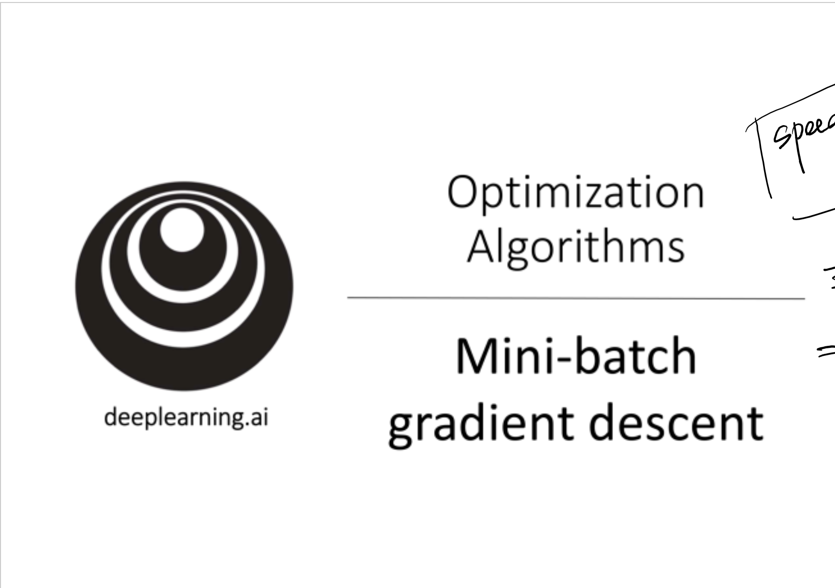
Optimization Algorithm

Monday, June 4, 2018 2:20 PM

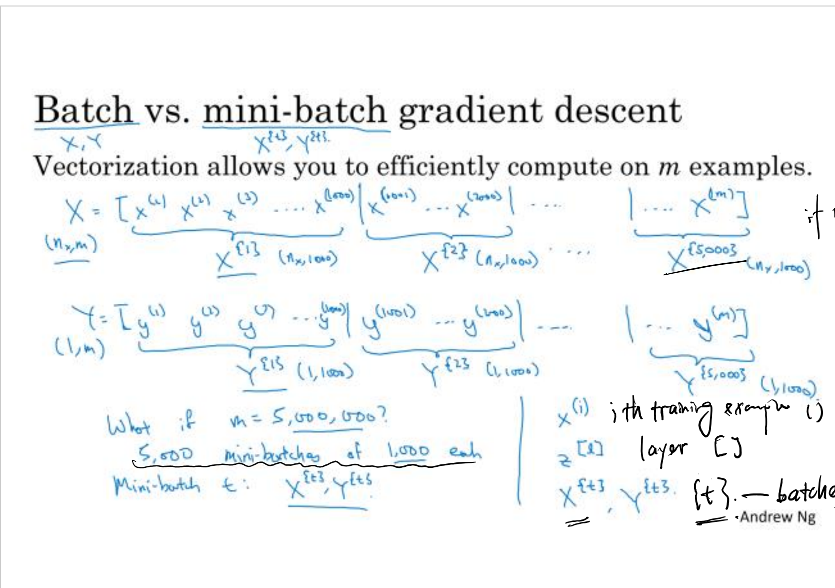
mini batch



speed up model training.

~~The~~ DL is a highly empirical subject

=> skills to speed up training



$X = [x^{(1)}\ x^{(2)}\ x^{(3)} \dots\ x^{(1000)}]$

if m is large, it can be slow.

Need to process the whole training set before gradient descent

5000 mini batches, 1000 each

ith training example ()

layer []

{t} — batches

$(n_x$, size of each batch)

Mini-batch gradient descent

\# of batches

repeat { for $t = 1, \ldots, 5000$ {

Forward prop on $X^{\{t\}}$.

$$Z^{[1]} = W^{[1]} X^{\{t\}} + b^{[1]}$$
$$A^{[1]} = g^{[1]}(Z^{[1]})$$
$$\vdots$$
$$A^{[L]} = g^{[L]}(Z^{[L]})$$

} Vectorized implementation (1000 examples)

Compute cost $J^{\{t\}} = \frac{1}{1000} \sum_{i=1}^{l} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2 \cdot 1000} \sum_{l} \|W^{[l]}\|_F^2$. ← from $X^{\{t\}}, Y^{\{t\}}$ → regularization

Backprop to compute gradients wrt $J^{\{t\}}$ (using $(X^{\{t\}}, Y^{\{t\}})$)

$W^{[l]} := W^{[l]} - \alpha dW^{[l]}$, $b^{[l]} := b^{[l]} - \alpha db^{[l]}$

} } "1 epoch" — pass through training set.

one pass through training set "one Epoch"

1 step of gradient descent using $X^{\{t\}}, Y^{\{t\}}$. (as if $m = 1000$)

X, Y

vectorization to process all 1000 examples in a certain batch

$$Z^{[1]} = W^{[1]} X^{\{t\}} + b^{[1]}$$
$$A^{[1]} = g^{[1]}(Z^{[1]})$$
$$\vdots$$
$$A^{[L]} = g^{[L]}(Z^{[L]})$$

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runs much faster. Everyone will use, with large datasets

minibatch2

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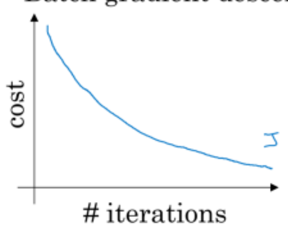
Understanding mini-batch gradient descent

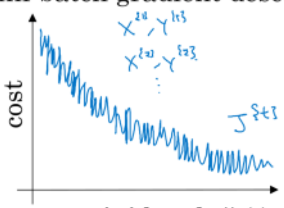
Training with mini batch gradient descent

Batch gradient descent



Always decreases (one every iteration).

Mini-batch gradient descent



Plot $J^{\{t\}}$ computed using $X^{\{t\}}, Y^{\{t\}}$

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Need not always decrease
should trend down.
but noisier!

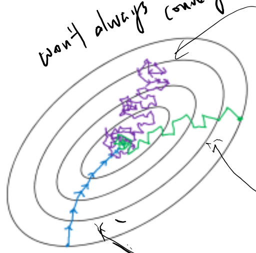
Choosing your mini-batch size

→ If mini-batch size = m : Batch gradient descent. $(X^{\{1\}}, Y^{\{1\}}) = (X, Y)$.

→ If mini-batch size = 1 : $\boxed{\text{Stochastic}}$ gradient descent. Every example is it own $(X^{\{1\}}, Y^{\{1\}}) = (x^{(1)}, y^{(1)})$ ... $(x^{(1)}, y^{(1)})$ mini-batch.

In practice: Somewhere in-between 1 and m

won't always converge



Stochastic gradient descent
↓
Lose speedup from vectorization

In-between (mini-batch size not too big/small)
↓
Fastest learning.
- Vectorization. (~1000)
- Make progress without processing entire training set.

Batch gradient descent (mini-batch size = m)
↓
Too long per iteration

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two extremes. 每个 batch → 全部数据
→ 一条数据.

choose something in between, not 1, not m.

Choosing your mini-batch size

If small training set: Use $\boxed{\text{batch}}$ gradient descent.
(m ≤ 2000)

Typical mini-batch sizes:

→ $\underbrace{\underset{2^6}{64}, \underset{2^7}{128}, \underset{2^8}{256}, \underset{2^9}{512}}$ $\underset{2^{10}}{1024}$

Make sure minibatch fit in CPU/GPU memory.
$X^{\{t\}}, Y^{\{t\}}$

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REMEMBER THESE RULES.

m. ≤ 2000. batch.

> 2000 sizes of mini-batch

因为内存. 2^n 的 batch size 更快.

(不能盲目学习).

$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
最常见.

$2^{10} = 1024$ 有点大.

Don't exceed CPU, GPU memory.

Approach: Try different power to 2. Use the most efficient one (size.)

up next: more efficient than mini-batch gd. "the most" efficient one (size.)

exp weighted average

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Exponentially weighted averages

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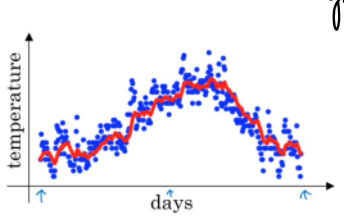
Temperature in London

get the trend of the temperature

$\theta_1 = 40°F$ 4°C ←
$\theta_2 = 49°F$ 9°C
$\theta_3 = 45°F$ ⋮
⋮
$\theta_{180} = 60°F$ 15°C
$\theta_{181} = 56°F$ ⋮
⋮



$V_0 = 0$
$V_1 = 0.9\,V_0 + 0.1\,\theta_1$
$V_2 = 0.9\,V_1 + 0.1\,\theta_2$
$V_3 = 0.9\,V_2 + 0.1\,\theta_3$
⋮
$\boxed{V_t = 0.9\,V_{t-1} + 0.1\,\theta_t}$

ARMA(1,1). ?

non-stationary 时序.

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Exponentially weighted (moving) averages

$V_t = \beta V_{t-1} + (1-\beta)\theta_t$

lots of weight of the past.

$\beta = 0.9$: ≈ 10 days' temperature

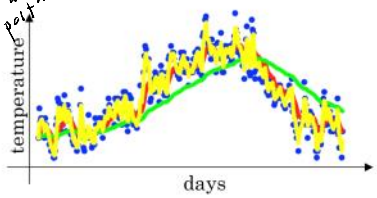
$\beta = 0.98$: ≈ 50 days

$\beta = 0.5$: ≈ 2 days

Smaller weights to the past.

V_t as approximately average over $\approx \frac{1}{1-\beta}$ days' temperature.

$\frac{1}{1-0.98} = 50$



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Why? 能解出来的吧?

Averaging over the last $\frac{1}{1-\beta}$ days temperature.

Next what this is doing

ewa2

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Understanding exponentially weighted averages

red. green. — yellow.

"Just a rule of thumb of how to think about it, not mathematical derivation"

?

?

$(1-\varepsilon)^{1/\varepsilon} = \frac{1}{e}$

怎么推出结果的呢?

$\varepsilon = 1-\beta.$

$\varepsilon = 0.1$

$(1-0.02)^{\frac{1}{0.02}} = 0.98^{50} \approx \frac{1}{e}$ about $\frac{1}{3}$

to talk about later

add up to 1

= bias correction

这就是用来算Average的

Implementing exponentially weighted averages

$v_0 = 0$

$v_1 = \beta v_0 + (1-\beta)\,\theta_1$

$v_2 = \beta v_1 + (1-\beta)\,\theta_2$

$v_3 = \beta v_2 + (1-\beta)\,\theta_3$

...

$v_\theta := 0$

$v_\theta := \beta v + (1-\beta)\theta_1$

$v_\theta := \beta v + (1-\beta)\theta_2$

⋮

$v_\theta = 0$

Repeat {

get next θ_t

$v_\theta := \beta v_\theta + (1-\beta)\theta_t$

}

repeat

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Advantage: takes very little memory

efficient. takes one line of code.

Not the most accurate way to compute the average

ewa-bias correction

ewa-bias correction

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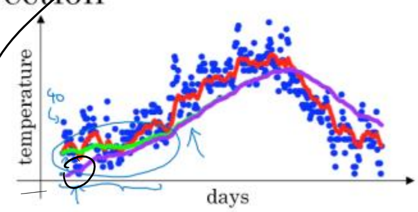
Bias correction in exponentially weighted average

Bias correction



$\beta = 0.98$

$\rightarrow v_t = \beta v_{t-1} + (1-\beta)\theta_t$

$V_0 = 0$

$V_1 = \cancel{0.98 V_0} + 0.02\,\theta_1$

$V_2 = 0.98\,V_1 + 0.02\,\theta_2$

$= 0.98 \times 0.02 \times \theta_1 + 0.02\,\theta_2$

$= 0.0196\,\theta_1 + 0.02\,\theta_2$

$\frac{V_t}{1-\beta^t}$

$t=2: \quad 1-\beta^t = 1-(0.98)^2 = 0.0396$

$\frac{V_2}{0.0396} = \frac{0.0196\,\theta_1 + 0.02\,\theta_2}{0.0396}$

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β large $\Rightarrow$ the initial value $v_0 = 0$ keeps the predicted value v. low for a while, not good ..

to modify this to make it better

$\frac{v_t}{1-\beta^t}$

$t=2: \; 1-\beta^t = 1-(0.98)^2 = 0.0396.$

As $t\uparrow$, $\Rightarrow \beta^t \Rightarrow 0$ this correction diminishes

only helpful for the "warm-up" period.

But people don't really bother to do this in ML

gd with momentum

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Algorithms

Gradient descent with momentum

→, works faster than gradient descent.

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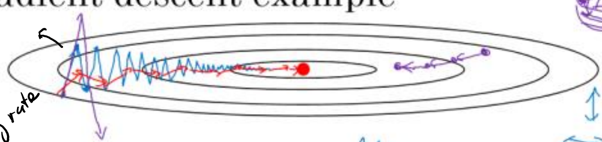
Gradient descent example

traction βV_{dw} + $(1-\beta)\, dW$.
velocity   acceleration



• slow down learning
• can't use larger learning rate

↕ slower learning
⟷ faster learning

"~~bowl~~ ball rolling down a bowl analogy"

Momentum:
On iteration t:
Compute dW, db on current mini-batch.
$V_{dw} = \beta V_{dw} + (1-\beta) dW$
$V_{db} = \beta V_{db} + (1-\beta) db$
friction → velocity   acceleration
$W = W - \alpha V_{dw}$, $b = b - \alpha V_{db}$

" $V_\theta = \beta V_\theta + (1-\beta)\theta_t$ "

moving average of the derivatives of W.

What is does average out (cancel out)
vertical 抵消 cancel out
horizontal 同方向. won't cancel out.

Want: larger learning rate horizontally.

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Implementation details

$V_{dw} = 0$, $V_{db} = 0$

On iteration t:

Compute dW, db on the current mini-batch

→ $v_{dW} = \beta v_{dW} + \cancel{(1-\beta)} dW$

→ $v_{db} = \beta v_{db} + \cancel{(1-\beta)} db$

$W = W - \alpha v_{dW}$, $b = b - \alpha v_{db}$

$V_{dw} = \beta V_{dw} + dW$

in the literature. A different type of parametrization.
Difference? scale α. just affect α's choice.
this way of parametrization is less intuitive.

$\cancel{\frac{V_{dw}}{1-\beta^t}}$

Hyperparameters: α, β

$\boxed{\beta = 0.9}$ → appear robust.
average over last ≈10 gradients

$\beta = 0.9$

$0.9^{\frac{1}{0.1}} = 0.9^{10} = \frac{1}{e}$

bias correction? Don't bother.
after just $\boxed{10}$ iteration. the moving average would have been warmed up.
the initial value won't cause much trouble.

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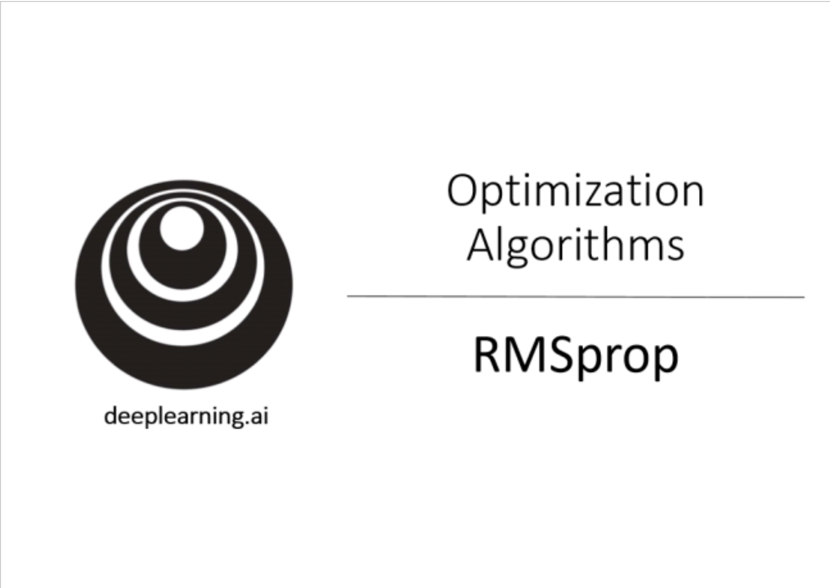
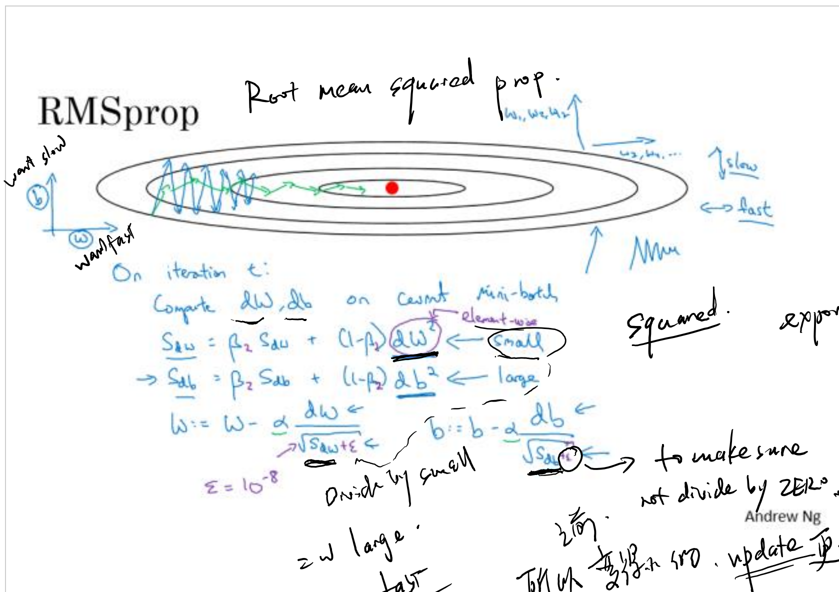
rmsprop





squared. exponentially weighted average of squared derivative.

In practice, high dimension
this is just a 2-dim example

→ to make sure not divide by ZERO → numerical stability

⇒ w large. ⇒ fast

之前. 所以 变得不那么 update 更快
变得不那么. update 慢. 打折.

Next video: Combine Momentum & RMSprop. ⇒ Adam optimization

adam

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Adam optimization algorithm

RMSprop & Adam are the two ~~of~~ optim algorithm that stand out among many new optimization algorithms

$\boxed{\text{proven effective}}$

Adam optimization algorithm

$V_{dw}=0,\ S_{dw}=0.\quad V_{db}=0,\ S_{db}=0$

On iteration t:

Compute dW, db using current mini-batch

$V_{dw}=\beta_1 V_{dw}+(1-\beta_1)dW,\quad V_{db}=\beta_1 V_{db}+(1-\beta_1)db \quad \leftarrow$ "momentum" β_1

$S_{dw}=\beta_2 S_{dw}+(1-\beta_2)dW^2,\quad S_{db}=\beta_2 S_{db}+(1-\beta_2)db \quad \leftarrow$ "RMSprop" β_2

yhat = np.array([.9, 0.2, 0.1, .4, .9])

bias correction

$V_{dw}^{corrected}=V_{dw}/(1-\beta_1^t),\quad V_{db}^{corrected}=V_{db}/(1-\beta_1^t)$

$S_{dw}^{corrected}=S_{dw}/(1-\beta_2^t),\quad S_{db}^{corrected}=S_{db}/(1-\beta_2^t)$

$W:=W-\alpha\frac{V_{dw}^{corrected}}{\sqrt{S_{dw}^{corrected}}+\varepsilon}\qquad b:=b-\alpha\frac{V_{db}^{corrected}}{\sqrt{S_{db}^{corrected}}+\varepsilon}$

一锅炖 ...

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Adaptive moment estimate.

Hyperparameters choice:

→ α : needs to be tune ← try a range of α to see which works

→ β_1 : 0.9 → (dw)

→ β_2 : 0.999 → (dw^2)

→ ε : 10^{-8} ← Doesn't matter much, no need to ~~tune~~ tune

真的有算 second moment 吗?

Adam: Adaptive moment estimation

Adam Coates

not related to this ~~guy~~ guy.

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lr decay

lr decay

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Learning rate decay

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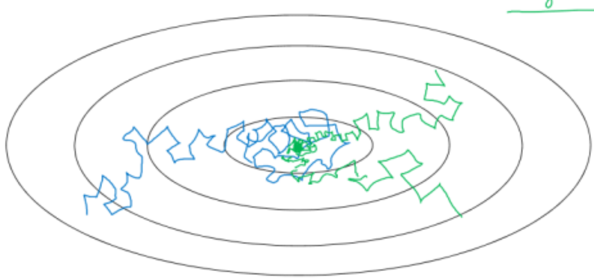
Learning rate decay

Slowly reduce α



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Learning rate decay

1 epoch = 1 pass through data.

$X^{\{1\}}$ | $X^{\{2\}}$ | ....

→ epoch 1

→ epoch 2

⋮

$$\alpha = \frac{1}{1 + \text{decay-rate} * \text{epoch-num}} \alpha_0$$

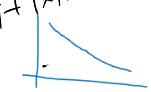
$\alpha_0 = 0.2$

decay.rate = 1

Epoch	α
1	0.1
2	0.67
3	0.5
4	0.4
⋮	⋮

first pass

$0.1 = \frac{1}{1+1 \times 1} \times 0.2$



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slowly reducing learning rate

start with high LR.

then when it is close to

convergence ⇒ reduce LR.

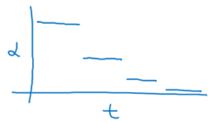
Other learning rate decay methods

看起来都好 ad hoc...

formula

$$\alpha = 0.95^{\text{epoch-num}} \cdot \alpha_0 \quad \text{– exponentially decay.}$$

$$\alpha = \frac{k}{\sqrt{\text{epoch-num}}} \cdot \alpha_0 \quad \text{or} \quad \frac{k}{\sqrt{t}} \cdot \alpha_0$$



discrete staircase

△ interesting. why not continuous?

Manual decay. If training only one model. works only if training only a few number of model.

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Next week: how to systematically tune param.

△ LR decay is not the top priority for param tuning

不是很重要

AN.

local optimal

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The problem of local optima

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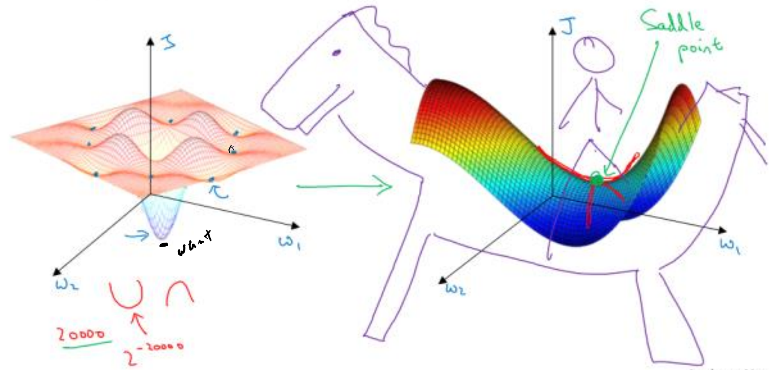
Local optima in neural networks



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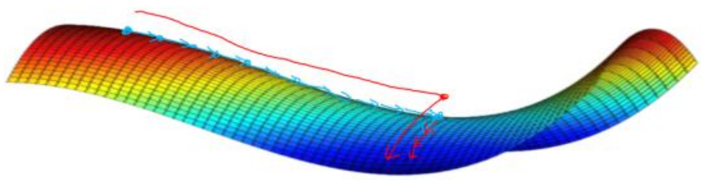
Derivative 0.

most points of zero gradient are not optimal.
They are saddle points.

* saddle points problem for high dim data

Problem of plateaus



- Unlikely to get stuck in a bad local optima
- Plateaus can make learning slow

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$V_1^{cor} =$

$v_1 = 0.5 \times 0 + 0.5 \times 10$
$= 5$
$v_2 = 0.5 \times 5 + 0.5 \times 10 = 7.5$

plateau
slow down training
take a long time to get to optimal.

Take away:

- local optimal not much a problem for DL tasks
- plateaus slow down learning

○ key: high dim spaces.
understanding on high-dim space is still evolving.