track lines and int

Figure 10: Trajectory Probability Distribution and Main Track Lines

considering the influence of the current on the trajectory and speed of the search UAV, in order to shorten the time of searching the submersible as much as possible, we may want to make the search UAV go to the initial deployment point in a straight line, in order to be able to better determine the search range according to the predicted trajectory, we need to find the predicted intersection point

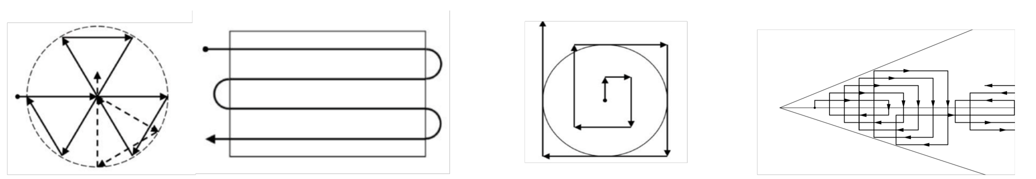
$$\frac{||r_s(t-\tau) - r(\tau)||_2}{v} - t \to 0 \tag{16}$$

Where $r_s = (x_s, y_s)$ is the two-dimensional coordinates of the search UAV, $r = (x, y)$ is the two-dimensional coordinates of the lost submersible, $v_s = (v_{sx}, v_{sy})$ is the speed of the search UAV, In the solution, since the data are stored discretely, only the following equations need to be satisfied above equation.There is $t \to t^*$, where the time lag $\tau =$

$10min$ when the main ship loses the submersible signal for 10min is recognized as its lost, solved $t^* = 201.334min$, at this time can determine the corresponding initial deployment point that is, the point of convergence for $r^* = (x*, y*) = (24032.5, 3458.7)$

5.2 Search Methods

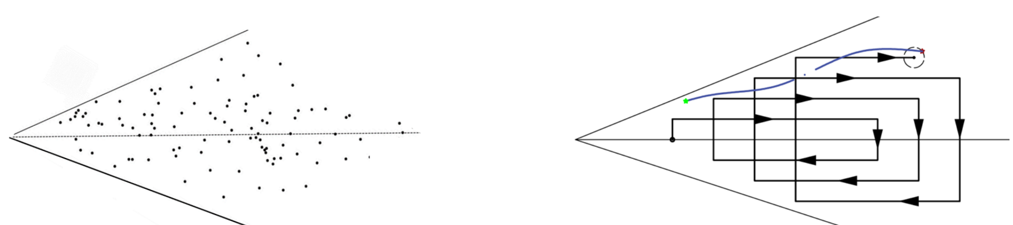
When the search UAV arrives at the initial deployment point (i.e., the intersection point), it enters the location with the highest probability of the location of the lost submersible, and launches a small-scope search with the initial deployment point as the origin, and the mainstream search modes, according to the division of the search trajectory, mainly include fan-shaped search, horizontal line search, extended rectangle search and moving rectangle search. Due to the influence of currents, it can be roughly judged that



(a) Sector Search (b) Horizontal Line Search (c) Extended Rectangle Search (d) Moving Rectangle Search

Figure 11: Search Methods

the final position will be distributed in a fan-shaped area with an axis along the tangent direction of the predicted main track line. After 200 simulations, the distribution density of the final position of the submersible is plotted using the frequency as the probability. Since the lost submersible is moving with the sea current, the moving rectangle search is a relatively more efficient search method. In the search model, we take the Bluefin-9



(a) fan-shaped probability distribution (b) Numerical simulation of moving rectangle search

Figure 12: Detailed search

AUV as an example, which can search a spherical range centered on itself with a radius of 100 meters. The cruising speed is 2m/s.

For this question, we first take the two-dimensional model as an example, easy to understand, the subsequent only need to add the z-axis motion prediction trajectory can be realized in the construction of 3-D model.

5.3 Probability Function for Searching

According to the initial deployment point and search mode determination, we will search the process is divided into two steps, because at this time the trapped submersible has lost power, the first step we combined with the speed of the ocean currents and the density of seawater and a series of impacts and according to the formula can be determined that we want to carry out the rescue of the direction and time, according to the calculation of the intersection coordinates of the (24032.5,3458.7), which is the starting point for the rescue vessel (0,978.5), the final time calculated as 201.334min. After reaching the rescue site, the positioning of the trapped submersible may have a slight deviation from the predicted value due to the influence of other external factors, so in the second step, we adopt the rectangular search mode proposed in the previous subsection to carry out precise rescue, and we get the corresponding search time and probability of rescue curves by simulating this process with the program. The curve of search time and rescue probability, and the result of its fitting function is

$$y = \frac{0.62}{1 + e^{(-4.54(x-4.94))}} + 1.22 \tag{17}$$

Its fitting curve is shown in Figure13 According to the image we can see that the final

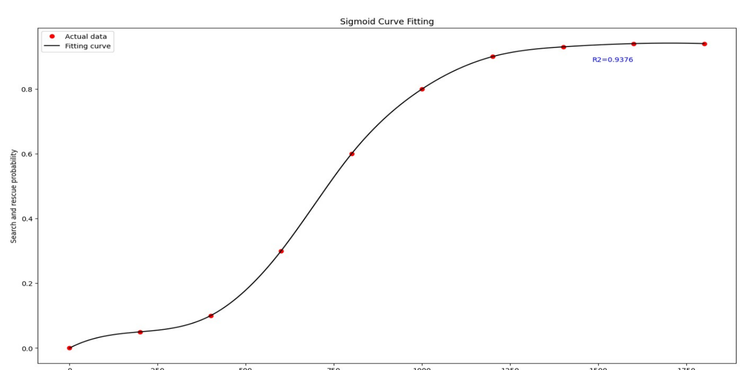


Figure 13: Probability function for searching

search and rescue probability is stabilized at about 94%, and its stabilization inflection point is roughly 1400s, plus the previously calculated optimal route time, so the total rescue time can be estimated as 224.667 min

6 Extrapolate: Multi-Target Localization Search Model

6.1 Extension of the Localization-Search Model

If the model is to be extended to other sea areas, it is only necessary to obtain the data of the sea area, and substituting these data into the model established in the first question for calculation, then the actual drift course of the submersible can be obtained. Take the Caribbean Sea as an example, find the various data in this sea area, as shown in

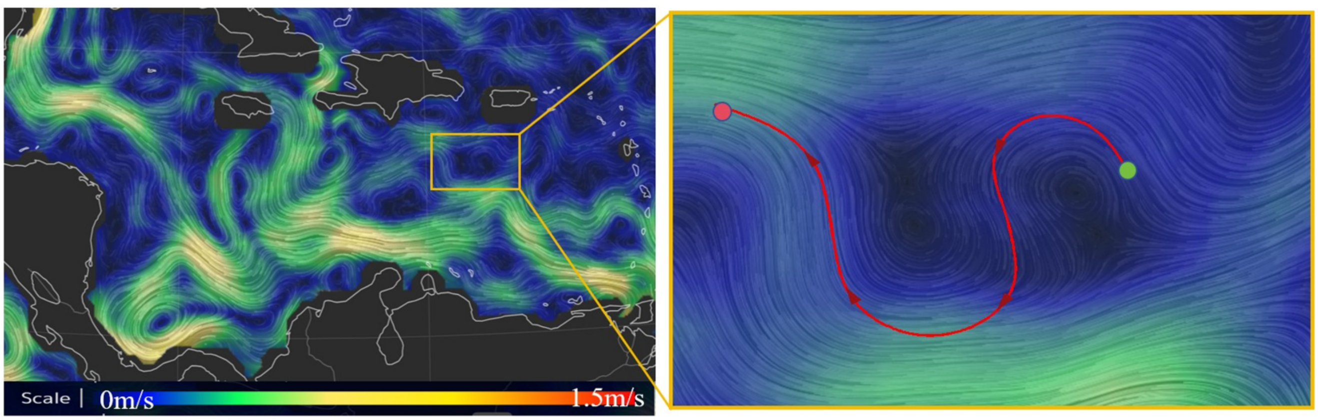


Figure 14: Caribbean Current

Figure (14), establish a database to get the velocity components of seawater flow velocity at different locations $r = (x, y, z)$ along different directions, seawater temperature trends with depth, etc., to facilitate the subsequent call.

A point is selected as the starting point, and the search UAV synchronizes its movement at the initial deployment point with the lost submersible at the green point in Fig. 14. Taking into account the energy constraints, it should minimize the work done to overcome the thrust of the ocean current, i.e., follow the direction of the ocean current, and thus the **tracking model** is proposed. Select a small time interval (1s, for example), at the end of each second according to the position of the submersible to call the data of the ocean under the position, brought into the established three-dimensional capsule dynamics model, while the search drone is in accordance with the moving rectangle search, when $t =$ **73.2**s, the lost submersible to achieve neutral buoyancy state, the depth tends to be stabilized, and the trajectory of the X, Y direction can also be determined. The trajectory in X, Y direction is also determined.

6.2 Multi-Target Locked Tracking Model

Multiple submersibles working in the same area scenario. Assuming that m submersibles are all lost at the same time, we assume that each submersible has exactly the same shape, size and mass, and because of the different orientations of the submersibles, the m submersibles are not subjected to the same currents in the horizontal direction under the same sea conditions, even if the starting points are the same, the routes are very different. Because of the same force in the plumb direction, and according to the three-dimensional capsule model, m lost submersibles will be stabilized at the same depth, so the problem can be simplified to a two-dimensional plane model. The following figure shows the course of a lost submersible under the influence of current and various other factors. Let's consider a search and rescue facility with three surrogate search and rescue

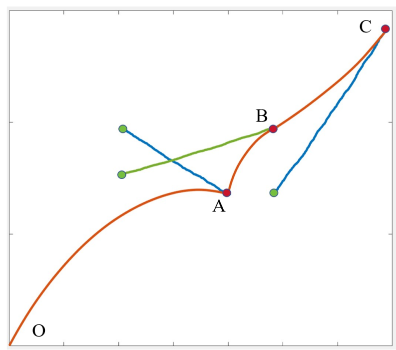


Figure 15: Order:$A \rightarrow B \rightarrow C$

submersibles. Assume that the three SAR submarines are lost at the same location and are moving in different directions. The search and rescue equipment receives the lost message at a distance of 1000m and immediately starts a search and rescue operation. The three to be searched and rescued equipment are labeled as A, B, C, then assuming that the order of search and rescue for the $A \rightarrow B \rightarrow C$, then the search and rescue equipment is always A's real-time prediction of the position of the speed direction (i.e., at a certain moment of time the speed of the search and rescue equipment is always pointing to the predicted position of the moment A, until the A appeared in the detection range of the search and rescue equipment), and then B's real-time prediction of the position of the speed direction until searching for the B, and so on. After that, the real-time predicted position of B is used as the speed direction until B is searched, and so on.。

However, we note that such a search order is not necessarily optimal. Under the assumption that there are three submersibles, there are A_3^3 kinds of search and rescue order, we now predict the main track line of each lost submersible, get three intersection points according to the formula (16), use the shortest path algorithm to find the search and rescue order that requires the shortest time in the case of sailing by a straight line, as shown in Figure 15, and then use the tracking search Then the search method is used to search according to the order, which optimizes the rescue time and also takes into account the continuation of the search and rescue ship.

7 Model Ecaluation and Furhter Discussion

7.1 Sensitivity Analysis

For the sensitivity analysis of the model, we chose seven of the parameters to carry out, which are P, R, δ, ρ, η, m, g. The global sensitivity analysis in the x, y, z 3- coordinate system is carried out by selecting the appropriate parameter ranges, where we chose Sobol's exponent as a statistic, which is designed to measure the contribution of the input parameters to the model outputs, and the results of the influence of each parameter on the prediction of the submersible's trajectory (i.e., the sensitivity) are shown in the following figure.

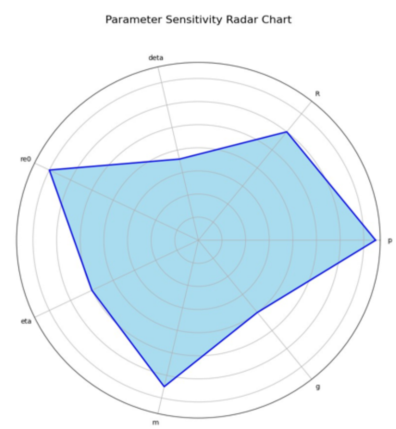


Figure 16: Sensitivity analysis of submersible trajectory prediction

By further analyzing the results, we see that the three variables that are more sensitive to the motion of the submersible are P,m,ρ.This suggests that more sophisticated instruments should be used to measure the power and mass of the submersible as well as the density of the sea water in the corresponding area when predicting the trajectory of the submersible. This sensitivity tuning will be a boost to the SAR mission and will increase the accuracy of the trajectory prediction.

In addition, in order to ensure that its trajectory can be accurately predicted by the main ship, the submersible should provide the seawater density of the region in which it is located as well as its ocean current speed in real time. With these two indicators, a more accurate trajectory prediction can be realized by the model we built above.

7.2 Strengths and Weaknesses

Strengths:

1. The model is simplified to a mass with only x, y, z three degrees of freedom without considering rotation when doing the kinematic analysis of the submersible and

search equipment. This simplifies the path simulation and search simulation and reduces the computational effort.

2. When performing the path simulation, the dynamics are analyzed according to the actual situation, and then the path is determined to be in accordance with the laws of physics rather than simply adding a random disturbance term.

3. Its kinematic trajectory prediction model has good prediction effect under different ocean data with certain generalization ability.

4. The sensitivity analysis of the parameters better captures the key factors affecting the trajectory, which is a significant improvement in terms of both cost savings and rescue efficiency.

Weaknesses:

1. While it is desirable to analyze the dynamics based on the actual situation, failure to account for all possible dynamical effects in the model may lead to inaccurate predictions of the motion pattern.

2. While sensitivity analyses of parameters can capture key factors, there is also a risk that some interaction effects or non-linear relationships may be overlooked, leading to an incomplete understanding of the influencing factors.

7.3 Further Discussion

1. Collect more rich data on the seafloor environment, including different geographic areas, depths and seafloor topography. Validate the applicability of the model in more scenarios to improve generalization.

2. Consider more accurate and complex kinematic modeling of submersibles and search equipment, including factors such as rotation, hydrodynamics, etc., to more accurately simulate real-world conditions.

3. Further optimize the search method by taking into account more factors such as underwater topography, sea currents, etc. to improve the efficiency and accuracy of the rescue process.

4. Integrate the different models to form a more comprehensive subsea rescue equipment purchase assessment model. Ensure that the sub-models work together to better reflect the reality.

7.4 Conclusion

Through comprehensive research and analysis of undersea rescue missions, we have succeeded in building a comprehensive solution to ensure accurate positioning of submersibles over time, safe deployment, and effective search and rescue when needed. The following are our main conclusions:

We built an accurate ocean dynamics model using a system of differential equations with full consideration of position and current parameters for predicting the position of the submersible over time, and also visualized its trajectory through numerical simulations. In order to reduce the uncertainty, the sensitivity analysis identifies the important factors affecting the trajectory of the submersible, and the prediction of the position is realized by the submersible periodically sending key information to the host ship, including real-time position, mechanical diagnosis and environmental conditions, where the recommended equipment includes advanced communication systems, GPS, anemometers, underwater sonar, and propulsion status sensors.

We also made recommendations for additional search equipment to be carried on board the host vessel, and identified the Saab Seaeye Leopard ROV as the optimal equipment to be carried using a hierarchical analysis combined with a consistency discriminant matrix, taking into account equipment availability, maintenance costs, and rescue efficiency.

In addition, for the search path planning, we built a search model and used the information from the localization prediction model to suggest the initial deployment point of the equipment and the search mode, and determined the two search steps and the rectangular moving search mode. Meanwhile, for the multi-target rescue, the shortest path algorithm was used to optimize it in order to better adapt to the situation of the coordinated action of multiple submersibles, and the rescue time was reduced to 224 minutes by the numerical simulation. Through numerical simulation, the rescue time is shortened to 224 minutes, and the successful rescue rate is as high as 94%, and through the time and cumulative search results, we fit the curve of the search and rescue success probability versus time and show its functional expression.

Finally, we consider how to extend the model to other tourist destinations, such as the Caribbean, to verify the robustness of the model as well as its generalization ability.

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